1

Mathematical Backgrund

set:

A set is a collection of object, which are the elemants of the set.

$S_1 = \{1, 2, 3, 4, 5, 6\}$ -> use six sided die

$S_2 = \{H, T\}$ -> Tossing coin

Let $S = \{x_1, x_2, \dots, x_n\}$

x_j in an elemant of s, $n \geq j \geq 1$

as same

$x_j \in S$, $n \geq j \geq 1$

2

If x_j is an elemant of S / in s / belongs to s

$\Rightarrow x_j \in s$

for $j > n \Rightarrow x_j \notin s$

3

Difinitions:-

1) Empty set :- The set that contains No elemant

2) universal set :- The set that contains all possible outcoms that could of interval in particolar contixt.

4

* set classifecation

1) finite set :- A set that has finite # of elemant

$S = \{H, T\}$

$S = \{1, 2, 3, 4\}$

2) countable infinit :-

The elemants of the set can be enumerated in a list

$S = \{1, 2, 3, 4, 5, \dots\}$ discret

3) uncontable set:-

the elemants can not be enumerated a list

$S = \{x \in S \mid 1 > x > 0\}$ continus

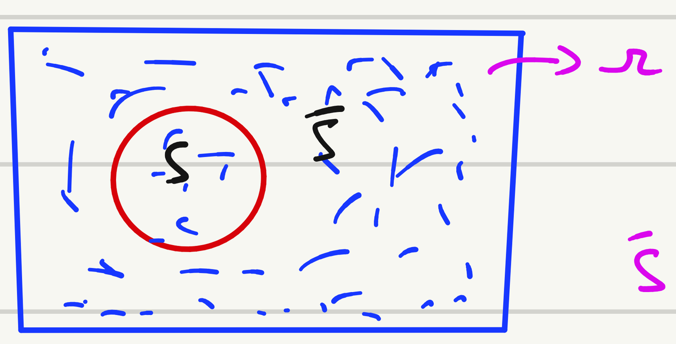
5

* set operations:-

1- complement of set:-

let S be a set then the complement of s (with respect universal set Ω) is defined as

$\bar{S} = \{x \in \Omega, x \notin s\}$



$\bar{S} = \Omega - S$

6

example:- Roling a die

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$S = \{1, 3, 5\}$

$\bar{S} = \{2, 4, 6\}$

5 تكملة.

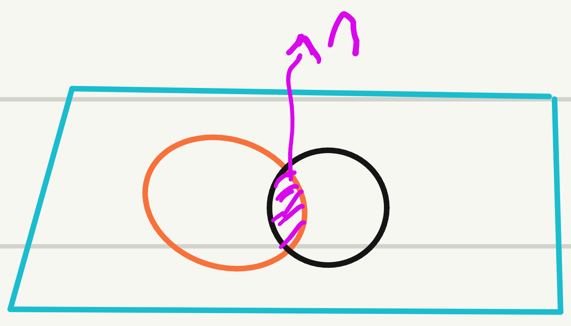
2 union of sets:-
The union of "S" and "T" is the set whose elements belong to S or T

$$S \cup T = \{x \mid x \in S \text{ or } x \in T\}$$

such That

3 intersection of sets:-
The intersection of S and T is the set that are in S and T

$$S \cap T = \{x \mid x \in S \text{ and } x \in T\}$$



For infintelly many sets:-

$S_1, S_2, S_3, \ldots$

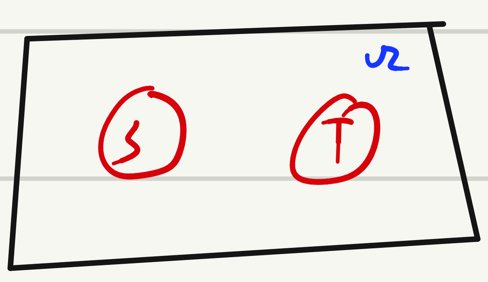
$S_1 \cup S_2 \cup S_3 \ldots = \bigcup_{i=1}^{\infty} S_i$

$= \{x \mid x \in S_i \text{ for some } i\}$

$S_1, S_2, S_3 \ldots\ldots = \bigcap_{i=1}^{\infty} S_i$

$= \{x \mid x \in S_i \text{ for all } i\}$

4 Disjoint sets:
S and T are said to be dissjoint iff $S \cap T = \varnothing$



⇒

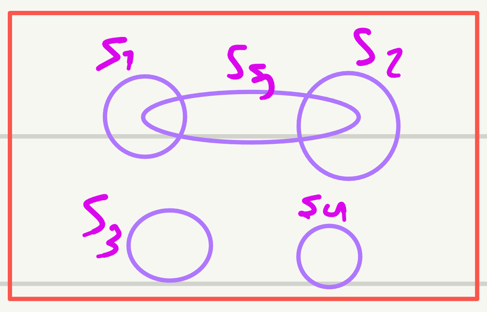
⇒ 4

For $S_1, S_2, S_3, \ldots\ldots, S_n$

we say that $S_1, S_2, S_3 \ldots, S_n$ are disjoint

if $S_j \cap S_i = \varnothing$, $i = 1,2,3 \ldots n$ $j \neq i$

$i = 1,2,3 \ldots\ldots, n$



$S_1 \cap S_2 = \varnothing$ $S_2 \cap S_1 = \varnothing$

$S_1 \cap S_3 = \varnothing$ $S_2 \cap S_3 = \varnothing$

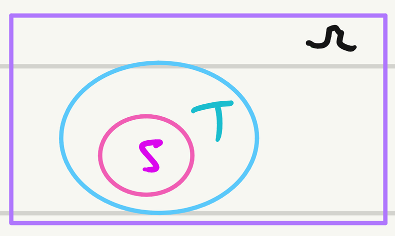
$S_1 \cap S_4 = \varnothing$ $S_2 \cap S_4 = \varnothing$

$S_3 \cap S_4 = \varnothing$

S_1, S_2, S_3, S_4 are disjoint

S_1, S_2, S_3, S_4, S_5 are joint

5 Subset:-
we say that "S" is subset of T if every element of "S" in an elemant of T



S is a subset of T

$S \subset T$ OR $T \supseteq S$

include

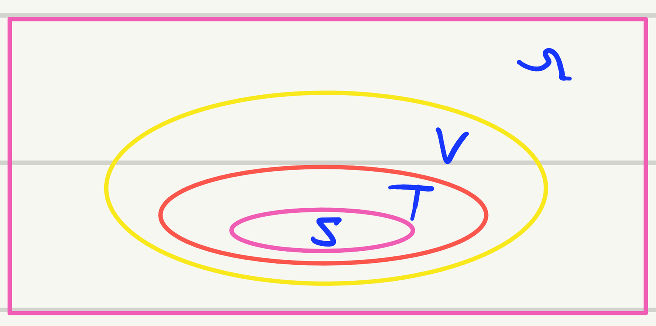
Propertes:-

1 Equality of sets:-

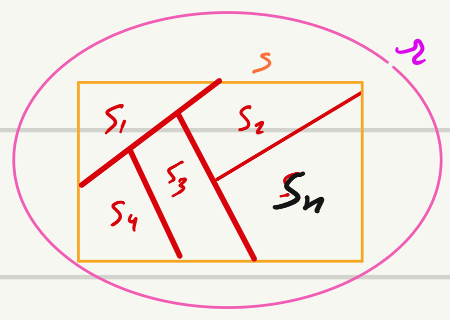
S and T are equal iff $S \subset T$ and $T \subset S$

