lecture 1

Reliability

→ in planning and operation.

* Definition: How much you rely on the system.

⊛ Assessments:
- event
- Importance
- uncertainty
- worst case
- planning
- decision
- cost

⊛ goal: ensure continuity of electricity.

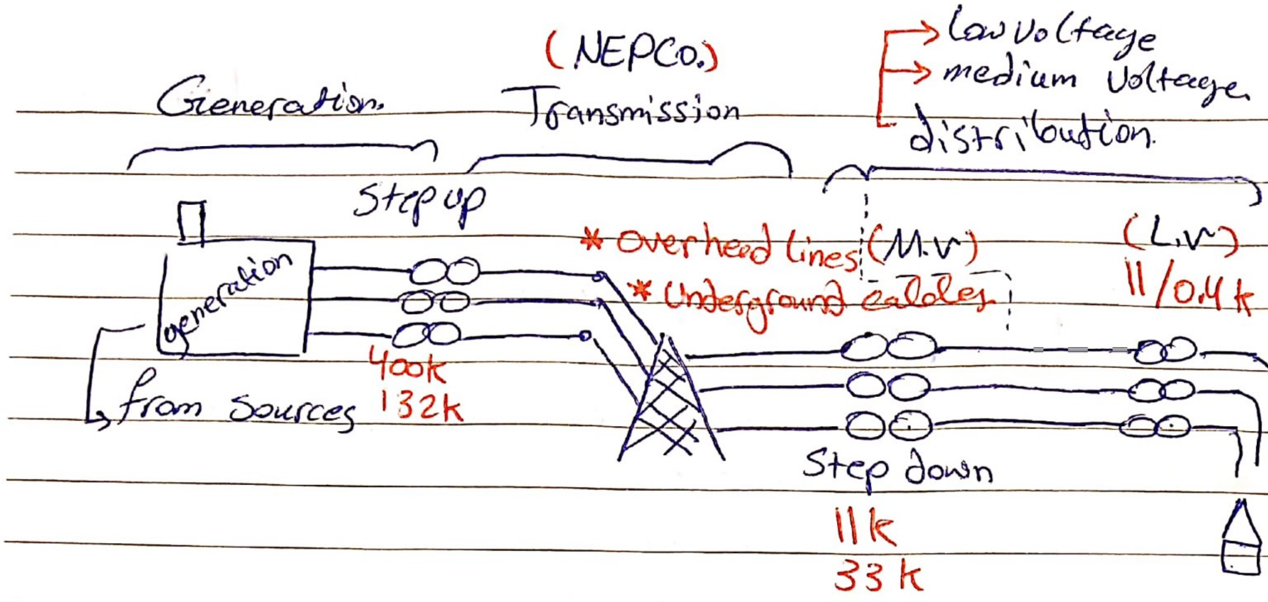
→ Decisions → before planning
→ while operation

$\boxed{1}$ * we need numbers to compare and compromise cost, reliability, and efficency.

$\boxed{2}$ → and mathmatical models

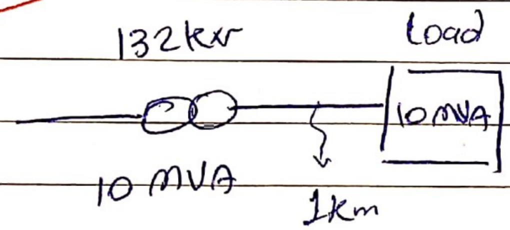
⊛ we must detrmine power system components & areas of uncertainty.

⊛ power system: Generation, distribution, and transmission.



- each part have its own uncertainties

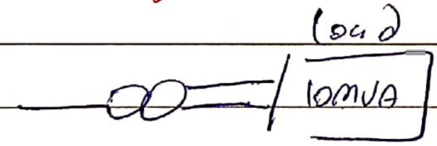
Example



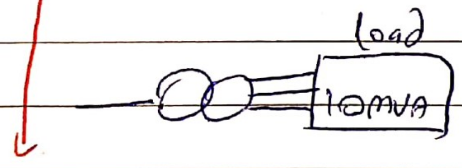
300k JD

This systems isn't reliable solution:

①



2-lines:
(n-1) circuits
300k + 150k → 450k JD



3-lines
(n-2) circuits

more reliable but also, more cost.

② add another Transformer

③ ring system

→ If decision is fixed ⇒ Deterministic approach
if not, probabilistic approach. (depends on the historical Data)

Ex) failure rate = 20%

cost = 300k + 150k + 150k → الخطة البديلة

⇓

0.2 * X → cost جديدة

احتمالية

⇒ I need to find the expected ⇒ Average cost.

- probabilistic approach : less cost & less reliable compared to deterministic approach.

lecture 2

- Reliability : The probability that the System won't fail. or The probability that the system will Survive.

- After placing models ⇝ take decision → Invesment → Minimum price

- If the approach wasn't deterministic, then the uncertainity is very high.

* probabilistic Model : Solves the uncertainity portion of the problem. ((can be applied on short or long term plannings))

- recall → probability : مقدار التكرار ل event معين بالنسبة لباقي الاحداث.

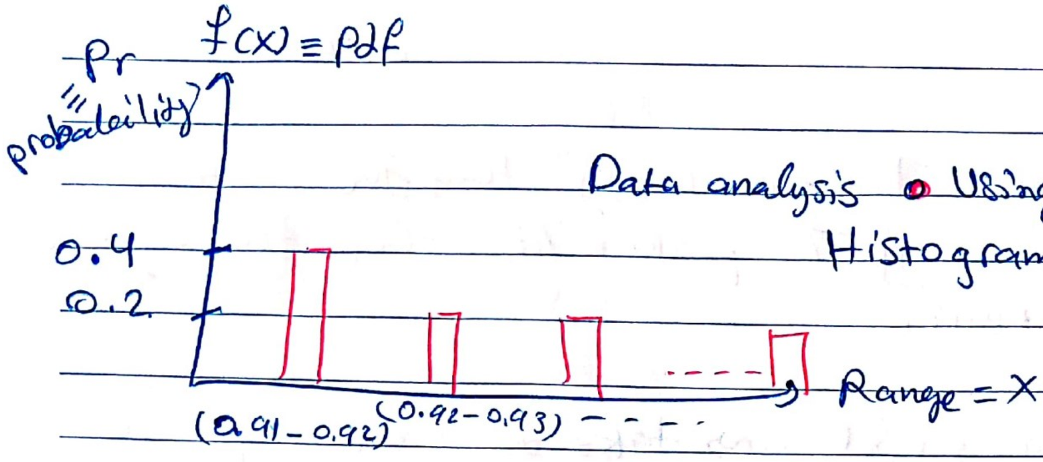
Examples بيدي Data ال Voltage مأخوذة
من Site معينة 5 - ساعات

1st hour	0.92
2nd hour	0.93
3rd hour	0.91
4th hour	0.91
5th hour	0.97

to find the probability density function (PdF)

Quantization : رح اعمل Ranges
↳ (Step 0.01)

0.91 → 0.92 → 2 (frequency) → 2/5
0.92 → 0.93 → 1 → 1/5
0.93 → 0.94 → 1 → 1/5
0.94 → 0.95 → 0 → 0
0.95 → 0.96 → 0 → 0
0.96 → 0.97 → 0 → 0
0.97 → 0.98 → 1 → 1/5



Data analysis • Using Excel
Histogram

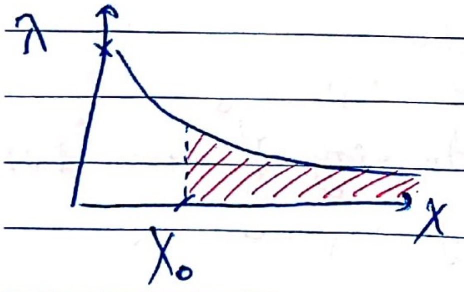
- $pr(X > X_o)$
  - continuous $\int_{X_o}^{\infty} f(x) \cdot dx$
  - Discrete $\sum_{X_o}^{X_N} pr$

* Failure Rate

يوجد pdfs مشهورة وتتناسب مع ال λ assessment power system :

1. Exponential pdf

$$f(x) = \lambda \cdot e^{-\lambda x}$$



$$P_r(X > X_o) = \int_{X_o}^{\infty} \lambda \cdot e^{-\lambda x} dX \Rightarrow e^{-\lambda X}$$

mean & Variance = λ.

- probability of failure & survival for the System is complematary

$$\underset{failure}{P_f} = 1 - \underset{survival}{P_s}$$

- failure rate $= (\lambda) \triangleq$ number of failure of the system given a period of time.

Example,

1000 transformers ~~~> Under test for one year time period $\Rightarrow$ (8760 hours)

working hours. $Tx_1 \Rightarrow$ { 500 hours } → بيستطاع من الخدمة

$Tx_2 \Rightarrow$ { 1000 hours } →

⋮ ,,

all remaing) transformers worked for 8760 hours.

$Tx_1 + Tx_2 + \dots$ كل

$500 + 1000 + 498 * (8760) = \#$

$$\text{failure rate} = \frac{\text{number of failure}}{\text{period}} = \frac{2}{\#}$$

$$\frac{\# \text{ failure}}{\text{unit} \cdot \text{houre}}$$ ← الوحدة

عدد Transformers

I can find the Epected # of transformers that will fail Criven # of transformers.

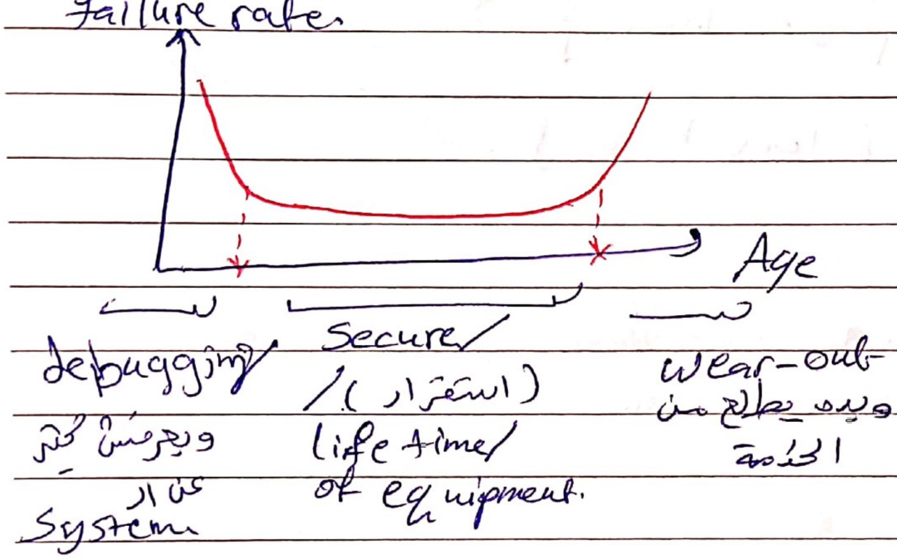
Ex) $\lambda = 0.01$, # of transformers = 2000

$2000 * 0.01 \Rightarrow 20$ failure / hour

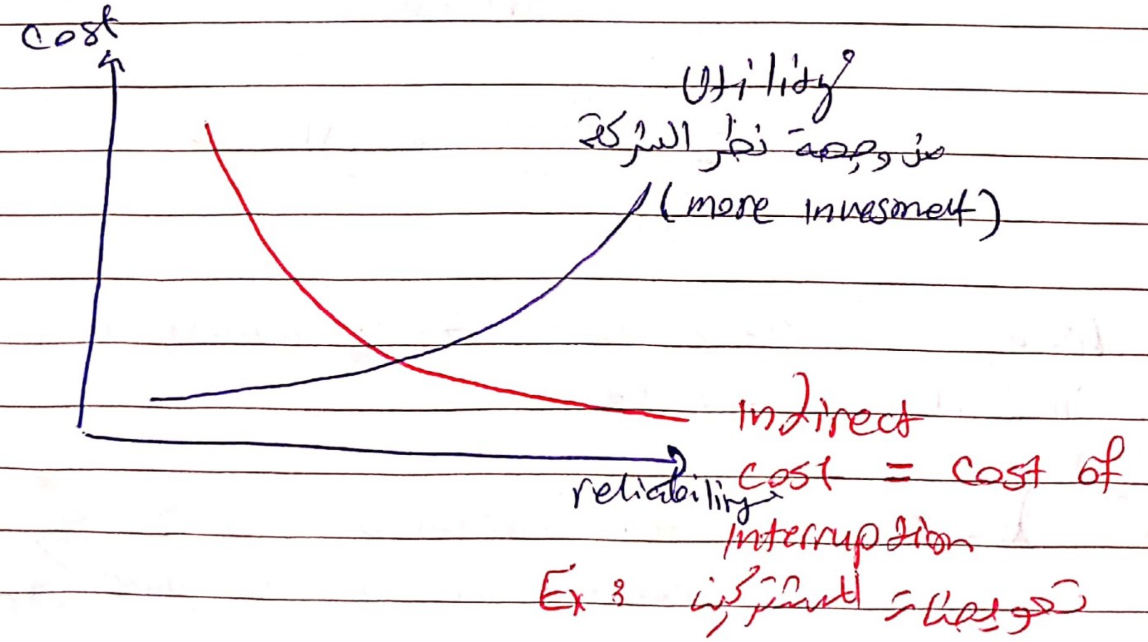
* Stochastic Analysis or Monticallo Analysis

↳ 2, Transformer اي بحطهم لانه ما يعرف
يصير عليه ال failure.

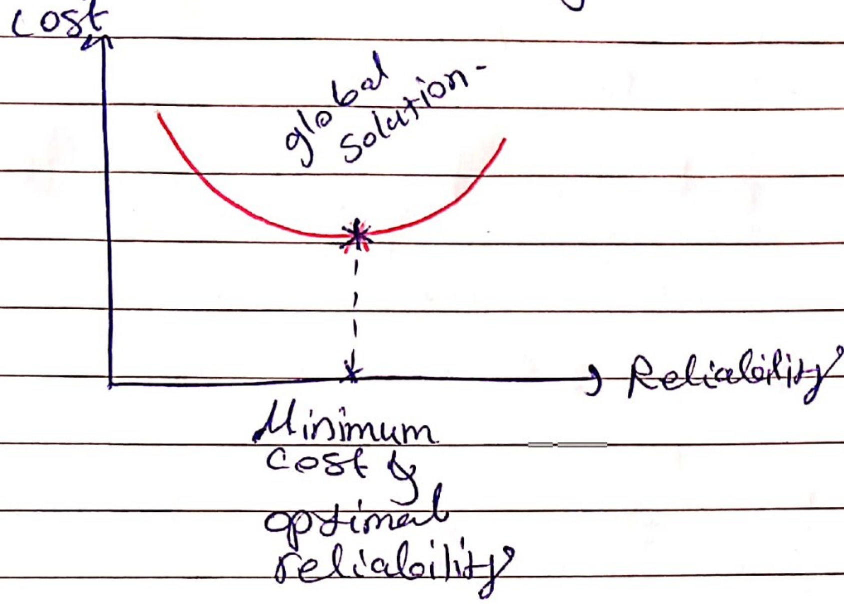
2 Bath-tub-curve: (relationship between Reliability & the System age/life)



3 Reliability cost-curve: (relationship between Reliability & cost.)



Adding the two curves together:



Reliability

Adequacy

ان يكون ال System sufficient.
بحيث يغطي متطلبات ال Demand.

