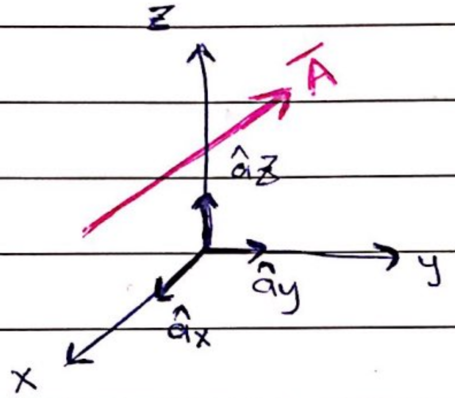
Ch1: Vector Algebra

Vector : magnitude and dir $\quad A, \bar{A}$

Scalar : magnitude only $\quad A, A$

in Cartesian Coordinates :-



unit vector ⇒ vector has a magnitude = 1

$\hat{a}_x \equiv$ unit vector

↳ باتجاه

$$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

⇓ Long format

A_x , A_y , A_z = Vector components

$\hat{a}_x , \hat{a}_y , \hat{a}_z$ = unit vectors

Short format

↳ $\bar{A} = (A_x , A_y , A_z)$

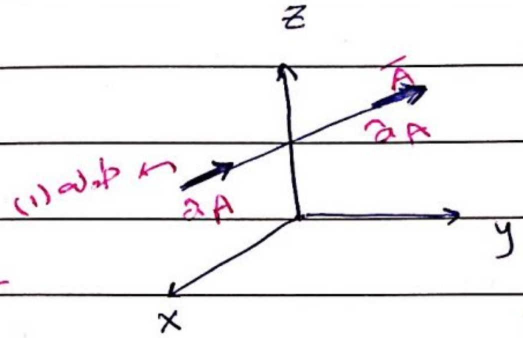
Vector magnitude (Norm)

$$|\bar{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

direction of vector A:-

$\hat{a}_A$ = unit vector A:-

$$\hat{a}_A = \frac{\bar{A}}{A}$$



operations on vectors:-

Addition and subtraction

$$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

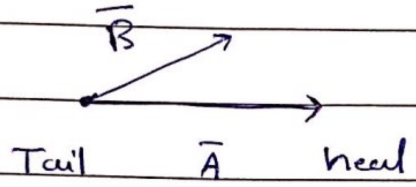
$$\bar{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$$

$$\bar{C} = \bar{A} + \bar{B} \Rightarrow (A_x + B_x)\hat{a}_x + (A_y + B_y)\hat{a}_y + (A_z + B_z)\hat{a}_z$$

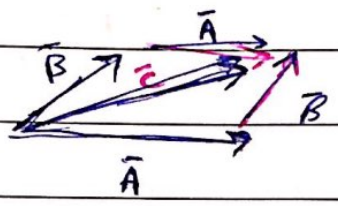
$$\bar{C} \Rightarrow C_x \hat{a}_x + C_y \hat{a}_y + C_z a_z$$

$$D = \bar{A} - \bar{B} \Rightarrow (A_x - B_x)\hat{a}_x + (A_y - B_y)\hat{a}_y + (A_z - B_z)\hat{a}_z$$

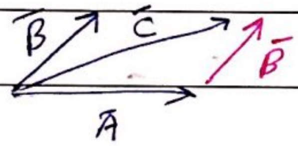
Graphical:-



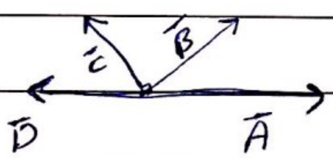
$\bar{C} = \bar{A} + \bar{B}$

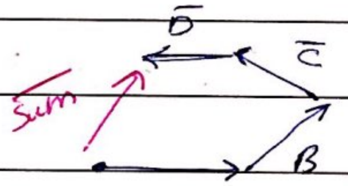


OR



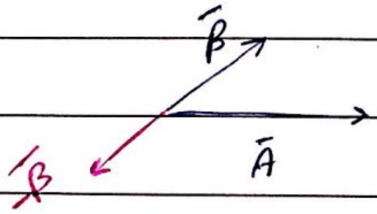
arrow method.

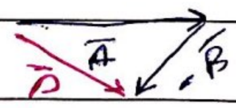




$\overline{Sum} = \bar{A} + \bar{B} + \bar{C} + \bar{D}$

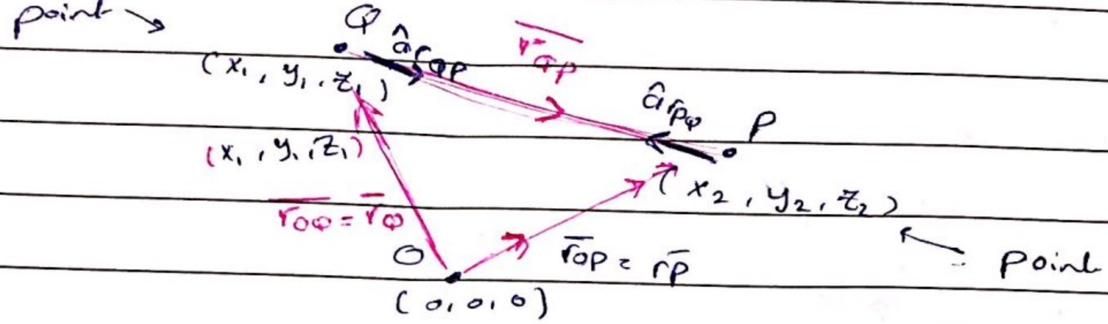
$\bar{D} = \bar{A} - \bar{B} \Rightarrow \bar{A} + (-\bar{B})$





subtraction $\Rightarrow$ Distance.

Distance:- vector



$\bar{r}_{QP} = ? = \bar{r}_P - \bar{r}_Q$ → vector not point

$= (x_2, y_2, z_2) - (x_1, y_1, z_1)$

$= (x_2 - x_1)\,\hat{a}_x + (y_2 - y_1)\,\hat{a}_y + (z_2 - z_1)\,\hat{a}_z$

$= -\bar{r}_{PQ}$

$r_{QP} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$$\boxed{\hat{a}_{rQP} = \frac{\bar{r}_{QP}}{r_{QP}}}$$ → unit vector

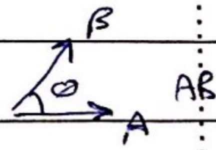
Multiplication

1) Dot product → scalar كمية

$\bar{A} \cdot \bar{B} = |\bar{A}|\,|\bar{B}| \cos\theta_{AB}$

i.e. $\bar{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z$

$\bar{B} = B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z$



$\bar{A}\cdot\bar{B} = A_x B_x + A_y B_y + A_z B_z$

$\hat{a}_x \cdot \hat{a}_x = (1)(1)\cos 0^\circ = 1$

$\hat{a}_x \cdot \hat{a}_y = (1)(1)\cos 90^\circ = 0$

$$\cos\theta_{AB} = \frac{\bar{A}\cdot\bar{B}}{AB}$$

$$\boxed{\theta_{AB} = \cos^{-1}\frac{\bar{A}\cdot\bar{B}}{AB}}$$

theta

note :-

1. $\bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{A}$
2. $A\cdot(\bar{B}+\bar{C}) = \bar{A}\cdot\bar{B} + \bar{A}\cdot\bar{C}$
3. $K(\bar{A}+\bar{B}) = K\bar{A} + K\bar{B}$
4. $\bar{A}\cdot\bar{A} = A^2 = A_x^2 + A_y^2 + A_z^2$

$\hat{a}_n \cdot \hat{a}_m = 0 \quad , \quad \text{if } n \neq m$

$\quad\quad = 1 \quad , \quad \text{if } n = m$

$\bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{A}$

2) Cross product → vector الناتج

$|\bar{A}\times\bar{B}| = AB\sin\theta_{AB}$

$$\boxed{\theta_{AB} = \sin^{-1}\left(\frac{|\bar{A}\times\bar{B}|}{AB}\right)}$$

$$\bar{A}\times\bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Row, Colound

$\bar{A}\times\bar{B} = (-1)^{1+1}(A_yB_z - B_yA_z)\,\hat{a}_x$

$+ (-1)^{1+2}(A_xB_z - A_zB_x)\,\hat{a}_y$

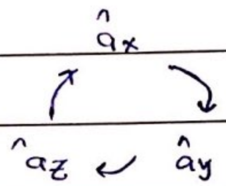
$+ (-1)^{1+3}(A_xB_y - A_yB_x)\,\hat{a}_z$

$\bar{A}\times\bar{B} \perp \bar{A}$ and $\bar{B}$

$|\hat{a}_x \times \hat{a}_x| = (1)(1)\sin 0^\circ = 0$

$|\hat{a}_x \times \hat{a}_y| = 1$

$\hat{a}_x \times \hat{a}_y = (1) = \hat{a}_z$



مع السهم ← موجب

عكس السهم ← سالب

$|\hat{a}_n \times \hat{a}_m| = 0$ if $n = m$

$\qquad\quad 1$ if $n \neq m$

$\bar{A} \times \bar{B} \neq \bar{B} \times \bar{A}$

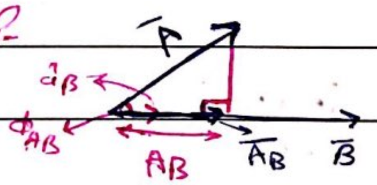
$\bar{A} \times \bar{B} = -\bar{B} \times \bar{A}$

$\bar{A} \times (B + C) = \bar{A} \times \bar{B} + \bar{A} \times \bar{C}$

Component of a vector:

$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

A_B = scalar projection of A along B



$\bar{A}_B$ = vector " "

" " "

$\cos\phi_{AB} = \dfrac{A_B}{A} \Rightarrow A_B = A\cos\theta_{AB}$

$\cos\phi_{AB} = \dfrac{\bar{A} \cdot \bar{B}}{AB}$, $\hat{a}_B = \dfrac{\bar{B}}{B}$

$= \dfrac{\bar{A} \cdot \hat{a}_B}{A}$

$A_B = A\cos\phi_{AB} = \cancel{A}\dfrac{\bar{A} \cdot \hat{a}_B}{\cancel{A}}$

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$$\boxed{A_B = \bar{A} \cdot \hat{a}_B}$$

$\overline{A_B} = A_B \, \hat{a}_B$

$(\bar{A} \cdot \hat{a}_B)\, \hat{a}_B$

scaler

vector

Ex:- Given $\bar{A} = 3\hat{a}_x + 4\hat{a}_y + \hat{a}_z$

$\bar{B} = 2\hat{a}_y - 5\hat{a}_z$

Find: Φ_{AB}, A_B, $\overline{A_B}$

$\Phi_{AB} = \cos^{-1} \dfrac{\bar{A} \cdot \bar{B}}{AB}$

$\bar{A} \cdot \bar{B} = 0 + 8 - 5 = 3$

$A = \sqrt{26}$

$B = \sqrt{29}$

$\Phi_{AB} = \cos^{-1}\left(\dfrac{3}{\sqrt{26}\sqrt{29}}\right) = 83.73^\circ$

$A_B = \bar{A} \cdot \hat{a}_B$, $\hat{a}_B = \dfrac{(0, 2, -5)}{\sqrt{29}}$

$= \dfrac{8 - 5}{\sqrt{29}} = \dfrac{3}{\sqrt{29}} = 0.557$

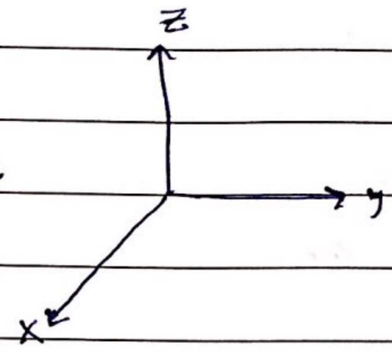
$\overline{A_B} = A_B \hat{a}_B$

$= \dfrac{3}{\sqrt{29}} \cdot \dfrac{(0, 2, -5)}{\sqrt{29}} = \dfrac{6\hat{a}_y - 15\hat{a}_z}{29} = \boxed{0.207\hat{a}_y - 0.517\hat{a}_z}$

Ch 2:- Coordinate systems and Transformation.

cartesian Coordinate:-

x, y, z



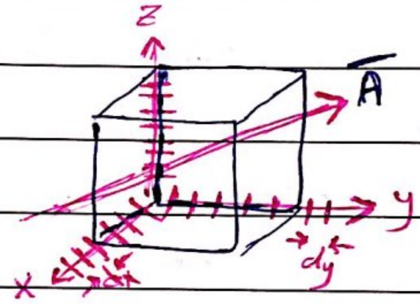
$-\infty < x < \infty$

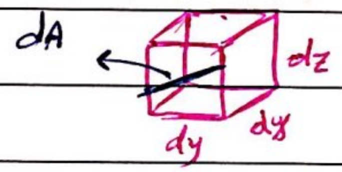
$-\infty < y < \infty$ } 3D object

$-\infty < z < \infty$ → infinite Box

⇒ unit vectors:-

$\hat{a}_x \ , \ \hat{a}_y \ , \ \hat{a}_z$





Differential Elements

$dx \ , \ dy \ , \ dz$

Differential length ($\bar{dl}$) "vector"

$$\bar{dl} = \underline{dx}\,\hat{a}_x + \underline{dy}\,\hat{a}_y + \underline{dz}\,\hat{a}_z$$

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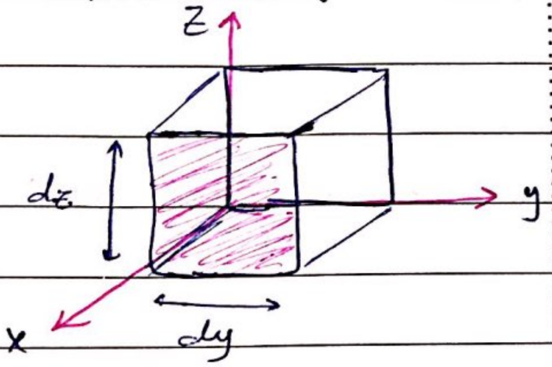
Differential Normal surface Area ($\bar{ds}$) "vector"



$\bar{ds}_{front} = dy\, dz\, \hat{a}_x$

$\bar{ds}_{back} = dy\, dz\, (-\hat{a}_x)$

$\bar{ds}_{right} = dx\, dz\, \hat{a}_y$ dx, $\cancel{dy}$, $\cancel{dz}$

$\bar{ds}_{left} = -dx\, dz\, \hat{a}_y$

$\bar{ds}_{Top} = dx\, dy\, \hat{a}_z$, $ds_{bot} = -dx\, dy\, \hat{a}_z$

Differential volume (dv) "scalar"

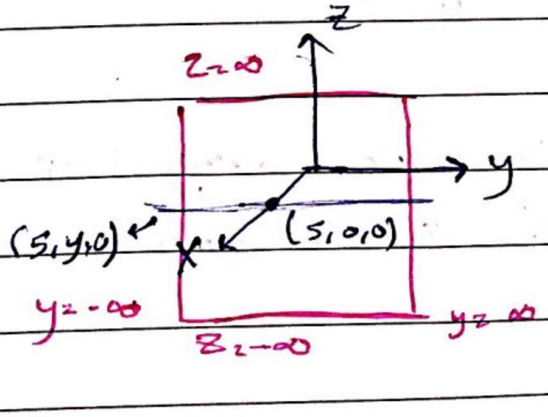
$dv = dx\, dy\, dz$

2D surfaces

by fixing one variable.

if x is constant

$\left(-\infty < \begin{pmatrix} y \\ z \end{pmatrix} < \infty\right)$



i.e. $= x = 5$

↳ infinite plane

// yz plane

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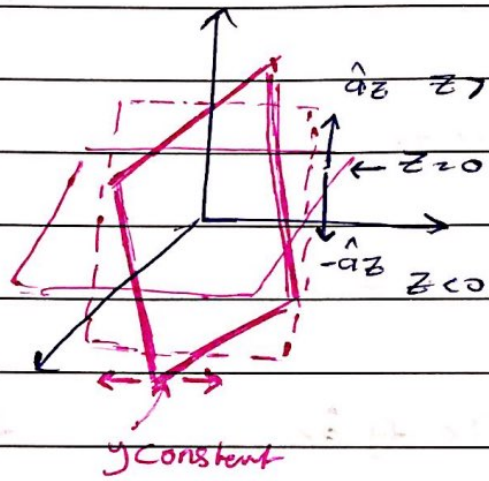
y = Constant ⇒ infinite plane along xz (y=0)
 " xz (y≠0)

z = Constant ⇒ infinite plane along xy (z=0)
 // xy (z≠0)

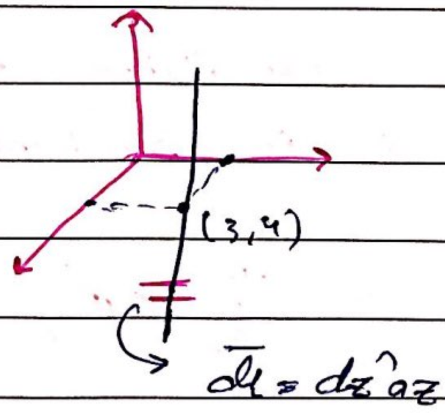


1D segment
by fixing two variables
⇒ if x, y are Constant

$(-\infty < z < \infty)$

x = 3 , y = 4

infinite line along z axis
if x=0 & y=0



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if y, z are constant
↳ infinite line along x- axis
if ($y=0$, $z=0$)
→ infinite line // x - axis
if ($y \neq 0$, OR $z \neq 0$)

if x, z are constants.
↳ infinite line along or
parallel to the y- axis

point → by fixing all variables.

