SIGNALS

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BY:AYA BASEM

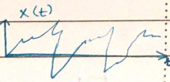
POWEROUNIT

Signals & systems.

x(t) —— | h(t) | —— y(t)
I/p — impulse response — o/p

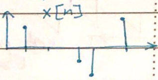
* classification of the signals.

1- Continuous ($x(t)$) or discrete ($x[n]$) sig. →



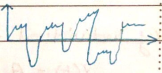
Cont.

2- Analog or digital sig.



discrete

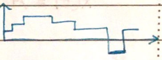
3- causal or non causal sig.



analog.

4- Deterministic or random sig.

5- odd or even sig.



digital

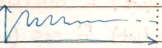
6- power or energy sig.

7- stable or unstable sig.

8- periodic or aperiodic sig.

• we can predict the future value.

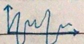
$x(t) = e^{-4t}$, $x(t) = 4\cos(4t)$.

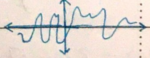


"overshort"

• $\lim_{t\to\infty} x(t) = K$ "stable sig" • $\lim_{t\to\infty} x(t) \to \pm\infty$ "unstable".

• $x(t) = \begin{cases} x(t), & t \geq 0 \\ 0, & t < 0 \end{cases}$ "causal" , $x(t) = \begin{cases} x_1(t), & t \geq 0 \\ x_2(t), & t < 0 \end{cases}$ "uncausal"

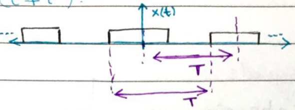


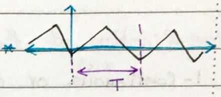


• periodic & aperiodic.

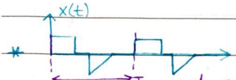
it repeats it self every "T" = periodic time $-\infty < t < \infty$.

$x(t) = x(t+T)$.





* $\omega = \frac{2\pi}{T}$ rad/s
* $f = \frac{1}{T}$ Hz.



not periodic
$t \geq 0$

* every individual sinusoidal sig. is aperiodic sig.

$$x(t) = A \cos(\omega t), \quad T = \frac{2\pi}{\omega}.$$

* $x(t) \overset{??}{=} x(t+T)$.

$$\cos(\omega t) \overset{?!}{=} \cos(\omega t + \omega T).$$

$$\cos(\omega t) \overset{?!}{=} \cos\left(\omega t + \cancel{\omega} \cdot \frac{2\pi}{\cancel{\omega}}\right)$$

$$\cos(\omega t) = \cos(\omega t + 2\pi).$$

$\rightarrow \cos\omega t \, \cancel{\cos 2\pi}^{\,1} + \sin\omega t \, \cancel{\sin 2\pi}^{\,0}$

$= \cos(\omega t)$.

* $x(t) = 4\cos(40\pi t)$

$A = 4$, $\omega = 40\pi$, $f = \frac{\omega}{2\pi} = \frac{40\pi}{2\pi} = 20$ Hz.

$T = \frac{1}{f} = \frac{1}{20}$ sec.

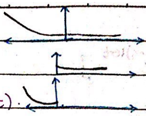
* $x(t) = 3\sin 5t$.

$A = 3$, $\omega = 5$, $T = \frac{2\pi}{\omega} = \frac{2\pi}{5}$ sec.

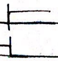
* $x_1(t) = e^{-3t}$

$x_2(t) = e^{-3t}\,u(t)$ $\quad t \geq 0$

$x_3(t) = e^{-3t}\,u(-t)$



- $u(t) =$
- $u(-t) =$



* $e^{j\omega t} = \cos \omega t + j \sin \omega t$.

* $x(t) = 3 \cos 10t + 4 \sin 20t$.

$T_1 = \frac{2\pi}{10}$, $T_2 = \frac{2\pi}{20}$, $\frac{T_1}{T_2} = \frac{2}{1}$ $\leftarrow$ int $\leftarrow$ int

* $\text{int}/\text{int} \rightarrow$ periodic signal.

* $y(t) = 4 \cos 10t - 6 \cos 20\pi t$.

* $T_0 = 2T_2$ OR $1.T_1$

- $T_1 = \frac{2\pi}{10}$, $T_2 = \frac{2\pi}{20\pi} = \frac{1}{10}$

- $\frac{T_1}{T_2} = \frac{\pi}{5} * 10 = \frac{2\pi}{1}$ $\leftarrow$ not int, $\leftarrow$ int $\rightarrow$ not periodic.

* $x(t) = 3\cos 10t + 4\sin 20t + 6\cos 50t$

* $\frac{T_1}{T_2} = \frac{0.02}{0.45} = \frac{2}{45}$ periodic

- $T_1 = \pi/5$, $T_2 = \frac{\pi}{10}$, $T_0 = \pi/5$, $T_3 = \frac{2\pi}{50}$

$\frac{T_0}{T_3} = \frac{5}{1}$ periodic

Ex: $x(t) = 4e^{-j12\pi t} + 4\cos 8\pi t$.

- $T_1 = \frac{2\pi}{12\pi} = 1/6$, $T_2 = \frac{2\pi}{8\pi} = 1/4$, $\frac{T_1}{T_2} = \frac{4}{6}$ periodic

fundimental periodic $T_0 = 3T_1 = \frac{1}{2}$ sec.

ex: $4 \cos 10t \sin 6t$.

$x(t) = \frac{1}{2} \sin(4t) + \frac{1}{2} \sin(16t)$

$T_1 = \frac{2\pi}{4} = \frac{\pi}{2}$ $\quad T_2 = \frac{2\pi}{6} = \frac{\pi}{3}$

- $\frac{T_1}{T_2} = \frac{3}{2}$ (Periodic sig) $\quad \cdot\, T_0 = 3T_2 = \pi$ non periodic

N O T E B O O K

Ex: $x(t) = e^{j8t} \cdot \cos 10t$

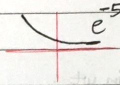
$$= e^{j8t}\left[\frac{1}{2}e^{j10t} + \frac{1}{2}e^{-j10t}\right]$$

$$= \frac{1}{2}e^{j18t} + \frac{1}{2}e^{-j2t}$$

* Complex or Real

- $e^{j5t} \rightarrow \cos 5 + j\sin 5$

$a + jb$



e^{-5t}

* power or energy signal * "Size of the signal"

- Energy sig. $E \rightarrow k$ "J".

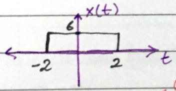
$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt \rightarrow \text{power} = 0$$

- power sig.

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt = k \rightarrow E = \infty.$$

(periodic = power "غالباً")
aperiodic = E or neither

Ex:



$T = 4$

- Find the size.

- non periodic

- $E = \int_{-2}^{2} 6^2\, dt = 36t\Big|_{-2}^{2} = 144\text{J}$. energy sig.

$T \rightarrow \infty$

- $P = \int |x(t)|^2 dt * \frac{1}{\cancel{T}\,\infty} = 0$

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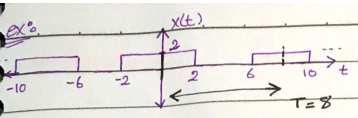
Ex:



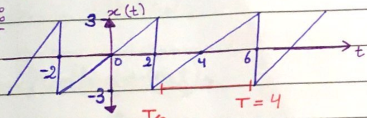
$$\frac{1}{8}\left[\cancel{\int_{-4}^{-2} 0\, dt} + \int_{-2}^{2} 2^2\, dt + \cancel{\int_{2}^{4} 0\, dt}\right]$$

$$\cdot\ P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt = \frac{1}{8}\int_{-2}^{2} 2^2\, dt = \frac{4t}{8}\Big|_{-2}^{2} = 2\,\omega$$

$$\cdot\ E = \int_{-\infty}^{\infty} = \infty$$

"power signal"

Ex:



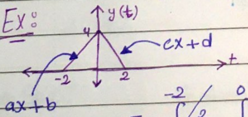
$$* \ \frac{3-(-3)}{2-(-2)} = \frac{6}{4}$$

$$* \ x(t) = 6/4\, t = 3/2\, t$$

$$* \ P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \frac{1}{4}\int_{-2}^{2} (3/2\, t)^2\, dt \Rightarrow \frac{\cancel{9}^{3}}{16} * \frac{t^3}{\cancel{3}}\Big|_{-2}^{2} \rightarrow \frac{3}{16}\left[2^3 - (-2)^3\right] = 3\,\omega.$$

Ex:



$$* \ E = \cancel{\int_{-\infty}^{-2} 0^2} + \int_{-2}^{0} + \int_{0}^{2} + \cancel{\int_{2}^{\infty} 0^2}.$$

Ex:

$x(t) = A\cos \omega t \qquad * P = A^2/2.$

Ex: $x(t) = 3\cos 10t$ periodic

$$* P = 3^2/2 = 4.5\ \omega.$$

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Ex: $3\sin 10t + 4\cos 20t$.

* $T_1 = \frac{2\pi}{10}$, $T_2 = \frac{2\pi}{20}$

* $\frac{T_1}{T_2} = \frac{2\pi}{10} * \frac{20}{2\pi} = \frac{2}{1}$ ∴ periodic

* $T_0 = T_1 = \frac{2\pi}{10}$

* $P = \frac{3^2}{2} + \frac{4^2}{2}$ w.

Ex: $x(t) = A e^{j\omega t}$ * (±∞) → neither, nor.

* $p = \frac{1}{2T} \int_{-T}^{T} |Ae^{j\omega t}|^2 \, dt$.

$= \frac{1}{2T} \int_{-T}^{T} A e^{j\omega t} \cdot A e^{-j\omega t} \, dt$.

$= \frac{1}{2T} A^2 \int \cancel{e}_1 \, dt = \frac{A^2}{2T} \cdot t \Big]_{-T}^{T} = A^2 \frac{\cancel{(2T)}}{\cancel{2T}} = A^2$.

* $y = 4\cos 3t - 6\sin 10t$.

$T_1 = \frac{2\pi}{3}$, $T_2 = \frac{2\pi}{10}$, $\frac{T_1}{T_2} = \frac{10}{3}$ periodic

$T_0 = 3T_1 = 2\pi$ * $P = \frac{4^2}{2} - \frac{6^2}{2}$ w.

* $y(t) = \cos^2 10t$.

$= \frac{1}{2} + \frac{1}{2}\cos 20t$. DC "shift up".

* $P = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 / 2$

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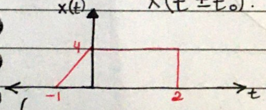
* Operations of the signals:

1- Time shift.
2- Time scale
3- Reflection / Reversal

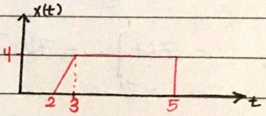
* Time shift:

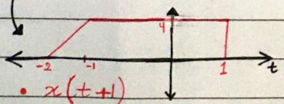
$x(t \pm t_0)$.



after the time shift

$x(t-3)$





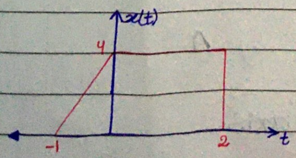
• $x(t+1)$

* Time-Scale: Compression or Expansion.

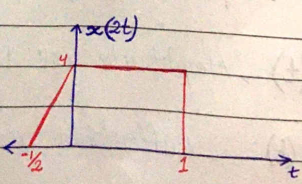
$x(at)$ ⇝ $|a| > 1$ $|a| < 1$

* Sketch.

$x(2t) \rightarrow |2| > 1$ ∴ Compression.



Time Scaled



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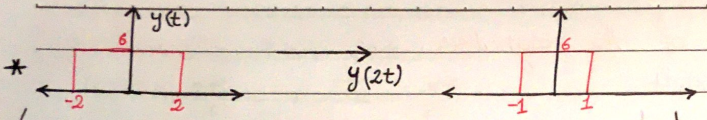
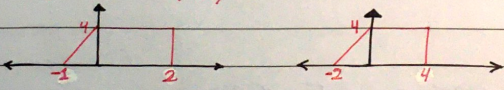


* Time shift doesn't affect the Energy.

. $E = \int_{-2}^{2} 6^2 \, dt = 36t \Big]_{-2}^{2} = 144\ J.$

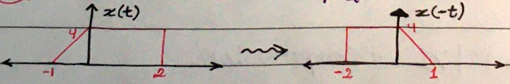
. $E = \int_{-1}^{1} 6^2 \, dt = 36t \Big]_{-1}^{1} = 72\ J.$

* $x(0.5t) = x(t/2).$



* Reflection / Reversed.

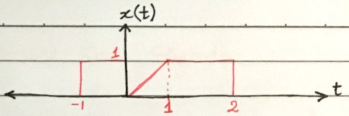
$x(-t) \rightarrow$ Reflection around the Y-axis.



* $Ax(t) \rightarrow$ Amplitude will be affected by A.

* $-x(t) \rightarrow$ Reflection around x-axis.