① Power: KW & kVAr

Voltage: ② ±3%, ±5%

③ Thermal capacity: Power System components must not be overloaded.

Security

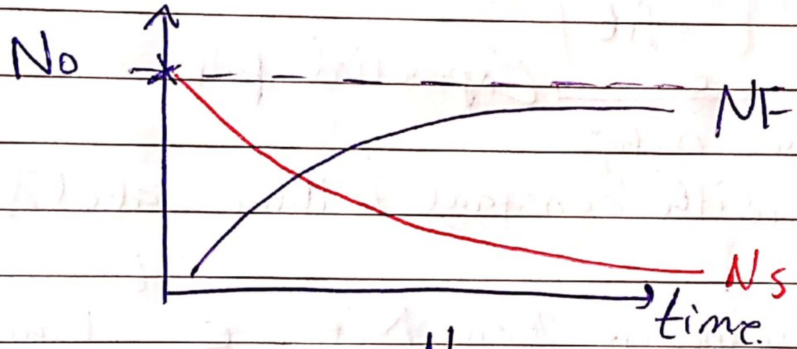
response of the System should be in any disturbance. (Fault) (transient period) → Dynamic Analysis

if all ③ network constrains are satisfied then the system is Adequate.

⟹ we will find ①, ②, & ③ in normal operation Analysis (steady state)

Lecture3

- $Q \equiv Pr$ to fail , $R = Pr$ to survive.
- Reliability is related to time.
- if you have a number of components: N_0. Number of survival N_s. Number of failure N_F:



- $R(t) = \dfrac{N_s}{N_0} = \dfrac{N_0 - N_F(t)}{N_0} = \boxed{1 - \dfrac{N_F}{N_0}}$

Reliability as a function of time

- $R(t) \Rightarrow$ is the cumulative of the pdf.

$$R(t) \Rightarrow F(x) = \int f(x) \cdot dx$$

$$\frac{dR}{dt} = \frac{-\partial N_F}{N_0\, dt}$$

- failure rate $(\lambda) = \dfrac{\text{\# of failures}}{\text{total \# of operatings}}$.

من الشكل السابق ← $\lambda \cdot N_s = \dfrac{\partial N_F}{dt}$

So → $\frac{dR}{dt} = -\frac{1}{N_0} \cdot \lambda \cdot N_s$

$$\frac{dR}{dt} = -\lambda R \rightsquigarrow \int \frac{dR}{R} = \int -\lambda \, dt$$

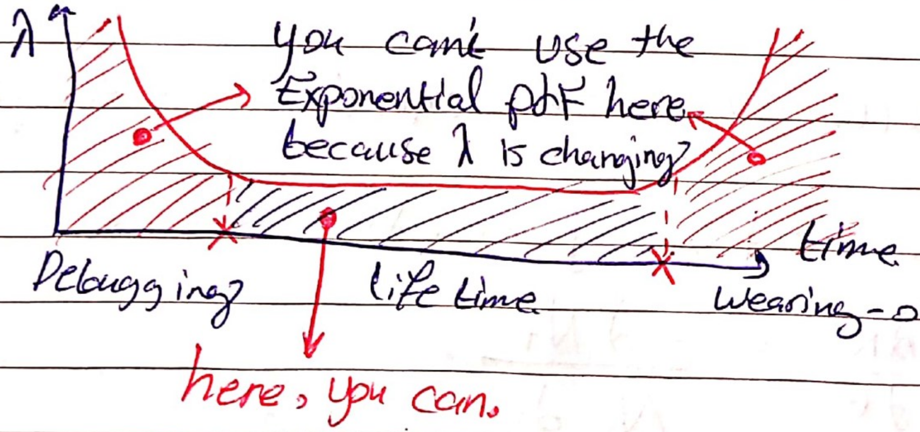
$$\ln R = -\lambda t$$

$\boxed{R(t) = e^{-\lambda t}}$ Exponetial pdF.

→ This applies only on systems with constant failure rate(λ).

→ If λ is changing then $R(t) = e^{-\lambda t}$ isn't valid.

⊗ Recall Bathtub curve.

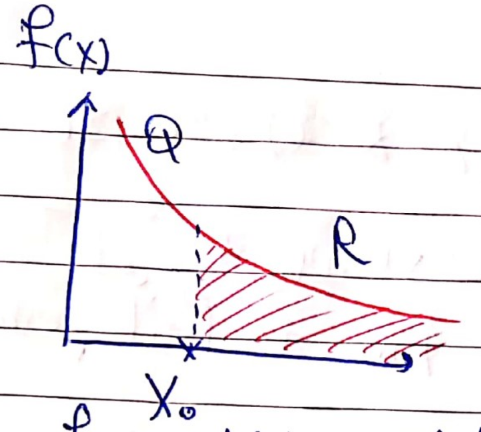


● $f(x) = $ pdF , $R = F(x) \Rightarrow f(x) = Q'$ مشتقة

$= (1-R)'$

$= (1 - e^{-\lambda t})'$

$= \lambda e^{-\lambda t}$ (positive because it's a probability).



$R = pr(X > X_0)$

→ from this model we can define a new Quantity: **Mean Time To Fail (MTTF)**

Expected الوقت اللازم اوال
(Average)

الي رح يصير بعده
Failure

- to calculate the Expected value:

$$E = \int x \cdot f(x) \cdot dx$$ Substituting $f(x) = \lambda e^{-\lambda t}$

$$MTTF = \int_0^{\infty} t(\lambda e^{-\lambda t}) \cdot dt$$

$$\boxed{MTTF = \frac{1}{\lambda}} \quad \#$$

- $MTTF = \int_0^{\infty} R(t) \cdot dt = 1/\lambda$ only on Exponential model at λ = constant.

Example

- Given that the failure rate is constant, Find the system Reliability @ $t = MTTF$.

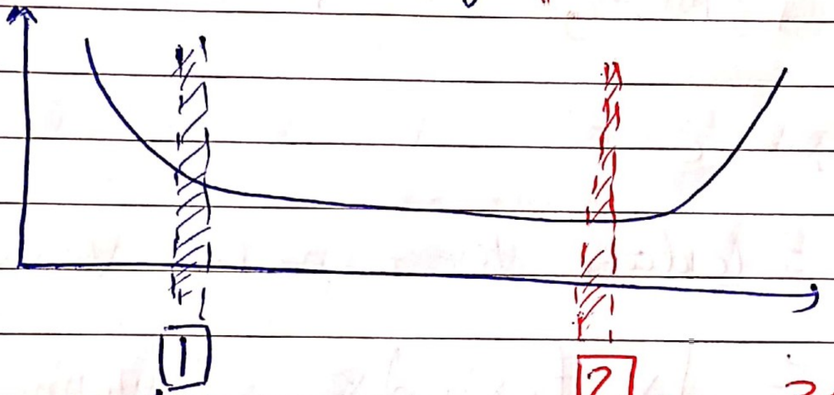
$$R(t) = e^{-\lambda t}\Big|_{t = MTTF}$$

$$= e^{-\lambda(1/\lambda)} = e^{-1} \Rightarrow R(t) = 0.36$$

- 36% تبعون failure ال اقضية

* [1] if system time to fail period is short ⇒ BAD.

* [2] if system time to fail period is Long ⇒ Good



36% ال من هاد ال MTTF is BAD.

36% ال من هاد ال MTTF is good.

- How to know MTTF? It's found from historical data & from the manufacturer.

Example, if a generating Station has 10 generators. each generator has a constant failure rate of 0.02/year. Find the MTTF for the Station: under the useful life او ضمن حياته time

$$\lambda_{gen} = 0.02/\text{year}$$

$$MTTF = 1/\lambda_{station}$$

$$\lambda_{station} = \lambda_{gen} * \# \text{ of Generators}$$

$$= 0.02 * 10 = 0.2$$

$MTTF = 1/0.2 = 5$, so after 5 years $MTTF = 36\%$

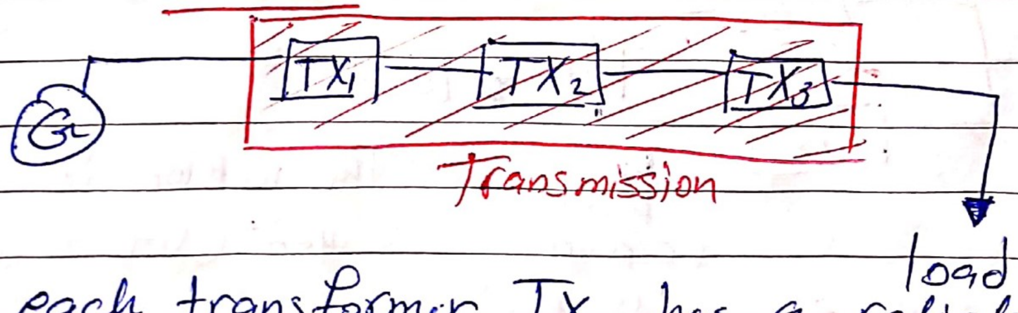
- Enviromental causes are calculated in the assessment not included when calculating the failure rate.

Reliability for complex systems.

⇝ two main methods:

1 Logic State: assume that the system is connected either series or parellel.

Ⓐ Series ⇒ also called radial System



each transformer TX has a reliability independent from the other transformers.

$TX_1 \rightarrow R_1$, $TX_2 \rightarrow R_2$, $TX_3 \rightarrow R_3$

$$R_{system} = R_1 * R_2 * R_3 * \dots * R_N$$

Ex) ① if $R_1 = 0.9$, $R_2 = 0.9$, $R_3 = 0.9$

$$R_{sys} = 0.9 * 0.9 * 0.9 = 0.73$$

Ex) ② if $\lambda_{1,2,3}$ for Tx_1, Tx_2, Tx_3 is constant

$$\Rightarrow R_1(t) = e^{-\lambda_1 t}, \quad R_2(t) = e^{-\lambda_2 t}, \quad R_3(t) = e^{-\lambda_3 t}$$

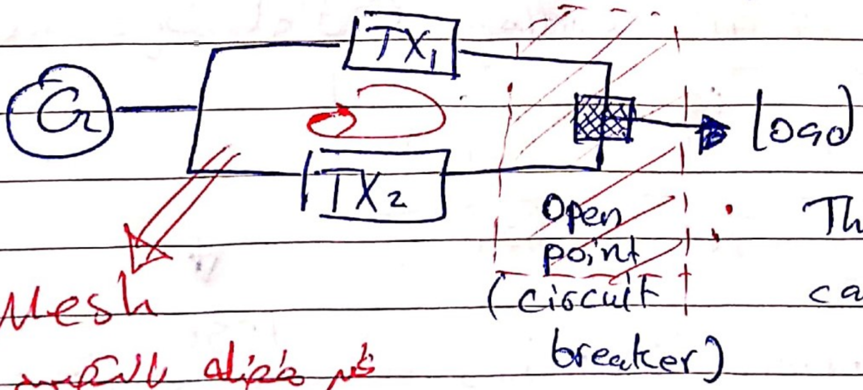
$$R_{sys}(t) = e^{\underbrace{-t(\lambda_1 + \lambda_2 + \lambda_3)}_{\lambda_{sys}}}$$

$$\lambda_{sys} = \lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_N$$

- So MTTF for system $= \int R(t) = \dfrac{1}{\lambda_1 + \lambda_2 + \lambda_3}$

Ⓑ parallel : Used to Improve reliability

also called $\Rightarrow$ Ring System (It's operated as radialy System.)



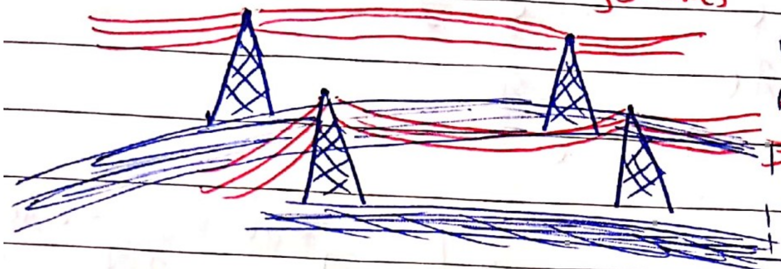
: This method is called: (N-1) connection.

غير مفضلة بالمصانع لأنه

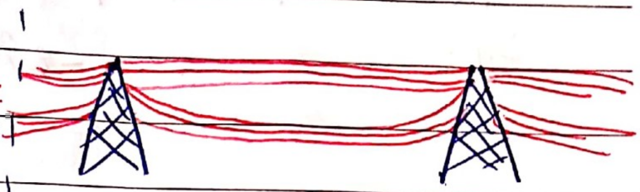
① صعب تحديد ال fault

② cannot/HARD to determine power flow path.

◉ الفرق بين ال Ring systems & double circuit :

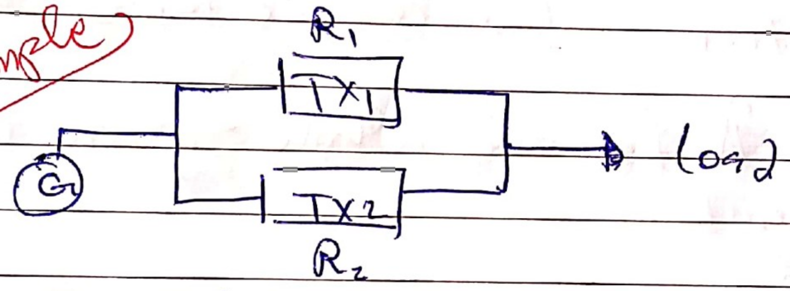


two 3 phase systems on two different towers / transmission lines.



two 3 phase systems on the same tower / transmission line.

