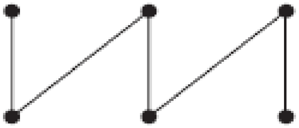
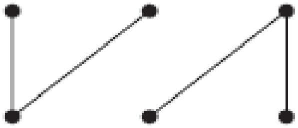
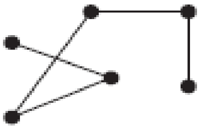
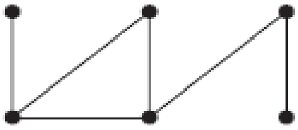
Trees

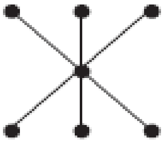
1. Which of these graphs are trees?

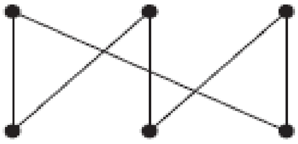
a) 

b) 

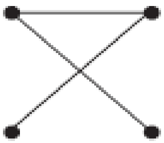
c) 

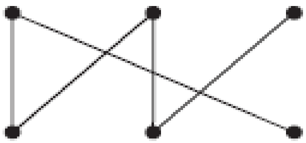
d) 

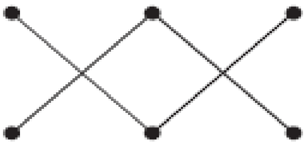
e) 

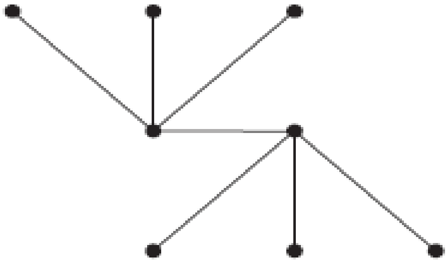
f) 

2. Which of these graphs are trees?

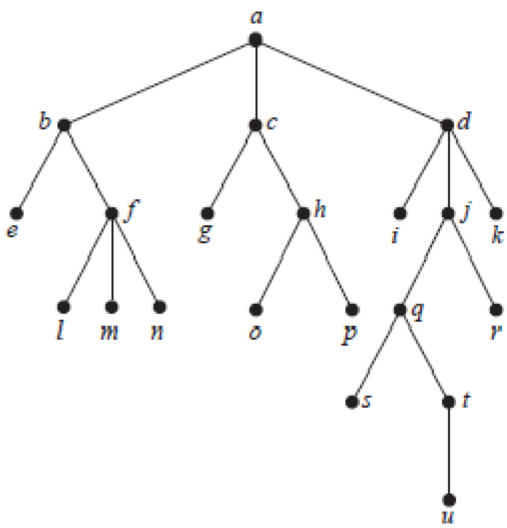
a) 

b) 

c) 

d) 

Answer these questions about the rooted tree illustrated.



Which vertex is the root?
Which vertices are internal?
Which vertices are leaves?
Which vertices are children of *j*?
Which vertex is the parent of *h*?
Which vertices are siblings of *o*?
Which vertices are descendants of *b*?
Which vertices are ancestors of *m*?
Is the rooted tree a full *m*-ary tree for some positive integer *m*?
What is the level of each vertex of the rooted tree?
10. Draw the subtree of the tree that is rooted At a) *a*. b) *c*.

POWERUNIT

ii) Represent the expressions (x + xy) + (x/y) and x + ((xy + x)/y) using binary trees.

Write these expressions in
b) prefix notation.
c) infix notation
d) postfix notation.

---

iii) What is the value of each of these prefix expressions?

i) − * 2 / 8 4 3

ii) ↑ − * 3 3 * 4 2 5

iii) + − ↑ 3 2 ↑ 2 3 / 6 − 4 2

.

Graph

i)Draw these graphs.

a)C_7 **b)** $K_{1,8}$ **c)** $K_{4,4}$

---

ii)How many vertices and how many edges do these graphs have?

