Trees

Q_1: a) Tree ⟹ [No simple closed path + Undirect graph + connected pathes]

b) not tree [not connected from (c) to (d)]

c) Tree [has no closed path + Undirect graph + connected]

d) Not a tree [contain closed path (b, c, d, b)

e) Tree [It has three reasons are mentioned]

f) Not tree [contain closed path (a, b, c, d, e, f, a)]

Q2: a) Tree

b) Tree

c) Not Tree [graph is not connected]

d) Tree [has no closed pathes + Undirect graph + every pair of vertices are connected]

Q3 :

- root ⇒ a

- internal ⇒ (a, b, c, d, f, h, j, q, t)
(vertices have children)

- Leaves ⇒ (e, g, i, k, L, m, n, O, p, r, s, u)
(Vertices with no children)

- children of "j" ⇒ (q , r)

- parent of "h" ⇒ "c"

- siblings of "O" ⇒ "p"
[vertices have same parent]

- ancestors of "m" ⇒ a, b, f
[vertices in the same path from the root to the selected vetices]

- descendants of "b" $\Rightarrow$ (f, L, m, n, e)

[all vertices whose ancestor is "b"]

- No, not m-ary because vertices have 2 children and other have three children

Levels:

root "a" $\Rightarrow$ level "0"

vertices [b, c, d] $\Rightarrow$ level "1"

vertices [e, f, g, h, i, j, k] $\Rightarrow$ level "2"

vertices [L, m, n, O, p, q, r] $\Rightarrow$ level "3"

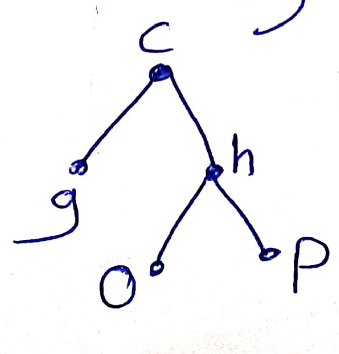
vertices [s, t] $\Rightarrow$ level "4"

~~vertices~~ vertex [u] $\Rightarrow$ level "5"

Subtrees:

a) since "a" root of tree So all entire tree are subtrees

c) assume that "C" is a root So the subtrees is



Represent expressions:

prefix notation =

first one: $+ + X * X y / X y$

second one: $+ X / + * X y X y$

infix notation =

f. one: $X X y * + X y / +$

second: $X X y * X + y / +$

postfix notation =

f. one = $(X + (X * y)) + (X / y))$

s. one = $(X + ((X * y) + X) / y))$

Value of prefix expressions

a) $- * 2 \underbrace{/ 8 4} 3$

$= - \underbrace{* 2 2} 3$

$\underbrace{- 4 3}$

$= 1$

b) $\uparrow - * 3 3 \underbrace{* 4 2} 5$

$\uparrow - \underbrace{* 3 3} 8 5$

$\uparrow \underbrace{- 9 8} 5$

$\underbrace{\uparrow 1 5}$

$= 1^{(5)} = 1$

c) $+ - \uparrow 3\ 2 \uparrow 2\ 3 / 6 \underbrace{- 4\ 2}_{4-2=2}$

$+ - \uparrow 3\ 2 \uparrow 2\ 3 \underbrace{/ 6\ 2}$

$+ - \uparrow 3\ 2 \underbrace{\uparrow 2\ 3}_{2^{(3)}=8}\ 3$

$+ - \underbrace{\uparrow 3\ 2}_{3^{(2)}=9}\ 8\ 3$

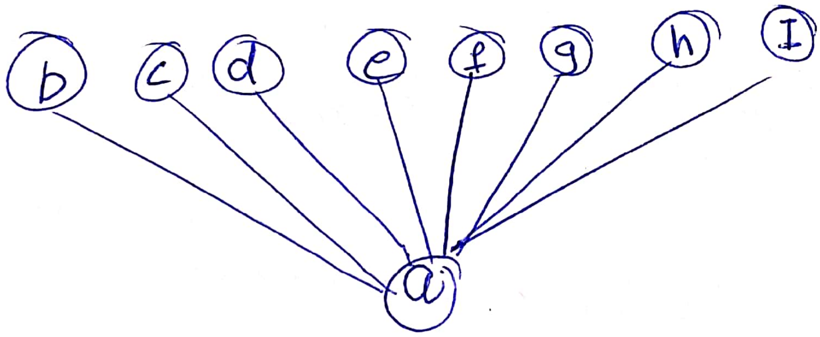
$+ \underbrace{- 9\ 8}\ 3$

$\underbrace{+ 1\ 3} = 4$

Graph :

