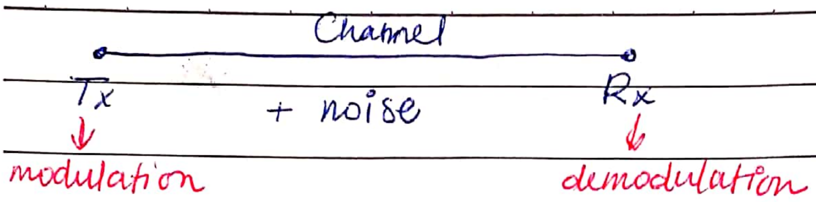
Subject Introduction to Communication 2 Date No.



Channels
- wired
- wireless (power + band width)

$n(t) = \sqrt{4KTB}$ where K = Boltzmann constant
T = temperature in Kelvin
B = bandwidth.

Functional Channels
- guided
  - directional
  - undirectional.
- unguided

Source of information:-
Multimedia
- Audio
- Video
- Data

* 1) Avalanche ثلجي , الزينا } أنواع لإنهيار → occurs because of the collision of free electrons with atom

2) Zenar breakdown } Channel في → occurs because of the high $\vec{E}$

\# Base Band Processor ⇒ manipulates the signal then modulates it energy is carried on the wire.

\# Signal coming from the source is mapped with signal coming from the output.

Standards give more flexibility to manipulate.

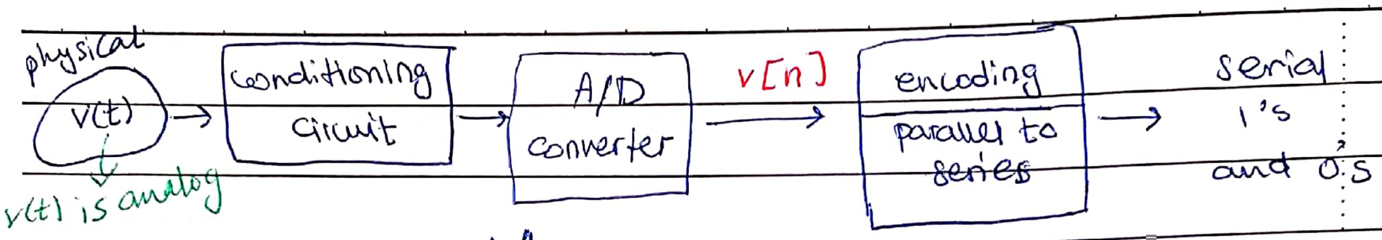
\# RF Processor = radio-frequency processors.

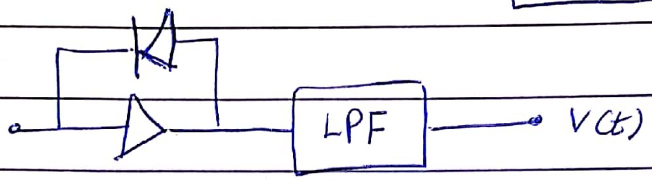
\# spectrum analyzer ⇒ shows the signal in frequency domain.

\# Transmission = Modulation = mapping from logical ~~state~~ signal into physical wave form.

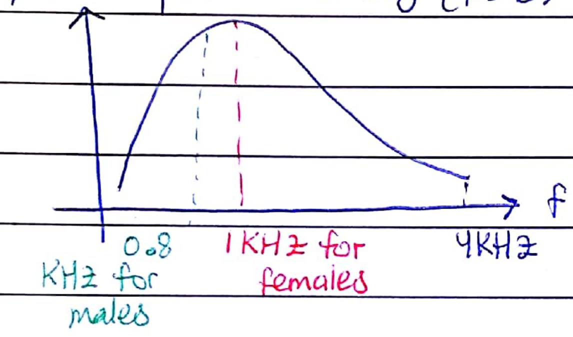
\# The condition that makes signal move from one point to another is energy. (not power because the signal is aperiodic, it won't repeat it self forever). periodic signals are power signals.

\# $\text{loss} \underset{\text{alpha}}{\underset{\downarrow}{\alpha}} 20 \log(\text{frequency}) + 10 \underset{\text{gama}}{\underset{\downarrow}{\gamma}} \log(\text{distance})$ [dB]





power spectrum density (PSD)



* For toll quality so

BW = 4KHz

mean of opinion score (MOS) = 3

* Deterministic signals carry no information.

probability of deterministic signal = $^{0}/_{100}$

* Random signal carry information.

* entropy $(H_i) = -\log_{②}(P_i)$; P_i = probability of that value.

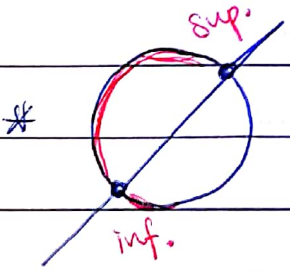
② → because it is in binary system.

entropy ; عشوائية ; thermodynamic quantity often intrepeted as the degree of randomness in the system.

entropy is measured in [bits]

entropy = minimum rate.

*



convex ⇒ بمرّ خط لازم يقطع نقطتين

* lossy ~~less~~ compression ⇒ I can use data from the source which leads to distortion.

* lossy compression ⇒ 1) Hufman code 2) LZ algorithm.

- loss

*R-D function (Rate-Distortion function) is used to measure the minimal number of bits per symbol, it provides the theoretical foundations for lossy compression.

* Quantization noise $D = \frac{\Delta^2}{12}$, $\Delta = \frac{V_{p-p}}{\text{number of levels}}$

Distortion $= \alpha\, 2^{-R} = \alpha\, 2^{-O(R^2)}$, O : order

$\therefore D = \frac{(V_{pp})^2}{12}\, 2^{-2R}$.

* Example; a source of 4 elements $S = \{a, b, c, d\}$ with probabilities $P = [0.1 \quad 0.5 \quad 0.2 \quad 0.2]$; 1) find each letter (element) entropy ? 2) find the source entropy?

element = letter = member

Sol Rate = R = 2 bits

1] $H_i = [-\log_2 0.1 \quad -\log_2 0.5 \quad -\log_2 0.2 \quad -\log_2 0.2]$

$= [\boxed{3.32} \quad 1 \quad 2.3 \quad 2.3]$

↓ letter a contains most information.

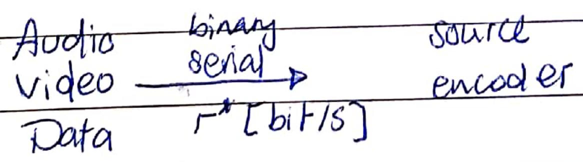
2] $H = [-P_i \log P_i] = (3.32)(1) + (1) \times (0.5) + (2.3)(0.2) + (2.3)(0.2)$

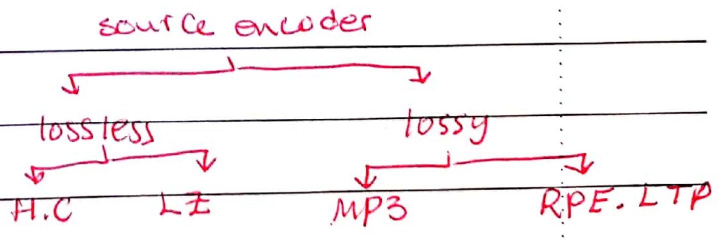
$= 0.332 + 0.5 + 0.46 + 0.46 = 1.79$ bits ها ع تطلع 2

لانه $H \leq R$

$\log_w(n) = \frac{\ln(n)}{\ln(w)}$

* optimal encoding of sources when R = H, this happens when all letters are equally likely.





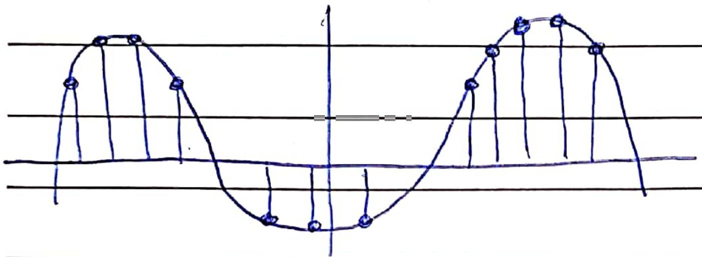
$r^* = R \cdot f_{sampling}$

audio is a signal
→ voice → spoken
→ audio → heard.

Random Signal : $X(t) = X_d(t) + X_r(t)$

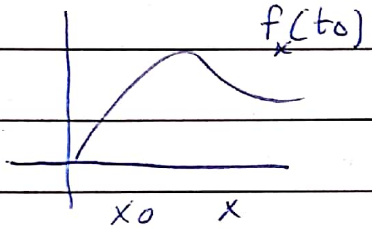
X_d → deterministic signal (احتمال وجودها أكيد = 1)

X_r → pure random signal



sampled signal

$t = t_0$



identical ∴ the same probability

auto correlation $R_{xx}(t_1, t_2) = \overset{\text{expectation}}{\boxed{E}}\{x_{*}(t_1) \cdot x(t_2)\}$

$R_{xx}(t_1, t_2) = R_{xx}(\tau)$; $\tau = t_2 - t_1$

تمشي ازاي t_1, t_2 بيتغيروا في الـ x مع بعض

$x(t_2) = a x(t_1) + b$. W.S.S (wide sense stationary)

correlation coefficient $\rho(x_1, x_2)$ → $\rho(x_1, x_2) = 0$ → uncorrelated.

→ $-1 \le \rho \le 1$ → correlated.

Ergodic random variable : random variable signals that can be written as a function of time.

$F.T\{R_{xx}(t)\} = S_{xx}(f)$ → power spectral density.

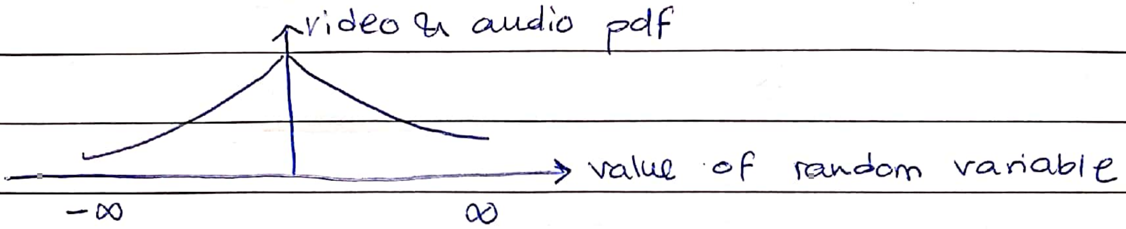
$\frac{1}{L} \underbrace{\sum^{L}}_{\text{large}} \underbrace{X(f)}_{F.T\{x(t)\}} \simeq S_{xx}(f)$ → (histogram)

~~PSD thermal~~ $= 4KT$, T= temperature [Kelvin]

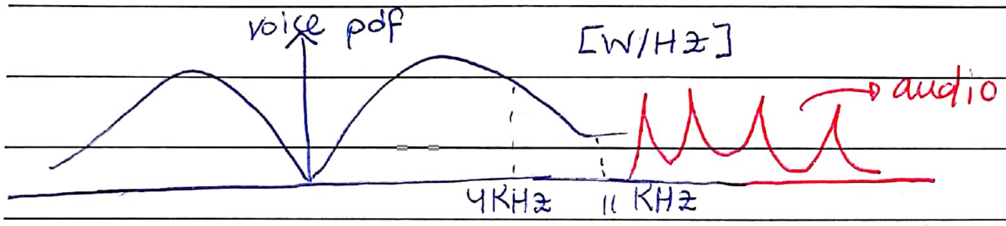
$P_{noise} = 4KTB = \sigma_{noise}^2$, B= bandwidth.

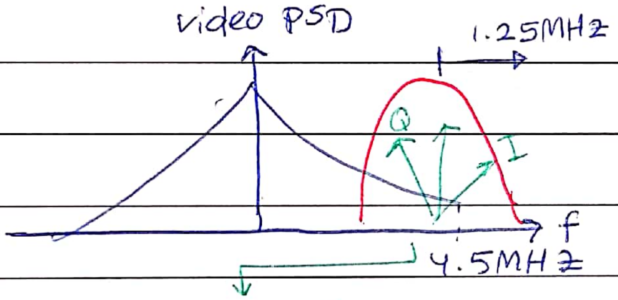
$V_{rms} = \sqrt{4KTB} = \sigma_{noise}$

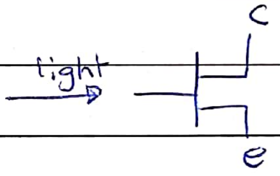
PDF: Probability Density Function.



$f_X(x) = \frac{1}{2} e^{-|x|}$







3.58 MHz
4.33 MHz

صور في الوان على اشارات الابيض والاسود

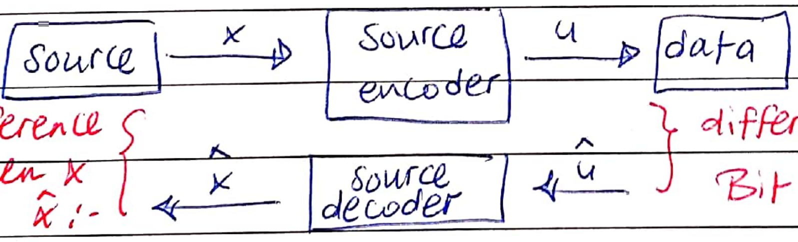
PAL / SECAM / NTSC

video :

CCD : Charge Couple Devices

$F_{clk} = 103 \text{ MHz}$

$r_b = 2.4 \text{ Gbps}$

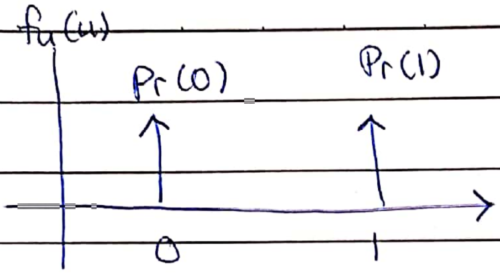


difference between x and $\hat{x}$:-
Distortion

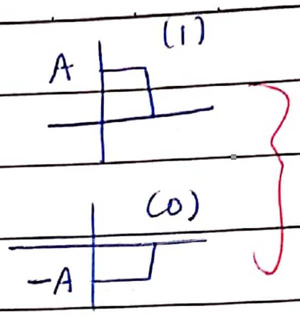
difference between u and $\hat{u}$:-
Bit Error Rate (BER)

u is a random process

HW write about DC restoration circuit & draw the circuit

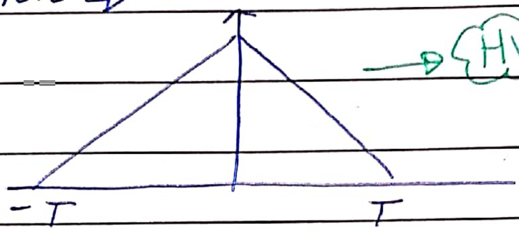


where



auto correlation.

auto correlation function ⇒ $R_{uu}(\tau)$



→ {HW} find F.T of this signal.

for good source encoders (data compression) we have the following features → 1] probability of (1) = probability of (0)

2] independent (uncorrelated)

$$E\{\underset{+\tau}{u(t)} \cdot \underset{+\tau}{u^*(t-\tau)}\} = R_{uu}(\tau)$$

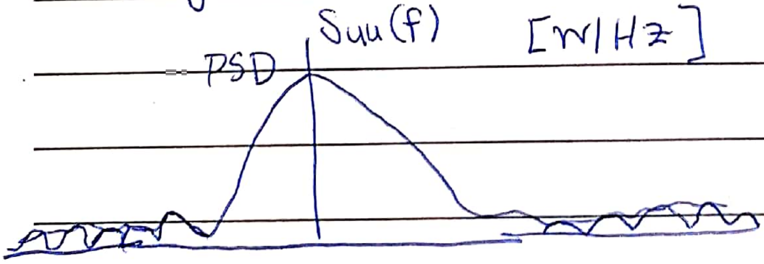
$$\cancel{E}\int_T u(t) \cdot u(t+\tau)\, dt$$

→ mean = best estimate of the signal

$$R_{uu}(0) = \underbrace{\sigma_u^2}_{\text{AC power}} + \underbrace{\mu_u^2}_{\text{DC power}}$$

$$\lim_{\tau \to \infty} R_{uu}(\tau) = \mu_u^2$$

taking the fourier transform of the signal ⇒ sinc function signal.

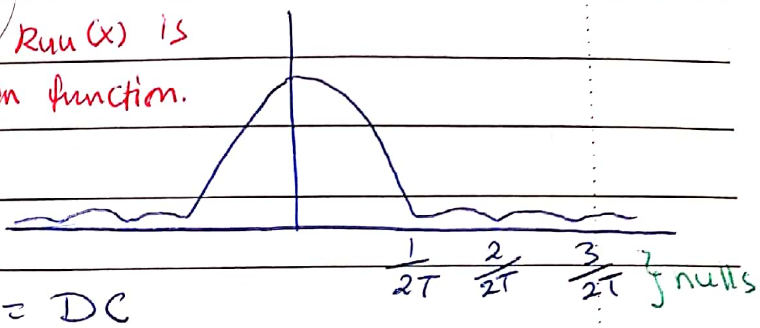


$|R_{uu}(\infty)| = \mu_u^2$

$S_{uu}(0) = \mu_u^2$

$\int_{-\infty}^{\infty} \text{sinc}(?) = \int_{-\infty}^{\infty} \text{sinc}^2(?) = 1$

usually $R_{uu}(x)$ is an even function.



for non zero ⇒ $A \cdot \frac{1}{2} + B \cdot \frac{1}{2} = DC$

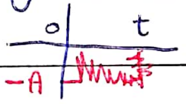
$BW = \frac{1}{2T} = \frac{r_b}{2}$ Hz where r_b = bit rate = $\frac{1}{T}$

We will use null-null bandwidth..

* We need a zero DC value for signals representing 1&0 logic to get rid of the line spectrum.

* Energy signal is time limited signal depending on bit rate.

* We can't make nulls inside the signal because the frequency of it is larger than the frequency of 0 & t so it will go and create the nulls outside ⇒



(Example)

*if t goes bigger (قاعدة المثلث صارت أكبر) → bandwidth decreases.
wider in time domain, narrower in bandwidth.

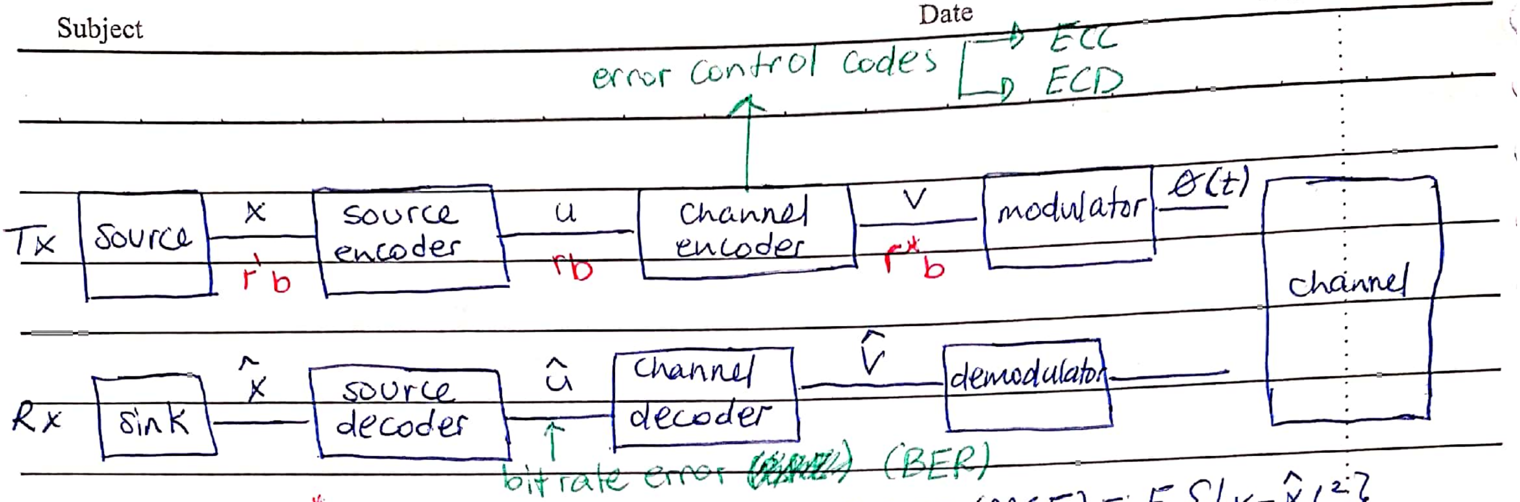
* The Smallest signal has zero bandwidth.
deterministic signals are smooth signals.

* if correlation decreases → the bandwidth decreases.

* As signal-to-noise-ratio (SNR) increases, the bit error rate decrease and the correct information delivered to the receiver is increase

* As bandwidth increases, the amount of information delivered to the receiver increases.

\# $C = BW \log_2(1+SNR)$ [bps] ⇒ Shannon's limit.



mean square error (MSE) = $E\{|x-\hat{x}|^2\}$

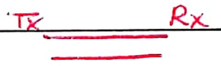
$r_b < r^*_b$

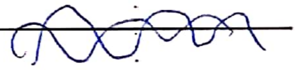
$r_b > r'_b$

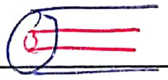
* Modulation is the mathematical process to map a logical signal (v) into a physical wave form (s(t)) such that it pass through the channel with the minimum distortion (error is meant here).

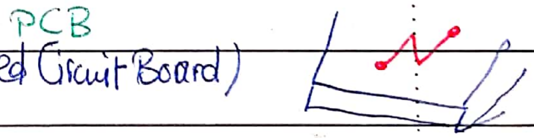
$BER_{channel} \simeq (BER_{channel})^{t+1}$, t = number of errors can be corrected

* Channel
- wired
  - open wire
  - twisted pair
    - unshielded (UTP)
    - shielded (STP)
  - coaxial
  - micro-strip (printed Circuit Board)
- wireless









Distortion
- Noise
  - AWGN (Thermal)
  - interferance
- Fading (when there is multipath)

. Multipath fading is having multiple replicas of the same transmitted signal arriving at the receiver with random phase shifts.