2 Transitivity:-

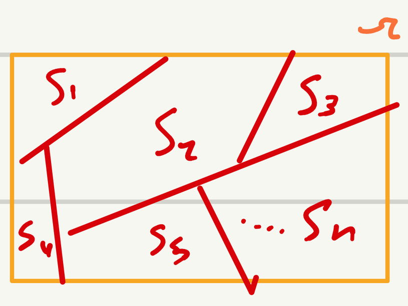
If $S \subset T$ and $T \subset V$, then $S \subset V$



6 Partiton:-
A colection of sets is said to be partiton of "S" if the set are disjoint and thir union is the set "S"



$S_1, S_2, S_3, \ldots, S_n$ is a partiton of "S"



$S_1, S_2, \ldots\ldots, S_n$ is a partition of Ω

8

* Algebra of sets :

1. $S \cup T = T \cup S$
2. $S \cap T = T \cap S$
3. $S \cap \Omega = S$
4. $S \cup \Omega = \Omega$
5. $S \cap \bar{S} = \emptyset$
6. $S \cup \bar{S} = \Omega$
7. $S \cap (T \cup A) = (S \cap A) \cup (S \cap A)$
8. $S \cup (T \cap A) = (S \cup T) \cap (S \cup A)$
9. $\bar{\bar{S}} = S$

---

* Ex:- an experiment of rolling a 4 side die

$\Omega = \{1, 2, 3, 4\}$

$A \triangleq$ set of even rolls $A = \{2, 4\}$

$B \triangleq$ set of all outcomes that losses than "3"

$B = \{1, 2\}$

Find

1. $A \cap B = \{2\}$
2. $A \cup B = \{1, 2, 4\}$
3. $A \cap \bar{B} = \{4\} = A - B \Rightarrow$ بشوف جميع العناصر في "A" والرقم المتكرر منها في "B" بشيلو
4. $B \cap \bar{A} = B - A = \{1\}$

---

9 Demorgan's law:-

1. $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$
2. $\overline{(A \cup B)} = \bar{A} \cap \bar{B} \Rightarrow$ we prove that by dewality

* prove 1

Let $x \in \overline{(A \cap B)}$

$x \notin (A \cap B)$

$x \notin A$ oR $x \notin B$

So $x \notin \bar{A}$ oR $x \in \bar{B}$

$\bar{A} \cup \bar{B}$

* Probabilistic Model :- (PM)

A mathmatical description of a random experiment

Element of PM:-

1 A sample space (univarsal set Ω)

2 probability low :- it assyins a non negative number that refar to likelihood of a particular event

Ex:- Tossing a coin:

1 $\Omega = \{H, T\}$ ← sample space

2 let $A = \{H\}$

$B = \{T\}$

find $P(A)$ & $P(B)$

$$P(A) = \frac{\#\text{Element on}(A)}{\#\text{Element on}(\Omega)} = \frac{1}{2}$$

$$P(B) = 1 - P(A) = \frac{1}{2}$$

2 probability axiom :-

1 nonnigativity:- $P(A) \geq 0$

2 normalization :- $P(\Omega) = 1$

3 additivity:-

Let A and A_2 be disjoint events

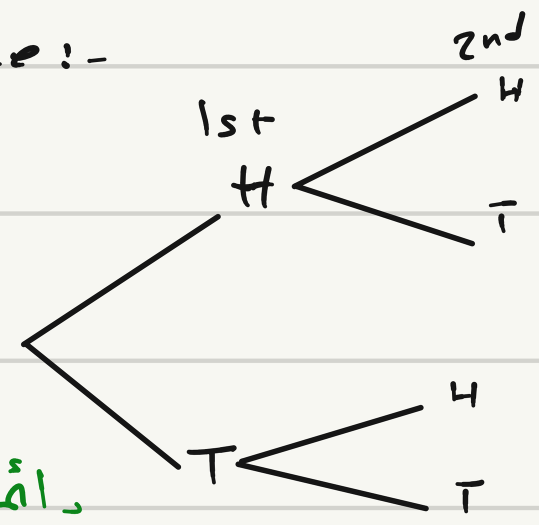
then $P(A_1 \cup A_2) = P(A_1) + P(A_2)$

Ex:- Tossing fair coin twice:-

1 $\Omega = \{HH, TH, HT\ TT\}$

$A = \{HH\}$, $B = \{HT\}$

$C = \{TH\}$, $D = \{TT\}$ } دائماً يكونو Disjoint



2 all events are equaly likely:-

$(P(A) = P(B) = P(C) = P(D)) \Rightarrow$ fair coin

$P(\Omega) = 1$

$\Omega = A \cup B \cup C \cup D$

$1 = P(A) + P(B) + P(C) + P(D) \Rightarrow$ Axiom 2

$1 = 4\,P(A) \Rightarrow$ Axiom(3)

$P(A) = \frac{1}{4}$

find $P(A \cup B) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

In general:- Let $A_1, A_2, \ldots\ A_n$ be disjoint events, then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

* consequnces of Axioms:-

1 $P(A) \leq$ for any event "A".