N O T E B O O K

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Cylindrical cordinates.

نصف القطر

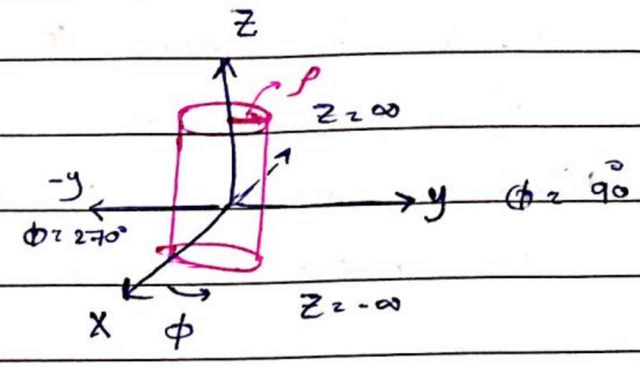
$0 \le \rho < \infty$

$0 \le \phi \le 2\pi$ → بالترتيب

ارتفاع الاسطوانة

$-\infty \le z < \infty$

$\underbrace{\phantom{-\infty \le z < \infty}}_{\text{3D object}}$



$\phi = 0°$, $\phi = 360$

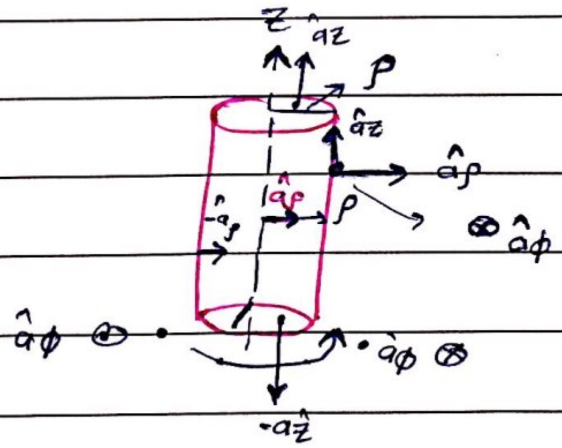
x-y plane

infinite solid cylinder.
↳ Filled from inside.

ρ, ϕ and z

unit vectors:

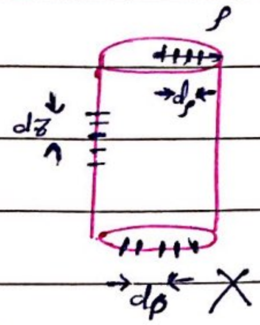
$\hat{a}_\rho$, $\hat{a}_\phi$, $\hat{a}_z$



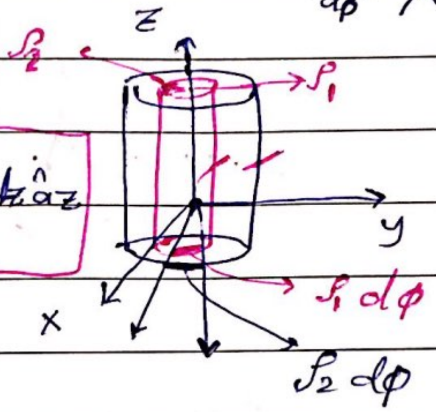
Differential Elements :-

$$\boxed{d\rho,\ \rho\, d\phi,\ dz}$$

$d\rho \rightarrow \hat{a}_\rho$, $\rho\, d\phi \rightarrow \hat{a}_\phi$, $dz \rightarrow \hat{a}_z$

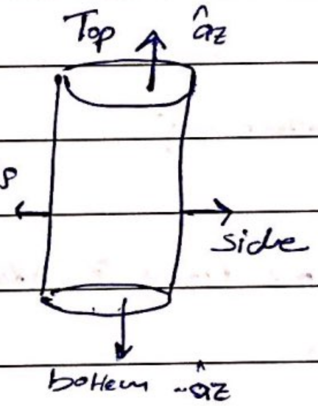


$$\boxed{\overline{dl} = d\rho\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + dz\, \hat{a}_z}$$



$\overline{ds}_{side} = \rho\, d\phi\, dz\, \hat{a}_\rho$

$\boxed{\rho = \text{Constant}}$



$\overline{ds}_{top} = \rho\, d\rho\, d\phi\, \hat{a}_z$

$\overline{ds}_{bot} = -\rho\, d\rho\, d\phi\, \hat{a}_z$

$\boxed{z = \text{Constant}}$

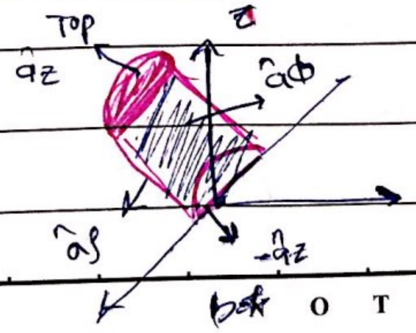
$\cancel{d\rho},\ \rho\, d\phi,\ dz$

$\overline{ds}_{cut} = d\rho\, dz\, \hat{a}_\phi \rightarrow \boxed{\phi = \text{Constant}}$

بنقطعه زي كيكة

$$\boxed{dv = \rho\, d\rho\, d\phi\, dz}$$

Scalar



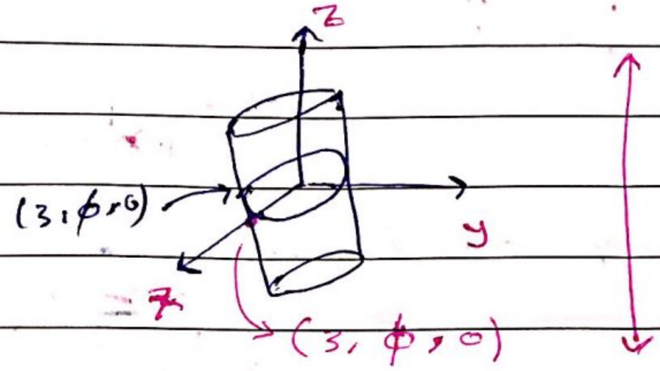
2D surfaces :-

ρ = Constant

$0 \leq \phi \leq 2\pi$

$-\infty < z < \infty$

i.e $\rho = 3$



$\underline{\rho = 3}$, $\underline{0 < \rho < 3}$

hollow Solid

infinite hollow cyl.

~~$d\rho$~~

~~$\rho\, d\phi$~~

dz

$\rho \neq 0$

infinite line long z-axis

$\rho = 0$

ϕ = Constant

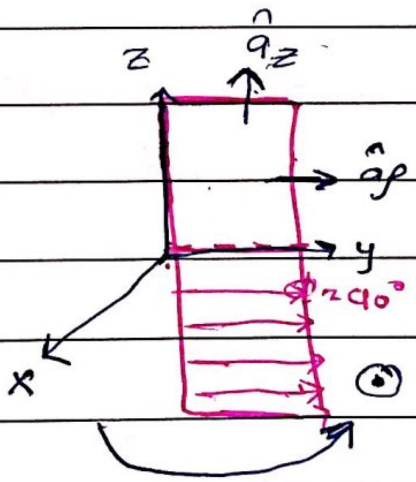
i.e $\phi = 90^\circ$

$\infty \leq \rho < \infty$

$-\infty < z < \infty$

inf. plane along

YZ plane

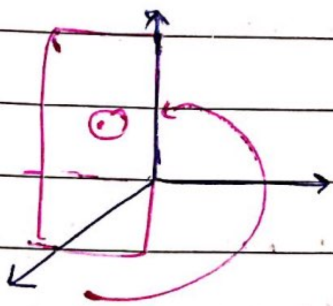


if $\phi = 0^\circ \rightarrow$ inf plane along XZ plane

if $\phi = 270^\circ$

$ds_{cut} = d\rho\, dz\, \hat{a}_\phi$

$\boxed{\text{Consul} = \phi}$ $\hat{a}_\rho \hat{z}$

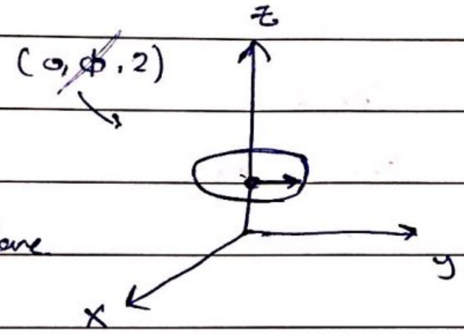


$Z =$ constant

i.e $Z = 2$

infinite disk

// parallel to xy plane



For $Z = 0$

infinite plane along xy plane

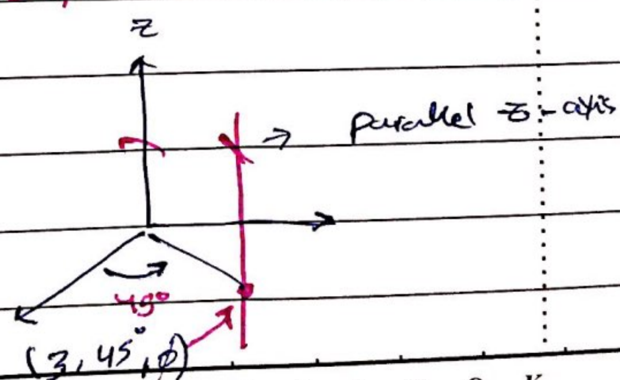
For 1D segment :-

ρ, ϕ are constant, $\rightarrow \vec{Z} \rightarrow \vec{dl} = dz\, \hat{a}_z$

inf line // Z-axis ($\rho \neq 0$)

or along Z-axis ($\rho = 0$)

i.e $\rho = 3, \quad \phi = 45^\circ$



\# if ρ, z are Constant

↳ circle (1D)

✗ disk (2D)

→ Circle // xy plane ($z \neq 0$), ($\rho \neq 0$)

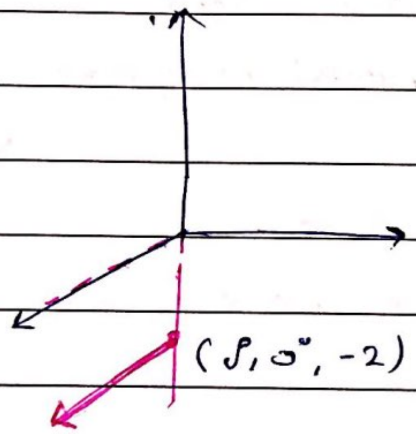
circle along xy plane ($z = 0$), ($\rho \neq 0$)

(point if $\rho = 0$)

\# if Φ, z are Const

↳ semi-infinite line (ray)

if $\phi = 0^\circ$, $z = -2$

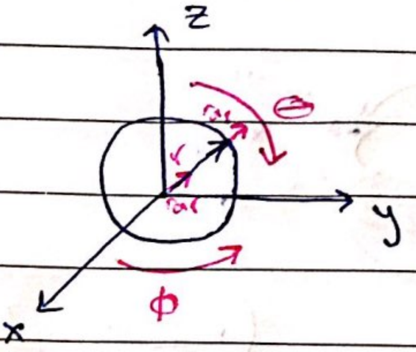


+ve x-axis → $\Phi = 0^\circ$, $z = 0$

point by fixing the 3-variable.

Spherical Coordinates

الشحنة النقطية ، النقطة حالة خاصة من ال spherical coordinates



$0 \leq r < \infty$

$0 \leq \theta \leq \pi$

$0 \leq \phi \leq 2\pi$

↓

infinite solid sphere

الكرة إلها اتجاه أفقي واتجاه عامودي
الاتجاه الأفقي مثله مثل الزاوية من ال x لل y
الاتجاه العامودي ماشي من z

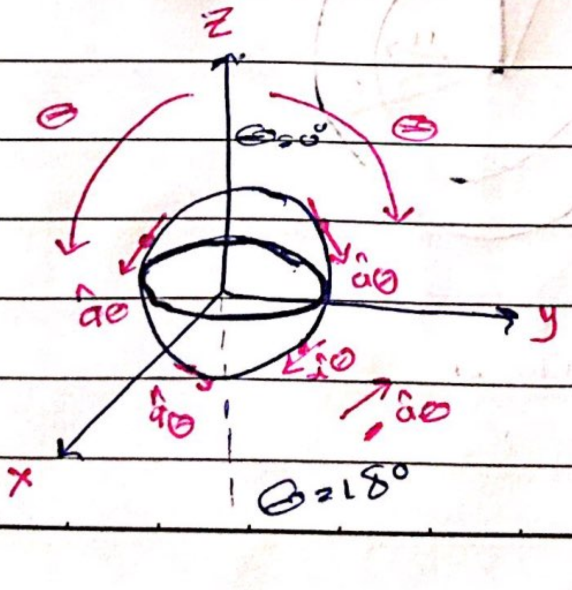
infinite → لأنه r (نصف) بيوصل إلى ال ∞

و solid لأنه r متغير

→ unit vectors :-

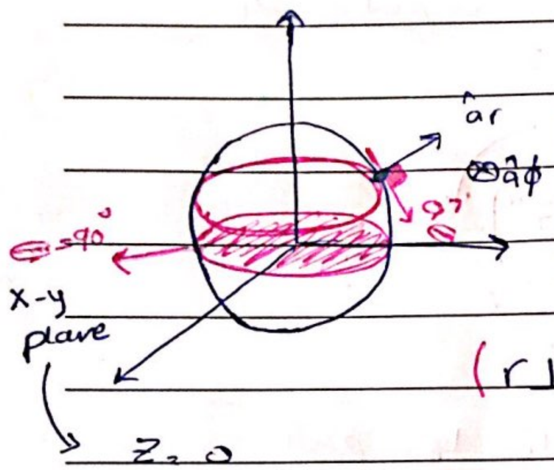
$\hat{a}_r$, $\hat{a}_\phi$, $\hat{a}_\theta$

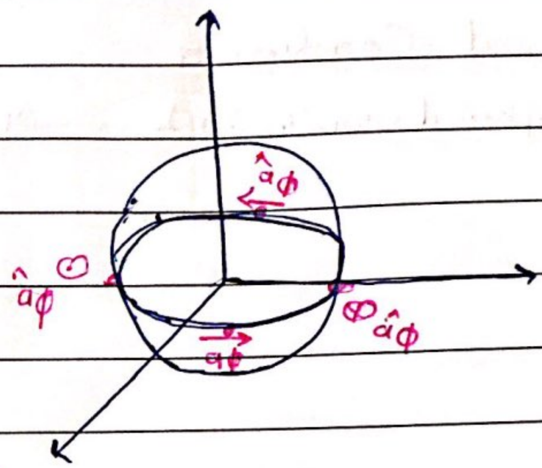
- أي خط طالع من ال origin هو Radius هو شرط يكون طالع من المحور.