- Physical enviroment ⤳ the media when sending from Tx to Rx.

all physical channels are band limited.

- When designing a communication system via wired channel, if the data rate (r_b) is less than channel bandwidth ($r_b \ll B.W$) then we treat the channel as infinite bandwidth channel, otherwise we must consider the channel bandwidth (bandlimited channels).

$$B = \frac{1}{2\pi\sqrt{LC}}$$

* I - Base-Band
- → energy signal
- → Centered around zero
- → time limited

wired communication / channel usually use BaseBand (Tx)

$$\begin{tikzcd} d \arrow[r] & \boxed{mod.} \arrow[r, "\varnothing_t(t)"] & \boxed{Ch.} \arrow[r] & \boxed{demod.} \arrow[r, "\hat{d}"] & {} \\ & & n \arrow[u] & & \end{tikzcd}$$

$\varnothing_r = \varnothing_t + \text{noise}$

((ادرسي نص لصفحة الي
لبعد هاي بالأول))

Data
- → Binary → $d \in \{0, 1\}$, $M = 2$
- → UnBinary → $d \in \{S_i , i = 0, 1 \cdots M-1\}$

we have M different data values ($M > 2$)

Symbols have 2 values → binary.

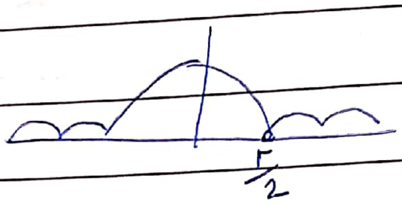
symbols have >2 values → unbinary.

$M = 2^m$

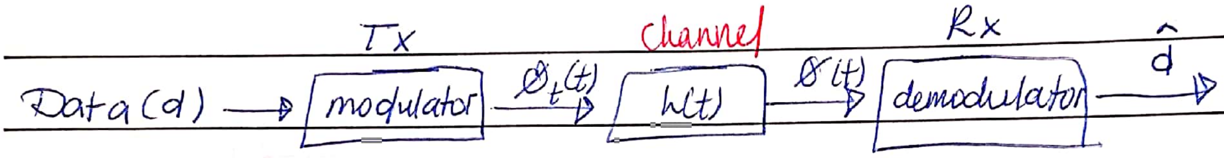
$S_i \leftarrow 2^m = M$

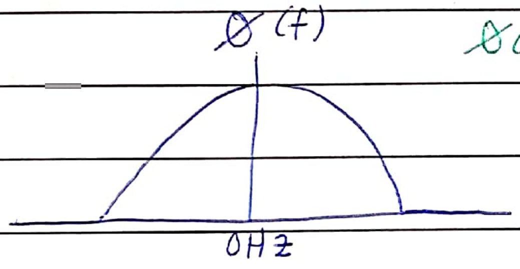
* Information has → period (T)
↳ rate (r) = $\frac{1}{T}$

$Bw = r/2$

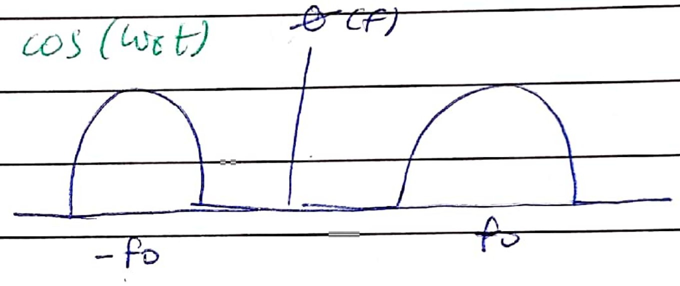


$V(t)$ → $\int V^2(t)\,dt < \infty$
↳ time limited (T)



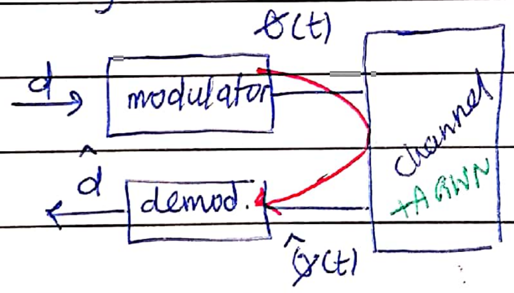


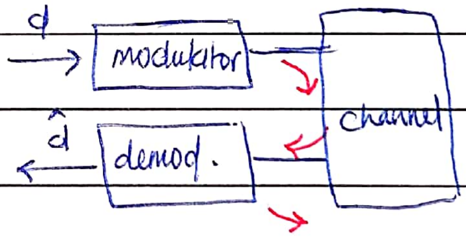
Base - Band



Band - Pass

* Binary Base Band ⇒ wired / unlimited bandwidth.





when it match ⇒ $\hat{\phi}(t) = \phi(t) + \text{noise}$

not match ⇒ reflect the signal (mismatch)

We prefer more immune channel signal (less noise)

intersymbol interference (تداخل اشارات ببعض)
"ISI"

if the signals are not matched ⇒ reflection

Wired channels → Additive White Gaussian Noise (AWGN)

Wireless channels → fading channels.

- Modulation is mapping a logical data into physical data that can pass the channel without distortion.

SNR $\left(\frac{\text{signal}}{\text{noise}}\right)$ → small → noisy signal.
→ large → clean signal.

$SNR = \frac{P_s}{P_n}$

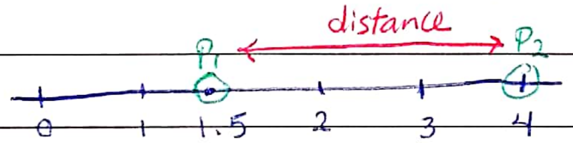
- We have 3 concepts → max / min
  → difference = distance
  → signal

as distance increases it is easier to distinguish between points (signals)

* Signal Space :

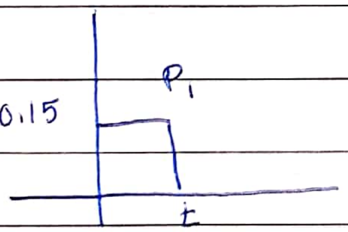
for space we need { axis
field (numbering system ie, binary)

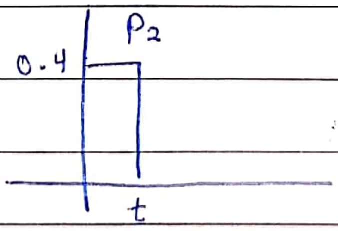
in 1-D ⇒

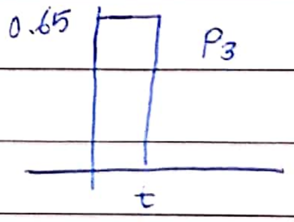


$|E_1| = |P_1|^2 = 1.5\overset{2}{a}_x$ [J]

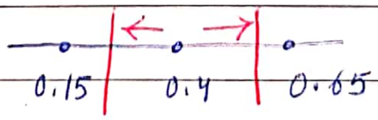
$P_1 = 1.5\,\hat{a}_x$



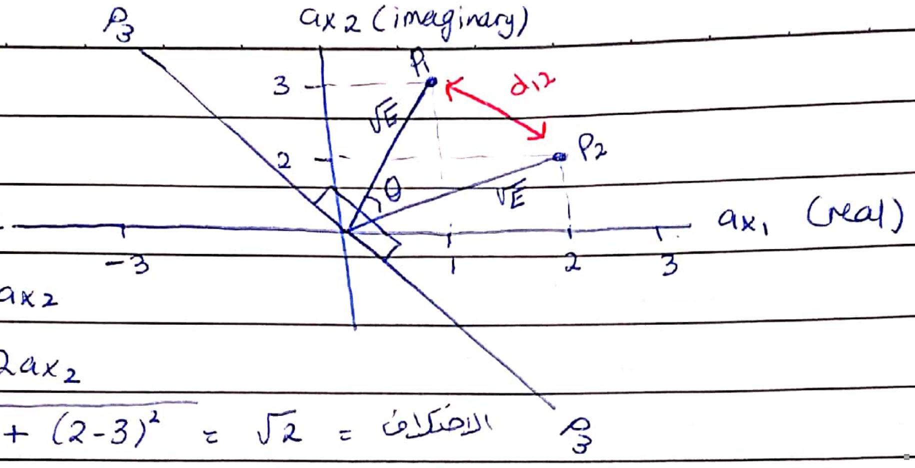




Noise Margin ⇒



in 2-D ⇒

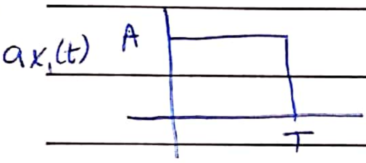


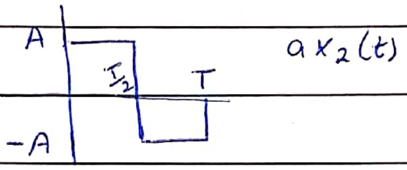
√energy = distance

$P_1 = ax_1 + 3ax_2$

$P_2 = 2ax_1 + 2ax_2$

$d_{12} = \sqrt{(2-1)^2 + (2-3)^2} = \sqrt{2}$ = الاختلاف





orthogonal ? $\langle ax_1(t), ax_2(t)\rangle = 0$; $\theta =$ مسقط الاول على التاني

$|ax_1(t)| \cdot |ax_2(t)| \cdot \cos(\theta_{12}) \triangleq \int_T ax_1(t) \cdot ax_2(t)\, dt$

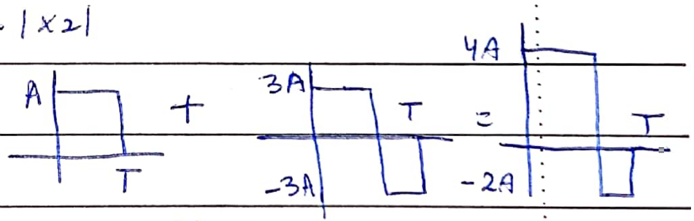
$\downarrow$

$\sqrt{ax_1(t)^2} = \sqrt{energy}$

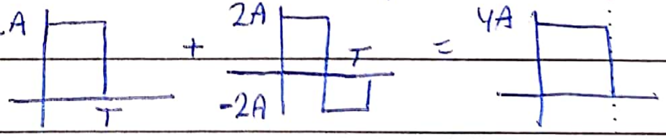
this is the corelation when shift = 0

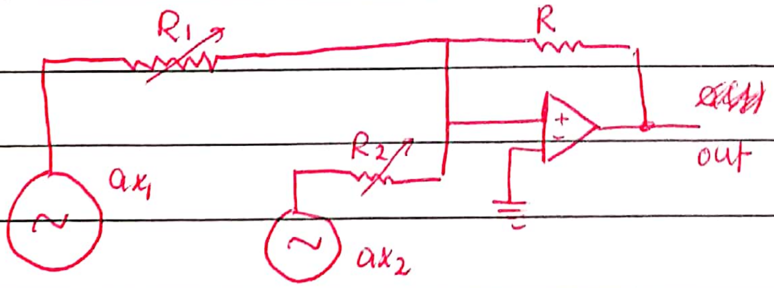
Coleration coefficient : $\rho_{12} = \frac{\langle x_1, x_2\rangle}{|x_1| \cdot |x_2|} = \cos(\theta_{12})$

$P_1, P_2 \rightarrow P_1 = ax_1 + 3ax_2 =$



$\rightarrow P_2 = 2ax_1 + 2ax_2 =$





$out = ax_1 \cdot \frac{R}{R_1} + ax_2 \frac{R}{R_2}$

$P = ax_1 + \textcircled{3} ax_2$

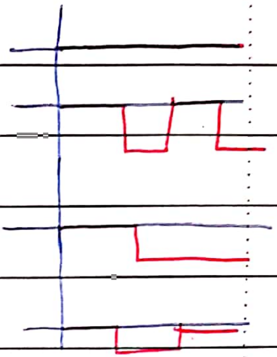
1] Gram schmidt

$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ $H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$

2] Orthogonal matrix space

→ Hadamard matrix

$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix}$



→ Discrete transform
- KL - T. F
- DST
- DCT
- DFT

$\hookrightarrow \vec{y} = A\vec{x}$ (N×N matrix)

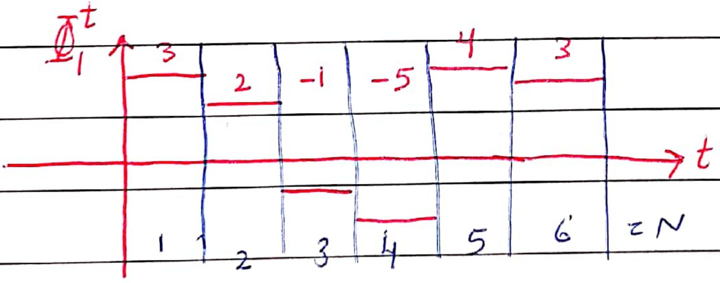
$A = \begin{bmatrix} c_1 & c_2 \\ a & b \\ c & d \end{bmatrix}$ $c_1 = \begin{bmatrix} a \\ c \end{bmatrix}$ $c_2 \begin{bmatrix} b \\ d \end{bmatrix}$ $\det(A) = \Delta A = ad - bc$

$\langle c_1, c_2 \rangle = c_1^T c_2 = ab + cd$; T = transpose

Note a matrix with inverse is non singular matrix.

$\underset{N \times N}{Eig(t)} = \bar{\Phi} = [\Phi_1, \Phi_2, \dots \Phi_N]$ $\bar{\Phi}$ eignvalue = λ's

$\langle \bar{\Phi}_i, \Phi_j \rangle = \delta_{ij}$ (δ = delta)

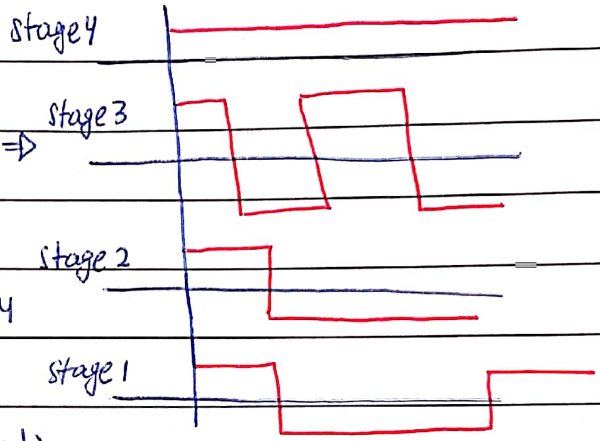


Note Discrete F.T ⇒ $Y[\phi] = \sum_{n=0}^{N-1} x[n] \, e^{-j\left(nK \frac{2\pi}{N}\right)}$

Hadamard Matrix ⇝

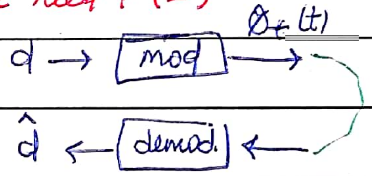
$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix}_{2\times2}$, $H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}_{4\times4}$ ⇒



* Binary Base Band (BW unlimited) :-

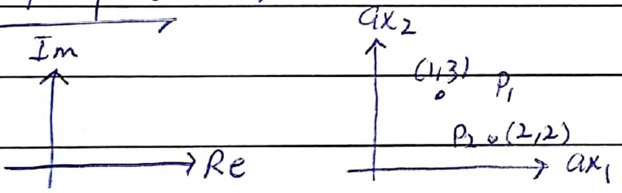
$d \in \{0,1\}$ → How many signals do we need ? (2)



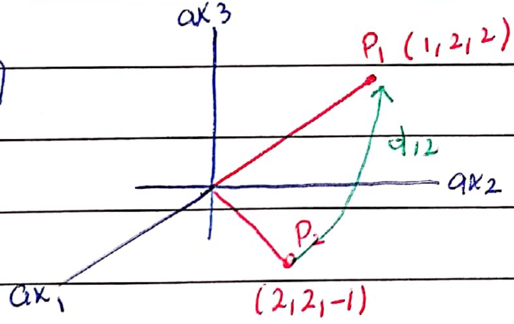
$\phi_r = \phi_t + \text{noise}$

signal space ⇝

2D



3D



$P_1 = \sqrt{\delta_1}$

P_1 in $ax_1 = \langle P_1, ax_1 \rangle$

P_2 in $ax_2 = \langle P_2, ax_2 \rangle$

P_1 in $ax_3 = \langle P_3, ax_3 \rangle$

ND N-axis → $ax_1, ax_2, \ldots ax_N$

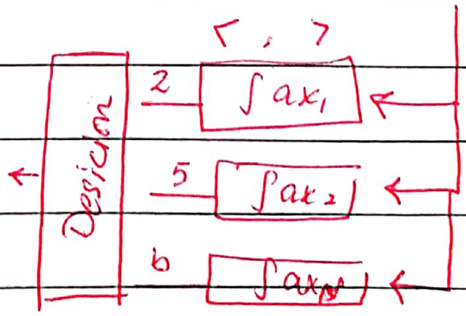
Demodulator :-

$P_1\ (2, 5, -7, 6, 1, 10)$

$2 = \langle P_1, ax_1 \rangle$

$5 = \langle P_1, ax_2 \rangle = \int_T P_1(t)\, ax_2(t)\, dt$

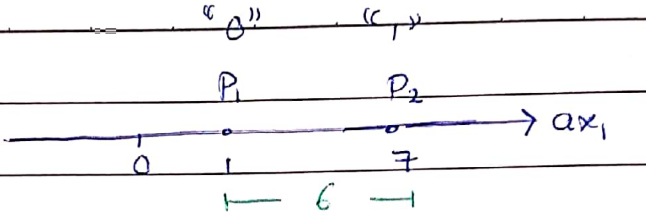
$-7 = \langle P_1, ax_3 \rangle$



1D design ⇒

Max. distance

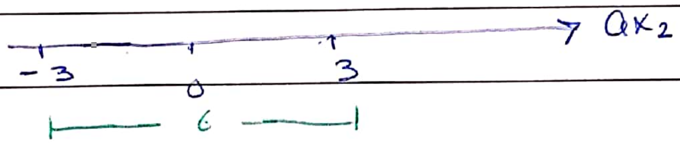
Min. energy.



$E = \frac{E_1 + E_2}{2}$

$= \frac{1 + 49}{2}$

$= 25\ J$



$E = 3^2 + (-3)^2$

$= 9\ J$

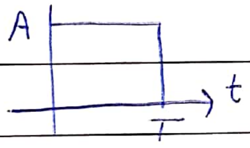
Criteria $\begin{cases} \text{Minimum energy} \\ \text{Maximum distance} \end{cases}$

min E ⇒ origin in center (CM) of points → point have the same probability.

BER α^{-1} distance (bit error rate α^{-1} distance)

Binary 1-D

ax (t)



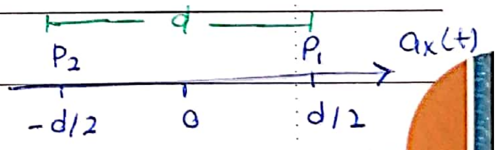
$E = A^2 T = 1 \rightarrow A = \frac{1}{\sqrt{T}}$

$\int_0^T |a_x(t)|^2 dt = A^2 T$

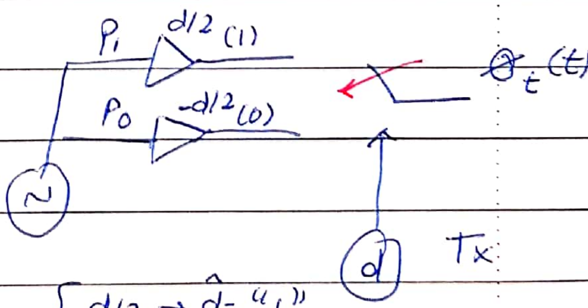
$T = \frac{1}{r_b} = T_{binary}$

$P_0 \Rightarrow$ "0" = d $\qquad P_0 \Rightarrow -d/2\ a_x(t)$

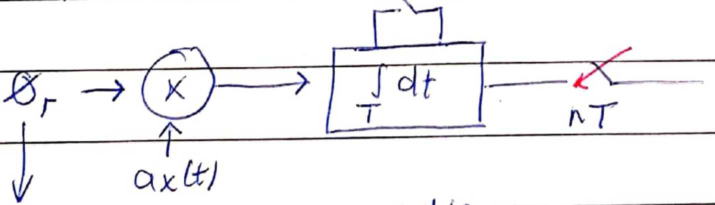
$P_1 \Rightarrow$ "1" = d $\qquad P_1 \Rightarrow +d/2\ a_x(t)$



$\vartheta_t(t) = \begin{cases} P_0(t), & d = \text{"0"} \\ P_1(t), & d = \text{"1"} \end{cases}$

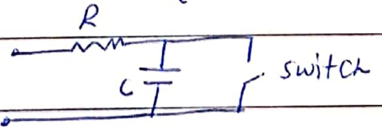


R_x ?? ⇒ P_0 or P_1



$V = \begin{cases} d/2 \rightarrow \hat{d} = \text{"1"} \\ -d/2 \rightarrow \hat{d} = \text{"0"} \end{cases}$ decision

$\int_0^T \vartheta_r\ a_x(t)\ dt = \begin{cases} +d/2 \\ -d/2 \end{cases}$



integrated & dump circuit.

$$\begin{tikzcd}\text{Integrate} \arrow[r] & \text{Read}(v) \arrow[r] & \text{Dump} \arrow[ll, bend left]\end{tikzcd}$$

$\theta_r = \theta_t + n$

$V = \pm d/2 + \int^T n(t)\, a_x(t)\, dt$

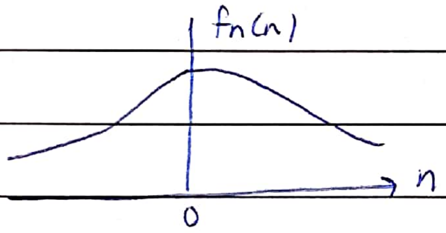
$f_n(n) = \frac{1}{\sqrt{2\pi}\, \sigma_n} e^{\frac{-1}{2\sigma_n^2}(n-\mu_n)^2}$

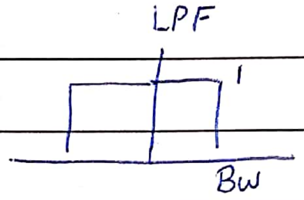
$V = \pm d/2 + \boxed{\text{noise}}$ → gaussian R.V

$\sigma_n = ?$, $\mu_n = ?$, $\mu_n = 0$ (zero)

σ_n^2 = power of noise = P_n = variance

$\sigma_n = \sqrt{P_n}$



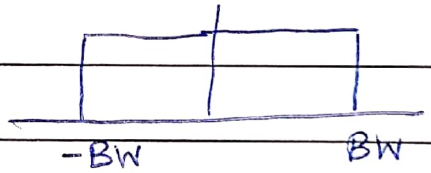


$n(t) \rightarrow \boxed{\int a_x(t)} \rightarrow$

random process ←

$n(t) \rightarrow \boxed{H(f)} \rightarrow$ Convolution $(h(t), n(t))$

so $PSD_{out} = \underset{\frac{N_o}{2}}{\boxed{PSD_{in}}} \cdot |H(t)|^2$



$\sigma_n^2 = BW$, $\frac{N_o}{2} \times 2$

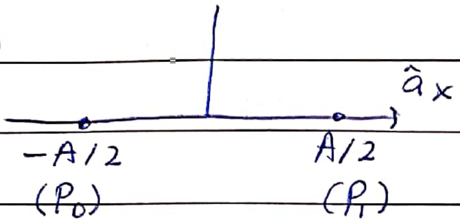
$\sigma_n^2 = BW \cdot N_o$

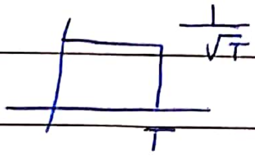
$BW = \frac{r_b}{2}$ (Base Band)

$\sigma_n^2 = \frac{N_o r_b}{2}$ W

Why to center the signal around the origin? to have minimum average energy & maximum distance.

