

إعداد
الطالبة :

بيان

شكري

المصغير

Problem 1. Solve the following short problems.

(6 points)

- a) Complete the following table to show the binary representation of the following number in signed-magnitude and 2's complement, assuming you have 7 bits.

Number	Signed Magnitude	2's Complement
-40		

- b) Given a 6-bit signed 2's complement number system, the maximum positive number that can be represented is _____, while the minimum negative number is _____.
- c) Consider the following operation of adding the following 5-bit 2's complement numbers:

$$(11100)_2 + (11001)_2 = (\quad)_{10}$$

1. Fill in the decimal value of the result in the equation above.
2. Does an overflow occur for this operation? Justify your answer.

second exam :-

{ logic second spring 0.15 }

problem 1

(A) Number

signed magnitude

2's complement

-40

1101000

1011000

64 32 16 8 4 2 1

(b) max positive number $\rightarrow 2^{n-1} - 1 = 2^5 - 1 = 31$.

min negative number $\rightarrow -2^{n-1} = -2^5 = -32$.

(c) $(11100)_2 + (11001)_2 = (-11)_{10}$

(1)

$$\begin{array}{r}
 11000 \\
 11100 \\
 11001 + \\
 \hline
 \end{array}$$

$(10101)_2 = (-11)_{10}$

$$\begin{array}{r}
 8 \ 4 \ 2 \ 1 \\
 (01011)_2 = (11)_{10}
 \end{array}$$

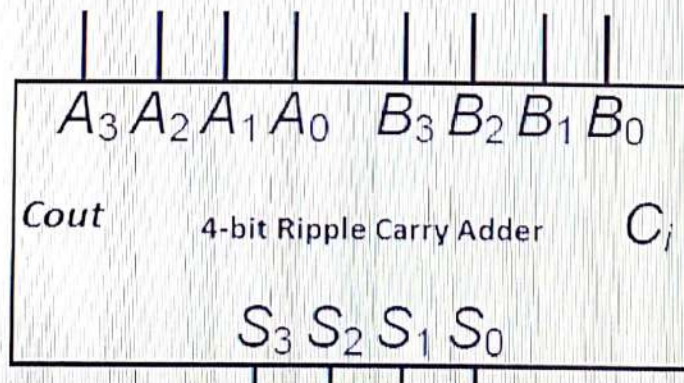
2

No overflow / overflow = 0

Problem 6. Assume x is a 4-bit 2's complement signed number. Given the following 4-bit ripple carry adder, design a circuit that outputs a 4 bit 2's complement signed number y . The circuit has a control bit S , when $S=0$ the output $y=2x$, when $S=1$ the output $y=-x$.

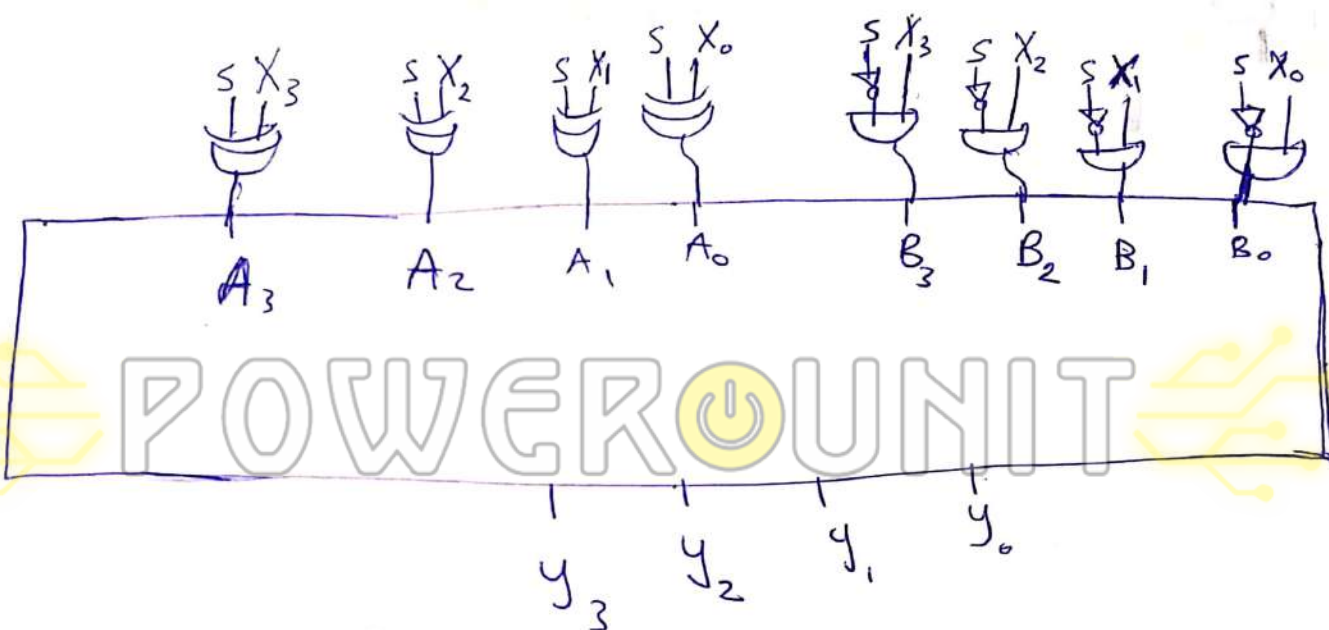
(Hint: remember that $2x=x+x$)