Ex:

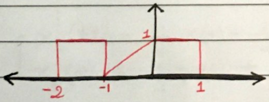


$x(-3) \neq -x(3)$

. Sketch $x(1-t/2)$

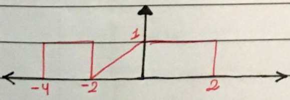
1. it has time shift $=1$.
2. time scale
3. Reflection around y.

الترتيب مهم



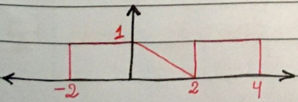
1- Time shift $=1$

$y(t) = x(t+1)$



2- Time Scale

expansion. $y(t/2)$.



3- Reflection.

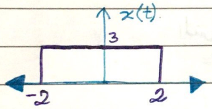
$x(1-t/2)$.

$$\begin{cases} 1-t/2 = \tau \\ t/2 = 1-\tau \\ t_{new} = 2-2\tau_{old} \end{cases}$$

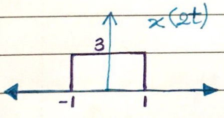
$$\begin{cases} \tau = 0 \\ t = 2-2(0) = 2. \end{cases}$$

* $x(t \pm t_0)$: Time shift.
* $x(at)$: Time scaling.
* $x(-t)$: Time reversal – y-axis
* $ax(t)$: Amplitude scaling.
* $-x(t)$: Amplitude reversal.

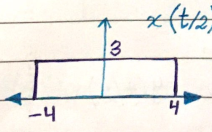
Ex:



$$E_1 = \int_{-2}^{2} 9 \, dt = 9t \Big|_{-2}^{2} = 36 \text{ J}$$



$$E_2 = \int_{-1}^{1} 9 \, dt = 9t \Big|_{-1}^{1} = 18 \text{ J}$$



$$E_3 = \int_{-4}^{4} 9 \, dt = 9t \Big|_{-4}^{4} = 72 \text{ J}.$$

* Odd and Even Signals:

<u>even</u>

$x(t) = x(-t)$

sin, cos, ...

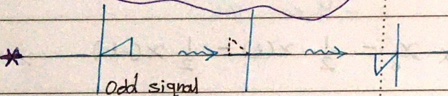
<u>odd</u>

$x(t) = -x(-t)$

* for Any Signal:

$$x(t) = x_{even}(t) + x_{odd}(t).$$

*



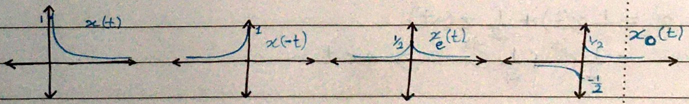
Odd signal

* Even → odd = ϕ
* Odd → Even = 0

$$x_o(t) = \frac{1}{2}x(t) - \frac{1}{2}x(-t)$$

$$x_e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(-t)$$

Ex: $x(t) = e^{-t}$, $t \geq 0$. even or odd or neither.



Ex: Check if $x(t) = 4\cos(10t) - 4\sin(6t)$.

* $x(-t) = 4\cos(-10t) - 4\sin(-6t)$.

$= 4\cos(10t + 4\sin 6t) \neq x(t)$

* $x(t) \stackrel{?}{=} -x(-t)$

$= -4\cos 10t - 4\sin 6t \neq x(t)$

→ Neither Odd Nor Even.

$* \; x_e = \frac{1}{2} x(t) + \frac{1}{2} x(-t).$

$= \frac{1}{2}[4\cos 10t - 4\sin 6t] + \frac{1}{2}[4\cos 10t + 4\sin 6t]$

$= 2\cos 10t - \cancel{2\sin 6t} + 2\cos 10t + \cancel{2\sin 6t}$

$= 4\cos 10t.$

$* \; x_o = \frac{1}{2} x(t) - \frac{1}{2} x(-t). \rightarrow -4\sin 6t.$

Ex:

$x(t) = e^{jt}.$

$x(t) \stackrel{?}{=} x(-t).$

$x(-t) = e^{-jt} \neq x(t).$

$-x(-t) = -e^{-jt} \neq x(t).$

$\cos x = \frac{1}{2} e^{jx} + \frac{1}{2} e^{-jx}.$

$\sin x = \frac{1}{2j} e^{jx} - \frac{1}{2j} e^{-jx}.$

$e^{\pm jx} = \cos x \pm j\sin x.$

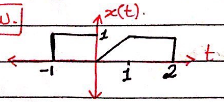
$* \; x_e = \frac{1}{2} x(t) + \frac{1}{2} x(-t)$

$= \frac{1}{2} e^{jt} + \frac{1}{2} e^{-jt} \rightsquigarrow \cos t.$

$* \; x_o = \frac{1}{2} x(t) - \frac{1}{2} x(-t)$

$= \frac{1}{2} e^{jt} - \frac{1}{2} e^{-jt} \rightsquigarrow j\sin t$

HW.



* Odd or Even, if nor find x_o, x_e.

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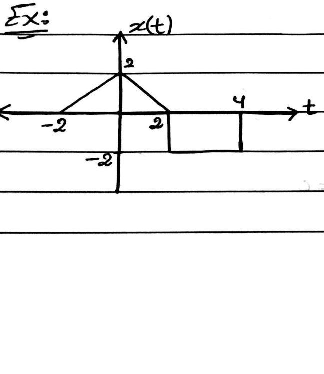
H.W

Ex:



a) Find $E_x(t)$

b) let $y(t) = -2x(-2-4t)$

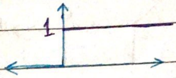
sketch $y(t)$ & find $E_y(t)$

c) Write your comment.

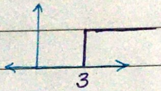
* Singularity, Elementary, Basic Signals.

▣ Unit-step-f^n.

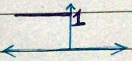
- $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \\ \text{discontinuous}, & t = 0 \end{cases}$



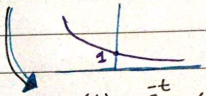
- $u(t-3) \rightarrow$



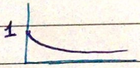
- $u(-t) \rightarrow$



* Sketch $x(t) = e^{-t}$.

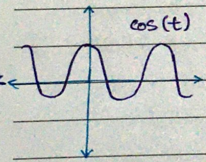


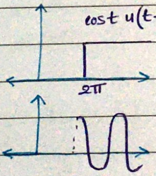
* $x(t) = e^{-t}\, u(t)$



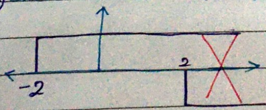
$t \geq 0$

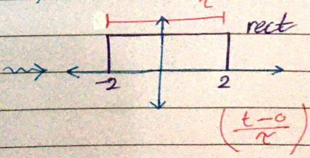
*





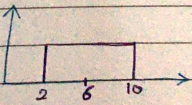
(*) sketch $x(t) = u(t+2) - u(t-2)$





* $\text{rect}\left(\dfrac{t-a}{\tau}\right)$ → center, → width

- sketch $4\,\text{rec}\left(\dfrac{t-6}{8}\right)$



- $x(t) = u(t-2) - u(t-10)$.

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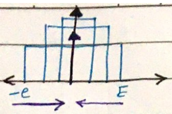
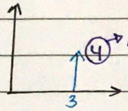


* Dirac, Delta, impulse f^n

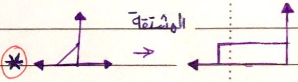
$$\delta(t) = \begin{cases} \delta(t) & t=0 \\ 0 & o.w \end{cases}$$

* $4\delta(t-3)$

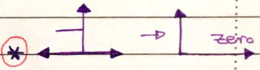


* Amplitude $= \infty$ "دايما"

* properties of $\delta(t)$.

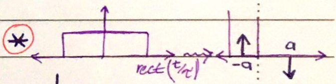


① $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$



② $\int_{-\infty}^{t} \delta(t)\, d\tau = u(t)$

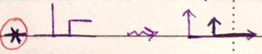
↳ $\frac{du(t)}{dt} = \delta(t)$.



↳ $\delta(t+a) - \delta(t-a)$

③ Sampling property.

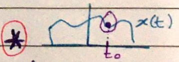
$x(t)\,\delta(t-t_0) = x(t_0)\,\underbrace{\delta(t-t_0)}$



"لازم نضرب الجواب في δ"

Ex: $\cos 4\pi t\, \delta(t-1)$

$\Rightarrow \cos 4\pi(1)\, \delta(t-1)$.



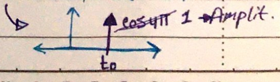
↳ $x(t) * \delta(t-t_0)$

④ sifting property.

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0) = x(t_0)$$

* ليس ضرب الناتج بال δ

لانه $\boxed{1 = \int \delta}$



N O T E B O O K

Ex:

* $\int_{-2}^{2} \cos 4\pi t \, \delta(t-1) \, dt \rightarrow t=1 \quad = \cos 4\pi(1).$

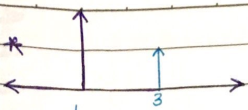
* $\int_{-2}^{3} \cos 4\pi t \, \delta(t-4) \, dt = 0$ خارج حدود التكامل

$t=4$

* $\int_{-3}^{3} \cos 4\pi t \, \delta(t-3) \, dt = 0$ لانه على حد التكامل بالضبط.

$t=3$

*

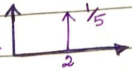


* Scaling properity:

• $\delta(at-b) = \left|\frac{1}{a}\right| \delta\left(t-\frac{b}{a}\right)$

$\delta(t-3)$

• $\delta(5t-10) = \frac{1}{5}\,\delta(t-2)$ ⇝



Ex:

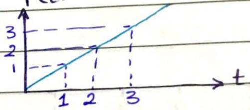
* $e^{-4t}\,\delta(t/3-4)$ ⇝ Sampling ...

$= e^{-4t}\,|3|\,\delta(t-12)$ $\quad t = 12$

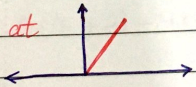
$= 3\,e^{-4(12)}\,\delta(t-12)$ ⇝ Scaling.

* Ramp f^{n}:

• $r(t) = \begin{cases} t & , t \geq 0 \\ 0 & , t < 0 \end{cases}$

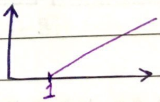


• at

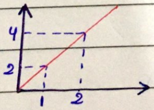


أقرب للـ y

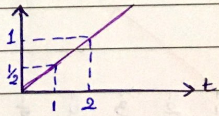
• (t−1)



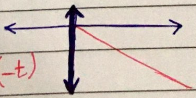
• 2t



• ½ t



• (−t)



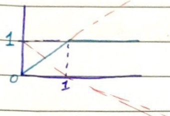
* $u(t) = \frac{dr(t)}{dt}$

$$\int_0^t u(\tau)\, d\tau = r(t).$$

* Sketch

$r(t) - r(t-1)$.

0 1

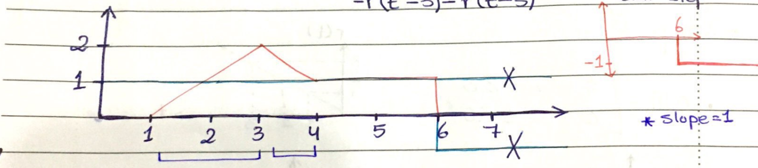


" محصلة قوة 1 + قوة 2 = Const. "

عشان هيك بتصير ——

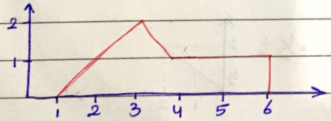
* Sketch $f(t) = r(t-1) - 2r(t-3) + r(t-4) - \underbrace{u(t-6)}_{\text{Unit-step}}$

$-r(t-3) - r(t-3)$

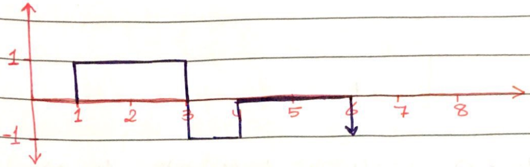


* slope = 1

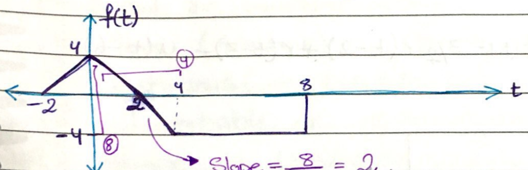
* derive $f(t)$ and sketch it



* $f(t) = u(t-1) - 2u(t-3) + u(t-4) - \delta(t-6)$.



* Describe the following function:



Slope $= \frac{8}{4} = 2$.

Slope

* $2r(t+2) - 2(r(t)) - 2r(t) + 2r(t-4) + 4u(t-8)$.

(مشيت 2 وطلعت 4)

* $2r(t+2) - 4r(t) + 2r(t-4) + 4u(t-8)$.

- $f(t) = 2u(t+2) - 4u(t) + 2u(t-4) + 4\delta(t-8)$.