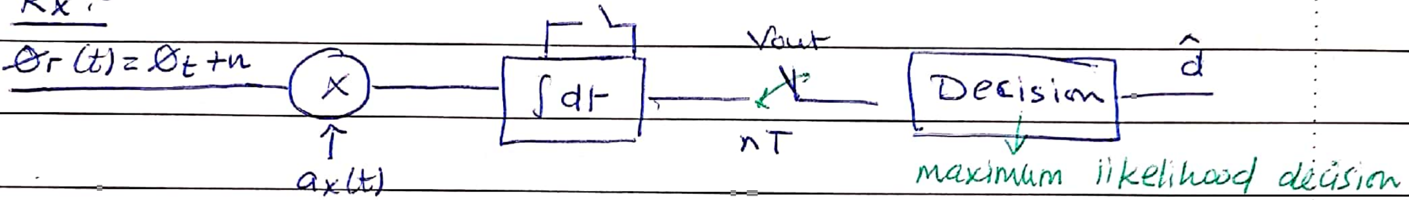
1-D



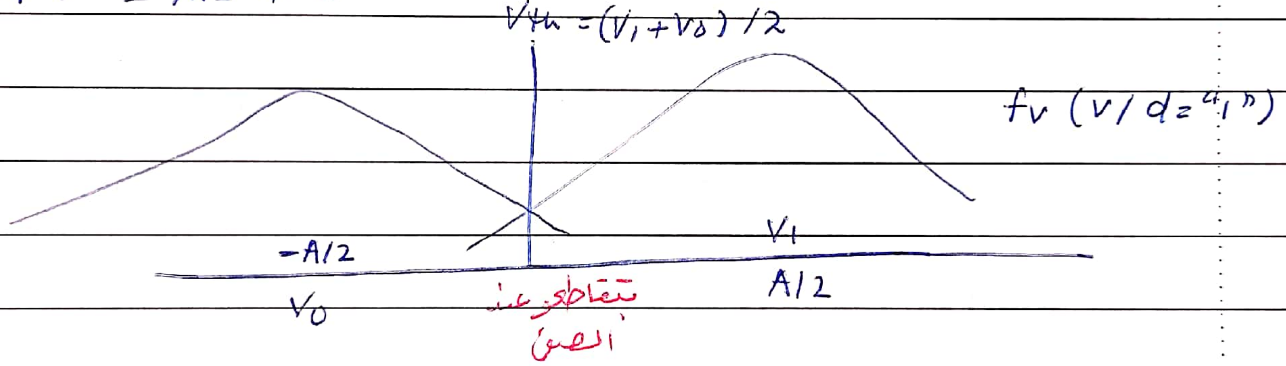


$$S_t(t) = \begin{cases} A/2 \cdot a_x(t) , & d = \text{"1"} \\ -A/2 \cdot a_x(t) , & d = \text{"0"} \end{cases}$$

Rx :-



$V_{out} = \pm A/2 + n$



average $V_{out} = \frac{(A/2 + (-A/2))}{2} = \text{zero}$

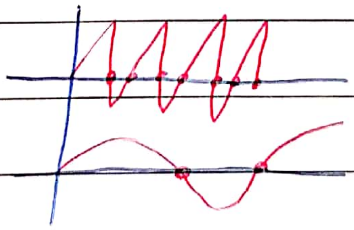
Synchronization has 3 levels :-

1) Carrier synchronization.

2) bit / symbol synchronization.

3) Frame synchronization.

Signals with more zero crossings (crossings with 0 v) are easier to synchronize



* Correlator is a synchronous receiver (coherent receiver).

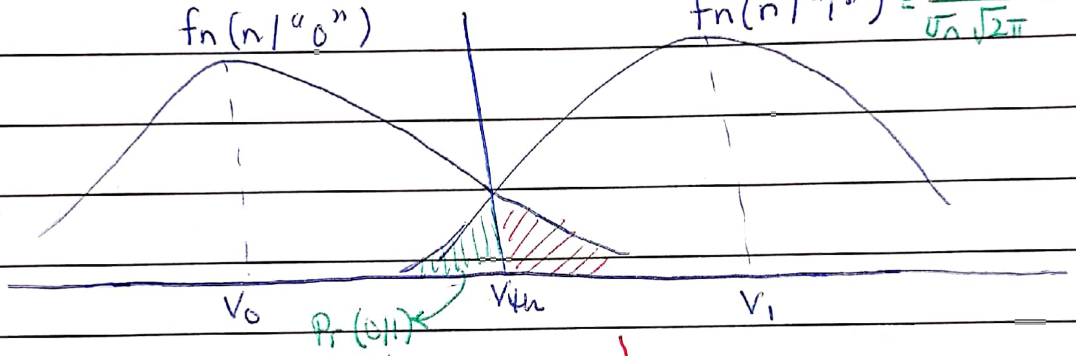
* Conditions to be synchronous $\rightarrow$ 1] minimum energy.
2] maximum distance.
3] minimum complexity.

* BER = Pr (data in error) = $\int_{-\infty}^{V_{Th}} \frac{1}{\sigma_n \sqrt{2\pi}} e^{-\frac{1}{2\sigma_n^2}(n-v_1)^2} dn$

$\downarrow$ error event

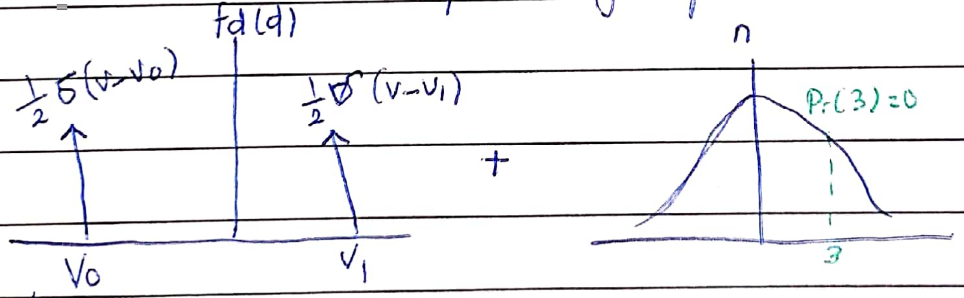
error event = Tx "1" Rx "0"
or Tx "0" Rx "1"

$f_n(n|"1") = \frac{1}{\sigma_n\sqrt{2\pi}} e^{-\frac{1}{2\sigma_n^2}(n-v_1)^2}$



المساحتين بساوو بعض لأنهم symmetric

* Random Variable (it's a function not a variable):
it is a function that maps many inputs into an output.



= output is convolution (shift)

$P_r(v_0) = \frac{1}{2}$
$P_r(v_1) = \frac{1}{2}$

[redacted] (v) + (n) [redacted] = (y)

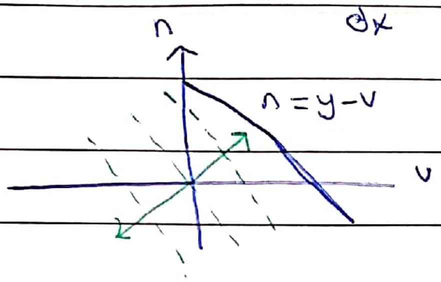
$\rightarrow$ probability density function.

P_r (event) = $\int_{e_0}^{e_1} f_x(x)\, dt = F_x(e_1) - F_x(e_0)$

probability of distribution function: $F_x(x) = \int_{-\infty}^{x} f_x(x)\, dx$

$F_x(x)$ is a non negative function.

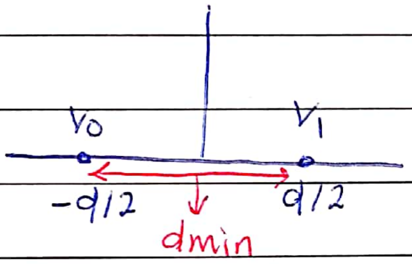
$$f_x(x) = \frac{dF_x(x)}{dx}$$



$$F_y(y) = \int_{-\infty}^{\infty} \int_{-\infty}^{v=y-n} f_n(n) \cdot f_v(v)\, dv\, dn$$

$$F_y(y) = \int_{-\infty}^{\infty} f_n(n)\, F_v(y-n)\, dn.$$

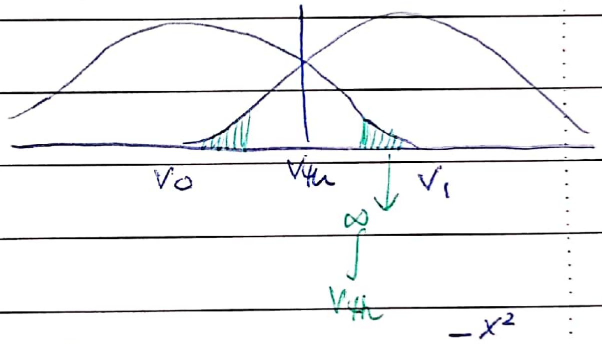
$$f_y(y) = \int_{-\infty}^{\infty} f_n(n)\, f_v(y-n)\, dn \leftarrow$$ convolution equation.



$$\text{"1"} \longrightarrow \frac{d}{2}\, a_x(t).$$

$$\text{"0"} \longrightarrow -\frac{d}{2}\, a_x(t).$$

$$BER = \int_{-\infty}^{V_{th}} \frac{1}{\boxed{\sigma_n}\sqrt{2\pi}}\, e^{-\frac{1}{2\sigma_n^2}(n-v_1)^2}\, \boxed{dn}$$



complementary error function:

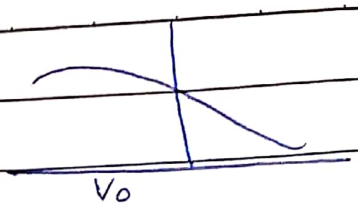
$$\operatorname{erfc}(x) = \frac{e^{-x^2}}{x\sqrt{\pi}}\left(1 - \frac{1}{2x^2} + \frac{3}{4x^4} - \frac{15}{86^6} \cdots\right) \simeq \frac{e^{-x^2}}{x\sqrt{\pi}}$$

$$Q(x) = \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right) \triangleq \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}\, dt$$

$$\text{let } t = \frac{n - V_1}{\sigma_n} \Rightarrow dt = \frac{dn}{dt}$$

$$BER = \int_{-\infty}^{?} \frac{1}{\sqrt{2\pi}}\, e^{\frac{-t^2}{2}}\, dt \qquad \left(t = \frac{n + v_0}{\sigma_n}\right)$$

$$BER = \int_{V_{th}}^{\infty} \frac{1}{\sigma_n \sqrt{2\pi}} e^{-\frac{1}{\sigma_n^2}(n+v_0)^2}$$



$$t = \frac{n+v_0}{\sigma_n}$$

$V_0 = -V_1$

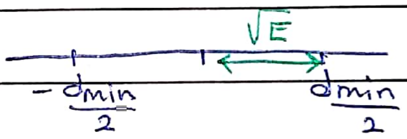
$$BER = \int_{?}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

$$BER = Q\left(\frac{V_{th}+v_0}{\sigma_n}\right) , \quad V_{th} = \frac{V_0+V_1}{2}$$

$$BER = Q\left(\frac{V_0}{\sigma_n}\right) = Q\left(\frac{d}{2\sigma_n}\right) , \quad d = d_{min}$$

$$BER = Q\left(\frac{d_{min}}{2\sigma_n}\right) = \frac{1}{2} erfc\left(\frac{d_{min}}{2\sqrt{2}\sigma_n}\right)$$

$$E = \frac{d_{min}^2}{4}$$



$$BER = \frac{1}{2} erfc\left(\sqrt{\frac{d^2_{min}}{4\times 2\,\sigma_n^2}}\right)$$

$E = P.T$

$$P = \frac{d_{min}^2}{4T}$$

$$SNR = \frac{d_{min}^2}{4T\sigma_n^2} = \gamma$$

$$\frac{d_{min}^2}{8\sigma_n^2} \cdot \frac{T}{1} = \frac{\gamma T}{2}$$

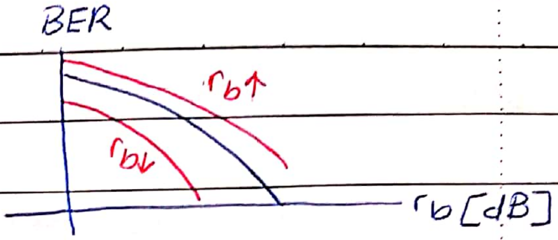
$$BER = \frac{1}{2} erfc\left(\sqrt{\frac{\gamma}{2} T}\right) , \quad T = \frac{1}{r_b}$$

$$BER = \frac{1}{2} erfc\left(\sqrt{\frac{\gamma}{2 r_b}}\right)$$

γ → SNR

r_b → data rate

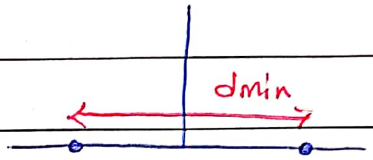
$$BER = \frac{1\, e^{-\frac{\gamma}{2r_b}}}{\sqrt{\frac{\gamma_b}{2r_b}}\sqrt{\pi}}$$



BER exponential scale

As bit rate (r_b) increases the required SNR to meet a certain BER should be increased.

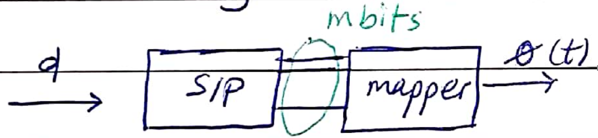
SNR ↑ ⇒ d_{min} ↑



signal space

(Binary needs only 1-D space)

* M-array op



$\theta(t) \in \{ S_i ,\ i = 0, 1, 2, \cdots, M-1 \}$ — signal

$M = 2^m$, number of different symbols

$T_s = m \cdot T_b$

$r_s = \frac{1}{T_b} = \frac{r_b}{m} = \frac{r_b}{\log_2 M}$

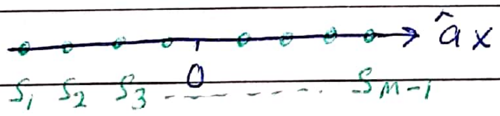
$\log_2 M = \eta$ = spectral efficiency [b/s/Hz]

↳ more efficient to transmit data.

Signal bandwidth = $\frac{r_s}{2}$

كلما كبرت كلما ضغطت bits آكتر

1-D



(signal space diagram = constellation)

$S_i = A_i \cdot a_x(t)$

Acceptable BER = $\frac{\%}{\text{file size [bits]}}$ used to find SNR, dmin positive

$$Y = \sum_{i=1}^{I} \alpha_i^{(2)}$$, what is the optimal solution?

when all $d_i = d_j$ ∀ i, j

Example; $\%_0 = 0.01 \Rightarrow BER = \frac{10^{-2}}{5\,10^{6} \times 8} = 2.5\,10^{-10}$

Mapping Table:

Gray code ⇝ adjacent symbols differ only in one bit

$$BER = \frac{SER}{m} = \frac{SER}{\log_2 M}$$

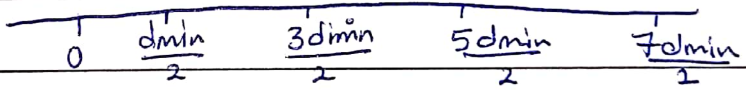
Example; $M = 4 \Rightarrow m = 2$

0 1
0 0
- - - - - - mirror
1 0
1 1

$M = 8$

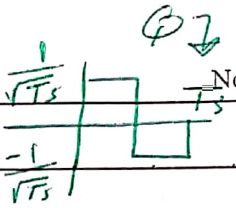
average energy

0 0 1
0 0 0
- - - - -
0 1 0
0 1 1
- - - - -
1 0 1
1 0 0
- - - - -
1 1 0
1 1 1



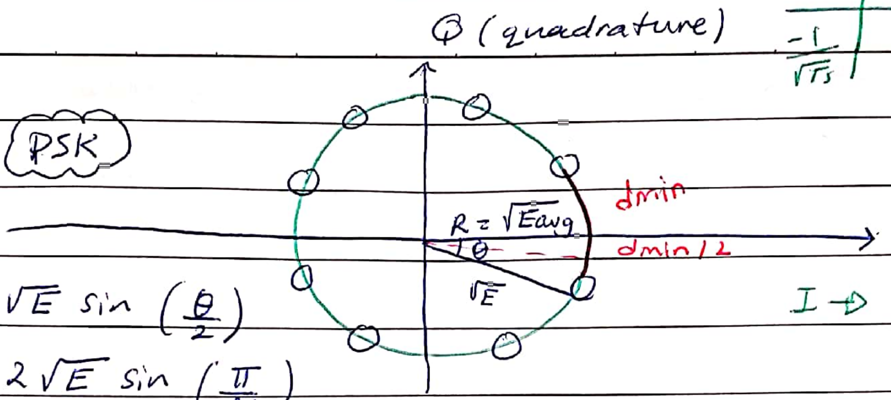
$$E_{av} = \frac{1}{4}\left[\left(\frac{d_{min}}{2}\right)^2 + \left(\frac{3d_{min}}{2}\right)^2 + \left(\frac{5d_{min}}{2}\right)^2 + \left(\frac{7d_{min}}{2}\right)^2\right]$$

$$= \frac{1}{M/2} \sum_{i=0}^{\frac{M}{2}-1} \left(\frac{2i+1}{2}\right)^2 T_s$$

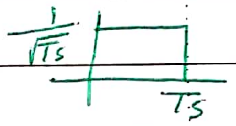


2-D

PSK



I →



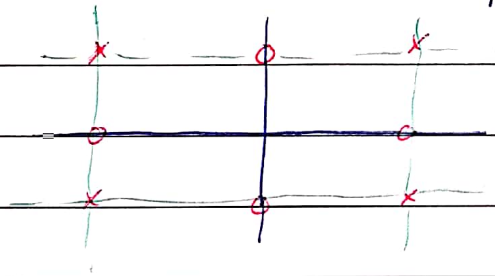
$$\frac{d_{min}}{2} = \sqrt{E} \sin\left(\frac{\theta}{2}\right)$$

$$d_{min} = 2\sqrt{E} \sin\left(\frac{\pi}{M}\right)$$

$$\sqrt{E} \angle 45^\circ = \sqrt{E} \cos(45^\circ) + j\sqrt{E} \sin(45)^\circ$$

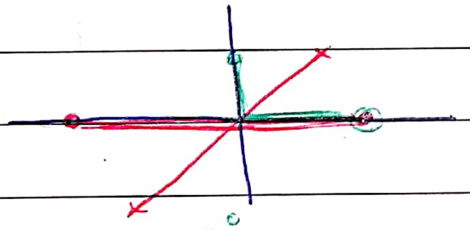
$$= I + Q$$

There is another shape:



QAM

* Constellation Design ::



orthogonal = 90°

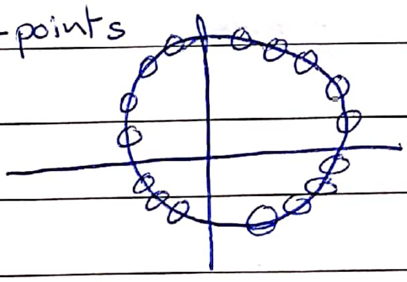
ρ = zero

to get max. distance & min. energy we need to have anti podal points

anti-podal = origin عكس بعض من خلال

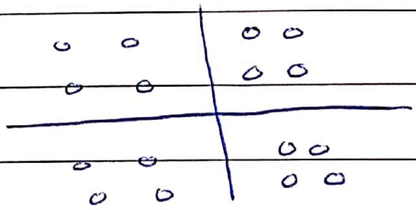
phase shift = 180° ⇒ ρ = −1

16-points



(constant envelope)

$$d_{min} = 2\sqrt{E} \sin\left(\frac{\pi}{16}\right)$$



(Grid)

$$E_{av} = \frac{d_{min}^2}{2}\left(2\left(\frac{1}{2}\right)^2 + 2\left(\frac{10}{4}\right) + \left(\frac{18}{4}\right) + 9\right)$$

$$E_{av} = 2.5\, d_{min}^2$$

$$d_{min} = \sqrt{\frac{E_{av}}{2.5}}$$

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d_{min} (grid) >> d_{min} (circle) for the same energy.