(4 points)



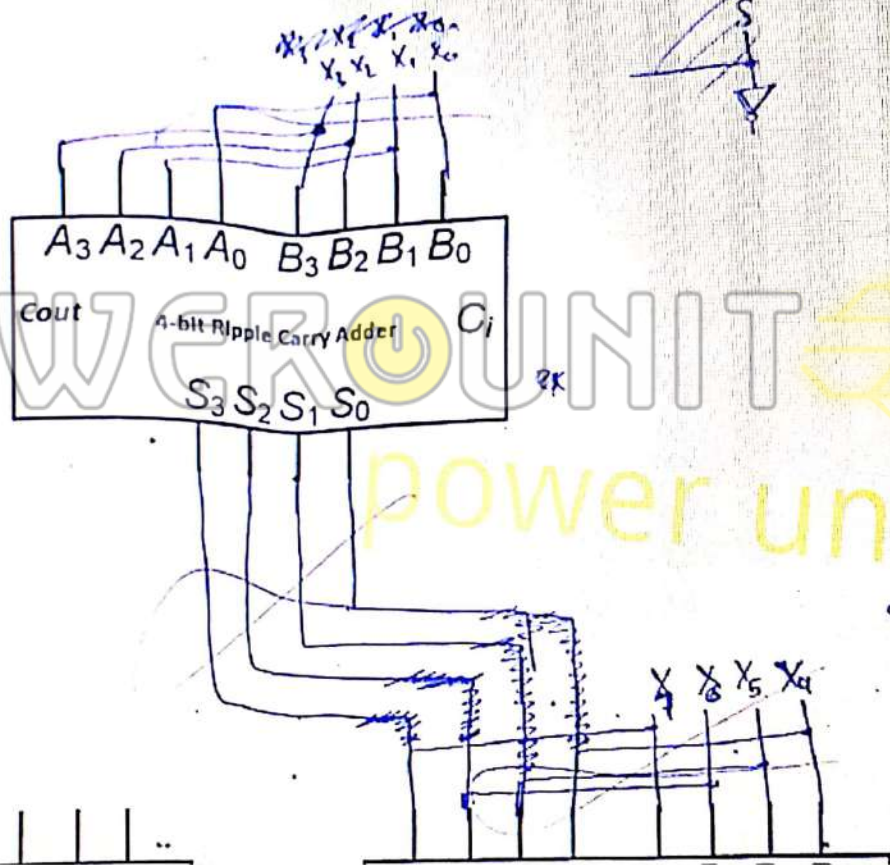
problem 6

11

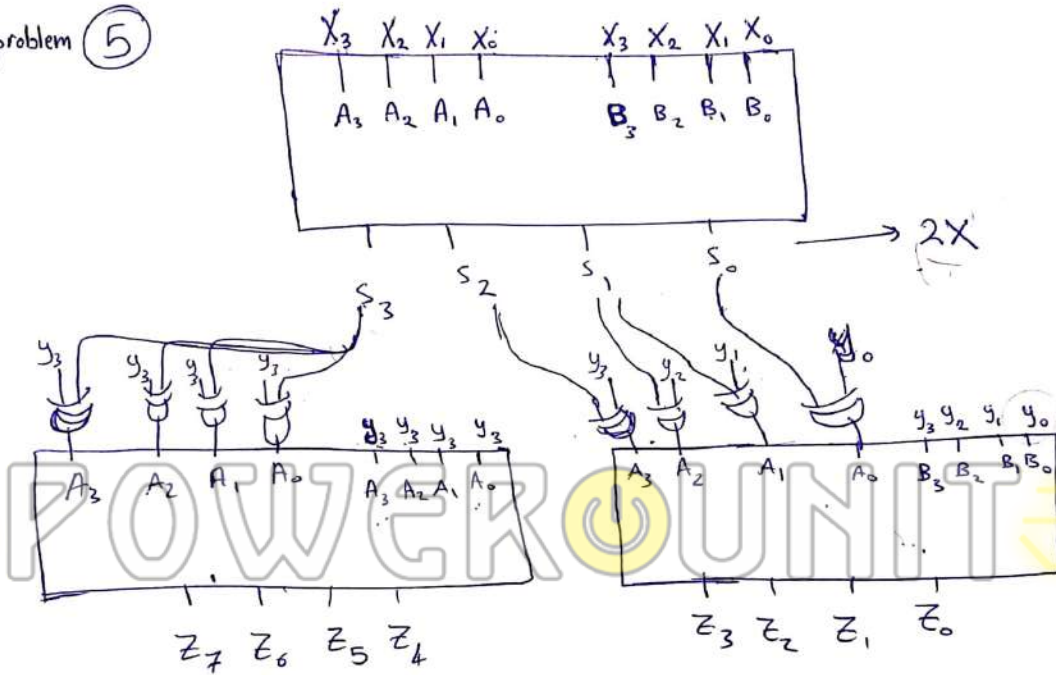


$$y = 2x$$
$$y = -x$$

Problem 5: Assume x and y are 4-bit 2's complement signed numbers. Using only the following three 4-bit ripple carry adders and any gates you need, design a circuit that outputs an 8-bit signed number z . The circuit has a control bit S , when $S=0$ the output $z=y+2x$, when $S=1$ the output $z=y-2x$. Your circuit should produce correct results for all possible input combinations of x and y . (Hint: remember that $2x=x+x$) (5 points)



problem (5)



$$\begin{array}{l}
 Z_7 - Z_0 \\
 y_3 y_2 y_1 y_0 \\
 X_3 X_2 X_1 X_0
 \end{array}
 \left\{
 \begin{array}{l}
 y = 2X \rightarrow S = 1 \\
 y + 2X \rightarrow S = 0
 \end{array}
 \right.$$

d. Represent the number $(-14)_{10}$ as an 8-bit binary number using signed-magnitude representation?

1110

Answer = 10001110 ✓

e. Given two 8-bit 2's complement signed numbers $A = (11010010)_2$ and $B = (00011101)_2$. In the given blank write the result of $A - B$?

$A - B = 10110101$ ✓

f. Given two 3-bit numbers $A = (011)_2$ and $B = (010)_2$. Determine the value of the overflow bit generated by the operation $A+B$ if the numbers are treated as 1) unsigned numbers and if they are treated as 2) 2's complement signed numbers?

$$\begin{array}{r} 011 \\ 010 \\ \hline 101 \end{array}$$

Unsigned Overflow = 0 ✓

Signed Overflow = 1 ✓

Logic second Fall 015

problem 1: (d) $(-14)_{10} \xrightarrow{\text{signed magnitude}} (10001110)_2$

unsigned number $\begin{matrix} 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 & + \\ (0 & 0 & 0 & 0 & 1 & 1 & 1 & 0) \end{matrix} \xrightarrow{(8 \text{ bits})} (10001110)_2$

(e) $(11010010)_2 - (00011101)_2 = (10110101)_2$

$$\begin{array}{r} 11000010 \\ 11010010 \\ 11100011 + \\ \hline 10110101 \end{array}$$

(f) $\begin{array}{r} 010 \\ 011 \\ 010 + \\ \hline 101 \end{array}$

unsigned overflow = 0

signed overflow = 1

Problem 1. Answer the following short questions.