- أي اتجاه طالع من سطح الكرة اسمه $\hat{a}_r$ ولكن اذا داخل من سطح الكرة اسمه $-\hat{a}_r$

90 درجة بين r وبين θ

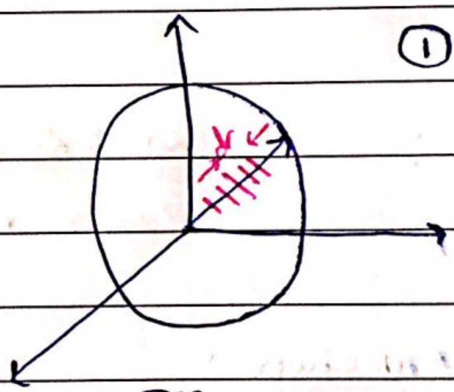


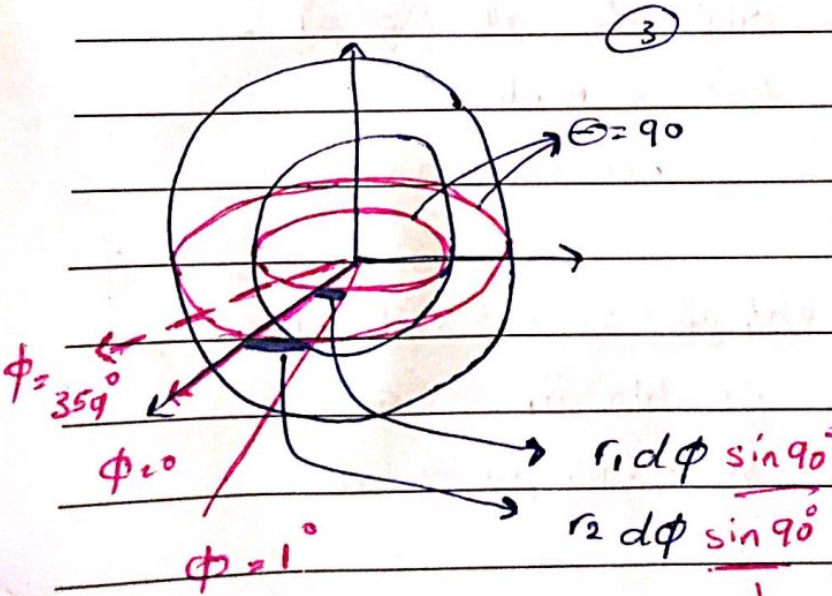


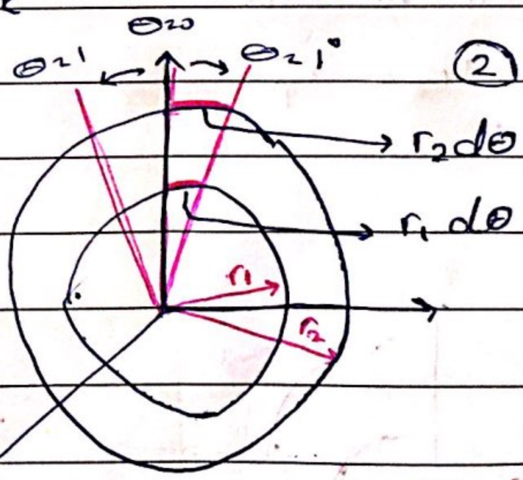
($r \perp \theta$ and ϕ)

Differential element's :-

dr , $r d\theta$, $r \sin\theta \, d\phi$







بوصلها كم بعدنا

عن الـ Z axis بالقطع

$\overline{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$

$\overline{ds}_{surface} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r \rightarrow r = \text{constant}$

$\overline{ds}_\theta = r\sin\theta\, dr\, d\phi\, \hat{a}_\theta \rightarrow \theta = \text{constant}$

$\overline{ds}_\phi = r\, dr\, d\theta\, \hat{a}_\phi \rightarrow \phi = \text{constant}$

$dv = r^2 \sin\theta\, dr\, d\phi\, d\theta$

2D surfaces :-

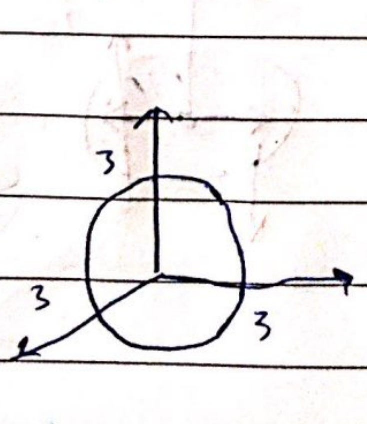
$r = \text{constant}$

$\left(\begin{array}{c} 0 \leq \theta \leq \pi \\ 0 \leq \phi \leq 2\pi \end{array} \right)$

→ hollow sphere $r \neq 0$

→ point ($r = 0$)

$r = 3$



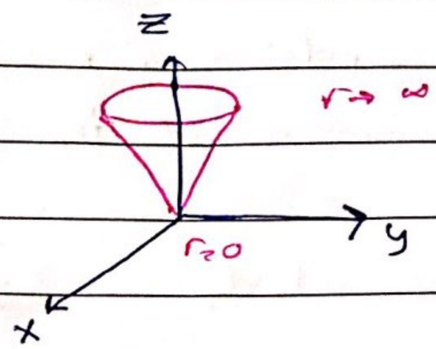
$r\,d$

$r\sin\theta$

N O T E B O O K

$\theta =$ constant

$\theta = 60°$



$0 < r < \infty$

$0 \leq \phi \leq 2\pi$

inf hollow cone

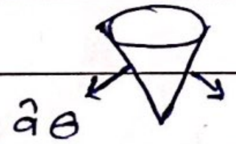
For any value

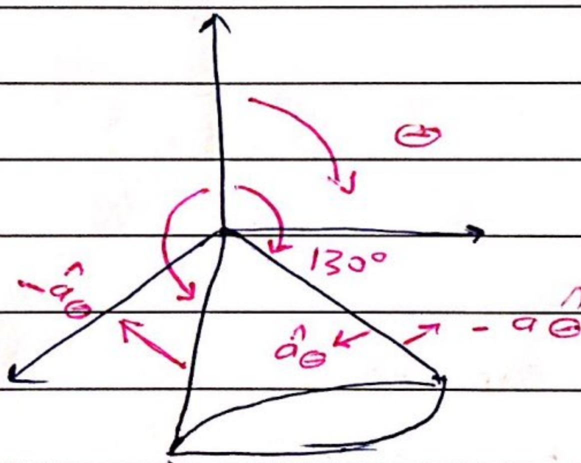
between 0 and 90

$\theta = 130°$

inf hollow cone

$90^+ \longrightarrow 180^-$



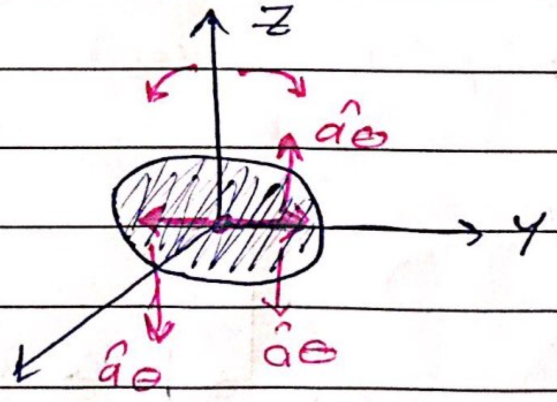


Special Cases :-

$\theta = 90°$

inf disk along

x-y plane



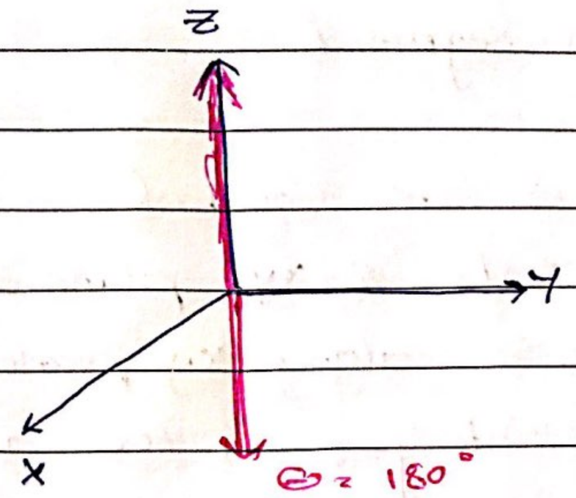
$\bar{ds} = r \sin\theta \, dr \, d\phi \, \hat{a}_\theta$

$\theta = 0 \rightarrow$ +ve z-axis

$\theta = 180$

↳ -ve z-axis

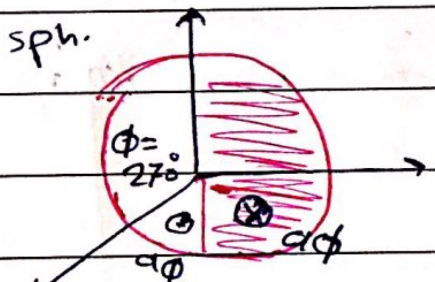
$r \sin\theta \, d\phi \leftarrow \phi$

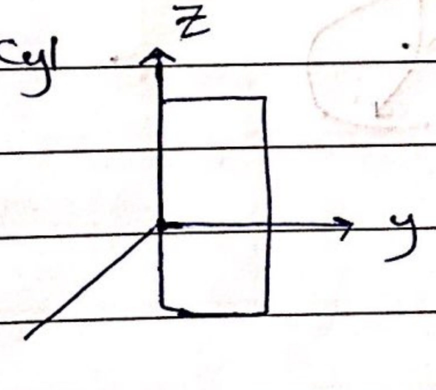


ϕ = constant $\quad \cancel{r}, \cancel{\theta}$

semi-inf Disk

$\phi = 90°$





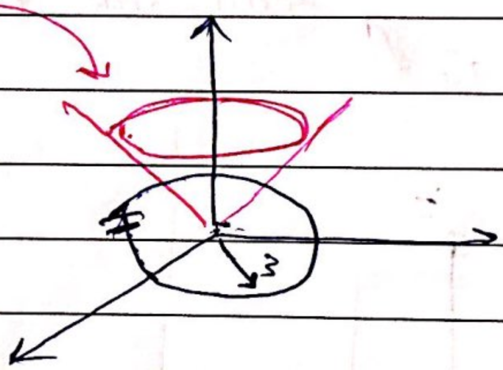
ID segment

r, θ are constant

- circle // xy plane (r ≠ 0, θ ≠ 90°)
- s along xy plane (r ≠ 0 θ = 90°)
- point
  - on the origin r = 0,
  - on the +ve z r ≠ 0,
  - on the -ve z, r ≠ 0,

r = 3, θ = 90°

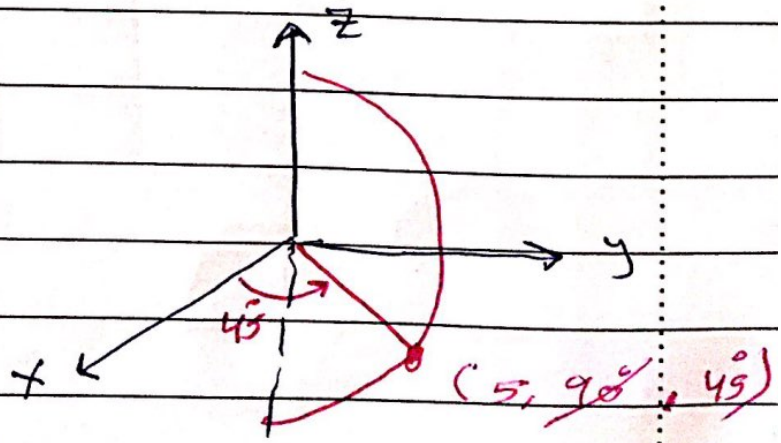
r = 3, θ = 60°



$\bar{dl} = r \sin d\phi \, \hat{a}_\phi$

r, φ are constant, φ

- half-circle
  - r = 5
  - φ = 45°
- point (r = 0)

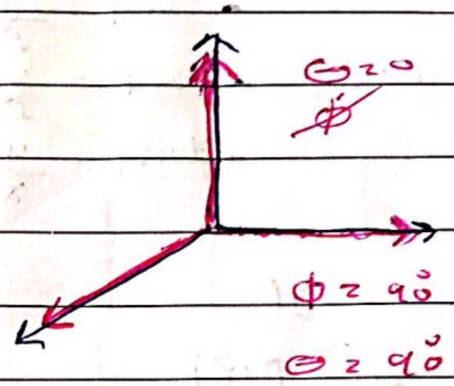


θ, ϕ are constants

Semi-inf. line ( )

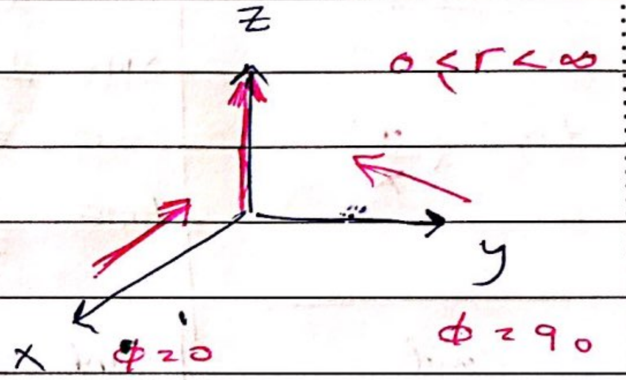
$\theta = 90^\circ$, $\phi = 90^\circ$

↳ +ve y.