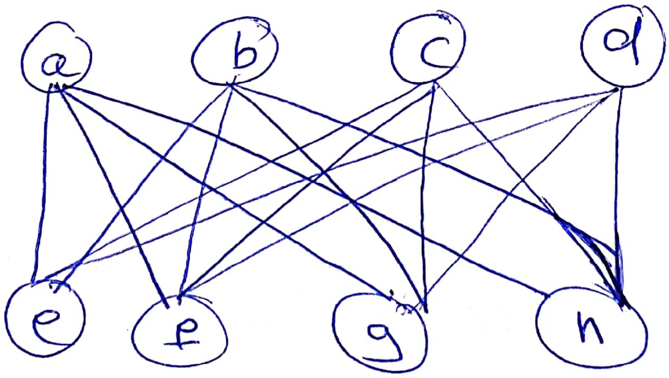
Q1 : $K(1,8)$ set(I) = {a}

set(II) = {b, c, d, e, f, g, H, I}

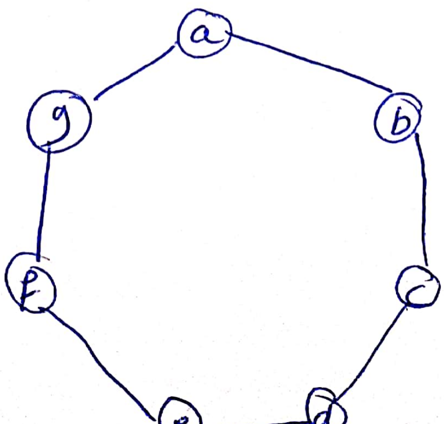


$K(4,4)$ ⟹ set(I) = {a, b, c, d}

set(II) = {e, f, g, h}



$C(7)$



Q2 :- Wheel "W_n"

no. vertices = $n+1$

no. edges = $2n$

Complete "$K_{(m,n)}$"

no. vertices = $m+n$

no. edges = $m*n$

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hypercube "Q_n"

no. vertices = $2^{(n)}$

no. edges = $n * 2^{(n-1)}$

Q3: (1)- num.verticies = 6

num.edges = 6

deg(a) = 2 / deg(b) = 4 / deg(c) = 1

deg(d) = 0 / deg(e) = 2 / deg(f) = 3

Isolated vertices = "d"

pendant vertices = "c"

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(3)- num.ver = 9

num.edges = 12

deg(a) = 3 / deg(b) = 2 / deg(c) = 4

deg(d) = 0 / deg(e) = 6 / deg(f) = 0

deg(g) = 4 / deg(h) = 2 / deg(i) = 3

Isolated.ver = "d", "f"

pendant.ver = Zero "None"

Q: Represent graphs with adjacency matrix

- $$\begin{array}{c|c} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$ first graph

- $$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$ Second graph

- $$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$$ Third

- $$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$ Forth

Q: graph one: Not connected

graph two: connected

Q5: a) not simple / not circuit / Just path

Length = 4

b) Not path [because of wrong connect]

Relations :

Q_1 : $\{(x,y) \mid xy \geq 1\}$

- Not reflexive
- Symmetric [$a*b \geq 1$ then $b*a \geq 1$]
- not antisymmetric [$1*3 \geq 1$ then $3*1 \geq 1$]

while $1 \neq 3$

- transitive (a,b) so $(2,1)$ $\Rightarrow (2,3)$ True
(b,c) $(1,3)$ (a,c)

$\{(x,y) \mid x$ is a multiple of $y\}$

- reflexive
- not symmetric [2 is multiple of 1, but 1 is not multiple of 2]
- not antisymmetric $(a,-a)$

a is mult of $-a$

and $-a$ is mult of a

- transitive

$\{(x,y) \mid x$ & y are both negative <u>or</u> both nonnegative

- reflexive sure ✓

- symmetric $(+x, +y) \Rightarrow (+y, +x)$ ✓

- not antisymmetric

- transitive $(a,b) \wedge (b,c) \Rightarrow (a,c)$ ✓

$\{(x,y) \mid x = y^2\}$

- not reflexive $\Rightarrow \overset{(a,a)}{(3,3)}$ $\quad 3 \neq 3^{(2)}$

- not symmetric $\Rightarrow (3,9), (9,3)$ $\quad \because 3 \neq 9^2$
  $(\cancel{y}, \cancel{x}), (x, y)$