BER (grid) << BER (circle)

SNR increases for the points with greater distance ⇒ unequal error protection

- Peak-Average-Power-Ratio (PAPR) = $\frac{E_{max}}{E_{avg.}}$ ≥ 1 always

PAPR is a parameter for constellation design.

PAPR (for envelope) = $\frac{4.5\, d^2_{min}}{2.5\, d^2_{min}}$ = 1.8

PAPR (for grid) = 1 PAPR = 1 → perfect

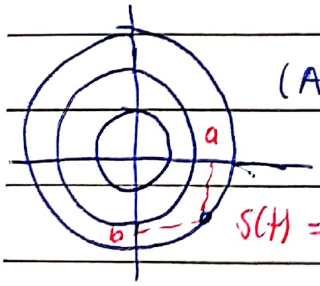
PAPR is an indecation for an unequal error protection.

- Power is more important than bandwidth in satellite applications. Cellular is (battery) power operated → so it is power limited (Eav is minimum so we better use QAM)

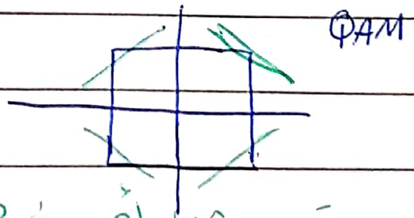
لايكون طالب مني بطارية ما بتستهلك كثير لازم system يكون power limited.

- Constant envelope modulations are immune against fading, because data is saved in phase (fading occurs on amplitude).

(APSK) like ← amplitude phase shift key.



$s(t) = a\,\hat{a}_x(t) + b\,\hat{a}_y(t)$



نعمل عشان أحسن PAPR

Modulation Constaraints ⇒ 1] E_{av} min. 2] d_{min} max.

3] PAPI ≃ 1 (min.) due to hardware utilization not to dominate the channel.

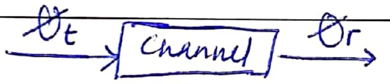
4] constant envelope (points have the same $\sqrt{E_i}$) $\sqrt{E_i}$ = distance from the origin

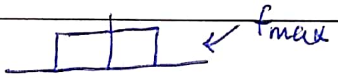
Signals that don't carry information in amplitude is better in fading channels.

* Band Limited Transmission :

بنثبت BW عند نقطة معينة فممنوع ال signal BW يزيد عنها .

BW is finite

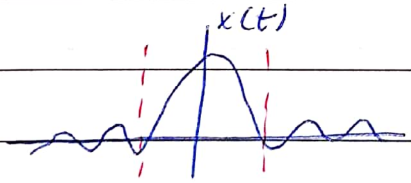




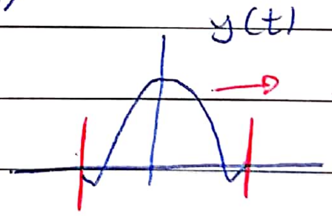
$BW(v) \leq BW(\text{channel})$

What happens if BW(signal) > BW (Channel) ??!

example; $x(t) \rightarrow \boxed{LPF} \rightarrow y(t)$

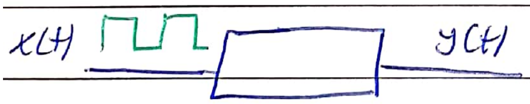


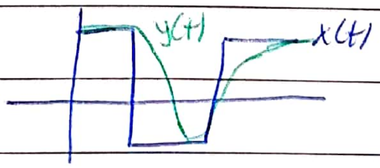
⇒



→ ال filter قطعها ، خلى هاد الجزء بس.

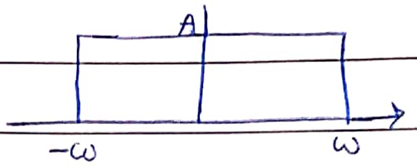
y(t) is a distorted version of x(t)





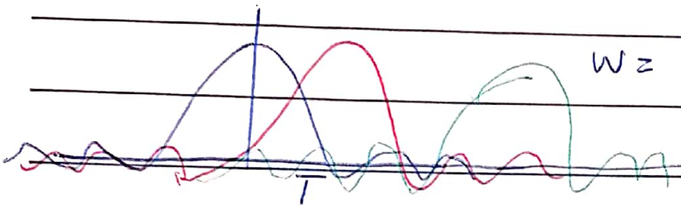
∴ when BW(signal) > BW(channel) ⇒ Intersymbol Interference (ISI) happens.

$a_x(t) = \text{Sinc}(\omega t)$



only 1 axis we can do because we only have 1 sinc function

Sinc is a non-causal signal (a signal that exists for positive as well for negative time) ما بقدر أعمل منها سيركت

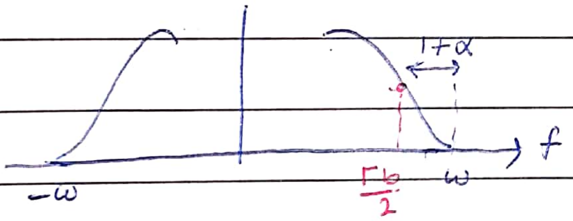


$W = \frac{1}{2T} = \frac{r_b}{2}$

sinc has value only in its center & zeros every enteger multiple of the period

at T → ISI = zero because all have zero crossings exept my signal (the green one)

Raised Cosine (RC) : Wave form shaping function



ω = channel bandwidth

$\omega = \frac{r_b}{2}(1+\alpha)$

α → roll off factor.

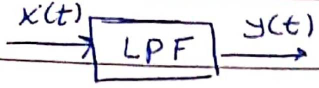
roll off factor = measure of the excess bandwidth of the filter.

in bandlimited signals we only have one axis (1 dimension) so only one raised cosine is available

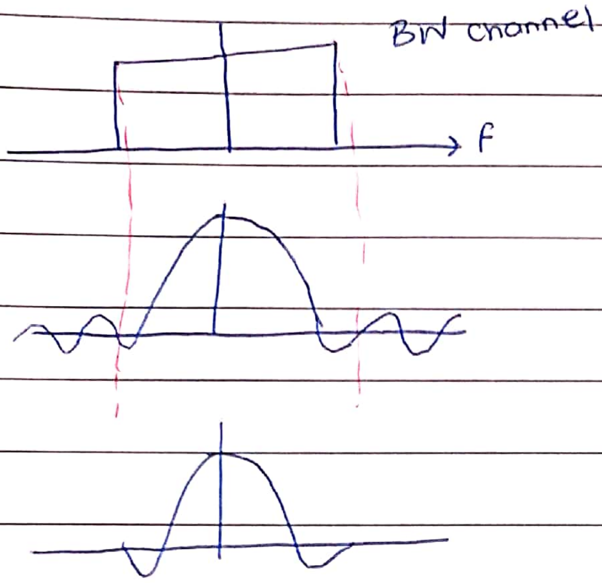
$BW = \frac{r_b}{2}(1+\alpha)$

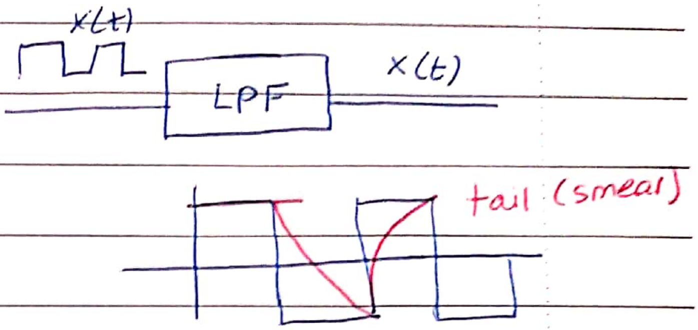
$BW = \frac{r_b}{2\log_2 M}(1+\alpha)$ if M-array (baseband)

$RC\alpha = p(t)$; $p(t)$ = wave form shaping function.



y(t) is a distorted version of X(F)

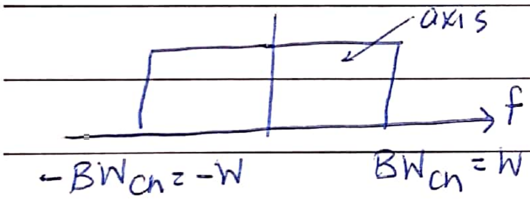




There is an ISI when

$BW_{signal} > BW_{channel}$

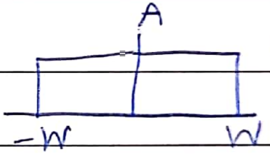
axis => Band limited



BW Ch = W

rect in frequency domain

sinc in time domain



$a_x(t) = sinc(Wt)$

Non Causal Signal :: (ie; sinc signal)

at T => ISI = 0

Example: for a 10KHz of channel bandwidth, find max r_b if M=16 and $\alpha = 0.1$

Sol $10K = \frac{r_b}{2*4}(1+0.1)$

$r_b = \frac{80K}{1.1} = 72.7$ Kbps

if $r_b = 100$ Kbps => assume $\alpha = 0$

you must write let / assume in the exam, otherwise → your answer is wrong

$$r_b = \frac{BW * 2\log_2 M}{1+\alpha}$$

$$100K = \frac{10K * 2 * \log_2 M}{1} \Rightarrow \log_2 M = 5$$

$$M = 32$$

if 4.5 take 5 then find α take $\log_2 M = 6$

if α is with the range → take 5

if not → 5+1 = 6

$$100k = \frac{10K * 2 * 6}{1+\alpha} \Rightarrow \alpha = 0.2$$

quality is always constant since we used the right data

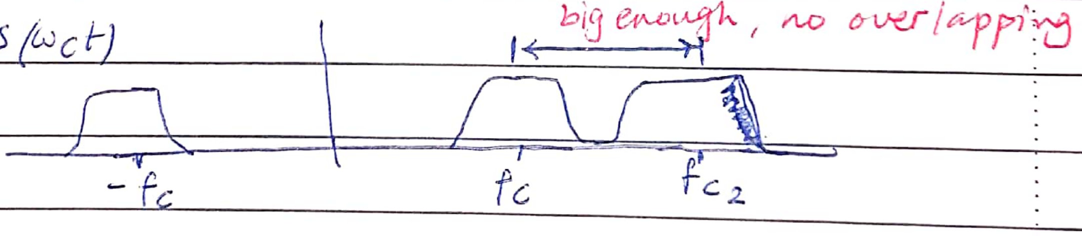
on earth → BW constant → power ↑

in deep space communication → power constant → BW ↑

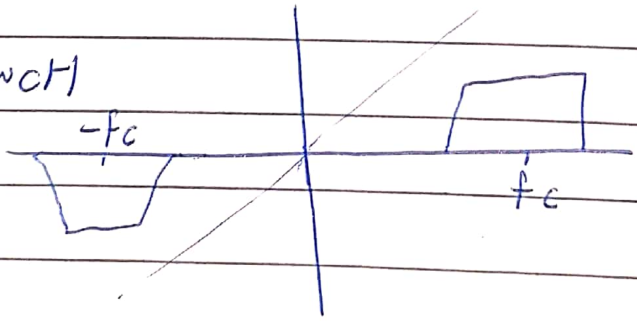
* BER depends on dmin

for the same dmin ⇒ BER are the same

$a_x(t) = p(t) * \cos(\omega_c t)$

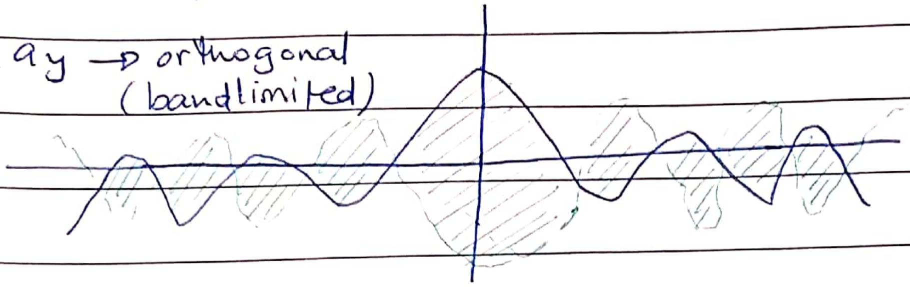


$a_y(t) = p(t) * \sin(\omega_c t)$



$\langle a_x , a_y \rangle = \int p(t)\cos(\omega_c t) \cdot p(t)\sin(\omega_c t)\, dt = 0$

$a_x \perp a_y$ → orthogonal
(bandlimited)

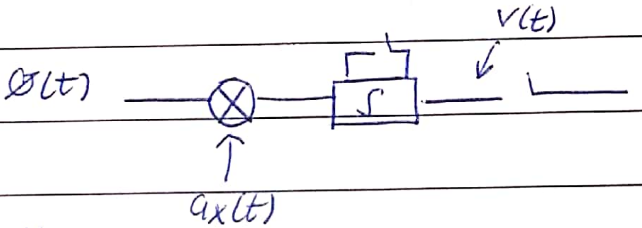


important $W_{ch} = \frac{r_b}{\log_2 M}(1+\alpha)$ "on 2 sides"

* optimal receiver :

$\phi(t) = s(t)$

at Rx



correlator
Rx

$v(t) = \int_0^T \phi(t) \cdot a_x(t)\, dt$

best v(t) (output) is when ~~SNR is maximum at output of~~ filter h(t)

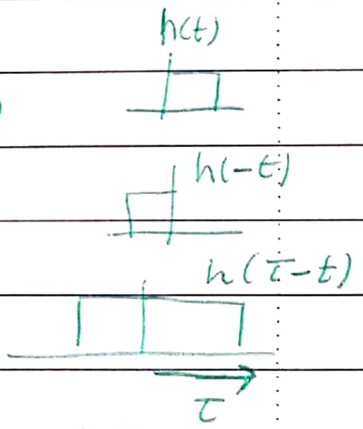
$\phi(t) - \overset{\text{filter}}{\boxed{h(t)}} - v(t)$

$\phi(t) = s(t) - \boxed{h(t)} - v(t)$

$v(t) = s(t) \overset{\text{convolution}}{\circledast} h(t)$

$v(\tau) = \int_{-\infty}^{\infty} s(t) \cdot h(\tau - t)\, dt$

$h(\tau - t)$



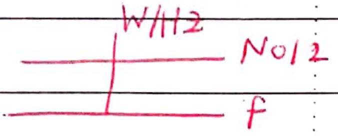
* Power of output of filter :

$P_{out} = \left| s(t) \cdot h(\tau - t)\, dt \right|^2 = P_s$

$SNR = \frac{P_s}{P_n}$

$P_n = \int_{-\infty}^{\infty} \underline{S_{nn}(f)} \cdot |H(f)|^2\, df$ → PSD

PSD ⇒



$P_n = \int_{-\infty}^{\infty} \frac{N_o}{2} |H(f)|^2\, df$

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$$SNR = \frac{|S(t)\cdot H(F)|^2}{\frac{No}{2}\int_{-\infty}^{\infty} |H(F)|^2\, df}$$ $\int |H(f)|^2$ should be finite

$\sigma_n^2 = \frac{No}{2}\int |H(f)|^2 df = No\cdot \underset{\text{noise equivalent BW}}{\underline{BW}} = P_n$

$SNR = \frac{|S(f)\cdot H(f)|^2}{No\cdot BW}$, we need to maximize it

$|V_1\cdot V_2|^2 = |V_1|^2\cdot |V_2|^2 - (2V_1V_2) \rightarrow$ this should equal zero so the term is maximum

This becomes max. when $S(f) = H(-f)^*$

$h(t) = s^*(T-t)$

Example; $\phi(t) = \begin{cases} s_0(t) & , d = \text{"0"} \\ s_1(t) & , d = \text{"1"} \end{cases}$

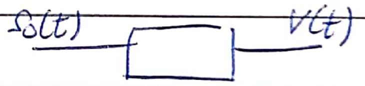
$h(t) = \begin{cases} s_0(T-t) & , h_0(t) \\ s_1(T-t) & , h_1(t) \end{cases}$

when S_0 arrives :

$v(t) = \int s_0(t)\, h(T-t)\, dt$

$= \int s_0(t)\cdot s_0(\tau-(T-t))\, dt$

$= \int s_0(t)\cdot s_0(\tau-T+t)\, dt$



when $T = \tau \Rightarrow v(t) = \int |s_0(t)|^2 dt = P_s$

if $s_1(t)$ arrived $\Rightarrow v(t) = \int s_1(t)\, s_0(\tau-T+t)\, dt$ (at $\tau = T$)

$= \int s_1(t)\, s_0(t)\, dt$

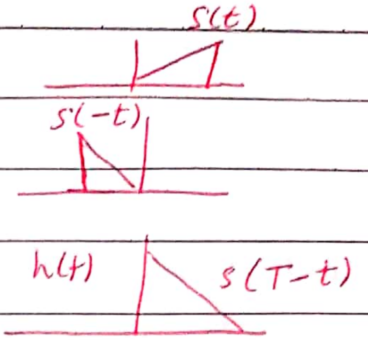
$= \langle s_1, s_0 \rangle = \rho_{01}\cdot\sqrt{E_1}\cdot\sqrt{E_0}$

N O T E B O O K

Scanned by CamScanner

$h(t) = s^*(T-t)$

Matched filter Rx , optimal detection



$$\phi(t) = \begin{cases} s_0(T-t), & h_0(t) \\ s_1(T-t), & h_1(t) \end{cases}$$

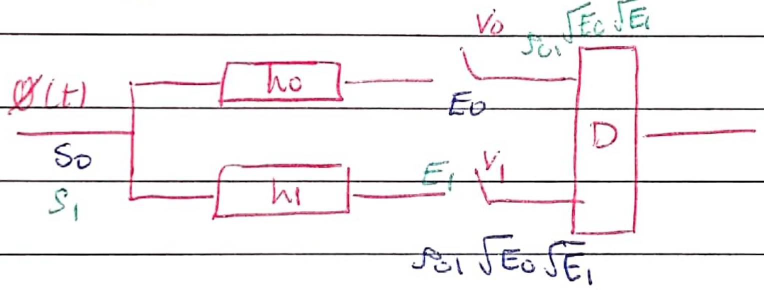
$$\begin{cases} V(t) = \begin{cases} E_0, & \phi(t) = s_0(t) \\ \rho_{01}\sqrt{E_0}\sqrt{E_1}, & \phi(t) = s_1(t) \end{cases} \\ h(t) = h_0 \end{cases}$$

$$\begin{cases} V(t) = \begin{cases} E_1, & \phi(t) = s_1(t) \\ \rho_{01}\sqrt{E_0}\sqrt{E_1}, & \phi(t) = s_0(t) \end{cases} \\ h(t) = h_1 \end{cases}$$

$\boxed{\rho_{01} = -1}$ antipodal

$\therefore s_0 = -s_1$

$E_0 = E_1 = E$



	V_0	V_1
s_0	E_0	$\rho_{01}\sqrt{E_0}\sqrt{E_1}$
s_1	$\rho_{01}\sqrt{E_0}\sqrt{E_1}$	E_1

	V_0	V_1
s_0	E	$-E$
s_1	$-E$	E

when

$\rho_{01} = -1$

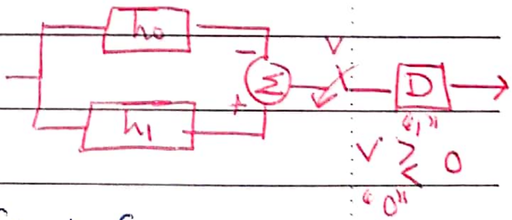
$\therefore E_0 = E_1 = E$

*Decision :-

$V = V_1 - V_0$

$V = -2E$, when s_0 was transmitted

$V = 2E$, when s_1 was transmitted



	s_0	s_1
V_0	E	$-E$
V_1	$-E$	E

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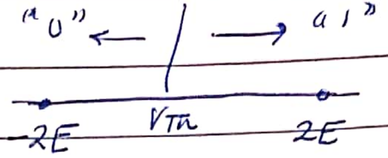
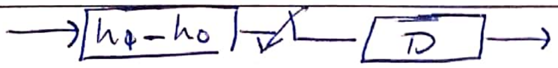
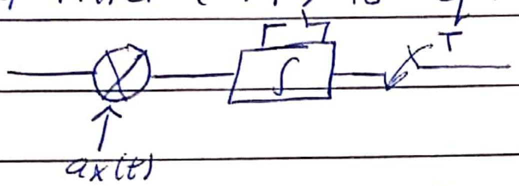
$$d = \begin{cases} 1, & \text{if } v \geq 0 \\ 0, & \text{if } v < 0 \end{cases} \Big] V_{Th}$$





* Matched Filter (M.F) is equivalent to a correlator Rx at $t=T$

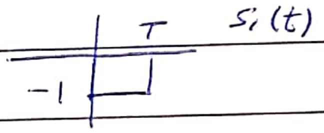


correlator Rx

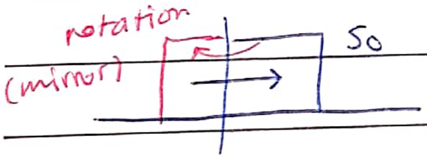
it must be synchronized { Synchronous Rx
Coherent Rx