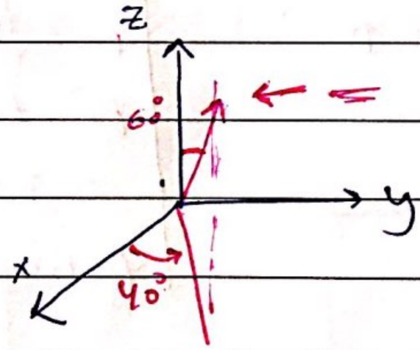


$\theta = 0^\circ$, $\phi = 90^\circ$

$\theta = 0^\circ$, $\phi = 0^\circ$



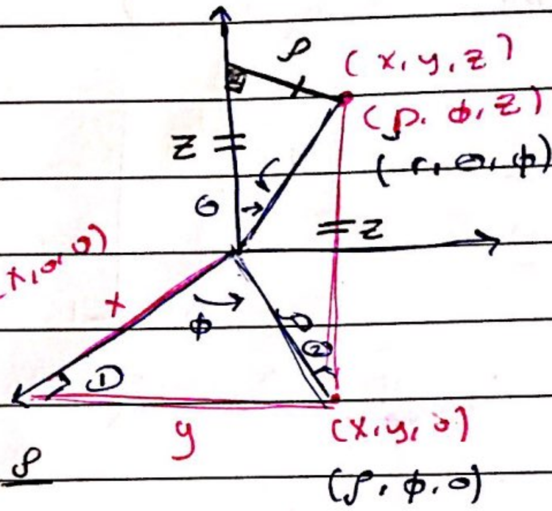
$\theta = 60$, $\phi = 40^\circ$



Transformation between coordinates

→ points:

- $\sin\phi = \frac{y}{\rho}$
- $\cos\phi = \frac{x}{\rho}$
- $\tan\phi = \frac{y}{x}$



Cart → cyl

$(x, y, z) \rightarrow (\rho, \phi, z)$

$\rho = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$z = z$

- $\sin\theta = \frac{\rho}{}$
- $\cos\theta = z/r$
- $\tan\theta = \frac{\rho}{z}$

cyl → cart

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$z = z$

Cart → sph

$(x, y, z) \rightarrow (r, \theta, \phi)$

$r = \sqrt{x^2 + y^2 + z^2}$

$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

cyl → sph

$(\rho, \phi, z) \rightarrow (r, \theta, \phi)$

$r = \sqrt{\rho^2 + z^2}$

$\theta = \tan^{-1}\frac{\rho}{z}$

$\phi = \phi$

sph $\rightarrow$ Cart

$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$

---

sph $\rightarrow$ cyl

$\rho = r \sin\theta$

$\phi = \phi$

$z = r \cos\theta$

$\sin\theta = \frac{\rho}{r}$

$\cos\theta = \frac{z}{r}$

$$\boxed{\begin{array}{l} y = \rho \sin\phi \\ x = \rho \cos\phi \end{array}}$$

Vectors :-

Cart $\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

cyl $\bar{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$

Conversion matrix

Cart $\rightarrow$ cyl

$(A_x, A_y, A_z) \rightarrow (\overset{?}{A_\rho}, \overset{?}{A_\phi}, \overset{?}{A_z})$

$\rightarrow$

Conversion matrix

Cart → cyl.

$(A_x, A_y, A_z) \rightarrow (\overset{?}{A_\rho}, \overset{?}{A_\phi}, \overset{?}{A_z})$

given.

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

3 X 1 $\qquad$ 3 x 3 $\qquad$ 3 x 1

Step 1.

$A_\rho = \cos\phi \, A_x + \sin\phi \, A_y$

$A_\phi = -\sin\phi \, A_x + \cos\phi \, A_y$

$A_z = A_z$

$x = \rho \cos\phi$

$y = \rho \sin\phi$

cyl → Cart

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Cart $\qquad$ cyl

$$\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\phi \\ \hat{a}_z \end{bmatrix}$$

$$\boxed{\hat{a}_y = \sin\phi \, \hat{a}_\rho + \cos\phi \, \hat{a}_\phi}$$

once you find A_ρ, A_ϕ, A_z

$$\bar{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

cart $\rightleftarrows$ sph

$(A_x, A_y, A_z) \longleftrightarrow (A_r, A_\theta, A_\phi)$

$$\bar{A}_{sph} = \underset{?}{A_r \hat{a}_r} + \underset{?}{A_\theta \hat{a}_\theta} + \underset{?}{A_\phi \hat{a}_\phi}$$

given

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

cyl $\longleftrightarrow$ sph $\quad (A_\rho, A_\phi, A_z) \longleftrightarrow (A_r, A_\theta, A_\phi)$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ \\ A_z \end{bmatrix}$$

NOTEBOOK

cyl $\rightarrow$ sph

$A_\phi = A_\phi$

Scanned with CamScanner

Ch3:- Vector Calculus:-

Integrals :-

* line Integral

$$\int_l \bar{A} \cdot \bar{dl}$$

i.e $\hat{A} = A_x \hat{a}_x + A_z \hat{a}_z$

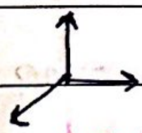
$$\bar{dl} = dx\,\hat{a}_x + \cancel{dy\,\hat{a}_y} + dz\,\hat{a}_z$$

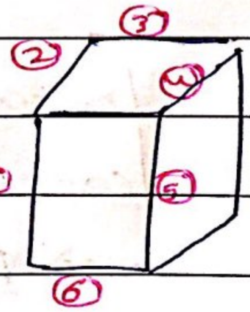
$$\int_l (A_x\,dx + A_z\,dz$$

$$\int_x (A_x\,dx + \int_z A_z\,dz$$

Closed line Integral

$$\oint_l \bar{A} \cdot \bar{dl}$$





$A_z \hat{a}_z$

$$\oint_l \bar{A} \cdot \bar{dl} = \int_{l_1} \bar{A} \cdot \bar{dl}_1 + \int \bar{A} \cdot \bar{dl}_2$$

$$+ \int_{l_3} \bar{A}\,\bar{dl}_3 + \int_{l_4} \bar{A} \cdot \bar{dl}_4$$

$$+ \int_{l_5} \bar{A} \cdot \bar{dl}_5 + \int_{l_6} \bar{A} \cdot \bar{dl}_6$$

$\overline{dl_1} = dz\,\hat{a}_z$

$\overline{dl_2} = dx\,\hat{a}_x$ (-ve)

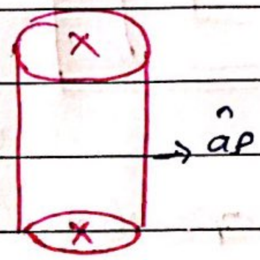
$\overline{dl_3} = dy\,\hat{a}_y$

$\overline{dl_4} = dx\,\hat{a}_x$,

$\overline{dl_5} = dz\,\hat{a}_z$

Surface Integral :-

$$\boxed{\int_S} \bar{A} \cdot \boxed{\overline{ds}}$$



hollow cylinder

$\overline{ds} = \rho\, d\phi\, dz\; \hat{a}_\rho$

$\bar{A}$ in Cart-

↳ must be converted to cyl

↳ $\bar{A} = A_\rho \hat{a}_\rho$ only

$$\iint_{\phi\, z} A_\rho \hat{a}_\rho \cdot \rho\, d\phi\, dz\, \hat{a}_\rho$$

closed surface Integral

$$\oint_S \bar{A} \cdot \overline{ds}$$

$$S = \int_{S_1} \bar{A} \cdot \overline{ds_1} + \int_{S_2} \bar{A} \cdot \overline{ds_2} + \int_{S_3} \bar{A} \cdot \overline{ds_3}$$

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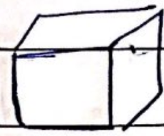
Volume Integral

$\int_v |\bar{A}| \, dv$ ← Scaler

*



$\int_z \int_y \int_x A \, dx \, dy \, dz$

Del operetor ∇
(vector)

in Cart $\quad \nabla \frac{d}{dx}\hat{a}_x + \frac{d}{dy}\hat{a}_y + \frac{d}{dz}\hat{a}_z$

P-E → $dx, \; dy, \; dz$ $\qquad d$

∂

in cyl

$$\nabla = \frac{\partial}{\partial \rho}\hat{a}_\rho + \frac{\partial}{\rho \partial \phi}\hat{a}_\phi + \frac{\partial}{dz}\hat{a}_z$$

in sph

$$\nabla = \frac{\partial}{\partial r}\hat{a}_r + \frac{\partial}{r\partial \theta}\hat{a}_\theta + \frac{\partial}{r\sin\theta \, d\phi}\hat{a}_\phi$$

Usage of ∇ :-

vector ← scalar

1. Gradient ∇V → ضرب بين vec و scalar
2. Divergence $\boxed{\nabla \cdot \bar{A}}$
3. Curl $\boxed{\nabla \times \bar{A}}$
4. Laplacian $\nabla \cdot (\nabla V) = \boxed{\nabla^2 V}$

↳ second partial

in Cart.

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

in cyl. ------ $\nabla V =$

in sph ------ $\nabla =$

Distance :-

Q ① (x_1, y_1, z_1) (ρ_1, ϕ_1, z_1)

لما يحول لـ Cart

r_{QP}

P (x_2, y_2, z_2) (ρ_2, ϕ_2, z_2)

$d = |r_{QP}|$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

in cyl

$$d^2 = \rho_2^2 + \rho_1^2 - 2\rho_1\rho_2 \cancel{\cos(\phi_2 - \phi_1)}$$

$$+ (z_2 - z_1)^2$$

if $\phi_2 = \phi_1$

② $\boxed{d^2 = (\rho_2 - \rho_1)^2 + (z_2 - z_1)}$

⇒ in sph :

$d^2 = (r_2 - r_1)^2$ if $\theta_1 = \theta_2$ and $\phi_1 = \phi_2$

NOTEBOOK

Ch4: Electrostatic Fields :

Sources :-

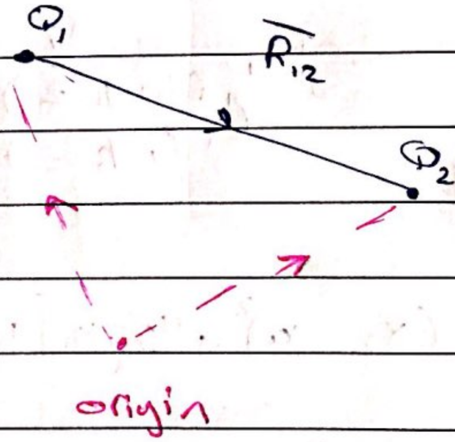
- point Charge (Q)
- line Charge dist (ρ_L)
- surface Charge dist (ρ_s)
- Electric Dipole
- Volume Charge dist (ρ_v)
- polarized Dielectric

Major laws:-

① Coulomb's Law

② Gauss's law

Coulomb's law!



$\overline{F}$ = Force (F_e)

$F \propto \frac{Q_1 Q_2}{R_{12}^2}$ relation

$F = \frac{K Q_1 Q_2}{R_{12}^2}$

K = Constant = $\frac{1}{4\pi \varepsilon_0}$ → Channel

unit itself

ε_0 : Free space permittivity

$= \frac{10^{-9}}{36\pi}$ F/m

$= 8.858 * 10^{-1}$ F/m

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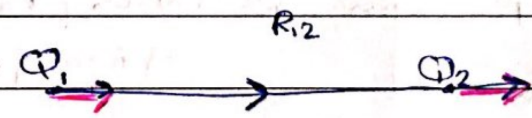
$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2} \ (N)$$ → magnitude only

To Find F as a vector

$\overline{F}_{12}$ ≡ The Force on Q_2 due to Q_1

$\overline{F}_{12} = -\overline{F}_{21}$, $|\overline{F}_{12}| = |\overline{F}_{21}|$

$$\boxed{\overline{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2} \hat{a}_{R_{12}} \ (N)} \quad (1)$$



$$\frac{\overline{R}_{12}}{R_{12}} = \hat{a}_{R_{12}}$$

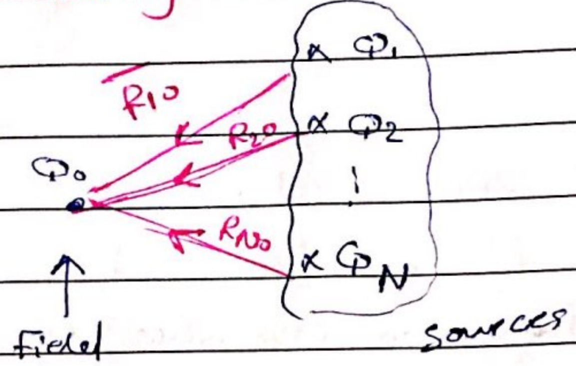
$$\boxed{F_{12} = \frac{Q_1 Q_2 \overline{R}_{12}}{4\pi\varepsilon_0 R_{12}^3} \ (N)}$$

$\overline{R}_{12} = \overline{r}_2 - \overline{r}_1$

$$\boxed{\overline{F}_{12} = \frac{Q_1 Q_2 (\overline{r}_2 - \overline{r}_1)}{4\pi\varepsilon_0 |\overline{r}_2 - \overline{r}_1|^3} \ N} \quad (3)$$

For N-charges affecting on a certain charge

Total Force on Q_0



$$\bar{F}_0 = \bar{F}_{10} + \bar{F}_{20} + \cdots + \bar{F}_{N0}$$

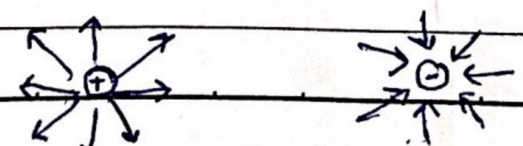
$$= \frac{Q_1 Q_0 (\bar{r}_0 - \bar{r}_1)}{4\pi\epsilon_0 |\bar{r}_0 - \bar{r}_1|^3} + \frac{Q_2 Q_0 (\bar{r}_0 - \bar{r}_2)}{4\pi\epsilon_0 |\bar{r}_0 - \bar{r}_2|^3}$$

$$+ \cdots + \frac{Q_N Q_0 (\bar{r}_0 - \bar{r}_N)}{4\pi\epsilon_0 |r_0 - \bar{r}_N|^3} \quad (N)$$

$$\boxed{\bar{F} = \frac{Q}{4\pi\epsilon_0} \sum_{k=0}^{N} \frac{Q_k (\bar{r}_0 - \bar{r}_k)}{|\bar{r}_0 - \bar{r}_k|^3} \quad (N)}$$

Electric Field Intensity $(\bar{E})$

$$(\bar{E}) = \frac{\text{Force}}{\text{charge (Field)}} = \frac{N}{C} \text{ or } \left(\frac{V}{m}\right)$$



$r \to 0,\ E \to \infty$ $\quad$ $r \to \infty$, $E \to 0$

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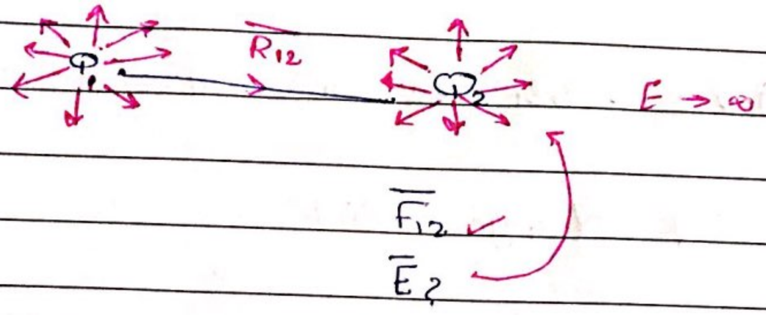

source

① $\overline{E} = \dfrac{Q}{4\pi\epsilon_0 R^2}\hat{a}_r$

$\rightarrow$

③ $E = \dfrac{Q(\overline{r}-\overline{r}'|^3}{4\pi\epsilon_0|\overline{r}-\overline{r}'|^3}$ V/m

Convention:-

use dashes to represent the source,
without dashes to " the field point

For N-Charges

$$E = \frac{1}{4\pi\epsilon_0}\sum_{k=1}^{N}\frac{Q_k(r-\overline{r}'_k)}{|\overline{r}-\overline{r}'_k|^3} \quad \text{V/m}$$