9

(14 points)

a. What is the decimal value of the binary number $(10111)_2$ if it represents:

1. Sign-Magnitude number: $1(0111) = -7$ ✓
2. 2's complement signed number: $(10111) = -9$ ✓

b. Does the following operation $(110001)_2 + (110111)_2$ result in an overflow or not when the number are: (You must justify your answer)

1. Unsigned numbers:
$$\begin{array}{r} 110001 \\ + 110111 \\ \hline 1101000 \end{array}$$
 overflow because the carry is 1 and ~~is~~ larger than the result the allowed bit

2's complement signed numbers:
$$\begin{array}{r} 110001 \\ + 110111 \\ \hline 1101000 \end{array}$$
 $C_{n-1} \oplus C_n = 0$ No over flow

c. Given two 2's complement signed numbers $A = (1010)_2$ and $B = (01011100)_2$. In the given blank write the result of $A + B$ using an 8-bit adder/subtractor?

Logic second spring 016.

problem ① A $(10111)_2$

sign magnitude number $\rightarrow (10111)_2 = (-7)_{10}$
2's complement signed number $\rightarrow (01001)_2 = (-9)_{10}$

$$\textcircled{b} \textcircled{1} (110001)_2 + (110111)_2 = (101000)_2$$
$$\begin{array}{r} 110111 \\ 110001 \\ 110111 + \\ \hline (101000)_2 \end{array}$$

unsigned number $\Rightarrow 1$

overflow \rightarrow 2's complement signed numbers $\Rightarrow 0$

$$\textcircled{c} \quad A = (1010)_2$$

$$B = (01011100)_2$$

$$A - B = A + 2's B$$

$$\begin{array}{r}
 1110000 \\
 1111010 + \\
 10100100 \\
 \hline
 10011110
 \end{array}$$

$$A - B = (10011110)_2$$

h. Draw the logic diagram of a combinational circuit that has a **4-bit unsigned** number (A) as an input and produces a **4-bit unsigned** number (B) as an output such that B equals the quotient of $(A \div 8)$.

h



- c. Variables **A** and **B** are 8-bit unsigned numbers with the following values: $A = (1000\ 1001)_2$ and $B = (1001\ 1010)_2$. An 8-bit adder/subtractor is used to perform the operation: $A - B$. Determine the 8-bit result of the adder/subtractor and Borrow bit.

$A - B =$ 11101111 Borrow bit = 1

$$\begin{array}{r}
 10001001 \\
 - 10011010 \\
 \hline
 11101111
 \end{array}$$

- d. The 2's complement for $(0010111)_2$ is (1101001)
- e. The 7's complement for $(735)_8$ is = *by complement* = (042)

Logic second spring 017

problem ① ③

$$A = (1000\ 100)_2, \quad B = (1001\ 1010)_2$$

$$A - B = A + 2' \text{ complement } B$$

$$\begin{array}{r} 00\ 00\ 00\ 00 \\ 1000\ 100 \\ 0110\ 0110 + \\ \hline (1110\ 1110)_2 \end{array}$$

$$\textcircled{d} (0010111)_2 \xrightarrow{2' \text{ complement}} (1101001)_2$$

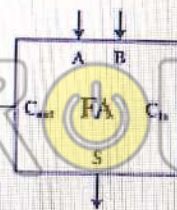
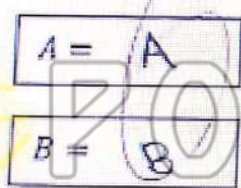
$$\textcircled{e} (735)_8 \xrightarrow{7' \text{ complement}} (\textcircled{042})_8$$

max(777)

↓

$$(042)_8$$

e. Given the following full adder cell, if $C_{in} = 0$ and $C_{out} = 1$, then the values of A and B are:



Problem 2. Show that the following circuit implements a 2-input XNOR gate. (2 points)

Logic second summer 018.