◉ Example



assuming $\lambda \equiv$ constant , dependent R_1 & R_2

(ع شان لازم يتعطلوا التنين حتى يصير في failure عال system

→ calculate the reliability from failure ↩

if TX_1 & TX_2 are off :

$$Q_{system} = Q_1 * Q_2 \Rightarrow R_{sys} = 1 - Q_{sys.}$$

Ⓐ if $R_1 = R_2 = e^{-\lambda t}$, λ constant

$$Q_{sys} = (1 - e^{\lambda t})^2 \quad , \quad R_{sys} = 1 - (1 - e^{-\lambda t})^2$$

calculate λ_{sys} , recall $\frac{1}{R} \cdot \frac{dR}{dt}$

$$R_{sys} = 1 - (1 + e^{-2\lambda t} - 2e^{-\lambda t})$$

$$= -e^{-2\lambda t} + 2e^{-\lambda t}$$

$$\lambda_{sys}(t) = \frac{-1}{2e^{-\lambda t} - e^{-2\lambda t}} * \left[-2\lambda e^{-\lambda t} + 2\lambda e^{-2\lambda t}\right]$$

⇒ λ for each TX is constant

But after calculation, we found out

that λ for the whole system isn't constant.

$$MTTF = \int_0^{\infty} R(t) \cdot dt = \left[\frac{3}{2\lambda} \text{ per element}\right]$$

continue if:

Ⓑ $R_1 = 0.9$, $R_2 = 0.9$, $R_3 = 0.9$, find Reliability for the systems:

$$Q = (1 - 0.9)^3$$

$$= (0.1)^3 = 10^{-3}$$

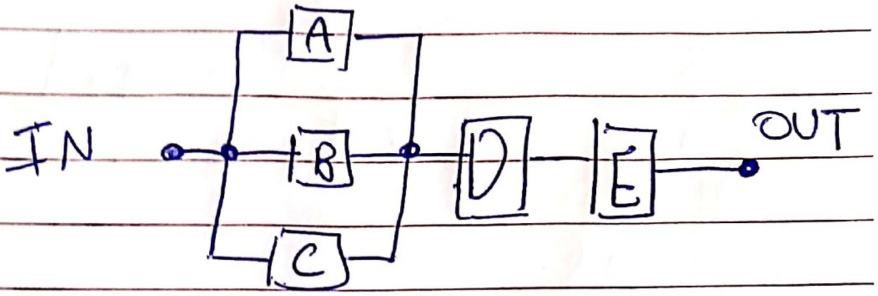
$$R_s = 1 - 10^{-3}$$

$$= 0.999$$

Scanned with CamScanner

Example: Find the Reliability for the whole following System:

* Given:



$R_A = R_B = R_c = 0.7$

$R_D = 0.9$, $R_E = 0.8$

$$Q_{A,B,C} = (1-0.7)^3 = (0.3)^3 = 0.027$$

$$R_{A,B,C} = (1-0.027) = 0.973$$

$$R_{Sys} = \overset{A,B,C}{0.973} * \overset{D}{0.9} * \overset{E}{0.8}$$

$$\boxed{R_{Sys} = 0.701}$$

Lecture 4

- Reliability generation

Planning → Adequacy

→ Gen = Load → maximum demand

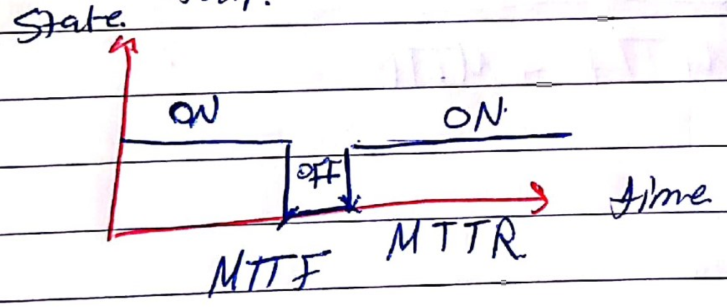
available capacity

- Objectives: to meet maximum demand
  1. generation type ⇒ { Renewables }
  2. failure ⇒ { Storage. Hydro generation }

- ① λ = failure rate × number of failure / time of operation.
  ↳ at usefull life time.

② MTTF $= 1/\lambda$

mean time to fail.



③ MTTR: mean time to repair. $= 1/\mu$

μ = repair rate

④ operating time: MTTR + MTTF

Ex) 1 year, 8760 hours, 10 transformers.

① → deterministic planning approach

$$\lambda = \frac{60}{8760}$$

fail

① 20 ← ساعة

② 40 استعطل

$\overline{\quad 60 \quad}$

$$\mu = \frac{16}{8760}$$

ساعات التي احتاجها ليتصلح

① 12

② 4

$\overline{\quad 16 \quad}$

Ex) if $\lambda = 0.01$
2
prob = 90%

probabilistic planning approach:

1 MTTF:

2 MTTR:

3 Availability = A = $\frac{MTTF}{MTTF + MTTR}$

$$\frac{\text{\# of hours/day}}{\text{total operating time}}$$

unavailability = $\bar{A}$ = $\frac{MTTR}{MTTF + MTTR}$

or (forced outage rate)

FOR

- To measure reliability:
  - → Deterministic Average
  - → probabilistic probability

Scanned with CamScanner

- planning?: output ⇒ Decision

1- invesment cost.
2- generation type.
3- when it has min. cost.

} if we do them: MASTER PLAN

- minimum cost ⇒ our objective.

constrains: generation's to meet demand.

index < Index X

Reliability generation.

↳ Deterministic indicies.

1) reserve margin (RM)
2) capacity (MW / GW) * available in the system.
* Excess over maximum demand.

Ex) * Maximum ⇒ 3.6 GW
* Installed ⇒ 6 GW
* Renewable
* Storage
} ⇒ 5 GW

or (worst case)

RM = available generation − Maximum demand

$5 - 3.6 = 1.4$ GW.

Lecture 5

- Deterministic indices :
  Reserve margin (RM) = Available capacity - Maximum Load.

$$RM(pu) = \frac{RM(GW)}{\text{maximum Load}}$$

- Largest unit (L.U) = RM largest unit in the system.

Ex) $G_1 = 1\,G$.
$G_2 = 2\,G$
$\boxed{G_3 = 3\,G}$

→ largest unit could be
L.U > 1
L.U < 1

If the system was working on full capacity (6 GW) (demand) & the reserved = 3 GW.
The Largest unit = 3. If it fails then Reserved can supply it.

→ If the Reserved = 2 GW and the L.U = 3 failed then there will be 1 GW not supplied

- Reserved margin must be ⇒ Larger or equal to the Largest unit.

Example) in a station there are the following units

G_I
- Thermal
- 10 GW capacity

G_{II}
- Hydro
- full capacity 3 GW max
- 1.3 GW (dry dam)

The largest unit = 1 GW (or generation الموقع)

Q₁) A) Find Reserved margin when the dam is full

B) Find Reserved margin when dam is dry.

Q₂) largest unit in part A & B ?

given that the maximum load is 11.5 GW

Sol) Q₁) A) RM(GW) = $(10+3) - 11.5 = \underbrace{1.5 \text{ GW}}_{\text{reserved}}$

→ in p.u $\frac{1.5}{11.5} = 13\%$ زيادة

B) RM(GW) = $(10+1.3) - 11.5 = \boxed{-}0.2$ GW

(unsupplied) defect

نقصان

Q₂) A) $L.U = \frac{1.5}{1 \to 1GW} = 1.5$

$1.5 > 1$ good system

B) $L.U = \frac{-0.2}{1} = -20\%$

$-0.2 < 1$ not good.

when these indices are according to time
probabilistic.

- probabilistic indices

I Loss of Load Expectation (LOLE)

Expected number of hours that the load isn't being supplied (insufficent generation)

number # of hours $G < load$ ⇒ per years

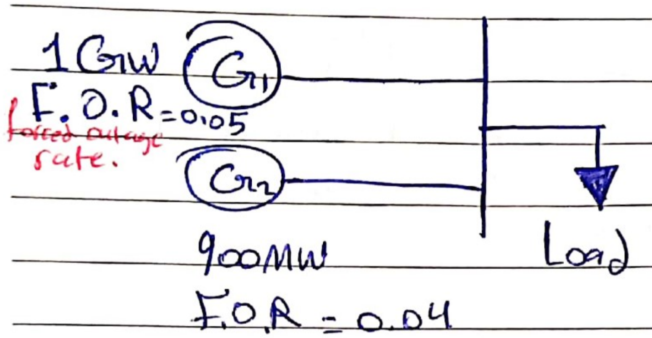
● LoLP = $\frac{\text{LoLE}}{\text{total number of hours The system is available}}$

loss of load probability

(8760)

→ conventional generation methods.

Ex) given two generation



Ⓐ 1100MW
Ⓑ 800 MW

Ⓐ Find Loss of load probability (LoLP):

→ case 1100MW load:

prob of 0.05 ⇐ لما ال G_1 تكون unavailable #1

OR

prob of 0.05 ⇐ لما ال G_2 تكون unavailable #2

OR

Both G_1 & G_2 are unavailable. #3

LoLP: $\underset{\#1}{(0.05)} + \underset{\#2}{\underline{(0.04)}} - \underset{\#3}{\underline{(0.05 * 0.04)}} = 0.088$

joint probability

8.8%

→ Case 800 MW:

يمكنني تزويد ال load من اي G والحالة الوحيدة

اللي ما بقدر ازوده فيه لما Both G_1 & G_2 aren't available.

LoLP: $0.05 * 0.04 = 0.002$

0.2%

To Find the Expectation:

probability

بنضرب بعدد الساعات 8760 عشان احتمالية

2 Loss of Energy Expectation ⇒ related to hours

GWh or TWh

➡ LoEE ≡ LoENS ≡ EENS

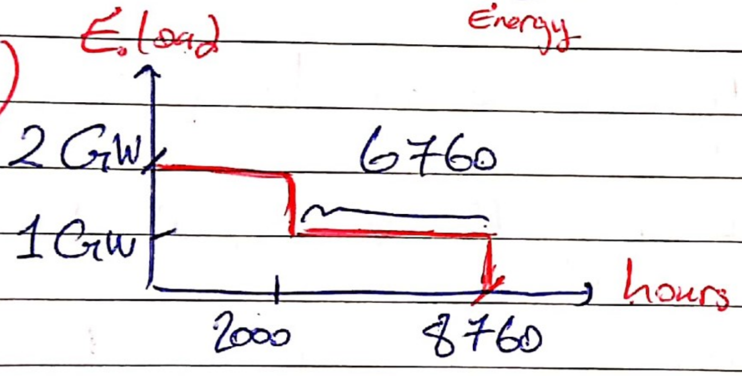
LoENS: loss of Energy not supplied

EENS: Expected energy not supplied.

$$\text{probability} = \frac{EENS}{E.\ load}$$

Energy

Ex)



⤳ E. Load = (2G * 2000) + (1G * 6760)

Annual Energy

3 Expected load not supplied (XLOL) (GW / MW)

Expected loss of load

$$\boxed{LoLE * XLOL = EENS}$$

LoLE: Number of hours

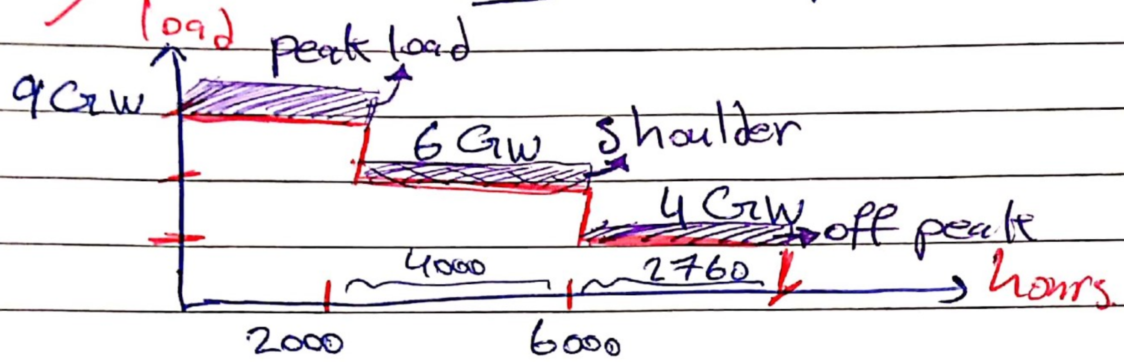
- Summary ? : Probabilistic.

[1] loss of load → hours

[2] Loss of Energy → Energy not supplied

[3] loss of load → (MW)

Ex) This is NOT a load profile:



two generations supplying:

→ G_1

- 5 GW (capacity) ?
- Thermal

→ G_2

- 5 GW , 10 TWh ? (capacity) (Energy)
- Hydro Storage.

Ⓐ Find R.M (MW/GW)

Ⓑ LOLE (hours)

Ⓒ LOLP (prob)

Ⓓ EENS (GWh/TWh)

Ⓔ LOEP (prob)

Ⓕ XLOL (MW/GW)

Lecture 6

Solution

Ⓐ R.M = 10 - 9 = 1 GW reserved

Ⓑ ما بتفرق من وين بتبلش، بس احنا
بلشنا من ال off peak

(last resort) used when it's needed!

	G1	G2	constraint (Energy)
off peak (4 GW)	4	0	0
Shoulder 6 GW	5	1	1 G * 4000 = $\underline{4}$ TWh used here This leaves us with 10-4 = $\underline{\underline{6 TWh}}$
peak 9 GW	5	4	4 * 2000 = 8 TWh & we only have 6 Twh left!!

⟹ 2 TWh defect

→ EENS: 2 TWh

→ $LOEP = \frac{2 \text{ TWh}}{\text{Load Energy}}$

$(9 * 2000) + (6 * 4000) + (2760 * 4)$

$\rightarrow$ $XLOL = \dfrac{2\ TW}{2000} = 1\ GW$

(Shoulder minimum) unstability

Solution : [1] shedding the load

[2] load side management $\rightarrow$ shift The load peak

بأن نشجع المستهلك بأسعار مختلفة.

$\hookrightarrow$ called Flatting the load.

$\rightarrow$ LOLE $\Rightarrow$ 2000 hours

$\rightarrow$ $LOLP = \dfrac{2000}{8760} = 0.228$

Ex) System 100 MW generation using 3 units.

Unit	rated	λ failure rate	μ repair
U1	25M	1%	0.49
U2	25M	1%	0.49
U3	50M	1%	0.49

(per day)

Daily peak load (MW)	57	52	46	34
Number of occurrence	12	83	107	47

total days 365

Calculate LOLE $\Rightarrow$ # of days

$F.O.R = \bar{A}$, $A = \dfrac{\mu}{\mu + \lambda} = \dfrac{0.49}{0.01 + 0.49}$

$A = 0.98$

$\bar{A} = 0.02 = F.O.R$

Load 57 ⇒ $\boxed{\text{or}}$ U_3 X fail and $(U_1 \& U_2)$ ✓ working

$\boxed{\text{or}}$ U_1 X & U_2 X and U_3 ✓

$\boxed{\text{or}}$ U_1 X & U_3 X and U_2 ✓

$\boxed{\text{or}}$ U_2 X & U_2 X and U_1 ✓

$\boxed{\text{or}}$ U_1 X & U_2 X & U_3 X

⇒ in a table

and

U_1	U_2	U_3			جمع
✓	✓	X	110	0.98*0.98*0.02	+
X	X	✓	001	0.02*0.02*0.98	
X	✓	X	010	0.02*0.98*0.02	+
✓	X	X	100	0.98*0.02*0.02	+
X	X	X	000	0.02*0.02*0.02	+

OR

total = 0.020392 * $\underline{\underline{12}}$ يوم ⇒ عدد الايام الي ما يقدر ازود فيها.

for 52 ⇒ نفس ال probabilites → لكن بيضرب ب 83 يوم!