Ex: point charges 1mC and -2mC located at S_1 (3,2,-1) and S_2 (-1,-1,4) Find the force and electric field at a 10nC charge located at (0,3,1) Field

$$\overline{F} = \frac{(9)\,1\times10^{-3} * 10\times10^{-9} * (-3,1,2)}{4\pi * \frac{10^{-9}}{36\pi} * (14)^{3/2}} + \frac{(9)\,-2\times10^{-3} * 10\times10^{-9} * (1,4,-3)}{4\pi * \frac{10^{-9}}{36\pi} * (26)^{3/2}}$$

$F = -6.507\,\hat{a}_x - 3.817\,\hat{a}_y + 7.506\,\hat{a}_z$ mN

$E = F/q = \overline{F}/10\times10^{-9} = \overline{F}*10^{8}$

$E = -650.7\,\hat{a}_x - 381.7\,\hat{a}_y + 750.6\,\hat{a}_z$ kV/m

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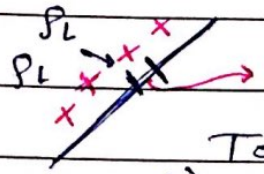
Electric Fields due to Continuous charge distribution

Cont charge dist :-

① line charge dist (1D segment)

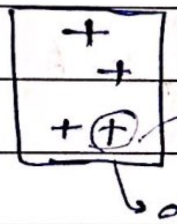


→ ρ_L = line charge density (C/m)

Total charge

$Q = \int_l \rho_L \, dl$ → scalar

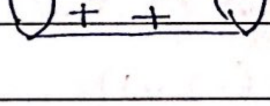
② Surface charge dist (2D surfaces)



ρ_s = Surface charge density C/m²

$Q = \int_S \rho_s \, ds$ → scalar

③ volume charge dist (3D object)



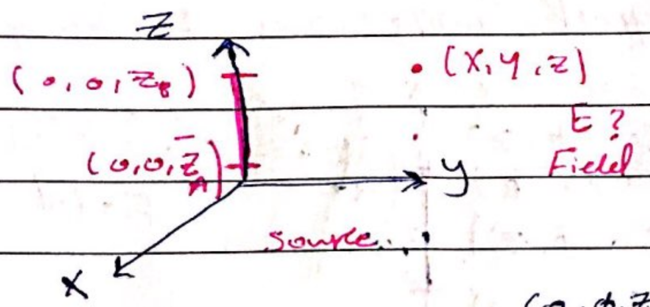
→ dv

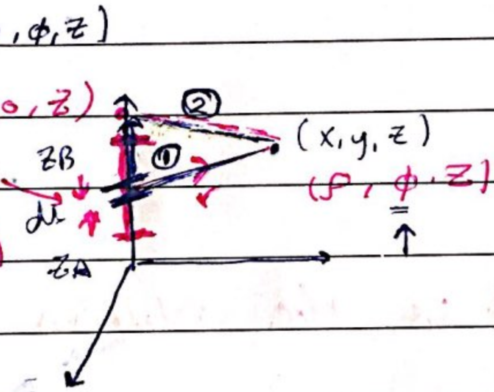
→ ρ_v = Volume charge density C/m³

$Q = \int_v \rho_v \, dv$

Ex. + derivation.

Consider a finite line along z-axis carry charge ρ_L (C/m). Find the $\bar{E}$ at point (x, y, z)





$$E = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r$$

$(0,0,z')$

$(0,\phi,z')$

point charge

$$Q = \int_L \rho_L \, dl$$

$$\boxed{E = \int_L \frac{\rho_L \, dl}{4\pi\epsilon_0 r^2}\,\hat{a}_r}$$

$\boxed{dl = dz}$ $\cancel{\hat{a}_z}$

$$\hat{a}_r = \frac{\bar{r}}{r}$$

$$\bar{r} = x\hat{a}_x + y\hat{a}_y + (z - z')\hat{a}_z$$

$$r = \sqrt{x^2 + y^2 + (z - z')^2}$$

$$E = \frac{\rho_L}{4\pi\epsilon_0} \int_{z_A}^{z_B} \frac{x\hat{a}_x + y\hat{a}_y + (z - z')\hat{a}_z \, dz}{\left[x^2 + y^2 + (z - z')^2\right]^{3/2}}$$

by converting to cylindrical coord.

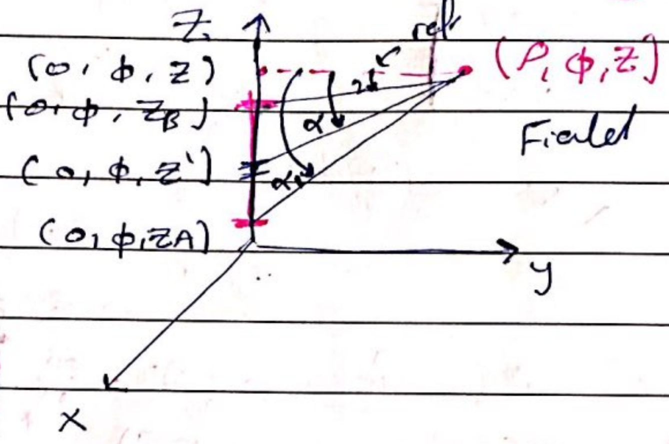
$$\bar{r} = \rho\hat{a}_\rho + (z - z')\hat{a}_z$$

$$r = \sqrt{\rho^2 + (z - z')^2} \quad , \quad dl = dz$$

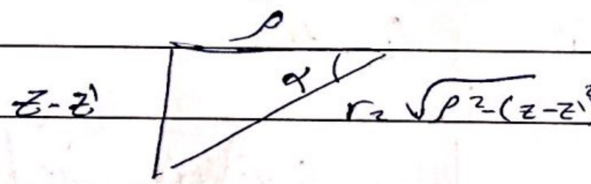
Date No.

$$\bar{E} = \frac{\rho}{4\pi\epsilon_0}\int_{Z_A}^{Z_B} \frac{\rho\,\hat{a}_\rho + (z-z')\,\hat{a}_z \; dz'}{\left[\rho^2 + (z-z')^2\right]^{3/2}}$$

Convert the integral limits :- by introducing $\alpha,\ \alpha_1,\ \alpha_2$



What is the relation between z and α?



$\sin\alpha = \frac{z-z'}{r}$

$\cos\alpha = \frac{\rho}{r} \rightarrow \rho = r\sin\alpha$

$\tan\alpha = \frac{z-z'}{\rho} \rightarrow z-z' = r\sin\alpha$

$= \rho\tan\alpha$

$z-z' = \rho\tan\alpha$

$-dz' = \rho\sec^2\alpha\, d\alpha$

$\boxed{dz' = -\rho\sec^2\alpha\, d\alpha}$

$r^2 = \rho^2 + (z-z')^2$

$= \rho^2 + \rho^2\tan^2\alpha$

$= \rho^2(1+\tan^2\alpha)$

$r^2 = \rho^2\sec^2\alpha$

$\boxed{r^3 = \rho^3\sec^3\alpha}$

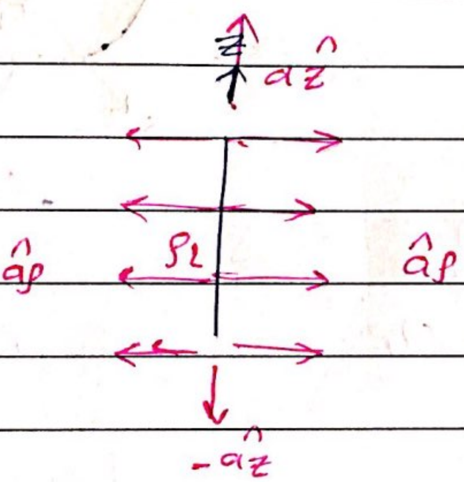
$= \left[(\rho^2 + (z-z')^2\right]^{3/2}$

$\rho = r\cos\alpha$, $z-z' = r\sin\alpha$

$r = \rho\sec\alpha$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2} \frac{\cancel{\rho}\,\cancel{\sec}\alpha\,(\cos\alpha\,\hat{a}_\rho + \sin\alpha\,\hat{a}_z) * -\cancel{\rho}\,\cancel{\sec^2\alpha}\,d\alpha}{\rho^2 \sec^2\alpha}$$

$$\boxed{E = \frac{-\rho_L}{4\pi\varepsilon\rho}\left[(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\rho - (\cos\alpha_2 - \cos\alpha_1)\hat{a}_z\right] \text{ V/m}}$$



For inf. line

① $\alpha_2 = 270 = -90$

② $\alpha_1 \simeq 90$

For an inf line

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho} \Rightarrow \bar{E} = \frac{\rho_L\,\bar{\rho}}{2\pi\varepsilon_0\rho^2} \text{ V/m}$$

always for any inf. line

$(0,\phi,z)$ ρ (ρ,ϕ,z)

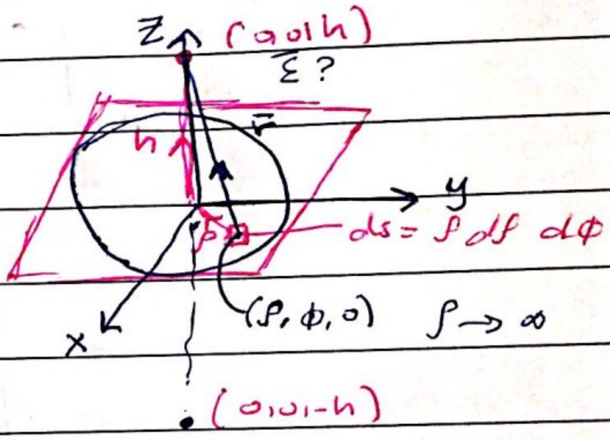
$\rho =$ is the shortest distance between the source or the extension of the source

1. E-Field for surface charge distribution:

Consider $\boxed{z=0}$ plane carry a charge dist of ρ_s C/m^2.

inf

Find the $\bar{E}$ at (0,0,h) and (0,0,-h)



$$\bar{E} = \frac{\alpha}{4\pi\epsilon_0 r^2} \hat{a}_r$$

z=0

$$Q = \int_s \rho_s \, ds$$

$$\rightarrow \bar{E} = \int_s \frac{\rho_s \, ds}{4\pi\epsilon_0 r^2} \hat{a}_r$$

$\bar{r} = -\rho \hat{a}_\rho + h\hat{a}_z$

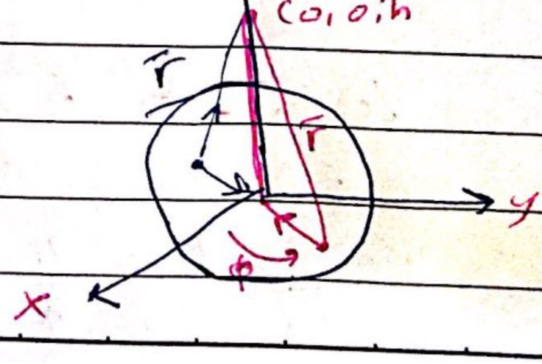
$$\bar{E} = \int_s \frac{\rho_s \, ds \, \bar{r}}{4\pi\epsilon_0 r^3} , \quad \bar{r} = -\rho \hat{a}_\rho + h\hat{a}_z$$

$r = \sqrt{\rho^2 + h^2}$.

$ds = \rho \, d\rho \, d\phi$

$$\bar{E} = \frac{\rho_s}{4\pi\epsilon_0} \int_0^{2\pi}\int_0^{\infty} \frac{-\rho \cancel{\hat{a}_\rho} + h\hat{a}_z}{[\rho^2 + h^2]^{3/2}} \rho \, d\rho \, d\phi$$

The ρ-component will be cancelled due to symmetry.



Subject

$$\bar{E} = \frac{\rho_s h (2\pi)}{4\pi \epsilon_0} \int_0^{\infty} \frac{1}{[\rho^2 + h^2]^{3/2}} \rho\, d\rho\ \hat{a}_z$$

let $u = \rho^2 + h^2 \longrightarrow \frac{du}{2} = \cancel{2} \rho\, d\rho$

$$\bar{E} = \frac{\rho_s h}{2\epsilon_0} \int \frac{du}{2 u^{3/2}}$$

$$\bar{E} = \frac{\rho_s h}{2\epsilon_0} \frac{\cancel{1}}{\cancel{2}} \frac{u^{-1/2}}{-\cancel{1/2}} \Big| \hat{a}_z$$

$$\bar{E} = \frac{-\rho_s h}{2\epsilon_0} \frac{1}{\sqrt{\rho^2 + h^2}} \Big|_0^{\infty} \hat{a}_z$$

$$\bar{E} = \frac{\cancel{*}\rho \cancel{h}}{2\epsilon_0}\left(0 + \frac{1}{h}\right) \hat{a}_z \longrightarrow \boxed{\bar{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_z \ \ v/m}$$

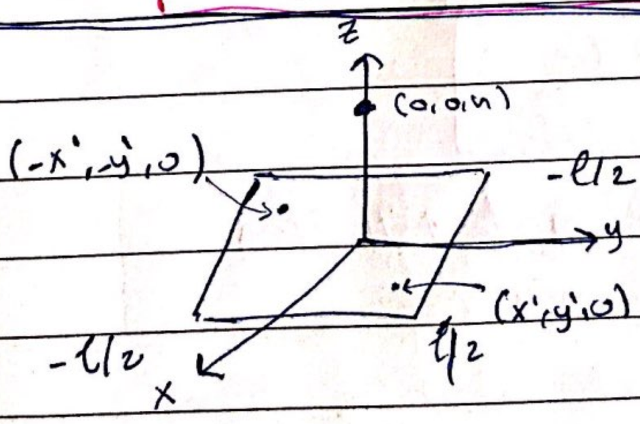
$\hookrightarrow (0,0,h)$

for $(0,0,-h)$

$\hat{a}_r = -\hat{a}_z$

For any inf sheet :-

$$\boxed{E = \frac{\rho_s}{2\epsilon_0} \hat{a}_r \ \ V/m} \longrightarrow$$ always for any inf sheet



Cart-

Symmetry $(x,y) \rightarrow$ both

$$\bar{E} = \int_s \frac{\rho_s\, ds\, \bar{r}}{4\pi\epsilon_0 r^3}$$

$ds = dx\,dy$

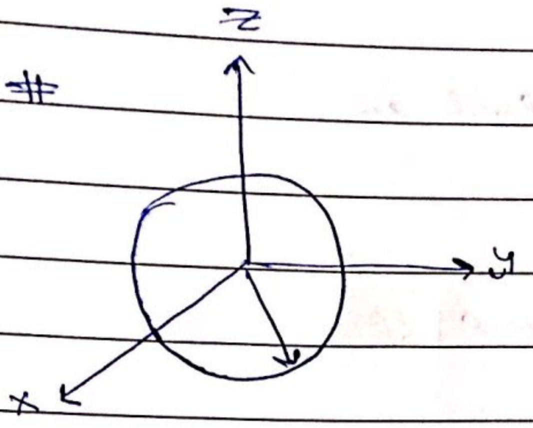
$\bar{r} = -\bar{x}\hat{a}_x - \bar{y}\hat{a}_y + h\hat{a}_z$

N O T E B O O K

$r = \sqrt{x^2 + y^2 + h^2}$

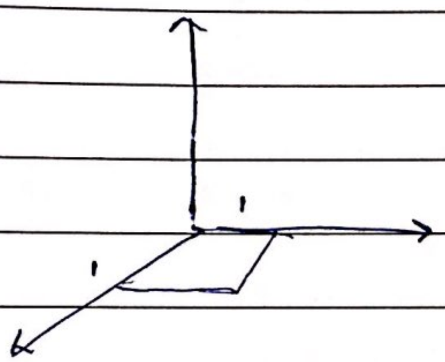
Scanned with CamScanner

#



cyl. & sym

ρx

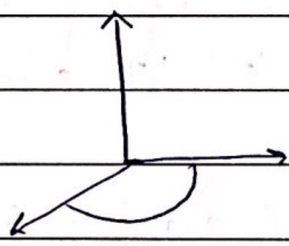


Cart, No sym

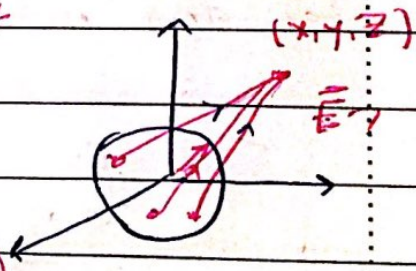
$\cancel{\bar{F} = -\rho \hat{a}_{\rho} + h\hat{a}_z}$

$\cancel{r = h}$

cyl., No sym



E-field due to a Volume charge dist



$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r$$

$$Q = \int \rho_v \, dv \quad , \quad \boxed{\bar{E} = \int_v \frac{\rho_v \, dv}{4\pi\epsilon_0 r^2}\,\hat{a}_r}$$

Volume charge → delayed till you study the Gauss's Law.