Example; $s_0(t)$

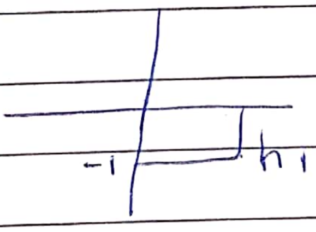
$S_1(t) = -S_0(t)$



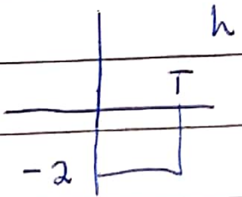
$h_0(t) = S_0(T-t)$



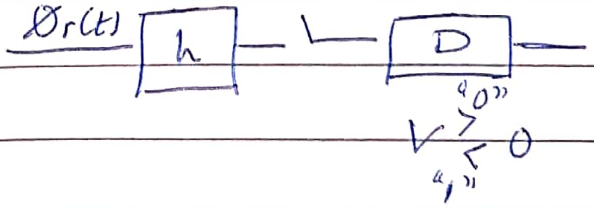
$h_1 = S_1(T-t)$

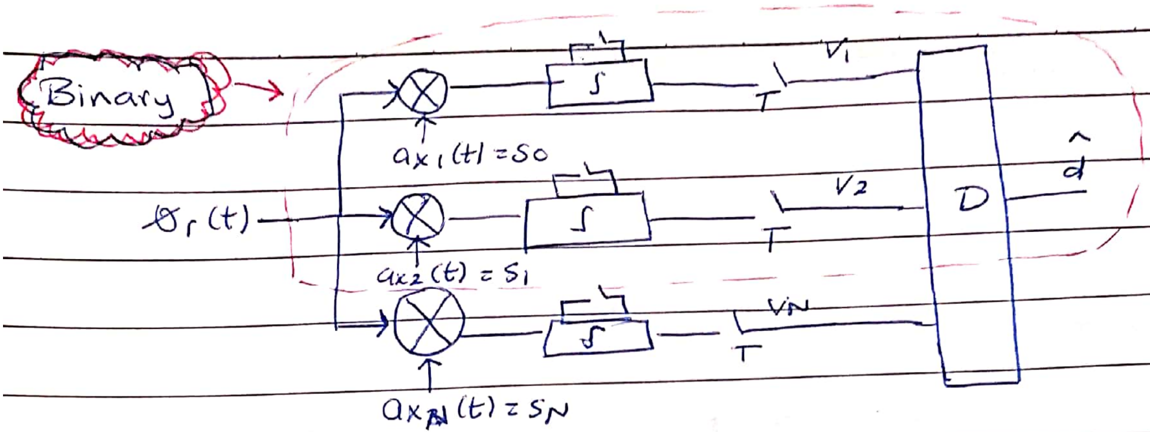


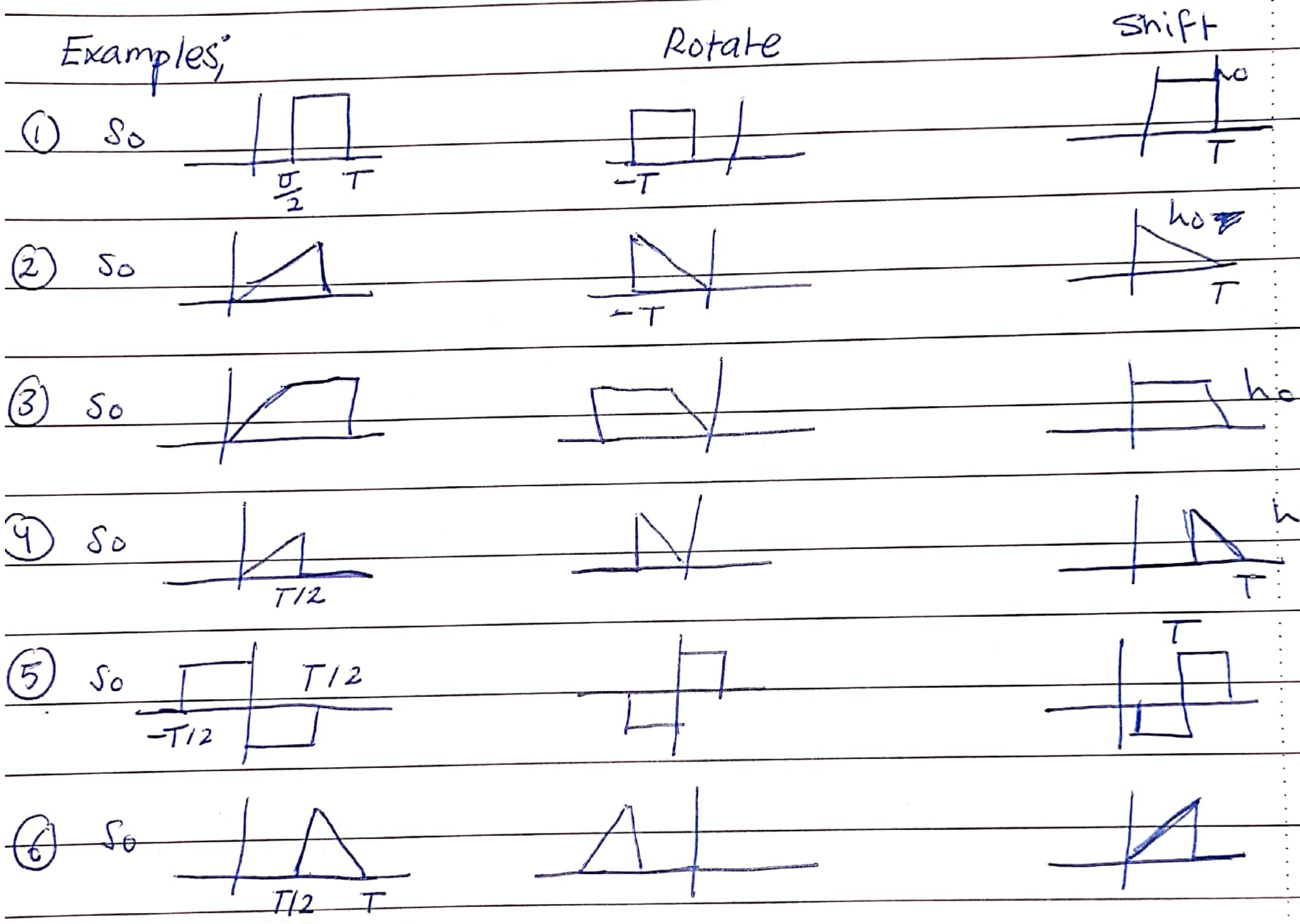
$h_1 - h_0 = 1$



Rx :

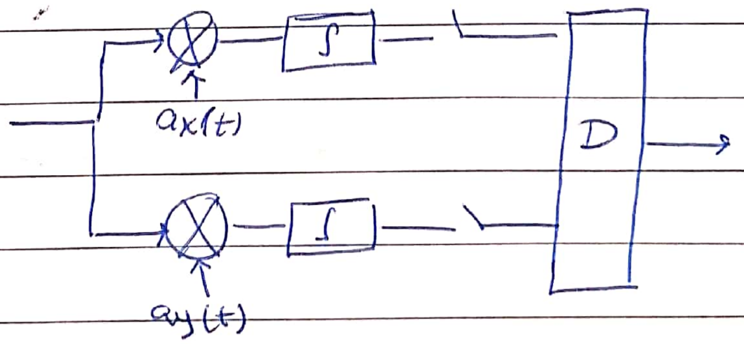






* 2D :

Quadrature Rx



orthogonal

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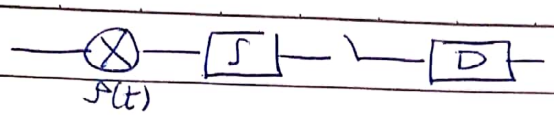
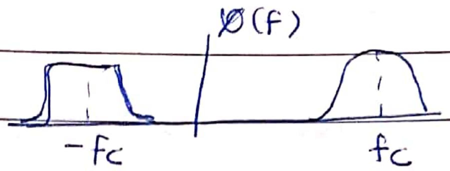
for RC_α



(one axis)

* Band Pass Comm. ∞

wireless channel
(no base band)



$$\boxed{\begin{array}{l}\lambda f = c \\ \lambda = \frac{3 10^8}{f}\end{array}}$$

$f = 10 KHz \rightarrow \lambda = \frac{3 10^8}{10 10^3} = 30 km$

f is large $\Rightarrow f = 1 GHz \rightarrow \lambda = \frac{3 10^8}{10^9} = 30 Cm$

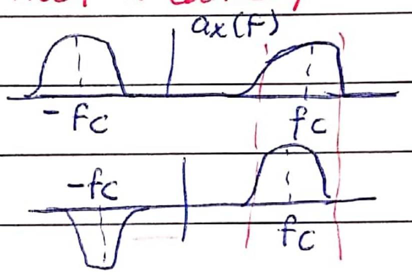
$B = \frac{r_b}{2 \log_2 M}(1+\alpha)$ BaseBand

multiple users $\rightarrow$ ✗

* Band Pass Tx ∞ (we need a carrier)

$a_x(t) = \underset{RC_\alpha}{p(t)} \cdot \cos(w_c t)$

$a_y(t) = p(t) \cdot \sin(w_c t)$

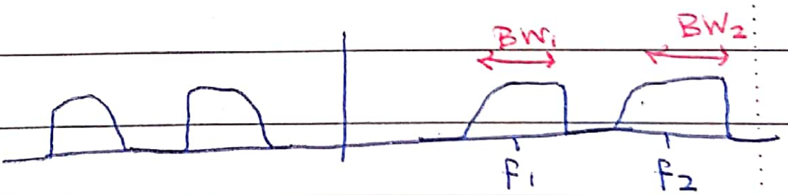


$BW = \frac{r_b}{\log_2 M}(1+\alpha)$

$f_c \gg BW$ of a signal

$a_{x_1}(t) = p(t) \cos(w_1 t)$

$a_{x_2}(t) = p(t) \cos(w_2 t)$



No overlapping in frequency domain
So they are orthogonal

$B = BW_1 + BW_2$

$BW_1 = \frac{r_b}{\log_2 M}(1+\alpha)$

$BW_2 = \frac{r_b}{\log_2 M}(1+\alpha)$

If we want 2D (2-axis) → cos & sin

If we want more than 2D → cos & change f

Band Pass :-

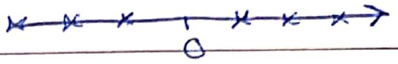
(ASK)

1-D

data

$$a_x(t) = A_i P(t) \cdot \cos(\omega_c t)$$

$$BW = \frac{r_b}{\log_2 M} (1+\alpha)$$

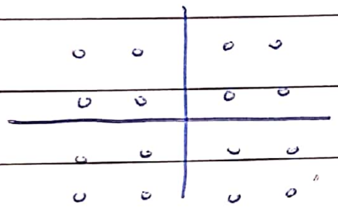


same constellation as Base-band

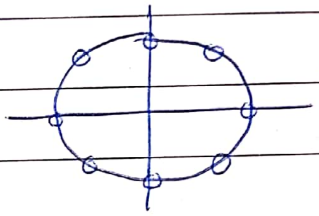
2-D

$$a_x(t) = \overset{\to I}{A_i} P(t) \cdot \cos(\omega_0 t)$$

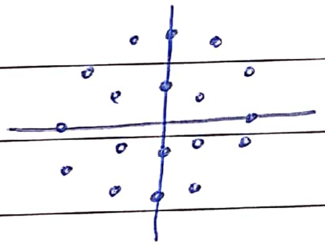
$$a_y(t) = \underset{\to \varphi}{B_i} P(t) \cdot \sin(\omega_0 t)$$



QAM



PSK



APSK

$$BW = \frac{r_b}{\log_2 M} (1+\alpha)$$

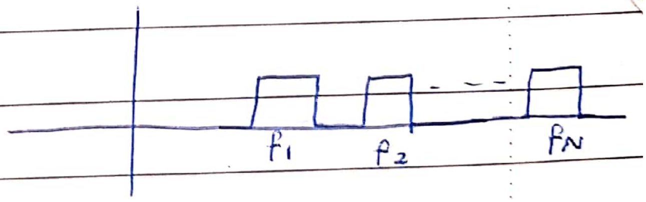
1-D & 2-D are Narrow Band Modulations

N-D

$$a_{x_1}(t) = P(t) \cos(\omega_1 t)$$

$$a_{x_2}(t) = P(t) \sin(\omega_2 t)$$

$$a_{x_N}(t) = P(t) \cos(\omega_N t)$$



$$BW_T = N \cdot \frac{r_b}{\log_2 M} (1+\alpha)$$

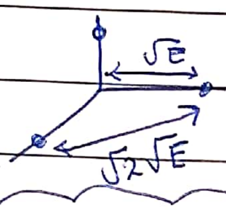
Wide Band Modulation is the best in fading channels.

Modulation $\Rightarrow$ 1 - FSK :

{NO GRAY CODE}

d	ω
00	ω_0
01	ω_1
10	ω_2
11	ω_3
⋮	⋮

d select ω



all points are at the same distance from each other = $\sqrt{2}\sqrt{E}$

important Notes

1) FSK $\rightarrow$ constellation $\rightarrow$ hypersphere.
2) all point at the same distance $\rightarrow$ NO GRAY CODE
3) equal (constant) envelope signals.

$$\phi_{FSK}(t) = A\,p(t)\cos(w_i t)$$

* Fading Channels :

When multiple replicas of same signal arrive at Rx:-

$$\phi_r(t) = \sum_{K=0}^{K-1} \alpha_K\, \phi_t(t-\tau_K) + n(t)$$

AWGN $\Rightarrow$ K=0

$$\phi_r = \underset{\text{constant}}{\alpha_0}\, \phi(t-\underset{\text{constant}}{\tau_0}) + n$$

$$\phi_r = \phi_t(t) + n(t)$$

Fading $\Rightarrow$ $\vec{\phi}_r = \vec{H}\,\vec{\phi}_t + \vec{n}$, $\vec{H} = \left\{\alpha_0 e^{-j\tau_0} \quad \alpha_1 e^{-j\tau_1} \cdots \alpha_{K-1} e^{-j\tau_K}\right\}$

$$\vec{\phi}_r = \vec{H}\,\vec{\phi}_t + \vec{n}$$

$$\vec{\phi}_t = (\vec{\phi}_r - \vec{n})\,\underset{\text{channel vector}}{H^{-1}}$$

$$H = \begin{bmatrix} \alpha_0 e^{-j\tau_0} & 0 & 0 & \cdots & 0 \\ 0 & \alpha_1 e^{-j\tau_1} & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & \\ \vdots & \vdots & & \ddots & \\ 0 & 0 & & & \alpha_{K-1} e^{-j\tau_{K-1}} \end{bmatrix}_{K\times K}$$

Linear Equation :

$\vec{\theta}_r = H^{-1}(\vec{\theta}_t - \vec{n})$ prediction of $n=0$

$\Rightarrow \mu_n$

$\vec{\theta}_r = H^{-1}\theta_t = H^H\vec{\theta}_t$

$\vec{\theta}_r = H^H\theta_t$ equalization equation of channel → minimize the effect of fading

H matrix is singular → no inverse.

$$\boxed{\vec{\theta}_r = H^{-1}\vec{\theta}_t}$$

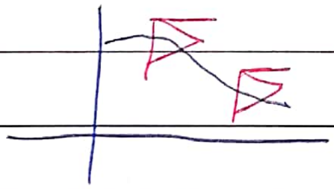
Advanced comm. theory :-

- psudo inverse
  - channel inversion
  - channel equalization (zero-forcing)
- Blue (best linear unbiased estimate) -WBlue.

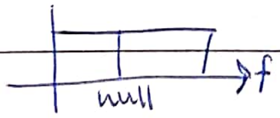
Indoor ; H → Nakgami
→ μ-α (pdf)

Outdoor : → Rician (los) → line of sight
→ Rayleigh (N los (worst case)) → non line of sight
→ log normal (Mixed)

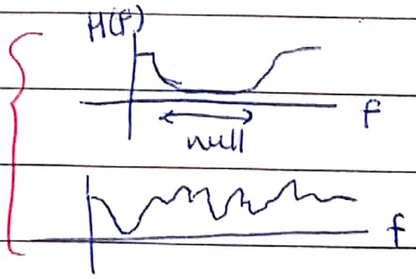
* Rayleigh with mean log normal (worst case)



(I) 1-Narrow Band Fading



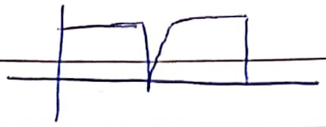
2-Wide Band Fading



Ⅱ) 1- Slow fading → Quzy fading (slow within one frame of data)

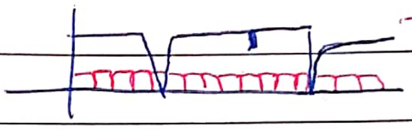
2- fast fading

- frequency non selective fading



- frequency selective fading

hard to deal with



$P_r = \frac{1}{N}$

In Fading Channels :

1)	FSK	⇒	The best	✓✓✓	B = NW
2)	PSK	⇒	very good	✓✓	W
3)	QAM	⇒	good	✓	W
4)	ASK	⇒	bad	✗	W

Tx :

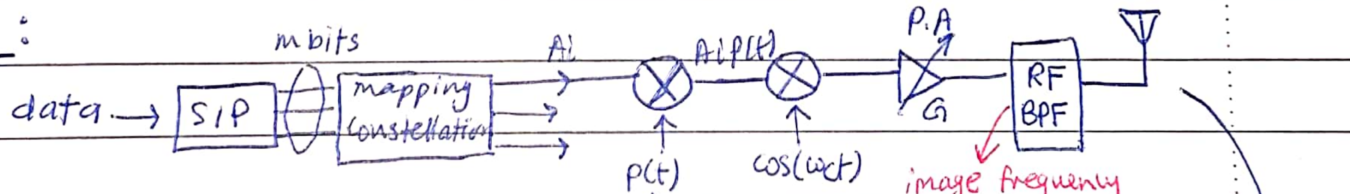


image frequency of generated from non linearity of mixed & P.A

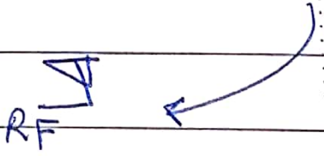
$\phi_t(t) = G A_i P(t) \cos(\omega_c t)$

$= \frac{(G A_i)^2}{2} = P$

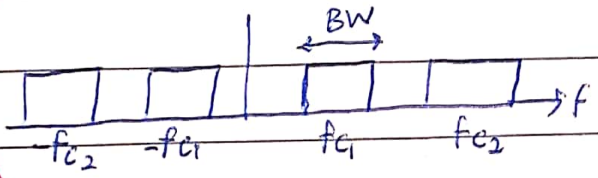
instead: $\boxed{RC_\alpha}$

Channel

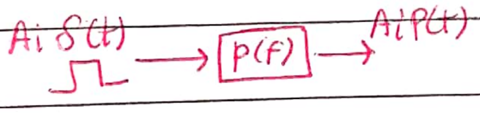
Rx :



RF = Radio Frequency

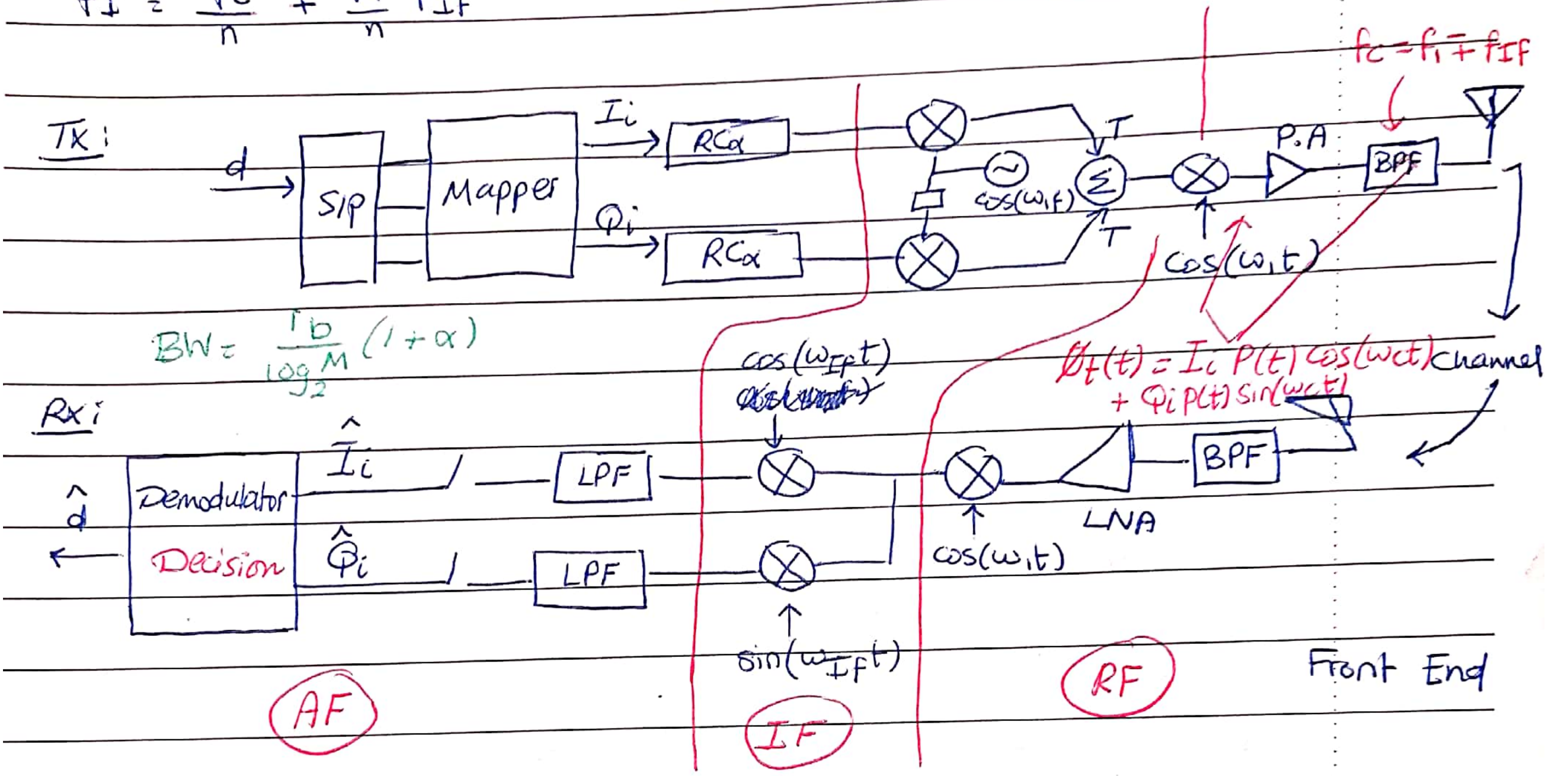


different channels must be orthogonal so we can sperate them.



RF filter ⇒ to get rid of any ~~[illegible]~~ non linearity → from P.A and mixer using the RF filter.

