* Current

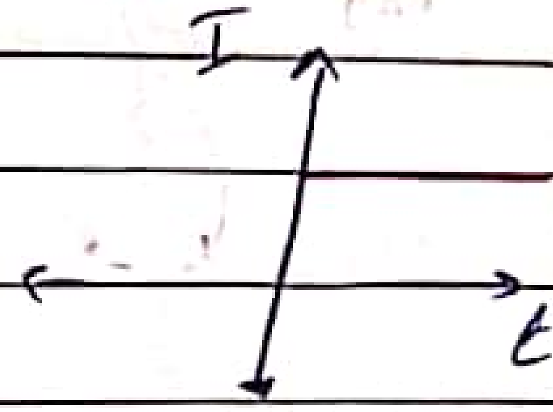
• Types :-

1. DC current (Direct constant) current :-

e.g. $I = 3A \rightarrow$ Amber, $I = -5A$, $I = 3.11\,A$, $I = 2.5A$.

↳ Doesn't Change with time
" Const. with time "

" Doesn't have to be integer value. ~~ikan~~



" negative current means that this current is moving in the opposite of real direction ".

in this direction

5A ←

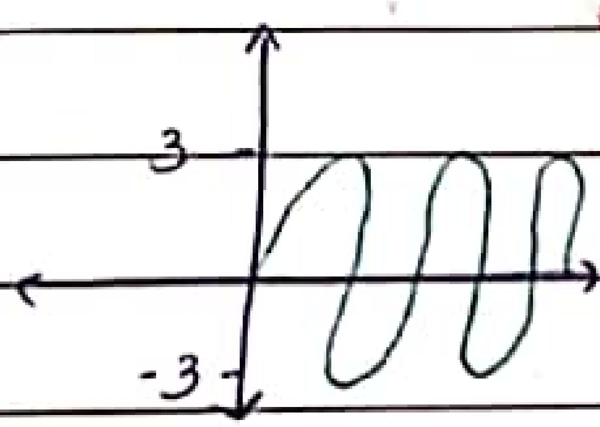
$I_x = -5A$

in this direction

2. AC current (Alternating current)

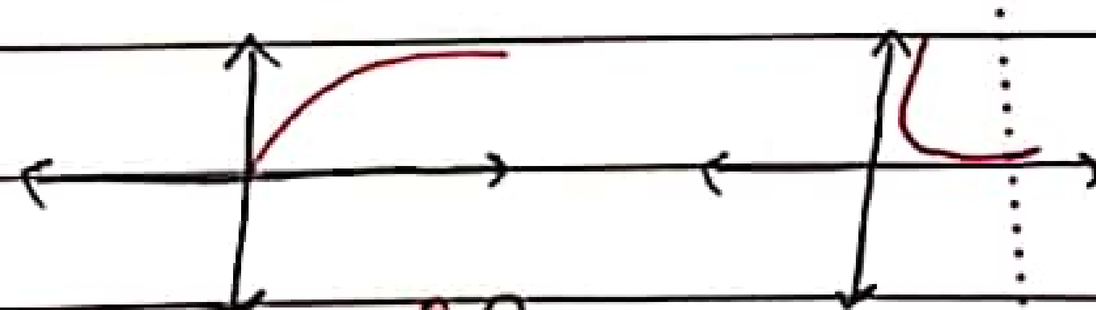
Simusaidly changes with time

e.g. $i(t) = 3\sin(wt)$, $i(t) = 4\cos(wt)$ " we write it as a funct. of sin and cos. "

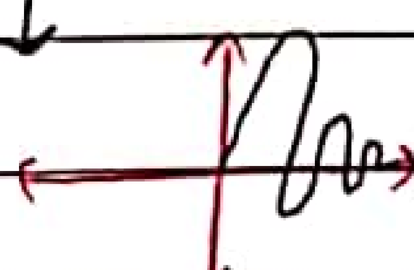


3. other types :-

e.g.s exponential current



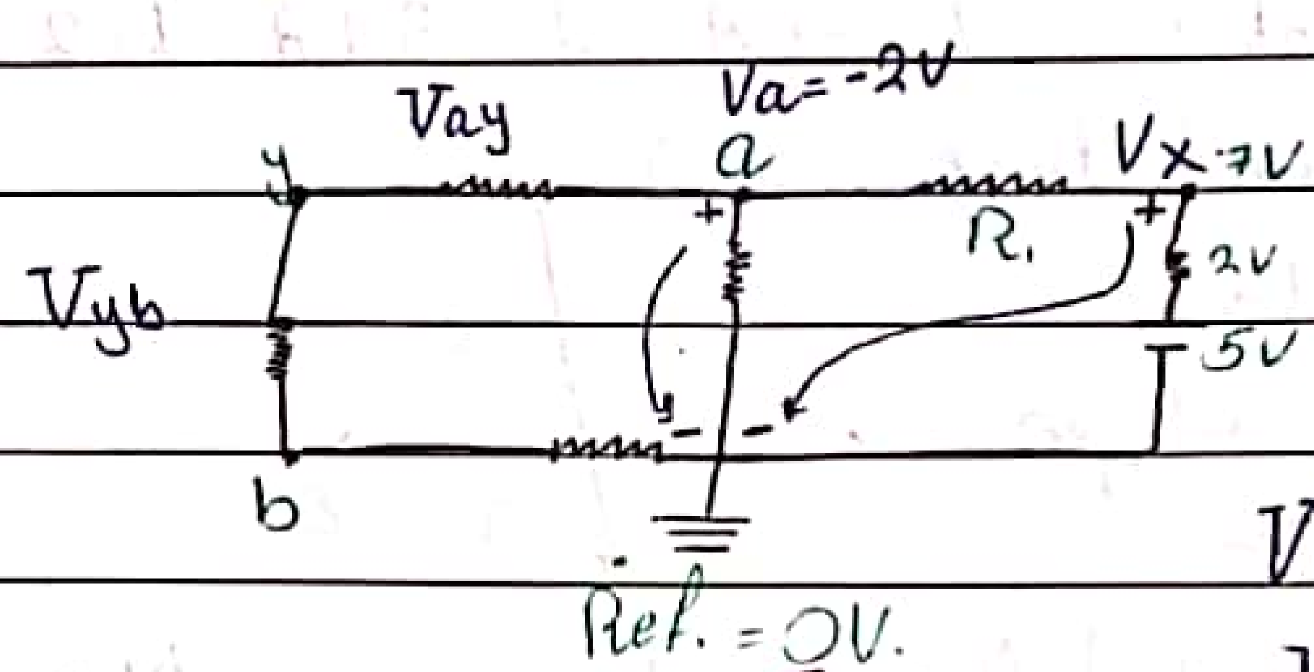
- dumped simusuidal current



* Voltage

→ DC Voltage $\underline{V} = 3v$, $\underline{V} = -10.4V$.

* V_{yb} + → −



V in Ref $>$ V_a by 2

V_a $>$ V in Ref by -2

V_{ab} . (a) positive , (b) negative

$V_{ax} = V_a - V_x$

$V_a{}^{b} > V_x$ By -9

→ AC voltage

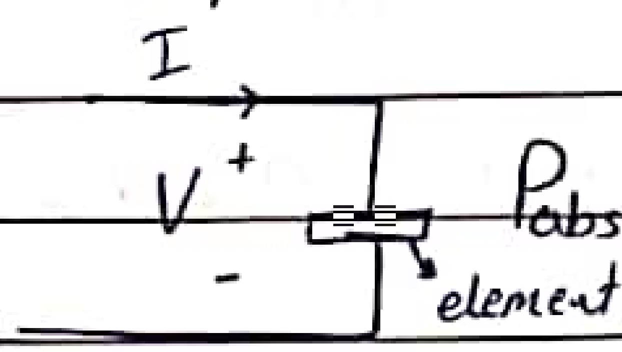
$V(t) = -9.1\cos(wt)$, $V(t) = 4.2\sin(wt)$ volt.

* Power

$P = I * V$ (W = watt) ← DC

$p(t) = i(t) * v(t)$ W ← AC

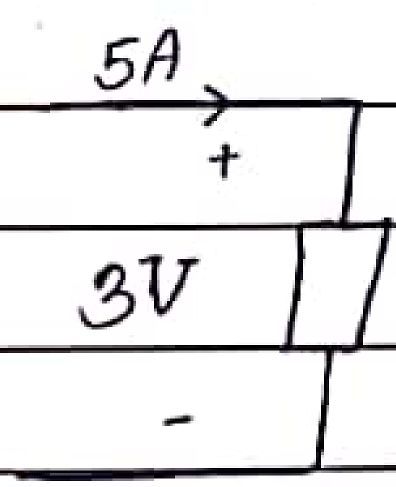
- Power could ~~by~~ be absorbed (dissipated) or generated by the element and this depends on the element.



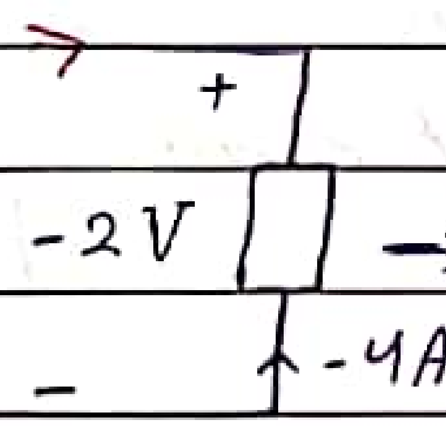
$P_{abs} = I * V$ watt — ⊕ absorbed, ⊖ generated

$P_{abs.} = -P_{gen}$

Ex :- find the P_{abs}.

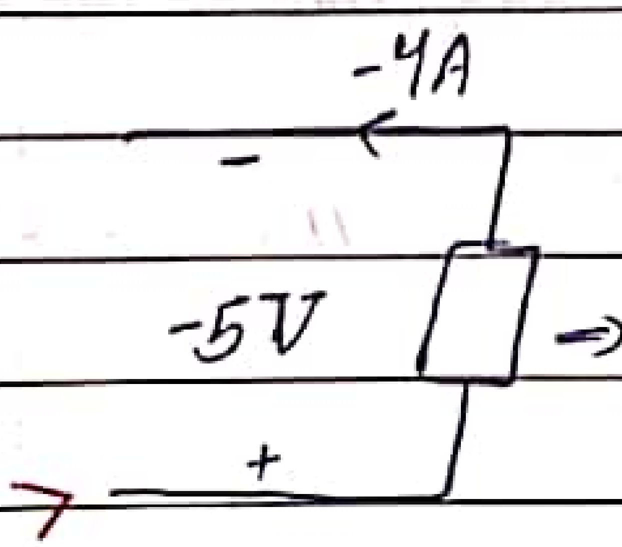


$\Rightarrow P_{abs.} = 5 \times 3 = 15$ W



$\rightarrow P_{abs.} = -2 * 4 = -8$ W

$P_{generated} = 8$ W



$\Rightarrow P_{abs.} = +20$ W "absorbing ~~generate~~ ~~the~~ the power"

$P_{generated} = -20$ W

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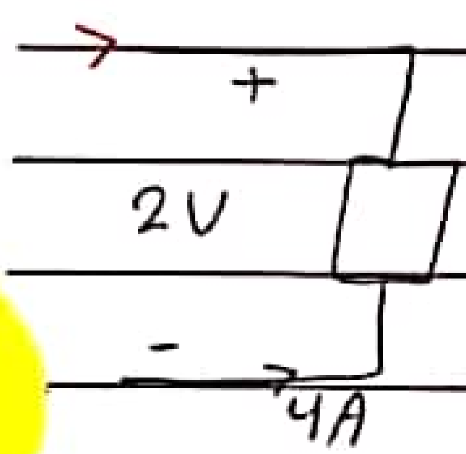
* Resistance always absorb the power.

→ Passive elements ≡ it doesn't give power by itself. "conductor, capacitor, Resistance".

→ Active elements ≡ it gives.

Ex: for this element, could be a Resistor



$P = 2 \times -4 = -8 \rightarrow P_g = 8\,W$

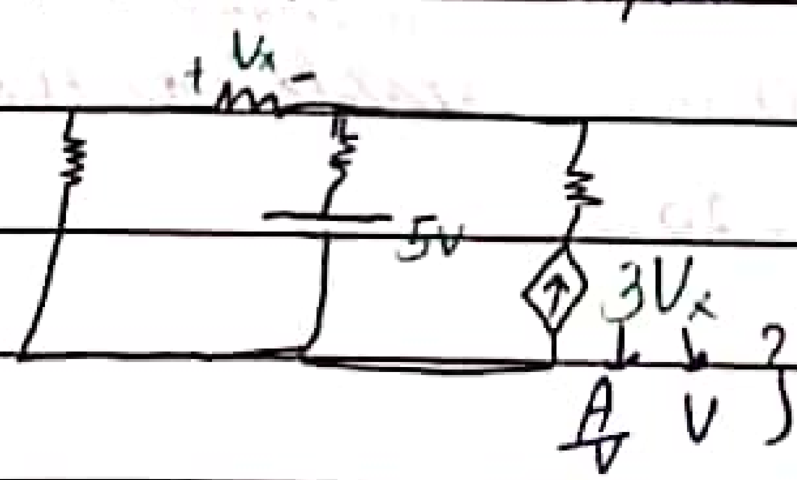
it actually generates Power ... so it could never be a resistor.

* Sources

1. Current Sources

↳ Independent sources → AC, DC "doesn't depend on any value on the circuit"

↳ Dependent sources " depend on other values on the circuit"



? → final result in Amber

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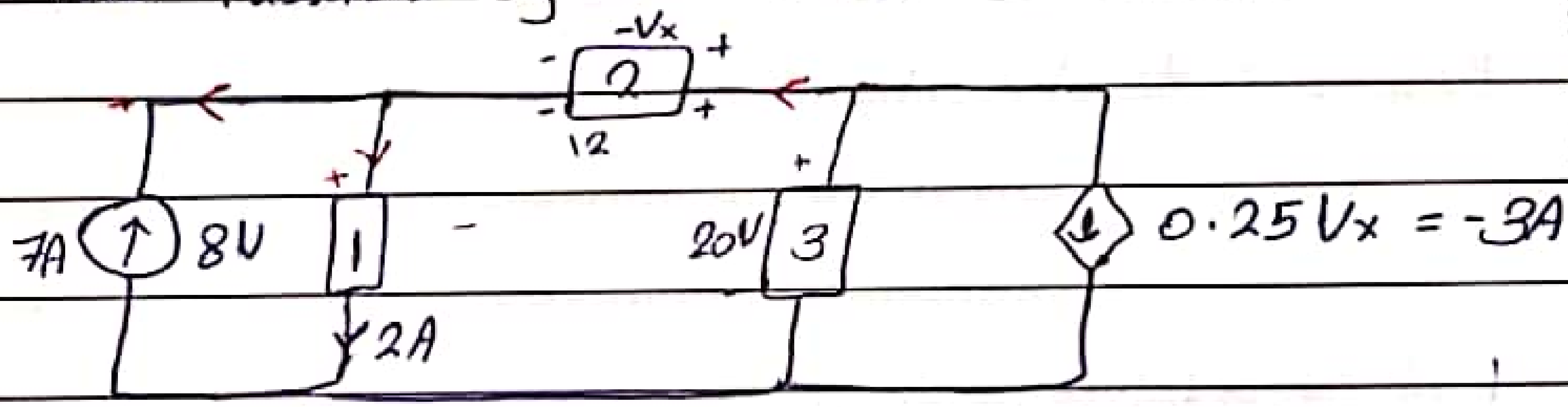
1

2. Voltage source

- Independent → Ac $V_s(t) = 7\sin(\omega t)\,V$
  - ↳ Dc $V_s = 5V$ $V_s = 3V$
- Dependent $3V_x$

Ex. Find Pabsorbed by each element in this circuit



$\boxed{7A} \rightarrow P_{abs.} = -7 \times 8 = -56\ W.$

$\boxed{2} \rightarrow P_{abso} = 12 \times -5 = -60\ W.$

$\boxed{1} \rightarrow P_{abs.} = 2 \times 8 = 16\ W$

$\boxed{3} \rightarrow P_{ab} = 20 \times 8 = 160\ W$

$\sum P_{abs} = \text{Zero}.$

$\sum P_g = \sum P_{abs}.$

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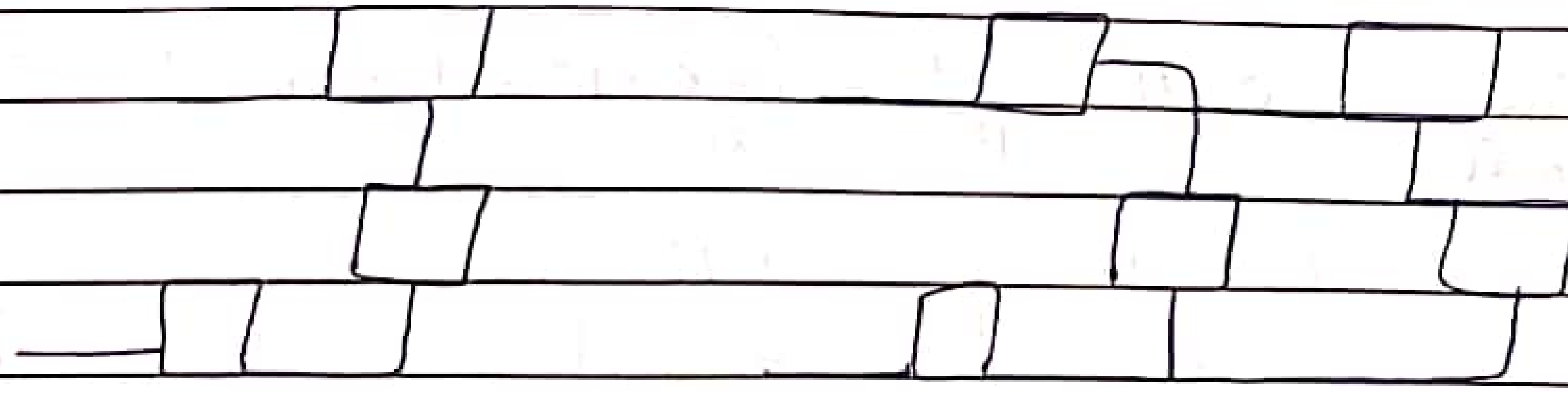
. Network :- two or more elements connected together.
. Circuit :- a network contains at least one closed path.



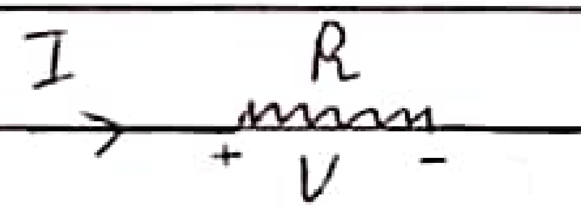
Network Circuit

* every circuit is a network but not every network is a circuit.

* Ohm's law :-

$V = R I$

Voltage Resistor current

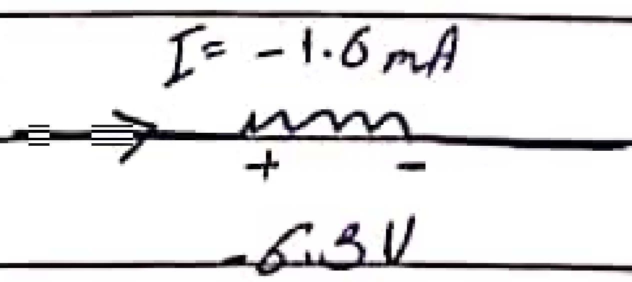


⊕ where the current enter.

$G = \frac{1}{R}$ ℧ or S

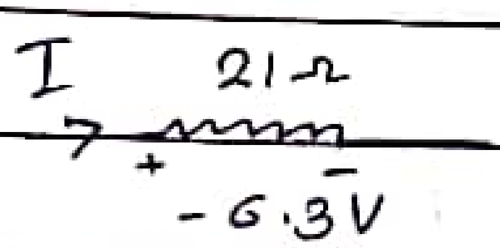
Conductance mho sienes

Ex :- find R



$R = \frac{V}{I} = 3.94 \, k\Omega$

Ex :- find the absorbed power by R. if $R = 21\,\Omega$ and $V = -6.3V$



$P = IV = \frac{V^2}{R} = \frac{(-6.3)^2}{21} = 1.89W$

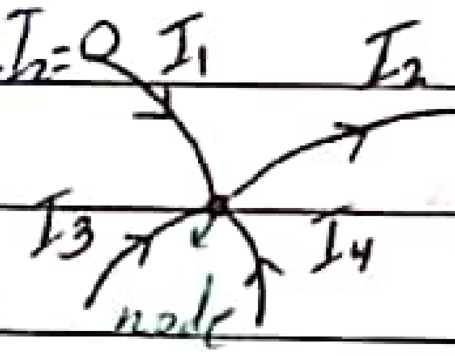
Note :- $P_{absorbed} = I.V = \frac{V^2}{R} = I^2 R$

* Kirchhoff's current law (KCL)

$\sum I_{in} = \sum I_{out}$ OR $\sum I_{in} = 0$

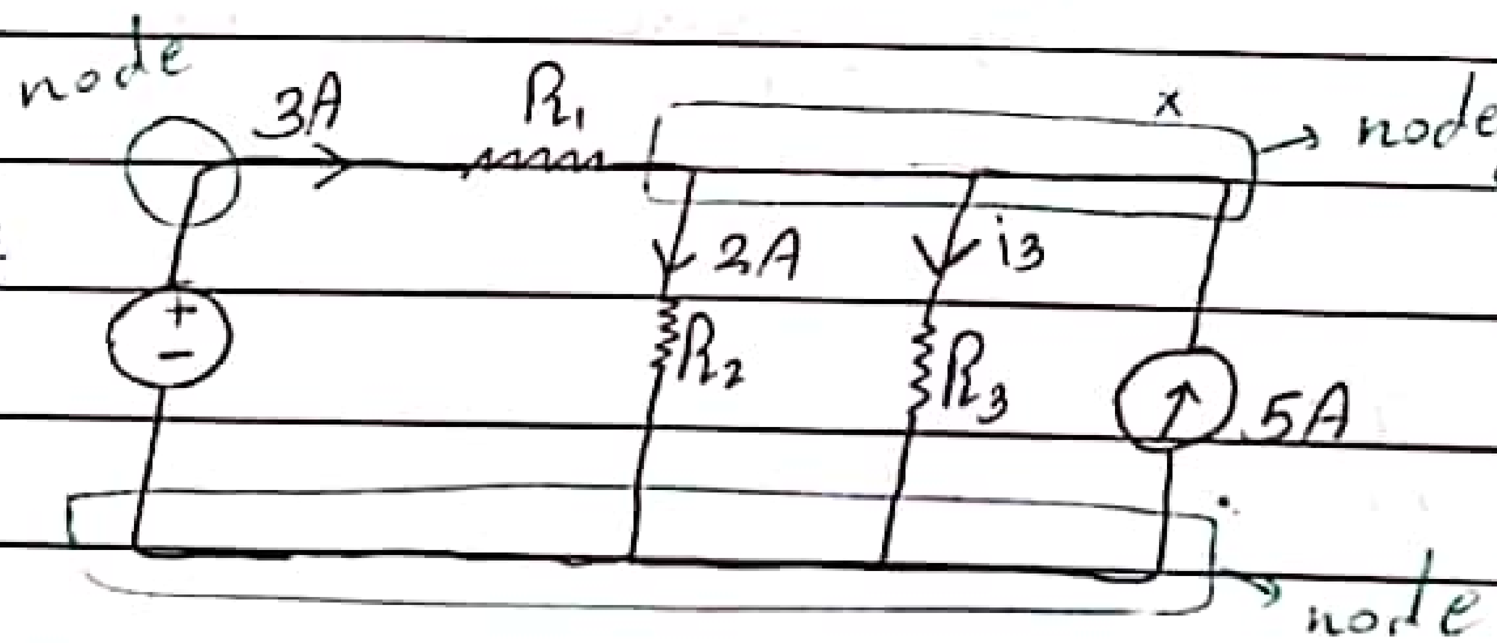
$I_1 + I_3 + I_4 = I_2$ $\quad I_1 + I_3 + I_4 - I_2 = 0$

OR $\sum I_{out} = 0$



Ex :-

find the current in R_3



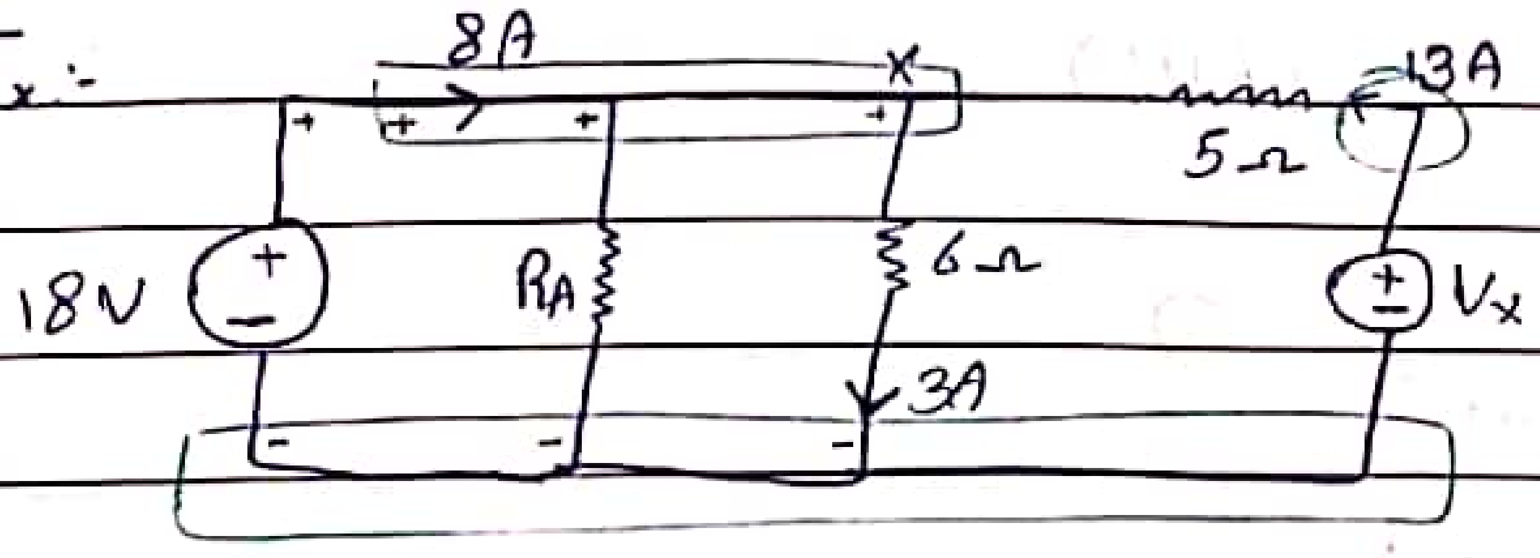
→ node :- where elements Connect together

* KCL at node x → $\sum I_{in} = 0$

$5 + 3 - 2 - I_3 = 0$

$I_3 = 6A$

Ex :-



a) find the no. of branches → 5   branches = elements

b) find the no. of nodes → 3

c) find the no. of elements → 5

d) find RA

$V_{RA} = 18V$, KCl at node x

$8+13-3-I_A$

$I = 18A$

$R_A = \frac{V_{RA}}{I_{RA}} = \frac{18}{18} = 1 \ \Omega$

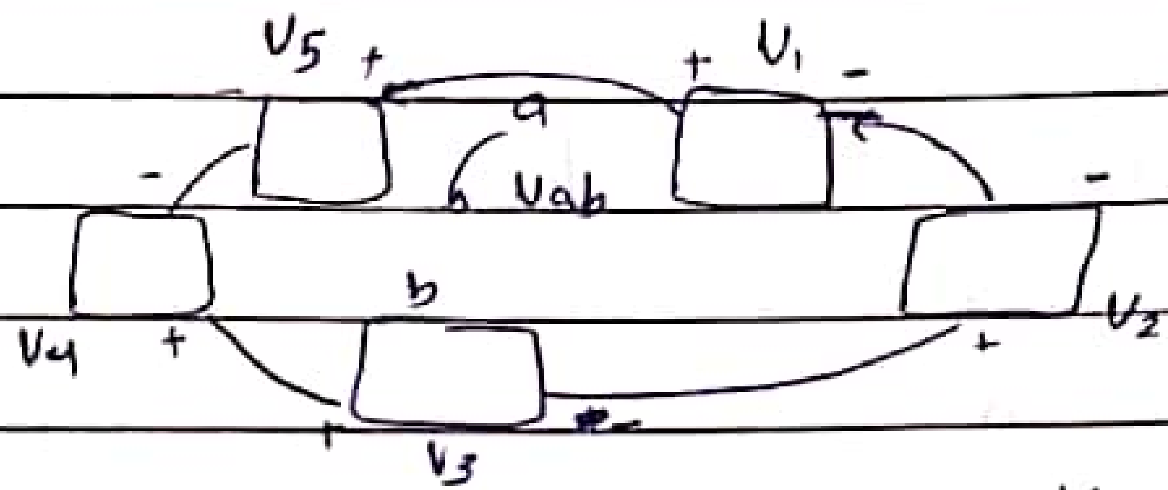
e) $P_{generated (18V)} = 8 \times 18 = +144 W$

↳ $P_{abs} = 18 \times -8 = -144 W$

f) $P_{absorbed (6\Omega)} = I^2 \times R = 9 \times 6 = 54 W$

• Kirchhoff's voltage law (KVL)

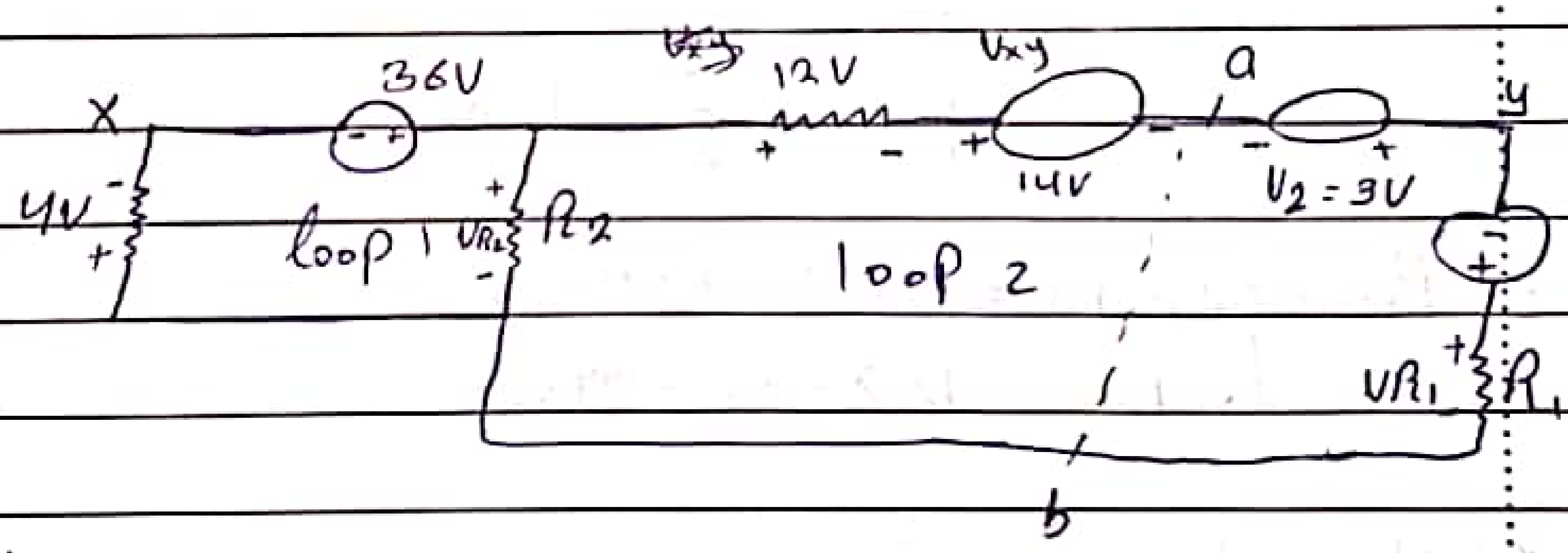
$$\sum V_{\text{in closed path}} = 0$$



• $V_1 - V_2 - V_3 + 4 - V_5 = 0$

* $V_1 - V_2 - V_3 - V_{ab} = 0$
• $V_5 - V_4 - V_{ab} = 0$

Ex



Find V_{R_2}, V_{ab}, V_{xy}

→ KVL at loop 1 :- $4 - 36 + V_{R_2} = 0$

$$V_{R_2} = 32\,V$$

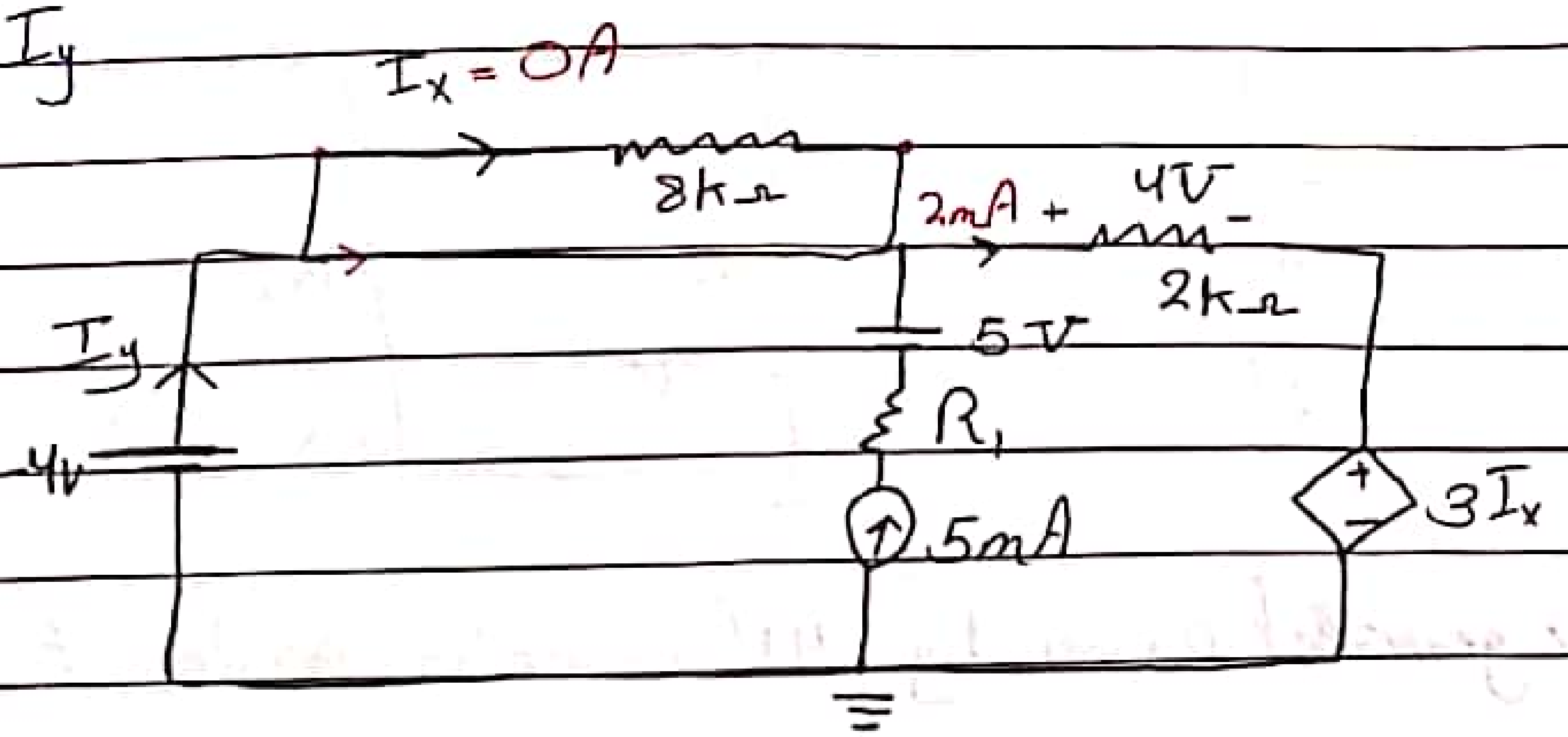
→ KVL at loop 2 :- $4 - 36 + 12 + 14 + V_{ab} = 0 \rightarrow V_{ab} = 6\,V$

→ KVL at loop 3 :- $V_{xy} + 3 - 14 - 12 + 36 = 0$

$$V_{xy} = -13\,V$$

③ Ms

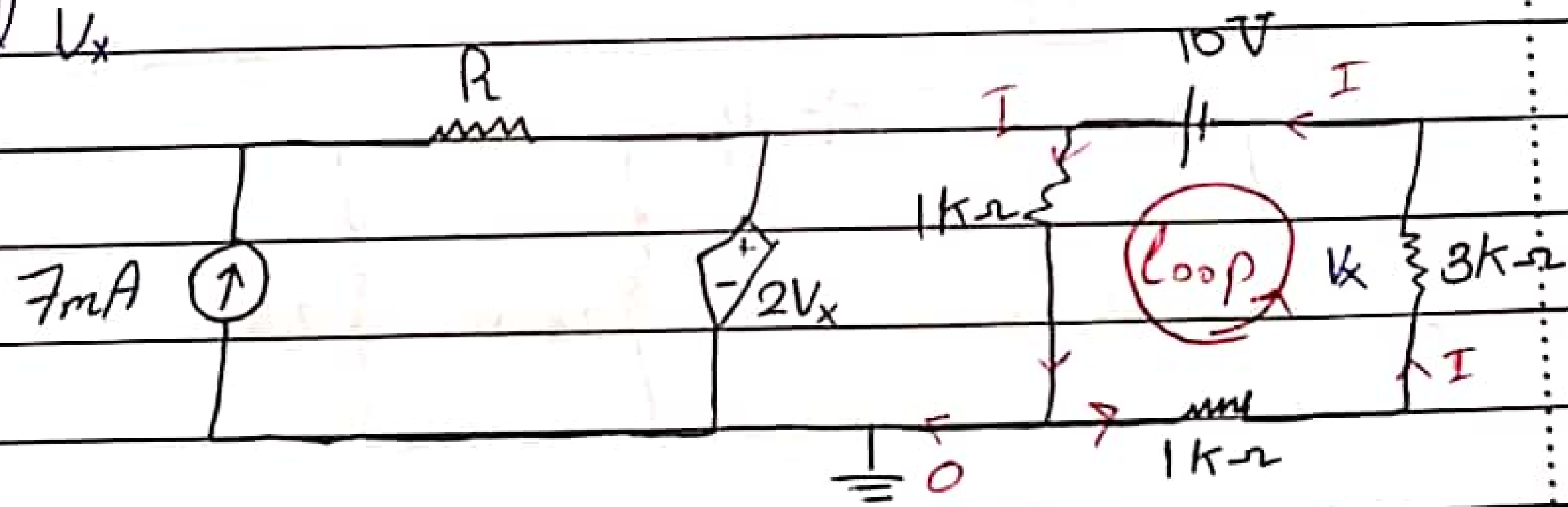
1) find I_y



$5 + I_y + 0 = 2$

$I_y = -3mA$

2) find V_x



$-10 + 1(I) + 3(I) + 1(I) = 0$

$I = 2mA$

$V_x = 6V$

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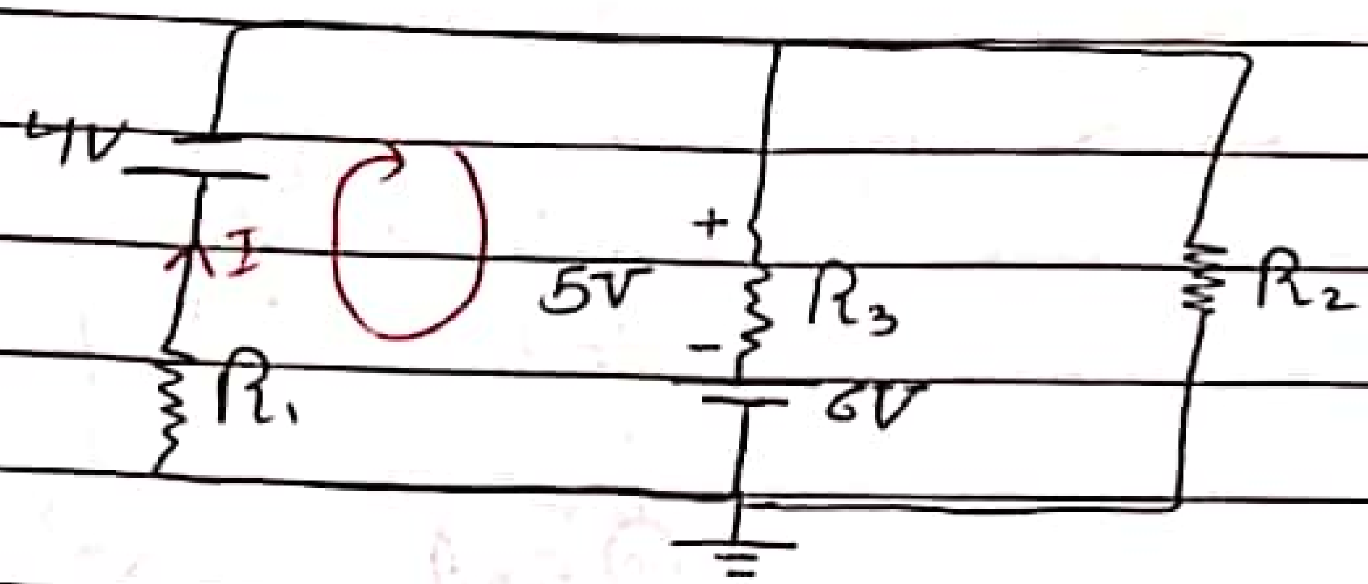
3)



If the generated power by 4V source is 20mW find R_1

$P_{abs} = I * V$

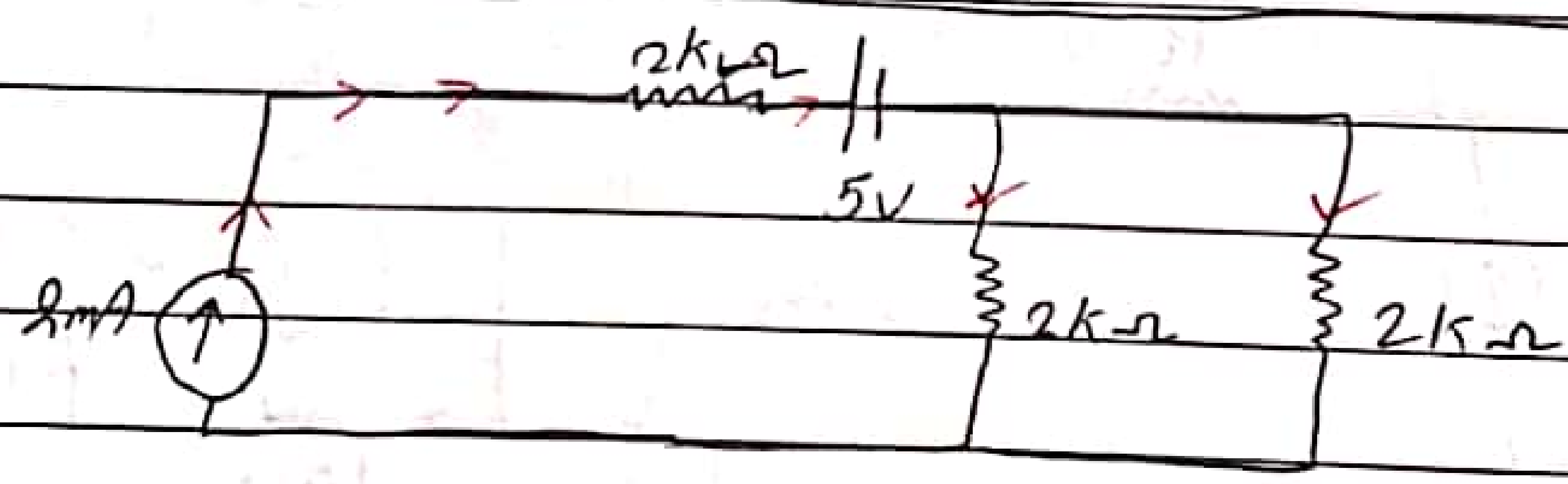
$-20 = 4I$

$\boxed{I = -5mA}$

$IR_1 + 4 + 5 + 6 = 0$

$R_1 = 3k\Omega$

4)



find the generated power by 5V source

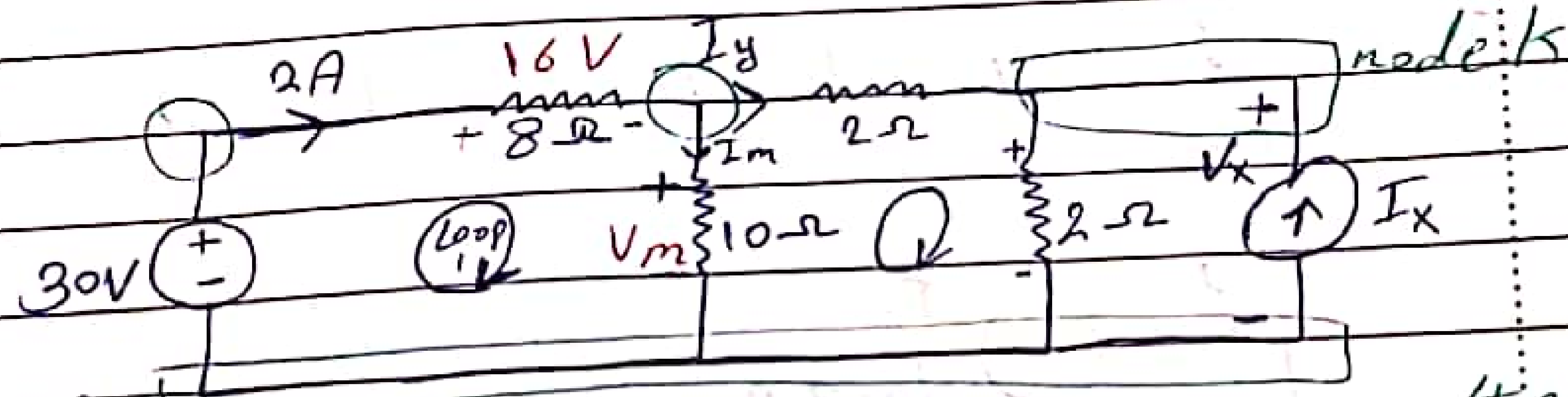
$P_{abs} = I \times V$

$2 \times 5 = 10mW$

$P_g = -10mW$

ject

④



find V_x and I_x

4 nodes

KCL at node k $\rightarrow I_x + I_y - \frac{V_x}{2} = 0$

$I_x = 5.8 A$

KVL loop 1 :- $-30 + 2*8 + V_m = 0$

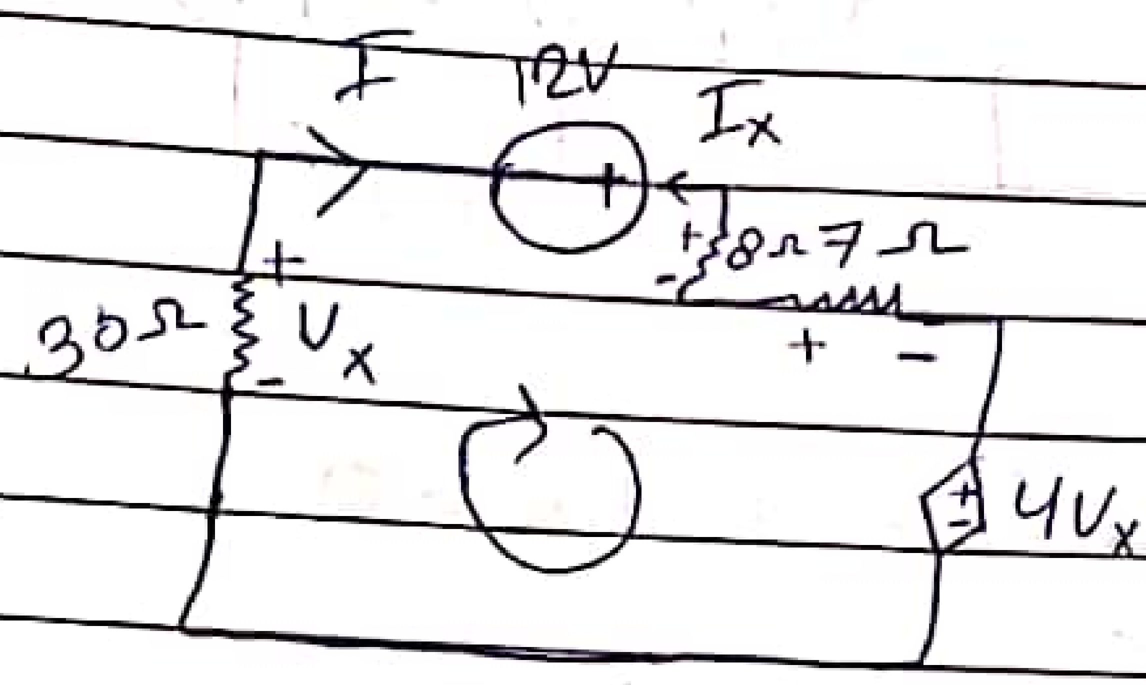
$V_m = 14 V$

$I_m = \frac{14}{10} = 1.4 A \rightarrow I_y = 2 - 1.4 = 0.6 A$

KVL loop 2 :- $-V_m + 2I_y + V_x = 0$

$V_x = 12.8 V$

Ex: find the absorbed power by each element :-



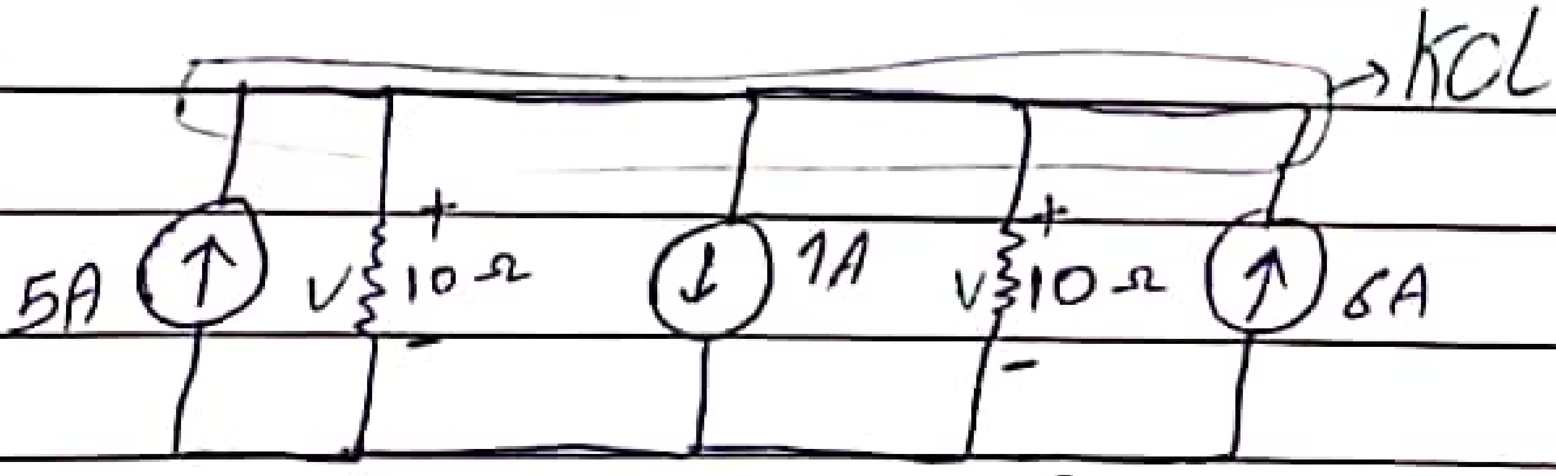
KVL $\rightarrow -12+8I+7I+4V_x-V_x=0$

$-12+8I+7I+3(-30I)=0$

$I=-160\,mA$

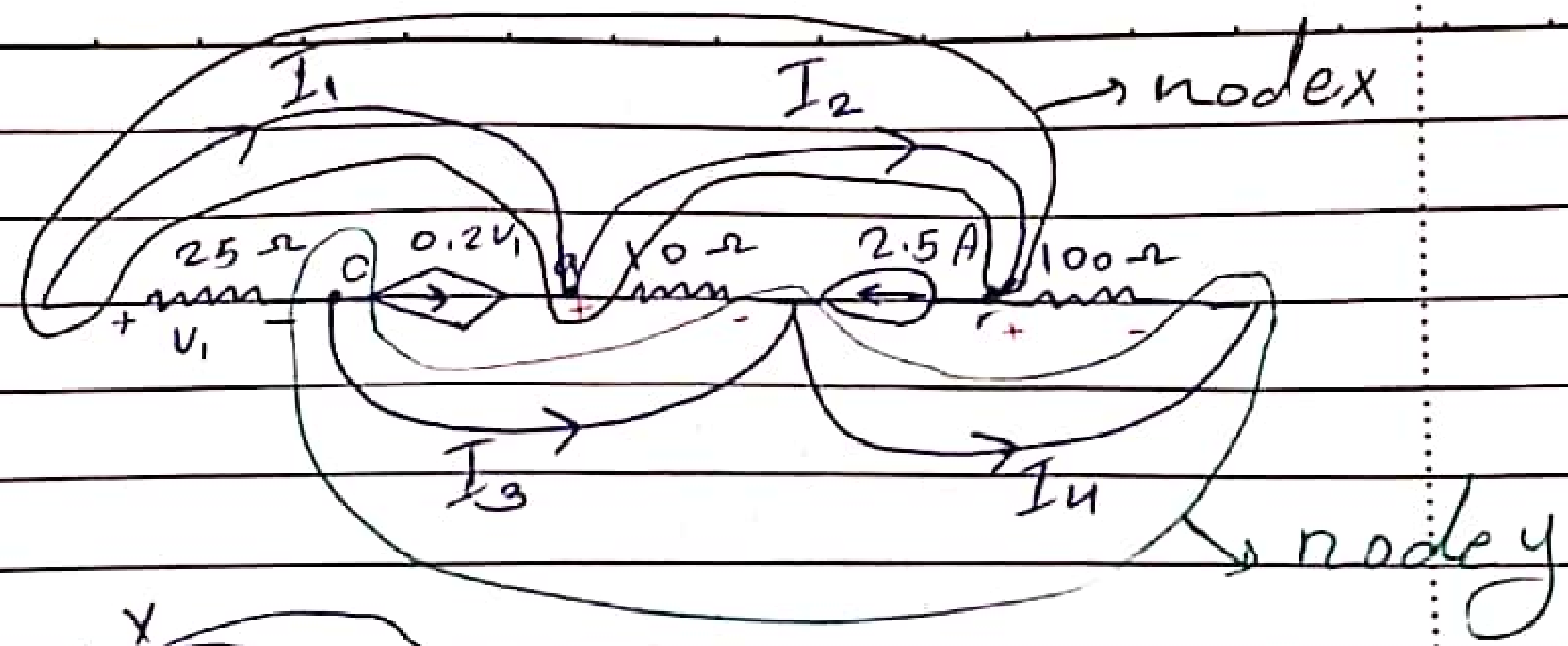
* $P_{abs(30\Omega)} = I^2R = (-0.16)^2\times 30 = 0.768\,W$
* $P_{abs(12V)} = I.V = 0.16\times 12 = 1.92\,W.$
* $8\Omega = P_{abs} = I^2R = (0.16)^2\times 8 = 0.2048\,W$
* $7\Omega = P_{abs} = I^2R = (0.16)^2\times 7 = 0.1792\,W$
* $4V_x \rightarrow P_{abs} = IV = (-0.16)(4\times -30I) = -3.072\,W.$

