SIGNALS

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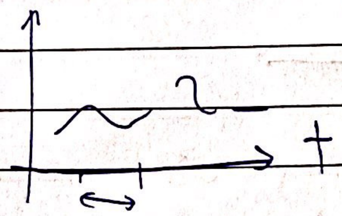
POWERUNIT

Signals

* Signal: function of independent variables $\overset{\text{مثل}}{\Rightarrow}$ $y(x) = x$

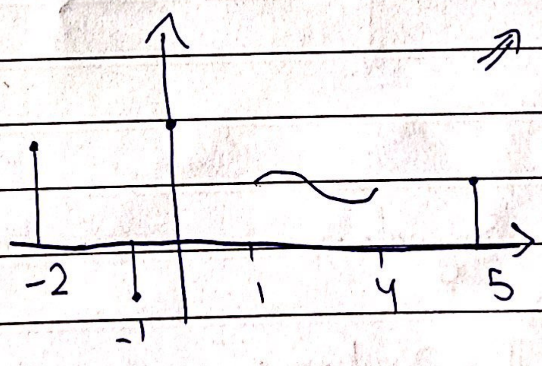
independent variable

* CT signal ⇒ تكون قيم معرّفة على فترة



CT Signal

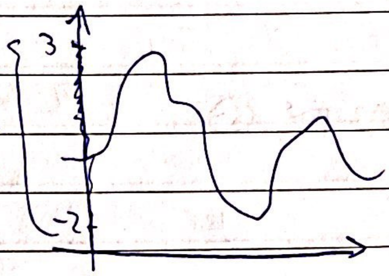
* DT signal ⇒ قيمة عند وقت محدد



⇗ CT Signal
عشان يوجد
فترة،
لو ما
كان
في فترة
بتكون DT

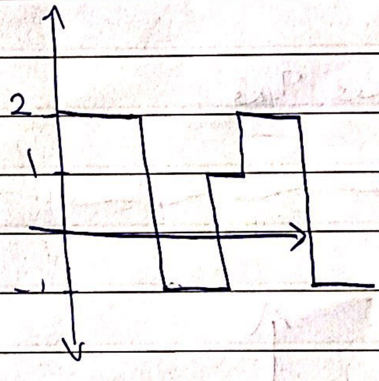
* Analog signal : تكون قيم الـ Signals على فترات من محور الصادات ولازم تكون CT
(continous amplitude)
قيم الصادات

فترة ⟸



$\Rightarrow$ Amplitude $\in \{-2 < A \leq 3\}$

* Digital signal : تكون قيم الـ Signals قيم محددة وليست على فترات
(Discrete amplitude)



$\Rightarrow$ amplitude $\in \{2, 1, 0, -1\}$

* Classification of Signals:

① Periodic & Non-Periodic :

• Periodic $\Rightarrow$ تكون الـ Signal بتكرر نفسها من $(-\infty, \infty)$ مثل اقتران $\cos(t)$

$T_0 \Rightarrow$ Fundemental
$T \Rightarrow$ Period (من مضاعفات T_0)

$x(t) = x(t + nT_0)$

$f_0 = \dfrac{1}{T_0}$

radian Frequency $= \omega_0 = 2\pi f_0$
$= \dfrac{2\pi}{T_0}$ rad/sec,

* Periodic (T_{01}) ± Periodic (T_{02}) = ?

rational $\left(\frac{T_1}{T_2}\right)$ ⇓ Periodic

not rational ⇓ non-periodic

Slide 12:

example: Is $x(t) = \cos\left(\frac{\pi t}{2}\right) + \cos\left(\frac{\pi t}{3}\right)$ periodic, find T_0?

$$T_{01} = \frac{2\pi}{\pi/2} = 4$$

$$T_{02} = \frac{2\pi}{\pi/3} = 6$$

$\frac{T_{01}}{T_{02}} = \frac{4}{6} = \frac{2}{3}$ ⇒ rational ⇒ $x(t)$ is periodic

$4 \times 6 = 6 \times 4 = 24 = T$ (Period)

$2 \times 6 = 3 \times 4 = 12 = T_0$ (fundemental period)

Slide 13: $T_0 = T_{01} \cdot L \Rightarrow L = Lcm$ للمقامات

يلي مكتوبة

بأبسط صورة

example 2: Is $x(t) = \cos(3.5t) + \sin(2t) + 2\cos\left(\frac{7t}{6}\right)$ periodic

find T_0?

Sol:

