General Circuits

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POWERUNIT

Introduction:

* Components of electric circuits:

  ① Source → voltage source — like batteries.
  → current source

  ② load → R (Resistor)
  → L (Inductors)
  → C (capacitor)

  ③ Interconnection (wires) : to connect the source with the load.

* The main unit to generate electricity is the Charge.

* Electron is the smallest charge $(1.602 * 10^{-19} C)$.

* Current: is the rate of change of charge < value, direction.

Note
خلال بكل خلي اتجاه التيار زي ما هو
وبوخذ القيمة والإشارة للتيار
كما هي.

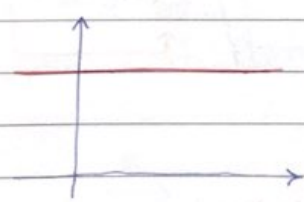
$$\boxed{i = \frac{dq}{dt}} \Rightarrow dq = i * dt$$

Coulomb = Ampere * second.

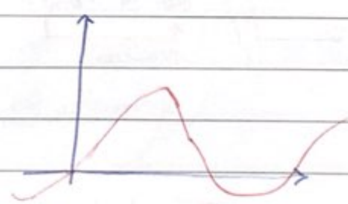
$$[C = A \cdot s]$$

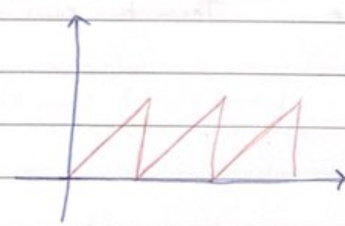
* Types of currents:

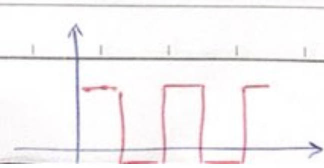
① Direct current (DC):



② Alternating current (AC): #periodic #







* Voltage :
  - value
  - polarity (عين موجب وعين سالب)

the energy required to move a unit charge through a circuit element.

* The source
  - → (Current), it is current source
  - → (voltage), it is voltage source

* The voltage doesn't change with time / And must be between two points.

$$\boxed{V = \frac{dw}{dq}}$$

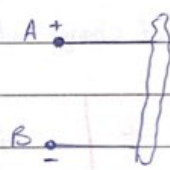
Note : Be carful to the polarity.

$V_{ab} = V_a - V_b$

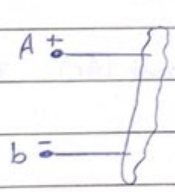
started with a because it is positive

$= (V_a - V_n) - (V_b - V_n)$

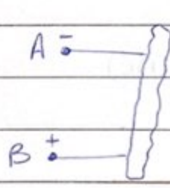
$\underbrace{V_n}_{\text{Reference} = \text{Zero}}$



Ex:



$V_{ab} = V_a - V_b = +9V$



$V_{ba} = V_b - V_a = -9V$

These two cases are equivilant $(V_a > V_b)$

* شو ما يطلع معي الجواب
بضل زي ما هو بإشارته
وبال polarity تبعته.

* Power:

$$P = \frac{dw}{dt} = \frac{dw}{dq} * \frac{dq}{dt}$$

$\frac{dw}{dq}$ → voltage, $\frac{dq}{dt}$ → current

$$\boxed{P = V \cdot i}$$

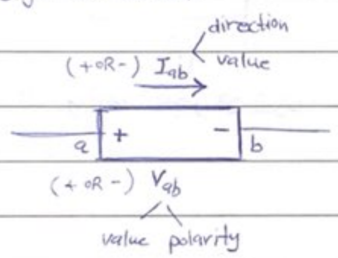
* when there is more than one element, $[\Sigma P = 0]$
(principle of conservation of the power).

* Elements in a circuit:

→ passive (load) / absorb or consume ⇒ it doesn't give power by itself.

→ active (source) / deliver or generate ⇒ it gives.

* Passive sign convention:



⇒ we can calculate P < positive / negative

$P = \pm V_{ab} \cdot I_{ab}$ ⟶ ال V وال I نعوض أرقامهم بإشاراتهم

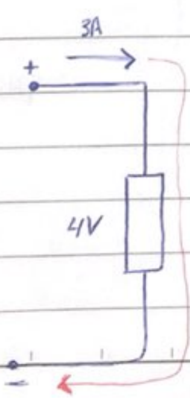
→ if (I) enters (+)

→ if (I) enters (−)

If $P \geq 0$ ⇒ absorb OR consume # $P_{absorbed} = P_{generated}$ #

$P < 0$ ⇒ deliver OR generate.

Ex:



طلع من (+)
دخل ب (−)

$P = \pm V_{ab} I_{ab}$

$= -(4)(3) = -12$ watt (supply power).

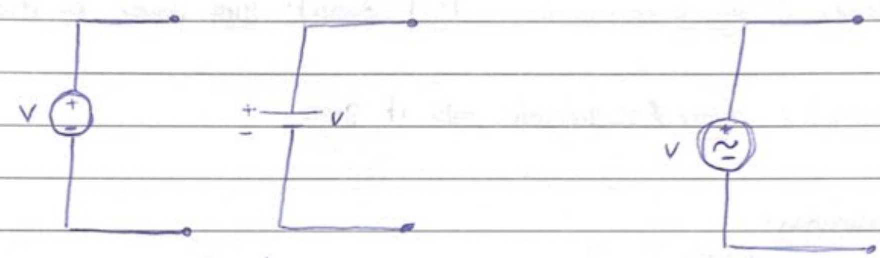
* Energy:

$$E = \sum_{i=0}^{i=t} P \Rightarrow$$ مجموع ال power خلال فترة من الزمن

unit ⇒ watt.hour (w.h) OR Kilowatt.hour (Kh.w).

* (Ideal) voltage and current sources:

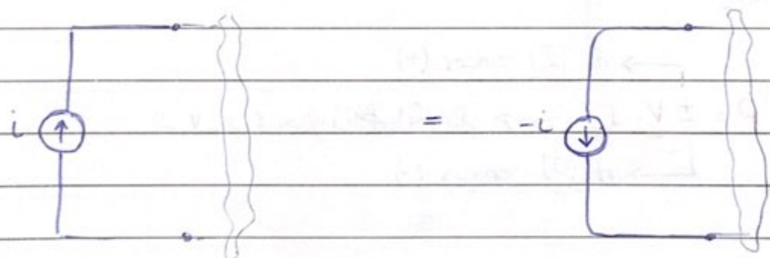
Voltage source:



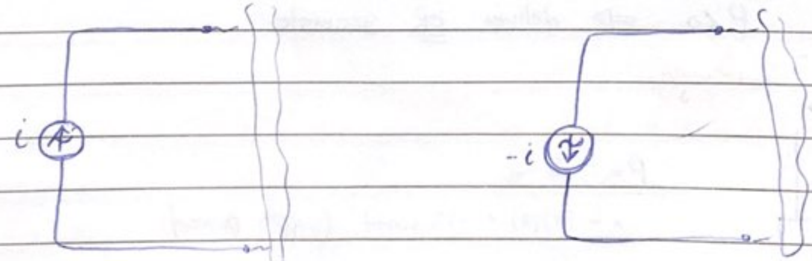
(DC) (AC)

ال voltage ثابت طول الوقت. ال voltage بتغير مع الزمن.

Current source:



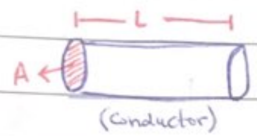
(DC)



(AC)

* Resistance :

A physical property or ability to resist current.



$R \propto \frac{L}{A} \Rightarrow \boxed{R = \rho \frac{L}{A}}$ (Ω)

ρ → Constant = resistivity (ρ)

The symbol is :

* Ohm's law : (R / I / V)

$$\boxed{I = \frac{V}{R}}$$

R → assume it is constant

So, the voltage across a resistor is directly proportional to the current i flowing through the resistor. (linear)

→ The slope = R

The instantaneous power dissipated in a resistor:

$$\boxed{P = vi = \frac{v^2}{R} = Ri^2} \geq 0 \Rightarrow \text{always positive}$$

always absorb power.

* Conductance :

is the ability of an element to conduct current.

$$\boxed{G = \frac{1}{R}} \Rightarrow \text{measured in simens } (S) \text{ OR mho } (\mho)$$

G → conductivity.

* Short and Open circuit :

- Short circuit :

  $R = 0 \Rightarrow V = 0$

  This doesn't mean that there is NO current.

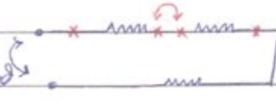
- Open circuit :

  $R = \infty \Rightarrow i = 0$ (no current)

  But there is voltage. (v).

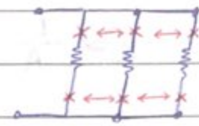
* Series and parallel :

- series : carry the same current not connected



$R_{eq} = R_1 + R_2 + R_3$

- parallel : have the same voltage



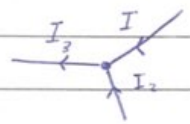
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

OR

for 2 resistances

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

* KCL :



$I_3 = I_1 + I_2$

$$\boxed{\sum I_{in} = \sum I_{out}}$$

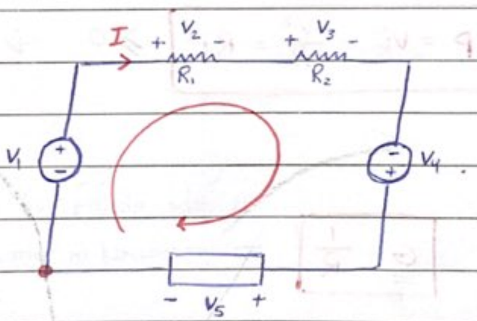
* KVL : Sum of voltage drops = Sum of voltage rises.

$\sum V = \text{Zero}$

loop ⇒ direction (clockwise OR counter clockwise)

If it enters (+) ⇒ (+)

" " " (−) ⇒ (−)

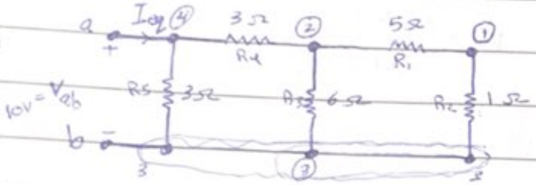


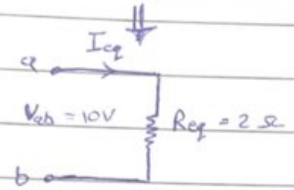
$-V_1 + V_2 + V_3 - V_4 = 0$

$-V_1 + R_1 I + R_2 I - V_4 = 0$

* Connection of Resistance:

- Equivelent Resistance:





$V_{ab} = + IR$

$10 = I(2)$

$\boxed{I = 5A}$

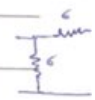
Ex:

Calculate R_{eq} between a and b:

$(R_1, R_2) \rightarrow$ series

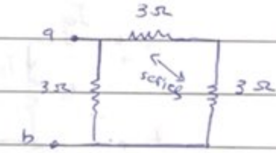
$R_1 + R_2 = 5 + 1 = 6\ \Omega$

$(R_1R_2, R_3) \rightarrow$ parallel (لأنهم شابكين على نفس ال nodes)

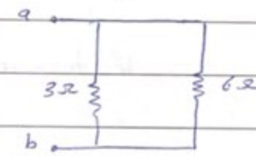


$\frac{(6)(6)}{6+6} = \frac{36}{12} = 3\ \Omega$

$3 + 3 = 6\ \Omega$

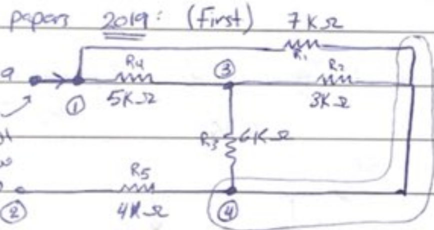


$R_{eq} = \frac{(3)(6)}{3+6} = \frac{18}{9} = 2\ \Omega$



* past papers 2019: (First)

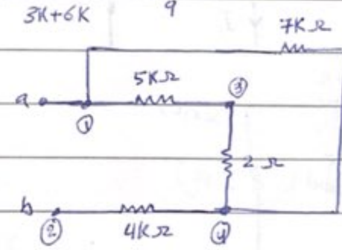
عشانها ي ال 5 وال 7 مش series



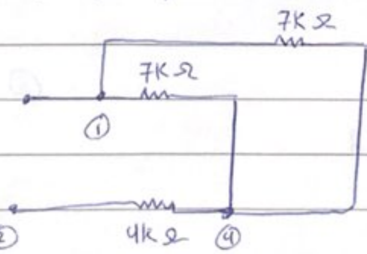
Calculate R_{eq} seen from a & b.

$(R_2, R_3) \rightarrow$ parallel

$\frac{(3K)(6K)}{3K+6K} = \frac{18}{9} = 2\ \Omega$

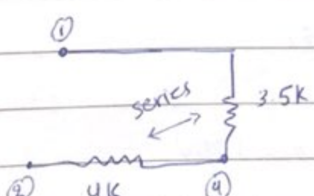


$(5K, 2K) \rightarrow$ series.



$(7K, 7K) \rightarrow$ parallel

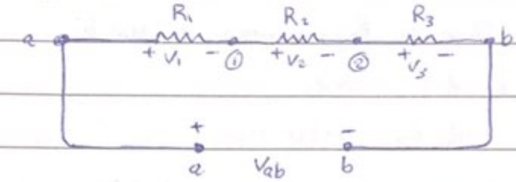
$\frac{(7K)(7K)}{7+7} = \frac{49}{14} = 3.5$



$R_{eq} = 3.5K + 4K = 7.5\ K\Omega$

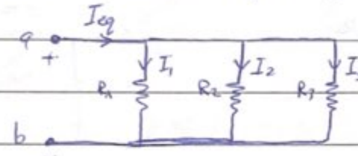
Voltage division and Current division:

- Voltage division ⟹ (Series)



$$V_1 = \left(\frac{V_{ab}}{R_1+R_2+R_3}\right) R_1$$

- Current division ⟹ (parallel)

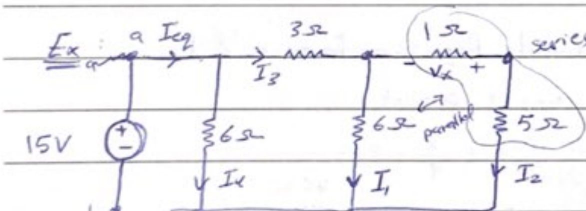


$$I_1 = I_{eq}\left(\frac{R_{eq}}{R_1}\right)$$

If there is just $\underline{\underline{2}}$ resistances:

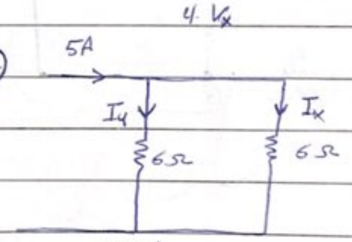
$$I_1 = I_{eq}\left(\frac{\frac{R_1R_2}{R_1+R_2}}{R_1}\right) = \frac{I_{eq}R_2}{R_1+R_2} \qquad I_2 = \frac{I_{eq}R_1}{R_1+R_2}$$

Ex:



Calculate: 1. I_{eq} 2. I_1 3. power absorbed by 5Ω 4. V_x

②



$$\underline{\underline{I_x}} = \frac{(5)(6)}{12} = 2.5A$$

$I_3 / I_1 / I_2$

$$I_1 = \frac{2.5(6)}{12} = 1.25A$$

① $R_{eq} = 3\Omega$

$V_{ab} = 15V$

$I_{eq} = \frac{15}{3} = 5A$

انتبه للإشارات.

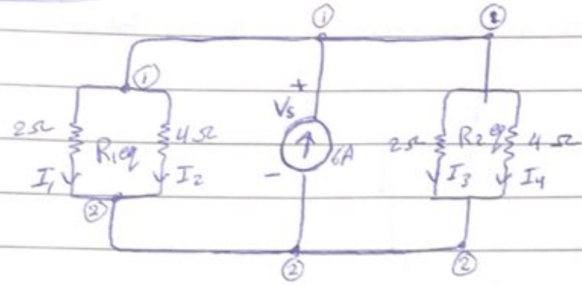
③ $P = +I_2^2R =$

$$I_2 = \frac{2.5(6)}{12} = 1.25A$$

$$P = (1.25)^2(5)$$

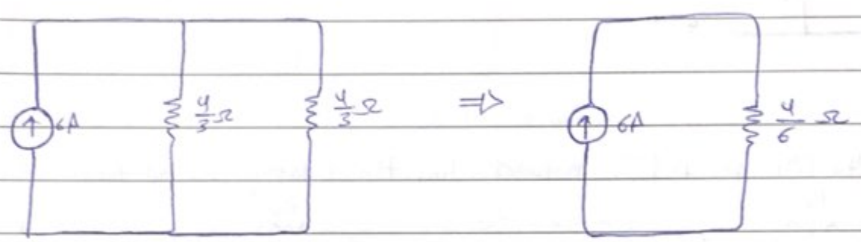
④ $V_x = +I_2 * 1 = -1.25V$

past paper



① Find I_1

② power generated by the current source



① $I_1 = \frac{6(4/6)}{2} = 2A$

② $P = -VI$ $\qquad V_s = \frac{4}{6} * 6 = 4$ volts

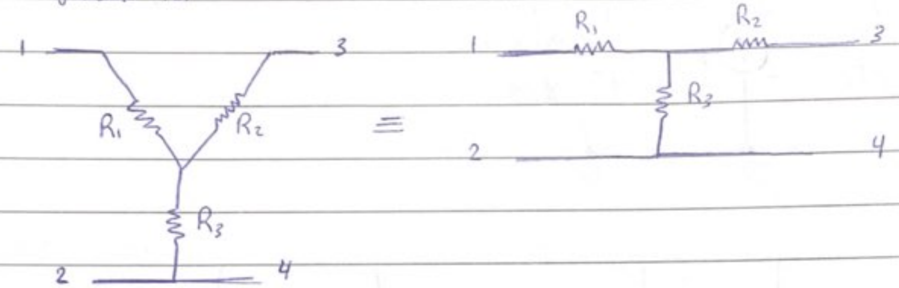
$= -V_s(6)$

$= -(4)(6) = -24$ watt

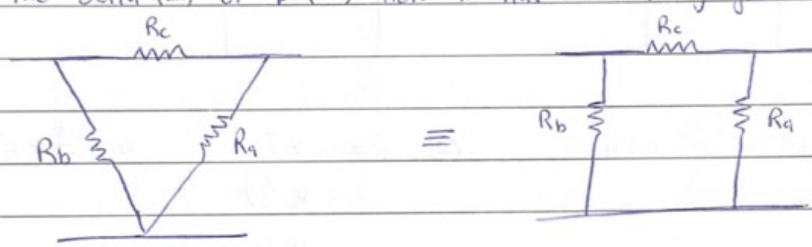
↳ generate power.

* Wye-Delta Transformations:

- A wye (Y) or tee (T) network is a three-terminal with the following general form:



- The delta (Δ) or pi (π) network has the following general form:

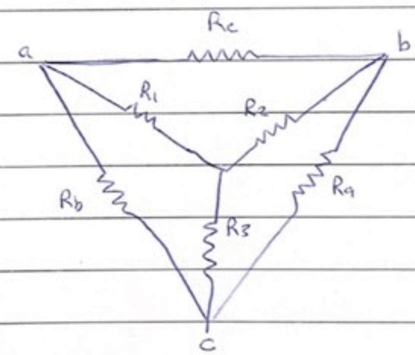


* Delta-Wye Conversion:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



* Wye-Delta Conversion:

$$R_a = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$R_b = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

$$R_c = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

* Y and ∇ are said to be balanced when:

$R_1 = R_2 = R_3 = R_Y$ and $R_a = R_b = R_c = R_\nabla$

$\Downarrow$

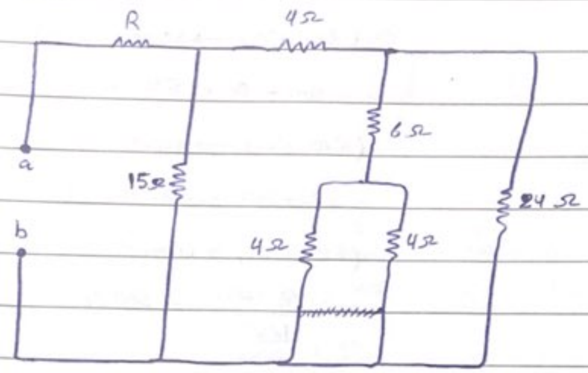
$R_Y = \frac{R_\nabla}{3}$ and $R_\nabla = 3R_Y$

Always $R_\Delta > R_Y$

Five Apple

Solving problems:

① In the circuit o figure below, if $R = 1\,\Omega$, Find R_{eq} seen from ~~open~~ the source open circuit:



(4,4) → parallel

$\frac{(4)(4)}{8} = 2\,\Omega$

(6,2) → series

$6 + 2 = 8\,\Omega$

(8,24) → parallel

$\frac{(8)(24)}{32} = 6\,\Omega$

(6,4) → series

$6 + 4 = 10\,\Omega$

(10,15) → parallel

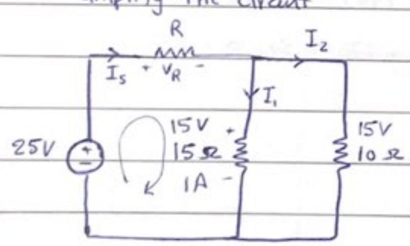
$\frac{(10)(15)}{25} = 6\,\Omega$

(6,1) → series

$R_{eq} = 7\,\Omega$.

② Q $\frac{31}{ch.2}$)

Simplify the circuit:



$P_{15\Omega} = RI^2 \Rightarrow I_1 = \sqrt{\frac{P}{R}} = \sqrt{\frac{15}{15}} = 1\,A$

$V_{15\Omega} = RI_1 = (15)(1) = 15$ Volts.

Because (10,15) are parallel, they have the same voltage.

$V_{10\Omega} = V_{15\Omega} = 15V$

$I_{10\Omega} = \frac{V}{R} = \frac{15}{10} = 1.5A$

$I_s = I_1 + I_2 = 1 + 1.5 = 2.5A$

$V_R = ? \Rightarrow$ based on KVL

$-25 + V_R + 15 = 0 \Rightarrow V_R = 10$ Volts

$R = \frac{V}{I} = \frac{10}{2.5} = 4\,\Omega$

③ $Q\frac{34}{Ch.2}$).

