Discrete Propositional Problems

A. Let p and q be the propositions p : It is below freezing. q : It is snowing. Write these propositions using p and q and logical connectives (including negations).

a) It is below freezing and snowing.

b) It is below freezing but not snowing.

c) It is not below freezing and it is not snowing.

d) It is either snowing or below freezing (or both).

e) If it is below freezing, it is also snowing.

f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.

g) That it is below freezing is necessary and sufficient for it to be snowing.

Solution:

a) $p \wedge q$

b) $p \wedge \neg q$

c) $\neg p \wedge \neg q$

d) $p \vee q$

e) $p \rightarrow q$

f) $(p \vee q) \wedge (p \rightarrow \neg q)$

g) $p \leftrightarrow q$

B. Let p and q be the propositions p : You drive over 65 miles per hour. q : You get a speeding ticket. Write these propositions using p and q and logical connectives (including negations).

a) You do not drive over 65 miles per hour.
b) You drive over 65 miles per hour, but you do not get a speeding ticket.
c) You will get a speeding ticket if you drive over 65 miles per hour.
d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
e) Driving over 65 miles per hour is sufficient for getting a speeding ticket. f) You get a speeding ticket, but you do not drive over 65 miles per hour. g) Whenever you get a speeding ticket, you are driving over 65 miles per hour

Solution:

a) $\neg p$
b) $p \Lambda \neg q$
c) $p \rightarrow q$
d) $\neg p \rightarrow \neg q$
e) $p \rightarrow q$
f) $q \Lambda \neg p$
g) $q \rightarrow p$

C. Let p, q, and r be the propositions p : You get an A on the final exam. q : You do every exercise in this book. r : You get an A in this class. Write these propositions using p, q, and r and logical connectives (including negations).

a) You get an A in this class, but you do not do every exercise in this book.
b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
c) To get an A in this class, it is necessary for you to get an A on the final.
d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Solution:

a) r Λ ¬p
b) p Λ q Λ r
c) p => r
d) p Λ ¬q Λ r
e) (p Λ q) => r
f) r ⇔ (p V q)

D. Determine whether these biconditionals are true or false.

a) 2 + 2 = 4 if and only if 1 + 1 = 2.
b) 1 + 1 = 2 if and only if 2 + 3 = 4.
c) 1 + 1 = 3 if and only if monkeys can fly.

d) 0 > 1 if and only if 2 > 1.

Solution:

a) True

b) False

c) True

d) False

E. Determine whether each of these conditional statements is true or false.

a) If 1 + 1 = 2, then 2 + 2 = 5.

b) If 1 + 1 = 3, then 2 + 2 = 4.

c) If 1 + 1 = 3, then 2 + 2 = 5.

d) If monkeys can fly, then 1 + 1 = 3.

Solution:

In each case, we simply need to determine the truth value of the hypothesis and the conclusion, and then use the definition of the truth table value of the conditional statement. The conditional statement is true in every case except when the hypothesis (the "if" part) is true and the conclusion (the "then" part) is false.

a) False

b) True

c) True

d) True

F. Determine whether each of these conditional statements is true or false.

a) If 1 + 1 = 3, then unicorns exist.
b) If 1 + 1 = 3, then dogs can fly.
c) If 1 + 1 = 2, then dogs can fly.
d) If 2 + 2 = 4, then 1 + 2 = 3.

Solution:

a) True
b) True
c) False
d) True