For 46 ⇒ 0,000792 * 107 يوم

وهكذا ... وآخر اشي بنجمع كل الاجابات

→ The final LOLE = 2.15 day/year

$$LoLp = \frac{2.15}{365} = 0.00589 \#$$

Generating Capacity Reliability Evaluation

- Deterministic (non-probabilistic)
  - Reserve Margin (RM)
  - Largest Unit (LU)
    - Deterministic → not consider different size and type units, or different load characteristic
- Probabilistic
  - LOLP
  - EENS
  - LOEP
  - XLOL
    - Probabilistic → consider the size and type of units, load profile → more accurate

Capacity Outage Probability Table

- What is the aim of the capacity outage table (Generation Model)?
  - What is the probability of the unavailable generation capacity → Capacity outage (e.g., GW)
    - Unavailability → (Forced Outage Rate (FOR))
  - In other words: What is the probability of the available generation capacity → Capacity in (e.g., GW)
    - Availability (A) = 1- FOR
  - Binomial distribution function is used:
    - 2 states → On and Off
  - Start with small units and add one unit each time until all units are considered

Case 1: Identical Generation Units 1/8

- Example 1:
  - Construct the capacity outage table for a system consists of three 25MW generation units each of which has FOR = 0.02
- Solution:
  - Step 1: Determine all the possible combinations of the units
    - All possible states → $2^3 = 8$

State	Unit 1	Unit 2	Unit 3
S1	0	0	0
S2	0	0	1
S3	0	1	0
S4	0	1	1
S5	1	0	0
S6	1	0	1
S7	1	1	0
S8	1	1	1

Case 1: Identical Generation Units 2/8

- Step 2: Define the corresponding capacity-out and capacity-in

State	Unit 1	Unit 2	Unit 3	Capacity-out (MW)	Capacity-in (MW)
S1	0	0	0	75	0
S2	0	0	1	50	25
S3	0	1	0	50	25
S4	0	1	1	25	50
S5	1	0	0	50	25
S6	1	0	1	25	50
S7	1	1	0	25	50
S8	1	1	1	0	75

Case 1: Identical Generation Units 3/8

- Step 3: Group the states with same capacity-in/capacity-out

State	Unit 1	Unit 2	Unit 3	Capacity-out (MW)	Capacity-in (MW)
S1	0	0	0	75	0
S2	0	0	1	50	25
S3	0	1	0	50	25
S4	0	1	1	25	50
S5	1	0	0	50	25
S6	1	0	1	25	50
S7	1	1	0	25	50
S8	1	1	1	0	75

Grouping will result in 4 states

Case 1: Identical Generation Units 4/8

- Step 3: Group the states with same capacity-in/capacity-out and start from small capacity-out

Num. unit out of service	State	Capacity-out (MW)	Capacity-in (MW)
None	S8 (all units are available)	0	75
1 or 2 or 3	S4 or S6 or S7	25	50
(1 **and** 2) or (1 **and** 3) or (2 **and** 3)	S2 or S3 or S5	50	25
1 **and** 2 **and** 3	S1 (all units are unavailable)	75	0

Case 1: Identical Generation Units 5/8

- Step 4: Find the corresponding probability
  - Pr (none) = Pr (capacity-out=0MW) = Pr (capacity-in=75MW)
    - Unit-1 available **AND** Unit-2 available **AND** Unit-3 available
    - Pr(Unit-1 available) **x** Pr(Unit-2 available) **x** Pr(Unit-3 available)
    - Pr (Unit-1 available) = Pr(Unit-2 available) = Pr(Unit-2 available) =

1 – FOR = 1- 0.02 = 0.98

→ Pr (none)=0.98 x 0.98 x 0.98 =**0.9412**

- Pr (capacity-out=25MW) = Pr (capacity-in=50MW)
  - Unit-1 unavailable **AND** Unit-2 available **AND** Unit-3 available **OR**
  - Unit-1 available **AND** Unit-2 unavailable **AND** Unit-3 available **OR**
  - Unit-1 available **AND** Unit-2 available **AND** Unit-3 unavailable

→ Pr (capacity-in=50MW)= (0.02 x 0.98 x 0.98) + (0.98 x .02 x 0.98) +(0.98 x 0.98 x 0.02) = **0.057624**

Case 1: Identical Generation Units 6/8

- Step 4: Find the corresponding probability
  - Pr (capacity-out=50MW) = Pr (capacity-in=25MW)
    - Unit-1 unavailable **AND** Unit-2 unavailable **AND** Unit-3 available **OR**
    - Unit-1 unavailable **AND** Unit-2 available **AND** Unit-3 unavailable **OR**
    - Unit-1 available **AND** Unit-2 unavailable **AND** Unit-3 unavailable

→ Pr (capacity-in=50MW)= (0.02 x 0.02 x 0.98) + (0.02x .98 x 0.02) + (0.98 x 0.02 x 0.02) = **0.0012**

- Pr (capacity-out=75MW) = Pr (capacity-in=0MW)
  - Unit-1 unavailable **AND** Unit-2 unavailable **AND** Unit-3 unavailable

→ Pr (capacity-in=0MW)= (0.02 x 0.02 x 0.02) = **0.000008 ~ zero**

All the units to be out of service at the same time has very low probability 8 x 10^{-6}

Case 1: Identical Generation Units 7/8

- Step 5: Find the corresponding cumulative probability of capacity outage
- Pr (capacity-in<=75MW) = Pr (capacity-in=75MW) + Pr (capacity-in=50MW) + Pr (capacity-in=25 MW) + Pr (capacity-in=0MW)

  → Pr (capacity-in<=75MW) = 0.9414+0.0576+0.0012+0.000008= 1

Capacity Outage Table: ⬇

Capacity-out (MW)	Capacity-in (MW)	Pr (capacity= capacity-in)	Cumulative. Pr (capacity<= capacity-in)
0	75	0.9412	1
25	50	0.0576	0.0588
50	25	0.0012	0.0012
75	0	0	0

Case 1: Identical Generation Units 8/8

- HW-1: Refer to example 1, if a new 25MW unit is added with FOR = 0.02 then update the capacity outage table (**Hint:** use the resulting capacity outage table from example-1 and conditional probability)

- **To be submitted by 25/3/2020 please upload it through the e-learning (5 marks)**

COLORS ARE SWITCHED BETWEEN 100MW AND 0MW

After adding the new unit (25MW capacity), state combinations will increase to: $2^4 = 16$.

State	Unit1	Unit2	Unit3	Unit4	Capacity- out	Capacity- in
S1	0	0	0	0	100	0
S2	0	0	0	1	75	25
S3	0	0	1	0	75	25
S4	0	0	1	1	50	50
S5	0	1	0	0	75	25
S6	0	1	0	1	50	50
S7	0	1	1	0	50	50
S8	0	1	1	1	25	75
S9	1	0	0	0	75	25
S10	1	0	0	1	50	50
S11	1	0	1	0	50	50
S12	1	0	1	1	25	75
S13	1	1	0	0	50	50
S14	1	1	0	1	25	75
S15	1	1	1	0	25	75
S16	1	1	1	1	0	100

We have 5 states:
100MW → 🟪 x1
75MW → 🟥 x4
50MW → 🟦 x6
25MW → 🟫 x4
0MW → 🟩 x1

1-FOR= 1- 0.02
=0.98

So,
Pr(1)=0.98
Pr(0)=0.02

Pr (capacity- in=100MW)= 0.98x0.98x0.98x0.98 =0.9223

Pr (capacity- in=75MW)=
(0.02x0.98x0.98x0.98)+(0.98x0.02x0.98x0.98)+(0.98x0.98x0.02x0.98)+(0.98x0.98x0.98x0.02)
=0.0564

Pr (capacity- in=50MW)=
(0.02x0.02x0.98x0.98)+(0.02x0.98x0.02x0.98)+(0.02x0.98x0.98x0.02)+(0.98x0.02x0.02x0.98)
+(0.98x0.02x0.98x0.02)+(0.98x0.98x0.02x0.02)
= 0.002304

Pr (capacity- in=25MW)=
(0.02x0.02x0.02x0.98)+(0.02x0.02x0.98x0.02)+(0.02x0.98x0.02x0.02)+(0.98x0.02x0.02x0.02)
=3.136×10^{-5}

Pr (capacity- in=0MW) =
0.02x0.02x0.02x0.02 $\cong zero$

Capacity Outage Table

Capacity- out	Capacity- in	Pr (capacity = capacity- in)	Cumulative .Pr (capacity <= capacity- in)
0	100	0.9223	1
25	75	0.0564	0.05873
50	50	0.002304	2.335×10^{-3}
75	25	3.136×10^{-5}	3.136×10^{-5}
100	0	0	0

Reliability Indices Calculation 1/6

- Example 2
  - Refer to example 1, if the load duration curve is given as below

	Load (MW)	Frequency (hours)
On Peak	70	3500
Off peak	40	5260

  - Find the following indices
    - Reserve Margin (RM)
    - Loss of Load Prob (LOLP)
    - Expected Energy not supplied (EENS)
    - Expected Loss of Load (XLOL)
    - Loss of Energy Prob (LOEP)

Reliability Indices Calculation 2/6

- RM →
  - Peak demand = 70MW
  - Installed capacity = 75 MW
  - RM = 75 – 70 = 5 MW
- LOLP →
  - Find number of hours where the generation cant meet the demand by Using resulting **capacity outage table** from example-1
    - On-peak = 70 MW for 3500 hours. The generation may not meet the 70MW demand if:
      - Capacity-in = 50 MW
      - Capacity-in = 25 MW
      - Capacity-in =0 MW

} from **capacity outage table** get the probability

→ 3500 x 0.0576 + 3500 x 0.0012 + 3500 x 0

Reliability Indices Calculation 3/6

- LOLP →
  - Off-peak = 40 MW for 5260 hours. The generation mayl not meet the 70MW demand if:
    - Capacity-in = 25 MW
    - Capacity-in =0 MW

    } from **capacity outage table** get the probability

→ 5260 x 0.0012 + 5260 x 0

→ Expected number of hours load not served =

3500 x 0.0576 + 3500 x 0.0012 + 3500 x 0 +5260 x 0.0012 + 5260 x 0 = 212 hours/year

→ LOLP = 212/8760 =**0.0242**

Reliability Indices Calculation 4/6

- EENS →
  - On-peak = 70 MW for 3500 hours. The generation may not meet the 70MW demand if:
    - Capacity-in = 50 MW → 70 - 50 = 20MW

      Expected Energy =20 x (3500 x 0.0576)
    - Capacity-in = 25 MW → 70 - 25 = 45MW

      Expected Energy =45 x (3500 x 0.0012)
  - Off-peak = 40 MW for 5260 hours. The generation may not meet the 40MW demand if:
    - Capacity-in = 25 MW → 40 - 25 = 15MW

      Expected Energy =15 x (5260 x 0.0012)
    - Capacity-in = 0 MW → 40 - 0 = 40MW

      Expected Energy =40 x (5260 x 0.0)

Reliability Indices Calculation 5/6

- EENS →
  - On-peak = 70 MW for 3500 hours. The generation may not meet the 70MW demand if:
    - Capacity-in = 50 MW → 70 - 50 = 20MW

      Expected Energy =20 x (3500 x 0.0576)
    - Capacity-in = 25 MW → 70 - 25 = 45MW

      Expected Energy =45 x (3500 x 0.0012)

  - Off-peak = 40 MW for 5260 hours. The generation may not meet the 40MW demand if:
  - Capacity-in = 25 MW → 40 - 25 = 15MW

    Expected Energy =15 x (5260 x 0.0012)
  - Capacity-in = 0 MW → 40 - 0 = 40MW

    Expected Energy =40 x (5260 x 0.0)

Reliability Indices Calculation 6/6

- EENS →

20 x (3500 x 0.0576) + 45 x (3500 x 0.0012) + 15 x (5260 x 0.0012) + 40 x (5260 x 0.0) = **4316 MWh**

- XLOL → EENS/8760 = 4316/8760 = **0.49 MW**
- LOEP → EENS/(total demand energy)
  - Total demand energy = 70 x 3500 + 40 x 5260 = 455400MWh

    → LOEP = 4316/455400 = **0.00948**

Lecture 2

22/3/2020-26/3/2020

Capacity Outage Table: Model-Case 1 – Identical Units

- A general model to find the capacity outage table for identical generation units is the binomial distribution:

$$pr\ (x = r,\ n,\ q) = \frac{n!}{r!\,(n-r)!} q^r p^{n-r}$$

  - *n= number of units*
  - *r = number of units on forced outage (unavailable)*
  - *p = availability (probability of having in-service)*
  - *q = unavailability (forced outage rate)*

Capacity Outage Table: Model-Case 1 – Identical Units

- Example: consider the following four generating systems:
  - System 1 - 24x10MW Units with FOR= 0.01
  - System 2 - 12x20MW Units with FOR= 0.01
  - System 3 - 12x20MW Units with FOR= 0.03
  - System 4 - 22x10MW Units with FOR= 0.01

a) For system 1 find the probability out of service of 30 MW

b) Find the capacity outage table for system 1-4

c) Assume that all the systems are designed to have reserve of 20% then find the probability of load loss (***risk to loss the load***)

Capacity Outage Table: Model-Case 1 – Identical Units

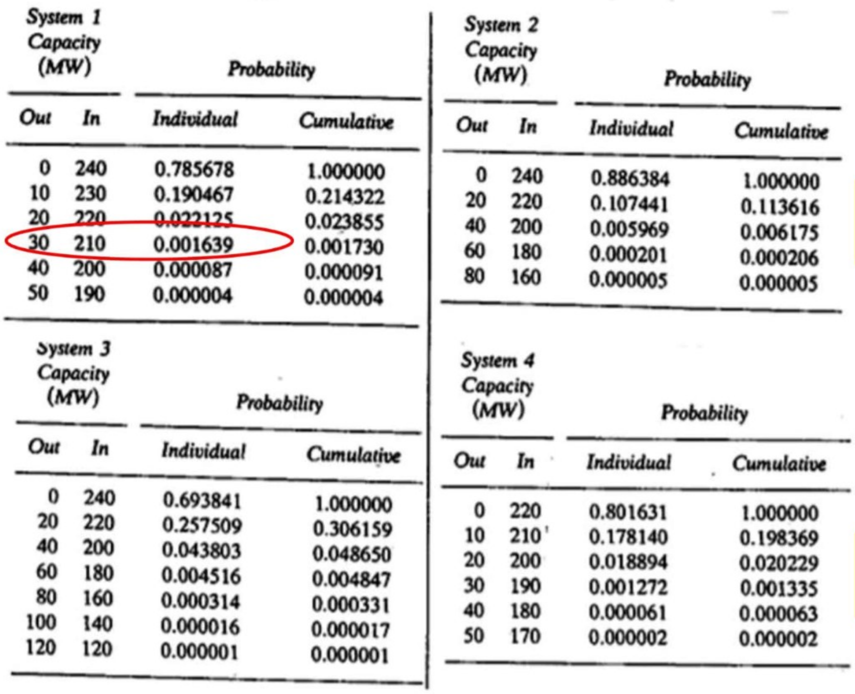
- a) System 1 - 24x10MW Units with FOR= 0.01
  - using the binomial distribution function

$$pr\,(x = r,\, n,\, q) = \frac{n!}{r!\,(n-r)!}\,q^r p^{n-r}$$

  - $n = 24$
  - $r = 3$ units ($3*10MW=30MW$)
  - $p = 1\text{-}0.01=0.99$
  - $q = 0.01$
- $pr\,(x = r,\, n,\, q) = \frac{24!}{3!(24-3)!}0.01^3 0.99^{24-3} = 0.001639$