$$T_{01} = \frac{2\pi}{3.5} = \frac{4\pi}{7}$$

$$T_{02} = \frac{2\pi}{2} = \pi$$

$$T_{03} = \frac{2\pi}{7/6} = \frac{12\pi}{7}$$

$$\frac{T_{01}}{T_{02}} = \frac{4\pi}{7\pi} = \frac{4}{7}$$

$$\frac{T_{01}}{T_{03}} = \frac{4}{12} = \frac{1}{3}$$

rational بما أنه القيمتين

$\rightarrow x(t)$ is periodic إذا

$L = LcM(7,3) = 21$

$$T_0 = L\, T_{01}$$

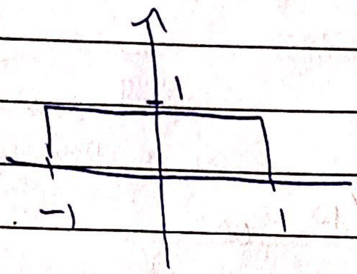
$$= 21 \times \frac{4}{7}$$

$$\boxed{T_0 = 12}$$

$2\cos(t) \Rightarrow$ Bounded signal / Time unlimited

$\text{rect}(t) \Rightarrow$ Bounded signal / Time limited

$\Downarrow$

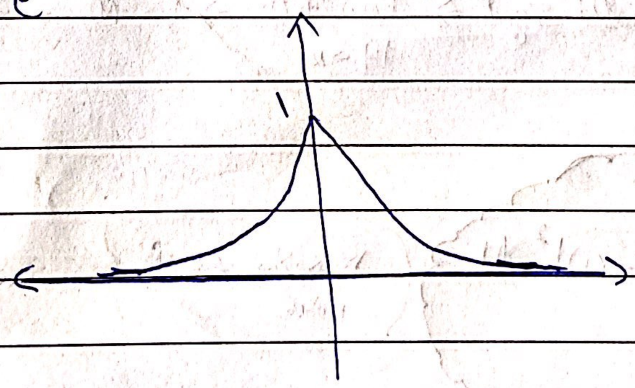


$e^{-t} \Rightarrow$ Unbounded signal / Time unlimited

$\tan(t) \Rightarrow$ Unbounded signal

$e^{t} \Rightarrow$ unbounded signal

$e^{-|t|} \Rightarrow$ bounded signal / Time unlimited



* Energy & Power Signal:

Energy Signal $\Rightarrow 0 < E < \infty \;/\; P = 0$

power Signal $\Rightarrow 0 < P < \infty \;/\; E = \infty$

ارجع للسلايد 9
Notes

Neither power or Energy Signal $\Rightarrow E = \infty \;/\; P = \infty$

- $$E = \int_{-\infty}^{\infty} x^2(t) \cdot dt$$

* unbounded signal $\Rightarrow E = \infty \Rightarrow$ مستحيل تكون Energy Signal

* Time limited / bounded signal $\Rightarrow 0 < E < \infty \Rightarrow$ Energy signal

* Time unlimited / bounded signal $\Rightarrow \lim_{|t| \to \infty} \neq 0 \Rightarrow E = \infty \Rightarrow$ Not energy signal
Cos sin مثل

- $$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \cdot dt$$

الرجوع للسلايد ١٣
Notes

$\Downarrow$ For periodic Signals

- $$P_{\text{periodic}} = \frac{1}{T_0} \int_{T_0} |x(t)|^2 \cdot dt$$

*

• If the signal is unbounded $\xrightarrow{\text{then}}$ $E = \infty$, $P = \infty$ $\xrightarrow{\text{so}}$ neither Power or Energy signal

• Slide 18

Slide 22 :

$$x(t) = A\cos(\omega_0 t + \phi)$$, energy or power?

Sol:

Power signal $\xRightarrow{\text{because}}$ bounded /periodic

$$P = \frac{1}{T_0}\int_0^{T_0} A^2\cos^2(\omega_0 t + \phi)\cdot dt = \frac{A^2}{2}$$

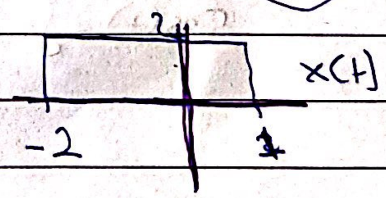
* odd & Even Signals:

• Sin is an odd signal if it's symmetrical about (0,0)

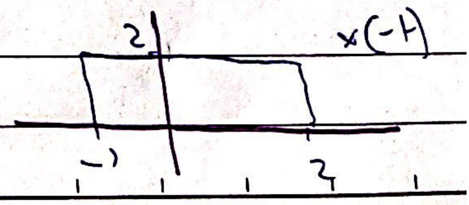
Composition in slide

• example slide 39 :

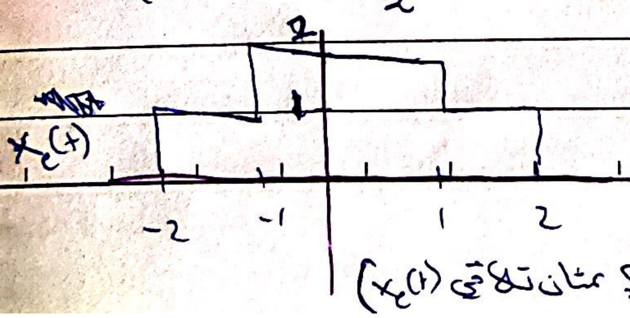
$$x(t) = x_e(t) + x_o(t)$$



$$x_e(t) = \frac{x(t) + x(-t)}{2}$$



ناتج الجمع



وبتقسم

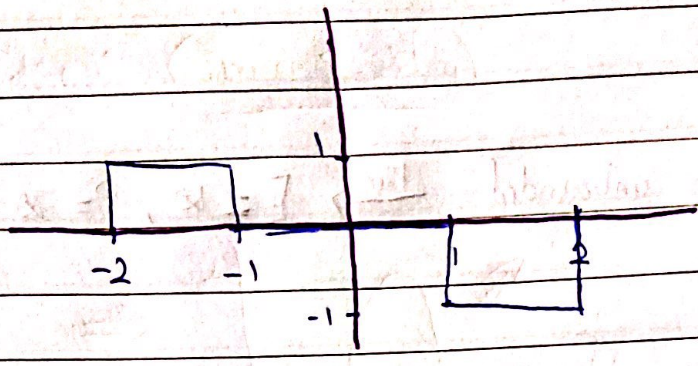
تابع

ناتج المجموع ÷ 2 بتلاقي ($x_e(t)$)