Ⓔ

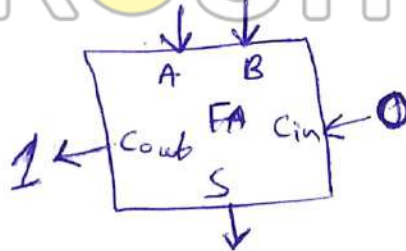
$C_{in} = 0$ $C_{out} = 1$

$$A = 1$$

$$B = 1$$

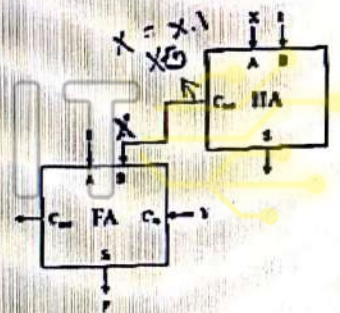
Ⓘ

$$\begin{array}{r} 1 \\ 1^+ \\ \hline 0 \end{array}$$



- c. Using 6-bits, the minimum negative number using Sign-Magnitude format is = ~~(11111)₂~~
- d. What is the decimal value of the 6-bits signed number (100111)₂ if it is represented using signed 2's complement format? (~~-25~~)₁₀
- e. What is the Boolean expression of output F in the figure below in terms of X and Y?

X	Y	F
0	0	0
0	1	1
1	0	1
1	1	0



$$F(X, Y) = \sum_m (1, 2) = \bar{X}Y + X\bar{Y}$$

$$F = X \oplus Y \oplus 0$$

\oplus
 \oplus
 \oplus
 \oplus . 1

Logic second spring 019

problem ©

1

6 bits, the minimum negative number

using Sign-magnitude $\rightarrow -(2^{n-1} - 1)$
 $= -(2^5 - 1)$
 $= \boxed{-31}$

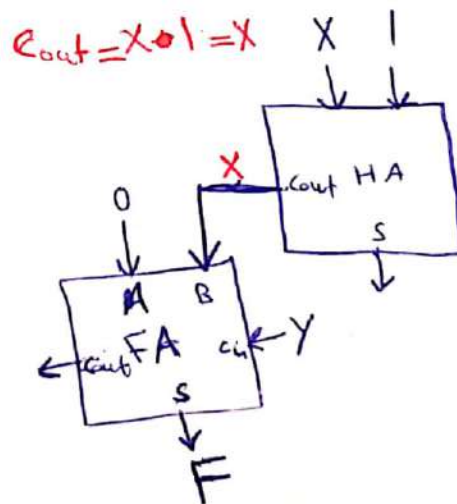
$(111111)_2$

d) $(100111)_2 \xrightarrow{\text{2's complement}} -(011001)_2 = -(25)_{10}$

e)

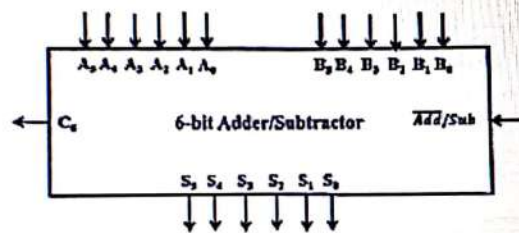
X	Y	A	F = (sum)
0	0	0	0
0	1	0	1
1	0	0	1
1	1	0	0

$F = \bar{X}Y + X\bar{Y}$



$\text{Sum} = X \oplus Y$

Problem 8. Given the following 6-bit adder/subtractor, answer the two questions below: (3 points)



- I. Assume that inputs A and B are unsigned numbers set to the following values: $A = (011010)_2$ and $B = (001100)_2$. The Add/Sub control is set to 0. Accordingly, compute the sum bits $S[5:0]$ and determine if there is an overflow or not.

$$\begin{array}{r} 011010 \\ + 001100 \\ \hline 100110 \end{array}$$

$S_5S_4S_3S_2S_1S_0 = 100110$

Is there an overflow? No.

- II. Assume that inputs A and B are signed numbers in 2's complement format set to the following values: $A = (110111)_2$ and $B = (111001)_2$. The Add/Sub control is set to 1. Accordingly, compute the sum bits $S[5:0]$ and determine if there is an overflow or not.

$$\begin{array}{r} 110111 \\ + 111001 \\ \hline 110100 \end{array}$$

$S_5S_4S_3S_2S_1S_0 = 110100$

Is there an overflow? No.

problem ⑧ ① $A = (011010)_2$, $B = (001100)_2$

set to 0 $\rightarrow s = 0 \rightarrow$ Adder.