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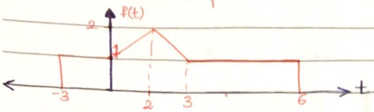
* Write the expressions



* $f(t) = 1\,u(t+3) + \underbrace{\frac{1}{2}}_{\frac{\Delta y}{\Delta x}} r(t) - \frac{1}{2} r(t-2) - 1\,r(t-2) + r(t-3) - u(t-6).$

$= u(t+3) + \frac{1}{2} r(t) - \frac{3}{2} r(t-2) + r(t-3) - u(t-6)$

H.W

* The system is processing the signals.

$$\xrightarrow[\text{I/P}]{\text{Input}} \boxed{\text{Sys.}} \xrightarrow[\text{O/P}]{\text{Output}}$$

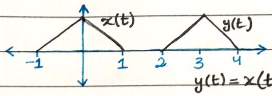
* Classification of sys:

1. linear or nonlinear sys.
2. Time varying or time invariant sys.
3. Causal or non Causal sys.
4. Memory (dynamic) or Memoryless (static) sys.
5. Continuous or Discrete sys.
6. Invertable or noninvertable sys.
7. stable or Unstable sys.

* $x(t) \to \boxed{\text{Sys.}} \to y(t)$ Continuous sys.

* $x(t) \to \boxed{\text{Sys.}} \to y[n]$ Discrete sys.

* $x(t) \to \boxed{\text{sys.}} \xrightarrow{y(t)} \boxed{\text{sys.}} \to x(t)$ invertable sys.

* 

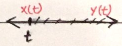
$y(t) = x(t-3)$

Causal sys. when the input proceeds the output, or at the same time

Ex:

* $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \to$ non-Causal

* $y(t) = \int_{t}^{\infty} x(\tau)\, d\tau \to$ Causal



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* $\lim_{t \to \infty} x(t) = K \rightarrow$ Bounded input (BI)

* $\lim_{t \to \infty} y(t) = N \rightarrow$ Bounded output (BO)

(K, N are constants)

$\rightarrow$ BIBO $\rightarrow$ Stable sys

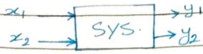
* إذا ال Output ظهر قبل أو بعد ال Input ← Memory sys (dynamic)

وإذا ال Input & Output ظهروا بنفس اللحظة ← Memoryless (Static)

- $x(t)$ without shifting & scalling $\rightarrow$ Memoryless (static).

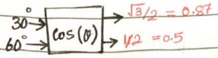
- $y(t) = x(t/4) \rightarrow$ dynamic
- $y(t) = 4 + x(t) \rightarrow$ static
- $y(t) = \sqrt{x(t)} \rightarrow$ static
- $y(t) = x^2(t) \rightarrow$ static
- $y(t) = x(t-2) \rightarrow$ dynamic
- $y(t) = e^{-x(t)} \rightarrow$ static

* linear or non-linear sys.



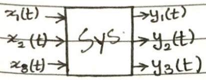
$\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha y_1(t) + \beta y_2(t)$.

Ex:



$(30+60) = 90 \longrightarrow \underset{\cos 90^\circ}{0} \neq 0.87 + 0.5$

∴ Non linear Sys.



$x_3 = \alpha x_1 + \beta x_2$

$y_3 = \alpha y_1 + \beta y_2$

$x_1 \rightarrow y_1$

$x_2 \rightarrow y_2$

$x_3 \rightarrow y_3$

$\alpha x_1 + \beta x_2 \overset{?}{\longrightarrow} \alpha y_1 + \beta y_2$

if yes ∴ the sys. is "linear", ow. it's "non linear."

Ex:

$y(t) = x(t) + a$

$y_1 = x_1 + a$

$y_2 = x_2 + a$ $\quad\rightarrow\quad \alpha y_1 + \beta y_2 = \alpha x_1 + \beta x_2 + a$

$y_3 = x_3 + a$

the sys. is non-linear.

Ex: $y(t) = a\,x(t)$.

$y_1 = a x_1$ $\quad\rightarrow\quad \alpha y_1 + \beta y_2 = a(\alpha x_1 + \beta x_2)$

$y_2 = a x_2$ $\quad\quad = a\alpha x_1 + a\beta x_2$

$y_3 = a x_3$

∴ the sys is linear.

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Ex: $y(t) = \frac{dx(t)}{dt} + x(t)$

$y_1 = \frac{dx_1}{dt} + x_1$

$y_2 = \frac{dx_2}{dt} + x_2$ $\rightarrow \alpha y_1 + \beta y_2 = \frac{d}{dt}(\alpha x_1 + \beta x_2) + (\alpha x_1 + \beta x_2)$

$y_3 = \frac{dx_3}{dt} + x_3$

$\alpha y_1 + \beta y_2 = \frac{d\alpha x_1}{dt} + \frac{d\beta x_2}{dt} + \alpha x_1 + \beta x_2$

∴ the sys is linear.

Ex: $y(t) = e^{x(t)}$

$y_1 = e^{x_1}$

$y_2 = e^{x_2}$ $\rightarrow \alpha y_1 + \beta y_2 = e^{\alpha x_1 + \beta x_2}$

$y_3 = e^{x_3}$ $= e^{\alpha x_1} \cdot e^{\beta x_2} \neq e^{\alpha x_1} + e^{\beta x_2}$

∴ The sys is non linear.

Ex: $y(t) = \sqrt{x(t)}$

$y_1 = \sqrt{x_1}$ $\rightarrow \alpha y_1 + \beta y_2 = \sqrt{\alpha x_1 + \beta x_2}$

$y_2 = \sqrt{x_2}$ $\neq \sqrt{\alpha x_1} + \sqrt{\beta x_2}$

$y_3 = \sqrt{x_3}$ ∴ Non-linear

Ex: $y(t) = \int x(t)\, dt$.

$y_1 = \int x_1$ $\rightarrow \alpha y_1 + \beta y_2 = \int \alpha x_1 + \beta x_2$

$y_2 = \int x_2$ linear

$y_3 = \int x_3$

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Ex:
$y(t) = x^2(t)$

$y_1 = x_1^2$

$y_2 = x_2^2$

$y_3 = x_3^2$

$\rightarrow \alpha y_1 + \beta y_2 = (\alpha x_1 + \beta x_2)^2$

$= \alpha^2 x_1^2 + 2\alpha\beta x_1 x_2 + \beta^2 x_2^2$

$\therefore$ non-linear

Ex:
$y(t) = x(t) \cos t$

linear

Ex:
$y(t) = \cos(x(t))$

non-linear

Ex:
$\frac{dy}{dt} + t^2 y(t) = (2t+3)\, x(t)$

- $\frac{dy_1}{dt} + t^2 y_1 = (2t+3)\, x_1$
- $\frac{dy_2}{dt} + t^2 y_2 = (2t+3)\, x_2$
- $\frac{dy_3}{dt} + t^2 y_3 = (2t+3)\, x_3$

$\rightarrow \alpha \frac{dy_1}{dt} + \beta \frac{dy_2}{dt} + t^2 \alpha y_1 + t^2 \beta y_2 = (2t+3)\alpha x_1 + (2t+3)\beta x_2$

$\therefore$ the sys. is linear.

Ex:
$y(t) = x(t) + 2$

$y_1 = x_1 + 2$

$y_2 = x_2 + 2$

$y_3 = x_3 + 2$

$\rightarrow \alpha y_1 + \beta y_2 = \alpha x_1 + \beta x_2 + 2$

$\therefore$ non-linear.

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Subject $\square * \square \rightarrow$ linear

$\square + \square \rightarrow$ non linear.

(لما يكون في t ضمن ال sys يطلع TV)

$\sin t$, $(\_\_)^t$, e^{-t}

Date No.

* Time Varying (TV.) & time invariant sys. (TIS)

$x(t) \rightarrow \boxed{\text{sys.}} \xrightarrow{y(t)} \boxed{\text{delay } t_0} \rightarrow y(t-t_0) = y_1$

$x(t) \rightarrow \boxed{\text{delay } t_0} \xrightarrow{x(t-t_0)} \boxed{\text{sys}} \rightarrow y_2$

* if $y_1 = y_2$

↳ The sys. called (TIS).

* $y_1 \neq y_2 \rightarrow$ TV.

Ex:

$y(t) = x(t) + 4$.

$x(t) - \boxed{\text{sys}} \xrightarrow{x(t)+4} \boxed{t_0} - x(t-t_0) + 4 = y_1$

$x(t) - \boxed{t_0} \xrightarrow{x(t-t_0)} \boxed{\text{sys}} - x(t-t_0) + 4 = y_2$

$y_1 = y_2 \therefore$ TIS.

Ex:

$y(t) = e^{-t} x(t)$.

$x(t) - \boxed{\text{sys}} \xrightarrow{e^{-t} x(t)} \boxed{t_0} - e^{-(t-t_0)} x(t-t_0) = y_1$

$x(t) - \boxed{t_0} \xrightarrow{x(t-t_0)} \boxed{\text{sys.}} - e^{-t} x(t-t_0) = y_2$

$\rightarrow y_1 \neq y_2$

$\therefore$ TV

Ex:

$y(t) = \sqrt{x(t)}$

$x(t) - \boxed{\text{sys}} \xrightarrow{\sqrt{x(t)}} \boxed{t_0} - \sqrt{x(t-t_0)} = y_1$

$x(t) - \boxed{t_0} \xrightarrow{x(t-t_0)} \boxed{\text{sys}} - \sqrt{x(t-t_0)} = y_2$

$\rightarrow y_1 = y_2$

$\therefore$ TIS

Ex:

$y(t) = \int x(t)\,dt$.

$x(t)$ — $\boxed{sys}$ — $\int x(t)dt$ — $\boxed{t_0}$ — $\int x(t-t_0)\,d(t-t_0) = y_1$

$x(t)$ — $\boxed{t_0}$ — $x(t-t_0)$ — $\boxed{sys}$ — $\int x(t-t_0)\,dt = y_2$

$y_1 \neq y_2$

$\therefore$ TV

* Continous linear time Invariant sys *

LTIS, CLTIS.

(*) $x(t)$ (I/P, Delta) — $\boxed{LTIS\ H(t)}$ — $y(t)$ (O/P, Response)

- $H(t)$: Impulse response.

* Convolution *

$x(t)$ — $\boxed{h(t)}$ — $y(t)$

- $y(t) = x(t) * h(t)$

↳ convolution sign

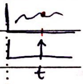
- $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$ → convulution integeration

① reflection
② shift

If $x(t) = \delta(t)$. for LTIS.

$\delta(t)$ — $\boxed{h(t)}$ — $y(t) \overset{??}{=} h(t)$

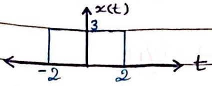
- $y(t) = \delta(t) * h(t)$.
- $y(t) = \int_{-\infty}^{\infty} \delta(t-\tau)\, h(\tau)\, d\tau$

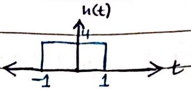


$= \int \delta(t-\tau)\, h(t)\, d\tau$.

$= \underbrace{h(t)}_{const.} \underbrace{\int \delta(t-\tau)\, d\tau}_{1}$ $\therefore h(t)$

Ex: Assume "LTIS" has



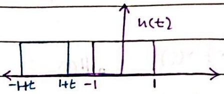


find the responce, (O/P), $y(t)$

$y(t) = x(t) * h(t)$

نقلب الأقل عرضاً

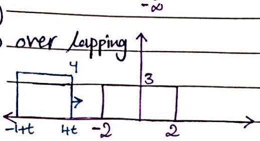
- $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$



① flip

② shift by t to the left

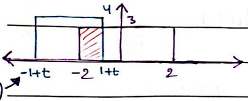
① no over lapping



$y(t) = 0$

$t+1 < -2 \rightarrow \boxed{t=-3}$

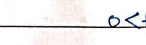
②



- $y(t) = \int_{-2}^{1+t} (3)(4)\, d\tau = 12\tau \Big]_{-2}^{1+t}$

$= 12(t+1) - 12(-2)$

$= 12t + 12 + 24 = 12t + 36$



$-2 < t+1 \leq 0$

$-3 < t \leq 1$

③

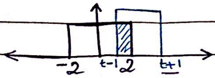
- $y(t) = \int_{t-1}^{1+t} (3)(4)\, d\tau$

$= 12\tau \Big]_{1-t}^{1+t} = 12(1+t) - 12(1-t)$

$= 24$

$0 < t+1 < 2 \rightarrow -1 < t < 1$

(4)



• $y(t) = \int_{t-1}^{2} (3)(4)\, d\tau$

$= 12\tau \Big]_{t-1}^{2} \rightarrow 12(2-t+1) = 36-12t$

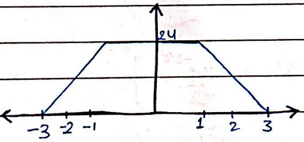
$2 < t+1 \leq 4$

$1 < t \leq 3$

عرضه 2

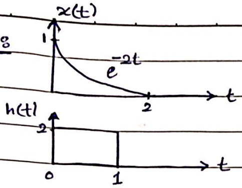
(5) $y(t) = 0 \qquad t+1 > 4 \rightarrow t > 3$

$$y(t) = \begin{cases} 0 & , t < -3 \\ 12t+36 & , -3 < t < -1 \\ 24 & , -1 < t < 1 \\ 36-12t & , 1 < t \leq 3 \\ 0 & , t > 3 \end{cases}$$