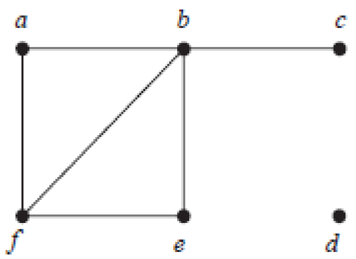
I)W_n II)Q_n III)$K_{(m,n)}$

---

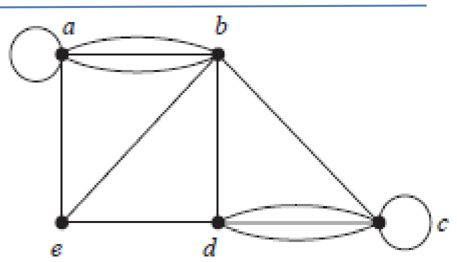
iii)find the number of vertices, the **number of edges**, and the **degree of each vertex** in the given undirected graph. Identify all **isolated** and **pendant** vertices.

POWERUNIT

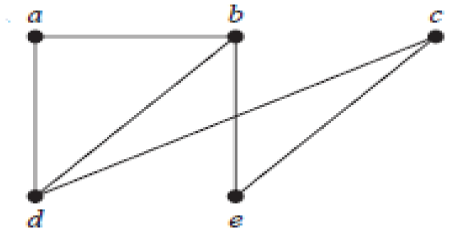
1.

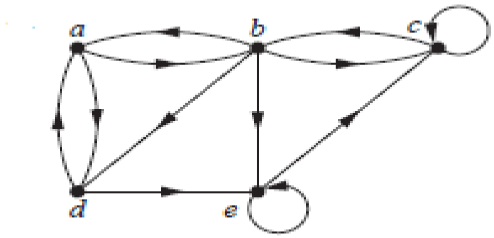


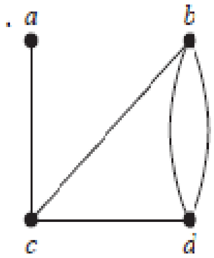
2.

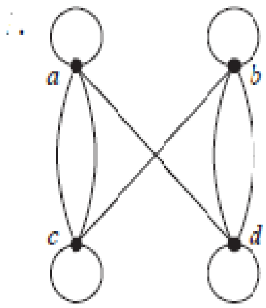


iv)Represent the graphs with an adjacency matrix.

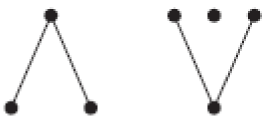


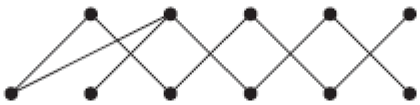






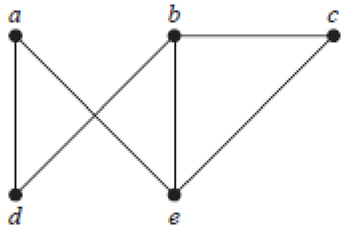
v)determine whether the given graph is connected





vi)Does each of these lists of vertices form a path or Simple path,or circuits?
What are the lengths of those that are paths?

i) ***a, e, b, c, b*** **ii)** ***a, e, a, d, b, c, a***



RELATIONS

I)Determine whether the relation*R* on the set of **all integers** is <u>reflexive, symmetric</u>, <u>antisymmetric</u>, and/or <u>transitive,</u> where ***(x, y)*** ∈ ***R*** if and only if

I) $xy \geq 1$..

II) x is a multiple of y.

III) x and y are both negative or both nonnegative.

IV) $x = y^2$

POWERUNIT

II) Let R be the relation $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$,

and let S be the relation $\{(2, 1), (3, 1), (3, 2), (4, 2)\}$.

Find $S \circ R$.

III)Let $R1 = \{(1, 2), (2, 3), (3, 4)\}$ and $R2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$ be relations from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$. Find

a) $R1 \cup R2$. **b) $R1 \cap R2$.**

c) $R1 - R2$ **. d) $R2 - R1$.**

IV)

List the ordered pairs in the relations on {1, 2, 3} corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ **b)** $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

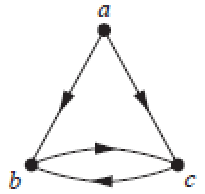
V)
Let R_1 and R_2 be relations on a set A represented by the matrices

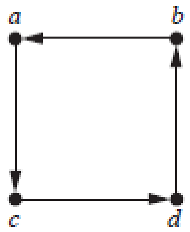
$$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find the matrices that represent

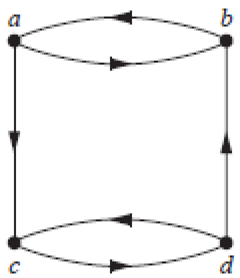
a) $R_1 \cup R_2$. **b)** $R_1 \cap R_2$. **c)** $R_2 \circ R_1$.
d) $R_1 \circ R_1$. **e)** $R_1 \oplus R_2$.

