Logic

- propositional logic
- predicate Logic

- A Propsition is a declarative sentence that is either true or false but not both. "it gives me information.

Ex :- Tornoto is the capital of canda X
1 + 1 = 2 ✓

- Which of these sentences are propositions

- 2 + 3 = 6 ✓ "it gives me an information"
- X + 5 = 7 X
- what time is it ? X
- Answer this question X
- Today is friday ✓

Propositional logic "compound proposition"

- A compound proposition is a proposition that is formed from an exiting proposition using logical operators.

logical operators → $\neg$, $\wedge$, $\vee$, $\oplus$, $\rightarrow$, $\leftrightarrow$
Not And OR XOR

- Negation "not" $\neg$

$$p \rightarrow \neg p$$

p	$\neg p$
T	F
F	T

p → Today is friday
$\neg$p → Today is not friday.

- Conjuction "$\wedge$" → let p and q be propositions. The conjuction of p and q is $p \wedge q$.

True when both of them are true

let p be :- Today is friday
let q be :- It is raining today
Then $p \wedge q$:- Today is friday and it is raining today.

p	q	$p \land q$
T	T	T
T	F	F
F	T	F
F	F	F

- Disjunction "or" "$\lor$"

- let p be :- Today is friday
- let q be :- it is raining today
- then $p \lor q$:- Today is friday or it is raining today

p	q	$p \lor q$
T	T	T
F	T	T
T	F	T
F	F	F

- Exclusive or " $\oplus$ " "XOR" "one of them only true"

- let p :- Soup comes with an entree
- let q :- salad comes with an entree
- Then $p \oplus q$:- Soup comes with an entree or salad but not both

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional statement

- Implication "⟶"

hypothesis
antecedent
premise

Conclusion/ consequence.

$$p \longrightarrow q$$

p is called hypothesis
q is called conclusion

let p be :- you get 100 on the final
let q be :- you will get an "A"
The $p \to q$:- If you get 100 on the final then you will get an "A"

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

* The Converse of $p \to q$ is ~~p~~ $q \to p$

* The Contrapositive of $p \to q$ is $\neg q \to \neg p$ "equivalent"

* The inverse of $p \to q$ is $\neg p \to \neg q$

- Bioconditional "$\longleftrightarrow$" "if and only if"

b bi-implications

let p :- you can take the flight

let q :- you buy a ticket

Then, $p \leftrightarrow q$:- you can take the flight if and only if you buy a ticket.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	F

$\to$ ~~$(p \wedge q)$~~ $(p \to q) \wedge (q \to p)$

Subject Date No.

D//

→ Compound propositions " one or more logical operators "

- Precedence of logical operators :-

* الأقواس اول اولوية :-

1 - ¬
2 - ∧
3 - ∨
4 - →
5 - ↔

Ex : construct the truth table of the following proposition

$(P \vee \neg q) \rightarrow (P \wedge q)$

1 2 3 4

# of rows = $2^n \rightarrow$ # of variables

P	q	¬q	P∨¬q
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

P	q	P∧q
T	T	T
T	F	F
F	T	F
F	F	F

P∨¬q	P∧q	(P∨¬q)→(P∧q)
T	T	T
T	F	F
F	F	T
T	F	F

Scanned with CamScanner

Tautology → a compound proposition that is always true

Contradiction → a compound proposition that is always false.

Contingency → a compound proposition that is neither tautology nor a contradiction

$(p \land q) \rightarrow p$

p	q	$p \land q$	$(p \land q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

" Tautology

$p \land \neg p$

p	$\neg p$	$p \land \neg p$
T	F	F
F	T	F

" contradiction '

- the compound propositions p and q are called logical equivalent if $p \leftrightarrow q$ is tautology

Ex : show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logical equivalet

p	q	$(p \vee q)$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	~~$\neg(p \vee q) \leftrightarrow \neg p$~~
T	T	T	F	F	F	F	~~F~~
T	F	T	F	F	T	F	~~F~~
F	T	T	F	T	F	F	~~F~~
F	F	F	T	T	T	T	~~T~~

$\equiv$

Prove that proposition $(p \to q) \wedge -q \to -p$ is tautology

p	q	$p \to q$	$-q$	$-p$	$(p \to q) \wedge -q$	
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

- Prove that the proposition $p \to q$ and $-q \to -p$ are equivalents

p	q	$p \to q$	$-q$	$-p$	$-q \to -p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Propositional Equivalences - Laws

- Identity laws
  - $P \wedge T \equiv P \longrightarrow$ "$\wedge$" عنصر محايد
  - $P \vee F \equiv P \rightarrow$ "$\vee$" عنصر محايد

- Dominations laws
  - $P \wedge F \equiv F$
  - $P \vee F \equiv T$

- Idempotent laws
  - $P \wedge P \equiv P$
  - $P \vee P \equiv P$

- Double negation law
  - $\neg(\neg P) \equiv P$

Subject Date No.

- Commutative laws
  - $p \wedge q \equiv q \wedge p$
  - $p \vee q \equiv q \vee p$
- Associative laws
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
- Distributive laws
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- De Morgans law
  - $\neg (p \wedge q) \equiv \neg p \vee \neg q$
  - $\neg (p \vee q) \equiv \neg p \wedge \neg q$

- Absorption laws
  - $p \wedge (p \vee q) \equiv p$
  - $p \vee (p \wedge q) \equiv p$

- Negation laws
  - $p \vee \neg p \equiv T$
  - $p \wedge \neg p \equiv F$

- Implication
  - $p \longrightarrow q \equiv \neg p \vee q$

- Biconditional
  - $p \longleftrightarrow q = (p \longrightarrow q) \wedge (q \longrightarrow p)$

- Show that the proposition $p \wedge q \longrightarrow p$ is tatology without using truth table.

$p \wedge q \longrightarrow p$

$\neg (p \wedge q) \vee p$

$(\neg p \vee \neg q) \vee p$

$(\neg p \vee p) \neg q$

$\equiv \overline{T} \vee \neg q \quad \boxed{T}$

- Determine if the proposition $P \vee q \rightarrow P$ is tautology or not without using truth table.

$(P \vee q) \rightarrow P$

$\neg(P \vee q) \vee P$

$(\neg P \wedge \neg q) \vee P$

$(\neg P \vee P) \wedge (\neg q \vee P)$

$T \wedge (\neg q \vee P)$

$\neg q \vee P \longrightarrow$ not tautology.

F

- Show that the proposition $\neg(P \rightarrow q) \wedge \neg P$ is Contradiction without using truth table.

$\neg(P \rightarrow q) \wedge \neg P$

$\neg(\neg P \vee q) \wedge \neg P$

$(P \wedge \neg q) \wedge \neg P$

$(P \wedge \neg P) \wedge \neg q$

$= F \wedge \neg q$

$\boxed{F} \longrightarrow$ Contradiction

- show that $(P \rightarrow q) \wedge (P \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logical equivalent without using truthtable.

1) $(P \rightarrow q) \wedge (P \rightarrow r)$

$(\neg P \vee q) \wedge (\neg P \vee r)$

عامل مش

$\neg P \vee (q \wedge r)$

equivalent.

2) $p \rightarrow (q \wedge r)$

$\neg p \vee (q \wedge r)$

- show that $\neg(p \vee q)$ and $\neg(p \vee (\neg P \wedge q))$ are logical equivalent without using truthtable.

1) $\neg(P \vee q) \rightarrow \neg P \wedge \neg q$

2) $\neg(P \vee (\neg P \wedge q))$

$\neg((P \vee \neg P) \wedge (P \vee q))$

$(\neg P \wedge P) \wedge (\neg P \wedge \neg q)$

$T \wedge (\neg P \wedge \neg q)$

$= \neg P \wedge \neg q$

* Translation

- Translate the given statement into Propositional logic

p

- A password must have at least three digits or be at leas 8 characters long.

q

$p \vee q$

p q

- It is freezing and snowing

$p \wedge q$

p and ¬ q

- It is below freezing but not snowing

$p \wedge \neg q$

- You don't drive over 65 miles per hour.

$\neg p$

p

- If you do not drive over 65 miles per hour, then you will not get a speeding ticket

q

$$\neg p \rightarrow \neg q$$

- ~~Whenever~~ whenever you get a speeding ticket, you are driving over 65 miles per hour

$$p \rightarrow q$$

p q

- Access is granted whenever the user has paid the subscription fee and enters a valid pass.

r

$$q \land r \rightarrow p$$

p q

- Access is denied if the user has not paid the subscription fee

$$\neg q \rightarrow p$$

p

- you can access the website only if you pay a ~~s~~ subscription fee

q

$$p \rightarrow q$$

implies ≡ only if

p

. you will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

q

r

$$p \leftrightarrow q \vee r$$

* System Specifications

- Determine whether these system specifications are, Consistent :- → not all false.

- The message is stored in the memory or it is retransmitted

$$p \vee q$$

- The message is not stored in the memory

$$\neg p$$

- If the message is ~~not~~ stored in the memory, then it is retransmitted

$$p \rightarrow q$$

→

Subject

Date No.

$\wedge$

p	q	$p \vee q$	$\neg p$	$p \to q$	$\wedge$
T	T	T	F	T	F
T	F	T	F	F	F
F	T	T	T	T	T
F	F	F	T	T	F

the message is not stored, it is ~~t~~ retransmitted.

* logical Puzzels.

- An island has two Kinds of residents, Knightes, who always tell the truth, and their opposits, Knaves, who always lie. you encounter two people A and B. what are A and B if A says B is a Knight and B says the two of us are opposite types.

p : A is Knight
q : B is Knight

what a said what p said $\wedge$

p	q	q	$p \oplus q$	$p \leftrightarrow q$	$q \leftrightarrow (p \oplus q)$	$\wedge$
T	T	T	F	T	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	F
F	F	F	F	T	T	T

N O T E B O O K

- A says the two of us are both Knights and B says A is a Knave

- p : A is Knight
- q : B is Knight

p	q	$p \land q$	$\neg p$	$p \Leftrightarrow (p \land q)$	$q \longleftrightarrow \neg p$	$\land$
T	T	T	F	T	F	F
T	F	F	F	F	T	F
F	T	F	T	T	T	T
F	F	F	T	T	F	F

Prove that the proposition $(p \rightarrow q) \wedge \neg q \rightarrow \neg p$ is tautology.

$$[(\neg p \vee q) \wedge \neg q] \rightarrow \neg p$$

$$[(\neg p \wedge \neg q) \vee (q \wedge \neg q)] \rightarrow \neg p$$

$$[(\neg p \wedge \neg q) \vee F] \rightarrow \neg p$$

$$(\neg p \wedge \neg q) \rightarrow \neg p$$

$$(p \vee q) \vee \neg p$$

$$(p \vee \neg p) \vee q$$

$$T \vee q = T$$

$(\neg p \vee q) \wedge (\neg q \vee r) \rightarrow (\neg p \vee r)$

~~$(\neg p \wedge \neg q) \vee (\neg p \wedge r) \vee (q \wedge \neg q) \vee (q \vee r)$~~

$$\neg[(\neg p \vee q) \wedge (\neg q \vee r)] \vee (\neg p \vee r)$$

$$[\neg(\neg p \vee q) \vee \neg(\neg q \vee r)] \vee (\neg q \vee r)$$

$$[(p \wedge \neg q) \vee (q \wedge \neg r)] \vee (\neg q \vee r) \longrightarrow$$

$[\overbrace{(p \land \neg q)}^{a} \lor \overbrace{(q \land \neg r)}^{b}] \lor (\overset{c}{\neg p} \lor \overset{d}{r})$

$\{(p \land \neg q) \lor \neg p\} \lor [(q \land \neg r) \lor r]$

$[\overbrace{(p \lor \neg p)}^{T} \lor (\neg q \lor \neg p)] \lor [(q \land \neg r) \land \overbrace{(\neg r \lor r)}^{T}]$

$(\neg q \lor \neg p) \lor (q \lor r)$

$(\neg q \lor q) \lor (\neg p \lor r)$

$T \lor (\neg p \lor r) \equiv T$

$\neg(p \overset{1}{\longleftrightarrow} q) \equiv p \overset{2}{\longleftrightarrow} \neg q$

1. $\neg((p \to q) \land (q \to p)) \equiv$

$\neg((\neg p \lor q) \land (\neg q \lor p))$

~~$(p \land q) \land$~~

$[(p \land \neg q) \lor (q \land \neg p)]$ ✓

2. $p \longleftrightarrow \neg q$

$(p \to \neg q) \land (\neg q \to p)$

$(\neg p \lor \neg q) \land (q \lor p)$

$(\neg p \land q) \lor (\neg p \lor p) \lor (\neg q \land q) \lor (\neg q \land p)$

$(\neg p \land q) \lor (\neg q \land p)$ ✓

Date No.

Note :- XOR → Bi-Conditional.

$p \to q$

p :- الشرط

q :- جواب الشرط.

- if p then q
- if p, q
- q if p
- p only if q → bi-Conditional

كلما p q

-(whever) you get a speeding ticket, you are driving over 65 miles per hour.

$p \to q$

is sufficient and necessary → if and only if

p q

- Getting A on the final and doing every exercise in this book is sufficient for getting an A in this class.

$p \wedge q \to r$

النتيجة

p

- to get an A in this class, it is suffiecent for you to get 50 on the final q

$q \to p$

p

- To get an A in this final, it is neccessary for you to get an 50 on final

q

$p \to q$

p q

- It is necessary to have a valid password to log on the server

$q \to p$ "الطريقة الوحيدة"

p $q \to$ النتيجة

- It is sufficient to have a valid password to log on to the serwen.

$p \to q$ "مجرد طرق"

p

- for hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area. $\neg q \wedge \neg r$

necessary suffiecient

$$(p \to (\neg q \wedge \neg r)) \wedge \neg[(\neg q \wedge \neg r) \to p]$$

p q

- If the water is not too cold, then Jan will go swimming

$\neg p \to q$

q $\neg p$

- Jan will go swimming unless the water is too cold

$\neg p \to q$

Subject Date No.

p q

you can acess the website only if you pay a subscription fee.

$p \longrightarrow q$

* في الجملة الشرطية إذا تحقق الشرط لازم يتحقق جواب الشرط

بس ممكن الشرط ما يتحقق وجواب الشرط يتحقق.

استنتاج

* Rules of Inference "شي موجود معطيات وبدي أستنتجها"

- Consider the following statments :-
  - If you have a current password, then you can log onto the network.
  - you have a current password
  - you can log onto the password

you can conclude

- Rules of inference

~~tautology~~

all of them are tautology

$T \leftarrow p$
$T \leftarrow q$ معطيات

$T \leftarrow p \wedge q$

$T \leftarrow p \wedge q \rightarrow$ معطيات
$T \leftarrow p$
$T \leftarrow q$

$T \leftarrow p \vee q$
$T \leftarrow \neg p$

$T \leftarrow q$

$p \rightarrow T$

$p \vee q$

$T \leftarrow p \vee q$
$T \leftarrow \neg p \vee r$

$T \leftarrow q \vee r$

$T \leftarrow p \rightarrow q$
$T \leftarrow p$

q

$p \rightarrow q$
$T \leftarrow \neg q$ [illegible]

$T \leftarrow \neg p$

$p \rightarrow q$
$q \rightarrow r$

$p \rightarrow r$

جواب الشرط للأول عبارة عن شرط للثاني

جواب الشرط هو شرط للتالي.

Example:-

- Show that the hypotheses, "It is $\overset{\neg p}{\text{not}}$ sunny this afternoon $\overset{\wedge}{\text{and}}$ it is $\overset{q}{\text{coolder}}$ than yesterday," "we will $\overset{r}{\text{go}}$ swimming only if it $\overset{p}{\text{is}}$ sunny," "if we do $\overset{\neg r}{\text{not}}$ go swimming, then we will $\overset{t}{\text{take}}$ a canoe trip," and "if we take a canoe trip, then we will be home by sunset" lead to the conclusion "we will be home by sunset."

↳ s is true.

$$1)\ \neg p \wedge q \qquad 3)\ \neg r \rightarrow t$$

$$2)\ r \rightarrow p \qquad 4)\ t \rightarrow s$$

$$1)\ \begin{array}{c} \neg p \wedge q \\ \hline \neg p \\ q \end{array} \qquad 2)\ \begin{array}{c} \neg r \rightarrow t \\ t \rightarrow s \\ \hline \neg r \rightarrow s \end{array}$$

$$3)\ \begin{array}{c} r \rightarrow p \\ \neg p \\ \hline \neg r \end{array} \qquad 4)\ \begin{array}{c} \neg r \rightarrow s \\ \neg r \\ \hline s \end{array} \ \#$$

- Example:-

show that the hypotheses "If you $\overset{p}{\text{send}}$ me an e-mail message, then I will finish writing $\overset{q}{\text{the}}$ program," "If you do not $\overset{\neg p}{\text{send}}$ me an e-mail message, then I will go $\overset{r}{\text{to}}$ sleep early." and "If I $\overset{r}{\text{go}}$ to sleep early then I will wake $\overset{s}{\text{up}}$ feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed.