$\Omega = A \cup \bar{A}$

$P(\Omega) = P(A \cup \bar{A}) = 1$ Ax #2 Axiom

$P(A \cup \bar{A}) = P(A) + P(\bar{A})$ Ax #2

$P(A) = 1 - P(\bar{A})$ by using the farst Axiom

So $0 < P(A) \leq 1$

2 $P(\phi) = 0$ Proove:-

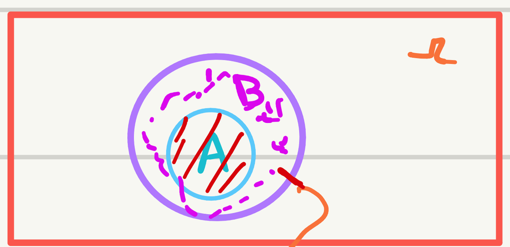
$\Omega = \phi \cup \Omega$

$P(\Omega) = P(\phi) + P(\Omega)$

$P(\phi) = P(\Omega) - P(\Omega) = 0$

* Additinal probabilty law:-

1 If $A \subset B$ then so $P(A) \leq P(B)$



this is aproove

$B - A$

$B \cap \bar{A}$

disjoint

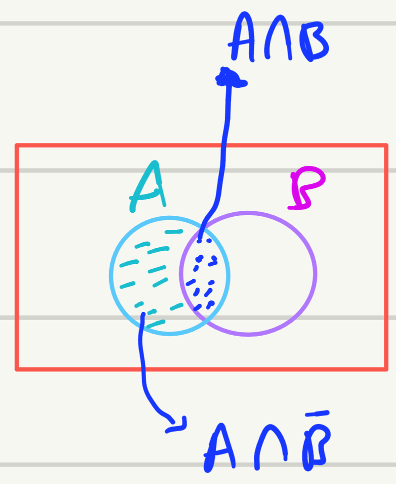
$B = (B \cap \bar{A}) \cup A$

So $P(B) = P(B \cap \bar{A}) + P(A)$ Ax #3

$P(B) \geq P(A)$

2 $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

so as the figer



$A = (A \cap \bar{B}) \cup (A \cap B)$

$P(A) = P(A \cap \bar{B}) + P(A \cap B)$

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

continue ⇒

[3] For any two events A and B,

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

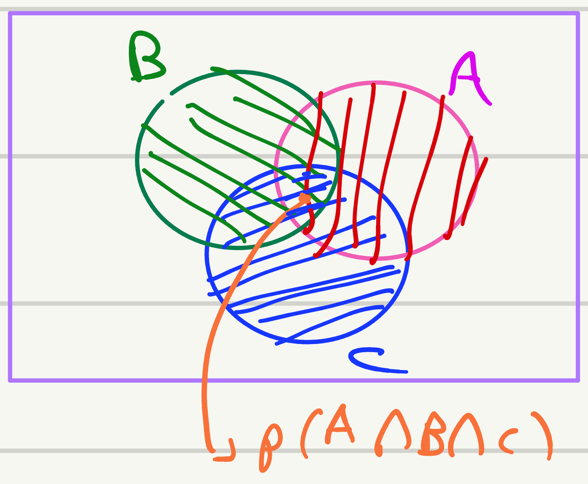
$$P(A\cup B) = P(B) \cup (A\cap \bar{B})$$

$$P(A\cup B) = P(B) + P(A\cap \bar{B})$$

$$= P(A) + P(B) - P(A\cap \bar{B})$$

* If we have 3 event

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$



prove:- let $P(V) = P(A\cup B)$

$$P(V) = P(A) + P(B) - P(A-B)$$

$$P(V\cap C) = P(C\cap A) \cup (C\cap B)$$

$$= P(C\cap A) + P(C\cap B) - P(C\cap A\cap B)$$

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

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* $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

* $P(A\cup B) \le P(A) + P(B)$ ⇒ طول اعقلا مهم

also * $P(A\cup B\cup C) \le P(A) + P(B) + P(C)$ ⇐ مهم

* For the events $A_1, A_2, A_3, \ldots, A_n$

$$P\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i)$$

Equality "=" hold when $A_1, A_2, \ldots A_n$ disjoint.

* Discrete probability law.

Let $\Omega = \{s_1, s_2, \ldots, s_n\} = |\Omega| = n$

$S = \{s_1, s_2, \ldots, s_k\} = |S| = k$

* The cardinality of a set S is

$|S| \triangleq$ # of element of S

$$P(S) = P(s_1) + P(s_2) + P(s_3) + \cdots P(s_k) \quad [2]$$

$$= \sum_{i=1}^{k} P_i$$

If all elements are equally likely (uniform)

$$P(\{s_i\}) = \frac{1}{n}, \quad i = 1, 2, 3, \ldots, n$$

$$\boxed{P(S) = \sum_{i=1}^{k} \frac{1}{n} = \frac{k}{n} = \frac{|S|}{|\Omega|}} \quad [1]$$