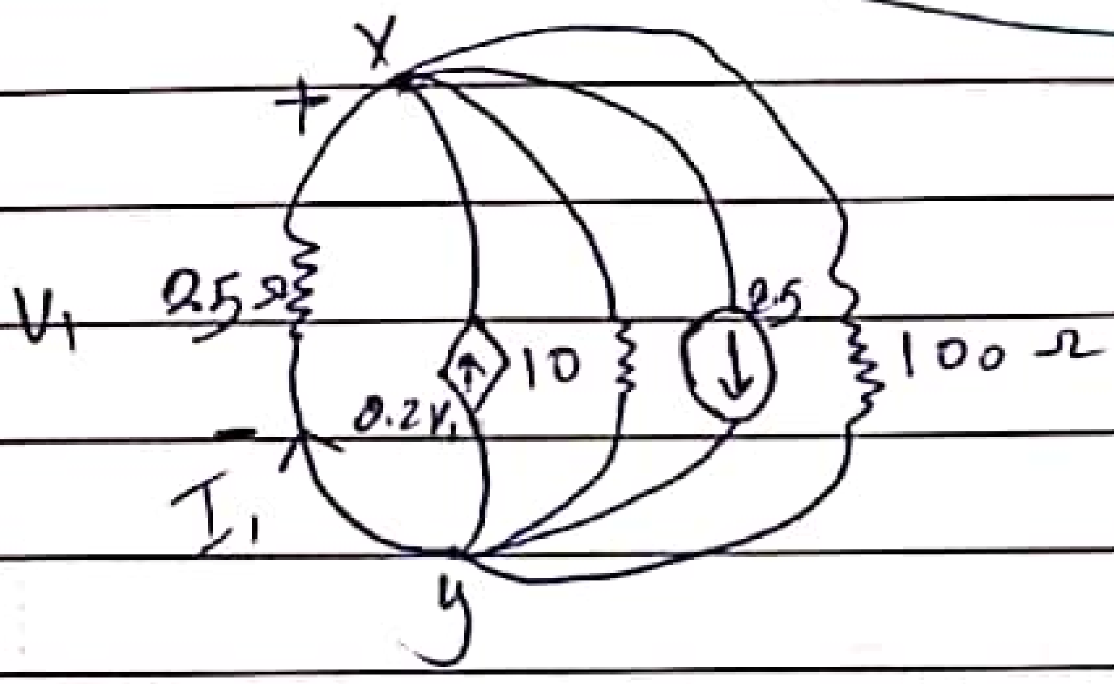
Ex



$P_{5A} = -250W$ / $P_{10\Omega} = 250W$ / $P_{1A} = 50W$ / $P_{6A} = -300W$

Ex





KCl at node x = $\frac{V_1}{25} - 0.2V_1 + \frac{V_1}{10} + 2.5 + \frac{V_1}{100} = 0$

$$V_1 = 50V$$

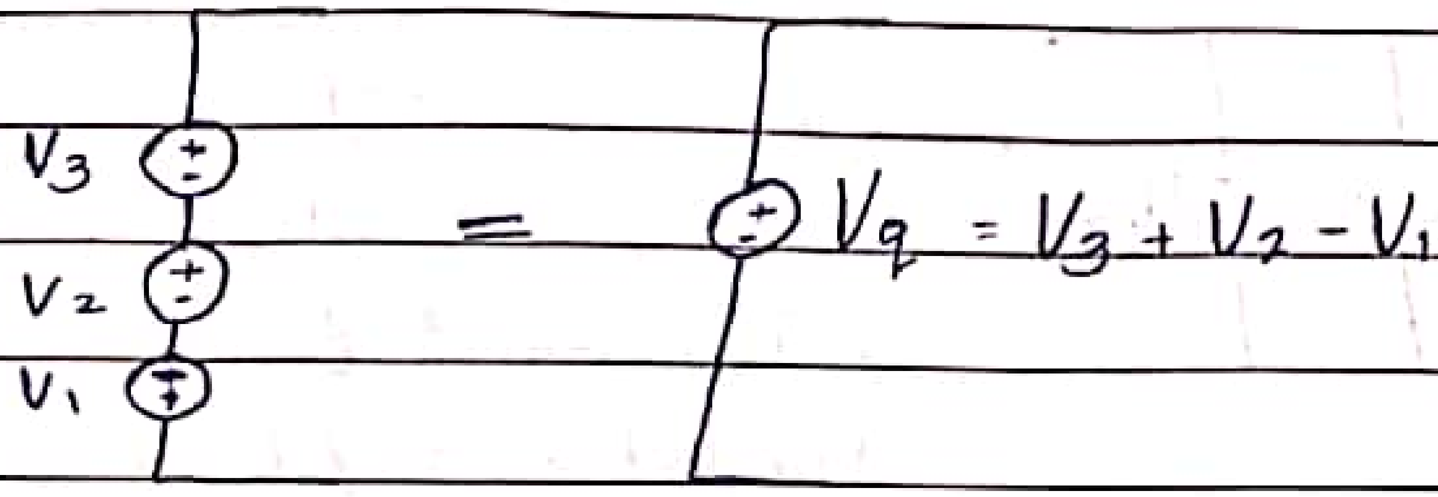
$$I_1 = \frac{-50}{25} = -2A$$

KCl at ~~node~~ a $\rightarrow I_1 + 0.2V_1 = \frac{V_1}{10} + I_2 \rightarrow I_2 = 3A$

KCl at c $\rightarrow I_1 + 0.2V_1 + I_3 = 0 \rightarrow I_3 = -8A$

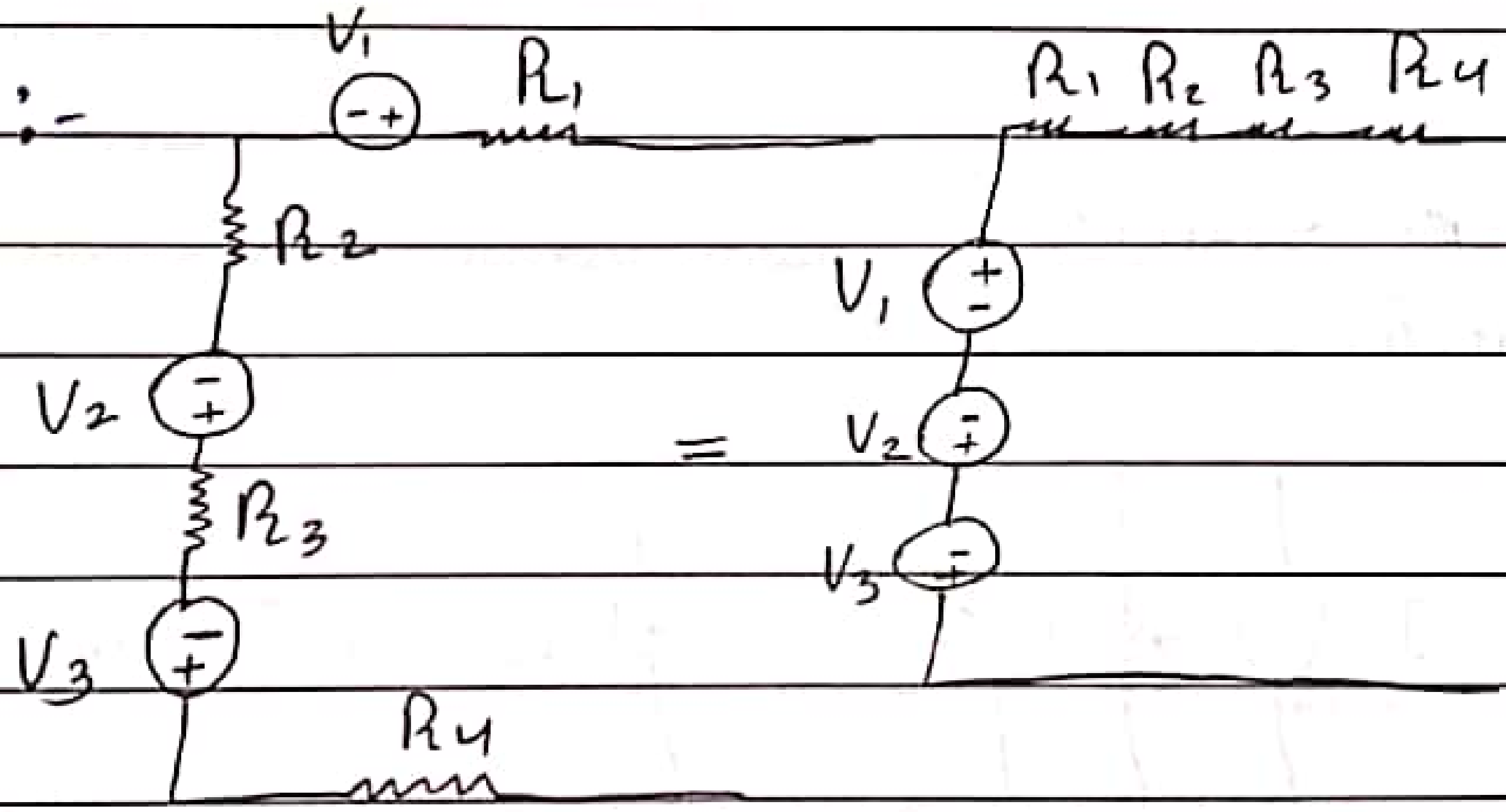
KCl at b $\rightarrow I_2 + I_4 = 2.5 \rightarrow I_4 = -0.5A$

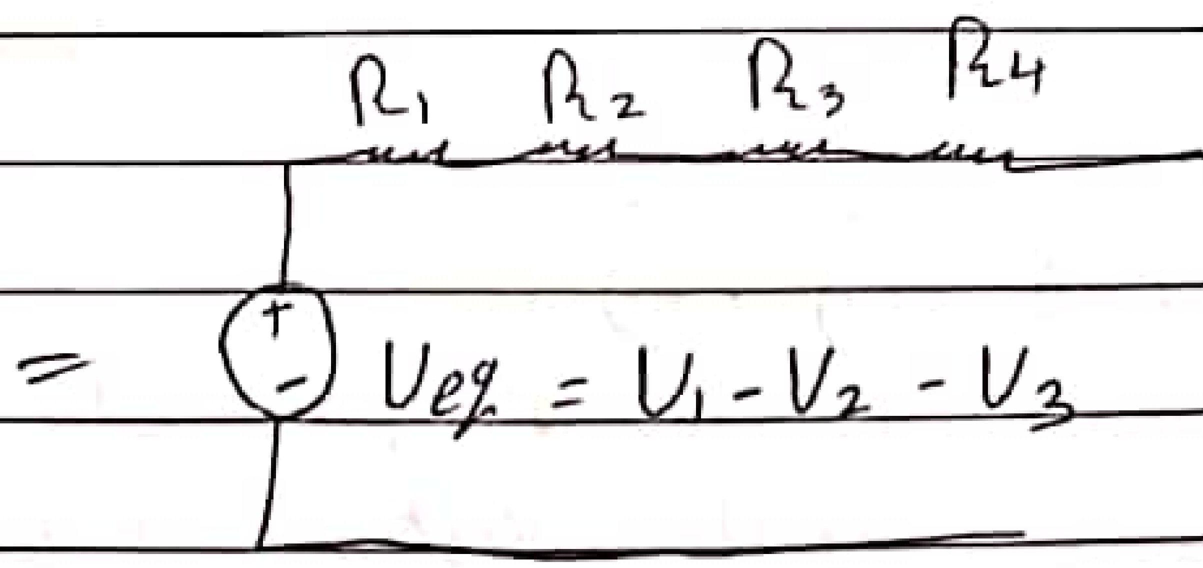
* Series and Parallel connections for sources.
  * Voltage sources



$V_q = V_3 + V_2 - V_1$

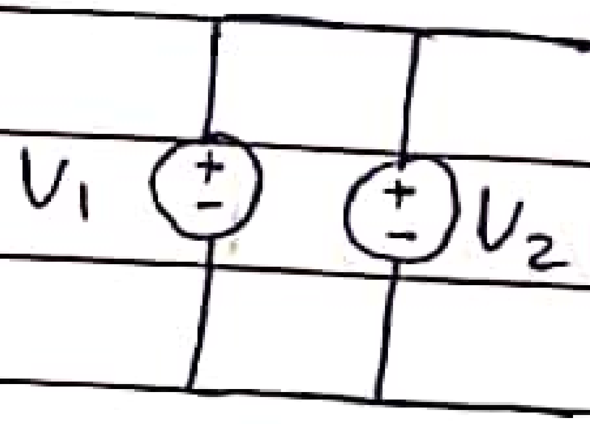
Ex:-





$V_{eq} = V_1 - V_2 - V_3$

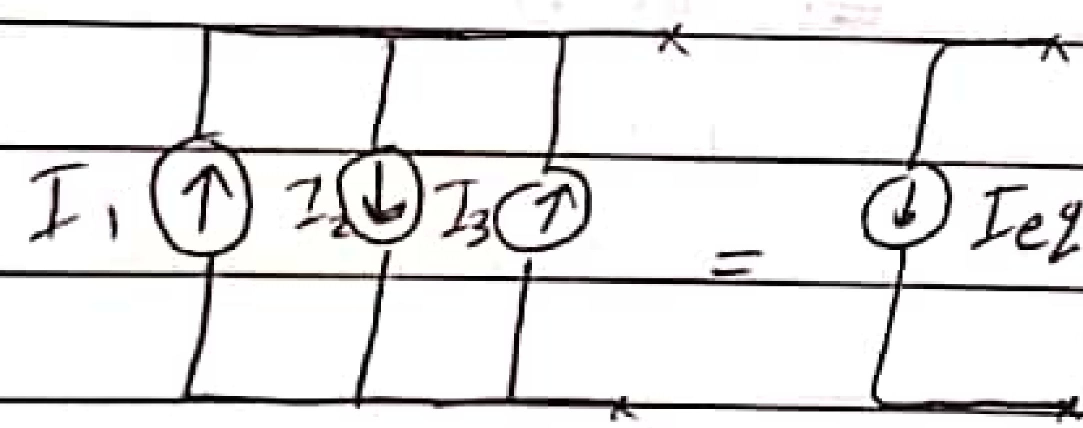
* Parallel



→ impossible!!

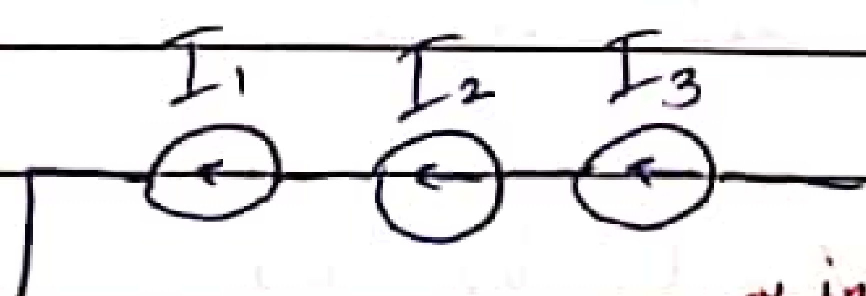
* it's correct only if $V_1 = V_2$.
* To connect two voltage sources parallel doesn't have advantage "Not practical"

* Current sources
  * parallel



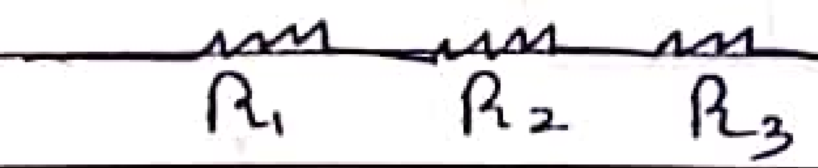
$I_{eq} = I_2 - I_1 - I_3$.

* series



* impossible!!
* it's correct only if $I_1 = I_2 = I_3 \cdots$!
* not practical connection.

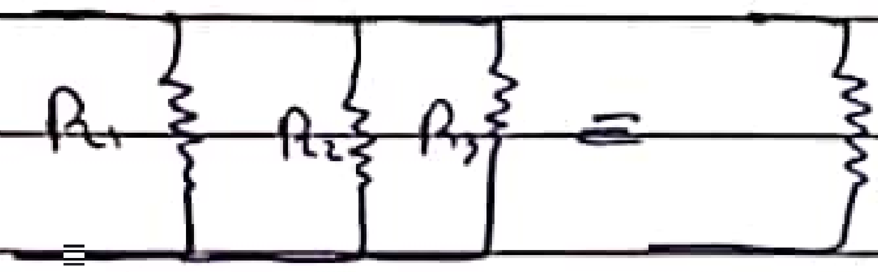
* Series and parallel connections for resistors
  * Series



$R_{eq} = R_1 + R_2 + R_3$

* Parallel

same V

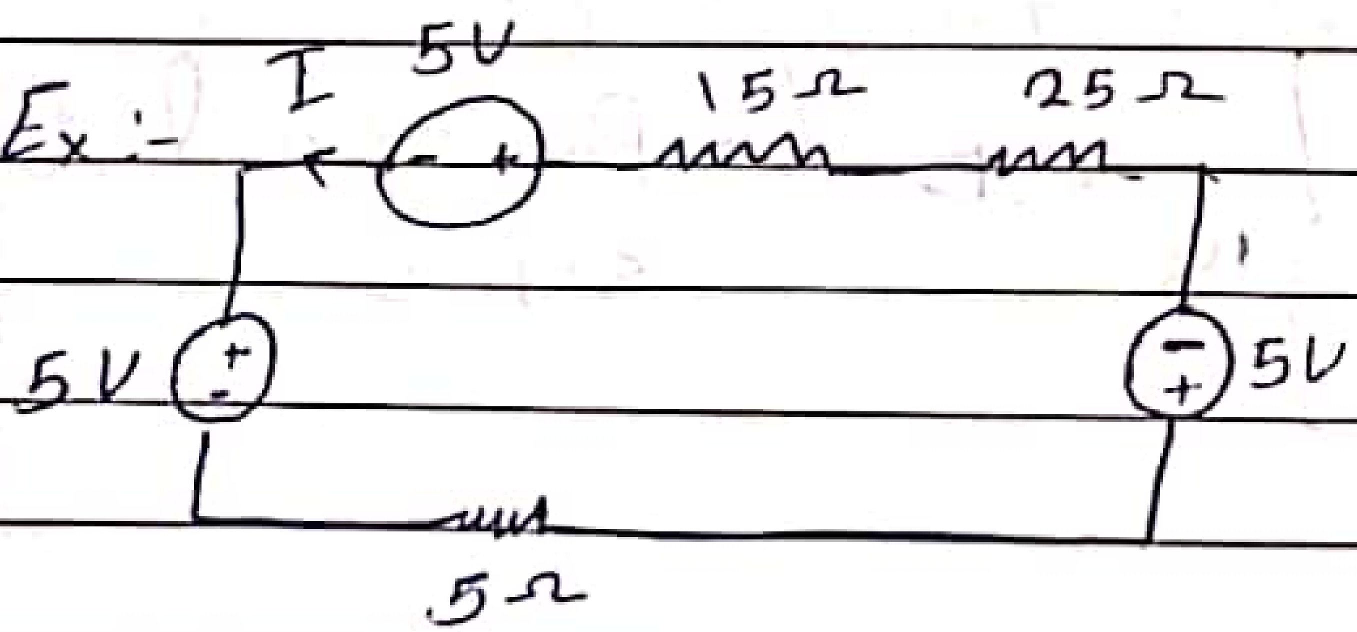


$R_{eq} = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots}$

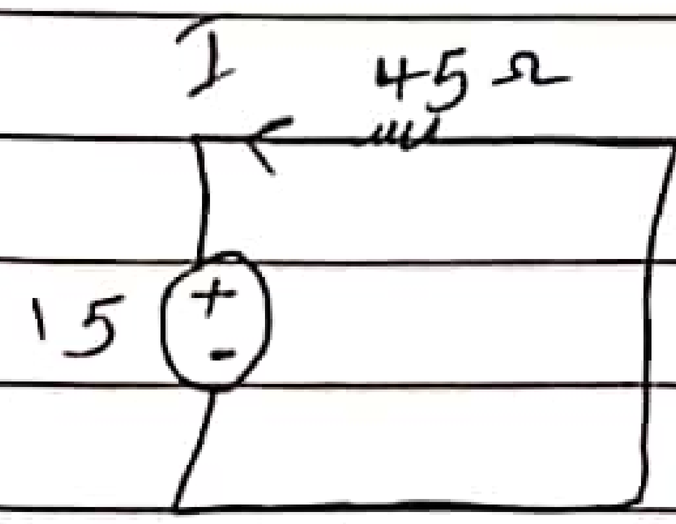
Note :- for 2 resistors in parallel

$R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2}$

Ex :-



simplify this circuit to one source and one resistor and find I



$I = \dfrac{V}{R} = \dfrac{-15}{45} \, A$

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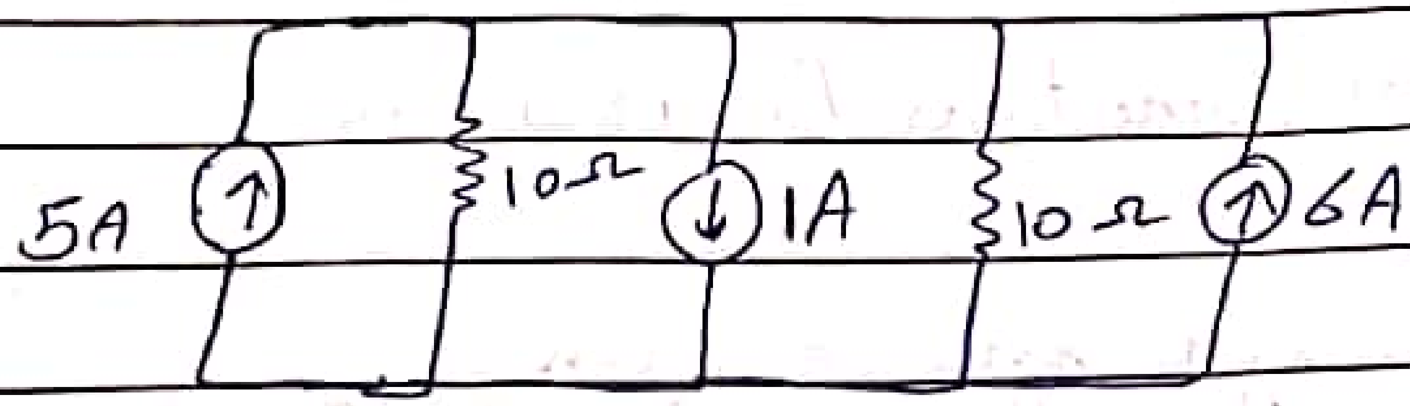
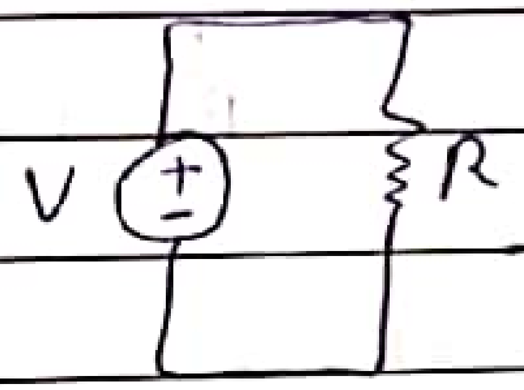
Ex

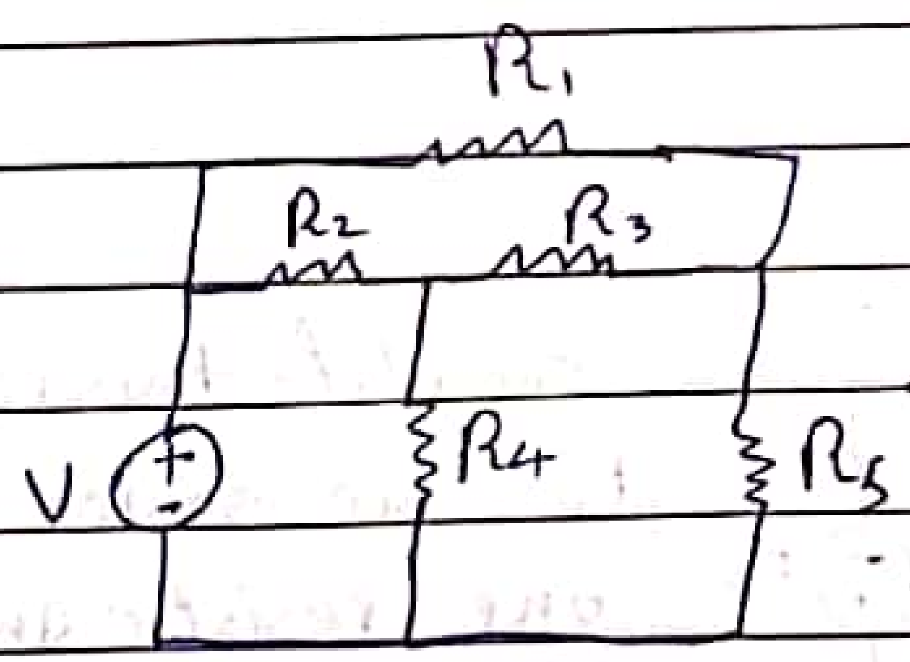


Sol:- $I = 10A$ $R = 5\Omega$

Ex



→ Parallel and series at the same time.



→ No Parallel and No series.

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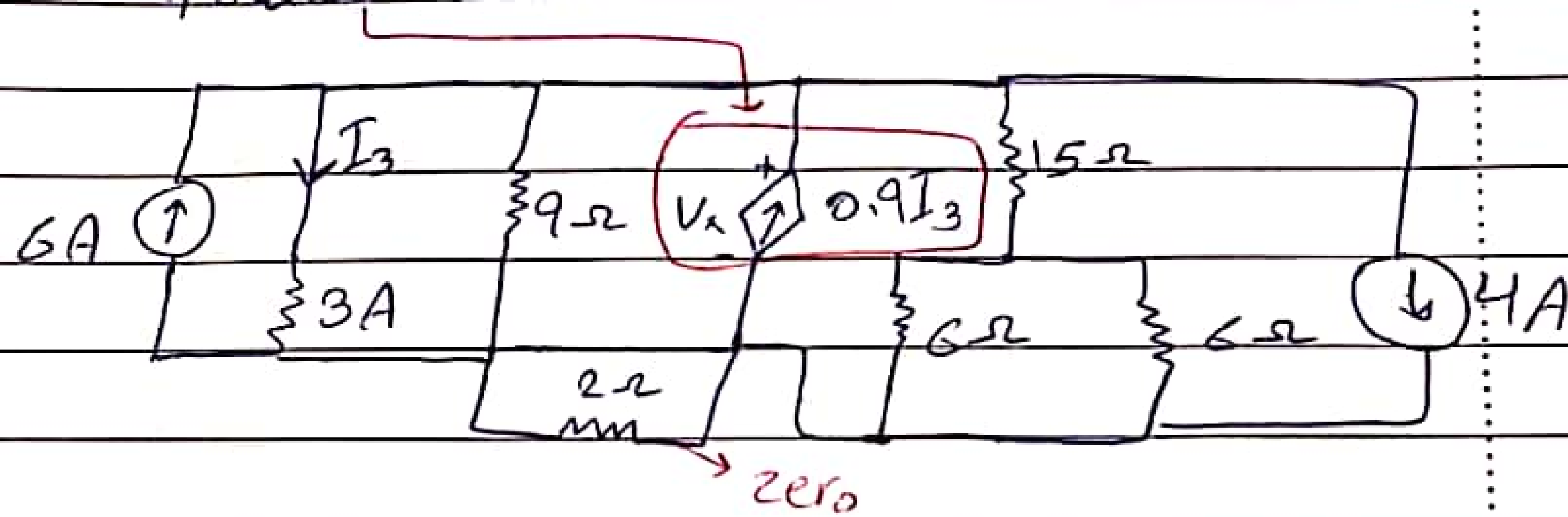
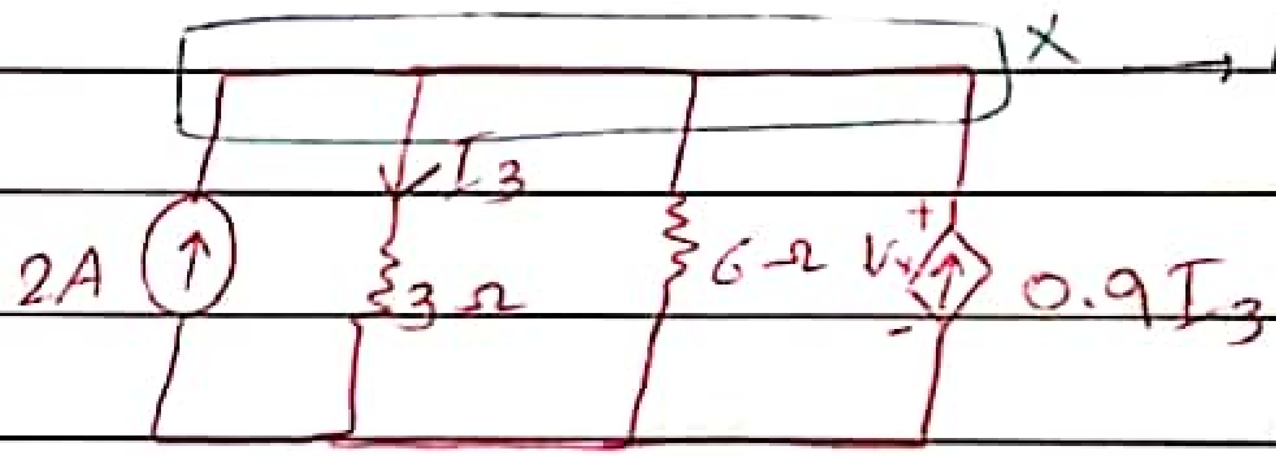
Ex find the power and voltage of the dependent source.





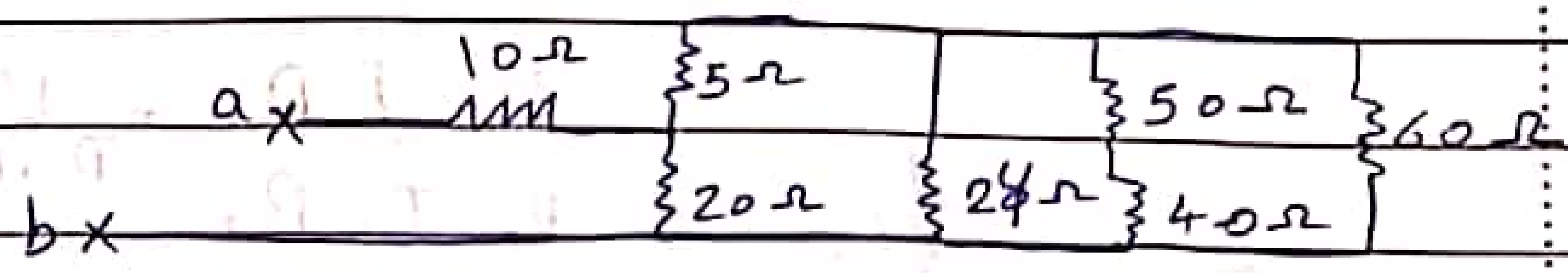
KCL: $2 + 0.9I_3 = I_3 + \frac{V_x}{6}$

$$2 + 0.9\,\frac{V_x}{3} = \frac{V_x}{3} + \frac{V_x}{6}$$

$$V_x = 10\,V,\quad I_3 = \frac{10}{3}\,A$$

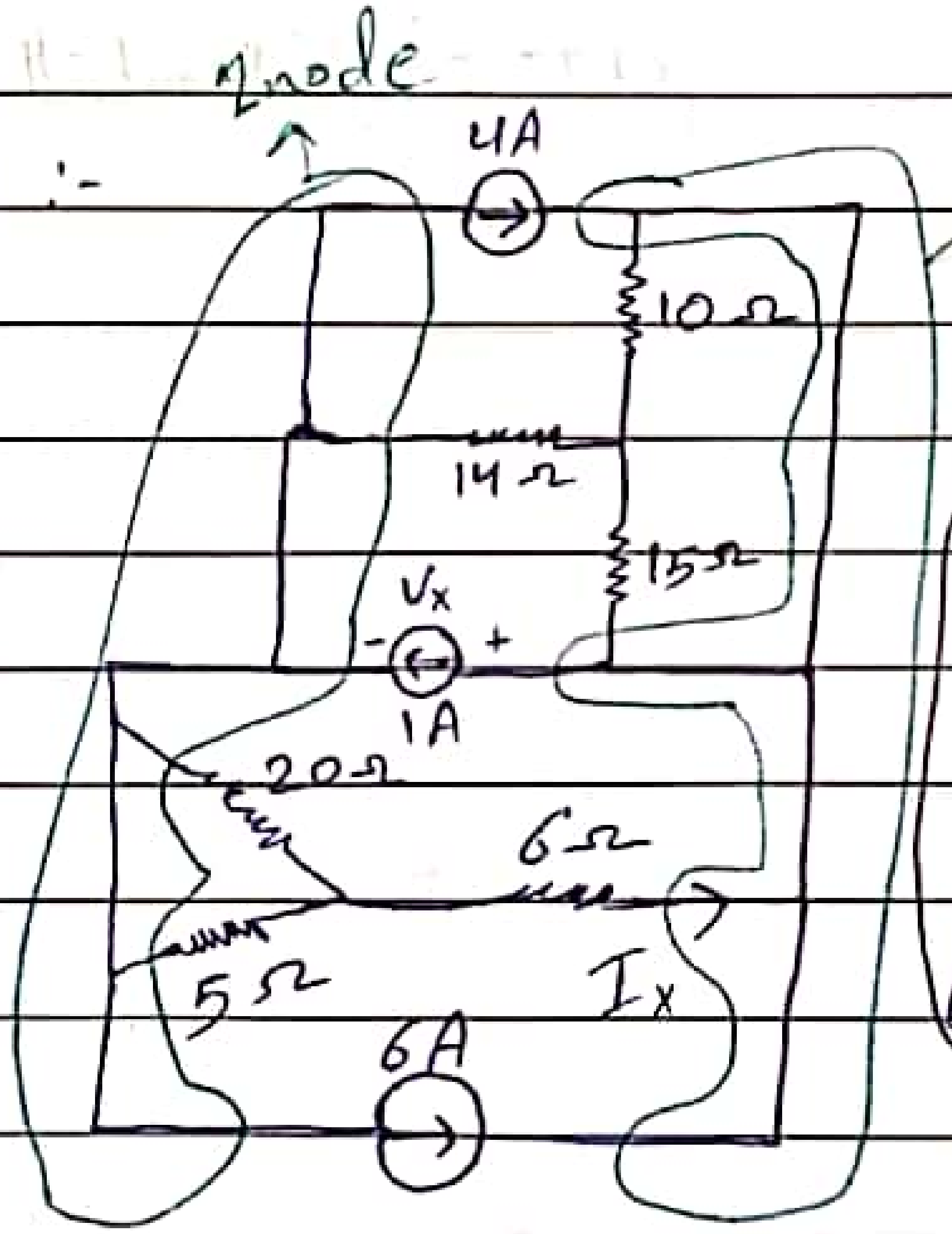
$$P_{abs} = 10 \times (-0.9 I_3) = -30\,W.$$

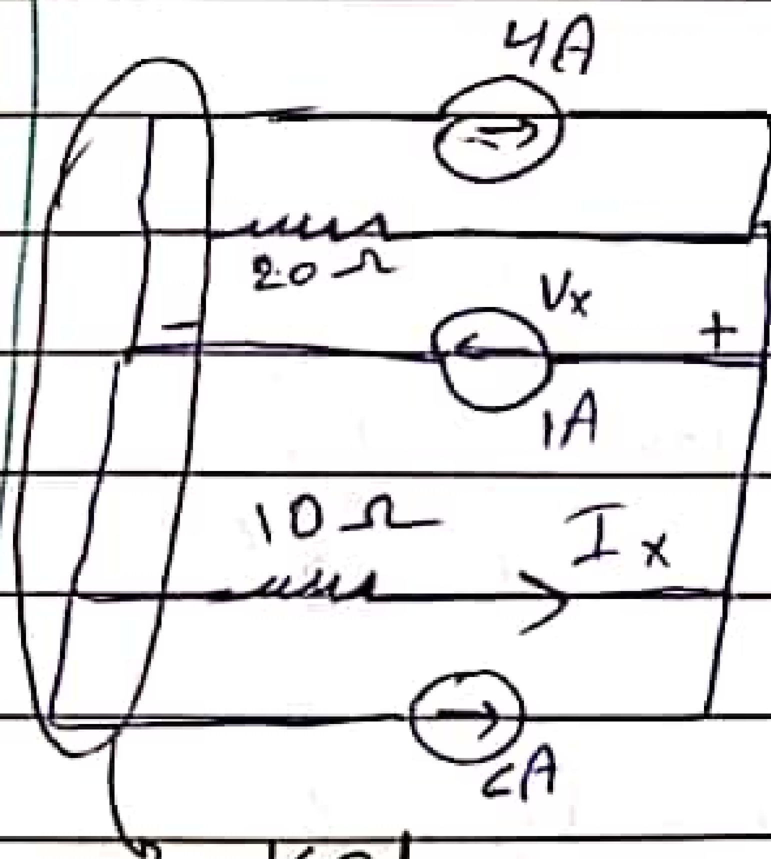
Ex :- find Req.



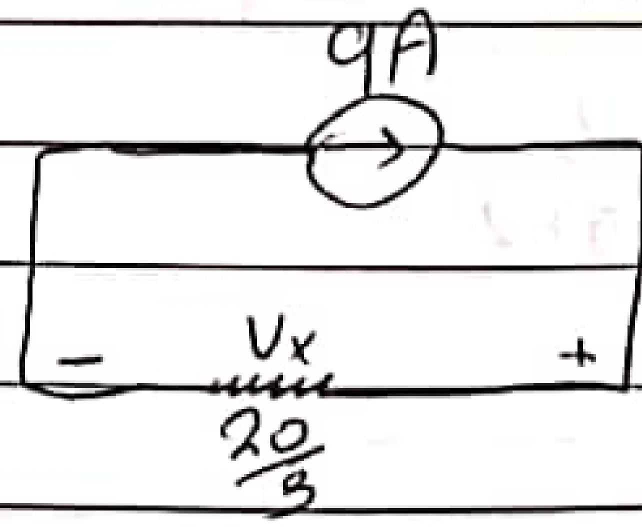
$$R_{eq} = (((50+40)\parallel 60 \parallel 24)+5)\parallel 20)+10 = 19.85\ \Omega$$

Ex :-





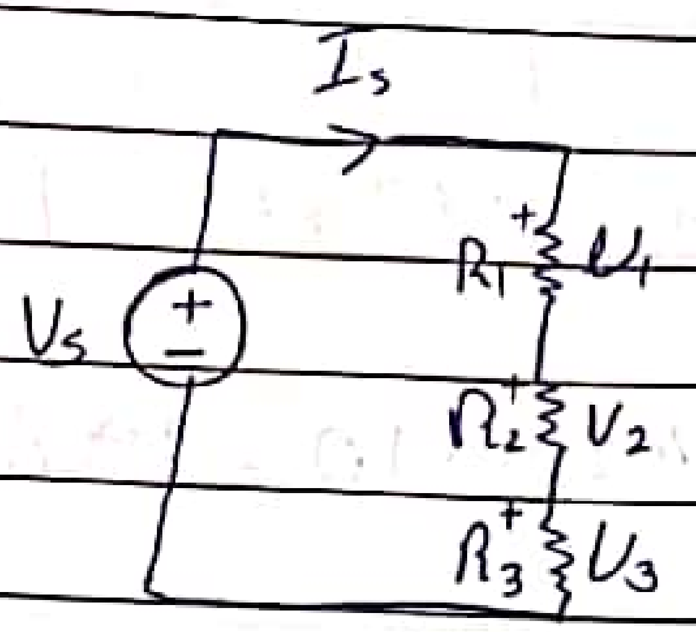
KCl



$V_x = 60\ \text{Volt}$

$I_x = \frac{-60}{10} = -6A$

* Voltage Division Rule :-



$V_1 = I_s R_1 = \frac{V_s}{R_1+R_2+R_3} = R_1$

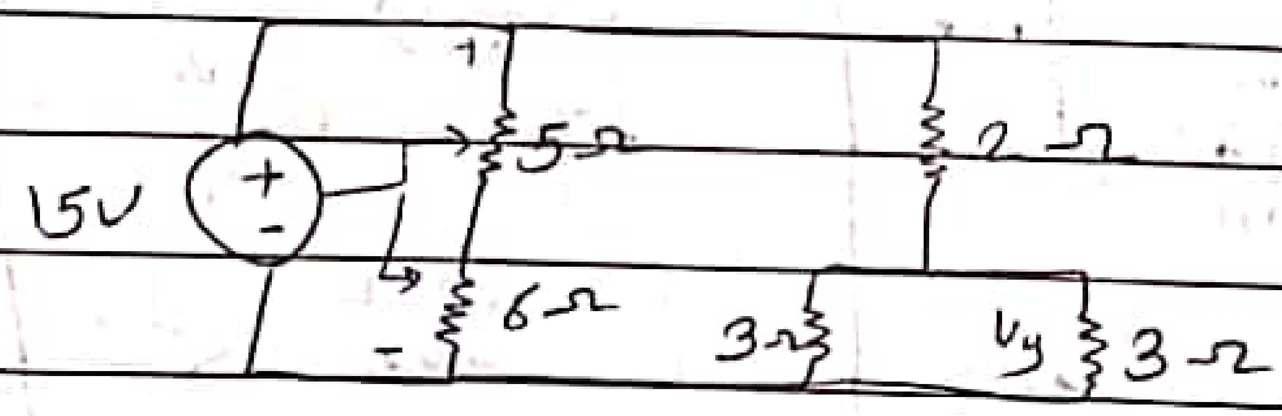
$V_2 \; I_s R_2$

$V_3 = I_s R_3$

* المقاومة الأكبر تأخذ V أكبر

$$V_K = V_s \frac{R_K}{R_1+R_2+R_3\ldots}$$

Ex :-



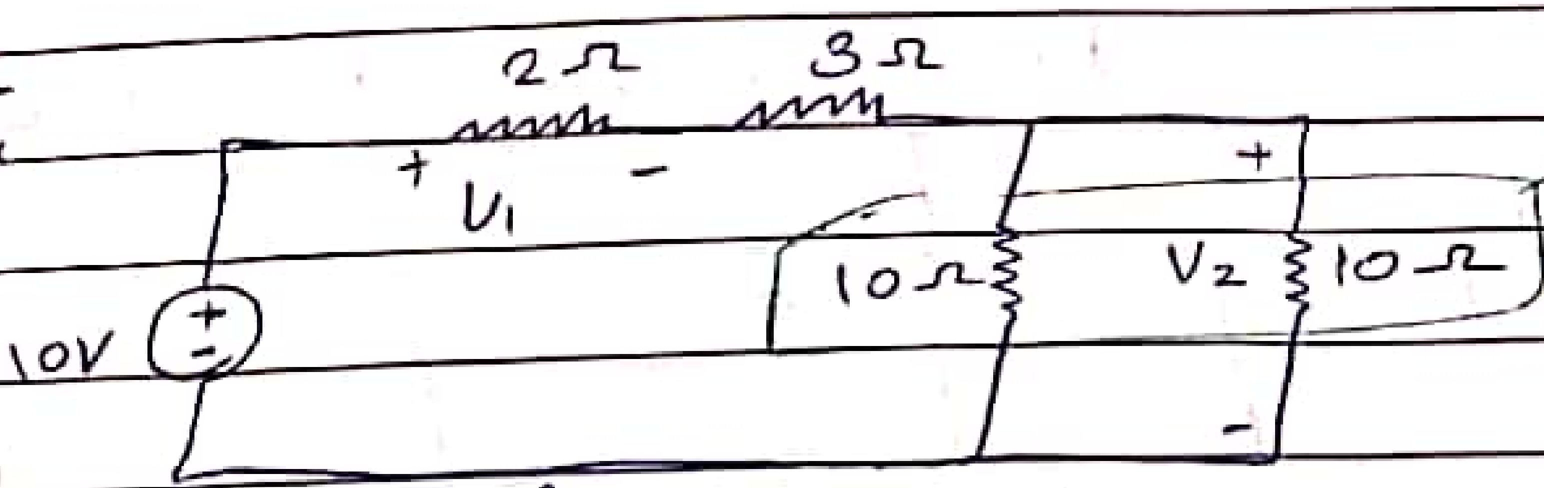
$$V_x = 15 \cdot \frac{6}{5+6} \text{ V.}$$

Parallel (3Ω, 3Ω)

$$V_y = 15 \cdot \frac{1.5}{1.5+2} \text{ V.}$$

Subject Date No.

Ex

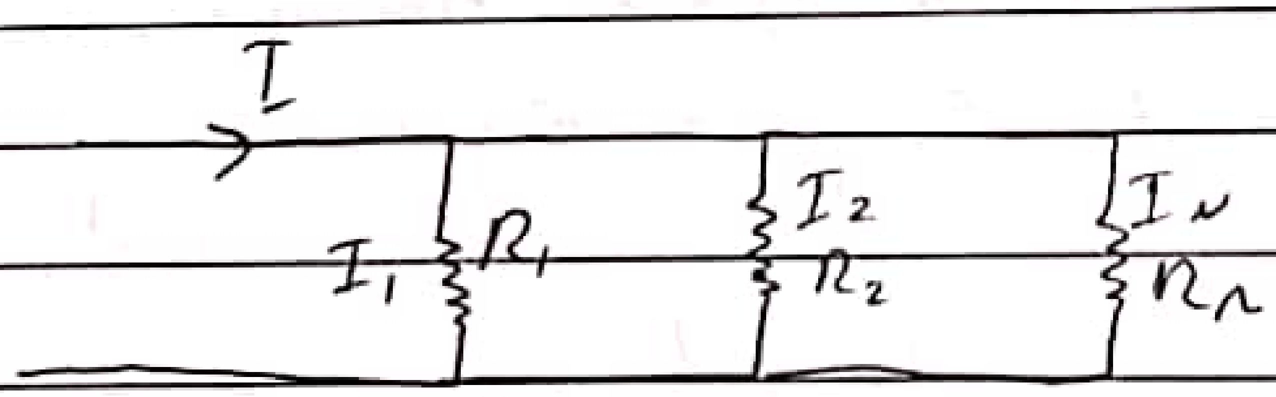


Parallel = 5

find V_1 and V_2 using voltage division rule

$V_1 = 2V$, $V_2 = 5V$

- Current division Rule



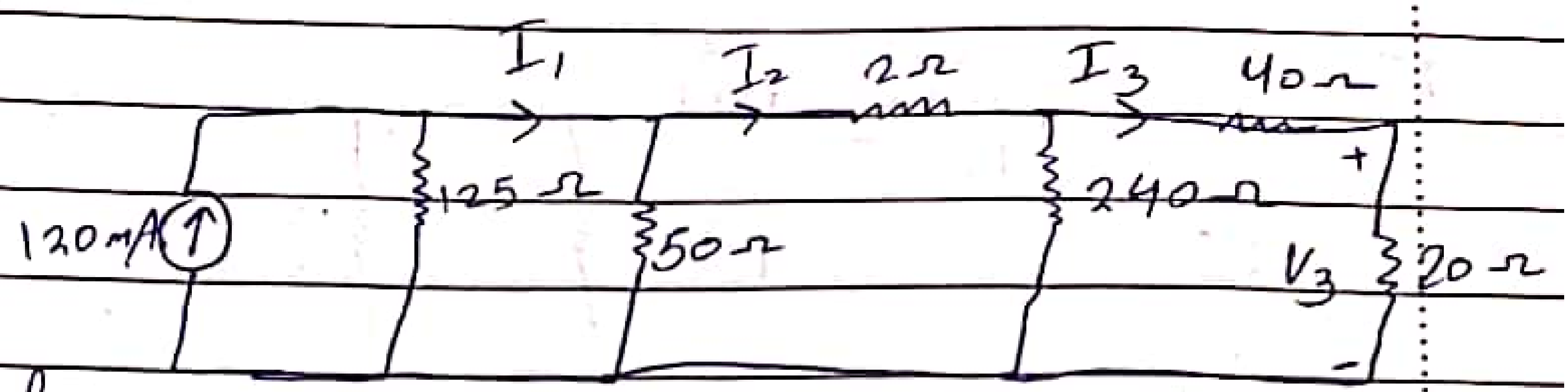
$$I_K = \frac{\frac{1}{R_K}}{\frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_n}}$$

for two resistors

$$\boxed{I_1 = I \frac{R_2}{R_1 + R_2} \quad , \quad I_2 = I \frac{R_1}{R_1 + R_2}}$$

Subject Date No.