$p \to q \equiv \neg q \to \neg p$

$$\begin{array}{l} \neg p \to r \\ r \to s \\ \hline \cancel{\neg p}\ \neg q \to s \end{array}$$

$$1)\ \begin{array}{l} \neg p \to r \\ r \to s \\ \hline \neg p \to s \end{array}$$

$$\begin{array}{l} \neg p \to s \\ \neg q \to \neg p \\ \hline \neg q \to s \ \# \end{array}$$

- from the following hypothesis conclude $\neg t$

- $p \wedge \neg q$
- $r \to q$
- $t \to r$

$$1)\ \begin{array}{c} p \wedge \neg q \\ \hline p \\ \neg q \end{array}$$

$$2)\ \begin{array}{l} t \to r \\ r \to q \\ \hline t \to q \end{array}$$

$$3)\ \begin{array}{l} t \to q \\ \neg q \\ \hline \neg t \ \# \end{array}$$

- from the following hypothesis conclude r

- $p \land \neg q$
- $s \lor q$
- $p \land s \to r$

1) $\begin{array}{c} p \land \neg q \\ \hline p \\ \neg q \end{array}$

2) $\begin{array}{c} s \lor q \\ \neg q \\ \hline s \end{array}$

3) $\begin{array}{c} p \\ s \\ \hline p \land s \end{array}$

4) $\begin{array}{c} p \land s \to r \\ p \land s \\ \hline r \ \# \end{array}$

- from the following hypothesis conclude s

- $p \land q$
- $p \lor r \to s \lor \neg t$
- $t \lor \neg q$

1) $\begin{array}{c} p \land q \\ \hline p \\ q \end{array}$

2) $\begin{array}{c} t \lor \neg q \\ q \\ \hline t \end{array}$

5) $\begin{array}{c} s \lor \neg t \\ t \\ \hline s \end{array}$

$\begin{array}{c} p \\ \hline p \lor r \end{array}$

4) $\begin{array}{c} p \lor r \to s \lor \neg t \\ p \lor r \\ \hline s \lor \neg t \end{array}$

- logic and proofs

- Propositional logic cannot adequately express: All discrete mathmatics students passed the final exam

- No rules of propositional logic allow us to conclude from Ahmad is a discrete mathmatics students that Ahmad passed the final exam properly.

- Predicate logic "statments involving variables"

ex. $X > 3$, $X = y + 3$

- We can represent the predicate $X > 3$ as $P(X)$

- Once value has been assigned ~~to~~ to the variable x, the statement $p(x)$ becomes a proposition and has a truth value.

City (x)

مدينة x

father (Ali, Ahmad)

Ali is a father of ahmad

Example:-

- let P(X) denote the statement X 73 what are the truth values of P(4) and &P(2)

P(4) = 473 T

P(2) = 273 F

- let Q(x,y) denotes the statment x = y + 3, what are the truth values of the propositions Q(1,2) and Q(3,0) ?

Q(1,2) = 1 = 2 + 3 F

Q(3,0) = 3 = 0+3 T

- Quantifiers

- Quantification is used to create a proposition from a propositional function.
- // express the extent to which a predicate is true over a range of elements
- we will focus on universal quantification and existential quantification.

for all

(Universal quantifiers $\forall$)

- The " " of $P(x)$ is the statement:-
  "$P(x)$ for all values of x in the domain."

- $\forall P(x)$ is the same as $P(x_1) \wedge P(x_2) \wedge ---- \wedge P(x_n)$

- An element for which $P(x)$ is false is called a counterexample of $p(x)$

Example:-

$P(x) : x > 3$

domain:- Z^+ "Positive integers"

$\forall x\, P(x) : P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5) ----$

Example

$P(x) : x > -1$

Domain:- Z^+

* Counter example:-
القيمة اللي بتخلي ال predicate غير صحيح.

$\forall x P(x) = T$

ال Predicate صحيح لكل قيم x

- $P(x) = x > 2$
  domain :- all real numbers
  $\forall x P(x) = F$

- $P(x) = x + 1 > x$
  Domain :- all real numbers
  $\forall x P(x) = T$

---

* Existential Quantification ($\exists$) "قيمة واحدة عالأقل تحقق الشرط"

- The // // of $P(x)$ is the statment :-
  "There ~~exists~~ exists an element x in the domain such that $P(x)$."

- $\exists P(x) = P(x_1) \vee P(x_2) \vee \ldots$

Example :-
$P(x) = x > 3$
Domain : Z^+
$\exists x P(x) = P(1) \vee P(2) \vee P(3) \vee P(4) \ldots$ etc.

$\boxed{T}$

قيمة واحدة ~~ع~~ عالأقل صح .

Ex :- $\boxed{M_s}$

$(\neg P \lor q) \rightarrow \neg s$

$s \land r$

$t \rightarrow q$

1) $$\begin{array}{l} s \land r \\ \hline s \\ r \end{array}$$

2) $$\begin{array}{l} (\neg P \lor q) \rightarrow \neg s \\ s \\ \hline P \land \neg q \end{array}$$

3) $$\begin{array}{l} P \land \neg q \\ \hline P \\ \neg q \end{array}$$

4) $$\begin{array}{l} t \rightarrow q \\ \neg q \\ \hline \neg t \end{array}$$

---

let $Q(x)$ be the statement "$x+1 > 2x$" If the domain consists of all integers, what are these truth values $\neg Q(x) = x+1 \leq 2x$

a) $Q(0) \rightarrow T$ b) $Q(-1) \rightarrow T$ c) $Q(1) \rightarrow F$

d) $\exists x\, Q(x) \rightarrow T$ e) $\forall x\, Q(x) \rightarrow F$ f) $\exists x\, \neg Q(x) \rightarrow T$

g) $\forall x\, \neg Q(x) \rightarrow F$

- Determine the truth value of each of these statements if the domain consists of all real numbers

الكسور تحقق الشرط

a) $\exists x\ (x^3 = -1) \rightarrow T$ b) $\exists x\ (x^4 < x^2) \rightarrow F$

c) $\forall(x)\ ((-x)^2 = x^2) \rightarrow T$ d) $\forall x\ (2x > x) \rightarrow F$

- Determine the truth value of each of these statements if the domain consists of all integers.

الصفر يحقق

a) $\forall n\ (n+1 > n) \rightarrow T$ b) $\exists n\ (2n = 3n) \rightarrow T$

c) $\exists n\ (n = -n) \rightarrow T$ d) $\forall n\ (3n \leq 4n) \rightarrow F$

← الصفر يحقق .

- Quantifiers

- What is the truth value for each of the following statements, The domain consists of integers

الصفر لا يتحقق

- $\forall_x (x > 0 \lor x < 0) \longrightarrow F$
- $\forall_x (x > 0 \longrightarrow x > 4) \longrightarrow F$

F يعني الشرط صح وجواب الشرط خطأ

- $\forall_x (x > 5 \longrightarrow x > 2) \longrightarrow T$
- $\forall_x (x > 1 \longrightarrow x^2 > x) \longrightarrow T$

- what is the truth value for each of the following statements, The domain consists of integers.

- $\exists_x (x > 4 \land x < 8) \longrightarrow T$
- $\exists_x (x > 2 \longrightarrow x^2 < 8) \longrightarrow F$
- $\exists_x (x = x^2) \longrightarrow T$
- $\exists_x (x < 10 \land x > 2 \land x^2 < 3x) \longrightarrow F$

---

* Nested quantifiers

- Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

- $\forall_x \forall_y ((x > 0) \land (y < 0) \longrightarrow (xy < 0)) \longrightarrow T$
- $\exists_x \exists_y (x + y \neq y + x) \longrightarrow F$
- $\exists_x \exists_y (x + y > yx) \longrightarrow T$

الترتيب مهم

- $\forall x \, \exists y \, (x+y=0) \rightarrow T$

$\downarrow$

بشكل عشوائي

$x=-1 \quad \exists y(-1+y=0) \rightarrow T$

$x=0 \quad \exists y(0+y=0) \rightarrow T$

$x=2 \quad \exists y(2+y=0) \rightarrow T$

- $\forall x \, \exists y \, (x+y=y+x) \rightarrow T$

$\downarrow$

$x=-2 \quad \exists y(-2+y=y-2) \rightarrow T$

$x=3 \quad \exists y(3+y=y+3) \rightarrow T$

$x=0 \quad \exists y(0+y=y+0) \rightarrow T$

- $\forall x \, \exists y \, (x^2=y) \rightarrow T$

$\downarrow$

$x=-1 \quad \exists y(1=y) \rightarrow T$

$x=0 \quad \exists y(0=y) \rightarrow T$

$x=4 \quad \exists y(16=y) \rightarrow T$

- $\forall x \, \exists y \, (x=y^2) \rightarrow F$

$x=-2 \quad \exists y(-2=y^2) \rightarrow F \rightarrow$ اذا قيمة وحدة F فالعبارة F كلها

~~$x=a \quad \exists y(0=$~~

هون بنختار قيمة واحدة ل x لأجرب

- $\exists x \forall y \, (x+y=x) \longrightarrow F$

$\downarrow$

$x=0 \quad \forall y \, (0+y=0) \longrightarrow F$

$x=1 \quad \forall y \, (1+y=1) \longrightarrow F$

- $\exists x \forall y \, (xy=x) \longrightarrow T$

$\downarrow$

$x=1 \quad \forall y \, (y=1) \longrightarrow F$

$x=0 \quad \forall y \, (0xy-0) \longrightarrow T$

- $\exists x \forall y \, (xy=1) \longrightarrow F$

$\downarrow$

$x=1 \quad \forall y \, (y=1) \longrightarrow F$

$x=0 \quad \forall y \, (0xy=1) \longrightarrow F$

- $\forall x \exists y \, (xy=1) \longrightarrow \overset{au}{F}$

$\downarrow$

$x=2 \quad \exists y \, (2y=1) \longrightarrow T$

$x=-3 \quad \exists y \, (-3y=1) \longrightarrow T$

$x=10 \quad \exists y \, (10y=1) \longrightarrow T$

$x=0 \quad \exists y \, (0xy=1) \longrightarrow F$

* Order of quantification

* الترتيب مهم

- $\forall x \exists y (x+y=0) \longrightarrow T$
- $\exists y \forall x (x+y=0) \longrightarrow F$

* Binding variables

"الvariable لازم تكون مربوطة
الـ quantifiers "

$\exists x \; (x+y=1)$. Here, x is bounded
but y is free.

"we can't determine its truth value".

$\exists x (P(x) \land Q(x)) \lor \forall x R(x)$

- $\exists x P(x) \land \underline{\underline{Q(x)}} \longrightarrow$ free

* تأثير الquantifier عاده
اللي بعدها مباشرة.

$\exists x (P(x) \land Q(x))$

- Logical equivalences involving quantifiers.

- $\forall_x (P(x) \wedge Q(x)) = \forall_x P(x) \wedge \forall Q(x)$

ex:- $\forall_x (x > 3 \wedge x^2 > 0) = \forall_x (x > 3) \wedge \forall_x (x^2 > 0) \xrightarrow{} F$

- $\exists_x (P(x) \vee Q(x)) = \exists_x P(x) \vee \exists_x Q(x)$

ex. $\exists x (x > 3 \vee x < 3) = \exists_x (x > 3) \vee \exists_x (x < 3)$

* Negating quantified expressions

- consider this statement "Every student in your class has taken a course in calculus"

- Its negation is :- "It is not the case that every student in your class has taken a course in calculus"

- or "There is a student in your class who has not taken a course in calculus."

$$\neg \forall_x P(x) = \exists_x \neg P(x)$$

- $\neg \forall_x P(x) \equiv \exists_x \neg P(x)$
- $\neg E_x P(x) = \forall_x \neg P(x)$

- prove that $\neg \forall x (P(x) \longrightarrow Q(x))$ and $\exists x (P(x) \land \neg Q(x))$ are logical equivalent.

$$\neg \forall x \exists y\, P(x,y) \equiv \exists_x \neg \exists y\, P(x,y)$$
$$\equiv \exists_x \forall y\, \neg P(x,y)$$

$$\neg \forall x (P(x) \longrightarrow Q(x)) \equiv \exists_x \neg (P(x) \longrightarrow Q(x))$$
$$\equiv \exists_x \neg (\neg P(x) \lor Q(x))$$
$$\exists_x (P(x) \land \neg Q(x))$$

* Translation "Translating from english into logical expression"

- let $I(x)$ be the statement "x has an Internet Connection", $C(x, y)$ be the statement "x and y have chatted over the Internet", and $S(x)$ be "x is a student", where the domain for the variables x and y consists of all people. Use Quantifiers to express each of these ~~item~~ statements.

- Bob has an internet connection.

  $I(Bob)$

- Mark does not have an internet connection.

  $\neg I(Mark)$

- Bob and Johan have an internet connection.

  $I(Bob) \land I(Johan)$

- Johan and Bob have chatted over the internet.

  $C(Johan, Bob)$

- Someone has an internet connection.

  $\exists_x I(x)$

- ~~every~~ A student has an internet connection.

  $\exists x\,(S(x) \land \cancel{S} I(x))$

- Everyone has an internet connection
  $\forall x I(x)$

- Everystudent has an internet connection.
  $\forall x \ (S(x) \rightarrow I(x))$

- Someone have shatted with Bob
  $\exists x \ C(x, Bob)$

- A student have chatted with Bob
  $\exists x \ (S(x) \wedge C(x, Bob))$

- Every studend hause chatted ~~with~~ by Bob
  $\forall x \ (S(x) \rightarrow C(x, Bob))$

- A student has not chatted with Bob
  $\exists x \ (S(x) \wedge \neg C(x, Bob))$

- No student has chatted with Bob.
  $\forall x \ (S(x) \rightarrow \neg C(x, Bob))$
  $\neg E x \ (S(x) \wedge C(x, Bob))$

- A student has chatted with everyone.

  $\exists x \forall y (S(x) \land C(x,y))$

- Every student has chatted with someone.

  $\forall x \exists y (S(x) \longrightarrow C(x,y))$

- Someone has chatted with everyone

  $\exists x \forall y \cancel{S} C(x,y)$

* Rules of Inference for Quantified statements.

$$\frac{\forall x\, P(x)}{P(c)} \longrightarrow \text{universal instantiation} \longrightarrow$$

بقدر اعوض اي قيمة
بدل ال x

$$\frac{\exists x\, \& P(x)}{P(c) \text{ for some element } c} \longrightarrow \text{Existential instantiation.}$$

→ اقيمة معينة

$$\frac{P(c) \text{ for an arbitary } c}{\forall x\, P(x)}$$

$$\frac{P(c) \text{ for some element } c}{\exists x\, P(x)}$$

$D(x)$

- "everyone in this discrete mathmatics class has taken a Course in computer science" and "Marla is a student in this class" imply the conclusion "Marla has taken a Course in Computer science."

$$\frac{\begin{array}{c}\forall x\,(D(x) \longrightarrow C(x)) \\ C(\text{Marla})\end{array}}{C(\text{Marla})}$$

1) $$\frac{\forall x\,(D(x) \longrightarrow C(x))}{P(\text{Marla}) \longrightarrow C(\text{Marla})}$$

2) $$\frac{\begin{array}{c}D(\text{Marla}) \longrightarrow C(\text{Marla}) \\ D(\text{Marla})\end{array}}{C(\text{Marla})}$$

Subject Date No.

$S(x)$

- Show that the premises "A student in this class has not
$R(x)$ read the book," and "Everyone in this class passed the $P(x)$
first exam" imply the conclusion "Someone who passed the
first exam has not read the book"

$$\exists_x (S(x) \land \neg R(x))$$
$$\underline{\forall x (S(x) \longrightarrow P(x))}$$
$$\exists_x (P(x) \land \neg R(x))$$

*عشان اوصل للنتيجة
بنبلش ب $\exists x$

$$1)\ \underline{\exists_x (S(x) \land \neg R(x))}$$
$$S(a) \land \neg R(a)$$

$$2)\ \underline{\forall x (S(x) \longrightarrow P(x))}$$
$$\cancel{\forall x}(S(a) \longrightarrow \cancel{\neg} P(a))$$

$$3)\ \underline{S(a) \land \neg R(a)}$$
$$S(a)$$
$$\neg R(a)$$

$$4)\ S(a) \rightarrow P(a)$$
$$\underline{S(a)}$$
$$P(a)$$

$$5)\ \underline{P(a) \land \neg R(a)}$$
$$\exists_x (P(x) \land \neg R(x))$$ → لانها من
الاســـ اثبت
من هون.