- Seen from ab if cd are open:

(360, 540) → series

$360 + 540 = 900\ \Omega$

(180, 540) → series

$180 + 540 = 720\ \Omega$

(900, 720) → parallel

$\frac{(900)(720)}{1620} = 400\ \Omega$

- Seen from cd if ab are open:

(360, 180) → series

$360 + 180 = 540\ \Omega$

(540, 540) → series

$540 + 540 = 1080\ \Omega$

(540, 1080) → parallel

$\frac{(540)(1080)}{1620} = 360\ \Omega$

- Seen from cd if ab short circuit:

(360, 540) → parallel

$\frac{(360)(540)}{900} = 216\ \Omega$

(180, 540) → parallel

$\frac{(180)(540)}{720} = 135\ \Omega$

(216, 135) → series

$216 + 135 = 351\ \Omega$

- Seen from ab if cd short circuit:

(360, 180) → parallel

$\frac{(360)(180)}{540} = 120\ \Omega$

(540, 540) → parallel

$\frac{(540)(540)}{1080} = 270\ \Omega$

(120, 270) → series

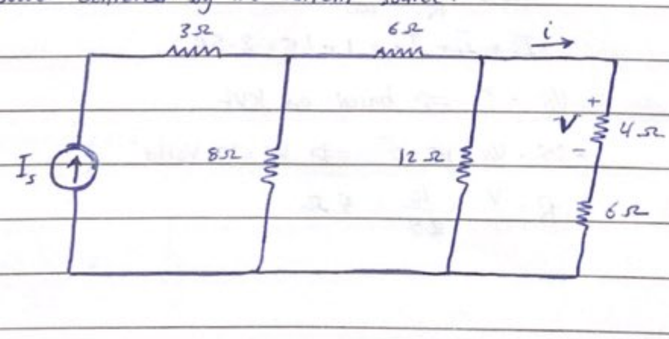
$120 + 270 = 390\ \Omega$

④ past paper: If $I_s = 1A$, then answer the following questions:

a. The value of i

b. The value of V

c. The power delivered by the current source.



$(4,6) \rightarrow$ series

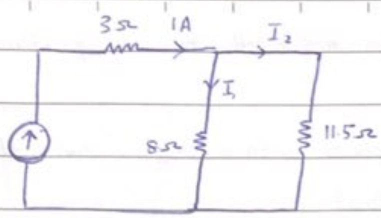
$4+6 = 10\,\Omega$

$(10, 12) \rightarrow$ parallel

$\frac{(10)(12)}{22} = 5.5\,\Omega$

$(5.5, 6) \rightarrow$ series

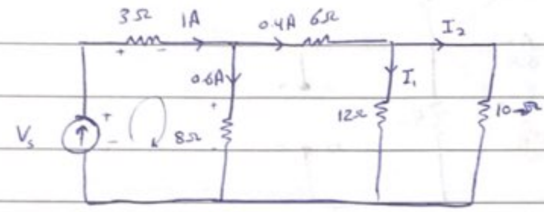
$5.5 + 6 = 11.5\,\Omega$



I_2 by current division:

$I_2 = 1\left(\frac{8}{8+11.5}\right) = \frac{8}{19.5} = 0.4A$

⇓



I_2 by current division:

ⓐ $I_2 = 0.4\left(\frac{12}{12+10}\right) = \boxed{0.2A}$

ⓑ $V = +RI = +(4)(0.2) = +0.8$ Volts.

ⓒ $P = -VI$

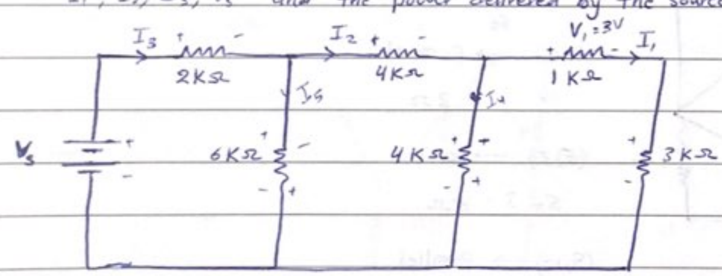
V_s by KVL $\Rightarrow$ $-V_s + 3*1 + 8*0.6 = 0$

$V_s = 7.8$ volts.

$P = -(7.8)(1) = -7.8$ Watt.

⑤ If $V_1 = 3V$, then answer the following questions:

I_1, I_2, I_3, V_s and the power delivered by the source.



$\left.\begin{matrix} R_1 = 1\,K\Omega \\ V_1 = 3V \end{matrix}\right] \Rightarrow I_1 = \frac{V_1}{R_1} = \frac{3}{1K} = 3mA$

by KVL

$-V_s + (2K)(12m) + (6K)(6m) = 0$

$V_s = 24 + 36 = 60$ volts

$I_4 = I_1 = 3mA$

$I_2 = I_1 + I_4 = 3mA + 3mA = 6mA$

$I_2 = I_5 = 6mA$

$I_3 = I_2 + I_5 = 6mA + 6mA = 12mA$

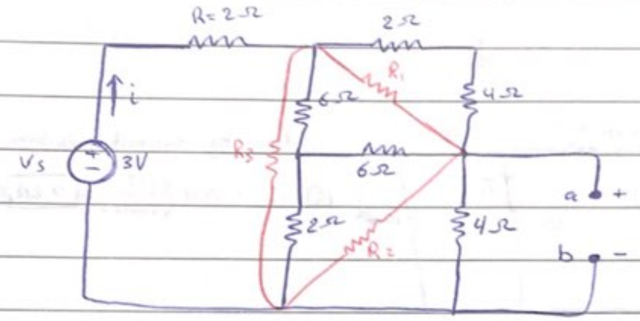
P delivered by the source:

$P = -VI = -(60)(12m) = -720$ mWatt.

⑥ If $V_s = 3V$ and $R = 2\,\Omega$, then answer the following questions:

a. The value of i

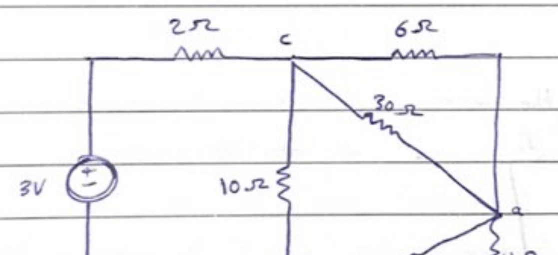
b. If the voltage source is replaced with a short circuit, then the equivalent resistance seen from a and b in (ohms) is:



$$R_1 = \frac{(6)(6) + (6)(2) + (2)(6)}{2} = \frac{60}{2} = 30\,\Omega$$

$$R_2 = \frac{(6)(6) + (6)(2) + (2)(6)}{6} = \frac{60}{6} = 10\,\Omega$$

$$R_3 = \frac{(6)(6) + (6)(2) + (2)(6)}{6} = \frac{60}{6} = 10\,\Omega$$



$(6, 30) \longrightarrow$ parallel

$$\frac{(6)(30)}{36} = 5\,\Omega$$

$(4, 10) \longrightarrow$ parallel

$$\frac{(4)(10)}{14} = 3\,\Omega$$

$(5, 3) \longrightarrow$ series

$5 + 3 = 8\,\Omega$

$(8, 10) \longrightarrow$ parallel

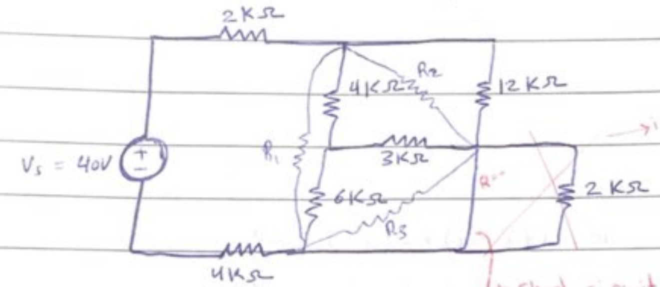
$$\frac{(8)(10)}{18} = 4\,\Omega$$

$(4, 2) \longrightarrow$ series

$4 + 2 = 6\,\Omega$ R_{eq}

$$i = \frac{V}{R} = \frac{3}{6} = 0.5A$$

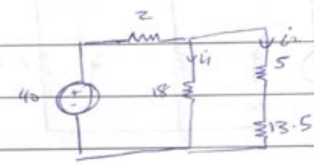
(7) If $V_s = 40\,V$ find the voltage across $R = 12\,K\Omega$ ~~9V??~~ 6.7volts



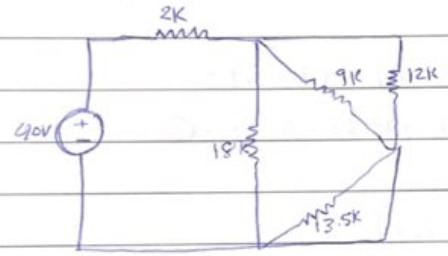
→ ignore all that part because the short circuit #

↳ short circuit

$$R_1 = \frac{(4)(3) + (4)(6) + (6)(3)}{3} = \frac{54}{3} = 18\,K\Omega$$



$$R_2 = \frac{54}{6} = 9\,K\Omega \qquad R_3 = \frac{54}{4} = 13.5\,K\Omega$$



$$12 \,//\, 9 \Rightarrow \frac{(12)(9)}{21} = 5\,K\Omega$$

$$5 + 13.5 = 18.5\,K\Omega$$

$$18.5 \,//\, 18 \Rightarrow \frac{(18.5)(18)}{36.5} = 9\,K\Omega$$

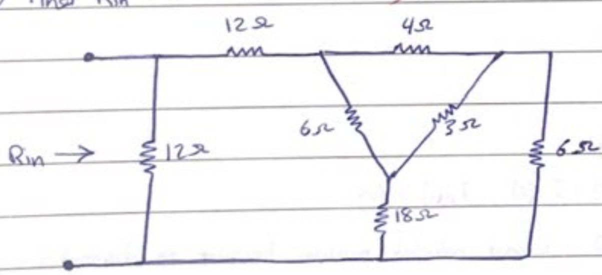
$$9 + 2 = 11\,K\Omega$$

$$I_{in} = \frac{V}{R} = \frac{40}{11} = 3.6\,A$$

$$i_2 = \left(\frac{18}{36.5}\right) 3.6 = 1.78\,A$$

$$V = (5)(1.78) = 8.9 \text{ Volts.}$$

(8) Find R_{in} = ? ~~6.67??~~ 7.3Ω

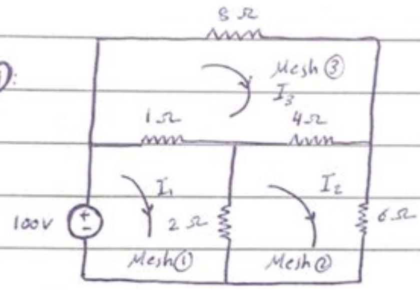


Chapter 3:

* Mesh analysis:

- Mesh loop
- Mesh current ⇒ clockwise

Ex ①:



Mesh ①:

$-100 + 1(I_1 - I_3) + 2(I_1 - I_2) = 0$

$100 = 3I_1 - 2I_2 - I_3 \quad \cdots ①$

Mesh ②:

$4(I_2 - I_3) + 6I_2 + 2(I_2 - I_1) = 0$

$-2I_1 + 12I_2 - 4I_3 = 0 \quad \cdots ②$

Mesh ③:

$8I_3 + 4(I_3 - I_2) + 1(I_3 - I_1) = 0$

$-I_1 - 4I_2 + 13I_3 = 0 \quad \cdots ③$

Mesh ①:

مجموع مقاومات mesh①   المقاومة التي بين ① و ②

$+ I_1(3) - I_2(2) - I_3(1) = 100$ OR

مع تيار mesh①   عكس تيار mesh② mesh③   المقاومة التي بين ① و ③

Mesh ②:

$+ I_2(12) - I_1(2) - I_3(4) = 0$

Mesh ③:

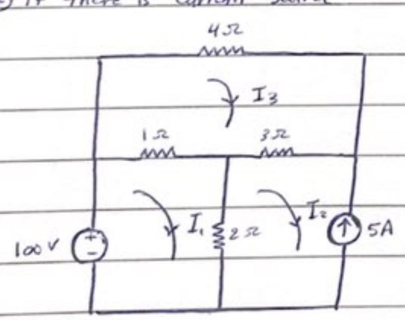
$+ I_3(13) - I_1(1) - I_2(4) = 0$

* $V_x = -I_x(6)$

$= -I_1(6)$

* $P_{2\Omega} = I^2(2) = (I_1 - I_2)^2(2)$

* If there is current source:



Mesh ①:

$+ I_1(3) - I_2(2) - I_3(1) = 100$

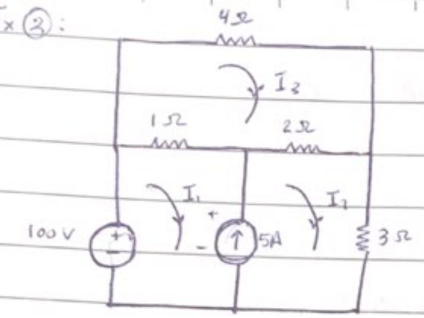
Mesh ②: Cannot calculate mesh② because the current source

$I_2 = -5A$ ⇒ (c.s عكس).

Mesh ③:

$+ I_3(8) - I_2(3) - I_1(1) = 0$

Ex ②:



Cannot calculate from Mesh ① and Mesh ②
because the current source

$5 = I_2 - I_1$

Mesh③:

$+ I_3(7) - I_2(2) - I_1(1) = 0$

OR (outer loop)

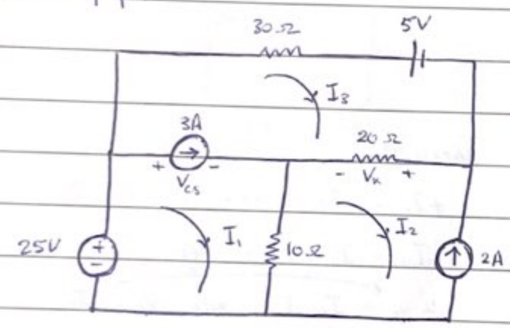
$-100 + 1(I_1 - I_3) + 2(I_2 - I_3) + 3I_2 = 0$

$-100 + 4I_3 + 3I_2 = 0$

$P_{cs} = \pm V.I = -5V_x$

$\hookrightarrow -100 + 1(I_1 - I_3) + V_x = 0$

⊛ Past paper:



$I_2 = -2A$

$3 = I_1 - I_3$

$-25 + 30I_3 + 5 + 20(I_3 - I_2) + 10(I_2 - I_2) = 0$

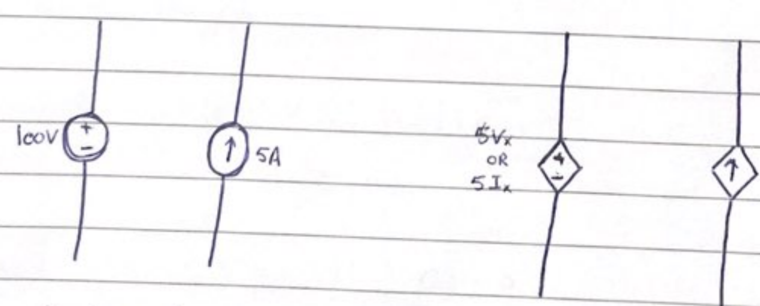
$\hookrightarrow I_3 = \frac{-70}{60} = \frac{-7}{6} \Rightarrow I_1 = 3 - \frac{7}{6}$

$V_x = RI = -(20)(I_x) = -(20)(I_2 - I_3)$

$P_{3A(cs)} = +3(V_{cs})$

$\xrightarrow{KVL} -25 + V_{c.s} + 10(I_1 - I_2) = 0$

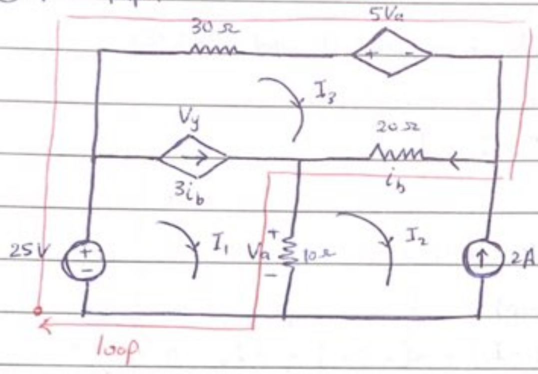
⊛ Dependent sources:



(Independent sources)

(dependent sources).

⊛ Past paper:



$I_2 = -2\,A \quad \cdots ①$

$3i_b = I_1 - I_2$

$\boxed{i_b = I_3 - I_2}$

$3(I_3 - I_2) = I_1 - I_3$

$4I_3 - 3I_2 - I_1 = 0$

$4I_3 - I_1 = -6 \quad \cdots ②$

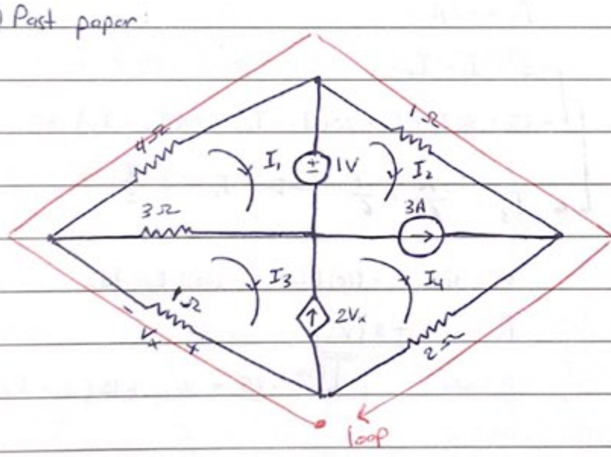
$\hookrightarrow -25 + 30I_3 + 5V_a + 20(I_3 - I_2) + 10(I_1 - I_2) = 0$

$V_a = +(I_1 - I_2)\,10$

$P = +V_y\,(3i_b)$

$i_b = I_3 - I_2$

⊛ Past paper:



Mesh ①:

$7I_1 - 3I_3 = -1 \quad \cdots ①$

$I_4 - I_2 = 3 \quad \cdots ②$

$2V_x = I_4 - I_3 \quad \Rightarrow \quad V_x = +I_3$

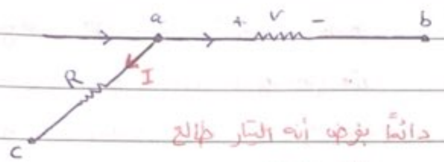
$2I_3 = I_4 - I_3$

$I_4 - 3I_3 = 0 \quad \cdots ③$

$I_3 - 4I_1 + I_2 + 2I_4 = 0 \quad \cdots ④$

(*) Nodal method $\Rightarrow$ No. of nodes $\Rightarrow$ KCL $(\sum I_{in} = \sum I_{out})$ $\Rightarrow$ نكتب بدلالة ال voltage

+ لازم أحدد Reference node $[V_{ref} = Zero]$.

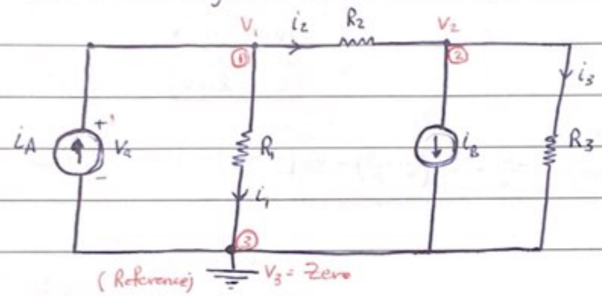


$V_{ac} = V_a - V_c = +IR$

دائماً بنفرض إنه التيار طالع من ال node.

$I = \frac{V_a - V_c}{R} = G(V_a - V_c)$

Example: Apply nodal analysis to write nodal equations:



لو بدي أحسب (i_2) بعد ما أجيب V_2 , V_1

$i_2 = \frac{V_1 - V_2}{R_2}$

$P_{i_A} = \pm VI = -V_A I_A$

$V_1 - V_3$

$i_3 = \frac{V_2 - \cancel{V_3}^{0}}{R_3} = \frac{V_2}{R_3}$

@ node 1:

$$\underbrace{\frac{V_1 - V_3}{R_1} + \frac{V_1 - V_2}{R_2}}_{I_{out}} = \underbrace{i_A}_{i_{in}} \Rightarrow V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) - \cancel{V_3\left(\frac{1}{R_1}\right)}^{Zero} = i_A \quad \cdots (1)$$

وحادلتين بمجهولين بطلع V_1 و V_2

@ node 2:

$$\frac{V_2 - V_1}{R_2} + \frac{V_2 - \cancel{V_3}^{Zero}}{R_3} + i_B = 0 \Rightarrow V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{V_1}{R_2} + i_B = 0 \quad \cdots (2)$$

OR

by inspection rule:

@ node 1: $+V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) - \cancel{V_3\left(\frac{1}{R_1}\right)}^{Zero} = i_A \rightarrow in$

نجمع مقلوب المقاومات الي شابكة على node(1)

مقلوب المقاومة الي بين (1) و (2)

مقلوب المقاومة الي بين (1) و (3)

(التيارات الداخلة ع النقطة)

لازم عند node(1)

لازم دش عند node(1)

@ node 2: $i_B + V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{R_2}\right) - \cancel{V_3\left(\frac{1}{R_3}\right)}^{Zero} = 0$

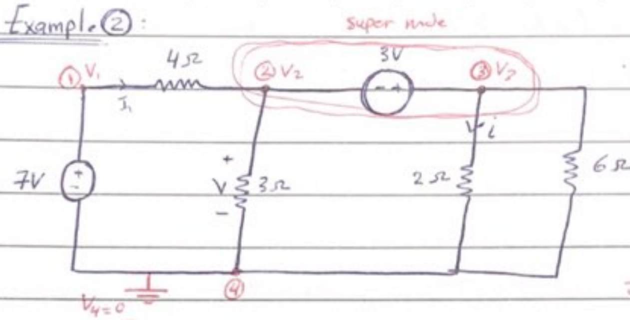
* هاي الطريقة باستخدمها باعتبار إنه جميع التيارات طالعة من ال node.

زي في ال mesh فرضنا إنه التيار مع عقارب الساعة.

وهون فقط بكون عندي current source

أما في ال mesh بطبقها لما يكون عندي voltage source

Example ②:



لا نستطيع أحلها بال mesh
بس هون طلب بال nodal .