Ex



find I_1, I_2, V_3

$$R_{eq} = [((20+40)//240)+2]//50 = 25\Omega$$

$$I = 120 \times \frac{125}{125+25} = 100mA$$

$$R_{eq} = ((20+40)//240)+2 = 50\Omega$$

$$I_2 = I_1 \frac{50}{50+50} = 50mA$$

$$I_3 = I_2 \frac{240}{240+60} = 40mA$$

$$V_3 = 40\times10^{-3} \times 20 = 0.8V.$$

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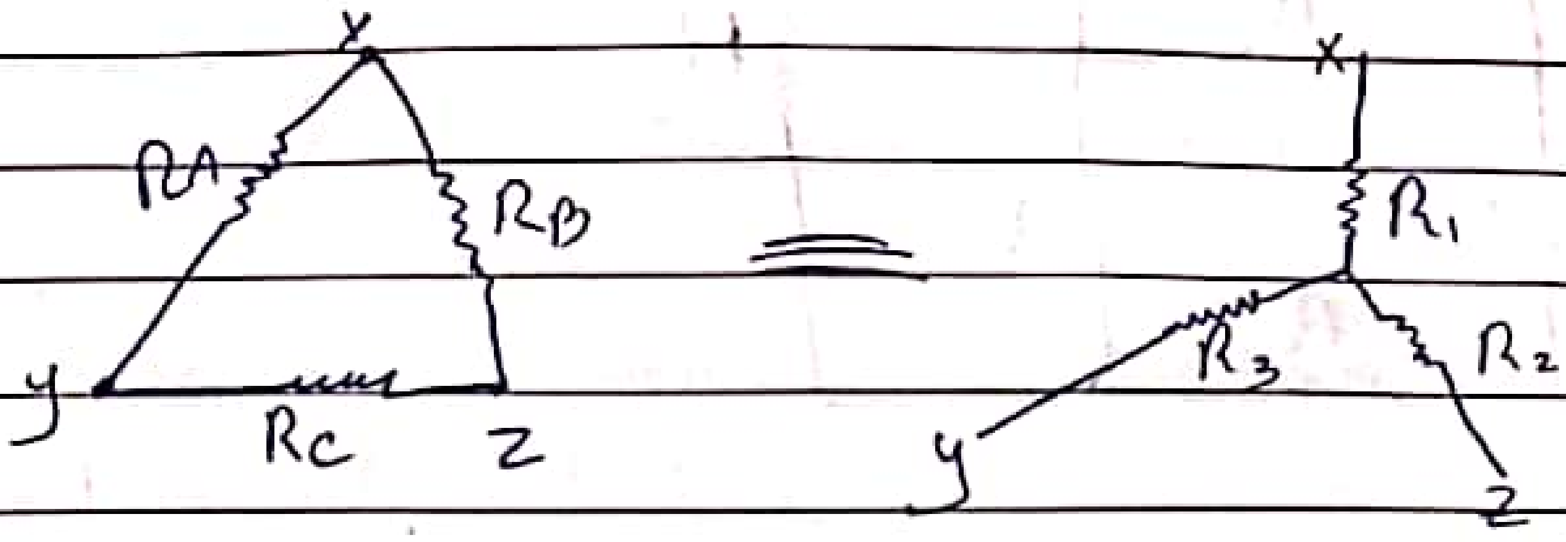
- $\Delta \rightarrow Y$ Conversion "no series, no parallel"



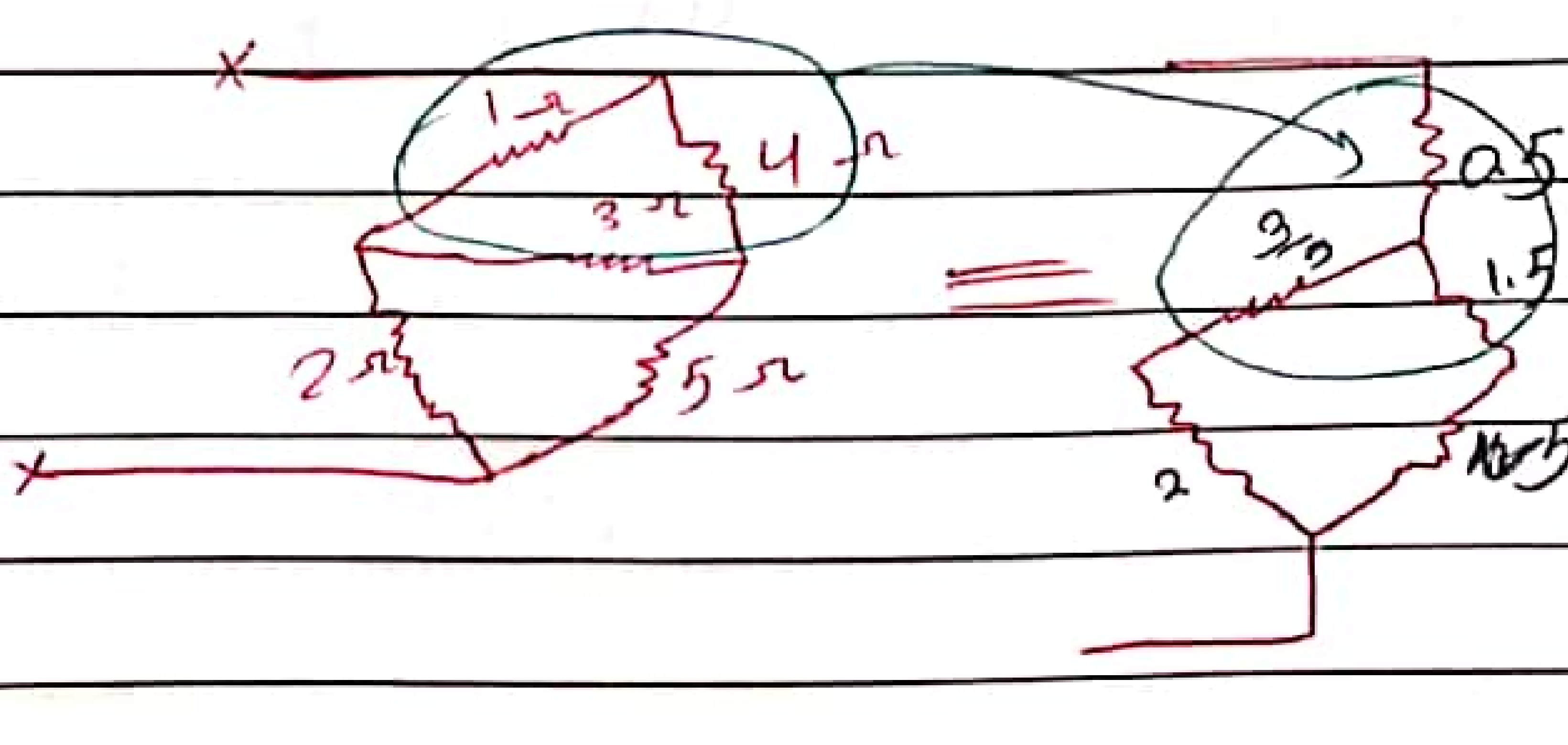
$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}, \quad R_2 = \frac{R_B R_C}{R_A + R_B + R_C}, \quad R_3 = \frac{R_A R_C}{R_A + R_B + R_C}$$

- $Y \rightarrow \Delta$ Conversion

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}, \quad R_B = \frac{R_1R_2 + R_2R_3 + R}{R_3}$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

Ex :-

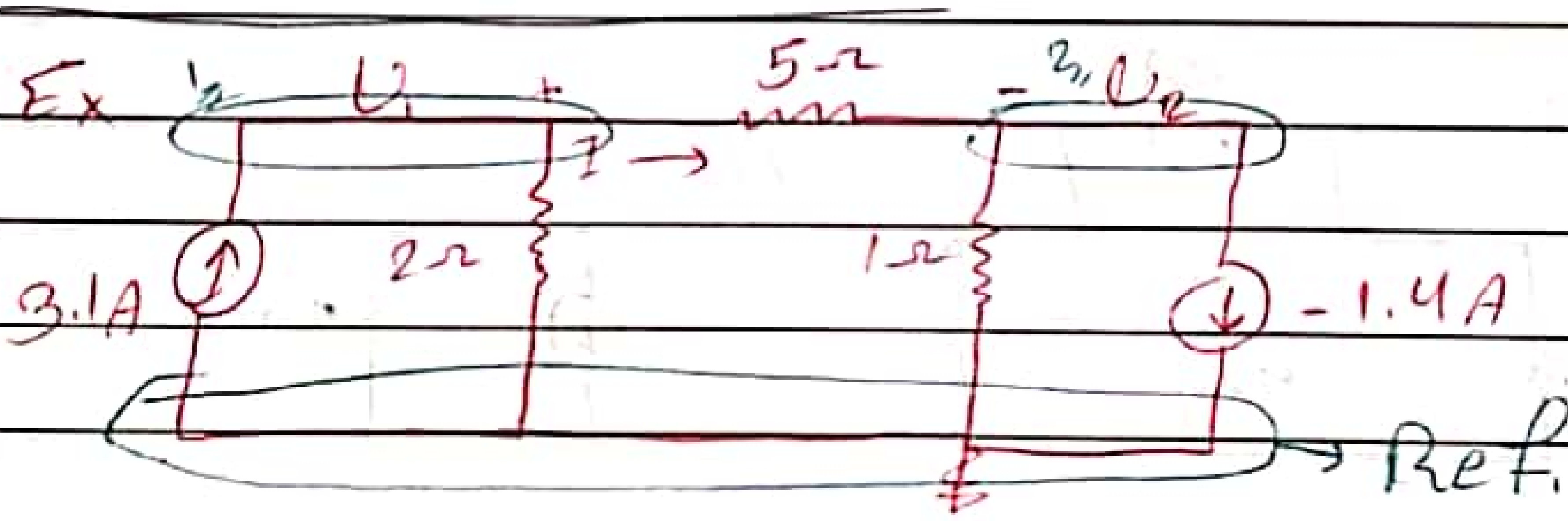


* Advanced Techniques :

- Nodal Analysis :- based on KCl
  - * Steps of Analysis
    1. Determine the nodes in the circuit
    2. select the node that has the higher number of connections as a Reference node.
    3. Apply KCL at the rest of nodes
    4. solve the equations → { no. of equ. = no. of nodes − 1 }

\# $\sum I$ out of node = 0

Ex



KCl at node 1 ⇒ $\sum I_{out} = 0 \Rightarrow -3.1 + \frac{V_1 - 0}{2} + \frac{V_1 - V_2}{5}$ ... 1

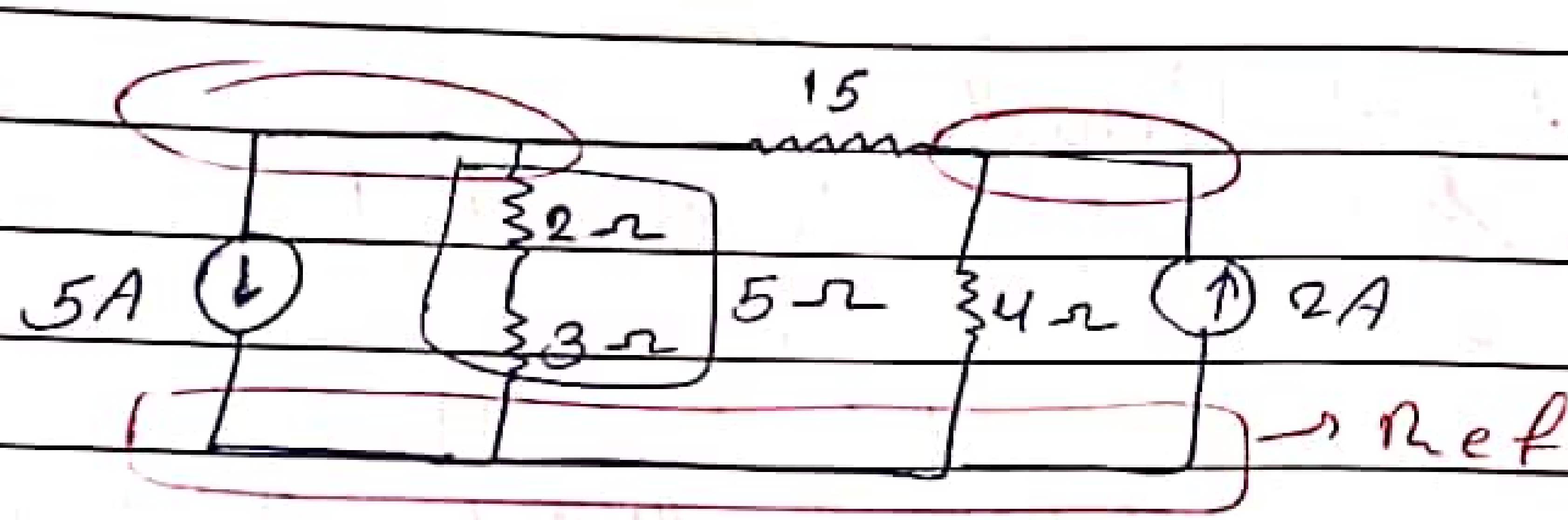
KCl at node 2 ⇒ $-1.4 + \frac{V_2}{1} + \frac{V_2 - V_1}{5} = 0$ ... eq 2

solve 1 and 2 $V_1 = 5V$

$V_2 = 2V$

$I = \frac{V_1 - V_2}{5} = \frac{5-2}{5} = \frac{3}{5}$ A

E_x

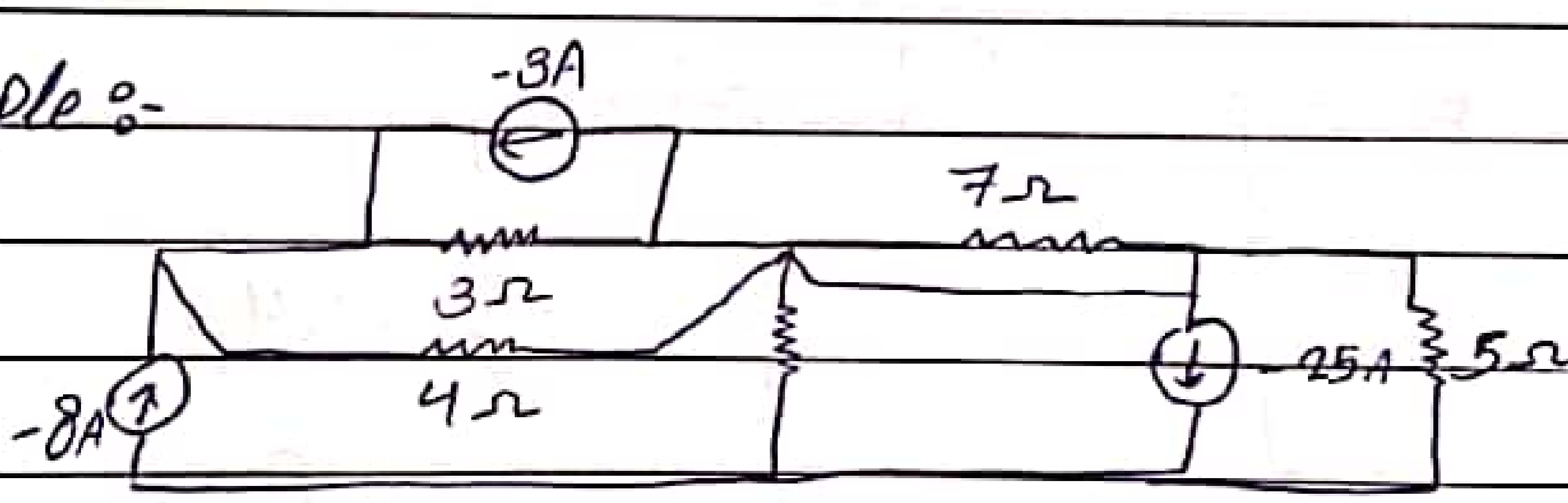


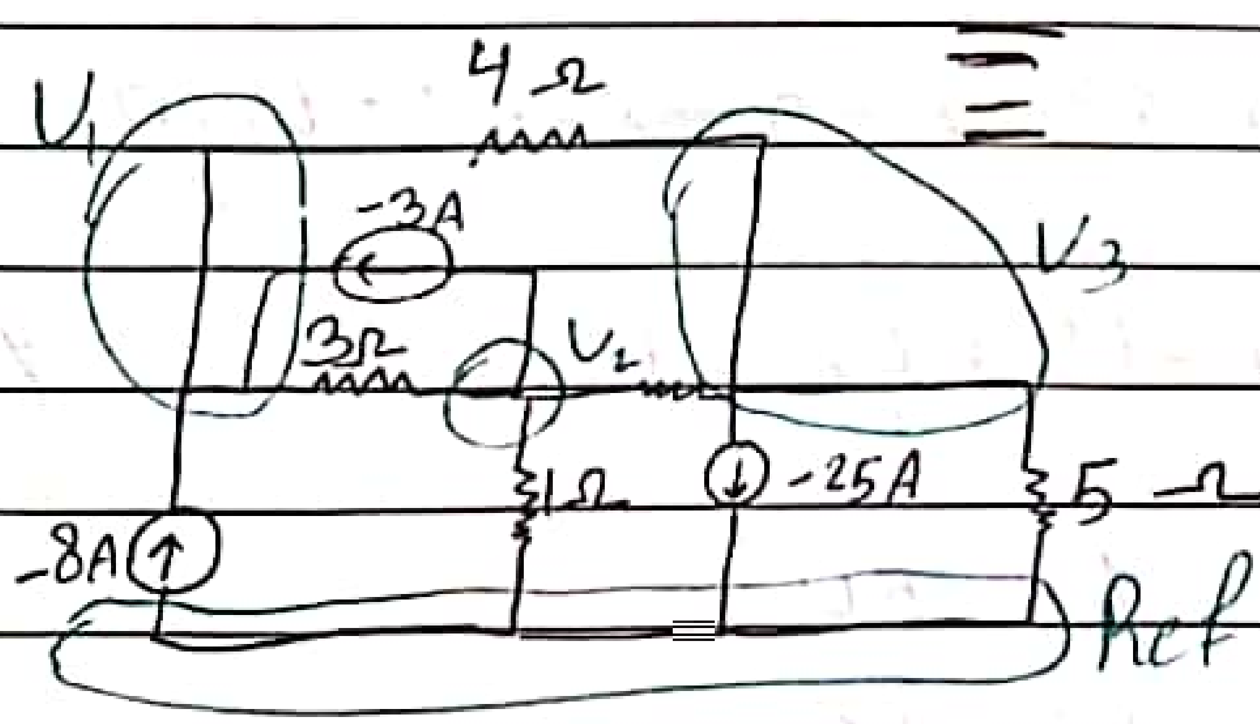
KCl at node 1 = $5 + \frac{V_1}{5} + \frac{V_1 - V_x}{15} = 0 \cdots 1$

KCl at node 2 : $-2 + \frac{V_x}{4} + \frac{V_x - V_1}{15} = 0 \cdots 2$

* لما تتساوى عدد الconnections في ال nodes بنختار، اي واحدة.

Example :-





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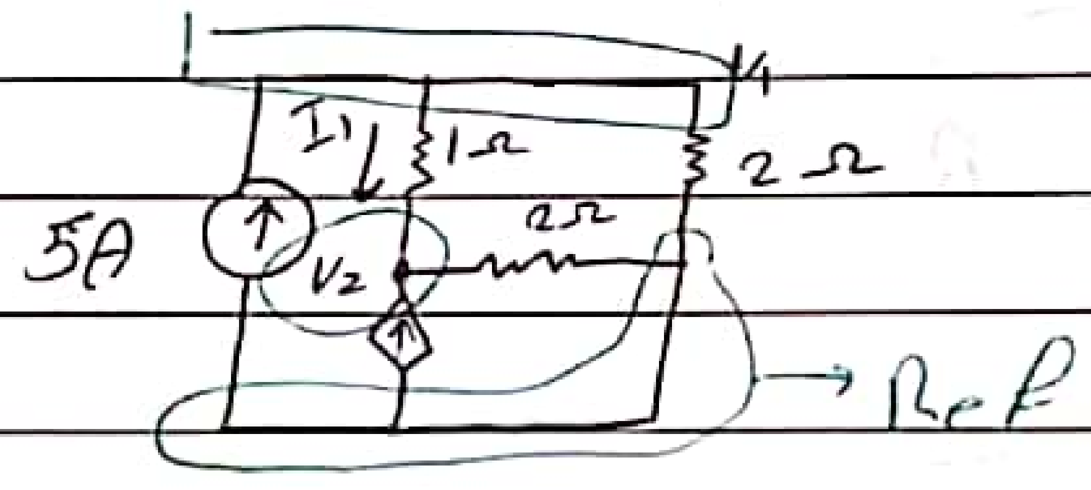
→ KCl at node 1 :- $8 + 3 + \frac{V - V}{3} + \frac{V - V_3}{4} = 0$ --- (1)

KCl at node 2 :- $-3 + \frac{V_2}{1} + \frac{V_2 - V_1}{3} + \frac{V_2 - V_3}{7} = 0$ - (2)

KCl at node 3 :- $-25 + \frac{V_3}{5} + \frac{V_3 - V_2}{7} + \frac{V_3 - V_1}{4} = 0$ --- (3)

. طلع يكون ٣ معادلات حلهم مش مطلوب .

Example :-



KCl node 1 : $\frac{V_1}{2} + \frac{V_1 - V_2}{1} - 5 = 0$ --- (1)

KCl node 2 :- $-2I + \frac{V_2}{2} + \frac{V_2 - V_1}{1} = 0$ --- (2)

$I = \frac{V_1 - V_2}{1}$

$V_1 = \frac{70}{9} V$ / $V_2 = \frac{20}{3} V$

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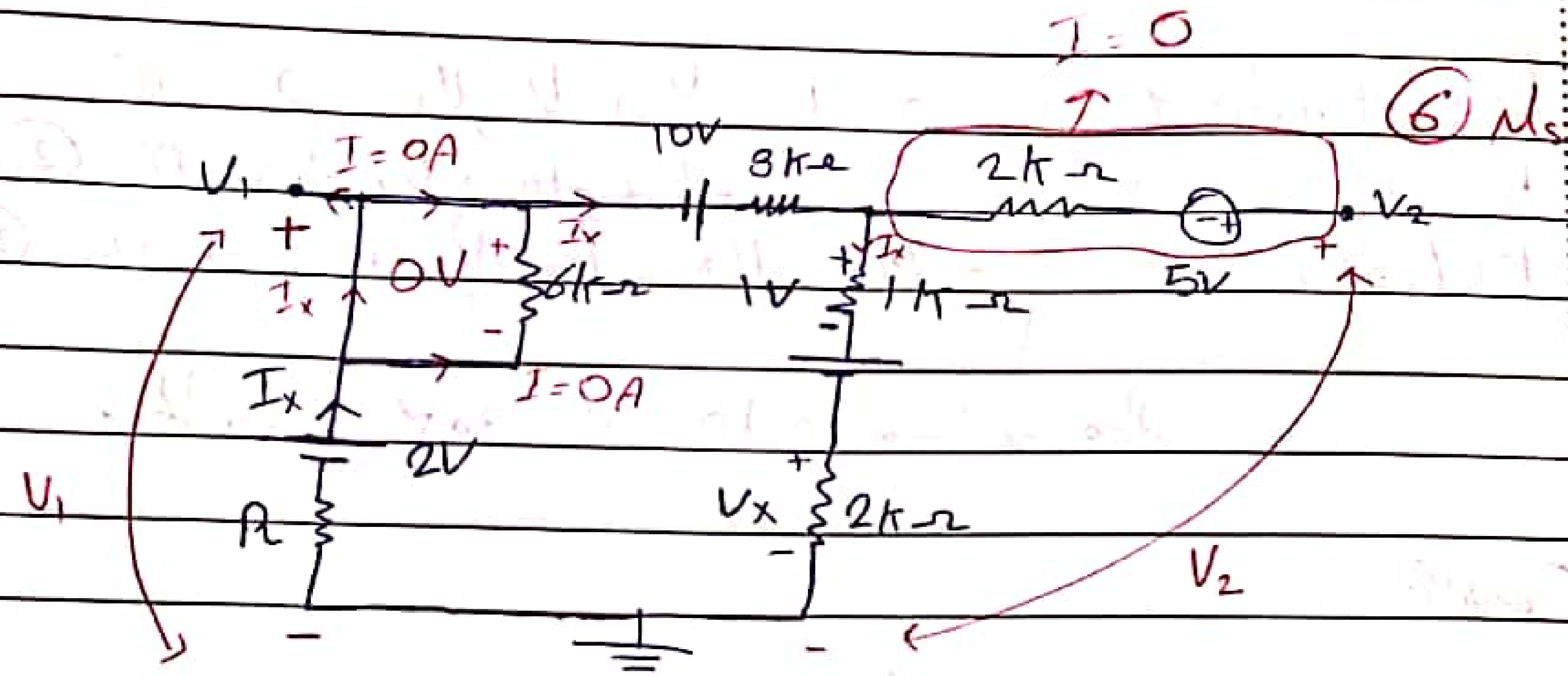


a) find V_1, V_2, I_x, V_x, R

$$I_x = \frac{1V}{1k\Omega} = \boxed{1mA}$$

$$V_x = 2 \times 1 = \boxed{2V}$$

loop

$$-V_1 - 10 + 3 + 1 + 3 + 2 = 0$$

$$V_1 = \boxed{-1\ V}$$

loop

$$V_2 - V_x - 3 - 1 - 5 = 0$$

$$V_2 = \boxed{11V}$$

$$R I_x - 2 - 10 + 3 + 1 + 3 + 2 = 0$$

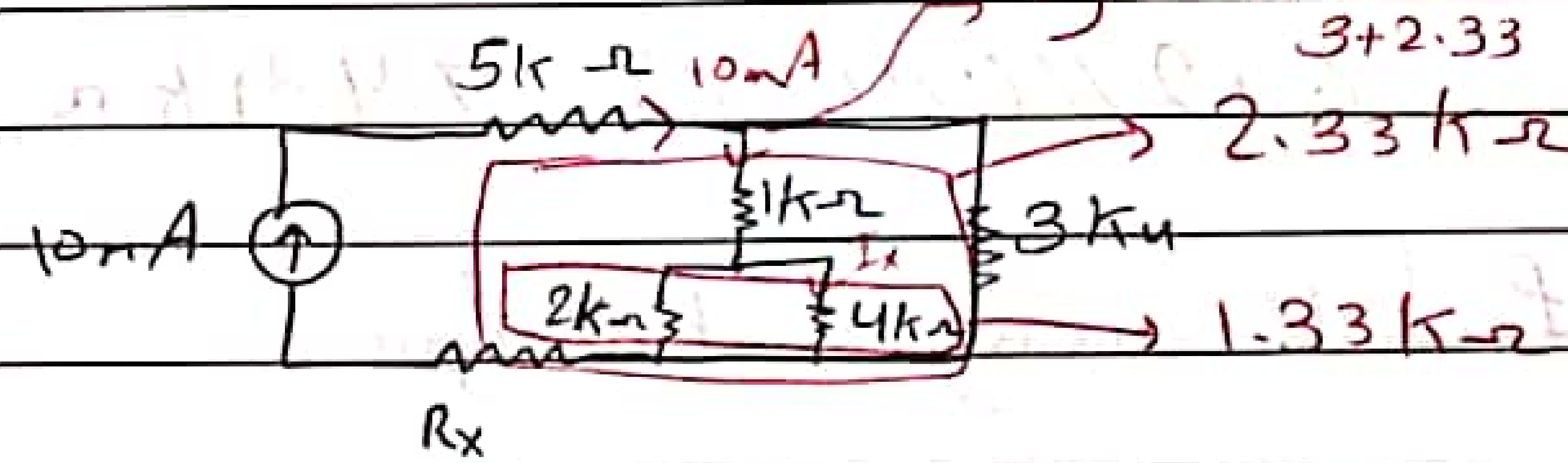
$$R = 3k\Omega$$

b) find P_{abs} by 10V and $P_{generated}$ by 3V source

$$P_{abs} = -1 \times 10 = -10\,mW$$
$$P_{generated} = -P_{abs} = -(1 \times 3) = -3\,mW$$

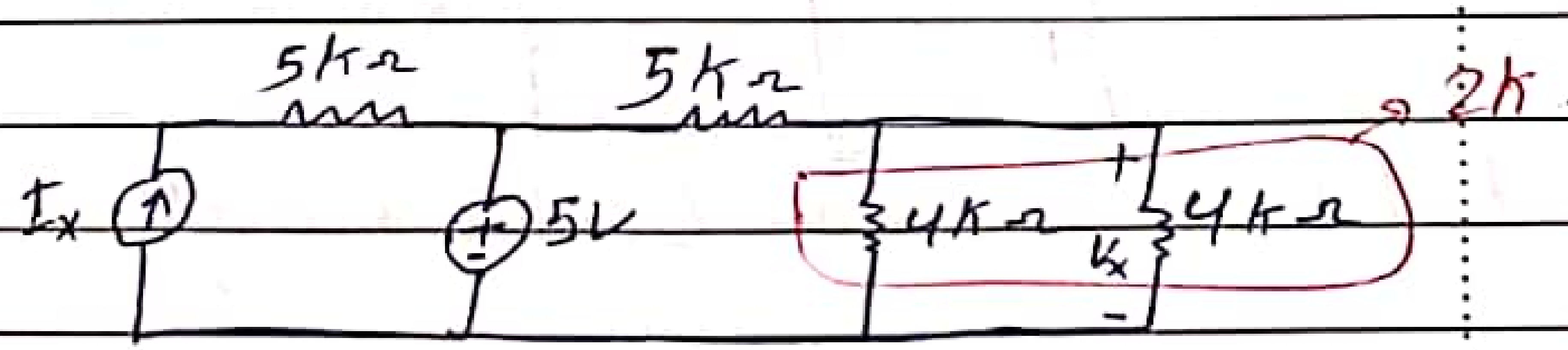
Q:- find I_x using current division rule

$$I_y = 10 \times \frac{3}{3+2.33} = 5.625\,mA$$



$$I_x = I_y \frac{2}{2+4} = 1.875\,mA$$

Q:- find V_x using voltage division rule



$$V_x = 5 \times \frac{2}{2+3} = 2V$$

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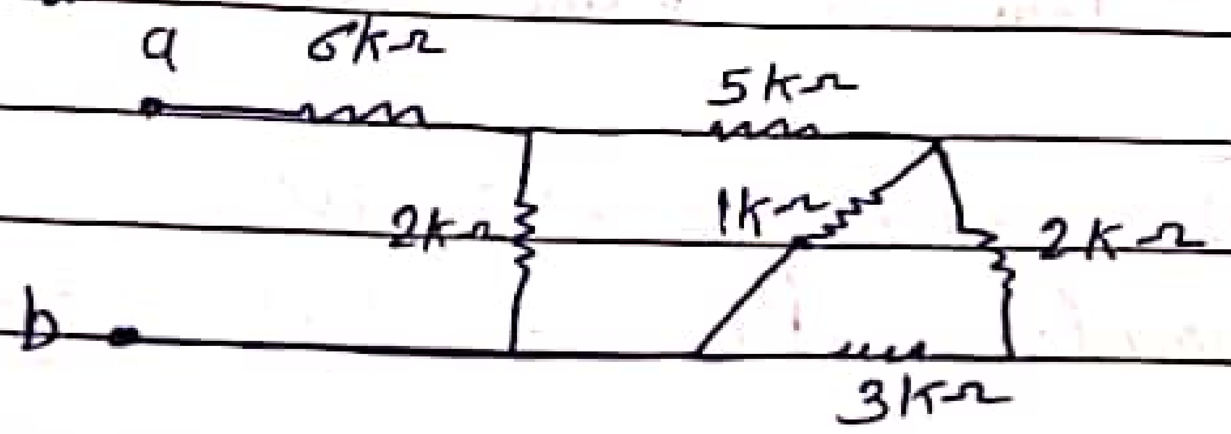
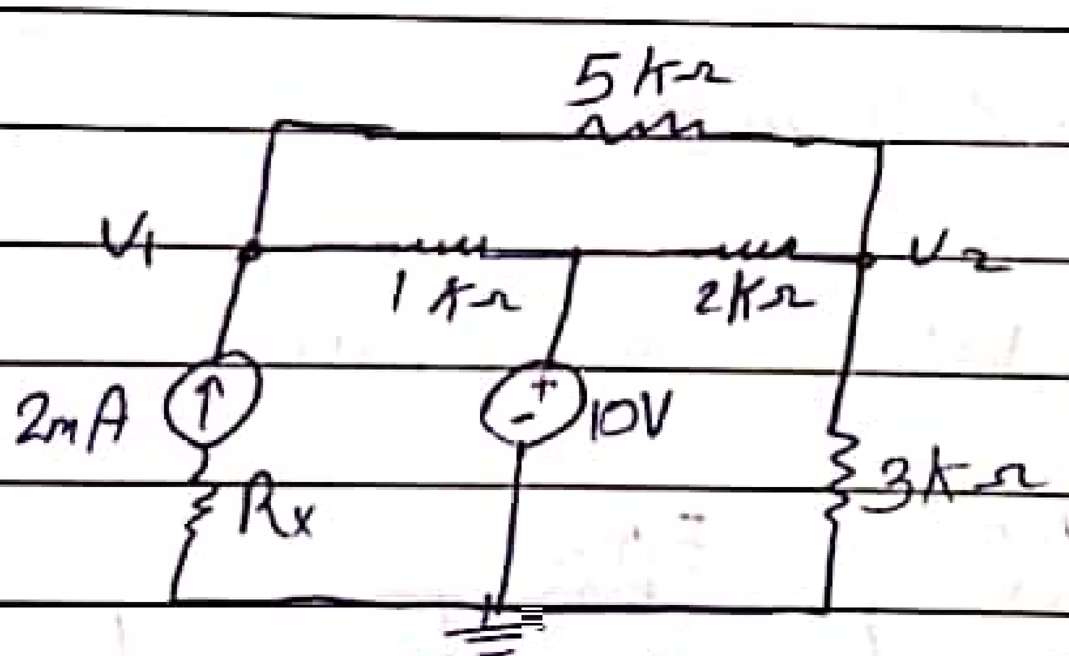
Q:- find R_{eq}.



$\rightarrow$ 2,3 $\rightarrow$ 5kΩ // 1 + 5

$6 + 2 // (1 // 5 + 5) = 7.489\ k\Omega$

Q:- find V_1 using nodal analysis



Kcl V_1 : $-2 + \frac{V_1 - 10}{1} + \frac{V_1 - V_2}{5} = 0$ --- (1)

Kcl V_2 :- $\frac{V_2 - 0}{3} + \frac{V_2 + 10}{2} + \frac{V_2 - V_1}{5} = 0$ --- (2)

$V_1 = 11.163\ V$

$V_2 = 6.978\ V$

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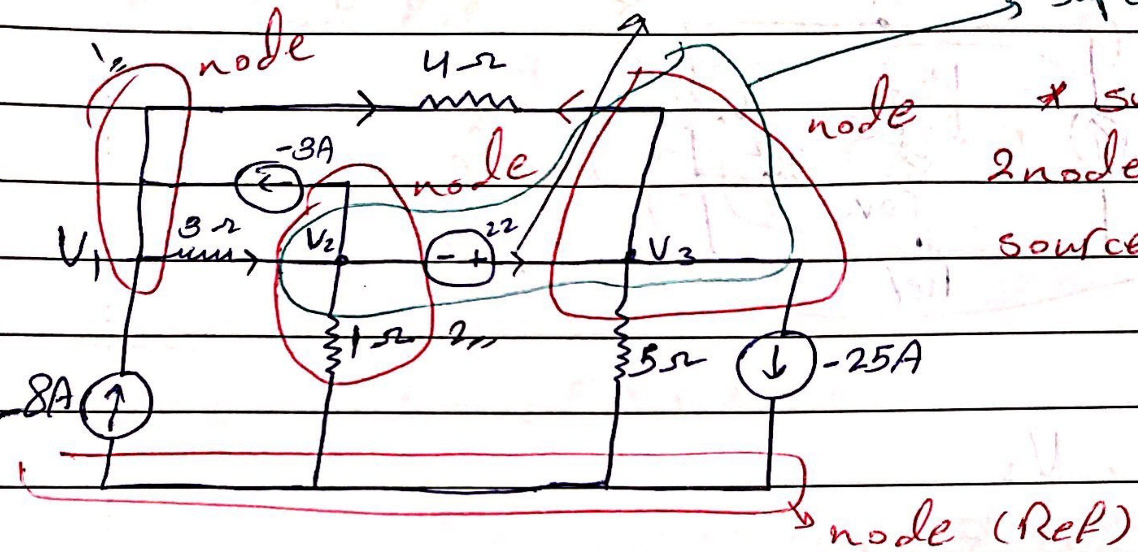
ما بنقدر نكتب قيمة التيار الخارج من node 2 لذلك نلجأ لل supernode



* supernode :- 2 nodes with voltage source between them

KCl node 1 :- $8 + 3 + \frac{V_1 - V_3}{4} + \frac{V_1 + V_2}{3} = 0$ --- (1)

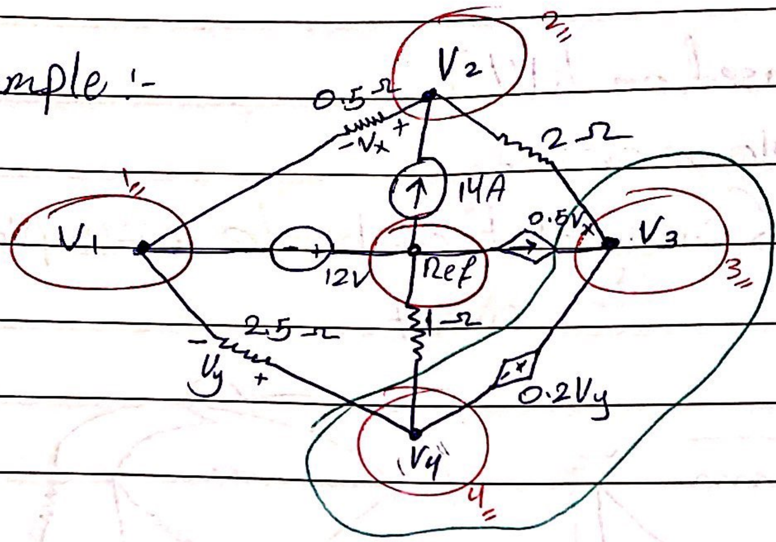
KCl node 2 :- $-3 + \frac{V_2}{1} + \frac{V_2 - V_1}{3}$ ... X

supernode :- $-25 + \frac{V_3}{5} + \frac{V_2}{1} + \frac{V_2 - V_1}{3} - 3 + \frac{V_3 - V_1}{4} = 0$ --- (2)

Inside supernode :- $V_3 - V_2 = 22$ ... (3)

$V_1 = 1.071V$, $V_2 = 10.5V$, $V_3 = 32.5V$

Example :-



another

* when the voltage source is between the Ref. and another node the another node = zero - Voltage source

$0 - 12 = -12V = V_1$

KCL $V_2 = -14 + \frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2} \cdots (1)$

~~KCL $V_3/7$~~ supernod → "inside equation" $= V_3 - V_4 = 0.5V_y \cdots$

where $V_y = V_4 - V_1 = V_4 + 12$

supernode → $\frac{V_3 - V_2}{2} - 0.2V_x$ "where $V_x = V_2 + 12$"

$\frac{V_4 - 0}{1} - \frac{V_4 - V_1}{2.5} = 0 \cdots (3)$

$V_1 = -12V$, $V_2 = -4V$, $V_3 = 1 \times 10^{-14} \approx 0$

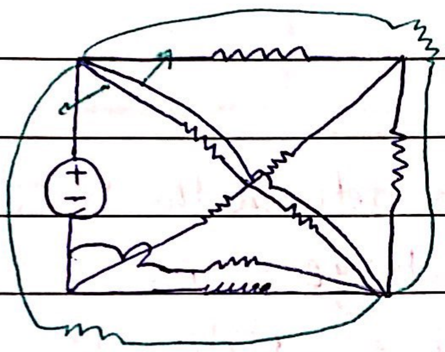
$V_4 = -2V$

• Mesh Analysis: (based on KVL)

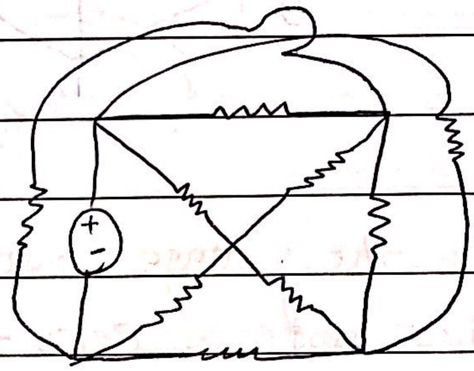
* It's applied on a (planer circuit)

↳ circuit that can be drawn in a plane circuit.

Example :-



Planer



not planer

"there will be an overlap if we reconnect it"

. steps of mesh Analysis

1- Locate all the meshs in the circuit. (mesh ≡ window) "closed loop"

2- Locate one current to each mesh.

3- Write the KVL equation for each mesh.

4- Solve the mesh equations.

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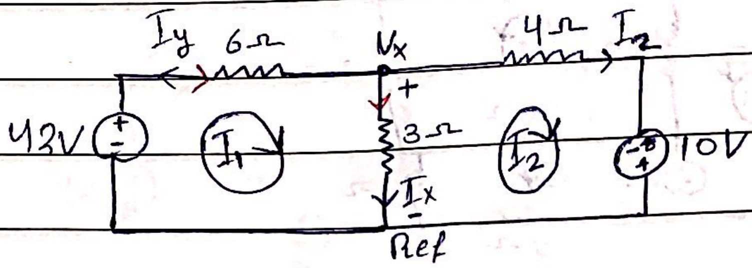
* مش شرط ال loops يكونوا بنفس الإتجاه.

Example :-



find all the currents

KVL at mesh 1 :- $-42 + 6I_1 + 3(I_1 - I_2) = 0$ ... (1)

KVL at mesh 2 :- $-10 + 3(I_2 - I_1) + 4I_2 = 0$ ... (2)

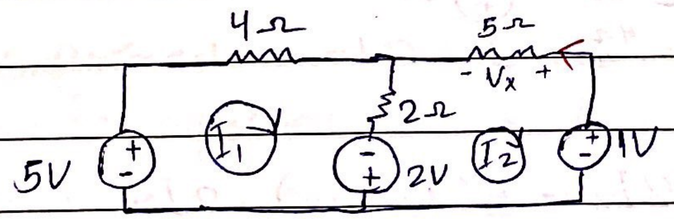
Solve 1 and 2 $\longrightarrow$ $I_1 = 6A$, $I_2 = 4A$

find I_x , I_y , I_z , V_x

$I_y = -I_1 = -6A$, $I_z = I_2 = 4A$

$I_x = I_1 - I_2 = 2A$, $V_x = 3I_x = 6V$

Example :-



using mesh analysis find
1. Pabs by 4Ω
2. V_x

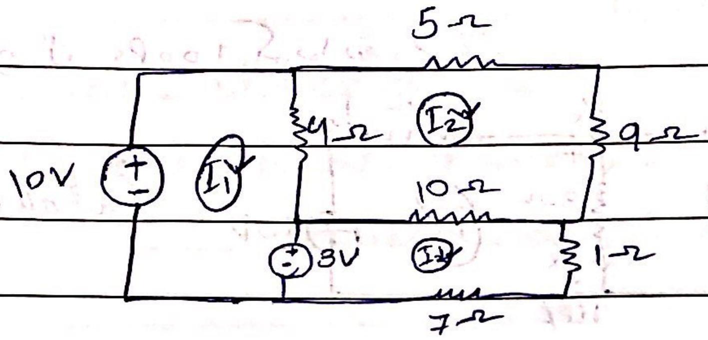
KVL at mesh 1 :- $-5 + 4I_1 + 2(I_1 - I_2) - 2 = 0$ ... (1) $\quad * P_{abs} = I^2 R$

KVL at mesh 2 :- $1 + 2 + 2(I_2 - I_1) + 5I_2 = 0$ ... (2)

$I_1 = \frac{43}{38} A$, $I_2 = \frac{2}{19} A$.

1) $P_{abs} = \left(\frac{43}{38}\right)^2 4$ W $\qquad$ 2) $V_x = -I_2 \times 5 = \frac{-2}{19} \times 5$ V

Example:-



find the current in each mesh:-

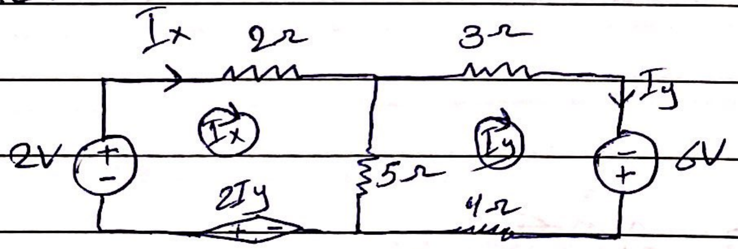
KVL mesh 1 :- $-10 + 4(I_1 - I_2) + 3 = 0$ --- (1)

KVL mesh 2 :- $5I_2 + 9I_2 + 10(I_2 - I_3) + 4(I_2 - I_1) = 0$ --- (2)

KVL mesh 3 :- $-3 + 10(I_3 - I_2) + I_3 + 7I_3 = 0$ --- (3)

$I_1 = 2.22\,A$, $I_2 = 470\,mA$, $I_3 = 427.7\,mA$

Example :-

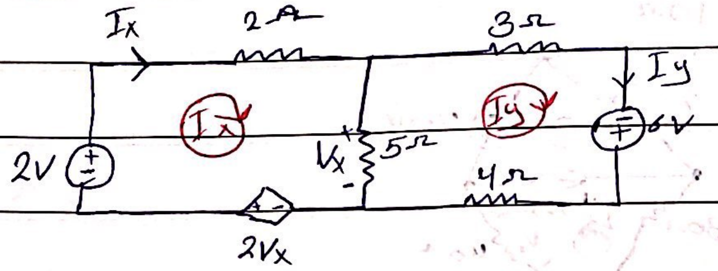


KVL mesh 1 :- $-2 + 2I_x + 5(I_x - I_y) - 2I_y = 0$ --- (1)

KVL mesh 2 :- $-6 + 3I_y + 4I_y + 5(I_y - I_x) = 0$ --- (2)

$I_x = \frac{66}{49}$, $I_y = \frac{52}{49}$

Example :-



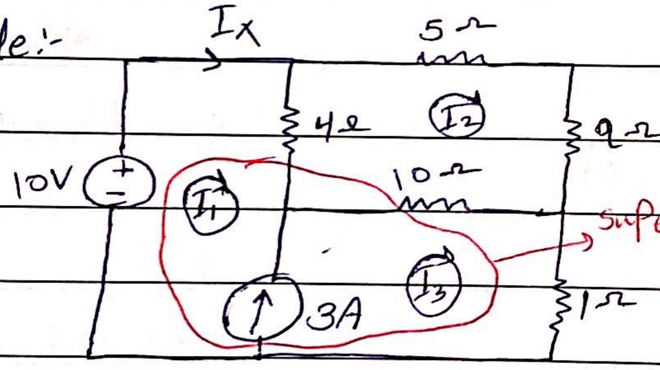
$5(I_x - I_y)$

KVL mesh 1 :- $-2 + 2I_x + 5(I_x - I_y) - 2V_x = 0$ ... (1)

KVL mesh 2 :- $-6 + 4I_y + 5(I_y - I_x) + 3I_y = 0$ ... (2)

$I_x = \frac{6}{11} A$, $I_y = \frac{8}{11} A$

Example :-



* supermesh :- two meshes and current source between them.

supermesh :- $-10 + 4(I_1 - I_2) + 10(I_3 - I_2) + 1(I_3) = 0$ -- (1)

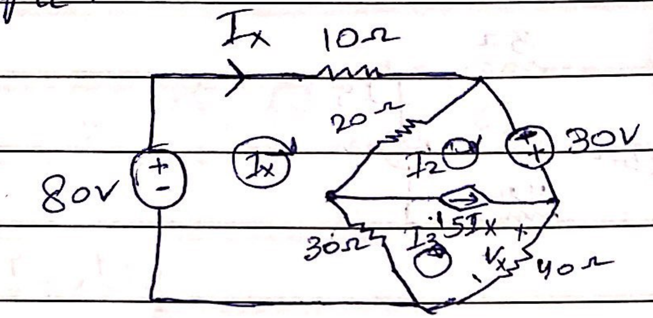
inside supermesh = $I_3 - I_1 = 3$ --- (2)

mesh 2 :- $5I_2 + 9I_2 + 4(I_2 - I_1) + 10(I_2 - I_3) = 0$ --- (3)

$I_1 = \frac{29}{15} A$, $I_2 = \frac{11}{105} A$, $I_3 = \frac{16}{15} A$

$I_1 = I_x$, current in 10Ω $= I_3 - I_2$

Example 1-



mesh 1 :- $-80 + 10 I_x + 20 (I_x - I_2) + 30 (I_x - I_3) = 0$

supermesh :- $-30 + 4 I_3 + 30 (I_3 - I_x) + 20 (I_2 - I_x) = 0 \rightarrow ②$

inside supermesh :- $I_3 - I_2 = .15 I_x$

[illegible]

$I_1 = \frac{87}{149}$ A , $I_2 = \frac{-917}{149}$, $I_3 = \frac{388}{149}$ A , $V_x = 40 I_3$

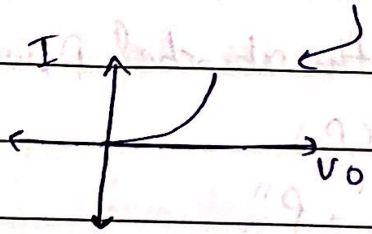
- Super Position

→ it's applied on linear circuits

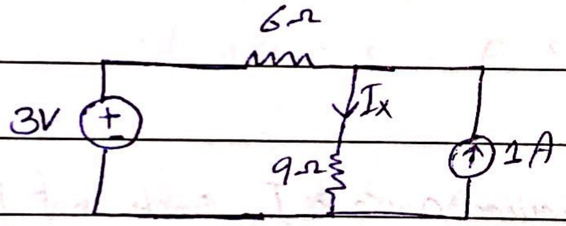
→ linear circuit :- all its elements are linear (V∝I is linear)

e.g., R, C, L → linear elements

Diode, transistor → not linear elements



* Example:-

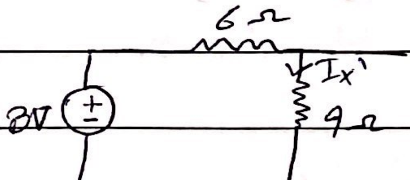


find I_x using superposition.