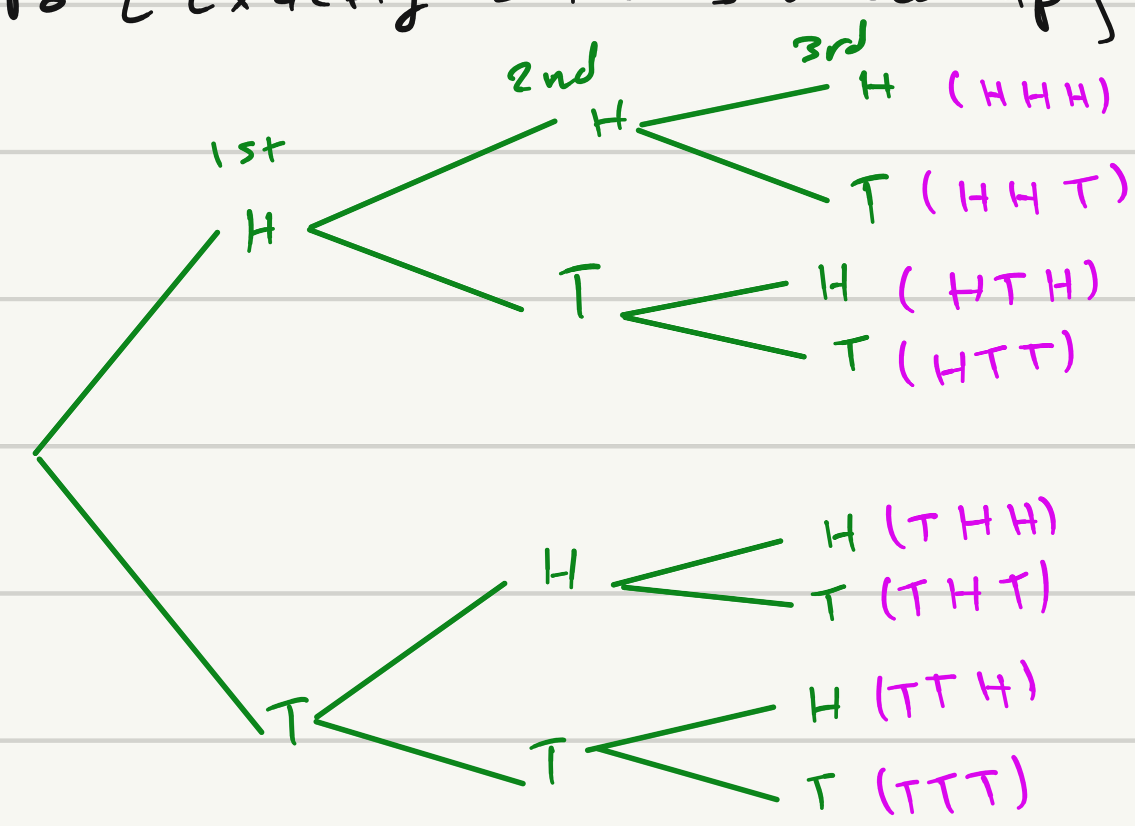
Discrete uniform probability low

[1] اذا كانو equally Likely بنستخدم السابقة

[2] اذا كانو مش equally likely نجمع كل وحده لحال

Ex:- Tossing a fair (⇒ equaly likly) coin thriee

P₀ {exactly 2 Hs showed up}



So

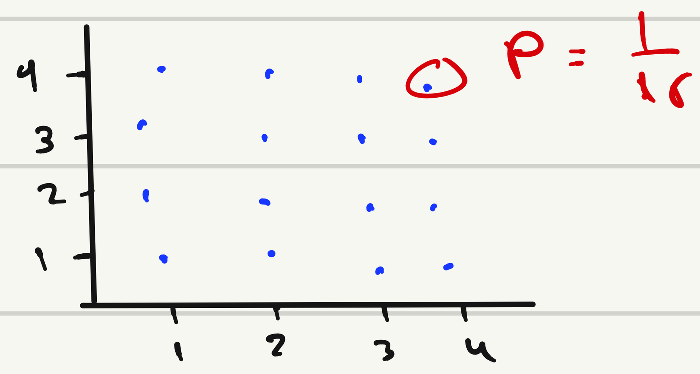
$\Omega = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$

since the coin is fair all outcomes are equaly likly

$A = \{(HHT), (HTH), (THH)\}$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{3}{8}$$

Ex: Rolling a fair 4-sided die twice



$\Omega = \{(1\ 1), (1\ 2), (1\ 3) \ldots\ldots (4\ 4)\} = |\Omega| = 16$

$Pr\{\text{sum of tow rolls in even}\} = \frac{|A|}{|\Omega|} = \frac{8}{16} = \frac{1}{2}$

$Pr(\text{sum of tow rolls is odd}) = 1 - P(A) = \frac{1}{2}$

$Pr\{\text{1st roll} = \text{second roll}\} = \frac{4}{16} = \frac{1}{4}$

$Pr\{\text{at lest 1 roll is 4}\} = \frac{7}{16}$

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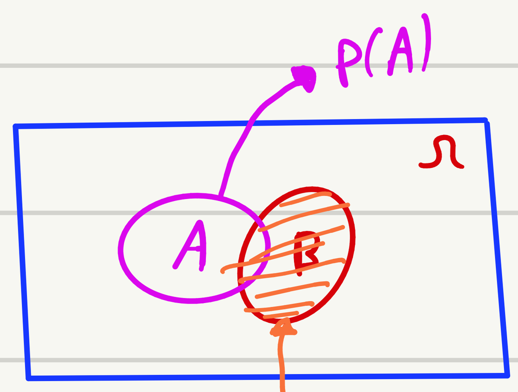
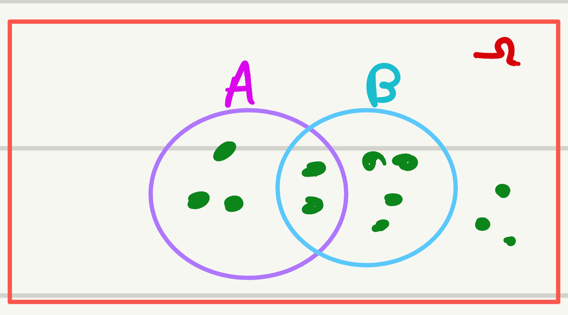
* conditional probabilty:

* $P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) > 0$



* Ex: assume all elemant are equally Likely



$P(A) = \frac{|A|}{|\Omega|} = \frac{5}{12}$

$P(B) = \frac{|B|}{|\Omega|} = \frac{6}{12} = \frac{1}{2}$

$P(A \cap B) = \frac{|A \cap B|}{|\Omega|} = \frac{2}{12}$

$P(A|B) = \frac{2}{6} = \frac{2/12}{6/12} = \frac{P(A \cap B)}{P(B)}$

supose that event B has occurred find the probability of A given B

conditinal prob $P(A|B)$

(new Ω) يعني : الذي حصل عندي فقط B وأصبح

و العناصر التي تدخل في الحساب من A هي التي تكون مشتركة مع B $(A \cap B)$

* $P(A|B) = \frac{P(A \cap B)}{P(B)}$, $P(B|A) = \frac{P(A \cap B)}{P(A)}$

so: $P(A \cap B) = P(A|B) * P(B) = P(B|A)\, P(A)$

* Multiplication Rule

* axioms Revisited:-

1) $P(A|B) \geq 0$ non negativity

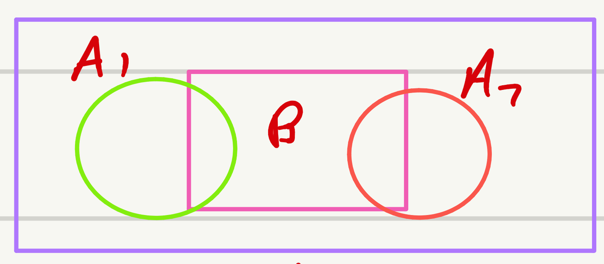
2) $P(A|A) = 1$ & $P(B|B) = 1$ normalization

$P(\Omega|A) = 1$

$P(A|\Omega) = P(A) \overset{Ex}{=} \frac{|A|}{\Omega}$

3) If A_1 & A_2 are disjoint

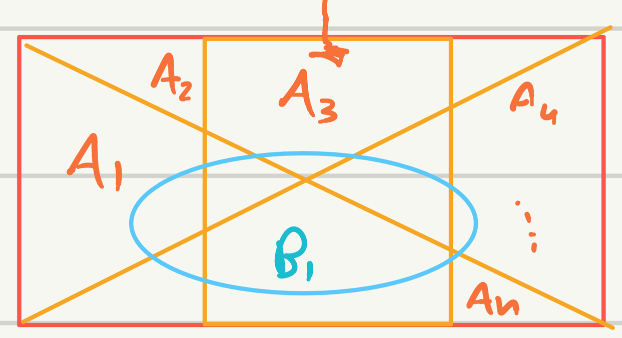
$P(A_1 \cup A_2 | B) = P(A_1|B) + P(A_2|B)$



disjoint

4) For $A_1, A_2, \ldots\ldots A_n$ all of thim are disjoint

$P(\bigcup_{i=1}^{n} A_i|B) \leq \sum_{i=1}^{n} P(A_i|B)$



comming

* multiplication Rule :-

$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot \underline{P(A_2 \mid A_1)} \cdot \underline{P(A_3 \mid A_1 \cap A_2)}$

$\cancel{P(A_1)} \cdot \dfrac{P(\cancel{A_1 \cap A_2})}{\cancel{P(A_1)}} \cdot \dfrac{P(A_2 \cap A_1 \cap A_3)}{P(\cancel{A_1 \cap A_2})}$