Scanned with CamScanner

- Conclude $\exists x(\neg P(x))$ from :-
  - $\forall x(P(x) \rightarrow Q(x))$
  - $\exists x(\neg Q(x))$

1) $$\frac{\exists x(\neg Q(x))}{\neg Q(a)}$$

2) $$\frac{\forall x(P(x) \rightarrow Q(x))}{P(a) \rightarrow Q(a)}$$

3) $$\begin{array}{c} P(a) \rightarrow Q(a) \\ \neg Q(a) \\ \hline \neg P(a) \end{array}$$

4) $$\frac{\neg P(a)}{\exists x(\neg P(x))}$$

- Conclude $\exists_x(\neg R(x))$ from
  - $\forall_x(P(x) \vee Q(x))$
  - $\forall_x(\neg Q(x) \vee S(x))$
  - $\exists_x(\neg P(x))$
  - $\forall_x(R(x) \longrightarrow \neg S(x))$

1) $\dfrac{\exists x(\neg P(x))}{\neg P(a)}$

2) $\dfrac{\forall x(P(x) \vee Q(x)}{P(a) \vee Q(a)}$

3) $\dfrac{\forall x(\neg Q(x) \vee S(x))}{\neg Q(a) \vee S(a)}$

4) $\dfrac{\forall x(R(x) \longrightarrow \neg S(x))}{R(a) \longrightarrow \neg S(a)}$

5) $\dfrac{\neg P(a) \quad P(a) \vee Q(a)}{Q(a)}$

6) $\dfrac{\neg Q(a) \vee S(a) \quad Q(a)}{S(a)}$

7) $\dfrac{R(a) \longrightarrow \neg S(a) \quad S(a)}{\neg R(a)}$

8) $\dfrac{\neg R(a)}{\exists(x)(\neg R(x))}$

- Conclude siblings (Ahmad, Ali) from

- $\forall x \forall y \forall z (father(x,y) \land father(x,z) \rightarrow siblings(y,z))$
- $father(Sami, Ahmad)$
- $father(Sami, Ali)$

$$\underline{\forall x \forall y \forall z (f(x,y) \land f(x,z) \rightarrow s(y,z))}$$
$$f(S,h)\ A\ F(S,L) \rightarrow F(h,L)$$

1) $F(S,h)$
$$\underline{f(S,A)}$$
$$f(S,h) \land f(S,L)$$

2) $f(S,h) \land f(S,L) \rightarrow \cancel{B} S(h,L$
$$\underline{f(S,h)\ A\ f(S,L)}$$
$$S(A,L)$$

True لكل قيم x

Ms

Lecture 5

Domain $\mathbb{Z}$

- $\forall_x (x > 0 \lor x < 2) \rightarrow T$

  $x = 1 \quad T \lor T \rightarrow T$

  $x = 5 \quad T \lor F \rightarrow T$

  $x = 0 \quad F \lor T \rightarrow T$

- $\forall_x (x > 0 \lor x < 0) \rightarrow F$

  $x = 0 \quad F \lor F \rightarrow F$

- $\forall_x (x > 0 \rightarrow x^2 > 0) \rightarrow T$

  $x = 1 \quad T \rightarrow T$

- $\forall_x (x^2 > 0 \rightarrow x > 0) \rightarrow F$

  $x = -2 \quad T \rightarrow F$

يكفي الاقي قيمة واحدة

- $\exists_x (x > 0 \lor x < 0) \rightarrow T$

- $\exists_x (x > 0 \land x < 0) \rightarrow F$

- $\exists_x (x > 0 \land x < 2) \rightarrow T$

* بالعادة $\exists_x$ ما بيجي implies مع

- $\exists_x (x > 0 \rightarrow x < 2) \rightarrow T$

- $\exists_x (x>0 \lor x<0)$

- $\exists_x \forall_x (x>0 \lor x<0)^{\times} \rightarrow$ we can't put one variable with two different quantifiers

- $\exists_x (x>0) \lor \forall_x (x<0)$ ✓

- $\exists_x (x>0 \land x<0) \rightarrow F$
- $\exists_x (x>0) \land \exists_x (x<0) \rightarrow T$
- $\exists_x (x>0) \land (x<0)$ ↘ free variable "we can't determine if it's ture or false"

- Domain integers

- $\forall_x \forall_y (x+y=y+x) \rightarrow T$
- $\forall_x \forall_y (x+y>0) \rightarrow F$
- $\forall_x \forall_y \forall_z ((x+y)+z=x+(y+z)) \rightarrow T$

- $\exists_x \exists_y (x+y>0) \rightarrow T$

- $\forall_x \exists_y (xy>0) \rightarrow F$

$x=-1$ $\exists_y (-y>0) \rightarrow T$

$x=0$ $\exists_y (0 \times y>0) \rightarrow F$

- $\forall x \exists y (x \neq 0 \rightarrow xy > 0) \rightarrow T$

  $x = 0 \quad \overset{\downarrow}{F} \longrightarrow \overset{\downarrow}{F} \longrightarrow T$

- $\exists x \forall y (xy > 0) \longrightarrow F$

  $x = 1 \quad \forall y (y > 0) \rightarrow F$

  $x = 0 \quad \forall y (0 > 0) \longrightarrow F$

  $x = 1 \quad \forall y (-1 \times y > 0) \longrightarrow F$

- $\exists x \forall y (xy = 0) \longrightarrow T$

  $x = 0 \quad \forall y (0y = 0) \longrightarrow T$

from the book:-

let $Q(x,y)$ be the statement "$x+y = x-y$". If the domain for both variables consists of all integers, what are the truth values.

a) $Q(1,1) \rightarrow F$ b) $Q(2,0) \longrightarrow T$
c) $\forall y\, Q(1,y) \rightarrow F$ d) $\exists x\, Q(x,2) \longrightarrow F$
e) $\exists x \exists y\, Q(x,y) \rightarrow T$ f) $\forall x \exists y (x+y = x-y) \rightarrow T$
g) $\exists y \forall x\, Q(x,y) \rightarrow T$ h) $\forall y \exists x\, Q(x,y) \rightarrow F$
i) $\forall x \forall y\, Q(x,y) \longrightarrow F$

- Determine the truth value of each of these statements, if the domain of each variable consists of all real numbers.

a) $\forall x \exists y (x^2 = y) \rightarrow T$ b) $\forall x \exists y (x = y^2) \rightarrow F$

c) $\exists x \forall y (xy = 0) \rightarrow T$ d) $\exists x \exists y (x + y \neq y + x) \rightarrow F$

e) $\forall x (x \neq 0 \rightarrow \exists y (xy = 1)) \rightarrow T$

لـ نحدد ال Domain $\forall x (x \neq 0) (Ey (xy = 1))$

f) $\exists x \forall y (y \neq 0 \rightarrow xy = 1) \rightarrow F$

g) $\forall x \exists y (x + y = 1) \rightarrow T$ h) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5) \rightarrow F$

i) $\forall x \exists y (x + y = 2 \wedge 2x - y = 1) \rightarrow F$

j) $\forall x \forall y \exists z (z = (x + y)/2) \rightarrow T$

الثبات

$$\neg \forall_x (P(x) \rightarrow \neg Q(x)) \equiv \exists_x (P(x) \land Q(x)) \quad [M_5]$$

$$\neg \forall_x (P(x) \rightarrow \neg Q(x)) \equiv \exists_x \neg (P(x) \rightarrow \neg Q(x))$$

$$\exists_x \neg (\neg P(x) \lor \neg Q(x)) \equiv \exists x (P(x) \land Q(x))$$

- Let S(x) be the predicate "x is a student," F(x) the predicate "x is a faculty member," And A(x,y) the predicate "x has asked y a Question," where the domain consists of all people associated with your school. Use Quantifiels to express each of these statement.

a) Lois has asked Professor Michael a Question.

A (Lois, Prof. Michael)

b) Every student has asked Professor Gross a Question.

$\forall x (S(x) \rightarrow A(x, Gross))$

c) Every factuly member has either asked professor Miller a Question or been asked a Question by proffessor Miller

$\forall x (f(x) \rightarrow A(x, miller) \lor A(Miller x))$

d) some student has not asked any faculty member a Question.

$\exists x (S(x) \land \forall y (f(y) \rightarrow \neg A(x,y))$

e) There is a faculty member who has never been asked a question by a student.

$$\exists x(F(x) \land \forall y(S(y) \rightarrow \neg A(y,x))$$

f) some student has asked every faculty member a question

$$\exists x(S(x) \land \forall y(S(y) \rightarrow A(x,y))$$

g) There is a faculty member who has asked every other faculty member a question

$$\exists x(F(x) \land \forall y(F(y) \land x \neq y \rightarrow A(x,y))$$

h) some student has never been asked a question by a faculty member.

$$\exists x(S(x) \land \forall y(F(y) \rightarrow \neg A(y,x))$$

i) There is a student who has asked a faculty member a question.

$$\exists x(S(x) \land \exists y(F(y) \land A(x,y))$$

$$\exists x\ \exists y(S(x) \land F(y) \land A(x,y))$$

j) Every student has asked a faculty member a question

$$\forall x\ \exists y(S(x) \rightarrow F(y) \land A(x,y))$$

k) Everyone has asked a faculty member a question

$$\forall x\ \exists y(F(y) \land A(x,y))$$

- Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domain of all

1) $\underline{\forall x (P(x) \vee Q(x))}$

$P(a) \vee Q(a)$

2) $\underline{\forall x (\neg P(x) \wedge Q(x) \rightarrow R(x))}$

$\neg P(a) \wedge Q(a) \rightarrow R(a)$

$= P(a) \vee \neg Q(a) \vee R(a)$

$\neg Q(a) \vee (P(a) \vee R(a)$

3) $P(a) \vee Q(a)$

$\underline{\neg Q(a) \vee (P(a) \vee R(a))}$

$P(a) \vee P(a) \vee R(a)$

$= P(a) \vee R(a)$

$= \neg R(a) \rightarrow P(a)$

$\underline{\neg R(a) \rightarrow P(a)}$

$\forall x (\neg R(x) \rightarrow P(x))$

* Direct proof

- give a direct proof of the theorem "If n is an odd integer, then n^2 is odd"

- Use direct proof to show that "the product of two rational numbers is rational." →

Definitions

- the integer n is even if there exists an integer k such that $n = 2k$ → عدد زوجي

- the integer n is odd if there exists an integer k such that $n = 2k+1$

assumption

n is odd

$n = 2k+1$ where k: integer number.

Proof :- prove that n^2 is odd

$$n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$$

عدد صحيح

n^2 is odd

* The real number is rational if there exist integers P and q with q ≠ 0 such that r = P/q

* A real ~~un~~ number that is not rational is called irrational.

- if n, m are rational numbers then nm is rational number.

assumption :- n is rational number

$= \frac{P}{q} \quad q \neq 0$

m is rational number

$= \frac{r}{s} \quad s \neq 0$

Proof :- $n \times m = \frac{P}{q} \times \frac{r}{s} = \frac{Pr}{qs} \quad qs \neq 0$

rational number.

- Proof by contrapositive "we can't prove using direct proof"

تكافؤ

$$p \to q \equiv ~~p~~ \neg q \to \neg p$$

- Prove that if n is integer and 3n+2 is odd, then n is odd

$$\underset{p}{3n+2 \text{ is odd}} \to \underset{q}{n \text{ is odd}}$$

$$p \to q \equiv \neg q \to \neg p$$

$$\underbrace{n \text{ is even}}_{p} \to \underbrace{3n+2 \text{ is even}}_{q}$$

~~assump~~ assumption:- n is even = 2k

proof:- $3n+2 = 3(2k)+2$

$6k+2 = 2(3k+1)$

عدد زوجي

- Show that if $x+y \geq 2$, where x and y are real numbers then $x \geq 1$ or $y \geq 1$

$$x+y \geq 2 \to x \geq 1 \vee y \geq 1$$

$$x<1 \wedge y<1 \to x+y<2$$

assumptions:- $\overset{1}{x<1}$, $\overset{2}{y<1}$

Proof:- $1+2 \to x+y<2$ ✓

- Proof by contradiction

* اول خطوة نفترض انه الجملة اللي بدي اثبتها خاطئة

* بدي اول معلومة تكون مناقضة لمعلومة معي.

- Example:- Show that at least ten of any 64 days chosen must fall on the same day of the week.

P

It's not a case that at least ten of any - - - - - - -

- at most nine days must fall on the same day of the week

$9 \times 7 = 63 \longrightarrow$ Contradiction!

then P cannot be false

- Prove that if n is a perfect square, then $n+2$ is not a perfect square.

P

q

assum. $\rightarrow P$

$\neg q$

n is a perfect square

$n = x^2$ where n:- integer

$n+2$ is a perfect square

$n+2 = y^2$ y:- integer

Proof:- $x^2 + 2 = y^2$

$2 = y^2 - x^2$

$(y+x)(y-x)$

$2 \quad 1$

$y + x = 2$

$y - x = 1$

$2y = 3$

$y = \frac{3}{}$

$\boxed{\text{isn't integer}}$

$P \rightarrow q$ is not false

- Proof by equivalence

- to Prove theorem that is a biconditional statement, that is, a statement of the form $p \leftrightarrow q$, we show that $p \to q$ and $q \to p$

For example, Prove the theorem "n is odd if and only if n^2 is odd." for positive numbers.

1) n is odd $\to$ n^2 is odd *if and only if

" الاثنين "

- assumption $\to$ n is odd

direct proof ← $n = 2k+1$ k:- is integer

- proof $n^2 = (2k+1)^2 = 4k^2 + 4k + 1$

$2(2k^2 + 2k) + 1$

integer ↲

$2L+1 \to$ odd

2) $ n^2 is odd $\to$ n is odd

contrapositive n is even $\to$ n^2 is even

assumption $\to$ n is even $\to$ $n = 2k$

Proof :- $n^2 = (2k)^2 = 4k^2 = 2(2k^2) = 2L$ #

طرق سـريعة للإثباتات نستخدمها في حالات معينة فقط.

- Vacuous Proof

↳ used to quickly prove that a Conditional statement $p \to q$ is true when we know that p is false.

p	q	$p \to q$
T	T	T
T	F	F
f	T	T
f	f	T

Example:- show that the proposition (P(0)) is true, where P(n) is "If $n > 1$, then $n^2 > n$" and the domain consists of all integers.

P(0)

$p \to q$

$n > 1 \to n^2 > n$

$\underline{\underline{0 > 1}} \to 0^2 > 0$ } → T الجملة الشرطية

↓

F

F لأنه الشرط

- Trivial proof

- used to quickly prove that a conditional statement $p \rightarrow q$ is true when we know that q is true

- let $p(n)$ be "If a and b are positive integers with $a \geq b$, then $a^n \geq b$," where the domain consists of all nonnegative integers. Show that $P(0)$ is true.

P	q	$P \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$a \geq b \longrightarrow a^n \geq b^n$

$P(0)$

$a^0 \geq b^0$

$1 \geq 1 \searrow$ T $\Big\}$ T الجملة الشرطية

T لأن جواب الشرط

-proof by canterexample

-used to prove that show that a statement of the form $\forall_x P(x)$ is false.

Ex - Show that a statement "Every positive integer is the sum of the squares of two integers" is false

$X=3$

$y=0$ or $y=1$

$0^2 = 0$ $\qquad 0+0 \rightarrow X$

$1^2 = 1$ $\qquad 0+1 \rightarrow X$

$1+1 \rightarrow X$

$1+0 \rightarrow X$

* اذاً العدد ٣ لا ينطبق عليه هذا الشرط.

* إذا العبارة ـــ false.

- Prove that if n is a positive integer then n is even if and only if 7n+4 is even $\boxed{Ms}$

n is even ⟷ 7n + 4 is even

$p \leftrightarrow q = p \overset{①}{\rightarrow} q \wedge q \overset{②}{\rightarrow} p$

* direct proof

1) n is even → 7n+4 is even.

* يفترض ان المعطى صحيح و نحاول اوصل للنتيجة.

assumption :- n is even

$n = 2k$ where k → integer

Proof :- 7n+4 is even

$7(2k) + 4$

$14 + 4 = 2(7k+2)$ → where k integer.

∴ even $2L$

---

2) $q \rightarrow p$ if 7n+4 is even then n is even

* $p \rightarrow q \equiv \neg q \rightarrow \neg p$

if n is odd then 7n+4 is odd.   * Contrapositive

assuption → n is odd → $n = 2k+1$

Proof → 7n + 4 is odd

$7(2k+1) + 4 = 14k + 7 + 4 = 14k + 11 = 14k + 10 +$

$2(7k+5) + 1$

- if n is even then $7n+4$ is even "using contradiction"

* دايمًا بالـ contradiction بنفترض انه المعطى خطأ وبدي اوصل لنتيجة معاكسة

assumption : n is even

$7n+4$ is even

Proof → $7n+4 = 7(2k)+4$

$14k+4$

$2(7k+2) = 2L$. " even "

" contradiction هذا المطلوب "

---

* $p \rightarrow q$

F → T :- T

F → F :- T

T → T :- T

F → T :- T

Ex prove that the proposition $P(0)$, where $P(n)$ is the proposition "if n is a positive integer greater than 1, then $n^2 > n$".

$n > 1 \longrightarrow n^2 > n$

$0 > 1 \longrightarrow 0 > 0$

F ⟶ ⟶ } T

* إذا الشرط F إذا الجملة صحيحة.

* ~~[illegible]~~ proof.

Nacuous

" Prove the proposition $P(1)$, where $P(n)$ is the proposition "if n is a positive integer, then $n^2 \geq n$."

$n > 0 \longrightarrow n^2 \geq n$

$1 > 0 \longrightarrow 1 \geq 1 \Big\} T$

$\longrightarrow T$

* إذا جواب شرط T

إذاً الجملة صحيحة .

* Trivial proof.

- if n is even then $7n+5$ is even

$n = 2$

2 is even $\rightarrow 14 + 5 = 19$

odd

$\therefore F$

* Counterexample → used to proof that the statement is false

How? by finding one value " F "

"لا تحقق الشرط."