- we will find the contribution of each element in the value of I_x

I_x' due to 3V only. (contribution of 3V in I_x)

→ kill 1A source

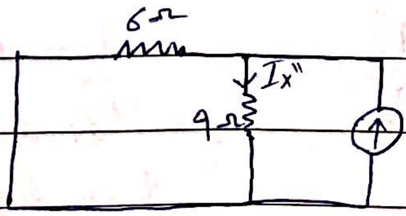


$I_x' = \frac{3}{9+6} = \frac{1}{5} = 0.2A$

* killing current soure :- open circuit.
* killing Voltage source short circuit.

I_x'' due to 1A only (contribution of ~~3V~~ 1A in I_x)

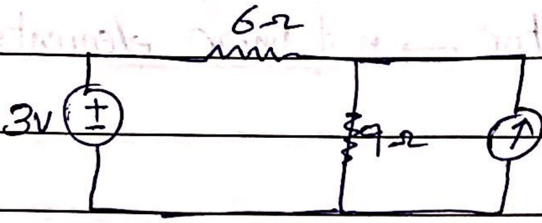
$I_x'' = 1 * \frac{6}{9+6} = \frac{6}{15}$



$I_x = I_x' + I_x'' = \frac{3}{5} = 0.6A$

* Note:- we can use superposition to find the current or the voltage but not the power.

Example:-



find the absorbed power by 9Ω (P_x)

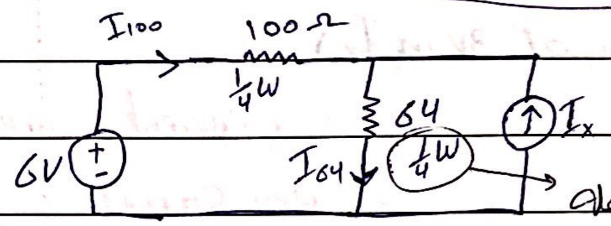
$P_x \neq P_x' + P_x''$

Sol:- find I_x then find P_x

→ $I_x' = 0.2A$ → $I_x'' = 0.4A$ → $I_x = 0.2 + 0.4 = 0.6A$

$P_x = I_x^2 R = 0.6^2 \cdot 9 = 3.2\,W$

Example:- find the maximum current I_x such that the resistors do not exceed their $\boxed{\text{power rating}}$ → maximum power that can be dissipated by the element without any damage



اكبر Power تتحمله المقاومة.

$P_{rating} = I_{100}^2 \times 100 = \frac{1}{4} \Rightarrow I_{100\,max} = 50mA$

$= I_{64}^2 \times 64 = \frac{1}{4} \Rightarrow I_{64\,max} = 62.5\,mA$

I_{100}' due to 6V only → $I_{100}' = \frac{6}{164} = 36.59mA$

$I_{100} = I_{100}' + I_{100}''$

I_{100}'' due to I_x only → $I_{100}'' = I_x \cdot \frac{64}{164}$

$I_{64} = I_{64}' + I_{64}''$

I_{64}' due to 6V only → $I_{64}' = \frac{6}{164} = 36.59m$

I_{64}'' due to I_x only → $I'' = I_x \frac{100}{164}$

$\rightarrow I_{64} < I_{64max} \rightarrow 36.59 + I_x \frac{100}{164} \leq 62.5 \rightarrow \boxed{I_x \leq 42.49 mA}$

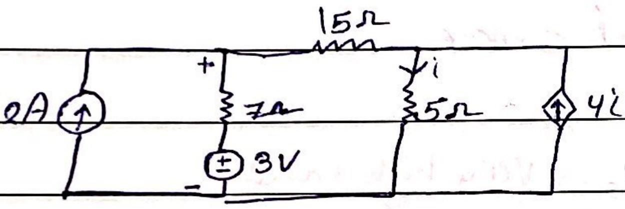
$\rightarrow I_{100} < I_{100max} \rightarrow 36.59 - I_x \frac{64}{164} \leq 50 \rightarrow \boxed{I_x \geq -34.36 mA}$

بعكس الاتجاه الأصلي　　　　بالاتجاه الأصلي

Range $\{ -34.36 \longleftrightarrow 42.49 \}$.

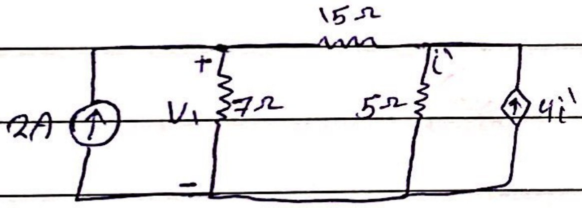
* Note:- We don't Kill dependent sources in superposition.

Example:-



find V_1 using superposition.

$\rightarrow V_1'$ due to 2A only

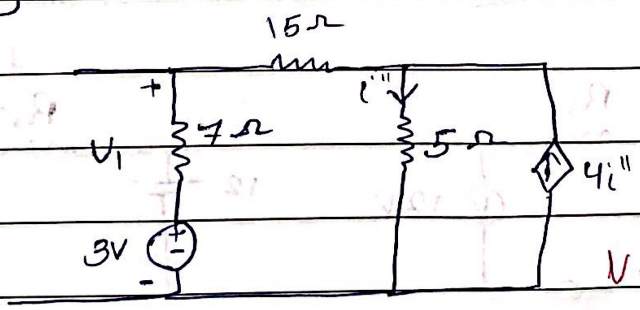


$V = 1.967$

~~$V_1 = 1.967 + 9.18 = 11.197 V$~~

$\rightarrow V_1''$ due to 3V only

Solve using nodal & for mesh

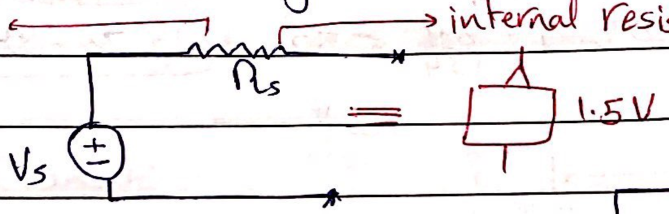


$V = 11.197$

$V_1 = 1.967 + 9.18 = 11.197 V$

- Practical Voltage source

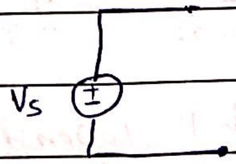
close to zero "small value" ← → internal resistance "for the voltage source"



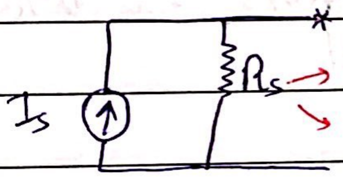
ideal Voltage source

$R_s = 0$

× no internal resistance.

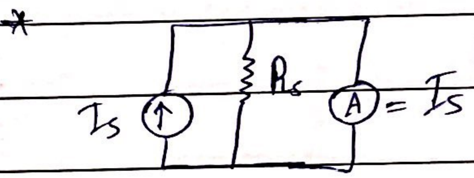


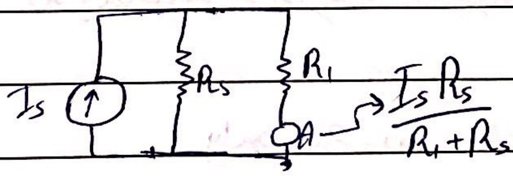
- Practical current source



→ very high value
→ in Parallel

* ideal current source ($R_s = \infty$)

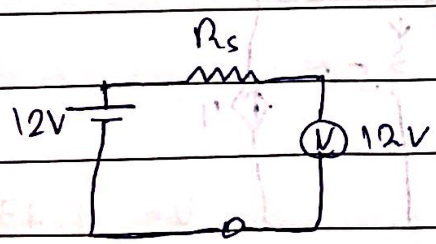


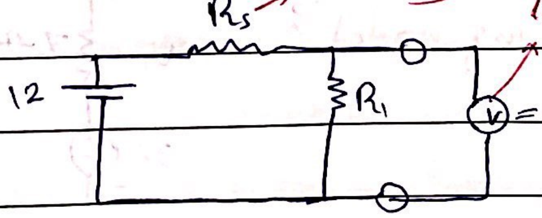


$\rightarrow \frac{I_s R_s}{R_1 + R_s}$

it make voltage division with the load

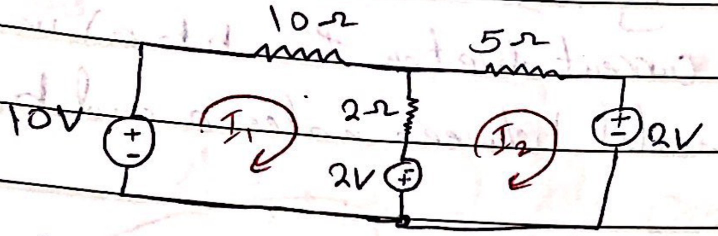
less than 12V





$V = 12 \cdot \frac{R_1}{R_1 + R_s}$

Q_1 find the mesh current equations for the circuit shown. $9 \_ M_s$



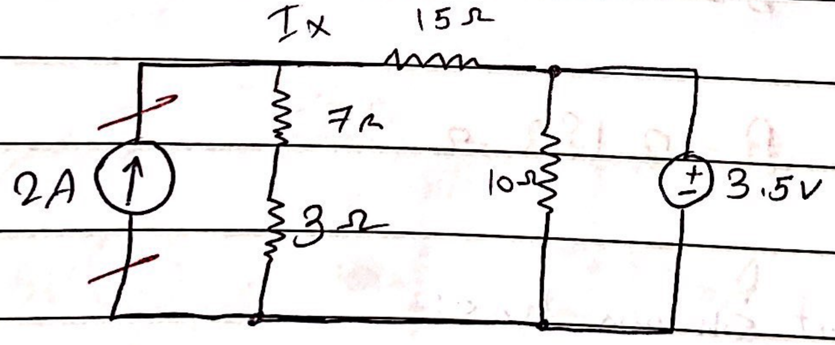
mesh 1 :- $-10 + 10I_1 + 2(I_1 - I_2) - 2 = 0 \quad \cdots \cdots (1)$

mesh 2 :- $2 + 2 + 2(I_2 - I_1) + 5I_2 = 0 \quad \cdots (2)$

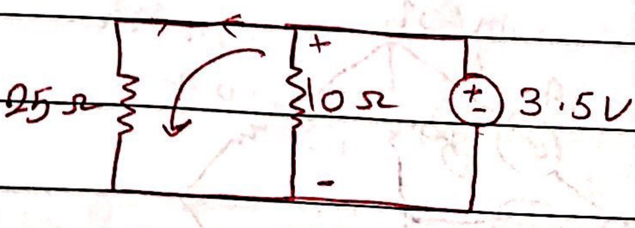
Ans :- $12I_1 - 2I_2 = 12$

$-2I_1 + 7I_2 = -4$

Q_2 for the circuit shown, and based on the superposition principle, the portion of I_x under the effect of the 3.5 Voltage source acting alone is :-



$$I_x = 2 * \frac{10}{10+15}$$



$$I_x = \frac{-3.5}{25} = -0.14A$$

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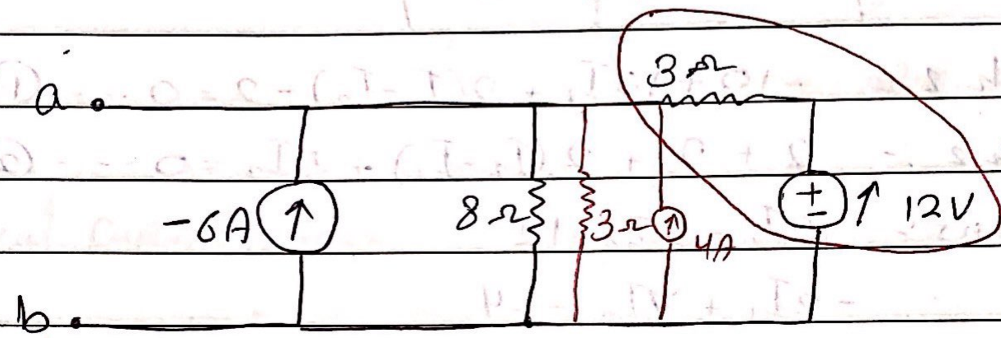
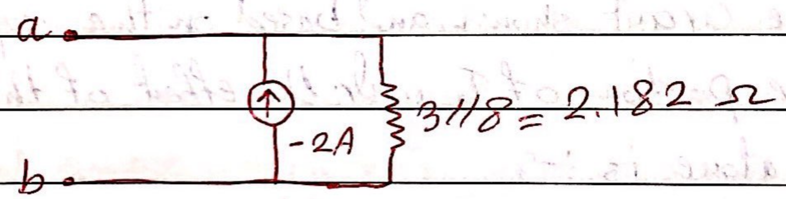
Subject Date No.

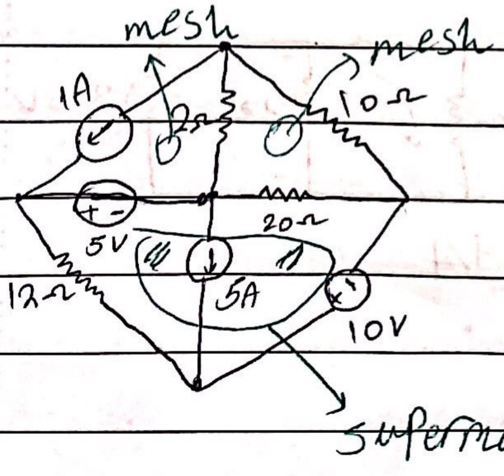
Q3 Using the principle of source transformation, convert the given circuit into a single ~~code~~ node circuit ~~ata~~ with current source i (with current direction from b to a) in parallel with resistor R and connected between nodes a and b, find i and R.





$I = -2A$, $R = 2.182\ \Omega$

Q4: The circuit shown has:-



2 meshs and 1 supermesh.

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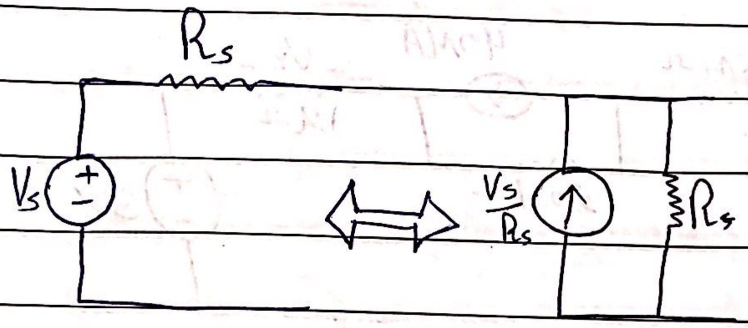
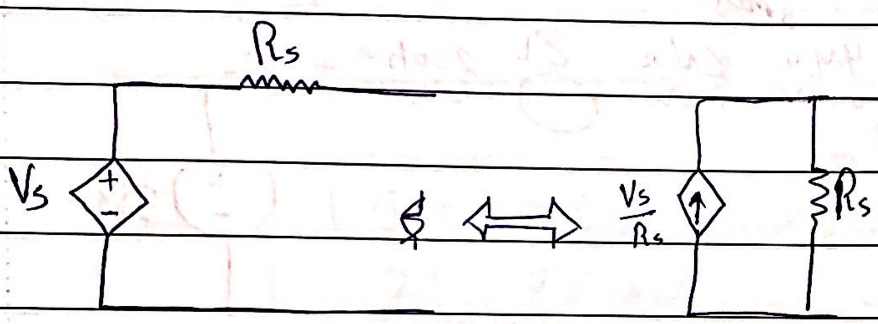
• Source Transformation:

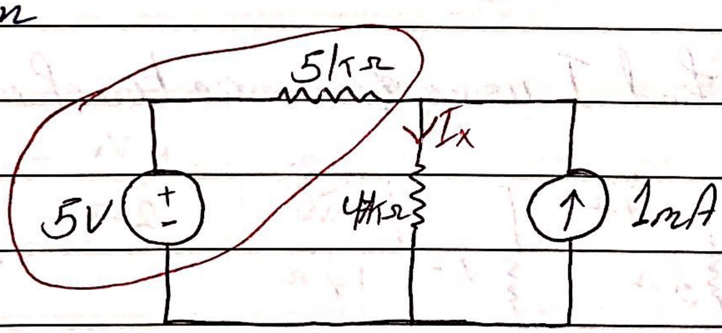


*Voltage source with resistor in series can be converted to current source $= \frac{V_s}{R_s}$

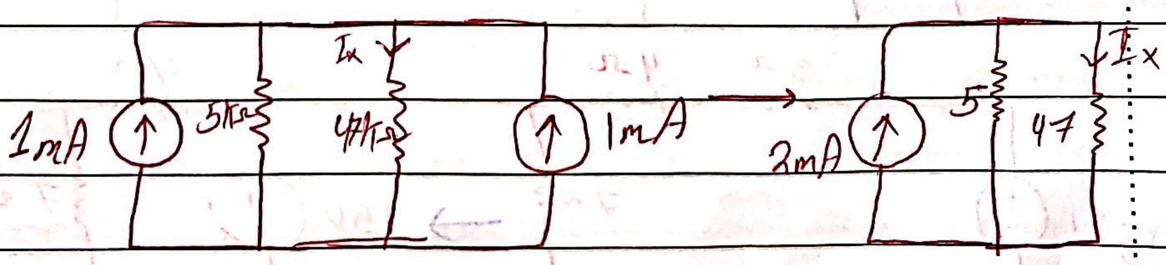


*useful in analysis.

Example :- Simplify the circuit then find I_x using source transformation

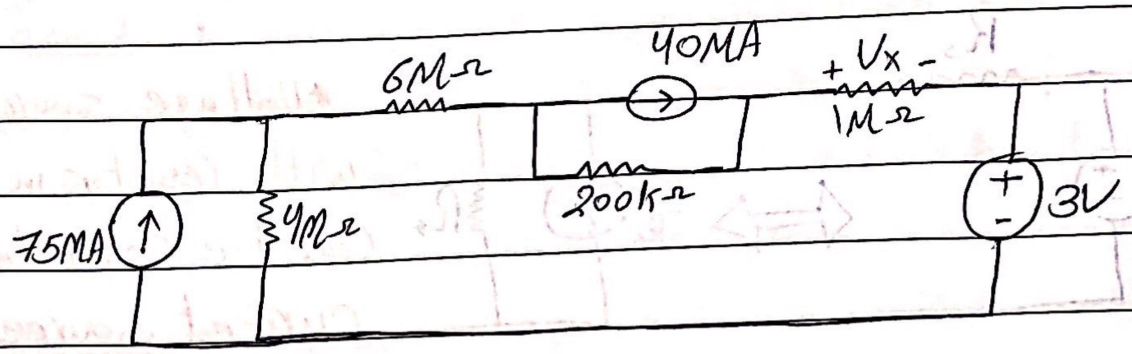


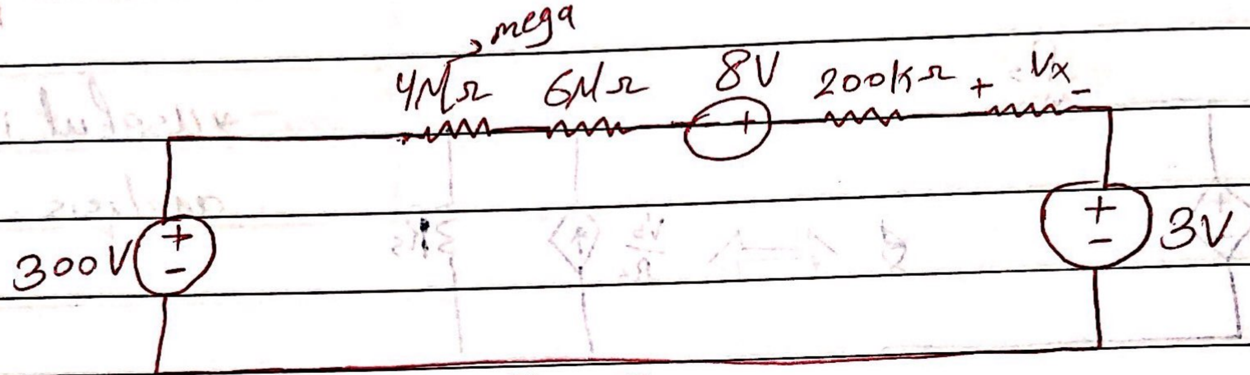
Sol :-



$$I_x = 2 * \frac{5}{5+47} = 196 \overset{\text{micro}}{m}A$$

Example:- Simplify this circuit and find V_x.

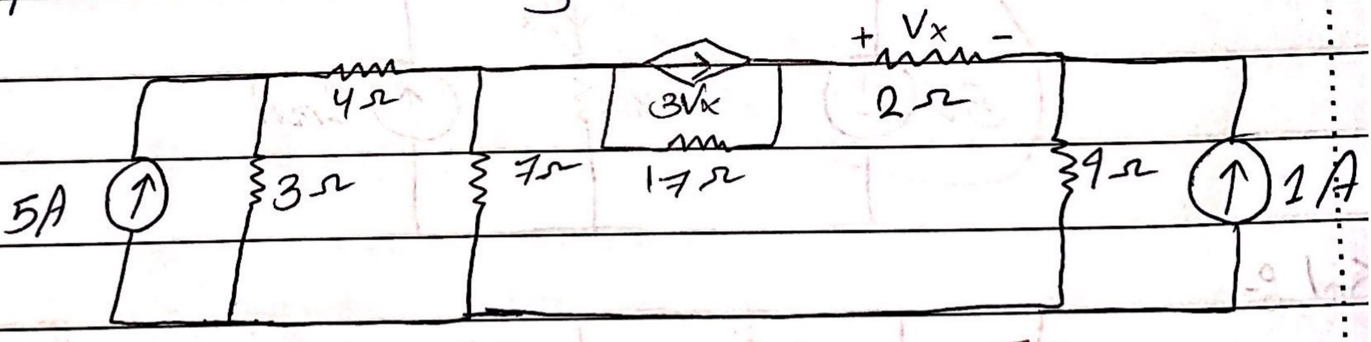


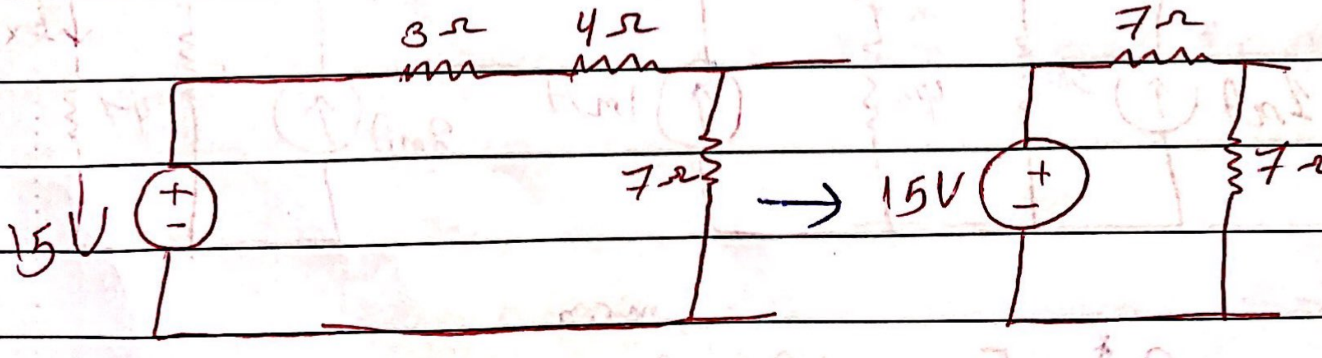


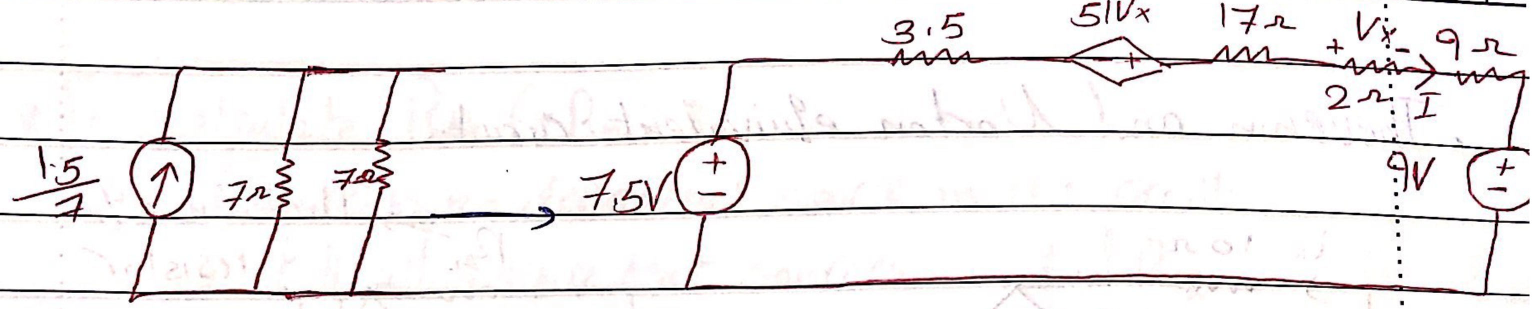
$308 - 3 = 305$

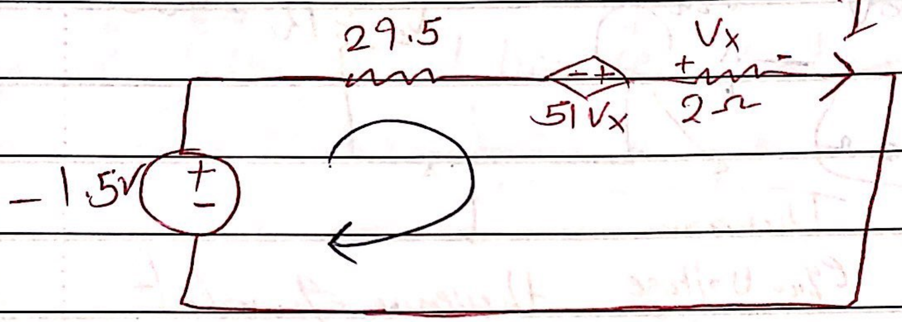
$$V_x = 305 * \frac{1}{10.2+1} = 27.23\,V.$$

Example:- find I using ~~&~~ source transformation





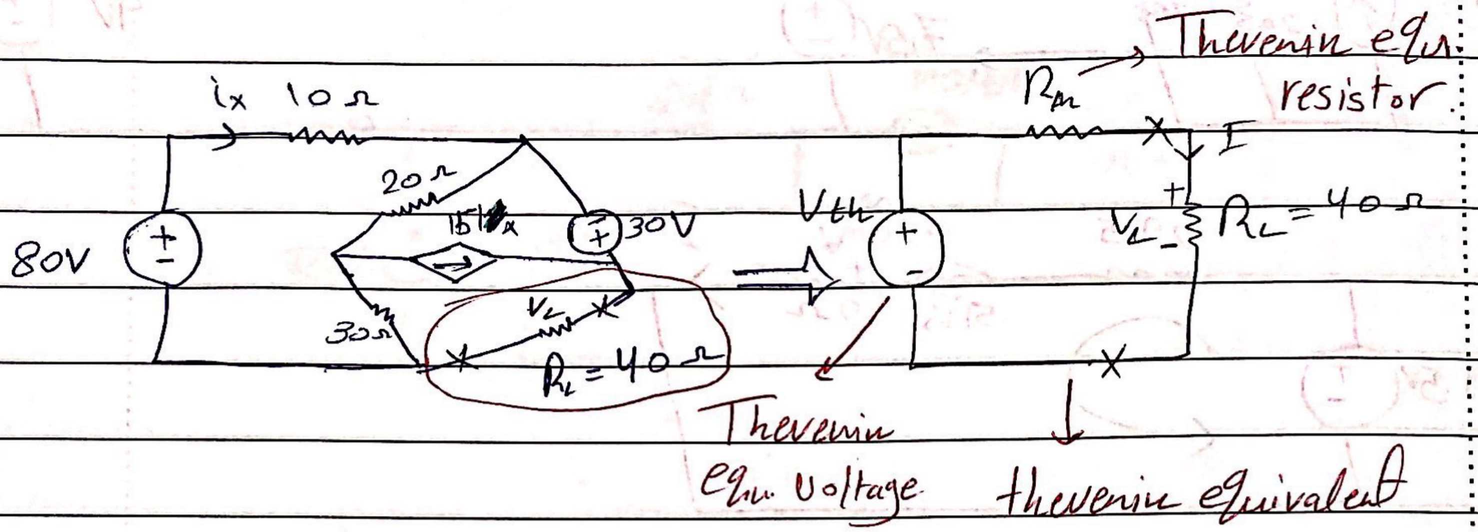


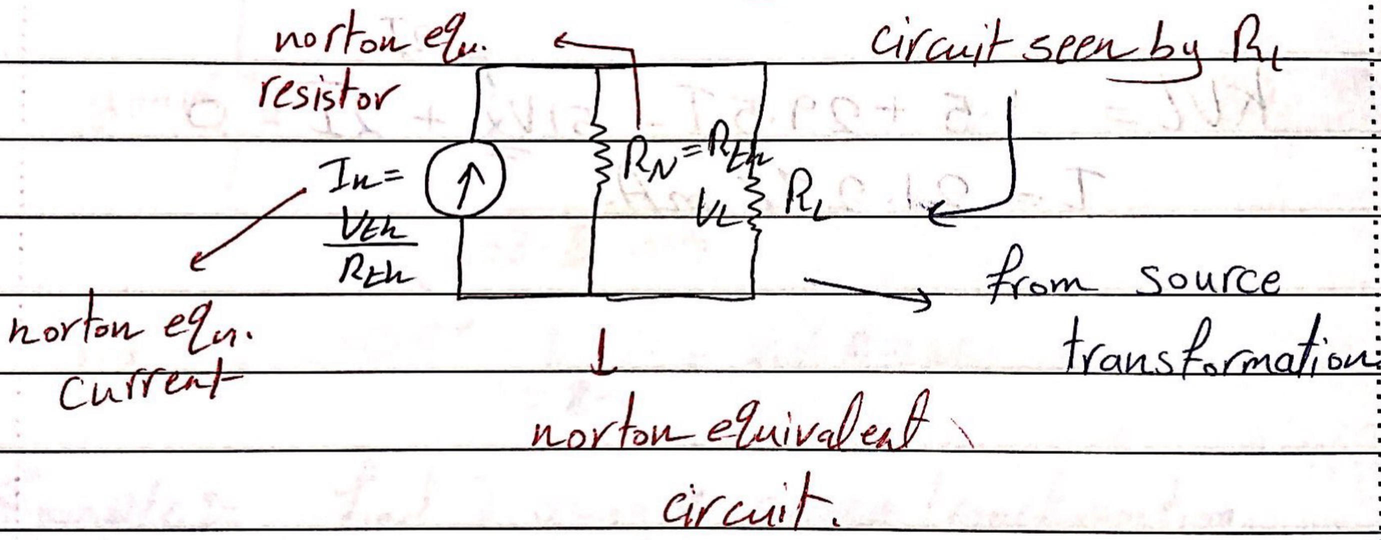


$$KVL = 1.5 + 29.5I - 51\overset{2I}{\underline{V_x}} + 2I = 0$$

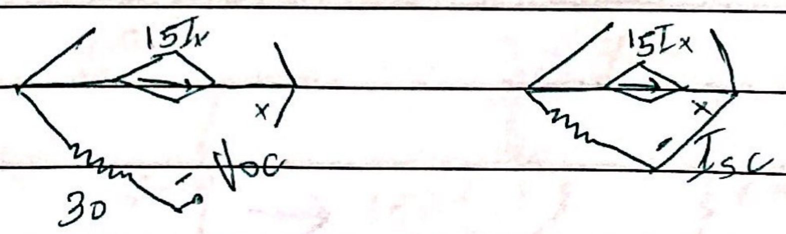
$$I = 21.276\ mA.$$

- Thevenin and Norton equivalent circuits.





* To find V_{th} in any circuit → $V_{th} = V_{o.c}$ (open circuit).
* To find I_n in any circuit → $I_n = I_{s.c}$ "short circuit"



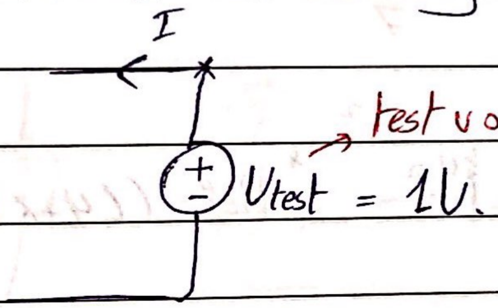
* To calculate R_{th} ($= R_n$) :-

1. If there is no dependent source in the circuit
   ↳ Kill all independent sources → find R_{eq} ..... $R_{eq} = R_{th}$

2. If the circuit has dependent and independent sources:
   ↳ $R_{th} = \dfrac{V_{oc} \rightarrow V_{th}}{I_{sc} \rightarrow I_N}$

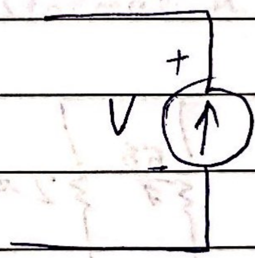
* R_{th} Could be (-ve) cause it means the ratio $\frac{V_{th}}{I_N}$ .. it's not a real value

3. If the circuit has only dependent source
   ↳



→ test voltage "could be any value"

$R_{th} = \dfrac{V_{test}}{I} = \dfrac{1}{I}\ \Omega$

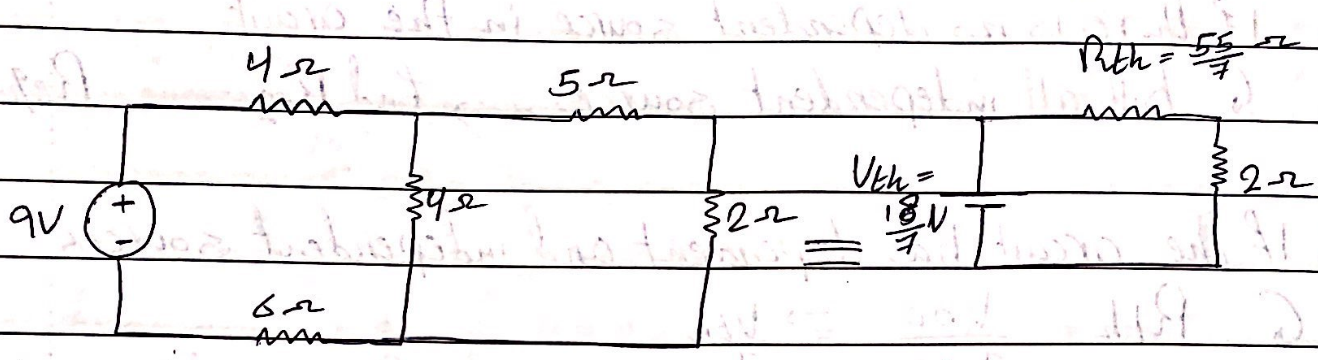


$I_{test} = 1A$
↳ test current "could be any value"

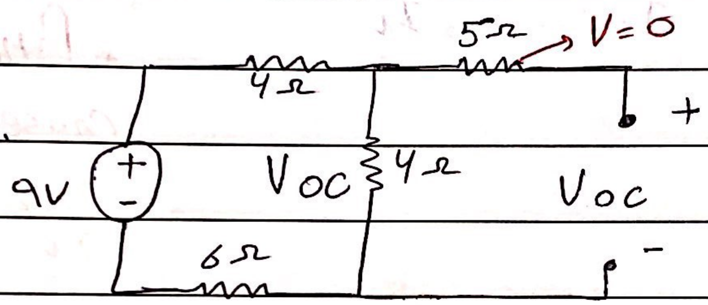
$R_{th} = \dfrac{V}{I} = \dfrac{V}{1}\ \Omega$

Note:- The method in type 3 can be used to solve type 1 and type 2 .. but we have to kill all independent sources.

Example :- Find Thevenin eq. circuit seen by 2 Ω

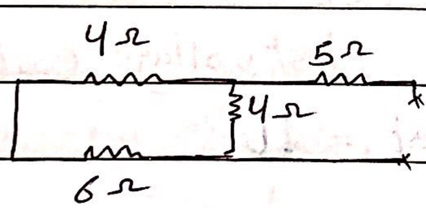


$V_{th} = V_{o.c.}$

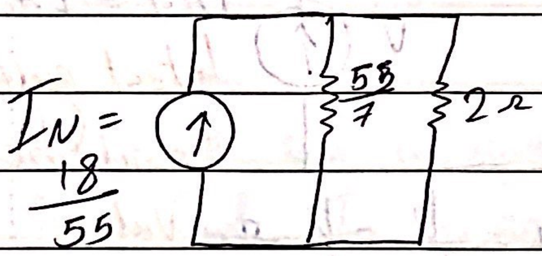


$$V_{th} = V_{oc} = \frac{9 * 4}{4+4+6} = \frac{36}{14} = \frac{18}{7}\ V.$$

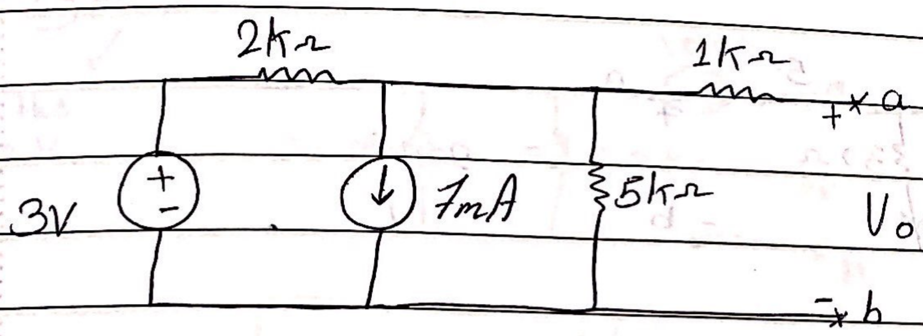
$R_{th} \longrightarrow$ Type 1



$R_{eq} = ((4+6)//4) + 5 = \frac{58}{7}$



Example:- find thevenin eq. circuit seen between a & b

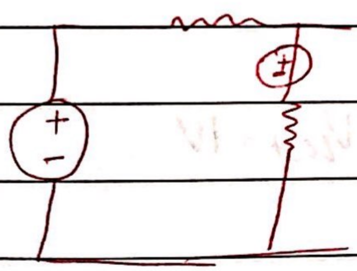


* We can use source transformation.

$V_{th} = V_{oc} \longrightarrow \frac{V_{oc}-3}{2} + 7 + \frac{V_{oc}}{5} \longrightarrow V_{oc}\left(\frac{1}{2}+\frac{1}{5}\right) = \frac{3}{2}$

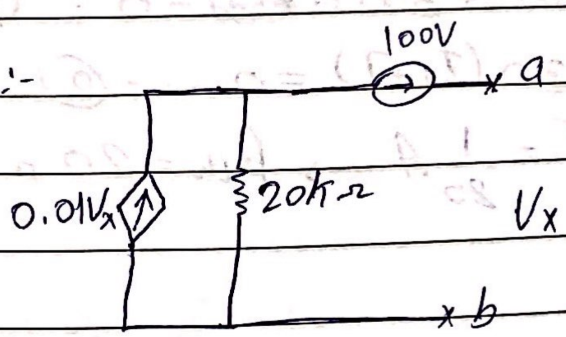
$V_{oc} = \frac{-55}{V}\ V.$

$R_{th} = R_{eq} = 2//5 + 1 = \frac{17}{7}\ k\Omega$



→ source transformation.

Example:-



$R_{th} = \frac{-502.5}{5} = -100.5$

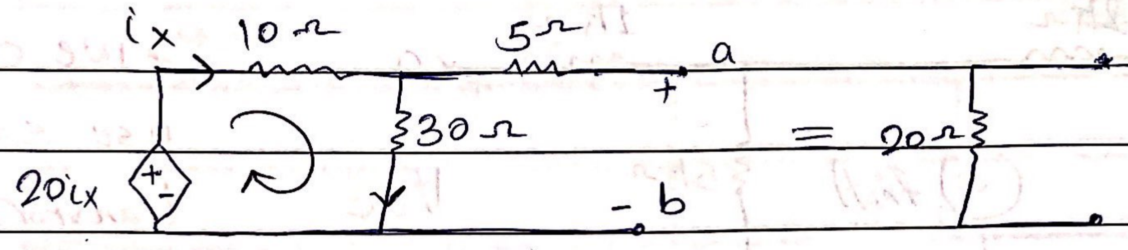
$V_{th} = V_x$

$V_{20k\Omega} = 0.01V_x * 20 = 200V_x$

$V_x - V_{20k\Omega} - 100 = 0 \longrightarrow V_x - 200V_x - 100 = 0$

$V_x = \frac{-100}{199} = -502.5\ mV$

Example :- find V_{th} and R_{th} seen between a & b



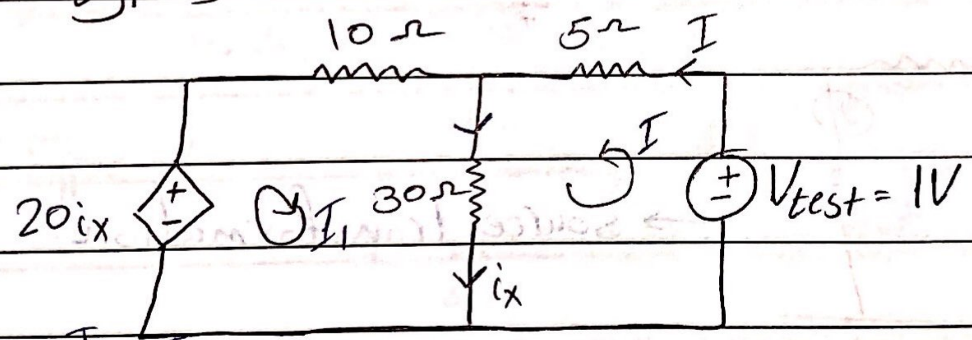
KVL : $-20i_x + 10i_x + 30i_x \rightarrow i_x = 0A$

$\Rightarrow V_{oc} = 0V \longrightarrow V_{th} = 0V \longrightarrow I_x = 0A.$

$\Rightarrow$ Dead circuit !!

* I_{sc} in this circuit = zero

R_{th} .. type 3



$I_1 + I$

mesh1 :- $-20i_x + 10I + 30(I_1 + I) = 0$ ... ①

mesh2 :- $-1 + 5I + 30(I_1 + I) = 0$ ... ②

$I_1 = \frac{-1}{40} A , I = \frac{1}{20} A \longrightarrow R_{th} = 20\,\Omega$

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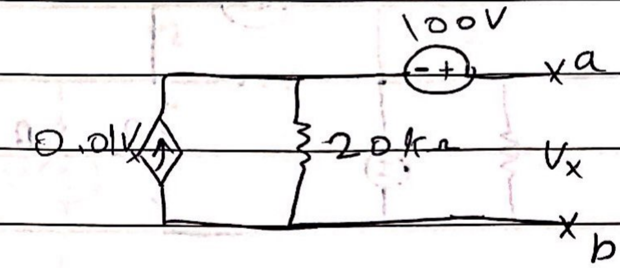
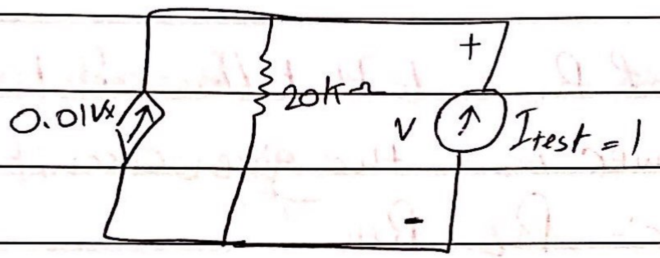
Subject Date No.

Example :- Find V_{th}, R_{th} seen between a and b using type 3

kill the 100 V





$$= 0.01V_x - 1 + \frac{V}{20} = 0$$

$$V = -100.5V$$

$$R_{th} = -100.5\,\Omega$$

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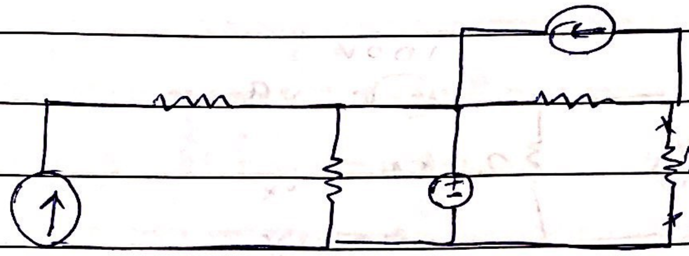
Subject Date No. 11

* Maximum power transfer

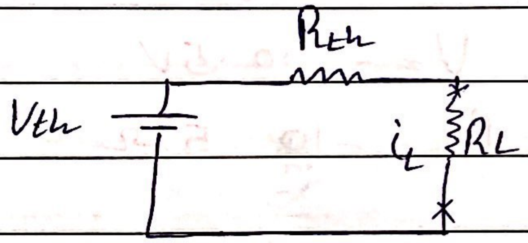


طبيعة السؤال

Q:- what is the value of R_L such that this resistor will absorb maximum power from the given circuit

answer:- $R_L = R_{th}$

why ?



$$P_{R_L} = I_L^2 R_L = \left(\frac{V_{th}}{R_L + R_{th}}\right)^2 R_L$$

$$\frac{dP_{R_L}}{dR_L} = 0 \rightarrow \frac{V_{th}^2 (R_L + R_{th})^2 - 2(R_L + R_{th}) V_{th}^2 R_L}{(R_L + R_{th})^4}$$

$$\Rightarrow V_{th}^2 (R_L + R_{th})^2 = 2(R_L + R_{th}) V_{th}^2 R_L$$

$$R_L + R_{th} = 2R_L \quad \therefore \quad R_L = R_{th}$$

$$\boxed{P_{max} = \frac{V_{th}^2}{4R_{th}} \, W} \quad \text{or} \quad \boxed{P_{max} = \frac{I_N^2 R_t}{4} \, W}$$

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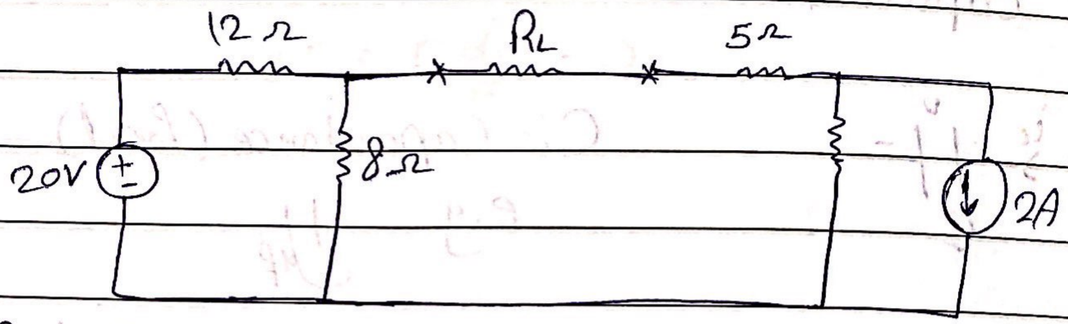
Subject　　Date　　No.

Ex:- find R_L that will absorb maximum power from the circuit.



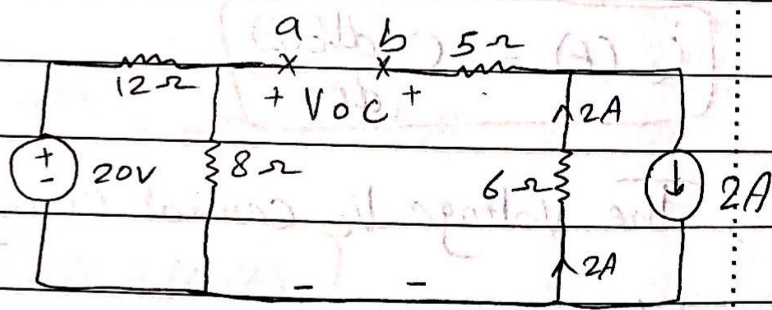
Sol:- $R_L = R_{th}$ - $R_{eq} = ((12 // 8) + 5) + 6 = 15.8\,\Omega$

$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

$$V_{th} = V_{oc} = V_a - V_b$$

voltage devision

$$V_a = 20 \times \frac{8}{12+5} \cancel{\;-\;(-12)}$$



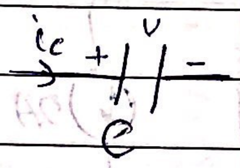
$$20 \times \frac{8}{12+5} - (-12)$$

$$= 20V$$

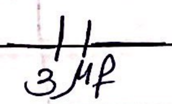
$$P_{max} = \frac{(20)^2}{4 \times 15.8} = 6.329\,W.$$ / or source transformatio

DC Circuit and analysis

- Capacitors and Inductors
  - Capacitors



C:- Capacitance (farad)

e.g 

→ linear element ($I_c \propto V_c$ linear) — Micro $= 10^{-6}$

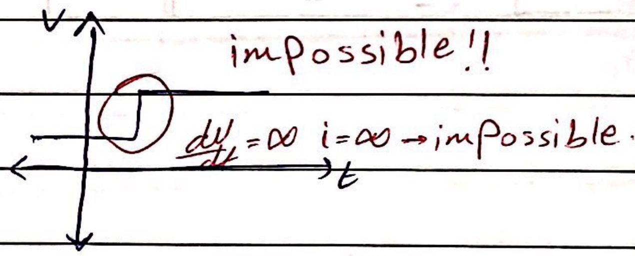
→ Passive element [P generates $\not< 0$] — Nano $= 10^{-9}$

over infinite time interval — biko $= 10^{-12}$

($P_{av} = 0$)

$$\boxed{i_c(t) = C \frac{dV_c(t)}{dt}}$$

→ The voltage V_c cannot change in zero time (suddenly)

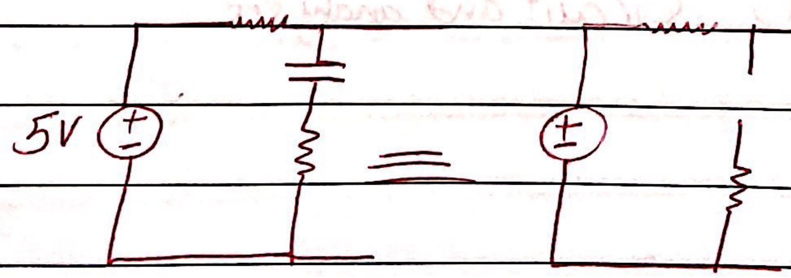


→ The capacitor in DC circuits can be replaced by open circuit

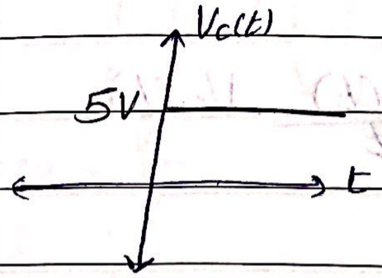
↳ Because $\frac{dV_c}{dt} = 0$

so $i_c = 0$

⟹ C is open circuit.