Capacity Outage Table: Model-Case 1 – Identical Units

b) Capacity outage



10MW

System 1 Capacity (MW)		Probability	
Out	In	Individual	Cumulative
0	240	0.785678	1.000000
10	230	0.190467	0.214322
20	220	0.022125	0.023855
30	210	0.001639	0.001730
40	200	0.000087	0.000091
50	190	0.000004	0.000004

20MW

System 2 Capacity (MW)		Probability	
Out	In	Individual	Cumulative
0	240	0.886384	1.000000
20	220	0.107441	0.113616
40	200	0.005969	0.006175
60	180	0.000201	0.000206
80	160	0.000005	0.000005

20MW

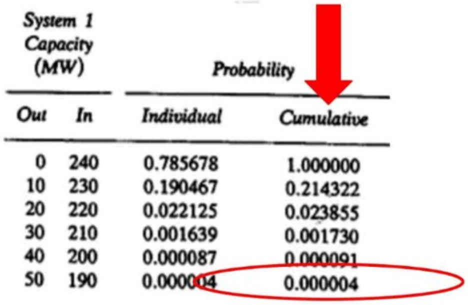
System 3 Capacity (MW)		Probability	
Out	In	Individual	Cumulative
0	240	0.693841	1.000000
20	220	0.257509	0.306159
40	200	0.043803	0.048650
60	180	0.004516	0.004847
80	160	0.000314	0.000331
100	140	0.000016	0.000017
120	120	0.000001	0.000001

10MW

System 4 Capacity (MW)		Probability	
Out	In	Individual	Cumulative
0	220	0.801631	1.000000
10	210	0.178140	0.198369
20	200	0.018894	0.020229
30	190	0.001272	0.001335
40	180	0.000061	0.000063
50	170	0.000002	0.000002

Capacity Outage Table: Model-Case 1 – Identical Units

- c) system 1→ Installed capacity = 24x10=240MW
  - Max demand → 240/1.2 = 200 MW → reserve=40MW
- Outage of more than (240MW – 200MW=40MW) → loss of load
- The probability to load loss = pr (outage>=50MW) → Find the cumulative probability for outage >=50M → Refer to the capacity outage table

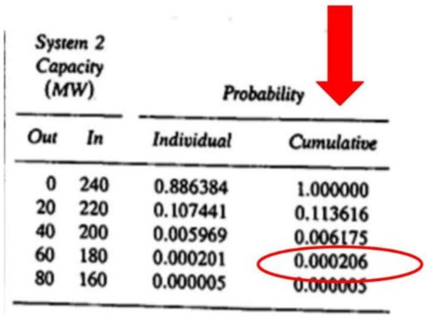


System 1 Capacity (MW)		Probability	
Out	In	Individual	Cumulative
0	240	0.785678	1.000000
10	230	0.190467	0.214322
20	220	0.022125	0.023855
30	210	0.001639	0.001730
40	200	0.000087	0.000091
50	190	0.000004	0.000004

The answer is 0.000004

Capacity Outage Table: Model-Case 1 – Identical Units

- c) system 2 → installed capacity = 12x20=240MW
  - Max demand=200MW → reserve=40MW **(same as system 1)**
- Outage of more than (240MW – 200MW=40MW) → loss of load
- The probability to load loss = pr (outage>=60MW)



System 2 Capacity (MW)		Probability	
Out	In	Individual	Cumulative
0	240	0.886384	1.000000
20	220	0.107441	0.113616
40	200	0.005969	0.006175
60	180	0.000201	0.000206
80	160	0.000005	0.000005

The answer is 0.000005+0.000201=.000206

Capacity Outage Table: Model- Case 1 – Identical Units

- System 3 → installed capacity = 12x20=240MW
  - Max demand=200MW, → reserve=40MW **(same as system 1 and 2) →**
  - Pr of load loss =0.004847
- System 4 → installed capacity = 22x10=220MW
  - Max demand=183MW, → reserve=37MW
  - Pr of load loss =0.000063

Capacity Outage Table: Model- Case 1 – Identical Units -Remarks

- The risk for load loss depends on:
  - FOR (Forced outage rate)
  - Number of units
  - Peak demand
- Adding new units is applied in generation expansion

Capacity Outage Table: Model- Case 2 – Non- Identical Units

- To produce the capacity outage table:
  - Produce the capacity outage tables for identical units ***(use binomial)***
  - Starting with the table with the smallest unit and start add the next unit to produce the new combinations of capacity outage and the corresponding probabilities
  - This method is called **recursive approach**
    - The following example illustrate the method

Capacity Outage Table: Model- Case 2 – Non- Identical Units

- Construct the capacity outage probability table for a system consists of: 2x3MW Units, each has FOR =0.02 and 1x5MW Unit, with FOR =0.02

Capacity out of service (MW)	Probability
0	0.98 x 0.98 =0.9604
3	0.98 x 0.02 x 2 = 0.0392
6	0.02 x 0.02 = 0.0004

Capacity out of service (MW)	Probability
0	0.98
5	0.02

Note: Omit all capacity outages for which the probabilities are less than 10^{-6}

Capacity Outage Table: Model-Case 2 – Non- Identical Units

Use Table 2x3MW with the first probability of 1x5MW (5MW unit in service)

Capacity out of service (MW)	Probability
0 +0	0.9604 x 0.98 = 0.941192
3 +0	0.0392 x 0.98 = 0.038416
6 +0	0.0004 x 0.98 = 0.000392

Capacity Outage Table: Model-Case 2 – Non- Identical Units

Capacity outage table

Capacity out of service (MW)	Probability	Cumulative
0	0.941192	1
3	0.038416	0.058808
5	0.019208	0.020392
6	0.000392	0.001184
8	0.000784	0.000008 + 0.000784 =0.000792
11	0.000008	0.000008

Capacity Outage Table: Recursive Model – No Derated States

- This model uses cumulative probability to find the capacity outage probability → recursive
- No derated states means 2 states (in service and out of service)
- The algorithm is applied as follows:
  - Step 1: Start with the smallest unit to produce the cumulative probability →
    - $Pr^{cum}(X \geq 0) = 1$
    - $Pr^{cum}(X \geq Unit\ capacity) = 0$
      - Where X: represent the outage capacity
      - Pr^{cum}: cumulative probability

Capacity Outage Table: Recursive Model – No Derated States

- Step 2: add the second unit to find the cumulative probability table using the following equation
  - $Pr_{new}{}^{cum}(X \geq outage\ capacity) =$

$$(1 - FOR_{\ new\ unit}) \times Pr_{old}{}^{cum}(X \geq outage\ capacity)$$
$$+$$
$$(FOR_{\ new\ unit})\ \times Pr_{old}{}^{cum}(X \geq outage\ capacity - new\ unit\ capacity)$$

Keep in mind to use the $Pr_{old}{}^{cum}$ from the table produced in the previous step

Capacity Outage Table: Recursive Model – No Derated States

- Step 3: Use step 2 to add the next unit and so on
- Step 4: Use the final table to produce the capacity outage probability from the resulting cumulative probability
  - The following example will illustrate the model

Capacity Outage Table: Recursive Model – No Derated States

- Example: Construct the capacity outage probability table for a system consists of: 2x25MW Units, and 1x50MW and each has FOR =0.02 using the **recursive model**
- Step 1: start with 25MW

Capacity out of service (MW)	Probability	Cumulative probability
0	0.98	1
25	0.02	0.02

Capacity Outage Table: Recursive Model – No Derated States

- Step 2: add 25MW (second unit) apply the following equation

$$Pr_{new}{}^{cum}(X \geq outage\ capacity) =$$
$$(1 - FOR_{\ new\ unit}) \times Pr_{old}{}^{cum}(X \geq outage\ capacity)$$
$$+$$
$$(FOR_{\ new\ unit})\ \times Pr_{old}{}^{cum}(X \geq outage\ capacity - new\ unit\ capacity)$$

$$Pr_{new}{}^{cum}(X \geq 0) =$$
$$(1 - 0.02) \times Pr_{old}{}^{cum}(X \geq 0)$$
$$+$$
$$(0.02)\ \times Pr_{old}{}^{cum}(X \geq 0 - 25)$$

From step 1: equals 1

From step 1: equals 1

$$Pr_{new}{}^{cum}(X \geq 0) = 1$$

Capacity Outage Table: Recursive Model – No Derated States

$$Pr_{new}{}^{cum}(X \geq 25) =$$
$$(1 - 0.02) \times Pr_{old}{}^{cum}(X \geq 25)$$
$$+$$
$$(0.02)\ \times Pr_{old}{}^{cum}(X \geq 25 - 25)$$

From step 1: equals 0.02

From step 1: equals 1

$$Pr_{new}{}^{cum}(X \geq 25) = (0.98)(0.02) + (0.02)(1) = 0.0396$$

Capacity Outage Table: Recursive Model – No Derated States

$$Pr_{new}^{\;cum}(X \geq 50) =$$

$$(1-0.02)\times Pr_{old}^{\;cum}(X \geq 50)$$

$$+$$

$$(0.02)\ \times Pr_{old}^{\;cum}(X \geq 50-25)$$

From step 1: equals 0

From step 1: equals 0.02

$$Pr_{new}^{\;cum}(X \geq 50) = (0.98)(0.) + (0.02)(0.02) = 0.0004$$

Continue to add the third unit

Capacity Outage Table: Recursive Model – No Derated States

Capacity out of service (MW)	Cumulative	Probability
0	1	1- 0.058808 = 0.941192
25	0.058808	0.058808- 0.020392 = 0.038416
50	0.020392	0.020392-0.000792= 0.0196
75	0.000792	0.000792-0.000008 = 0.000784
100	0.000008	0.000008

Capacity Outage Table: Recursive Model – Unit Removal

- The cumulative probability table is given then you need to remove a unit → Maintenance
- Rearrange the following equation to update the cumulative table

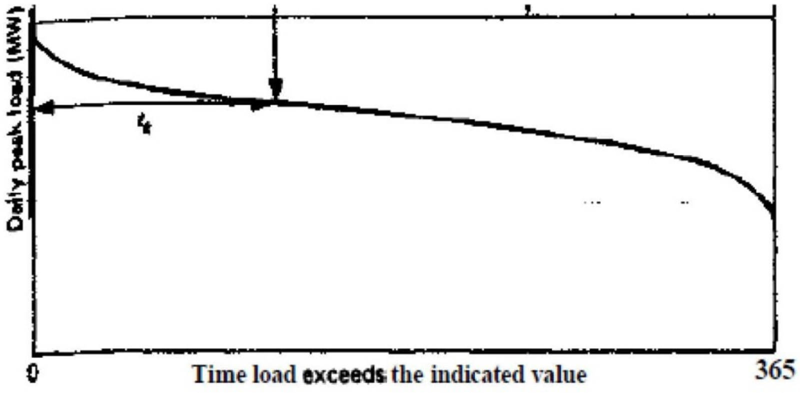
$$Pr_{new}{}^{cum}(X \geq outage\ capacity) =$$
$$(1 - FOR_{\ new\ unit}) \times Pr_{old}{}^{cum}(X \geq outage\ capacity)$$
$$+$$
$$(FOR_{\ new\ unit})\ \times Pr_{old}{}^{cum}(X \geq outage\ capacity - new\ unit\ capacity)$$

After rearrangement

$$Pr_{old}{}^{cum}(X \geq outage\ capacity) = [Pr_{new}{}^{cum}(X \geq outage\ capacity) - (FOR_{\ new\ unit})\ \times Pr_{old}{}^{cum}(X \geq outage\ capacity - new\ unit\ capacity)]/(1 - FOR_{\ new\ unit})$$

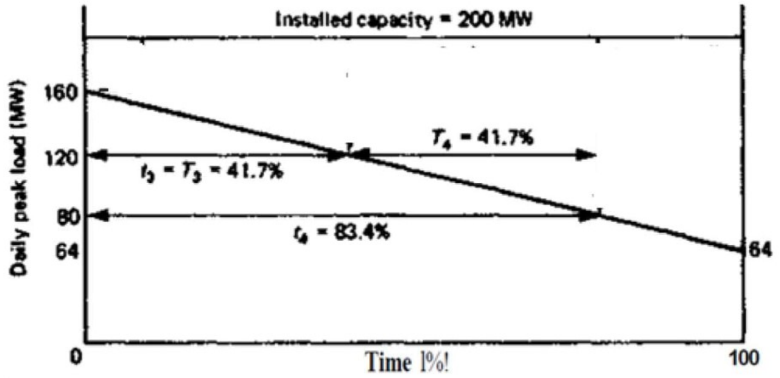
Loss of Load – Peak Load Variation Curve

- Peak Load Variation Curve represents the number of days (hours/year) the load exceed a certain value
  - Yaxis → Peak load
  - X axis → number of days/year or hours/year or a percentage



Loss of Load – Peak Load Variation Curve - Example

- Consider 5 x 40 MW units each with FOR = 0.01. If the forecast peak load for this system is 160MW and the peak load variation curve is given in figure below then find the following:
  - a) Capacity outage table
  - b) Loss of load probability (LOLP) for this system



Loss of Load – Peak Load Variation Curve - Example

- a) Capacity outage table

Capacity out of service (MW)	Probability	Cumulative
0	0.950991	1
40	0.048029	0.049009
80	0.000971	0.000980
120	0.000009	.000009

Note: Omit all capacity outages for which the probabilities are less than 10^{-6}

Loss of Load – Peak Load Variation Curve - Example

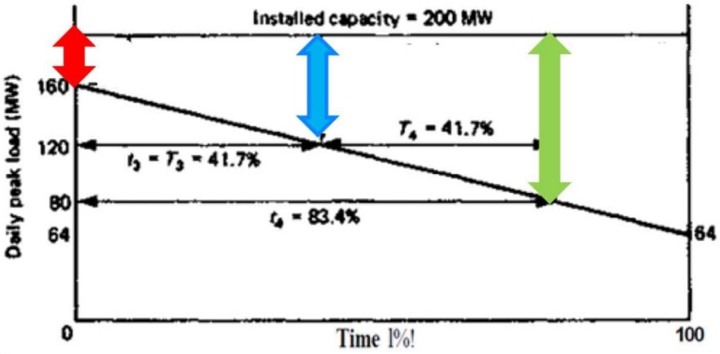
- b) LOLP
  - Installed capacity = 5 x 40 = 200MW
  - Peak demand = 160 MW
  - Loss of load occurs if the load exceeds the generation
    - If the capacity out = 40MW then the capacity in (200- 40 = 160MW) which equals the peak demand → no load loss
    - If the capacity out = 80MW then the capacity in =120MW → the load above 120MW will not be served (load loss)
      - LOLP1 = $Pr(X = 80MW) \times time\% \ the \ load \ exceed \ 120MW$