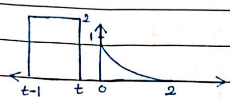


* نهاية ال O/P ← h نهاية + نهاية I/P
بداية ال O/P ← h بداية + بداية I/P

Ex:

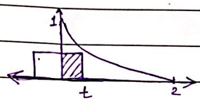


(1)



$y(t) = 0 \quad , \; t<0$

(2)



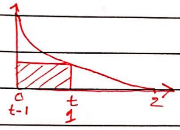
* إذا كانوا نفس الـ width ماني داخل للإنطباق التام

$$\bullet\; y(t) = \int_0^t e^{-2\tau} \cdot 2 \, d\tau$$

$$= \frac{2e^{-2\tau}}{-2}\Bigg]_0^t = 1 - e^{-2t}$$

$0<t<1$

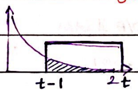
(3)



$$\bullet\; y(t) = \int_0^1 e^{-2\tau}(2)\, d\tau = 1 - e^{-2}$$

$1<t\leq 2$

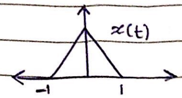
(4)

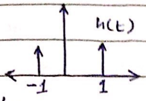


$$\bullet\; y(t) = \int_{t-1}^{2} e^{-2\tau}(2)\, d\tau = \frac{2e^{-2\tau}}{-2}\Bigg]_2^{t-1}$$

$2<t\leq 3$

Ex:





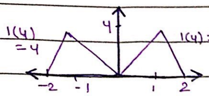
conv. with δ

↳ shift (δ تبعد)

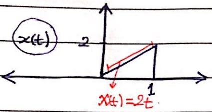
$y(t) = x(t) * h(t)$

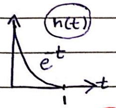
$\bullet\ h(t) = \delta(t-1) + \delta(t+1)$

$= x(t) * [\delta(t-1) + \delta(t+1)]$



Ex: LTIS

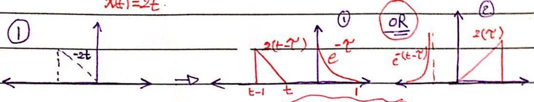




* لا يكون نفس الـ width
اح يتداخل بنقطة

* find $y(t)$!

①

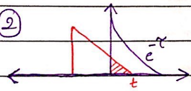


$\left[\begin{array}{l} t \to t-\tau \\ 2(t) \to 2(t-\tau) \end{array}\right]$

$(\bullet$ اللي بيظل نفسه $t \to \tau)$

$y(t) = 0 \ , t < 0$

②



$y(t) = \int_0^t e^{-\tau}(2(t-\tau))\, d\tau$

$= \int_0^t 2te^{-\tau} - \int_0^t 2\tau e^{-\tau}\, d\tau = \left.\frac{2te^{-\tau}}{-1}\right]_0^t -$

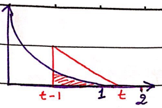
by parts ...

$\left(-2te^{-t} - 2e^{-t} - (-2)\right)$

$0 < t < 1$

2τ (+)	$e^{-\tau}$
2 (−)	$-e^{-\tau}$
0	$e^{-\tau}$

(3)

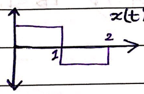


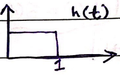
$$y(t) = \int_{t-1}^{1} e^{-\tau} \cdot (2t-2\tau) \, d\tau = \dots$$

$1 < t \leq 2$

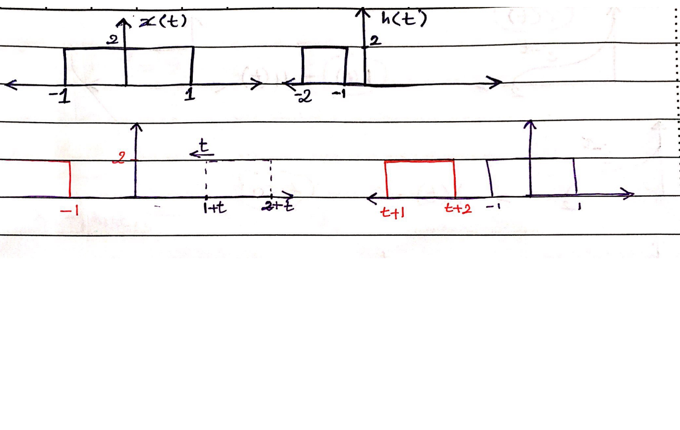
(4) $y(t) = 0 \rightarrow t > 2$

Ex:

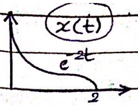


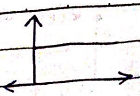


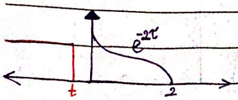
Ex:



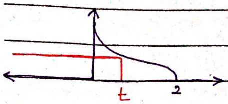
Ex:







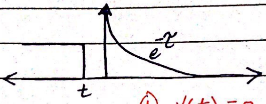
• $y(t) = 0$ $\quad (t<0)$



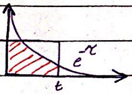
• $y(t) = \int_0^t (1)e^{-2\tau}\,d\tau = \left.\frac{e^{-2\tau}}{-2}\right]_0^t = \frac{1}{2} - \frac{1}{2}e^{-2t}$ $\quad (t>0)$

Ex: $x(t) = e^{-t}u(t)$

$h(t) = u(t)$.

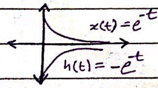


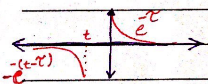
① $y(t) = 0$



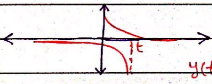
• $y(t) = \int_0^t (1)(e^{-\tau})\,d\tau = -e^{-\tau}\Big]_0^t = 1 - e^{-t}$ $\quad (t>0)$

Ex:





$= -e^{\tau}\cdot e^{-t}$



$y(t) = \int_0^t e^{-\tau}(-e^{\tau})(e^{-t})\,d\tau$

$= -\int_0^t e^{-t}\,d\tau = -e^{-t}\tau\Big]_0^t$

$= 0 - te^{-t} = -te^{-t}$ $\quad (t>0)$

$x \text{ — } \boxed{h(t)} \text{ — } y = x * h(t)$

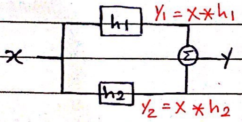
* Cascade *

$x \text{ — } \boxed{h_1(t)} \xrightarrow{y_1} \boxed{h_2(t)} \text{ — } y$

$$y_1 = x * h_1(t)$$
$$y = y_1 * h_2(t)$$
$$= x * h_1(t) * h_2(t).$$

$x \text{ — } \boxed{h_1 * h_2} \text{ — } y$

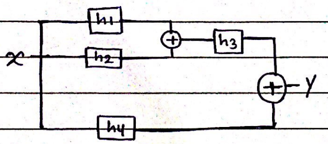
Ex:

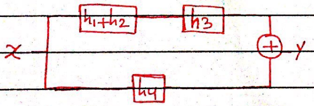


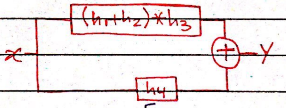
- $y = y_1 + y_2$
  $= x * [h_1 + h_2]$

$x \text{ — } \boxed{h_1 + h_2} \text{ — } y$

Ex:







$$y = x * [((h_1+h_2) * h_3) + h_4]$$

$x \text{ — } \boxed{[(h_1+h_2) * h_3] + h_4} \text{ — } y$

* $\sin(-\theta) = -\sin\theta$
* $\cos(-\theta) = \cos\theta$
* $\tan(-\theta) = -\tan\theta$

$\pm \cos 3t =$

* $\sin(A+B) = \sin A \cos B + \cos A \sin B$
* $\cos(A \mp B) = \cos A \cos B \pm \sin A \sin B$
* $\sin 2x = 2 \sin x \cos x$
* $\cos 2x = \cos^2 x - \sin^2 x$
  $= 2\cos^2 x - 1$
  $= 1 - 2\sin^2 x$
* $\sin A + \sin B = 2 \sin \frac{(A+B)}{2} \cos \frac{(A-B)}{2}$
* $\sin A - \sin B = 2 \cos \frac{(A+B)}{2} \sin \frac{(A-B)}{2}$
* $\cos A - \cos B = 2 \cos \frac{(A+B)}{2} \cos \frac{(A-B)}{2}$
* $\cos A + \cos B = -2 \sin \frac{(A+B)}{2} \sin \frac{(A-B)}{2}$
* $\sin A \cos B = [\sin(A+B) + \sin(A-B)]/2$
* $\cos A \sin B = [\sin(A+B) - \sin(A-B)]/2$
* $\sin A \sin B = -[\cos(A+B) - \cos(A-B)]/2$
* $\cos A \cos B = [\cos(A+B) + \cos(A-B)]/2$
* $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

Scanned with CamScanner

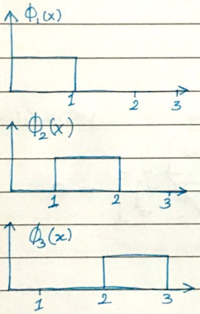
* Fourier Series * (FS)

. ∏ Basis :

• let $\phi_1(x), \phi_2(x), \phi_3(x)$

if $\int_a^b \phi_n(x)\cdot\phi_m(x)\,dx = \begin{cases} 0 & n\neq m \\ 1 & n=m \end{cases}$

Then --- ϕ_1, ϕ_2, ϕ_3 are to be ((Orthonormal basis))



• $\int_0^3 \phi_1\cdot\phi_2\,dx = \int_0^1 (1)(0)\,dx + \int_1^2 0(1)\,dx + \int_2^3 (0)(0) = 0.$

• $\int_0^3 \phi_1\cdot\phi_3 = 0.$

• $\int_0^3 \phi_2\cdot\phi_3 = 0.$

"Then they are Orthogonal Basis"

* Orthogonal functions *

Ex: $\{\cos x, \cos 2x, \cos 3x, \ldots, \cos nx\}$, $\{-\pi, \pi\}$

نقارن أول مرة.

$$\rightarrow \int_{-\infty}^{\infty} \cos x \cdot \cos 2x \, dx = \int_{-\pi}^{\pi} \frac{1}{2}\cos(2x-x) + \frac{1}{2}\cos(2x+x) \, dx.$$

$$= \frac{1}{2}\sin x + \frac{1}{2}\sin\frac{3x}{3}\Big]_{-\pi}^{\pi} \rightarrow 0+0=0$$

* Orthogonal. → تكامله مع غيره = 0
* Ortho Normal → تكامله مع نفسه = 1

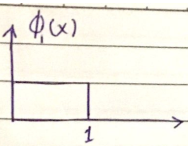
$$\int_{-\pi}^{\pi} \cos nx \cdot \cos mx \, dx \quad , \; m \,\&\, n \text{ are int num.}$$

هون لما أعيد بالتكامل --

$$= \frac{1}{2}\left[\int_{-\pi}^{\pi} \cos(n-m)x + \cos(n+m)x \, dx\right]$$

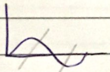
$$= \frac{1}{2}\left[\sin\frac{\cancel{(n-m)}}{\cancel{(n-m)}}x + \sin\frac{\cancel{(n+m)}}{\cancel{(n+m)}}x\Big]_{-\pi}^{\pi}\right] = 0.$$

Ex: $\phi_1(x)$



$\int \phi_1(x) \cdot \phi_1(x)\,dx = \int (1)\,(1)\,dx = 1$ ∴ OrthoNormal

Ex:



$$\int_{-\pi}^{\pi} \cos x \cdot \cos x \, dx.$$

$$= \int \frac{1}{2} + \cancel{\frac{1}{2}\cos 2x}\, dx = \frac{1}{2}x\Big]_{-\pi}^{\pi} = \pi$$

"orthogonal but ~~not orthonormal~~"

$\hookrightarrow = E$

Ex: $\{\cos x, \cos 2x, ---\}$

$\left(\frac{1}{\sqrt{\pi}}\cos x, \frac{1}{\sqrt{\pi}}\cos 2x, ---\right.$ عشان نخليها (orthonormal) $\frac{1}{\sqrt{E}}$

* periodic sig → set of orthogonal sig

Ex: $\{e^{j\omega x}, e^{j2\omega x}, e^{j3\omega x}, ---\}$

$\int e^{j\omega x} \cdot e^{-j\omega 2x}\,dx$

$\int e^{j\omega x} \cdot e^{-j\omega x}\,dx.$

"تغيير اشارة لـ J"

complex^2
$\hookrightarrow n \times n$
الاشي ضرب نفسه
مش comp^2.

* FS → exponential.
→ Trigonometric.
→ compact.