$x_o(t)$

$\boxed{\text{نطرح}}$

ونقسم على ٢



$x(t) = x_o(t) + x_e(t)$ بعدين بنجمع

بطلع الجواب = صفر

neither signal

- ex slide 43 :

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$= \frac{e^{jt} + e^{-jt}}{2}$$

$$= \cos(t)$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{e^{jt} - e^{-jt}}{2}$$

$$= j\sin(t)$$

$$\cos(t) + j\sin(t) = e^{jt}$$

• Practice slide 45 :

$$x(t) = \underbrace{\cos(t)}_{\text{even}} + \underbrace{2e^{-t}}_{\text{neither}}$$

$$= x_e(t) + x_o(t)$$

$$x_e(t) = \frac{\cos(t) + 2e^{-t} + \cos(t) + 2e^{t}}{2}$$

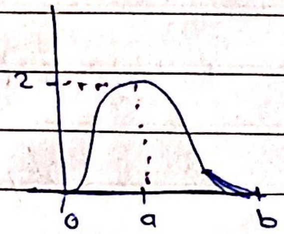
$$= \cos(t) + e^{-t} + e^{t}$$

$$x_o(t) = \frac{\cos(t) + 2e^{-t} - \cos(t) - 2e^{(t)}}{2}$$

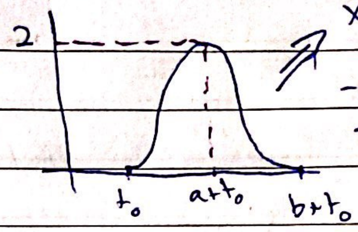
$$= e^{-t} - e^{t}$$

odd signal بطلع الجواب

* Time Transformation : (Time Shifting)



$\Longrightarrow$

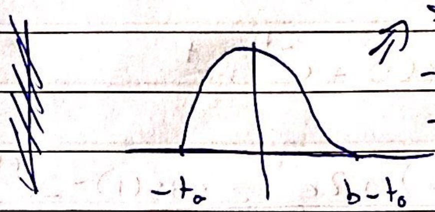


$\nearrow x(t-t_0)$

- Shift to the right
- delayed

$$\boxed{\phi(t+t_0) = x(t)}$$

$\Longrightarrow$



$\nearrow x(t+t_0)$

- shift to the left
- advanced

(Time Scaling)

Slide 14 : approach 1 : ~~(Shifting/X(a/t))~~

Shifting last :

$$x(t) \xrightarrow[\text{Scaling}]{t\to at} x(at) \xrightarrow[\text{reflection}]{t\to -t} x(-at) \xrightarrow[\text{Shifting}]{t\to t\pm t_0} x(-a(t\pm t_0))$$

$\neq y(t)$

إذاً خطأ ال shifting يكون آخر اشي

Shifting first:

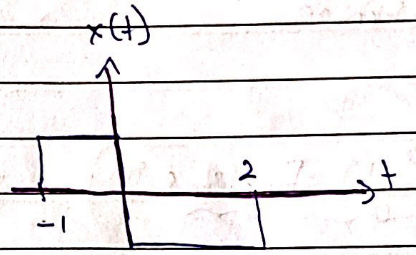
$$x(t) \xrightarrow[\text{Shifting}]{t \to t \pm t_0} x(t \pm t_0) \xrightarrow{t \to at} x(at \pm t_0) \xrightarrow{t \to -t} x(-at \pm t_0)$$

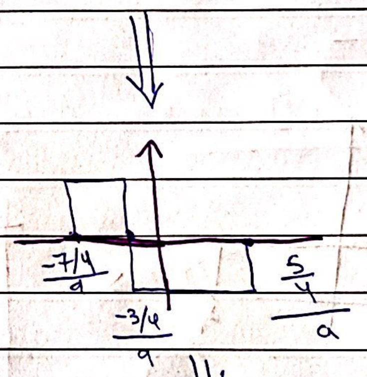
$$= y(t) \checkmark$$

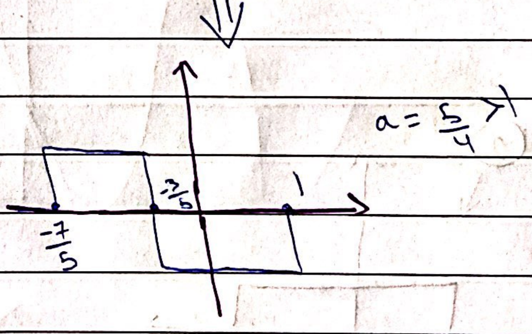
Slide 26:

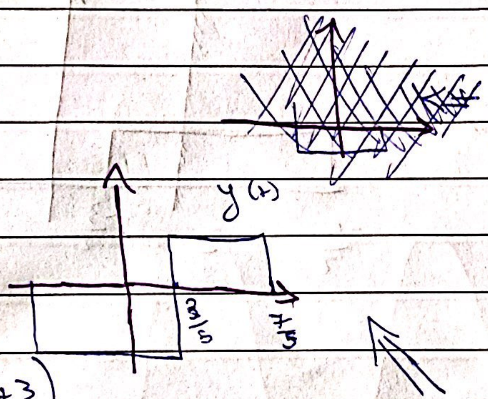
$$y(t) = x\left(\frac{3-5t}{4}\right)$$

$$= x\left(-\frac{5}{4}t + \frac{3}{4}\right)$$









$x\left(\frac{-5t+3}{4}\right)$

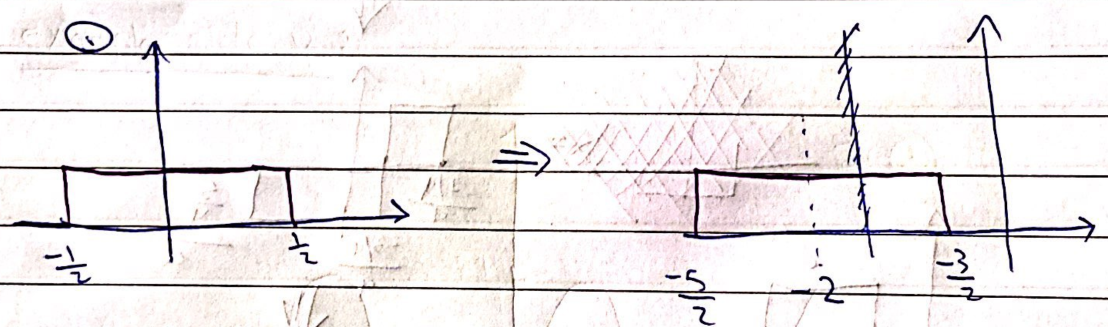
1. Scaling by $a = \frac{1}{4}$
2. Shifting to left by 3
3. scaling by $a_2 = 5$
4. reflection

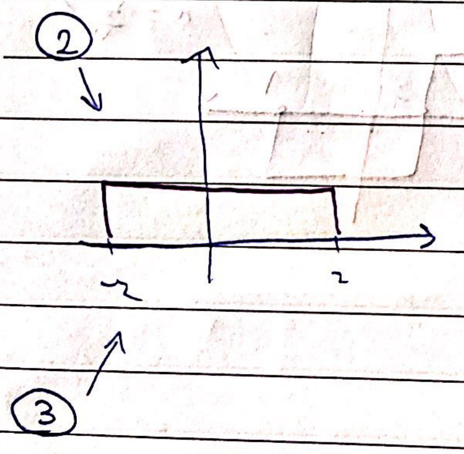
$y(t) = x\left(\frac{1-t}{2}\right)$

1. Scaling by $\frac{1}{2}$
2. Shift to the left by 1
3. Reflection

*** Rectangle Function:**

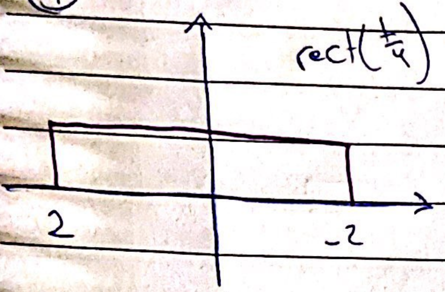
Slide 5:



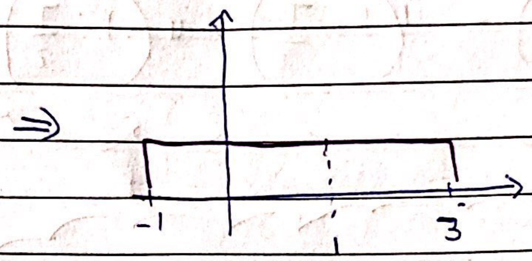


4

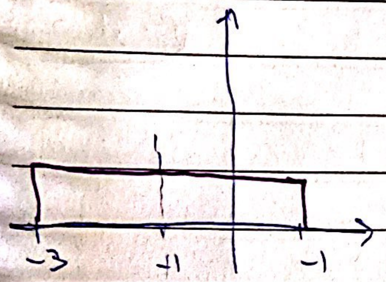
$\text{rect}\left(\frac{t}{4}\right)$



$\Rightarrow$

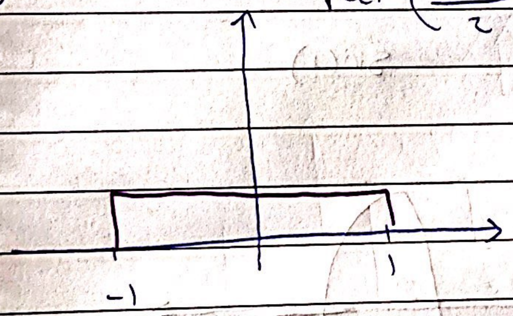


5

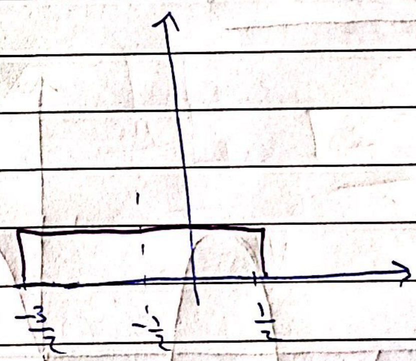


6

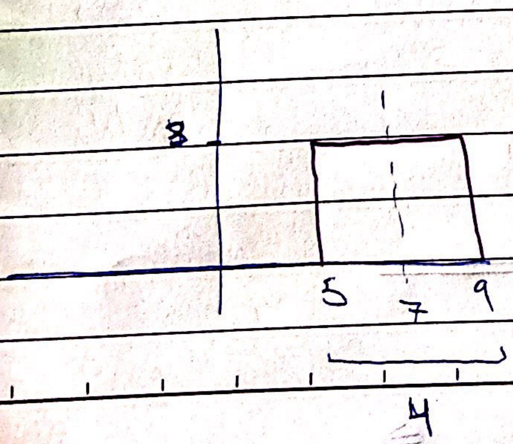
$\text{rect}\left(\frac{t+\frac{1}{2}}{2}\right)$



$\Rightarrow$



example: تحويل من رسم لمعادلة



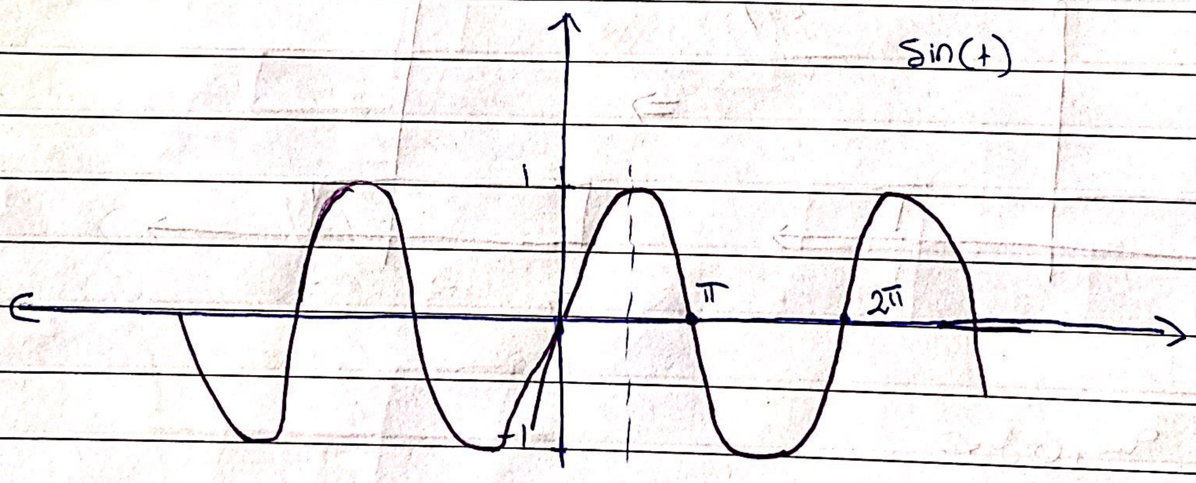
$\Rightarrow x(t) = 3\,\text{rect}\left(\frac{t-7}{4}\right)$

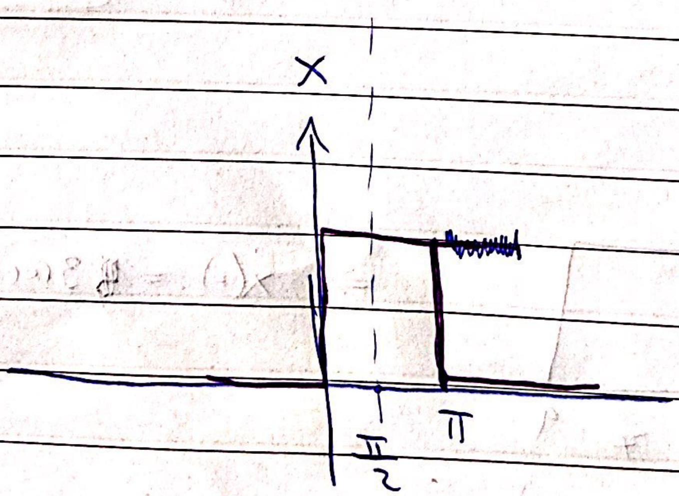
- $\mathrm{rect}\left(\frac{t}{T}\right) = u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right)$

