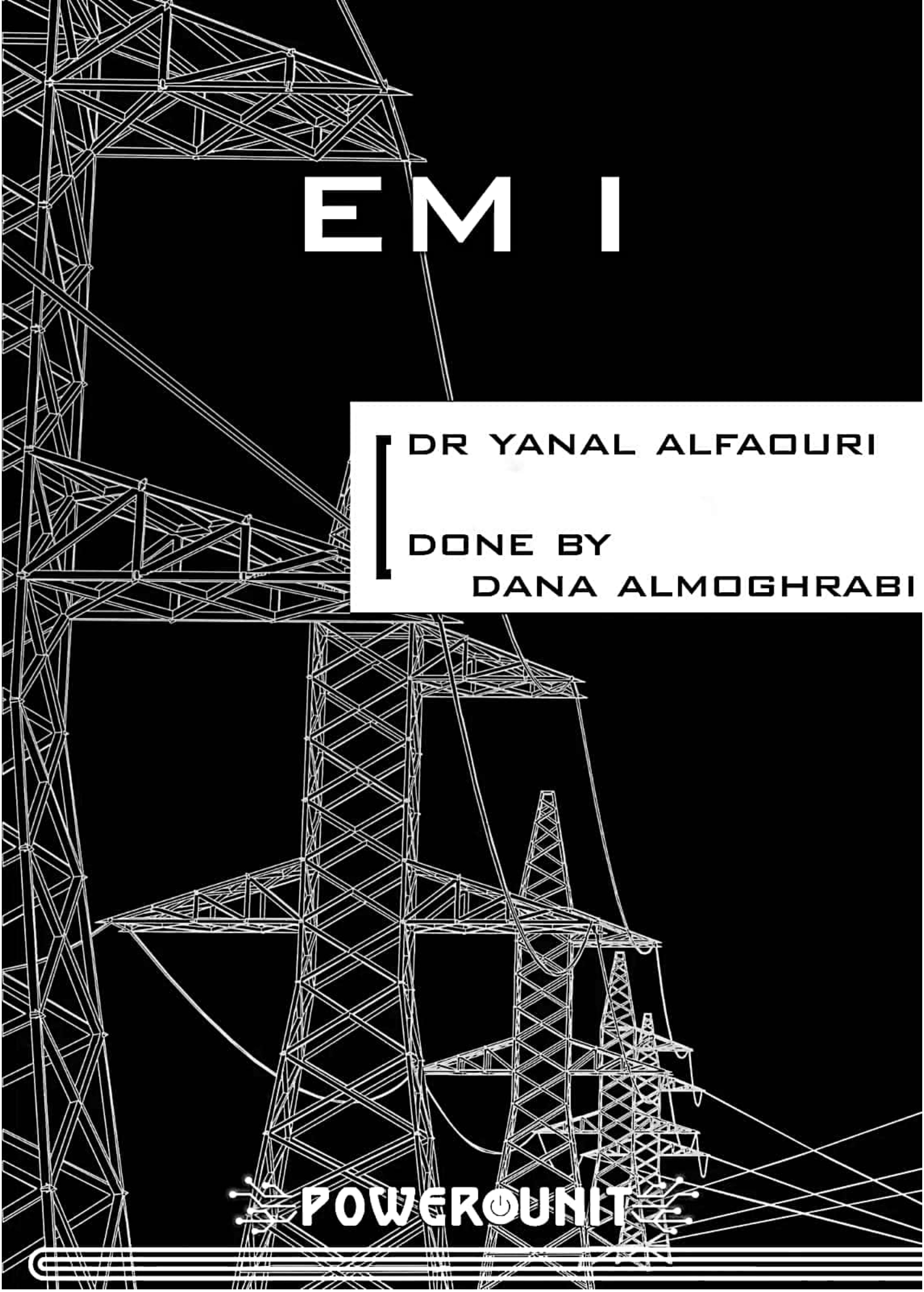


EM I

DR YANAL ALFAOURI

DONE BY
DANA ALMOGHRABI

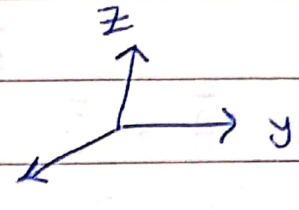
POWEROUNIT

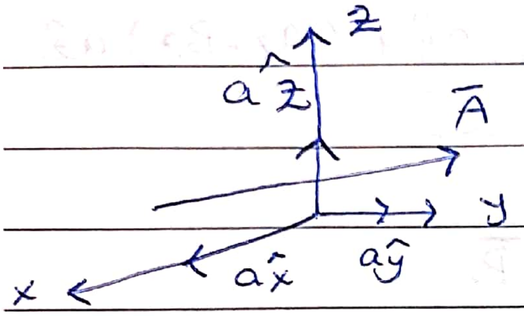
Subject Vector Algebra Day ________ Date ________

Vector → magnitude and direction. ($\bar{A}$)

Scalar → magnitude only. (A)

in cartesian coordinate :-





mutually orthogonal axes

$\hat{ax}$ = unit vector, has a magnitude that equals (1) and its direction is towards the x-axis.

$\bar{A} = A_x \hat{ax} + A_y \hat{ay} + A_z \hat{az}$ (this is the long format)

A_x, A_y, A_z → vector components

$\hat{ax}, \hat{ay}, \hat{az}$ → unit vectors

$\bar{A} = (A_x, A_y, A_z)$, (this is the short format)

vector magnitude $|\bar{A}|$:-

$$|\bar{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

vector direction :-

$\hat{a}_A$ = unit vector along $\bar{A}$ = $\frac{\bar{A}}{A}$

Subject ____________ Day ____________ Date ____________

* Operations on vectors :-

$\boxed{1}$ Addition $\Rightarrow$ $\bar{A} = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$

$$\bar{B} = B_x \hat{a_x} + B_y \hat{a_y} + B_z \hat{a_z}$$

$$\bar{C} = \bar{A} + \bar{B}$$

$$= (A_x + B_x)\hat{a_x} + (A_y + B_y)\hat{a_y} + (A_z + B_z)\hat{a_z}$$

$$= C_x \hat{a_x} + C_y \hat{a_y} + C_z \hat{a_z}$$

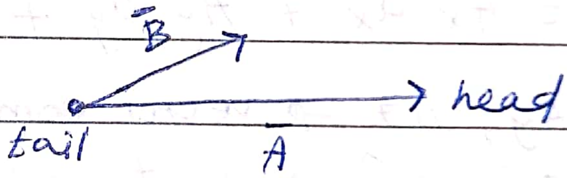
$\boxed{2}$ Substraction $\Rightarrow$ $\bar{D} = \bar{A} - \bar{B}$

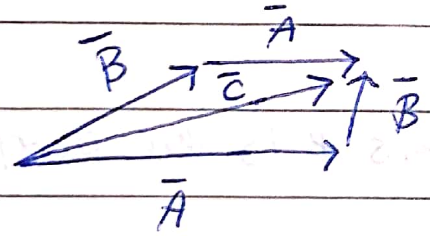
$$= (A_x - B_x)\hat{a_x} + (A_y - B_y)\hat{a_y} + (A_z - B_z)\hat{a_z}$$

$$= D_x \hat{a_x} + D_y \hat{a_y} + D_z \hat{a_z}$$

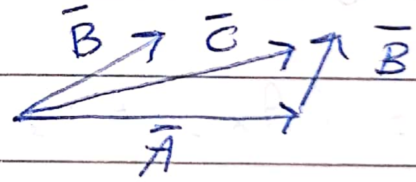
Graphical :-

$\bar{C} = \bar{A} + \bar{B}$

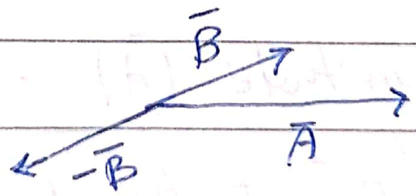


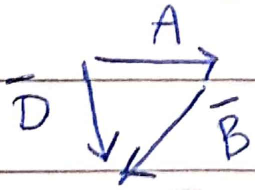


or



$\bar{D} = \bar{A} - \bar{B}$





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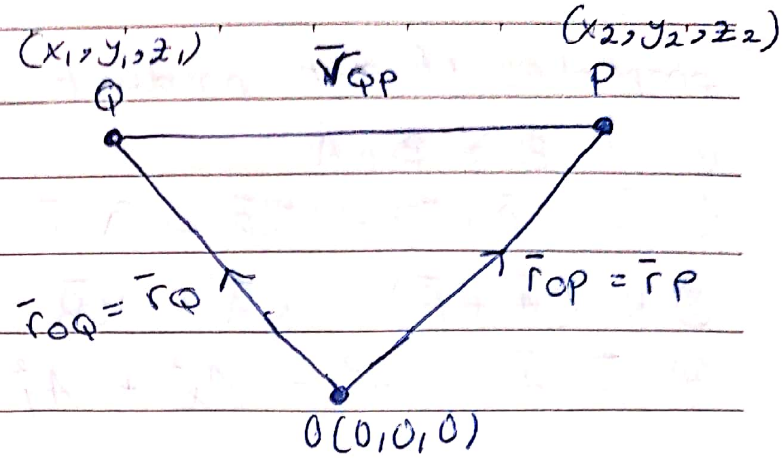
Distance :-



$\bar{r}_{QP} = \bar{r}_P - \bar{r}_Q$

$= (x_2, y_2, z_2) - (x_1, y_1, z_1)$

$= (x_2 - x_1)\hat{a_x} + (y_2 - y_1)\hat{a_y} + (z_2 - z_1)\hat{a_z}$

$= -\bar{r}_{PQ}$

$r_{QP} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$\hat{a}_{r_{QP}} = \dfrac{\bar{r}_{QP}}{r_{QP}}$

$\boxed{3}$ Multiplication $\Rightarrow$ < Dot product (scalar) / cross product (vector)

(A) Dot product :- $\bar{A} \cdot \bar{B} = |\bar{A}|\,|\bar{B}| \cos\theta_{AB}$

if $\bar{A} = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$

$\bar{B} = B_x \hat{a_x} + B_y \hat{a_y} + B_z \hat{a_z}$

then $\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z$

$\hat{a_x} \cdot \hat{a_x} = (1)(1) \cos 0^\circ = 1$

$\hat{a_x} \cdot \hat{a_y} = (1)(1) \cos 90^\circ = 0$

$\cos\theta_{AB} = \dfrac{\bar{A} \cdot \bar{B}}{AB}$ then $\theta_{AB} = \cos^{-1} \dfrac{\bar{A} \cdot \bar{B}}{AB}$

$\hat{a}_n \cdot \hat{a}_m = 0$ if $n \neq m$

$= 1$ if $n = m$

Subject ______ Day ______ Date ______

properties of dot product :-

① $\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$

② $\bar{A} \cdot (\bar{B} + \bar{C}) = \bar{A} \cdot \bar{B} + \bar{A} \cdot \bar{C}$

③ $K(\bar{A} + \bar{B}) = K\bar{A} + K\bar{B}$

④ $\bar{A} \cdot \bar{A} = A^2 = A_x^2 + A_y^2 + A_z^2$

(B) Cross product :- $|\bar{A} \times \bar{B}| = AB \sin \theta_{AB}$

$$\theta_{AB} = \sin^{-1} \frac{|\bar{A} \times \bar{B}|}{AB}$$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{a_x}(A_yB_z - A_zB_y) - \hat{a_y}(A_xB_z - A_zB_x) + \hat{a_z}(A_xB_y - A_yB_x)$$

$|\hat{a_x} \times \hat{a_x}| = (1)(1) \sin 0° = 0$

$|\hat{a_x} \times \hat{a_y}| = 1$

properties of cross product :-

① $\bar{A} \times \bar{B} \neq \bar{B} \times \bar{A}$

② $\bar{A} \times \bar{B} = -\bar{B} \times \bar{A}$

③ $\bar{A} \times (\bar{B} + \bar{C}) = (\bar{A} \times \bar{B}) + (\bar{A} \times \bar{C})$

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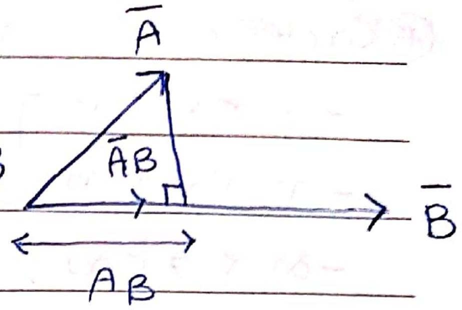
* Components of a vector :-



A_B = scalar projection of A along B

$\bar{A}_B$ = vector projection of A along B

$$\cos\theta_{AB} = \frac{A_B}{A}$$

$$A_B = A\cos\theta_{AB}$$

$$\cos\theta_{AB} = \frac{\bar{A}\cdot\bar{B}}{AB} \quad , \quad \hat{a_B} = \frac{\bar{B}}{B}$$

$$= \frac{\bar{A}\cdot\hat{a_B}}{A}$$

$$A_B = \bar{A}\cdot\hat{a_B}$$

so $\bar{A}_B = A_B\,\hat{a_B}$

$= (\bar{A}\cdot\hat{a_B})$, if scalar

$= (\bar{A}\cdot\hat{a_B})\,\hat{a_B}$, if vector

Example :- Given $\bar{A} = 3\hat{a_x} + 4\hat{a_y} + \hat{a_z}$

$\bar{B} = 2\hat{a_y} - 5\hat{a_z}$

find : θ_{AB}, A_B, $\bar{A}_B$?

Sol $\theta_{AB} = \cos^{-1}\dfrac{\bar{A}\cdot\bar{B}}{AB} = \cos^{-1}\left(\dfrac{3}{\sqrt{26}\sqrt{29}}\right) = 83.73^{\circ}$

$\bar{A}\cdot\bar{B} = 0 + 8 + -5 = 3$, $A = \sqrt{26}$, $B = \sqrt{29}$

$A_B = \bar{A}\cdot\hat{a_B} = \dfrac{8-5}{\sqrt{29}} = \dfrac{3}{\sqrt{29}} = 0.557$, $\hat{a_B} = \dfrac{(0,2,-5)}{\sqrt{29}}$

$\bar{A}_B = A_B\,\hat{a_B} = \dfrac{3}{\sqrt{29}}\left(\dfrac{(0,2,-5)}{\sqrt{29}}\right) = \dfrac{6\hat{a_y} - 15\hat{a_z}}{29}$

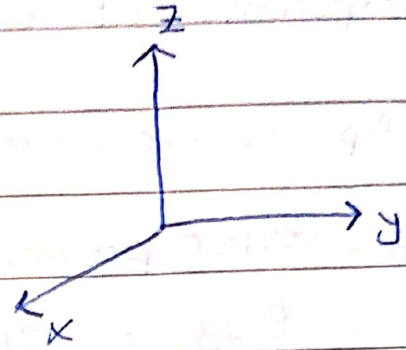
$= 0.207\hat{a_y} - 0.517\hat{a_z}$

Subject Coordinate System & Transformation.   Day ______   Date ______

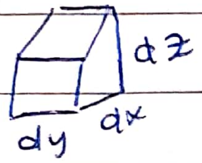
Cartesian coordinate :-

$-\infty < x < \infty$
$-\infty < y < \infty$
$-\infty < z < \infty$

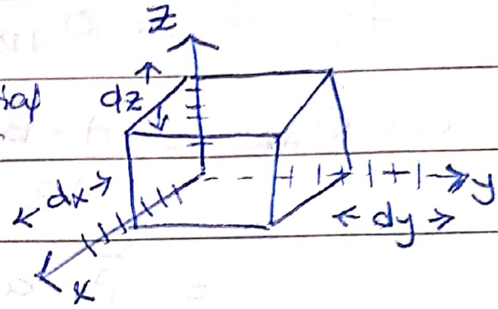
3D object
infinite solid
box



unit vectors $\Rightarrow$ $a\hat{x}$, $a\hat{y}$, $a\hat{z}$



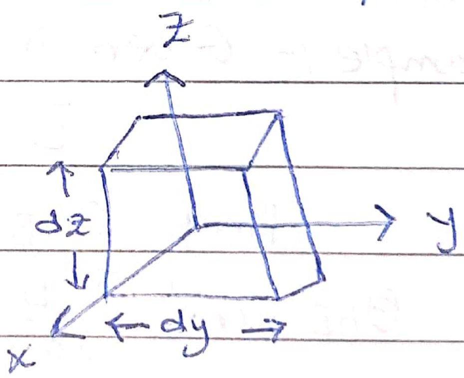
dx, dy, dz :- differential elements



Differential length $\Rightarrow$ vector quantity.
$(\bar{dl})$

$$\bar{dl} = dx\, a\hat{x} + dy\, a\hat{y} + dz\, a\hat{z}$$

Differential normal surface area $\Rightarrow$ vector quantity.
$(\bar{ds})$



$\bar{ds}$ front $= dy\,dz\, a\hat{x}$

$\bar{ds}$ back $= -dy\,dz\, a\hat{x}$

$\bar{ds}$ right $= dx\,dz\, a\hat{y}$

$\bar{ds}$ left $= -dx\,dz\, a\hat{y}$

$\bar{ds}$ top $= dx\,dy\, a\hat{z}$

$\bar{ds}$ bottom $= -dx\,dy\, a\hat{z}$

الاشارة السابقة تدل على الاتجاه.

Subject ________ Day ________ Date ________

Differential volume (dv) $\Rightarrow$ scalar quantity.

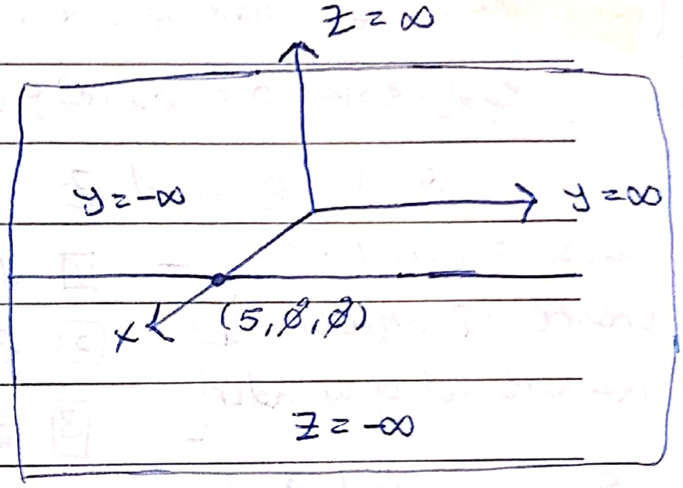
$dv = dx\,dy\,dz$

* 2D Surface :- by fixing one variable, if x is constant then $-\infty < \frac{y}{z} < \infty$

example: $x = 5$, infinite plane parallel to yz plane.

parallel $\to$ $x \neq 0$

along $\to$ $x = 0$



y = constant $\to$ infinite plane along xz ($y = 0$)

$\to$ infinite plane parallel to xz ($y \neq 0$)

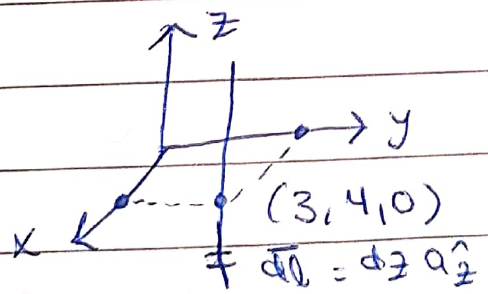
z = constant $\to$ infinite plane along xy ($z = 0$)

$\to$ infinite plane // xz ($z \neq 0$)

* 1D surface :- by fixing two variables

if x, y are constant then $-\infty < z < \infty$

$x = 3$, $y = 4$



infinite line // z axis if $x \neq 0$, $y \neq 0$

infinite line along x axis if $x = 0$ and $y = 0$

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Subject ⊛ Cylindrical Coordinate Day Date

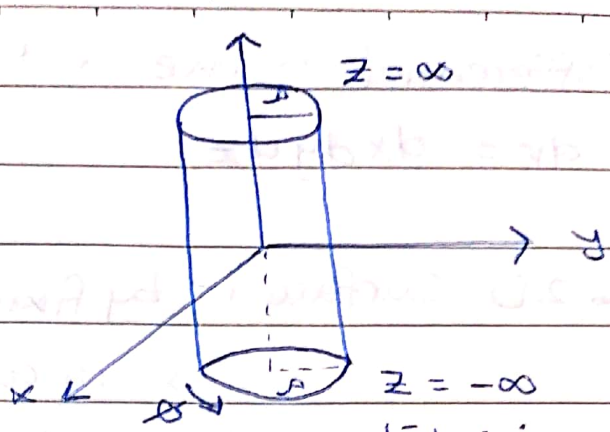
ما في نص قطر سالب $0 \le \rho \le \infty$

$0 \le \theta \le 2\pi$

$-\infty \le z \le \infty$

$\theta = 0^\circ$ at x-axis

$\theta = 90^\circ$ at y-axis



الحركة تبدأ من the x axis +ve الى the y axis +ve

ثم -ve x axis الى -ve y axis (x-y plane)

نصف القطر : ρ

الارتفاع : z

قاعدة الاسطوانة فيها جزء منحني curvature عشان نوصفه احنا بحاجة لزاوية

$\rho \perp \theta$ and z

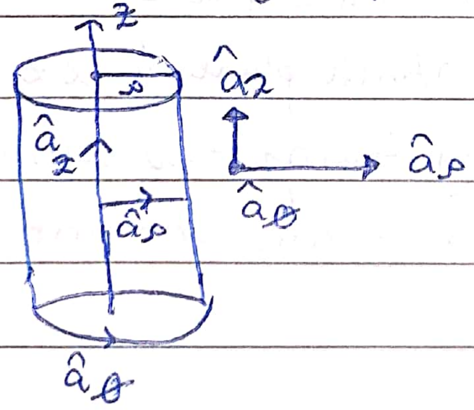
those 3 variables create 3D object; infinite solid cylinder

1. ρ
2. θ
3. z

لازم نحافظ على الترتيب

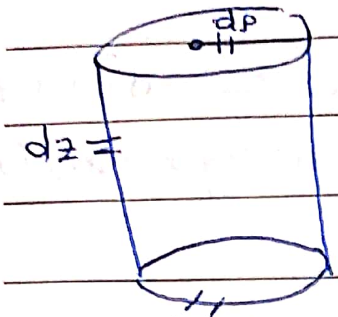
solid = filled from the inside.

unit vectors : $\hat{a}_\rho$, $\hat{a}_\theta$, $\hat{a}_z$

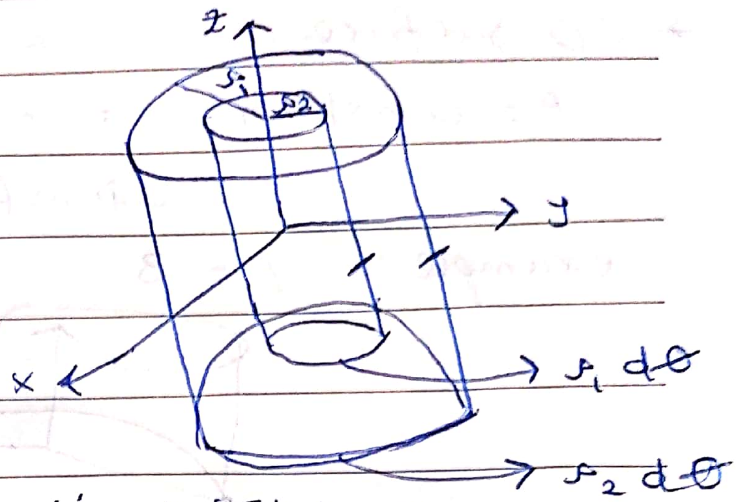


Differential elements $\Rightarrow d\rho \,,\, \boxed{\rho\, d\theta} \,,\, dz$

curvature لانه فيه $d\theta$ ما بنفع تكون



$d\theta$ X ما بنفع



$z_1 = z_2$ الاسطوانتين الهم نفس الارتفاع

بس الاختلاف بنصف القطر $\rho_2 > \rho_1$

المقطع الداخلي طوله $= \rho_1\, d\theta$

المقطع الخارجي طوله $= \rho_2\, d\theta$

بسبب اختلاف نصف القطر

$\bar{dl} = d\rho\, \hat{a}_\rho + \rho\, d\theta\, \hat{a}_\theta + dz\, \hat{a}_z$ [vector]

