Applications Logic Problems

A. You can see the movie only if you are over 18 years old or you have the permission of a parent. Express your answer in terms of m: "You can see the movie," e: "You are over 18 years old," and p: "You have the permission of a parent."

<u>Solution:</u>

Interpretation symbols

Negation $\neg p$: not p

Disjunction $p \lor q$: p or q

Conjunction $p \land q$: p and q

Conditional statement $p \rightarrow q$: if p, then q

Biconditional statement $p \leftrightarrow q$: if p, then q

"p only if q" can be rewritten as "if p, then q"

"You can see the movie only if you are over 18 years old or you have the permission of a parent" can then be rewritten as "if m, then (e or p)", or rewritten using the above symbols: <u>$m \rightarrow (e \lor p)$</u>

B. You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book. Express your answer in terms of g: "You can graduate," m: "You owe money to the university," r:

"You have completed the requirements of your major,"
and b: "You have an overdue library book."
Solution:
$g \rightarrow (\ r \wedge \neg m \wedge \neg b)$
g only if (r and not m not b)

C. Express these system specifications using the propositions p "The message is scanned for viruses" and q "The message was sent from an unknown system" together with logical connectives (including negations).

a. "The message is scanned for viruses whenever the message was sent from an unknown system."

b. "The message was sent from an unknown system but it was not scanned for viruses."

c. "It is necessary to scan the message for viruses whenever it was sent from an unknown system."

d. "When a message is not sent from an unknown system it is not scanned for viruses."

Solution:

a. $q \rightarrow p$

b. $q \wedge \neg p$

c. $q \rightarrow p$

d. $\neg q \rightarrow \neg p$

D. Are these system specifications consistent? "Whenever the system software is being upgraded, users cannot

access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded."

Solution:

"whenever p, q" in a sentence implies "if p, then q".

We can then rewrite the given sentences as:
(1)
$$p \rightarrow \neg q$$

(2)
$$q \rightarrow r$$

(3)
$$\neg r \rightarrow \neg p$$

FIRST CASE If we assume that q is false, then $\neg q$ is true.
By proposition (1), we then know that p has to be true and thus $\neg p$ is false.
By proposition (3), we know that $\neg r$ has to be false and thus r has to be true.
Proposition (2) then states that q is false and r is true, thus this conditional statement is true as well.

SECOND CASE If we assume that q is true, then $\neg q$ is false.
By proposition (2), we then know that r has to be true and thus $\neg r$ is false.
By proposition (1), we know that p has to be false and thus $\neq p$ has to be true.
Proposition (3) then states that $\neg r$ is false and $\neq p$ is true, thus this conditional statement is true as well.

CONCLUSION We note that both cases led to all three propositions to be true and thus the system is consistent.