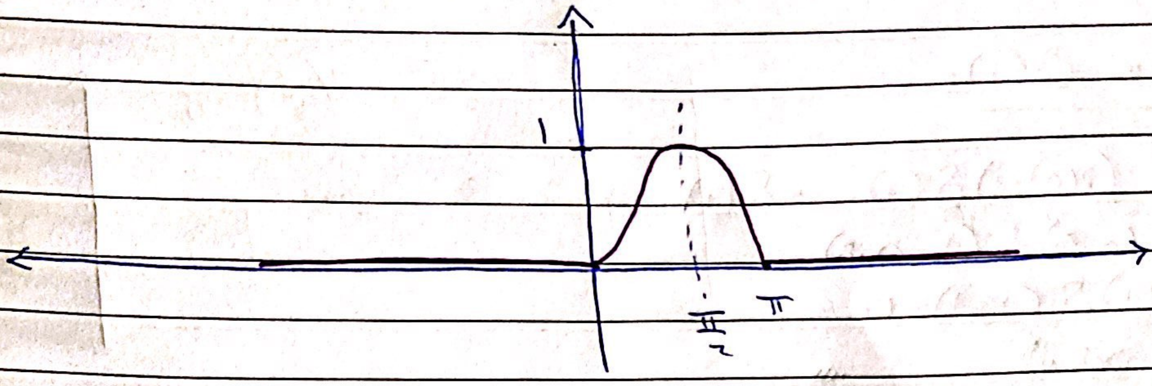
- $\mathrm{rect}\left(\frac{t}{T}\right) = u\left(\frac{T}{2}-t\right) - u\left(-\frac{T}{2}-t\right)$

- $\mathrm{rect}\left(\frac{t}{T}\right) = u\left(t+\frac{T}{2}\right) \cdot u\left(\frac{T}{2}-t\right)$

<u>example slide 12 :</u>

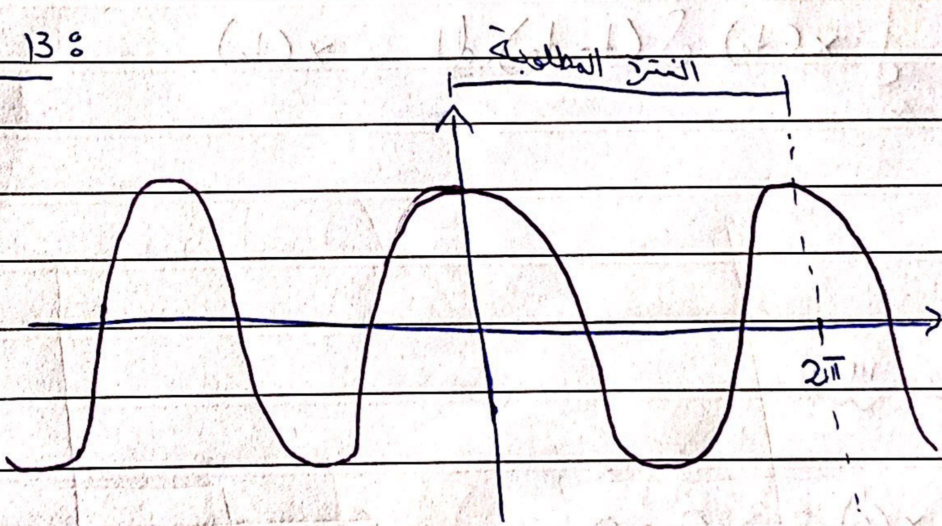






$$x(t) = \sin(t)\,\text{rect}\left(\frac{t - \frac{\pi}{2}}{\pi}\right) = \begin{cases} \sin(t), & 0 < t < \pi \\ 0, & \text{otherwise} \end{cases}$$

example slide 13:



$$x(t) = \cos(t)\,\text{rect}\left(\frac{t - \pi}{2\pi}\right)$$

Slide 15 $\delta(t)$:

① → $(2(0)-1)\,\delta(t) = -\delta(t)$
② $\sin(0.5)\,\delta(t-0.5)$
③ $u(-1)\,\delta(t+1) = 0$
④ Not Defined

* Shifting property $\delta(t)$:

proof:

$$\int_{-\infty}^{\infty} x(t_0)\,\delta(t-t_0)\,dt = x(t_0)$$

example slide 19:

① → $x(0) = 3*0 = 0$
② $= x(2) = 3*2 = 6$
③ $= x(2) = 3*2 = 6$
④ zero (خارج الفترة)

* Scaling property:

$$\delta(4t) = \frac{1}{|4|}\delta(t)$$

example slide 26:

① $= \frac{1}{3}\delta(t)$

② $4\delta(t)$

③ $\frac{1}{2}\delta(t)$

ex slide 29:

① $= \frac{1}{3}\delta(t)$

② $= \delta\left(\frac{1}{2}t - \frac{1}{2}\right) = \frac{1}{\left|\frac{1}{2}\right|}\delta\left(t - \frac{1/2}{1/2}\right) = 2\delta(t-1)$

③ $= \delta\left(\frac{1}{2}t - 1\right) = \frac{1}{|1/2|}\delta\left(t - \frac{1}{1/2}\right) = 2\delta(t-2)$

Five

example slide 31 :

using ~~sh~~ shifting property $\Rightarrow x(3) = 3 + 3^2 = 12$

Slide 33 :

$\alpha = 2 \quad \beta = 4$

$$= \int_0^3 \underbrace{e^{(t-2)} \frac{1}{2}}_{x(t)} \delta\left(t - \frac{4}{2}\right) . dt$$

$$= x(2) = \frac{1}{2} e^{2-2} = \frac{1}{2}$$

Slide 35 :