Electric Flux Density :-

$$\boxed{\bar{D} = \epsilon_0 \bar{E}} \quad C/m^2$$

$$\frac{F}{m} \cdot \frac{V}{m} = \frac{F \cdot V}{m^2} = \frac{C}{m^2}$$

For a point charge

$$\bar{D} = \frac{Q}{4\pi r^2} \hat{a}_r \quad \frac{C}{m^2}$$

For an inf line $\rightarrow$ $\bar{D} = \frac{\rho_L}{2\pi\rho} \hat{a}_\rho$

For an inf sheet $\rightarrow$ $\bar{D} = \frac{\rho_s}{2} \hat{a}_n \rightarrow C/m^2$

Electric Flux $\rightarrow$ scalar

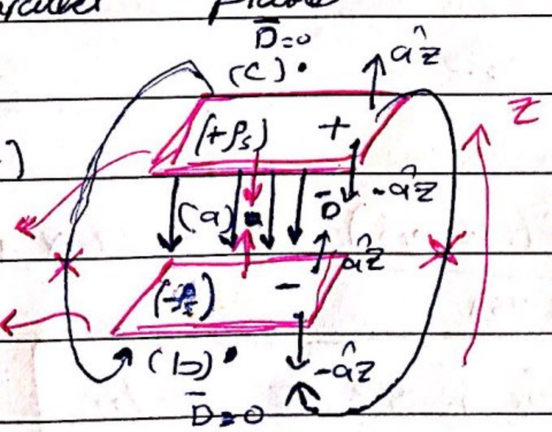
Ψ (later Ψ_e) $\rightarrow$ "C"

$\Psi = Q_{enc.}$

$$\boxed{\Psi = \int_S \bar{D} \cdot \bar{ds}}$$

Ex For parallel plate capacitor

$\bar{D}$ at (a) (b) (c)



$\bar{D} = \frac{\rho_s}{2} \hat{a}_n$

$\bar{D} = \frac{-\rho_s}{2} \hat{a}_n$

$\bar{D}_{1\,(a)} = \bar{D}_+ + \bar{D}_-$

$= \frac{\rho_s}{2}(-\hat{a}_z) + \frac{-\rho_s}{2}\hat{a}_z = \rho_s(-\hat{a}_z) \quad C/m^2$

NOTEBOOK

$\bar{D} = \rho \, \hat{a}_r$

$\bar{D} \cdot \hat{a}_r = \rho_s \, \cancel{\hat{a}_r} \cdot \cancel{\hat{a}_n}$

$$\boxed{\underbrace{\bar{D}}_{\text{field}} \cdot \hat{a}_n = \underbrace{\rho_s}_{\text{source}}}$$

Ex:- Find $\bar{D}$ at $\underbrace{(4, 0, 3)}_{\text{field}}$ if there is a point charge -5π mC at $(4, 0, 0)$ and a line charge 3π mC/m along y-axis

$\bar{D} = \bar{D}_Q + \bar{D}_\ell$

$\bar{D}_Q = \frac{Q}{4\pi r^2} \hat{a}_r = \frac{Q\bar{r}}{4\pi r^3} = \frac{-5\pi * 10^{-3} (3 \hat{a}_z)}{4\pi \cdot 27} = -0.138 \hat{a}_z$ C/m²

$\bar{D}_\ell = \frac{\rho_L}{2\pi\rho} \hat{a}_\rho = \frac{\rho_L \bar{\rho}}{2\pi\rho^2}$

$\bar{\rho} = (4, 0, 3) - (0, \cancel{y}, 0)$ shortest distance when y=0

$\bar{\rho} = 4\hat{a}_x + 3\hat{a}_z$

$\rho = 5$

$\bar{D}_\ell = \frac{3\pi * 10^{-3} (4, 0, 3)}{2\pi (25)} = 0.24 \hat{a}_x + 0.18 \hat{a}_z$ mC/m²

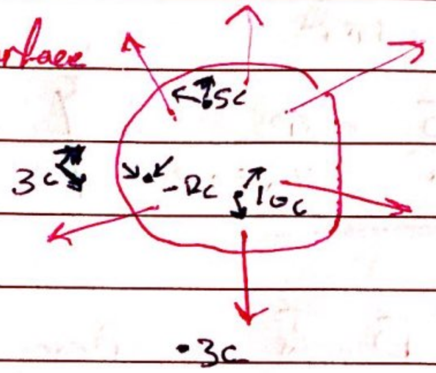
$$\boxed{\bar{D}_{total} = 240 \hat{a}_x + 42 \hat{a}_z \ \mu C/m^2}$$

$\bar{E} = \frac{\bar{D}}{\epsilon_0}$

Gauss's law :-

$$\boxed{\oint_S \bar{D}\cdot\bar{ds} = Q_{enc}} \quad , \quad \bar{D} = \epsilon_0 \bar{E}$$

Gaussian surface



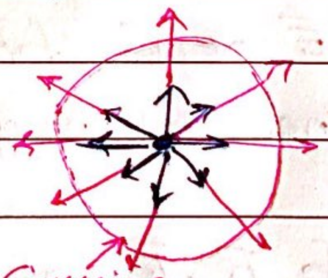
→ 1'st maxwell's eq in integral form

$Q_{enc} = Q \; , \; \int_L \rho_L \, dl \; , \; \int_S \rho_s \, ds \; , \; \int_v \rho_v \, dv$

inf ✓ inf ✓

Applications of Gauss's law

[1] $\bar{E}$ or $\bar{D}$ for a point charge



$$\oint_S \bar{D}\cdot\bar{ds} = Q_{enc} = Q$$

?

$\bar{D} = D_r \hat{a}_r + D_\theta \hat{a}_\theta + D_\phi \hat{a}_\phi$

Gaussian surface (hollow sphere)

$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$

$r =$ Constant

$$\int_0^{2\pi}\int_0^{\pi} D_r \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r = Q$$

$$D r^2 \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\theta = Q$$

$-\cos\theta \Big|_0^{\pi}$

$\pi - (-1-1)$

$\int_0^{\pi} \sin\theta \, d\theta = 2$

$$\boxed{4\pi r^2 D_r = Q}$$

\# $D_r = \dfrac{Q}{4\pi r^2}$

✓ $\bar{D} = \dfrac{Q}{4\pi r^2}\,\hat{a}_r$

✓ $\bar{E} = \dfrac{\bar{D}}{\epsilon_0} = \dfrac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r$ like Colomb's law

$\oint_S \bar{D}\cdot\bar{ds} = Q_{enc} = Q$

$\dfrac{Q}{4\pi r^2}\,4\pi r^2 = Q$ \#

(c) $\leftarrow \Psi = \int_S \bar{D}\cdot\bar{ds}$

$\Psi = \oint \bar{D}\cdot\bar{ds} = Q_{enc}$

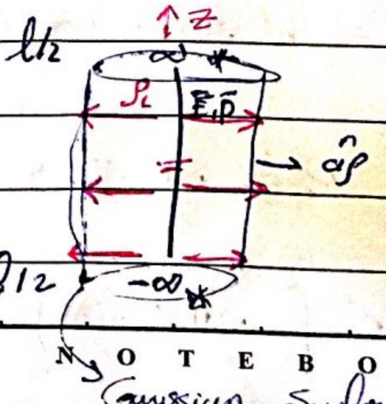
2) E or $\bar{D}$ for an infinite line of change

$$\boxed{E = \frac{\rho_L}{2\pi\epsilon_0\,\rho}\,\hat{a}_\rho}$$

$\oint_S \bar{D}\cdot\bar{ds} = Q_{enc}$

$= \int_\ell \rho_L\,dl$

$\bar{D} = D_\rho\,\hat{a}_\rho$



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Gaussian Surface

✓ $\overline{ds} = \rho \, d\phi \, dz \, \hat{a}_\rho$

$dl = dz$

$$\int_{-l/2}^{l/2} \int_{0}^{2\pi} D_\rho \, \hat{a}_\rho \cdot \rho \, d\phi \, dz \, \hat{a}_\rho$$

$$= \int_{-l/2}^{l/2} \rho_L \, dz$$

$D_\rho \, \rho (2\pi) l = \rho_L \, l$

$\boxed{2\pi \rho \cancel{l}} D_\rho = \rho_L \cancel{l}$

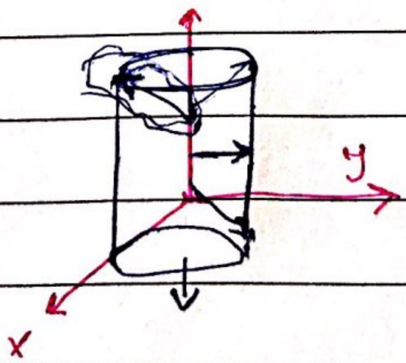
Area $\rightarrow D_\rho = \dfrac{\rho_L}{2\pi \rho}$

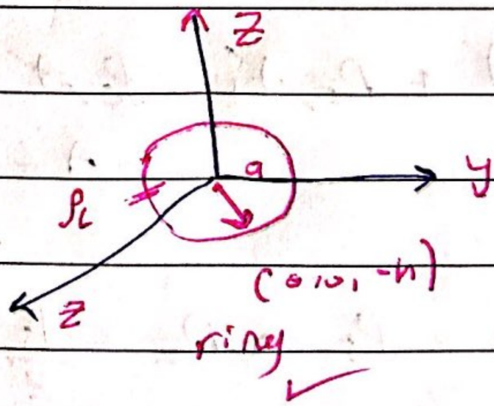
$$\overline{D} = \frac{\rho_L}{2\pi \rho} \hat{a}_\rho$$

$$\overline{E} = \frac{\overline{D}}{\epsilon_0} = \boxed{\frac{\rho_L}{2\pi \epsilon_0 \rho} \hat{a}_\rho}$$

---

if the line in finite or with curvature then use Coulomb's Law to find $\overline{E}$ or $\overline{D}$ ---



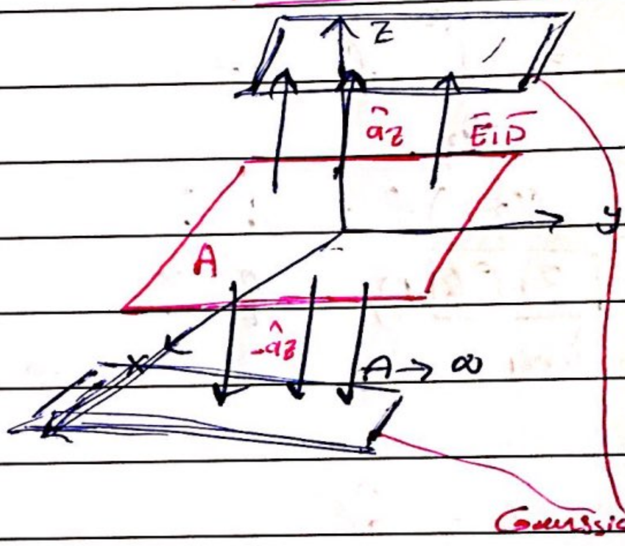


Gauss's law (2)

3) $\bar{E}$ or $\bar{D}$ for an infinite sheet of charge:

$$\bar{D} = \begin{cases} D_z \hat{a}_z, & z > 0 \\ D_z(-\hat{a}_z), & z < 0 \end{cases}$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0} \hat{a}_r}$$



$\bar{ds}_{top} = dx\,dy\,\hat{a}_z$

$\bar{ds}_{bot} = -dx\,dy\,\hat{a}_z$

$ds = dx\,dy$

$$\int_{S_{top}} \bar{D} \cdot \bar{ds}_{top} + \int_{S_{bot}} \bar{D}\, \bar{ds}_{bot} = \int_S \rho_s\, ds$$

$$\iint_{-\infty}^{\infty} D_z\, \hat{a}_z \cdot dx\,dy\,\hat{a}_z + \iint_{-\infty}^{\infty} D_z\, \cancel{(-\hat{a}_z)}\, dx\,dy\, \cancel{(-\hat{a}_z)}$$

$$= \iint_{-\infty}^{\infty} \rho_s\, dx\,dy$$

$$D_z \cancel{A} + D_z \cancel{A} = \rho_s \cancel{A}$$

where $\quad A = \iint_{-\infty}^{\infty} dx\,dy$

$$2D_z = \rho_s \rightarrow D_z = \frac{\rho_s}{2}$$

NOTEBOOK

$$2D_z = \rho_s \longrightarrow D_z = \frac{\rho_s}{2}$$

$$\bar{D} = \begin{cases} \frac{\rho_s}{2}\,\hat{a}_z , & z > 0 \\ \frac{\rho_s}{2}(-\hat{a}_z) , & z < 0 \end{cases}$$

$$\boxed{\bar{D} = \frac{\rho_s}{2}\,\hat{a}_n}$$

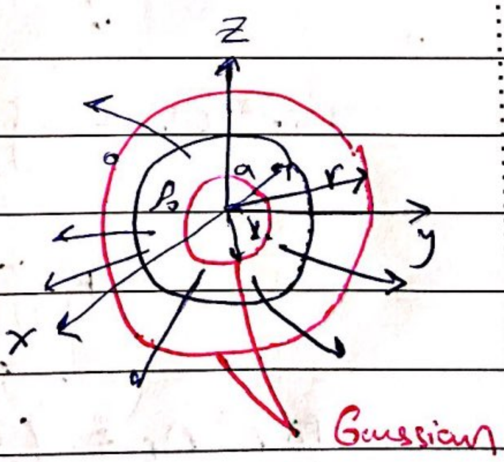
$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon_0}\,\hat{a}_n}$$

4) $\bar{E}$ or $\bar{D}$ for a uniform volume charge distribution.

