LINEAR ALGEBRA

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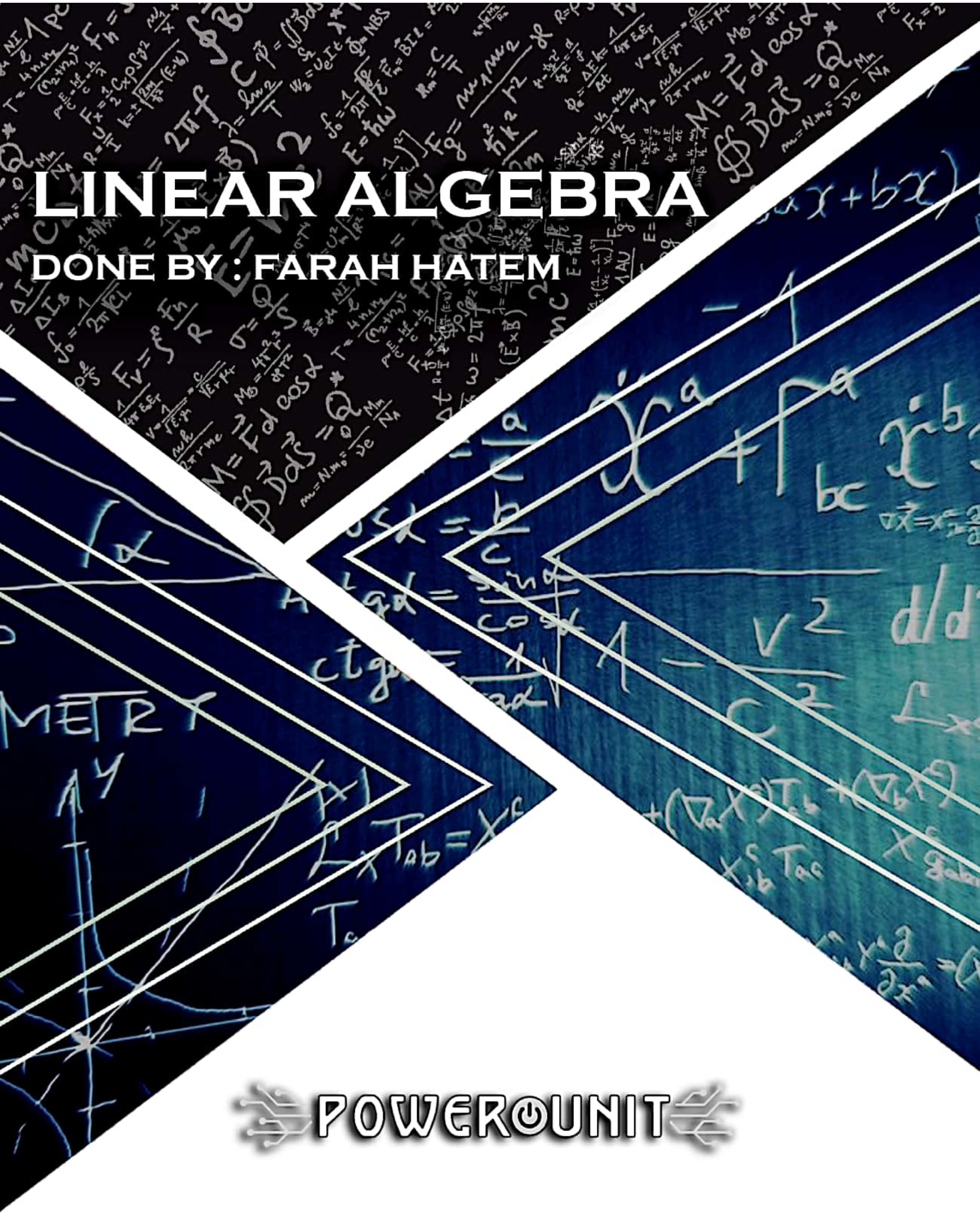

POWERUNIT

Solve

let $A = \begin{bmatrix} 3 & 6 \\ 2 & 3 \end{bmatrix}$

① Find A^{-1}

$$A^{-1} = \frac{1}{9-12}\begin{bmatrix} 3 & -6 \\ -2 & 3 \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} 3 & -6 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2/3 & -1 \end{bmatrix}$$

② If $(3B-2I)^{-1} = A \rightarrow$ find (B)

$(3B-2I)^{-1} = A \rightarrow$ take inverse to both sides.

$$3B - 2I = A^{-1}$$

$$3B = A^{-1} + 2I \Rightarrow 3B = \begin{bmatrix} -1 & 2 \\ 2/3 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2/3 & 1 \end{bmatrix}$$

$$B = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ 2/3 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 2/3 \\ 2/9 & 1/3 \end{bmatrix}$$

③ If $AX = \begin{bmatrix} 2 & 1 & 4 \\ -1 & -2 & 3 \end{bmatrix} \rightarrow$ find X

لما أقسم مصفوفة كأني
باخذ ال inverse تبعها

$$X = \begin{bmatrix} 2 & 1 & 4 \\ -1 & -2 & 3 \end{bmatrix} A^{-1} = \text{-------} = \begin{bmatrix} -4 & -5 & 2 \\ \frac{4}{3}+1 & \frac{2}{3}+2 & \frac{8}{3}-3 \end{bmatrix}$$

Doing the multiplication

Solve

Use the Gaussian elimination to solve the following system.