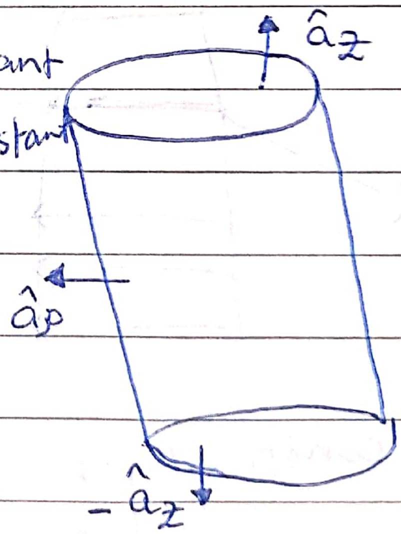
Differential normal surface area ($\bar{ds}$)

$\bar{ds}_{side} = \rho\, d\theta\, dz\, \hat{a}_\rho$, ρ = constant

$\bar{ds}_{top} = \rho\, d\rho\, d\theta\, \hat{a}_z$

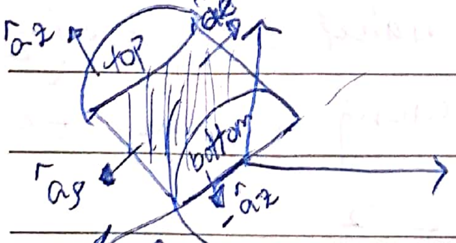
$\bar{ds}_{bottom} = -\rho\, d\rho\, d\theta\, \hat{a}_z$

z = constant



if θ = constant, then:

$\bar{ds}_{cut} = d\rho\, dz\, \hat{a}_\theta$



هنا، عنا سطح جديد خارج لقطع بس صار يعوض $\hat{a}_\rho$ ، ع يعوض

$\hat{a}_\theta$

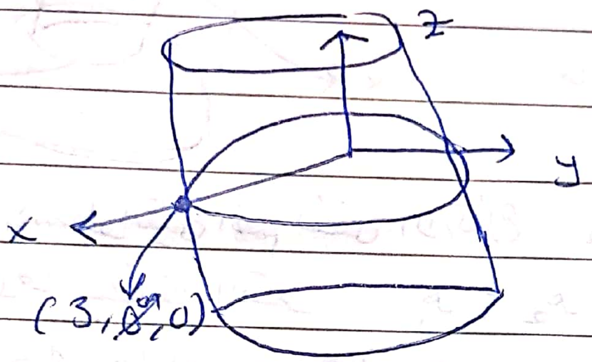
Subject ______ Day ______ Date ______

$dv = \rho \, d\rho \, \theta \, dz$ [scalar]

* 2D surface :-

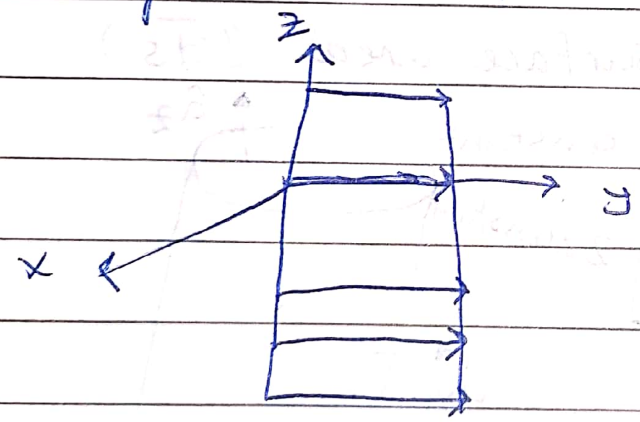
ρ = constant → infinite hollow cylinder ($\rho \neq 0$)
→ infinite line along z-axis ($\rho = 0$)

example :- $\rho = 3$



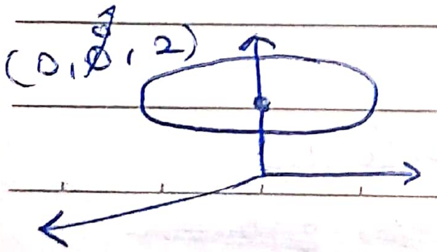
θ = constant → infinite plane along yz plane ($\theta \neq 0°$)
→ infinite plane along xz plane ($\theta = 0°$)

example :- $\theta = 90°$



z = constant → infinite disk parallel to xyplane ($z \neq 0$)
→ infinite plane along xy plane ($z = 0$)

example : z = 2

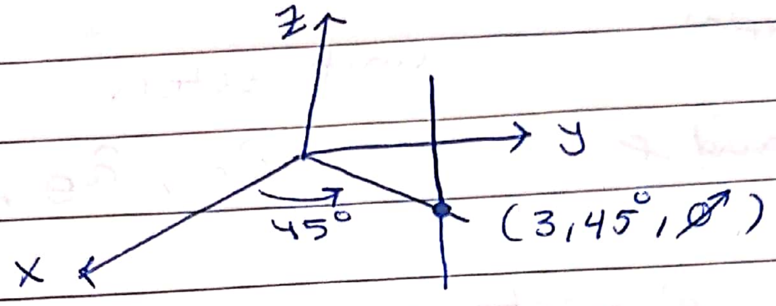


Subject ______ Day ______ Date ______

* 1D segment :

ρ, θ are constant → infinite line // z-axis ($\rho \neq 0$)
→ infinite line along z-axis ($\rho = 0$)

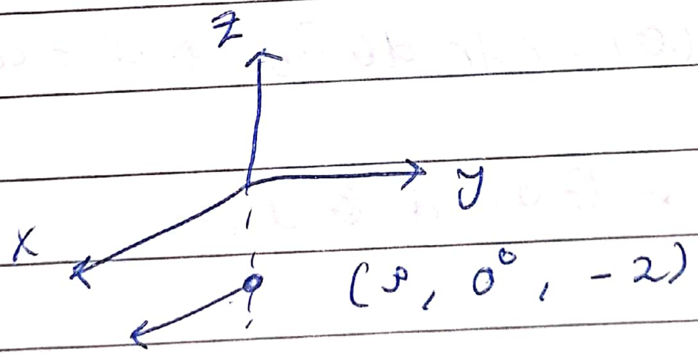
example : $\rho = 3$, $\theta = 45°$



ρ, z are constant → circle // xy plane ($z \neq 0$, $\rho \neq 0$)
→ circle along xy plane ($z = 0$, $\rho = 0$)
→ point if $\rho = 0$

θ, z are constant → semi-infinite line (ray)

example : $\theta = 0°$, $z = -2$



* by fixing 3 variables → point

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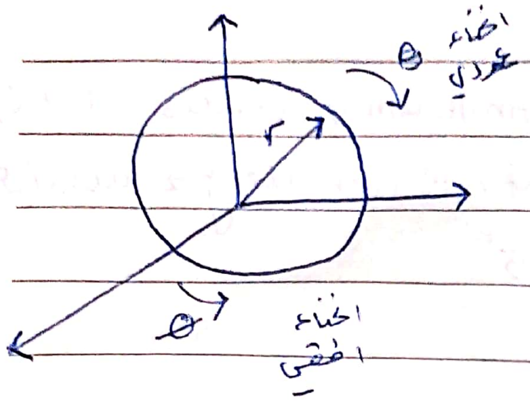
Subject (*) Spherical coordinate Day ______ Date ______



$0 \leq r \leq \infty$
$0 \leq \theta \leq \pi$
$0 \leq \phi \leq 2\pi$
} 3D object: infinite solid sphere

$r \perp \theta$ and ϕ

unit vectors: $\hat{a}_r, \hat{a}_\theta, \hat{a}_\phi$

Differential elements :-

dr , $r\,d\theta$, $r\sin\,d\phi$

$r\,d\theta$ ← بسبب اختلاف نصف القطر (كل ما كبر نصف القطر بيزيد طول arch)

$r\sin\,d\phi$ ← بسبب اختلاف مساحة القطع ونصف القطر

$\bar{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$

$\bar{ds}$ surface $= r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r \rightarrow r = \text{constant}$

$\bar{ds}$ horizontal cut $(\theta) = r\sin\theta\,dr\,d\phi\,\hat{a}_\theta \rightarrow \theta = \text{constant}$

$\bar{ds}$ vertical cut $(\phi) = r\,dr\,d\theta\,\hat{a}_\phi \rightarrow \phi = \text{constant}$

$dv = r^2\sin\theta\,dr\,d\theta\,d\phi$

* 2D surface :-

$r = \text{constant}$ $\left(\begin{array}{l} 0 \leq \theta \leq \pi \\ 0 \leq \phi \leq 2\pi \end{array}\right)$
→ Hollow sphere $(r \neq 0)$
→ point $(r = 0)$

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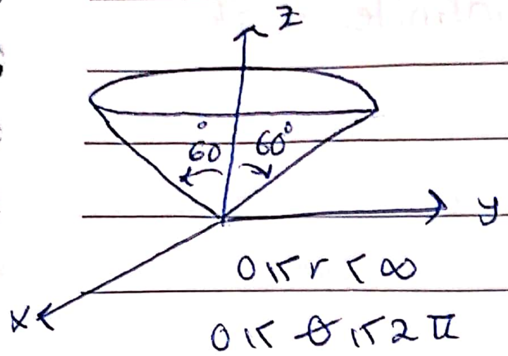
θ = constant $\rightarrow$ infinite hollow cone

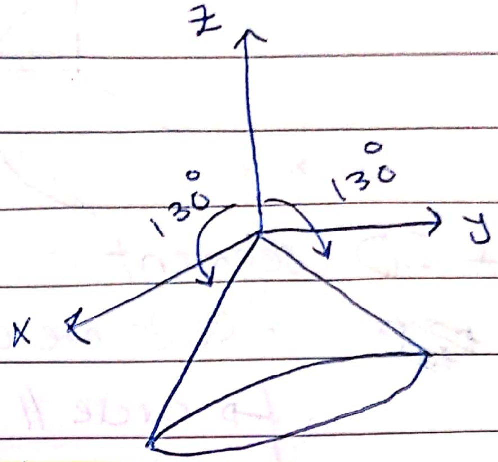
examples:

$\theta = 60^\circ$ $\qquad\qquad\qquad\qquad$ $\theta = 130^\circ$



$0 \leq r \leq \infty$

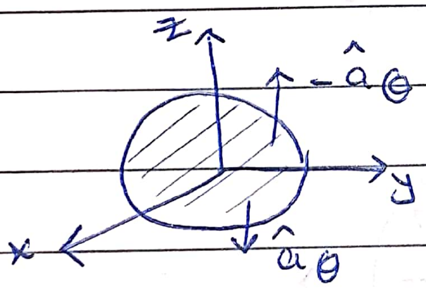
$0 \leq \phi \leq 2\pi$



Note:- if θ is constant $\rightarrow$ hollow
if θ varies $\rightarrow$ solid

Special Cases:

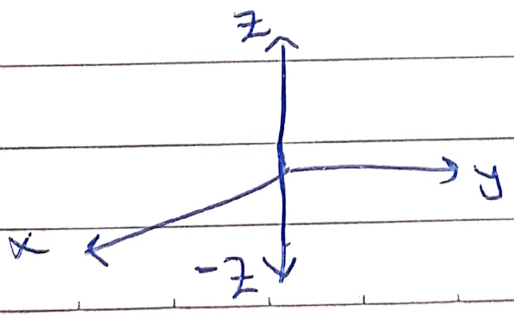
$\boxed{1}$ $\theta = 90^\circ \rightarrow$ infinite disk along xy-plane



θ [illegible]

$\bar{ds} = r \sin\theta \, dr \, d\phi \, \hat{a}_\theta$

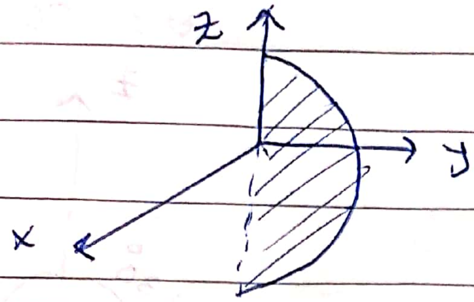
$\boxed{2}$ $\theta = 0^\circ \rightarrow$ positive z-axis
$\theta = 180^\circ \rightarrow$ negative z-axis



Subject ______ Day ______ Date ______

$\theta =$ constant

example : $\theta = 90^\circ$



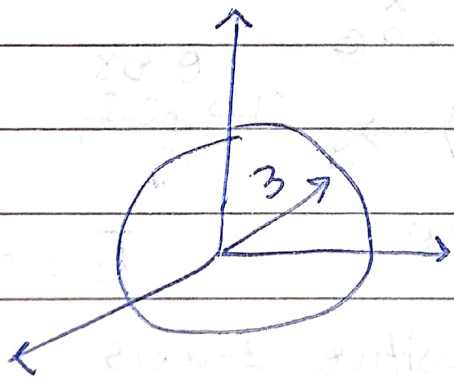
semi infinite disk

* 1D segment :-

r & θ are constants

- circle // xy plane ($r \neq 0$, $\theta \neq 90^\circ$)
- circle along xy plane ($r = 0$, $\theta = 90^\circ$)
- point on the origin ($r = 0$, ~~θ ?~~)
  - on the positive z axis ($\theta = 0^\circ$)
  - on the negative z axis ($\theta = 180^\circ$)

example : $r = 3$, $\theta = 90^\circ$



$\bar{dl} = r \sin\theta \, d\theta \, \hat{a}_\theta$

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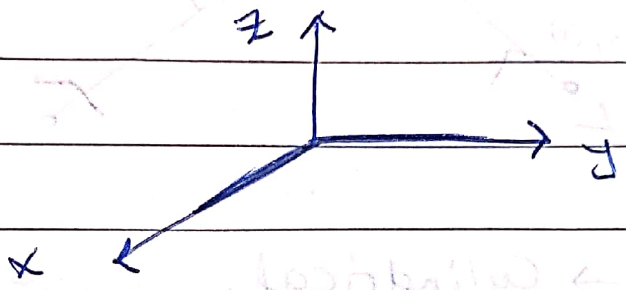
r & ϕ are constants → half circle $(r \neq 0)$
→ point $(r = 0)$

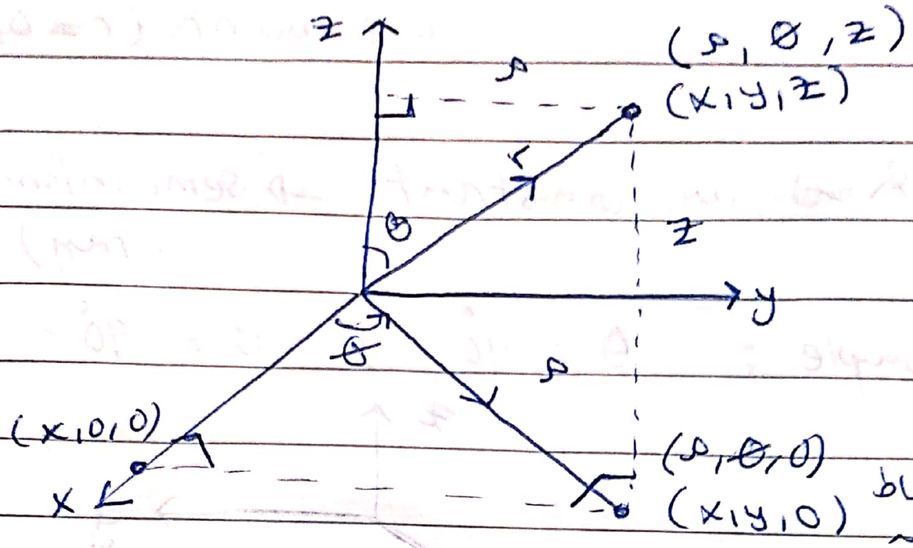
θ & ϕ are constants → semi infinite line (ray)

example : $\theta = 90^\circ$, $\phi = 90^\circ$



Subject Transformation Between Coordinates Day ________ Date ________

(*) Point conversion :-



عملنا اسقاط
عمودي لشان
خلص من الارتفاع
z

1] Cartesian → Cylindrical.

$(x, y, z) \rightarrow (\rho, \theta, z)$

$\rho = \sqrt{x^2 + y^2}$

$\theta \Rightarrow \sin\theta = \frac{y}{\rho}$

$\cos\theta = \frac{x}{\rho}$

$\tan\theta = \frac{y}{x}$ ✓

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$z = z$

2] Cylindrical → Cartesian

$(\rho, \theta, z) \rightarrow (x, y, z)$

$x = \rho \cos\theta$

$y = \rho \sin\theta$

$z = z$

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3] Cartesian → Spherical

$(x, y, z) \rightarrow (r, \theta, \phi)$

$r = \sqrt{z^2 + \rho^2} = \sqrt{x^2 + y^2 + z^2}$

$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$\sin\theta = \frac{\rho}{r}$

$\cos\theta = \frac{z}{r}$

$\tan\theta = \frac{\rho}{z}$

4] Spherical → Cartesian

$(r, \theta, \phi) \rightarrow (x, y, z)$

$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$

5] Cylindrical → Spherical

$(\rho, \theta, z) \rightarrow (r, \theta, \phi)$

$r = \sqrt{\rho^2 + z^2}$

$\phi = \tan^{-1}\left(\frac{\rho}{z}\right)$

$\theta = \theta$

6] Spherical → Cylindrical

$(r, \theta, \phi) \rightarrow (\rho, \theta, z)$

$\rho = r \sin\theta$

$\theta = \theta$

$z = r \cos\theta$

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⊛ Vector conversion :-

1] Cartesian → Cylindrical

$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + \boxed{A_z \hat{a}_z}$ remains the same

$\bar{A} = A_\rho \hat{a}_\rho + A_\theta \hat{a}_\theta + \boxed{A_z \hat{a}_z}$

$$\underbrace{\begin{bmatrix} A_\rho \\ A_\theta \\ A_z \end{bmatrix}_{3\times1}}_{\text{unknown}} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}}_{\text{conversion matrix}} \underbrace{\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}}_{\text{given}}$$

Step ① $A_\rho = \cos\theta \, A_x + \sin\theta \, A_y$

$A_\theta = -\sin\theta \, A_x + \cos\theta \, A_y$

$A_z = A_z$

Step ② replace x, y by their point laws (in the previous pages)

2] Cylindrical → Cartesian

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\theta \\ A_z \end{bmatrix}$$

3] Cartesian → Spherical

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\theta \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\theta & \sin\theta\sin\theta & \cos\theta \\ \cos\theta\cos\theta & \cos\theta\sin\theta & -\sin\theta \\ -\sin\theta & \cos\theta & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

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4) Cylindrical → Spherical

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

(*) Distance :-

① Cartesian ⇒ $Q \leftarrow d \rightarrow P$

$Q\,(x_1, y_1, z_1)$ ———— $P\,(x_2, y_2, z_2)$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

② Cylindrical ⇒ $Q\,(\rho_1, \phi_1, z_1)$ ———— $P\,(\rho_2, \phi_2, z_2)$

$$d^2 = \rho_1^2+\rho_2^2 - 2\rho_1\rho_2\cos(\phi_2-\phi_1)+(z_2-z_1)^2$$

$z, \rho \to m$

$\phi \to$ degree

ما بنفع نستخدم تعويض cartesian الا لو الوحدات ثابتة

بنقدر نستخدم فقط اذا $\phi_1 = \phi_2$: $d = \sqrt{(\rho_2-\rho_1)^2+(z_2-z_1)^2}$

③ Spherical ⇒ $Q\,(r_1, \theta_1, \phi_1)$ ———— $P\,(r_2, \theta_2, \phi_2)$

if $\phi_1 = \phi_2$ and $\theta_1 = \theta_2$ → $d = \sqrt{(r_2-r_1)^2}$

if they are not → $d^2 = r_2^2 + r_1^2 - 2r_1r_2\cos\theta_2\cos\theta_1 - 2r_1r_2\sin\theta_2\sin\theta_1\cos(\phi_2-\phi_1)$

Note:

بنقدر، كحالة spherical , cylindrical

نحول لـcartesian بالاول بعدين

نحسب distance .

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* Integrals

[I] Line integral :-

differentiallength تعنى line integral

أقصر طول ممكن الطول كامل

$\int_l \bar{A} \cdot \bar{dl}$ → we use dot product so the result is scalar.

example : $\bar{A} = A_x \hat{a}_x + A_z \hat{a}_z$

$\bar{dl} = dx\,\hat{a}_x + \cancel{dy}^{\,zero}\,\hat{a}_y + dz\,\hat{a}_z$

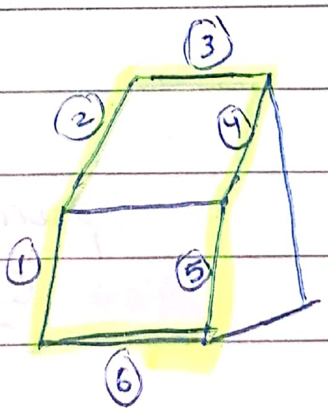
so $\int_l (A_x dx + A_z dz) = \int_x A_x dx + \int_z A_z dz$

* special case ⇒ Closed Line Integral :

difference between closed line and Line is that Line is applied on a branch and closed line is applied on a loop.

طريقة الحل نفس طريقة line بس خطوة الإشارة انه لازم خاطئ من circulation.

example :-



لا تضيع اشارة

السالب فقط

نعوض عنها

بعد التكامل

$\bar{dl}_1 = dz\,\hat{a}_z$

$\bar{dl}_2 = dx\,\hat{a}_x$

$\bar{dl}_3 = dy\,\hat{a}_y$

$\bar{dl}_4 = dx\,\hat{a}_x$

$\bar{dl}_5 = dz\,\hat{a}_z$

$\bar{dl}_6 = dy\,\hat{a}_y$

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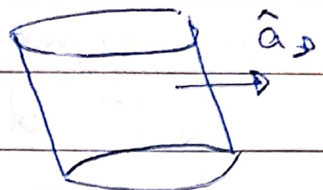
$$\oint_{l} \bar{A}\cdot\bar{dl} = \int_{l_1}\bar{A}\cdot\bar{dl}_1 + \int_{l_2}\bar{A}\cdot\bar{dl}_2 + \int_{l_3}\bar{A}\cdot\bar{dl}_3$$

$$+ \int_{l_4}\bar{A}\cdot\bar{dl}_4 + \int_{l_5}\bar{A}\cdot\bar{dl}_5 + \int_{l_6}\bar{A}\cdot\bar{dl}_6$$

$\boxed{2}$ Surface integral :-



hollow cylinder

$$\int_{S} \bar{A}\cdot\bar{ds}$$

$$\bar{ds} = \rho\, d\theta\, dz\, \hat{a}_\rho$$

if A is in Cartesian then it should be converted to cylindrical.

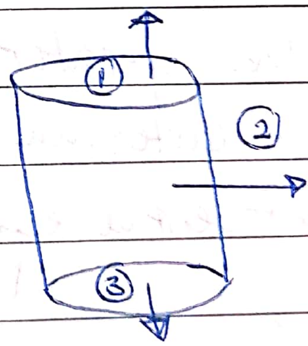
$$\bar{A} = A_\rho\, \hat{a}_\rho \quad \text{only.}$$

$$\to \int_{\theta}\int_{z} A_\rho\, \hat{a}_\rho \cdot \rho\, d\theta\, dz\, \hat{a}_\rho$$

Note عدد التكاملات = عدد d

* special case ⇒ Closed Surface Integral :

$$\oint \bar{A}\cdot\bar{ds}$$



$$S = \int_{S_1}\bar{A}\cdot\bar{ds}_1 + \int_{S_2}\bar{A}\cdot\bar{ds}_2 + \int_{S_3}\bar{A}\cdot\bar{ds}_3$$

$$\bar{ds}_1 = \rho\, d\rho\, d\theta\, \hat{a}_z$$

$$\bar{ds}_2 = \rho\, d\theta\, dz\, \hat{a}_\rho$$

$$\bar{ds}_3 = -\rho\, d\rho\, d\theta\, \hat{a}_z$$

بالك بتحط اشارة السالب اما l & .