$\text{OR} = P(A_3) \cdot P(A_2 \mid A_3) \cdot P(A_1 \mid A_2 \cap A_3)$

in general

$P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1) \cdot P(A_2 \mid A_1) \cdot P(A_3 \mid A_1 \cap A_2) \cdots\cdots$

$* P(A_n \mid A_1 \cap A_2 \cap \cdots A_{n-1})$

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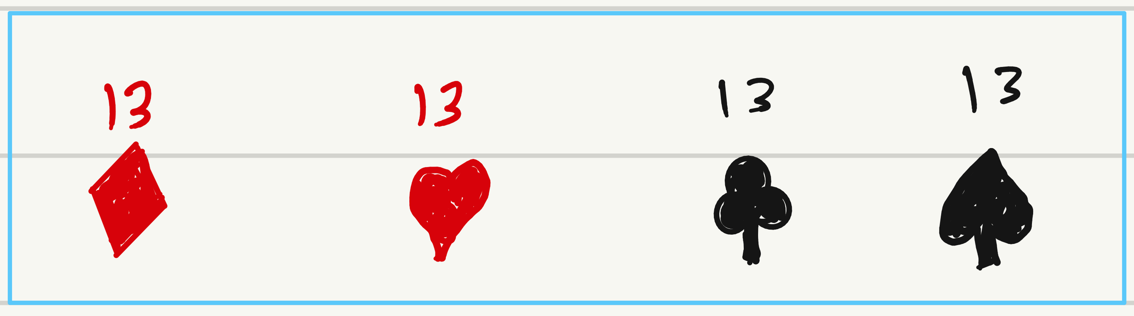
52 card deck

* all card are equally Likely &

will - shufeled deck



Ex :- Three card are drawn form a well shaffled 52 - card deck , find the probabilty that non of of these cards

$\boxed{1}$ is not a Heart with out replacement :-

solution :-

1st card is not Hart $= A_1$

2nd // // // // $= A_2$

3ed // // // // $= A_3$

$P(A_1 \cap A_2 \cap A_3) = P(A_1)\, P(A_2 \mid A_1)\, P(A_3 \mid A_1 \cap A_2)$

$= \dfrac{39}{52} \cdot \dfrac{38}{51} \cdot \dfrac{37}{50}$

$P(A_1 \cap A_2 \cap \bar{A}_3) = P(A_1) \cdot P(A_2 \mid A_1)\, P(\bar{A}_3 \mid A_1 \cap A_2)$

=>

For the last example :-

$\boxed{2}$ P { only thrid card is a hart }

1st is not H A_1

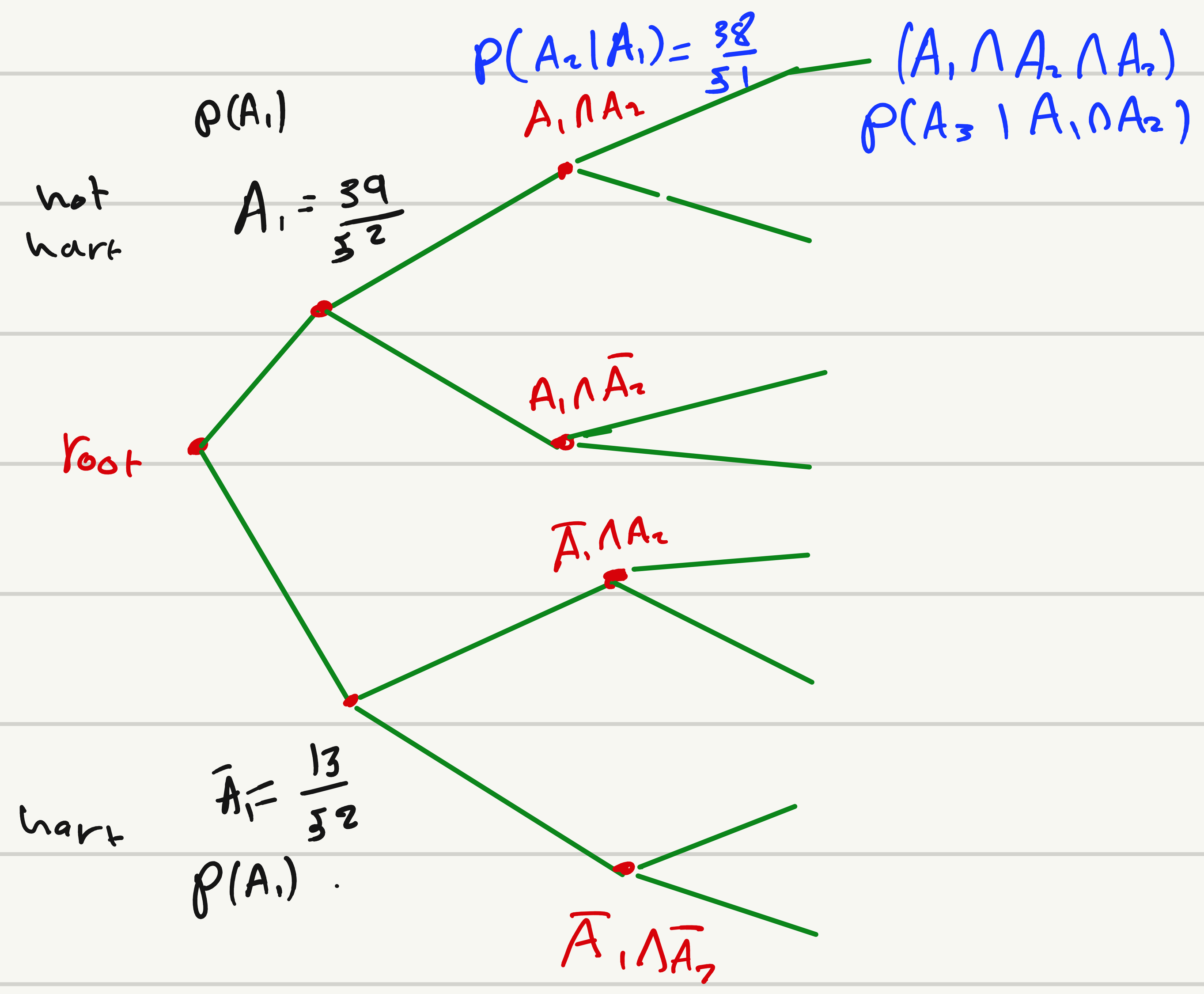
2nd is not H A_2

3rd is a hart $\bar{A}_3$ OR B

$P(A_1 \cap A_2 \cap \bar{A}_3) = P(A_1)\, P(A_2 \mid A_1)\, P(\bar{A}_3 \mid A_1 \cap A_2)$