* الـ (super node) بستخدمها لما يكون في عندي (voltage source) فإحنا ما بقدر أطبق الـ nodal فبنفرض انه تيار وزي كأنه ال(voltage source) مش موجود ونحل عادي (nodal) وبعتبر V_2 و V_3 node وحدة بسميها super node

أنا زرعتها ↗

@ node 1:

$V_1 - V_4 = 7 \Rightarrow V_1 - 0 = 7 \Rightarrow \boxed{V_1 = 7V}$

$V_3 - V_2 = 3 \quad \cdots ①$

$\boxed{V_2 = -0.2V}$

$\boxed{V_3 = 2.8V}$

@ node 2:

$-V_1\left(\frac{1}{4}\right) + V_2\left(\frac{1}{4}+\frac{1}{3}\right) + i_v = \text{Zero}$

$\Rightarrow \frac{-V_1}{4} + V_2\left(\frac{1}{4}+\frac{1}{3}\right) + V_3\left(\frac{1}{2}+\frac{1}{6}\right) = \text{Zero} \quad \cdots ②$

@ node 3:

$i_v = +V_3\left(\frac{1}{2}+\frac{1}{6}\right)$

$I_1 = \frac{V_1 - V_2}{R} = \frac{7+0.2}{4} = 1.8A$

* $V = V_2 - V_4 = -0.2 - 0 = -0.2V$
* $i = \frac{V_3 - V_4}{R} = \frac{2.8-0}{2} = 1.4A$

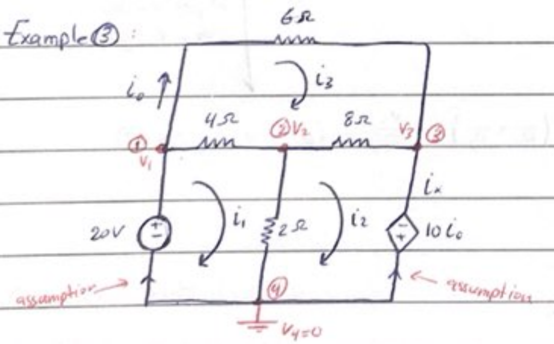
* $P_{7V} = \pm VI = \pm 7I = -7I_1 = -7(1.8) = -12.6$ watt (generator)
* $P_{3V} = -I_v(3) = -(1.9)(3) = 5.7$ watt

I_v by nodal (KCL)

i_v @ node 2 $\Rightarrow i_v + V_2\left(\frac{1}{4}+\frac{1}{3}\right) - \frac{V_1}{4} = 0$

$i_v = 1.9A$

Example ③:



@ node 2:

$-V_1\left(\frac{1}{2}\right) + V_2\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{2}\right) - V_3\left(\frac{1}{8}\right) = \text{Zero}$

$-\frac{V_1}{2} + V_2\left(\frac{7}{8}\right) - \frac{V_3}{8} = \text{Zero} \quad \cdots \cancel{①}$

$V_1 - \underbrace{V_4}_{\text{Zero}} = 20 \Rightarrow \boxed{V_1 = 20V}$

بعد تعويض V_1 $\quad V_2\left(\frac{7}{8}\right) - \frac{V_3}{8} = 10 \quad \cdots ①$

$V_2\left(\frac{7}{8}\right) - \frac{50}{8} = 10$

$\boxed{V_2 = 18.6V}$

$10i_o = \underbrace{V_4}_{\text{Zero}} - V_3 \Rightarrow -V_3 = 10i_o$

$i_o = \frac{V_1 - V_3}{R} = \frac{20 - V_3}{6}$

بعد تعويض i_o $\quad -V_3 = 10\left(\frac{20-V_3}{6}\right)$

$\boxed{V_3 = 50V}$

$P_{\text{ind. source}} = -i_1(20) = (-4.65)(-20) = +93$ watt

KCL → $i_1 = V_1\left(\frac{1}{4}+\frac{1}{6}\right) - V_2\left(\frac{1}{4}\right) - V_3\left(\frac{1}{6}\right)$

$i_1 = -4.65$

$P_{\text{dep. source}} = +(-i_2)(10i_o)$

$= +i_x(10i_o) \Rightarrow i_o = \frac{V_1 - V_3}{6} = \frac{20-50}{6} = -5A$

$= (8.9)(10)(-5)$ $\quad i_x$ by KCL $= V_3\left(\frac{1}{8}+\frac{1}{6}\right) - V_2\left(\frac{1}{8}\right) - V_1\left(\frac{1}{6}\right)$

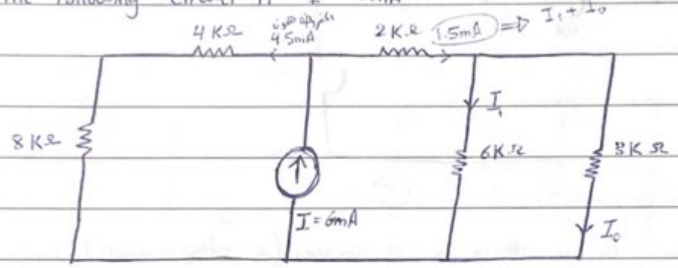
$= -445$ watt $\quad i_x = 8.925A$

* Superposition: $\Rightarrow$ ohm's law

$V = I\underset{\hookrightarrow \text{constant}}{R} \longrightarrow V$ and I is $\overset{\text{directly}}{\uparrow}$ proportional (linear)

- Just on voltage and current, cannot apply it to calculate the power because it is not directly proportional with I ($P = I^2R$).

Example: Use linearity and the assumption that $I_o = 1mA$ to compute the correct value of I_o in the following circuit if $I = 6mA$.



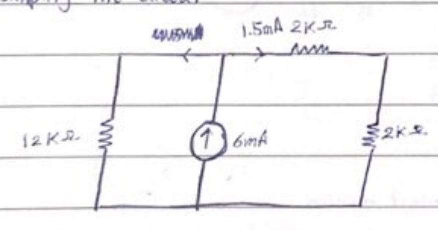
To prove the linearity JUST

$$\left.\begin{array}{l} R = 3K\Omega \\ I_o = 1mA \end{array}\right] \Rightarrow V_{3K\Omega} = RI = (3k)(1m) = 3V$$

$V_{6K\Omega} = V_{3K\Omega} = 3V \Rightarrow$ parallel

$I_{6K\Omega} = \frac{V}{R} = \frac{3}{6k} = 0.5mA = I_1$

* Simplify the circuit:



$$\left.\begin{array}{l} R = 4K\Omega \\ V = ? \\ I = 1.5mA \end{array}\right] \Rightarrow V = (4K)(1.5m) = 6V$$

$V_{12K\Omega} = 6V$

$I = \frac{V}{R} = \frac{6}{12K} = 0.5mA$

* المفروض إنه ال (6mA) تنقسم ثلثين بـ ثلث

لأنه عندي مقاومتين parallel وحدة 4KΩ

ووحدة 12KΩ فالنسبة بينهم $(\frac{1}{3})$.

وعشان ال assumption غلط

$6mA <$ $\frac{2}{3} \Rightarrow$ ال 4KΩ لازم يروح فيها 4mA

$\frac{1}{3} \Rightarrow$ وال 12KΩ لازم يروح فيها 2mA

القيمة الصح اللي المفروض تطلع

القيمة الغلط اللي طلعت بسبب ال assumption

$4mA \longrightarrow 1.5mA$

$?? \longrightarrow 1mA$

فقيمة المفروض بكون ال assumption

ال assumption الغلط

if $I_o = 1mA \Rightarrow I_2 = 1.5mA$ (wrong)

$I_o = ?? \Rightarrow I_2 = 4mA$

$x = \frac{4}{1.5} = \frac{8}{3}$

$\hookrightarrow I_o = \frac{8}{3}$

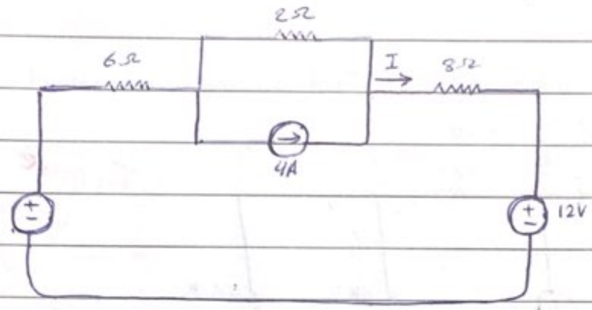
Superposition property:

- Turn off independent sources except one source

~~VS~~ ⟹ to kill the voltage source, replace it with a short circuit.

~~CS~~ ⟹ to kill the current source, replace it with an open circuit.

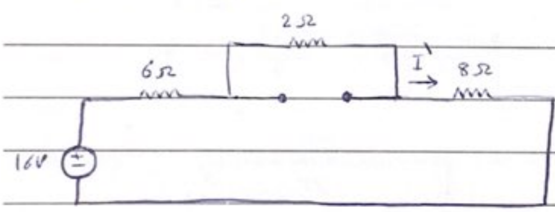
Example:



Find I ?

* we will solve it 3 times because there is 3 sources (2 voltage and 1 current)
  ↳ Everytime we will kill 2 of these sources and keep the third one

#1 (Keep 16V)

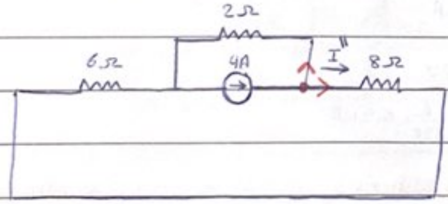


$R_{eq} = 16\,\Omega$

$V = 16\,V$

$I' = \frac{V}{R} = \frac{16}{16} = 1A$

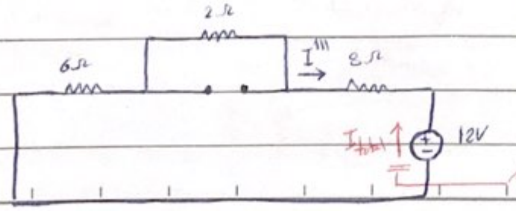
#2 (Keep 4A)



by current division:

$I'' = 4\left(\frac{2}{16}\right) = 0.5A$

#3 (Keep 12V)



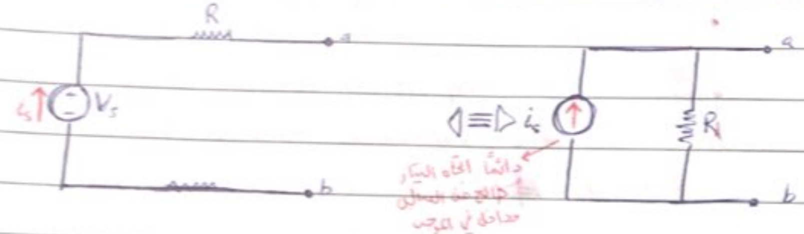
$R_{eq} = 16\,\Omega$

$I''' = \frac{-12}{16} = \frac{-3}{4} = -0.75A$

$I = 1 + 0.5 - 0.75 = \underline{0.75A}$

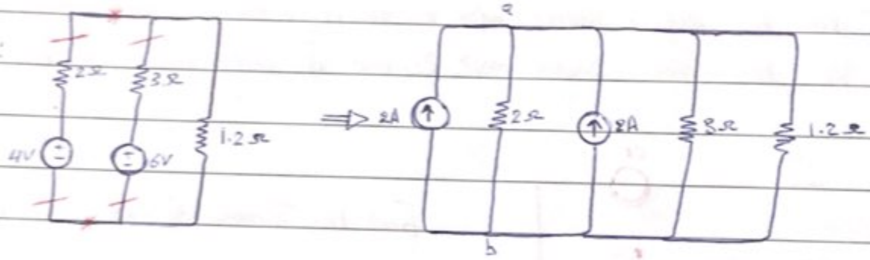
* Source transformation:

↳ refers to the process of replacing a voltage source in series with a resistor R with a current source in parallel with the same resistor and vice versa.



$$\left[\, i_s = \frac{V_s}{R} \,\right]$$

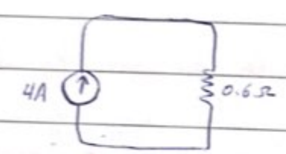
Example:



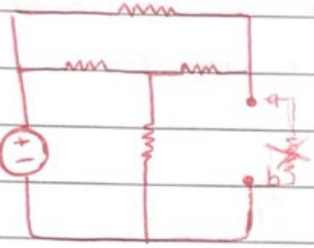
So,

$(2, 3, 1.2) \rightarrow$ parallel

$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{1.2} \Rightarrow R_{eq} = 0.6\,\Omega$ $\qquad I_{eq} = 2 + 2 = 4A$ (going in the same node)



* Thevinin Eq. CKt:



Find V_{th} seen from a b:

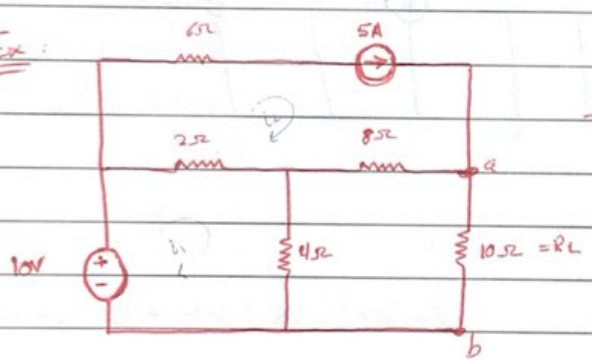
$\Rightarrow$ Remove all elements between a and b and keep it as open circuit and find $V_{open\ circuit}$ which is V_{th}.

$R_{th} \Rightarrow R_{eq}$ seen from ab

* We have to kill all the sources (must be independent)

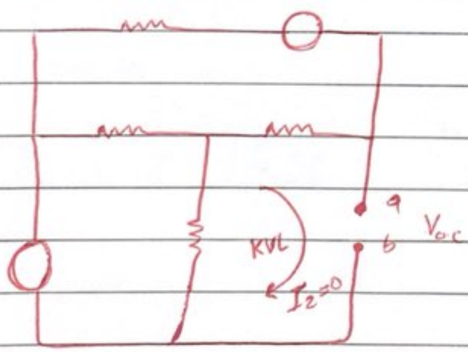
- To kill the voltage source $\Rightarrow$ Replace it with a short circuit
- To kill the current source $\Rightarrow$ Replace it with an open circuit.

Ex:



Find the Thevinin Eq CKt seen from ab:

* Remove R_L and replace it with an open circuit:

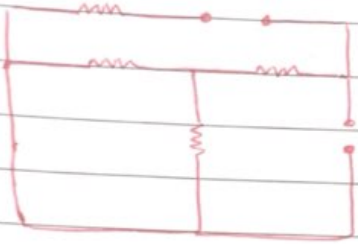


$10 = 6I_1 - 2(5) - 4(0) \Rightarrow I_1 = \frac{20}{6} A$

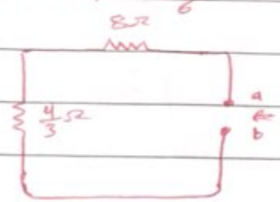
$8(0-5) + V_{o.c} + 4\left(I_2 - \frac{20}{6}\right) = 0$

$V_{oc} = 40 + \frac{80}{6} \cong 53$ Volt

* Remove the voltage and current sources:

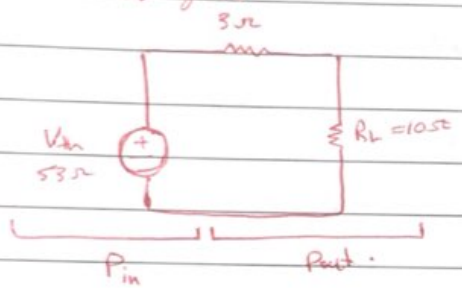


$2 \parallel 4 \Rightarrow \frac{(2)(4)}{6} = \frac{4}{3}\,\Omega$



$R_{th} = \left(8 + \frac{4}{3}\right)\,\Omega$

* Put R_L again:



$P_{RL} = I^2 R_L$

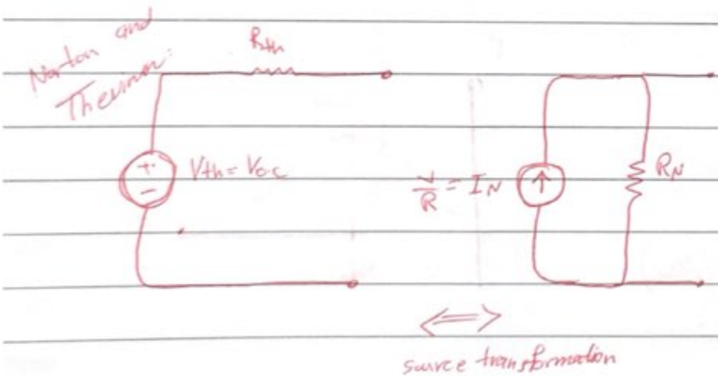
$\hookrightarrow I = \frac{53}{9.3+10}$

$P_{in} = -VI$

$P_{in} > P_{out} \Rightarrow$ loss of energy because of the 3Ω resistor

So, $P_{in} = P_{lost} + P_{RL}$

* efficiency $(\eta) = \frac{P_{out}}{P_{in}}$ always < 1



$R_n = R_{th}$ # $\Rightarrow \frac{V_{o.c}}{I_{s.c}} = \frac{V_{th}}{I_n}$

So, $I_n = \frac{V_{oc}}{R_{th}}$

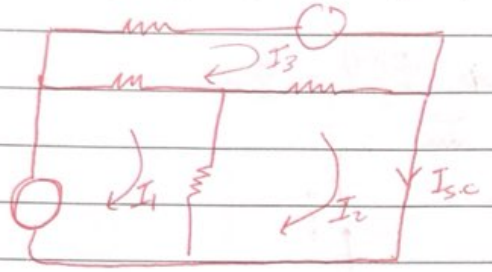
OR

Remove R_L and put short circuit then calculate the current in the short circuit which is $I_{sc} = I_N$

$\hookrightarrow$ If I don't calculate R_{th}.

~~For the same example~~

~~$I_N = V/R$~~



$$I_N = \frac{53}{9.3}$$

by (Mesh)

$$I_{s.c} = I_2$$

$$I_3 = 5$$

2// $10 = 6I_1 - 4I_2 - 2(5)$

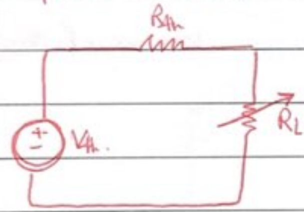
3// $0 = -4I_1 + 12I_2 - 8(5)$

=

$$20 = 28I_2 - 20 - 120$$

$$I_2 = \frac{160}{28}$$

* Maximum power transfer:



$R_L = ??$

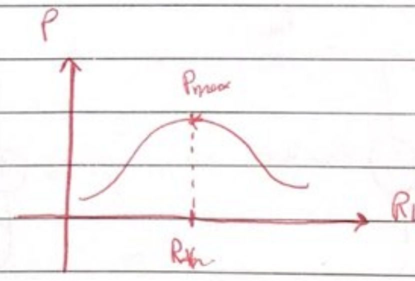
① $P_{R_L} = Max$

② $P_{transfered}$ from the source Max

③ P_{loss} Min

$$P_{R_L} = I^2 R_L$$

$$(f(x))\ \underset{\text{Max}}{\underset{\downarrow}{P_{R_L}}} = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 * R_L$$



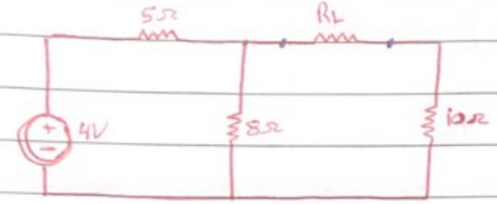
$f'(x) = 0 \Rightarrow$ τ find x (Max)

$R_L = R_{th}$

P_{R_L} Max

$$P_{Max} = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_{th} = \frac{V_{th}^2}{4R_{th}^2} R_{th} = \frac{V_{th}^2}{4R_{th}}\ \#$$

* Past paper:



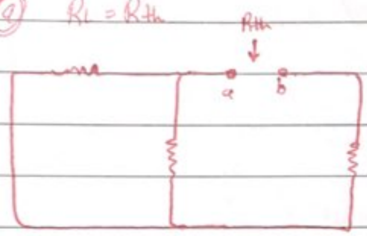
(a) Select a value for R_L to absorb Max power from the ckt

(b) V_{th}

(c) P_{max}

(d) $R_{L2} = 10\Omega \Rightarrow P_{R_L}$

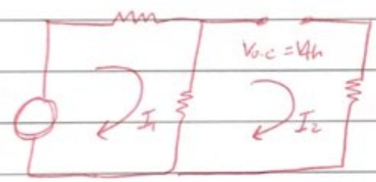
(a) $R_L = R_{th}$



$5 // 8 \Rightarrow \frac{(5)(8)}{13} = \frac{40}{13}$

$\underline{R_{th}} = 10 + \frac{40}{13} = 13$

(b) Put the voltage again

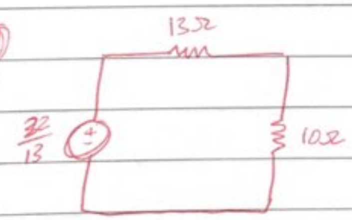


$4 = 13 I_1 \Rightarrow I_1 = \frac{4}{13} A$

$+V_{oc} - 8\left(\frac{4}{13}\right) = 0 \Rightarrow V_{oc} = \frac{32}{13}$

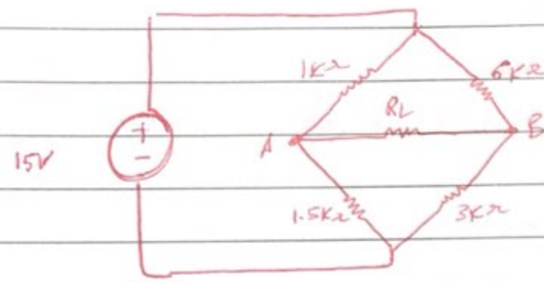
(c) $P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{(32/13)^2}{4(13)} = \frac{(32)(8)}{(13)(13)}$ watt.

(d)



$P_{R_L} = I^2 R = \left(\frac{\frac{32}{13}}{13+10}\right)^2 10$

Past paper:



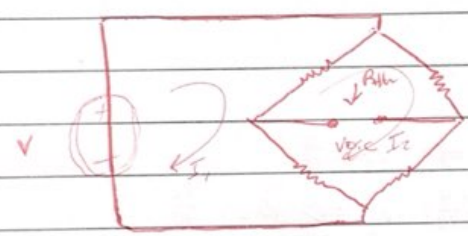
ⓐ $R_L \Rightarrow$ Max power

ⓑ P_{max}

ⓒ If the voltage source is replaced with a current source (15A)

Repeat a, b.

ⓐ $R_L = R_{th}$



$(1, 1.5) \rightarrow$ parallel

$\frac{(1)(1.5)}{2.5} = 0.6\ K\Omega$

$(6, 3) \rightarrow$ parallel

$\frac{(6)(3)}{9} = 2K\Omega$

$(0.6, 2) \rightarrow$ series

$R_{th} = 0.6 + 2 = 2.6 K\Omega$

ⓑ $P_{max} = \frac{V_{th}^2}{4R_{th}} =$

V_{th} = by mesh

voltage source

$15 = (2.5K)(I_1) - (2.5K)(I_2)$ ... ①

$0 = (-2.5K)(I_1) + (11.5K)(I_2)$ ... ②

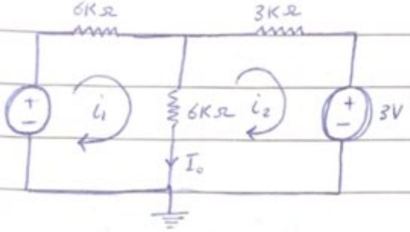
$15 = 9K I_2 \Rightarrow I_2 = \frac{15}{9} = \frac{5}{3} mA$

$6K I_2 - V_{oc} + 1K(I_2 - I_1) = 0 \Rightarrow V_{oc} = V_{th}$

$P_{max} = \frac{V_{th}^2}{4R_{th}}$

Solving problems (Mesh and Nodal).