i.e consider a sphere of radius (a) carry a charge

$$\rho_v = \begin{cases} \rho_0 , & r < a \\ 0 , & r \geq a \end{cases}$$

Find $\bar{E}$ or $\bar{D}$ everywhere



for $(0 \leq r < a)$, $r = a$

$$\oint_S \bar{D}\cdot\bar{ds} = Q_{enc} = \int_V \rho_v \, dv$$

$\bar{D} = D_r \cdot \hat{a}_r$

$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$

$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$

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$$\oint_S \bar{D} \cdot \bar{ds} = \int_v \rho_v \, dv$$

$$\int_0^{2\pi}\int_0^{\pi} D_r \hat{a_r} \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r} = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho_v r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\cancel{4\pi} \cancel{r^2} D_r = \rho_0 \left(\frac{\cancel{4\pi} \cancel{r^3}}{3} \right)$$

$$D_r = \frac{\rho_0 r}{3}$$

$$\boxed{\begin{aligned} \bar{D} &= \frac{\rho_0 r}{3} \hat{a_r} \\ \bar{E} &= \frac{\rho_0 r}{3\epsilon_0} \hat{a_r} \end{aligned}} \rightarrow 0 \leq r \leq a$$

at $r = a \rightarrow \bar{D} = \dfrac{\rho_0 a}{3} \hat{a_r}$

$$\bar{E} = \frac{\rho_0}{3\epsilon_0} \hat{a_r}$$

For $a < r < \infty$

$$\rho_v = \begin{cases} \rho_0 \; ; & r \leq a \\ 0 \; , & r \geq 0 \end{cases}$$

$$\oint_S \bar{D} \cdot \bar{ds} = \int_v \rho_v \, dv$$

$$4\pi r^2 D_r = \iiint_0^a \rho_0 r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$+ \iiint_a^r 0 \, r^2 \sin\theta \, dr \, d\theta \, d\phi \rightarrow 0$$

$$\cancel{4\pi} r^2 D_r = \left(\frac{\cancel{4\pi} a^2}{3} \right) \rho_0$$

$$D_r = \frac{\rho_0 a^3}{3 r^2}$$

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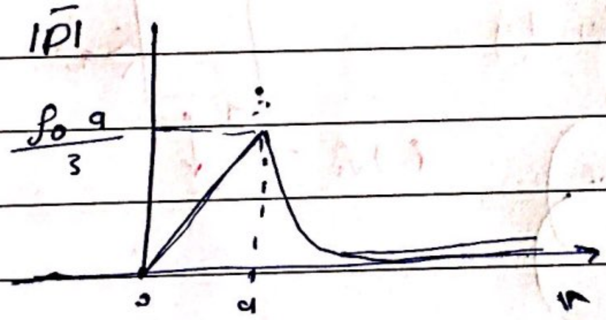
$\boxed{\bar{D} = \frac{\rho_0 a^3}{3r^2}\,\hat{a}_r}$, $\boxed{\bar{E} = \frac{\rho_0 a^3}{3\epsilon_0 r^2}\,\hat{a}_r}$

$a < r < \infty$

at $r = a \rightarrow \bar{D} = \frac{\rho_0 a}{3}\,\hat{a}_r$.

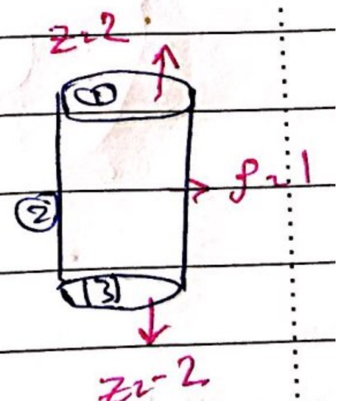
$$\bar{D} = \begin{cases} \frac{\rho_0}{3} r\,\hat{a}_r , & 0 \le r \le a \\ \frac{\rho_0 a^3}{3r^2}\,\hat{a}_r , & a \le r \le \infty \end{cases}$$



Ex Given $\bar{D} = z\rho\cos^2\theta\,\hat{a}_z$

Calculate the total charge enclosed by the cylinder of radius 1m with $-2 \le z \le 2$

$= Q_{enc} = \oint_s \bar{D}\cdot\bar{ds} = \int_v \rho_v\,dv = Q$



$\bar{ds_1} = \rho\,d\rho\,d\phi\,\hat{a}_z$, $\bar{ds_2} = \rho\,d\phi\,dz\,\hat{a}_z$

$\bar{ds_3} = -\rho\,d\rho\,d\phi\,\hat{a}_z$

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$$\Phi = \int_{S1} \overline{D} \cdot \overline{ds} + \int_{S2} \overline{D} \cdot \overline{ds_2} + \int_{S3} \overline{D} \cdot \overline{ds_3}$$

$$\Phi = \int_0^{2\pi}\int_0^1 Z\rho \cos^2\phi \, \rho \, d\rho \, d\phi + \int_0^{2\pi}\int_0^1 -Z\rho \cos^2\phi \, \rho \, d\rho \, d\phi$$

$$\boxed{\Phi = \frac{2\pi}{3} + \frac{2\pi}{3} = \frac{4\pi}{3} \, C}$$

Find surface charge density

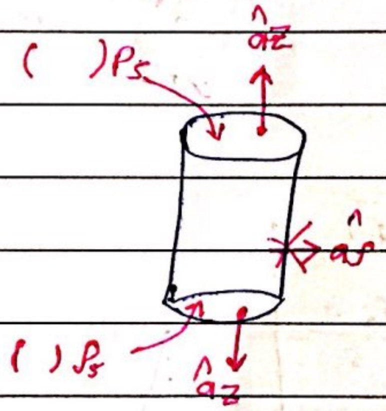
$$\rho_s = \overline{D} \cdot \hat{a}_r$$

$\hat{a}_z$ Top — $z = 2$

$-\hat{a}_z \to$ Bot — $z = -2$

at $\left(1, \frac{\pi}{4}\right)$

ρ ϕ



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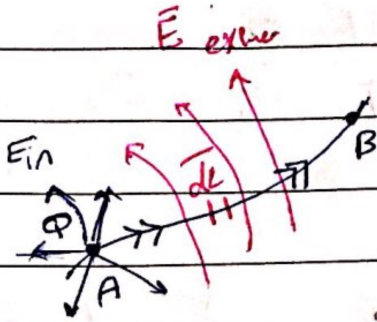
Subject Date No.

Electric potential

$\bar{F} = Q\bar{E} \rightarrow \bar{E} = \frac{\bar{F}}{Q}$



Work = Force * distance.

$w = \bar{F} \cdot \bar{l}$ in (J)

$dw = -\bar{F} \cdot \overline{dl}$ → The work is done by an extend agent.

$\int dw = -\int Q\bar{E} \cdot \overline{dl}$

$$\boxed{w = -Q\int_l \bar{E} \cdot \overline{dl}}$$

$\frac{w}{Q} = \boxed{V_{AB} = -\oint_l \bar{E}_{ext} \cdot \overline{dl}}$ in (V)

$= V_B - V_A$

$V_{AB} = -\int_l \bar{E} \cdot \overline{dl}$

→ if VAB is positive.
- gain in potential
- The work is done by the External field

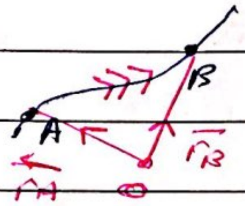
→ if VAB is negative
- drop in potential
- The work is done by the field it self

N O T E B O O K

→ For a point charge

$V_{AB} = -\int_L \bar{E}\cdot d\bar{L}$, $\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r$

$$V_{AB} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon_0 r^2}\,\cancel{\hat{a}_r}\cdot dr\,\cancel{\hat{a}_r}$$



$$V_{AB} = +\frac{Q}{4\pi\epsilon_0}\frac{1}{r}\Big|_{r_A}^{r_B}$$

$$V_{AB} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_B}-\frac{1}{r_A}\right) V$$

$$V_{AB} = \frac{Q}{4\pi\epsilon_0 r_B} - \frac{Q}{4\pi\epsilon_0 r_A}$$

$$= V_B - V_A$$

if point A is locate at ∞

$r_A \to \infty$, $V_A = 0$

$V_{AB} = A\infty B = V_B - V_\infty = V_B - 0 = V_B$

$\boxed{V_\infty \text{ when } r_\infty \to \infty}$ → Ref

$\boxed{V = \frac{Q}{4\pi\epsilon_0 r}}$ if ref is at ∞

→ For point charge

For line Charge

$$\boxed{V = \int_{l} \frac{\rho_L \, dl}{4\pi\epsilon_0 r}}$$

For Surface charge:

$$\boxed{V = \int_{S} \frac{\rho_s \, ds}{4\pi\epsilon_0 r}}$$

For Volume charge!

$$\boxed{\underset{\text{potential}}{V} = \int_{\underset{\text{vol}}{v}} \frac{\rho_v \, dv \leftarrow \text{vol}}{4\pi\epsilon_0 r}}$$

$$V(\bar{r}) = \int_{v'} \frac{\rho_v(\bar{r}') \, d\bar{r}'}{4\pi\epsilon_0 |\bar{r} - \bar{r}'|}$$

For N-point charges

$$V = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^{N} \frac{Q_k}{|\bar{r} - \bar{r}'_k|}$$

For point charge.

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r \quad , \quad V = \frac{Q}{4\pi\epsilon_0 r}$$

$$\hat{a}_r = \frac{\bar{r}}{r}$$

N O T E B O O K

Subject   Date   No.

Ex Two point charges $-4\mu c$ and $5\mu c$ are located at $(2,-1,3)$ and $(0,4,-2)$. Find the potential at $(1,0,1)$

↳ ref is at ∞ $\boxed{V_\infty = 0}$

$V = V_1 + V_2$, $V = \frac{Q}{4\pi\epsilon_o r}$

$$= \frac{-4 * 10^{-6}}{4\pi * \frac{10^{-9}}{36\pi} * |\underbrace{(-1,1,-2)}_{\sqrt{6}}|}$$

$$+ \frac{5 * 10^{-6}}{4\pi * \frac{10^{-9}}{36\pi} * |\underbrace{(1,-4,3)}_{\sqrt{26}}|}$$

$V_1 = -5.872$ KV
$(1,0,1)$

work is done by the field it self

Electric potential (2)

Ex :- A point charge of 5nc Located at $(-3,4,0)$ and a line $y=1$, $z=1$ carries a uniform charge of $2nc/m$

Ref

(b) IF $\boxed{V = 100V \text{ at } B(1,2,1)}$, find V at point $C(-3,5,3)$

$V_{BC} = V_C - V_B = V_C - 100$

$\boxed{V_C = V_B + 100}$

N O T E B O O K

Subject Date

$$V_{BC} = V_Q + V_L$$

$$V_Q = -\int_L \bar{E} \cdot \bar{dl} = -\int_{r_B}^{r_C} \frac{Q}{4\pi\epsilon_0 r^2}\, \hat{a_r} \cdot dr\, \hat{a_r}$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_C} - \frac{1}{r_B}\right) V$$

$r_C = |(-2,5,3) - (-3,4,0)| = |(1,1,3)| = \sqrt{11}$

$r_B = |(1,2,1) - (-3,4,0)| = |(4,-2,1)| = \sqrt{21}$

$$V_\ell = -\int_L \bar{E} \cdot \bar{dl} = \int_{\rho_B}^{\rho_C} \frac{\rho_L}{2\pi\epsilon_0 \rho}\, \cancel{\hat{a_\rho}}\, d\rho\, \cancel{\hat{a_\rho}}$$

$$= \frac{-\rho_L}{2\pi\epsilon_0}\left(\ln \rho_C - \ln \rho_B\right)$$

$$= -\frac{\rho_L}{2\pi\epsilon_0}\left(\ln\left(\frac{\rho_C}{\rho_B}\right)\right) V$$

$= \rho_C = |(-2,5,3) - (\underset{-2}{X},1,1)| = |(0,4,2)|$

$= \sqrt{20}$

$\rho_B = |(1,2,1) - (\underset{1}{X},1,1)| = |(0,1,0)| = 1$

$$\boxed{V_{BC} = -50.175\, V}$$

$V_C = V_{BC} + V_B = 49.825\, V$

N O T E B O O K

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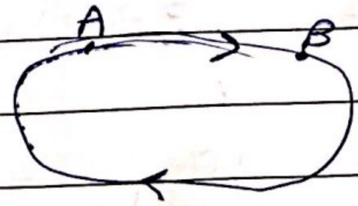
How to find $\bar{E}$ from V?!

$$V_{AB} = -\int \bar{E} \cdot \bar{dl}$$



$$V = -\oint \bar{E} \cdot \bar{dl}$$

$$\oint \bar{E} \cdot \bar{dl} = -\int_A^B \bar{E} \cdot \bar{dl} + \int_B^A \bar{E} \cdot \bar{dl}$$

$$= -\int_A^B \bar{E} \cdot \bar{dl} + \int_A^B \bar{E} \cdot \bar{dl}$$

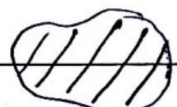
$$\boxed{\oint_l \bar{E} \bar{dl} = 0}$$ always

→ equi- potential

$V_{AA} = V_A - V_A = 0$

→ 2^{nd} maxwell's eq. is integral

$$\oint \bar{E} \cdot \bar{dl} = 0$$



by applying the stocke's Theorem

$$\oint_l \bar{E} \cdot \bar{dl} = \int_s \nabla \times \bar{E} \cdot \bar{ds} = 0$$

$$\boxed{\nabla \times \bar{E} = 0}$$

→ 2^{nd} maxwell's eq. in diff form

→ $\bar{E}$-field is irrotational

($\bar{E}$-field is conservative)

$$U = -\int_l \bar{E}\cdot\bar{dl}$$

let: $\bar{E} = E_x\,\hat{a}_x + E_y\,\hat{a}_y + E_z\,\hat{a}_z$

$\bar{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

$$dU = -\bar{E}\cdot\bar{dl}$$

$$dU = -(E_x\,dx + E_y\,dy + E_z\,dz)$$

Assume, $dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz$

$$\frac{dU}{\partial x}\cancel{dx} = -E_x\cancel{dx}$$

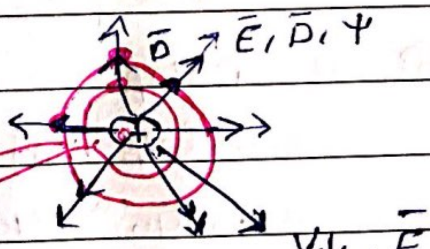
$$E_x = -\frac{\partial U}{\partial x}, \quad E_y = \frac{\partial U}{\partial y}, \quad E_z = -\frac{\partial U}{\partial z}$$

$$\bar{E} = -\left(\frac{\partial U}{\partial x}\hat{a}_x + \frac{\partial U}{\partial y}\hat{a}_y + \frac{\partial U}{\partial z}\hat{a}_z\right) = -\nabla U$$

$U = -\int_l \bar{E}\cdot\bar{dl}$, $\boxed{\bar{E} = -\nabla U}$

$U \qquad m \qquad \frac{V}{m} = \frac{1}{m}\; V$

$$V = \frac{Q}{4\pi\epsilon_0 r}$$



equi-potential surface

Subject

Date No.

