* The source free $\boxed{RC}$ Circuits

tau

$\tau = RC$

$v(t) = V(0)e^{-t/\tau}$

$i(t) = i(0)\,e^{-t/\tau}$

* it takes 5τ for the circuit to reach its final state or steady state.
  - life time of the circuit $= 5\tau$
* if the value of τ is ↑ then → more lifetime

* $\tau\uparrow \longrightarrow$ more energy stored in capacitor.
* $\tau\uparrow \longrightarrow$ less power absorbed or generated by R.

* $V_c = V(t) = V(0)$

* 3 steps solution

⇒ step 1 at $t = \infty$ " determine the type of the circuit and find τ and write the solution form.

step 2 → at $t = 0^-$ " switching اللحظة ما قبل عملية "

↳ draw the circuit at $t = 0^-$

↳ find $i(0^-)$, $V(0^-)$, $V_c(0^-)$

step 3 → at $t = 0^+ = 0$ " switching بعد عملية "

↳ Draw the circuit at $t = 0^+$

↳ Replace each capacitor by a voltage source.

↳ find $i(0^+)$, $V(0^+)$ = $i(0)$, $V(0)$

to use them in the solution

* $V_c(0^-) = V_c(0^+) = V_c(0)$

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* Source free RL circuits.

$$\tau = \frac{L_{eq}}{R_{eq}}$$

$$i(t) = I(0^+) e^{-t/\tau}$$

$$V(t) = V(0^+) e^{-\frac{t}{\tau}}$$

→ step 1 :- at $t = \infty$ → find the type of the circuit then find τ, and write the solution form.

→ step 2 :- at $t = 0^-$:- Draw the circuit

↳ find $i(0^-)$, $V(0^-)$, $i_L(0^-)$

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→ step 3 → at $t = 0^+$:- Draw the circuit
↳ replace each inductor by
a current source $i_L(0^-)$

* $i_L(0^-) = i_L(0^+) = i_L(0)$
find $i(0^+)$, $V(0^+)$ to use them in the
solution.

* $i_L(t) = i(t) = i(0)e^{-t/\tau}$

Remember: in DC circuits we replace inductor with short circuit, and capacitors with open circuits.

* Driven RC and RL Circuits

$$V(t) = V(\infty) + (V(0) - V(\infty))e^{-t/\tau} \quad \text{for } t > 0$$

$$V(t) = V(0) \quad \text{for } t < 0$$

$$i(t) = i(\infty) + (i(0) - i(\infty))e^{-\frac{t}{\tau}}, \quad t > 0$$

$$i(t) = i(0), \quad t < 0$$

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⇒ step 1: determine the type at $\boxed{t=\infty}$ and find "τ" and find $i(\infty)$ or $V(\infty)$ or both. then write the solution form.

⇒ step 2:- at $t=0^-$ find $i_L(0^-)$, $i(s)$, $V_c(0^-)$

⇒ step 3:- at $t=0^+$

$$V_c(0^+) = V_c(0^-)$$

$$i_L(0^+) = i_L(0^-)$$

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* Source free RLC Circuit

* series

$$\alpha = \frac{R_{eq}}{2L}$$

parallel

$$\alpha = \frac{1}{2RC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Characteristic roots.

$$\begin{cases} s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\ s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \end{cases}$$

Solution :-

* if $\alpha > w_o \Rightarrow$ over damped response

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

* if $\alpha = w_o \Rightarrow$ Critically damped response

$$i(t) = A_1 t e^{-\alpha t} + A_2 e^{-\alpha t}$$

* if $\alpha < w_o \Rightarrow$ underdamped response

$$i(t) = e^{-\alpha t} [A_1 \cos(w_d t) + A_2 \sin(w_d t)]$$

$$w_d = \sqrt{w_o^2 - \alpha^2}$$

* same solution for $V(t)$.

* Driven RLC Circuits

$i(t) = i(\infty)$ + solution in source-free.

$V(t) = V(\infty)$ + solution in source-free

Note :- $S_1 = S_2 \longrightarrow$ Critical damped

S_1, S_2 (real numbers) $\longrightarrow$ overdamped.

S_1, S_2 (complex numbers) $\longrightarrow$ underdamped