* Any periodic Signal can describe as:

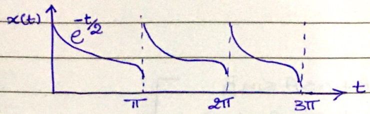
* $x(t) = C_o + \sum_{n=-\infty}^{\infty} C_n . e^{j\omega_o n t}$

we have to find C_o & C_n

• $C_o = \frac{1}{T} \int_{-T}^{T} x(t) \, dt$ ((zero term DC-value))

• $C_n = \frac{1}{T} \int_{0}^{T} x(t) . e^{-j\omega_o n t} \, dt$.

Ex: Find the exp



- $T_0 = \pi$
- $\omega_0 = \frac{2\pi}{T_0} = \boxed{2}$
- $C_0 = \frac{1}{T}\int_0^T x(t)\, dt$

$$= \frac{1}{\pi}\int_0^{\pi} e^{-t/2}\, dt = \frac{1}{\pi}\left.\frac{e^{-t/2}}{-1/2}\right|_0^{\pi} = \frac{1}{\pi}\left[-2e^{-t/2}\Big|_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-2 - e^{-\pi/2}\right] = K_{\text{const.}}$$

- $C_n = \frac{1}{T}\int_0^T x(t) \cdot e^{-j\omega_0 n t}\, dt$

$$= \frac{1}{\pi}\int_0^{\pi} e^{-t/2} \cdot e^{-j2nt}\, dt \rightarrow \frac{1}{\pi}\int_0^{\pi} e^{-t[\frac{1}{2}+j2n]}\, dt$$

$$= \frac{1}{\pi}\left[\left.\frac{e^{-t(\frac{1}{2}+j2n)}}{-[\frac{1}{2}+2jn]}\right|_0^{\pi}\right] =$$

$$= C_0 + \sum_{n=-\infty}^{\infty} C_n \cdot e^{-j\pi n t}$$

$$= C_0 + e^{-j\pi t} + e^{j\pi t} + e^{j2\pi t} + e^{-j2\pi t} + \text{- - - -}$$

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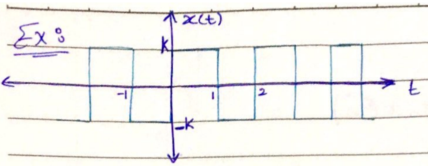
Ex:



find the exp f^n.

* $T_0 = 2$ * $\omega_0 = \frac{2\pi}{2} = \pi$

* $C_n = \frac{1}{2}\int_{-1}^{1} x(t)\, e^{-j\omega n t}\, dt$

$$= \frac{1}{2}\int_{-1}^{0} (-k)\, e^{-j\pi n t}\, dt + \frac{1}{2}\int_{0}^{1} (k)\, e^{-j\pi n t}\, dt.$$

$$= \frac{-k}{2}\left[\frac{e^{-jn\pi t}}{-jn\pi}\right]_{-1}^{0} + \frac{k}{2}\left[\frac{e^{-j\pi n t}}{-j\pi n}\right]$$

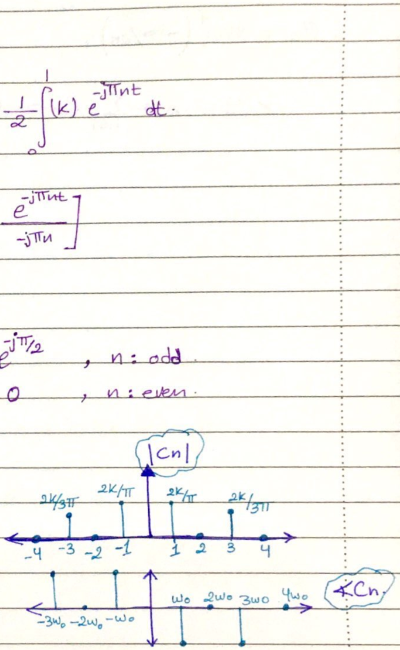
$$= \frac{k}{jn\pi}\left[1 - \cos(n\pi)\right]$$

$$= \begin{cases} \frac{2K}{jn\pi} = \frac{-j2K}{n\pi} = \frac{2K}{n\pi} e^{-j\pi/2} & , n: \text{odd} \\ 0 & , n: \text{even} \end{cases}$$

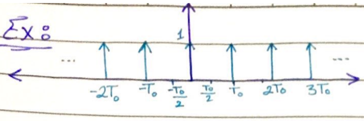
* $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

* $|C_n| = \left|\frac{2K}{n\pi}\right|$, n: odd

* $\measuredangle C_n = \theta_n = \begin{cases} \pm\pi/2 & , n: \text{odd} \\ 0 & , n: \text{even} \end{cases}$



Ex:

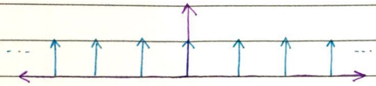


$\delta_{T_0}(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_0)$

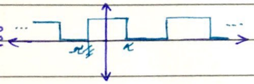
$*\; C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta_{T_0}(t)\cdot e^{-jn\omega_0 t}\,dt$

$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)\, e^{-jn\omega_0 t}\,dt = \frac{1}{T_0} e^{0} = \frac{1}{T_0}$

$\rightarrow \delta_{T_0}(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T_0} e^{jn\omega_0 t} = \sum_{-\infty}^{\infty} \delta(t-nT_0)$.



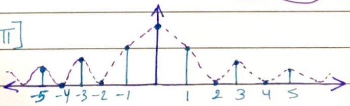
Ex:

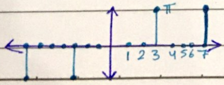


slide 25

$\left(\mathrm{sinc} = \frac{\sin x}{x}\right)$

- $C_0 = \frac{1}{T_0}\int_{-\tau}^{\tau} 1\,dt = \frac{2\tau}{T_0}$
- $C_n = \frac{1}{T_0}\int_{-\tau}^{\tau} (1)\, e^{-j\omega_0 n t}\,dt = \frac{1}{n\pi}\sin\left(\frac{n2\pi\tau}{T_0}\right) \rightsquigarrow \frac{1}{2}\mathrm{sinc}\left(\frac{n}{2}\right)$
- $\theta_n = \angle C_n = [0, \pm\pi]$





* Trig. F.S. *

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cdot \cos \omega_0 n t + \sum_{n=1}^{\infty} b_n \sin \omega_0 n t .$$

F.S. Coefficient $\rightarrow a_0, a_n, b_n$.

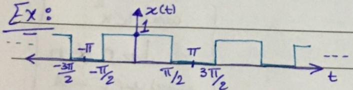
- $a_0 = \frac{1}{T} \int_0^T x(t)\, dt.$
- $a_n = \frac{2}{T} \int_0^T x(t) \cdot \cos \omega_0 n t \; dt.$
- $b_n = \frac{2}{T} \int_0^T x(t) \cdot \sin \omega_0 n t \; dt.$

(*) If $x(t)$ is an $\boxed{\text{even}}$ f^n $\rightarrow$ $b_n = 0$

& if $x(t)$ is an $\boxed{\text{odd}}$ f^n $\rightarrow$ $a_n = 0$

غالباً وليس دائماً. $a_0 = 0$

Ex:



(*) This is even f^n

$\hookrightarrow b_n = 0$

- $T_0 = 2\pi$
- $\omega_0 = \frac{2\pi}{T_0} = 1.$

- $a_0 = \frac{1}{T_0} \int_0^T x(t)\, dt \rightarrow \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} (1)\, dt \rightarrow \frac{1}{2\pi} t \Big|_{-\pi/2}^{\pi/2} \rightarrow = \frac{1}{2}$

- $a_n = \frac{2}{T} \int_0^T x(t) \cos \omega_0 n t \; dt$

$$= \frac{2}{2\pi} \int_{-\pi/2}^{\pi/2} (1) \cos(1) n t \; dt \rightarrow \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos n t \; dt = \frac{2}{n\pi} \sin \frac{n\pi}{2}$$

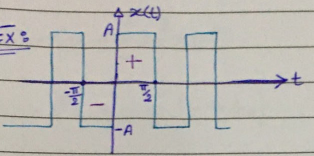
$$a_n = \begin{cases} 0 & , n: \text{even} \\ 2/\pi n & , n: \text{odd } 1, 5, 9, \ldots \\ -2/\pi n & , n: \text{odd } 3, 7, 11, \ldots \end{cases}$$

NOTEBOOK

$$\rightarrow x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \omega_0 n t + \cancel{\sum_{n=1}^{\infty} b_n \sin \omega_0 n t}^{\,0}$$

$$= \underbrace{\tfrac{1}{2}}_{} + \underbrace{\tfrac{2}{\pi}\cos(1)(1)t}_{n=1} + \underbrace{0}_{n=2} + \underbrace{\tfrac{-2}{3\pi}\cos 3t}_{n=3} + \underbrace{0}_{n=4} + \underbrace{\tfrac{2}{5\pi}\cos 5t}_{n=5} + \cdots$$

Ex:



the are in one period

* Odd Sig. $\longrightarrow$ $a_n = 0 \longrightarrow a_0 = 0$

* $T_0 =$

$$\boxed{\begin{aligned} T_0 &= T_0 \\ \omega_0 &= \frac{2\pi}{T_0} \end{aligned}}$$

* $b_n = \dfrac{2}{T}\displaystyle\int_0^T x(t) \sin \omega_0 n t \, dt$

$$= \frac{2}{T}\left[\int_{-T/2}^{0} (-A) \sin \frac{2\pi}{T_0} n t \, dt + \int_0^{T/2} (A) \sin \frac{2\pi}{T_0} n t \, dt\right]$$

$$= \frac{2}{T}\left[\left[A \; \frac{-\cos \omega_0 n t}{\omega_0 n}\Big|_0^{T/2}\right] - \left[A \; - \frac{\cos \omega_0 n t}{\omega_0 n}\Big|_{-T/2}^{0}\right]\right] = \cdots$$

$$= \frac{2A}{n\pi}\left[1 - \cos n\pi\right] \rightarrow \text{نعوض}$$

n	b_n
1	$\frac{2A}{\pi}[2] = \frac{4A}{\pi}$
2	0

• $b_n = \begin{cases} 0 & n: \text{even} \\ \dfrac{4A}{n\pi} & n: \text{odd} \end{cases}$

• $x(t) = \cancel{a_0}^{\,0} + \cancel{\sum a_n \cos \omega_0 n t}^{\,0} + \sum b_n \sin \omega_0 n t$

$\quad \hookrightarrow = \dfrac{4A}{\pi} \sin \omega_0 t + 0 + \dfrac{4A}{3\pi} \sin 3\omega_0 t + 0 + \dfrac{4A}{5\pi} \sin 5\omega t \; \cdots$

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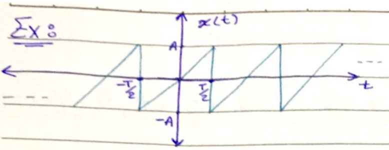
Ex:



* معادلة خط مستقيم ---

. $x(t) = \left(\frac{2A}{T} t\right)$

* $T_o = T$ * $\omega_o = \frac{2\pi}{T}$

* Odd Sig.

↳ $a_n = 0$ & $a_o = 0$.

* $b_n = \frac{2}{T}\int_0^T x(t)\sin \omega_o n t \; dt$.

$= \frac{2}{T}\int_{-T/2}^{T/2}\left(\frac{2A}{T}t\right)\sin \omega_o n t \; dt$

↳ $= \frac{-2A}{n\pi} \cos n\pi$.

. $b_n =$

$x(t) = \cancel{a_o}^{\,0} + \cancel{\sum a_n \cos}^{\,0} \omega_o n t + \sum b_n \sin \omega_o n t$.

$= \frac{-2A}{\pi}\cos\pi . \sin \omega_o t + ----$

* Compact F.S. *

• $C_n \cos(n\omega_o t + \theta_n)$.

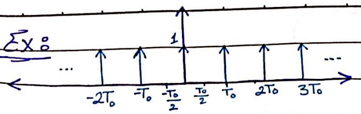
• $C_n = \sqrt{a_n^2 + b_n^2}$.

• $C_o = a_o = \frac{1}{T}\int_0^T x(t)\, dt$.

• $\theta_n = \tan^{-1}(-b_n/a_n)$.

$$x(t) = C_o + \sum_{n=1}^{\infty} C_n \cos(n\omega_o t + \theta_n)$$

Ex:

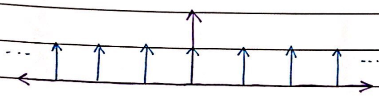


$$\delta_{T_0}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0).$$

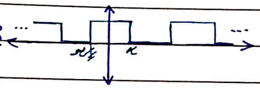
$$* \; C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta_{T_0}(t)\cdot e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{T_0} e^{0} = \frac{1}{T_0}$$

$$\rightarrow \delta_{T_0}(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T_0} e^{jn\omega_0 t} = \sum_{-\infty}^{\infty} \delta(t - nT_0).$$



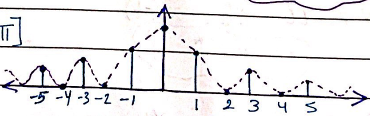
Ex:

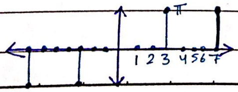


slide 25

$$\left(\text{sinc} = \frac{\sin x}{x}\right)$$

- $C_0 = \frac{1}{T_0}\int_{-\tau}^{\tau} 1\, dt = \frac{2\tau}{T_0}$
- $C_n = \frac{1}{T_0}\int_{-\tau}^{\tau} (1)\, e^{-j\omega_0 n t}\, dt = \frac{1}{n\pi}\sin\left(\frac{n2\pi\tau}{T_0}\right) \rightsquigarrow \frac{1}{2}\text{sinc}\left(\frac{n}{2}\right)$
- $\theta_n = \angle C_n = \boxed{[0, \pm\pi]}$





*properties of fourier series:

- let $x(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{jn\omega_o t}$

$\hookrightarrow \alpha_n = \frac{1}{T_o} \int x(t) e^{-jn\omega_o t} dt$

[1] shifting $y(t) = x(t) + B$.