$= \dfrac{39}{52} \cdot \dfrac{38}{51} \cdot \dfrac{13}{50}$

another solution in Tree Diagramt



$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2 \mid A_1) \cdot P(A_3 \mid A_1 \cap A_2)$

$\boxed{3}$ P [The Third card is a hart }

$P(\bar{A}_3) \Rightarrow$ منشتغلي

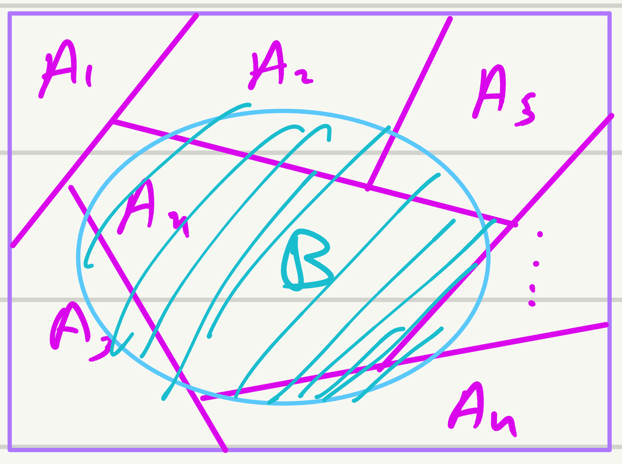
$= P(A_1 \cap A_2 \cap \bar{A}_3) \cup P(A_1 \cap \bar{A}_2 \cap \bar{A}_3) \cap P(\bar{A}_1 \cap A_2 \cap \bar{A}_3)$

$\cup\ P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3)$

* Total pobability theorem:-

let, $A_1, A_2, A_3, \dots\dots A_n$ be partition of Ω let B an event, then

$$P(B) = \sum_{i=1}^{n} P(B|A_i)\, P(A_i)$$



Proof:-

$$B = (B\cap A_1) \cup (B\cap A_2) \cup (B\cap A_3) \cup \dots (B\cap A_n)$$

$$P(B) = \sum_{i=1}^{n} P(B\cap A_i)$$

$$= \sum_{i=1}^{n} P(B|A_i)\cdot P(A_i)$$

* $P(A_i\cap B) = \dfrac{P(B|A_i)\, P(A_i)}{P(B)}$

* $P(A_i\cap B) = \dfrac{P(B|A_i)\, P(A_i)}{\sum P(B|A_i)P(A_i)}$ Bayes Rule

* Ex:- we have tow coin and we Bick one of them randomly

the 1st H & T equaly likly

the 2nd H & H

* Ex:- pr{$\overbrace{\text{generating}}^{B}$ asignal given $\overbrace{\text{aircraft}}^{A}$ presents} = 0,99

Pr{ $\overbrace{\text{"} \quad \text{"}}^{B}$ " $\overbrace{\text{No air craft}}^{\bar{A}}$ in sky}

Pr{$\underbrace{\text{Aircraft presents}}_{A}$} = 0,05

B = asignal has been generating

$P(B|A) = 0,99 \qquad P(A) = 0,05$

$P(B|\bar{A}) = 0,1$

[1] probability of detection $(A\cap B)$

Aircraft and somthing regester

$$P(A\cap B) = P(B|A)\cdot P(A)$$

$$= 0,99 * 0,05$$

[2] probabilitty missdetection $P(\bar{B}\cap A)$

$$P(A\cap\bar{B}) = P(\bar{B}|A)\cdot P(A)$$

$$= 0,01 * 0,05$$

[3] False alarm:- $P(\bar{A}\cap B)$

$$P(\bar{A}\cap B) = 0,1 * 0,95$$

[4] $P(B) = P(A\cap B) + P(\bar{A}\cap B)$

$$= P(B|A)\, P(A) + P(B|\bar{A})\cdot P(\bar{A})$$

$$= 0,99 * 0,05 + 0,1 * 0,05$$

[5]

$$P(B|A) = \frac{P(B|A)\cdot P(A)}{P(B)}$$

نفس [1] بس بنعكس الكلام

Ex you roll a fair 4-sided die If the outcome is "1" or "2" you roll once more

otherwise you stop.

مرة واحدة فقط

$Pr\{\underline{\text{sum of your rolls is at least "4"}}\}$

B

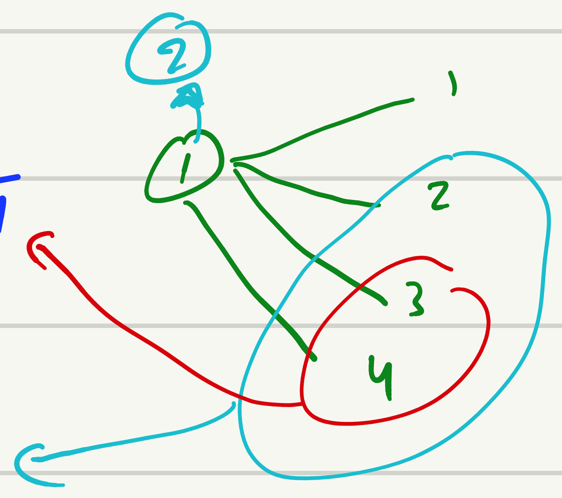
Define $A_i = \{$the outcome 1st Roll is $i\}$

$p(B) = \sum_{i=1}^{4} p(B|A_i)\,p(A_i)$ $\qquad P(A_i) = \frac{1}{4}$

$p(B|A_3) = 0$

$p(B|A_4) = 1$

$p(B|A_1) = \frac{2}{4}$



$p(B|A_2) = \frac{3}{4}$

$= \frac{1}{4}(1 + 0 + 2/4 + 3/4) = 9/16$

$$P(A_1|B) = \frac{P(B|A_1)\,P(A_1)}{P(B)}$$

---

* independent of events:-

consider two events A and B we say that A is independent (ind.) of B if the occurence of B provides no information about the likelihood of A That is

$P(A|B) = P(A)$ * $\quad$ A is ind of B, B is ind of A

$P(A\cap B)$ are ind.

Prove

$P(A|B) = \frac{P(A\cap B)}{P(B)} = P(A)$ using *

$So \Rightarrow P(A\cap B) = P(A)\cdot P(B)$

* Note :- disjoint $\Rightarrow$ is fully dependant

لانه واحد منهم بلغي التاني

* For A_1, A_2, we say that A_1 and A_2 are ind. if $P(A_1\cap A_2) = P(A_1)\cdot P(A_2)$

For A_1, A_2, A_3 we say that they are ind. if :-

$P(A_1\cap A_2) = P(A_1)\cdot P(A_2)$

$P(A_1\cap A_3) = P(A_1)\cdot P(A_3)$

$P(A_2\cap A_3) = P(A_2)\cdot P(A_3)$

$P(A_1\cap A_2\cap A_3) = P(A_1)\cdot P(A_2)\cdot P(A_3)$

الشروط حتى نحكي عنه ind.