Ex ①:



$i_1(6K+6K) - i_2(6K) - 12 = 0$

$12K\,i_1 - 6K\,i_2 = 12 \quad \cdots ①$

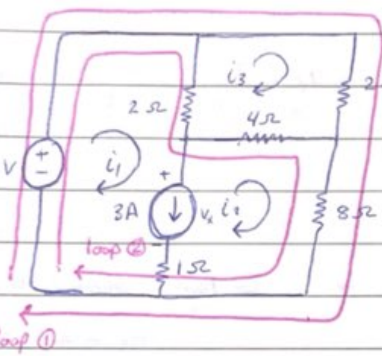
$i_2(6K+3K) - i_1(6K) + 3 = 0$

$9K\,i_2 - 6K\,i_1 = -3 \quad \cdots ②$

$\Rightarrow i_1 = 1.25mA$, $i_2 = 0.5mA$

$I_o = i_1 - i_2 = 1.25 - 0.5 = 0.75mA$

Ex ②:



$i_3(2+4+2) - i_2(4) - i_1(2) = 0$ (Mesh 3)

$8i_3 - 4i_2 - 2i_1 = 0 \quad \cdots ①$

$-6 + 2i_3 + 8i_2 = 0 \quad \cdots ②$ (by supermesh loop 1)

OR

$-6 + 2(i_2 - i_3) + 4(i_2 - i_3) + 8i_2 = 0 \quad \cdots ②$ (by loop 2)

$3 = i_1 - i_2 \quad \cdots ③$

$i_1 = 1.10A \quad i_2 = 0.47A \quad i_3 = 3.47A$

* Power consumed or delivered by current source (3A):

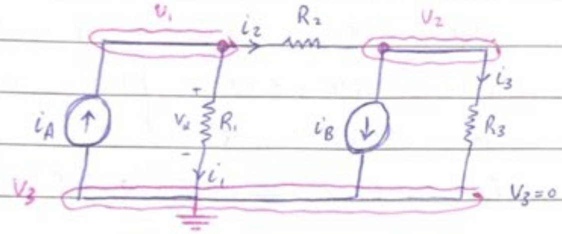
$P = +3V_x$

$\xrightarrow[\text{by KVL}]{} \; 4(i_2 - i_3) + 8i_2 + 1(i_2 - i_1) - V_x = 0$

$V_x = -8.87$ Volts.

$P = 3(-8.87) = -26.61$ watt.

Ex ③:



at V_1: $i_1 = \frac{V_1 - V_3}{R_1}$ $\quad i_2 = \frac{V_1 - V_2}{R_2}$ $\quad i_A = i_1 + i_2$

$$i_A = \frac{V_1 - V_3}{R_1} + \frac{V_1 - V_2}{R_2} \quad \cdots ①$$

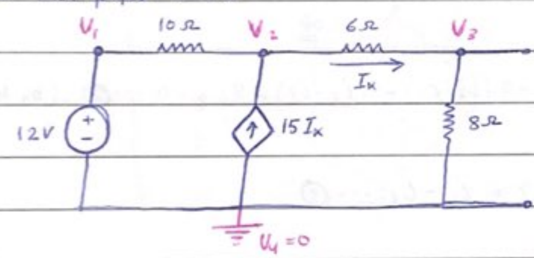
at V_2: $i_2 = \frac{V_2 - V_1}{R_2}$ $\quad i_3 = \frac{V_2 - V_3}{R_3}$ $\quad i_B = -i_3 - i_2$

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) - i_A = 0 \quad \cdots ①$$

OR $\quad i_B = \frac{V_3 - V_2}{R_3} + \frac{V_1 - V_2}{R_2} \quad \cdots ②$

$$-\underbrace{\cancel{V_3\left(\frac{1}{R_3}\right)}}_{\text{zero}} + V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{R_2}\right) + i_B = 0 \quad \cdots ②$$

* Past paper (2019):



Find: ① I_x $\quad$ ② Power delivered/absorbed by the dependent source

Can solve it by Mesh analysis → straightforward

by Nodal:

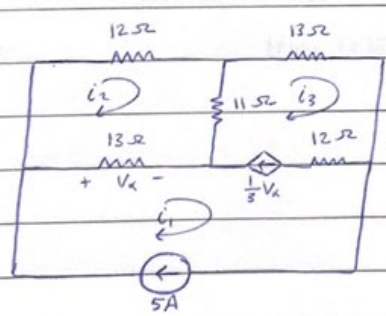
① $\boxed{V_1 = 12V}$

$$V_2\left(\frac{1}{10} + \frac{1}{6}\right) - V_3\left(\frac{1}{6}\right) - V_1\left(\frac{1}{10}\right) - 15 I_x = 0 \qquad I_x = \frac{V_2 - V_3}{6} \quad \cdots ①$$

$$V_3\left(\frac{1}{6} + \frac{1}{8}\right) - V_2\left(\frac{1}{6}\right) = 0 \quad \cdots ②$$

② $P = -15 I_x (V_2 - 0)$

* Past paper:



Find: i_1, i_2, i_3 and V_x

by Mesh analysis:

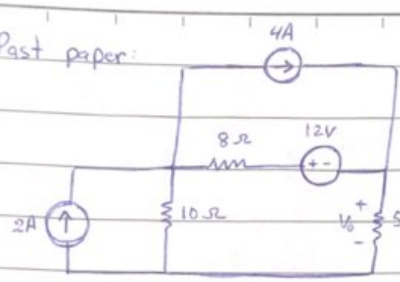
$i_1 = 5A$

$$-i_1(13) + i_2(12 + 11 + 13) - i_3(12) = 0 \quad \cdots ①$$

$$\frac{1}{3}V_x = i_3 - i_2 \quad \cdots ③$$

$\hookrightarrow V_x = 13(i_1 - i_2)$

* Past paper:



by superposition:

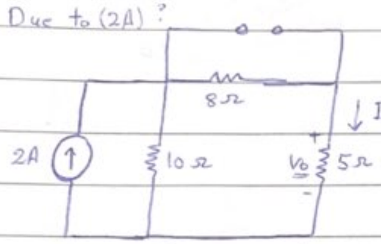
the component V_o due to 2A current source

↳ by killing all the sources except the (2A) current source.

* we can use the superposition to find V $(V'+V''+V''')$ and I $(I'+I''+I''')$ because they are linear but the power non-linear so we cannot use the superposition to find it by $(P'+P''+P''')$ ↳ wrong

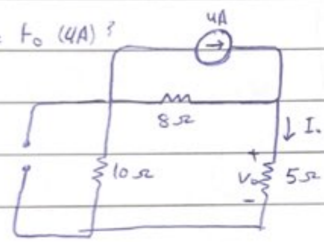
* DONT USE superposition if there is an independent source.

- Due to (2A)?



$I_o' = 2\left(\frac{10}{23}\right) \Rightarrow V_o' = 5I_o = \frac{100}{23}$

- Due to (4A)?

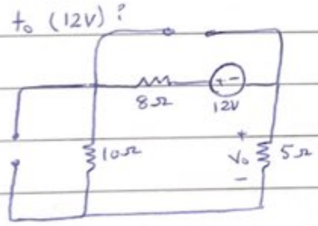


$I_o'' = \frac{-12}{23} * 5 \Rightarrow V_o'' = \frac{-60}{23}$

$V_{out} = V_o' + V_o'' + V_o'''$

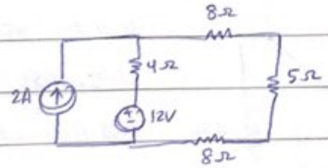
$P = \frac{V^2}{R} = I^2R.$

- Due to (12V)?

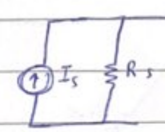


$V_o''' = \left(\frac{4*8}{23}\right) * 5$

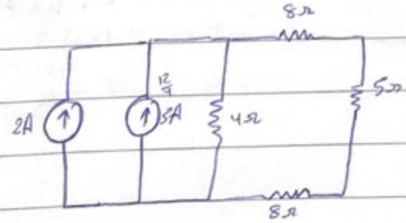
* Past paper:



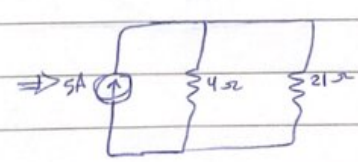
⟺ equivalent to



Find: I_s and R_s.



$\Rightarrow$

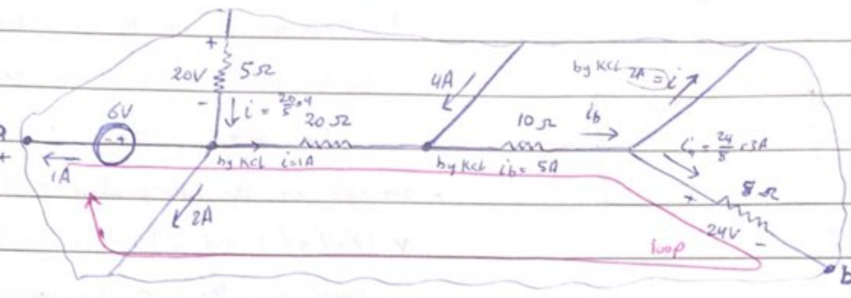


$(4//21) \longrightarrow$ parallel

$R_s = \frac{(4)(21)}{25} = 33.6Ω$

$I_s = 5A.$

* Find $V_{ab} = ?$ $V_a - V_b$

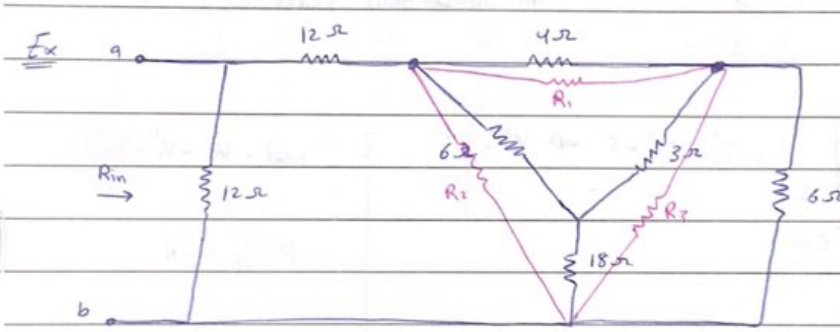


$-6 + 20 I_{20} + 10 i_b + 20 = V_{ab}$

$-6 + 20(1) + 10(5) + 20 = V_{ab}$

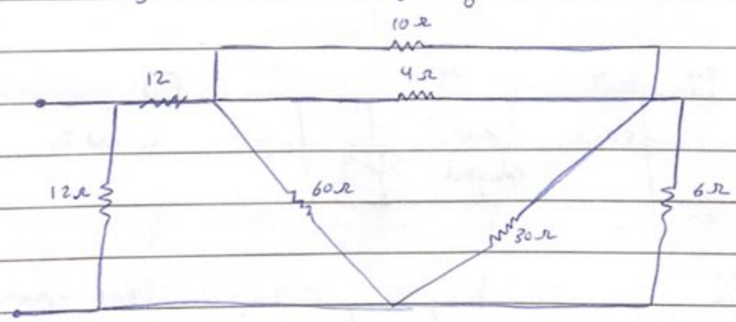
$-6 + 20 + 50 + 20 = V_{ab} \Rightarrow V_{ab} = 84$ Volts.

Ex



$$R_1 = \frac{(3)(18) + (3)(6) + (18)(6)}{18} = \frac{180}{18} = 10\,\Omega$$

$$R_2 = \frac{180}{3} = 60\,\Omega \qquad R_3 = \frac{180}{6} = 30\,\Omega$$



$6 \parallel 30 \Rightarrow \frac{(6)(30)}{36} = 5\,\Omega$

$4 \parallel 10 \Rightarrow \frac{(4)(10)}{14} = 2.86\,\Omega$

$5 + 2.86 = 7.86\,\Omega$

$7.86 \parallel 60 \Rightarrow \frac{(7.86)(60)}{67.86} = 6.95\,\Omega$

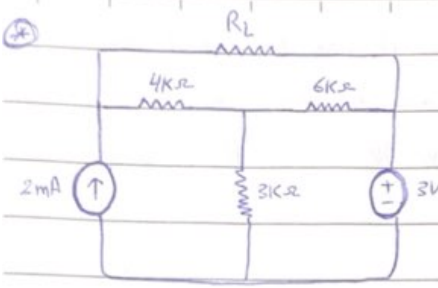
~~6.95 // 12 ⇒ (6.95)(12)/18.95 = 4.4~~

$6.95 + 12 = 18.95\,\Omega$

$18.95 \parallel 12 \Rightarrow \frac{(18.95)(12)}{30.95} = 7.35\,\Omega$

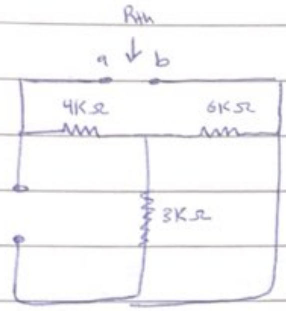
problems (Thevenin and Norton) .

(*)



Find the thevenin equivalent circuit seen by R_L ?

↳ Find R_{th} and V_{th}.

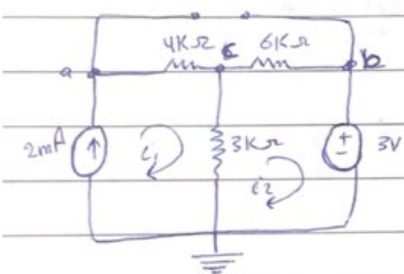


$6 \parallel 3 \rightarrow \frac{(6)(3)}{9} = 2K\Omega$

$(2, 4) \rightarrow$ series

$2+4 = 6K\Omega$

$\therefore R_{th} = 6K\Omega$



by Nodal

$V_a\left(\frac{1}{4K}\right) - V_c\left(\frac{1}{4K}\right) = 2m$ . . . ①

$-V_a\left(\frac{1}{4K}\right) - V_b\left(\frac{1}{6K}\right) + V_c\left(\frac{1}{4K} + \frac{1}{3K} + \frac{1}{6K}\right) = 0$ . . . ②

$\boxed{V_b = 3V}$ # $V_{oc} = V_a - V_b$ #

by Mesh.

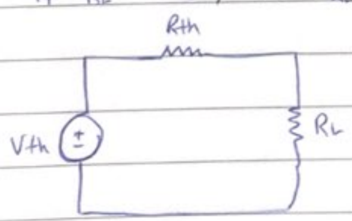
$-2m(3K) + 9K\,I_2 = -3$

$+V_{oc} - 6K\,I_2 - 4K\,I_1 = 0$

$P_{max} = \frac{V_{th}^2}{4R_{th}}$ # $\Rightarrow$ Just if $R_{th} = R_L$.

If not $P = I^2 R_L$.

If $R_L = 10\Omega$, Find $I_{R_L} = ?$

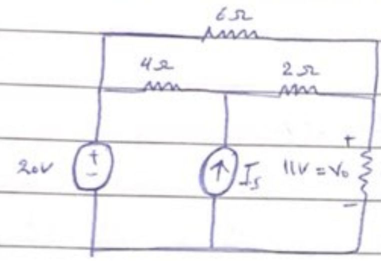


$I = \frac{V_{th}}{R_{th} + R_L}$

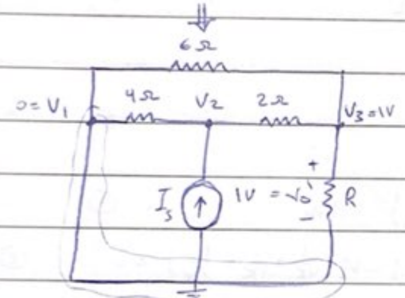
$P_{absorb \text{ by } R_L} = I^2 R_L$. not $\frac{V_{th}^2}{4R_{th}}$

$I_N = I_{sc} = \frac{V_{th}}{R_{th}}$.

⊛ Using superposition method for the circuit shown below, it was found that the total value of the voltage $V_o = 11\,V$, and when disconnecting the voltage source (i.e. replaced by short circuit) the voltage was found to be $V_o = 1V$. Hence, the value of the current source (I_s) would have to be:



⇓



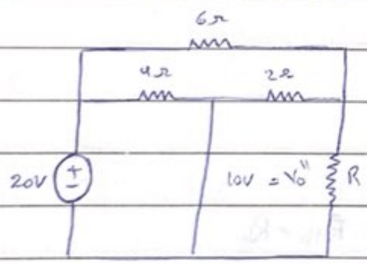
$$\cancel{V_1\left(\frac{1}{4}+\frac{1}{6}\right)} - V_2\left(\frac{1}{4}\right) - 1\left(\frac{1}{6}\right) = 0$$

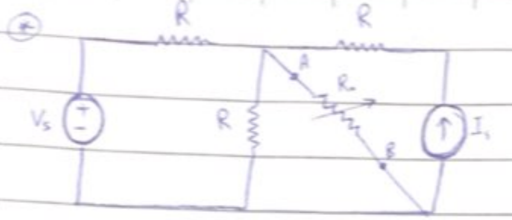
$$V_2 = -\frac{4}{6}\text{ Volts.}$$

$$\underline{\underline{V_2}}\left(\frac{1}{4}+\frac{1}{2}\right) \overset{zero}{\cancel{-V_1\left(\frac{1}{4}\right)}} - \underbrace{V_3}_{1V}\left(\frac{1}{2}\right) = I_s$$

⇓

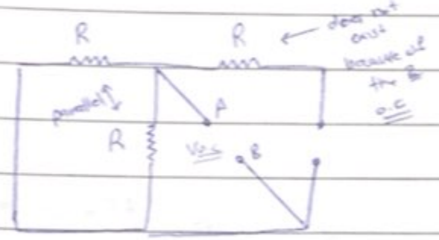
$V_o'' = V_o - V_o' = 11 - 1 = 10V$



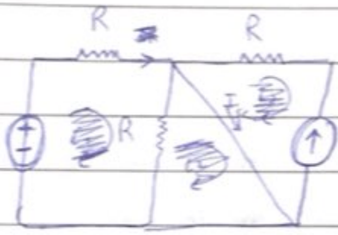


- The Norton resistance:
- The Norton current:
- The maximum power transferred to R_L

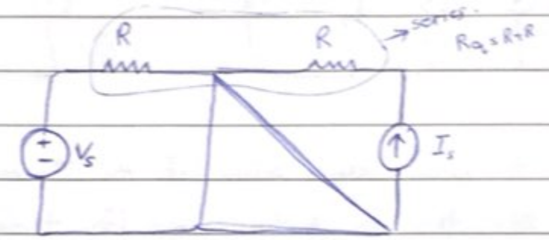
Seen from R_L

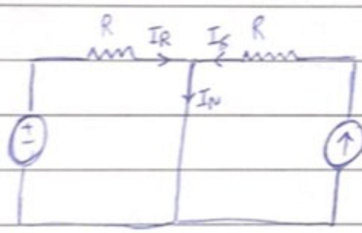


$$R_N = \frac{R}{2}$$



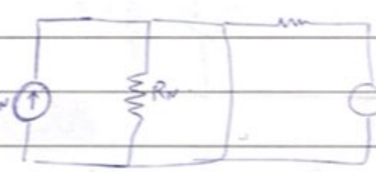
$\Rightarrow$





$\underline{I_N} = I_s + I_R$

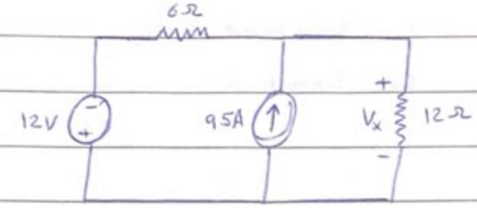
$\hookrightarrow I_R = \frac{V_s}{R}$



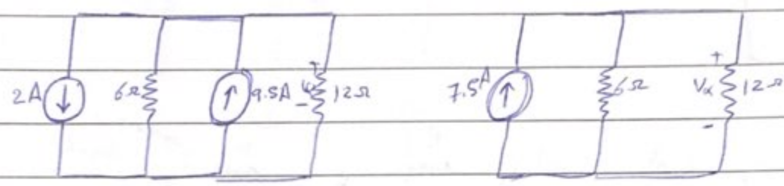
by source transformation

- $P_{max} = \frac{V_{th}^2}{4R_{th}^2} \rightarrow (V_{th} = R_{th} I_N)$

(*) Using the source transformation method, the value of the voltage V_x (in volt) in the following circuit:

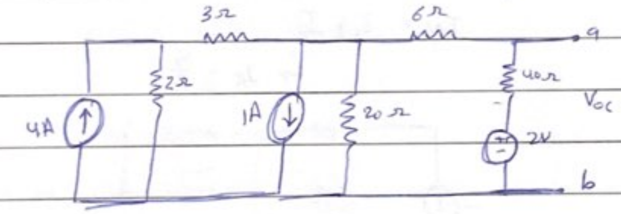


⇓

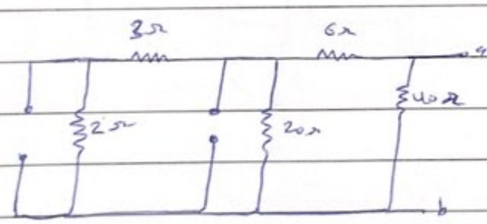


$V_x = \left(7.5 \times \left(\frac{6}{18}\right)\right) \times 12$

(*) For the circuit shown below, if the Thevenin voltage (V_{th}) across the terminals (a) and (b) is 24V, then the Norton current (I_N) through the terminals (a) and (b) is:



$V_{oc} = V_{th} = 24V$



$I_N = \frac{V_{th}}{R_{th}}$

$(2,3) \rightarrow$ series

$2+3 = 5\,\Omega$

$(5,20) \rightarrow$ parallel

$\frac{(5)(20)}{25} = 4\,\Omega$

$(4,6) \rightarrow$ series

$4+6 = 10\,\Omega$

$(10,40) \rightarrow$ parallel

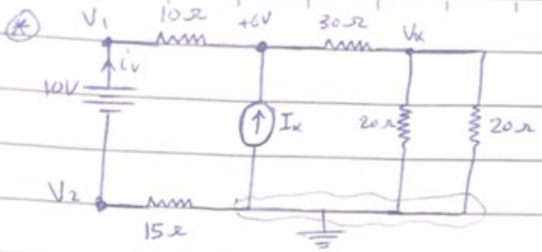
$\frac{(10)(40)}{50} = 8\,\Omega$

$R_{th} = 8\,\Omega$

$V_{th} = 24V$

$\Rightarrow I_N = \frac{24}{8} = 3A.$

(*)



Find I_x and V_x

$V_1 - V_2 = 10$

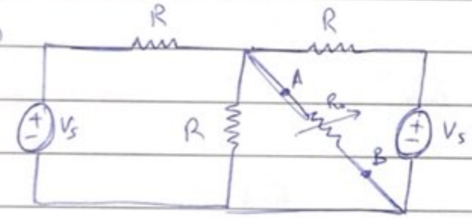
$-V_1\left(\frac{1}{10}\right) + 6\left(\frac{1}{10} + \frac{1}{30}\right) - V_x\left(\frac{1}{30}\right) = I_x$

$V_x\left(\frac{1}{30} + \frac{1}{20} + \frac{1}{20}\right) - 6\left(\frac{1}{30}\right) = 0$

Supernode: $i_v = V_1\left(\frac{1}{10}\right) - 6\left(\frac{1}{10}\right)$

$V_2\left(\frac{1}{15}\right) + i_v = 0 \Rightarrow i_v = \frac{-V_2}{15}$

(*)



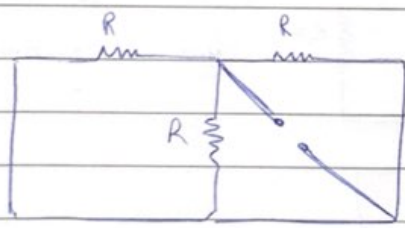
Find R_{th} and V_{th} ?