Ex : Given the potential $V = \frac{10}{r^2} \sin\theta \cos\theta$

(a) find $\bar{D}$ at $(2, \frac{\pi}{2}, 10)$ — r, θ, ϕ

(b) Calculate the work done in moving $10\ \mu C$ charge from $A(1, 30^\circ, 120^\circ)$ to $B(4, 90^\circ, 60^\circ)$

(a) $\bar{E} = -\nabla V$, $\bar{D} = \epsilon_0 \bar{E}$

$\bar{D} = -\epsilon_0 \nabla V$

$$E = -\left[\frac{\partial V}{\partial r} \hat{a}_r + \frac{\partial V}{r \partial \theta} \hat{a}_\theta + \frac{\partial V}{r \sin\theta\, d\phi} \hat{a}_\phi \right]$$

$$\bar{E} = \frac{20}{r^3} \sin\theta \cos\phi\ \hat{a}_r - \frac{10}{r^3} \cos\theta \cos\phi\ \hat{a}_\theta$$

$$+ \frac{10}{r^3} \sin\phi\ \hat{a}_\phi]\ V/m$$

$$\bar{D}_{1\,(2, \frac{\pi}{2}, 0)} = \epsilon_0 \bar{E} = \frac{20}{8} \hat{a}_r\ C/m^2$$

$$= 22.1\ \hat{a}_r\ pC/m^2$$

(b) $W = Q V_{AB}$

$= Q (V_B - V_A)$

$= 10 \times 10^{-6} \left(\frac{10}{16} (1)\left(\frac{1}{2}\right) - \frac{10}{1} \left(\frac{1}{2}\right)\left(\frac{-1}{2}\right) \right)$

$= 10 \times 10^{-6} \left(\frac{45}{16} \right)$ V_{AB}

$= \boxed{28.125\ \mu J}$

Date No.

method (2) :-

$$w = -Q\int_{L} \bar{E}\cdot \bar{dl}$$

$$= -Q\left[\int_{r_A}^{r_B} E_r\,dr + \int_{\theta_A \to 30^\circ}^{\theta_B \to 90^\circ} E_\theta\, r\,d\theta + \int_{\phi_A}^{\phi_B} E_\phi\, r\sin\theta\, d\phi\right]$$

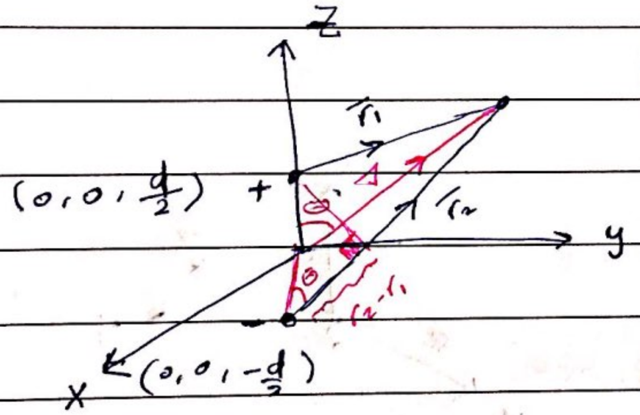
$A(1, 30^\circ, 120^\circ)$ ∵ $B(4, 90^\circ, 60^\circ)$

r ↳ $(4, 30^\circ, 120^\circ)$ → $(4, 90^\circ, 120^\circ)$ ϕ

θ

Electric Dipole :-

↳ $\bar{E}, \bar{D}, V$?



Q ⊕
$\bar{d}$
$-Q$ ⊖

(r, θ, ϕ) $\bar{E}, \bar{D}, \bar{V}$?

$\bar{E} = -\nabla V$

$\bar{D} = \epsilon_0 \bar{E}$

$\boxed{d \lll r}$

$V = V_+ + V_-$, $\boxed{V_\infty = 0}$

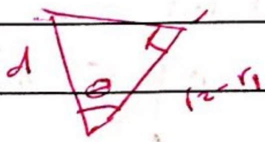
$$V = \frac{Q}{4\pi\epsilon_0 r_1} + \frac{-Q}{4\pi\epsilon_0 r_2}$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) V$$

$V = \dfrac{Q(r_2 - r_1)}{4\pi\epsilon_0 r_1 r_2}$, Q and d are not known

$\boxed{r_1 r_2 = r^2}$

$\cos\theta = \frac{r_2 - r_1}{d}$



$r_2 - r_1 = d\cos\theta$

$r_1 r_2 = r^2$

$$\boxed{V = \frac{Q\, d\cos\theta}{4\pi\epsilon_0 r^2}} \quad (v) \quad (2)$$

define dipole moment

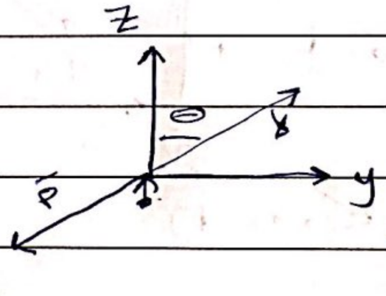
$\bar{P} = Q\bar{d} \Rightarrow$ C.m vector

$P = Qd$

$$\boxed{V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2}} \quad (v) \quad (3)$$

$$\boxed{V = \frac{\bar{P}\cdot\hat{a}_r}{4\pi\epsilon_0 r^2}} \quad (3')$$

$\bar{P}\cdot\hat{a}_r = p(1)\cos\theta$



$$V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2} = \frac{P\cdot\hat{a}_r}{4\pi\epsilon_0 r^2} = \frac{\bar{P}\cdot(\bar{r}-\bar{r}')}{4\pi\epsilon_0|\bar{r}-\bar{r}'|^3} \quad (3)$$

For N-dipoles field center

$$r = \frac{1}{4\pi\epsilon_0}\sum_{k=1}^{N}\frac{\bar{P}_k\cdot(\bar{r}-r'_k)}{|\bar{r}-\bar{r}'|^3} \quad (V)$$

center of the dipole

Subject Date No.

$\bar{E} = \bar{E}_+ + \bar{E}_-$

or

$\bar{E} = -\nabla V$ sph

$\bar{E} = -\left(\frac{\partial V}{\partial r}\hat{a}_r + \frac{\partial V}{r\partial\theta}\hat{a}_\theta\right)$

since (V) is ind on ϕ

scaler

$\bar{E} = \frac{P}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right)$ V/m

vector

$\bar{D} = \epsilon_0\bar{E}$, $V = \frac{\bar{P}\cdot\hat{a}_r}{4\pi\epsilon_0 r^2}$ scaler

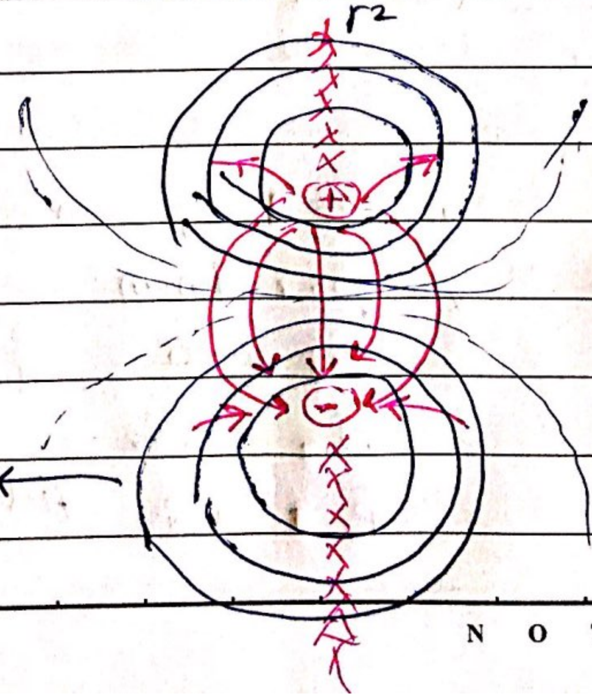
point Charge E. Dipole

$Q\cdot$ $E \propto \frac{1}{r^2}$ $+\cdot$ $E \propto \frac{1}{r^3}$

$V \propto \frac{1}{r}$ $-\cdot$ $V \propto \frac{1}{r^2}$

Flux lines

$V_\infty = 0$



$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

equi-po

Ex. Two dipoles with dipole moments $-5\,\hat{a}_z$ nCm and $9\,\hat{a}_z$ nCm are located at $(0,0,-2)$ and $(0,0,3)$. Find the potential and $\bar{E}$ at the origin

ref is at ∞

$$V = \frac{\bar{P}(\bar{r}-\bar{r}')}{4\pi\epsilon_0 |\bar{r}-\bar{r}'|^3},$$

$$V = \frac{(9)\,-5\hat{a}_z * 10^{-9} \cdot (2\hat{a}_z)}{4\pi * \frac{10^{-9}}{36\pi}\,(8)\,4}$$

$$+ \frac{(9)\; 9\,\hat{a}_z * 10^{-9} \cdot (-3\hat{a}_z)}{4\pi * \frac{10^{-9}}{36\pi}\,(27)}$$

$$\boxed{V = -20.25\,V}$$

$\bar{E}_1 = \frac{P_1}{4\pi\epsilon_0 r_1^3}\,(2\cos\theta_1\,\hat{a}_r + \sin\theta_1\,\hat{a}_\theta)$

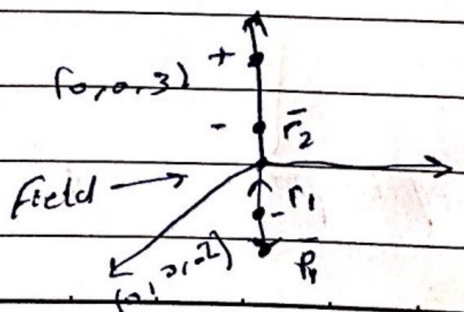
$\bar{E} = -\nabla V$, $r_1 = 2m$, $r_2 = 3m$

$P_1 = 5n.m$, $P_2 = 9n.m$

? $\theta_1 = 180$, $\theta_2 = 180$

$\bar{r}_1 = 2\hat{a}_z$, $\bar{r}_2 = -3\hat{a}_z$

$\bar{P}_1 = -5\hat{a}_z$, $\bar{P}_2 = 9\hat{a}_z$



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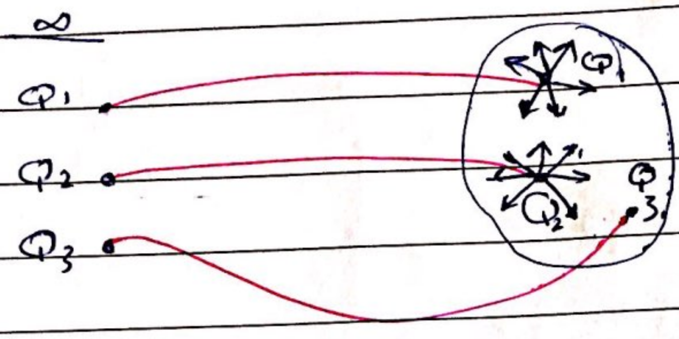
Energy Density in Electrostatic Fields:



$$W = Q\,V_{diff}$$

$$Q_1 \rightarrow Q_2 \rightarrow Q_3$$

$W_E \equiv$ Electrical Energy

$$W_E = W_1 + W_2 + W_3$$

$$= Q(0) + Q_2 V_{12} + Q_3(V_{13} + V_{23}) \quad \text{--- (1)}$$

$$Q_3 \rightarrow Q_2 \rightarrow Q_1$$

$$W_E = W_1 + W_2 + W_3$$

$$= Q_1(V_{31} + V_{21}) + Q_2 V_{32} + 0 \quad \text{--- (2)}$$

$$2W_E = Q\underbrace{(V_{21} + V_{31})}_{V_1} + Q\underbrace{(V_{12} + V_{32})}_{V_2} + Q_3\underbrace{(V_{13} + V_{23})}_{V_3}$$

$$W_E = \frac{1}{2}\left[Q_1 V_1 + Q_2 V_2 + Q_3 V_3\right]$$

$$\boxed{W_E = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k \quad \text{in (J)}}$$

$$W = QV$$

For line charge

$$W_E = \frac{1}{2} \int_l \rho_l \, V \, dl$$ potential

For surface charge.

$$W_E = \frac{1}{2} \int_s \rho_s \, V \, ds \qquad \text{in } J$$

for Volume

charge $\rightarrow$ $W_e = \frac{1}{2} \int_v \rho_v V \, dv$

$$W_E = \frac{1}{2} \int_v \rho_v \, V \, dv$$

$$\rho_v = \nabla \cdot \bar{D} \Rightarrow W_E = \frac{1}{2} \int_v (\nabla \overset{*}{\cdot} \bar{D}) V \, dv$$

in general

$$\underset{(1)}{\nabla \cdot V\bar{A}} = \underset{(2)}{\bar{A} \cdot \nabla V} + V(\nabla \cdot \bar{A})$$

$$W_E = \frac{1}{2} \left[\int_v \underset{(1)}{\nabla \cdot (V\bar{D})} \, dv - \int_v \underset{(2)}{\bar{D} \cdot \nabla V} \, dv \right]$$

Apply the divergence theorem on integral (1)

$$W_E = \frac{1}{2} \left[\overset{0}{\underset{(1)}{\oint_s (V\bar{D}) \cdot \bar{ds}}} - \int_v \bar{D} \cdot \nabla V \, dv \right]$$

Subject

$V = \frac{Q}{4\pi\epsilon_0 r} \leftarrow V \propto \frac{1}{r}$

$\frac{1}{r^3}$

$\frac{1}{r} \Rightarrow 0$ as $r \to \infty$

$\bar{D} = \frac{Q}{4\pi r^2} a_r \leftarrow \bar{D} \propto \frac{1}{r^2}$

$ds \propto r^2$

$W_E = \frac{1}{2}\int_v \bar{E}\cdot\bar{D}\, dv$, $\bar{D} = \epsilon_0 \bar{E}$

Field

$= \frac{1}{2}\int_v \epsilon E^2 dv$, $\bar{E} = \frac{\bar{D}}{\epsilon_0}$

$= \frac{1}{2}\int_v \frac{D^2}{\epsilon_0} dv$

$$\boxed{W_E = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k} \rightarrow \text{Source}$$

Energy Density w_E

$w_E = \frac{W_E}{\text{Volume}}$ in J/m^3

$= \frac{1}{2}\bar{E}\cdot\bar{D}$

$= \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\frac{D^2}{\epsilon_0}$

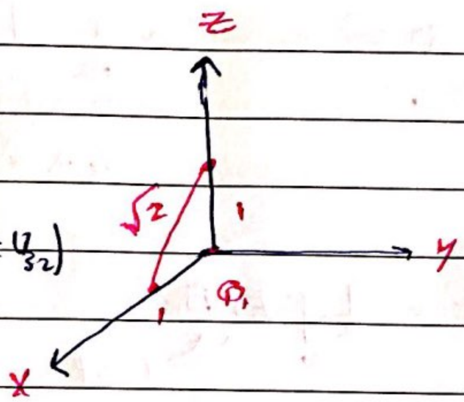
$$\boxed{W_E = \int_v w_E\, dv}$$

N O T E B O O K

Subject

Date No.

Ex: The point charges $-1nC$, $4nC$ and $3nC$ are located at $(0,0,0)$, $(0,0,1)$ and $(1,0,0)$. Find the energy in the system.



$$W_E = \frac{1}{2} \sum_{k=1}^{N} Q_k V_k$$

$$W_E = \frac{1}{2} [Q_1 V_1 + Q_2 V_2 + Q_3 V_3]$$

$$W_E = \frac{1}{2} [Q_1 (V_{21} + V_{31}) + Q_2 (V_{12} + V_{32}) + Q_3 (V_{13} + V_{23})]$$

$$W_E = \frac{1}{2} \Big[Q_1 \left(\frac{Q_2}{4\pi\epsilon_0 (1)} + \frac{Q_3}{4\pi\epsilon_0 (1)} \right) + Q_2 \left(\frac{Q_1}{4\pi\epsilon_0 (1)} + \frac{Q_3}{4\pi\epsilon_0 \sqrt{2}} \right) + Q_3 \left(\frac{Q_1}{4\pi\epsilon_0 (1)} + \frac{Q_3}{4\pi\epsilon_0 \sqrt{2}} \right) \Big]$$

$$W_E = 13.37 \ nJ$$

N O T E B O O K