Sets.

- A set is an unordered collection of objects "الترتيب مش مهم"
- The objects in a set are called the elements, or members, of the set. A set is said to contain its elements.

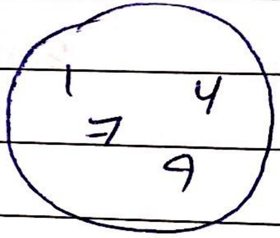
- $a \in S$:- a is an element of the set S.

  " a تنتمي للمجموعة S"

- $a \notin S$:- a is not an element of the set S.

  " a لا تنتمي للمجموعة S"

Example :- $S = \{1, 4, 7, 9\}$



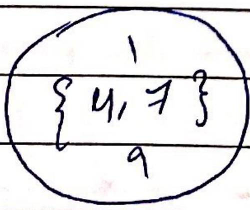
- $1 \in S$, $4 \in S$

$7 \in S$, $9 \in S$

$2 \notin S$, $5 \notin S$

$10 \notin S$

$S = \{1, \{4, 7\}, 9\}$ "ثلاث عناصر"



$1 \in S$, $\{4, 7\} \in S$

$9 \in S$

$4 \notin S$, $\{4\} \notin S$

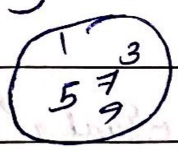
$\{1\} \notin S$

note :- $4 \neq \{4\}$.

- N :- set of natural numbers "من صفر إلى مالا نهاية"
- Z :- set of integers "الأعداد السالبة والموجبة"
- Z^+ :- set of positive integers "الأعداد الموجبة" " بدون الصفر
- Q :- set of rational numbers " بسط ومقام"
- R :- set of real numbers " الجذور / Q / الأعداد الموجبة"
- Z^- :- set of negative integers " الأعداد السالبة"

* we characterice all those elements in the set by stating the property or properties they must have to be members

* Example:- the set of positive $\overset{\text{odd}}{\uparrow}$integers less than 10 is $\{1, 3, 5, 7, 9\}$



- $\{\underset{\text{domain}}{x \in Z^+} | \underset{x \bmod 2 = 1}{x \text{ is odd}} \land x < 10\} \longrightarrow$ خصائص المجموعة

Example :- $\{1, 4, 7, 10, 13\} \rightarrow$ set A

$$A = \{3x+1 \mid x \geq 0 \land x \leq 4 \land x \in Z^+\}$$

or

$$\{3x+1 \mid x \leq 4 \land x \in N\}$$

- $\{1, 4, 9, 16, 25\} \rightarrow$ set B.

$$B = \{x^2 \mid x \geq 1 \land x \leq 5 \land x \in Z^+\}$$

or $x \in N$, $x \in Z$.

المجموعة الأولى

- $\{\underbrace{1, 3, 5, 7, 9}, \underbrace{10, 12, 14, 16, 8},\} \rightarrow$ set C

$$C = \{x \mid (x \geq 1 \land x \leq 9 \land x \text{ is odd}) \cancel{\}}$$
$$\lor (x \geq 10 \land x \leq 18 \land x \text{ is even})\}$$

---

* Two sets are equal if they have the same elements

$$A = B = \forall x (x \in A \longleftrightarrow x \in B)$$

- $\{1, 2, 3\} = \{1, 3, 2\}$ الترتيب غير مهم.
- $\{1, 2, 3\} = \{1, 2, 2, 3, 3\}$ التكرار غير مهم.

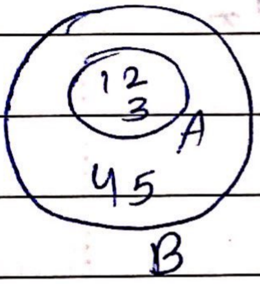
- Singleton set :- Contains only one element.

- Empty set :- $\emptyset$, { } "doesn't have any element."

- Subset : $A \subseteq B$, $\forall x (x \in A \rightarrow x \in B)$

* كل العناصر التي تنتمي إلى المجموعة A تنتمي إلى B .

Example :- - $\{1,2,3\} \subseteq \{1,2,3,4,5\}$



- $\{1,2,3\} \subseteq \{1,2,3\}$

* كل مجموعة هي subset من نفسها

- $\emptyset \subseteq \{1,2,3\}$

* كل empty set عبارة عن subset من اي مجموعة .

Example :- $S = \{1, \{4,7\}, 9\}$

- $\{1, \{4,7\}\} \subseteq S$
- $\{1,9\} \subseteq S$
- $\{\{4,7\}\} \subseteq S$
- $\{1,4,7\} \not\subseteq S$
- $\{\{1\}, \{4,7\}\} \not\subseteq S$
- $\{4,7\} \not\subseteq S$

* Proper Subset : $A \subset B$: $A \subseteq B \land A \neq B$

"المساواة غير مسموحة"

- $\{1,2\} \subsetneq \{1,2,3\}$
- $\{1,2,3\} \not\subset \{1,2,3\}$

* Cardinality (number of element) $|S|$

* التكرار لا يُحسب

- $S = \{1,2,3\}$, $|S| = 3$
- $S = \{1,2,3,3\}$, $|S| = 3$
- $S = \{1,2,3,4,1,3\}$, $|S| = 4$.

- $\{x \in Z^{+} \mid x \text{ is odd} \land x < 10\}$, $|S| = 5$
- $\{x \in Z^{-} \mid x \text{ is odd} \land x > 10\}$, $|S| = 0$
- $\{x \mid x \text{ is an English Alphabet}\}$, $|S| = 26$
- Infinite set : a set that is not finite

  ex:- positive integers.

* Powerset: $P(S)$: is the set of all subsets of the set of S.

* جميع المجموعات الجزئية التي يمكن تكوينها من مجموعة معينة.

$P(S) = \{X \mid X \subset S\}$, $P(S) = 2^{|S|}$

Example:- ① what is the powerset of $\{1,2,3\}$

$S = \{1,2,3\}$

$P(S) = \{ \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}$
$\{1,2,3\}, \{\emptyset\} \}$

$P(S) = 2^{|S|} = 8$

② $S = \{1\}$

$P(S) = 2^1 = 2$

③ $S = \{\emptyset\}$

$P(S) = \cancel{5}\ 2^1 = 2$

④ $S = \emptyset$

$P(S) = 2^0 = 1$

* Note :- $A = \emptyset$ $B = \{\emptyset\}$

$|A| = 0$ $|B| = 1$

$\emptyset \notin A$ $\emptyset \in B$

$\emptyset \subseteq A$ $\emptyset \subseteq B$

$\emptyset \not\subset A$ $\emptyset \subset B$

- Cartesian product

Cartesian product of A and B : $A \times B$, is the set of all ordered pairs (a,b), where $a \in A$ and $b \in B$

$$A \times B = \{(a,b) \mid a \in A \land b \in B\}$$

- Note that $(a,b) \neq (b,a)$ الترتيب مهم داخل الازدواج.

$A = \{1, 2, 3\}$, $B = \{a, b\}$, $C = \{x\}$

find $A \times B$, $B \times A$, $C \times A$, $A \times B \times C$

$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

الترتيب بداخل المجموعة غير مهم

$A \times B \times C = \{(1,a,x), (1,b,x), (2,a,x), (2,b,x), (3,a,x), (3,b,x)\}$

- $|A \times B| = |A| \times |B|$

- Set operators

(Union and Intersection)

- $A \cup B = \{x \mid x \in A \lor x \in B\}$

- $A \cap B = \{x \mid x \in A \land x \in B\}$

- (Difference and Complement)

- $\bar{A} = \{x \mid x \notin A\}$
- $A - B = \{x \mid x \in A \land x \notin B\} = A \cap \bar{B}$

Example:- $A = \{1, 3, 5, 7, 9\}$

$B = \{6, 7, 8, 9, 10\}$

universal set $\leftarrow U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

- find 1) $A \cup B \rightarrow$ ~~[illegible]~~ $\{1, 3, 5, 7, 9, 6, 8, 10\}$

2) $A \cap B \rightarrow \{7, 9\}$

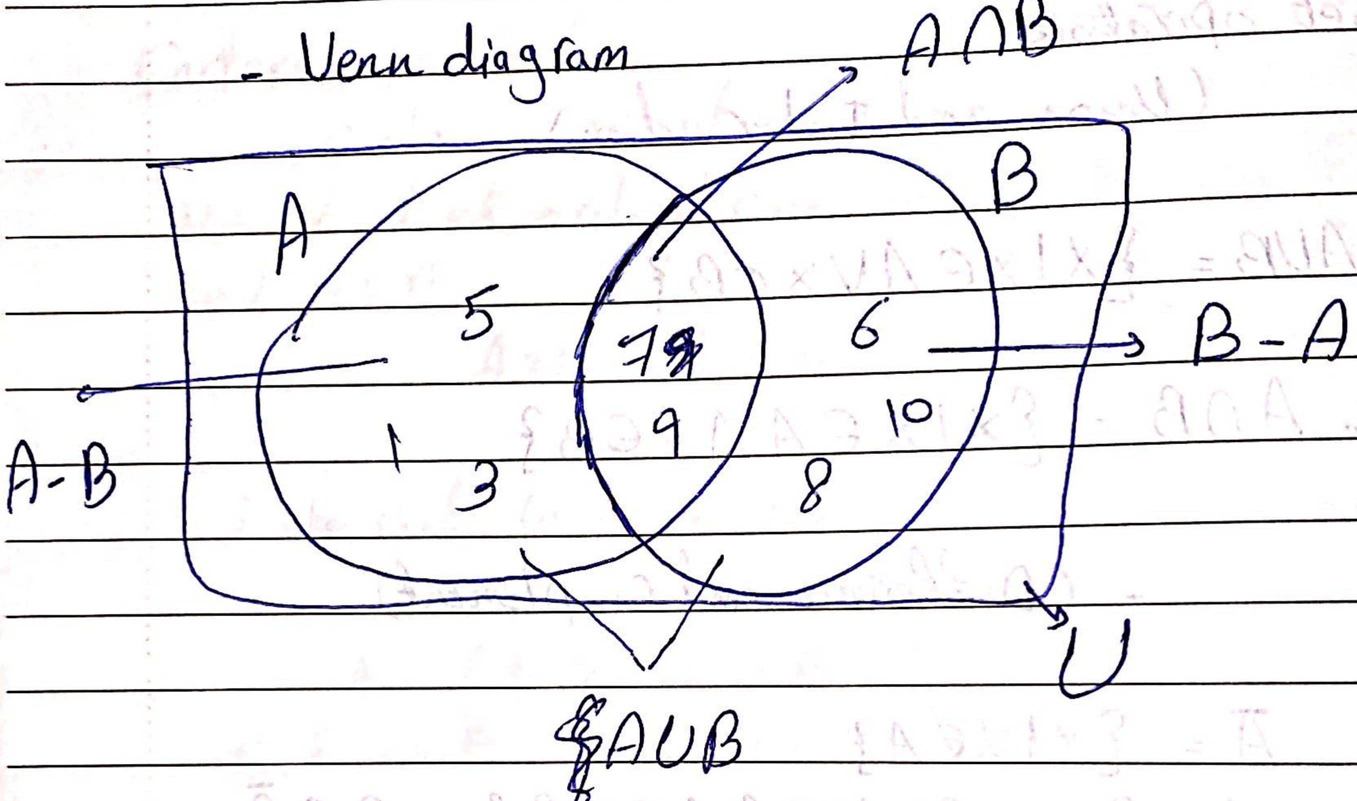
3) $A - B \rightarrow \{1, 3, 5\}$

4) $B - A \rightarrow \{6, 8, 10\}$

$U - A \leftarrow$

5) $\bar{A} \rightarrow \{2, 4, 6, 8, 10\}$

- Venn diagram



* Two sets are called disjoint if their intersection is the empty set

- $|A \cup B| = |A| + |B| - |A \cap B|$

- let $|A \cup B| = 10$, $|A| = 7$, $|B| = 6$ find $|A \cap B|$

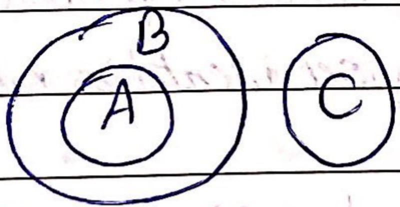
$10 = 7 + 6 - |A \cap B|$

$10 = 13 - |A \cap B|$

$|A \cap B| = 3$

* let $A \subseteq B$, B and C are disjoint, $|A| = 5$, $|B| = 8$

$|C| = 4$ find:-



1) $|A \cup B| \rightarrow 8$

2) $|A \cap B| \rightarrow 5$

3) $|A - B| \rightarrow 0$

4) $|B - A| \rightarrow 3$

5) $|B - C| \rightarrow 8$

6) $|A \cup C| \rightarrow 9$

* set Identities "two sets are equal"

1) using set builder notation.
2) using membership table
3) using set identities rules.

and $(\wedge) \equiv \cap$ "intersection"
or $(\vee) \equiv \cup$ "union"
negation $(\neg) \equiv -$ "complement"
$T \equiv U$ "universal set"
$F \equiv \emptyset$ "empty set"

---

ex:- using set builder notation and membership table prove $\overline{(A \cap B)} = (\bar{A} \cup \bar{B})$

- ~~At~~ $\overline{(A \cap B)}$ $\{x \mid x \notin A \cap B\}$

$= \{x \mid \neg(x \in A \cap B)\}$

$= \{x \mid \neg(x \in A \wedge x \in B)\}$

$= \{x (\neg x \in A \vee \neg x \in B)\}$ → Demorgan's law.

$= \{x \mid x \notin A \cup x \notin B\} = \{x \mid x \in \bar{A} \vee x \in \bar{B}\}$

$= \{x \mid x \in \bar{A} \cup \bar{B}\}$

* membership table

②

A	B	$A \cap B$	$\overline{A \cup B}$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	1

1 → العنصر ينتمي

0 → العنصر لا ينتمي

$\bar{A}$	$\bar{B}$	$\bar{A} \cup \bar{B}$
0	0	0
0	1	1
1	0	1
1	1	1

نفس العناصر :: المجموعتين
متساويتين.

(Laws) :- $A \cap U = A$ $A \cup U = U$

$A \cup \emptyset = A$ $A \cap \emptyset = \emptyset$

Identity laws Demonation laws

$A \cup A = A$ $\overline{(\bar{A})} = (A)$

$A \cap A = A$ Complementation law

Idempotent laws

$A \cup B \equiv B \cup A$ $A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap B \equiv B \cap A$ $A \cap (B \cap C) \equiv (A \cap B) \cap C$

Commutative laws Assosiative laws

Subject Date

$A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Distributive laws. $\longrightarrow$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

Demorgn's law. $\longrightarrow$

$A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

Apsorption law $\longrightarrow$

$A \cup \bar{A} = U$

$A \cap \bar{A} = \emptyset$

Complement laws $\longrightarrow$

Subject Date No.

- Prove that $(A \cap U) \cup (A \cap B) = A$

$= A \cup (A \cap B) = A$

- simplify $(\bar{A} \cup \bar{B}) \cup (A \cap B)$

$= (\bar{A} \cup \bar{B} \cup A) \cap (\bar{A} \cup \bar{B} \cup B) =$

$(U \cup \bar{B}) \cap (U \cup \bar{A})$

$U \cap U = U$

- simplify the set:-

$(A - C) \cap (C - B)$

$= (A \cap \bar{C}) \cap (C \cap \bar{B})$

$(C \cap \bar{C}) \cap (A \cap \bar{B})$

$= \emptyset \cap (A \cap \bar{B})$

$= \emptyset$

sequences.

* a sequence is a function from a subset of the set of integers (usually either the set $\{0, 1, 2 \cdots\}$ or the set $\{1, 2, 3, \dots\}$) to a set S.

* we use the notation a_n to denote the image of the integer n. where n is the index.

* we call a_n a term of the sequence.

Example:-

* let $\{a_n\} = 2n + 1$

$a_0 = 1$

$a_1 = 3$

$a_5 = 11$

* the ~~elements~~ the elements are $\{1, 3, 5, 7, 9, 11 \cdots\}$

---

1) Arithmetic Progression

It is a sequence of the form $a, a+d, a+2d, \cdots a+nd$. where the initial term a and the common difference d are real numbers.

* الفرق بين كل حدين عبارة عن مقدار ثابت.

"$a + nd$" → الصيغة العامة

Examples:-

$\underset{\text{initial term}}{S_n = -1} + 4\underset{d}{n}$, $t_n = 7 - 3n \rightarrow d = -3$

* find the formula for the sequences.

* 4, 7, 10, 13, 16, 19

$d = 7 - 4$ or $10 - 7$ . . . . .

$\boxed{d = 3}$

initial term $= a_0 = 4$

formula $\longrightarrow a_0 + nd$

$4 + 3n$

* $a_5 = 3, \quad a_{11} = 15$

$$d = \cancel{\frac{15-3}{6}} = \frac{a_{11} - a_5}{11 - 5} = \boxed{2}$$

$a_n = a_0 + dn$

$a_5 = a_0 + 2(5)$

$a_5 = a_0 + 10$

$3 = a_0 + 10$

$a_0 = -7$

$\boxed{a_n = -7 + 2n}$

* $a_3 = 7, \quad d = 5$

$a_n = a_0 + dn$

$a_3 = a_0 + 5(3)$

$7 = a_0 + 15$

$a_0 = -8$

$a_n = -8 + 5n$

* A room has 15 chairs in the first row. The row number i has 2 chairs more than the row number i - 1. what is the total number of chairs at the 10th row.