- $\beta_n = \alpha_n \quad , n \neq 0$
- $\beta_o = \alpha_o + B$

((only DC-value will be affected)).

[2] Scaling. $y(t) = Ax(t)$.

- $\beta_n = A\alpha_n$.

→ Shift & scale $y(t) = Ax(t) + B$.

- $\beta_o = A\alpha_o + B$
- $\beta_n = A\alpha_n$

[3] Shifting ((time shift)). $y(t) = x(t - t_o)$

phase shift

- $\beta_n = \alpha_n e^{-jn\omega_o t_o}$

[4] Reversal. $y(t) = x(-t)$.

- $\beta_n = \alpha_n^*$ "conj" $\alpha_n \neq -\alpha_n$

[5] Scaling "x-axis" $y(t) = x(at)$

- $\beta_n = \alpha_n$.

* linearity :

- $z(t) = k_1 x(t) + k_2 y(t)$. , k_1 & k_2 are constants.

- $y_n = k_1 \alpha_n + k_2 \beta_n$.

* Multiplication :

- $z(t) = x(t)\, y(t)$.

- $y_n = \alpha_n * \beta_n$ "convolution"

* → Mul. in time domain give us → conv. in phase domain.
  → & conv. in time domain → Mul. in phase domain.

* differentiation :

- $y(t) = \frac{dx(t)}{dt}$

- $\beta_n = j n \omega_0 \alpha_n$.

$\boxed{\angle \alpha_n + \frac{\pi}{2}}$

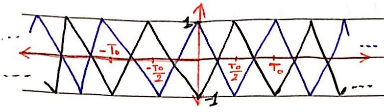
* Integration :

- $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$.

- $\beta_n = \alpha_n / j n \omega_0$

$\boxed{\angle \alpha_n - \frac{\pi}{2}}$

* half wave symmetrical.



- $b_n = 0$
- $\alpha_n = \frac{2}{\frac{T_o}{2}} \int_0^{T_o/2} x(t) \cos(n\omega_o t)\, dt.$

$\frac{4}{T_o}$

* parseval's theorem:

- power Sig. [1] $\bullet\ P_x = \frac{1}{T_o} \int |x(t)|^2 dt$
- power in time domain
  =
  power in phase domain.

[2] $\bullet\ P_x = \sum_{-\infty}^{\infty} |\alpha_n|^2$ → exp. FS.

[3] $\bullet\ P_x = A_o^2 + \frac{1}{2} \sum_{n=1}^{\infty} A_n^2 + \frac{1}{2} B_n^2$ → trigonometric. FS.

* System Response.

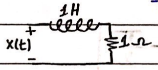
↳ periodic input

$x(t): T_o$ → $h(t)$ → $Y(t): T_o$

FS. $H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt.$

* if periodic:
  $Y(t) = X(t)\, H(\omega)$

Ex:



$x(t) = 4\cos(t) - 2\cos(2t)$, Find $y(t) = ?!$

$\boxed{\omega_1 = 1}$ $\boxed{\omega_2 = 2}$

- $H(\omega) = \frac{1}{1+j\omega}$
- $Y(t) = x(t).H(\omega) = \frac{4\cos(t)}{1+j\omega_1} - \frac{2\cos(2t)}{1+j\omega_2}$

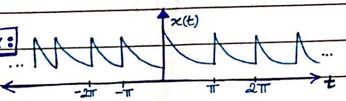
بنحول → magnitude & angle

$= 2\sqrt{2}\cos(t - 45^\circ) - \frac{2}{\sqrt{5}}\cos(2t - 63^\circ).$

* Examples :

Series form	Coefficient Computation	Conversion formulas
1- Trigonometric	$a_0=\frac{1}{T_0}\int f(t)\,dt$	$a_0=C_0=D_0$
2- $f(t)=a_0+\sum^{\infty} a_n\cos n\omega_0 t + b_n\sin n\omega_0 t$	$a_n=\frac{2}{T_0}\int f(t)\cos n\omega_0 t\,dt$	$a_n-jb_n=C_n e^{j\theta_n}=2D_n$
3-	$b_n=\frac{2}{T_0}\int f(t)\sin n\omega_0 t\,dt$	$a_n+jb_n=C_n e^{-j\theta_n}=2D_{-n}$
4- Compact trigonometric	$C_0=a_0$	$C_0=D_0$
5- $f(t)=C_0+\sum_{n=1}^{\infty} C_n\cos(n\omega_0 t+\theta_n)$	$C_n=\sqrt{a_n^2+b_n^2}$	$C_n=2\lvert D_n\rvert \quad , n\geq 1$
6-	$\theta_n=\tan^{-1}\left(\frac{-b_n}{a_n}\right)$	$\theta_n=\angle D_n$
7- exponential		
8- $f(t)=\sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$	$D_n=\frac{1}{T_0}\int f(t)\,e^{-jn\omega_0 t}\,dt$	

Ex:



Sol.

- $T_0=\pi \quad , \quad \omega_0=\frac{2\pi}{\pi}=2$.
- $x(t)=\sum_{n=-\infty}^{\infty} D_n e^{j2nt}$.
- $D_n=\frac{1}{T_0}\int f(t)\,e^{-jn2t}\,dt$

$$=\frac{1}{\pi}\int_0^{\pi} e^{-t/2}\cdot e^{-jn2t}\,dt = \frac{1}{\pi}\int_0^{\pi} e^{-t(\frac{1}{2}+j2n)}\,dt = \left.\frac{-e^{-t(\frac{1}{2}+j2n)}}{\pi(\frac{1}{2}+j2n)}\right|_0^{\pi} = \frac{0.504}{1+j4n}$$

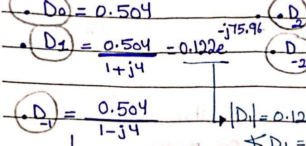
$$=0.504\sum_{n=-\infty}^{\infty}\frac{e^{-jn2t}}{1+j4n}$$

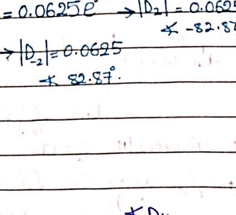
$$=0.504\left[1+\frac{e^{j2t}}{1+j4}+\frac{e^{j4t}}{1+j8}+\frac{e^{j6t}}{1+j12}+\frac{e^{-j2t}}{1-4j}+\frac{e^{-j4t}}{1-j8}+\frac{e^{-j6t}}{1-12j}+\cdots\right]$$

D_0

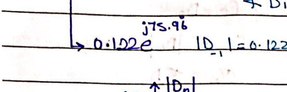
$$D_n = \frac{0.504}{1+j4n}$$

- $D_0 = 0.504$
- $D_1 = \frac{0.504}{1+j4} = 0.122e^{-j75.96}$ → $|D_1| = 0.122$, $\angle D_1 = -75.96°$
- $D_{-1} = \frac{0.504}{1-j4}$ → $0.122e^{j75.96}$, $|D_{-1}| = 0.122$, $\angle 75.96°$
- $D_2 = \frac{0.504}{1+j8} = 0.0625e^{-j82.87°}$ → $|D_2| = 0.0625$, $\angle -82.87°$
- $D_{-2} = \frac{0.504}{1-j8}$ → $|D_{-2}| = 0.0625$, $\angle 82.87°$





Ex8



- $V(t) = \frac{V_m}{T} t$

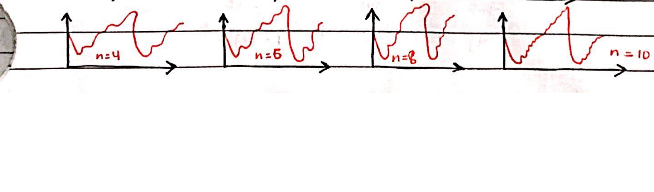
- $a_0 = \frac{1}{T}\int_0^T \left(\frac{V_m}{T}t\right) dt = \frac{V_m}{2}$. "Avg. Value"

- $a_n = \frac{2}{T}\int_0^T \left(\frac{V_m}{T}t\right)\cos(\omega_0 n t)\, dt = 0.$

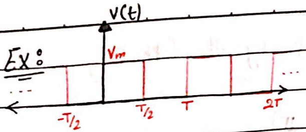
- $b_n = \frac{2}{T}\int_0^T \left(\frac{V_m}{T}t\right)\sin(n\omega_0 t)\, dt = \frac{-V_m}{n\pi}$

$C_n = b_n$, $\theta_n = 90°$.

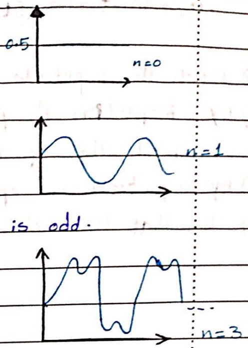
- $V(t) = \frac{V_m}{2} - \frac{V_m}{\pi}\sum_{n=1}^{\infty} \frac{\sin(n\omega_0 t)}{n}$.



Ex:



$c_n = b_n$
$\theta_n = 90^\circ$



$v(t) = V_m \quad 0 \leq T < T/2$

$\quad\;\; = 0 \quad T/2 \leq T < T$

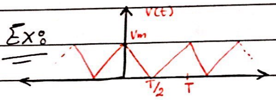
- $a_0 = \frac{1}{T}\int_0^{T/2} V_m \, dt \rightarrow \frac{V_m}{2}$ Avg. Value.

- $a_n = 0$.

- $b_n = \frac{2}{T}\int_0^{T/2} V_m \sin(n\omega_0 t)\, dt = \frac{2V_m}{n\pi}$ if n is odd.

$\rightarrow v(t) = \frac{V_m}{2} + \frac{2V_m}{\pi}\sum_{n \text{ odd}} \frac{\sin(n\omega_0 t)}{n}$

Ex:



$c_n = a_n$
$\theta_n = 0$

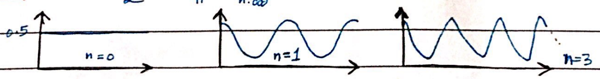
- $v(t) = \begin{cases} \frac{V_m}{T}(T-2T), & 0 \leq t \leq T/2 \\ \frac{V_m}{T}(2T-T), & T/2 \leq t \leq T \end{cases}$

- $b_n = 0$.

- $a_0 = \frac{1}{T}\int_0^{T/2} \frac{V_m}{T}(T-2T)\, dt + \frac{1}{T}\int_{T/2}^{T} \frac{V_m}{T}(2T-T)\, dt = \frac{V_m}{2}$.

- $a_n = \frac{2}{T}\int_0^{T/2} \frac{V_m}{T}(T-2T)\cos(n\omega_0 t)\, dt + \frac{2}{T}\int_{T/2}^{T} \frac{V_m}{T}(2T-T)\cos(n\omega_0 t)\, dt = \begin{cases} 0, & n: \text{even} \\ \frac{4V_m}{n^2\pi^2}, & n: \text{odd} \end{cases}$

$\rightarrow v(t) = \frac{V_m}{2} + \frac{4V_m}{\pi^2}\sum_{n:\text{odd}} \frac{\cos(n\omega_0 t)}{n^2}$.



Q: A periodic signal $x(t)$ is expressed by the following Fourier series:

$$x(t) = 3\cos t + \sin\left(5t - \frac{\pi}{6}\right) - 2\cos\left(8t - \frac{\pi}{3}\right)$$

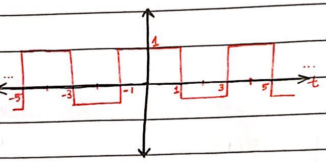
(a) Sketch the amplitude and phase spectra for the trigonometric series.

(b) By inspection of spectra in part (a), sketch the exp. FS spectra.

(c) By inspection of spectra in part (b), write exp. FS. for $x(t)$.

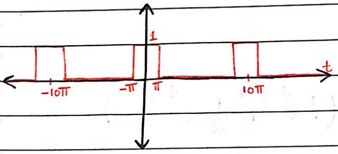
(d) Show that the series found in (c) is equivalent to trigonometric series for $x(t)$.

Q: For each of the periodic sig. shown. Find the compact trigonometric F.S. and sketch the amplitude and phase spectra. If either the sine or cosine terms are absent in the FS. explain why?