$f_I = \frac{f_c}{n} \mp \frac{m}{n} f_{IF}$ $m, n = 1, 2, \dots, N$ → order



$BW = \frac{T_b}{\log_2 M}(1+\alpha)$

$\phi_t(t) = I_i P(t)\cos(\omega_c t) + Q_i p(t)\sin(\omega_c t)$

$\phi_t(t) = G_i \cdot A_i \, p(t) \cos(\omega_c t)$

$\frac{(G_i A_i)^2}{2} = P_{av}$

$P_r = K P_t d^{-3}$ → all components generate noise (Thermal noise; heat)

noise $= \sqrt{4KTB}$

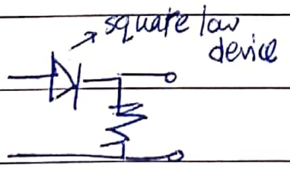
low noise ⇒ 1) small bias Vcc (low Vcc voltage)

2) dual supply +Vcc & −Vcc (dual polarity)

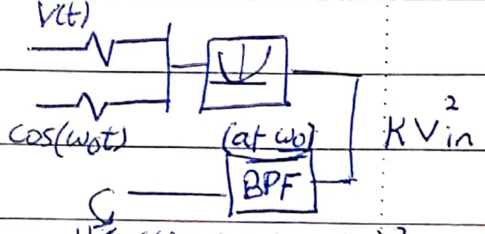
3) zero Q point current.

Mixer ⇒ $v(t) \rightarrow \otimes \rightarrow v(t) \cdot \cos(\omega_0 t)$

$\uparrow \cos(\omega_0 t)$



$f_0 = \frac{1}{2\pi\sqrt{LC}}$



$K V_{in}^2$

$= K(v(t) + \cos(\omega_0 t))^2$

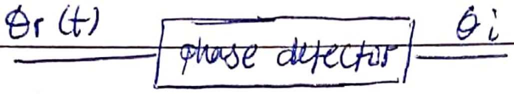
$= \cancel{K v^2(t)} + \cancel{K \cos^2(\omega_0 t)} + 2K v(t) \cos(\omega_0 t)$

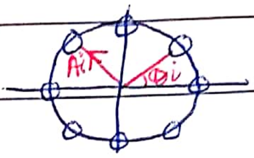
Subject Date No.

1-D → $A_i P(t) \cos(w_c t)$ (Gray)

2-D $I_i P(t) \cos(w_c t) + Q_i P(t) \sin(w_c t)$ (Gray)

$= A_i \angle \theta_i$





N-D 1) $A P(t) \cos(w_i t)$

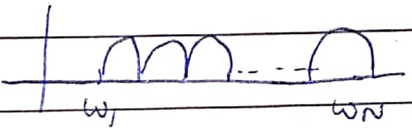
9 data → w_i → FSK

2) $I_R P(t) \cos(w_i t) + Q_R P(t) \sin(w_i t)$ (2N-D)

MC (2D Mod)

MT (−)

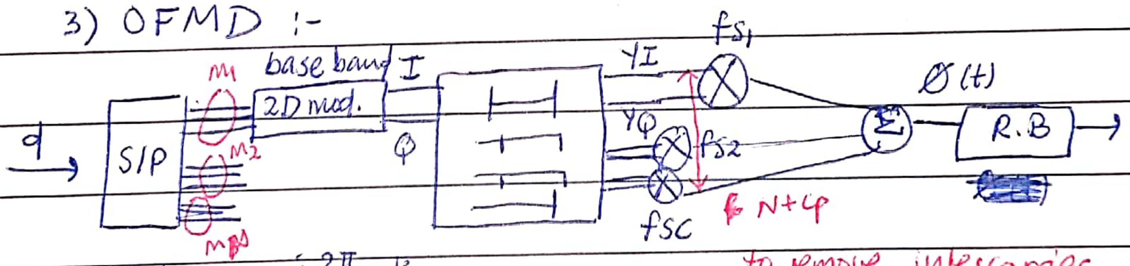
$BW = N \frac{r_b}{\log_2 M}(1+\alpha)$



pilot sub carriers → power ↑

constant amplitude

3) OFMD :-



to remove intercarrier interference (ICI)

$$Y(K) = \sum_{n=0}^{N-1} X_n e^{j \frac{2\pi}{N} nK}$$

$$\Delta f = \frac{f_s}{N} \quad f_s \to 2f_{max} \quad , \quad f_{max} = \frac{r_b}{2} \to f_s = r_b$$

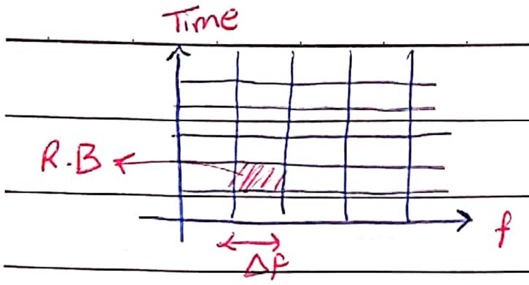
$$= \frac{r_b}{N}$$

CP = cyclic prefix

$$BW = (N + CP)\,\Delta F = \frac{r_b}{\log_2 M_i}\left(1 + \frac{CP}{N}\right) = \sum_{i=1}^{N} \frac{r_b}{\log_2 M_i}\left(1 + \frac{C.P}{N}\right)$$

C.P = 8-12% of N

BW demand concept solution to OFDM



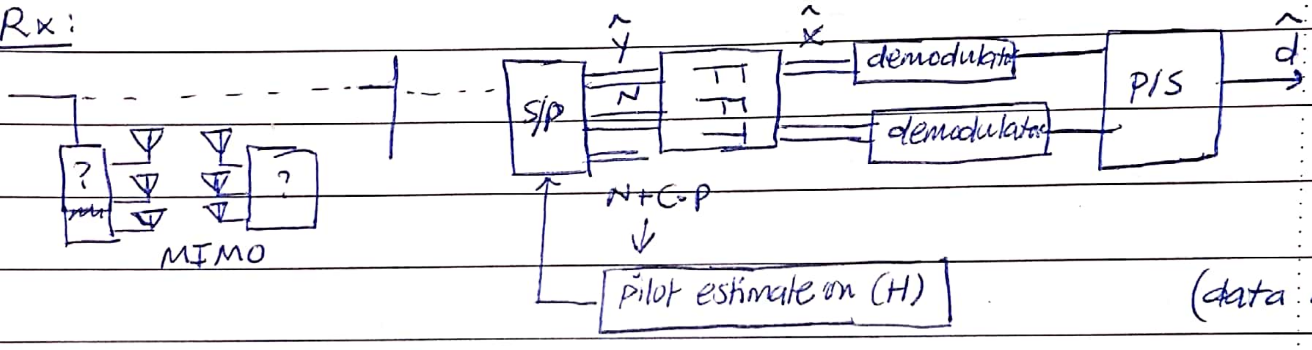
Resource Block

$\Delta f = I f_c \cdot \frac{V\cos\theta}{c}$,
c = speed of light (310^{8})
V = mechanical speed
Δf = ~~solution~~ frequency resolution.

✱ Doppler Shift :-

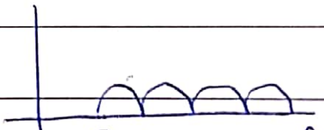
Solution to doppler effect by using guard.

Rx:

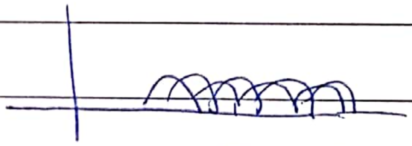


(data rate↑)

MIMO ⇒ can create $\boxed{\text{diversity}}$ or increase capacity
(Waterfall Algorithm) ↳ best solution for fading channels



$BW = N * \frac{r_b}{\log_2 M}(1+\alpha)$



$BW = \frac{r_b}{\log_2 M}\left(1+\frac{C.P}{N}\right)$

Sub carrier → $\underbrace{N}_{\text{data}} + \underbrace{C.P}_{}$ ↳ extra replicas
- pilot estimation
- deep fading N_d

Actual Tx data = $N - N_p - N_d$
→ 8-12% of N

Advantages ⇒ 1) communication management
→ multi users
→ BW on demand
→ get rid of deep fading R.B

2) cover frequency selective fading channel into flat fading channel

Disadvantage ⇒ Synchronization : error in oscillators
→ solution → add guard band (GB) doppler

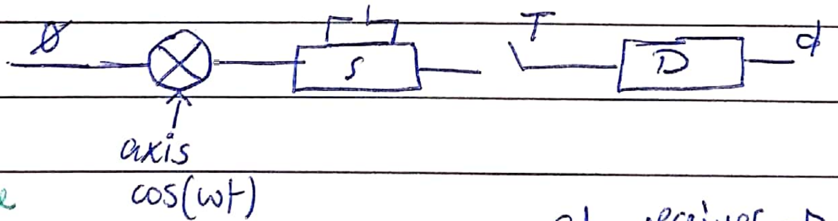
$BW_T > \frac{f_b}{\log_2 M} (1+ \frac{C.P}{N})$

1-D	2-D	N-D
ASK	PSK	FSK
	QAM	MC
	APSK	OFDM

* Introduction to coding ⇒ spread spectrum (SS)
direct sequence spread spectrum (DS-SS)
F&F - SS

Rx → coherent → (M.F) ~~correlator~~ Corelator
→ non coherent (no need to regenerate the carrier) → (ASK & FSK)

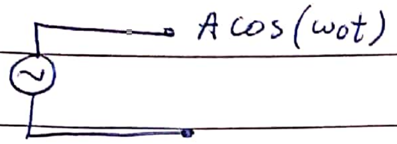
correlator
↓
coherent
↓
regenerate the carrier frequency of the receiver

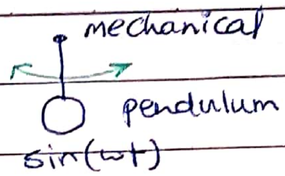


at receiver ⇒ $W = W_{at}$ Tx

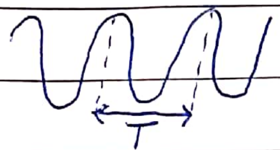
carrier synch. { $W = W_s + \Delta W$, random $\cos(\Delta W)$
error in phase $\Delta\theta$, $\cos(\Delta\theta)$

* Oscillators :-



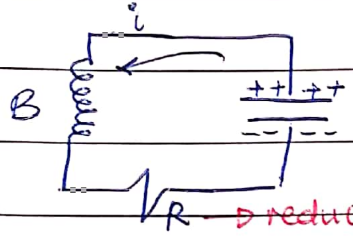


V =



= $V_o \sin(wt)$

Electronics :-



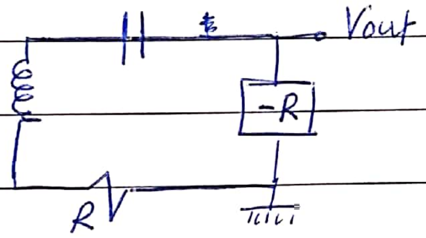
resonance : $f_r = \frac{1}{2\pi\sqrt{LC}}$

$\frac{L}{R} \to \infty$

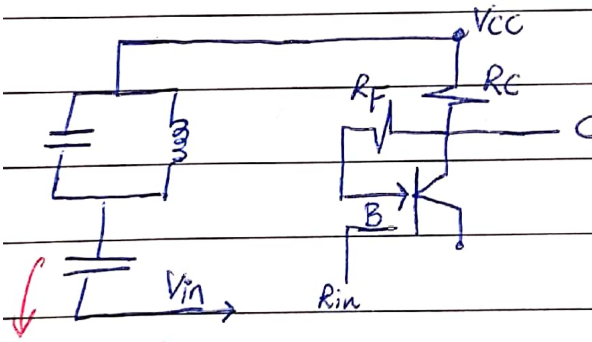
→ reduce the amount of charges

Types of oscillators
1) Cobelt.
2) Clapp
3) Tunnel

negative resistance oscillator :



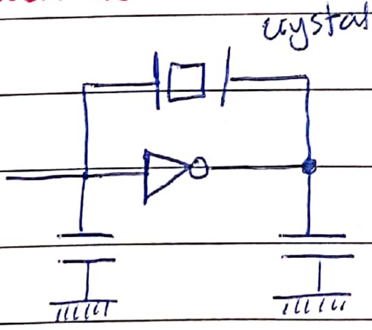
resistance = $\frac{V}{I}$



block dc

out = $\frac{-R_s}{R_{in}}$

$f_o = \frac{1}{2\pi\sqrt{LC_{eq}}}$



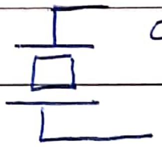
part per million

error (ppm)

Nuclear → more accurate

Irridium ppm (10^{-9} - 10^{-12})

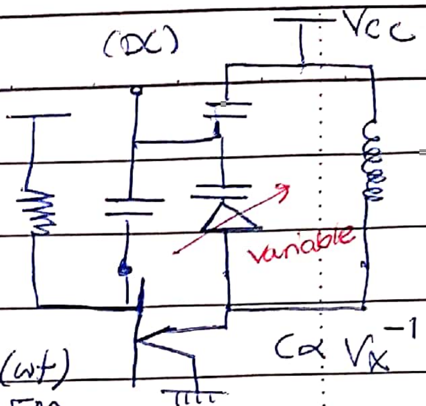
cesium (10^{-15} - 10^{-18})



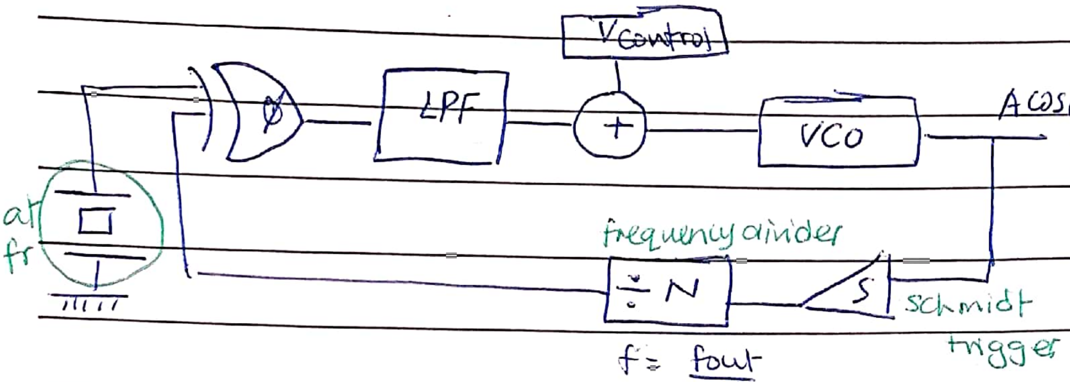
reference oscillator in phase-locked-loops (PLL)

$f_r = \frac{1}{2\pi\sqrt{LC}}$ (reference frequency)

$C = \frac{K}{V_x}$ $f_r \propto \sqrt{V_x}$



$C \propto V_x^{-1}$



$A\cos(\omega t)$
FM
PM(with d/dt)

$\omega \propto V_{in}$

$f = \frac{f_{out}}{N}$

control N

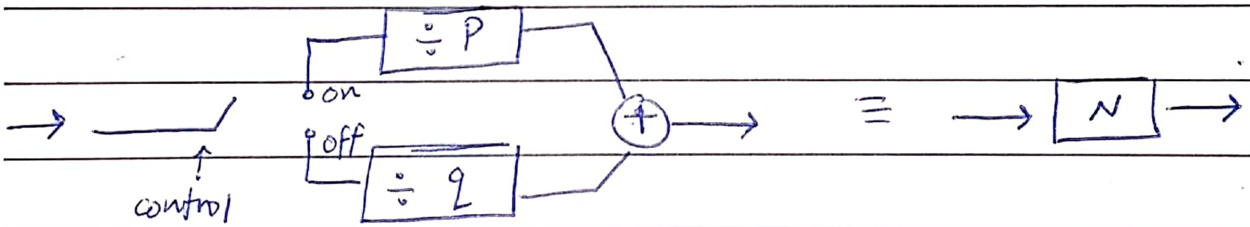
at lock state → $V_{error} = 0$ volt

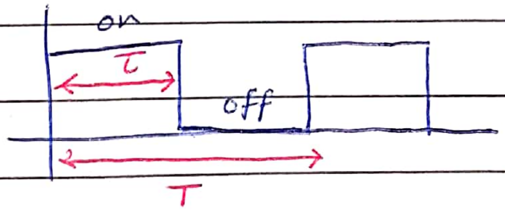
$f_{out} = f_o$

$f_o = Nf = Nf_r$

LPF has small bandwidth → control variation

* Prescaller :





$\frac{\tau}{T}$ = duty cycle

$N = \frac{\tau \cdot P}{T} + \frac{(T-\tau)q}{T}$

Example; $f_o(min) = 88MHz$, $f_o(max) = 108 MHz$

$\frac{20}{0.1} = 200$ steps $\frac{20}{0.2} = 100$ steps

at min → $\tau = 0$ → $N = q$

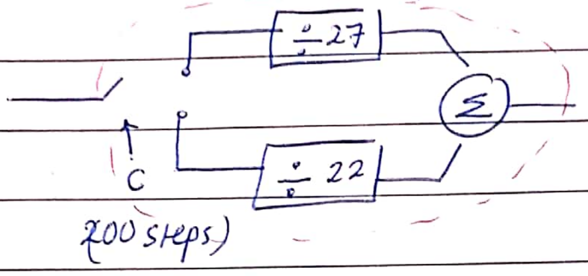
at max → $\tau = T$, $N = p$

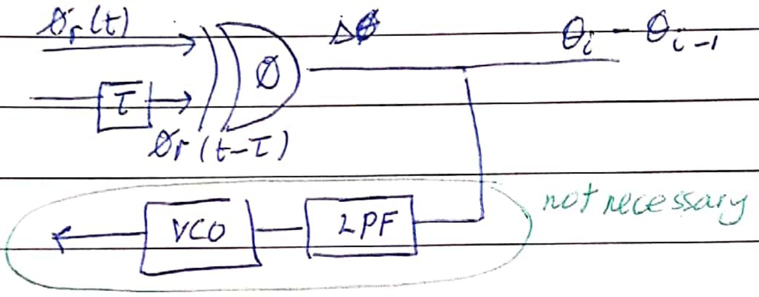
Choose fr = 8 MHz → $\frac{88}{8} = 11 = q$

$\frac{108}{8} = 13.5 = P$

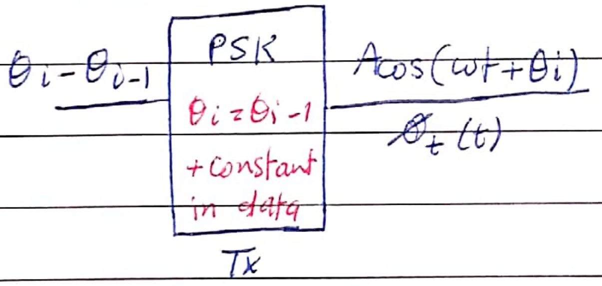
Choose fr = 4 MHz → $\frac{88}{4} = 22 = q$

$\frac{108}{4} = 27 = P$ ✓ integer





Differentially Coherant Receiver.

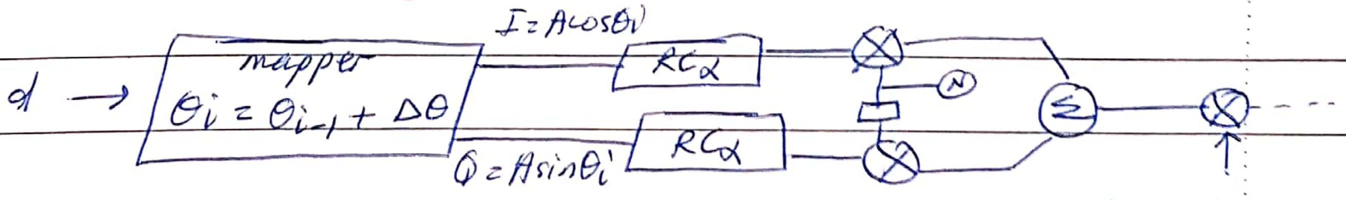


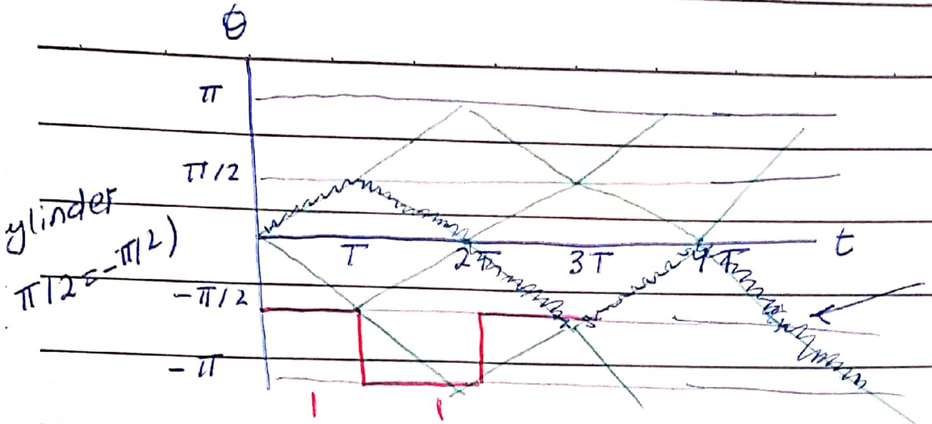
$\theta_i - \theta_{i-1}$ = constant (K_{data}) in differential PSK.