$a_n = a_0 + dn$

$a_n = 13 + 2n$

$a(10) = 13 + 2(10)$

$= 33$ chairs.

$\boxed{d = 2 , a_1 = 15 \quad a_0 = 13}$

2) geometric progression.

It is a sequence of the form $a, ar, ar^2 \ldots ar^n$ where the initial term a, and the common ratio r are real numbers. $ar^n \longrightarrow$ الصيغة العامة

Example:- $b_n = (-1)^n$, $C_n = 2 * 5^n$

$r = -1 , a = 1$ $\qquad$ $a = 2 , r = 5$

* find the formula ....

* $1, 1/2, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

$a = 1 \quad r = \frac{1}{2} \div 1 = \frac{1}{2}$

$1 \times \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2$

Scanned with CamScanner

* $\frac{2}{9}, \frac{2}{3}, 2, 6, 18$

$a = \frac{2}{9}, \quad r = 3$

$\frac{2}{9}(3)^n$

---

* $a_5 = \frac{5}{4}, \quad a_{12} = 160$

$160 \div \frac{5}{4} = r^{7} \leftarrow \boxed{12-5}$

$128 = r^7$

$\boxed{r = 2}$

$a_n = a_0 r^n$

$a_5 = a_0 r^5$

$\frac{5}{4} = a_0 2^5$

$a_0 = \frac{5}{128}$

$a_n = \frac{5}{128} \times 2^n$

* A recurrence Relations

* A recurrence relation for the sequence $\{a_n\}$ is an equation that expresses a_n in terms of one or more of the previous terms of the sequence, namely, $a_0, a_1, \ldots a_{n-1}$, for all integers n with $n \geq n_0$, where n_0 is a nonnegative integer.

Examples :-

1) let $\{a_n\}$ be a sequence that satisfies the recurrence relation $\underline{a_n = a_{n-1} + 3}$ for $n = 1, 2, 3 \ldots$ and suppose that $\underline{a_0 = 2}$. what are a_1, a_2, a_3?

$a_1 = a_0 + 3 = 2 + 3 = 5$

$a_2 = a_1 + 3 = 8$

$a_3 = a_2 + 3 = 11$

2) $a_n = a_{n-1} - a_{n-2}$, $a_0 = 3$ $a_1 = 5$, $a_2 = ??$

$a_2 = a_1 - a_0 - \cancel{5}\ 5 - 3 = 2$

$a_3 = a_2 - a_1 = 2 - 5 = -3$

- The fibonacci sequence → كل عدد يعتمد على العددين قبله

The fibonacci sequence $f_0, f_1, f_2 \ldots$ is defined by the initial conditions $f_0 = 0$, $f_1 = 1$ and the recurrence relation $f_n = f_{n-1} + f_{n-2}$ for $n = 2, 3, 4 \ldots$

- find the fibonacci numbers f_2, f_3, f_4, f_5 and f_6.

$f_2 = 0 + 1 = 1$ $f_4 = 2 + 1 = 3$

$f_3 = 1 + 1 = 2$ $f_5 = 3 + 2 = 5$

$f_6 = 8$, $f_7 = 11$

- We say that we have solved the recurrence relation together with the initial conditions when we find an explicit formula, called a closed formula, for the ~~#~~ terms of the sequence.

$a_n = 2a_{n-1} - a_{n-2}$

- Determine whether the sequence $\{a_n\}$, where $a_n = 3n$ for every nonnegative integer n, is a solution of the

$n = 2, 3, 4, \ldots$

$a_n = 3n$

$a_n = 2a_{n-1} - a_{n-2}$

$= 2(3(n-1)) - 3(n-2)$

$6n - 6 - 3n + 6$

$6n - 3n = 3n$ ✓

Answer the same question where $a_n = 2^n$ and where $a_n = 5$

$a_n = 2a_{n-1} - a_{n-2}$

$a_n = 2^n$

$2(2^{n-1}) - (2^{n-2})$

$\neq 2^n - 2^{n-2}$ X

$a_n = 5$

$a_n = 2(a_{n-1}) - a_{n-2}$

$= 2(5) - 5 = 5$ ✓

Solve the recurrence relation $a_n = a_{n-1} + 3$ where $a_0 = 2$

$a_n = a_{n-1} + 3$

$a_0 = 2$

$a_1 = a_0 + 3 = 2 + 3 \times$ [illegible]

$a_2 = a_1 + 3$

$2 + 3 + 3 = 2 + 3 \times 2$

$a_3 = 2 + 3 \times 2 + 3$

$2 + 3 \times 3$

$a_4 = 2 + 3 \times 3 + 3 = 2 + 3 \times 4$

$\vdots$

$a_n = 2 + 3n$

Date No.

- special integer sequences.

- How can we produce the terms of a sequence if the first 10 terms are:-

- 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5

- 1, 2, 4, 7, 11, 16, 22, . . .

- 1, 2, 2, 3, 4, 4, 5, 6, 6, 7, 8, 8, 9, 10, 10

* Summation.

- $\sum_{j=m}^{n} aj = am + am+1 + \cdots an$

*j is called the index of summation.

*m is the lower limit and n is the upper limit.

Examples:-

what is the value of :-

1) $\sum_{j=1}^{5} j^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$= 1 + 4 + 9 + 16 + 25 = 55$

2) $\sum_{k=4}^{8} (-1)^k = (-1)^4 + (-1)^5 + (-1)^6 + (-1)^7 + (-1)^8$

$1 - 1 + 1 - 1 + 1 = 1$

3) $\sum_{i=1}^{4} \sum_{j=1}^{3} ij =$

nested summation.

$\sum_{j=1}^{3} ij = i + 2i + 3i = 6i$

$\sum_{i=1}^{4} 6i = 6 + 12 + 18 + 24 = 60.$

4) $\sum_{s=(0,2,4)^s} = 0 + 2 + 4 = 6$

Scanned with CamScanner

* useful laws :-

* Suppose we have the sum $\sum_{j=1}^{5} j^2$. we can shift the index to run between 0 and 4.

$$\sum_{j=1}^{5} j^2 = \sum_{j=0}^{4} (j+1)^2$$

* إذا غيرت بال lower
بغير بال upper
وبعوض العكس بال function

* $\sum_{j=1}^{n} a x_j = a \sum_{j=1}^{n} x_j$ where a is constant.

* $\sum_{j=1}^{n} (x_j + y_j) = \sum_{j=1}^{n} x_j + \sum_{j=1}^{n} y_j \rightarrow$ عملية توزيع

* The summation of constant.

* if a is constant then $\sum_{j=n}^{m} a = ((m-n)+1)\, a$

$$\sum_{j=3}^{6} 5 = ((6-3)+1)\, a$$

$$4 \times 5 = 20$$

Some useful summation formulae.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{j=1}^{10} j = \frac{10 \times 11}{2} = 55$$

$$\sum_{j=10}^{50} j = \sum_{j=1}^{50} j - \sum_{j=1}^{9} j =$$

$$\frac{50 \times 51}{2} - \frac{9 \times 10}{2} = 1275 - 45 = 1230$$

$$\sum_{j=10}^{50} j = \sum_{1}^{41} j + 9 = \sum_{j=1}^{41} j + \sum_{1}^{41} 9$$

Shift the index.

$$= \frac{41 \times 42}{2} + 41 \times 9 = 861 + 369 = 123$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{j=5}^{20} j^2 = \sum_{j=1}^{20} j^2 - \sum_{j=1}^{4} j^2$$

$$\frac{20 \times 21 \times 41}{6} - \frac{4 \times 5 \times 9}{6} = 2870 - 3$$

$$= 2840$$

$$\sum_{j=5}^{20} j^2 = \sum_{j=1}^{16} (j+4)^2 = \sum_{j=1}^{16} (j^2 + 8j + 16)$$

$\sum_{j=1}^{10} 2j + j^2$

$\sum_{j=1}^{10} 2j + \sum_{j=1}^{10} j^2$

$\frac{2\times10\times11}{2} + \frac{10\times11\times21}{6} = 110 + 385 = 495$

$\sum_{j=1}^{10} 2j + 7 =$

$\sum_{j=1}^{10} 2j + \sum_{j=1} 7 = \frac{2\times10\times11}{2} + 7\times10 = 110 + 70 = 180$

~~$\sum_{j=0}^{5} 3\times 2^j$~~

$\sum_{k=0}^{n} ar^k \ (r\neq0) = \frac{ar^{n+1} - a}{r-1}, \ r\neq1$

$\sum_{j=0}^{5} 3\times2^j = \frac{3\times2^6 - 3}{2-1} = 3\times64 - 3$

$= 192 - 3 = 189$

[illegible] $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

$\sum_{j=1}^{5} j^3 = \frac{25 + 36}{4} = 225$

Scanned with CamScanner

* $\sum_{k=0}^{\infty} x^{k}, |x|<1$

* $\sum_{k=1}^{\infty} k x^{k-1}, |x|<1$

* $\sum_{k=0}^{\infty} (\frac{1}{2})^{k} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

* $\sum_{k=1}^{\infty} k (\frac{2}{3})^{k-1} = \frac{1}{(1-\frac{2}{3})^{2}} = \frac{1}{(\frac{1}{3})^{2}} = 9$

---

* Matrices

- a matrix is a rectangular array of numbers. A matrix with "n rows" and "n columns" is called an mXn matri

- The plural of matrix is matrices

* type or Size = mXn

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$$

is a 3X2 matrix.

Subject Date No.

- A matrix with the same number of rows as Columns is called $\boxed{\text{square}}$

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = 2 \times 2$$

$$[1] = 1 \times 1$$

- two matrices are equal if they have the same number of rows and the same number of Columns and the Corresponding entiers in every position are equal

- $$\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \neq \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

- $$\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \neq \begin{bmatrix} 1 & 3 & 0 \\ 4 & 2 & 0 \end{bmatrix}$$

- $$\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

∘ Summation

* The two matrics should be the same size.

$$\begin{bmatrix} 1 & 5 \\ 0 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & 5 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 0 & 7 \\ 0 & 9 \end{bmatrix}$$

∘ نجمع كل رقم مع الرقم المقابل.

- let the size of A be := $m \times n$ and $B = n \times k$, let $C = AB$, the size of $C =$ ~~A~~ $m \times k$.

• بالضرب ← عدد أعمدة المصفوفة الأولى يساوي عدد صفوف المصفوفة الثانية

* Matrix multiplication is associative but not commutative.

- Identity matrix

"I_n" is a square matrix $n \times n$

- let A be a matrix of size $n \times m$, $AI_m = I_n A = A$

$$I_1 = \begin{bmatrix} 1 \end{bmatrix}$$

$$I_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

• القطر عبارة عن "1"
وباقي القيم = صفر

• إذا ضربت اي مصفوفة بالمصفوفة المحايدة فالناتج نفس المصفوفة.

- Power of matrices

- let A be nXn matrix, then $A^0 = I_n$, $A^r = AAAA \dots r$ times.

- let $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

find :-

$A^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$ $A_1 = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

$A^2_{\cancel{2}} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 0 & 9 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 8 \\ 0 & 9 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 26 \\ 0 & 27 \end{bmatrix}$

$A \times A \times A$

$= A^2 \times A$

- Transpose of matrix

- what is the transpose A^t of the following matrix

$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix} \longrightarrow A^t = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

3×2 نعكس 2×3

* A matrix is symmetric if $A = A^t$ → transpose.

* which of the following matrices is symmetric.

- $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, $A^t = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

not symmetric.

- $A = \begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix} \longrightarrow A^t = \begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}$

symmetric

- $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 5 & 2 \\ 4 & 2 & 6 \end{bmatrix} \longrightarrow A^t = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 5 & 2 \\ 4 & 2 & 6 \end{bmatrix}$

symmetric.

+ الأرقام حول القطر تكون متناظرة.

* Zero - One Matrix.

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 \end{bmatrix}$

- Boolean Operations on zero-one Matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \overset{\overset{\text{or, join}}{\uparrow}}{\vee} \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$\vee$ ← لازم يكونوا نفس ال size.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \overset{\text{meet}}{\wedge} \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$\wedge$ ← لازم يكونوا نفس الـ size.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$(1 \wedge 1) \vee (0 \wedge 0) = 1$

* صف و عمود.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

- let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$A^{0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$A^{t} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

الأس بين اقواس

$A^{[2]} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$A^{2} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

- find the first six terms of the sequence by each of these recurrence relations and initial conditions. $\boxed{M_s}$

- $a_n = -2a_{n-1}$, $a_0 = -1$

$a_1 = -2(-1) = 2$

$a_2 = -2(2) = -4$

$a_3 = -2(-4) = 8$

$a_4 = -2(8) = -16$

- $a_n = a_{n-1} - a_{n-2}$, $a_0 = 2$, $a_1 = -1$

$a_2 = -1-(-2) = -3$

$a_3 = -3-(-1) = -2$

$a_4 = -2-(-3) = 1$

Show that the sequences $[a_n]$ is a solution of the recurrence relation $\boxed{a_n = -3a_{n-1} + 4a_{n-2}}$ if

a) $a_n = 0$

$a_n = -3(0) + 4(0) = 0$ ✓

b) $a_n = 1 \longrightarrow$ كل قيمة داخل seq = 0

$a_n = -3(1) + 4(1) = -3+4 = 1$ ✓

c) $a_n = -4^n$

$a_n = -3(-4)^{n-1} + 4(-4)^{n-2}$

$= -3 \times -4(-4)^{n-2} + 4(-4)^{n-2}$

عامل مشترك ← $= (-4)^{n-2}[12 + 4]$

$= -4^{n-2} \times 16$

$= -4^{n-2} \times -4^2 = -4^n$ ✓

- Is the sequence $\{a_n\}$ solution of the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ if

a) $a_n = 0$

$$8(0) - 16(0) = 0 \checkmark$$

b) $a_n = 1$

$$8(1) - 16(1) = -8 \times$$

c) $a_n = 2^n$

$a_n = 8a_{n-1} - 16a_{n-2}$

$= 8(2)^{n-1} - 16(2)^{n-2}$

$= 8 \times 2(2)^{n-2} - 16(2)^{n-2}$

= [illegible]

$16(2)^{n-2} - 16(2)^{n-2} = 0$ X

- find the solution to each of these recurrence relations with the given conditions.

a) $a_n = -a_{n-1}$, $a_0 = 5$

$a_1 = -5$

$a_2 = 5$ → geometric sequence.

$a_3 = -5$ $r = -1$, $a_0 = 5$

$$\boxed{a(-1)^n}$$

b) $a_n = a_{n-1} + 3 \ , \ a_0 = 1$

$a_1 = 4$

$a_2 = 7$ Arithmatic sequence

$a_3 = 10$ $a_0 = 1 \ , \ d = 3$

$$\boxed{a_n = 1 + 3n}$$

---

nested summation $\leftarrow$ $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i + 3j)$ * نبدأ بال Summation الداخلي

$$\sum_{j=0}^{3} (2i + 3j) = (2i+0) + (2i+3) + (2i+6) + (2i+9)$$

$$\cancel{2i +} \ \boxed{8i+18}$$

$$\sum_{i=0}^{2} (8i+18) = (0+18) + (8+18) + (16+18)$$

$$18 + 26 + 34 = 78$$

$$\sum_{k=10}^{100} k = \sum_{k=1}^{100} k - \sum_{k=1}^{9} k$$

or $\rightarrow$ shift the index

$$\sum_{k=1}^{91} (k+9) = \sum_{k=1}^{91} k + \sum_{k=1}^{91} 9$$

* Integers

*If a and b are integers with $a \neq 0$, we say that a divides b if there is an integer c such that $b = ac$. when a divides b we say that a is a factor of b and that b is a multiple of a.

* the notation $a \mid b$ denotes that a divides b.
* the notation $a \nmid b$ denotes that a does not divide b.

* Example: $3 \nmid 7$, $5 \nmid 12$, $4 \nmid 14$

لو قسمنا 8 على 2 باقي القسمة يساوي صفر. ← $2 \mid 8$, $3 \mid 12$, $5 \mid 15$

* Division

- let a, b, and c be integers. Then
  * if $a \mid b$ and $a \mid c$, then $a \mid (b + c)$ $\rightarrow b + c = 6 + 9$

    ex. $3 \mid 6$ and $3 \mid 9$, then $3 \mid 15$

    ٣ من عوامل ٩

  * if $a \mid b$ and $b \mid c$, then $a \mid c$

    $3 \mid 6$, and $6 \mid 24$ then $3 \mid 24$

* if $a \mid b$, then $a \mid bc$ for all integers c

  $3 \mid 6$ then $3 \mid 12$, $3 \mid 18$, $3 \mid 24$.