From capacity outage table: 0.000971

From peak load variation curve: 41.7%

- LOLP1 = 0.000971 x 41.7%= 0.000404907

Loss of Load – Peak Load Variation Curve - Example



🟥 → 40MW generation capacity outage

🟦 → 80MW generation capacity outage

🟩 → 120MW generation capacity outage

Loss of Load – Peak Load Variation Curve - Example

- b) LOLP
  - Loss of load occurs if the load exceeds the generation
    - If the capacity out = 120MW then the capacity in =80MW → the load above 80MW will not be served (load loss)
      - $LOLP_2$ = $Pr(X = 120MW) \times time\%\ the\ load\ exceed\ 80MW$

From capacity outage table: 0.000009

From peak load variation curve: 83.4%

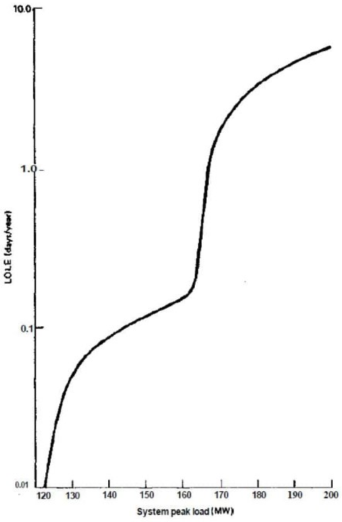
      - $LOLP_2 = 0.000009 \times 83.4\% = 0.000007506$
    - LOLP (LOLE) $= LOLP_1 + LOLP_2$
    - **LOLP= 0.000412413**
    - **0.000412413 x 365 = 0.1505 days/year**

Lecture 3

29/3/2020-2/4/2020

Loss of Load – Sensitivity studies – System Peak load

- Repeat the previous example and change the system peak load (sensitivity analysis) the results are shown as below (using logarithmic scale)

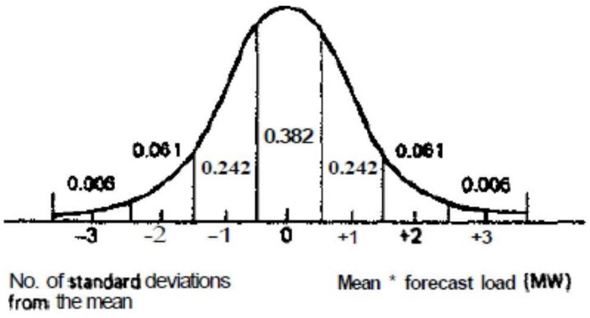


Using this curve you can design your system with the desired LOLP

Using the same approach you can produce a sensitivity analysis curve for different FOR

Load Forecast Uncertainty

- Finding the LOLE is based on the forecast. However uncertainty in load forecast will change the value of the LOLE
- Refer to the previous example if the load forecast follow the normal pdf (shown below) with variance = 20MW, then find the value of LOLE



Load Forecast Uncertainty

- Produce the LOLE for the possible values of load peak

Peak load (MW)	LOLE (days/year)	Scenario Probability	Col_2 x Col_3
120	0.002005	0.061	0.000122305
140	0.08686	0.242	0.02102012
160	0.1506	0.382	0.0575292
180	3.447	0.242	0.834174
200	6.083	0.067	0.371063

- LOLE=$\sum LOLE$ =1.284 compared to 0.1506 days if uncertainty is not considered

Capacity Expansion Analysis

- Capacity expansion → at which year/years new units have to be added to meet accepted system risk level in other words accepted LOLP
- Capacity expansion → How much capacity to be added to meet accepted system risk level in other words accepted LOLP → This capacity is called Peak Load Carrying Capability (PLCC)
  - Inputs:
    - Current installed capacity (MW) at Year 0
    - Current peak load (MW) at Year 0
    - Annual load growth (% peak load from the previous year)
    - Accepted LOLP (days/year) or (hours/year)

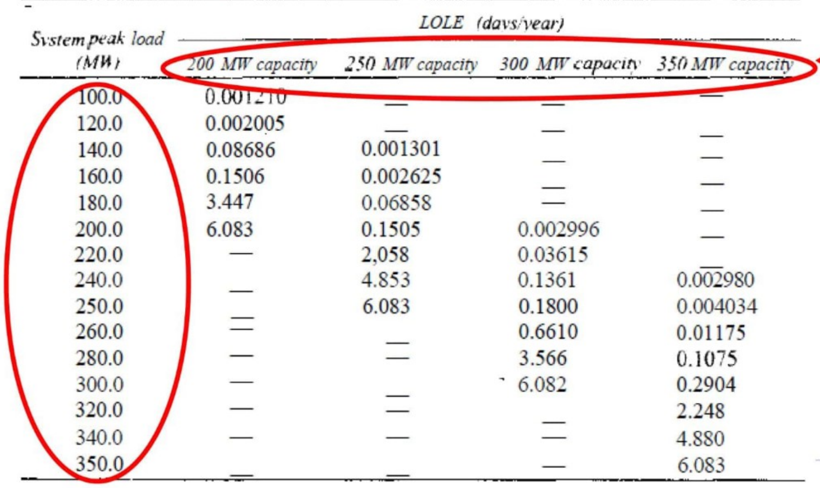
Capacity Expansion Analysis - Example

- Consider 5 x 40 MW units each with FOR = 0.01. **Do the capacity expansion analysis** assuming that:
  - Additional 50MW units to be added with FOR = 0.01
  - The load growth is assumed to be 10% per year
  - The current peak load is 160MW
  - Accepted LOLP = 0.15days/year

Capacity Expansion Analysis - Example

- Step 1: Produce LOLE (days/year) for different peak load → Refer to the previous example
- Step 2: extend the LOLE table to add 50MW, 100, 150, and so on

Table 2.17 LOLE in generation expansion



System peak load (MW)	LOLE (days/year)			
	200 MW capacity	250 MW capacity	300 MW capacity	350 MW capacity
100.0	0.001210	—	—	—
120.0	0.002005	—	—	—
140.0	0.08686	0.001301	—	—
160.0	0.1506	0.002625	—	—
180.0	3.447	0.06858	—	—
200.0	6.083	0.1505	0.002996	—
220.0	—	2,058	0.03615	—
240.0	—	4.853	0.1361	0.002980
250.0	—	6.083	0.1800	0.004034
260.0	—	—	0.6610	0.01175
280.0	—	—	3.566	0.1075
300.0	—	—	6.082	0.2904
320.0	—	—	—	2.248
340.0	—	—	—	4.880
350.0	—	—	—	6.083

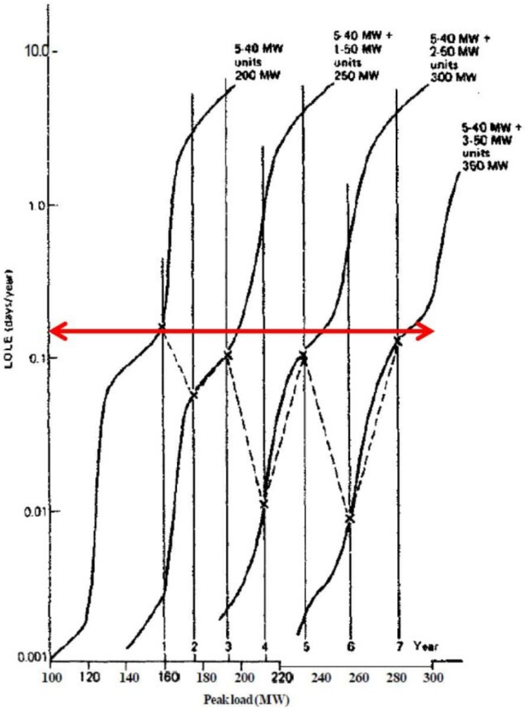
Generation capacity start from 5 x 40 =200MW

Then add 50 → 250MW and so on

LOLE = LOLP

Capacity Expansion Analysis - Example

- Step 3: Draw the LOLP (days/year) versus the peak demand



Capacity Expansion Analysis - Example

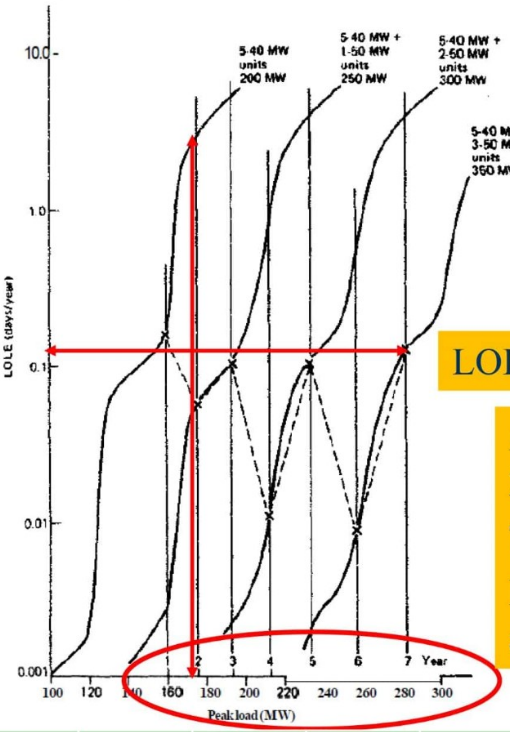
- Step 4: Produce peak load for the upcoming years

Year	Forecast Peak load (MW)
1	160
2	160 x 1.1 = 176
3	176 x 1.1 = 193.6
4	213
5	234.3
6	257.5
7	283.1
8	311.4

Capacity Expansion Analysis - Example

- Step 5: From the graph and peak demand at each year determine at which year new generation has to be commissioned to respect the desired LOLP (0.15days/year)

Capacity Expansion Analysis - Example



LOLP =0.15

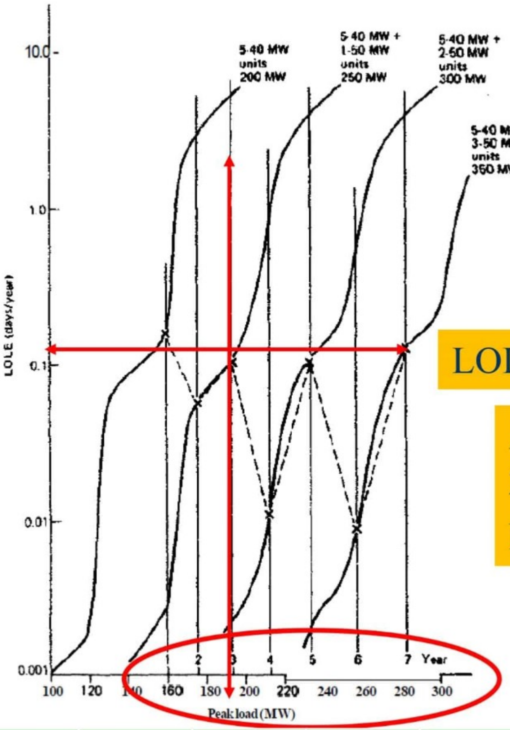
At year 2 if the capacity remains 200MW → LOLE = 5 → not acceptable