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$\boxed{3}$ Volume integral :-

$\int_V A \, dv$

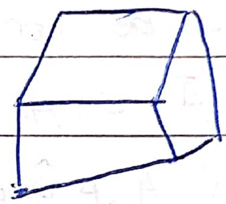
dv → scalar quantity so we do not need dot product

if A was given as a vector then we need to find its magnitude first then integrate,

$\int_V |\bar{A}| \, dv$

example :

$\int_z \int_y \int_x A \, dx \, dy \, dz$



(*) Del Operator (∇)

Del operator is a vector quantity.

- in cartesian : $\nabla = \frac{\partial}{\partial x} \hat{a}_x + \frac{\partial}{\partial y} \hat{a}_y + \frac{\partial}{\partial z} \hat{a}_z$

differential elements (D.E) → dx, dy, dz

كل D.E بنحولها ل partial و بكتبها تماماً

Note: ∂ : partial , d : differential

- in cylindrical : $\nabla = \frac{\partial}{\partial \rho} \hat{a}_\rho + \frac{\partial}{\rho \partial \phi} \hat{a}_\phi + \frac{\partial}{\partial z} \hat{a}_z$

- in spherical : $\nabla = \frac{\partial}{\partial r} \hat{a}_r + \frac{\partial}{r d\theta} \hat{a}_\theta + \frac{\partial}{r \sin\theta \partial \phi} \hat{a}_\phi$

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Usage of Del operator $\Rightarrow$ ① Gradiant (∇V)
② Divergence $(\nabla \cdot \bar{A})$
③ Curl $(\nabla \times \bar{A})$
④ Laplacian $(\nabla \cdot (\nabla V) = \nabla^2 V)$

* Gradiant (∇V) :-

V : scalar , ∇ : vector $\Rightarrow$ vector quantity

in Cartesian : $\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$

(as well as for spherical and cylindrical.)

* Divergence $(\nabla \cdot \bar{A})$:-

∇ : vector , $\bar{A}$: vector , dot product $\Rightarrow$ scalar

in cartesian : $\nabla \cdot \bar{A} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)(A_x, A_y, A_z)$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

in cylindrical : $\nabla \cdot \bar{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{\partial A_\theta}{\rho\, d\theta} + \frac{\partial A_z}{\partial z}$

in spherical : $\nabla \cdot \bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, A_\theta)$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}$$

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* Curl $(\nabla \times \bar{A})$:-

∇ : vector , $\bar{A}$: vector , cross product $\Rightarrow$ vector

in cartesian : $\nabla \times \bar{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

in cylindrical : $\nabla \times \bar{A} = \frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$

in spherical : $\nabla \times \bar{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & A_\theta & r\sin\theta A_\phi \end{vmatrix}$

* Distance :-

$Q \xrightarrow{\bar{r}_{QP}} P$

Q: (x_1, y_1, z_1), (ρ_1, ϕ_1, z_1) P: (x_2, y_2, z_2), (ρ_2, ϕ_2, z_2)

$d = |\bar{r}_{QP}|$

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$ in cartesian

in cylindrical, $d^2 = \rho_2^2 + \rho_1^2 - 2\rho_1\rho_2 \cos(\phi_2 - \phi_1) + (z_2 - z_1)^2$

if $\phi_2 = \phi_1 \rightarrow d^2 = (\rho_2 - \rho_1)^2 + (z_2 - z_1)^2$

in spherical, $d^2 = (r_2 - r_1)^2$ if $\theta_1 = \theta_2$

and $\phi_1 = \phi_2$.

Chapter "4"

Subject Coulomb's Law Day ______ Date ______

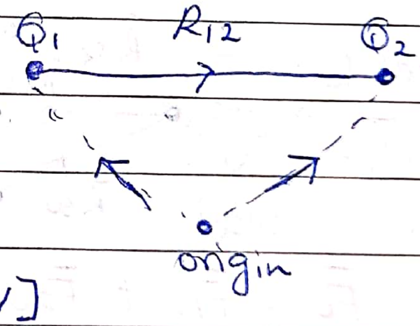
* Sources of electrostatics:

1) Point charge (Q) [coulomb]
2) Line charge distribution (ρ_L) → 1D segment
3) Surface charge distribution (ρ_s) → 2D surface
4) Volume charge distribution (ρ_v) → 3D object
5) Electric dipole.
6) Polarized dielectric.

* Major Laws ⇒ 1) Coulomb's law.
2) Gauss's law (special case of coulomb's law)

* Coulomb's Law:-

$\bar{F}$ ($\bar{F}e$) : fource



- the relation → $F \propto \dfrac{Q_1 Q_2}{R_{12}^2}$

- the law → $F = \dfrac{K\, Q_1 Q_2}{R_{12}^2}$ [N]

→ the proportionality constant

$$K = \frac{1}{4\pi\, \epsilon_0}$$

4π → depends on the units used

ϵ_0 → depends on the channel (الوسط)

ϵ_0 = free space permittivity

$= \dfrac{10^{-9}}{36\pi} = 8.85 \times 10^{-12} \left[\dfrac{F}{m}\right]$

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- F as a magnitude $\Rightarrow$ $F = \dfrac{Q_1 Q_2}{4\pi \epsilon_0 R_{12}^2}$ [N]

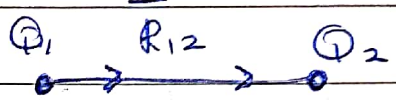
- F as a vector $\Rightarrow$ $\overline{F}_{12}$ = the force on Q_2 due to Q_1

$$\overline{F}_{12} = -\overline{F}_{21}$$

$$|\overline{F}_{12}| = |\overline{F}_{21}|$$

$$\overline{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\,\hat{a}_{R_{12}} \rightarrow \boxed{1}$$

$$\overline{F}_{12} = \frac{Q_1 Q_2 \overline{R}_{12}}{4\pi\epsilon_0 R_{12}^3} \rightarrow \boxed{2}$$

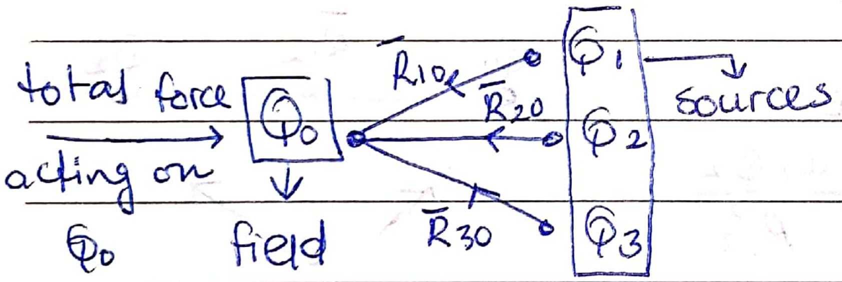


$$\frac{\overline{R}_{12}}{R_{12}} = \hat{a}_{R_{12}}$$

$$\overline{F}_{12} = \frac{Q_1 Q_2 (\overline{r}_2 - \overline{r}_1)}{4\pi\epsilon_0 |\overline{r}_2 - \overline{r}_1|^3} \rightarrow \boxed{3}$$

$$\overline{R}_{12} = \overline{r}_2 - \overline{r}_1$$

- For N-charges affecting on a certain charge :-



$$\overline{F}_0 = \overline{F}_{10} + \overline{F}_{20} + \text{----} + \overline{F}_{N0}$$

$$\overline{F}_0 = \frac{Q_1 Q_0 (\overline{r}_0 - \overline{r}_1)}{4\pi\epsilon_0 |\overline{r}_0 - \overline{r}_1|^3} + \frac{Q_2 Q_0 (\overline{r}_0 - \overline{r}_2)}{4\pi\epsilon_0 |\overline{r}_0 - \overline{r}_2|^3} + \cdots + \frac{Q_N Q_0 (\overline{r}_0 - \overline{r}_N)}{4\pi\epsilon_0 |\overline{r}_0 - \overline{r}_N|^3}$$

$$\overline{F}_0 = \frac{Q}{4\pi\epsilon_0} \sum_{K=0}^{N} \frac{Q_K (\overline{r}_0 - \overline{r}_K)}{|\overline{r}_0 - \overline{r}_K|^3} \quad [N]$$

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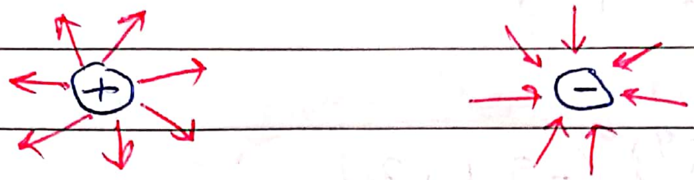
- Electric Field Intensity ($\bar{E}$) :-

$$\bar{E} = \frac{\text{force}}{\text{charge (field)}} = \frac{\bar{F}}{Q_{\text{field}}} \quad \left[\frac{N}{C}\right] \left[\frac{V}{m}\right]$$

$\left[\frac{V}{m}\right]$ → most used.

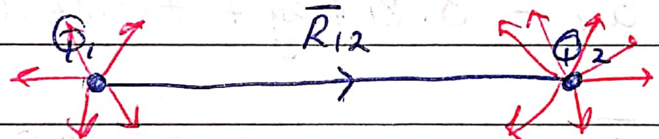


always exiting positive charge and entering negative charge

$\bar{E} \gg \rightarrow$ $\bar{E}$ اكبر قيمة كلما اقتربنا من الشحنة

$\bar{E} \ll \rightarrow$ $\bar{E}$ اقل قيمة كل ما ابتعدنا عن الشحنة

(as $r \to 0$, $E \to \infty$) (as $r \to \infty$, $E \to 0$)



في قوة متبادلة بينهم بس انا احسب مجال لبشوف مين الشحنة اي بتأثر عشان احصل بالقانون.

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{a}_r$$

$$= \frac{Q\,(\bar{r} - \bar{r}')}{4\pi\epsilon_0 |r - r'|^3}$$

the dash refers to source (without dash refers to field)

Convention :- use dashes to represent the source and without dashes to represend the field point.

- For N-charges :- $\frac{1}{4\pi\epsilon_0} \sum_{k=1}^{N} \frac{Q_k (\bar{r} - \bar{r}'_k)}{|\bar{r} - \bar{r}'_k|^3}$

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Example :- point charges 1 mC and −2 mC located at $\underset{S_1}{(3, 2, -1)}$ and $\underset{S_2}{(-1, -1, 4)}$. Find the force and electric field at a 10 nC charge located at $(0, 3, 1)$?

field

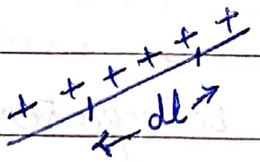
Sol $\bar{F} = \dfrac{(1\,10^{-3})(10\,10^{-9})(-3, 1, 2)}{4\pi \frac{10^{-9}}{36\pi}\sqrt{(14)^3}} + \dfrac{(-2\,10^{-3})(10\,10^{-9})(1, 4, -3)}{4\pi \frac{10^{-9}}{36\pi}\sqrt{(26)^3}}$ = field − source

$\bar{F} = -6.507\,\hat{a}_x - 3.817\,\hat{a}_y + 7.506\,\hat{a}_z$ mN

$\bar{E} = \dfrac{\bar{F}}{Q_{field}} = \dfrac{\bar{F}}{10\,10^{-9}} = -650.7\hat{a}_x - 381.7\hat{a}_y + 750.6\hat{a}_z$ KV/m.

(*) Electric Field Due to Continuous Charge Distribution :-

1) Line charge distribution (1D segment)



ρ_L : line charge density $[\frac{C}{m}]$

$Q = \int_l \rho_l \, dl \rightarrow$ لازم تكون scalar

عمره Q مستحيل تكون vector

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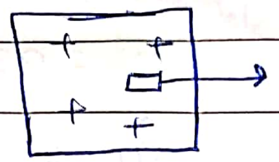
2) Surface charge distribution (2D surface)



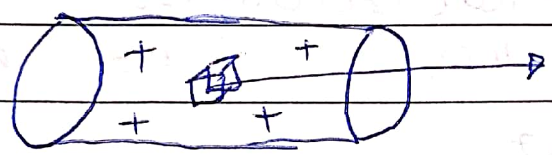
ρ_s : surface charge density $\left[\frac{c}{m^2}\right]$

$Q = \int_s \rho_s \, ds$

كان لازم تكون scalar

عشوع يكون vector

3) Volume charge distribution (3D object)

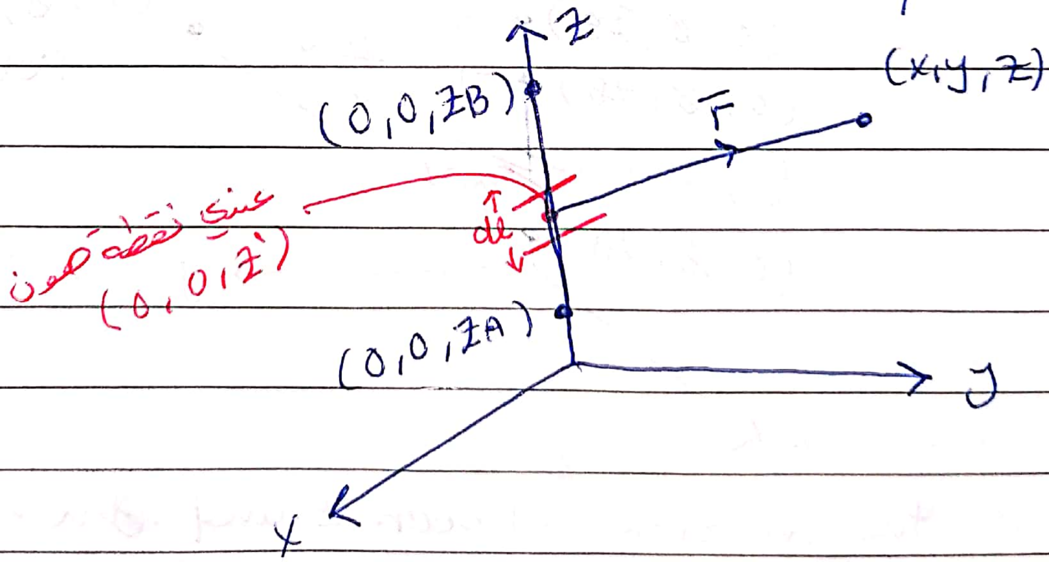


ρ_v : volume charge density $\left[\frac{c}{m^3}\right]$

$Q = \int_v \rho_v \, dv$.

Example (+ Derivation) :-

consider a finite line along z-axis carry charge ρ_L (c/m). Find the electric field at point (x, y, z)



$$Q = \int_l \rho_L \, dl$$

$$\bar{E} = \int_l \frac{\rho_L \, dl}{4\pi\epsilon_0 \, r^2}$$

constant

ما بنشتغلها لأنه بالإشي scalar

$dl = dz \, \boxed{\hat{a}_z}$

$\hat{a}_r = \dfrac{\bar{r}}{r}$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{zA}^{zB} \frac{x\hat{a}_x + y\hat{a}_y + (z-\acute{z})\hat{a}_z \, dz}{\left(\sqrt{x^2+y^2+(z-\acute{z})^2}\right)^3}$$

$\bar{r} = x\hat{a}_x + y\hat{a}_y + (z-\acute{z})\hat{a}_z$

$r = \sqrt{x^2+y^2+(z-z)^2}$

by converting to cylindrical coordinate

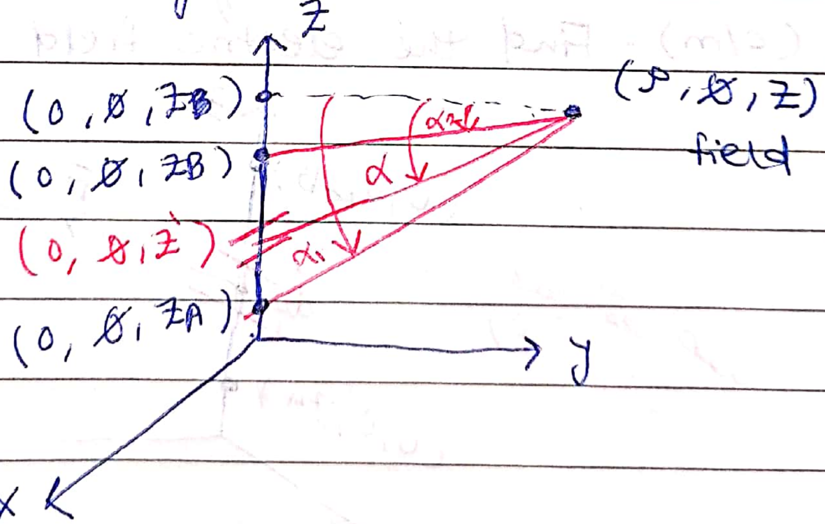
(لأنه line الها علاقة من cylindrical.)

$\bar{r} = \rho \, \hat{a}_\rho + (z-\acute{z})\hat{a}_z$

$r = \sqrt{\rho^2 + (z-\acute{z})^2}$, $dl = d\acute{z}$

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0} \int_{zA}^{zB} \frac{\rho\hat{a}_\rho + (z-\acute{z})\hat{a}_z}{[\rho^2 + (z-\acute{z})^2]^{3/2}} \, d\acute{z}$$

convert the integral limits :-

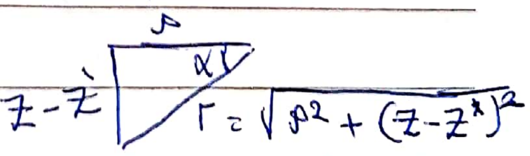


What is the relation between z and α ?

$\sin\alpha = (z-\acute{z})/r$

$\cos\alpha = \rho/r$

$\tan\alpha = (z-\acute{z})/\rho$



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اسهل اشي اشتق $\tan\alpha$ ضمنياً عشان اصل لـ dz و $d\alpha$

$$z - z' = r\sin\alpha = \rho\tan\alpha$$

from $z - z' = \rho\tan\alpha$

$$-dz' = \rho\sec^2\alpha\, d\alpha$$

$$\boxed{dz' = -\rho\sec^2\alpha\, d\alpha}$$

$$r^2 = \rho^2 + (z - z')^2$$
$$= \rho^2 + \rho^2\tan^2\alpha$$
$$= \rho^2(1 + \tan^2\alpha)$$
$$= \rho^2\sec^2\alpha$$

$$r^3 = \rho^3\sec^3\alpha = \left[\rho^2 + (z - z')^2\right]^{3/2}$$

$$\boxed{\begin{aligned} \text{So } \rho &= r\cos\alpha \\ z - z' &= r\sin\alpha \\ r &= \rho\sec\alpha \end{aligned}}$$

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0}\int_{\alpha_1}^{\alpha_2} \frac{\overbrace{\rho\sec\alpha(\cos\alpha\,\hat{a}_\rho + \sin\alpha\,\hat{a}_z)}^{\rho\hat{a}_\rho}\overbrace{(-\rho\sec^2\alpha\, d\alpha)}^{dz'}}{\underbrace{\rho^3\sec^3\alpha}_{r^3}}$$

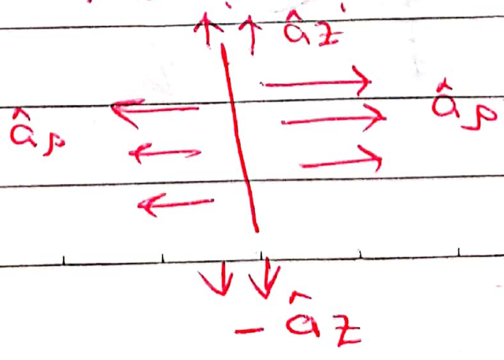
$$\bar{E} = \frac{-\rho_L}{4\pi\epsilon_0\rho}\left[(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\rho - (\cos\alpha_2 - \cos\alpha_1)\hat{a}_z\right] \frac{V}{m}$$

ليه طلع عنا الجواب الـ 2 components ؟

لما يكون عندك الـ line along z - axis مش منتهي بتنتج مركبة الـ electric field

و يكون طارح من الـ line بكل الاتجاهات .

This is finite line.



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Special case: infinite line

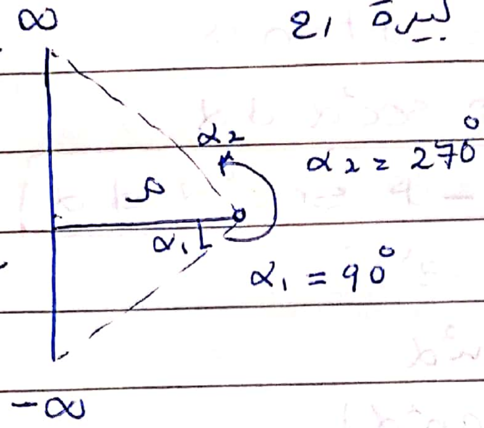
المسافة بين line ونقطة field

بغض النظر عن اديا تكون كبيرة ، ع

تضل مهملة مقارنة

مع طول line

ρ: the shortest distance between the field and the source or the extension of the source.



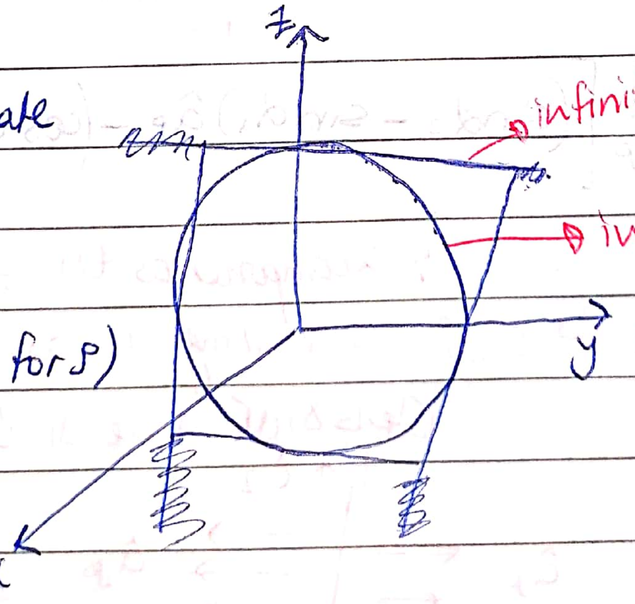
$$\boxed{\bar{E} \text{ for infinite line} = \frac{\rho_l \bar{\rho}}{2\pi\epsilon_0 \rho^2} = \frac{\rho_l}{2\pi\epsilon_0} \hat{a}_\rho}$$

always for any infinite line.