* if a, b and c are integers such that $a|b$ and $a|c$; then $a|(mb+mc)$

$3|6$, and $3|9$ then $3|15$, $3|30$, $3|45$

~~*let a be~~

* Division Algorithm

* let "a" be an integer and "d" a positive integer.
Then there are unique integers "q" and r, with $0 \leq r < d$
such that $\boxed{a = dq + r}$ الشرط لناتج القسمة

* d is called the **divisor**, a is called the **dividend**, q is called the **quotient**, and r is called the **remainder**.

- $q = a \text{ div } d \rightarrow$ ناتج القسمة
- $r = a \text{ mod } d \rightarrow$ الباقي.

Examples:-

* what are the quotient and remainder when 101 is divided by 11 $q = 9$ $r = 2$

?? $a = dq + r$

$= 11 \times 9 + 2 = 101$ ✓

* what are the quotient and remainder when -11 is divided by 3

$q = -4 \quad r = 1$

$-11 = 3(-4) + 1 = -11$

* $18 \text{ div } 4 = 4$
* $18 \bmod 4 = 2$
* $-14 \text{ div } 5 = -3$
* $-14 \bmod 5 = 1$

* $22 \bmod 5 = 2$
* $7 \bmod 5 = 2$

} نفس ال mod

* $22 \equiv 7 \pmod{5}$ indicates that 22 is congruent to 7 modulo 5

* Modular Arithmetic

* $a \equiv b \pmod{m}$ indicates that a is congruent to b modulo m.

* $a \equiv b \pmod{m}$ iff $a \bmod m = b \bmod m$

* $a \equiv b \pmod{m}$ if m divides $a - b$

Examples:-

- Determine whether 17 is congruent to 5 module 6

$17 \equiv 5 \pmod 6$

$17 \bmod 6 = 5$

$5 \bmod 6 = 5$

- Determine whether 24 is congruent to 14 module 6

$24 \equiv 14 \pmod 6$

$24 - 14 = 10$

$6 \nmid 10 \longrightarrow \not\equiv$

$24 \bmod 6 = 0$

$14 \bmod 6 = 2$

- Determine whether $22 \equiv 13 \pmod 4$

$22 - 13 = 9$

$4 \nmid 9$

$22 \bmod 4 = 2$

$13 \bmod 4 = 1$

- Determine whether $15 \equiv 7 \pmod 4$

$15 - 7 = 8$

$4 \mid 8$ ✓

* Modular Arithmetic.

- let m be a positive integer. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then.

* $a + c \equiv b + d \pmod{m}$
* $ac \equiv bd \pmod{m}$

* Example:-

from $7 \equiv 2 \pmod 5$ and $11 \equiv 1 \pmod 5$, then:

$18 \equiv 3 \pmod 5$

$77 \equiv 2 \pmod 5$

* Arithmetic Modulo 'm'

- let Z_m be the set of nonnegative integers less than m
- we define addition of these integers, denoted by $+_m$

as:- $a +_m b = (a + b) \bmod m$.

- we define addition of these integers, denoted by $\cdot_m$ as:-

$a \cdot_m b = (a \cdot b) \bmod m$.

- Example:-

* $7 +_{11} 9 = (7 + 9) \bmod 11 = 16 \bmod 11 = 5$

* $7 \cdot_{11} 9 = 63 \bmod 11 = 8$

* Arithmetic Modulo property :- "$+_m$ and $\cdot_m$"

1) Closure → "Z_m" الجواب ينتمي لنفس المجموعة
2) Associativity → تجميعية
3) Commutativity → ابدالية
4) Identity elements → العنصر المحايد بالجمع = صفر ، بالضرب = 1
5) Additive inverse → العملية الجمع
6) Distributivity → توزيع الضرب على الجمع.

assume $+_{11}$

$(2)^{-1} = 9$ why?? → $2+9=11$ → $11 \bmod 11 = 0$, $(2 +_{11} 9) = 0$

$(1)^{-1} = 10$ → $10+1=11$ → $11 \bmod 11 = 0$

---

* Primes. → لا تقسم على نفسه وعلى 1

* A positive integer p is greater than 1 is called prime if the only positive ~~integer~~ factors of p are 1 and p. A positive integer is greater than 1 and is not prime is called composite.

* 7 is a prime number while 4 is a composite number.

- The primes less than 100 are :-

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, ~~71~~, 73, 79, 83, 89, 97.

Date

No.

- The Fundamental theorem of Arithmetic.

- every positive integer greater than 1 can be written uniquely as a prime or as the product of two or more primes where the prime factors are written in order of nondecreasing size. "اي عدد نستطيع كتابته على شكل مضروب مجموعة من الأعداد الأولية."

ex:- what are the prime factorizations of 100, 49 and 60.

$100 = 2 \times 5 \times 2 \times 5 = 2^2 \times 5^2$

$49 = 7 \times 7 = 7^2$

$11 = 11$

$60 = 6 \times 10 = 3 \times 2 \times 2 \times 5 = 2^2 \times 3 \times 5$

* greatest common divisors → اكبر عدد يقسم العدد a والعدد b

- let a and b be integers, not both zero. The largest integer d such that $d \mid a$ and $d \mid b$ is called the greatest common divisor of a and b. The greatest common divisor of a and b is denoted by $\gcd(a, b)$

ex:- What is the gcd of 24 and 36?

$24 = 6 \times 4 = 2^3 \times 3$

$36 = 6 \times 6 = 2 \times 3 \times 2 \times 3 = 2^2 \times 3^2$

* ناخذ العدد المشترك والأس الأصغر.

$\gcd = 2^2 \times 3 = 12$

Scanned with CamScanner

ex:- find gcd for 100 and 30

$100 = 2 \times 5 \times 2 \times 5 = 2^2 \times 5^2$

$30 = 2 \times 5 \times 3$

$\gcd(100, 30) = 2 \times 5 = 10$

ex:- $\gcd(17, 22)$

$17 = 17$

$22 = 2 \times 11$

$\gcd = 1$

* The integers a and b are relatively prime if their greatest common divisor is 1.

ex:- $\gcd(8, 9) = 1$

* Pairwise Relatively prime

- The integers $a_1, a_2, \dots, a_n$ are pairwise relatively prime if $\gcd(a_i, a_j) = 1$ whenever $1 \leq i < j \leq n$

ex:- Determine whether the integers 10, 17 and 21 are pairwaise relatively prime and whethere the integers 10, 19, 24 - - -

$\gcd(10, 17) = 1$, $\gcd(10, 21) = 1$, $\gcd(17, 21) = 1$ ✓

$\gcd(10, 19) = 1$, $\gcd(10, 24) = 2$ X x least Common Multiple:- → اصغر عدد يقسم b ، a وباقي القسمة صفر

- LCM of the positive integers a and b is the smallest positive integer that is divisible by both a and b. The least Common multiple of a and b is denoted by lcm (a, b)

ex:- what is lcm of 24 and 36?

$24 = 2 \times 4 \times 3 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$36 = 2^2 \times 3^2$

- نأخذ كل الأعداد ونأخذ الأس الأكبر من الأعداد المشتركة.

$lcm = 2^3 \times 3^2 = 72.$

ex:- lcm (100, 30)

$100 = 2^2 \times 5^2$ $30 = 2 \times 3 \times 5$

$lcm(100, 30) = 2^2 \times 3 \times 5^2 = 300$

---

x let a and b be positive integers. Then

$$ab = gcd(a, b) \cdot lcm(a, b)$$

ex:- $a = 100$ $b = 30$

$lcm = 300$

$gcd = 10$

$ab = 3000$

$gcd \cdot lcm = 3000$

* Mathematical Induction.

* Mathematical Induction can be used to prove statements that assert that P(n) is true for all positive integers n, where P(n) is a propositional function.

* A proof by mathematical induction has two parts, a ① basis step, where we ~~know~~ show that P(1) is true, and an ② inductive step, where we show that for all positive integers k, if P(k) is true, then P(k+1) is true.

مهم!

$P(k) \longrightarrow P(k+1)$

↳ using direct Proof

examples :-

show that if n is a positive integer, then

$1+2+\cdots+n = n(n+1)/2$

بشكل آخر

Proof $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

⇒ step 1 :- P(1) , $1 \overset{?}{=} \frac{1(2)}{2}$ ✓

⇒ step 2 :- $P(k) \longrightarrow P(k+1)$

assumption :- $1+2+\cdots+k = \frac{k(k+1)}{2}$

Prove :- P(k+1)

$\underbrace{1+2\cdots+k+(k+1)} = \frac{(k+1)(k+2)}{2}$

عامل مشترك

$\frac{k(k+1)}{2}+(k+1) = \frac{k(k+1)}{2}+\frac{2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$ ✓

ex:- Show that $1+2+2^2+\cdots+2^n=2^{n+1}-1$ for all nonnegative integer n.

$n \geq 0$

1 $P(0) \longrightarrow 1 \overset{?}{=} 2^1-1$ ✓

2 $P(k) \longrightarrow P(k+1)$

assumption $\longrightarrow$ $1+2+2^2+\cdots+2^k = 2^{k+1}-1$

prove $\longrightarrow$ $P(k+1)$

L.H.S R.H.S.

$1+2+2^2+\cdots\cdots+2^{k+1} = 2^{k+2}-1$

$= 1+2+2^2+\cdots\cdots 2^k+2^{k+1}$

$= 2^{k+1}-1+2^{k+1}$

$= 2 \cdot 2^{k+1}-1$

$2^{k+2}-1$ # R.H.S.

ex: $\sum_{K=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}$ $\boxed{Ms}$

بشكل آخر

$1+4+9 \dots\dots n^2 = \frac{n(n+1)(2n+1)}{6}$

$n \geq 1$

1- $P(1)$ $1 \overset{?}{=} \frac{1(2)(3)}{6} = \frac{6}{6} = 1$ ✓

2- $P(K) \longrightarrow P(K+1)$ ↷ direct Proof

$\Rightarrow$ assumption $p(K)$ is true

$1+4+9+\dots\dots+K^2 = \frac{K(K+1)(2K+1)}{6}$

$\Rightarrow$ Proof $P(K+1)$

L.H.S R.H.S.

$\underbrace{1+4+9+\dots+K^2+(K+1)^2} = \underbrace{\frac{(K+1)(K+2)(2(K+1)+1)}{6}}$

$\frac{K(K+1)(2K+1)}{6} + (K+1)^2$ بدنا نثبت انه يساوي ال R.H.S

$\frac{K(K+1)(2K+1)+6(K+1)^2}{6}$

عامل مشترك

$\frac{(K+1)[K(2K+1)+6(K+1)]}{6}$

$\frac{(K+1)[2K^2+K+6K+6]}{6} = \frac{(K+1)(2K^2+7K+6)}{6}$

R.H.S $\Rightarrow \frac{(K+1)(K+2)(2K+3)}{6} = \frac{(K+1)(2K^2+\overset{7K}{\boxed{3K+4K}}+6)}{6}$ //

* -17 div $3 = -6$

* -17 mod $3 = 1$

* باقي القسمة r: يكون عدد موجب

$0 \le r < d$

* $14 \bmod 12 = 2 \bmod 12$

$$\underset{2}{\underset{\downarrow}{14 \bmod 12}} \qquad \underset{2}{\underset{\downarrow}{2 \bmod 12}}$$

so, $14 \equiv 2 \pmod{12}$

* $15 \equiv 4 \pmod{3}$ X

$\rightarrow 15 \bmod 3 = 0$

$4 \bmod 3 = 1$

other way, $3 | (15-4) = 3 | (11)$ X

* $(a+b) \bmod 9 = a +_9 b$ "نفس الخاصية للضرب"

ex: $4 +_9 8 = 3$

$4 \cdot_9 8 = 7$

---

* inverse العدد في الجمع هو عبارة عن عدد يجمع للعدد الأصلي وبأخذ ال mod
والناتج = 0 "العنصر المحايد"

* inverse العدد في الضرب هو عبارة عن عدد يضربه بالعدد الأصلي وبأخذ ال mo
والناتج = "1" "العنصر المحايد".

* Relations

- Let A and B be sets. A binary relation from A to B is a subset of $\underbrace{A \times B}$

- Relations are a generalization of function.

- A relation on the set A is a relation from A to A.

* Representation of Relations

* Set of ordered pairs

* Set builder notation

* Zero-one matrix

* Directed graph.

ex:- Let $A = \{1,2,3,4\}$, R is defined on A.

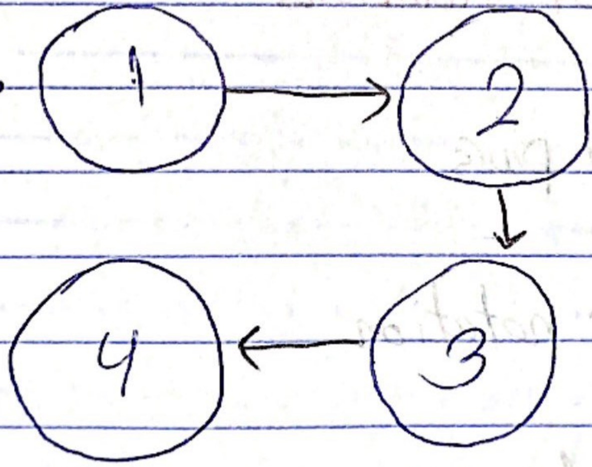
- $R = \{(1,2), (2,3), (3,4)\}$

- $R = \{(x,y) \mid x = y-1\}$

- $$\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

الصفوف ← domain

الأعمدة ← Codomain

- 

* عناصر A هي عبارة عن الدوائر

* الأسهم هي عبارة عن ال relations

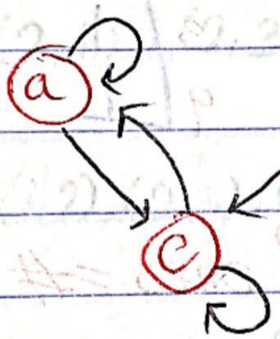
- let $A = \{a, b, c\}$ is defined on A such that $R = \{(a,a), (a,c), (b,c), (c,a), (c,c)\}$ Represent the relation R as:-

- Matrix.
- Graph.

1- Matrix

$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 0 & 1 \\ b & 0 & 0 & 1 \\ c & 1 & 0 & 1 \end{array}$$

2- Graph



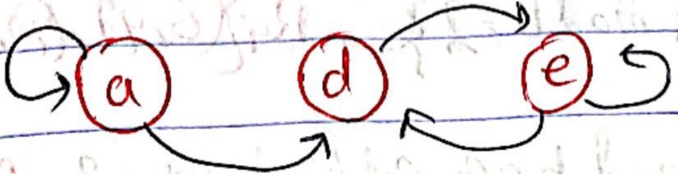
* let $A = \{a, d, e\}$. R is defined on A by the followi
Matrix.

$$\begin{array}{c|ccc} & a & d & e \\ \hline a & 1 & 1 & 0 \\ d & 0 & 0 & 1 \\ e & 0 & 1 & 1 \end{array}$$

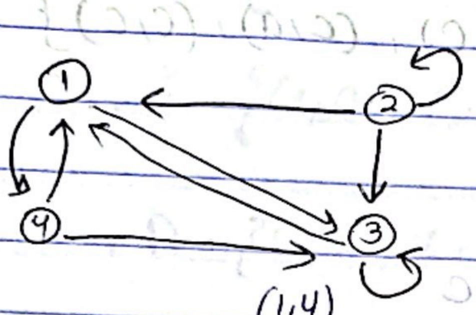
find 1) the relation
2) the representation of R as graph.

1) $R = \{(a,a), (a,d), (d,e), (e,d), (e,e)\}$

2)



- Let R be defined by the following graph.



find:- 1) The relation
2) Matrix

1) $R = \{(1,3) \overset{(1,4)}{\uparrow}, (2,1), (2,2), (2,3), (3,1), (3,3), (4,3), (4,1)\}$.

2)
$$\cancel{\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 1 \\ 2 & 1\,0 & 1\,0 & 1\,0 & \\ 3 & 1 & 0\,0 & 0 & 1\,0 \\ 4 & 1 & & 0 & 1\,0 \end{array}}$$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 1 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 & 0 \\ 4 & 1 & 0 & 1 & 0 \end{array}$$

\# of ones = \# of lines on graph.