$V_s = 2R\,I_1 - R\,I_2$

$-V_s = -R\,I_1 - 2R\,I_2$

$\Rightarrow I_1 = \frac{3}{5R}V_s$

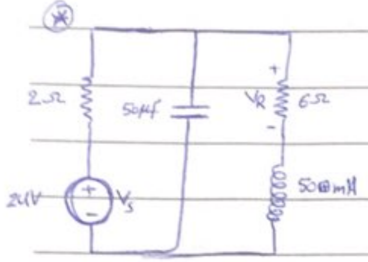
$I_2 = \dots$



$V_{oc} = R(I_1 - I_2) = V_{th}$.

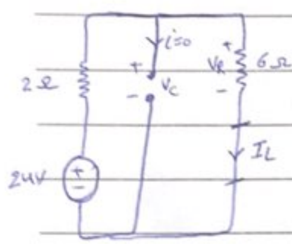
$R_{th} = \frac{R}{3}$

Problems (Conductors and Inductors).



find:

E_c , E_l , V_R under DC condition.



$V_s = I_L R_{eq}$

$24 = I_L (8) \Rightarrow I_L = \frac{24}{8} = 3A$

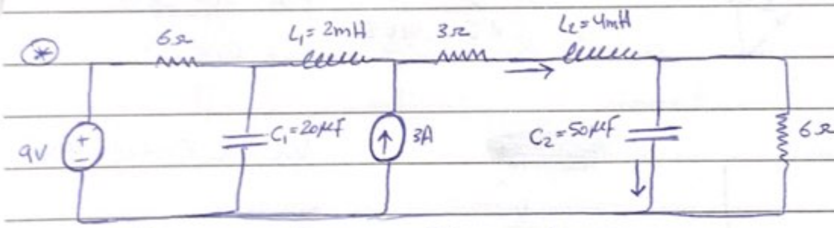
$V_R = I_L R$

$= (3)(6) = 18$ Volts.

$E_c = \frac{1}{2} C V_c^2 = \frac{1}{2}(50\mu)(18)^2$ Jouls.

by KVL $\hookrightarrow -V_c + IR = 0 \Rightarrow V_c = V_R = 18$ Volts.

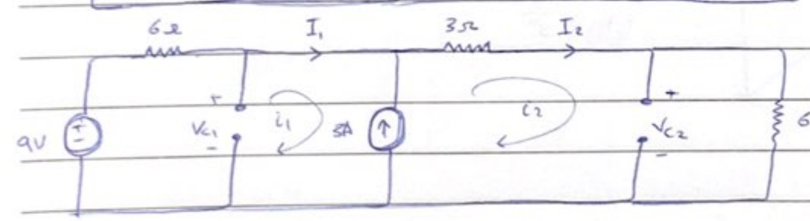
$E_L = \frac{1}{2} L i^2 = \frac{1}{2}(500m)(3)^2 =$ Jouls.



find:

E_{L_1} , E_{L_2} , E_{C_1} , E_{C_2}

under DC condition.



by super Mesh:

$9 = 6 I_1 + 9 I_2 \quad \cdots (1)$

$3 = -I_1 + I_2 \quad \cdots (2)$ $\Rightarrow$

$I_2 = 1.8 A$

$E_{L2} = \frac{1}{2}(4m)(1.8)^2$

$V_{C_2} = 6 I_2 = 6(1.8) = 10.8$

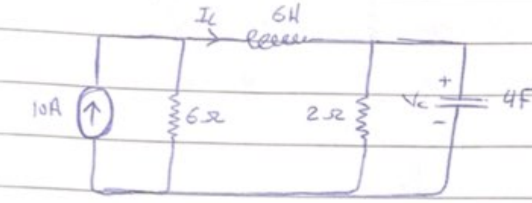
$E_{C_2} = \frac{1}{2}(50\mu)(10.8)^2$

$I_1 = -1.2A$

$E_{L2} = \frac{1}{2}(2m)(-1.2)^2$

$-9 + 6(-1.8) + V_{C_1} = 0$

㊟ Past paper:

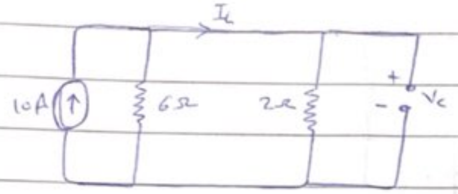


Find I_L and E_C:

@ DC conditions OR @ Steady state.

→ Cap. ⇒ O.P

Ind. ⇒ S.C

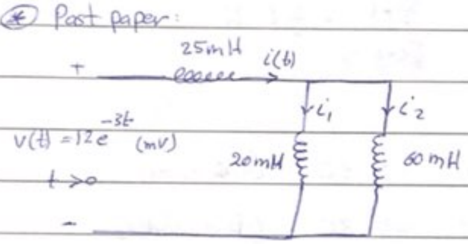


$$I_L = 10\left(\frac{6}{8}\right) = \frac{60}{8}$$

$$V_c = V_{2\Omega} = RI_L = (2)\left(\frac{60}{8}\right)$$

$$E_C = \frac{1}{2}CV^2 = \frac{1}{2}(4)(15)^2$$

㊟ Past paper:



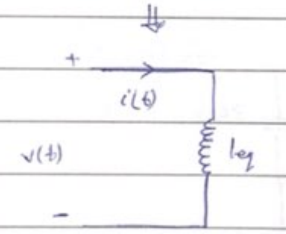
$i_1(0) = -30mA$ → إذا ما أعطاك إياها. فرضها صفر

Note that $i_1(0^-) = i_1(0^+)$

Find: $i_2(0)$, $i_1(0)$

$i_1(t)$, $i_2(t)$

⇓



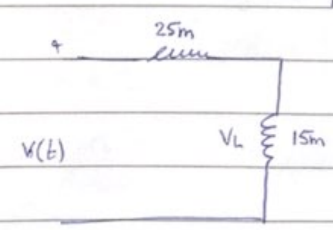
$$V_L = L\frac{di}{dt} \Rightarrow \int_0^t \frac{V_L}{L}dt = \int_0^t di$$

$$\frac{1}{L}\int_0^t V_L(t)\,dt = I(t) - I(t=0) \quad \#$$

$$L_{eq} = \frac{20 * 60}{80} = 15mH + 25mH = 40mH$$

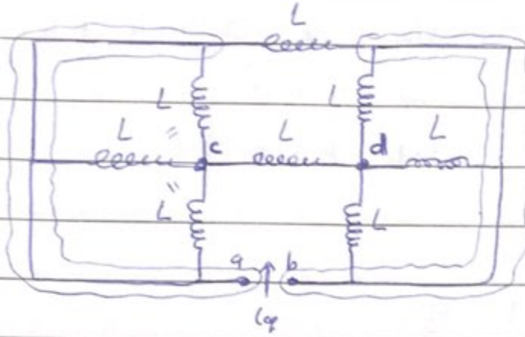
$$\frac{1}{40m}\int_0^t 12e^{-3t}dt + (-30m) = i(t)$$

$$i(0) = i_1(0) + i_2(0)$$



$$V_L(t) = L\frac{di}{dt}$$

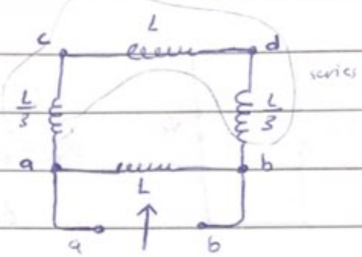
※ past paper



$L \parallel L \parallel L \Rightarrow \frac{L}{3}$ (a,c nodes)

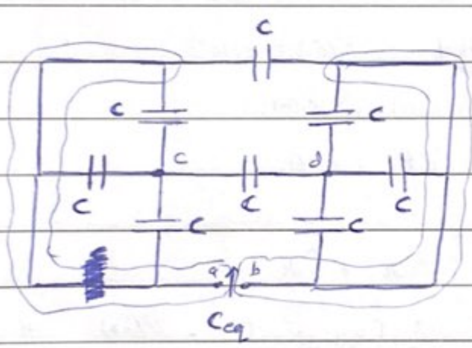
$L \parallel L \parallel L \Rightarrow \frac{L}{3}$ (b,d nodes)

$\Downarrow$



$\frac{L}{3} + L + \frac{L}{3} = \frac{5L}{3}$

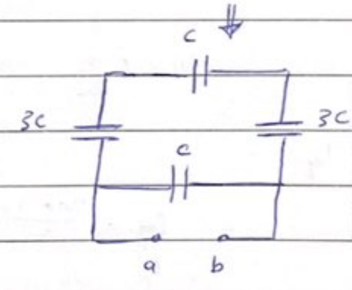
$\frac{5L}{3} \parallel L \Rightarrow \dfrac{\frac{5L}{3} \cdot L}{\frac{5}{3}L + L} = \frac{5}{8}L$ (H)



$C \parallel C \parallel C \Rightarrow 3C$ (a,c nodes)

$C \parallel C \parallel C \Rightarrow 3C$ (b,d nodes)

$\Downarrow$



$(3C, C, 3C) \rightarrow$ series

$\frac{1}{3C} + \frac{3}{3C} + \frac{1}{3C} = \frac{5}{3C} \Rightarrow \frac{3C}{5}$

$\frac{3C}{5} \parallel C \Rightarrow \frac{8}{5}C$

Transient.

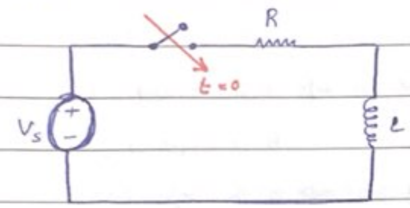
⇒ We use differential equations

1^{st} order ⇒ $a\dot{x} + bx + c = 0$

$\frac{dx}{dt}$ ↵ ↳ $x(t) = A\, e^{-t/\tau} + B$

↳ from the circuit

⊛ RL ckt :



$t < 0$ ⇒ switch is open.

$t \geq 0$ ⇒ switch is closed.

$i_L(0^-) = i_L(0^+)$ ⇒ Intial condition.

$$i(t) = \begin{cases} 0 & , t < 0 \Rightarrow i_L(0^-) = 0 \text{ because the switch is open.} \\ ?! & , t \geq 0 \end{cases}$$

↳ by KVL ⇒ $V_s = V_R + V_L$

$V_s = Ri + L\frac{di}{dt}$ ⇒ $\boxed{L\frac{di}{dt} + Ri - V_s = 0}$ diff. equation

$i(t) = A\, e^{-t/\tau} + B$

for RL ckt ⇒ $\tau = \frac{L_{eq}}{R_{th}}$ sec

R_{th} → Thevinin equavalent resistance seen from L

To find A and B:

① Intial Condition :

$i(0^-) = i(0^+) = 0$

when $t \geq 0$ $\boxed{A + B = 0}$

② Final Condition :

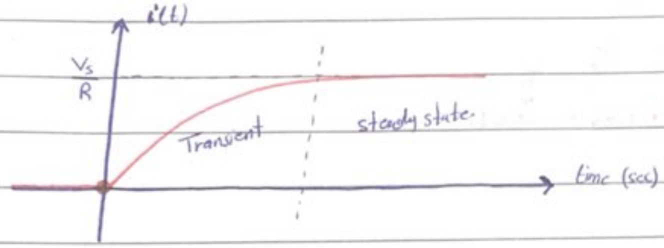
when $t \to \infty$ ⇒ steady state $(L \to s.c)$

$i(t \to \infty) = \frac{V_s}{R}$

$\frac{V_s}{R} = \cancel{A e^{-\infty}} + B$ (zero)

$\boxed{B = \frac{V_s}{R}}$

$$i(t) = \frac{-V_s}{R} * e^{-\frac{t}{\tau}} + \frac{V_s}{R} = \frac{V_s}{R}\left[1 - e^{-t/\tau}\right]$$

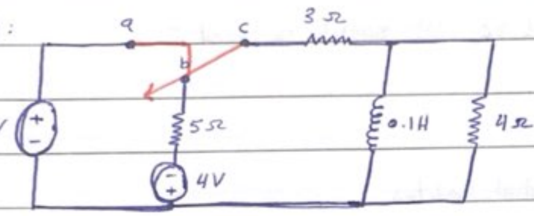


$$V_L = L\frac{di}{dt}$$
$$= L\left(\frac{-V_s}{R}\left(\frac{1}{-\tau}\right)e^{-t/\tau}\right)$$
$$V_L = V_s\, e^{-t/\tau}$$

@ $t = 0$

$V_L = V_s$

Ex ①:



$t < 0 \Rightarrow$ a, b closed

b, c opened

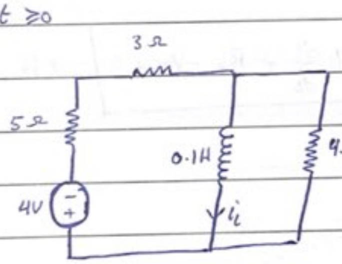
$t \geq 0 \Rightarrow$ a, b opened

b, c closed

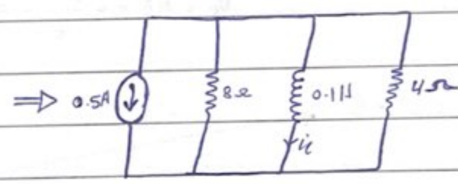
@ $t < 0 \Rightarrow$ Intial condition

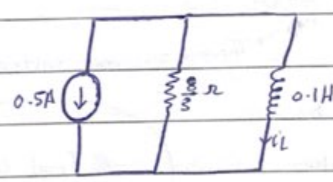
$i(0^-) = i(0^+) = 0$

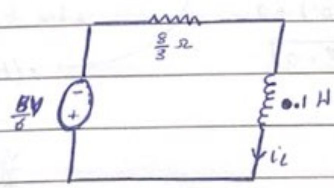
@ $t \geq 0$



$\Rightarrow$







by KVL $\Rightarrow \frac{8}{3} + \frac{8}{3}i + 0.1\frac{di}{dt} = 0$

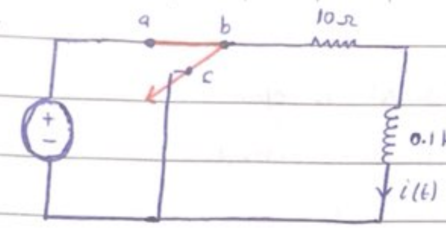
$i = Ae^{-t/\tau} + B \Rightarrow \tau = \frac{0.1}{\frac{8}{3}} = \frac{0.3}{8}$ sec

@ $t = 0 \rightarrow A + B = 0 \Rightarrow A = 0.5$

@ $t = \infty \rightarrow B = -0.5$

$$i = (0.5)e^{-t/0.3/8} + 0.5$$

Ex ②:



@ $t<0$ ⇒ a , b closed

@ $t \geq 0$ ⇒ a , b opened

b , c closed.

@ $t<0$ ⇒ steady state

$i(0^-) = \frac{10}{10} = 1A$

$i(0^-) = i(0^+) = 1$ ⇒ Intial condition.

@ $t \to \infty$ ⇒ steady state.

$i(\infty) = 0$ ⇒ final condition.

@ $t \geq 0$ ⇒ $10i + 0.1\frac{di}{dt} = 0$ ⇒ $Ae^{-t/\tau} + B = i(t)$

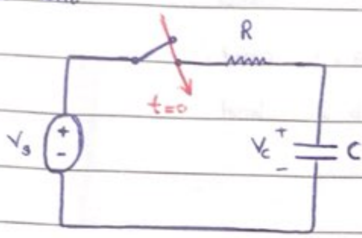
$\tau = \frac{0.1}{10} = 0.01\ sec$

Intial condition ⇒ $A+B=1$ ⇒ $A=1$

final condition ⇒ $B=0$

$i(t) = e^{-t/0.01}$

* RC Ckt :



$t < 0 \Rightarrow$ switch is open

$t \geq 0 \Rightarrow$ switch is closed

$\left[i_c = C \frac{dV}{dt} \right]$

$V_c(0^-) = V_c(0^+)$ $\Rightarrow$ Intial condition.

by $\underline{\underline{KVL}}$: $V_s = R i_c + V_c$

$V_s = RC \frac{dV}{dt} + V_c$

$\hookrightarrow V_c(t) = A e^{-t/\tau} + B \quad \Rightarrow \quad \tau = RC$ sec

* To find A & B :

① Intial Condition

$V_c(0^-) = V_c(0^+)$

@ $t = 0 \Rightarrow V_c(0) = A e^{-0/\tau} + B = 0$

$\boxed{A = -B}$

$A = -V_s$

② final Condition

when $(t \to \infty) \Rightarrow V_c(\infty) = V_s$

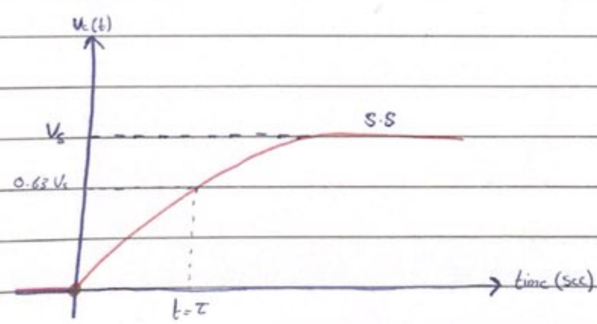
@ $t = \infty \Rightarrow V_c(\infty) = A \underbrace{e^{-\infty}}_{\text{zero}} + B = V_s$

$\boxed{B = V_s}$

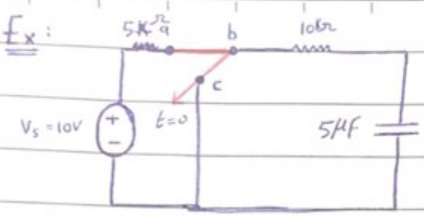
$\boxed{V_c(t) = -V_s e^{-t/\tau} + V_s}$

$i_R(t) = i_C(t) = C \frac{dV}{dt} = \frac{C V_s}{\tau} e^{-t/\tau}$

$V_c(t) = V_s (1 - e^{-t/\tau})$



Ex:



@ $t = 0 \Rightarrow$ a, b (open)

b, c (closed)

Find ⓐ $V_c(t)$ and ⓑ $i(t)$

ⓒ @ $t = 5$ → نفس الحل بس بنستبدل (t) بـ (t-5)

ⓐ by KVL $\Rightarrow$ $10 i_c + V_c = 0$ @ $t \geq 0$

$$10 * 5\mu * \frac{dV_c}{dt} + V_c = 0$$

$$\rightarrow V_c(t) = A e^{-t/\tau} + B \quad \#$$

@ $t \geq 0 \Rightarrow \tau = \frac{1}{RC} = \frac{1}{10^4 * 5\mu} = 20$ sec

* Intial condition

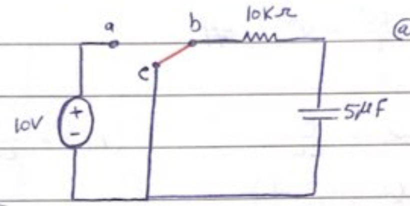
$V_c(0^-) = V_c(0^+) = 10$ Volts

@ $t = 0 \Rightarrow V_c(0) = A e^0 + B = 10$

$\boxed{A + B = 10}$

$A = 10$

* Final condition: $V_c(t \to \infty) = $ Zero

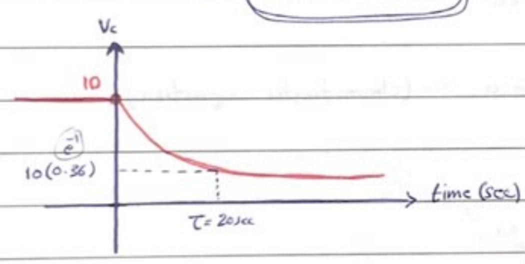


@ $t = \infty \Rightarrow$ zero

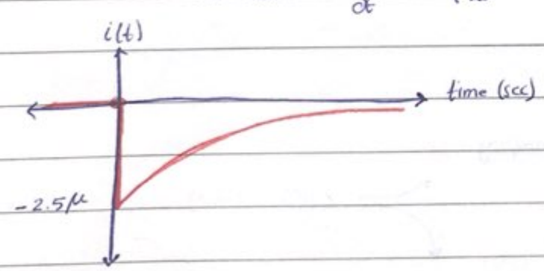
$V_c(\infty) = \cancel{A e^{-\infty}} + B = 0$

$\boxed{B = 0}$

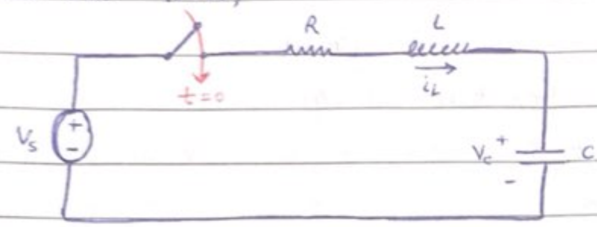
ⓐ $\boxed{V_c(t) = 10 e^{-t/20}}$



ⓑ $i_c(t) = C \frac{dV}{dt} = 5\mu \left(\frac{-10}{20} e^{-t/20} \right) = -2.5\mu e^{-t/20}$



* RLC ckt (series)



by KVL

$i_L = C\frac{dV}{dt} = i_c$

$V_s = V_R + V_c + V_L \rightarrow V_L = L\frac{di}{dt} = LC\frac{d^2V}{dt^2}$

$V_s = Ri + RC\frac{dV}{dt} + LC\frac{d^2V}{dt^2} \Rightarrow 2^{nd}$ order

$\frac{V_s}{LC} = \frac{V_R}{LC} + \frac{RV'}{L} + V''$

$\alpha = \frac{R}{2L}$ (damping rate) $\omega_o^2 = \frac{1}{LC} \Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$ rad/sec (natural frequency)

$$\omega_o^2 V_s = V'' + 2\alpha V' + \frac{V_R}{LC}$$

$\Rightarrow s^2 + 2\alpha s + \omega_o^2 = 0$ (characteristic equation)

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$$\Rightarrow V_c = V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

* If $\alpha > \omega_o$

$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$

$V_c(t) = V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \Rightarrow$ over damping.

$i_L(0^-) = i_L(0^+)$ & $V_c(0^-) = V_c(0^+)$

* If $\alpha = \omega_o$

$s_1, s_2 = -\alpha$

$V_c(t) = V_s + (A_1 + A_2 t)e^{-\alpha t} \Rightarrow$ critically damping

⊛ If $\alpha < \omega_0$

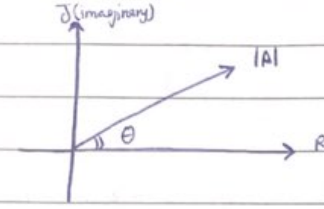
$\sqrt{-\text{number}}$ ⟹ imaginary number or complex number (not Real number).