- Electric field for surface charge distribution :-

Example; consider infinite $z=0$ plane carry a charge distribution of ρ_s $\frac{C}{m^2}$. find $\bar{E}$ at $(0,0,h)$ and $(0,0,-h)$?

cartesian coordinate has 6 ∞ but cylindrical has 3 (2 for z, 1 for ρ) so its easier to use the disk.



infinite plane (cartesian)

infinite disk (cylindrical)

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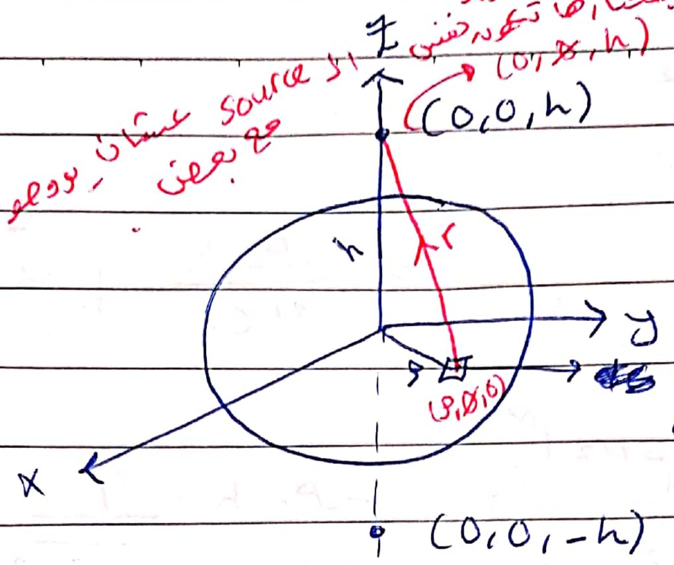
اي قيمة هي ∅ ممكن تكون فبختار اها تكون صفر $(0,\phi,h)$



$\bar{r} = -\rho\hat{a}_\rho + h\hat{a}_z$

ds (scalar quantity)

$ds = \rho\, d\rho\, d\phi$

$$\bar{E}_{\substack{point\\charge}} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r \rightarrow Q = \int_{\substack{s\\surface}} \rho_s\, ds$$

$$\text{so} \quad \bar{E}_{surface} = \int_s \frac{\rho_s\, ds}{\underbrace{4\pi\epsilon_0}_{constant} r^2}\hat{a}_r$$

$$\bar{E} = \int_\sigma \frac{\rho_s\, ds\, \bar{r}}{4\pi\epsilon_0 r^3}, \quad \bar{r} = -\rho\hat{a}_\rho + h\hat{a}_z$$

$$r = \sqrt{\rho^2 + h^2}$$

$$d\sigma = \rho\, d\rho\, d\phi$$

constant

$$\bar{E} = \frac{\rho_s}{4\pi\epsilon_0}\underbrace{\int_0^{2\pi}}_{\phi}\underbrace{\int_0^{\infty}}_{\rho} \frac{\cancel{-\rho\hat{a}_\rho} + h\hat{a}_z}{[\rho^2+h^2]^{3/2}}\rho\, d\rho\, d\phi$$

ناخذه بعيد عن التكامل

the ρ component will be cancelled due to symmetry (only when the graph is symmetrical around the origin)

$$\bar{E} = \frac{\rho_s h (2\pi)}{{}_2 4\pi\epsilon_0}\int_0^\infty \frac{1}{[\rho^2+h^2]^{3/2}}\rho\, d\rho\, \hat{a}_z$$

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let $u = \rho^2 + h^2$

$du = 2\rho\, d\rho$

$$\bar{E} = \frac{\rho_s h}{2\epsilon_0} \int \frac{du}{2(u)^{3/2}} = \frac{\rho_s h}{4\epsilon_0} \int u^{-3/2}\, du$$

$$\bar{E} = \frac{\rho_s h}{\cancel{2}4\epsilon_0} \left. \frac{u^{-1/2}}{-\frac{1}{2}} \right]_0^{\infty} \hat{a}_z = \frac{-\rho_s h}{2\epsilon_0} \left. \frac{1}{\sqrt{\rho^2 + h^2}} \right]_0^{\infty} \hat{a}_z$$

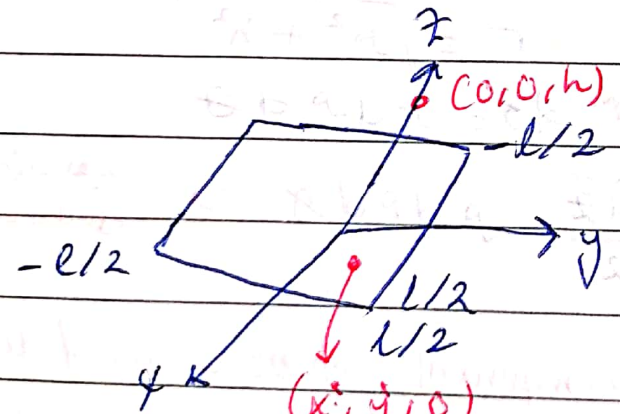
$$\bar{E} = \frac{-\rho_s h}{2\epsilon_0} \left(0 - \frac{1}{h}\right) \hat{a}_z = \frac{\rho_s}{2\epsilon_0} \hat{a}_z \ \ V/m$$

→ for $(0,0,h)$

for $(0,0,-h) \rightarrow -\hat{a}_z$

$$\boxed{\text{For any infinite sheet :- } \quad \bar{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_{normal}}$$

always for any infinite sheet.



sheet is not infinite it must be solved in cartesian coordinate

x,y both cancel due to symmetry.

$$\bar{E} = \int_s \frac{\rho_s\, ds\, \bar{r}}{4\pi\epsilon_0 r^3},$$

$ds = dx\, dy$

$\bar{r} = -x'\hat{a}_x - y'\hat{a}_y + h\hat{a}_z$

$r = \sqrt{x^2 + y^2 + h^2}$

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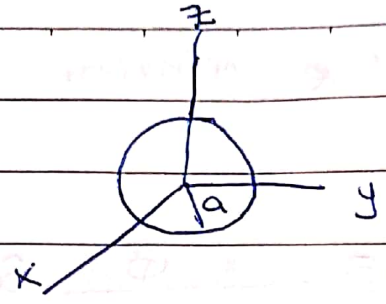
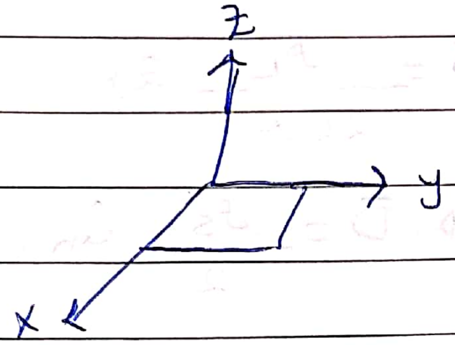
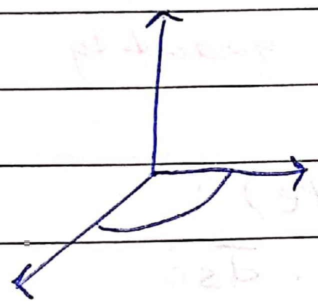


finite disk of radius (a)
solved on cylindrical coordinate
there's symmetry on ρ component

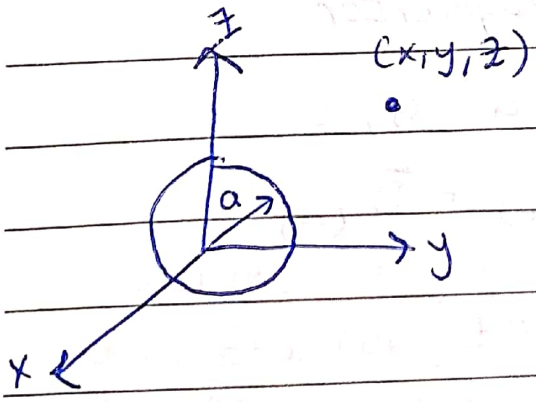


solved on Cartesian
there is no symmetry



~~quarter disk~~
solved on cylindrical
there is no symmetry

- Electric Field Due to Volume Charge Distribution :-



$$Q = \int \rho_v \, dv$$

$$\bar{E} = \int_V \frac{\rho_v \, dv}{4\pi \epsilon_0 r^2} \hat{a}_r$$

volume charge delayed till we study Gauss law.

Subject Electric Flux Density ($\bar{D}$)　　**Day**　　**Date**

$\boxed{\bar{D} = \epsilon_0 \bar{E}}$ → constitutive relation

$[\bar{D}] = \frac{C}{m^2}$

for a point charge → $\bar{D} = \frac{Q}{4\pi r^2} \hat{a}_r$

for an infinite line → $\bar{D} = \frac{\rho_L}{2\pi\rho} \hat{a}_\rho$

for an infinite sheet → $\bar{D} = \frac{\rho_s}{2} \hat{a}_n$

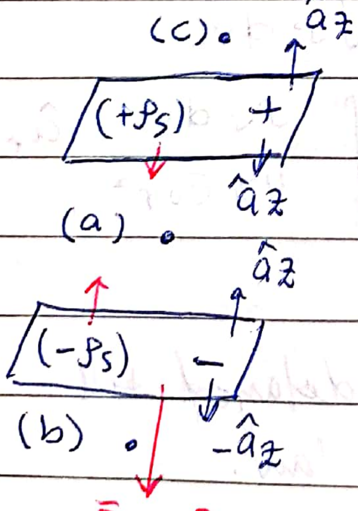
electric flux density ⇒ vector quantity

electric flux ⇒ scalar quantity

Electric flux Ψ (later Ψ_e):

$$\Psi = Q_{enclosed} = \int_S \bar{D} \cdot \bar{ds}$$

example: for parallel plate capacitor



$\bar{D} = 0$

لأن مجال كهربائي محصور
فقط داخل اقطاب البطارية

$\bar{D}$ at a, b, c

$$D_a = \bar{D}_{(+)} + \bar{D}_{(-)}$$

ناخذ العمودي من source لل field

عند نقطة a الصفيحة الفوق باتجاه $-\hat{a}_z$

الصفيحة اللي تحت باتجاه $+\hat{a}_z$

$$= \frac{\rho_s}{2}(-\hat{a}_z) + \frac{-\rho_s}{2}(\hat{a}_z)$$

$$= \rho_s(-\hat{a}_z) \ \frac{C}{m^2}$$

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$\bar{D} = \rho_s \hat{a}_n$

$\bar{D} \cdot \hat{a}_n = \rho_s \hat{a}_n \cdot \hat{a}_n$

$$\boxed{\underset{\text{field}}{\bar{D}} \cdot \hat{a}_n = \underset{\text{source}}{\rho_s}}$$

example: find $\bar{D}$ at $\underset{\text{field}}{(4,0,3)}$ if there is a point charge -5π mC at $(4,0,0)$ and a line charge 3π mC/m along y-axis ?

infinite :- بنعتبر طوله من طول y-axis [illegible]

Sol $\bar{D} = \bar{D}_Q + \bar{D}_\ell$

$\bar{D}_Q = \frac{Q}{4\pi r^2}\hat{a}_r = \frac{Q\bar{r}}{4\pi r^3} = \frac{-5\pi 10^{-3}(3\hat{a}_z)}{(4\pi)(27)}$

$= -0.138\,\hat{a}_z \ \frac{C}{m^3}$

$\bar{D}_\ell = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho = \frac{\rho_l \bar{\rho}}{2\pi\rho^2}$

$= \frac{3\pi 10^{-3}(4,0,3)}{2\pi(25)}$

$= 0.24\,\hat{a}_x + 0.18\,\hat{a}_z \ \frac{mC}{m^2}$

$\bar{\rho} = (4,0,3) - (0,y,0)$

Shortest distance is when $y = y = 0$

$\bar{\rho} = 4\hat{a}_x + 3\hat{a}_z$

$\rho = 5$

$\bar{D}_{total} = 240\,\hat{a}_x + 42\,\hat{a}_z \ \frac{\mu C}{m^2}$

Subject: Gauss's Law Day ______ Date ______

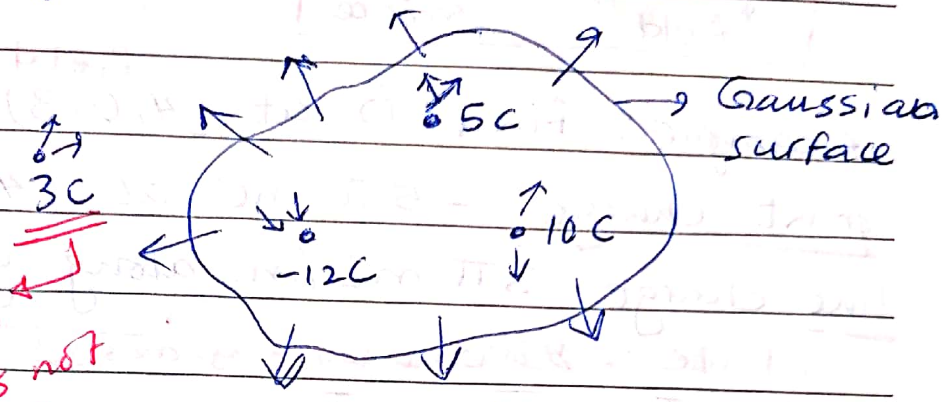
Gauss's law states that:

$$\oint_S \bar{D} \cdot \bar{ds} = Q_{enclosed}$$

الشحنة المحصورة داخل سطح gauss

$\bar{D} = \bar{E}\epsilon_0 \rightarrow \oint_S \bar{E}\epsilon_0 \cdot \bar{ds} = Q_{enclosed}$

example:



not included because it is not in the Gaussian surface

equivalent to 3C charge

→ this law is also called "Maxwell's 1st equation in integral form"

$Q_{enclosed}$ has four states
- → Q (point charge)
- → $\int_l \rho_l \, dl$ (line charge)
- → $\int_s \rho_s \, ds$ (surface charge)
- → $\int_v \rho_v \, dv$ (volume charge)

Gauss's law only works on infinite line and infinite sheet, but both of them have laws derivated from Coloumb's law so it is redundant to apply Gauss's law on them.

The main goal from Gauss law is to solve Volume charge density distribution questions.

* يفضل ان يكون شكل سطح غاوس من شكل الـ source *

Subject ______ Day ______ Date ______

$$\oint_S \bar{D} \cdot \bar{ds} = Q_{enclosed} = \int \rho_v \, dv$$

by applying the $\boxed{\text{divergence theorem}}$ → للتخلص من التكامل على سطح مغلق

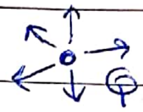
$$\oint_S \bar{D} \cdot \bar{ds} = \cancel{\int_v} \nabla \cdot \bar{D} \, \cancel{dv} = \cancel{\int} \rho_v \, \cancel{dv}$$

$$\boxed{\begin{aligned} \nabla \cdot \bar{D} &= \rho_v \\ \rho_s &= \bar{D} \cdot \hat{a}_n \end{aligned}}$$

⇒ Maxwell's 1^{st} equation in differential form (point form)

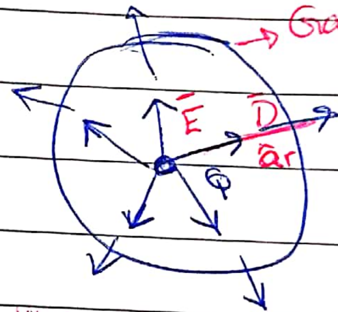
* Applications of Gauss's law :-

1) $\bar{E}$ or $\bar{D}$ for a point charge



$$\oint_S \bar{D} \cdot \bar{ds} = Q_{enclosed}$$

$$Q_{enclosed} = Q$$



$$\bar{D} = D_r \hat{a}_r + \underbrace{\cancel{D_\theta \hat{a}_\theta} + \cancel{D_\phi \hat{a}_\phi}}$$

because $\bar{D}$ will be in r direction only

$\bar{ds}$ (العمودي على سطح غاوس) ⇒ we get hollow sphere when r = constant

so the normal will be $\hat{a}_r$

the sphere must be hollow because Gaussian surface is always 2D

hollow → 2D

solid → 3D

$$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$\int_0^{2\pi}\int_0^{\pi} D_r \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r = Q$$

$$D_r \, r^2 \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi = Q$$

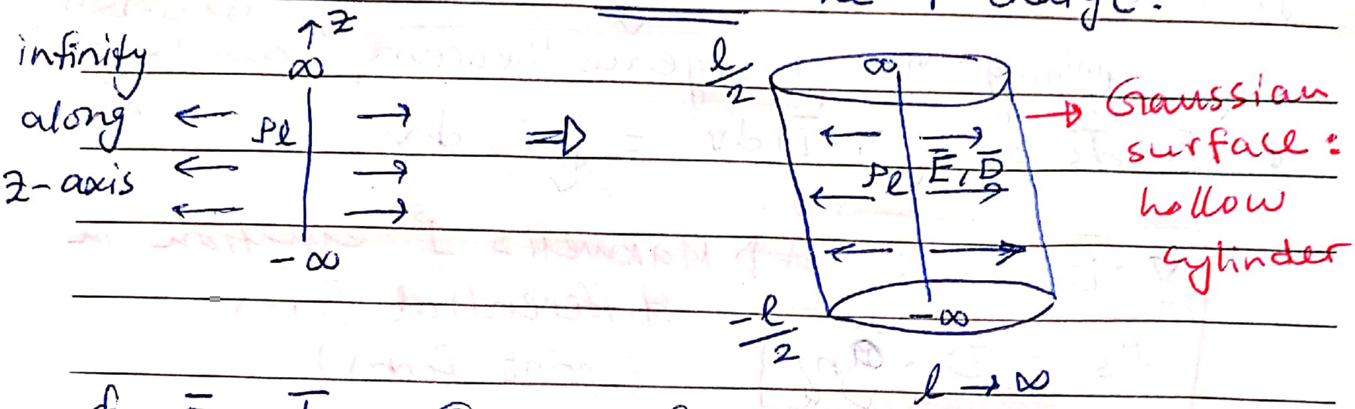
$$D_r \, r^2 \left(-\cos\theta \Big]_0^{\pi}\right)\left(\phi \Big]_0^{2\pi}\right) = Q$$

$$4\pi r^2 \, D_r = Q \rightarrow D_r = \frac{Q}{4\pi r^2}$$

$$\bar{D} = \frac{Q}{4\pi r^2} \hat{a}_r \quad , \quad \bar{E} = \frac{\bar{D}}{\epsilon_0} = \frac{Q}{4\pi \epsilon_0 r^2} \hat{a}_r$$

مساحة الكرة

2) $\bar{E}$ or $\bar{D}$ for an infinite line of charge.



$$\oint_s \bar{D} \cdot \bar{ds} = Q_{enc.} = \int_l \rho_l \, dl$$

$$\bar{D} = D_\rho \hat{a}_\rho + \underbrace{\cancel{D_\theta \hat{a}_\theta} + \cancel{D_z \hat{a}_z}}_{\text{لأنه، خطوط فقط باتجاه } \rho}$$

$dl = dz$ (scalar because of Q)

$$\bar{ds} = \rho \, d\theta \, dz \, \hat{a}_\rho$$

$$\int_{-l/2}^{l/2} \int_0^{2\pi} D_\rho \hat{a}_\rho \cdot \rho \, d\theta \, dz \, \hat{a}_\rho = \int_{-l/2}^{l/2} \rho_l \, dz$$

$$D_\rho \, \rho \, (2\pi)(l) = \rho_l \, l$$

المساحة الجانبية للأسطوانة $\boxed{2\pi\rho \cancel{l}} \, D_\rho = \rho_l \cancel{l}$ (وعطينا l قيمتين $\frac{l}{2}, -\frac{l}{2}$ عشان نشطبهم مع بعض لأنه لو $l = \infty$ ما بنفع نشطبهم مع بعض)

$$D_\rho = \frac{\rho_l}{2\pi\rho}$$

$$\bar{D} = \frac{\rho_l}{2\pi\rho} \hat{a}_\rho$$

$$\bar{E} = \frac{\rho_l}{2\pi\epsilon_0\rho} \hat{a}_\rho$$

if the line is finite or with curvature, then use Coloumb's law to find $\bar{E}, \bar{D}, \ldots$

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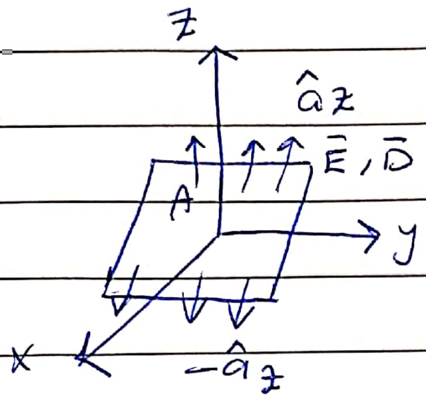
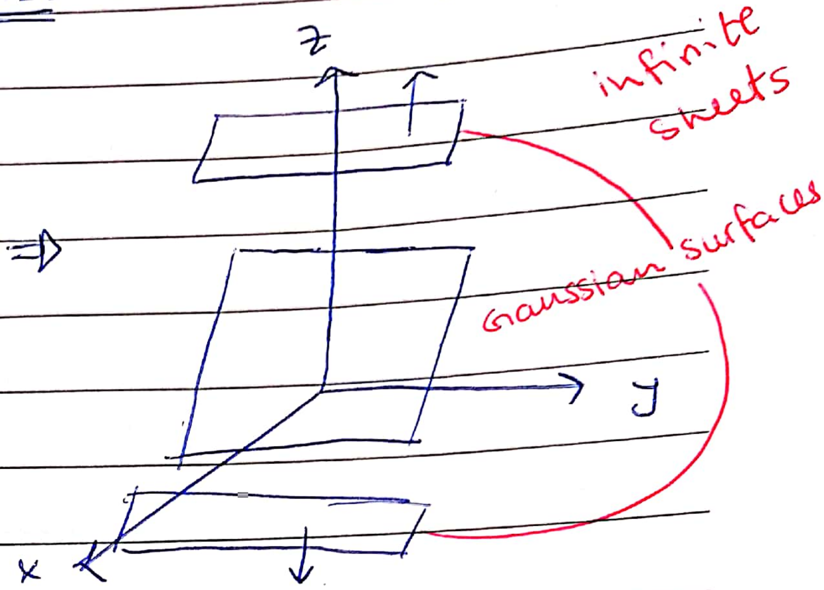
3) $\bar{E}$ or $\bar{D}$ for an infinite sheet of charge.



$A \to \infty$

$\Rightarrow$



$$\oint_S \bar{D} \cdot \bar{ds} = Q_{enc.} = \int_S \rho_s \, ds$$

$$\bar{D} = \begin{cases} D_z \hat{a}_z, & z > 0 \\ D_z(-\hat{a}_z), & z < 0 \end{cases} \qquad ds = dx\,dy$$

$$\bar{ds}_{top} = dx\,dy\,\hat{a}_z \qquad \bar{ds}_{bottom} = -dx\,dy\,\hat{a}_z$$

$$\int_{S_{top}} \bar{D} \cdot \bar{ds}_{top} + \int_{S_{bot.}} \bar{D} \cdot \bar{ds}_{bot.} = \int_S \rho_s \, ds$$

$$\iint_{-\infty}^{\infty} D_z \hat{a}_z \cdot dx\,dy\,\hat{a}_z + \iint_{-\infty}^{\infty} D_z(-\hat{a}_z) \cdot dx\,dy(-\hat{a}_z) = \iint_{-\infty}^{\infty} \rho_s \, dx\,dy$$

$$D_z \cancel{A} + D_z \cancel{A} = \rho_s \cancel{A}, \quad \text{where } A = \iint_{-\infty}^{\infty} dx\,dy$$

$$2D_z = \rho_s \rightarrow D_z = \frac{\rho_s}{2}$$

$$\bar{D} = \begin{cases} \frac{\rho_s}{2} \hat{a}_z, & z > 0 \\ \frac{\rho_s}{2}(-\hat{a}_z), & z < 0 \end{cases} = \frac{\rho_s}{2} \hat{a}_n$$

$$\bar{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_n$$

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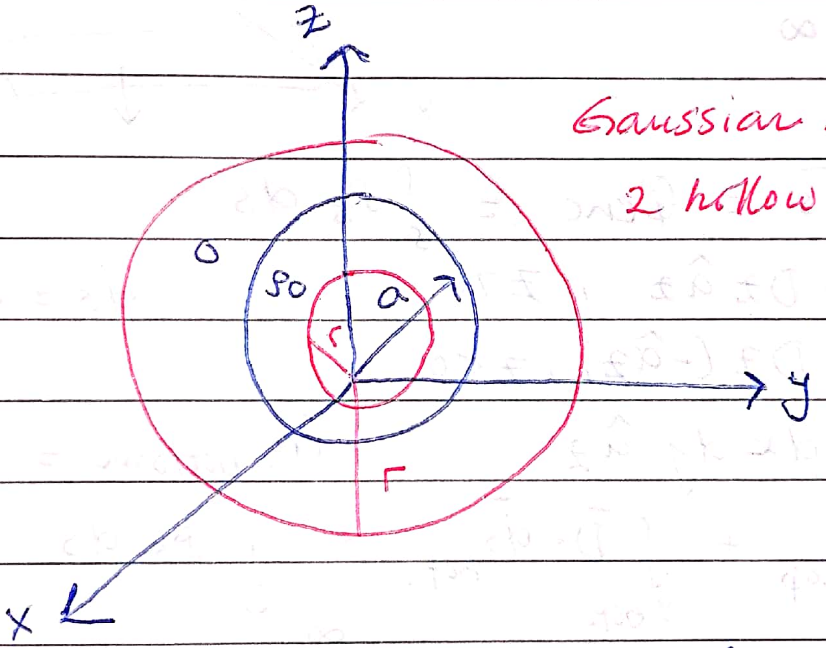
4) $\bar{E}$ or $\bar{D}$ for a uniform volume charge distribution :-

example; consider a sphere of radius a that carries a charge $\rho_v = \begin{cases} \rho_o , & r \leq a \\ 0 , & r \geq a \end{cases}$

find $\bar{E}$ or $\bar{D}$ everywhere ?