- let R be defined on Z
- $R_1 = \{(a,b) \mid a \mid b\} \rightarrow R_1 = \{(2,4), (3,6), (3,9) \dots\}$

$R_2 = \{(a,b) \mid a > b\} \rightarrow R_2 = \{(5,1), (0,-2), (7,4)\}$

$R_3 = \{(a,b) \mid (a+b) > 3\} \rightarrow R_3 = \{(7,-1), (0,4) \dots\}$

$R_4 = \{(a,b) \mid a \bmod b = 2\} \rightarrow R_4 = \{(6,4), (10,4), (12,5) \dots\}$

$R_5 = \{(a,b) \mid$ a and b are relatively Prime$\} \rightarrow R_5 = \{(5,7), (8,9)\}$

* Operations on Relations

- * $R_1 \cup R_2$
- * $R_1 \cap R_2$
- $R_1 - R_2$
- $R_1 \oplus R_2$
- $\overline{R_1}$
- R^{-1}
- $R_1 \circ R_2$
- R_1 "identity"
- R^n

ex:- R_1 and R_2 are defined on $A = \{1,2,3\}$

$R_1 = \{(1,1), (1,2), (2,3), (3,1), (3,3)\}$

$R_2 = \{(1,2), (2,1), (2,3), (3,1)\}$

- * $R_1 \cup R_2 = \{(1,1), (1,2), (2,3), (3,1), (3,3), (2,1)\}$
- * $R_1 \cap R_2 = \{(1,2), (2,3), (3,1)\}$
- $R_1 - R_2 = \{(1,1), (3,3)\}$
- $R_2 - R_1 = \{(2,1)\}$
- $R_1 \oplus R_2 = \{(1,1), (3,3), (2,1)\}$
- $\overline{R_1} = \{(1,3), (2,1), (2,2), (3,2)\}$

- $\overline{R_2} = \{(1,1),(1,3),(2,2),(3,2),(3,3)\}$
- $R_1^{-1} = \{(1,1),(2,1),(3,2),(1,3),(3,3)\}$
- $R_2^{-1} = \{(2,1),(1,2),(3,2),(1,3)\}$
- $R_2 \circ R_1 = \{(1,2),(1,1),(1,3),(2,1),(3,2),(3,1)\}$
- $R_1 \circ R_2 = \{(1,3),(2,1),(2,2),(2,3),(3,1),(3,2)\}$
- $R_1^0 = \{(1,1),(2,2),(3,3)\} \rightarrow$ identity relation
- $R_1^2 = R_1 \circ R_1 = \{(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(3,3)\}$
- $R_1^3 = R_1 \circ R_1^2$

$$M_{R_1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

- $M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

- $M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

- $M_{R_1 \oplus R_2} = M_{R_1} \odot M_{R_2} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- $M_{\overline{R_1}} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

- $M_{R_1^{-1}} = M_{R_1^{+}} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

- $M_{R_2 \odot \underline{\underline{R_1}}} = M_{\underline{\underline{R_1}}} \odot M_{R_2} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

- $M_{R_1^{0}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ identity Matrix.

- $M_{R_1}^2 = M_{R_1}^{(2)} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$M_{R_1 - R_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Properties of relations

*Reflexive :- A relation R on a set A is called reflexive if $(a,a) \in R$ for every element $a \in A$, $R, \subseteq A$

* Symmetric :- A relation R on a set A is called symmetric if $(b,a) \in R$ where $(a,b) \in R$, for all $a,b \in A$, $\underline{A^{-1} = A}$

* Antisymmetric :- A relation R on a set A such that for all $a,b \in A$, if $(a,b) \in R$ and $(b,a) \in R$, then $a=b$ is called antisymmetric, $(A-R) \cap A^{-1} = \phi$

* Transitive :- A relation R on a set A is called transitive if whenever $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$, for all $a,b,c \in A$, ~~$A^2 \subseteq A$~~ $R^2 \subseteq R$

---

ex:- what are the properties of the following relations that are defined on $A = \{a,b,c\}$

* $R_1 = \{(a,a), (a,b), (b,a), (c,c)\}$ → symmetric

* $R_2 = \{(a,a), (b,b), (c,c)\}$ → Reflexive, symmetric, Antisymmetric, Transitive

* $R_3 = \{(a,c), (b,c)\}$ → Antisymmetric, Transitive

$R_4 = \{\ \ \} \longrightarrow$ symmetric, Antisymmetric, Transitive

---

ex: what are the properties of the following relations that are represented as matrices.

$$\cdot M_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad \cdot M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\cdot M_3 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

القطر عبارة عن 1

العناصر حول القطر متساوية (المتناظرة)

ممنوع (1) يناظر (1) حول القطر

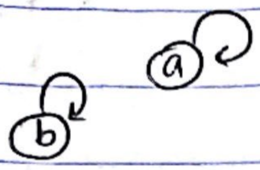
	Reflexive	symmetric	Antisymmetric	Transitive
R_1	✗	✗	✗	✗
R_2	✗	✓	✗	✗
R_3	✓	✗	✓	✓

لازم تكون نفس الأصلية (1)

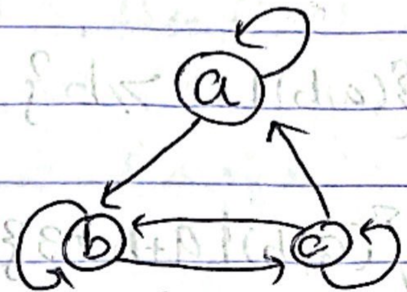
ط و

$$M_1^2 \begin{bmatrix} 1 & 1 & 1 \\ & & \\ & & \end{bmatrix}, \quad M_2^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \\ & & \end{bmatrix}, \quad M_3^2 \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

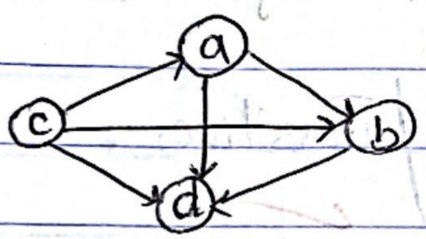
ex: what are the properties of the following relations that are represented as graphs.



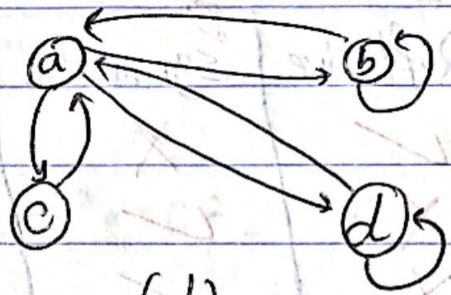
(a)



(b)



(c)



(d)

loop على كل عنصر.

بين كل نقطتين سهمين او ولا سهم؟

سهم واحد او ولا سهم.

	Reflexive	Symmetric	Antisymmetric	Transitive
a R₁	✓	~~X~~ ✓	✓	✓
b R₂	✓	X	X	X
c R₃	X	X	✓	~~X~~ ✓
d R₄	X	✓	X	X

Transitive → إذا في path بين نقطتين
لازم يكون في سهم واصل بينهم مباشرة

d) path → from b to a to d

ex:- what are the properties of the following relations that are defined on Z.

- $R_1 = \{(a,b) \mid a \geq b\}$
- $R_2 = \{(a,b) \mid a+b > 3\}$
- $R_3 = \{(a,b) \mid a \text{ and } b \text{ are relatively prime}\}$.

بعوض عناصر متساوية ←
x,x

	Reflexive	Symmetric	Antisymmetric	Transitive.
R_1	✓	✗	✓	✓
R_2	✗	✓	✗	✗
R_1	✗	✓	✗	✗

$a \geq b \overset{?}{=} a < b$ | $a \geq b \neq a < b$

$a+b > 3 \overset{?}{=} b+a > 3$ | $a+b > 3 = b+a > 3$

b and a relatively Prime | b and a relatively Prime

Transitive → $(a,b) \in R \wedge (b,c) \in R \Rightarrow (a,c) \in R$

$R_1 = a \geq b \wedge b \geq c \overset{?}{\Rightarrow} a \geq c$ ✓

$R_2 : a+b > 3 \wedge b+c > 3 \rightarrow a+c \not> 3$ ✗

$R_3 = GCD(a,b) = 1 \wedge gcd(b,c) \overset{?}{=} 1 \rightarrow gcd(a,c) = 1$

✗

* Closures of Relations

• If R is a relation on a set A, then the closure of R with respect to p, if it exists is the relation S on A with property P that contains R and is a subset of every subset of A×A containing R with property P.

- Reflexive closure :- $R \cup R_1$ ↳ identify relation

- symmetric closure :- $R \cup R^{-1}$

- Transitive closure :- $R \cup R^1 \cup R^2 --- R^n$ ↳ size of set.

ex: find the reflexive, symmetric, and transitive closure for each of the following relation.

- $R = \{(a,a), (a,b), (b,c)\}$, $A = \{a,b,c\} \rightarrow n=3$

- Reflexive closure of $R = \{(a,a), (a,b), (b,c), (b,b), (c,c)\}$
- symmetric closure of $R = \{(a,a), (a,b), (b,c), (b,a), (c,b)\}$
- Transitive closure of $R = \{(a,a), (a,b), (b,c), (a,c)\}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- Reflexive Closure

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- Symmetric Closure

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

لو القطر عبارة عن ١

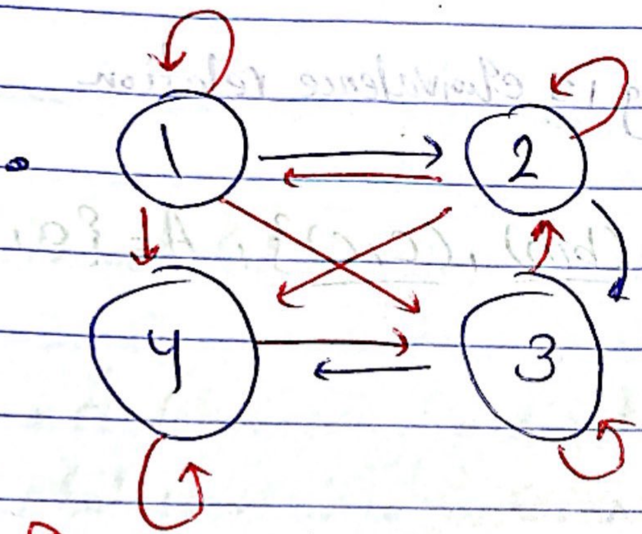
- Transitive Closure

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \overset{R^2}{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \underset{R^3}{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}}$$

→

$$M_{R^1} \vee M_{R^2} \vee M_{R^3} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$



Reflexive closure → loop

Symmetric closure → two way

Transitive closure → edge.

B

→ Transitive closure.

- $R = \{(a,b) \mid a > b\}$, R is defined on Z

Reflexive closure → $\{(a,b) \mid a \geq b\}$

Symmetric closure → $\{(a,b) \mid a > b \vee b > a\}$

Transitive closure → $\{(a,b) \mid a > b\}$.

equivalence Relations

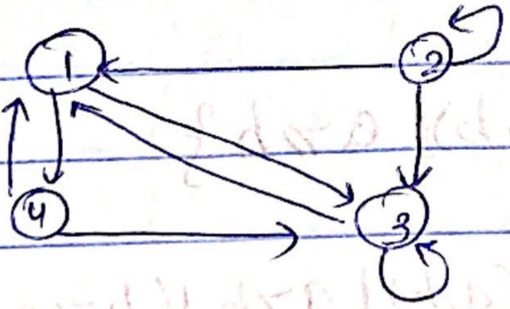
- A relation on a set A is called equivalence if it is, reflexive, symmetric and transitive,

ex:- which of the following is equivalence relation

$R_1 = \{(a,a), (a,b), (b,a), (b,b), (c,c)\}, A = \{a,b,c\}$

equivalence.

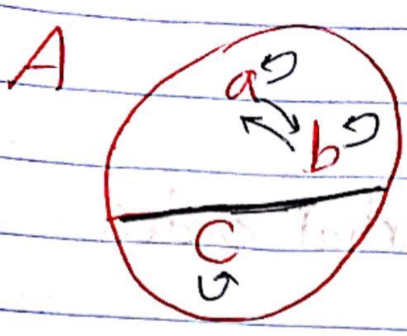
- $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ not equivalence.

- 

not equivalence.

- $R_2 = \{(a,b) | a = b\}$, R is defined on Z.

equivalence.



الأهمية

relations تعطيك تجزئة الdomain

إلى Classes.

"equivalence relations."

---

* Partial Orderings

- A relation R on a set S is called partial ordering or partial order if it is reflexive, antisymmetric and transitive.

- A set S together with a partial ordering R is called a partially ordered set, or poset.

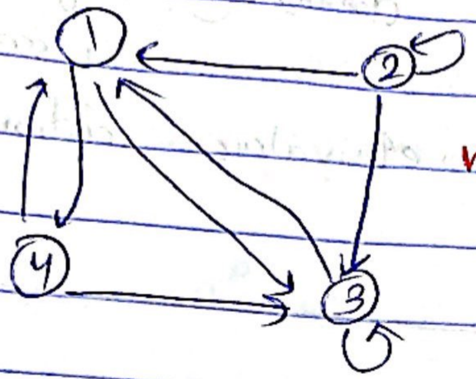
ex:- which of the following is an partial order.

$R_1 = \{(a,a), (a,b), (b,a), (b,b), (c,c)\}, A = \{a,b,c\}$

not partial order.

$$\begin{bmatrix} 1 & 1 & \underset{=}{0} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ \underline{0} & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & \underset{=}{1} \\ & & \\ & & \end{bmatrix}$$

not partial order.

- 

not Partial order.

- $R_2 = \{(a,b) \mid a \leq b\}$, R is defined on Z.

Partial order.

- Total ordering relations → نستطيع المقارنة بين اي عنصرين في المجال.

ex: $\{(a,b) \mid a \leq b\}$ defined on $\underline{\underline{Z}}$

- Partial ordering relations → لا نستطيع مقارنة جميع العناصر -

ex:- $R = \{(a,b) \mid a \leq b\}$ defined on

$\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Graph

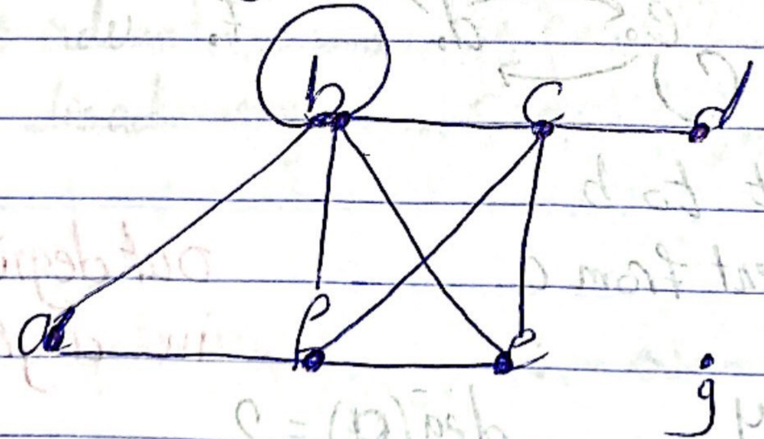
* A graph $G = (V, E)$ consists of V, a nonempty set of vertices (or nodes) and E, a set of edges

* Graph has many applications such as networking.

Graph
- Undirected graph → مافي اتجاهات للاسهم
- Directed graph → في اتجاهات

Undirected graph

ex:-



*degree of a node = # of edges, ~~edge~~ in or out the node.

a f are adjacent

deg(a) = 2

deg(f) = 4

اللوب بتحسب مرتين ← deg(b) = 6

deg(d) = 1 → pendant

deg(g) = 0 → isolated

/ /

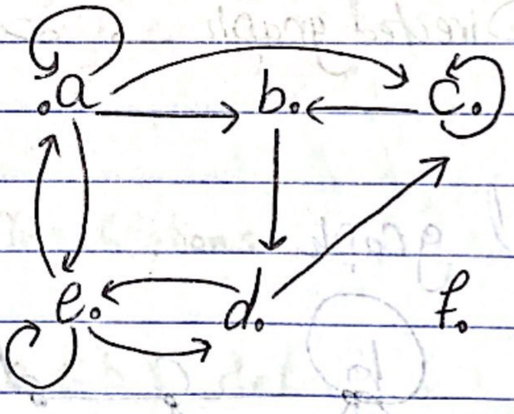
Σ of degrees = 2(# of edges)

ex: How many edges are there in graph with 10 vertices each of degree 6 ?

$$\Sigma \deg = 2e$$
$$60 = 2e$$
$$e = 30$$

---

directed graph



- a is adjacent to b
- b is adjacent from a

Out degree ($\deg^+$)
in degree ($\deg^-$)

$\deg^+(a) = 4$ $\deg^-(a) = 2$
$\deg^+(b) = 1$ $\deg^-(b) = 2$
$\deg^+(c) = 2$ $\deg^-(c) = 3$
$\deg^+(e) = 3$ $\deg^-(f) = 0$
$\deg^+(d) = 2$ $\deg^-(d) = 2$
$\deg^+(f) = 0$ $\deg^-(e) = 3$

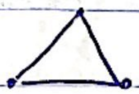
$\boxed{\Sigma \text{ in degree} = \Sigma \text{ out degree} = \#\text{ of edges}}$

* Special undirected graphs

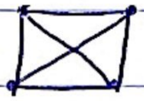
- (complete Graph) "K" → nodes [illegible] node [illegible]

K_1

K_2



K_3



K_4

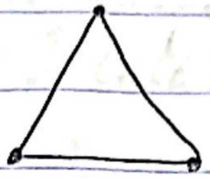
K_5

K_n :- The number of nodes is 'n'
The degree of each node = n-1
The number of edges is n(n-1)/2

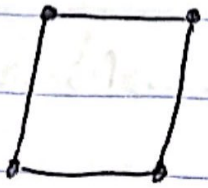
* advantage → easy connection
* disadvantage → Complexity → high degree for each node.