- antisymmtric

- not transitive $\underset{(a,b)}{(16,4)} \wedge \underset{(b,c)}{(4,2)} \Rightarrow (16,2)$ $\quad 16 \neq 2^2$ X

Q2: $R = \{(1,2),(1,3),(2,3),(2,4),(3,1)\}$

$S = \{(2,1),(3,1),(3,2),(4,2)\}$

$S \circ R = \{(1,1),(1,2),(2,1),(2,2)\}$

Q3: $R_1 \cup R_2 = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\ (3,4)\}$

$R_1 \cap R_2 = \{(1,2),(2,3),(3,4)\}$

$R_1 - R_2 = \emptyset$

$R_2 - R_1 = \{(1,1),(2,1),(2,2)\ (3,1)\ (3,2)\ (3,3)\}$

Q5: $R_1 \cup R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $R_1 \circ R_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

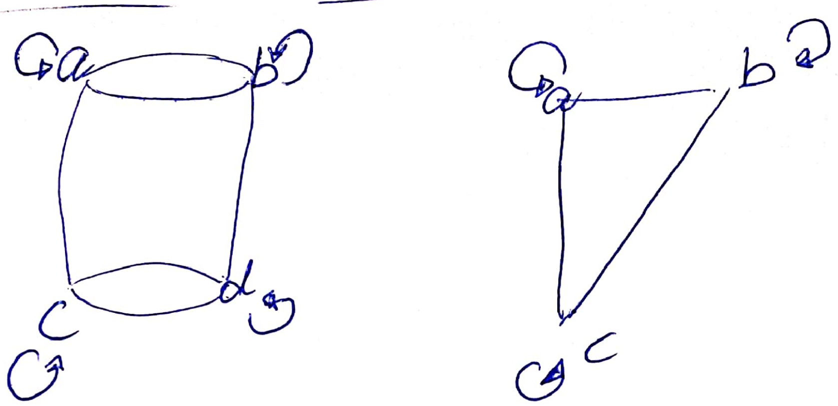
$R_1 \cap R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ $R_1 \oplus R_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$R_2 \circ R_1 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Q6: A) $\{(a,b)(a,c)(b,c)(c,b)\}$

B) $= \{(a,c),(b,a),(c,d),(d,b)\}$

Q7



Q: transitive closures of $\{1,2,3,4\}$

$\Rightarrow \{(1,2)\ (2,1)(2,3)\ (3,4)\ (4,1)\}$

$\Downarrow$

$$B_1 = A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$M_R \odot M_R = A_1^2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$A_1 \cup A_1^2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} = B_2$$

$$A_1^3 = A_1^2 \odot A_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$A_1^3 \cup B_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \Rightarrow B_3$$

$$A_1^4 = A_1^3 \odot A_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$A_1^{(4)} \cup B_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = B4 \checkmark$$

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result #

$\boxed{Q9}$ equivalence relation → symmetric
→ reflexive
→ Transitive

Divisibility and Modular arithmetic

a) $\underset{a}{44}$ divided by $\underset{d}{8}$?

$= 5d + 4$

b) $\underset{a}{777}$ divided by $\underset{d}{21}$?

$= 37d + 0$

c) 0 divided by 17?

$= 0d + 0$

d) -100 div 101?

$= -1(d) + 1$

Q.

a) $-1 \bmod 23 = 22$

b) $-97 \bmod 11 = 2$

c) $-155 \bmod 19 = 3$

d) $-221 \bmod 23 = 9$

Q $[(\underline{177 \bmod 31} + 270 \bmod 31) \bmod 31]$

$22 + 22$

$=$

$44 \bmod 31$

* بطرح $\boxed{31}$ تبسطني بالحل زي ماتعلمنا

$44 - 31 \bmod 31$

$13 \bmod 31 = \boxed{13}$

Sequences :

Q1 : $\{a_n\}$ a) 2^{n-1} $2^7 = 128$

b) 7

c) $1+(-1)^8 = 1+1 = 2$

d) $-(-2)^8 = -256$

Q2 : $2^n + 1$ $\equiv$ $2^0 + 1 = 2$

$2^1 + 1 = 3$

$2^2 + 1 = 5$

$2^3 + 1 = 9$

b) $(n+1)^{n+1}$ $\equiv$ $(0+1)^1 = 1$

$(1+1)^2 = 4$

$(2+1)^3 = 27$

$(3+1)^4 = 256$

c] $\lceil n/2 \rceil \equiv \lceil 0/2 \rceil = 0$

$\lceil 1/2 \rceil = 0$

$\lceil 2/2 \rceil = 1$

$\lceil 3/2 \rceil = 1$

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d) $\lfloor n/2 \rfloor + \lceil n/2 \rceil \equiv a_0 = 0$