→ this indicates that we need to use Gauss's law.

Sol



Gaussian surface : 2 hollow spheres

for $0 \leq r \leq a$ $(r = \bar{a}) \Rightarrow \oint \bar{D} \cdot \bar{ds} = Q_{enc.} = \int \rho_v \, dv$

$\bar{D} = D_r \, \hat{a}_r$

$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$

$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$

$\oint_s \bar{D} \cdot \bar{ds} = \int_v \rho_v \, dv$ → $\rho_v = \rho_o , r \leq a$

$\int_0^{2\pi}\int_0^{\pi} D_r \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho_v \, r^2 \sin\theta \, dr \, d\theta \, d\phi$

$4\pi r^2 D_r = \rho_o \left(\frac{4\pi r^3}{3}\right)$ → حجم كرة

$D_r = \frac{\rho_o r}{3}$

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Subject ______ Day ______ Date ______

$$\bar{D} = \frac{\rho_0 r}{3}\hat{a}_r$$
$$\bar{E} = \frac{\rho_0 r}{3\epsilon_0}\hat{a}_r$$
$\rightarrow 0 \leq r \leq a$

at $r = a \rightarrow \bar{D} = \frac{\rho_0 a}{3}\hat{a}_r$

$$\bar{E} = \frac{\rho_0 a}{3\epsilon_0}\hat{a}_r$$

ازا طلع الجواب نفسه
عند $r \geq a$ يكون
الاقتران متصل .

for $a \leq r \leq \infty \Rightarrow \oint_s \bar{D}\cdot\bar{ds} = \int_v \rho_v\, dv$

$$4\pi r^2 D_r = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho_0 r^2 \sin\theta\, dr\, d\theta\, d\phi + \cancel{\int_0^{2\pi}\int_0^{\pi}\int_a^{r} \text{zero}}$$

$$4\pi r^2 D_r = \left(\frac{4\pi a^3}{3}\right)\rho_0$$

$$D_r = \frac{\rho_0 a^3}{3r^2}$$

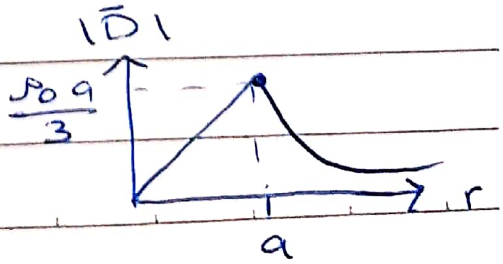
$$\bar{D} = \frac{\rho_0 a^3}{3r^2}\hat{a}_r$$
$$\bar{E} = \frac{\rho_0 a^3}{3\epsilon_0 r^2}\hat{a}_r$$
$\rightarrow a \leq r < \infty$

at $r = a \rightarrow \bar{D} = \frac{\rho_0 a}{3}\hat{a}_r$

$$\bar{E} = \frac{\rho_0 a}{3\epsilon_0}\hat{a}_r$$

طلع نفس الي
فوق معناها
متصل .

$$\bar{D} = \begin{cases} \frac{\rho_0 r}{3}\hat{a}_r, & 0 \leq r \leq a \\ \frac{\rho_0 a^3}{3r^2}\hat{a}_r, & a \leq r \leq \infty \end{cases}$$



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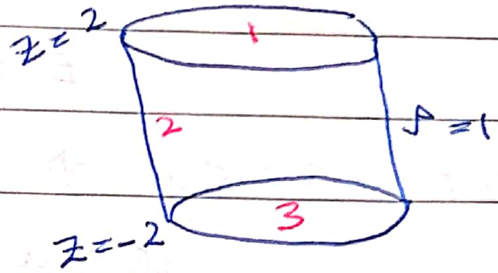
Example; Given $\bar{D} = z\rho\cos^2\phi\,\hat{a}_z \;\; C/m^2$, calculate the total charge enclosed by the cylinder of radius 1m with $-2 \leq z \leq 2$?

Sol $Q_{enclosed} = \oint_S \bar{D}\cdot\bar{ds} = \int_V \rho_v\, dv = Q$



$\bar{ds}_1 = \rho\, d\phi\, d\rho\, \hat{a}_z$

$\bar{ds}_2 = \rho\, d\phi\, dz\, \hat{a}_\rho$

$\bar{ds}_3 = -\rho\, d\rho\, d\phi\, \hat{a}_z$

$$Q = \int_{S_1} \bar{D}\cdot\bar{ds}_1 + \int_{S_2} \bar{D}\cdot\bar{ds}_2 + \int_{S_3} \bar{D}\cdot\bar{ds}_3$$

equal zero because $\bar{D}$ given is in the direction of $\hat{a}_z$

$$Q = \int_0^{2\pi}\int_0^{1} z\rho\cos^2\phi\,\rho\,d\rho\,d\phi\,\Big|_{z=2} + \int_0^{2\pi}\int_0^{1} -z\rho\cos^2\phi\,\rho\,d\rho\,d\phi\,\Big|_{z=-2}$$

$$Q = \frac{2\pi}{3} + \frac{2\pi}{3} = \frac{4\pi}{3}\ C$$

Surface charge density $\rightarrow \rho_s = \bar{D}\cdot\hat{a}_n$ $\rightarrow \hat{a}_z$ (top), $\rightarrow -\hat{a}_z$ (bottom)

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Subject: Electric Potential | Day | Date

$\bar{F} = Q E$

Work (W) = Force $(\bar{F})$ * Distance $(\bar{l})$

$dW = - \bar{F} \cdot \overline{dl}$

→ the work is done by an external agent

$\int dW = -\int Q\bar{E} \cdot \overline{dl}$

$W = -Q \int_l \bar{E} \cdot \overline{dl}$

$\frac{W}{Q} = V = -\int_l \bar{E} \cdot \overline{dl}$

عشان نقدر نحصل على فرق جهد
معناه 1V لازم نبذل شغل
معناه 1J لنقل شحنة مقدارها 1C

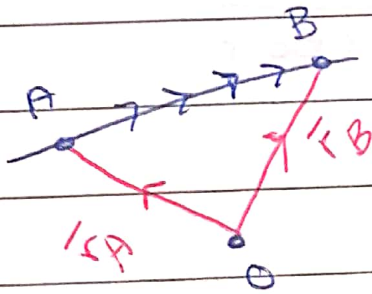
$V_{AB} = V_B - V_A$

→ if V_{AB} is positive → 1) gain in potential.
2) work is done by the external field.

→ if V_{AB} is negative → 1) drop in potential.
2) work is done by the field itself.

For a point charge ⇒ $V_{AB} = -\int_l \bar{E} \cdot \overline{dl}$

$= -\int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r \cdot dr\hat{a}_r$



$= \frac{Q}{4\pi\epsilon_0} \left.\frac{1}{r}\right]_{r_A}^{r_B}$

$= \frac{Q}{4\pi\epsilon_0 r_B} - \frac{Q}{4\pi\epsilon_0 r_A}$

$= V_B - V_A$

if point A is located at ∞ then $V_A = 0$, $r_A \to \infty$

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$$V_{AB} = V_{\infty B} = V_B - V_\infty = V_B - 0 = V_B$$

$\boxed{V_\infty = 0 \text{ when } r_\infty \to \infty}$ → ~~reference~~ referance

(لوجود نقطة معلومة احنا اعتبرناها ، معلومة potential بتعها .

$$V = \frac{Q}{4\pi\epsilon_0 r}$$ if referance is at ∞ (point charge case)

For line charge ⇒ $$V = \int_l \frac{\rho_l \, dl}{4\pi\epsilon_0 r}$$

For surface charge ⇒ $$V = \int_s \frac{\rho_s \, ds}{4\pi\epsilon_0 r}$$

For volume charge ⇒ $$V = \int_v \frac{\rho_v \, dv}{4\pi\epsilon_0 r}$$

potential

volume

in the book dash is used to refer to the volume, $$V(\vec{r}) = \int_{v'} \frac{\rho_v(\vec{r}\,') \, dr'}{4\pi\epsilon_0 |\vec{r} - \vec{r}\,'|}$$

For N-point charges ⇒ $$V = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^{N} \frac{Q_k}{|r - r_k|}$$

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r \quad \text{but} \quad V = \frac{Q}{4\pi\epsilon_0 r}$$

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Example; Two point charges $-4\mu C$ and $5\mu C$ are located at $(2,-1,3)$ and $(0,4,-2)$. find the potential at $(1,0,1)$

Sol reference is at ∞, V_∞ = zero

$$V = V_1 + V_2$$

$$= \frac{4 \cdot 10^{-6}}{4\pi\left(\frac{10^{-9}}{36\pi}\right)|(-1,1,-2)|} + \frac{5 \cdot 10^{-6}}{4\pi\left(\frac{10^{-9}}{36\pi}\right)|(1,-4,3)|}$$

$$V = -5.872 \text{ KV}$$

work is done by the field itself.

Example; a point charge of $5nC$ located at $(-3,4,0)$ and a line $y=1$, $z=1$ carries a uniform charge of $2nC/m$. if $\boxed{V = 100\,V \text{ at } B\ (1,2,1)}$. find V at point C $(-2,5,3)$

reference

Sol $V_{BC} = V_C - V_B$

$$= V_C - 100 \rightarrow V_C = V_{BC} + 100$$

$$V_{BC} = V_Q + V_\ell$$

$$V_Q = -\int_\ell \bar{E}\cdot d\bar{\ell} = -\int_{r_B}^{r_C} \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r \cdot dr\,\hat{a}_r$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_C} - \frac{1}{r_B}\right)$$

$$r_C = |(-2,5,3) - (-3,4,0)| = |(1,1,3)| = \sqrt{11}$$

$$r_B = |(1,2,1) - (-3,4,0)| = |(4,-2,1)| = \sqrt{21}$$

$$V_\ell = -\int_\ell \bar{E}\cdot d\bar{\ell} = -\int_{\rho_B}^{\rho_C} \frac{\rho_\ell}{2\pi\epsilon_0 \rho}\,\hat{a}_\rho \cdot d\rho\,\hat{a}_\rho$$

$$= \frac{-\rho_\ell}{2\pi\epsilon_0}\left(\ln \rho_C - \ln \rho_B\right)$$

$$= \frac{-\rho_l}{2\pi\epsilon_0} \ln\left(\frac{\rho_C}{\rho_B}\right)$$

$$\rho_C = |(-2,5,3) - (x,1,1)| = |(0,4,2)| = \sqrt{20}$$

عشان تكون اقصر مسافة $x = -2$

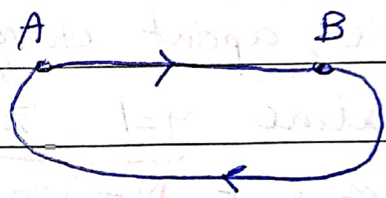
$$\rho_B = |(1,2,1) - (1,1,1)| = |(0,1,0)| = 1$$

$$V_{BC} = -50.175 \text{ V}$$

$$V_C = V_{BC} + V_B = 49.825 \text{ V}$$

How to find $\bar{E}$ from V ?

$$V_{AB} = -\int_l \bar{E} \cdot \bar{dl}$$



$$= -\oint_l \bar{E} \cdot \bar{dl}$$ for closed path

$$\oint \bar{E} \cdot \bar{dl} = -\int_A^B \bar{E} \cdot \bar{dl} + -\int_B^A \bar{E} \cdot \bar{dl}$$

$$= -\int_A^B \bar{E} \cdot \bar{dl} + \int_A^B \bar{E} \cdot \bar{dl}$$

$$= \text{zero}$$

$$\boxed{\oint \bar{E} \cdot \bar{dl} = \text{zero}}$$ always

→ equi-potential surface

→ 2^{nd} Maxwell's equation in integral form

$$V_{AA} = V_A - V_A = 0$$

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to convert it into differential form, we use Stocke's theorem $\Rightarrow \oint_l \bar{E} \cdot \bar{dl} = \int_s \nabla \cdot \bar{E} \cdot \bar{ds} = 0$

electric field is irrotational (conservative) $\leftarrow$ $\nabla \times \bar{E} = 0$ $\rightarrow$ 2nd Maxwell's equation in differential form

let : $\bar{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z$

$\bar{dl} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$

$V = -\int_l \bar{E} \cdot \bar{dl} \longrightarrow dv = -\bar{E} \cdot \bar{dl}$

$dv = -(E_x dx + E_y dy + E_z dz)$

assume , $dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy + \frac{\partial v}{\partial z} dz$

$\frac{\partial v}{\partial x} \cancel{dx} = -E_x \cancel{dx} \rightarrow E_x = -\frac{\partial v}{\partial x}$

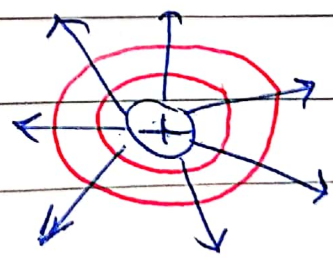
$E_y = -\frac{\partial v}{\partial y}$

$E_z = \frac{-\partial v}{\partial x}$

$\bar{E} = -\left(\frac{\partial v}{\partial x} \hat{a}_x + \frac{\partial v}{\partial y} \hat{a}_y + \frac{\partial v}{\partial x} \hat{a}_x\right) = -\nabla v$ (gradient)

$$\boxed{\bar{E} = -\nabla v}$$

Equi-Potential Surface :-



كل ما بعدنا عن الشحنة بقل potential

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Example; Given the potential $V = \frac{10}{r^2} \sin\theta \cos\phi$

ⓐ find $\bar{D}$ at $(\overset{r}{2}, \overset{\theta}{\frac{\pi}{2}}, \overset{\phi}{0})$

ⓑ calculate the work done in moving a $10\,\mu C$ charge from $A\,(1, 30^\circ, 120^\circ)$ to $B\,(4, 90^\circ, 60^\circ)$

Sol a) $\bar{E} = -\nabla V$

$\bar{D} = \epsilon_0 E = -\epsilon_0 \nabla V$

the source is unknown so we can't use Coloumb or Gauss

$$\bar{E} = -\left(\frac{\partial v}{\partial r}\hat{a}_r + \frac{\partial v}{r\,\partial\theta}\hat{a}_\theta + \frac{\partial v}{r\sin\theta\,\partial\phi}\hat{a}_\phi\right)$$

$$= \frac{20}{r^3}\sin\theta\cos\phi\,\hat{a}_r - \frac{10}{r^3}\cos\theta\cos\phi\,\hat{a}_\theta + \frac{10}{r^3}\sin\phi\,\hat{a}_\phi \quad \frac{V}{m}$$

$\bar{D} = \epsilon_0 \bar{E} = \epsilon_0 \frac{20}{8}\hat{a}_r = 22.1\ pC/m^2$

at $(2, \frac{\pi}{2}, 0)$

b) $W = Q\,V_{AB} = Q\,(V_B - V_A)$

$$= 10\,10^{-6}\left(\frac{10}{16}(1)\left(\frac{1}{2}\right) - \frac{10}{1}\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\right)$$

$$= 10\,10^{-6}\left(\frac{45}{16}\right) = 28.125\ \mu J$$

Method 2 $\Rightarrow$ $W = -Q \int_L \bar{E} \cdot \overline{dl}$

$$= -Q\left[\int_{r_A}^{r_B} E_r\,dr + \int_{\theta_A}^{\theta_B} E_\theta\, r\,d\theta + \int_{\phi_A}^{\phi_B} E_\phi\, r\sin\theta\,d\phi\right]$$

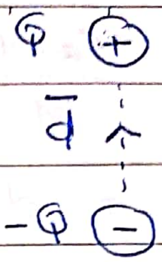
هون تكامل بالنسبة لـ r فقط
بنقل r من 1 لـ 4
θ و ϕ ثوابت
بحصل على نقطة جديدة
$(4, 30^\circ, 120^\circ)$

r, ϕ ثوابت
بنقل θ فقط
بحصل على نقطة جديدة
$(4, 90^\circ, 120^\circ)$

$(4, 90^\circ, 60^\circ)$

r متساويين B لأنه
نقلتها ، خلصت

Subject: Electric Dipole   Day ______   Date ______

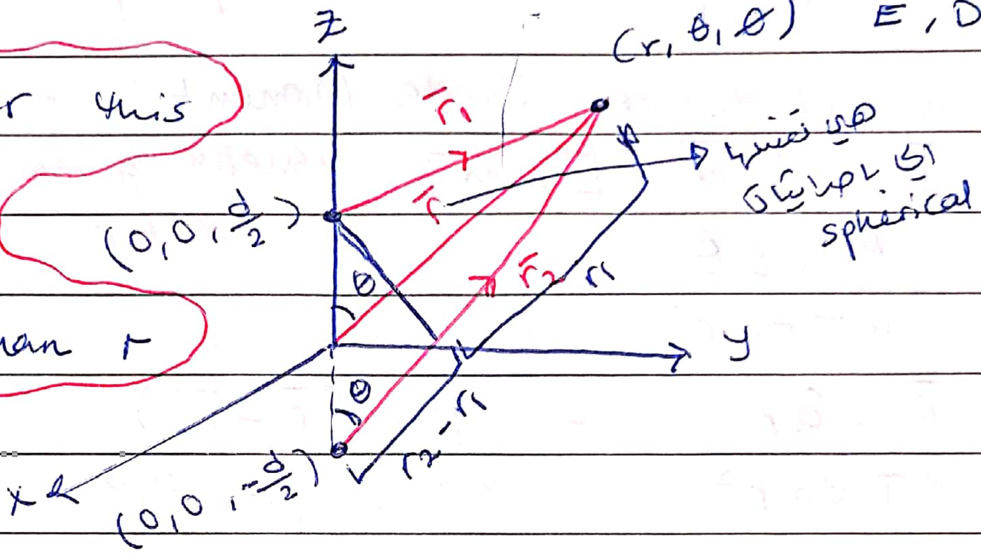


very small distance → $\bar{d}$

Q and $-Q$ are equal in magnitude, different in polarity.

(r, θ, ϕ) $\bar{E}, \bar{D}, V$?

To consider this as a dipole d must be much less than r



$$V = V_+ + V_-$$

$V_\infty = 0$ (reference)

$$= \frac{Q}{4\pi\varepsilon_0 r_1} + \frac{-Q}{4\pi\varepsilon_0 r_2}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{Q(r_2 - r_1)}{4\pi\varepsilon_0 r_1 r_2}$$

!! usually Q and d are not known

ع ا علك اسقاط عمودي من r_1 على r_2

بنعرف θ من z-axis extension لل z-axis بدل z-axis, وبرده r_2

بدل ماتكون هاي r

$$r_1 r_2 = r^2$$

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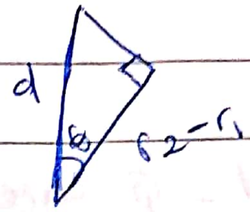
$$\cos\theta = \frac{r_2 - r_1}{d}$$



$$r_2 - r_1 = d\cos\theta$$

$$V = \frac{Q\,(d\cos\theta)}{4\pi\epsilon_0 r^2}$$ we still don't know θ and d

so we will define "Dipole Moment" :-

$$\bar{P} = Q\,\bar{d} \quad [C\cdot m]$$ usually given as a vector

$$V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2}$$

$$V = \frac{\bar{P}\cdot\hat{a}_r}{4\pi\epsilon_0 r^2} = \frac{\bar{P}\cdot(\bar{r}-\bar{r}')}{4\pi\epsilon_0 |\bar{r}-\bar{r}'|^3}$$

For N-dipoles :

field center of the dipole

$$V = \frac{1}{4\pi\epsilon_0}\sum_{K=1}^{N}\frac{\bar{P}_K\cdot(\bar{r}-\bar{r}_K)}{|\bar{r}-\bar{r}_K'|^3}$$

$$\bar{E} = \bar{E}_+ + \bar{E}_-$$

or $\bar{E} = -\nabla V$ in spherical coordinate

$$= -\left(\frac{\partial v}{\partial r}\hat{a}_r + \frac{\partial v}{r\,\partial\theta}\hat{a}_\theta\right)$$ since V is independent on ϕ

scalar

$$\bar{E} = \frac{P}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right)\ \frac{V}{m}$$ (vector)

$$\bar{D} = \bar{E}\epsilon_0$$

vector

$$V = \frac{\bar{P}\cdot\hat{a}_r}{4\pi\epsilon_0 r^2}$$ (~~scalar~~)

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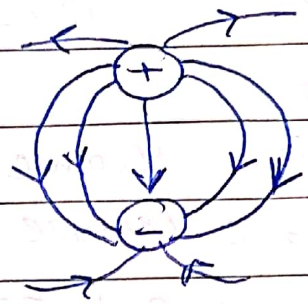
point charge	electric dipole
$E \propto \frac{1}{r^2}$	$E \propto \frac{1}{r^3}$
$V \propto \frac{1}{r}$	$V \propto \frac{1}{r^2}$

Flux Lines :



Example ; Two dipoles with dipole moments $-5\hat{a}_z$ nC.m and $9\hat{a}_z$ nC.m are located at $(0,0,-2)$ and $(0,0,3)$ find the potential and $\bar{E}$ at the origin ?

Sol $\quad V = \dfrac{\bar{P}\cdot(\bar{r}-\bar{r}')}{4\pi\epsilon_0 |\bar{r}-\bar{r}'|^3}$ $\qquad$ reference is at ∞

$$= \frac{(-5\hat{a}_z 10^{-9})(2\hat{a}_z)}{4\pi \frac{10^{-9}}{36\pi}(8)} + \frac{(9\hat{a}_z 10^{-9})(-3\hat{a}_z)}{4\pi\frac{10^{-9}}{36\pi}(27)}$$

$$= -20.25\ V$$

$$\bar{E}_1 = \frac{P_1}{4\pi\epsilon_0 r_1^3}(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta)$$

scalar

$r_1 = 2m$	$P_1 = 5$ n.m	$\theta_1 = 180^\circ$
$r_2 = 3m$	$P_2 = 9$ n.m	$\theta_2 = 180^\circ$

θ is the angle between $\bar{P}_{1,2}$ and $\bar{r}$

$\left.\begin{matrix}\bar{r}_1 = 2\hat{a}_z \\ \bar{P}_1 = (-)5\hat{a}_z\end{matrix}\right\} 180^\circ$ $\qquad$ $\left.\begin{matrix}\bar{r}_2 = (-)3\hat{a}_z \\ \bar{P}_2 = 9\hat{a}_z\end{matrix}\right\} 180^\circ$

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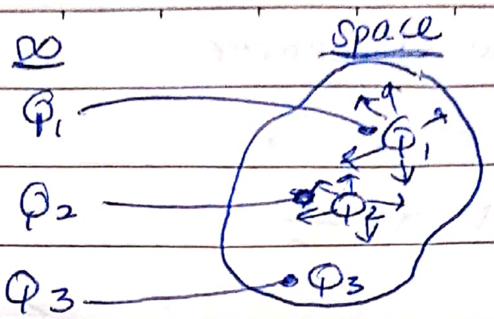
Subject: Energy Density in Electrostatic Fields   Day ______ Date ______



لما ننقل Q_1 من ∞ الى space
ما رح يكون في صعوبة لانه
space فاضي ما في force تأثر
عليه .