$S_1, S_2 = -\alpha \pm \sqrt{-\alpha^2 - \omega_0^2} = -\alpha \pm \sqrt{-1}\,\sqrt{\alpha^2+\omega_0^2} = -\alpha \pm J\omega_d$ such that

$J = \sqrt{-1}$

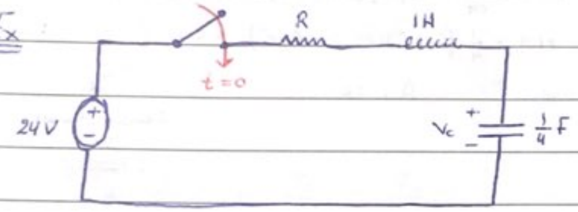
$\omega_d = \sqrt{\alpha^2+\omega_0^2}$

$V_c(t) = (A_1 \cos\omega_d t + A_2 \sin\omega_d t)\, e^{-\alpha t}$



$|A|\cos\theta + J|A|\sin\theta$.

Ex:



when $R = 5$, $R = 4$??

⊛ When $R = 5$:

$\alpha = \frac{R}{2L} = \frac{5}{2} = 2.5$ $\qquad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}}} = 2$

$\alpha > \omega_0$ ⟹ over damping.

$V_c(t) = 24 + A_1 e^{S_1 t} + A_2 e^{S_2 t}$ ⟹ $V_c(t) = 24 + A_1 e^{-t} + A_2 e^{-4t}$

$S_1, S_2 = -2.5 \pm \sqrt{(2.5)^2 - (2)^2}$

$S_1 = -1$, $S_2 = -4$

$V_c(0^-) = V_c(0^+) = 0$

$V_c(0) = 24 + A_1 e^0 + A_2 e^0 = 0$

$\boxed{A_1 + A_2 = -24}$

Find A_1 and A_2

$i_L(0^-) = i_L(0^+)$

$i_c = i_L = C\frac{dv}{dt} = \frac{1}{4}(-A e^{-t} - 4A_2 e^{-4t})$

$i(0) = \frac{1}{4}(-A_1 e^0 - 4A_2 e^0) = 0$

$(-\frac{1}{4}A_1 - 1A_2 = 0) * 4$

$-A_1 - 4A_2 = 0$

$\boxed{A_1 = -4A_2}$

* when $R = 4$

$$\alpha = \frac{4}{2} = 2 \qquad \omega_0 = 2$$

$$\alpha = \omega \Rightarrow \text{critically damping.}$$

$$V_c(t) = (A_1 + A_2 t)\, e^{-\alpha t}$$

$$V_c(t) = (A_1 + A_2 t)\, e^{-2t}$$

$$V_c(0^-) = V_c(0^+) = 0$$

$$V(0) = A_1 e^0 = 0$$

$$A_1 = 0$$

$$i_L = i_c = i_c(0^-) = i_c(0^+) = 0$$

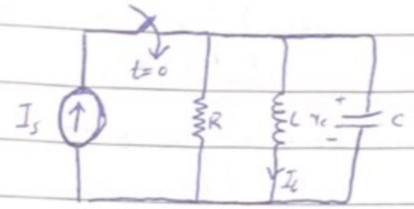
$$i_c = C\frac{dV_c}{dt} = \frac{1}{4}\left(\left(-2e^{-2t}(A_1 + A_2 t)\right) + \left(A_2 e^{-2t}\right)\right)$$

$$i(0) = \frac{1}{4}\left((-2e^{0})(0) + A_2 e^{0}\right) = 0$$

$$A_2 = 0$$

السؤال نفسه
غلط .

* RLC ckt (parallel)



ⓐ $v_c(t)$ $t \geq 0$

ⓑ $I_L(t)$ $t \geq 0$

ⓒ $I_R(t)$ $t \geq 0$

by KCL

$I_s = I_R + I_L + I_c$

$v_L = L \frac{di_L}{dt}$ $\qquad i_c = C \frac{dv_c}{dt}$

Note that $V_c = V_L = V_R$

$I_R = \frac{V_L}{R} = \frac{L\frac{di}{dt}}{R}$

$i_c = C\frac{dv}{dt} = CL\frac{di^2}{dt^2}$

$I_s = \frac{L}{R} i_L' + I l + CL\, i_L''$ → 2^{nd} order diff.

$\Rightarrow i'' + \frac{L}{RCL} i' + \frac{Il}{CL} = \frac{I_s}{CL}$

$i'' + \frac{1}{RC} i' + \frac{Il}{LC} = \frac{I_s}{LC}$

$\alpha_{//} = \frac{1}{2RC}$ (Neper freq.)

$\omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec (Natural freq.)

$i_L'' + 2\alpha i_L' + \omega_0^2 i_L = \omega_0^2 I_s$

⊛ To find $i_L(t) \Rightarrow$ use characteristic equation

$s^2 + 2\alpha s + \omega_0^2 = 0$

$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

① $i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $\Rightarrow$ overdamping. $(\alpha > \omega_0)$

② $(\alpha = \omega_0) \Rightarrow i_L(t) = I_s + (A_1 + A_2 t)\, e^{-\alpha t}$ $\Rightarrow$ Critical damping.

③ $(\alpha < \omega_0) \Rightarrow i_L(t) = I_s + (A_1 \cos \omega_d t + A_2 \omega_0 dt)\, e^{-\alpha t}$ $\qquad \omega_d = \sqrt{\omega_0^2 + \alpha^2}$

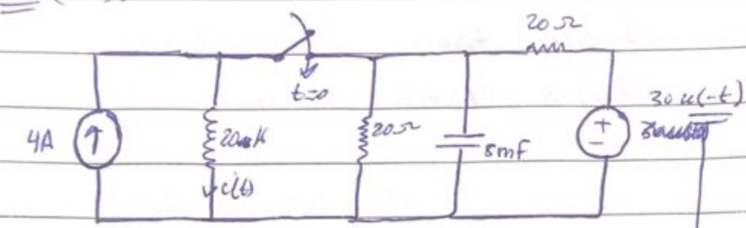
↳ underdamping.

$V_L = L\frac{di}{dt} = V_c = V_R$

$I_R = \frac{V_R}{R} = \frac{V_L}{R}$

s/A

Ex (8.8) book



$u(t) \Rightarrow$ unit step function

$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$

$u(t-5) = \begin{cases} 0, & t < 5 \\ 1, & t > 5 \end{cases}$

$u(-t) = \begin{cases} 0, & t > 0 \\ 1, & t < 0 \end{cases}$

@ $t > 0$

$\Rightarrow$ close the switch

$\Rightarrow$ voltage source $= 0 \Rightarrow$ short circuit

$20 \parallel 20 \Rightarrow \frac{(20)(20)}{40} = 10\,\Omega = R_{eq}$

$\alpha_{\parallel} = \frac{1}{2RC} = \frac{1}{2(10)(8m)} = \frac{1000}{160} = 6.25$ Neper

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{8m * 20}} = 2.5$ rad/sec

$\Rightarrow \alpha > \omega_0$

overdamping.

$i_L(t) = 4 + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$s_1 = -12$ $s_2 = -0.5$

$\Rightarrow i_L(t) = 4 + A_1 e^{-12t} + A_2 e^{-0.5t}$

@ Initial condition

$i_L(0^-) = i_L(0^+)$

$t<0 \leftarrow$

$\Rightarrow$ switch open

$\Rightarrow$ voltage source $= 30V$

$i_L(0) = 4A$

$i(0) = 4 + A_1 e^{-12t} + A_2 e^{-0.5t} = 4$

$\boxed{A_1 e^{-12t} + A_2 e^{-0.5t} = 0}$ #

$V_c(0^-) = V_c(0^+)$

$t<0 \leftarrow$

$V_c(0) = 30V$

$V_c = V_L = L\frac{di}{dt} = 20\left(-12A_1 e^{-12t} - 0.5 A_2 e^{-0.5t}\right)$

$V_c(0) = 20(-12A_1 - 0.5A_2) = 30$ #

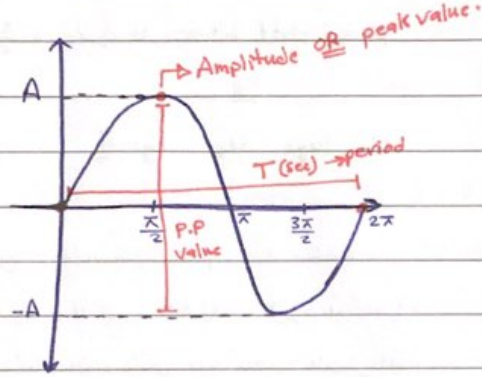
find A_1 && A_2

Five Apple

AC analysis :

↳ periodic

$f(x) = A\sin(x)$



Amplitude = (from avg. → peak).

peak value = A

Peak to peak value = 2A

↳ (Max → Min).

period = T (sec)

↳ frequency = $f = \frac{1}{T}$ (# of periods) sec^{-1} (H

1 period → T sec

f (??) → 1 sec

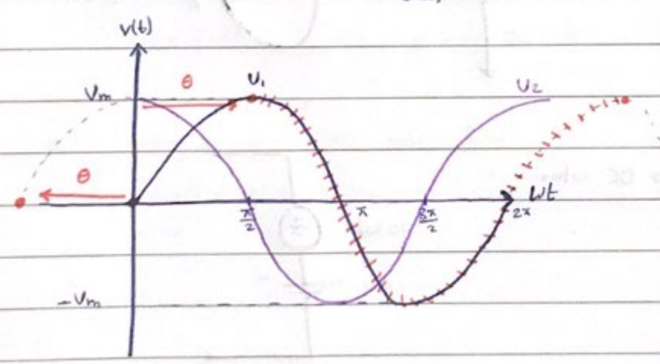
Avg. value = $\frac{1}{T}\int_0^T v(t)\,dt$ = Zero

↳ when it's pure sinusoidal functio

⊛ Angular speed = Angular frequency = $\underline{\omega} = \frac{2\pi}{T} = 2\pi f$ (rad/sec)

↳ constant.

$\omega = \frac{\alpha\,(rad)}{T\,(sec)} \Rightarrow \alpha = \omega t$



$V_1 = V_m \sin(\omega t)$

$V_1 = V_m \cos(\omega t - \theta)$

$V_1 = V_m \cos(\omega t - \frac{\pi}{2})$

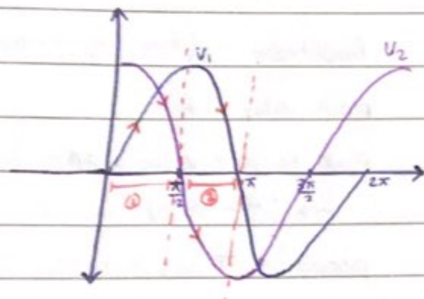
$\sin(x) = \cos(x - 90)$

$V_2 = V_m \cos(\omega t)$

$V_2 = V_m \sin(\omega t + \theta)$

$V_2 = V_m \sin(\omega t + \frac{\pi}{2})$

* To find the phase shift :



V_2 leads V_1 by $\frac{\pi}{2}$

OR

phase shift between V_1 & V_2 = $\frac{\pi}{2}$

OR

V_1 lags V_2 by $\frac{\pi}{2}$

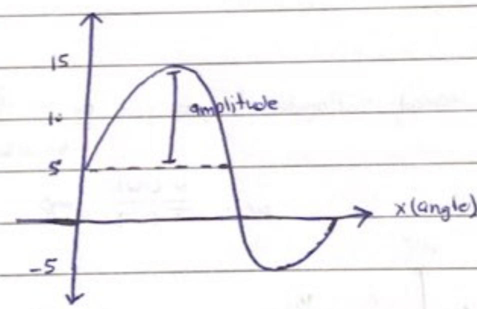
* نختار فترة ، هاي الفترة لازم الـ 2 function
يتصرفوا نفس التصرف (يا الاثنين لفوق أو لتحت).
يعني بنشوف بهاي الفترة مين بيضرب الصفر بالأول
هو اللي بيكون (leads).
* لازم يكون بالرسم نفس (ω).

* Ex : $f(x) = 5 + 10\sin x$

peak value = 15

p.p value = 15 - (-5) = 20

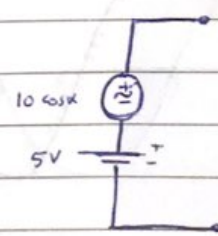
amplitude = 10



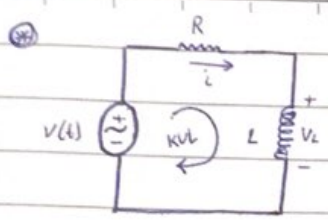
$$V_{avg} = \frac{1}{T}\int_0^{period} v(x)\,dx$$

$$= \frac{1}{T}\int_0^{T}(5 + 10\cos x)\,dx = \underline{\underline{5}}$$

↳ DC value.



⊛



$V(t) = \cos(\omega t)$

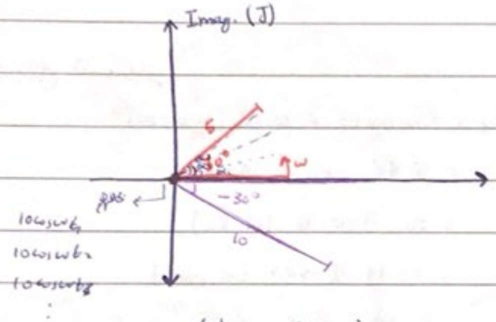
$V(t) = iR + V_L = iR + L\frac{di}{dt}$

$\cos(\omega t) = iR + L\frac{di}{dt}$

~~time~~ ~~Cos~~ ⟹ phasor

$V(t) = V_m \cos(\omega t + \theta) \Rightarrow |\ | \angle\theta$ (phasor). θ → phase shift.

(time domain) $|\ |$ → magnitude = V_m (Cos ...)



(phasor diagram)

$5\angle 30° \Rightarrow 5\cos(\omega t + 30)$

$10\angle -30° \Rightarrow 10\cos(\omega t - 30)$

↳ (polar form).

⊛ Polar form ⟹ Rectangular form

$a\angle\theta \Rightarrow a\cos\theta + Ja\sin\theta$

⊛ Rectangular form ⟹ polar form

$\underbrace{a\cos\theta}_{real} + \underbrace{Ja\sin\theta}_{img} \Rightarrow$

$V_m = \sqrt{r^2 + img^2}$

$\theta = \tan^{-1}\left(\frac{img.}{real}\right)$

⊛ Rectangular form:

$V_1 = a + Jb$

$V_2 = c + Jd$

$\Rightarrow V_1 \pm V_2 = (a \pm c) + (Jb \pm Jd) = (a \pm c) + J(b \pm d)$

$V_1 V_2 = (a + Jb)(c + Jd) = ac + Jad + Jbc + JJbd$ $\quad JJ = \sqrt{-1}\sqrt{-1} = -1$

$= ac + Jad + Jbc - bd$

$= (ac - bd) + J(ad + bc)$

$V_1/V_2 = \frac{a + Jb}{c + Jd} \times \frac{c - Jd}{c - Jd} = \frac{(a + Jb)(c - Jd)}{c^2 + d^2}$ → Real number $\quad (c + Jd)^* = c - Jd$

Five Apple

⊛ polar form:

$$V_1 = |V_{m_1}| \measuredangle \theta_1 \qquad V_2 = |V_{m_2}| \measuredangle \theta_2 \;\Big]\Rightarrow\; V_1V_2 = |V_{m_1}||V_{m_2}| \measuredangle(\theta_1+\theta_2), \quad V_1/V_2 = \frac{|V_{m_1}|}{|V_{m_2}|} \measuredangle(\theta_1-\theta_2)$$

$V_1 \pm V_2 \Rightarrow$ ① polar form → Rectangular form

$V_{1} \pm V_2$ (in Rect)

Rectangular form → polar form.

Ex: $v(t) = 5\cos(100t) + 30\sin(100t + 30°)$

$\sin \hookrightarrow \cos(x - 90°)$

لازم يكون إلهم نفس (ω)

$= 5\cos(100t) + 30\cos(100t + 30 - 90) = 5\cos(\underline{\underline{100}}t) + 30\cos(\underline{\underline{100}}t - 60)$

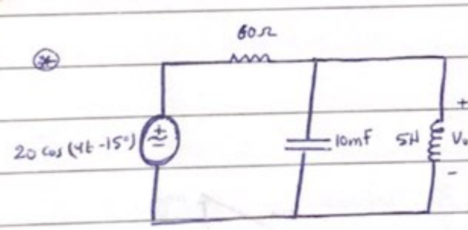
$= 5\measuredangle 0° + 30\measuredangle -60°$

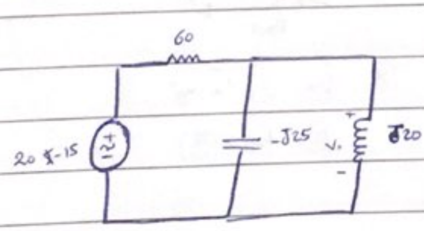
$= 20 - j25.98$ (in Re)

$= 32.78 \measuredangle -52.4$ (in polar)

$= 32.78\cos(100t - 52.4)$ (in time domain).

⊛





phasor:

source $\Rightarrow 20\measuredangle -15°$

impedance $= Z\ (\Omega)$

$R \Rightarrow Z_R = R\ \Omega = 60\ \Omega$

بصير نقطي / زي ال $\underline{\underline{R}}$

$C \Rightarrow Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}\ \Omega = -j25\ \Omega$

$L \Rightarrow Z_L = j\omega L = +j20\ \Omega$

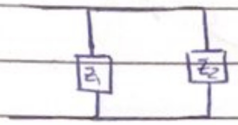
$L \parallel C \Rightarrow \frac{(j20)(-j25)}{-j5} = \frac{100}{-j} = j100$

Remember: $\frac{1}{j} = -j$

$$V_0 = \frac{20\measuredangle -15\ (j100)}{60 + j100} = \frac{2000\measuredangle 75}{60 + j100} = \frac{2000\measuredangle 75}{\sqrt{3600+10000}\ \measuredangle \tan^{-1}\left(\frac{100}{60}\right)}$$

⊛ To find Z_{eq} ⇒ same as R_{eq}.

Ex



$$\Rightarrow Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(5+J5)(10-J4)}{(5+J5)+(10-J4)} = \underbrace{\frac{460}{113}}_{\text{Real}} + J\underbrace{\frac{106}{113}}_{\text{Imag.}}$$

$Z_1 = 5 + J5$

$Z_2 = 10 - J4$

In general:

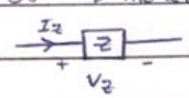
$$Z = \underbrace{R}_{\substack{\text{resistance} \\ (\Omega) \\ \text{always positive}}} \pm J \underbrace{X}_{\substack{\text{reactance} \\ (\Omega) \\ \text{always positive}}}$$

→ if (+) ⇒ inductive load

(−) ⇒ capacitive load

$X = 0$ ⇒ Resistive load

⊛ $Z = R + JX$ ⇒ Inductive load

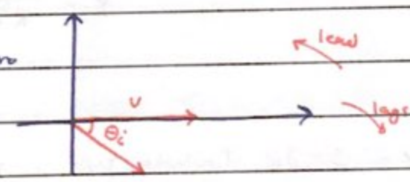


I_Z lags V_Z by $\theta_Z = \theta_v - \theta_i$

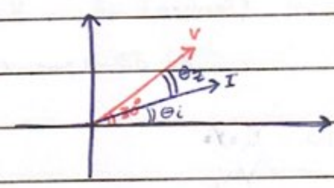
$V = IZ \Rightarrow |V| \angle\theta_v = (|I| \angle\theta_I)(|Z| \angle\theta_Z)$

$$\frac{|V|}{|I|} \angle\theta_v - \theta_i = |Z| \angle\theta_Z$$

if θ_v = Zero



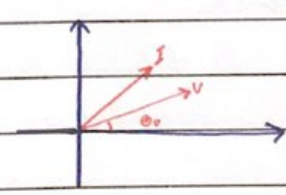
if $\theta_v = 30°$



⊛ $Z = R - JX$ ⇒ capacitive load

$= |Z| \angle\theta_Z$
↳ <0

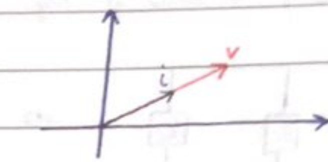
I_Z leads V_Z by $\theta_Z = \theta_v - \theta_i$



* $Z = R \Rightarrow$ Resistive load

I_Z in phase $V_Z \Rightarrow \theta_v - \theta_i = 0$

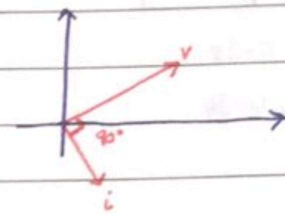
$\theta_v = \theta_i$



* $Z = JX \Rightarrow$ pure inductor

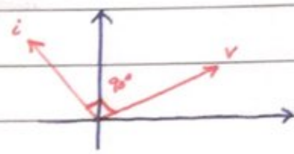
$\underline{Z} \to X \measuredangle 90°$

I_Z lags V by $90°$



* $Z = -JX \Rightarrow$ pure capacitor

$\underline{Z} \to X \measuredangle -90°$ I leads V by $90°$



* Admittance $= Y = \frac{1}{Z}$

$Z = R + JX$ (inductive) $\Rightarrow Y = \frac{1}{R+JX} * \frac{R-JX}{R-JX}$

$$Y = \frac{R-JX}{R^2+X^2} = \underbrace{\frac{R}{R^2+X^2}}_{G \text{ (Conductance)}} - J\underbrace{\frac{X}{R^2+X^2}}_{B} \quad \text{(inductive)}$$

$Y = G - JB$ (inductive) $\qquad Y = G + JB$ (capacitive).

$Y = G$ (Resistive) $\qquad Y = -JB$ (pure inductor)

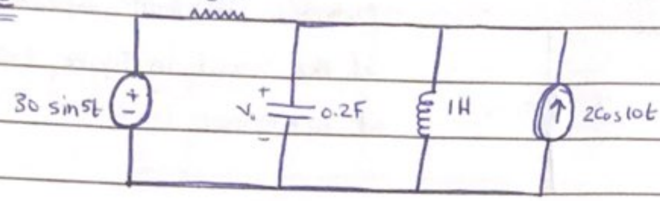
$Y = JB$ (pure capacitor)

$Y_{eq} \to$ parallel $= Y_1 + Y_2$

$Y_{eq} \to$ series $= \frac{Y_1 Y_2}{Y_1 + Y_2}$

* $f =$ Zero $\Rightarrow$ DC

Example:



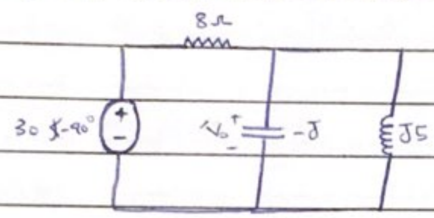
Calculate V_o ?

We can use the superposition method ONLY because the sources are with different frequencies.

Answer:

phasor:

① Kill current source ⇒ open circuit.