---

* we say that $A_1, A_2, \ldots\ldots A_n$ are ind. if

$P(\bigcap_{i\in S} A_i) = \prod_{i\in S} P(A_i)$, For every subset

$S, S\subset \Omega = \{1, 2, \ldots n\}$

$\boxed{(2^n - 1 - n)\ \text{تكون}\ \text{عدد الشروط}}$

Ex:- A Fair coin is tossed twice

$P(1^{st}\text{ is H and }2^{nd}\text{ is H}) = \frac{1}{4}$

$P(1^{st} = H) = \frac{1}{2}$

$P(2^{nd} = H) = \frac{1}{2}$

$P(H^1\cap H^2) = P(H^1)\cdot P(H^2)$

so H^1 & H^2 ind

Ex:- Two successive Rolls of a fair 4-sided die :-

متعاقبة

$A_i = \{1^{st}\text{ roll is } i\}$, $i = 1, \ldots\ldots, 4$

$B_j = \{2^{nd}\text{ roll is } j\}$, $j = 1, \ldots\ldots, 4$

Are A_i and B_j ind ??

$P(A_i\cap B_j) = P(1^{st} = i, 2^{nd} = j) = \frac{1}{16}$

* $P(A_i) = \frac{1}{4}$ $\quad$ * $P(B_j) = \frac{1}{4}$

$P(A_i\cap B_j) = P(A_i) * P(B_j)$

A_i and B_j are ind. $\Rightarrow$ rare continue =)

(2) $A = \{1^{st} \text{ roll is } 1\}$

$B = \{\text{sum of the two Rolls is } 5\}$

$P(A) = \frac{1}{4}$

$P(B) = \{(4,1), (1,4), (2,3), (3,2)\} = \frac{4}{16}$

$P(A \cap B) = \frac{1}{16} = P(A)\, P(B)$ A and B are ind.

(3) $A = \{\text{the max of two rolls is } 2\}$

$B = \{\text{the min of two rolls is } 2\}$

$A = \{(1,2), (2,1), (2,2)\} = \frac{3}{16}$

$B = \{(2,3), (2,4), (3,2), (2,2), (4,2)\} = \frac{5}{16}$

$P(A \cap B) = \frac{1}{16} \neq P(A).P(B)$

A and B are not ind.

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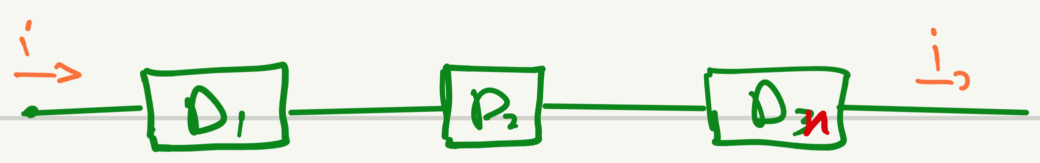
* Applications on ind.

(1) consider n devices. up دائماً بنقل انه



series

$A_i = \{D_i \text{ is up}\},\ i = 1, \dots, n$

$P(A_i) = P_i,\ i = 1, 2, 3, \dots, n$

$P(\text{system is up}) = P(\bigcap_{i=1}^{n} A_i)$

each device fails independently of other diveces.

$P(\bigcap_{i=1}^{n} A_i) = \prod_{i=1}^{n} P(A_i) = \prod_{i=1}^{n} P_i$ هاي اقل

P system is Down $= 1 - \prod_{i=1}^{n} P_i$ هاي لازم تكون اكبر

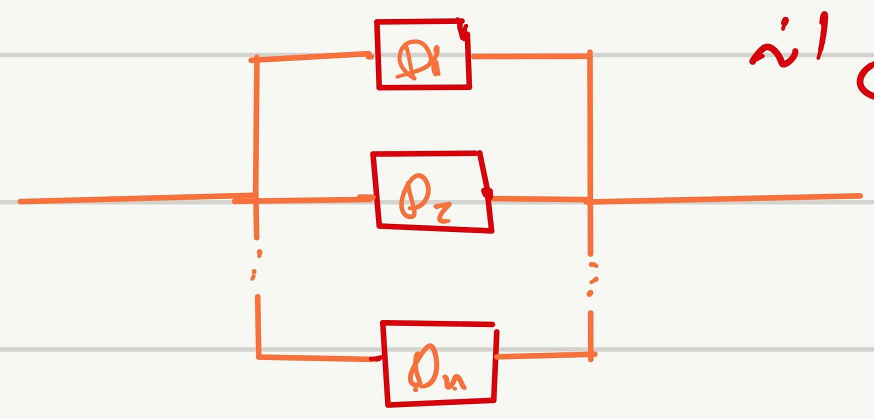
(at least (1) of the devises is down) طريقة ثانية للتعبير