$Q_1 \to Q_2 \to Q_3$

$W = QV = \text{zero}$

↳ potential difference = zero because $V_\infty = 0$ and $V_{space} = 0$ because the space is empty

لما ننقل Q_2 بنجد في force ناتجة عن electric field intensity for Q_1

لما ننقل Q_3 بنجد في force ناتجة عن Q_1 & Q_2 " " " "

W_E = electrical energy

$$W_E = W_1 + W_2 + W_3$$

$$= Q_1(0) + Q_2 V_{12} + Q_3(V_{13} + V_{23}) \quad \cdots (1)$$

$(\text{if } Q_1 \to Q_2 \to Q_3)$

if $(Q_3 \to Q_2 \to Q_1) \Rightarrow W_E = W_1 + W_2 + W_3$

$$= Q_1(V_{31} + V_{21}) + Q_2 V_{32} + 0 \quad \cdots (2)$$

(1) + (2) ... $2W_E = Q_1(V_{21} + V_{31}) + Q_2(V_{12} + V_{32}) + Q_3(V_{13} + V_{23})$

$$W_E = \frac{1}{2}\left[Q_1 V_1 + Q_2 V_2 + Q_3 V_3\right]$$

$$W_E = \frac{1}{2}\sum_{K=1}^{N} Q_K V_K \quad [\text{Joule}]$$

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for line charge $\Rightarrow W_E = \frac{1}{2}\int_{l} \rho_l \, V \, dl$ potential

for surface charge $\Rightarrow W_E = \frac{1}{2}\int_{S} \rho_s \, V \, ds$

for volume charge $\Rightarrow W_E = \frac{1}{2}\int_{V} \rho_v \, V \, dv$

$\rho_v = \nabla \cdot \bar{D} \rightarrow W_E = \frac{1}{2}\int_{V} (\nabla \cdot \bar{D}) \, V \, dv$

in general : $\nabla \cdot V\bar{A} = \bar{A} \cdot \nabla V + V(\nabla \cdot \bar{A})$ الحدين نفس بعض

$$W_E = \frac{1}{2}\Big[\underbrace{\int_V \nabla \cdot V\bar{D} \, dv}_{1} - \underbrace{\int_V \bar{D} \cdot \nabla V \, dv}_{2}\Big]$$

applying the divergence theorem on 1 :

$$W_E = \frac{1}{2}\Big[\underbrace{\cancel{\oint_S (V\bar{D}) \cdot \bar{ds}}}_{1 \;=\; \text{zero}} - \int_V \bar{D} \cdot \nabla V \, dv\Big]$$

$V = \frac{Q}{4\pi\epsilon_0 r} \leftarrow V \propto \frac{1}{r}$

$D = \frac{Q}{4\pi r^2}\hat{a}_r \leftarrow \bar{D} \propto \frac{1}{r^2}$

$ds = r^2 \sin\theta \, d\theta \, d\phi \; \hat{a}_r \leftarrow ds \propto r^2$

$\Big\} \; \frac{1}{r^3} \; \Big\} \; \frac{1}{r} \rightarrow 0$ as $r \rightarrow \infty$

$$W_E = \frac{1}{2}\Big[\underbrace{\int_V \bar{D} \cdot \nabla V \, dv}_{2}\Big]$$

$\bar{E} = -\nabla V$

$\bar{D} = \epsilon_0 \bar{E}$

$\bar{E} = \frac{\bar{D}}{\epsilon_0}$

$$W_E = \frac{1}{2}\int_V \bar{E} \cdot \bar{D} \, dv$$

$$= \frac{1}{2}\int_V \epsilon_0 E^2 \, dv$$

$$= \frac{1}{2}\int_V \frac{D^2}{\epsilon_0} \, dv$$

$$W_E = \frac{1}{2}\sum_{K=1}^{N} Q_K V_K$$

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Energy Density (ω_E) :- → ω is small while in ~~...~~ energy W is capital

$$\omega_E = \frac{W_E}{\text{volume}} \left[\frac{J}{m^3}\right]$$

$$= \frac{1}{2} \bar{E} \cdot \bar{D} = \frac{1}{2} E^2 \epsilon_0 = \frac{1}{2} \frac{D^2}{\epsilon_0}$$

$$W_E = \int_V \omega_E \, dV$$

Example; The point charges $-1nC$, $4nC$ and $3nC$ are located at $(0,0,0)$, $(0,0,1)$ and $(1,0,0)$ find the energy in the system?

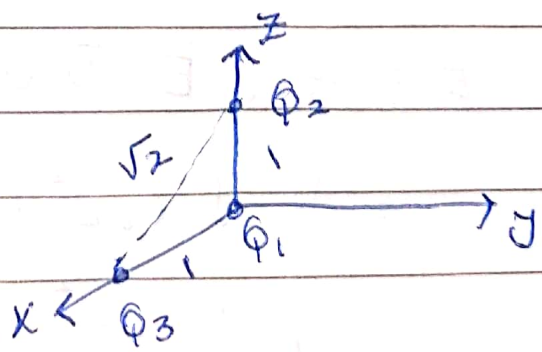
Sol $W_E = \frac{1}{2} \sum_{k=1}^{N} Q_k V_k$

$$= \frac{1}{2} (Q_1 V_1 + Q_2 V_2 + Q_3 V_3)$$

$$= \frac{1}{2} \left[Q_1 (V_{21} + V_{31}) + Q_2 (V_{12} + V_{32}) + Q_3 (V_{13} + V_{23}) \right]$$

$$= \frac{1}{2} \left[Q_1 \left(\frac{Q_2}{4\pi\epsilon_0(1)} + \frac{Q_3}{4\pi\epsilon_0(1)} \right) + Q_2 \left(\frac{Q_1}{4\pi\epsilon_0(1)} + \frac{Q_3}{4\pi\epsilon_0(\sqrt{2})} \right) + Q_3 \left(\frac{Q_1}{4\pi\epsilon_0(1)} + \frac{Q_2}{4\pi\epsilon_0(\sqrt{2})} \right) \right]$$

$$= 13.37 \, nJ$$



Chapter "5"

Subject Classifications of Materials Day ______ Date ______

[1] Conductors (Metals) $\Rightarrow$ σ is much bigger than 1

(when $\sigma = \infty$: perfect conductor)

$\hookrightarrow$ very very very big.

examples: Aluminum, Silver, Gold, Copper, Lead ... etc

[2] Semi-conductors $\Rightarrow$ σ is bigger than 1

examples: Silicon, Germanium, Indium

[3] Dielectrics (Insulators) $\Rightarrow$ $0 < \sigma < 1$

examples: Wood, Glass, Carbon, Lithium

σ is the conductivity $= \frac{1}{\rho}$ $\left[\frac{S}{m}\right]$

$\hookrightarrow$ it represents the number of electrons that can move freely.

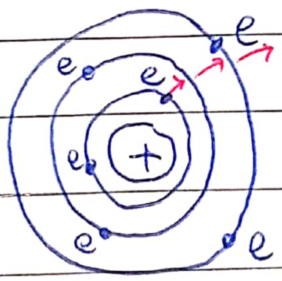
Types of Currents:

[1] Conduction Current $\rightarrow$ AC / DC

[2] Convection Current $\rightarrow$ DC تيار اصطلاحي (يمر بالمادة الحارة)

[3] Displacement Current $\rightarrow$ AC يمر بالمادة الحارة في حالة AC فقط

Atom : اذا وضعنا الحرارة بتزداد الطاقة الحرارية اي الـ e2 تكتسبها على شكل طاقة تمكنها من الانتقال من مداراتها الى مدارات أعلى



الالكترونات موزعة حول النواة بناء على طاقة كل الكترون

$T \uparrow$ $\sigma \downarrow$

الحرارة اع ترتفع (مولدة e تطلع من الذرة) تصير e تتحرك عكسيا منقل التيار

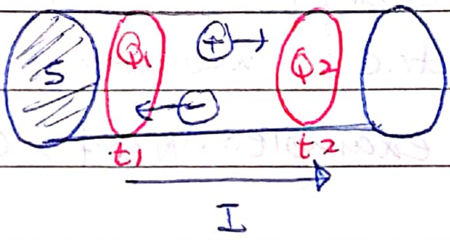
Subject ______ Day ______ Date ______

Electric Current :

$I = \frac{dQ}{dt}$ to get (1 A) current, you need to move a charge of (1C) in duration of (1 S)

for uniform cross section we use delta, if not we use ds

$J = \frac{I}{\Delta s}$ $\left[\frac{A}{m^2}\right]$



→ Current density

$J = \frac{dI}{ds}$

$\int dI = \int J \, ds$

$I = \int_S \bar{J} \cdot \bar{ds}$ → scalar

$\varphi = \int_S \bar{D} \cdot \bar{ds}$

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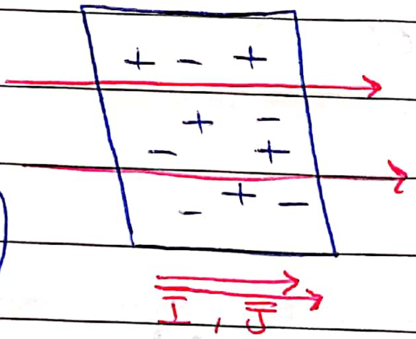
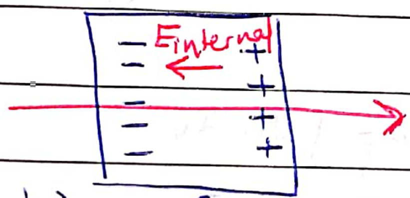
Subject Conductors **Day** ______ **Date** ______



external electric field

ع بسبب الشحنة الموجبة باتجاهها

وهاي الحالة بتحصل عنا توصيل كهربائي



$\bar{E}$ external

an internal E was generated from the relation

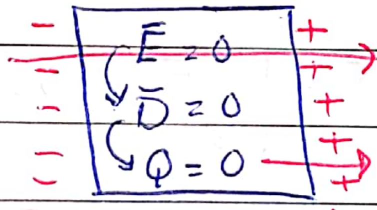
$$\bar{E} = -\nabla V$$

صار عنا فرقا جهد (طرف موجب وطرف سالب)

اتجاه E باتجاه تناقص V

$\bar{E}_{total} = zero$ $\leftarrow$ $\bar{E}_{int.} = \bar{E}_{ext.}$ داخل المادة

جاهزة لتوصيل التيار الكهربائي



Eext.

تجمعت الشحنات على السطح الخارجي

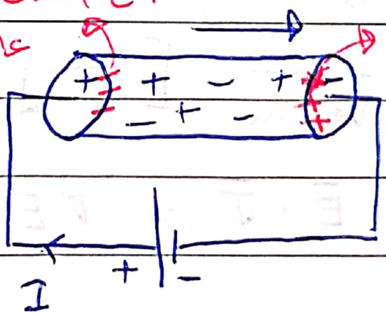
$V_{difference} = 0$ المادة عشان تحافظ على الاتزان

So the conductor became equipotential body.

* Resistance :-

ع تتجمع الشحنات السالبة على القطب الموجب

ع تتجمع الشحنات الموجبة على القطب السالب



$$\boxed{R = \frac{V}{I} = \frac{-\int_l \bar{E} \cdot \bar{dl}}{\int_s \bar{J} \cdot \bar{ds}}}$$

$$R = \frac{l}{\sigma A} = \frac{\rho l}{A} \quad \text{where} \quad \rho = \frac{1}{\sigma}$$

→ for uniform cross section wire

Subject ______ Day ______ Date ______

$$R = \frac{El}{Js} = \frac{El}{\sigma E S} = \frac{l}{\sigma S} \quad \text{(because its uniform)}$$

* Power :

$$P = \frac{\Delta W}{\Delta t} \quad \left[\frac{J}{S} \text{ or } W\right]$$

$$= \frac{F \Delta l}{\Delta t} = \bar{F} \cdot \bar{u} \quad , \quad \frac{\Delta l}{\Delta t} = u$$

$$= Q \bar{E} \cdot \bar{u} \quad , \quad Q = \int_V \rho_v \, dv$$

$$= \int \rho_v \bar{E} \cdot \bar{u} \, dv \quad , \quad \bar{J} = \rho_v \bar{u}$$

$$= \boxed{\int_V \bar{E} \cdot \bar{J} \, dv} \rightarrow \text{Joule's law}$$

for uniform wire $\Rightarrow P = \int_l \int_S \bar{E} \cdot \bar{J} \, ds \, dl$

$$= \underbrace{- \int_l \bar{E} \cdot \bar{dl}}_{V} \; \underbrace{\int_S \bar{J} \cdot \bar{ds}}_{I}$$

$$P = V \cdot I = I^2 R$$

* Power Density :-

$$\omega_p = \frac{\text{power}}{\text{volume}} \quad \left[\frac{W}{m^3}\right]$$

$$= \bar{E} \cdot \bar{J} = \sigma E^2 = \frac{J^2}{\sigma}$$

Example, if $J = \frac{1}{r^3}(2\cos\theta\, \hat{a}_r + \sin\theta\, \hat{a}_\theta)$ A/m²
find I for:

(a) a hemi-spherical shell of radius 20 cm with
$0 < \theta < \frac{\pi}{2}$

hemi (نصف) → 3D
half → 1D

(b) a spherical shell of radius 10 cm.

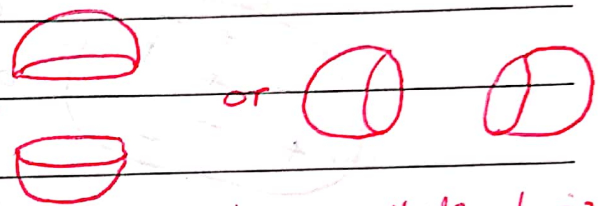
Sol (a) $I = \int_S \bar{J} \cdot \bar{ds}$

نصف الكرة ممكن تجي بالشكلين

$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$

$I = \int_0^{2\pi}\int_0^{\pi/2} \frac{1}{r} \sin 2\theta \, d\theta \, d\phi \Big|_{r=0.2m}$

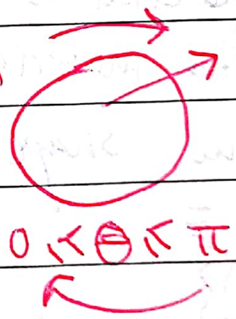
$= 10\pi$ A



or

but ϕ is complete horizontal rotation $(0 < \phi < 2\pi)$
→ so this shape that should be used.

(b) $I = \int \bar{J} \cdot \bar{ds}$



التيار اي J عيمشي على الجزء العلوي على الكرة J,
يمشي نفسه بس بعكس

$I = zero$

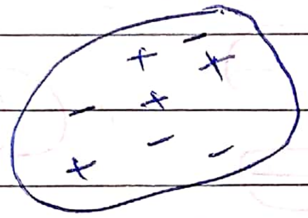
$0 < \theta < \pi$ الاتجاه على السطح السفلي للكرة.

Subject Polarization in Dielectrics ($\bar{P}$) Day ________ Date ________

Dielectrics → non polar
Dielectrics → polar (permenant dipoles)

$$\bar{P} = \lim_{\Delta v \to 0} \frac{\sum_{K=1}^{N} \bar{P}_K}{\Delta v}$$

example ; non polar dielectric (H_2, O_2, N_2, rare gases)

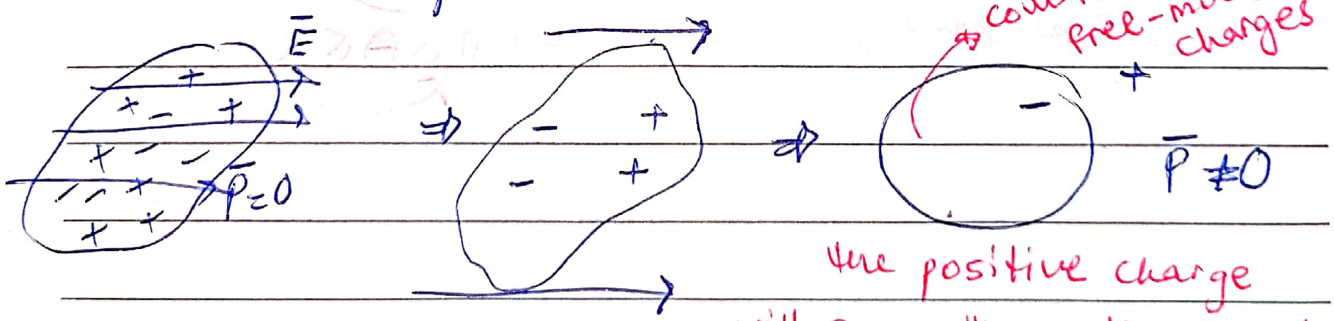


charges are not distributed on the sides of the material → $Q_{total} = 0$

so $\bar{P} = 0$

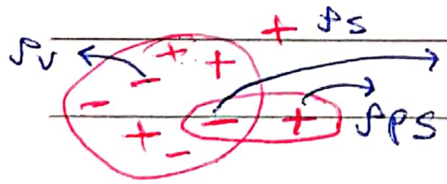
$\bar{P} = 0$ → has dipoles with dipole moment $= 0$
$\bar{P} = 0$ → has no dipoles at all (nonpolar material)

if this material is put under an external electric field, the material will exhibit a force that will effect the shape due to pressure.



the positive charge will go on the outer peremeter of the surface

while the -ve charge will remain inside the surface

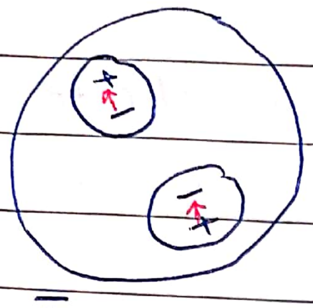


ρ_s : free moving charge

ρ_{pv} / ρ_{ps} : polarization

Subject ______ Day ______ Date ______

Polar :-

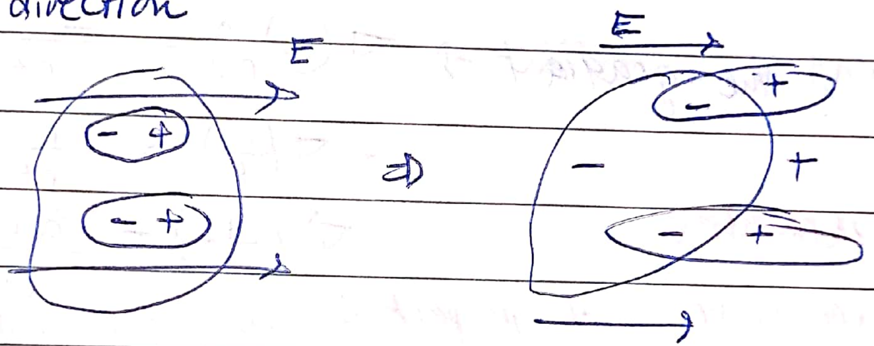


at equilibrum → Q total $= 0$

these two dipoles have dipole moments in opposite directions so the sum equals zero.

$\bar{P} = 0$

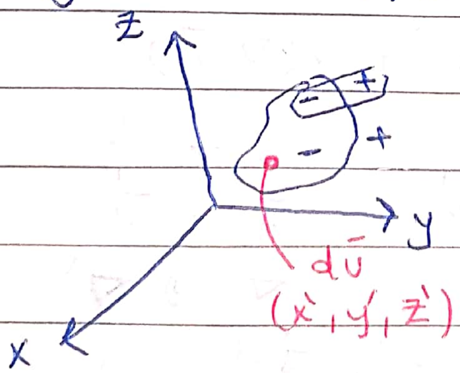
if this material is put under an external E.F, it will effect the positive charges to be alligned in the E.F direction



$\bar{P} \neq 0$

So, whatever the material was polar or nonpolar, the result after electric field is applied will be always (Polarized Dielectric)

* Finding the Potential / Electric Field ⟨ density / intensity :-



• (x, y, z)

the potential at (x, y, z) will be due to bound surface charge distribution and free surface distribution and bound volume C.D and free volume C.D

So we will need to see the effect of these four charge indivisually

Subject ______ Day ______ Date ______

$dV = \frac{\bar{P} \cdot \hat{a}_r}{4\pi\epsilon_0 r^2}$ (due to one dipole)

So we need to integrate both sides in order to consider the whole volume

$$\int_{v'} dV = \int_{v'} \frac{\bar{P} \cdot \hat{a}_r}{4\pi\epsilon_0 r^2}$$

$$V = \int_{v'} \frac{\bar{P} \cdot \hat{a}_r}{4\pi\epsilon_0 r^2} \, dv'$$

applying the gradiant $\rightarrow \nabla\left(\frac{1}{r}\right) = \frac{-1}{r^2} \hat{a}_r$

$$-\nabla\left(\frac{1}{r}\right) = \frac{1}{r^2} \hat{a}_r$$

~~the derivative~~

$$\nabla'\left(\frac{1}{r}\right) = \frac{\hat{a}_r}{r^2}$$

∇ : derivative with respect to the field point (x, y, z)

∇' : " " " " (x', y', z')

$$V = \int_{v'} \frac{\bar{P} \cdot \nabla'\left(\frac{1}{r}\right)}{4\pi\epsilon_0} \, dv'$$

in order to solve this integral, we need a certain identity $\rightarrow$ $\nabla \cdot V\bar{A} = \bar{A} \cdot \nabla V + V(\nabla \cdot \bar{A})$

$$V = \int_{v'} \frac{\bar{P} \cdot \nabla'\left(\frac{1}{r}\right)}{4\pi\epsilon_0} \, dv'$$

$V \rightarrow \frac{1}{r}$

$\bar{A} \rightarrow \bar{P}$

$\nabla \rightarrow \nabla'$

$$= \underbrace{\int_{v'} \frac{\nabla' \cdot \frac{\bar{P}}{r}}{4\pi\epsilon_0} \, dv'}_{1} + \underbrace{\int_{v'} \frac{-\frac{1}{r} \nabla' \cdot \bar{P}}{4\pi\epsilon_0} \, dv'}_{2}$$

Subject ______ Day ______ Date ______

by applying the divergence theorem into integration ①:-

$$V = \oint_{s'} \frac{\frac{\bar{P}}{r} \cdot \bar{ds}}{4\pi\epsilon_0} + \int_{v'} \frac{-\nabla' \cdot \bar{P}}{4\pi\epsilon_0 r} , \quad \bar{ds} = ds\,\hat{a}_n$$

$$= \oint_{s'} \frac{\bar{P} \cdot \hat{a}_n}{(r)\, 4\pi\epsilon_0} ds' + \int_{v'} \frac{-\nabla \cdot \bar{P}}{4\pi\epsilon_0 r} dv'$$

these two terms are used to find the potential due to bound charges (polarized)

For free charges (if exists):-

$$V = \int_s \frac{\rho_s\, ds}{4\pi\epsilon_0 r} + \int_v \frac{\rho_v\, dv}{4\pi\epsilon_0 r}$$

$\rho_{ps} = \bar{P} \cdot \hat{a}_n$ (equal to $\rho_s = \bar{D} \cdot \hat{a}_n$)

$\rho_{pv} = -\nabla \cdot \bar{P}$ ($\rho_v = \nabla \cdot \bar{D}$)

Bound Charges (Q_b) :-

$$Q_{b+} = \int_s \rho_{ps}\, ds$$

$$Q_{b-} = \int_v \rho_{pv}\, dv$$

Remember that for free charges:

$$Q_{b+} = \int_s \rho_s\, ds$$

$$Q_{b-} = \int_v \rho_v\, dv$$

$$Q_{b\,total} = Q_{b+} + Q_{b-}$$

if $Q_{b\,total} = 0 \rightarrow$ electrically neuteralized

$$\rho_{v\,total} = \rho_v + \rho_{pv} = \nabla \cdot \epsilon_0 \bar{E}$$

$$\rho_v = \rho_{v\,total} - \rho_{pv}$$

$$= \nabla \cdot \epsilon_0 \bar{E} - (-\nabla \cdot \bar{P}) = \nabla \cdot \bar{D}$$

$\bar{D} = \epsilon_o E + \bar{P}$

another way to calculate polarization :-

$\bar{P} = X_e \, \epsilon_o \bar{E}$ → this equation is valid for linear materials

since the relation between $\bar{E}$ & $\bar{P}$ is linear.