$30 \sin 5t \Rightarrow \omega = 5 \text{ rad/sec}$

$30 \sin 5t = 30 \cos(5t - 90) = 30 \angle -90^\circ$

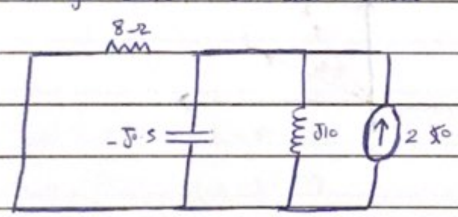
$C = 0.2F \Rightarrow Z_c = \frac{1}{J(5)(0.2)} = -J\ \Omega$

$L = 1H \Rightarrow Z_L = J(5) = J5\ \Omega$

$$(-J \parallel J5) \Rightarrow \frac{(-J)(J5)}{J4} = -\frac{J5}{4} = -J1.25\ \Omega$$

$$V_o' = \left(\frac{-J1.25}{8 - J1.25}\right)(30 \angle -90^\circ) = 4.631 \angle -171.12$$

② Kill voltage source ⇒ short circuit.



$2\cos 10t \Rightarrow \omega = 10 \text{ rad/s}$

$2\cos 10t = 2 \angle 0$

$Z_c = \frac{-J}{(10)(0.2)} = -J0.5\ \Omega$

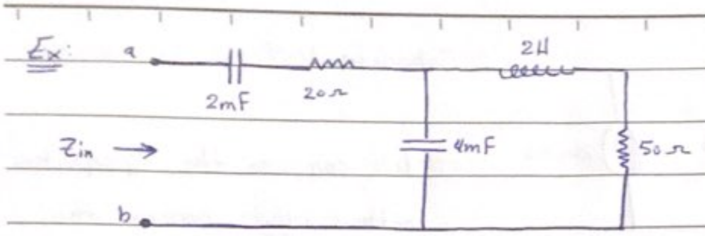
$Z_L = J10\ \Omega$

$$(8 \parallel J10) \Rightarrow \frac{(8)(J10)}{8 + J10} = \frac{J80}{8 + J10}$$

$$V_o'' = \left(\frac{J80/8+10J}{-J0.5}\right)(2 \angle 0) = 1.05 \angle -86.24$$

$V_o = V_o' + V_o''$ ⇒ ما بصير أجمعهم هيك بال phasor لازم أحولهم لـ time domain لأنه إلهم frequencies مختلفة.

Ex:



Determine the input impedance of the circuit in figure below at $\omega = 10$ rad/s.

$\circledast$ $2mF \Rightarrow \frac{1}{j\omega c} = \frac{1}{j(10)(2m)} = -j50$

$\circledast$ $2H \Rightarrow j\omega L = j(10)(2) = j20$

$\circledast$ $4mF \Rightarrow \frac{1}{j\omega c} = \frac{1}{j(10)(4m)} = -j25$

$(50 \,,\, j20) \Rightarrow$ series

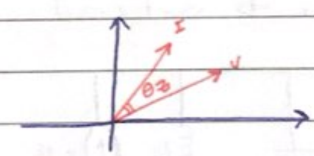
$50 + j20$

$(50 + j20 \,,\, -j25) \Rightarrow$ parallel

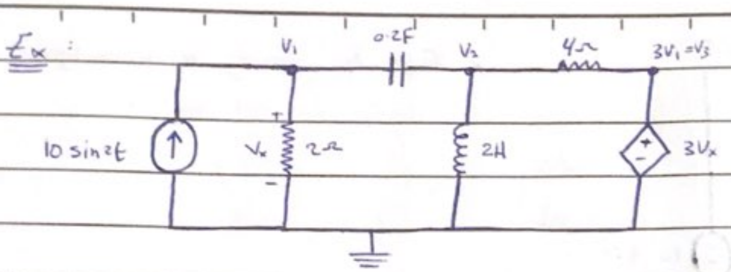
$\frac{(50+j20)(-j25)}{(50+j20)+(-j25)} \rightsquigarrow$ series with $20\,\Omega$ & $-j50$

$Z_{eq} = 32.38 - j73.76 \Rightarrow I_Z$ lead V_Z by $\tan^{-1} \frac{73.76}{32.38}$

↳ capacitive load



Ex :



* Using nodal analysis, find V_1 and V_2 ?

Answer

$10 \sin 2t \Rightarrow \omega = 2$ rad/s

$10 \sin 2t = 10 \cos(2t - 90) = 10 \angle -90^\circ$

$0.2F \Rightarrow \frac{1}{J(0.2)(2)} = -J2.5\ \Omega$

$2H \Rightarrow J(2)(2) = J4$

by nodal:

$V_1\left(\frac{1}{2} + \frac{1}{-J2.5}\right) - V_2\left(\frac{-1}{J2.5}\right) = 10 \angle -90^\circ \quad \cdots ①$

$V_1(0.5 + J2.5) - J0.4\,V_2 = -J10$

$V_2 = \frac{V_1(0.5 + J2.5) + J10}{J0.4}$

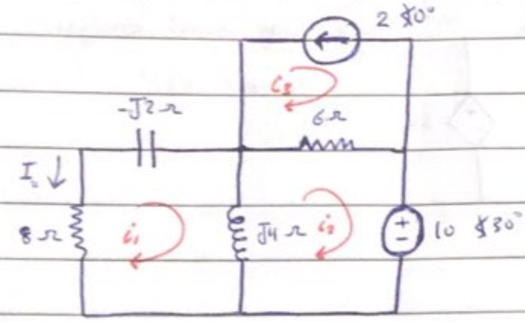
$V_2\left(\frac{1}{-J2.5} + \frac{1}{J4} + \frac{1}{4}\right) - V_1\left(\frac{1}{-J2.5}\right) - 3V_1\left(\frac{1}{4}\right) = 0 \quad \cdots ②$

⋮

$V_1(t) = 11.32 \sin(2t + 60.01^\circ)$

$V_2(t) = 33.02 \sin(2t + 57.12^\circ)$

Ex:



* Find I_o using mesh analysis:

Answer:

$$i_3 = -2\angle 0°$$

$$i_1(8 - j2 + j4) - i_2(j4) = 0$$

$$i_1(8 + j2) - i_2(j4) = 0$$

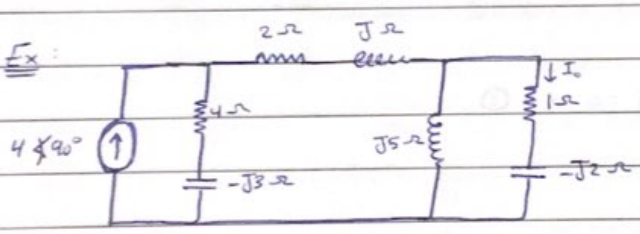
$$i_2 = \frac{i_1(8+j2)}{j4} = i_1(-j2 + 0.5) \quad \cdots \quad (1)$$

$$i_2(j4+6) - i_1(j4) - i_3(6) = -10\angle 30°$$

→ find i_1

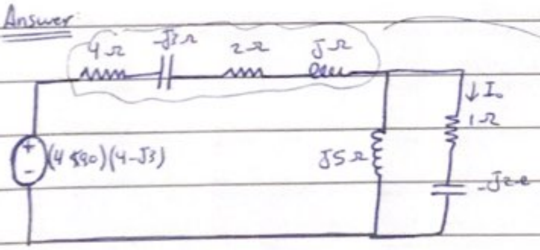
$$\Rightarrow I_o = -i_1 = 1.194\angle 65.44°$$

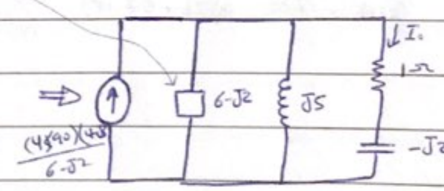
Ex:



* Find I_o using source transformation:

Answer:





by current divisio find I_o

$$I_o = 3.288\angle 99.46°$$

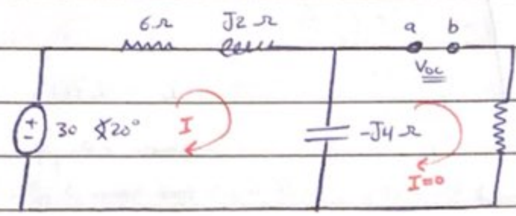
* Thevenin and Norton equivalent:

$Z_{th} \Rightarrow$ Kill all sources

$V.S \rightarrow S.C$

$C.S \rightarrow O.C$ $\Big] \Rightarrow$ $V_{th} = V_{o.c}$, $I_N = I_{s.c}$ $\Rightarrow \frac{V_{o.c}}{I_{s.c}} = Z_{th} = Z_N$.

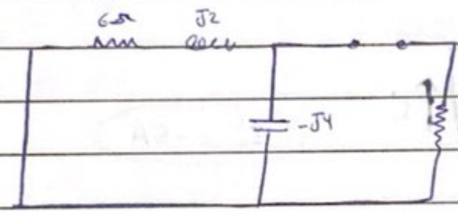
<u>Example:</u>



* Find the Thevenin equivalent at terminals a-b.

<u>Answer:</u>

$Z_{th} \Rightarrow$



$(6, J2) \Rightarrow$ series

$6+J2$

$(6+J2, -J4) \Rightarrow$ parallel $\Rightarrow$ series with 10Ω

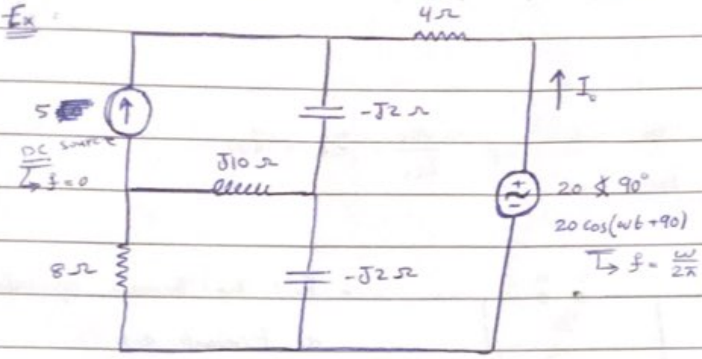
$$Z_{th} = \frac{(6+J2)(-J4)}{6-J2} + 10 = 12.4 - J3.2$$

$V_{th} \Rightarrow$ by KCL

$$I = \frac{30 \angle 20^\circ}{6-2J}$$

$$V_{th} = V_{oc} = V_{-J4} = (-J4)\left(\frac{30 \angle 20^\circ}{6-2J}\right) = 18.97 \angle -51.57^\circ$$

Ex:



find I_o:

Note that the 2 sources have different frequencies.
↳ by superposition only

$I_o = I_o'(t) + I_o''(t)$

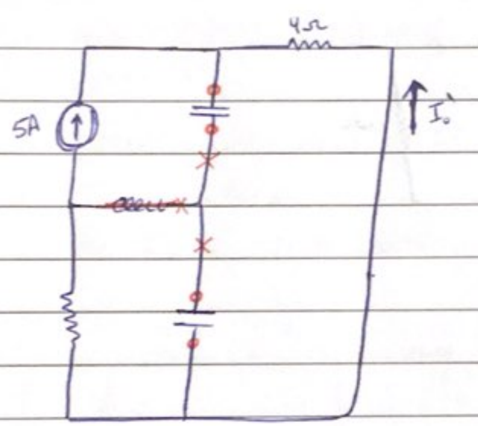
phasor ما بقدر أجمعهم ك

time domain لازم أحولهم لـ

بعدين أجمعهم

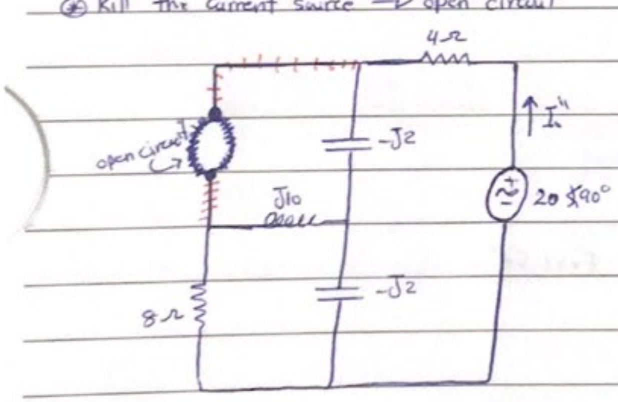
* Kill the voltage source ⟹ short circuit.

L ⟹ S.C C ⟹ O.C



$I_o' = -5A$

* Kill the current source ⟹ open circuit



(8, J10) → series

8 + J10

(8 + J10, -J2) → parallel

$\frac{(8+J10)(-J2)}{8+J8}$ ⟹ series with -J2 & 4Ω

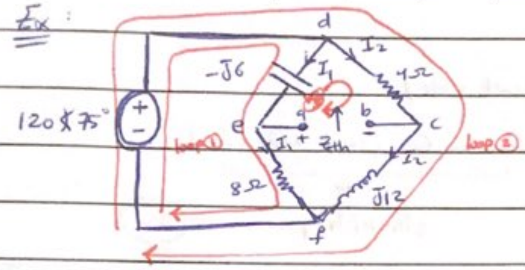
$Z = 0.25 - 2.25J - 2J + 4$

$Z = 4.25 - J4.25$

$I_o'' = \frac{20 \angle 90°}{4.25 - J4.25} = -2.35 + J2.35 = 3.3 \angle 135.3$

$I_o = I_o' + I_o'' = -5 + 3.3 \angle 135.3°$

Ex:



find Z_{th} & V_{oc} seen from ab:

$(-J6, 8) \rightarrow$ parallel $\quad$ $(4, J12) \rightarrow$ parallel

$\frac{(-J6)(8)}{8-J6} = 2.88 - J3.84$ $\quad$ series $\Longleftrightarrow$ $\quad$ $\frac{(4)(J12)}{4+J12} = 3.6 + J1.2$

$Z_{th} = 2.88 - J3.84 + 3.6 + J1.2 = 6.48 - J2.64$

Loop ① $\Rightarrow$ $I_1 = \frac{120 \measuredangle 75}{8-J6} = -4.47 + J11.1$ $\quad$ Loop ② $\Rightarrow$ $I_2 = \frac{120 \measuredangle 75°}{4+J12} = 9.47 + J0.57$

loop ③ $\Rightarrow$ $I_2(4) - V_{oc} - I_1(-J6) = 0$

$V_{oc} = 4(9.47 + J0.57) - (-4.47 + J11.1)(-J6)$

$= (37.88 + J2.28) - (66.6 + J26.82) = -28.72 - J24.54$

$I_N = I_{sc} = \frac{V_{th}}{Z_{th}} = \frac{-28.72 - J24.54}{6.48 - J2.64} = -5.12 - J1.7$.

* Instantaneous and Average power:

$$p(t) = v(t)\, i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

$$= \underbrace{\tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{\text{Constant power}} + \underbrace{\tfrac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}_{\text{sinusoidal power at } 2\omega}$$

2ω → 2 period → 1T

same as P_{avg}. θ_z

$$P_{avg} = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

(active or real power). between 90 / −90

$$P_{max} = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \tfrac{1}{2} V_m I_m \quad \text{such that } [\cos(2\omega t + \theta_v + \theta_i) = 1]$$

$$P_{min} = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i) - \tfrac{1}{2} V_m I_m \quad \text{such that } [\cos(2\omega t + \theta_v + \theta_i) = 1]$$

peak to peak value $= P_{max} - P_{min} = V_m I_m$

$p(t) > 0 \Rightarrow$ absorbed power by the circuit $\quad p(t) < 0 \Rightarrow$ absorbed power by the source.

* In P_{avg}: $P_{avg} = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$\theta_v - \theta_i > 0 \Rightarrow i$ lags $v \Rightarrow$ inductive load $\Rightarrow P_{avg.} > 0$

$\theta_v - \theta_i < 0 \Rightarrow i$ leads $v \Rightarrow$ capacitive load $\Rightarrow P_{avg.} > 0$

$\theta_v = \theta_i \Rightarrow i$ & v inphase $\Rightarrow$ resistive load $\Rightarrow P_{avg} = \tfrac{1}{2} V_m I_m$

$\theta_v - \theta_i = \pm 90 \Rightarrow$ (+) pure inductive / (−) pure conductor $\Rightarrow P_{avg} =$ Zero.

$Z_L = Z_{th}^{*} \Rightarrow$ maximum average power

$$P_{max} = \frac{V_{th}^2}{8 R_{th}}$$

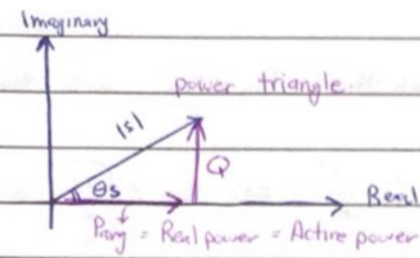
⊛ Complex power:

$\theta_v - \theta_i = \theta_Z$

S = Complex power $= |S| \measuredangle \theta_s \Rightarrow$ unit $(V.A)$

$\hookrightarrow$ Apparent power $= \frac{1}{2}|V_m||I_m| \measuredangle (\theta_v - \theta_i) = \frac{1}{2}\vec{V}\vec{I}^*$

in Rectangular form $\Rightarrow S = |S| \cos\theta_s + J|S| \sin\theta_s$



$|S| = \sqrt{P^2 + Q^2}$

$$S = \underbrace{\tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{P_{avg} \Rightarrow \text{unit (watt)}} + J \underbrace{\tfrac{1}{2} V_m I_m \sin(\theta_s)}_{Q = \text{reactive power}}$$

$\Rightarrow$ unit $(V.Ar)$

⊛ Power factor $= P.f \triangleq \frac{P_{avg.}}{|S|} = \frac{\frac{1}{2}\cancel{V_m I_m}\cos(\theta_v - \theta_i)}{\frac{1}{2}\cancel{V_m I_m}} = \cos(\theta_v - \theta_i)$

$-90^\circ \leq \theta_v - \theta_i \leq 90^\circ \Rightarrow 0 \leq$ power factor ≤ 1

⊛ if power factor $= 1 \Rightarrow$ unity power factor

$\theta_v - \theta_i > 0 \Rightarrow i$ lags $v \Rightarrow$ Inductive $\Rightarrow P.f \geq 0$

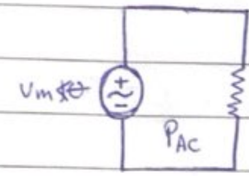
$\theta_v - \theta_i < 0 \Rightarrow i$ leads $v \Rightarrow$ capacitive $\Rightarrow P.f \geq 0$

] $\Rightarrow$ عشان أفرق بينهم لازم أطلع قيمة ال P.f وأحط جنبها يا lagging أو leading

$\theta_v - \theta_i = 0 \Rightarrow$ pure resistive $\Rightarrow P.f = 1$

$\theta_v - \theta_i = \pm 90 \Rightarrow P.f = 0$ (the worst case)

⊛ Root mean square = RMS $= \sqrt{\frac{1}{T}\int_0^T v^2 dt}$ = Volts (V_{rms})

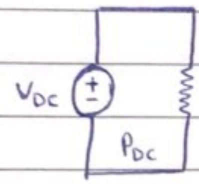


$$\Rightarrow \quad P_{avg} = \frac{1}{2} V I \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} V \left(\frac{V}{R}\right)(1)$$

→ because it is a pure resistance.

$$P_{avg} = \frac{1}{2}\frac{V^2}{R}$$



Find (V_{DC}) such that $P_{AC} = P_{DC}$ with the same R ?

$$P_{DC} = \frac{V_{DC}^2}{\cancel{R}} = \frac{1}{2}\frac{V^2}{\cancel{R}} \Rightarrow V_{DC} = \sqrt{\frac{V^2}{2}} = \frac{V_m}{\sqrt{2}} = V_{rms}$$

⊛ RMS:

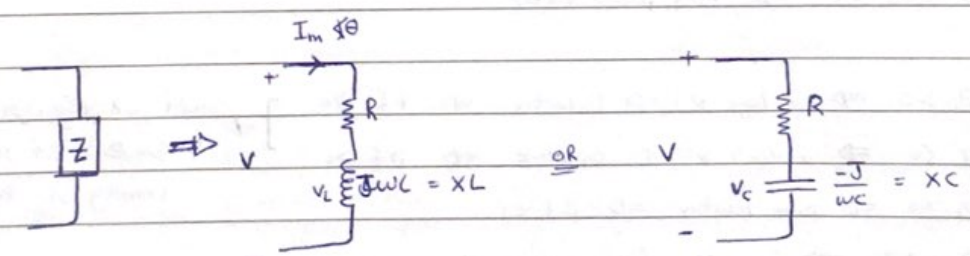
$$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$|S| = \frac{1}{2} V_m I_m = |V_{rms}||I_{rms}|$$

$$Q = \frac{1}{2} V_m I_m \sin\theta_s = V_{rms} I_{rms} \sin\theta_r .$$

⊛



$$P_{avg} = \frac{1}{2}|I_m|^2 R = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

$$Q = \frac{1}{2}|I_m|^2 X = \underset{\to C}{\overset{\to L}{\pm}} I_{rms}^2 X = \frac{|V_{L\,rms}|^2}{X}$$

OR

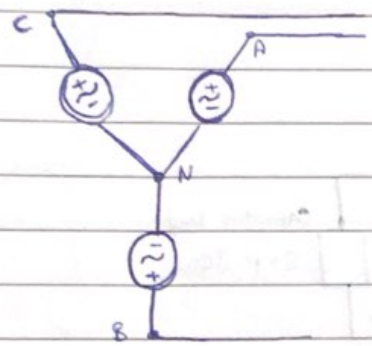
$$= \frac{-|V_{C\,rms}|^2}{X}$$

$$S = \left(|V_{rms}| \angle\theta_v\right)\left(|I_{rms}| \angle\theta_i\right) = V_{rms} I_{rms} \angle\theta_v - \theta_i$$

* أسئلة ال power طويلة بالامتحان *

Three phase

Y - Connection sources:

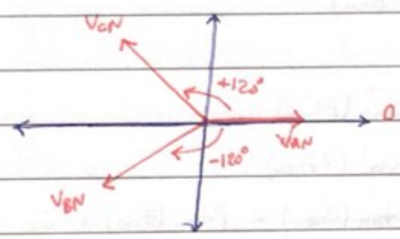


V_{AN} = V_{phase}

V_{AB} = V_{LL} (line to line).