① $= f(0) = 2$

② $= f(-1) = 1$

③ ~~[illegible]~~ $f(t-1)\, \delta(t-1)\, dt = f(0) = 2$

④ $f(1) = 3$

⑤ $= \int_{-\infty}^{\infty} f(t) . \frac{1}{4} \delta(t) . dt = \frac{1}{4} f(0) = \frac{2}{4} = \frac{1}{2}$

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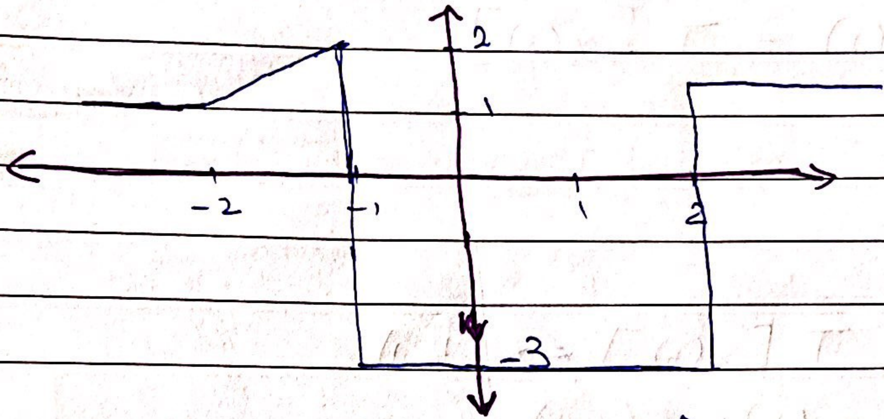
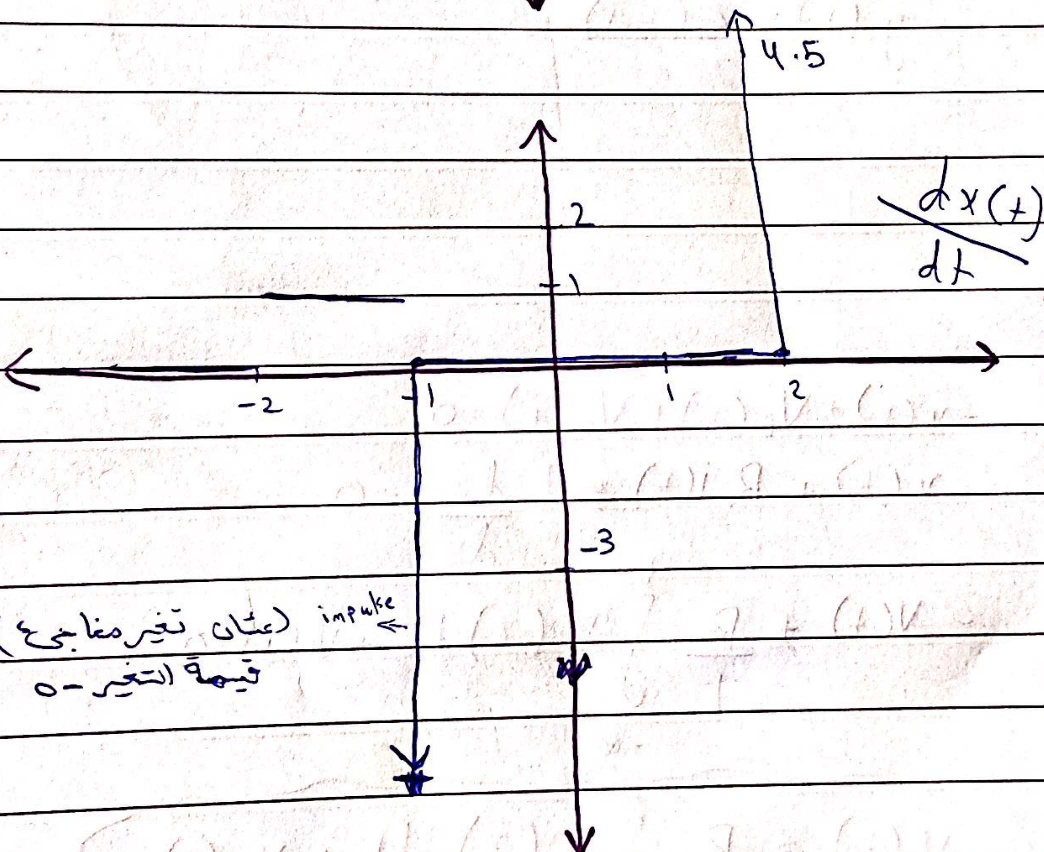
example slide 39 :





* Systems:

example slide 4:

$$y(t) = T[x(t)] = 5x(t)$$

slide 6:

$$y(t) = x^2(t) = T[x(t)]$$

slide 8:

$$y(t) = T[x(t)] = x(t-1)$$

$$y(t) = x(t-1)$$

slide 10:

KVL:

$$-v(t) + V_R(t) + V_L(t) = 0$$

$$-v(t) + R\,i(t) + L\frac{di}{dt} = 0$$

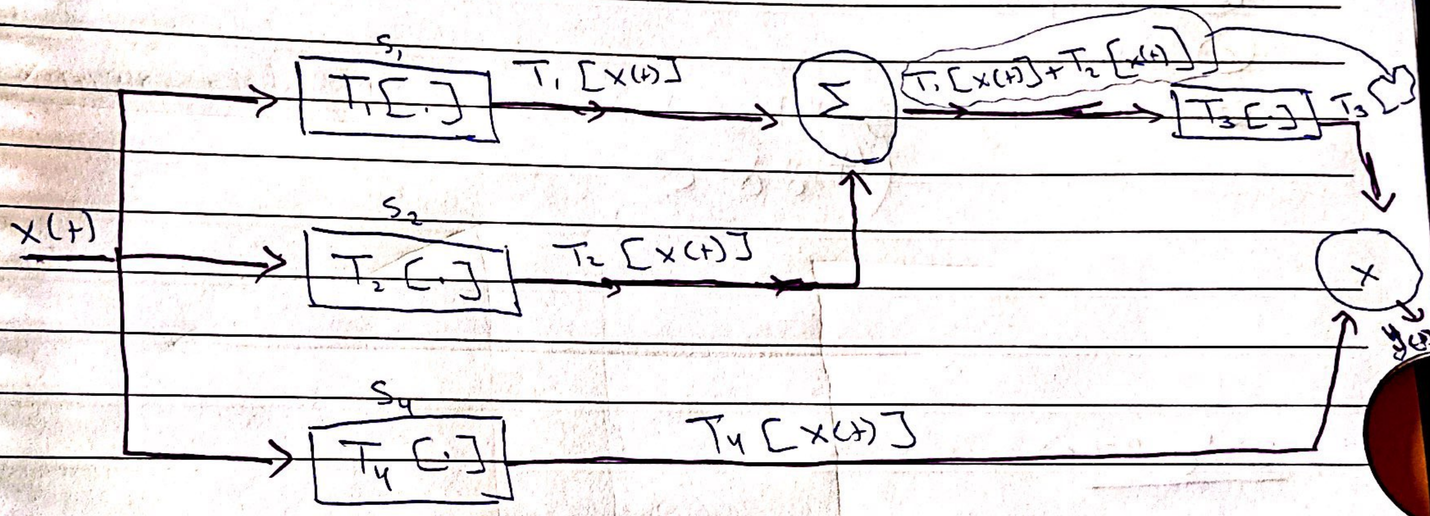
$$-V(t) + \frac{R}{L}\int_{-\infty}^{t} V_L(\tau)\,d\tau + V_L(t) = 0$$

$$V(t) = \frac{R}{L}\int_{-\infty}^{t} V_L(\tau)\cdot dt + V_L(t)$$

Slide 16:

$$y(t) = T_2\left[T_1[x(t)]\right]$$

Slide 18:



$$y(t) = T_3\left[T_1[x(t)] + T_2[x(t)]\right] \times T_4[x(t)]$$

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Slide 24 examples:

1. with memory
2. ~~without~~ memoryless
3. with memory
4. memoryless
5. Memoryless
6. with memory
7. with memory — (عشان تشملنا كل الفترة يلي قبل ال (t))

Slide 27:

3. with memory
4. without memory

Slide 32 :

1. with memory / causal (depends on present & past and not on future)
2. with memory / causal (depends on past & present)
3. with memory / non causal (depends on future value)
4. with memory / non causal (depends on future value)
5. with memory / non causal (

$y(t_0) = x(-t_0)$

$y(0) = x(0)$ present

$y(1) = x(-1)$ past

$y(-1) = x(1)$ Future

* Summary :

- Memoryless → must be causal
- With memory → causal or non-causal
- Causal → Memoryless or with memory (past)
- Non-causal → must be with memory (future)

example slide 36 :

system 1 → $x(t+1) = y(t)$ non-causal

system 2 → ~~[illegible]~~ causal

$y(t) = x(t-2+1)$

$= x(t-1) \Rightarrow \boxed{\text{causal}}$

* Causal signal ⇒ no values before 0

Slide 43 :

1-1 mapping ⇒ Invertible

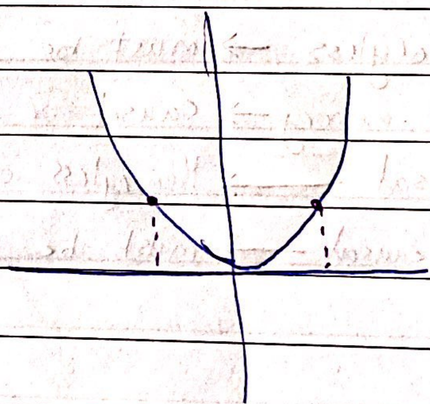
$x(t) = \frac{1}{5} y(t)$

Slide 45 :

not 1-1 mapping (cos [illegible])
not invertible

Slide 46 : ~~not invertible~~

not invertible



Slide 48 ⇒ invertible

* Stable vs unstable :

bounded $\xrightarrow{\text{means}}$ the amplitude values are not ∞ or $-\infty$

- The system is stable when the inputs are bounded and gives an output which is bounded

example slide 54 :

sol:

Bounded input

assume $|x(t)| \leq M$ $(M < \infty)$

$$y(t) = x^2(t)$$

$$|y(t)| = |x^2(t)| = |x(t)|^2 \leq M^2$$

$$\xRightarrow{\text{is}} |y(t)| \leq M^2$$

$|y(t)| \leq N$ (Bounded output)

Stable System