Ex:- find $i_c(t)$, $C = 2F$

1) 

$$i_c = C\frac{dv}{dt} = 0$$

2) $V_c = 5\sin(\pi t)\ V.$

$i_c = 2\,[5\pi\cos\pi t]$

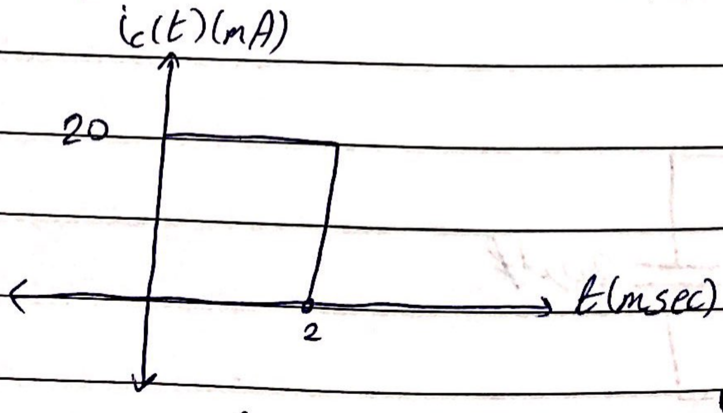
$= 10\pi\cos\pi t\ A$

3) $V_c(t) = 2e^{-5t}$

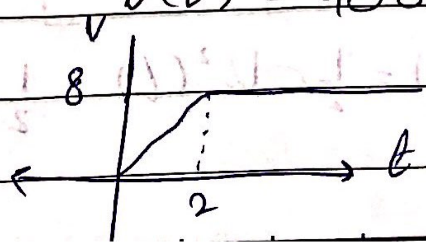
$i_c(t) = 2(-10e^{-5t}) = -20e^{-5t}\ A$

$$\rightarrow \boxed{V_c(t) = \frac{1}{C}\int_{t_0}^{t} i_c(t)\,dt + V_c(t_0)}$$

Ex: find and draw $V_c(t)$, $C = 5\mu F$.



* for $-\infty < t < 0$ msec
  $V(t) = 0\,V$
* for $0 < t < $ msec
  $V(t) = 4000t$
* for $2 < t < \infty$
  $V_c(t) = 8V$



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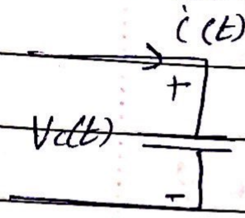
$\rightarrow$ Joule (J)

* Stored energy in capacitor : $w(t)$



$$P(t) = i_c(t)\, V_c(t) = C \frac{dV_c(t)}{dt} \cdot V_c(t)$$

$$P(t) = \frac{dw(t)}{dt}$$

$$w(t) = \int_{t_0}^{t} P(\tau)\, d\tau = C \int_{t_0}^{t} V_c(\tau) \frac{dV_c(\tau)}{d\tau}\, d\tau$$

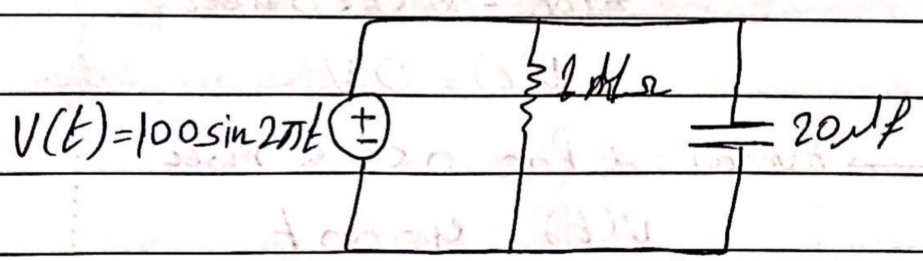
$$w_c(t) - w_c(t_0) = \frac{1}{2} C \left(V_c^2(t) - V_c^2(t_0)\right)$$

$$\boxed{w_c(t) = \frac{1}{2} C \left[V_c^2(t) - V_c^2(t_0)\right] + w_c(t_0)}$$

if $w_c(t_0) = 0 \rightarrow V_c(t_0) = 0$ * initial value = 0

$$\boxed{\therefore w_c(t) = \frac{1}{2} C V_c^2(t)}$$

Example:- find the maximum energy stored in the capacitor over the interval $0 < t < 0.5$ sec, also find the dissipated energy in the resistor



Sol :- $w_c(t) = \frac{1}{2} C V_c^2(t) = \frac{1}{2} \times 20 \times 10^{-6} \times 10^4 \sin^2(2\pi t) = 0.1 \sin^2(2\pi t)$

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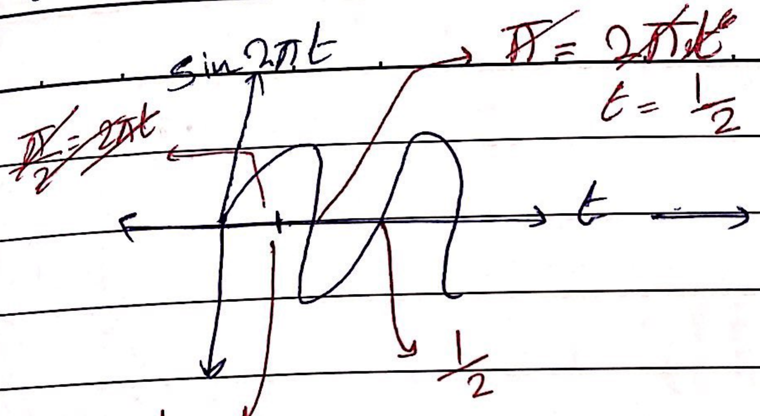
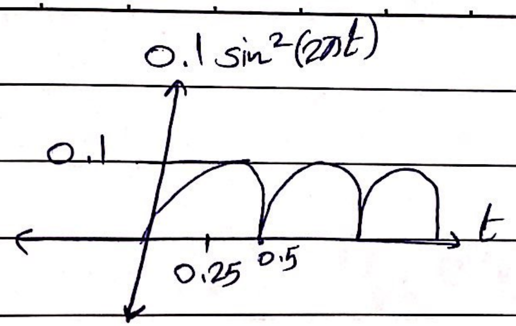




$W_{C_{max}} = 0.1\,J$ at $t = \frac{1}{4}$ sec.

$$P_{Rt} = i_{R(t)} V_{R(t)} = \frac{V_R^2(t)}{R} = \frac{(100)^2 \sin^2(2\pi t)}{1\times 10^6} = 10^{-2}\sin^2(2\pi t)$$

$$W_R = \int_0^{0.5} P(t)\,dt = \int_0^{0.5} 10^{-2}\sin^2(2\pi t)\,dt = \int_0^{0.5} 10^{-2}\times\frac{1}{2}(1-\cos 4\pi t)$$

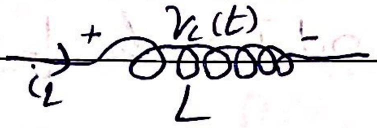
$$= 2.5\,mJ$$

12/1

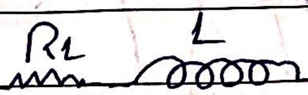
• Inductor

→ linear element

→ Passive element

→ 

(Ideal)



Practical

L:- Inductance

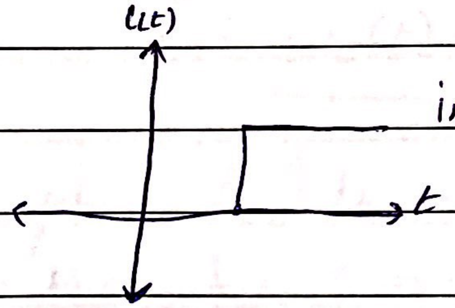
(H) Henry

* $V_L(t) = L\frac{di_L(t)}{dt}$ ... ①

* $i_L(t) = \left(\int_{t_0}^{t} V_L(\tau)\,d\tau\right)\frac{1}{L} + i_L(t_0)$ ... ②

I → from ①, the inductor in DC circuits can be reblaced by short circuit, since $V_L(t) = L\frac{di(t)}{dt}$ → DC = 0

→ The current in the inductor cannot change in 0 time!



impossible!!

* ال inductor يُستبدل بـ short circuit

* ال capacitor يُستبدل بـ open circuit.

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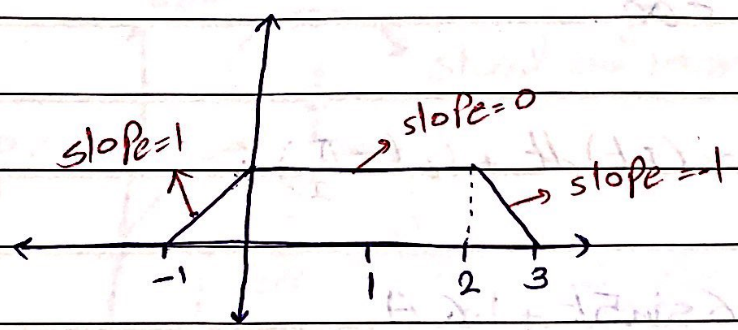
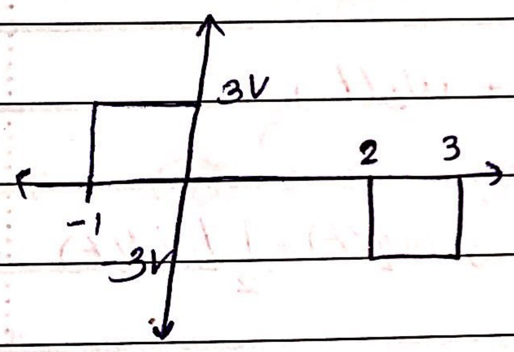
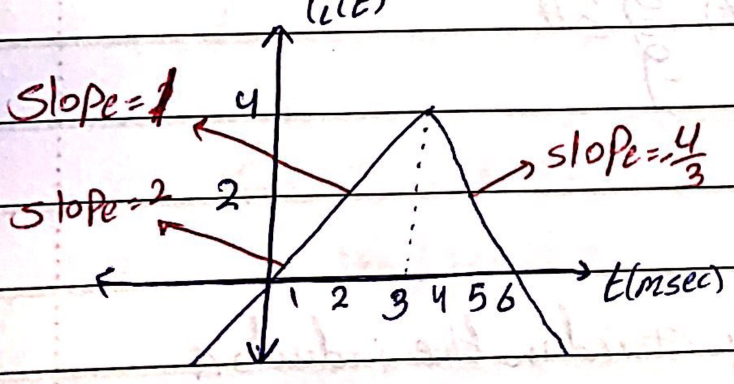
Example :- If $L = 3H$, find $V_L(t)$

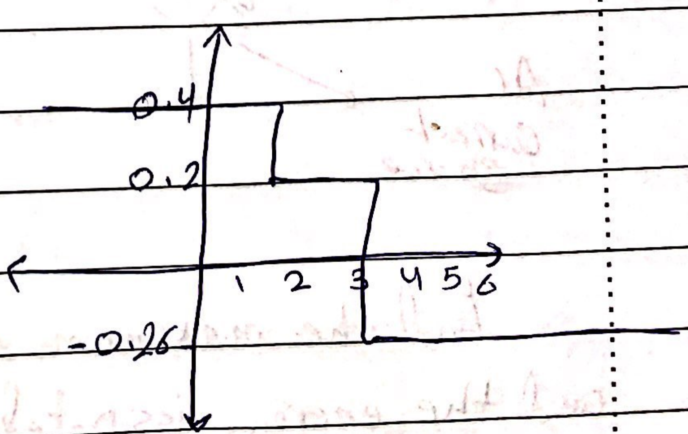


$$V_L(t) = L \times \text{slope} = \{3, -3\}$$



Example :- $L = 200mH$ .. find $V_L(t)$ where $i_L(t)$:-





Example:- if $V_L(t) = 6\cos(5t)\,V$, $L = 2H$, $i(t=-\frac{\pi}{2}) = 1A$
find $i(t)$ from $-\frac{\pi}{2} \le t < \infty$.

$$i_L(t) = \frac{1}{L}\int_{-\frac{\pi}{2}}^{t} 6\cos(5t)\,dt + i_L(-\tfrac{\pi}{2}).$$

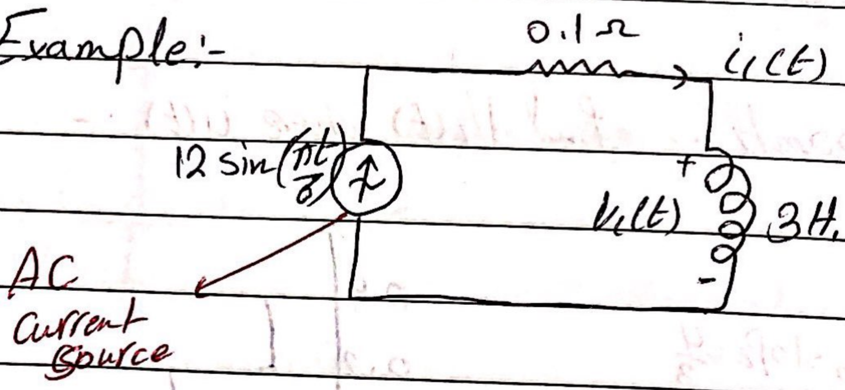
$$i_L(t) = 0.6\sin 5t + 1.6\,A$$

* The stored energy in Inductor

$$W_L(t) = \frac{1}{2}L\left[i_L^2(t) - i_L^2(t_0)\right] + W_L(t_0)$$

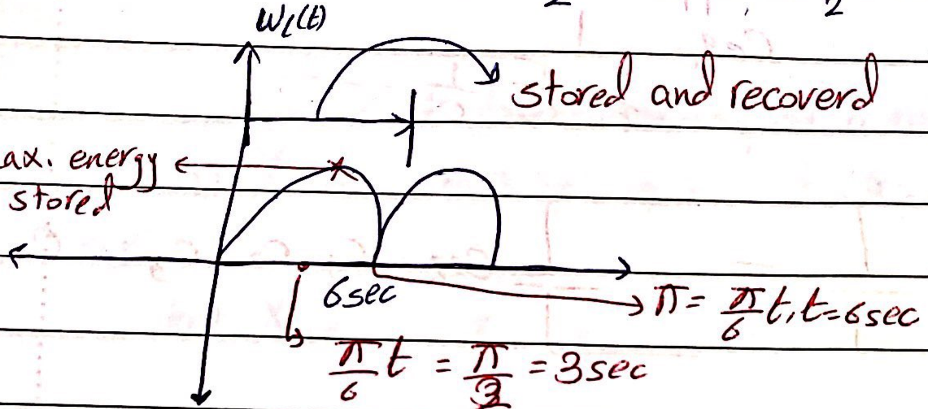
if $w_L(t_0) = 0 \longrightarrow i_L(t_0) = 0 \longrightarrow W_L(t) = \frac{1}{2}L\,i_L^2(t)$

Example:-



- find the maximum energy stored in the inductor, and the energy dissipated in the resistor in the time during which the energy is being stored and then recoverd from the inductor. given that $W_L(-\infty) = 0$.

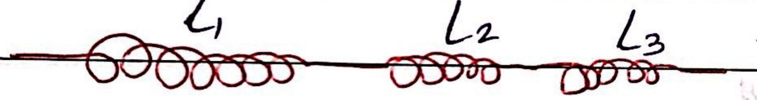
Sol:- $W_L(t) = \frac{1}{2} L\, i_L^2(t) = \frac{1}{2} \times 3 \times (144 \sin^2(\frac{\pi}{6}t))\ J.$



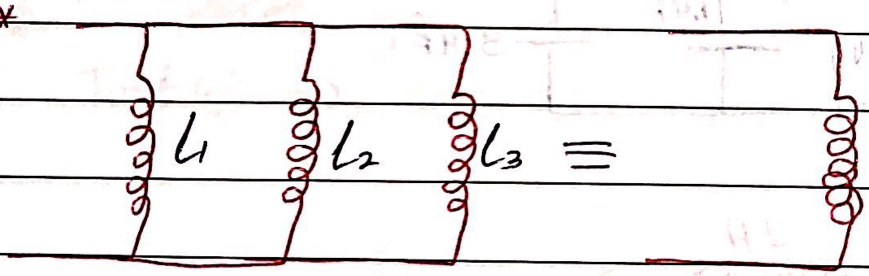
$\frac{\pi}{6}t = \frac{\pi}{2} = 3sec$

$W_L = 216\ J$

$$W_R(t) = \int_0^6 P_R(t)\,dt = \int_0^6 0.1(144 \sin^2(\frac{\pi}{6}t))\,dt = 43.2\ J$$

* 

$L_{eq} = L_1 + L_2 + L_3 \ldots$

* 

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \cdots \frac{1}{L_n}}$$

for 2 inductors in parallel, $\frac{L_1 L_2}{L_1 + L_2}$

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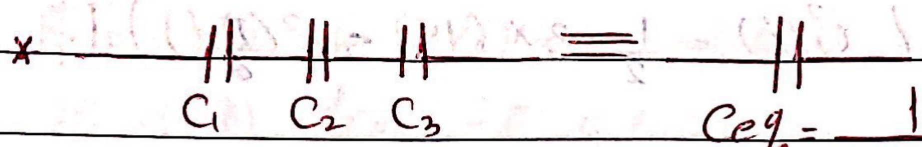
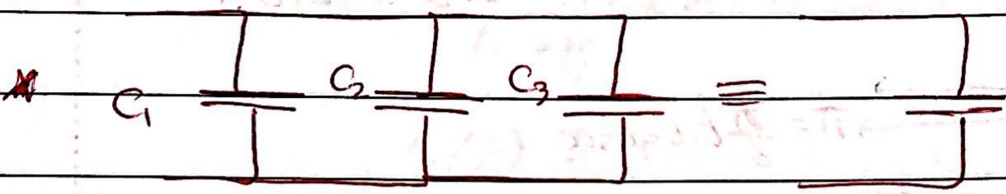


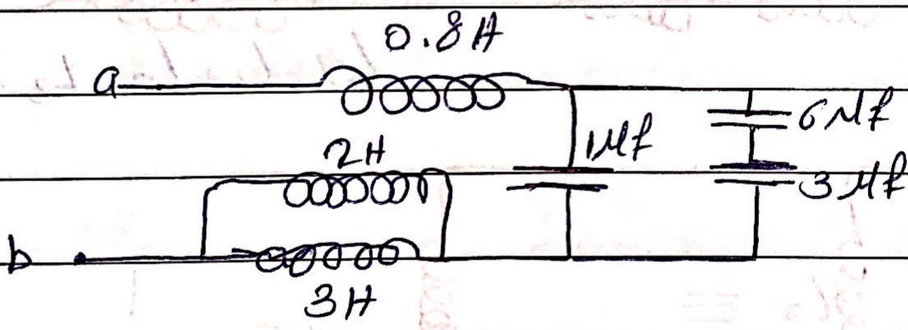
$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \cdots}$$

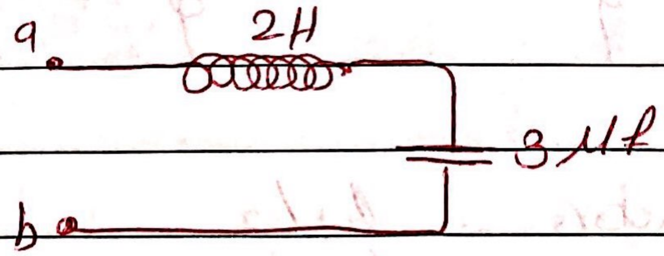


$C_{eq} = C_1 + C_2 + C_3 \cdots C_n$

for 2 capacitors in ~~parallel~~ series $\quad C_{eq} = \frac{C_1 \times C_2}{C_1 + C_2}$

Ex





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* Linearity in Capacitors and Inductors

To test the linearity of a relationship:- $y(x)$

a) $X_{new} = k_1 X \Rightarrow y_{new} = k_2 X_{new}$

or

b) $X_{new} = X_1 + X_2 \Rightarrow y_{new} = y_1 + y_2$

Ex:- $y = X + 3$

Test (a) $\Rightarrow X_{new} = CX \Rightarrow y_{new} = X_{new} + 3$

$CX + 3 \neq Ky$ Const.

* not linera relationship

E:- ~~V(t)~~ $y = 5X$ Const.

Test (a) $\Rightarrow X_{new} = CX \longrightarrow y_{new} = 5X_{new}$

$= 5(CX)$

$= C(5X) \rightarrow Cy$ Const.

* linear relationship

Ex:- $V_L(t) = L\frac{di_L(t)}{dt}$

Test (a) $\Rightarrow i_{Lnew} = K i_L(t) \Rightarrow V_{Lnew} = L\frac{di_{Lnew}}{dt}$

$= L\frac{dK i_L(t)}{dt}$

linear $= K\left(L\frac{dK i_L}{dt}\right)$

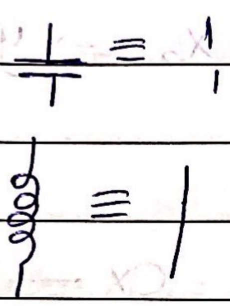
NOTEBOOK

Ex :- $y = 5x^2$

test b) $\rightarrow 5(X_1^2 + X_2^2 + 2X_1X_2)$

* not linear.

→ In DC circuit →



open circuit

short circuit

→ In AC circuits

↳ Direct approach:-

in solving AC circuits that have capacitors and inductors will result in integradifferential equations

→ difficult to be solved.

Note:- All techniques studied before can be used to solve AC circuits that have capacitors and inductors.

"if the circuit has an AC sourse then it's AC circuit."

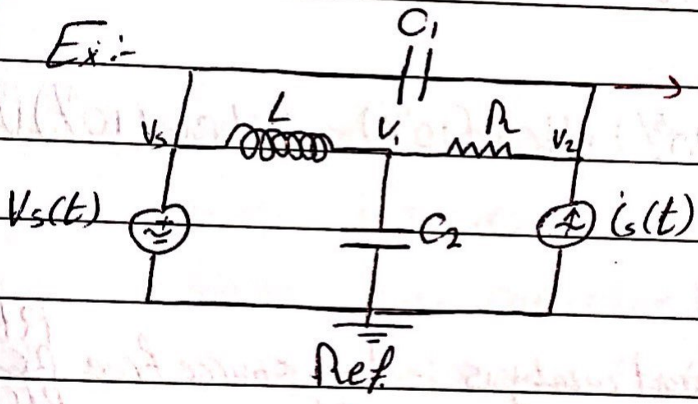
"if it has both AC and DC then it's AC circuit."

"if it hasn't any AC or DC sources then it's DC circuit."

- Types of analysis for circuits

1) DC analysis → for DC circuits "No switching"
2) Transient analysis → for DC circuits with switching process. → 3 steps solution
3) steady state analysis → for AC circuits "No switching"
   - direct approach → (integrodifferential equations) → hard to solve
   - indirect approach.

Ex:-



→ AC circuit → steady state analysis.
- direct approach.

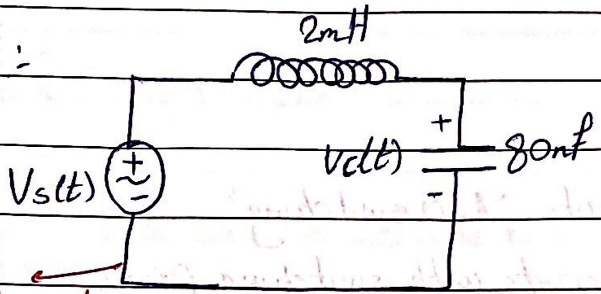
node 1:- $\frac{V_1 - V_2}{R} + C_2 \frac{dV_1(t)}{dt} + \frac{1}{L}\int_{t_0}^{t} (V_1(\tau) - V_s(\tau))\, d\tau + i_L(t_0) = 0$ --- (1)

node 2:- $-i_s(t) + \frac{V_2 - V_1}{R} + C_1 \frac{d(V_2(t) - V_s(t))}{dt} = 0$ --- (2)

These are integradifferential equations → difficult to be solved.

* نكتفي فقط بكتابة المعادلات بالـ (direct approach)

Ex:-



given $V_c(t) = 4\cos 10^5 t$ v
find $V_s(t)$

steady state analysis.

$$i_c(t) = C\frac{dV}{dt} = 80 * 10^{-9}(-4 * 10^5 \sin 10^5 t) = -32 * 10^{-3} \sin(10^5 t)\, A$$

$$KVL = -V_s(t) + L\frac{di(t)}{dt} + V_c(t) = 0$$

$$V_s(t) = -6.4\cos(10^5 t) + 4\cos(10^5 t) = -2.4\cos(10^5 t)\, V.$$

* Transient Analysis

* Types of circuits in transient analysis :- 1- source free RL / RC / RLC circuits.
2- Driven RL / RC / RLC circuits.

* Solution → 3 steps solution
→ step 1 :- at $t = \infty$ → find they type of the circuit.
find τ → $\tau = R_{eq} C_{eq}$ sec / $\tau = \frac{L_{eq}}{R_{eq}}$ sec.
time constant

↳ solution * source free RC → Resistors and capacitors → $i(t) = i(0^+) e^{\frac{-t}{\tau}}$
RL → R and inductors $\underbrace{V(t) = V(0^+) e^{\frac{-t}{\tau}}}_{\text{natural response}}$

no source after switching

final result.

0^+ → from step 3

* Driven $\left.\begin{matrix} RC \\ RL \end{matrix}\right\}$ $i(t) = \underbrace{i(\infty)}_{\text{forced response}} + \underbrace{\left(i(0^+) - i(\infty)\right) e^{\frac{-t}{\tau}}}_{\text{natural response}}$

$$V(t) = V(\infty) + \left(V(0^+) - V(\infty)\right) e^{\frac{-t}{\tau}}$$

→ step 2: at $t = 0^-$ اللحظة ما قبل عملية ال switching

↳ Draw the circuit at $t = 0^-$

↳ find $i(0^-)$, $V(0^-)$, $V_c(0^-)$, $i_L(0^-)$

إذا كان في capacitor فرح نوجد الجهد عليه.

إذا كان في inductor فرح نوجد ال current عليه

→ step 3: at $t = 0^+ = 0$ اللحظة بعد عملية ال switching

→ Draw the circuit at $t = 0^+$

↳ Replace each capacitor by a voltage source of $V_c(0^-)$

↳ Replace each inductor by a current source of $i_L(0^-)$

Because $V_c(0^-) = V_c(0^+) = V_c(0)$

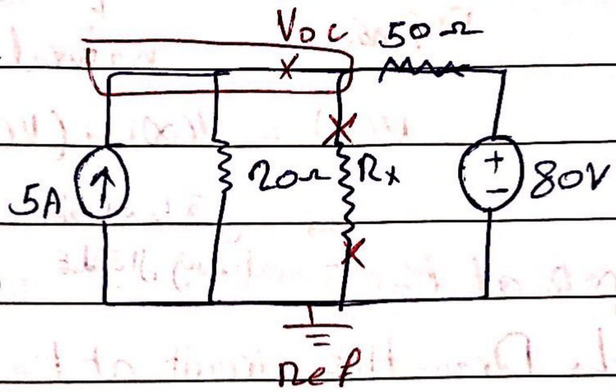
Because $i_L(0^-) = i_L(0^+) = i_L(0)$

↳ find $i(0^+)$, $V(0^+)$

Q1 Find the maximum power absorbed by R_x $\boxed{M_s}$

$R_x = R_{th} = 20 // 50 = 14.29\,\Omega$

$P_{max} = \dfrac{V_{th}^2}{4R_{th}}$

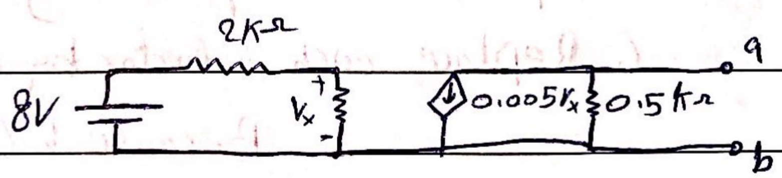


$-5 + \dfrac{V_{oc}}{20} + \dfrac{V_{oc} - 80}{50} = 0$

$V_{oc} = 94.29\,V = V_{th}$

$P_{max} = 155.54\,W.$

Q2 Find R_{th} between points a and b



$R_{th} = \dfrac{V_{oc}}{I_{sc}} = 0.5\,k\Omega$

$V_{oc} = -0.005 V_x \times 0.5$

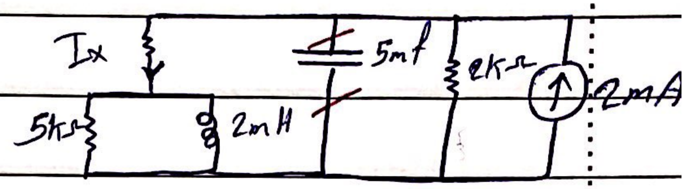
$V_x = 4V \rightarrow$ Voltage division.

$V_{oc} = -10\,V$

$I_{sc} = -0.005 V_x$

$= -20\,mA$

find I_x



Q3

$$I_x = 2 * \frac{2}{2+3} \rightarrow \text{Current division}$$

$$= 0.8\,mA$$

Q4 find the stored energy in a capacitor at $t=2$ seconds given that the initial voltage on the capacitor is 0V, $C = 5\mu F$, and the applied voltage on the capacitor is $V_c(t) = 2\cos(3\pi t)$ V.

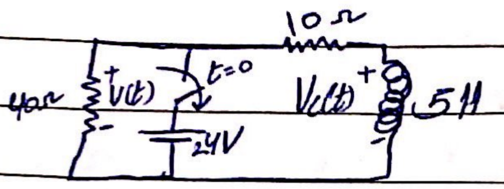
$$W_c(t) = \frac{1}{2} C \left[V_c^2(t) - V_c^2(t_0)\right]$$

$$= \frac{1}{5} \times 5 \times \left(4\cos^2(3\pi t)\right) \mu J$$

$$W_c(t) = 10\,\mu J$$

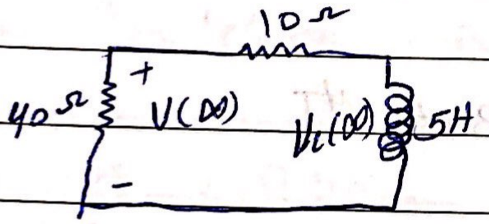
Example:- Find $V(t)$ and $V_L(t)$ for $-\infty < t < \infty$.. (14)
then find $V(t = 200\ msec)$

* في حال طلب السؤال V(t=200) فقط
يجب ان نجد ال voltage او current
ثم نعوض بالقيمة المطلوبة.



* Switching → ∴ Transient analysis. → 3 steps solution.

- step 1: at $t = \infty$ → switch (open)



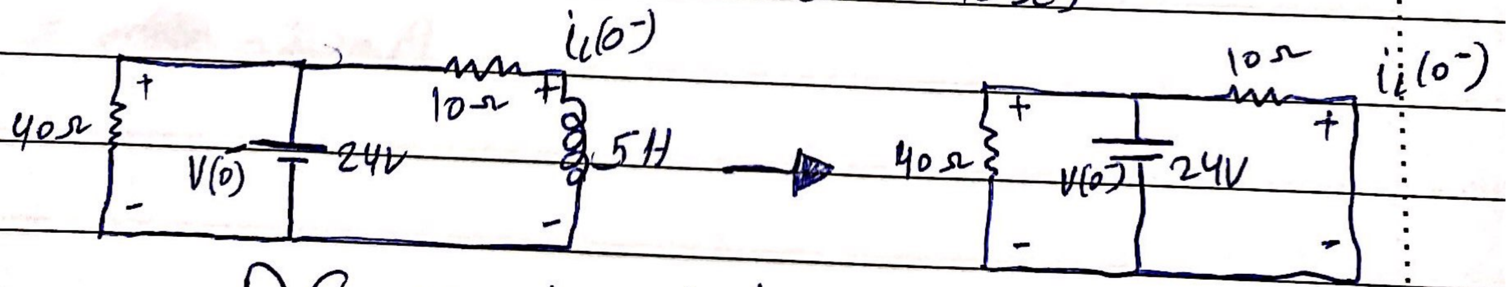
1. Type :- source free RL
2. solution:- $V(t) = V(0)\, e^{-\frac{t}{\tau}}$

$V_L(t) = V_L(0)\, e^{-\frac{t}{\tau}}$

* كونها RL يجب ان نجد قيمة τ

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{5}{50} = 0.1\ sec$$

step 2:- at $t = 0^-$ switch (close)

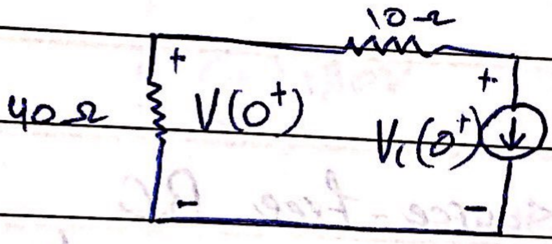


DC circuit → L = | short circuit.

$$V_c(0^-) = 0 \quad , \quad V(0^-) = 24\,V$$

$$i_L(0^-) = \frac{24}{10} = 2.4\,A$$

- step 3 at $t = 0^+$ switch (open)



* نستبدل ال inductor ب current source اتجاهه نفس اتجاه $i_L(0^-)$

$$i_L(0^+) = i_L(0^-) = 2.4\,A$$

$$V(0^+) = -2.4 * 40 = -96\,V$$

$$V_c(0^+) \Rightarrow -(-96) + 10 * 2.4 + V_c(0^+) = 0$$

$$V_c(0^+) = -120\,V$$

$$V(t) = \begin{cases} 24 , & t < 0 \\ -96 e^{-\frac{t}{0.1}} & t > 0 \end{cases}$$

$V(0^-)$

$V(0^+) , V(0)$

$$V_c(t) = \begin{cases} 0 , & t < 0 \\ -120 e^{-\frac{t}{0.1}} , & t \geq 0 \end{cases}$$

$$i_L(t) = \begin{cases} 2.4 , & t < 0 \\ 2.4 e^{-t/0.1} & t \geq 0 \end{cases}$$

لو طلبه مني احسبه

$V(t = 200 mse) \rightarrow V(t)$ at $t > 0$

$$= -96 e^{-\frac{200}{0.1}}$$

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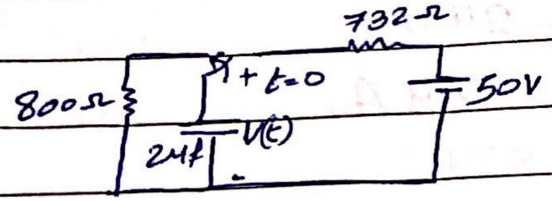
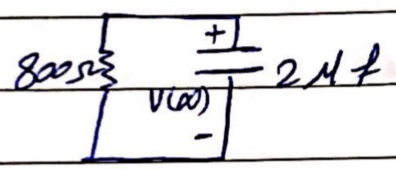
Example:-



find $v(t)$ for all time.

switching → transiet analysis → 3 steps solution.

step 1 at $t=\infty$



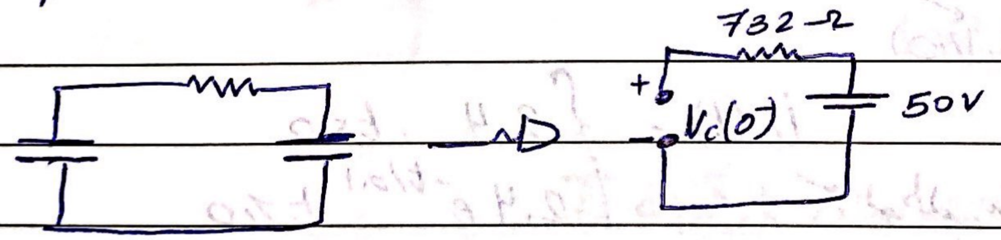
Type → source-free RC

solution → $V_c(t) = V_c(0)\, e^{-t/\tau}$

* $V_c(0^+) = V_c(0^-) = V_c(0)$ in capacitor
* so we don't have to use step 3.

$\tau = RC = 800 * 2 * 10^{-6} = 1.6\,msec.$

step 2 at $t = 0^-$

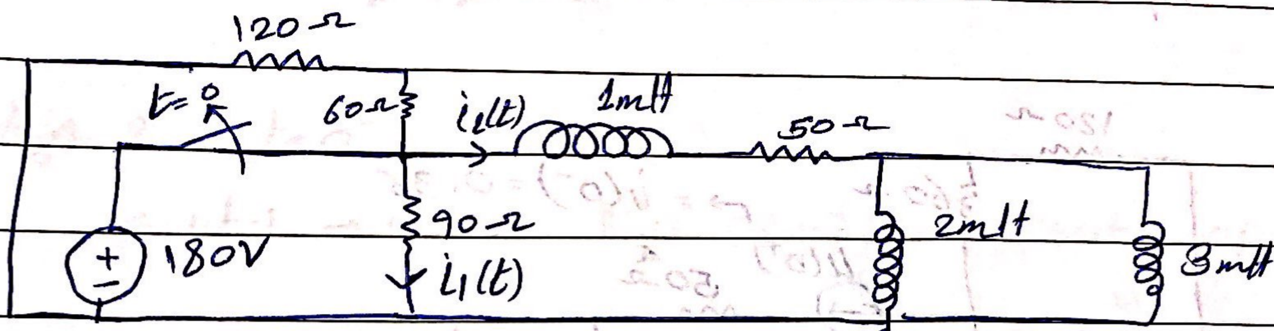


DC circuit → capacitor (open circuit)

$\therefore\ V_c(0^-) = 50V.$

$$\therefore\ V_c(t) = \begin{cases} 50V & t < 0 \\ 50e^{\frac{t}{1.6 * 10^{-6}}} & t \geq 0 \end{cases}$$

Example:- find $i_1(t)$ and $i_L(t)$



switch → 3 steps solution

step 1 : $t = \infty$ type → source free RL circuit

swith (open) solution: $i_1(t) = i_1(0)\, e^{\frac{-t}{\tau}}$

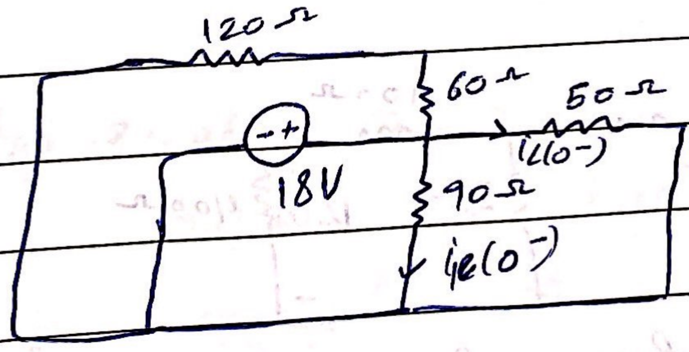
$i_L(t) = i_L(0)\, e^{\frac{-t}{\tau}}$

$i_L(0) = i_L(0^+) = i_L(0^-)$ for inductor.

$\Rightarrow \tau = \frac{L_{eq}}{R_{eq}} = \frac{2.2mH}{50+90 \parallel (60+120)} = 20 \mu sec.$

step 2: at $t = 0^-$

switch (closed), inductors → short circuits "DC"



$i_1(0^-) = \frac{18}{90} = 0.2$

$i_L(0^-) = \frac{18}{50} = 0.36$

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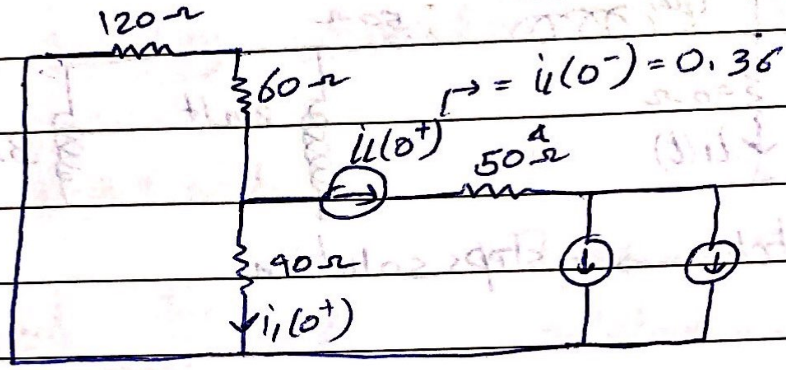
step 3 :- $t = 0^+ = 0$

replace ever inductor → current source $i_L(0^+)$

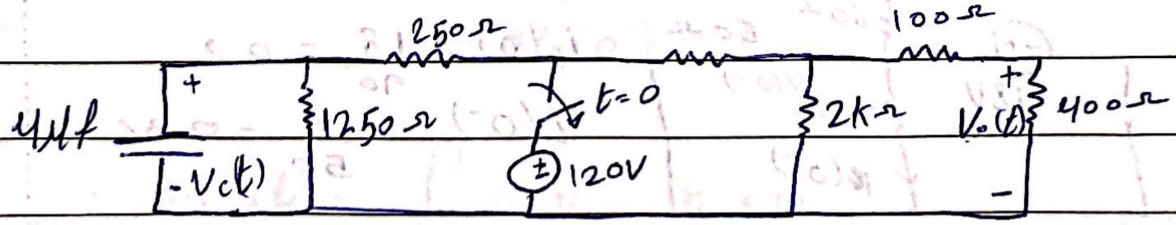


$$i_1(0^+) = -0.36 \times \frac{180}{180+90} = -0.24A$$ "curren division"

$$i_1(t) = \begin{cases} 0.2 & , t<0 \\ -0.24e^{-\frac{t}{\tau}} & , t \geq 0 \end{cases}$$

$$i_L(t) = \begin{cases} 0.36 & t<0 \\ 0.36\, e^{-\frac{t}{\tau}} & , t \geq 0 \end{cases}$$

Ex:- find $V_c(t)$ and $V_o(t)$



step 1 :- $t = \infty$ source free → RC circuit

solution :- $V_c(t) = V_c(0)e^{-\frac{t}{\tau}}, t \geq 0$

$V_o(t) = V_o(0)\, e^{-\frac{t}{\tau}}, t \geq 0$

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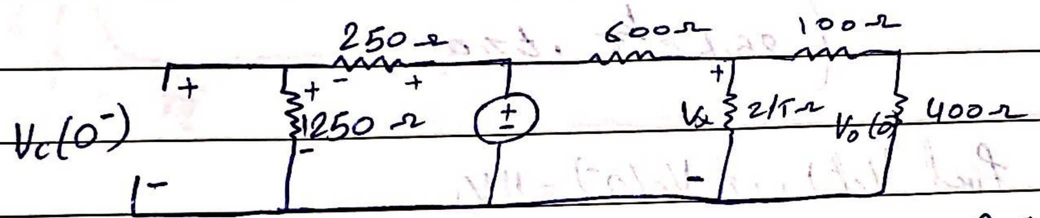
$R_{eq} = (((100+400) \parallel 2K+600)+250) \parallel 1250 = 625\ \Omega$

$\tau = R_{eq} C_{eq} = 625 * 4 = 2.5\ ms.$

-step 2 :- $t=0^-$

switch → closed (DC circuit) → capacitor (open circuit)



* $R_{eq} = 400\ \Omega$

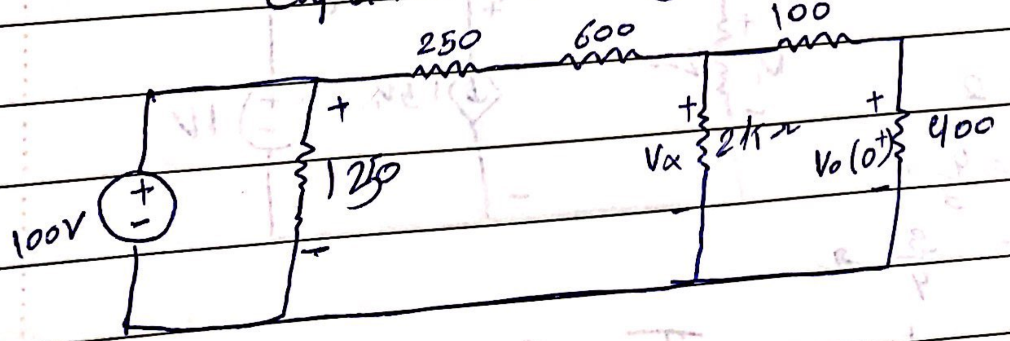
$V_c(0^-) = 120 * \frac{1250}{1250+250} = 100$ → Voltage division

$V_x(0^-) = 120 * \frac{400}{400+600} = 48V.$

$V_o(0^-) = 48 * \frac{400}{400+100} = 38.4$

Step 3 :- at $t = 0^+ = 0$

capacitor → Voltage sorce $(V_c(0^-))$



$V_x(0^+) = 100 * \frac{400}{400+250+600}$

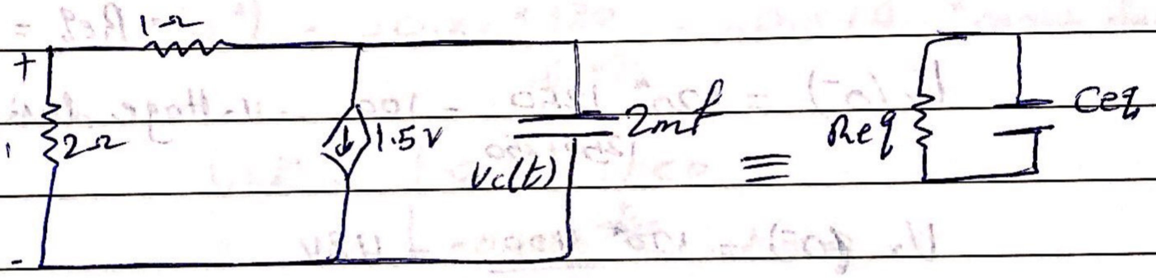
$= \frac{40000}{1250} = 32V$

$$V_o(0^+) = 32 * \frac{400}{400+100} = 25.6V$$

$$V_c(t) = \begin{cases} 100, & t<0 \\ 100e^{-\frac{t}{\tau}}, & t \geq 0 \end{cases}$$

$$V_o(t) = \begin{cases} 38.4, & t<0 \\ 25.6e^{-\frac{t}{\tau}}, & t \geq 0 \end{cases}$$

Ex: find $V_c(t)$, if $V_c(0^-) = 11V$.



* من المعطى بعرف انه في عملية switching والدارة الموجودة بعد عملية switching

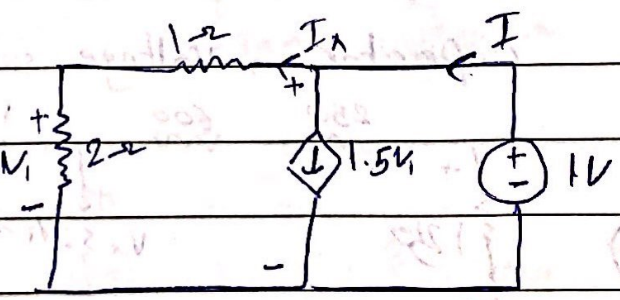
step 1: $t = \infty$ type:- source free RC circuit.

solution $V_c(t) = V_c(0^+)e^{-\frac{t}{\tau}}$

$V_c(t) = 11e^{-\frac{t}{\tau}}$

Rth seen by C

$\tau = R_{eq} C_{eq}$



$V_1 = 1 * \frac{2}{3} = \frac{2}{3}$

$I = I_x + 1.5V_1 = \frac{4}{3} A$

$R_{eq} = \frac{1}{I} = \frac{3}{4} \Omega$

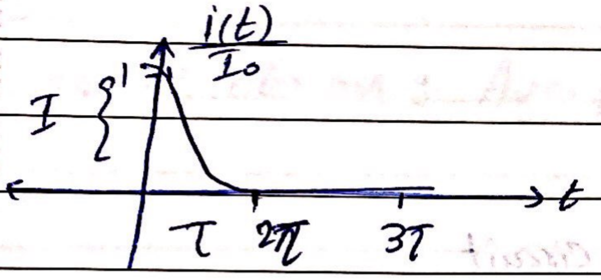
$\tau = \frac{3}{4} \times 2 = 1.5 ms$

$V_c(t) = 11e^{\frac{-t}{1.5 \times 10^{-3}}}$

* Time constant (τ)

- for RL circuits $\tau = \frac{L_{eq}}{R_{eq}}$ -- why?

$$i(t) = I_0 e^{-\frac{R}{L}t}$$



τ :- time requied for the current to drop to zero if it continues to drop at its initial rate,

↓

slope $\left. \frac{di(t)}{dt} \right|_{t=0}$

$$\left. \frac{d(i(t)/I_0)}{dt} \right|_{t=0} = \left. \frac{-R}{L} e^{-\frac{R}{L}t} \right|_{t_1=0} = \frac{-R}{L} = \text{initial rate of decaying}$$

$$-\frac{R}{L} = \frac{1-0}{0-\tau} \quad \therefore \tau = \frac{L}{R}$$

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$i(t=\tau) = I_o e^{-\frac{\tau}{\tau}} = I_o e^{-1} = 0.3679 I_o$

$1\tau \rightarrow 36\% I_o$

$2\tau \rightarrow 13.5\% I_o$

$3\tau \rightarrow 4.9\% I_o$

$4\tau \rightarrow 0.832\% I_o$

$5\tau \rightarrow 0.6738\% I_o \simeq 0$

↳ life time of the circuit

* إذا طلب مني احسب lifetime of the circuit نحسب τ في البداية ثم 5τ.

RC circuit $\Rightarrow \tau = RC$

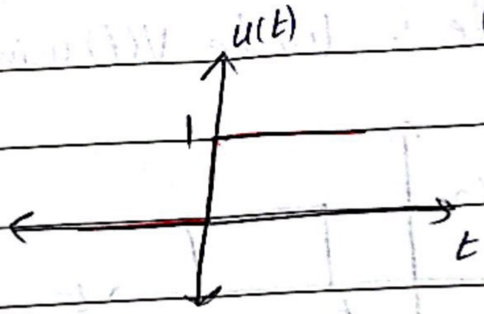
$\tau\uparrow$ = more lifetime ← قيمة كبيرة.