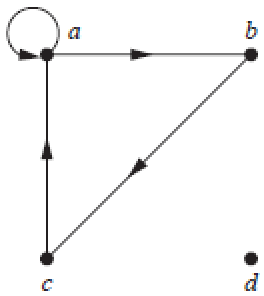
VI)list the ordered pairs in the relations represented by the directed graphs.





VII)draw the directed graph of the reflexive closure of the relations with the directed graph shown





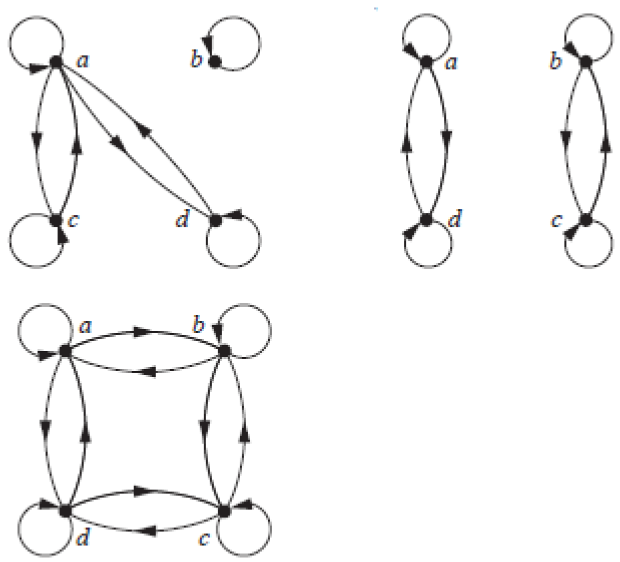
VIII) Use Algorithm 1 to find the transitive closures of these relations on {*a, b, c, d, e*}.

a) {*(a, c), (b, d), (c, a), (d, b), (e, d)*}

b) {*(b, c), (b, e), (c, e), (d, a), (e, b), (e, c)*}

C) {*(a, e), (b, a), (b, d), (c,d), (d,a), (d, c), (e,a), (e,b), (e, c), (e, e)*}

II)determine whether the relation with the directed graph shown is an equivalence relation.



a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Divisibility and Modular Arithmetic

I)What are the quotient and remainder when

i) 44 is divided by 8?
ii) 777 is divided by 21?
iii)) 0 is divided by 17?

iv) −100 is divided by 101?

v) −2002 is divided by 87?

II) Evaluate these quantities.

a) -1 **mod** 23 **b)** −97 **mod** 11

c) 155 **mod** 19 **d)** −221 **mod** 23

III) Find value of .

[(177 **mod** 31 **+** 270 **mod** 31) **mod** 31]

IV) Find the integer a such that

i) a ≡ −15 (mod 27) and −26 ≤ a ≤ 0.

ii) a ≡ 24 (mod 31) and −15 ≤ a ≤ 15.

iii) a ≡ 99 (mod 41) and 100 ≤ a ≤ 140.

POWERUNIT

IV) Find the prime factorization of each of these integers.

i) 39 **ii)** 81 **iii)** 101 **iv)** 289

V)Determine whether the integers in each of these sets are pairwise relatively prime.

i) 11, 15, 19 **ii)** 14, 15, 21

iii) 12, 17, 31, 37 **iv)** 7, 8, 9, 11

V)Find gcd(1000, 625) * lcm(1000, 625)

VI) What are the greatest common divisors of these pairs of integers?

$3^7 \cdot 5^3 \cdot 7^3, 2^{11} \cdot 3^5 \cdot 5^9$

$11 \cdot 13 \cdot 17, 2^9 \cdot 3^7 \cdot 5^5 \cdot 7^3$

$23^{31}, 23^{17}$

(41 · 43 · 53) , (41 · 43 · 53)

(1111) , 0

Functions

I)Determine whether each of these functions is a bijection from **R** to **R**.

a) f (x) = −3x + 4

b) f (x) = −3x2 + 7

c) f (x) = (x + 1)/(x + 2)

d) f (x) = x5 + 1

II) Find these values**.(CEIL& FLOOR**)

a) [1.1] **b)** [1.1]

c) [−0.1] **d)** [−0.1]

e)[2.99] **f)** [−2.99]

POWERUNIT

III) Determine whether the function f : **Z** × **Z** → **Z** is onto if

a) f (m, n) = m + n.

b) f (m, n) = m**2 + n**2 .**note that (**) mean power to**

c) f (m, n) = m.

d) f (m, n) = |n|.

e) f (m, n) = m − n.