$(\bar{A}_1 \cup \bar{A}_2 \cup \bar{A}_3 \cup \dots \bar{A}_n)$

$= \overline{P(A_1 \cap A_2 \cap \dots A_n)} = 1 - P(\bigcap_{i=1}^{n} A_i)$

$= 1 - \prod_{i=1}^{n} P_i$

(2) parallel connection



دائماً بنقل انه Down

$A_i : \{D_i \text{ is up}\}$

$P(A_i) = P_i,\ i = P_i,\ i = 1, 2, \dots, n$

$P(\text{system is down}) = P(\bar{A}_1 \cap \bar{A}_2 \cap \dots \bar{A}_n)$

$= \prod_{i=1}^{n} P(\bar{A}_i)$

$= \prod_{i=1}^{n} (1 - P_i)$

Prove :- $P(\bar{A}_1 \cap \bar{A}_2) = \overline{P(A_1 \cup A_2)}$

$= 1 - P(A_1 \cup A_2)$

$= 1 - P_{A_1} - P_{A_2} + P(A_1 \cap A_2)$

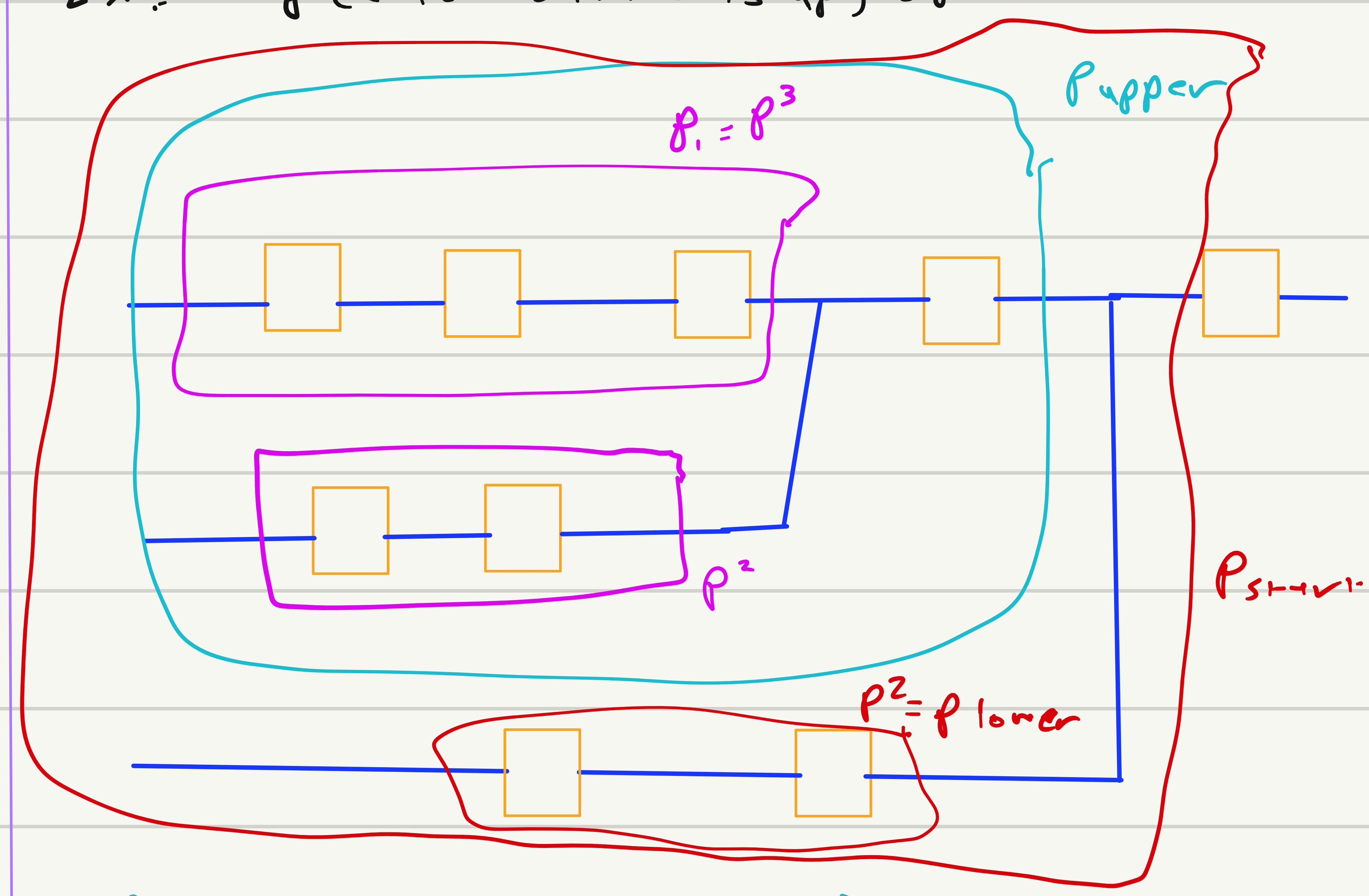
$= 1 - P_{A_1} - P_{A_2} + P_{A_1} P_{A_2}$

$= (1 - P(A_1)).(1 - P(A_2))$

$P(\text{system is up})$

$= 1 - \prod_{i=1}^{n} (1 - P_i)$

Ex: $P(\text{each divice is up}) = P$



$P_{upper} = [1 - (1 - P_1)(1 - P_2)]P$

$= [1 - (1 - P^3)(1 - P^2)]P$

$P_{start} = 1 - (1 - P_{lower})(1 - P_{upper})$

$P_{system} = P(1 - (1 - P_{lower})(1 - P_{upper}))$

Counting principle:

If we have 2 shirts and 3 Jackets:

$\{(s_1,j_1),(s_1,j_2),(s_1,j_3),(s_2,j_1),(s_2,j_2),(s_2,j_3)\}$

note: we have 6 outcom this mean's it's

$2 * 3 = 6$

* conside a process that involves r stages than the total # of ways in which The proses can be completed is is

$$\boxed{n = \prod_{i=1}^{r} n_i , \text{ wher } = n_1 * n_2 * n_3 \cdots n_r}$$

$n_i \triangleq$ # of ways in which the stage i can be completed.

Ex: If we have 4 shirts, 2 Jacket, 3 ties

the nomper outcom

$n = 4 * 2 * 3 = 24$

Ex: The nnmber of License plate can be formed from 2 letters followed by 3 Digits.

نمرة السيارة

L	L	D_1	D_2	D_3
26	26	10	10	10

L: A - Z => 26

D: 0 — 10 => 10

# of License plate = 26 * 26 * 10 * 10 * 10

repitition Alowed * ممنوع التكرار

License plate = 26 * 25 * 10 * 9 * 8

with out reitiron

$\Omega = \{1,2,3\} \Rightarrow n=3$

$\phi \rightarrow 1$ $\quad \{1,2,3\} \Rightarrow 1$

$\{1\}\ \{2\}\ \{3\} \Rightarrow 3$ $\quad$ the total number of subset

$\{1,2\}\ \{1,3\}\ \{2,3\} \Rightarrow 3$ $\quad = 2^n = 2^3 = 8$

$1+3+3+1 = $

Permutation: The # of ways of ordering n elemants in asiquence of n slots

n-element

# of ways $= n(n-1)(n-2)\cdots 1$

$= n!$

1	2	3	...	n
n	n-1	n-2	...	1

numper of choise

Ex:

$\Omega = \{S_1, S_2, P\}$

①	②	3

S_1, S_2, P

S_1, P, S_2 ~ # of permution

P, S_1, S_2 $\quad = 3!$

P, S_2, S_1 $\quad$ (عدد الترتيبات الممكنة)

S_2, S_1, P

S_2, P, S_1

K - permation $\quad$ على سبيل المثال عدد المقاعد

أقل من عدد الطلاب حيث عدد الطلاب (n)

n-elemaunt

①	②	③	...	k
n	n-1	n-2	...	n-k+1

# of ways $= n(n-1)(n-2)\ (n-k+1)$

$$= \frac{n(n-1)(n-2)\cdots(n-k+1)\cancel{(n-k)\cdots 1}}{\cancel{(n-k)\cdots 1}}$$

as same

So => $\boxed{P^n_k = \dfrac{n!}{(n-k)!}}$ $\quad$ اثباتها

Ex: $\Omega = \{S_1, S_2, P\}$

$n=3 \quad K = 2 \Rightarrow$ عدد المقاعد

①	②

S_1, P	P, S_2
S_2, P	S_1, S_2
P, S_2	S_2, S_1

في حال كانا عندنا عدد K اكبر من

عدد ال n في القانون الاساسي

بصير عندنا بدل n<=K و بدل

$P^k_n = \dfrac{K!}{(K-n)!}$ $\quad n <= K$