$\tau\uparrow \Rightarrow R\uparrow$ or $C\uparrow$

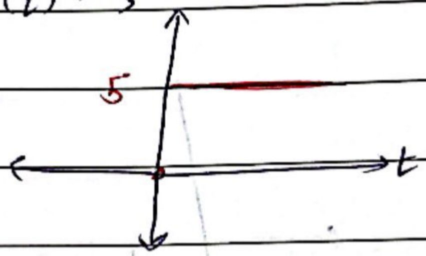
$P_{R\downarrow} = \frac{V_o^2}{R\uparrow}$ more energy stored

* The unit step function

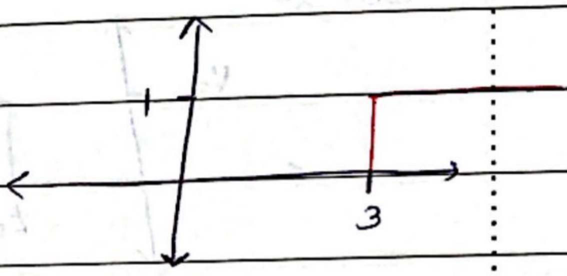
* $u(t) = \begin{cases} 0, & t<0 \\ 1, & t>0 \quad \leftarrow \text{قيمة} \\ \text{undifined} & t=0 \end{cases}$



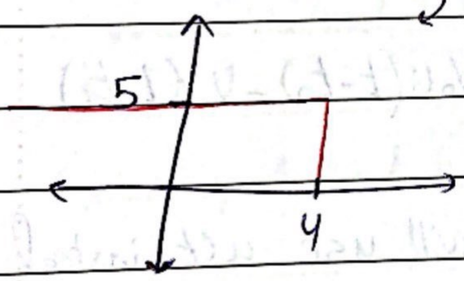
* $5u(t) \rightarrow$



* $u(t-3) \rightarrow t=3$



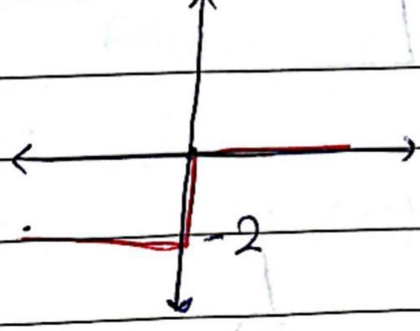
* $5u(4-t)$



$$\boxed{\begin{array}{c} u(-Ve) = 0 \\ u(+Ve) = 1 \end{array}}$$

* نعوض قيمة أقل من ٣ إذا كانت سالبة فالقيمة صفر وإذا كانت موجبة فالقيمة (١) ويكون اتجاه الرسمة باتجاه الذي ظهرت فيه القيمة (١)

* $-2u(-t)$



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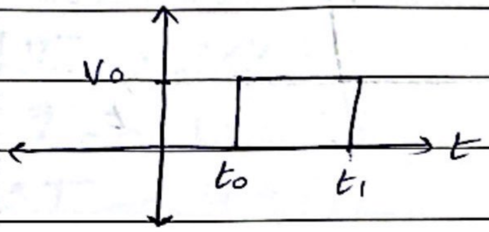
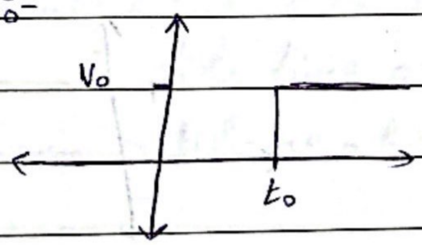
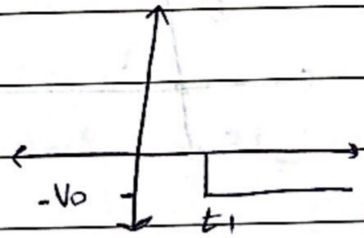
Example :- Write $V(t)$ using $u(t)$



Sol :-



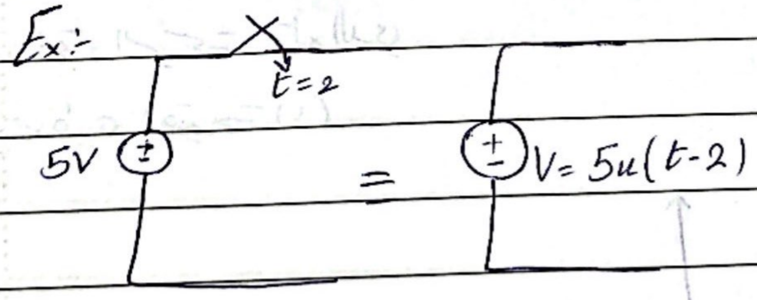


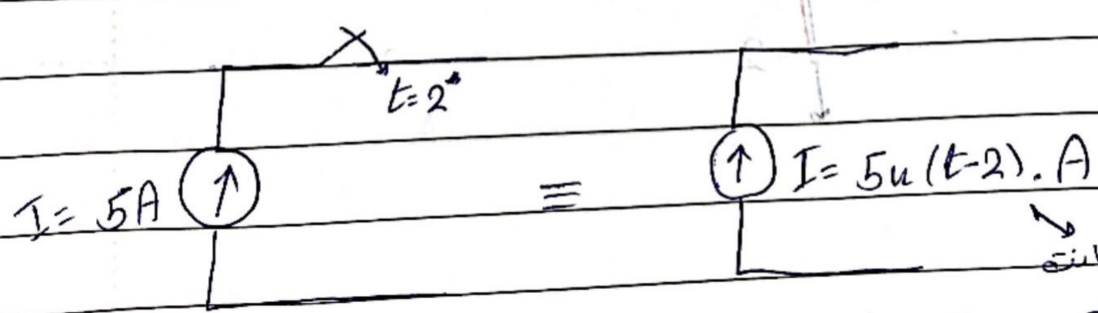
$V_0 u(t-t_0)$ $-V_0 u(t-t_1)$

$$V(t) = V_0 u(t-t_0) - V_0 u(t-t_1)$$

Ex :-

* we will use $u(t)$ instead of switch in some circuits.

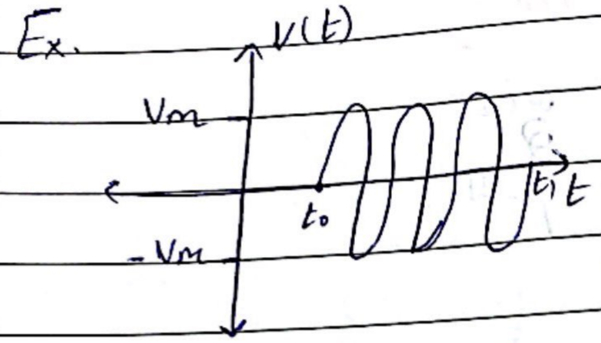




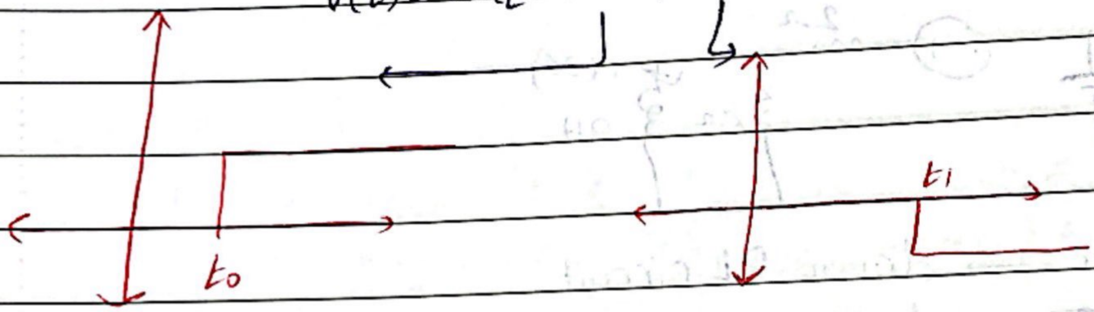
اقل من 2 كانت

Open circuit , Zero = Current قيمة ال

Ex.



$V(t) = V_m[u(t-t_0) - u(t-t_1)]\sin\omega t$



* Driven RL and RC circuits "there is independent source when we apply step 1"

→ step 1: at $t = \infty$

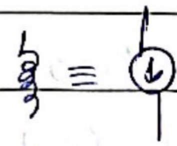
type: ✓ τ: ✓

solution: $i(t) = \underbrace{i(\infty)}_{\text{forced response}} + \underbrace{\left(i(0)^+ - i(\infty)\right) e^{-\frac{t}{\tau}}}_{\text{natural responce.}}$

→ step 2: $t = 0^-$

→ $i_L(0^-)$, $i(s)$, $V_c(0^-)$

→ step 3: $t = 0^+$

 $V_c(0^-) = V_c(0^+)$,  $i_L(0^-) = i_L(0^+) = i_L(0^+)$

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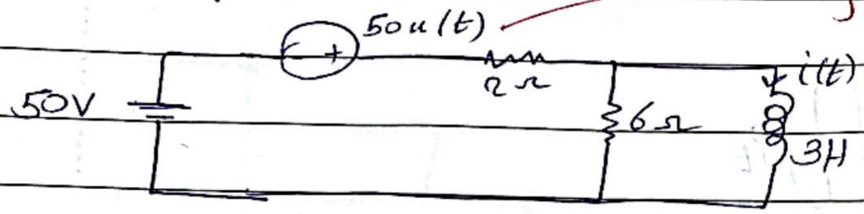
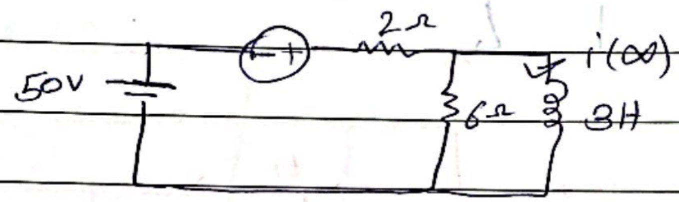
* Example:- find $i(t)$ → switching.



→ step 1 :- $t = \infty$



- type → driven RL circuit

- $\tau = \dfrac{L_{eq}}{R_{eq}} = \dfrac{3}{2 \parallel 6} = 2\,sec$

$i(\infty) = \dfrac{100V}{2\Omega} = 50A$ → "inductor = short circuit"

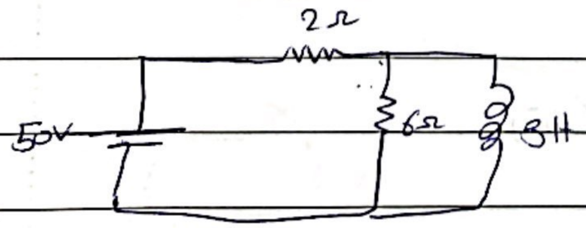
"current through 6Ω = zero"

Solution:- $i(t) = i(\infty) + (i(0) - i(\infty))e^{-\frac{t}{\tau}}, \ t \geq 0$

$= i_L(0^-)$

$= i_L(0^+)$

→ step 2 at $t = (0^-)$



$i_L(0^-) = \dfrac{50}{2} = 25\,A.$

$i_L(0^+) = i_L(0)$

$$i_L(t) = \begin{cases} 25A, & t < 0 \\ 50 + (25 - 50)e^{-\frac{t}{2}}, & t \geq 0 \end{cases}$$

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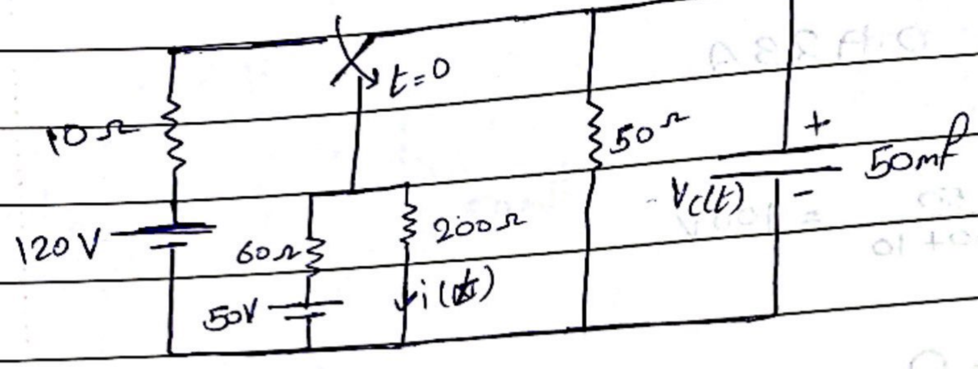
(17)

Ex :- find $i(t)$ and $V_c(t)$



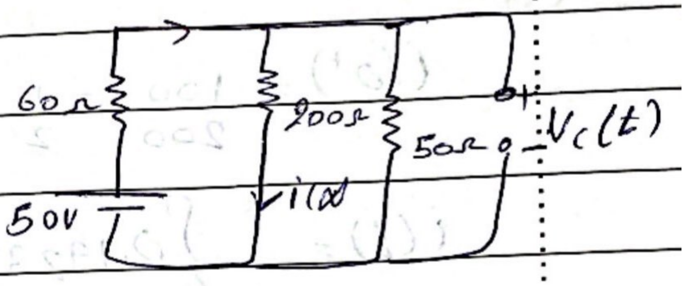
step 1 :- $t=\infty$

type :- driven RC circuit → بنجيب الجزء اللي فيه المطلوب .

$\tau = R_{eq} C_{eq} = (60 \parallel 200 \parallel 50) * 50mf$

$\tau = 1.2 sec$

$i(\infty) = \frac{50}{60} \cdot \frac{\frac{1}{200}}{\frac{1}{200}+\frac{1}{60}+\frac{1}{50}}$



$V_c(\infty) = 0.1 \times 200 = 20V.$

→ $i(t) = 0.1 + (i(0) - 0.1) e^{\frac{-t}{1.2}}, t \geq 0$

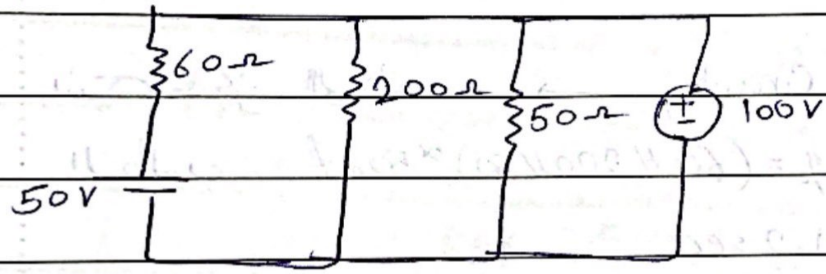
$V_c(t) = 20 + (V_c(0) - 20) e^{\frac{-t}{1.2}}, t \geq 0.$

→ Step 2 :- $t = 0^-$

$$i(0^-) = \frac{50}{260} = 0.1923\,A$$

$$V_c(0^-) = 120 \times \frac{50}{50+10} = 100\,V$$

→ step 3 $t = 0^+ = 0$



$$i(0^+) = \frac{100}{200} = \frac{1}{2}\,A$$

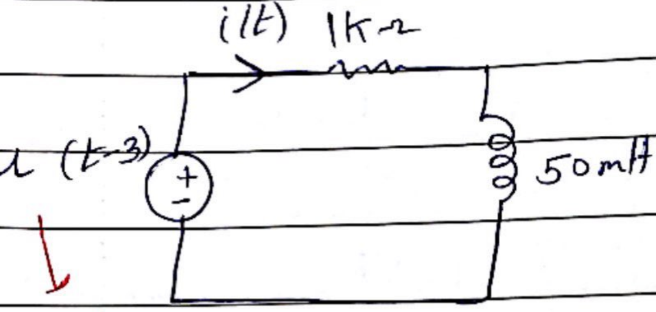
$$i(t) = \begin{cases} 0.1923\,A, & t < 0 \\ 0.1 + \left(\frac{1}{2} - 0.1\right) e^{\frac{-t}{1.2}}, & t \geq 0 \end{cases}$$

$$V_c(t) = \begin{cases} 100, & t < 0 \\ 20 + (100 - 20) e^{\frac{-t}{1.2}}, & t \geq 0 \end{cases}$$

$$i(t=5) = 0.1 + 0.4\, e^{\frac{-5}{1.2}}$$
$$i(t=0) = 0.1 + 0.4 = 0.5\,A$$

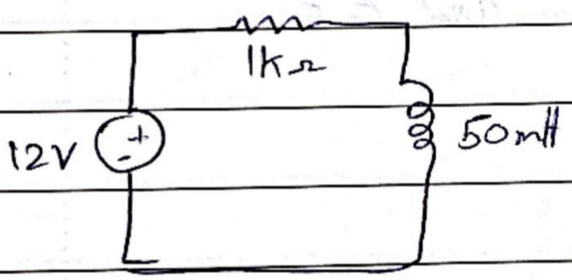
ubject

Example:- find $i(t)$



Switching $= 12u(t')$

→ step 1: $t' = \infty$



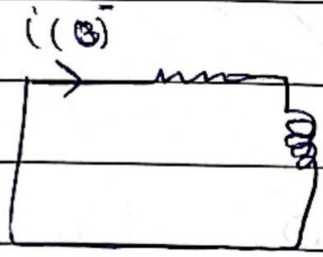
type:- driven RL Circuit.

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{50 \times 10^{-3}}{1000} = 50\,\mu sec.$$

$$i(\infty) = \frac{12}{1000} = 12mA$$

$$i(t') = 12 + (i(0) - 12)\, e^{\frac{-t'}{50 \times 10^{-6}}}$$

→ step 2:- at $t' = 0$

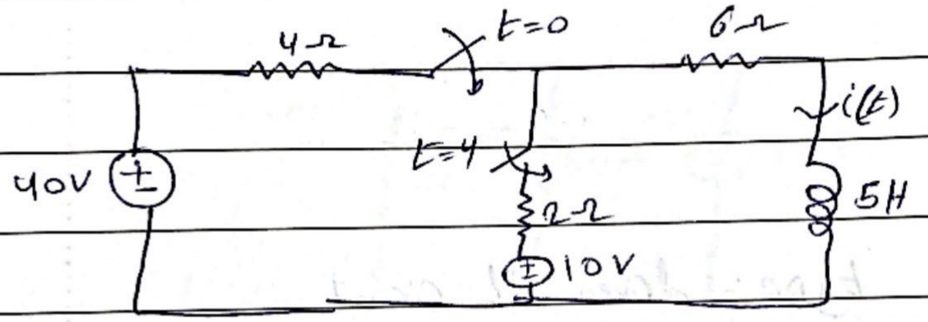


$$i(0^-) = i(0^+) = 0A$$

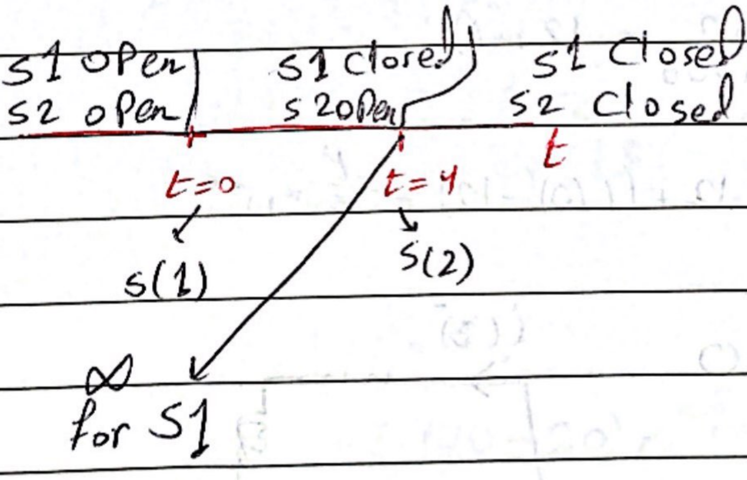
$$i(t') = \begin{cases} 0, & t' < 0 \\ 12 - 12e^{\frac{-t'}{50 \times 10^{-6}}}, & t \geq 0 \end{cases}$$

$$i(t) = \begin{cases} 0, & t-3<0 = t<3 \\ 12-12e^{\frac{-(t-3)}{50\times10^{-6}}} = & t-3\geq 0 = t\geq 3 \end{cases}$$

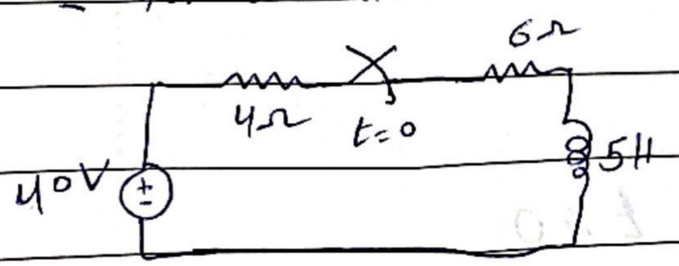
Example:-



find i(t) at t=2sec and t=5sec



for s1 at t=0



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→ step 1 :- at $t=\infty$ $(t<4sec)$

Type → Driven RL circuit.

Solution $i(t) = i(\infty) + (i(0^-) - i(\infty))\, e^{-\frac{t}{\tau}}$

$\tau = \frac{5}{10} = 0.5$ sec

$i(\infty) = \frac{40}{10} = 4A$

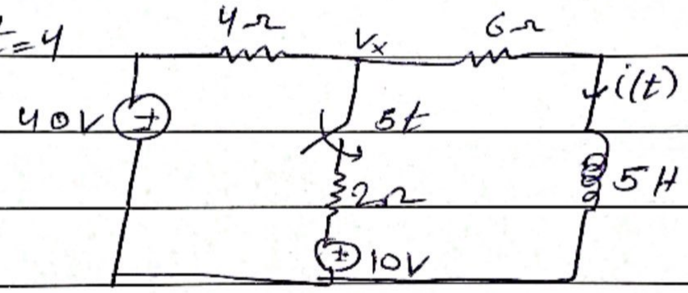
→ step 2 at $t=0^-$

$i(0^-) = 0A$

$6\, i_L$

$i(t) = \begin{cases} 0, & t<0 \\ 4+(0-4)e^{-\frac{t}{0.3}}, & 0 \le t \le 4 \end{cases}$

for s2 $t=4$



→ step 1 $t=\infty$ $(t>4)$

Type ← Driven RL Circuit

Solution → $i(t) = i(\infty) + (i(0) - i(\infty))\, e^{-\frac{t}{\tau}}$

$\tau = \frac{5}{2//4+6} = \frac{15}{25}$ sec

$i(\infty) \rightarrow \frac{V_x - 40}{4} + \frac{V_x - 10}{2} + \frac{V_x}{6} - 0 \rightarrow \cancel{V_x = 10+5}$

nodal analysis

$V_x = \frac{108}{11}$ V

$i(\infty) = \frac{V_x}{6} = 2.72A$

* اول switch نأخذ
الفترة قبل و بعد
* ثاني switch نأخذ الفترة
بعد وفقط

step 2 $t' = 0^-$

$$i(0^-) = \frac{40}{10} = 4A$$

$$i(t') = 2.72 + (4-2.72)e^{-\frac{t'}{\tau}}, \quad t' \geq 0$$

$$i(t) = 2.72 + (4-2.72)e^{-\frac{(t-4)}{\tau}}, \quad t-4 \geq 0 = t \geq 4$$

$$i(t) = \begin{cases} 0A, & t \leq 0 \\ 4-4e^{-\frac{t}{0.5}}, & 0 \leq t \leq 4 \\ 2.72, 1.273e^{-\frac{(t-4)}{15}}, & t \geq 4 \end{cases}$$

s1

s2

* RLC circuit.

* step 1 :- at $t=\infty$

Type :- ① source free RLC circuit
↳ Parallel
↳ series
② Driven RLC circuit
↳ parallel
↳ series

⇒ find ① $\omega_0 = \frac{1}{\sqrt{LC}}$ (resonant frequency) (rad/sec)

Neper frequency / exponential dumping coficient > ② $\alpha = \frac{1}{2RC}$ (in parallel) , $\frac{R}{2L}$ (in series)

③ $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

④ $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

⇒ solution (for driven RLC circuit)

$i(t) = i(\infty) +$ solutions in source free.

⇒ solution (for source free)

* if $\alpha > \omega_0$ ⇒ over damp response

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

* if $\alpha = \omega_0$ ⇒ critical damp response.

$$i(t) = A_1 t e^{-\alpha t} + A_2^{-\alpha t}$$

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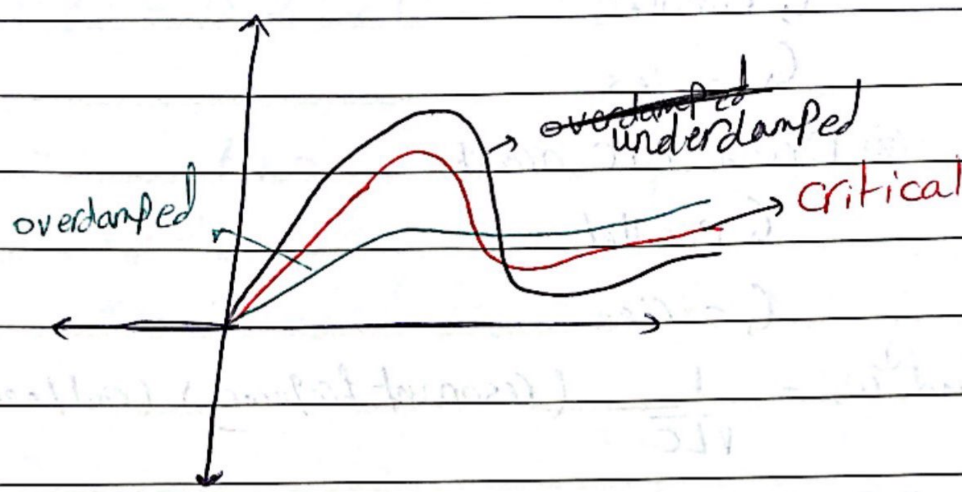
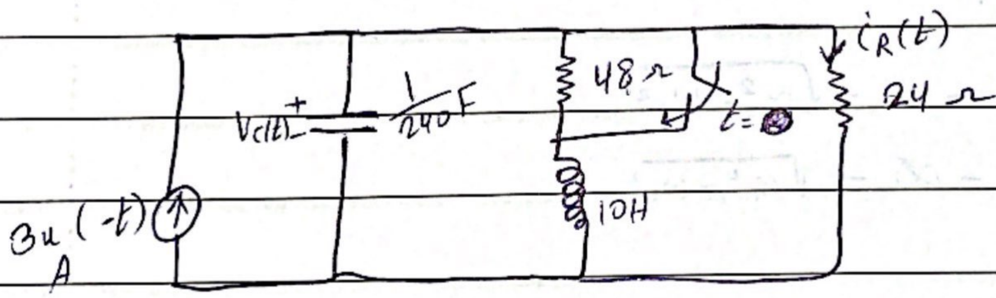
* if $\alpha < w_0$ → underdamped response

$$i(t) = e^{-\alpha t}\left[A_1 \cos(w_d t) + A_2 \sin(w_d t)\right]$$

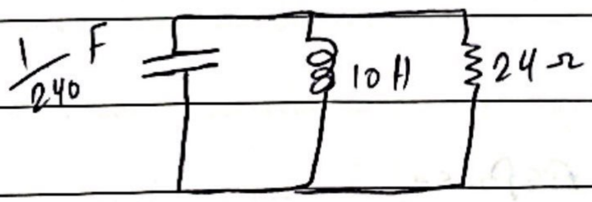
$w_d = \sqrt{w_0^2 - \alpha^2}$ (natural resonant frequency)



Example:- find $V_c(t)$ and $i_R(t)$



step1 : $t=\infty$ Type :- source-free parallel RLC circuit



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$$\alpha = \frac{1}{2RC} = \frac{1}{2 * 24 * \frac{1}{240}} = 5 \sec^{-1} (Hz)$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{24} \text{ rad/sec}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0} = -4 \sec^{-1}$$

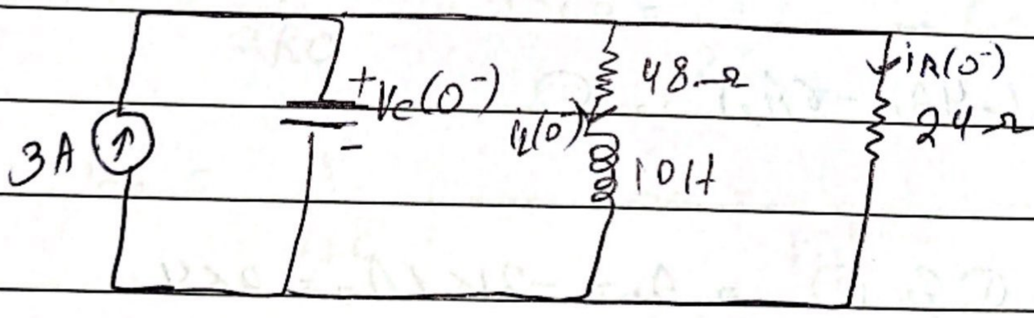
$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -6 \sec^{2}$$

Because $\alpha > \omega_0 \Rightarrow$ overdamped

$$V_c(t) = A_1 e^{-4t} + A_2 e^{-6t}$$

$$i_R(t) = B_1 e^{-4t} + B_2 e^{-6t}$$

$\Rightarrow$ step 2 :- $t = 0^-$



$$V_c(0^-) = 3(24 // 48) = 48 \text{ V.}$$

$$i_R(0^-) = \frac{48}{24} = 2A$$

$$i_L(0^-) = \frac{48}{48} = 1A$$

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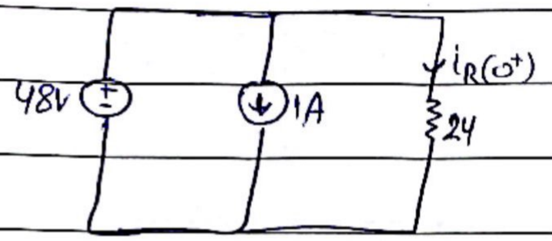
⟹ step 3: $(t=0^+)$



$$V_c(0^+) = V_c(0^-) = 48V = V_c(0)$$

$$i_R(0^+) = i_R(0) = \frac{48}{24} = 2A$$

* To find A_1 and A_2

$$V_c(0) = 48 = A_1 + A_2 \quad \ldots ①$$

$$i_c(t) = C\frac{dV(t)}{dt} = \frac{1}{240} * (-4A_1e^{-4t} - 6A_2e^{-6t})$$

$$i_c(0) = \frac{1}{240} = (-4A_1 - 6A_2) \quad \ldots ②$$

Solve ① & ② → $A_1 = -216 \,/\, A_2 = 264$

$$\therefore V_c(t) = \begin{cases} 48V, & t<0 \\ -216e^{-4t} + 264e^{-6t}, & t \geq 0 \end{cases}$$

$$i_R(t) = \frac{V_c(t)}{24} = \begin{cases} 2A, & t<0 \\ \frac{216}{24}e^{-4t} + \frac{264}{24}e^{-6t}, & t \geq 0 \end{cases}$$

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Example :- A parallel RLC circuit, $L = 10mH$, $C = 100\mu f$. find R that will lead to :- 1) overdamped response. 2) Underdamped response 3) Critical damped response.

$\Rightarrow$ overdamped :- $\alpha > \omega_0$

$$\frac{1}{2RC} > \frac{1}{\sqrt{LC}} \rightarrow \frac{1}{2R \times 100 \times 10^{-6}} > 100$$

$$R < 5\,\Omega$$

1. $R < 5$ 2. $R > 5$ 3. $R = 5$

Ex :- Parallel RLC circuit, $R = 100\,\Omega$, $\alpha = 1000\ sec^{-1}$, $\omega_0 = 800\ rad/sec$. find L, C, S_1, S_2.

$$\alpha = \frac{1}{2RC} \rightarrow 1000 = \frac{1}{2 \times 100 C} \rightarrow C = 5\mu f$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow 800 = \frac{1}{\sqrt{L \times 5 \times 10^{-6}}} \rightarrow L = 312.5\,mH$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -400\,sec^{-1}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -1600\,sec^{-1}$$

- $S_1 = S_2 \longrightarrow$ Critical damped
- S_1 and $S_2 \longrightarrow$ real numbers $\longrightarrow$ over damped
- S_1 and $S_2 \longrightarrow$ Complex $\longrightarrow$ underdamped.

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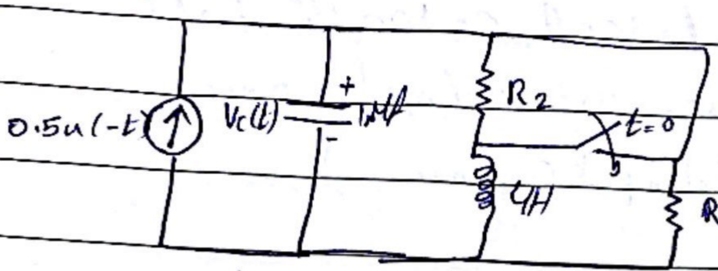
1a



a) select R_1 so that the responce after $t=0$ will be critically damped.

b) Select R_2 to obtain $V_c(0) = 100V$

c) find $V_c(t)$ at $t=1$ msec.

Sol:- step 1: at $t=\infty$

↳ source-free parallel RLC circuit.

$$\alpha = \frac{1}{2RC} \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

for critical damping → $\alpha = \omega_o$

$$\frac{1}{R_1 C} = \frac{1}{\sqrt{LC}} \rightarrow \frac{1}{2R_1 \times 10^{-6}} = \frac{1}{\sqrt{4 \times 10^{-6}}}$$

$\therefore R_1 = 1k\Omega$, $\alpha = 500 \sec^{-1}$

$$V_c(t) = A_1 t e^{-\alpha t} + A_2 e^{-\alpha t}$$

$$= A_1 t e^{-500t} + A_2 e^{-500t}$$

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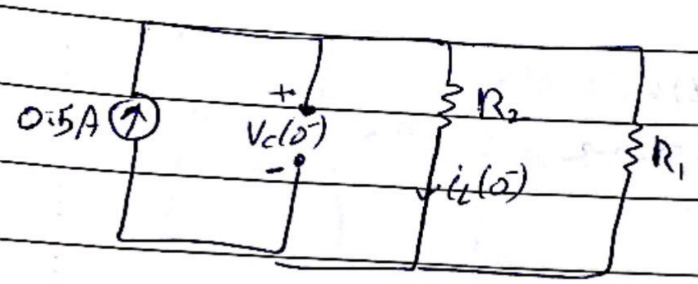
⟹ step 2 :- $t = 0^-$

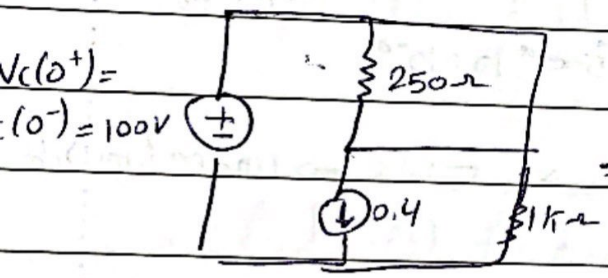


$V_c(0^-) = 100 = 0.5\ R_1 // R_2$

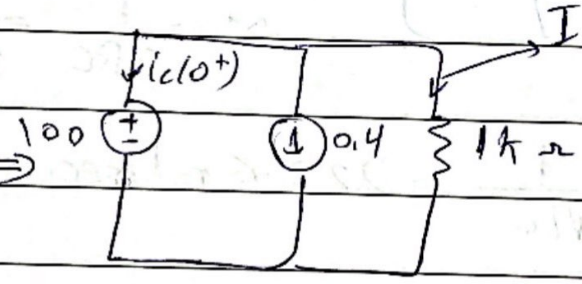
$R_2 = 250\,\Omega$

$i_L(0^-) = 0.5 \frac{R_1}{R_1 + R_2} = 0.4A$

⟹ step 3 : at $t = 0^+ = 0$



⟹



$I_R = \frac{100}{1000} = 0.1A$

$i_c(0^+) = -0.5A$

$V_c(0) = A_2 \ \ldots ① \longrightarrow A_2 = 100$

$i_c(0^+) = C \frac{dV_c(t)}{dt}\Big|_{t=0} \ \ldots ②$

$-0.5 = 1 * 10^{-6} [A_1 e^{-500t} - 500 A_1 t e^{-500t} - 500 A_2 e^{-500t}]$

$A_1 = -45 * 10^4$

$\therefore V_c(t) = \begin{cases} -45*10^4 t e^{-500t} + 100 e^{-500t}, & t \geq 0 \\ 100, & t < 0 \end{cases}$

~~Vc(t)~~ $V_c(t = 1msec) = -212V.$

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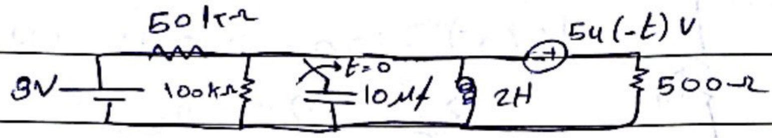
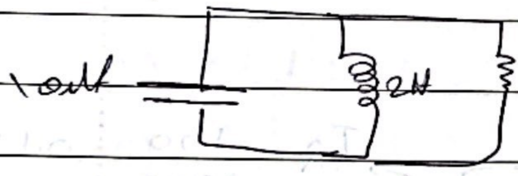
Example:- Find $V_c(t)$ for all t.



⟹ step 1 : $t = \infty$



* source-free parallel RLC circuit.

$$\alpha = \frac{1}{2RC} = \frac{1}{2 * 500 * 10 * 10^{-6}} = 100 \sec^{-1}$$

$$w_0 = \frac{1}{\sqrt{LC}} = 223.6 \text{ rad/sec.} \qquad \alpha < w_0 \Rightarrow \text{underdamped}$$

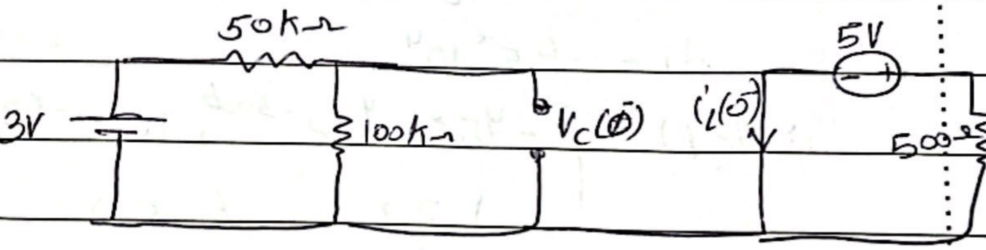
$$V_c(t) = [A_1 \cos(w_0 t) + A_2 \sin(w_0 t)\, e^{-\alpha t}$$

$$w_d = \sqrt{w_0^2 - \alpha^2} = 200 \text{ rad/sec.}$$

$$V_c(t) = [A_1 \cos(200t) + A_2 \sin(200t)]\, e^{-100t}$$

$$i_L'(0) = C \frac{dV_c(t)}{dt}$$

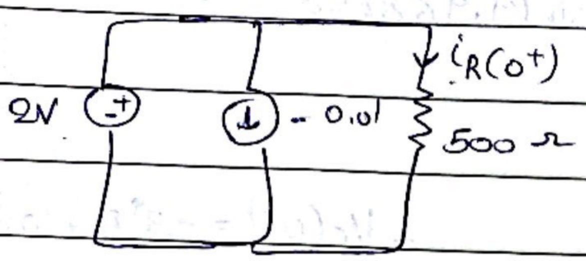
⟹ step 2 : $t = 0^-$



voltage division.

$$V_c(0^-) = \frac{3 * 100}{100+5} = 2 \text{ Volt.} \qquad i_L(0^-) = \frac{-5}{500} = -0.01 \text{ A.}$$

$\Rightarrow$ step 3 $t=0^+$



$i_R(0^+) = \frac{2}{500}\,A$

$i_c(0^+) = -(-0.01 + \frac{2}{500}) = 0.006\,A$

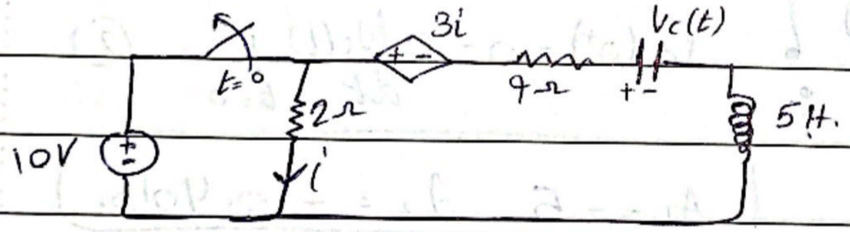
$V_c(0) = 2 = A_1 \quad \ldots ① \rightarrow A_1 = 2$

$i_L(0) = 0.006 = 10^{x}\,10^{-6}\,[[-A\,200\sin(200t)]e^{-100t}\,[A_1\cos \ldots$

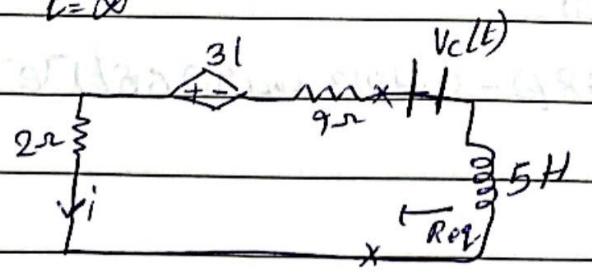
$0.006 = 200A_2 - 100A_1 \quad \ldots ②$

$\therefore V_c(t) = \begin{cases} 2V, & t<0 \\ [2\cos(200t) + 4\sin(200t)]e^{-100t}, & t \geq 0 \end{cases}$

Ex :- find $V_c(t)$ for this circuit.



$\Rightarrow$ step 1 :- $t=\infty$



$\Rightarrow$ source-free series RLC circuit

$\alpha = \frac{Req}{2L} = 0.8\,sec^{-1}$

$Req = R_{th}$

test voltage

$R_{th} = \frac{1}{i} = 8\,\Omega$

$-1 + 9i - 3i + 2i = 0$

$i = \frac{1}{8}$

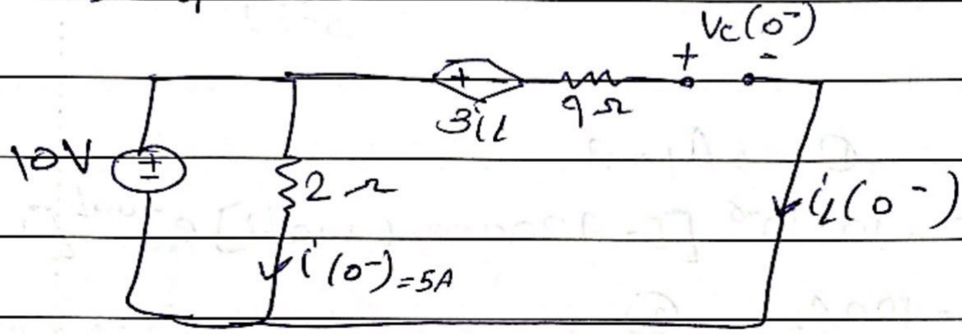
$w_0 = \frac{1}{\sqrt{LC}} = 10\,rad/sec$

$\alpha < w_0 \Rightarrow$ underdamped

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 9.968$ rad/sec

$V_c(t) = [A_1 \cos(9.968t) + A_2 \sin(9.968t)]e^{-0.8t}$

⇒ step 2 $t = 0^-$



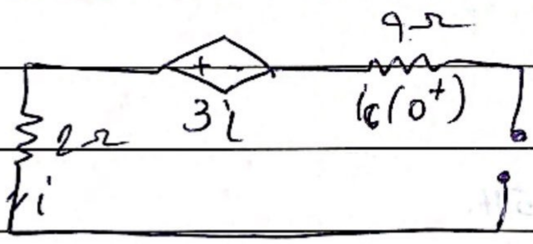
$-V_c(0^-) = -3*5 + 10 = 0$

$V_c(0^-) = -5V$ ... (1)

$i = \frac{10}{2} = 5$

$i_L(0^-) = 0A.$

⇒ step 3 $t = 0^+ = 0$



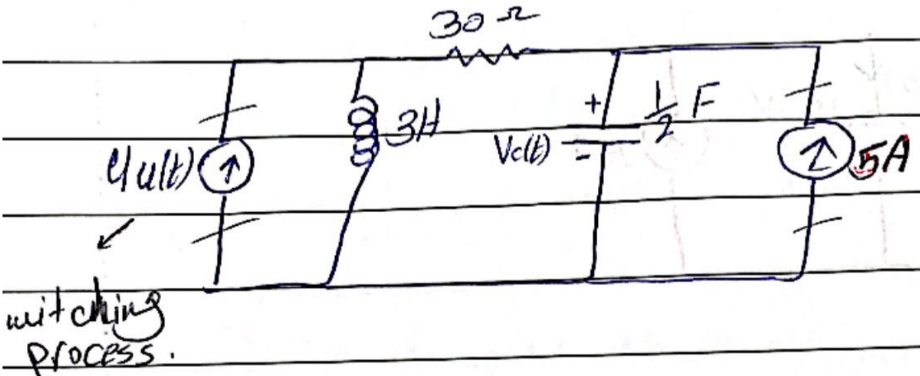
$i_c(0^+) = 0 = C\frac{dV_c(t)}{dt}\Big|_{t=0}$ ... (2)

$i_c(0^+) = 0A$

$\boxed{A_1 = -5, \; A_2 = -0.4013.}$

$$V_c(t) = \begin{cases} -5, & t < 0 \\ [-5\cos(9.968t) - 0.4013\sin(9.968t)]e^{-0.8t}, & t \geq 0 \end{cases}$$

Ex :- find $V_c(t)$ for all time



* to determine whether the RLC driven circuit is parallel or series Kill all the indpendent sources.

⟹ step 1 :- at $t = \infty$

type :- Driven series RLC

solution :- $V_c(t) = V_{cf} + V_{cn}$

$= V_c(\infty) + \underbrace{A_1 e^{s_1 t} + A_2 e^{s_2 t}}$ according to type of damping.

V_{cf} :- force responce

V_{cn} :- natural response

$V_c(\infty) = 5 * 30 = 150V$

$\alpha = \frac{R}{2L} = 5 \sec^{-1}$

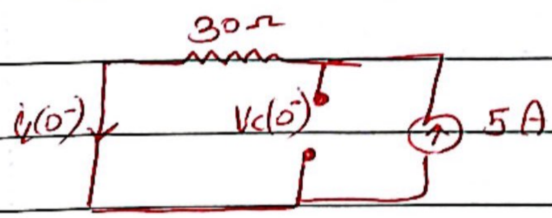
$\omega_0 = \frac{1}{\sqrt{LC}} = 3$ rad/sec

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -1 \sec^{-1}$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -9 \sec^{-1}$

* $\alpha > \omega_0 \Rightarrow$ overdamping.

⟹ step 2 :- at $t = 0^-$

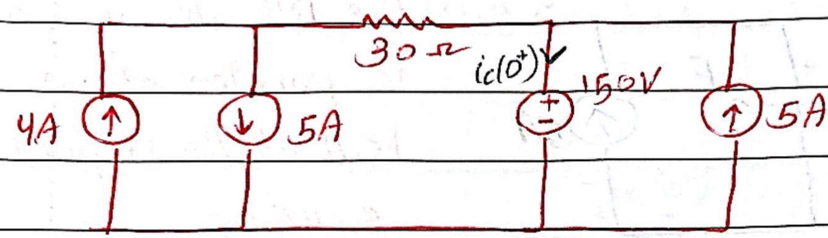


$V_c(0^-) = 5 * 30 = 150V$

$i_L(0^-) = 5A$

• remember : if there is an inductor we have to find $i_L(0^-)$, and if there is a capacitor we have to find $V_c(0^-)$

$\Rightarrow$ step 3 :- at $t = 0^+ = 0$



$$i_c(0^+) = i_c(0) = 4A$$

- to find A_1 and A_2

تعويض في المعادلة الأصلية.

$$V_c(0) = 150 = 150 + A_1 + A_2 \quad \ldots\ldots (1)$$

$$i_c(0) = C \frac{dV_c(t)}{dt}\bigg|_{t=0} \Rightarrow 4 = \frac{1}{27}\left[-A_1 - 9A_2\right] \ldots (2)$$

$A_1 = 13.5$, $A_2 = -13.5$

$$\therefore V_c(t) = \begin{cases} 150 & , t \le 0 \\ 150 + 13.5e^{-t} - 13.5e^{-9t} & t \ge 0 \end{cases}$$

Note:- lossless LC circuit

↳ parallel RLC $\Rightarrow$ if $R=\infty$, then lossless LC circuit

↳ series RLC $\Rightarrow$ if $R=0$ " " " ".

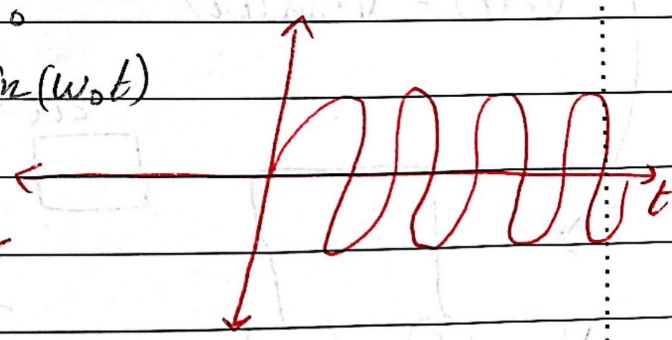
$$\alpha = \frac{1}{2RC} = 0 \Rightarrow \alpha < \omega_0$$

$$\alpha = \frac{R}{2L} = 0 \Rightarrow \alpha < \omega_0$$

underdamping

$$V(t) = \underbrace{\cancel{e^{-\alpha t}}}_{1}\left[A_1 \cos(\underbrace{\omega_d}_{\omega_0} t) + A_2 \sin(\omega_d t)\right]$$

$$\therefore V(t) = A_1 \cos(\omega_0 t) + A_2 \sin(\omega_0 t)$$



$\boxed{\text{Quiz 2}}$

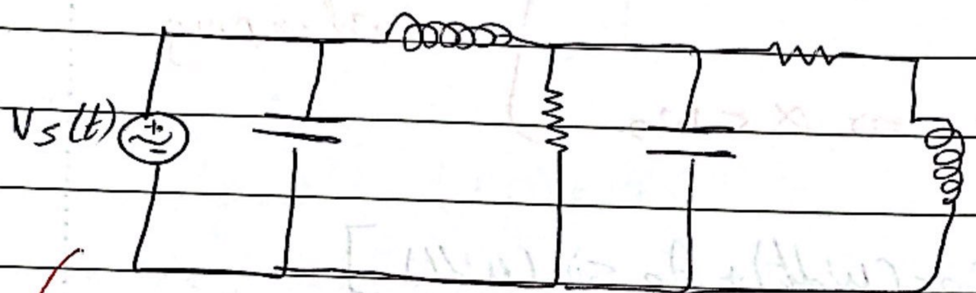
→ AC circuit without switching.

* Sinusoidal steady-state Analysis

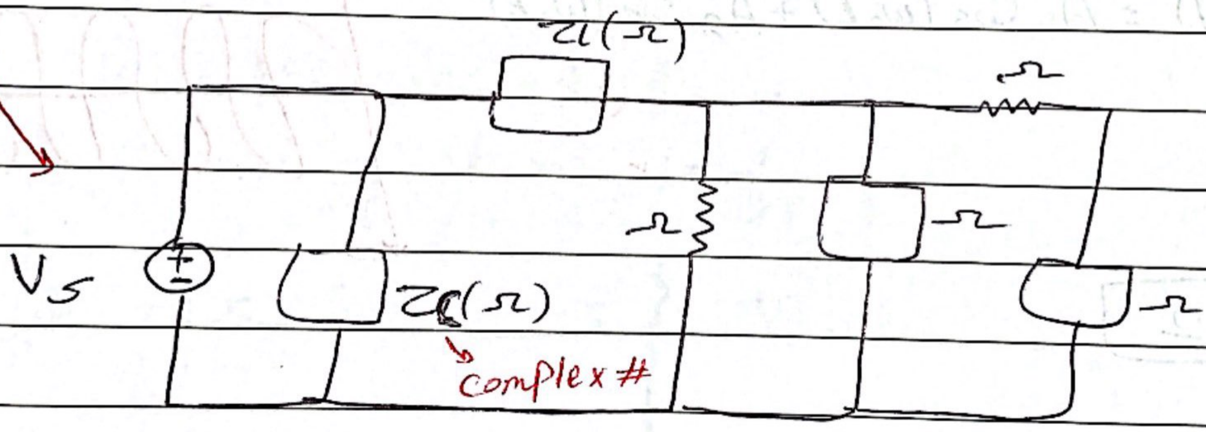
$\boxed{21}$

* Solution → sources as a function of time

time domain $\xrightarrow{\text{إلى}}$ frequency-domain (or phaser-domain)



$V_s(t) = V\sin(\omega t)$



in phaser domain we represent (capcitors and inductors) as a blocks (Z_L) "Ω" and (Z_C) "Ω"

inductors — capacitors

* Complex numbers :- Real number + imaginary number

ex:- $\bar{X} = 5 + j3$

→ jx imaginary part
- imaginary operator

$\Rightarrow j = \sqrt{-1}$

$j^2 = -1 \quad j^3 = -j \quad , \frac{1}{j} = -j$

$j^4 = 1$

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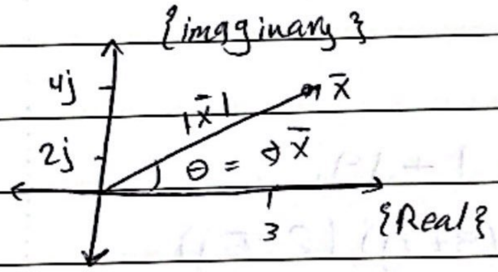
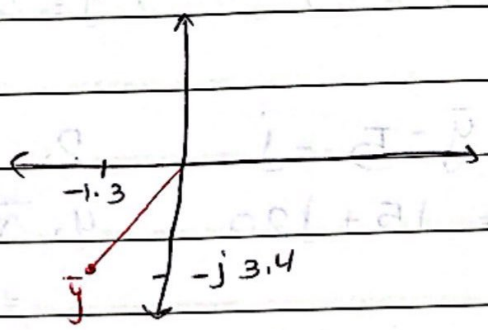
* How to represent Complex number

$\boxed{1}$ Rectangular form

Ex :- $\bar{X} = 3 + j4$ $\bar{y} = -1.3 - j3.4$





$\boxed{2}$ polar form (exponental form)

$$\bar{X} = |\bar{X}| \angle \theta.$$
$$= |\bar{X}| e^{j\theta}$$

* if $\bar{X} = a + jb$
$\hookrightarrow |\bar{X}| = \sqrt{a^2 + b^2}$, $\theta = \tan^{-1} \frac{b}{a}$
$a = |X| \cos\theta$, $b = |X| \sin\theta$

Ex :- ① if $\bar{A} = 4 + j2$ write A in Polar form.