$$-x - 10y - 10z = -31$$
$$2x - y + z = -1$$
$$3x + 2y - 2z = 9$$

$$\begin{bmatrix} -1 & -10 & -10 & -31 \\ 2 & -1 & 1 & -1 \\ 3 & 2 & -2 & 9 \end{bmatrix} \; -R_1$$

$$\begin{bmatrix} 1 & 10 & -10 & 31 \\ 2 & -1 & 1 & -1 \\ 3 & 2 & -2 & 9 \end{bmatrix} \; \begin{matrix} 2R_1 + R_2 \\ -3R_1 + R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 10 & -10 & 31 \\ 0 & -21 & 21 & -63 \\ 0 & -20 & 28 & -34 \end{bmatrix} \; -\tfrac{1}{21} R_2$$

$$\begin{bmatrix} 1 & 10 & -10 & 31 \\ 0 & 1 & -1 & 3 \\ 0 & -28 & 28 & -34 \end{bmatrix} \; 28R_2 + R_3$$

$$\begin{bmatrix} 1 & 10 & -10 & 31 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \text{Backward substitution.}$$

$y - z = 3 \longrightarrow \overset{\text{assume}}{z = t} \rightarrow t \in R$

$y = 3 + t$

$x + 10y - 10z = 31$

$x = 1$

$\{(1, 3+t, t) : t \in R\}$

Solve

let A be square matrix such that $A^2+5A-I=0$ show that A is invertible and $A^{-1}=A+5I$

$A^2+5A-I=0$

$A^2+5A=I$

$A(A+5I)=I \rightarrow B=A+5I$

$\rightarrow AB=I$

then $B=A^{-1}=A+5I$

then $A^{-1}=A+5I$

Solve

If $A=\begin{bmatrix}-3 & 1\\ 0 & 2\end{bmatrix}$ and $p(x)=x^2-2x-1$, then $p(A)=?$

follow the equation $\rightarrow$ the result is $p(A)=\begin{bmatrix}2 & -3\\ 0 & -1\end{bmatrix}$

Solve

which one of these matrices is elementary?

(A) $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 4 & 1\end{bmatrix}$ (B) $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & 0 & -1\end{bmatrix}$ (C) $\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}$

(D) $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}$

the correct answer is (D)

Solve

what conditions on b_1, b_2 and b_3 in order for the following system to be consistent.

system $\rightarrow$ $\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ -2 & -4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

solution is: use Gaussian elemination to solve it

the condition is $b_3 = -2b_1$

موجود مثال مطابق إله
في الشرح لسكشن 6
على موقع Powerunit

← شرح النقطة هاي موجود في ملف:
chapter(1) Comprehensive explanation Part 2

Solve

the solution of the following system is?

$x_1 + 2x_2 - x_3 - x_4 + x_5 = 3$

$x_1 + x_2 + x_3 + x_4 + 2x_5 = 0$

$x_1 + -x_2 + 3x_3 + 3x_4 + x_5 = -4$

$3x_1 + 2x_2 + 3x_3 + 3x_4 + 4x_5 = -1$

\# Try it yourself

Find answer $\Rightarrow$

The matrix resulting after doing Gaussian elemination

$$\begin{bmatrix} 1 & 2 & -1 & -1 & 1 & 3 \\ 0 & 1 & -2 & -2 & -1 & 3 \\ 0 & 0 & -2 & -2 & -3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By doing backward substitution:

① $-2x_3 - 2x_4 - 3x_5 = 2$ assume $x_4 = t$ $x_5 = S$ for $t, S \in R$ $\rightarrow$ then $x_3 = -1 - t - \frac{3}{2}S$

② $x_2 - 2x_3 - 2x_4 - 1x_5 = 3$

$x_2 = 1 - 2S$

③ $x_1 + 2x_2 - 1x_3 - 1x_4 + 1x_5 = 3$

$x_1 = \frac{3}{2}S.$

Solve

If $A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & -1 \\ 4 & 1 & 2 \end{bmatrix}$, the find A^{-1}.

using elementary matrix method

starting with → $\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 \\ 4 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$

doing row operations in between you finish solution with

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 1 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & 1 & -3 & -1 \end{array}\right]$$

Thus $A^{-1} = \begin{bmatrix} -1 & 2 & 1 \\ 2 & -2 & -1 \\ 1 & -3 & -1 \end{bmatrix}$

طريقة الحل هاي مشروحة
برضه على موقع Powerunit
في ملف منفصل اسمه:
chapter (1) Extenstion (1)

Solve

Suppose that A is a square matrix with
$A^3 + 18A^2 - 6I = 0$
show that A is invertible and find A^{-1} in terms of A

from the equation

$A^3 + 12A^2 - 6I = 0$

$A^3 + 18A^2 = 6I$

$A\left(\frac{A^2}{6} + 3A\right) = I$

so A is invertible if and only if $A^{-1} = \left(\frac{A^2}{6} + 3A\right)$ for $AA^{-1} = I$ and A^{-1} here should be $\left(\frac{A^2}{6} + 3A\right)$ so when the equation is satisfied A is invertible and the inverse of $A = A^{-1} = \left(\frac{A^2}{6} + 3A\right)$

Solve

If A is a 5x3 ~~an~~ matrix and B is a 5x6, then find the size of the matrix

A is 5x3 B is 5x6

Find the size of $3A^TB$

→ size of $A^T = 3\times5$

→ size of $A^TB = \underset{3\times5}{A^T} \times B_{5\times6}$ is → 3×6 = size of $3A^TB$

Solve

For which values of (a) and (b) the following system is inconsistent?

$x+2y+z=4$
$y-z=3$
$(a-3)z=(b-8)$

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & (a-3) & (b-8) \end{bmatrix}$$

$(a-3)=(b-8)$

$\boxed{a=3}$

when $(a-3)z=b-8$ turns into $0z=k$ then it is inconsistent and then when $a=3$

$0=b-8 \rightarrow b=8$ / but b should not equal 8 to make the system inconsistent and satisfy my statement above. then the system has no solution when:

$a=3 \rightarrow B \in R-\{8\}$