$$A+B \Rightarrow \begin{array}{r} \boxed{0}11000 \\ 011010 \\ 001100 \\ \hline (100110)_2 \end{array} + \Rightarrow (100110)_2$$

No overflow

② $A = (110111)_2$, $B = (111001)_2$

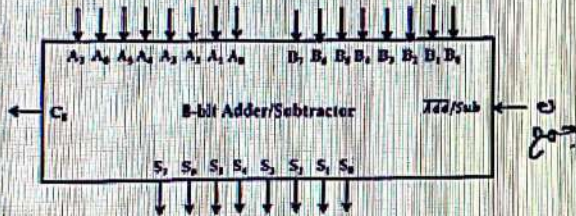
set to 1 $\rightarrow s = 1 \rightarrow$ subtraction.

$A - B = A + 2's B = (11110)_2$

$$\begin{array}{r} \boxed{00}01111 \\ 110111 \\ 000111 \\ \hline (111110)_2 \end{array} +$$

No overflow

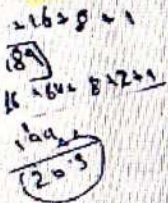
Problem 8. Given the following 8-bit adder/subtractor, answer the two questions below: (3 points)



I. Assume that inputs A and B are unsigned numbers set to the following values: $A = (10101101)_2$ and $B = (00111111)_2$. The *Add/Sub* control is set to 0. Accordingly, compute the sum bits $S[7:0]$ and determine if there is an overflow or not.

$S_7 S_6 S_5 S_4 S_3 S_2 S_1 S_0 = 11101100$

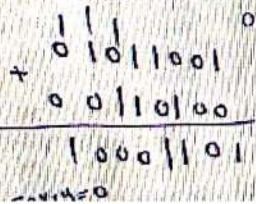
Is there an overflow? No
no carry



II. Assume that inputs A and B are signed numbers in 2's complement format set to the following values: $A = (01011001)_2$ and $B = (11001011)_2$. Use the 8-bit adder/subtractor above to compute $A - B$ and specify the value of sum bits $S[7:0]$ and determine if there is an overflow or not.

$S_7 S_6 S_5 S_4 S_3 S_2 S_1 S_0 = 10001011$

Is there an overflow? yes



⑧ □ $A = (10101101)_2$, $B = (00111111)_2$

$s = 0 \rightarrow$ Adder

$$\begin{array}{r} 00111111 \\ 10101101 \\ \hline 00111111 \\ \hline (11101100)_2 \end{array} +$$

overflow = 0
No overflow.

$(11101100)_2 \rightarrow s_7 s_6 s_5 s_4 s_3 s_2 s_1 s_0$

⑨ $A = (01011001)_2$, $B = (11001011)_2$

A - B ??

$$\begin{array}{r} 01110001 \\ 01011001 \\ \hline 00110101 \\ \hline (1000110)_2 \end{array} +$$

overflow = 1

POWERGUNIT

Problem 5: Assume m and n are 4-bit unsigned numbers. Using only the following three 4-bit ripple carry adders and any logic gates you need, design a circuit that outputs an 8-bit unsigned number k such that: (5 points)

$$k = (6 \times m) - n$$

$m_3 \ m_2 \ m_1 \ m_0$

أول خطوة (6m)

يحول الضرب إلى جمع

$$K = 6m - n$$

$$m \rightarrow m_3 m_2 m_1 m_0$$

$$n \rightarrow n_3 n_2 n_1 n_0$$

$$K \rightarrow k_7 k_6 k_5 k_4 \dots k_0$$

	m_3	m_2	m_1	m_0		
	1	1	0	0	X	
	0	0	0	0	0	
	0	m_3	m_2	m_1	m_0	0
	m_3	m_2	m_1	m_0	0	0
	S_3	S_2	S_1	S_0	m_0	0

بعطوا (S)

إذا قدم

حاصل جمعهم بالaddress

← 2 طرحهم مع n أول address بوضع 4 bits ($S_3 S_2 S_1 S_0$) وبطرحهم مع bit 4 ($N_3 \dots N_0$) [N] و الثاني address بوضع 4 bits (الكلمة $S_3 S_3 S_3 S_2$ بتكرارها 2 مرات بطرحهم مع 4-bits [N] (N) فقط بتكرارها وبكررها 3) $6m - n$