$a_1 = 1$

$a_2 = 2$

$a_3 = 3$

Q_3: $\sum_{j=0}^{8} [1 + (-1)^j]$

$\sum_{j=0}^{8} (1) + \sum_{j=0}^{8} (-1)^j$

$a_0(n+1) + a_0 \left[\frac{r^{n+1} - 1}{r - 1} \right]$

$1(8+1) + 1 \left[\frac{(-1)^9 - 1}{-1 - 1} \right]$

$= 9 + 1 = 10$

d) $\sum_{j=0}^{8}(2^{j+1}-2^{j})$

$= (2^1-2^0)+(2^2-2^1)+(2^3-2^2)+(2^4-2^3)$
$+(2^5-2^4)+(2^6-2^5)+(2^7-2^6)+(2^8-2^7)$

$+(2^9-2^8)$

$= -2^0+2^9 = 511$

$\sum_{j=0}^{8}[2(3^j)+3(2^j)]$

$= 2\sum_{j=0}^{8}(3^j)+3\sum_{j=0}^{8}(2^j)$

$= a_0\left[\frac{r^{n+1}-1}{r-1}\right]+a_0\left[\frac{r^{n+1}-1}{r-1}\right]$

$= 2\left[\frac{3^9-1}{3-1}\right]+3\left[\frac{2^9-1}{2-1}\right]$

$= 21215$

Functions:

$f(x) = -3x+4 \Rightarrow f(x) = fy$

$-3x+4 = -3y$ it's one to one

It is onto as $f\left(\frac{4-x}{3}\right) = x$

$f(x) = -3x^2+7 \quad f(-x) = f(x)$ it's not bijection

$f(x) = \frac{x+1}{x+2}$ So the function is not bijection

$f(x) = x^5+1$ ✓ bijection ("increasing function")

Q: Find these values =

$[1.1] = 1$

$[1.1] = 2$

$[-0.1] = -1$

$[-0.1] = 0$

$[2.99] = 3$

$[-2.99] = -2$

Q: a) $f = m+n$ Onto

b) $f = m^2+n^2$ Not onto

c) $f = m$ onto

d) $f = |n|$ Not onto

e) $f = m-n$ onto

Q:

$f(n) = n - 1$ One to one

$f(n) = n^2 + 1$ Not one to one

$f(n) = n^3$ One to one

$f(n) = \lceil n/2 \rceil$ Not one to one

Sets:

Q1: a) 1, -1

b) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

c) 1, 4, 9, 16, 25, 36, 49, 64, 81

d) null set

Q2: $\{(x \in N) \mid x \text{ multiple of } 3 \text{ and } x \leq 12\}$

b) $\{(x \in Z) \mid -3 \leq x \leq 3\}$

c) $\{x \mid x \text{ is a letter in alphabet from "m" to "p"}\}$

Q3: Subsets:

$A \subseteq A$ $\quad$ $C \subseteq A$ $\quad$ $D \subseteq D$

$B \subseteq A$ $\quad$ $C \subseteq C$

$B \subseteq B$ $\quad$ $C \subseteq D$

Q4:

a) False

b) False

c) False

d) False

e) True

Q7:

a) $\bigcup_{i=1}^{n} A_i \Rightarrow A_i \subset A_n$

$$\bigcup_{i=1}^{n} A_i \subseteq \bigcup_{i=1}^{n} A_n \quad \boxed{i \leq n}$$

$$= A_n$$

because $\bigcup_{i=1}^{n} A_i \subseteq A_n$ and $A_n \subseteq \bigcup_{i=1}^{n}$

b) $\bigcap_{i=1}^{n} A_i \quad i \geq 1 \qquad A_1 \subset A_i$

$$\Downarrow$$

$$\bigcap_{i=1}^{n} A_1 \subseteq \bigcap_{i=1}^{n} A_i$$

$$= A_1$$

Q8: a) $A \cup B \stackrel{?}{=} A$

$$A \cup B = \{X \mid X \in A \vee X \in B\}$$

$$X \in (A \cup B)$$

$$\Downarrow$$

$$X \in A$$

then we conclude that $B \subseteq A$

b) $A \cap B = \{X \mid (X \in A) \wedge (X \in B)\}$

So $X \in (A \cap B)$

then we conclude that $X \in B$

So $A \subseteq B$

c) $A - B = \{X \mid (X \in A) \wedge (X \notin B)\}$

So $A \cap B = \emptyset$