+ Cycle graph

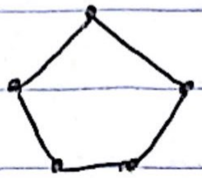
→ 2 [illegible] node [illegible] nodes "C"



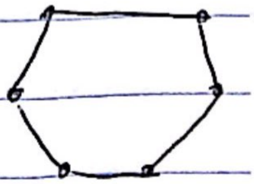
C_3



C_4



C_5



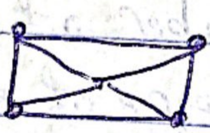
C_6

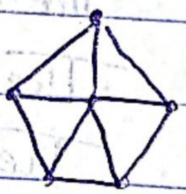
- The degree of each node = 2
- The number of nodes = n → C_n
- The number of edges = n

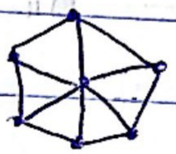
- advantage → less complexity
- disadvantage → all the network will be affected if one ~~node~~ a problem occurs in one ~~node~~ edge

- Wheel graph → نفس ال cycle مع اضافة node في الداخل . "W"


W_3


W_4


W_5

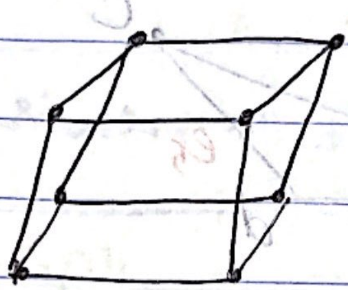

W_6

- The number of nodes = n+1
- The number of edges = 2n
- The degree of each node is 3 except the central node its degree is n

* n Cube Graph "Q" $Q_n \rightarrow$ degree for node.

Q_1

Q_2



Q_3

- The number of nodes is 2^n
- The number of edges is $2^{n-1} n$
- The degree of each node is n.

* Representing graphs

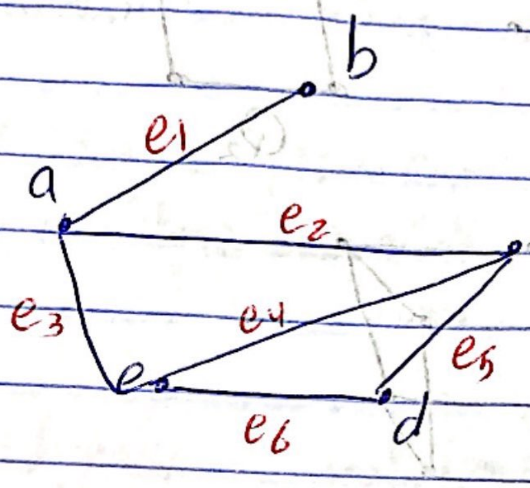
1) Adjacency list

2) Adjacency matrix

3) ~~Path~~ Incidence matrix.

Undirected graph → Adjacency list

↳ كل node
عنده جنبه ايه.



a	b, c, e
b	a
c	a, e, d
d	c, e
e	a, c, d

// → adjacency matrix

$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 1 & 1 & 0 & 1 \\ b & 1 & 0 & 0 & 0 & 0 \\ c & 1 & 0 & 0 & 1 & 1 \\ d & 0 & 0 & 1 & 0 & 1 \\ e & 1 & 0 & 1 & 0 & 1 \end{array}$$

→ symmetric matrix

deg(a) = 3

deg(b) = 1

deg(c) = 3

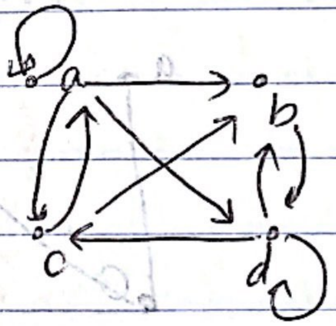
/ /

$$\begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline a & 1 & 1 & 1 & 0 & 0 & 0 \\ b & 1 & 0 & 0 & 0 & 0 & 0 \\ c & 0 & 1 & 0 & 1 & 1 & 0 \\ d & 0 & 0 & 0 & 0 & 1 & 1 \\ e & 0 & 0 & 1 & 1 & 0 & 1 \end{array}$$

* الصف بعطيني degree لكل node ، ومن العمود بعرف على الـ nodes المتصلة .

Directed graph

- Adjacency list



out degree

a	a , b , c , d
b	d
c	a , b
d	c , b , d

- adjacency matrix.

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 1 \\ b & 0 & 0 & 0 & 1 \\ c & 1 & 1 & 0 & 0 \\ d & 0 & 1 & 1 & 1 \end{array}$$

$deg^-(a) = 2$
$deg^-(b) = 3$
$deg^-(c) = 2$
$deg^-(d) = 3$

$deg^+(a) = 4$
$deg^+(b) = 1$
$deg^+(c) = 2$
$deg^+(d) = 3$

/ /

* Connectivity

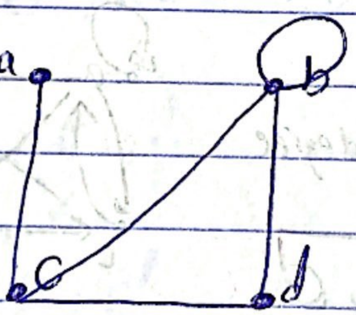
* A path of length "n" from u to V is a sequence of adjacent edges going from vertex u to vertex v.

* A path is a circuit if u=V

* A path is simple if it contains no edge more than once.

- Undirected graph

path length = # of edges



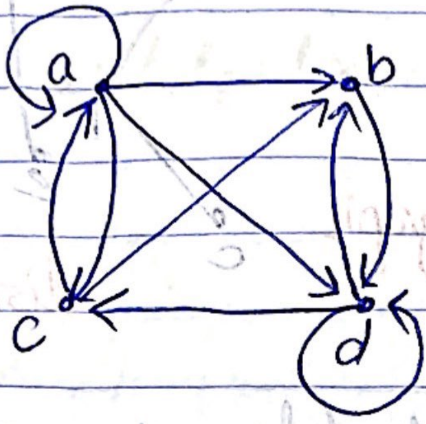
* cycle path :-
نقطة البداية نفس نقطة النهاية

- which of the following is a path, simple path, cycle path.
(S)
(C)

- a c b simple path (L=2) - a c b b d (L=4) S

- b d b cycle path (L=2) - a c a C (L=2)

* directed graph



- a b d b (l=3) S

- a b a a € (not path)

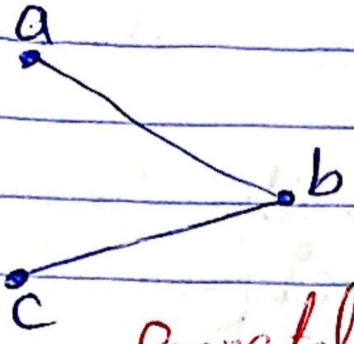
- a b c (not a path)

- c b d c C (l=2)

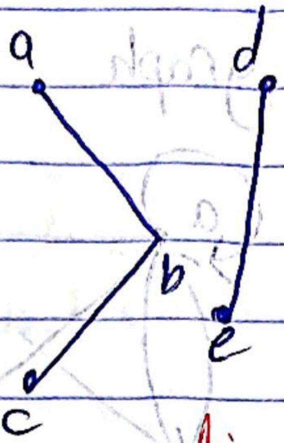
* Connectedness in undirected graph

* An undirected graph is called connected if there is a path between every pair of distinct vertices of the graph.

- An undirected graph that is not connected is called disconnected.



Connected graph



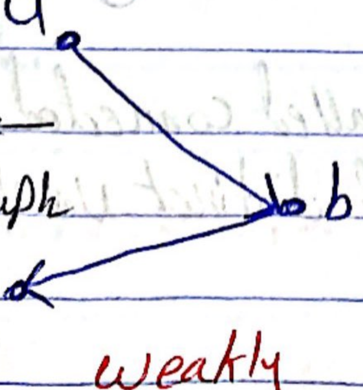
disconnected graph

---

* Connectedness in directed graph

- A directed graph is strongly connected if there is a path from a to b and from b to a whenever a and b are vertices in the graph.

- A directed graph is weakly connected if there is a path between every two vertices in the underlying undirected graph.



لو افترضنا انه ←
undirected graph
Connected. بكون

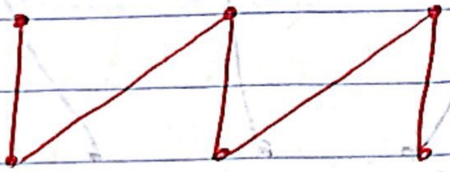
weakly Connected.

Trees

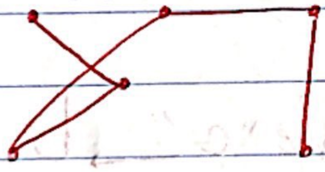
A tree is connected undirected graph with no simple Circuits

Example:-

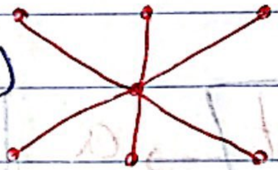
a)



b)



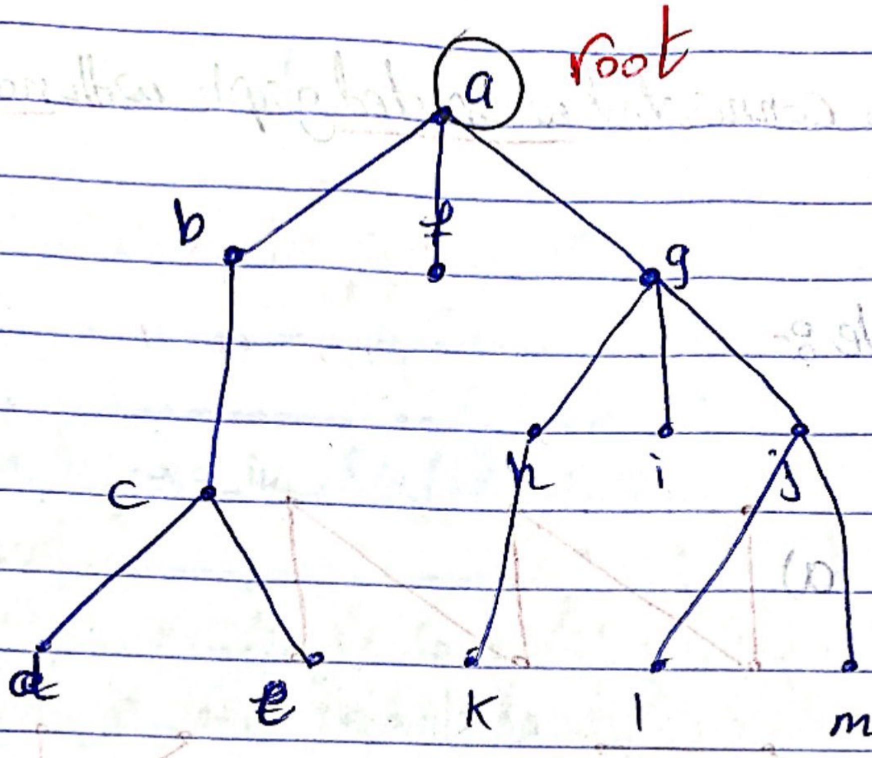
c)



× Rooted tree:- is a tree in which one vertex has been designated as the root and every edge is directed away from the root.

× Terminology:- Root, parent, child, sibilings, Descendants, Ancestors, leaf, internal vertices, Subtree, level, height, m-ary tree, full m-ary tree.

• Rooted tree example:-



root → a , parent for node "c" → b.
" " " "f" → a

Child → ex:- "e" is a child of "c"

sibilings → two nodes with the same parent
ex :- (f,b,g) / (h,i,j)

Descendants → for b → (c,d,e)
" g → (h,i,j,k,l,m)

Ancestor → for "d" → (c,b,a)
" "f" → a
" "i" → (g,a)

leaf → node ~~without~~ hasn't children. "f,d,e,k,l,m,i"

- Internal vertices → node has children.

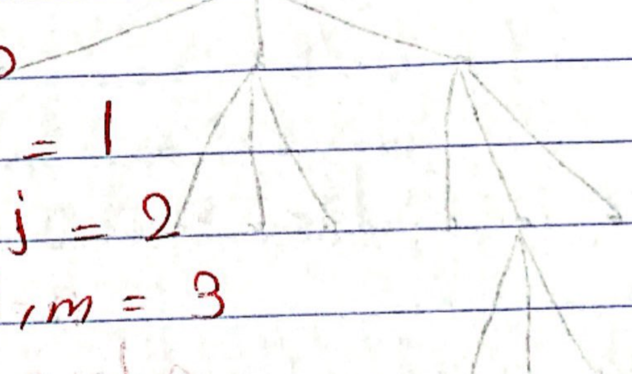
"a, b, g, c, h, j"

- subtree → {c, d, e} / (g, h, i, j, k, l, m)

(h, k) / {L}

- level → distance between node and root.

a = 0
b, f, j = 1
c, h, i, j = 2
d, e, k, l, m = 3



- hight → highest level → "3"

- m-ary-tree → largest number of children.

ex → "3"
3-ary tree

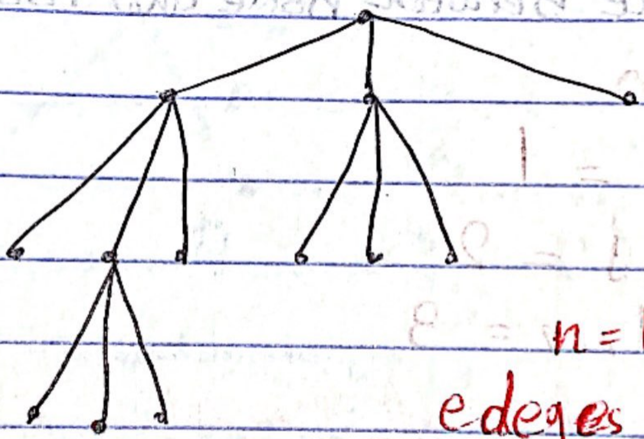
- if all the internal nodes have the same number of children → full m-ary tree

ex:- full 3-ary tree.

* Properties of trees.

node

- A tree with n vertices has n-1 degree
- A full m-ary tree with i internal vertices contains $n = mi+1$ vertices.



$n = 13$

edeges $= 13-1 = 12$

$13 = 3(i)+1$

$\therefore i = 4$

* internal nodes + leaf = total nods

$4 + l = 13$

$l = 9$

* $(i+l) = mi+1$

- A rooted m-ary tree of hight h is <u>balanced</u> if all leaves are at levels h or h-1

- There are at most m^h leaves in an m-ary tree of hight h.

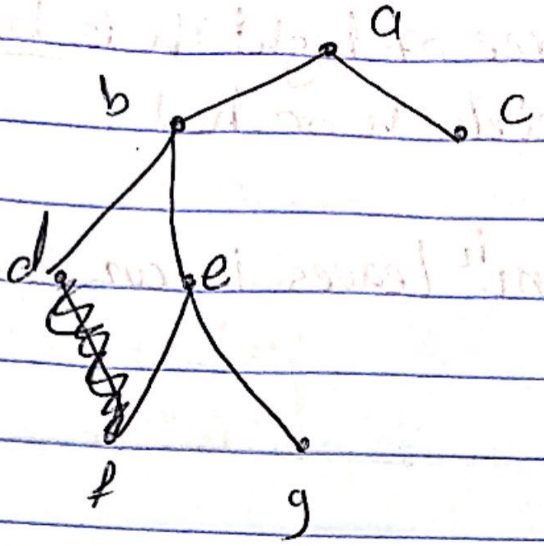
- Ordered Rooted tree

* An ordered rooted tree is a rooted tree where the children of each internal node are ordered.

- In ordered <u>binary</u> trees, we can define left child, right child.

- Tree Traversal "Traversal Algorithm:"

1) Preorder Traversal (root → left → right)
2) Inorder Traversal (left → root → right)
3) Postorder Traversal. (left → right → root)



Preorder → (abdefgc)

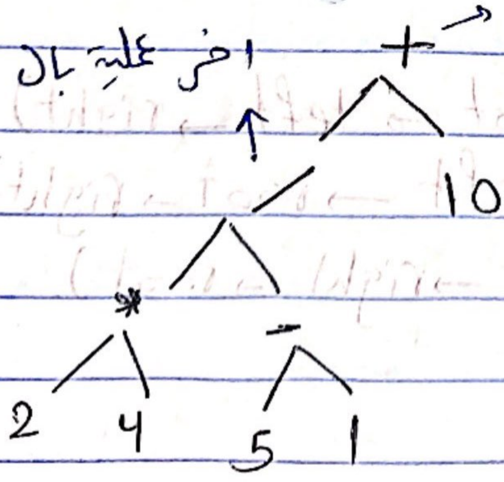
Inorder → (dbfegac)

Postorder → (dfgebca)

- Infix :- 2*4/(5-1)+10 = 8/4+10 = 12

افرع عليه بالأولويات

left افرع عليه بال



Post fix ($\underbrace{2\ 4\ *}$ 5 1 - / 10 +)

8 $\underbrace{5\ 1\ -}$ / 10 +

$\underbrace{8\ 4\ /}$ 10 +

2 10 +

= 12

Prefix ([illegible] 2 4 ~~- 5 1 10~~)

prefix $(+ / * 2\ 4 - \underbrace{5\ 1}\ \overset{\leftarrow}{10})$

~~+ [illegible] 4~~

$(+ / \underbrace{*\ 2\ 4},\ 4\ \overset{\leftarrow}{10})$

$+ \underbrace{/\ 8\ 4},\ 10$

$+\ 2\ 10$

$= 12$