Try 250MW → LOLE < 0.1

Decision → At year 2 do expansion with adding new 50MW

Year1	2	3	4	5	6	7	8
160	176	194	213	234	258	283	311

Capacity Expansion Analysis - Example



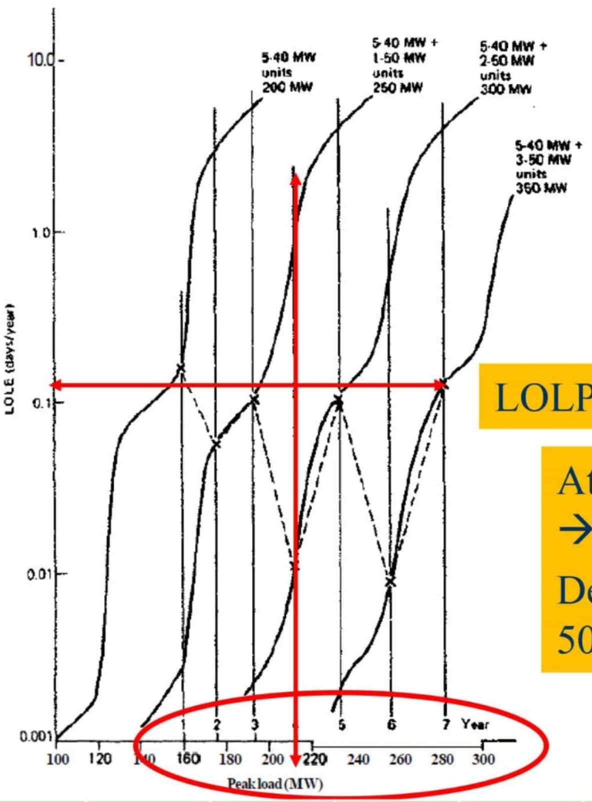
LOLP =0.15

At year 3 if the capacity remains 250MW → LOLE = 0.1 → acceptable

Decision → No expansion

Year1	2	3	4	5	6	7	8
160	176	194	213	234	258	283	311

Capacity Expansion Analysis - Example



LOLP =0.15

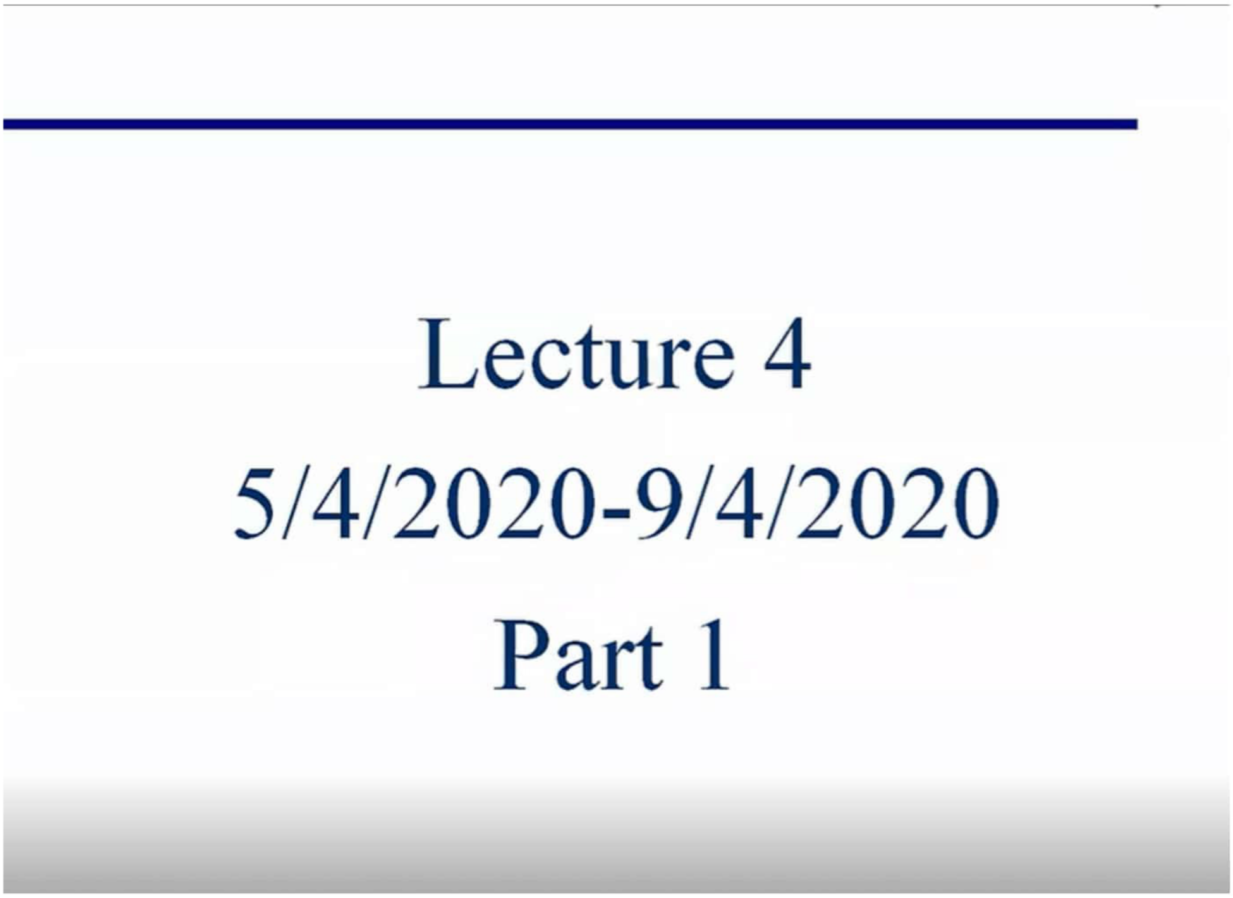
At year 4 if the capacity remains 250MW → LOLE > 0.15 → not acceptable

Decision → Do expansion by adding new 50MW the new capacity = 300MW

Year1	2	3	4	5	6	7	8
160	176	194	213	234	258	283	311

Capacity Expansion Analysis - Example

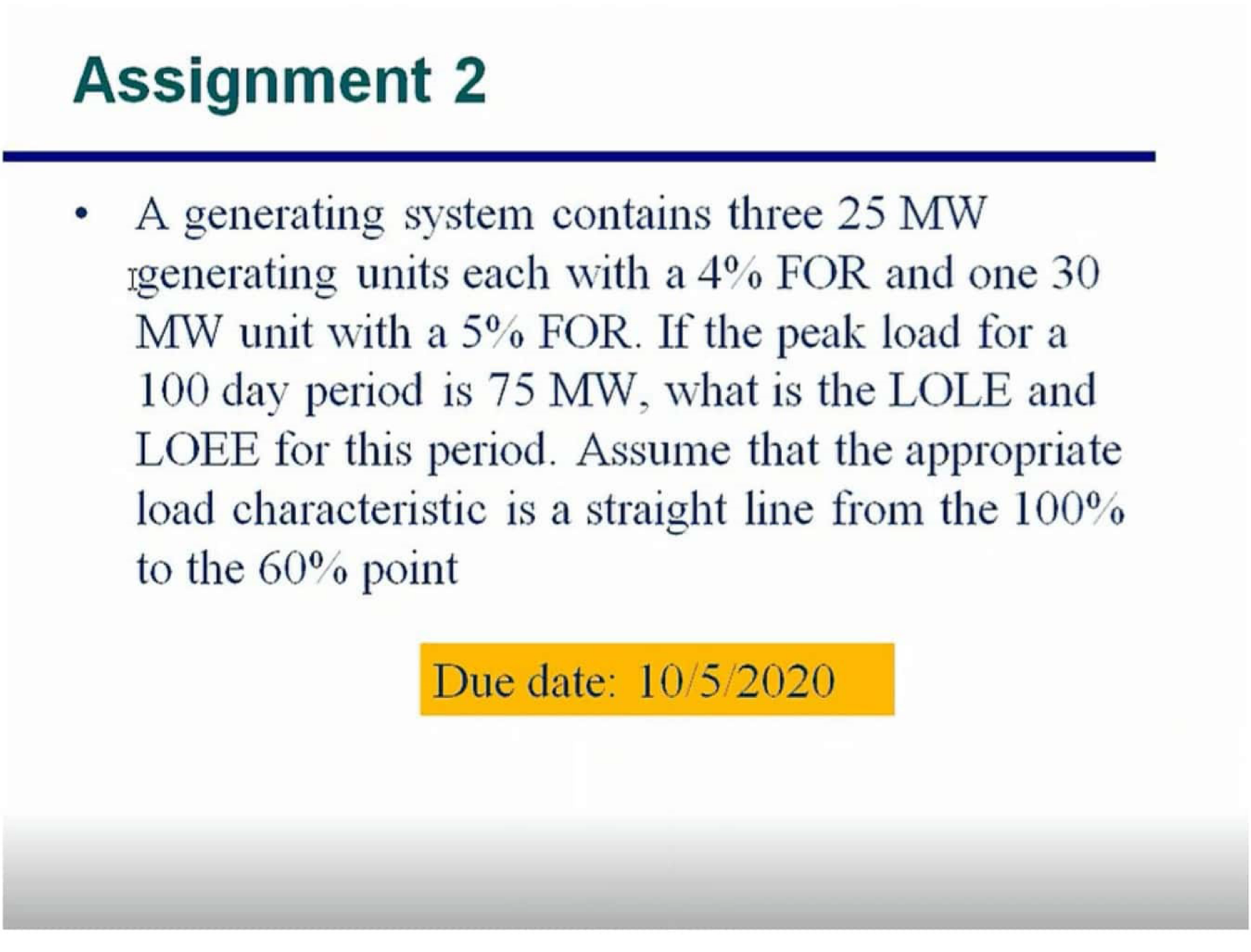
- Expansion to be carried out at year 2, 4, 6 and 8 with adding 50MW each year end with 400MW installed capacity by the end of year 8
- Generation expansion plan is carried out over 20-30 years time horizon



Lecture 4

5/4/2020-9/4/2020

Part 1



Assignment 2

- A generating system contains three 25 MW generating units each with a 4% FOR and one 30 MW unit with a 5% FOR. If the peak load for a 100 day period is 75 MW, what is the LOLE and LOEE for this period. Assume that the appropriate load characteristic is a straight line from the 100% to the 60% point

Due date: 10/5/2020

Problem: System with **((3- 25 MW))** each with **((4%))** FOR and **((1- 30 MW))** with **((5%))**FOR. Peak load for **100** days is **((75 MW))**

- What's the LOLE and LOEE for this period? (Assume load characteristic is a straight line from the 100% to the 60%)

Solution:

- For the 3- 25 MW generators:

FOR= 0.04
1-FOR= 0.96
P(0)=0.04, P(1)=0.96
Number of states= 2^3=8

G1-25	G2-25	G3-25	IN	OUT
0	0	0	0	75
0	0	1	25	50
0	1	0	25	50
0	1	1	50	25
1	0	0	25	50
1	0	1	50	25
1	1	0	50	25
1	1	1	75	0

Capacity Out	Probability
0	0.884736
25	0.110592
50	0.004608
75	0.000064

- For the 1- 30 MW generator:

FOR= 0.05
1-FOR= 0.95
P(0)=0.05, P(1)=0.95
Number of states= 2^1=2

G1-30	IN	OUT
0	0	30
1	30	0

Capacity Out	Probability
0	0.05
30	0.95

- Entire System:

G1-25	G2-25	G3-25	G1-30	IN	OUT
0	0	0	0	0	105
0	0	0	1	30	75
0	0	1	0	25	80
0	0	1	1	55	50
0	1	0	0	25	80
0	1	0	1	55	50
0	1	1	0	50	55
0	1	1	1	80	25
1	0	0	0	25	80
1	0	0	1	50	55
1	0	1	0	50	55
1	0	1	1	80	25
1	1	0	0	50	55
1	1	0	1	80	25
1	1	1	0	75	30
1	1	1	1	105	105

Capacity out	Probability	hours	MWh
0	0.84049	-	-
25	0.10506	-	-
30	0.04423	-	-
50	0.00437	1600	16,000
55	0.00553	2000	25,000
75	0.00006	2400	72,000
80	0.00023	2400	84,000
105	0.000003	2400	144,000

- Loss of Load Expectation (LOLE):

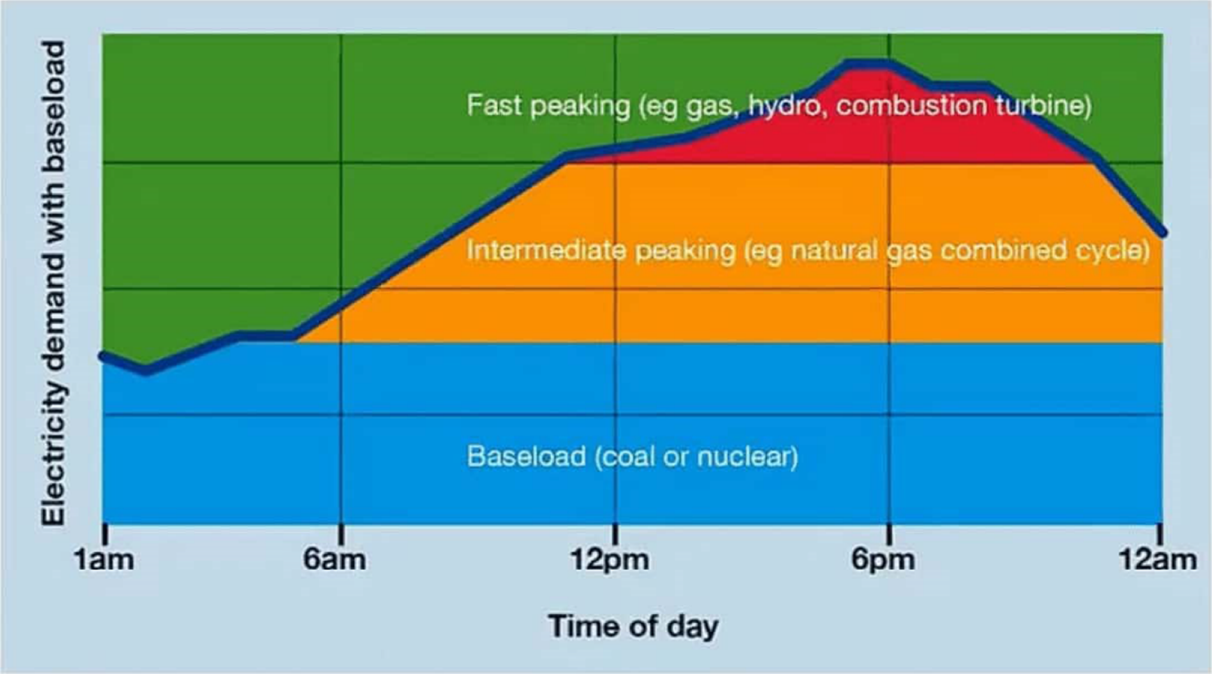
$$\sum_{k=1}^{n} \mathrm{P_k t_k} = \sum_{\mathrm{k=1}}^{\mathrm{n}} (\mathrm{t_k} - \mathrm{t_{k-1}})\mathrm{P_k}$$

$$= 18.77 \text{ hrs}/100\text{d}$$

- Loss of Energy Expectation (LOEE):

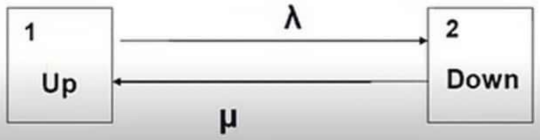
$$\sum_{k=1}^{n} \mathrm{P_k E_k} = 232.44 \text{ MWh}/100\text{d}$$

Generating Capacity



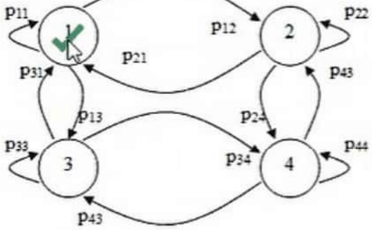
Generating Capacity: Frequency and Duration Method (Markov Model)

- Two- state model adopted so far (Up (working), down (failed)) is only appropriate to determine the capacity outage for base load units
- Two- state model is not appropriate to find the capacity outage for peak or reserve load units neither considering maintenance intervals why?
  - It does not consider the de-rated state of the unit (not operate at full capacity or being shut in purpose to be used through reserve or peak time periods)



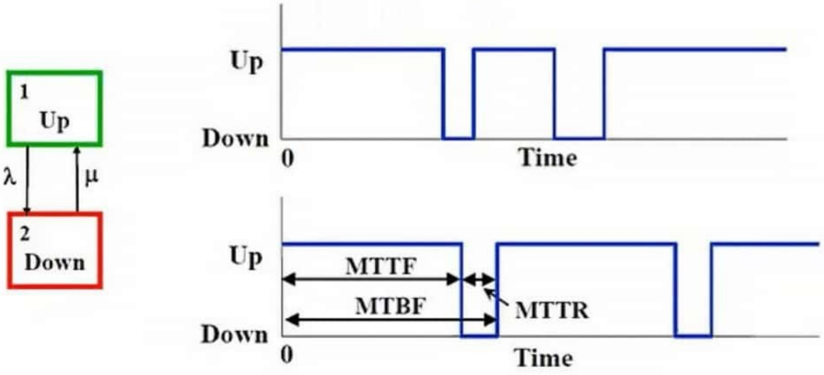
Generating Capacity: Frequency and Duration Method (Markov Model)

- Three states: S1 (working well), S2 (failed), S3 (working with deterioration)
- Four states: S1 (working well) S2 (failed), S3(working with minor deterioration), S4 (working with major deterioration)



Generating Capacity: Frequency and Duration Method (Two-State Model)

System Availability

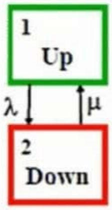


$$\text{MTTF} = \frac{\text{total up time}}{\text{\# of failures}} = \frac{1}{\lambda} \qquad \text{MTBF} = 1/F$$

$$\text{MTTR} = \text{average repair time} = r = \frac{\text{total down time}}{\text{\# of failures}} = \frac{1}{\mu}$$

Generating Capacity: Frequency and Duration Method (Two-State Model)