* Differential Modulation :

ifferentially coherant receiver is better than non coherent receiver ecause you don't need coherent receiver at the output, so it is easier

ost systems nowadays use differential PSK.





path must be continuous at the receiver, otherwise → error

path representation to the data

phase tree: all probabilities that data can make.

d	Δθ
0	π/2
1	−π/2

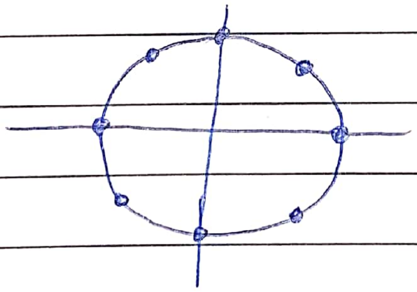
} encoding data table

example;

d	Δθ
0 0	π/4
0 1	−π/4
1 0	3π/4
1 1	−3π/4

encoding table

representation →



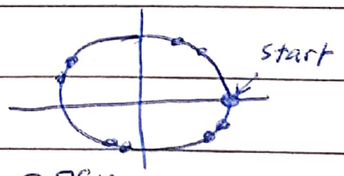
π/4 DPSK

example;

d	Δθ
0	17π/37
1	−13π/23

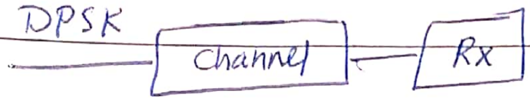
] prime numbers

constellation →



GCD

{ No data on DPSK
No zeroes & ones



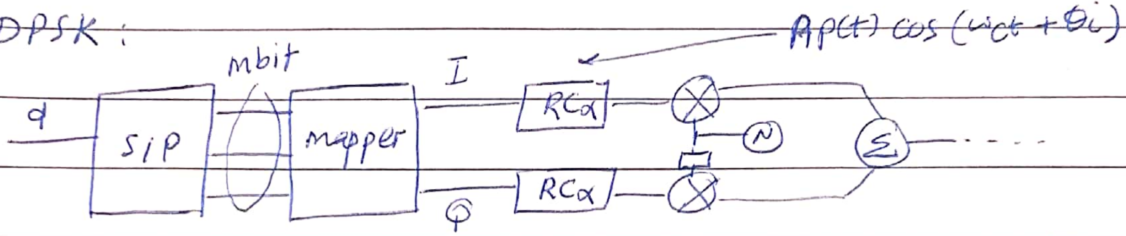
partial information

partial response signaling

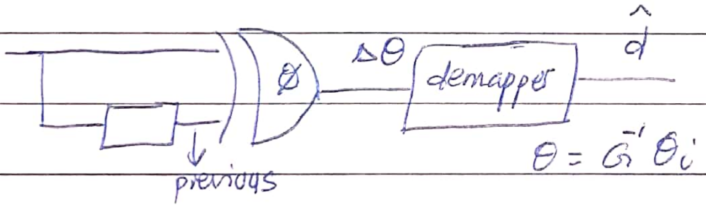
in $\left\{\begin{matrix} PSK \\ FSK \\ QAM \end{matrix}\right\}$ => full response signaling
(Multi-h Mod.)

CPFSK = Continuous phase frequency shift keying

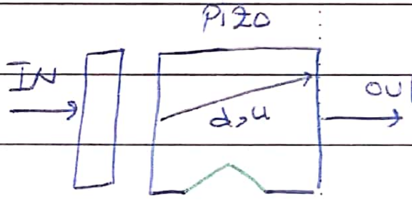
* DPSK :



$d \mid \Delta\theta$ } DPSK $d \mid \Delta\theta_i$ } PSK $\theta_i = \theta_{i-1} + \Delta\theta$



$\theta = G^{-1}\theta_i$



$\frac{\text{distance}}{\text{speed}} = \frac{d}{u} = \tau$

$\theta_i = g(\theta_{i-1}, \theta_{i-2}, \cdots, \theta_{i-L})$ $out = \sum a_i s(t-\tau_i)$

beasewise linear (FIR Filter)

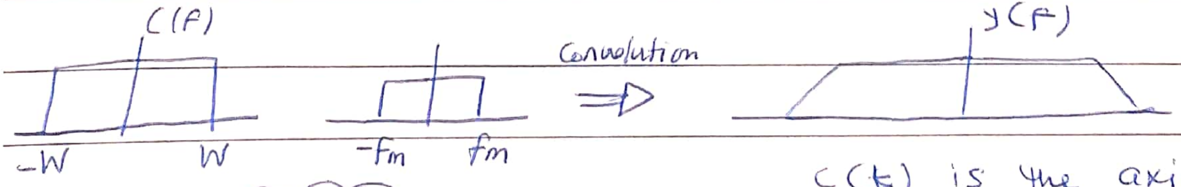
$\theta_i = G\vec{\theta}$, $\theta = \begin{bmatrix} \theta_{i-1} \\ \theta_{i-2} \\ \vdots \\ \theta_{i-L} \end{bmatrix}$

matrix [ ]

G is a code
so this is a coded modulation
TCM DPSK

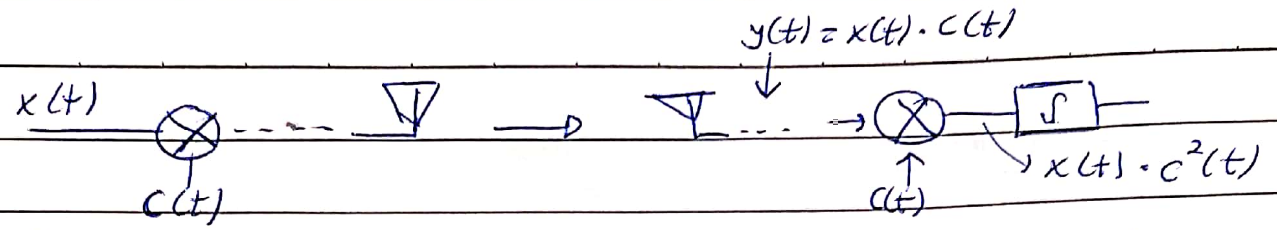
* Spread Spectrum :

$x(t) \cdot c(t) \xrightarrow{\text{convolve}} X(f) * C(t)$

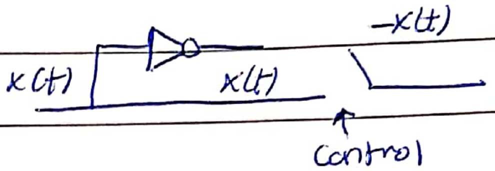


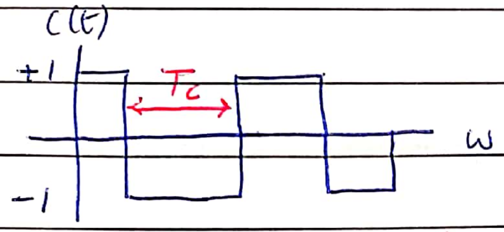
c(t) is the axis

W >> fm

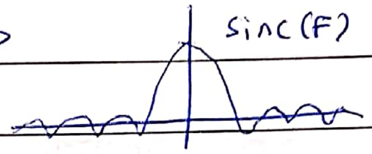


$c^2(t) = 1$; $c(t) \in \mp 1$





⇒



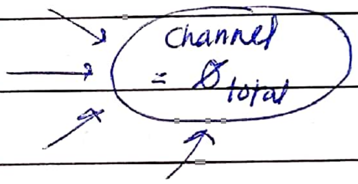
Tc = chip interval

$w = \frac{1}{2T_c}$

N-axis ⇒ $c_1(t), c_2(t), ----, c_N(t)$

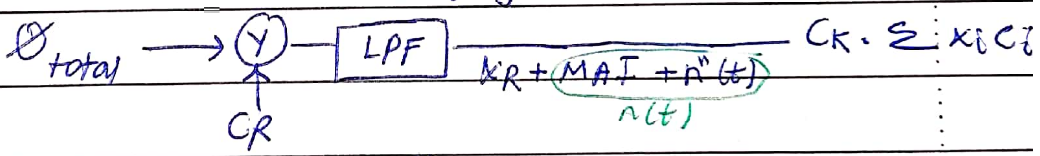
$x_1 c_1$ $x_2 c_2$

$\phi_{total} = \sum_{i=1}^{N} x_i \cdot c_i$



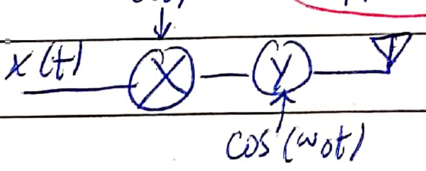
orthogonal $\langle c_i, c_j \rangle = \delta_{ij}$; when $c_i = c_j \Rightarrow \delta_{ij} = 1$

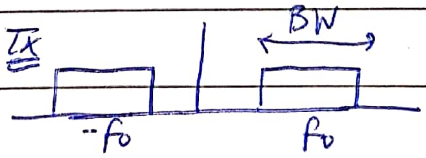
at receiver Rx ⇒



$C_K \cdot \sum x_i c_i = \cancel{C_K} \cdot x_K \cdot \cancel{C_K} + \sum_{i \neq K}^{N} C_K x_i c_i$

$= c_K + \boxed{\sum_{i \neq K}^{N} x_i c_i C_K}$ → multiple access interference (MAI)



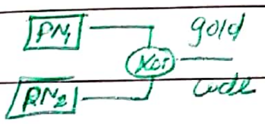


$BW = 2W = \frac{1}{2T_c} = r_c$

in fading channels we need a RAKE receiver

$T^3 = KR^2$, R = ارتفاع القمر الصناعي من مركز الارض

c(t) → Hadmard → $<\ ,\ > = \{ \frac{-1}{N}$
→ maximum length sequence (PN codes); [ML-sequence] } codes
→ Gold codes → $<\ ,\ > = \frac{3}{N^2}$



$\int c_i c_j = \sum_{i=1}^{N} c_i c_j = \delta_{ij}$

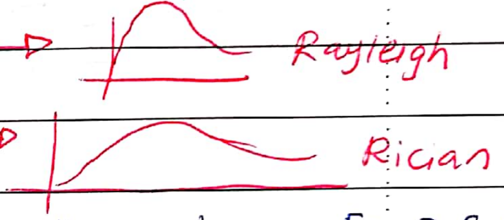
* Importance of channel coding:- → * ادرسي لصفحة الي بعد هاي بالأول

1] Communication system becomes interference limited not noise limited.
S/I (signal interference ratio) is low (ECC is the solution.)

2] Carrier frequency [GHz] $BER = \frac{1}{2} erfc\left(\sqrt{\frac{\gamma_b}{2r_b}}\right)$

3] Fading channels

$\gamma_{fading} = \boxed{\xi_D} \gamma$



عدة
(burrest of errors)

$\gamma_{coded} = C.G\ \gamma \xi_D$, $\left\{ C.G = \text{coding gain} = \frac{r_c}{r_b} \right\} S.F$
(DS-SS)

The best code we can get is the direct sequence spread spectrum.

S.F = spreading factor → inhance performance

DS-SS = direct sequence spread spectrum

* Introduction to Number Theory:-

fields (نظام العد) ⇒ { G element (group of elements)
+ , x

Any field is equivalent to any field else

The easiest field to deal with is binary → $G = \{0,1\}$

identity for + = zero (0) ; $0 + a = a$
identity for x = one (1) ; $a \times 1 = a$, $a \in G$

$G = \{0,1\}$

$G_N = \{0,1,2\}$

G_4 = { red, green, blue, white, black }

$GF\ (\underbrace{\text{number}}_{q})$

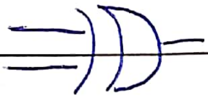
GF (10) ⇒ decimal   GF(2) ⇒ binary

- Extended Field ⇒ $\underbrace{0\ 0\ 0 ---- 0}_{m \text{ bits}}$ $GF(2^m)$

m = عدد البتات

addition   (XOR)

	0	1
0	0	1
1	1	0



multiplication   (AND)

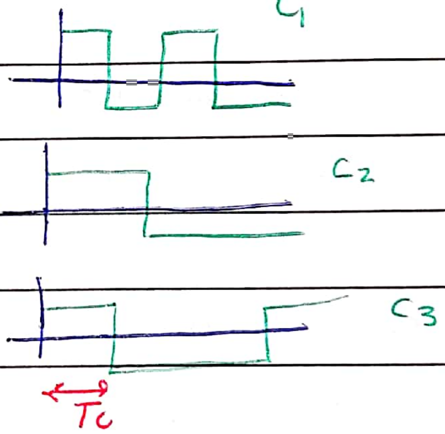
	0	1
0	0	0
1	0	1

Thursday Additional lecture (19/5)

* Spread Spectrum :-

Example: Hadmard (s-axis) ⇒ 3 codes

$$\text{use } H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$



$$u_1 \rightarrow c_1 d_1 \cdot \begin{pmatrix} r_{b1} \\ r_{b2} \\ r_{b3} \end{pmatrix}$$

$u_2 \rightarrow c_2 d_2$

$u_3 \rightarrow c_3 d_3$

Chip rate : $r_c = \frac{1}{T_c} >> r_{bi}$

Spread Rate Factor = $SF_i = \frac{r_c}{r_{bi}}$

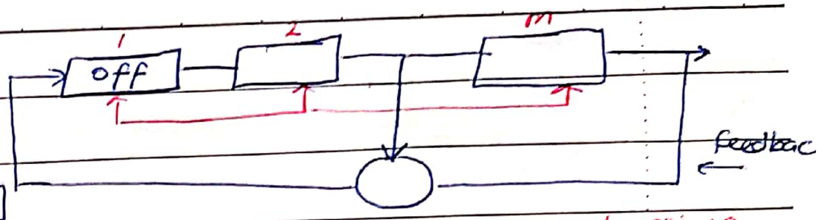
* How to generate PN ?

PN = P(sudo - Noise)   (ML-sequence)   (m-sequence)

PN = psudo noise

Subject Date No.

Example; FBSR
(m-registers)



random number : PN = [1001⓪11]

C = [1 -1 -1 -1 1 -① 11] =D shift

N bits

short register (SR)

$$\boxed{N = 2^m - 1}$$ Max-length

SR1	SR2	SR3	PN
0	0	1	1
1	0	0	0
0	1	0	0
1	0	1	1
1	1	0	0
1	1	1	1
0	1	1	1
0	0	1	1

الارسي بعرضا
ا الصفحة الثانية
(الرسمات بعدين
١٢ صفحة تالي)

$\frac{13}{11} = xxxx$

reminder = random number

take first prime = P_1
take second prime = P_2 } $P_1 > P_2$

$\frac{P_1}{P_2} = \underline{xxxxx}$ → random

* Chinese Remainder Theorem (CRT) :-

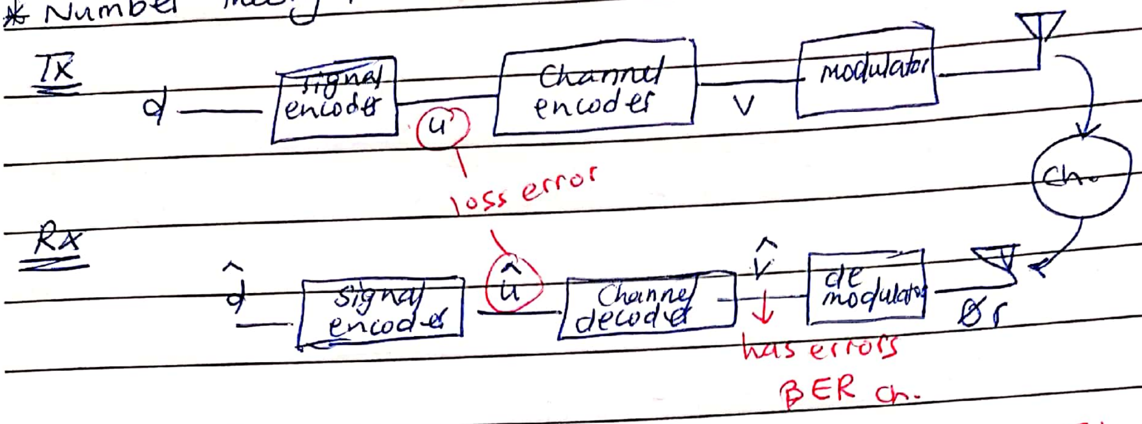
$m = 160 \rightarrow N = 2^{160} - 1$

prime : $(2^{160} - 1)$

N O T E B O O K

(all solution)

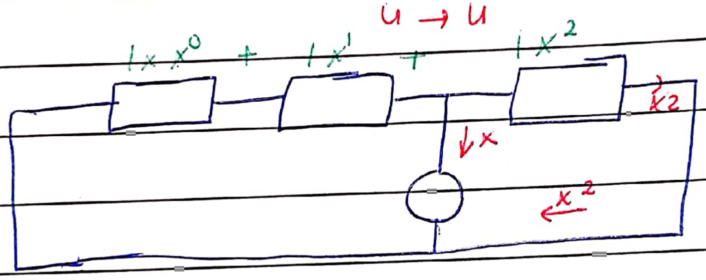
* Number Theory :-



Error correcting codes $V \rightarrow \hat{V}$ as of error Ist

$u \rightarrow \hat{u}$

①



$x^2 + x^1 = x^0 = 1$

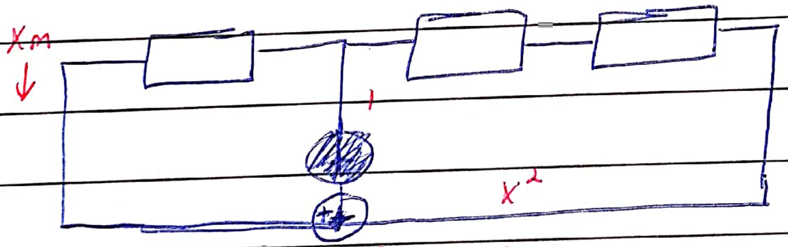
Connection

F.B $+ x + x^2$

$g(x) = 1x + x^2 = 0$

$x^3 = x + x^2$

②



$g(x) = 1 + x^2 + x^3$

$x^3 = 1 + x^2$

$N = 1 + x^2 + x^3$

$x^3 = 1 + x^2 = \overset{2}{\underline{(1+x)}}(1 + x^2 + x^3)$

$N = 2^m - 1 = 2^3 - 1 = 7$

$= 1 + x + x^3$ (primary eq.)

Random :

Prime 13

11, 7, 5, 3, ②, 1 → the only even prime

BER coded = ~~$(BER)^{t+1}$ channel~~ $(BER_{ch})^{t+1}$

Example: $BER_{ch} = 10^{-2} = 0.01$, $t = 5$

$BER_{coded} = (10^{-2})^{\gamma} = 10^{-12}$

Example; $GF(2^3)$

should be in the same field

$GF(2^3)$						
0	0	0	0	0	0	0
1	1	1	1	1	0	0
2	α	x	x	0	1	0
3	α^2	x^2	x^2	0	0	1
4	α^3	x^3	$x^3 = 1 + x^2$	1	0	1
5	α^4	x^4	$x^4 = x \cdot x^3 = 1 + x + x^2$	1	1	1
6	α^5	x^5	$x^4 \cdot x = 1 + x$	1	1	0
7	α^6	x^6	$x + x^2$	0	1	1

Representation ⇒ $GF(2^m)$

No carry $V_1 = [1\ 0\ 1\ 1] \in GF(2^4)$ $[1\ 0\ 1\ 1\ \ 0\ 0\ 0\ 0]$

$V_2 = [0\ 0\ 1\ 1\ 0\ 1\ 1\ 0] \in GF(2^2)$

$V_1 + V_2 = [1\ 0\ 0\ 0\ 0\ 1\ 1\ 0]$ vector

polynomial $V_1 = [\underset{x^0}{1}\ \underset{x^1}{0}\ \underset{x^2}{1}\ \underset{x^3}{1}]$ ⇒ vector

$V_1 = 1 + x^2 + x^3$ ⇒ polynomial

$V_2 = x^2 + x^3 + x^5 + x^6$

$V_1 + V_2 = 1 + 0 + 0 + x^5 + x^6 = 1 + x^5 + x^6$

$V_1 - V_2$ = same answer as $V_1 + V_2$

- Multiplication ⇒ $V_1 \times V_2 = x^2 + x^3 + x^5 + x^6$

$+ x^4 + x^5 + x^7 + x^8$

$+ x^5 + x^6 + x^8 + x^9$

~~Mod (2)~~

$1+1=0$

$1+1+1=0$

$V_1 \cdot \cancel{X}_2 = x^2 + x^3 + x^4 + x^5 + x^7 + x^9$

- Division ⇒

$x^3 \rightarrow q(t)$

$1+x^2+x^3 \overline{\big)\, x^6+x^5+x^3+x^2}$

$x^6+x^5+x^3$

$x^2 \rightarrow r(t)$

q(t) = quotient

r(t) = remainder

$\frac{V2}{V_1} = x^3 + \frac{x^2}{1+x^2+x^3}$

m = 3 $\quad 1+x+x^3 \quad\quad 1+x^2+x^3$

① $(0,1,3)$ ③ $(0,2,3)$

② is the same as ①

$\emptyset i = \emptyset i.2$; $\emptyset$ = prime polynomial

$\emptyset$'s are the roots of $(x^{2^m-1}+1)$

$(x^7+1) = (1+x)(1+x+x^3)(1+x^2+x^3)$

↓ special case (represents 2 in decimals)

m = 3, g(x): generator polynomial

① $(0,1,3)$ ③ $(0,2,3)$

$g(x) = 1+x^2+x^3$

$0 = 1+x^2+x^3$

$x^3 = 1+x^2$

$x+1$

$1+x^2+x^3 \overline{\big)\, x^4}$

x^4+x^3+x

x^3+x

x^3+x^2+1

$x^2+x+1 = r(x)$

prime polynomials will span the whole field span (it can generate)

$\alpha^3\alpha^4 = \alpha^7 = 1$

$x^{-e} = x^{N-e}$ equivalent both prime number but different organization

$N = 2^m - 1$

$x^L = x^{L-N.t}$ range 25-21 should be with 0-6

$x^{25} = x^{23-\textcircled{21}} = x^4$ → 7*3

Multiplication by residue.