$|\bar{A}| = \sqrt{16+4} = \sqrt{20} = 4.47$

$\theta = \tan^{-1} \frac{2}{4} = 26.6°$ $\bar{A} = 4.47 \angle 26.6°$

② if $\bar{X} = 5 \angle -36.9°$ write X in the rectangular form.

$\bar{X} = 5\cos\theta(-36.9) + j5\sin(-36.9) = \cancel{4-j3}$

$4 - j3$

Ex:- $\vec{X} = e^{j90^\circ} \longrightarrow \bar{X} = j.$

Ex:- $\vec{X} = e^{-j90^\circ} \longrightarrow \bar{X} = -j$

* Mathematical operations. for Complex numbers.

ex:- if $\bar{X} = 3+j4$, $\bar{y} = 2-j5$

1. $\bar{X} + \bar{y} = 5 - j$ 2. $\bar{X} - \bar{y} = 1 + j9.$

3. $5\bar{X} = 15 + j20$ 4. $\bar{X} * \bar{y} = (3+4j)(2-5j)$

$= 6 - j15 + j8 + 20 = 26 - j7$

5. Conjugate of $\bar{X} \Rightarrow \bar{X}^* = 3 - j4 \rightarrow$ نقلب اشارة [illegible] imaginary.

$\bar{y} \Rightarrow \bar{y}^* = 2 + j5.$

Ex:- $\vec{X} + \vec{X}^* = 6$, $\vec{X} - X^* = j8$, $\vec{X} * X^* = 3^2 + 4^2 = 25.$

$(\vec{X}^*)^* = \vec{X}.$

6. $$\frac{\vec{X}}{\bar{y}} = \frac{(3+j4)}{(2-j5)}\left(\frac{2+j5}{2+j5}\right) = \frac{6+15j+8j-20}{4+25}$$

$$= \frac{-14+23j}{29} = \frac{-14}{29} + \frac{23}{29}j$$

Polar form. $\leftarrow$

OR $$\frac{\vec{X}}{\bar{y}} = \frac{5e^{j\tan^{-1}(\frac{4}{3})}}{\sqrt{29}\, e^{j\tan^{-1}(2.5)}} = \frac{5}{\sqrt{29}} e^{j(\tan^{-1}(\frac{4}{3}) - \tan^{-1}(2.5$$

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* Integers

* If a and b are integers with $a \neq 0$, we say that a divides b if there is an integer c such that $b = ac$. when a divides b we say that a is a factor of b and that b is a multiple of a.

* the notation $a|b$ denotes that a divides b.
* the notation $a \nmid b$ denotes that a does not divide b.

* Example: $3 \nmid 7$, $5 \nmid 12$, $4 \nmid 14$

لو قسمنا 8 على 2 باقي القسمة بيساوي صفر

$2|8$, $3|12$, $5|15$

* Division
- let a, b, and c be integers. Then
  * if $a|b$ and $a|c$, then $a|(b+c)$ → $b + c = 6 + 9$
    ex. $3|6$ and $3|9$, then $3|15$
    ٣ من عوامل ٩
  * if $a|b$ and $b|c$, then $a|c$
    $3|6$, and $6|24$ then $3|24$

* if $a|b$, then $a|bc$ for all integers c
  $3|6$ then $3|12$, $3|18$, $3|24$.

* if a, b and c are integers such that $a|b$ and $a|c$ then $a|(mb+mc)$

$3|6$, and $3|9$ then $3|15$, $3|30$, $3|45$

~~* let a be~~

* Division Algorithm

* let "a" be an integer and "d" a positive integer. Then there are unique integers "q" and r, with $0 \le r < d$ such that $\boxed{a = dq + r}$ لي شرط باقي القسمة

* d is called the divisor, a is called the dividend, q is called the quotient, and r is called the remainder.

* $q = a \text{ div } d \longrightarrow$ ناتج القسمة

* $r = a \bmod d \longrightarrow$ الباقي

Examples:-

* what are the quotient and remainder when 101 is divided by 11 $\quad q = 9 \quad r = 2$

?? $a = dq + r$

$= 11 \times 9 + 2 = 101$ ✓

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* what are the quotient and remainder when -11 is divided by 3

$q = -4 \quad r = 1$

$-11 = 3(-4) + 1 = -11$

* $18 \text{ div } 4 = 4$
* $18 \bmod 4 = 2$
* $-14 \text{ div } 5 = -3$
* $-14 \bmod 5 = 1$

$$\left.\begin{array}{l} 22 \bmod 5 = 2 \\ 7 \bmod 5 = 2 \end{array}\right\} \text{mod نفس ال}$$

* $22 \equiv 7 \pmod 5$ indicates that 22 is congruent to 7 modulo 5

* Modular Arithmetic

* $a \equiv b \pmod m$ indicates that a is congruent to b modulo m.

* $a \equiv b \pmod m$ iff $a \bmod m = b \bmod m$

* $a \equiv b \pmod m$ if m divides $a-b$

Subject

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Examples:-

* Determine whether 17 is congruent to 5 module 6

$17 \equiv 5 \pmod 6$

$17 \bmod 6 = 5$

$5 \bmod 6 = 5$

* Determine whether 24 is congruent to 14 module 6

$24 \equiv 14 \pmod 6$

$24 - 14 = 10$

$6 \nmid 10 \longrightarrow \not\equiv$

$24 \bmod 6 = 0$

$14 \bmod 6 = 2$

* Determine whether $22 \equiv 13 \pmod 4$

$22 - 13 = 9$

$4 \nmid 9$

$22 \bmod 4 = 2$

$13 \bmod 4 = 1$

* Determine whether $15 \equiv 7 \pmod 4$

$15 - 7 = 8$

$4 \mid 8$ ✓

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* Modular Arithmetic.

- let m be a positive integer. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then.

* $a + c \equiv b + d \pmod{m}$
* $ac \equiv bd \pmod{m}$

* Example:-

from $7 \equiv 2 \pmod{5}$ and $11 \equiv 1 \pmod{5}$, then:

$18 \equiv 3 \pmod{5}$

$77 \equiv 2 \pmod{5}$

* Arithmetic Modulo "m"

- let Z_m be the set of nonnegative integers less than m
- we define addition of these integers, denoted by $+_m$ as:- $a +_m b = (a + b) \bmod m$.
- we define addition of these integers, denoted by $\cdot_m$ as:- $a \cdot_m b = (a \cdot b) \bmod m$.

- Example:-

* $7 +_{11} 9 = (7+9) \bmod 11 = 16 \bmod 11 = 5$

* $7 \cdot_{11} 9 = 63 \bmod 11 = 8$

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* Arithmetic Modulo property: "$+_m$ and $\cdot_m$"

1) Closure → "Z_m" الجواب ينتمي لنفس المجموعة
2) Associativity → تجميعية
3) Commutativity → الإبدالية
4) Identity elements → العنصر المحايد بالجمع = صفر ، بالضرب = 1
5) Additive inverse → العملية الجمع
6) Distributivity → توزيع الضرب على الجمع.

assume $+_{11}$

$(2)^{-1} = 9$ why?? → $2+9=11$ → $11 \bmod 11 = 0$, $(2 +_{11} 9) = 0$

$(1)^{-1} = 10$ → $10+1=11$ → $11 \bmod 11 = 0$

---

* Primes. → لا تقسم على نفسه وعلى 1

* A positive integer p is greater than 1 is called prime if the only positive ~~integer~~ factors of p are 1 and p.
A positive integer is greater than 1 and is not prime is called composite.

* 7 is a prime number while 4 is a composite number.

- The primes less than 100 are:-

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

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- The Fundamental theorem of Arithmetic.

- every positive integer greater than 1 can be written uniquely as a prime or as the product of two or more primes where the prime factors are written in order of nondecreasing size. "اي عدد نستطيع ~~كتابته~~ كتابته على شكل مضروب مجموعة من الأعداد الأولية."

ex:- what are the prime factorizations of 100, 49 and 60.

$100 = 2 \times 5 \times 2 \times 5 = 2^2 \times 5^2$

$49 = 7 \times 7 = 7^2$

$11 = 11$

$60 = 6 \times 10 = 3 \times 2 \times 2 \times 5 = 2^2 \times 3 \times 5$

* greatest common divisors → اكبر عدد يقسم العدد a والعدد b

- let a and b be integers, not both zero. The largest integer d such that $d | a$ and $d | b$ is called the greatest common divisor of a and b. The greatest common divisor of a and b is denoted by $\gcd(a, b)$

ex:- what is the gcd of 24 and 36?

$24 = 6 \times 4 = 2^3 \times 3$

$36 = 6 \times 6 = 2 \times 3 \times 2 \times 3 = 2^2 \times 3^2$

* نأخذ العدد المشترك والأس الأصغر.

$\gcd = 2^2 \times 3 = 12$

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ex:- find gcd for 100 and 30

$100 = 2 \times 5 \times 2 \times 5 = 2^2 \times 5^2$

$30 = 2 \times 5 \times 3$

$gcd(100, 30) = 2 \times 5 = 10$

ex:- $gcd(17, 22)$

$17 = 17$

$22 = 2 \times 11$

$gcd = 1$

* The integers a and b are $\boxed{\text{relatively prime}}$ if their greatest common divisor is 1.

ex:- $gcd(8, 9) = 1$

* Pairwise Relatively prime

- The integers $a_1, a_2, \dots, a_n$ are pairwise relatively prime if $gcd(a_i, a_j) = 1$ whenever $1 \le i < j \le n$

ex:- Determine whether the integers 10, 17 and 21 are pairwaise relatively prime and whethere the integers 10, 19, 24 ---

$gcd(10, 17) = 1$, $gcd(10, 21) = 1$, $gcd(17, 21) = 1$ ✓

$gcd(10, 19) = 1$, $gcd(10, 24) = 2$ X

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* least Common Multiple:- ← المضاعف المشترك الاصغر للعددين a , b وباقي المشتقة مثلها

- LCM of the positive integers a and b is the smallest positive integer that is divisible by both a and b. The least common multiple of a and b is denoted by lcm (a, b)

ex:- what is lcm of 24 and 36?

$24 = 2 \times 4 \times 3 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$36 = 2^2 \times 3^2$

- نأخذ كل الأعداد و نأخذ الأس الأكبر من الأعداد المشتركة.

$lcm = 2^3 \times 3^2 = 72.$

ex:- lcm(100, 30)

$100 = 2^2 \times 5^2$ $30 = 2 \times 3 \times 5$

$lcm(100, 30) = 2^2 \times 3 \times 5^2 = 300$

---

* let a and b be positive integers. Then

$$ab = gcd(a, b) \cdot lcm(a, b)$$

ex:- $a = 100$ $b = 30$

$lCm = 300$

$gcd = 10$

$ab = 3000$

$gcd \cdot lcm = 3000$

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* Mathematical Induction.

* Mathematical Induction can be used to prove statements that assert that P(n) is true for all positive integers n, where P(n) is a propositional function.

* A proof by mathematical induction has two parts, a (1) $\boxed{\text{basis step}}$, where we ~~know~~ show that P(1) is true, and an (2) $\boxed{\text{inductive step}}$, where we show that for all positive integers k, if P(k) is true, then P(k+1) is true.

مهم جدا!

$P(k) \longrightarrow P(k+1)$

↳ using direct Proof

examples :-

show that if n is a positive integer, then

بشكل آخر

$1 + 2 + \cdots + n = n(n+1)/2$

Proof $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$\Rightarrow$ step 1:- $P(1) \longrightarrow 1 \overset{?}{=} \frac{1(2)}{2}$ ✓

$\Rightarrow$ step 2 :- $P(k) \longrightarrow P(k+1)$

assumption :- $1 + 2 + \cdots + k = \frac{k(k+1)}{2}$

Prove :- $P(k+1)$

عامل مشترك

$\underbrace{1 + 2 \cdots k}_{} + (k+1) = \frac{(k+1)(k+2)}{2}$

$\frac{k(k+1)}{2} + (k+1) = \frac{k(k+1)}{2} + \frac{2(k+1)}{2} \overset{?}{=} \frac{(k+1)(k+2)}{2}$ ✓

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ex:- Show that $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$ for all nonnegative integer n.

$n \geq 0$

1. $P(0) \longrightarrow 1 \stackrel{?}{=} 2^1 - 1$ ✓

2. $P(k) \longrightarrow P(k+1)$

assumption $\longrightarrow 1 + 2 + 2^2 + \cdots + 2^k = 2^{k+1} - 1$

prove $\longrightarrow P(k+1)$

L.H.S R.H.S.

$1 + 2 + 2^2 + \cdots + 2^{k+1} = 2^{k+2} - 1$

$= 1 + 2 + 2^2 + \cdots 2^k + 2^{k+1}$

$= 2^{k+1} - 1 + 2^{k+1}$

$= 2 \cdot 2^{k+1} - 1$

$2^{k+2} - 1 \#$ R.H.S.

Date No.

ex. $\sum_{K=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}$ Ms

بشكل آخر

$1+4+9 \cdots n^2 = \frac{n(n+1)(2n+1)}{6}$

$n \geq 1$

1. $P(1)$ $1 \stackrel{?}{=} \frac{1(2)(3)}{6} = \frac{6}{6} = 1$ ✓

2. $P(K) \longrightarrow P(K+1)$ → direct Proof

$\Rightarrow$ assumption $p(K)$ is true

$1+4+9+\cdots+K^2 = \frac{K(K+1)(2K+1)}{6}$

$\Rightarrow$ Proof $P(K+1)$

L.H.S R.H.S.

$\underbrace{1+4+9+\cdots+K^2+(K+1)^2}_{} = \underbrace{\frac{(K+1)(K+2)(2(K+1)+1)}{6}}_{}$

$\frac{K(K+1)(2K+1)}{6} + (K+1)^2$ ← لازم نثبت انه يساوي R.H.S

$\frac{K(K+1)(2K+1)+6(K+1)^2}{6}$

عامل مشترك

$\frac{(K+1)[K(2K+1)+6(K+1)]}{6}$

$\frac{(K+1)[2K^2+K+6K+6]}{6} = \frac{(K+1)(2K^2+7K+6)}{6}$

$R.H.S \Rightarrow \frac{(K+1)(K+2)(2K+3)}{6} = \frac{(K+1)(2K^2+\overbrace{\boxed{3K+4K}}^{7K}+6)}{6}$ //

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* $-17 \text{ div } 3 = -6$
* $-17 \text{ mod } 3 = 1$

* باقي القسمة r: يكون عدد موجب

$0 \leq r < d$

* $14 \text{ mod } 12 = 2 \text{ mod } 12$

$\downarrow 2 \qquad \downarrow 2$

So, $14 \equiv 2 \pmod{12}$

* $15 \equiv 4 \pmod{3}$ X

$\rightarrow 15 \text{ mod } 3 = 0$

$4 \text{ mod } 3 = 1$

other way, $3 \mid (15-4) = 3 \mid (11)$ X

* $(a+b) \text{ mod } 9 = a +_9 b$ "نفس الخاصية للضرب"

ex: $4 +_9 8 = 3$

$4 \cdot_9 8 = 7$

* inverse العدد في الجمع هو عبارة عن عدد يجمع للعدد الأصلي وبأخذ له mod والناتج = 0 "العنصر المحايد"

* inverse العدد في الضرب هو عبارة عن عدد نضربه بالعدد الأصلي وبأخذ له mod والناتج = "1" "العنصر المحايد".

* Sine wave signals "we use it to convert time domain to frequency domain"

ex:

$$V(t) = \underbrace{V_m}_{\text{Amplitude "Peak value"}} \underbrace{\sin \omega t}_{\text{Argument "Angle"}}$$

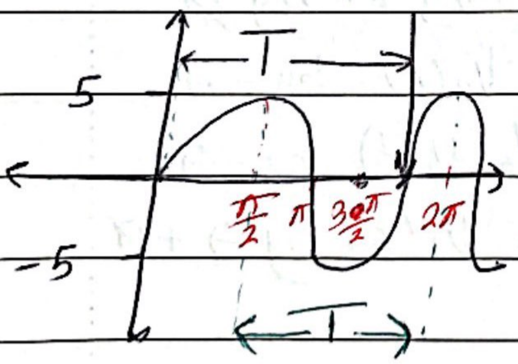
ω :- radian frequency "(angular frequency)" [rad / sec]

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{where } f = \frac{1}{T}$$

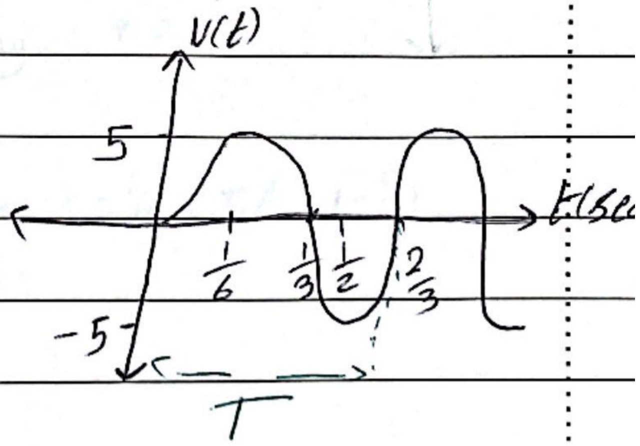
T → Period of the signal. (sec) → المسافة بين نقطتين متشابهات

f → frequency of the signal. (HZ) sine waves

Ex :- $V(t) = 5\sin(3\pi t)$



time domain



$3\pi t = \frac{\pi}{2} \rightarrow t = \frac{1}{6}$

$3\pi t = \pi \rightarrow t = \frac{1}{3}$

$\omega = 2\pi f$

$3\pi = 2\pi f$

$f = 1.5$ Hz

$T = \frac{2}{3}$

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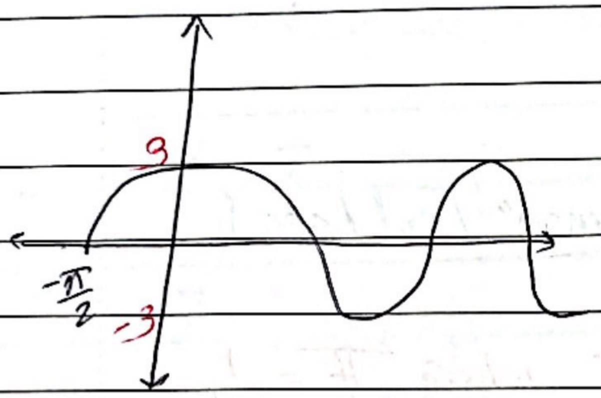
* In general, $V(t) = V_m(\sin(\omega t + \theta))$ Phase angle

shift to the left.

ex:- $V(t) = 3\sin(\underbrace{2\pi t}_{\omega_0} + 90°)$

* "−" to shift to the right

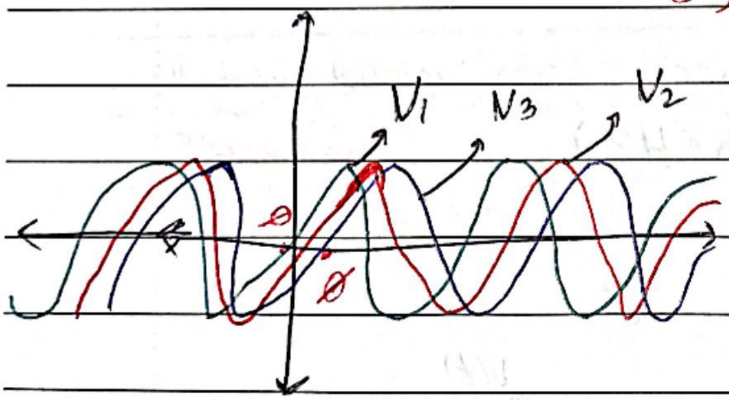


$2\pi = 2\pi f$

$f = 1\,Hz$

$T = 1\,sec.$

يعني $V_1(t)$ سبقت $V_2(t)$ ال x-axis بمقدار θ



$V_1(t)$ leads $V_2(t)$ by θ

$V_2(t)$ lags $V_1(t)$ by θ

تأخر

$V_2(t)$ leads $V_3(t)$ by $\emptyset$

$V_1(t)$ lags $V_2(t)$ by $-\theta$

$V_2(t)$ leads $V_1(t)$ by $-\theta$

$V_3(t)$ lags $V_1(t)$ by $\theta + \emptyset$

* let $V_1(t) = V_m \sin(\omega t + \theta)$, $V_2(t) = V_m \sin(\omega t + \cancel{\theta})$

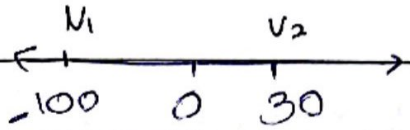
↳ if $\theta = \emptyset$, then $V_1(t)$ and $V_2(t)$ are inphase
if $\theta \neq \emptyset$ then $V_1(t)$ and $V_2(t)$ are out of phase

* To compute V_1 and V_2
1. Must be positive sine or Cosine] "sine, sine" or "Cos, Cos"
2. Must be positive amplitude
3. Same frequency

$+\sin(\omega t \pm 180) \equiv -\sin\omega t$
$+\cos(\omega t \pm 180) \equiv -\cos\omega t$.
$\cos(\omega t \pm 90) \equiv \mp \sin\omega t$
$\sin(\omega t \pm 90) \equiv \pm \cos\omega t$.

ex:- if $V_1(t) = 3\cos(5t + 10°)$, $V_2(t) = 4\sin(5t - 30°)$
then $V_1(t)$ leads $V_2(t)$ by ??

$V_1(t) = 3\sin(5t + 10° + 90) = 3\sin(5t + 100°)$
$\omega t + 100 = 0$, $\omega t = 0 = V_1$
$\omega t - 30 = 0 \longrightarrow \omega t = 30 = V_2$



V_1 leads V_2 by 130°
V_2 lags V_1 by 130°

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Ex:- if $v_1(t) = -3\cos(4t-30^\circ)$, $v_2(t) = -2\sin(4t+40^\circ)$

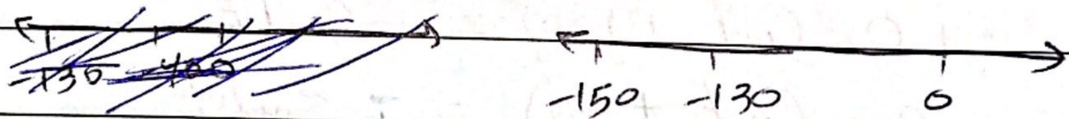
$v_2(t)$ lags $v_1(t)$ by ?? ______

$$v_1(t) = 3\cos(4t-30^\circ+180) = 3\cos(4t+150^\circ)$$

$$v_2(t) = 2\cos(4t+40^\circ+90^\circ) = 2\cos(4t+130^\circ)$$

$$\omega t + 40 = 0 \rightarrow \omega t = -150 = v_1$$

$$\omega t + 130 = 0 \rightarrow \omega t = -130 = v_2$$



v_2 lags v_1 by 20°

v_1 leads v_2 by 20°

v_2 leads v_1 by $-20^\circ \equiv 340^\circ$

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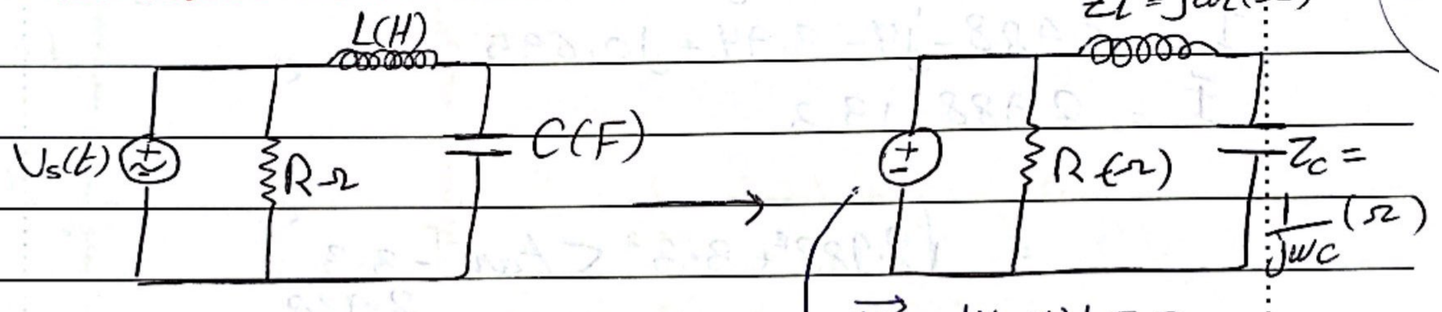
(23)

* Sinusoidal steady-state Analysis

→ step 1 :- Time-domain ⟶ frequency domain (phaser domain)

→ step 2 :- Solve the circuit in phaser domain using all techniques studied before.



$\vec{V_s} = |V_s(t)| \angle \theta$

↳ phaser form of $V_s(t)$

* $\omega = 2\pi f$

* The phaser.

→ To write the phaser form of a given signal :-

1. Write the given signal using positive cosine function

   ↳ $V_s(t) = V_m \cos(\omega t + \theta)$

2. $\vec{V_s} = V_m \angle \theta$

ex:- if $V(t) = 100 \cos(400t - 30°)$

$\vec{V} = 100 \angle -30$

ex:- $i(t) = -5\sin(580t - 110°)$

$= 5\cos(580t - 110° + 90°) = 5\cos(580t - 20°)$

$\vec{I} = 5\angle -20°$

NOTEBOOK

ex:- $i(t) = 8\cos(4t-30^\circ) + 4\sin(4t-100^\circ)$

$= 8\cos(4t-30^\circ) + 4\cos(4t-100^\circ-90^\circ)$

$= 8\cos(4t-30) + 4\cos(4t-190^\circ)$

$\vec{I} = 8\angle-30 + 4\angle-19$

$= 8\cos(-30) + j8\sin(-30^\circ) + 4\cos(190^\circ) + j4\sin(-190^\circ)$

$\vec{I} = 6.928 - j4 - 3.94 + j0.695$

$\vec{I} = 2.988 - j3.3$

$$= \sqrt{2.988^2 + 3.3^2} \angle \tan^{-1}\frac{-3.3}{2.988}$$

$\vec{I} = 4.45 \angle -47.8^\circ$

ex:- find $i(t)$ if $I = 20 + j10$ A. $\omega = 2000$ rad/sec

$$\vec{I} = \sqrt{20^2+10^2} \angle \tan^{-1}\frac{10}{20}$$

$= 22.36 \angle 26.56^\circ$

$i(t) = 22.36\cos(200t + 26.57)$ A.

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* Time - domain $\longrightarrow$ frequency domain

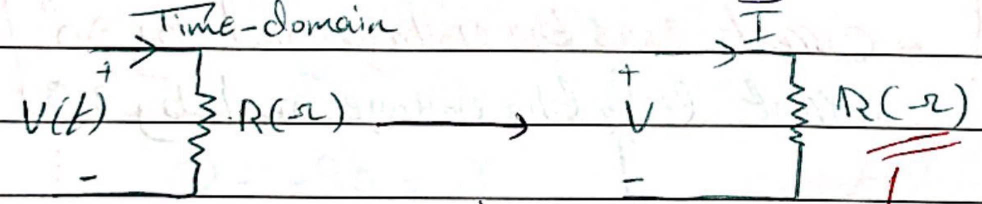
$v_s(t)$ $\longrightarrow$ $\vec{V_s}$ (phaser)

$R(\Omega)$ $\longrightarrow$ $R(\Omega)$

$C(F)$ $\longrightarrow$ $Z_c = \frac{1}{j\omega C}(\Omega)$ impedance.

$L(H)$ $\longrightarrow$ $Z_L = j\omega L\,(\Omega)$ impedance.

* Resistor: $R \longrightarrow R$ ... why?



let $V(t) = V_m e^{j(\omega t+\theta)}$

$i(t) = I_m e^{j(\omega t+\theta)}$

$V(t) = R\, i(t)$

$V_m e^{j(\omega t+\theta)} = R I_m e^{j(\omega t+\theta)}$

$V_m e^{j\theta} \cancel{e^{j\omega t}} = R I_m e^{j\phi} \cancel{e^{\omega t}}$

$\vec{V} = R\vec{I}$

$\phi = \theta \Rightarrow$ The current and voltage on R are inphase

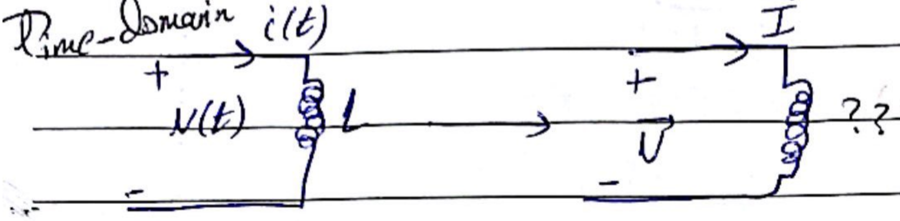
* general form of $V(t)$

$V(t) = \cancel{\quad}$

$V_m \cos(\omega t+\theta) + j V_m \sin(\omega t+\theta)$

$= V_m e^{j(\omega t+\theta)}$

* Inductor (H) $\longrightarrow$ $j\omega L$ (Ω)



let $V(t) = V_m e^{j(\omega t + \theta)}$

$i(t) = I_m e^{j(\omega t + \phi)}$

$V(t) = L \frac{di}{dt}$

$V_m e^{j(\omega t + \theta)} = j\omega L I_m e^{j(\omega t + \phi)}$

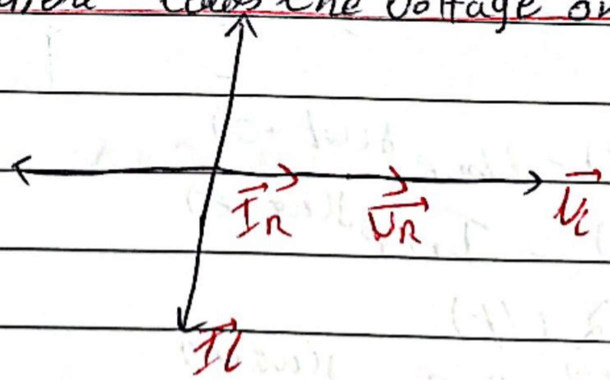
$V_m e^{j\omega t} \cdot e^{j\theta} = j\omega L I_m e^{j\omega t} e^{j\phi}$

$\vec{V} = j\omega L \cdot \vec{I}$

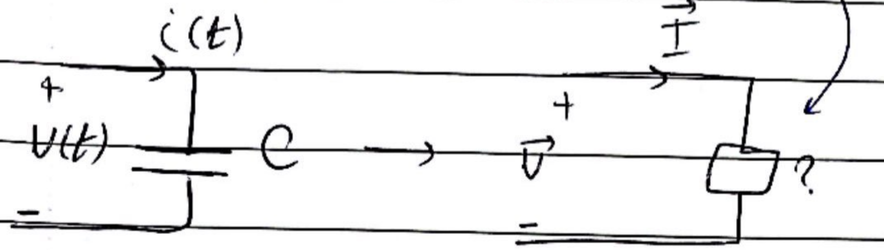
$\theta = 90^\circ + \phi$

Rule.

$\boxed{\begin{array}{l}\text{* Current lags the voltage on } L \text{ by } 90^\circ \\ \text{current leads the voltage on } L \text{ by } 270\end{array}}$



* Capacitor. $C(F) \rightarrow Z_c = \frac{1}{j\omega C}$ (Ω)



$$v(t) = V_m e^{j(\omega t+\theta)}$$

$$i(t) = I_m e^{j(\omega t+\phi)}$$

$$i(t) = C\frac{dv}{dt}$$

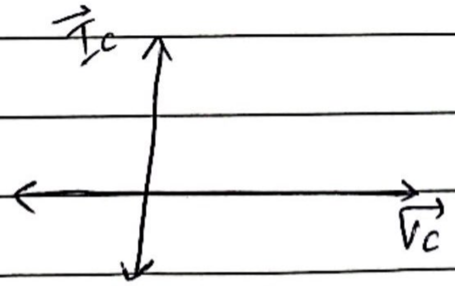
$$I_m e^{j(\omega t+\phi)} = C\left[j\omega e^{j(\omega t+\theta)}\right]$$

$$I_m e^{j\phi} = Cj\omega V_m e^{j\theta}$$

$$\vec{I} = j\omega C\vec{V}$$

$$\vec{V} = \frac{1}{j\omega C}\vec{I} \quad (\Omega)$$

$\theta = -90 + \phi$ * current leads the voltage on C by 90°



• **Impedence**

Resistance "real Part"

$\vec{Z} = R + jX$ → Reactance (Ω) $= |Z| \angle \theta$

impedence (Ω) "imaginary Part"

$\vec{Z} = \frac{\vec{V}}{\vec{I}} = \frac{|V| \angle \theta_1}{I \angle \phi}$

* if $\theta = 0 \Rightarrow$ Resistive load. (I and V are inphase)
* if $\theta = 90° \Rightarrow$ purely inductive load. (I lags V by $90°$)
* if $\theta = -90° \Rightarrow$ purely capacitive load. (I leads V by $90°$)
* $90° \neq$ if θ is positive $\Rightarrow$ inductive load (not purely). (I lags V by θ)
* $-90° \neq$ if θ is negative $\Rightarrow$ Capacitive Load (I leads V by θ)

ex:

$\vec{Z} = 5\,\Omega$ → Purely resistive load.

$\vec{Z} = j20\,\Omega$ → Purely inductive load.

$\vec{Z} = \frac{1}{j20}\,\Omega = -j\frac{1}{20}$ → Purely capacitive load.

$\vec{Z} = 4 + j3 \rightarrow \tan^{-1}\frac{3}{4}$ → inductive load.

$\vec{Z} = 2 - j2 \rightarrow \tan^{-1} -1$ → Capacitive load.

* **Admittance**:

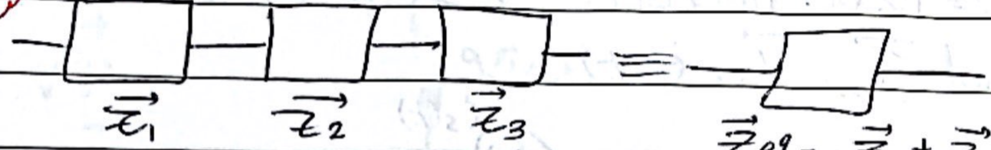
$\vec{Y} = \frac{1}{\vec{Z}}$ ($\overset{S}{\mho}$)

Conductance

$\vec{Y} = G + jB$ → susceptance

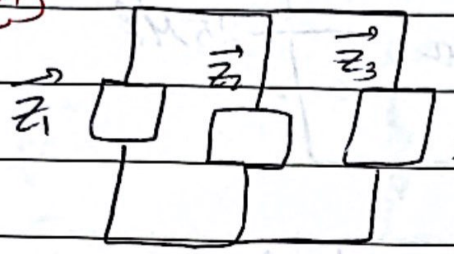
Admittance

Series



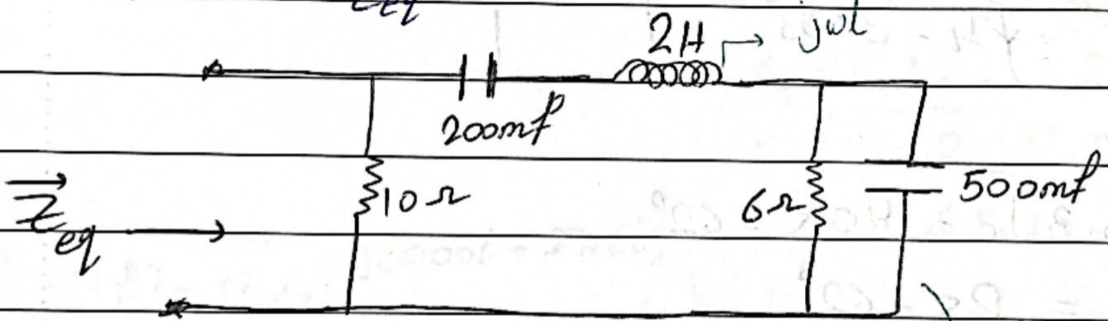
$$\vec{Z}_{eq} = \vec{Z}_1 + \vec{Z}_2 + \vec{Z}_3$$

Parallel



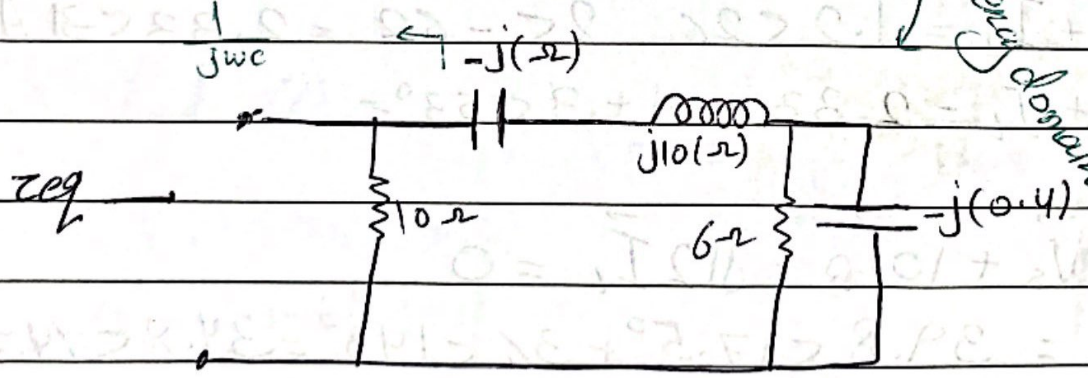
$$\equiv \frac{1}{\frac{1}{\vec{Z}_1} + \frac{1}{\vec{Z}_2} + \frac{1}{\vec{Z}_3}} = \vec{Z}_{eq}$$

ex:- find $\vec{Z}_{eq}$



frequency domain

$\omega = 5 \text{ rad/sec.}$

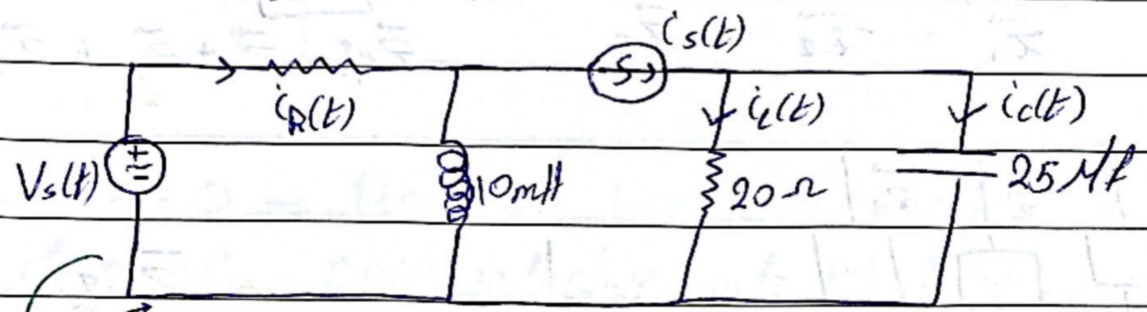


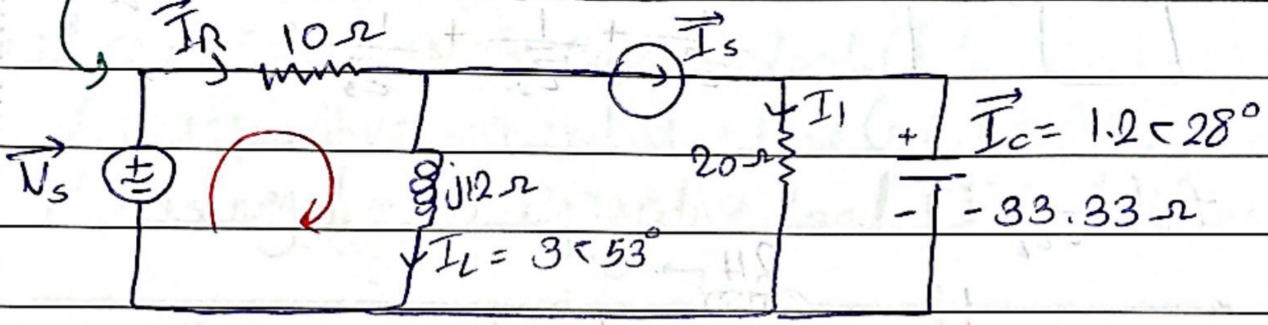
$$Z_{eq} = [(-j0.4 // 6) + 10j - j] // 10 = \left(\frac{-j2.4}{6-j0.4} + 9j\right) // 10$$

$$= 4.256 + j4.929\ \Omega$$

ex:- $\omega = 1200$ rad/sec , $\vec{I}_c = 1.2\angle 28^\circ$, $\vec{I}_L = 3\angle 53^\circ$

find $\vec{I}_s$, $\vec{V}_s$, $i_R(t)$





$\vec{V}_c = -j33.3\,\vec{I}_c = 40\angle -62^\circ$

$\vec{I}_1 = \dfrac{V_c}{20} = 2\angle -62^\circ$

$\vec{I}_s = \vec{I}_c + \vec{I}_1 = 1.2\angle 26 \quad 2\angle -62 = 2.33\angle -31\ A$

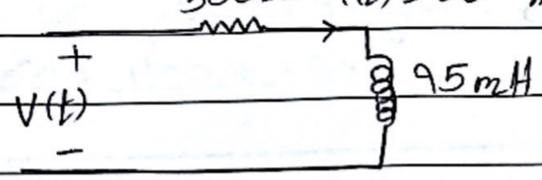
$I_R = \vec{I}_s + I_L = 2.33\angle -31 + 3\angle 53^\circ =$

KVL :- $-\vec{V}_s + 10\vec{I}_R + j12\vec{I}_L = 0$

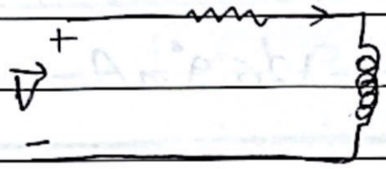
$\vec{V}_s = 39.8\angle 17.5^\circ + 36\angle 143^\circ = 34.8\angle 74.7^\circ\ V$

$V_s(t) = 34.8\cos(1200t + 74.7)\ V$

ex:- $i(t) = 8e^{j300t}$ mA $\rightarrow$ $w = 300$ rad/sec



$\vec{I} = 8\angle 0^\circ$



$j3000 * 95 * 10^{-3} = j285\ \Omega$

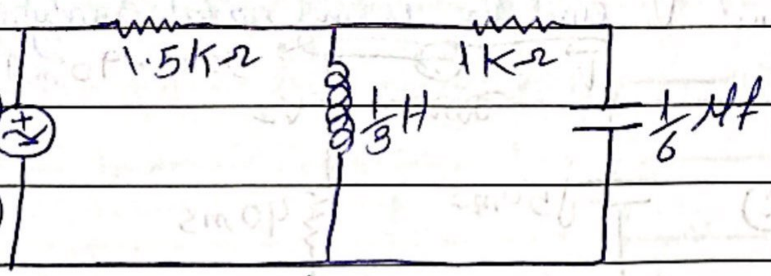
$$\vec{V} = \vec{I}\,(500 + j285)$$
$$= 8\angle 20^\circ (500 + j285)$$
$$= 4 + j2.280\ V$$
$$= \sqrt{16 + 2.28^2} \angle \tan^{-1} \frac{2.28}{4}$$
$$= 4.6 \angle 29.63^\circ$$

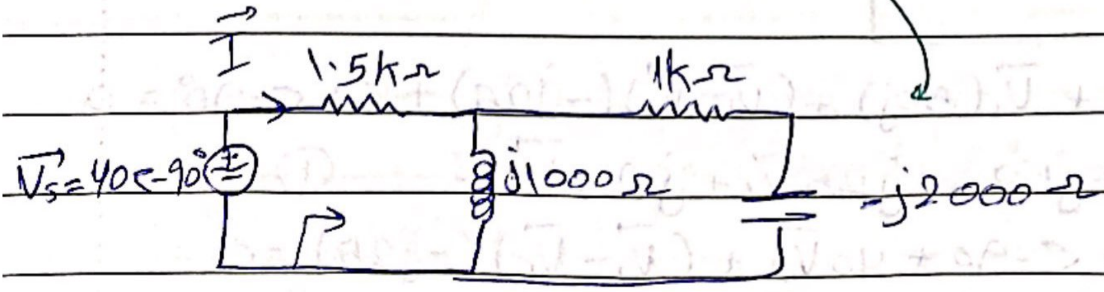
$$V(t) = 4.6e^{(j3000t + 29.68j)}$$
$$= 4.6\cos(3000t + 29.68) + j4.6\underset{\sin}{\underbrace{\ }}(3000t + 29.68^\circ)$$

ex: find $i(t)$

$V_s(t) = 40\sin(3000t)$

$= 40\cos(3000t - 90^\circ)$





zeq

$\vec{Z}_{eq} = (1-j2) // j + 1.5$

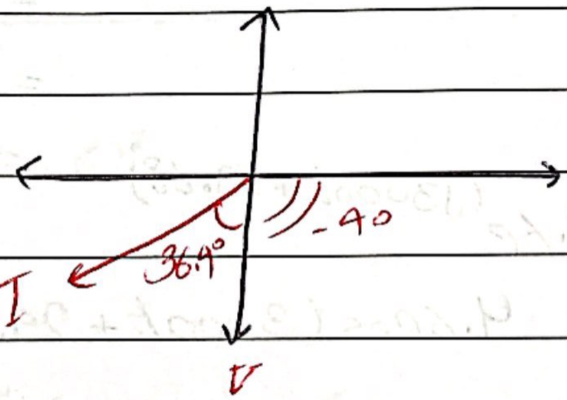
$= 2 + j1.5 \ k\Omega$

$= 2.5 \angle 36.9^\circ \ k\Omega$

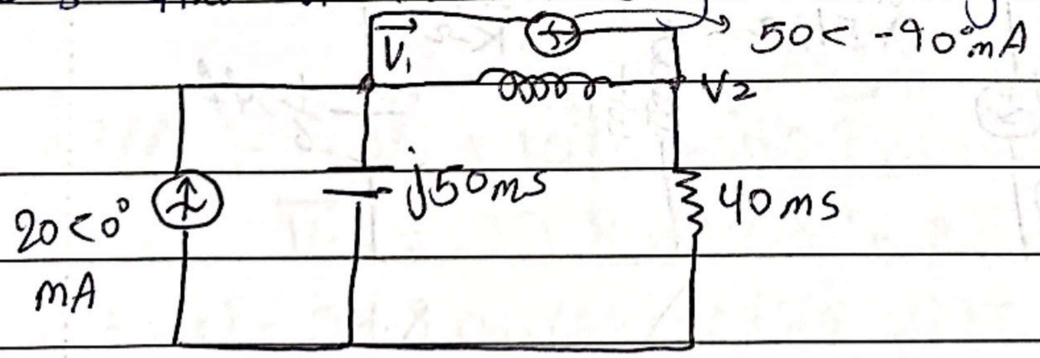
$\vec{I} = \dfrac{\vec{V}}{\vec{Z}_{eq}} = \dfrac{40 \angle -90^\circ}{2 + j1.5} = 16 \angle -126.9^\circ \ mA$

$\vec{I}(t) = 16 \cos(3000t - 126.9) \ mA$

$\vec{I}$ Lags V by 36.9°



ex:- find $\vec{V}_1$ and $\vec{V}_2$ using nodal analysis



node 1 :- $-20 + \vec{V}_1(50j) + (\vec{V}_1 - \vec{V}_2)(-j25) + 50 \angle -90^\circ = 0$

$20 + j50 = j25\vec{V}_1 + j25\vec{V}_2$ - - - - (1)

node 2 :- $-50 \angle -90 + 40\vec{V}_2 + (\vec{V}_2 - \vec{V}_1)(-j25) = 0$

$j50 + (40 - j25)\vec{V}_2 + j25\vec{V}_1 = 0$ - - - (2)

$j25\vec{V}_1 = -j50 - (40 - j25)\vec{V}_2$

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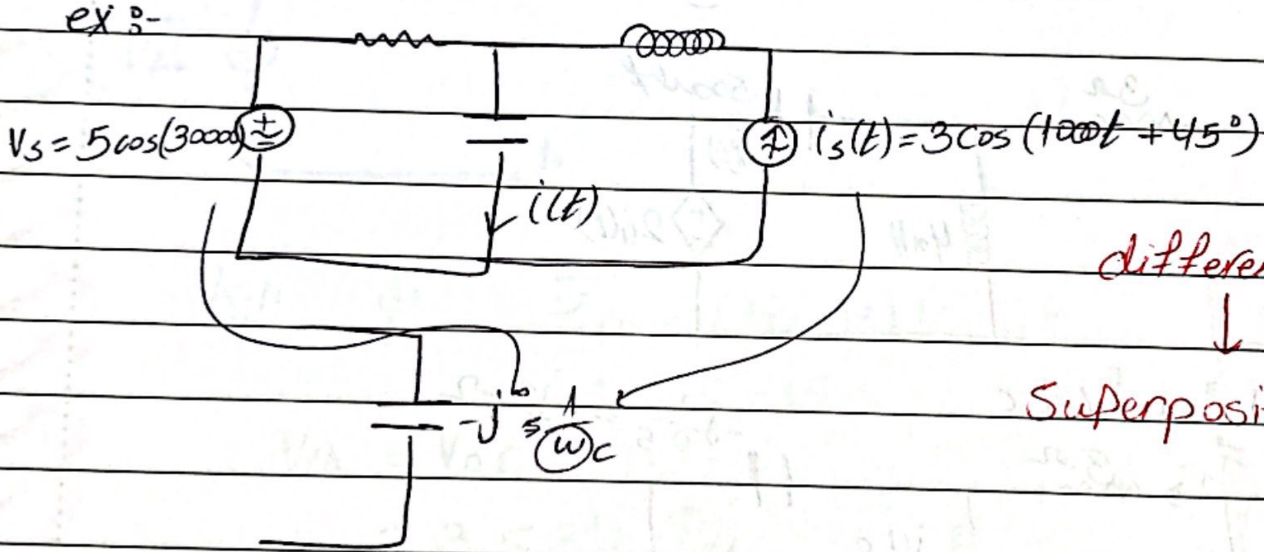
$$20 + j50 = -j50 - (40 - j25)\vec{V_2} + j25\vec{V_2}$$

$$60 + j100 = j50\vec{V_2}$$

$$\vec{V_2} = \frac{(60 + j100) - j50}{j50^{2} - j50} = \frac{j3000 + 5000}{2500} = 1.593\angle{-80^\circ}$$

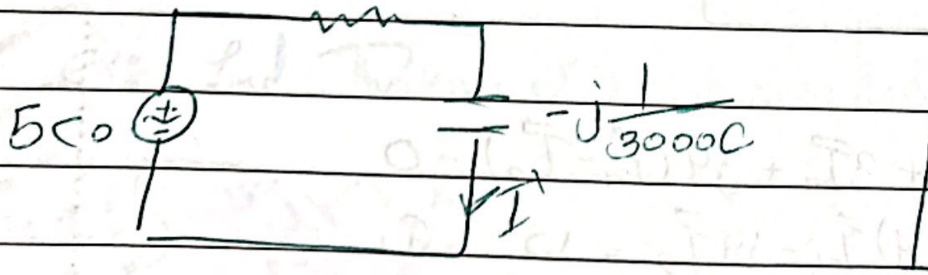
$$\vec{V_1} = 1.062\angle 23.3^\circ\ V$$

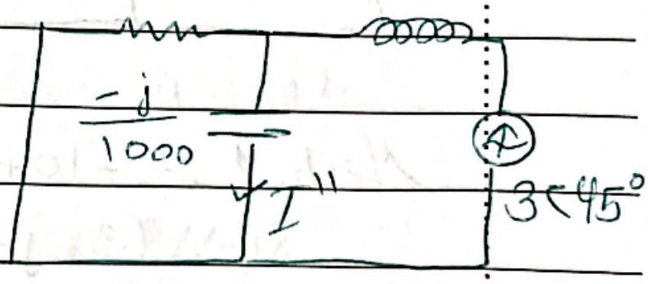
ex :-



different frequency
↓
Superposition!