X_e = electrical susceptibility → $X_e = 0$ for air and conductive materials

→ $X_e > 1$ for dielectrics

$D_{1\,\text{dielectric}} > D_{1\,\text{air}} , D_{1\,\text{conductive}}$

$\rho_v = \nabla \cdot \bar{D}$

$\bar{D} = \epsilon_o \bar{E} + \bar{P} \quad , \quad \bar{P} = X_e \epsilon_o \bar{E}$

$\bar{D} = \epsilon_o \bar{E} + X_e \epsilon_o \bar{E}$

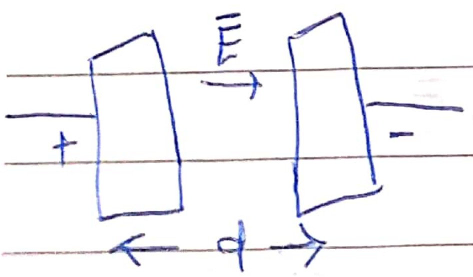
$= (1 + X_e) \epsilon_o \bar{E}$

$= \epsilon_r \, \epsilon_o \, \bar{E}$, ϵ_r = relative permittivity

$\epsilon_r = 1$ → for free space

$\epsilon_r = 1 + X_e = \dfrac{\epsilon}{\epsilon_o}$ (ϵ → total permittivity, ϵ_o → free space permittivity)

to measure ϵ_r experemintally ,



$C_o = \dfrac{\epsilon_o S}{d}$

$C = \dfrac{\epsilon_o \epsilon_r S}{d}$

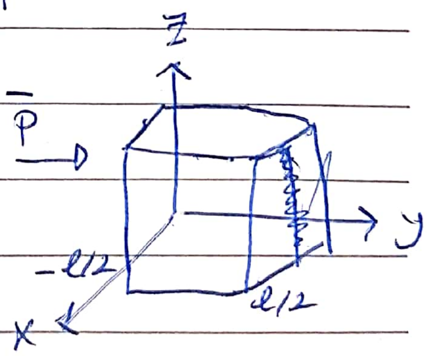
$\dfrac{C}{C_o} = \epsilon_r$

Subject ______ Day ______ Date ______

* Dielectric Breakdown :-

Factors ⇒ 1) Nature of the dielectric material
the dielectrics that we use are in this range → $0 < \sigma << 1$
2) Temperature (increasing Temp → decreasing conductivity)
3) Humidity (as it increase, σ decrease)
4) Applied electric field (as it increase, σ increase)
every material has maximum E-field & this is called "Dielectric Strength"
5) Time the $\bar{E}$ is applied

Example; a dielectric cube of length (ℓ) centered at the origin has $\bar{P} = a\bar{r}$ (a: constant) and $\bar{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$. Find all bound charge densities and the total charge?



Sol $\rho_{ps} = \bar{P}\cdot\hat{a}_n$ (we have 6 faces)

ρ_{ps} front $= ax\,\hat{a}_x\cdot\hat{a}_x = ax = \dfrac{a\ell}{2}$

ρ_{ps} back $= \dfrac{a\ell}{2}$

because : $ax\,\hat{a}_x\cdot -\hat{a}_x \Big|_{x=-\frac{\ell}{2}} = \dfrac{a\ell}{2}$

all faces ⇒ $\rho_{ps} = \dfrac{a\ell}{2}$ C/m^2 since the position vector is identical in all directions

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$\rho_{ps\,total} = -\nabla \cdot \bar{P} = -\frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z}$

$= -(a+a+a) = -3a \quad C/m^3$

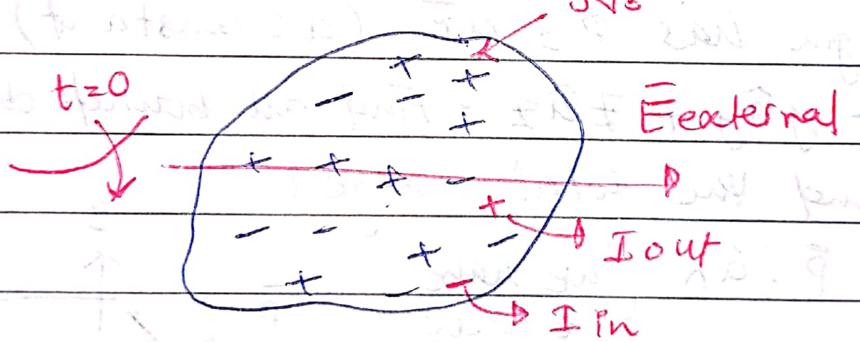
$Q_{b+} = \oint_S \rho_{ps}\, ds = 6 \int_S \rho_{ps}\, ds = 6\, \frac{al}{2}\, l^2$

$= 3\, a\, l^3\, C$

$Q_{b-} = \int_V \rho_{pv}\, dv = -3a\, l^3\, C$

$Q_{total} = Q_{b+} + Q_{b-} = \text{zero}$ (electrically neutralized)

* Continuity Equation :-



KCl : $I_{out} = I_{in}$

$\oint_S \bar{J} \cdot \bar{dS} = -\frac{dQ_{in}}{dt} = -\frac{d}{dt}\int_V \rho_v\, dv = -\int \frac{\partial \rho_v}{\partial t}\, dv$

↳ the charges inside the volume are decreasing

$$\boxed{\oint_S \bar{J} \cdot \bar{dS} = \int_V -\frac{\partial \rho_v}{\partial t}\, dv}$$ → continuity equation in integral form.

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$$\oint_S \bar{J} \cdot \bar{ds} = -\int_V \frac{\partial \rho_v}{\partial t}\, dv$$

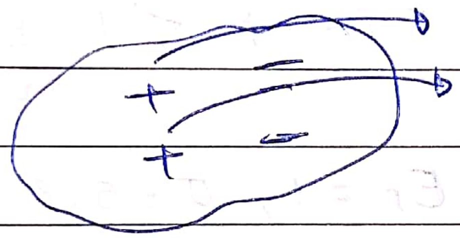
↳ applying the divergence theorem:

$$\int_V \nabla \cdot \bar{J}\, dv = -\int_V \frac{\partial \rho_v}{\partial t}\, dv$$

$$\boxed{\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}}$$ → continuity equation in differential form.

* Relaxation Time (T_r) :-

the time needed for the charge to appear on the surface in order that the material become conductor



$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}$$

$\bar{J} = \sigma \bar{E}$

$\bar{D} = \epsilon_0 \bar{E}$

$$\nabla \cdot \sigma \bar{E} = \frac{\sigma \rho_v}{\epsilon_0} = -\frac{\partial \rho_v}{\partial t}$$

$\rho_v = \nabla \cdot \bar{D}$

$\rho_v = \nabla \cdot \epsilon_0 \bar{E}$

$\frac{\rho_v}{\epsilon_0} = \nabla \cdot \bar{E}$

$$\int_{\rho_{v0}}^{\rho_v} + \frac{\partial \rho_v}{\rho_v} = \int_0^t \frac{-\sigma}{\epsilon_0}\, dt$$

$$\ln(\rho_v - \rho_{v0}) = \frac{-\sigma}{\epsilon_0} t = \frac{-\sigma}{\epsilon} t$$

↳ for all materials

$$\ln\left(\frac{\rho_v}{\rho_{v0}}\right) = \frac{-\sigma}{\epsilon} t$$

$$\rho_v = \rho_{v0}\, e^{\frac{-\sigma}{\epsilon} t}$$

→ الشحنات داخل المادة في البداية

↳ الشحنات التي بقيت داخل المادة بعد فترة من الزمن

$$\boxed{\rho_v = \rho_{v0}, \; t = 0}$$

Subject ____________ Day ____________ Date ____________

$\rho_v = \rho_{vo} \, e^{-\frac{\sigma}{\epsilon} t}$

$\rho_v = \rho_{vo} \, e^{-t/T_r}$

$T_r = \dfrac{\epsilon}{\sigma} = \dfrac{\epsilon_0 \, \epsilon_r}{\sigma}$ → Relaxation time. [s]

at $t = 1\,T_r$ → $\rho_v = \rho_{vo} \, e^{-1} = 0.368 \, \rho_{vo}$

→ %63.2

%36.8

1 Tr

$2\,T_r$ → %14

$3\,T_r$ → % 5

$4\,T_r$ → %1.8

$5\,T_r$ → $<$%1 المادة سوف تصبح in steady state

Example; * for copper $\epsilon_r = 1$, $\sigma = 5.8 \, 10^{+7}$ S/m

so find T_r:

<u>Sol</u> $T_r = \dfrac{\epsilon}{\sigma} = \dfrac{\frac{10^{-9}}{36\pi}}{5.8 \, 10^{7}} = 1.53 \, 10^{-19}$ s

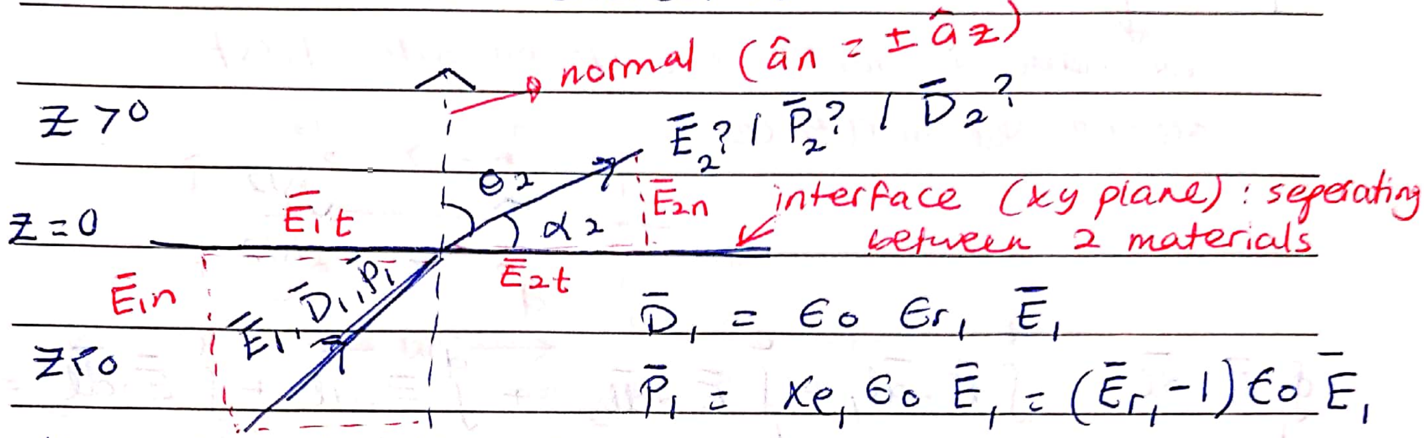
* for fused quartz $\epsilon_r = 5$, $\sigma \simeq 10^{-17}$ S/m.

<u>Sol</u> $T_r = \dfrac{\epsilon}{\sigma} = 51.2$ Days.

Subject: Boundary Conditions Day ______ Date ______

Purpose :- عشان نوصف شخصية المجال الكهربائي لما ينتقل من وسط له مواصفات معينة إلى وسط آخر مختلف معه في الخصائص.

1] Dielectric to Dielectric



$\bar{D}_1 = \epsilon_0 \epsilon_{r_1} \bar{E}_1$

$\bar{P}_1 = \chi e_1 \epsilon_0 \bar{E}_1 = (\epsilon_{r_1} - 1)\epsilon_0 \bar{E}_1$

the characteristic of the milieu $(z < 0)$ is ϵ_{r_1} and for $(z > 0)$ is ϵ_{r_2}, where ϵ_{r_1} and ϵ_{r_2} are both greater than 1 (non of these materials is free space).

* We start with the given $\bar{E}$;

$\bar{E}_1 = \bar{E}_{1n} + \bar{E}_{1t}$

$n \rightarrow$ normal

$t \rightarrow$ tangent

$\hat{a}_n = \hat{a}_z$

$\bar{E}_{1n} = (\bar{E}_1 \cdot \hat{a}_n)\, \hat{a}_n$

$\bar{E}_{1t} = \bar{E}_1 - \bar{E}_{1n}$

$\theta_1 = \sin^{-1} \dfrac{|\bar{E}_{1t}|}{E_1} = \cos^{-1} \dfrac{|\bar{E}_{1n}|}{E_1} = \tan^{-1} \dfrac{E_{1t}}{E_{1n}}$

$\alpha_1 = 90^\circ - \theta_1$

* To find the field in region 2 ;

$\bar{E}_2 = \bar{E}_{2n} + \bar{E}_{2t}$

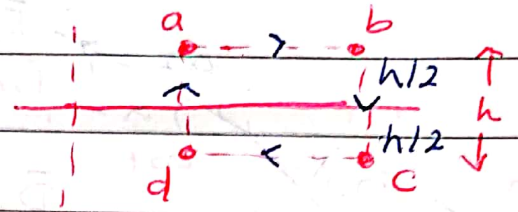
we need to apply 2 equations which are 1st and 2nd Maxwell's equations.

Subject ____________ Day ____________ Date ____________

$$\oint_s \bar{D} \cdot ds = Q_{enclosed} = \int_s \rho_s \, ds \quad (\text{Gauss})$$

$$\oint_l \bar{E} \cdot \bar{dl} = 0 \quad (\text{equipotential surface})$$

we have to make the closed path first around the interface



$$\oint_l \bar{E} \cdot \bar{dl} = \int_a^b \bar{E} \cdot \bar{dl}_1 + \int_b^c \bar{E} \cdot \bar{dl}_2 + \int_c^d \bar{E} \cdot \bar{dl}_3 + \int_d^a \bar{E} \cdot \bar{dl}_4 = 0$$

$$0 = E_{2t} w - \cancel{E_{2n} \frac{h}{2}} - \cancel{E_{1n} \frac{h}{2}} - E_{1t} w + \cancel{E_{1n} \frac{h}{2}} + \cancel{E_{2n} \frac{h}{2}}$$

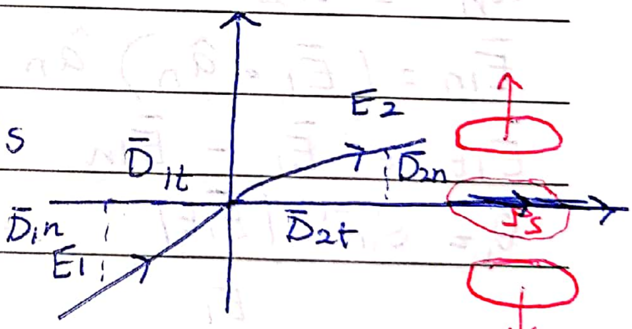
$$E_{2t} w - E_t w = 0$$

$$\boxed{\bar{E}_{2t} = \bar{E}_{1t}}$$ → the tangential component of electric field must be continuous

To find E_{2n} :-

$$\oint_s \bar{D} \cdot \bar{ds} = \int_s \rho_s \, ds \quad (\text{at } z = 0)$$

$$\int_{S_{top}} \bar{D} \cdot \bar{ds}_{top} + \int_{S_{bottom}} \bar{D} \cdot \bar{ds}_{bottom} = \rho_s \, ds$$



$$D_{2n} \cancel{\Delta s} - \cancel{\Delta s} D_{1n} = \rho_s \cancel{\Delta s}$$

$$\boxed{D_{2n} - D_{1n} = \rho_s}$$

assume of the area for all of them $= \Delta s$

$\Delta s \to \infty$

if $\rho_s = 0$ then $\bar{D}_{2n} = \bar{D}_{1n}$

so $\epsilon_0 \epsilon_{r2} E_{2n} = \epsilon_0 \epsilon_{r1} \bar{E}_{1n}$

$$\boxed{\bar{E}_{2n} = \frac{\epsilon_{r1}}{\epsilon_{r2}} \bar{E}_{1n}}$$

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$$\theta_2 = \sin^{-1}\left(\frac{E_{2t}}{E_2}\right)$$

$$\alpha_2 = 90^\circ - \theta_2$$

or $\bar{E}_{1t} = \bar{E}_{2t}$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2 \quad \text{--- (1)}$$

if $\rho_s = 0 \rightarrow D_{2n} - D_{1n} = 0$ $\qquad \epsilon_{r1} E_1 \cos\theta_1 = \epsilon_{r2} E_2 \cos\theta_2$

$$\bar{D}_{2n} = \bar{D}_{1n}$$

$$\epsilon_0 \epsilon_{r1} E_{1n} = \epsilon_0 \epsilon_{r2} E_{2n} \quad \text{--- (2)}$$

divide (1) by (2) $\rightarrow \dfrac{\tan\theta_1}{\epsilon_{r1}} = \dfrac{\tan\theta_2}{\epsilon_{r2}}$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$ only if $\rho_s = 0$

2] Conductor to Dielectric

$\sigma_2 \simeq \infty$ (perfect conductor)

$\epsilon_{r2} = 1$

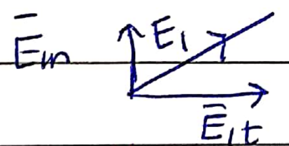
$\bar{E}_2 = 0 \rightarrow \bar{E}_{2n} = 0,\ \bar{E}_{2t} = 0$ ~~conductor~~

$\bar{D}_2 = 0 \rightarrow \bar{D}_{2t} = 0,\ \bar{D}_{2n} = 0$

interface

$z = 0$

$\epsilon_{r1} > 1$ ~~dielectric~~



E_1 hasn't reached the interface yet.

$\bar{E}_{1t} = \bar{E}_{2t} \rightarrow \bar{E}_{1t} = 0$ at $z = 0$ So the electric field ~~will enter the interface~~ with 90° (normal)

$\cancel{D_{2n}} - D_{1n} = \rho_s$

$D_{1n} = -\rho_s = \rho_s(-\hat{a}_n)$ $\qquad$ θ_2, α_2 doesn't exist

$\bar{E}_{1n} = \dfrac{-\rho_s\ \hat{a}_n}{\epsilon_0 \epsilon_{r1}}$ $\qquad$ $\theta_1 = 0^\circ$

$\alpha_1 = 90^\circ$

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isotropic : permittivity doesn't change with direction.

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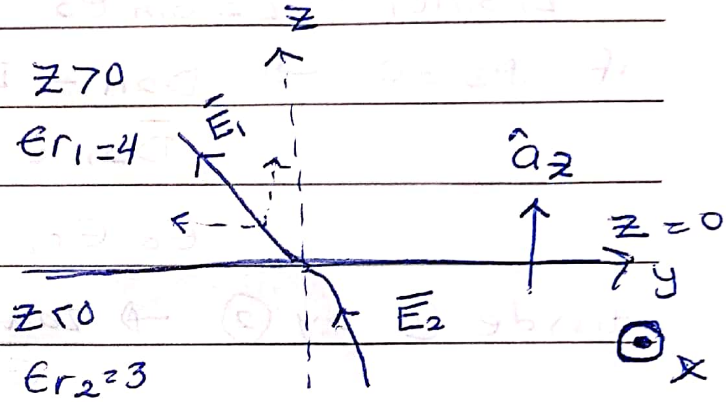
ϵ_r is constant everywhere inside the region

Example; Two homogeneous isotropic dielectrics meet on plane $z=0$, for $z>0$, $\epsilon_{r_1}=4$ and for $z<0$, $\epsilon_{r_2}=3$. if $\bar{E}_1 = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$ KV/m exists for $z \geq 0$. find:

(a) $\bar{E}_2$ for $z \leq 0$?



Sol $\hat{a}_n = \hat{a}_z$

$\bar{E}_{1n} = (\bar{E}_1 \cdot \hat{a}_n)\,\hat{a}_n$

$= 3\hat{a}_z$ KV/m

$\bar{E}_{1t} = \bar{E}_1 - \bar{E}_{1n}$

$= 5\hat{a}_x - 2\hat{a}_y$ KV/m $= \bar{E}_{2t}$

$D_{2n} - D_{1n} = \rho_s = 0$ since ρ_s is not mentioned in the question and it said dielectric

Most likely between dielectrics there is no surface charge because the charges inside dielectrics need large relaxation time to reach the surface so usually there is no surface charge on the interface

$\bar{E}_{2n} = \dfrac{\epsilon_{r_1}}{\epsilon_{r_2}} \bar{E}_{1n} = \left(\dfrac{4}{3}\right) 3\hat{a}_z = 4\hat{a}_z$ KV/m

$\bar{E}_2 = \bar{E}_{2n} + \bar{E}_{2t} = 5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z$ KV/m

(b) the angles E_1 and E_2 makes with the interface?

Sol $\theta_1 = \sin^{-1} \dfrac{E_{1t}}{E_1} = \sin^{-1} \dfrac{\sqrt{29}}{\sqrt{38}} = 60.9°$

$\alpha_1 = 90° - \theta_1 = 29.1°$

$\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{4}{3} \rightarrow \theta_2 = 53.4°$

$\alpha_2 = 36.6°$

Note:-
angles with the normal $\rightarrow \theta_1 / \theta_2$
angles with the interface $\rightarrow \alpha_1 / \alpha_2$

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ⓒ Energy densities in both dielectrics ?-

Sol $w_{E1} = \frac{1}{2} \epsilon_1 E_1^2 = \frac{1}{2} \epsilon_0 \epsilon_{r1} E_1^2 = \frac{1}{2} \frac{10^{-9}}{36\pi} (4)\, 38\, 10^6$

$= 672 \frac{\mu J}{m^3}$

$w_{E2} = \frac{1}{2} \epsilon_0 \epsilon_{r2} E_2^2 = \frac{1}{2} \frac{10^{-9}}{36\pi} (3)\, 45\, 10^6 = 597 \frac{\mu J}{m^3}$

$|E_2| = \sqrt{45} \frac{KV}{m}$

$|E_1| = \sqrt{38} \frac{KV}{m}$

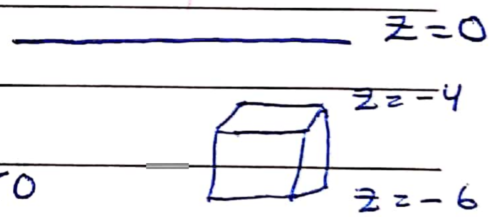
$E_2 > E_1$

$w_{E2} < w_{E1}$ because ϵ_{r2} is greater than ϵ_{r1}

ⓓ The energy within a cube of side 2m centered at (3, 4, -5) ?

$z > 0$

Sol $W_E = \int_v w_{E2}\, dv$ $z < 0$



$= (597\, 10^{-6})(8)$

$z < 0$

$= 4.776$ mJ

if the cube is centered at the origin → WE equals $(w_{E1} \times 4) + (w_{E2} \times 4)$ since half the cube will be in region 1 and the other half will be in region 2.

Subject: Boundary Value Problems Day: (B.V.P) Date:

Purpose :- we will use it as a fourth way to find electric field.

we can find $\bar{E}$ using → 1) Coulomb's law } charge must
2) Gauss law } be known
3) potential → $\bar{E} = -\nabla V$

problem : if the charge or the potential are known for some part of the surface (not for the whole surface) so we can't use coloumb, gauss & potential → we will use B.V.P.

$\rho_v = \nabla \cdot \bar{D}$, $\bar{D} = \epsilon \bar{E}$, $\bar{E} = -\nabla V$

$\nabla \cdot (\epsilon \bar{E}) = \rho_v$

$\nabla \cdot (-\epsilon \nabla V) = \rho_v$

$\nabla \cdot \epsilon \nabla V = -\rho_v$ ---- (1) Poisson's equation for non homogeneous media.