$\left[V_{LL} > V_{phase} \right]$

$$\left.\begin{aligned} V_{phase} \Rightarrow V_{AN} &= |V_P| \angle \theta_P \\ = V_{BN} &= |V_P| \angle \theta_P - 120^\circ \\ = V_{CN} &= |V_P| \angle \theta_P + 120^\circ \end{aligned}\right\} \Rightarrow \sum = \text{Zero}$$



ABC $\Rightarrow$ positive sequence

(الترتيب مهم)

$$V_{LL} \Rightarrow V_{AB} = |V_{LL}| \angle \theta_{AB}$$

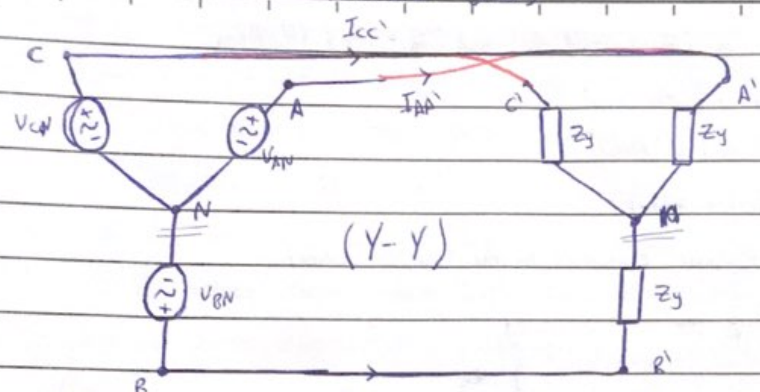
$$V_{BC} = |V_{LL}| \angle \theta_{AB} - 120^\circ$$

$$V_{CA} = |V_{LL}| \angle \theta_{AB} + 120$$

In a balanced system:

$\underline{\underline{V_P}}$		$\underline{\underline{V_{LL}}}$
$V_{AN} = \lvert V_P \rvert \angle \theta_P$	$\Rightarrow$	$V_{AB} = \sqrt{3}\, \lvert V_P \rvert \angle \theta_P + 30^\circ$
$V_{BN} = \lvert V_P \rvert \angle \theta_P - 120^\circ$	$\Rightarrow$	$V_{BC} = \sqrt{3}\, \lvert V_P \rvert \angle \theta_P - 120 + 30^\circ$
$V_{CN} = \lvert V_P \rvert \angle \theta_P + 120^\circ$	$\Rightarrow$	$V_{CA} = \sqrt{3}\, \lvert V_P \rvert \angle \theta_P + 120^\circ + 30^\circ$

(Balanced system)



(Balanced source) $I_{BB'}$ (line current) (Balanced load).

3∅ system ⇒ Balanced

$[V_N = V_n]$

V_{Nn} = Zero #

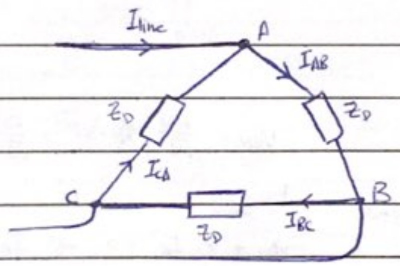
* Line current:

$I_{AA'} = |I_{line}| \measuredangle \theta_{line}$

$I_{BB'} = |I_{line}| \measuredangle \theta_{line} - 120^\circ$

$I_{CC'} = |I_{line}| \measuredangle \theta_{line} + 120^\circ$

* phase current ⇒ in delta connection



$I_{AB} = |I_\Delta| \measuredangle \theta_\Delta$

$I_{BC} = |I_\Delta| \measuredangle \theta_\Delta - 120^\circ$

$I_{CA} = |I_\Delta| \measuredangle \theta_\Delta + 120^\circ$

I_{phase}

$I_{ab} = |I_{ab}| \measuredangle \theta_{ab}$ ⇒

$I_{bc} = |I_{bc}| \measuredangle \theta_{ab} - 120^\circ$ ⇒

$I_{ca} = |I_{ca}| \measuredangle \theta_{ab} + 120^\circ$ ⇒

I_{line}

$I_{AA'} = \sqrt{3}\,|I_{ab}| \measuredangle \theta_{ab} - 30^\circ$

$I_{BB'} = \sqrt{3}\,|I_{bc}| \measuredangle \theta_{bc} - 120^\circ - 30^\circ$

$I_{CC'} = \sqrt{3}\,|I_{ca}| \measuredangle \theta_{ab} + 120^\circ - 30^\circ$

by KCL @ node A:

$I_{line} = I_{AB} - I_{CA}$

Power in 3phase:

$P_{3\phi} = 3 P_{1\phi}$

$Q_{3\phi} = 3 Q_{1\phi}$

$S_{3\phi} = 3 S_{1\phi}$

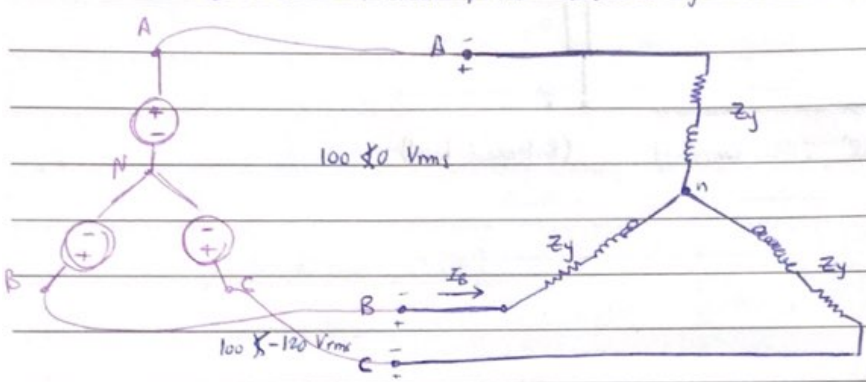
Ex For the circuit shown, if ($Z_y = 4 + J3$) Ω and ($I_B = |I_B| \angle(\theta)$) Arms.

Find: ① The power factor of the load.

② The value of $|I_B|$ (in Arms)

③ The value of θ (in degree).

④ The total active power consumed by the load (in watt).

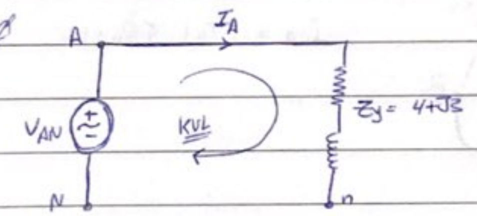


Answer:

① $Pf = \cos(\theta_Z)$ $\qquad Z_y = 5 \angle 36.87^\circ$

$= \cos(36.87^\circ) = 0.8$ lagging.

② 1ϕ



$V_{AN} = \frac{100}{\sqrt{3}} \angle 0 - 30^\circ = \frac{100}{\sqrt{3}} \angle -30^\circ$

by KVL

$-V_{AN} + Z_y I_A = 0 \Rightarrow I_A = \frac{V_{AN}}{Z}$

$I_A = \dfrac{\frac{100}{\sqrt{3}} \angle -30^\circ}{4 + J3} = 11.46 - J1.38$

$I_A = 11.54 \angle -6.87^\circ$

$I_B = |I_A| \angle \theta_A - 120^\circ = 11.54 \angle -126.87^\circ$

$P_A = \cancel{\tfrac{1}{2}} V_{rms} I_{rms} \cos(\theta_v - \theta_i) = \cancel{\tfrac{1}{2}} (\frac{100}{\sqrt{3}})(11.54) \cos(-30 + 6.87) = 612.7$ watt.

P for $3\phi = 3P_A = 3(612.7) = 1838$ watt.

OR $P_{1\phi} = I_{rms}^2 R = (\frac{100}{5\sqrt{3}})^2 (4)$

$P_{3\phi} = 3P_{1\phi}$

$Q_{1\phi} = |I_{rms}|^2 X$

$Q_{3\phi} = 3 Q_{1\phi}$

$S_{1\phi} = VI^* = \left(\frac{100}{\sqrt{3}} \angle -30^\circ\right)\left(\frac{100}{5\sqrt{3}} \angle (30 - \tan^{-1}(\frac{3}{4})\right)^*$

$S_{3\phi} = 3 S_{1\phi} = 3 (\qquad)$

$S = P + JQ$

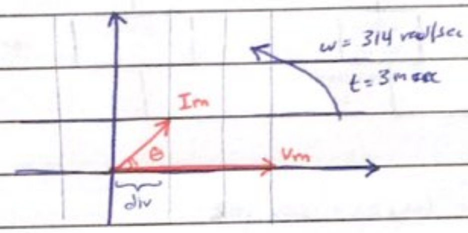
Ex: The figure shows a phasor diagram for a circuit. The voltage and the current scales are 1V/division and 1A/division respectively and $i(t) = I\cos(\omega t + \theta_i)$ and $v(t) = V\cos(\omega t + \theta_v)$.

Find: ① The value of I.

② The value of θ_i

③ The apparent power consumed by the circuit (in VA)

④ The power factor of the circuit



$V_m = 3V$

$I_m = \sqrt{2}\ A$

① $I = \sqrt{2}\ A$

② $\omega t + \theta_v = 0\Big|_{t=3ms}$

$\theta_v = -\omega t = -(314)(3m) = -0.942\ rad * \frac{180}{\pi} = -54^\circ$.

$\theta_Z = \theta_v - \theta_i = -45^\circ$ (i leads V by 45°)

$\theta_i = -54 + 45 = -9^\circ$

OR

$\omega t + \theta_i = 45$

$(314)(3m)\left(\frac{180}{\pi}\right) + \theta_i = 45$

$\theta_i = 45 - 54 = -9^\circ$

③ $|S| = \frac{1}{2}|V_m||I_m| = \frac{1}{2}(3)(\sqrt{2}) = \frac{3}{\sqrt{2}}$

$\vec{S} = \frac{1}{2}\vec{V}\vec{I}^* = \frac{1}{2}(3\angle -54)(\sqrt{2}\angle -9)^* = 1.5\sqrt{2}\ \angle -45$ $\quad \theta_v - \theta_i = \theta_Z = \theta$

$P = 1.5\sqrt{2}\cos(-45) = 1.5$ watt

$Q = 1.5\sqrt{2}\sin(-45) = -1.5$ ~~watt~~ VAR $\Rightarrow$ capacitive load

④ $pf = \cos(-45) = 0.707$ leading

$Z = \frac{V}{I} = \frac{3\angle -54}{\sqrt{2}\angle -9} = \frac{3}{\sqrt{2}}\angle -45^\circ$

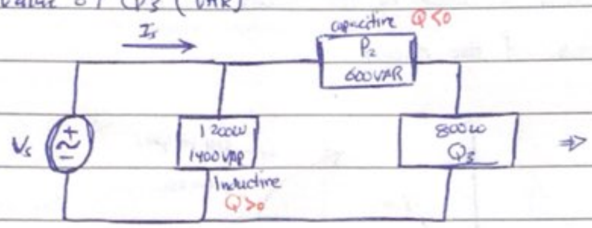
Ex: For the circuit shown, if $V_s = 100 \angle 0°$ Vrms and $I_s = 30 \angle -30°$ Arms

Find: ① The apparent power supplied by the source

② The reactive power supplied by the source

③ The value of P_1 (watt)

④ The value of Q_3 (VAR)



$\Rightarrow$ $S = 800 + J700$

$S = V I_2^*$

$V = \frac{S}{I_2^*}$

Answer:

① $|S| = V_{rms} * I_{rms} = (100)(30) = 3000$ VA

② $\vec{S} = \vec{V_{rms}} * \vec{I_{rms}}^*$

$= (100 \angle 0°)(30 \angle -30°)^* = 3000 \angle 30°$ VA

$\underset{source}{S} = 2598 + J1500$

③ $\Sigma P_{supplied} = \Sigma P_{consumed}$

$2598 = 1200 + P_2 + 800$

$P_2 = 598$ watt

④ $\Sigma Q_{supplied} = \Sigma Q_{consumed}$

$1500 = 1400 - 600 + Q_3$

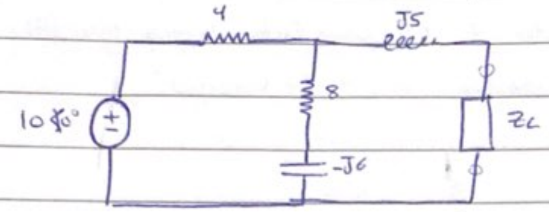
$Q_3 = +700$ VAR $\Rightarrow$ ~~capacitive~~ inductive load

$I_1 \Rightarrow S_1 = 1200 + J1400 = 1844 \angle 49.4° = V_{rms} I_{rms}^*$

$\Rightarrow I_{rms} = \left(\frac{S_1}{V}\right)^* = \left(\frac{1844 \angle 49.4°}{100 \angle 0°}\right)^* = 18.44 \angle -49.4°$

$I_2 = I_s - I_1 = (30 \angle 30) - (18.44 \angle 49.4°)$

Ex: Find Z_L which transfer the maximum power from ckt. & find the P_{max}?



Answer:

① $Z_L = Z_{th}^*$

② $P_{max} = \frac{|V_{th(m)}|^2}{8 R_{th}}$ ⇒ Note that $V_{th} = V_m$ NOT V_{rms}

$(4 \parallel 8-J6)$

$\frac{(4)(8-J6)}{12-J6} + J5 = 2.93 + J4.47$

$V_{oc} = \left(\frac{8-J6}{12-J6}\right)(10 \angle 0°) = 7.33 - J1.33$

$= 7.45 \angle -10.23°$

$Z_L = Z_{th}^* = \underline{\underline{2.93}} - J4.47$

$\hookrightarrow R_{th}$

$P_{max} = \frac{(7.45)^2}{8(2.93)} = 2.37$ watt.

$(Y-Y)$ connection

V_{phase} ↕ $\sqrt{3}$ $| \; | \measuredangle \theta + 30^\circ$

V_{line}

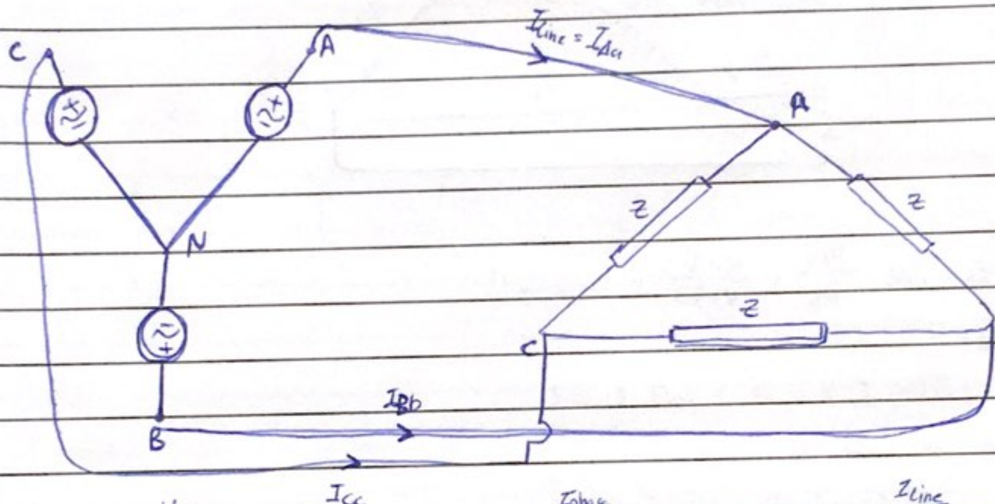
I_{line}

$(Y-\Delta)$ connection

V_{Line}

I_{Line}

I_{phase}



V_{line}	I_{phase}	I_{line}
$V_{ab} = \| \; \| \measuredangle \theta$	$I_{ab} = \| \; \| \measuredangle \theta_{ab}$	$I_{Aa} = \|I_{ab}\| * \sqrt{3} \; \measuredangle \theta_{ab} - 30^\circ$
$V_{bc} = \| \; \| \measuredangle \theta - 120^\circ$	$I_{bc} = \| \; \| \measuredangle \theta_{ab} - 120^\circ$	$I_{Bb} = \|I_{bc}\| * \sqrt{3} \; \measuredangle \theta_{ab} - 120^\circ - 30^\circ$
$V_{ca} = \| \; \| \measuredangle \theta + 120^\circ$	$I_{ca} = \| \; \| \measuredangle \theta_{ab} + 120^\circ$	$I_{Cc} = \|I_{ca}\| * \sqrt{3} \; \measuredangle \theta_{ab} + 120^\circ - 30^\circ$

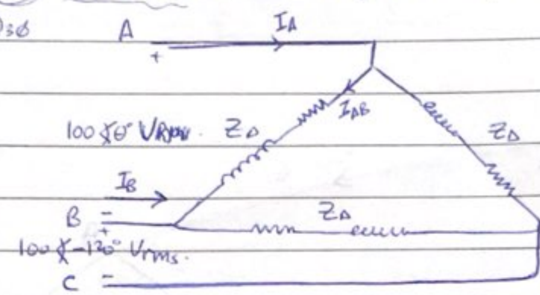
Ex For the circuit shown, if ($Z_\Delta = 4+J3$) Ω and ($I_B = |I_B| \angle \theta$) Arms.

find: ① The value of $|I_B|$ (in Arms).

② The value of θ (in degree).

③ The total reactive power consumed by the load (in VAr)

$Q_{3\phi}$



Answer:

$$I_\Delta = I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{100 \angle 0°}{4+J3} = 20 \angle -36.7°$$

$$I_A = \sqrt{3}|I_\Delta| \angle \theta_{I_\Delta} - 30°$$

$$= \sqrt{3}(20) \angle -36.7 - 30 = 20\sqrt{3} \angle -66.7°$$

①+②: $I_B = |I_A| \angle \theta_{I_A} - 120°$

$$= 20\sqrt{3} \angle -66.7 - 120 = 20\sqrt{3} \angle -186.7°$$

③ $Q_{1\phi} = I_\Delta^2 X = (20)^2(3) = 1200$ VAr

$Q_{3\phi} = 3\, Q_{1\phi} = 3(1200) = 3600$ VAr

④ Total complex power.

$$S_{3\phi} = P_{3\phi} + J Q_{3\phi}$$

$P_{1\phi} = I^2 R = (20)^2(4) = 1600$ watt

$P_{3\phi} = 3 P_{1\phi} = 3(1600) = 4800$ watt.

$$S_{3\phi} = 4800 + J3600$$

OR

$$S_{1\phi} = VI^* = (100 \angle 0°)(20 \angle -36.7)^*$$

$$= 2000 \angle 36.7°$$

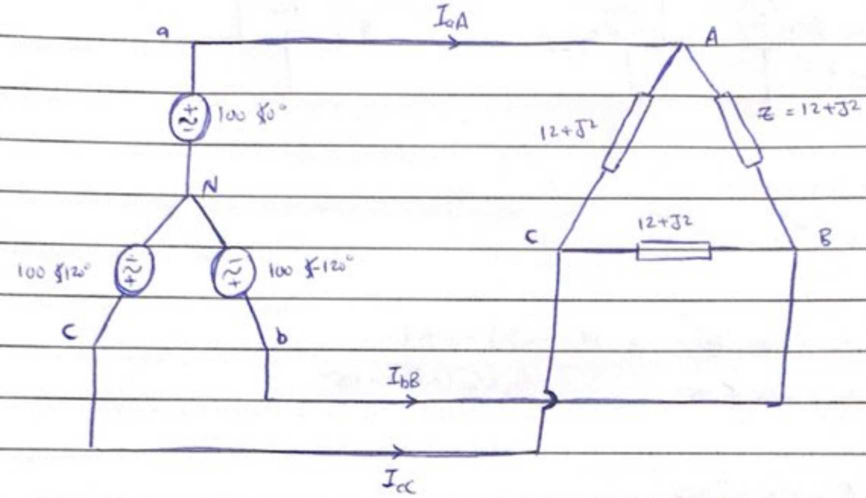
$$S_{3\phi} = 3S_{1\phi} = 3\,(2000 \angle 36.7°) = 6000 \angle 36.7°$$

⑤ Power factor of the load ⇒ $pf = \cos(\theta_Z) = \cos(36.7) = 0.8$ lagging.

⑥ apparent power = $|S|$ = 6000 VA

Ex: From the circuit below:

Find: ① I_{aA} ② I_{cC} ③ Complex power consumed by the load ④ pf load



Answer:

① $V_{an} = 100\angle 0^\circ \Rightarrow V_{AB} = |V_{an}| * \sqrt{3}\ \angle\theta_{an} + 30^\circ$

$= 100\sqrt{3}\ \angle 30^\circ$

$I_{AB} = \frac{V_{AB}}{Z} = \frac{100\sqrt{3}\angle 30^\circ}{12+j2} = \frac{100\sqrt{3}}{12.2}\angle 20.5^\circ$

$I_{aA} = \sqrt{3}\ I_{AB}\ \angle\theta_{AB} - 30^\circ$

$= \sqrt{3}\left(\frac{100\sqrt{3}}{12.2}\right)\angle 20.5 - 30^\circ$

(*) Find $I_{AC} = ?$

$I_{AC} = -I_{CA}$

$I_{CA} = |I_{AB}|\ \angle\theta_{ab} + 120^\circ$

$= \frac{100\sqrt{3}}{12.2}\angle 20.5 + 120^\circ$

$I_{AC} = (-)\frac{100\sqrt{3}}{12.2}\angle 20.5 + 120^\circ$ $\pm 180^\circ$

$I_{AC} = \frac{100\sqrt{3}}{12.2}\angle 20.5 + 120 - 180^\circ$

② $I_{cC} = |I_{aA}|\ \angle\theta_{aA} + 120^\circ$

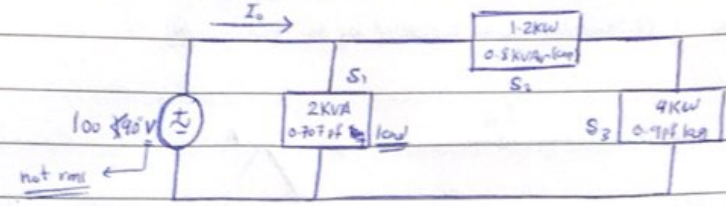
$= \sqrt{3}\left(\frac{100\sqrt{3}}{12.2}\right)\angle -9.5 + 120^\circ$

③ $S_{1\phi} = V_{AB} I_{AB}^* = (100\sqrt{3}\angle 30^\circ)\left(\frac{100\sqrt{3}}{12.2}\angle 20.5\right)^*$

$S_{3\phi} = 3 S_{1\phi} = 3(100\sqrt{3}\angle 30)\left(\frac{100\sqrt{3}}{12.2}\angle 20.5\right)^*$ VA

④ $pf = \cos(\theta_Z) = \cos(9.5) = 0.986$ lagging.

Ex: From the circuit below, find I_o and the overall complex power supplied.



$S_{total} = S_1 + S_2 + S_3$

$\underline{S_{total}} = V I_o^*$

$\rightarrow S_1 = 2000 \angle \theta_s \longrightarrow pf = \cos(\theta_s) = 0.707$

$\theta_s = -\cos^{-1}(.707) = -45°$

$S_1 = 2000 \angle -45°$

$S_2 = 1200 - j800$

$S_2 = 1442.2 \angle -33.69°$

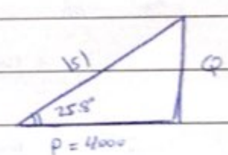
$S_3 \Rightarrow \quad P = 4000 \text{ watt}$

$pf = +\cos^{-1}(0.9) = +25.8°$

$\cos(25.8) = \dfrac{4000}{|S|}$

$|S| = 4442.87$

$S_3 = 4442.87 \angle 25.8°$



$S_{total} = (2000 \angle -45) + (1442.2 \angle -33.69) + (4442.87 \angle 25.8)$

$I_o = \left(\dfrac{S_{total}}{V}\right)^*$