* Hamming :>

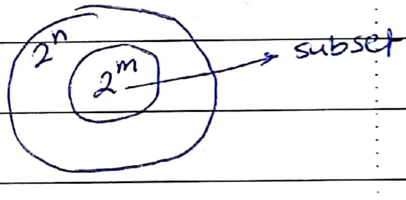
$1\ 0\ 1\ 1\ 0\ 1 \quad \} \ 4$ weight = Σ ones

$1\ 1\ 0\ 1\ 0\ 1 \quad 4$

$d_H = 2$

Transmit data : $\begin{cases} 0\ 1\ 0 \\ 1\ 1\ 0 \end{cases}$ min d_H

if $d_H = 1$ ⇒ there is error



output → n-bit $GF(2^n)$

input → m-bit $GF(2^m)$

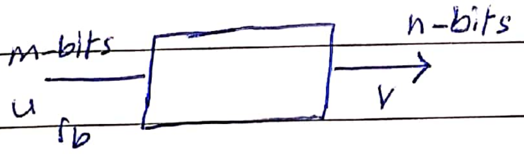
$\boxed{n > m}$

Channel Coding ⇒ it is to select 2^m code words out of a possible 2^n words

such that all output code words have max d_{min}^{H}

Max Hamming distance

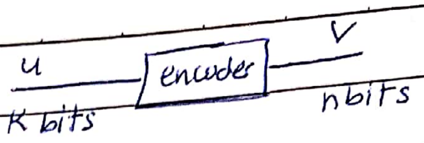
Difference between n & m is minimum.



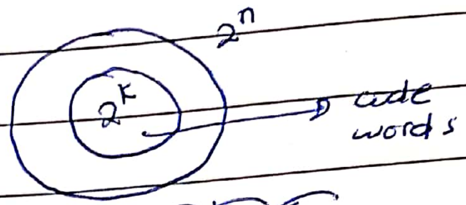
$r_b' = \frac{n}{m} r_b$

$\frac{u}{B}$ (BW expansion factor)

Subject Field Arithmetic Date No.



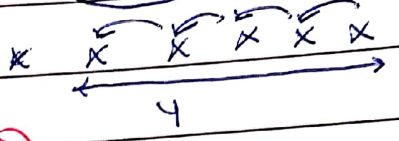
Select 2^k, out of possible 2^n $(n > k)$ such that we have max. d^H_{min}



1 0 1 1 0 1 1

$\underline{1\ 1\ 0\ 1\ 1\ 0\ 1}$

$d^H = 4$



$$\left[\frac{d^H_{min} - 1}{2} \right] = t \Rightarrow \boxed{d^H_{min} = 2t + 1}$$

* Linear Block Codes :-

1 - Hamming Codes

$V = u\,G$ ← generator matrix (it will spand the whole field)

$d^H_{min} = 3 \rightarrow t = 1$, $g = 1 + x^2 + x^3 \rightarrow$ order $(m = 3)$

$$\begin{cases} n = 2^m - 1 \\ t = 1 \qquad d^H_{min} = 3 \\ K = n - m \end{cases}$$

Example ; $m = 3$

$n = 7$

$K = 4$

0	0 0 0
1	1 0 0
α	0 1 0
α^2	0 0 1
α^3	1 0 1
α^4	1 1 1
α^5	1 1 0
α^6	0 1 1

transpos

Sol (n, K) Hamming Code

$$H = \left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ \underbrace{0 \quad 0 \quad 1}_{I_{3x3}} & & & \underbrace{1 \quad 1 \quad 0 \quad 1}_{P^T} & & & \end{array} \right]$$

(parity check)

$G = [I_{K \times K} : P]$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ \underbrace{0 & 0 & 0 & 1}_{I_{4\times4}} & 0 & 1 & 1 \end{bmatrix}$$

generator

G&H ⇒ dual space to $GF(2^n)$

$H = [I_{n-k \times n-k} : P^T]$ parity check

at Tx $V = uG$ $u = [1\ 1\ 0\ 0]$

at Rx $\hat{V} = V + \text{error}$

Tx $$V = [1\ 1\ 0\ 0] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} = \underbrace{[\underbrace{1\ 1\ 0\ 0}_{u},\ \underbrace{0\ 1\ 0}_{\text{parity}}]}_{V}$$

$K \rightarrow n$

$\text{rate} = \frac{n}{K} r_b = r'_b$

$BW = \frac{r'_b}{\log_2 M}(1+\alpha) = \frac{n}{K} \cdot \frac{r_b}{\log_2 M}(1+\alpha)$ $\frac{n}{K}$ = bandwidth expansion factor

$\text{code rate} = \frac{K}{n}$

Rx $\hat{V} = [1\ 1\ 0\ 0\ 0\ 1\ 0]_{1\times7}$

Syndrom ⇒ $S = VH^T = [0\ 0\ 0]$ $S_{1\times n-k} = V_{1\times m} H^T_{n\times n-k}$

if $S=0$ → No errors

otherwise → S^T = column of H corresponding to error location

$e = [0\ 0\ 0\ 0\ 0\ 1\ 0]$, $S = [1\ 1\ 0]$

$\hat{v} = [1\ 1\ 0\ \overset{\overset{\text{error}}{\downarrow}}{1}\ 0\ 1\ 0]$

$$S = [1\,1\,0\,1\,0\,1\,0] \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}^T = [1\ 0\ 1]$$

$$e = [0\ 0\ 0\ 1\ 0\ 0\ 0]$$

To correct the error:-

$v = \hat{v} + e = [1\,1\,0\,1\,0\,1\,0] + [0\,0\,0\,1\,0\,0\,0]$

$= [\underline{1\ 1\ 0\ 0}\ \ 0\ 1\ 0]$

$u = [1\ 1\ 0\ 0]$ error corrected

$BER_{coded} \cong (BER_{channel})^{t^2+1}$; correct t errors every n-bits

detect $d^H_{min} - 1$ errors

ECC → Error correcting → LBC
→ C.C

→ Error detecting code (CRC)

CRC = cyclic redundan check

LBC ⇒ 1] Hamming Code $(t=1)$

2] Cyclic Code $(t=1)$

3] BCH $(t>1)$ → Golay Code
→ Reed-Solomon Code (RS)

4] LDPC (low density parity check) → the best.

Cyclic Codes :

$u(x) \circ g(x) = v(x)$

$$\frac{v(x)}{g(x)} = u(x)$$

$e(x) + v(x) = \hat{v}(x)$

$\frac{e(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$ $n = 2^m - 1$

any polynomial (single element) doesn't divide g(x) without remainder

$g(x) = 1 + \cdots + x^m$

$$\boxed{\frac{\hat{v}(x)}{g(x)} = \underbrace{u(x) + q(x)}_{\hat{u}(x)} + \frac{r(x)}{g(x)}}$$

$\hat{u}(x) = u(x) + q(x)$

if $r(x) = 0 \to g(x) = 0$

$\hat{u}(x) = u(x) \to$ no error

every $r(x) \to$ one $e(x)$

q(x)	e(x)	r(x)
0	0	0
0	1	1
0	x	x
0	x^2	x^2
1	x^3	$1+x^2$
$1+x$	x^4	$1+x+x^2$
$1+x+x^2$	x^5	$1+x$
$x+x^2+x^3$	x^6	$x+x^2$

$1+x^2+x^3 \,)\, x^3$ quotient 1

x^3+x^2+1

$1+x^2$

$1+x^2+x^3 \,)\, x^4$ quotient $x+1$

x^4+x^3+x

x^3+x

x^3+x^2+1

x^2+x+1

$1+x^2+x^3 \,)\, x^5$ quotient x^2+x+1

$x^5+x^4+x^2$

x^4+x^2

x^4+x^3+x

x^3+x^2+x

x^3+x^2+1

$1+x$

$1+x^2+x^3 \,)\, x^6$ quotient x^3+x^2+x

$x^6+x^5+x^3$

x^5+x^3

$x^5+x^4+x^2$

$x^4+x^3+x^2$

x^4+x^3+x

x^2+x

$1+x^2+x^3 \,)\, x^5+x^4+x^3+1$ quotient x^2+1

$x^5+x^4+x^2$

x^3+x^2+1

x^3+x^2+1

0

Example; $u(x) = 1 + x^2$ $[1 \ 0 \ 1 \ 0]$

$m = 3$, $n =$, $(t = 1)$

$V(x) = u(x) \cdot g(x) = (1 + x^2)(1 + x^2 + x^3)$

$= 1 + x^2 + x^3 + x^2 + x^4 + x^5$

$= 1 + x^3 + x^4 + x^5$

$u(x) = 1 + x^2$ no errors

$V(x) = 1 + x^3 + x^4 + x^5 + e(x)$

$e(x) = x^3$

$\hat{V} = 1 + x^4 + x^5$

$\frac{\hat{V}(x)}{g(x)} = \hat{u}(x) + \frac{r(x)}{g(x)}$

$u = \hat{u} + q(x)$ $u = x^2 + 1$

$\hat{u}(x)$

$\boxed{x^2}$

$1 + x^2 + x^3 \overline{\big) x^5 + x^4 + 1}$

$\underline{x^5 + x^4 + x^2}$

$r(x)$ $\boxed{x^2 + 1}$

error ?

* CRC ⁚ detect d^H_{min} errors$_2$ (even) (special number)

$g(x)_{crc} = g(x)_{cyclic} \cdot (1 + x)$

$g(x)_{crc} = 1 + \cdots + x^{m+1}$

order of remainder < order of g_{crc}

Max remainder order $= m$

$\Rightarrow m + 1$ bits $[\underbrace{x \quad x \quad x \quad x}_{m+1}]$

Step ① → 1) length of transmitted packet (Lp)

example; Lp = 200 bits

cyclic → $d^H_{min} = 3$

$g_{crc} = g_{cyclic} (1 + x)$

2) output data = $[\underset{k'}{u} \;\vdots\; \overset{\to \text{remainder}}{\underset{m+1 \text{ bits}}{r}}]$

k' = number of data bits for CRC

$k' = Lp - m - 1$

find m such that $\underset{\downarrow\, 200-m-1}{k'} \leq \underset{\hookrightarrow\, n-m}{2^m - m - 1}$

$2^m - 1 > 200$

$\boxed{m = 8}$

3) from table $g(x)$ at $m=8$

(0, 2, 3, 4, 8) → order should equal 8

$g(x) = 1 + x^2 + x^3 + x^4 + x^8$

$g_{CRC}(x) = (1+x^2+x^3+x^4+x^8)(1+x)$

at Tx

$V = [u : r]$

$$g_{CRC}\,\overline{)\,u\,}$$
r

$m+1$ bit , $m = 8$
↓
9 bits

$k' = 200 - 9 = 191$ bits

at Rx

$\hat{V} = [\xleftarrow{\hat{u}} \,|\, \xrightarrow{m+1 = r'}]$

$$g_{CRC}\,\overline{)\,u'\,}$$
r^*

if $r^* = r'$ → no errors

? → error

$* \quad g_{CRC}(x) = \overbrace{g_{cyclic}(x)}^{1+\cdots+x^m} \cdot (1+x) = 1 + \cdots + x^{m+1}$

↑ $\phi(x)$ "minimal polynomial"

$g_{CRC}\,\overline{)\,\text{polynomial}\,}$

remainder → x^m (m+1) bits

least common multiplier

For $2t+1$ errors detection CRC code we find $g_{CRC}(x) = \text{LCM}\{\phi_i, i=1,2,\dots 2t\} \cdot (1+x)$

Example; design a 7 error detection CRC code ?

Sol $t=3$

for a packet of 200 bits   it is important to find m.

$\boxed{2^m \geq L\ packet}$ $m=8 \Rightarrow GF(2^8)$

ϕ_1, ϕ_3, ϕ_5 (from table)

$(0,2,3,4,8)(0,1,2,4,5,6,8)(0,1,4,5,6,7,8)$

$=(1+x^2+x^3+x^4+x^8)(1+x+x^2+x^4+x^5+x^6+x^8)(1+x+x^4+x^5,x^6+x^7+x^8)(1+x)$

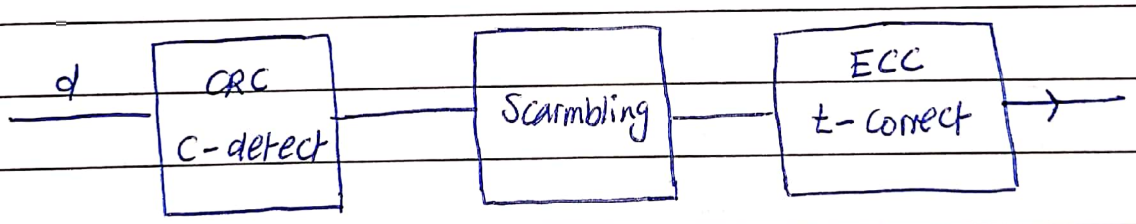
remainder = order of g_{CRC} = 25

data = 200 − 25 = 175 $[175 : 25]$ parity check
$d \quad r$

$d[175\ bits]\ [r_{Tx}]$ 200 bits

at Tx $g_{CRC}(x) \overline{)\ 1+ \dots + x^{174}}$ r_{Tx}

$g_{CRC} \overline{)\ \hat{d}}$ $\hat{r}_{Rx}$ at Rx

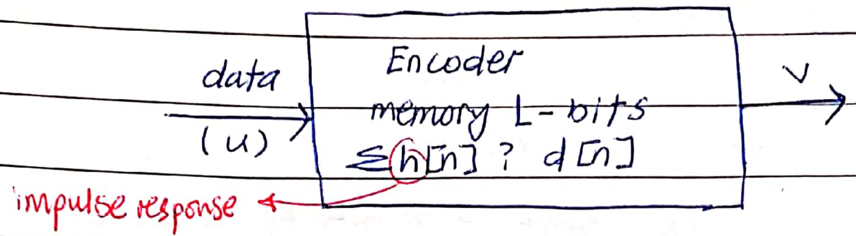


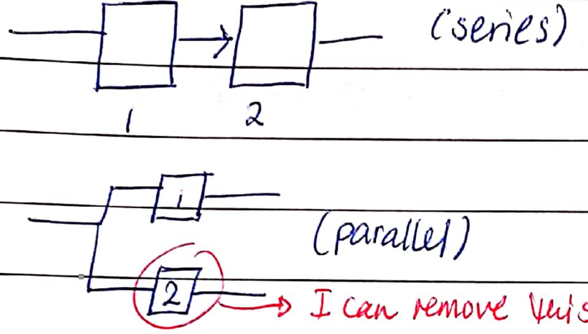
$$BER = (BER_{ch}^{t+1})^{C+1} = (BER_{ch})^{(t+1)(C+1)}$$

* Convolutional Code :

convolution = delay + add

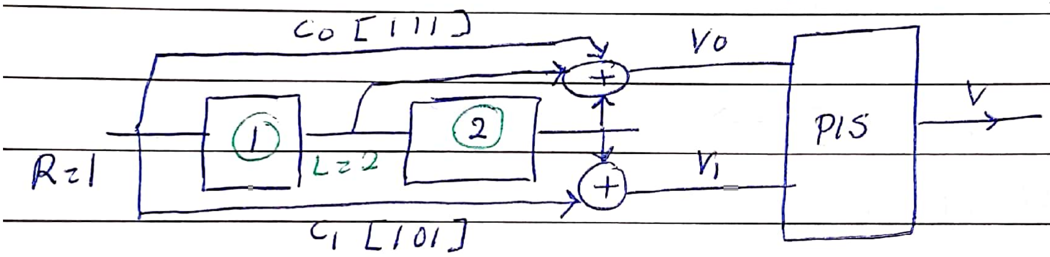
add $\Sigma \leftarrow \int x(t) \cdot h(\tau - t)\, dt$
shift (delay)





I can remove this because parallel will cause redundancy.

so always in series

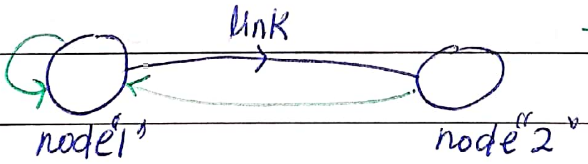


2^L – states

C_0 , C_1 ⇒ Connection vector (n=2)

State Machine = a machine that convolves input with a memory state.

Graph :

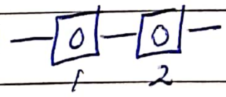


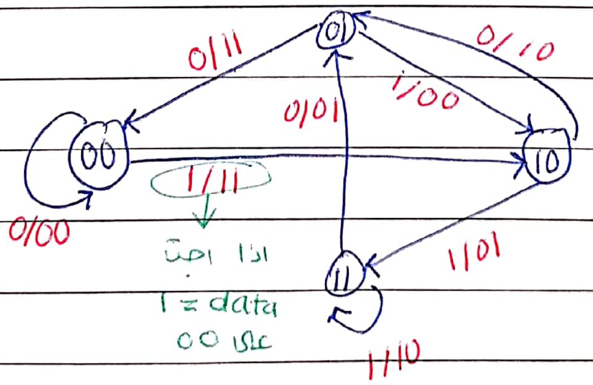
State = node

There might be more than one arrow from the same node.

State Diagram :

Start from





اذا اجت
1 = data
00 علك

output = [V_0 V_1]

1 / 01
input
output

* Directives to choose directional vectors :

1) input data should directly pass to output data.
2) no self loop with weight equals (0) other than all zero states.
3) the free distance d_{free} equals the weight of all outputs in the minimum error event loop

$d_{free} = 5$

$= \left[\frac{2t+1}{2}\right]$ min error event loop → it can convert 2 errors every 6 output bits.

Example; $d = [0\ 1\ 1\ 0\ 1\ 1\ 1]$

so $V_{out} = [00\ 11\ 01\ 01\ 00\ 01\ 10]$

meanwhile $d = [0\ 1\ 1\ 1\ 1\ 1\ 1]$ → 1 bit difference

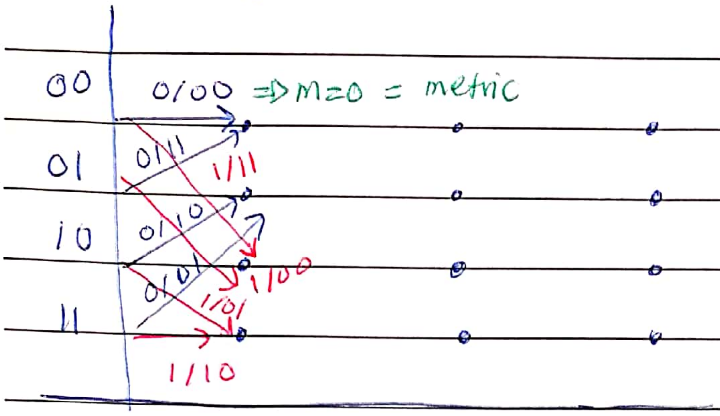
$V_{out} = [00\ 11\ 01\ 10\ 10\ 10\ 10]$ → $d_{free} = 5$ bits

(5 bits difference)

$d_{min} = 5$ → error correcting = 2. insert 2 zeroes at the end of the packet.

* To reset code; we need to enter L - zeroes at the end of the packet.

* Trillis Diagram :

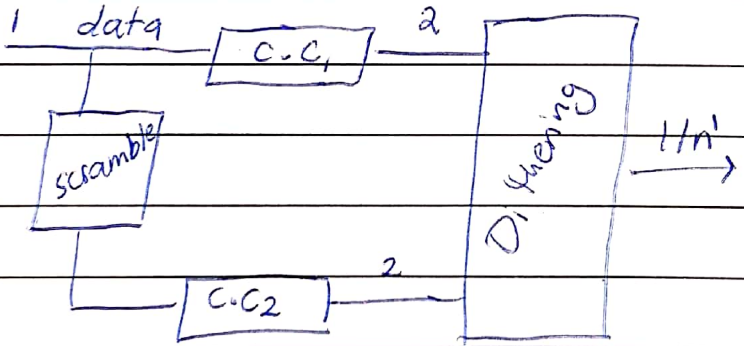


metric is the measure of accumalated error for each possible branch. مقياس errors

* Viterbi Algorithm :

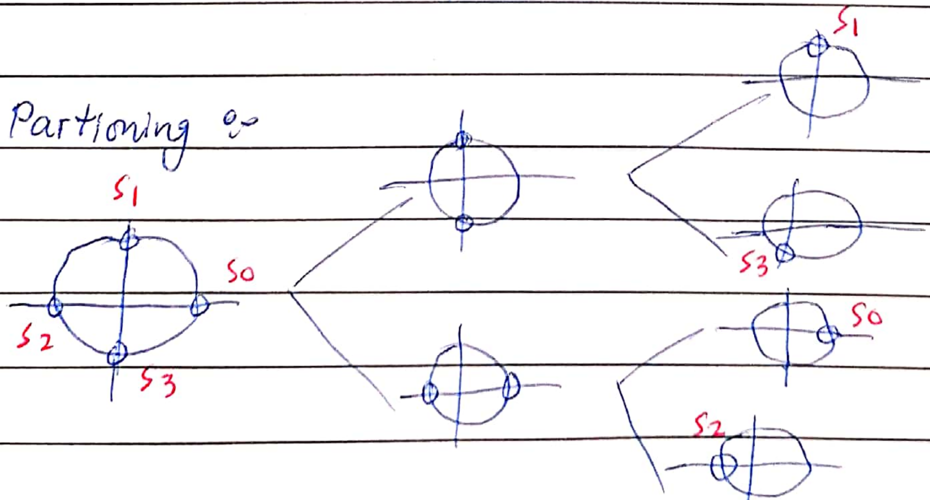
1) find branch metric (m) for each possible state.
2) set (m) and keep the least (r) branches.
3) end ⇒ select min (m) so this is the correct path.

* Turbo Code :



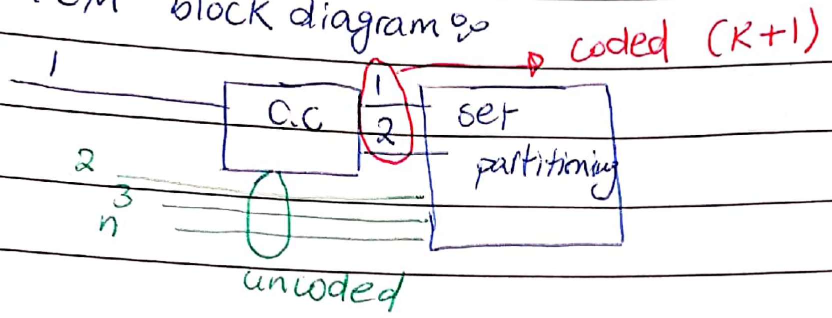
dithering → باخد bit وهيا
باخد bit بعدها

* Partioning :



partitioning & Trillis Diagram ≡ TCM

TCM block diagram:



Example;