IV) Determine whether each of these functions from {*a, b, c, d*} to itself is one-to-one.

a) *f (a) = b, f (b) = a, f (c) = c, f (d) = d*

b) *f (a) = b, f (b) = b, f (c) = d, f (d) = c*

c) *f (a) = d, f (b) = b, f (c) = c, f (d) = d*

V) Determine whether each of these functions from **Z** to **Z** is one-to-one.

a) f (n) = n – 1 **b)** f (n) =(n**2) + 1

c) f (n) = n**3 **d)** f (n) = [n/2]

Sequences

I)

What is the term a_8 of the sequence $\{a_n\}$ if a_n equals

a) 2^{n-1}?

b) 7?

c) $1 + (-1)^n$?

d) $-(-2)^n$?

What are the terms a_0, a_1, a_2, and a_3 of the sequence $\{a_n\}$, where a_n equals

a) $2^n + 1$?

b) $(n + 1)^{n+1}$?

c) $\lfloor n/2 \rfloor$?

d) $\lfloor n/2 \rfloor + \lceil n/2 \rceil$?

II)

Find the value of each of these sums.

a) $\sum_{j=0}^{8} (1 + (-1)^j)$

b) $\sum_{j=0}^{8} (3^j - 2^j)$

c) $\sum_{j=0}^{8} (2 \cdot 3^j + 3 \cdot 2^j)$

d) $\sum_{j=0}^{8} (2^{j+1} - 2^j)$

NIT

Compute each of these double sums.

a) $\sum_{i=1}^{2} \sum_{j=1}^{3} (i + j)$

b) $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i + 3j)$

c) $\sum_{i=1}^{3} \sum_{j=0}^{2} i$

d) $\sum_{i=0}^{2} \sum_{j=1}^{3} ij$

SETS

I)List the members of these sets.

a) {x | x is a real number such that (x**2)= 1}

b) {x | x is a positive integer less than 12}

c) {x | x is the square of an integer and x < 100}

d) {x | x is an integer such that (x**2) = 2}

II) Use set builder notation to give a description of each of these sets.

a) $\{0, 3, 6, 9, 12\}$

b) $\{-3, -2, -1, 0, 1, 2, 3\}$

c) $\{m, n, o, p\}$

III) Suppose that $A = \{2, 4, 6\}$, $B = \{2, 6\}$, $C = \{4, 6\}$, and $D = \{4, 6, 8\}$. Determine which of these sets are subsets of which other of these sets.

III) Determine whether each of these statements is true or false.

i) $0 \in \varnothing$

ii) $\varnothing \in \{0\}$

iii) $\{0\} \subset \varnothing$

iv) $\{0\} \in \{0$

v) $\{\varnothing\} \subseteq \{\varnothing\}$

POWERUNIT

IV) What is the cardinality of each of these sets?

a) $\{a\}$ **b)** $\{\{a\}\}$

c) $\{a, \{a\}\}$ **d)** $\{a, \{a\}, \{a, \{a\}\}\}$

V) Determine whether each of these sets is the power set of a set, where a and b are distinct elements.

i) $\varnothing$ **ii)** $\{\varnothing, \{a\}, \{\varnothing, a\}\}$ **iii)** $\{\varnothing, \{a\}, \{b\}, \{a, b\}\}$

IV) Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find

&) $A \times B$. **&)** $B \times A$.

V)Let A = {a, b, c}, B = {x, y}, and C = {0, 1}. Find

O) A × B × C. **O)** C × B × A.

O) C × A × B. **O)** B × B × B.

VI) Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}. Find

a) A ∪ B. **b)** A ∩ B.

c) A − B. **d)** B − A.

VII)

Let $A_i = \{1, 2, 3, \ldots, i\}$ for $i = 1, 2, 3, \ldots$. Find

a) $\bigcup_{i=1}^{n} A_i$. b) $\bigcap_{i=1}^{n} A_i$.

VIII)

What can you say about the sets A and B if we know that

0) A ∪ B = A? **0)** A ∩ B = A?

0) A − B = A? **0)** A ∩ B = B ∩ A?

Best Wishes!

SONDOS IDAQ