So, solution is using superposition





$I' \longrightarrow i'(t)$ time-domain

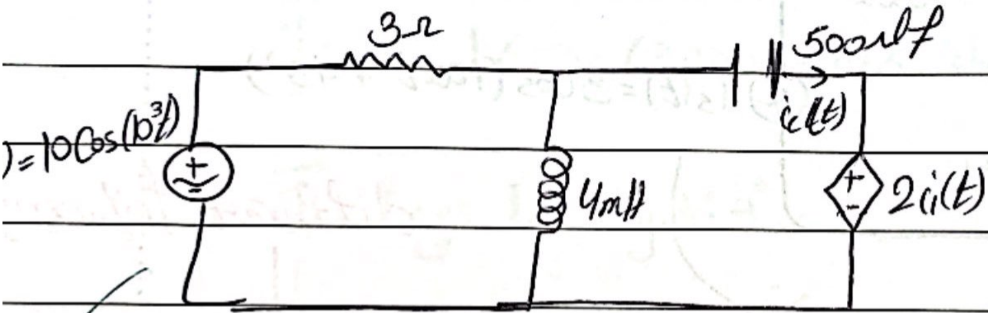
$I'' \longrightarrow i''(t)$

$$i(t) = i'(t) + i''(t) = 5\cos(3000t + 30^\circ) + 6\cos(1000t + 20^\circ)$$

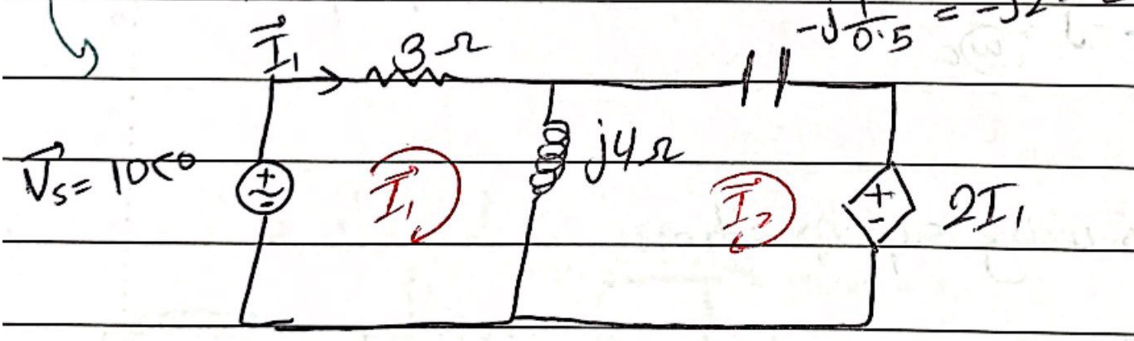
* 2 sources one of them is AC and the other DC
$\Rightarrow$ superposition.

* C in DC $\rightarrow$ open circuit.
L in DC $\rightarrow$ short circuit.

ex: find $i_1(t)$ and $i_2(t)$ using mesh analysis



$\omega = 10^3$ rad/sec



Mesh 1 :- $-10 + 3\vec{I}_1 + j4(\vec{I}_1 - \vec{I}_2) = 0$

$$(3+j4)\vec{I}_1 - j4\vec{I}_2 = 10 \quad \ldots (1)$$

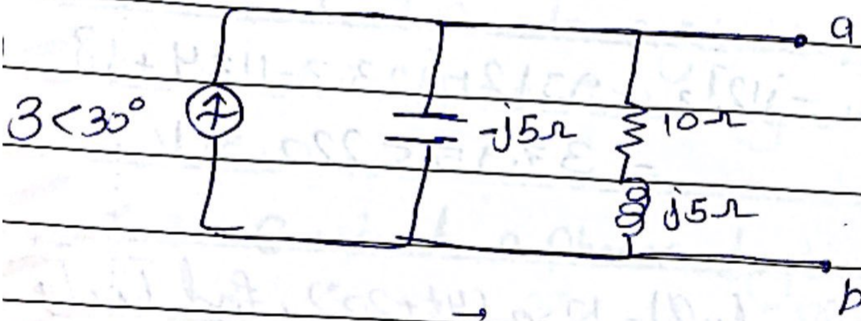
Mesh 2 :- $-j2I_2 + 2I_1 + j4(I_2 - I_1) = 0$

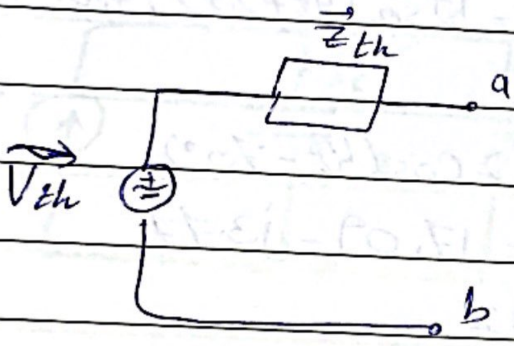
$$(2-4j)I_1 + 2jI_2 = 0 \quad \ldots (2)$$

$I_1 = 1.24\angle 29.7\,A \Rightarrow i_1(t) = 1.24\cos(10^3t + 29.7^\circ)$

$I_2 = 2.77\angle 56.3\,A \Rightarrow i_2(t) = 2.77\cos(10^3t + 56.3^\circ)\,A$

ex: find Thevenin eq. circuit.





Kill  $\vec{Z}_{th} = (10 + j5) // -j5$

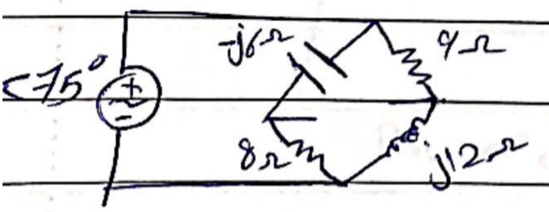
$= 2.5 - j5\,\Omega$

$V_{th} = V_{oC}$

$= 3 \angle 3^\circ \times \vec{Z}_{th}$

$= 16.77 \angle -33.47^\circ$

ex: find Thevenin eq. circuit seen between a and b



$\vec{Z}_{th} = 4 // j12 + 8 // -j6$

$= 6.48 - j2.64\,\Omega$

$\vec{I}_1 = \dfrac{120 \angle 75^\circ}{8 - j6} = 11.64 + j2.913$

$\vec{I}_2 = \dfrac{120 \angle 75^\circ}{4 + j12} = 0.724 - j9.45$

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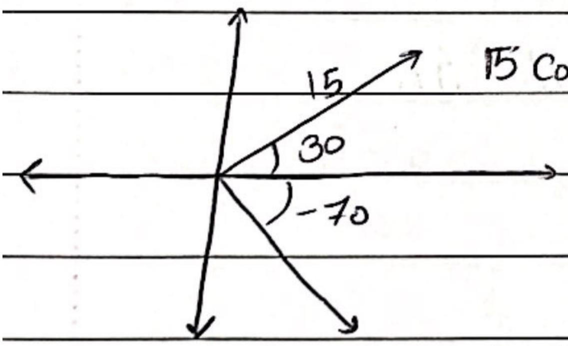
$V_a = 8I_1$, $\vec{V}_b = j12\vec{I}_2$

$V_{oc} = \vec{V}_a - \vec{V}_b = 8\vec{I}_1 - j12\vec{I}_2 = 93.12 + j23.3 - 113.4 + j8$
$= 37.95 \angle 220.31 \text{ V}.$

ex:- $i_x(t) = 15\cos(4t + 30^\circ)$. $i_y(t) = 12\sin(4t + 20^\circ)$, find $\vec{I}_x, \vec{I}_y$

$i_y(t) = 12\cos(4t + 20 - 90) = 12\cos(4t - 70^\circ)$

$15 \angle 30 + 12 \angle -70 = 17.09 - j3.77$
$= 17.5 \angle -12$
$17.5\cos(4t - 12^\circ)$



$15\cos 30^\circ + j15\sin(30^\circ)$

$12\cos(-70) + j12\sin(-70^\circ)$

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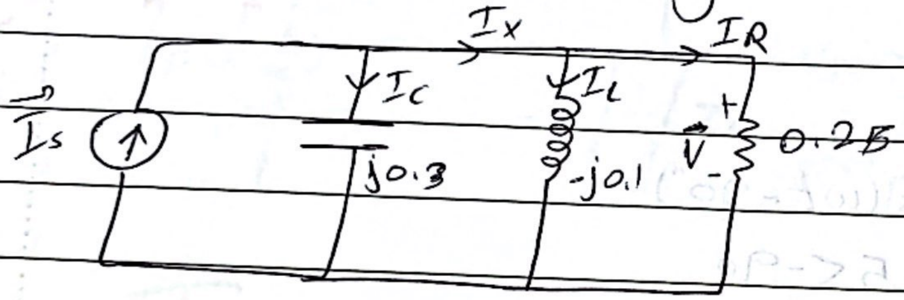
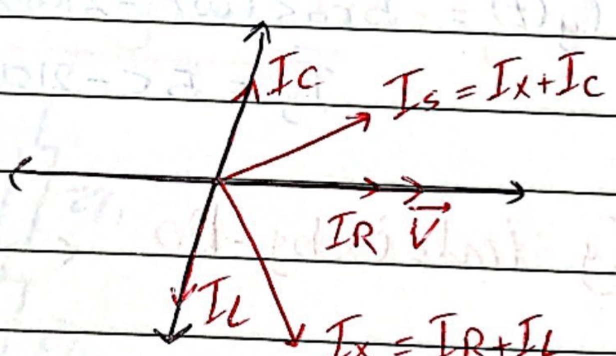
* Phasor Diagram
  ↳ it provides a graphical method for solving certain Problems which may be used to check more exact methods.

ex: Construct a phasor diagram showing $\vec{I}_R, \vec{I}_L, \vec{I}_C, \vec{I}_s, \vec{I}_x, V$



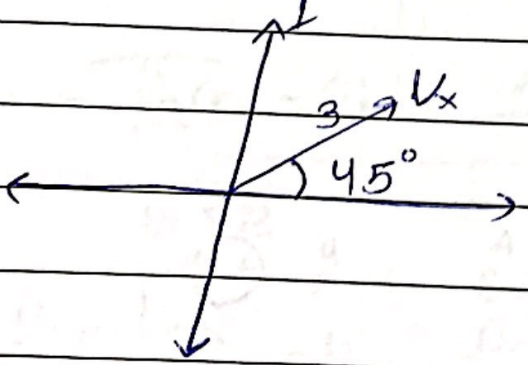
let $\vec{V} = 1\angle 0$

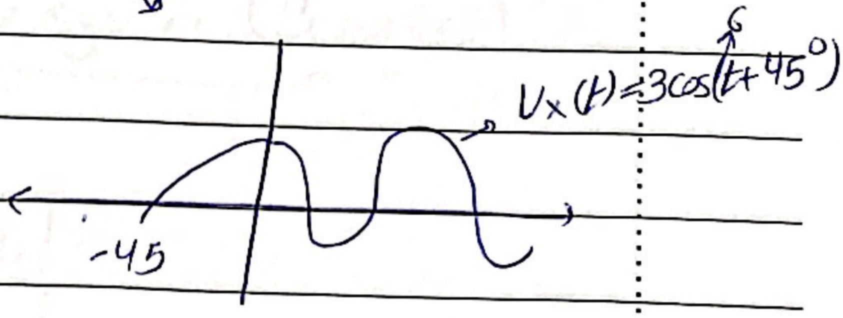


~~↳ make sure~~ :-

ex: $V_x = 3\cos(6t+45^\circ)$

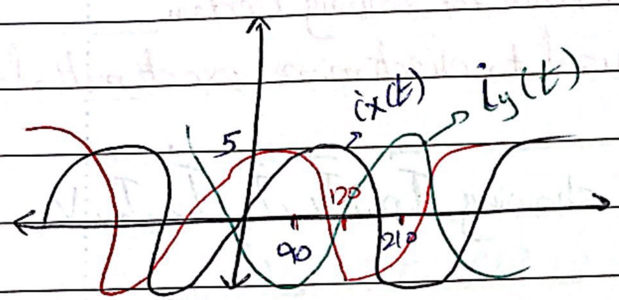
time domain





Phasor diagram.

ex: Draw ~~is~~ i_x in phasor diagram



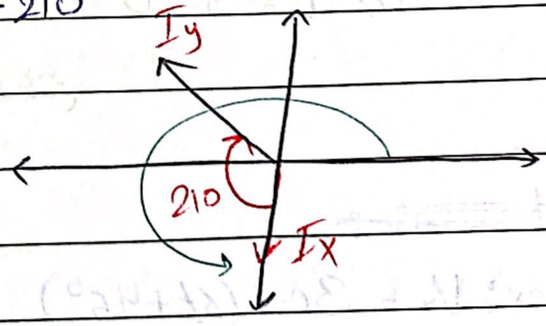
$$i_x(t) = 5\cos(\omega t - 90^\circ)$$

$$I_x = 5\angle -90$$

$$i_y(t) = 5\cos(\omega t - 210^\circ)$$

$$\vec{I}_y = 5\angle -210^\circ$$

I_y leads i_x by -120

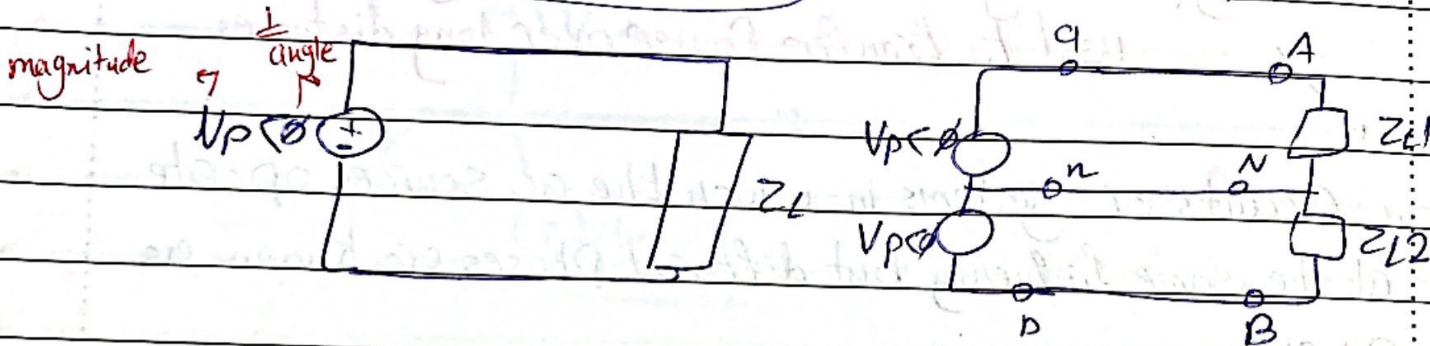


* Three-phase Systems.

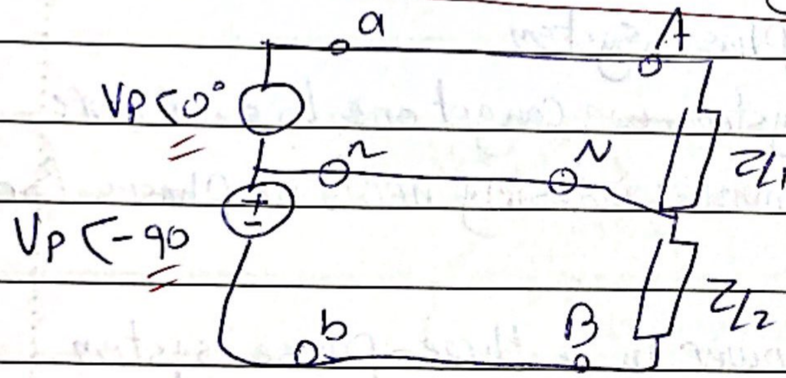
$\boxed{25}$

- system: 1) source
  2) load
  3) wire

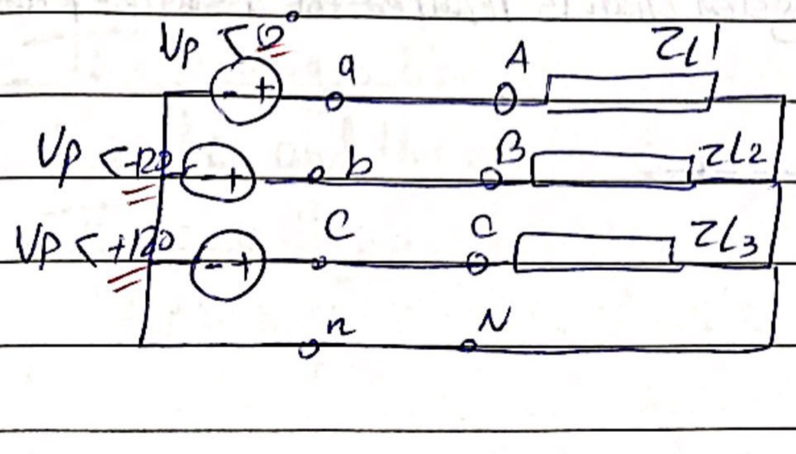
- Single phase system (1 angle)



- Two-phase Three-wire system (Two angles)



- Three-phase four-wire system (3 angles)



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* Single phase system → 1 angle (used for light loads) like lighting and heating.

* Polyphase systems → more than 1 angle. used to tranfer Power over long distances.

* Circuits or systems in which the ac sources operate at the same frequency but different phases are known as Polyphase.

* Advantages of Three-Phase System

1- from three-Phase system, we can get one, two, or more than three phases (aluminum industry needs 48 phases for melting.

2- The instantaneous power in a three-Phase system can be constant (not pulsating)

3- Less material is needed to deliver the same power with a three-phase system than is required for a single-Phase system

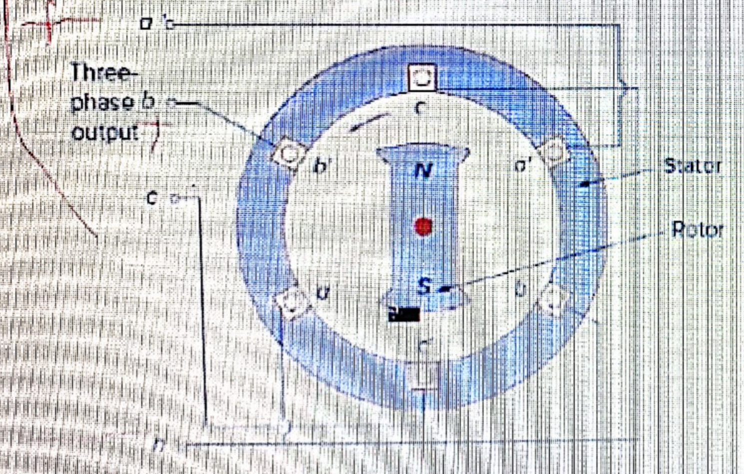
• ~~So, we will study it in this~~

Three-Phase Voltages

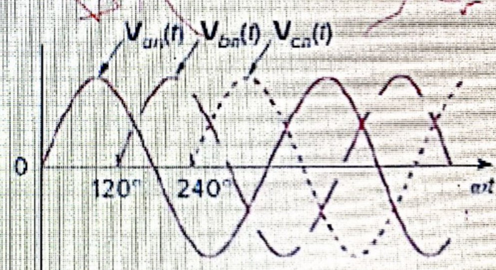
$= V_m \cos(\omega t)$

$V_m \cos(\omega t - 120°)$

$V_m \cos(\omega t + 120°)$



A three-phase generator



The generated voltages are 120° apart from each other.

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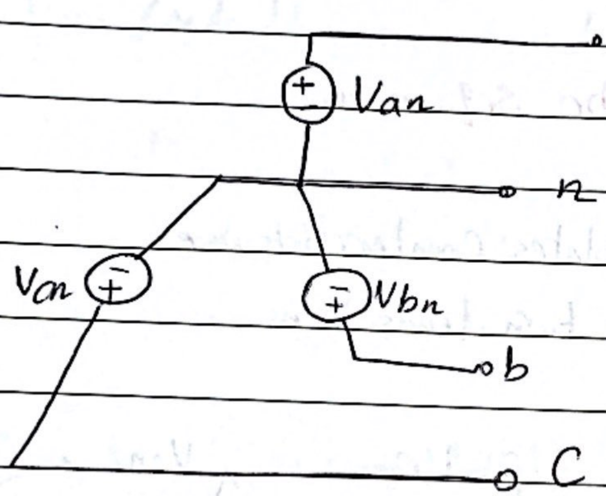
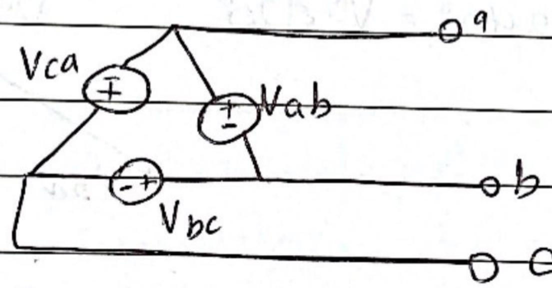
* Three-phase system (generator) is equivalent to three-single-Phase circuits.



y-connected.
4 wires



Δ-connected
3 wires

* Balanced Three-phase Voltages.

- The voltages Van, Vbn, Vcn are respectively between lines a, b and c, and the neutral line n.
- These voltages are called phase voltages.
- If the voltage source have the same amplitude and frequency ω and are out of Phase with each other by 120°, the voltages are said to be balanced. (Balanced three-Phase voltages.)

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$$V_{an} + V_{bn} + V_{cn} = V_p \angle 0° + V_p \angle 120° + V_p \angle 120°$$
$$= V_p (1.0 - 0.5 - j0.866 - 0.5 + j0.866)$$
$$= 0$$

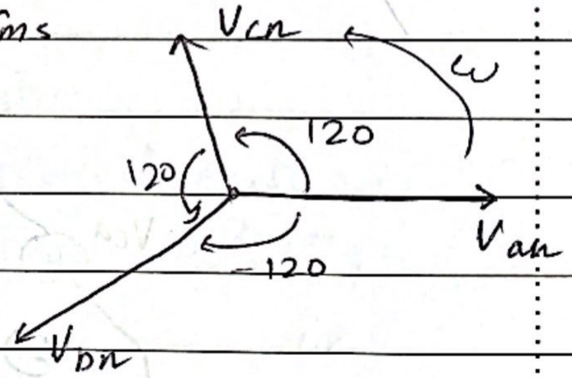
- Positive sequence or abc sequences

- Produced when the rotor rotates Counterclockwise
- Van leads Vbn, which in turn leads Vcn

$V_{an} = V_p \angle 0$ $\quad V_p = V_{rms}$

$V_{bn} = V_p \angle -120$

$V_{cn} = V_p \angle -240° = V_p \angle 120°$



- As the phasors rotate in the Counterclockwise direction with frequency ω, they pass through the horizantal axis in a sequence abcabca ---. Thus, the sequence is abc or bca or cab.

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* Negative sequence or acb sequence

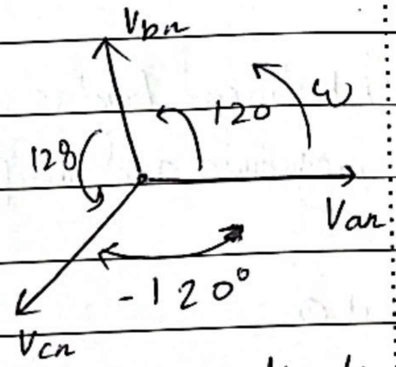
- Produced when the rotor rotates clockwise
- Van leads Vcn, which in turn leads Vbn

$V_p = V_{rms}$

$V_{an} = V_p \angle 0^\circ$

$V_{cn} = V_p \angle -120^\circ$

$V_{bn} = V_p \angle -240 = V_p \angle 120^\circ$



- for the phasors, as they rotate in the counterclockwise direction, they pass the horizontal axis in a sequence acbacb ... This describes the acb sequence.

---

ex: Determine the phase sequence of the set of voltages.

$$V_{an} = 200\cos(\omega t + 10^\circ)$$

$$V_{bn} = 200\cos(\omega t - 230^\circ), \quad V_{cn} = 200\cos(\omega t - 110^\circ)$$

Solution :- The voltages can be expressed in phasor form as:

$V_{an} = 200 \angle 10^\circ V$, $\quad V_{bn} = 200 \angle -230^\circ V$

$V_{cn} = 200 \angle -110^\circ V$

We notice that V_{an} leads V_{cn} by 120° and V_{cn} in turns leads V_{bn} by 120°.

So, we have an acb sequence.

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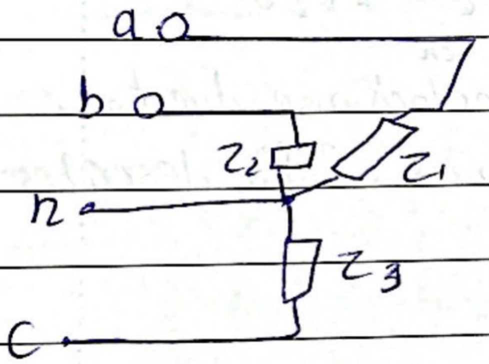
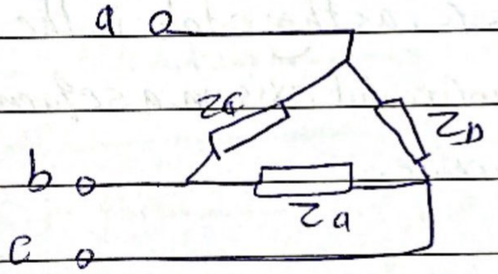
* Three-phase Balanced Load

- Three phase load can be either Y-connected or delta-connected depending on the end application.

- A balanced load is one in which the phase impedances are equal in magnitude and in phase.





$$Z_a = Z_b = Z_c = Z_\Delta$$

$$Z_1 = Z_2 = Z_3 = Z_y$$

$$Z_y = \frac{1}{3} Z_\Delta$$

* Possible connection between sources and loads.

1) Y-Y connection. ex:- (Y-Connected source with a Y-connected load).
2) Y-Δ connection
3) Δ-Δ connection
4) Δ-Y connection

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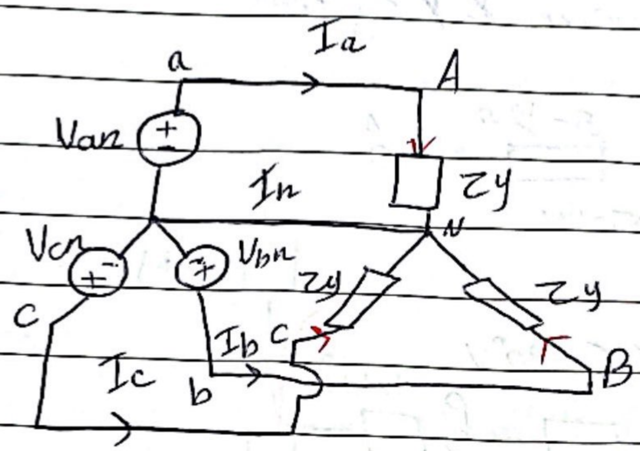
Balanced Y-Y Connection



Currents :- line current
phase current.

line current = phase current

$$Z_y = Z_s + Z_l + Z_L$$

* Assuming the positive sequence :

Connection	Phase voltage/currents	Line " / "
Y–Y	$V_{an} = V_p \angle 0^\circ$ $\times\sqrt{3} + 3^\circ$	$V_{ab} = \sqrt{3} V_p \angle 30^\circ$
	$V_{bn} = V_p \angle -120^\circ$	-120° $V_{bc} = V_{ab} \angle -120^\circ$
if (−) عكس	$V_{cn} = V_p \angle +120^\circ$	$V_{ca} = V_{ab} \angle +120^\circ$
	same as line currents.	$I_a = V_{an} / Z_y$
		$I_b = I_a \angle -120$ $\frac{V_{bn}}{Z_y}$
		$I_c = I_a \angle +120$ $\frac{V_{cn}}{Z_y}$

$$I_a + I_b + I_c = 0$$
$$I_n = -(I_a + I_b + I_c) = 0$$
$$V_{nN} = Z_n I_n = 0$$

✓ The ~~neutral~~ neutral line can thus be removed without affecting the system.

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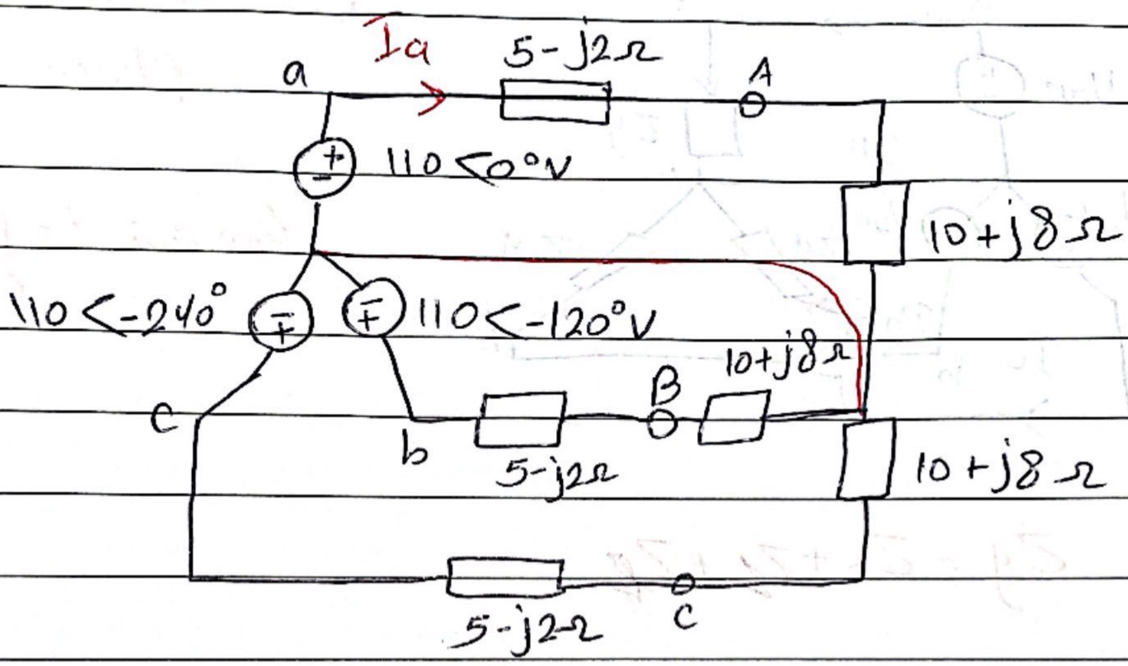
Example :- Calculate the line currents in the three-wire Y-Y system.



$$Z_y = (5-j2) + (10+j8) = 15 + j6 = 16.155 \angle 21.8°$$

$$I_a = \frac{110 \angle 0°}{16.155 \angle 21.8} = 6.81 \angle -21.8 \ A$$

$$I_b = I_a \angle -120 = 6.81 \angle -141.8° \ A$$

$$I_c = I_a \angle -240° = 6.81 \angle -261.8° A = 6.81 \angle 98.2° A$$

b

+120 gl

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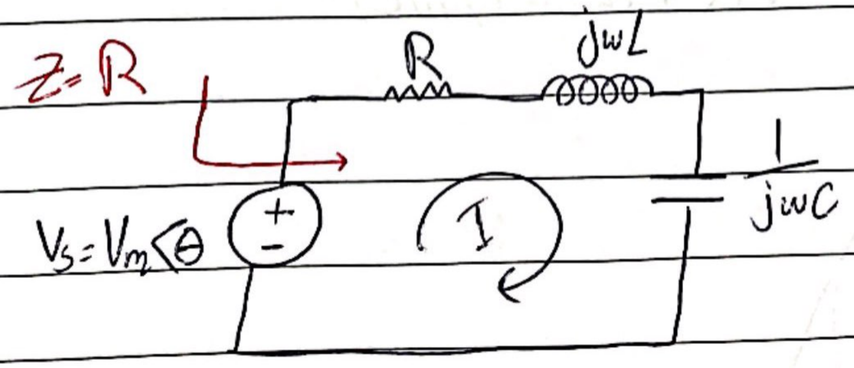
Parallel and Series Resonance

- Resonance occurs in any system that has a complex conjugate pair of poles; it is the cause of oscillations of stored energy from one to another.
- The concept of resonance applies in several areas of science and engineering.
- Resonance occurs in any circuit that has at least one inductor and one capacitor.
- Resonance is a condition in an RLC circuit in which the capacitive and inductive reactances are equal in magnitude, thereby in a purely resistive impedance.
- Resonant circuit (series or parallel) are useful for constructin filters as their transfer functions can be highly frequency selective. They are used in many applications such as selecting the desired stations in radio and TV receivers.

* Series Resonant RLC circuit



$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

* Resonance results when imaginary part of the transfer function is zero, or

$$Im(Z) = \omega L - \frac{1}{\omega C} = 0$$

* The value of ω that satisfies this condition is called the resonant frequency ω_0. Thus, the resonance condition is

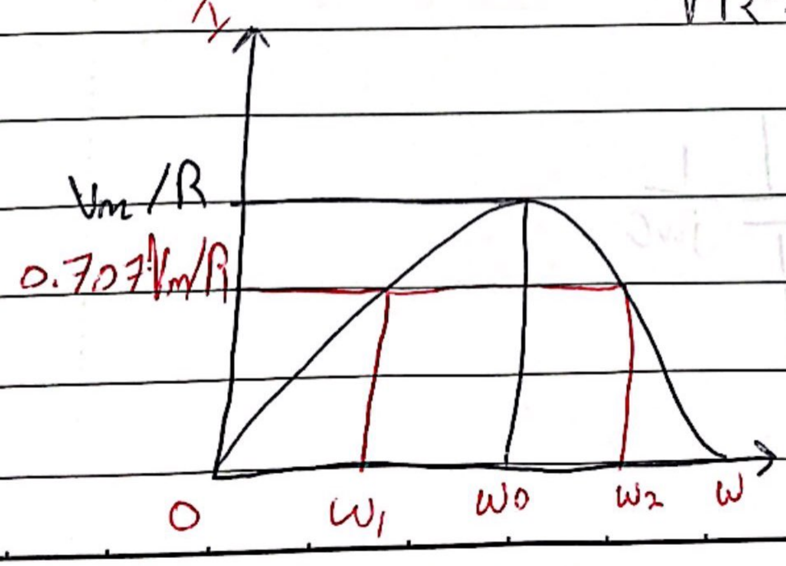
$$\omega_0 L = \frac{1}{\omega_0 C} \quad or \quad \omega_0 = \frac{1}{\sqrt{LC}} \ rad/s$$

since $\omega_0 = 2\pi f_0$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \ Hz.$$

* The current amplitude versus frequency for the series RLC circuit

$$I = |I| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$



NOTEBOOK

* The average power dissipated by the RLC circuit

$$P(\omega) = \frac{1}{2} I^2 R$$

* The highest power dissipated occurs at resonance, when $I = V_m / R$ so that

$$P(\omega_0) = \frac{1}{2} \frac{V_m^2}{R}$$

* At certain frequencies $\omega = \omega_1, \omega_2$, the dissipated Power is half the maximum value; that is

$$P(\omega_1) = P(\omega_2) = \frac{(V_m / \sqrt{2})^2}{2R} = \frac{V_m^2}{4R}$$

Hence, ω_1 and ω_2 are called the half power frequencies.

Bandwidth $= \omega_2 - \omega_1$

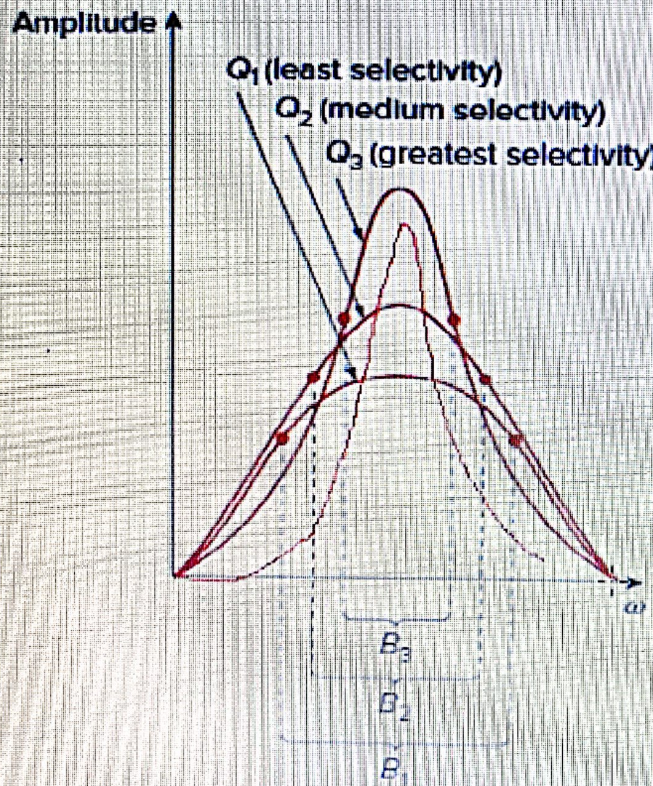
* average power on inductor or capacitor $= 0$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

- **<u>The "sharpness" or selectivity of the resonance in a resonant circuit is measured quantitatively by the quality factor Q</u>**



The higher the circuit Q, the smaller the bandwidth.

$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}}$$

$$\boxed{B = \frac{R}{L} = \frac{\omega_0}{Q}}$$

Quality factor Q

$$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o C R}$$

$$B = \frac{R}{L} = \frac{\omega_o}{Q}$$

sharp response

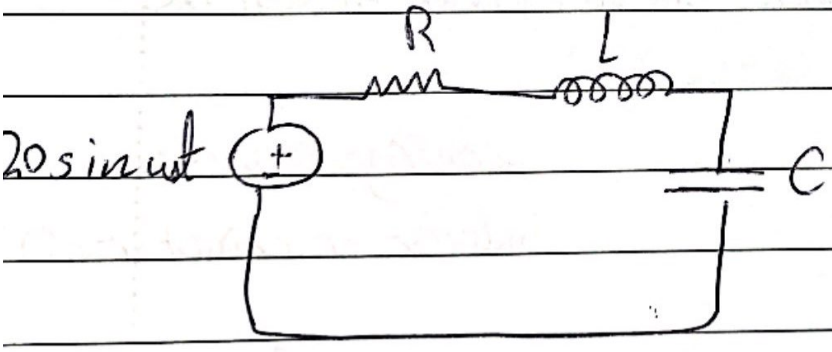
• The higher the circuit Q, the smaller the bandwidth.

Examples:- In the circuit $R = 2\,\Omega$, $L = 1\,mH$, $C = 0.4\,\mu f$

a) find the resonant frequency and half Power frequencies.

b) Calculate the quality factor and bandwidth.

c) Determine the amplitude of the current at $\omega_o, \omega_1, \omega_2$



a) $\omega_o = \frac{1}{\sqrt{LC}} = 50\, krad/s$

Solution:

(a) The resonant frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 0.4 \times 10^{-6}}} = 50 \text{ krad/s}$$

METHOD 1 The lower half-power frequency is

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$= -\frac{2}{2 \times 10^{-3}} + \sqrt{(10^3)^2 + (50 \times 10^3)^2}$$

$$= -1 + \sqrt{1 + 2500} \text{ krad/s} = 49 \text{ krad/s}$$

Similarly, the upper half-power frequency is

$$\omega_2 = 1 + \sqrt{1 + 2500} \text{ krad/s} = 51 \text{ krad/s}$$

(b) The bandwidth is

$$B = \omega_2 - \omega_1 = 2 \text{ krad/s}$$

or

$$B = \frac{R}{L} = \frac{2}{10^{-3}} = 2 \text{ krad/s}$$

The quality factor is

$$Q = \frac{\omega_0}{B} = \frac{50}{2} = 25$$

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■ **METHOD 2** Alternatively, we could find

$$Q = \frac{\omega_0 L}{R} = \frac{50 \times 10^3 \times 10^{-3}}{2} = 25$$

From Q, we find

$$B = \frac{\omega_0}{Q} = \frac{50 \times 10^3}{25} = 2 \text{ krad/s}$$

Since $Q > 10$, this is a high-Q circuit and we can obtain the half-power frequencies as

$$\omega_1 = \omega_0 - \frac{B}{2} = 50 - 1 = 49 \text{ krad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 50 + 1 = 51 \text{ krad/s}$$

as obtained earlier.

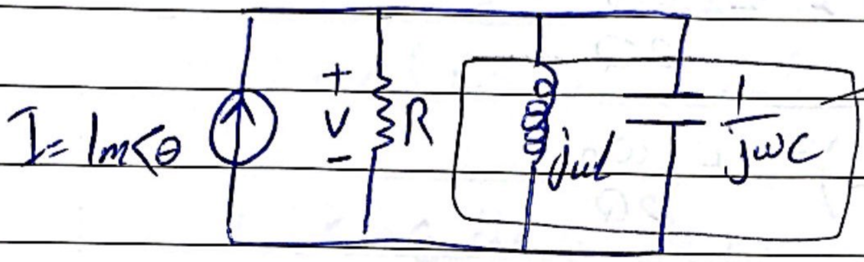
(c) At $\omega = \omega_0$,

$$I = \frac{V_m}{R} = \frac{20}{2} = 10 \text{ A}$$

At $\omega = \omega_1, \omega_2$,

$$I = \frac{V_m}{\sqrt{2}\,R} = \frac{10}{\sqrt{2}} = 7.071 \text{ A}$$

* Parallel Resonant RLC circuit



$Z = \infty$

$Y = \frac{1}{Z} = 0$

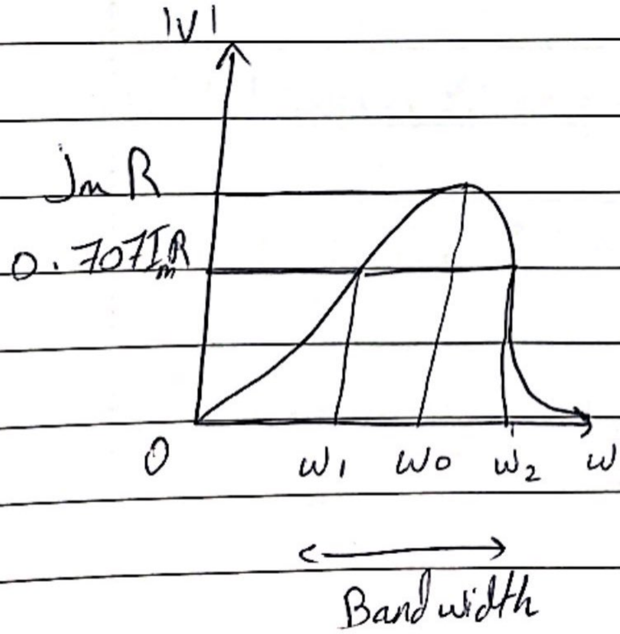
$$Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

Resonance occurs when the imaginary ~~pat~~ part of

$Y =$ zero

$$\omega C - \frac{1}{\omega L} = 0$$

or $$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$



$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

$$\omega_1 = \omega_0 \sqrt{1 + \left(\frac{1}{Q}\right)^2} - \frac{\omega_0}{2Q}$$

$$\omega_2 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$

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Summary of the characteristics of resonant *RLC* circuits.

Characteristic	Series circuit	Parallel circuit
Resonant frequency, ω_0	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
Quality factor, Q	$\frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 RC}$	$\frac{R}{\omega_0 L}$ or $\omega_0 RC$
Bandwidth, B	$\frac{\omega_0}{Q}$	$\frac{\omega_0}{Q}$
Half-power frequencies, ω_1, ω_2	$\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$	$\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$
For $Q \geq 10$, ω_1, ω_2	$\omega_0 \pm \frac{B}{2}$	$\omega_0 \pm \frac{B}{2}$

· Filters

- A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others

- filters are the circuits used in radio and TV reciever to allow us to select one desired signal out of multitude of broadcast signals in the environment.

- A filter is passive filter if it consists of only passive elements R, L, C. It is said to be an active filter if it consists of active elements (such as transistors and op amps) in addition to passive elements R, L, C.

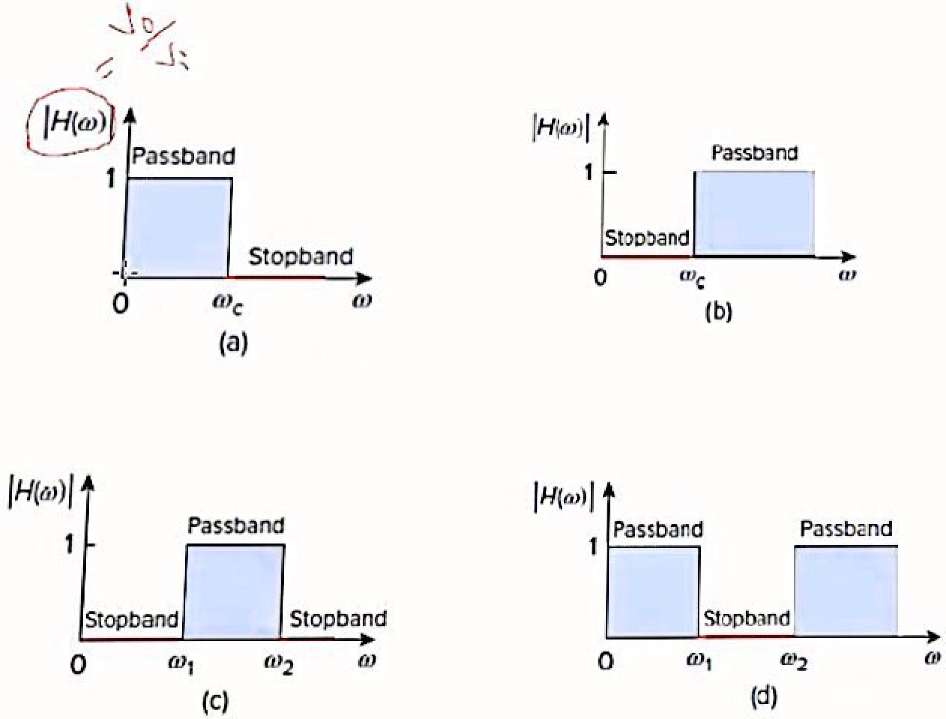
- Types of passive filters:

1- A low-pass filter passes law frequencies and stops high frequencies.

2. A high-pass filter passes high frequencies and rejects low frequencies.

3. A band-pass filter pass frequencies within a frequency band and blocks or attenuates frequencies outside the band.

4- A band-stop filter passes frequencies outside a frequency band and locks or attenuates frequencies within the band.


|H(ω)|
Passband
1
Stopband
0
ωc
ω
(a)
|H(ω)|
1
Passband
Stopband
0
ωc
ω
(b)
|H(ω)|
1
Passband
Stopband
Stopband
0
ω1
ω2
ω
(c)
|H(ω)|
1
Passband
Stopband
Passband
0
ω1
ω2
ω
(d)

Low-Pass Filter

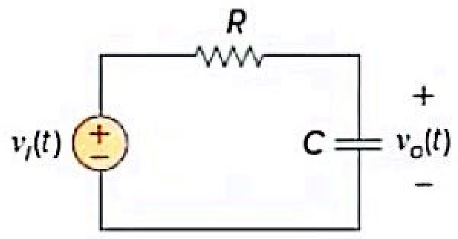
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

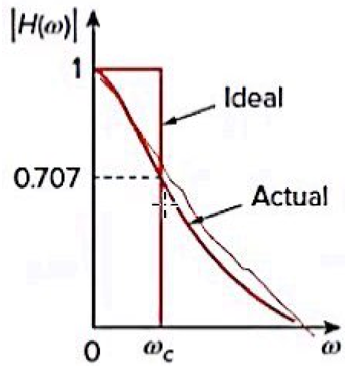
$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_c = \frac{1}{RC}$$

ωc: cutoff frequency or rolloff frequency



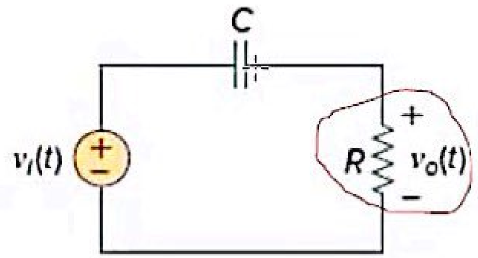


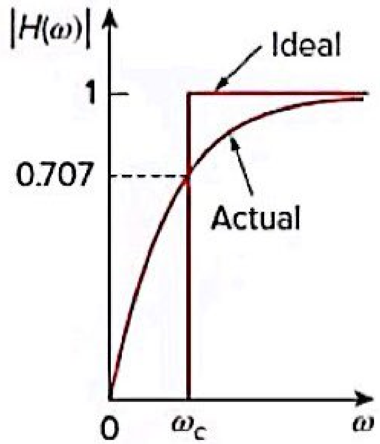
High-Pass Filter

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

$$\omega_c = \frac{1}{RC}$$

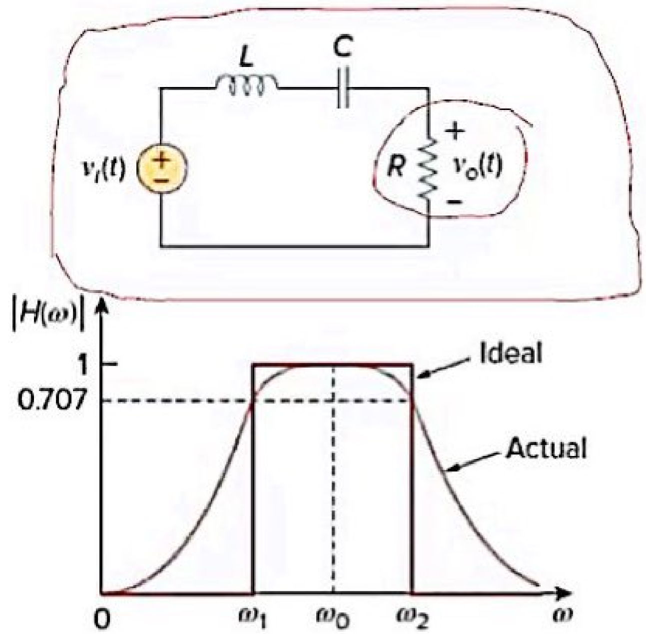




Band-Pass Filter

$$H(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



Band-Stop Filter

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$