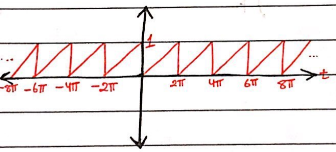
- even sig. $\rightarrow b_n = 0$, $a_0 = 0$.
- $T = 4 \rightarrow \omega = \frac{2\pi}{4} = \pi/2$.
- $a_0 = \frac{1}{\cancel{T}\,4}\int_0^T x(t)\,dt$

$$= \frac{1}{4}\left[\int_{-1}^{1}(1)\,dt + \int_{1}^{3}(-1)\,dt\right] = 0.$$



- $b_n = 0$.
- $a_n = \frac{2}{T}\int_0^T x(t)\cos\omega n t\,dt$

$$= \frac{2}{4}\left[\int_{-1}^{1} 1\cos\frac{\pi}{2}nt\,dt + \int_{1}^{3} -\cos\frac{\pi}{2}nt\,dt\right]$$



$$= \frac{1}{2}\left[\left.\frac{\sin\frac{\pi}{2}nt}{\frac{\pi}{2}n}\right|_{-1}^{1} - \left.\frac{\sin\frac{\pi}{2}nt}{\frac{\pi}{2}n}\right|_{1}^{3}\right]$$

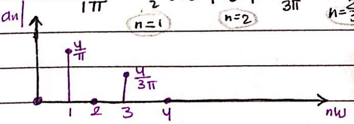
$$= \frac{2}{2\pi n}\left[\sin\frac{\pi}{2}n + \sin\frac{\pi}{2}n - \sin\frac{\pi}{2}3n + \sin\frac{\pi}{2}n\right]$$

$$= \frac{1}{n\pi}\left[3\sin\frac{\pi}{2}n - \sin\frac{3\pi}{2}n\right]$$

$$* \; x(t) = \cancel{a_0} + \sum_{n=1}^{\infty} a_n\cos\omega nt + \cancel{\sum_{n=1}^{\infty} b_n\sin\omega nt}$$

$$= \underbrace{\frac{4}{1\pi}\cos\frac{\pi}{2}(1)t}_{n=1} + \underbrace{0}_{n=2} + \underbrace{\frac{-4}{3\pi}\cos\frac{\pi}{2}3t}_{n=3} + 0 + \cdots$$

$$a_n = \begin{cases} 0, & n \text{ even} \\ \frac{4}{\pi n}, & n: 1, 5, 9, \ldots \\ \frac{-4}{\pi n}, & n: 3, 7, 11, \ldots \end{cases}$$



→ parseval $(P_t = P_f)$.

power in $x(t) = \frac{1}{T} \int x^2(t)\, dt$

$$= \frac{1}{4}\left[\int_{-1}^{1} (1)^2\, dt + \int_{1}^{3} (-1)^2\, dt\right] = 1\ \omega.$$

• $P_f \to \frac{1}{2} \sum |a_n|^2$

$$\to \frac{1}{2}\left[\left(\frac{4}{\pi}\right)^2 + \left(\frac{4}{3\pi}\right)^2 + \left(\frac{4}{5\pi}\right)^2 + \text{-----}\right] \approx \longrightarrow 1$$

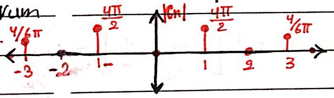
* write $x(t)$ in exp F.S.

• $\cos wt = \frac{1}{2} e^{jwt} + \frac{1}{2} e^{-jwt}$

$$X(t) = \frac{4}{2\pi} e^{j\frac{\pi}{2}t} + \frac{4}{2\pi} e^{-j\frac{\pi}{2}t} - \frac{4}{3\pi}\left(\frac{1}{2} e^{j\frac{3\pi}{2}t} + \frac{1}{2} e^{-j\frac{3\pi}{2}t}\right).$$

$$+ \frac{4}{5\pi}\left(\frac{1}{2} e^{j\frac{5\pi}{2}t} + \frac{1}{2} e^{-j\frac{5\pi}{2}t}\right) + \text{---}$$

• Spectrum



نفس الرسمة

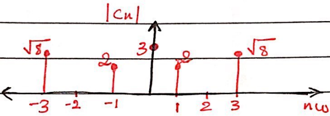
$\ast \frac{1}{2}$ مثل

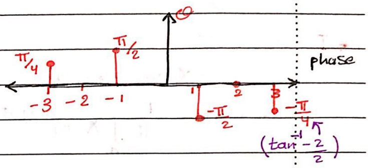
• $P = \sum_{n=-\infty}^{\infty} |C_n|$

ex: $x(t) = (2+j2)e^{-j3t} + j2e^{-jt} + 3 - j2e^{jt} + (2-j2)e^{j3t}$.

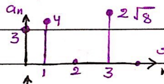
$x(t) = 3 + j2e^{-jt} - j2e^{jt} + \underbrace{(2+2j)}_{\sqrt{8}}e^{-j3t} + (2-2j)e^{j3t}$.

$C_0 \quad C_{-1} \quad C_1 \quad C_{-3} \quad C_3$

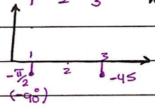




* Write $x(t)$ in Trig. F-S.



كأني حطيت الرسمة الأولى بجهة وحدة.



• $x(t) = 3 + \underbrace{4\cos(t-90)}_{=4\sin t} + 0 + 2\sqrt{8}\cos(3t-45)$

$\omega = 1$
$n = 1$

(FT)

- Convert signal $x(t)$ to $X(t)$ from Time domain to freq domain.

$$X(t) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

* Inverse F.T

$$X(\omega) \rightarrow x(t)$$

$$\therefore x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{+j\omega t}\, d\omega.$$

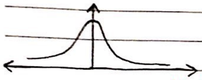
$$x(t) \rightleftharpoons X(\omega).$$

ex: $x(t) = e^{-at}u(t)$

* find $X(\omega)$, spectral of $x(t)$, F.T.

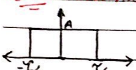
$$X(\omega) = \int_{0}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-t(a+j\omega)}\, dt$$



$$= \left.\frac{e^{-t(a+j\omega)}}{-(a+j\omega)}\right|_{0}^{\infty} = \frac{1}{a+j\omega}$$

ex: $x(t) = A\,\mathrm{rect}\left(\frac{t}{\tau}\right)$



$$\bullet\ X(\omega) = \int_{-\tau/2}^{\tau/2} A\, e^{-j\omega t}\, dt = \left.\frac{A\, e^{-j\omega t}}{-j\omega}\right|_{-\tau/2}^{\tau/2}$$

$$\sin x = \frac{e^{jx}}{2j} - \frac{e^{-jx}}{2j}$$

$$= \frac{A}{j\omega}\left(e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right) \rightarrow \frac{2jA}{j\omega}\left(\frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{2j}\right)$$

$$\boxed{= \frac{2A}{\omega}\sin \omega\tau/2}$$

8 →

$$\frac{\sin x}{x} = \mathrm{Sin}\,x$$

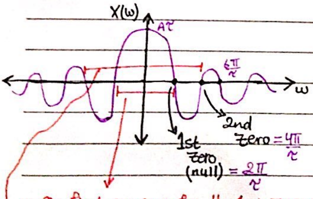
$$\bullet\ X(\omega) = \frac{2A}{\omega}\sin\frac{\omega\tau}{2}$$

$$= \frac{\cancel{2}A}{\cancel{\omega}} \cdot \frac{\cancel{\omega}\tau}{\cancel{2}} \cdot \frac{\sin\frac{\omega\tau}{2}}{\omega\tau/2}$$

$$= A\tau\, \mathrm{sinc}\,\frac{\omega\tau}{2}.$$

$$* A\,\mathrm{rect}\left(\frac{t}{\tau}\right) \longleftrightarrow \tau A\, \mathrm{sinc}\,\frac{\omega\tau}{2}$$

Imp



* To find 1st zero:

$$\frac{\omega\tau}{2} = \pi$$

$$\omega = \frac{2\pi}{\tau},$$

* 2nd zero:

$$\omega\tau/2 = 2\pi$$

$$\omega = \frac{4\pi}{\tau}$$

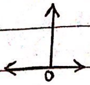
⋮

* Q: find energy for the 1st zero.

→ " 2nd zero

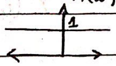
ex:

$x(t) = \delta(t)$



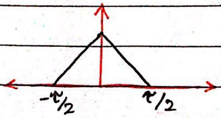
$$X(\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

$(t=0)$



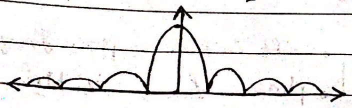
$$\int_{-\infty}^{\infty} \delta(t)\,(1)\, dt = 1$$

* $\Delta\left(\frac{t}{\tau}\right) \rightarrow$



$\rightarrow \text{sinc}^2 \ldots$

(موجودين بالجدول كلهم.)



* $X(\omega) = \delta(\omega)$

find $x(t)$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega)\, e^{+j\omega t}\, d\omega$$

$\omega = 0$

$$= \frac{1}{2\pi} \cdot 1$$

* properties of F.T:

① linearity.

$$x(t) = x_1(t) + x_2(t) \longrightarrow X(\omega) = X_1(\omega) + X_2(\omega)$$

ex: $x(t) = \text{rect}\left(\frac{t}{6}\right) + \delta(t)$

$$X(\omega) = 6\,\text{sinc}\,\frac{\omega 6}{2} + 1 \rightarrow 6\,\text{sinc}\,3\omega + 1.$$

② Time shift.

- $x(t) \longleftrightarrow X(\omega)$
- $x(t \mp t_0) \longleftrightarrow X(\omega)\, e^{\mp j\omega t_0}$

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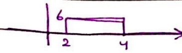
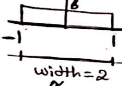
$\{x(t-t_0) \rightarrow X(\omega)e^{-j\omega t_0}$

ex:



find $X(\omega)$

$x(t-3)$



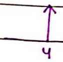
width = 2

τ

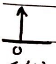
$FT \rightarrow 2\,\text{sinc}\,\frac{2\omega}{2} \rightarrow 2\,\text{sinc}\,\omega$

$X(\omega) = 2\,\text{sinc}\,\omega\, e^{-j3\omega}$.

ex:



$\delta(t-4)$



$\delta(t) = 1$

$X(\omega) = 1 \cdot e^{-j4\omega}$.

- a (constant) $\rightarrow a2\pi * \delta(\omega)$ مش كضرب

- shift in time $\rightarrow$ phase freq.
- phase " " $\rightarrow$ shift f.

③ phase - shift:

$x(t)e^{\pm j\omega_0 t} \longleftrightarrow X(\omega \mp \omega_0)$.

عكس الإشارة

ex: $x(t) = \cos \omega_0 t$.

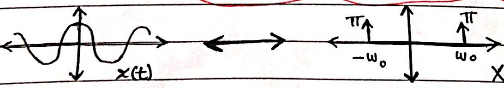
$\cos \omega_0 t = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{j\omega_0 t}$.

$\frac{2\pi}{2}\delta(\omega) \rightarrow \pi\delta(\omega-\omega_0)$

$\rightarrow \pi\delta(\omega+\omega_0)$.

مهم

$\cos \omega_0 t \longleftrightarrow \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$.



$x(t)$

$X(\omega)$

ex: $\cos 10t \longleftrightarrow \pi[\delta(\omega-10)+\delta(\omega+10)]$.

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• $x^*(t) \longleftrightarrow X^*(-w)$.

* Convolution prop.

$$x_1(t) * x_2(t) \longleftrightarrow X_1(w) \cdot X_2(w).$$

• $u(t) \longleftrightarrow \frac{1}{jw} + \pi\delta(w)$

* Multiplication pro.

$$x_1(t) \cdot x_2(t) \longleftrightarrow \frac{X_1(w) * X_2(w)}{2\pi}.$$

* differentiation pro.

$$\frac{d}{dt} x(t) \longleftrightarrow jw\, X(w).$$

* Integration pro

$$\int x(t)\, dt \longleftrightarrow \frac{X(w)}{jw} + \pi X(0)\, \delta(w).$$

* Scaling pro.

$$x(at) \longleftrightarrow X\left(\frac{w}{a}\right) \left|\frac{1}{a}\right|$$

* duality pro.

$$x(t) \longleftrightarrow X(w)$$

$$x(w_0) \longleftrightarrow 2\pi X(-t)$$

ex: $x(t) = rect(t/10) \longleftrightarrow X(w) = 10\, sinc\left(\frac{10w}{2}\right) = 10\, sinc(5w)$

* If $x(t) = 10\, sinc(5w)$ find $X(w)$

$$X(w) = 2\pi\, rect\left(\frac{-w}{10}\right).$$

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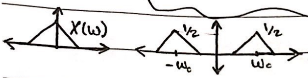
Ex: Modulation

$\underbrace{x(t)}_{\text{Data}} \cdot \underbrace{\cos w_o t}_{\text{Carrier}} = Y(t)$. Find $Y(\omega)$.

$$Y(\omega) = \frac{1}{2\pi}\left(X(\omega) * \pi\left[\delta(\omega - w_o) + \delta(\omega + w_o)\right]\right).$$

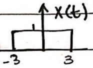
$$= \frac{1}{2} X(\omega - w_o) + \frac{1}{2} X(\omega + w_o).$$



Ex: $x(t) = rect\left(\frac{t}{6}\right)$, carrier $= 4\cos 10t$.