ϵ is not a constant
ما بنقدر نطلعها برا المشتقة

$\epsilon \nabla \cdot \nabla V = -\rho_v$

scalar quantity $\nabla^2 V = \dfrac{-\rho_v}{\epsilon}$ --- (2) Poisson's equation for homogeneous media

if $\rho_v = 0$ → source free region

$\nabla \cdot \epsilon \nabla V = 0$ ---- (3) Laplace's equation for non homogeneous media

$\nabla^2 V = 0$ --- (4) Laplace's equation for homogeneous media

ϵ ذكر

function → non homogeneous for laplace

ثابت → homogeneous

اذا ذكر ρ_v بالسؤال شحنات ← poisson

اذا ذكر ρ_v بالسؤال ← laplace

اذا ما ذكر ϵ ← homogeneous

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Cartesian :-

$$\nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}$$

Cylindrical :- ((this equation is given))

$$\nabla^2 v = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\frac{\rho\, \partial v}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 v}{\partial \phi^2} + \frac{\partial^2 v}{\partial z^2}$$

الاولوية لداخل القوس بنشتق $\frac{\partial v}{\partial \rho}$ بعدين نضرب ب

ب ρ بعدين نشتق بالنسبة لـ ρ وبعدين نقسم على ρ

Spherical :-

$$\nabla^2 v = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial v}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial v}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v}{\partial \phi^2}$$

* Uniqueness Theorem :-

اذا كان حل intial conditions two ← بتقدر توجد قيمة 2 unknowns

B.V.P + 2 intial conditions → unique

يعني مش عليك تطلع كل السؤال جوابين لازم جواب واحد فقط

* General procedure to solve B.V.P :-

1) solve laplace's or poisson's equation using ⇒

(a) direct integration if (v) is function of one variable

(b) separation of variables if (v) is function of two or three variables.

2) apply the two boundary conditions to make the solution unique

3) find electric field ($\bar{E} = -\nabla v$).

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4) find other quantities ⟹

drift velocity, mobility

$\bar{D} = \epsilon \bar{E}$, $\bar{J} = \sigma \bar{E}$, $\bar{u} = \mu \bar{E}$

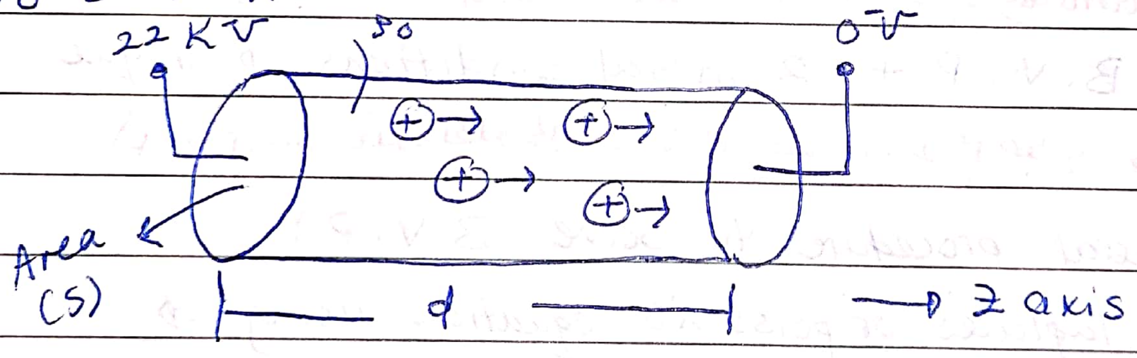
$W_E = \frac{1}{2} \int_v \epsilon E^2 \, dv$, $F = Q\bar{E}$

$\text{pressure} = \frac{F}{\text{area}}$, $Q = \oint_s \epsilon \bar{E} \cdot \bar{ds}$, $R = \frac{V}{I} = \frac{-\int_l \bar{E} \cdot \bar{dl}}{\int_s \sigma \bar{E} \cdot \bar{ds}}$

$C = \frac{Q}{V} = \frac{\oint_s \epsilon \bar{E} \cdot \bar{ds}}{-\int_l \bar{E} \cdot \bar{dl}}$, $P = \int_v \sigma E^2 \, dv$

Example; an electrohydrodynamic (EHD) pumping has a uniform charge ρ_o c/m^3 exists between the two electrods which is generated in the left electrode and collected at the right electrode calculate the pressure of the pump if $\rho_o = 25 \, m\frac{c}{m^3}$

$V_o = 22KV$



Sol $\nabla^2 V = \frac{-\rho_v}{\epsilon} = \frac{-\rho_o}{\epsilon}$

$\frac{d^2 V}{dz^2} = \frac{-\rho_o}{\epsilon}$

integrating → $\frac{dv}{dz} = \frac{-\rho_o}{\epsilon} z + A$ integration constant

$V = \frac{-\rho_o}{\epsilon} \frac{z^2}{2} + Az + B$ → General Solution.

integration constant

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$$V = \frac{-\rho_0}{\epsilon}\frac{z^2}{2} + Az + B$$

$V\big|_{z=0} = V_0$, $V\big|_{z=d} = 0$

$\downarrow$ $\boxed{B = V_0}$

$\downarrow$ $0 = \frac{-\rho_0}{\epsilon}\frac{d^2}{2} + Ad + V_0$

$$\boxed{A = \left[\frac{\rho_0 d^2}{2\epsilon} - V_0\right]\frac{1}{d}}$$

$$V = \frac{-\rho_0 z^2}{2\epsilon} + \left(\frac{\rho_0 d}{2\epsilon} - \frac{V_0}{d}\right) z^2 + V_0 \rightarrow \text{Unique Solution}$$

$$\bar{E} = -\nabla V = -\frac{dV}{dz}\hat{a}_z = \frac{\rho_0 z}{\epsilon} - \frac{\rho_0 d}{2\epsilon} + \frac{V_0}{d}\hat{a}_z$$

$$\text{pressure} = \frac{|F|}{\text{area}} = \frac{Q|E|}{\text{area}} \quad , \quad Q = \int_V \rho_V \, dV$$

scalar quantity

$$\bar{E} = \frac{\rho_0 z}{\epsilon} - \frac{\rho_0 d}{2\epsilon} + \frac{V_0}{d}\hat{a}_z$$

$$\bar{F} = Q\bar{E} = \int_V \rho_V \bar{E}\, dV = \rho_0 \int_S\int_z \bar{E}\, ds\, dz \quad , \quad \int_S ds = S$$

$$= \rho_0 S \int_z \bar{E}\, dz = \rho_0 S \left(\frac{\rho_0 z^2}{2\epsilon} - \frac{\rho_0 d}{2\epsilon} z + \frac{V_0}{d} z\right)\Bigg]_{z=0}^{z=d}$$

$$= \rho_0 S \left(\cancel{\frac{\rho_0 d^2}{2\epsilon}} - \cancel{\frac{\rho_0 d^2}{2\epsilon}} + V_0 \frac{d}{d}\right)\hat{a}_z$$

$$= \rho_0 S V_0 \,\hat{a}_z \ \text{N}$$

$$\text{pressure} = \frac{\rho_0 S V_0}{S} = \rho_0 V_0 = 550 \ \text{N/m}^2$$

Subject Resistance and Capacitance Day ______ Date ______

$$R = \frac{V}{I} = \frac{-\int_l \bar{E}\cdot\bar{dl}}{\oint_s \sigma \bar{E}\cdot\bar{ds}}$$

always positive

$$C = \frac{Q}{V} = \frac{\oint_s \bar{E}\epsilon\cdot ds}{-\int_l \bar{E}\cdot dl}$$

It means that the integration must be from the lower potential to the higher potential

Procedure to find R and C :-

1] choose a suitable coordinate system.

2] assume either Vo difference or a positive or negative charges

$\downarrow$ B.V.P

Gauss method

3] solve B.V.P and apply the boundary conditions

4] $\bar{E} = -\nabla V$

5] = $R = \dfrac{V_o}{\oint_s \sigma \bar{E}\cdot\bar{ds}}$, $C = \dfrac{\oint_s \epsilon \bar{E}\cdot\bar{ds}}{V_o}$

for B.V.P

for Gauss method :-

3] $Q = \oint_s \epsilon \bar{E}\cdot\bar{ds}$

4] $V = -\oint_l \bar{E}\cdot\bar{dl}$

5] $R = \dfrac{V}{I}$, $C = \dfrac{Q}{V}$

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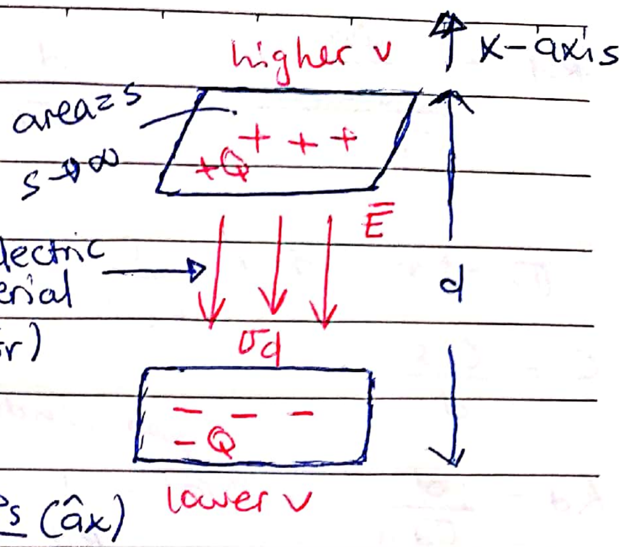
Subject ① Parallel Plate Capacitor Day ______ Date ______

Gauss :-



$$\Phi = \oint_s \epsilon \bar{E} \cdot \bar{ds}$$

$$\rho_s = \frac{\Phi}{s}$$

$$\bar{E}_{\text{infinite plate}} = \frac{\rho_s}{2\epsilon}(-\hat{a}_x)$$

two plate $\rightarrow$ $\bar{E} = \frac{\rho_s}{2\epsilon_0}(-\hat{a}_x) + \frac{-\rho_s}{2\epsilon_0}(\hat{a}_x)$

$$= \frac{\rho_s}{\epsilon}(-\hat{a}_x) \quad , \quad \rho_s = \frac{\Phi}{s}$$

$$= \frac{\Phi}{s\epsilon}(-\hat{a}_x) \ \frac{V}{m}$$ step ③

$$V = -\int_l \bar{E} \cdot \bar{dl} = -\int \frac{\Phi}{s\epsilon}(-\hat{a}_x) \cdot dx\,\hat{a}_x$$ step ④

$$= \frac{\Phi}{s\epsilon} d \ V$$

$$C = \frac{\Phi}{V} = \frac{\Phi}{\frac{\Phi d}{s\epsilon}} = \frac{\epsilon s}{d} \ F \quad , \quad \epsilon = \epsilon_0 \epsilon_r$$

$$R = \frac{V}{I} \quad , \quad I = \oint_s \bar{J} \cdot \bar{ds} = \oint_s \sigma_d \bar{E} \cdot \bar{ds}$$

$$I = \iint_{z\ y} \sigma_d \frac{\Phi}{s\epsilon}(-\hat{a}_x) \cdot dy\,dz\,(-\hat{a}_x)$$

$\rightarrow \left[\frac{s}{m}\right]$, $s = \frac{1}{\Omega}$

$$I = \frac{\sigma_d \ \Phi}{s \ \epsilon} s \Rightarrow I = \frac{\sigma_d \Phi}{\epsilon} \ A$$

convection current

$$\text{Volt} = \left[\frac{C}{F}\right] = \frac{\Phi}{c}$$

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$$R_d = \frac{\frac{\phi d}{S\epsilon}}{\frac{\sigma_d \phi}{E}} = \frac{d}{S\sigma_d} \ \Omega$$ (dielectric resistance)

$\sigma_c \to \infty \Rightarrow R = \frac{l}{\sigma_c A} = zero$ (ideal conductor)

$C = \frac{\epsilon S}{d}$

conductance

$R_d = \frac{d}{\sigma_d S}$, $G = \frac{1}{R_d} = \frac{\sigma_d S}{d}$ [S]

R نفس الشيء
و C بس نقلب
وبدل كل ϵ بـ σ

$W_E = \frac{1}{2}\int_V \epsilon E^2 dv = \frac{1}{2}\epsilon \frac{\phi^2}{S^2\epsilon^2} (Sd)$

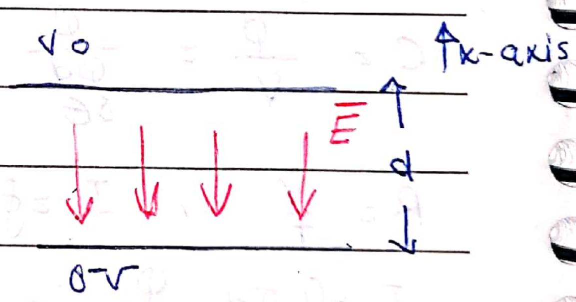
capacitor حجم = المساحة × ارتفاع = $d \times S$

$= \frac{\phi^2 d}{2\epsilon S}$ $\frac{1}{C}$ $= \frac{\phi^2}{2C}$, $C = \frac{\phi}{V}$

$= \frac{1}{2}\phi V$, $\phi = CV$

$= \boxed{\frac{1}{2} C V^2 \ [J]}$ ~~if uniform~~

B.V.P :-



cartesian $\to \nabla^2 v = 0$

$\frac{d^2 v}{dx^2} = 0$

$\frac{dv}{dx} = A \Rightarrow V = Ax + b$

$V|_{x=0} = 0 \Rightarrow B = 0$

$V|_{x=d} = V_o \Rightarrow A = \frac{V_o}{d}$

$\boxed{V = \frac{V_o}{d} x}$ General more than Gauss

غاوس بتعطي V بس عند اللوحين لأنه ما فيه المتغير X

B.V.P بتعطي V عند اي مكان بين اللوحين .

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$$\bar{E} = -\nabla V = \frac{-dV}{dx}\hat{a}_x = \frac{-V_0}{d}\hat{a}_x \quad \frac{V}{m}$$

$$\Phi = \int_S \epsilon \bar{E} \cdot d\bar{s} = \int_z\int_y \epsilon \left(\frac{-V_0}{d}\right)\cancel{\hat{a}_x} \cdot dy\,dz(\cancel{-\hat{a}_x})$$

$$= \frac{\epsilon V_0 S}{d}$$

$$C = \frac{\Phi}{V_0} = \frac{\epsilon S}{d}$$

$$I = \int_S \sigma_d \bar{E} \cdot \bar{ds} = \int_z\int_y \sigma_d \left(\frac{-V_0}{d}\right)\hat{a}_x \cdot dy\,dz(-\hat{a}_x)$$

$$= \frac{\sigma_d V_0 S}{d}$$

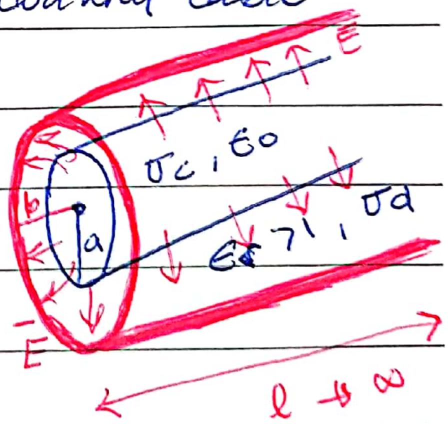
$$R_d = \frac{V_0}{\frac{\sigma_d V_0 S}{d}} = \frac{d}{\sigma_d S}$$

② Cylindrical Capacitor :-

also known as Coaxial capacitor / Coaxial cable

Gauss :

practically
- → inner cable → $+\Phi$
- → external cable → $-\Phi$



$$\Phi = \oint \epsilon \bar{E} \cdot \bar{ds}$$

$$= \int_0^l\int_0^{2\pi}\epsilon \bar{E}_\rho \hat{a}_\rho \cdot \rho\,d\phi\,dz\,\hat{a}_\rho$$

$$= \epsilon E_\rho \rho (2\pi)(l) \Rightarrow \bar{E} = \frac{\Phi}{2\pi\epsilon\rho l}\hat{a}_\rho$$

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$V = -\frac{Q}{2\pi\epsilon l} \ln\left(\frac{a}{b}\right)$ V $= \frac{Q}{2\pi\epsilon l} \ln\left(\frac{b}{a}\right)$ V

$C = \frac{Q}{V} = \frac{2\pi\epsilon l}{\ln\left(\frac{b}{a}\right)}$ F $\quad (l \to \infty)$

$\frac{C}{l} = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)}$ $\frac{F}{m}$

$R_d = \frac{V}{I}$, $I = \int \sigma_d \bar{E} \cdot \bar{ds}$

$= \int_0^l \int_0^{2\pi} \sigma_d \frac{Q}{2\pi\epsilon\rho l} \hat{a}_\rho \cdot \rho \, d\phi \, dz \, \hat{a}_\rho$

$= \frac{\ln\left(\frac{b}{a}\right)}{2\pi \sigma_d l}$

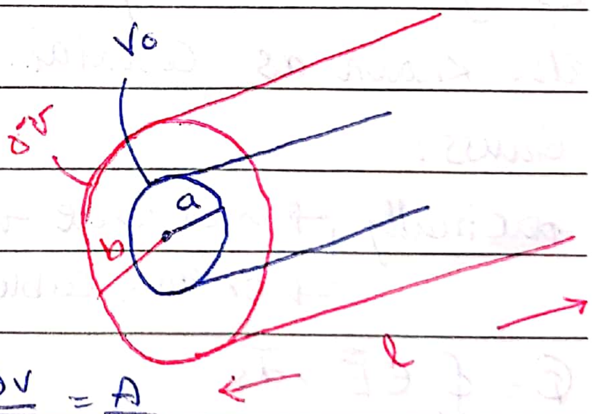
$= \frac{\sigma_d Q}{\epsilon}$ A

$W_E = \frac{1}{2} C V^2$, $W_E = \frac{1}{2} \int_v \epsilon E^2 dv$

$\downarrow$

$C = \frac{2\pi\epsilon l}{\ln\left(\frac{b}{a}\right)}$ F

B.V.P :-



$\nabla^2 V = 0$

$\frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0$

integrating $\to \rho \frac{dV}{d\rho} = A \to \frac{dV}{d\rho} = \frac{A}{\rho}$

integrating $\to A \ln \rho + B$

$V(\rho = a) = V_0$, $V(\rho = b) = 0$

$V_0 = A \ln a - A \ln b$ $\qquad 0 = A \ln b + B$

$= A \ln \frac{a}{b}$ $\qquad \boxed{B = -A \ln b}$

$\boxed{A = \frac{V_0}{\ln \frac{a}{b}}}$

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$$V = \frac{V_0}{\ln\left(\frac{a}{b}\right)} \ln\left(\frac{\rho}{b}\right)$$ → unique potential

$$\bar{E} = -\nabla V = \frac{-\partial V}{\partial \rho} \hat{a}_\rho = \frac{-V_0}{\rho \ln\left(\frac{a}{b}\right)} \hat{a}_\rho$$

$$Q = \oint_S \epsilon \bar{E} \cdot \bar{ds} = \int_0^{l}\int_0^{2\pi} \frac{\epsilon}{\rho}\left(\frac{-V_0}{\ln\frac{a}{b}} \hat{a}_\rho\right) \cdot \rho \, d\phi \, dz \, \hat{a}_\rho$$

$$= \frac{-\epsilon V_0}{\ln\frac{a}{b}} 2\pi l$$

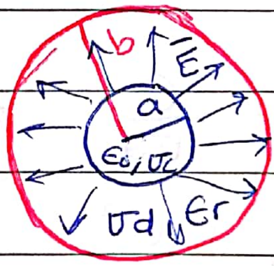
$$C = \frac{Q}{V_0} = \frac{2\pi \epsilon l}{\ln\frac{b}{a}} \quad F$$

$$I = \oint_S \sigma_d \bar{E} \cdot \bar{ds}$$

$$R = \frac{V_0}{I}$$

③ Spherical Capacitor

Gauss :-



$$Q = \oint_S \epsilon \bar{E} \cdot \bar{ds}$$

$$= \int_0^{2\pi}\int_0^{\pi} \epsilon E_r \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$= \epsilon E_r r^2 (2)(2\pi)$$

$$\bar{E} = \frac{Q}{4\pi \epsilon r^2} \hat{a}_r$$

$$V = -\int_l \bar{E} \cdot \bar{dl} = -\int_{r=b}^{r=a} \frac{Q}{4\pi\epsilon r^2} \hat{a}_r \cdot dr \, \hat{a}_r$$

$$= \frac{Q}{4\pi\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right)$$

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$$C = \frac{Q}{V} = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}} \quad F$$

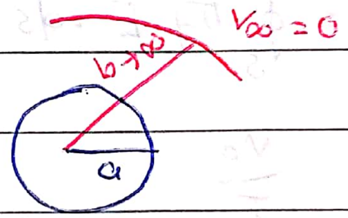
$$I = \oint_S \sigma_d \bar{E} \cdot \bar{ds} = \int_0^{2\pi}\int_0^{\pi} \sigma_d \frac{Q}{4\pi\epsilon r^2} \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$= \frac{\sigma_d Q}{\epsilon} \quad A$$

$$R_d = \frac{\frac{Q}{4\pi\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right)}{\frac{\sigma_d Q}{\epsilon}} = \frac{\frac{1}{a} - \frac{1}{b}}{4\pi\sigma_d}$$

(4) Isolated Capacitor (Isolated Sphere) :-

$$C = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{\infty}} = 4\pi\epsilon a \quad F$$

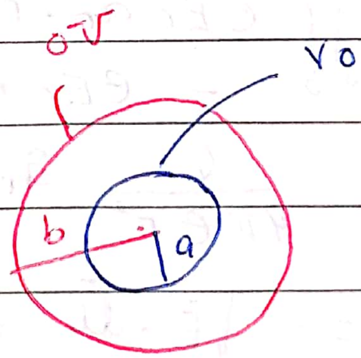


$$R_d = \frac{1}{4\pi\sigma_d a} \quad \Omega$$

$$G = 4\pi\sigma_d a \quad S$$

$$R_d C = \frac{\epsilon}{\sigma_d} = Tr$$

$$W_E = \frac{1}{2} C V^2$$



B.V.P :-

$$\nabla^2 V = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

integrating → $\int \frac{\partial V}{\partial r} = \int \frac{A}{r^2}$

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$V = \frac{-A}{r} + B \rightarrow$ general solution

$V(r=a) = V_0$ $\quad\quad$ $V(r=b) = 0$

$V_0 = \frac{-A}{a} + \frac{A}{b}$ $\quad\quad$ $-\frac{A}{b} = -B$

$V_0 = A\left(\frac{1}{b} - \frac{1}{a}\right)$ $\quad\quad$ $\boxed{B = \frac{A}{b}}$

$$\boxed{A = \frac{V_0}{\frac{1}{b} - \frac{1}{a}}}$$

$$V = \frac{V_0\left(\frac{1}{r} - \frac{1}{b}\right)}{\frac{1}{a} - \frac{1}{b}}$$

$$\bar{E} = -\nabla V = \frac{-dv}{dr}\hat{a}_r = \frac{-V_0}{\frac{1}{a} - \frac{1}{b}}\left(\frac{-1}{r^2}\right)\hat{a}_r$$

$$= \frac{V_0}{\left(\frac{1}{a} - \frac{1}{b}\right)r^2}\hat{a}_r \quad V/m$$

$$\Phi = \oint_s \epsilon\bar{E}\cdot\bar{ds} = \int_0^{2\pi}\int_0^{\pi} \epsilon \frac{V_0}{\frac{1}{a} - \frac{1}{b}} \frac{1}{r^2}\hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$= \frac{\epsilon V_0}{\frac{1}{a} - \frac{1}{b}}(4\pi) \; C$$

$$C = \frac{\Phi}{V_0} = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}} \; F$$