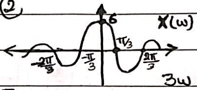
let $Y(t) = x(t) \cdot 4\cos 10t$.

1. what we call the process? Modulation process.
2. sketch $X(\omega)$.
3. sketch $F.T\{4\cos 10t\}$.
4. sketch $Y(\omega)$
5. Write your comment.



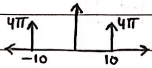
$rect(t/6) \leftrightarrow 6\,\text{sinc}\,3\omega$

(2)



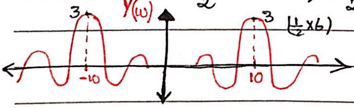
$3\omega = \pi \rightarrow \omega = \pi/3$

[3] $4\cos 10t \leftrightarrow$



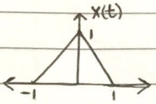
[4] $Y(\omega) = \frac{1}{2\pi}\left(X(\omega) * \pi\left[\delta(\omega - 10) + \delta(\omega + 10)\right]\right.$

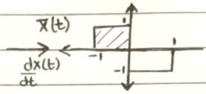
$$= \frac{1}{2} X(\omega - 10) + \frac{1}{2} X(\omega + 10)$$

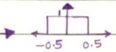


* if we MUL any sig. $x(t)$ by $\cos w_o t$ in FT. it will be shifted by w_o.

Ex: sketch $x(t) = \Delta\left(\frac{t}{\tau}\right)$, let $\tau = 2$.
find $X(\omega)$, let $y(t) = x(t)\cos\omega_0 t$ sketch $Y(\omega)$.







$\mathrm{rect}\left(\frac{t}{1}\right)$
$\updownarrow$
$\mathrm{sinc}\left(\frac{\omega}{2}\right)$

F.T. $\frac{dx(t)}{dt} \longleftrightarrow j\omega X(\omega)$

$* \; x(t-t_0) = X(\omega)e^{-j\omega_0 t}$

$\rightarrow \mathrm{sinc}\left(\frac{\omega}{2}\right)e^{+j0.5\omega} - \mathrm{sinc}\left(\frac{\omega}{2}\right)e^{-j0.5\omega}$

$= \mathrm{sinc}\left(\frac{\omega}{2}\right)\left[e^{j0.5\omega} - e^{-j0.5\omega}\right]$

$X(\omega) = \dfrac{\frac{dx(t)}{dt}}{j\omega}$

$= 2\,\mathrm{sinc}\left(\frac{\omega}{2}\right)\dfrac{e^{j0.5\omega} - e^{-j0.5\omega}}{2j\omega}$

$* \; \sin x \rightarrow \dfrac{e^{jx} - e^{-jx}}{2}$

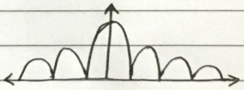
$* \; \mathrm{sinc}\,x \rightarrow \dfrac{\sin x}{x}$

$\frac{\omega/2}{}$ *

$= \dfrac{2}{\omega}\mathrm{sinc}\left(\frac{\omega}{2}\right)\cdot \dfrac{\sin\left(\frac{\omega}{2}\right)}{\omega/2}$

$= \dfrac{2}{\omega}\dfrac{\omega}{2}\,\mathrm{sinc}\left(\frac{\omega}{2}\right)\cdot \mathrm{sinc}\left(\frac{\omega}{2}\right)$

$= \mathrm{sinc}^2\left(\frac{\omega}{2}\right)$

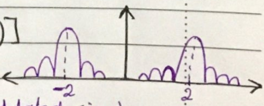


$\boxed{\Delta\left(\frac{t}{\tau}\right) \longleftrightarrow \mathrm{sinc}^2\left(\frac{\omega}{2}\right)}$

$y(t) = x(t)\cos\omega_0 t$, $\omega_0 = 2$

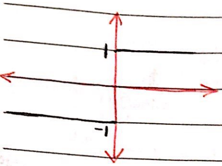
$Y(\omega) = \frac{1}{2\pi}X(\omega) * \pi\left[\delta(t-2) + \delta(t+2)\right]$

$= \frac{1}{2}X(\omega-2) + \frac{1}{2}(\omega+2)$



(y(t) modulated sig.)

* $sgn(t) = \begin{cases} 1 & t>0 \\ -1 & t<0 \end{cases}$



$x(t) = sgn(t)$

$X(\omega) = \frac{2}{j\omega}$

* parseval theorm.

- for periodic sig $P_t = P_f$.
- for nonperiodic " $E_t = E_f$.

$\bullet\ E_t = \int_{-\infty}^{\infty} x^2(t)\, dt.$ $\quad \bullet\ E_f = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega.$

Ex: if $x(t) = e^{-at} u(t)$

verify parseval's theorm: $E_t = E_f = E_\omega$.

$\bullet\ X(\omega) = \frac{1}{a+j\omega}$

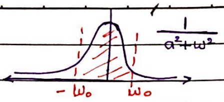
① $E_t = \int_{0}^{\infty} (e^{-at})^2\, dt = \frac{e^{-2at}}{-2a} \Big|_0^{\infty} = \frac{1}{2a}$ J.

② $E_f = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{1}{a+j\omega} \right|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{a^2+\omega^2}\, d\omega$

$= \frac{1}{2\pi} \cdot \boxed{\frac{1}{a} \tan^{-1}\left(\frac{\omega}{a}\right)} = \frac{1}{2a}$ J

$\tan^{-1}(\infty) = \frac{\pi}{2}$

$\tan^{-1}(-\infty) = -\frac{\pi}{2}$



* what is the bandwidth to restore 90% of energy?
$(w_o, -w_o)$

$$\bullet\ E_f = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{a^2+w^2}\, dw = 0.9 \cdot \frac{1}{2a}$$

$$= \frac{1}{2\pi} \cdot \frac{1}{a} \tan^{-1}\left(\frac{w}{a}\right)\Bigg]_{-w_o}^{w_o} = \frac{0.9}{2a}$$

$$= \frac{1}{2\pi a}\left[\tan^{-1}\left(\frac{w_o}{a}\right) - \tan^{-1}\left(\frac{-w_o}{a}\right)\right] = \frac{0.9}{2a}$$

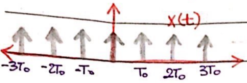
$$\rightarrow \frac{1}{\pi}\left[2\tan^{-1}\frac{w_o}{a}\right] = 0.9$$

$$\rightarrow \frac{0.9\pi}{2} = \tan^{-1}\frac{w_o}{a}$$

$$\rightarrow \tan\left(\frac{0.9\pi}{2}\right) = \frac{w_o}{a}$$

$$\boxed{w_o = a\tan\left(\frac{0.9\pi}{2}\right).}$$

* Train of Impulses *



$$x(t) = \sum_{-\infty}^{\infty} \delta(t - nT_0)$$

$$X(\omega) = \frac{2\pi}{T_0} \sum_{-\infty}^{\infty} \delta(\omega - n\omega_0)$$

$\hookrightarrow \omega_0 = \frac{2\pi}{T_0}$.

* $x(t) \to \boxed{h(t)} \to y(t)$

$y(t) = x(t) * h(t)$

$y(\omega) = X(\omega) \cdot h(\omega)$

$$\boxed{h(\omega) = \frac{Y(\omega)}{X(\omega)}}$$

Ex: $\frac{dy(t)}{dt} + y(t) = 3x(t)$

find $H(\omega)$ "transfer f^n"

$h(t) \rightarrow$ impulse response.

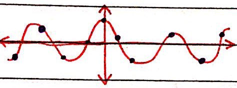
$j\omega Y(\omega) + Y(\omega) = 3X(\omega)$

$Y(\omega)[j\omega + 1] = 3X(\omega) \rightarrow \underbrace{\frac{Y(\omega)}{X(\omega)}}_{H(\omega)} = \frac{3}{1 + j\omega}$.

$\rightarrow e^{-at} u(t) = \frac{1}{a + j\omega}$

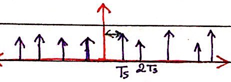
$\hookrightarrow h(t) = 3e^{-t} u(t)$.

* Sampling *



$f_m = \beta$ "bandwidth".

(We use Train of impulses to perform the sampling)



$\frac{1}{T_s} = f_s$ sampling f.

$f_s > f_m$ (($f_s = 2f_m \rightarrow$ Nyquist Rate)).

$f_s \not> f_m$ (Alaising) تداخل

$f_s \geq 2f_m$

- $\delta(t - nT_s) \longleftrightarrow \omega_s \sum \delta(\omega - n\omega_s)$.

$\omega_s = \frac{2\pi}{T_s}$.

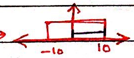
- $x(t) \cdot h(t) \rightarrow \frac{1}{2\pi} H(\omega) * X(\omega)$.

- $x(t) = rect\left(\frac{t}{\tau}\right)$

$X(\omega) = \tau\, sinc\left(\frac{\omega\tau}{2}\right)$.

Ex:

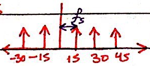
(*) $X(\omega) \rightarrow$

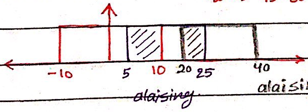


$f_m = 10$ Hz.

"conv. with δ = we shift the signal"

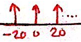
① $f_s < 2f_m$, $f_s = 15$

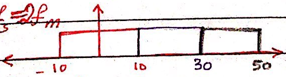




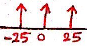
fail.

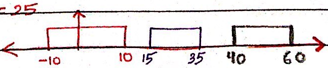
② $f_s = 2f_m$





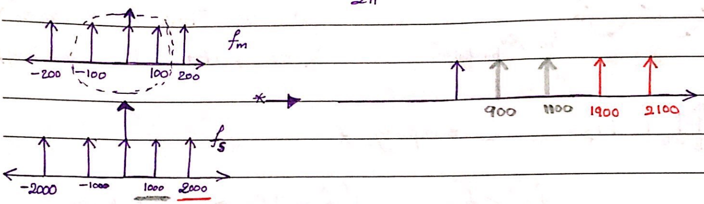
③ $f_s > 2f_m$, $f_s = 25$





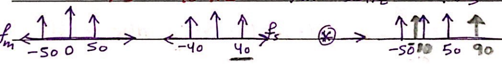
Ex: $x(t) = \cos(\underbrace{200\pi}_{w_m} t)$ sampled at rate $f_s = 1000$ Hz.

$f_m = \frac{w_m}{2\pi} = 100$ $\quad \therefore f_s > f_m$.



Ex: $x(t) = \cos(\underbrace{100\pi}_{w_m} t)$

$f_s = 40$ Hz. $\quad f_m = 50$ Hz $\quad \therefore f_s < f_m$

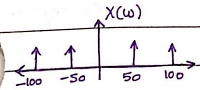


Alaising

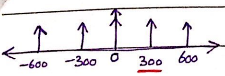
Ex: $x(t) = \cos(100\pi t) + 20\cos(200\pi t)$.

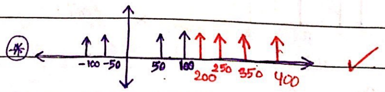
$f_{m_1} = 50$ Hz $\quad f_{m_2} = 100$ Hz

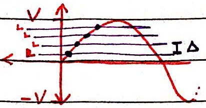
$\boxed{\therefore f_m = 100}$ في حالة وجود أكثر من f_m نختار الأكبر



if $f_s = 300$ Hz, $f_s > f_m$.



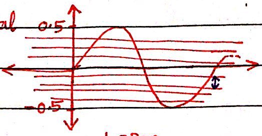


* 

• Analog sig.

$\Delta = \dfrac{2V}{L}$ → # of levels.

$\boxed{f_s > 2f_m}$

* signal 

L=200

$2V = 2(0.5) = 1.$

$\Delta = \dfrac{1}{200}$

Ex: bandwidth $f_m = 3KHz$.

sampled at rate 33.5% over the Nyquist.

↳ $f_s = 2f_m$

$f_s' = (3000 \times 2) \times 1\frac{1}{3} = 8000\ Hz$

↳ $= f_s(1 + 0.335) = 6000(1.335) = 8000 Hz$.

Analog → digital

$L \rightarrow 2^n$

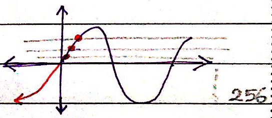
* Δ delta, tri, quantization step.

ex: $\Delta = 0.0005$

$L = \dfrac{2V}{\Delta} = \dfrac{2(1)}{0.0005} = 200$ level 2^n

$\approx 2^8 = 256$.

$\boxed{n=8}$



every sample should be (8 bits).

* bit rate = $f_s' * n$

$= \underset{\#\text{ samples}}{8000} \times \underset{\text{bits}}{8} = 64K$ bit per seconds.