Frequency and Duration Evaluation



Frequency of encountering State *i*
= P(being in State *i*) x (rate of departure from State *i*)
= P(not being in State *i*) x (rate of entry into State *i*)

$$P_1.\lambda = P_2.\mu \quad .. \quad \text{Eq. 1}$$

$$P_1 + P_2 = 1 \quad .. \quad \text{Eq. 2}$$

Solving Equations 1 and 2, $P_1 = \frac{\mu}{\lambda+\mu} = A$ and $P_2 = \frac{\lambda}{\lambda+\mu} = U$

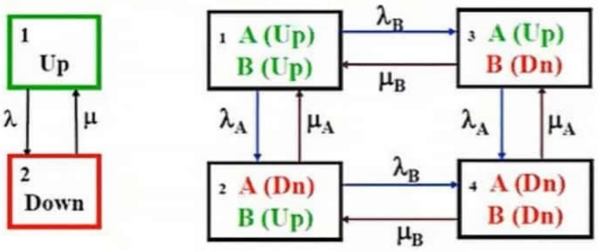
Frequency of encountering the Down State,

$$F_{Down} = P_2 \text{ x (rate of departure from State 2)} = \frac{\lambda}{\lambda+\mu}\mu$$

Mean Duration in the Down State $= U / F_{Down} = 1/\mu$

Generating Capacity: Frequency and Duration Method (Two-state Model)

Frequency and Duration Evaluation



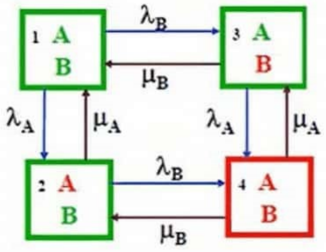
Probability of being in State *i* → Availability, Unavailability

Frequency of encountering State *i*
= P(being in State *i*) x (rate of departure from State *i*)

$$\text{Mean Duration in State } i = \frac{\text{Probabilit y of being in State } i}{\text{Frequency of encountering State } i}$$

Generating Capacity: Frequency and Duration Method (Two-State Model)

Parallel System Evaluation



$$P_1 = \left(\frac{\mu_A}{\lambda_A + \mu_A}\right)\left(\frac{\mu_B}{\lambda_B + \mu_B}\right)$$

$$P_2 = \left(\frac{\lambda_A}{\lambda_A + \mu_A}\right)\left(\frac{\mu_B}{\lambda_B + \mu_B}\right)$$

$$P_3 = \left(\frac{\mu_A}{\lambda_A + \mu_A}\right)\left(\frac{\lambda_B}{\lambda_B + \mu_B}\right)$$

$$P_4 = \left(\frac{\lambda_A}{\lambda_A + \mu_A}\right)\left(\frac{\lambda_B}{\lambda_B + \mu_B}\right)$$

System Unavailability, $U = P_4 = \left(\frac{\lambda_A}{\lambda_A + \mu_A}\right)\left(\frac{\lambda_B}{\lambda_B + \mu_B}\right)$

Frequency of Failure
= (P_4).(rate of departure from State 4) = $U.(\mu_A + \mu_B)$

Mean Duration of Failure = $U / F_{failure} = 1/ (\mu_A + \mu_B)$

Generating Capacity: Frequency and Duration Method (Two-State Model)

Parallel System Example

A customer is supplied by a distribution system that consists of an underground cable in parallel with an overhead line. The failure rate and the average repair time of the cable are 1 failure/year and 100 hours respectively, and that of the overhead line are 2 failure/year and 10 hours respectively. Evaluate the unavailability, frequency and the mean duration of failure of the distribution system.

Underground Cable:	**Overhead Line:**
$\lambda_A = 1$ f/yr	$\lambda_B = 2$ f/yr
$\mu_A = 1/r_1 = 8760/100 = 87.6$ rep/yr	$\mu_B = 1/r_2 = 8760/10 = 876$ rep/yr

System Unavailability, U $= P_4 = \left(\frac{\lambda_A}{\lambda_A + \mu_A}\right)\left(\frac{\lambda_B}{\lambda_B + \mu_B}\right)$

$= [1/(1+87.6)].[2/(2+876)] = 0.000026$

$= 0.000026 \times 8760 = 0.2252$ hr/yr

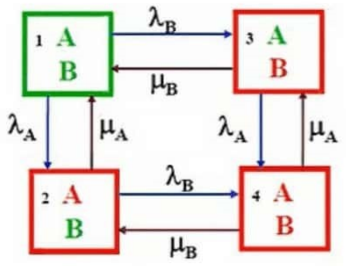
Frequency of Failure $= U.(\mu_A + \mu_B) = 0.000026 \times (87.6 + 876) = 0.0251$ f/yr

Mean Duration of Failure $= 1/(\mu_A + \mu_B) = 1/(87.6 + 876) = 0.001$ yr $= 9.09$ hr

---

Generating Capacity: Frequency and Duration Method (Two-State Model)

Series System Evaluation



Component A: $\lambda_A = 1$ f/yr, $\mu_A = 87.6$ r/yr

Component B: $\lambda_B = 2$ f/yr, $\mu_B = 876$ r/yr

$P_1 = 0.986461$

$P_2 = 0.011261$

$P_3 = 0.002252$

$P_4 = 0.000026$

System Unavailability, $U = P_2 + P_3 + P_4 = 0.013539$

$= 0.013539 \times 8760 = 118.60$ hr/yr

Frequency of Failure, $F_{failure} = P_2.\mu_A + P_3.\mu_B$

$= 0.011261 \times 87.6 + 0.002252 \times 876 = 2.96$ f/yr

Mean Duration of Failure $= U / F_{failure} = 0.013539 / 2.96 = 0.004575$ yr

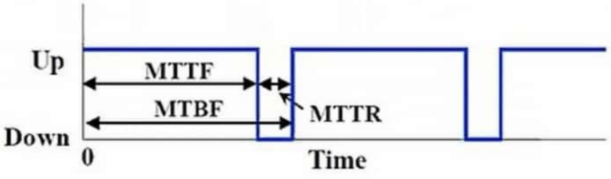
$= 0.004575 \times 8760 = 40.08$ hr

Generating Capacity: Frequency and Duration Method

F & D Using Approximate Equation

$U = F_{failure} .r$

$\approx \lambda.r$ for MTTF $(1/\lambda) \approx$ MTBF $(1/ F_{failure})$



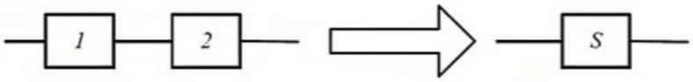
Generating Capacity: Frequency and Duration Method

Practical Adequacy Indices

- Failure rate (or frequency)
  - λ= failures/operating time
  - f = failures/time
- Average outage time
  - r = time/failure
- Average annual outage time
  - $U = f.r \approx \lambda.r$

Generating Capacity: Frequency and Duration Method

Series Systems



$$\lambda_s = \lambda_1 + \lambda_2 = \sum \lambda_i$$

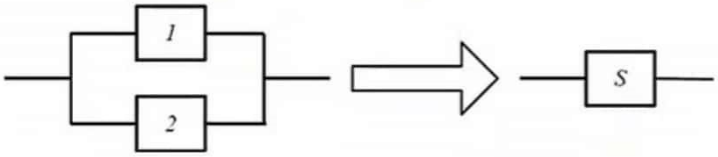
$$r_s = \frac{\lambda_1 r_1 + \lambda_2 r_2 + \lambda_1 \lambda_2 r_1 r_2}{\lambda_1 + \lambda_2}$$

$$\approx \frac{\lambda_1 r_1 + \lambda_2 r_2}{\lambda_1 + \lambda_2} = \frac{\sum \lambda_i r_i}{\sum \lambda_i}$$

$$U_s \approx \lambda_s r_s$$

Generating Capacity: Frequency and Duration Method

Parallel Systems



$$\lambda_s = \frac{\lambda_1 \lambda_2 (r_1 + r_2)}{1 + \lambda_1 r_1 + \lambda_2 r_2}$$

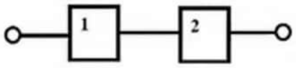
$$\approx \lambda_1 \lambda_2 (r_1 + r_2)$$

$$r_s = \frac{r_1 r_2}{r_1 + r_2}$$

$$U_s \approx \lambda_s r_s$$

Generating Capacity: Frequency and Duration Method

Availability, F & D – Series System



Component 1:
$\lambda_1 = 1$ f/yr
$r_1 = 100$ hr

Component 2:
$\lambda_2 = 2$ f/yr
$r_2 = 10$ hr

System failure rate, $\lambda_s = \Sigma \lambda_i = \lambda_1 + \lambda_2 = 1 + 2 = 3$ f/yr

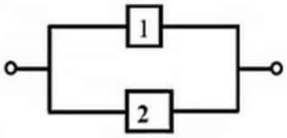
System unavailability, $U_s = \Sigma \lambda_i r_i = 1 \times 100 + 2 \times 10 = 120$ hr/yr

System average down time, $r_s = U_s / \lambda_s = 120/3 = 40$ hr

---

Generating Capacity: Frequency and Duration Method

Availability, F & D – Parallel System



Component 1:
$\lambda_1 = 1$ f/yr
$r_1 = 100$ hr

Component 2:
$\lambda_2 = 2$ f/yr
$r_2 = 10$ hr

System failure rate, $\lambda_s = \lambda_1 . \lambda_2 (r_1 + r_2)$

$= 1 \times 2 \times (100 + 10)/8760 = 0.0251$ f/yr

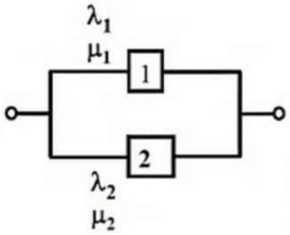
System average down time, $r_s = r_1 . r_2 / (r_1 + r_2)$

$= 100 \times 10 / (100 + 10) = 9.09$ hr

System unavailability, $U_s = \lambda_s r_s = 0.025 \times 9.09 = 0.228$ hr/yr

Generating Capacity: Frequency and Duration Method

Approximate Equations for Parallel Systems



For a 2-component parallel system,

$\lambda_s \approx \lambda_1.\lambda_2\,(r_1 + r_2)$ for $\lambda_i\,.r_i << 1$ → $\lambda_s = \lambda_1(\lambda_2\, r_1) + \lambda_2(\lambda_1\, r_2)$

$r_s = r_1.r_2 \,/\, (r_1 + r_2)$ → $\mu_s = \Sigma\, \mu_i$

$U_s = \lambda_s\,.r_s$

Lecture 4

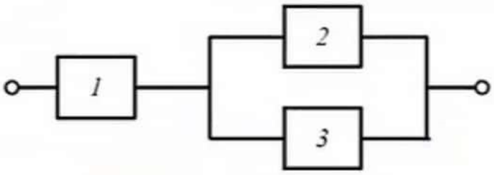
12/4/2020-16/4/2020

Part 2

Network Analysis Techniques

Basic Network Analysis Techniques

- Series / Parallel Reduction
- Minimal Cut Set Analysis



Transmission System Reliability Evaluation? Minimal Cut Set Method

Minimal Cut Set Method

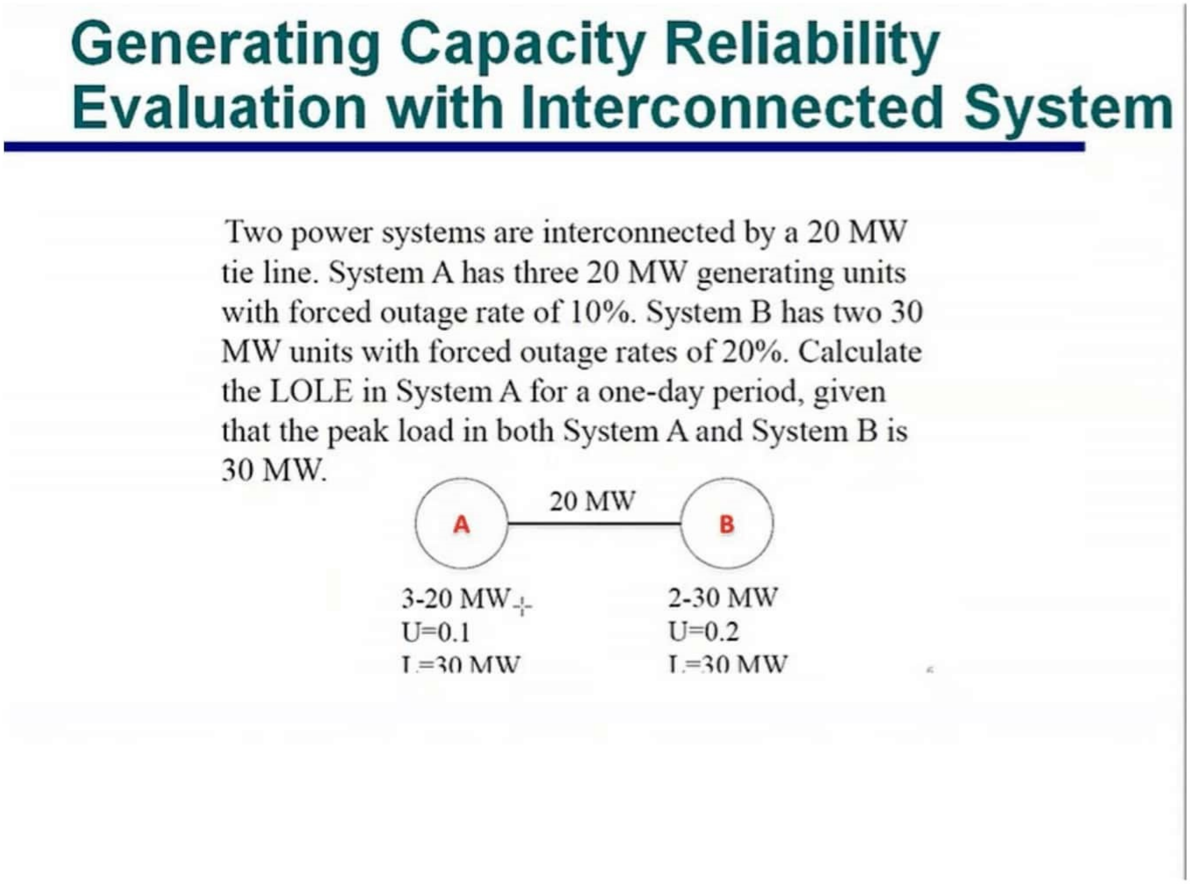
Cut Set – A set of components which if removed from the network separate the input from the output.

i.e. cause the network to fail.

Minimal Cut Set – Any cut set which does not contain any other cut sets as subsets.

$$\begin{aligned} P\{\text{System Failure}\} &= P\{\text{Union of All Cut Sets}\} \\ &= P\{\text{Union of All Minimal Cut Sets}\} \\ &\leq \sum P\{\text{Min Cut Sets}\} \end{aligned}$$

This is a good approximation for highly reliable components.

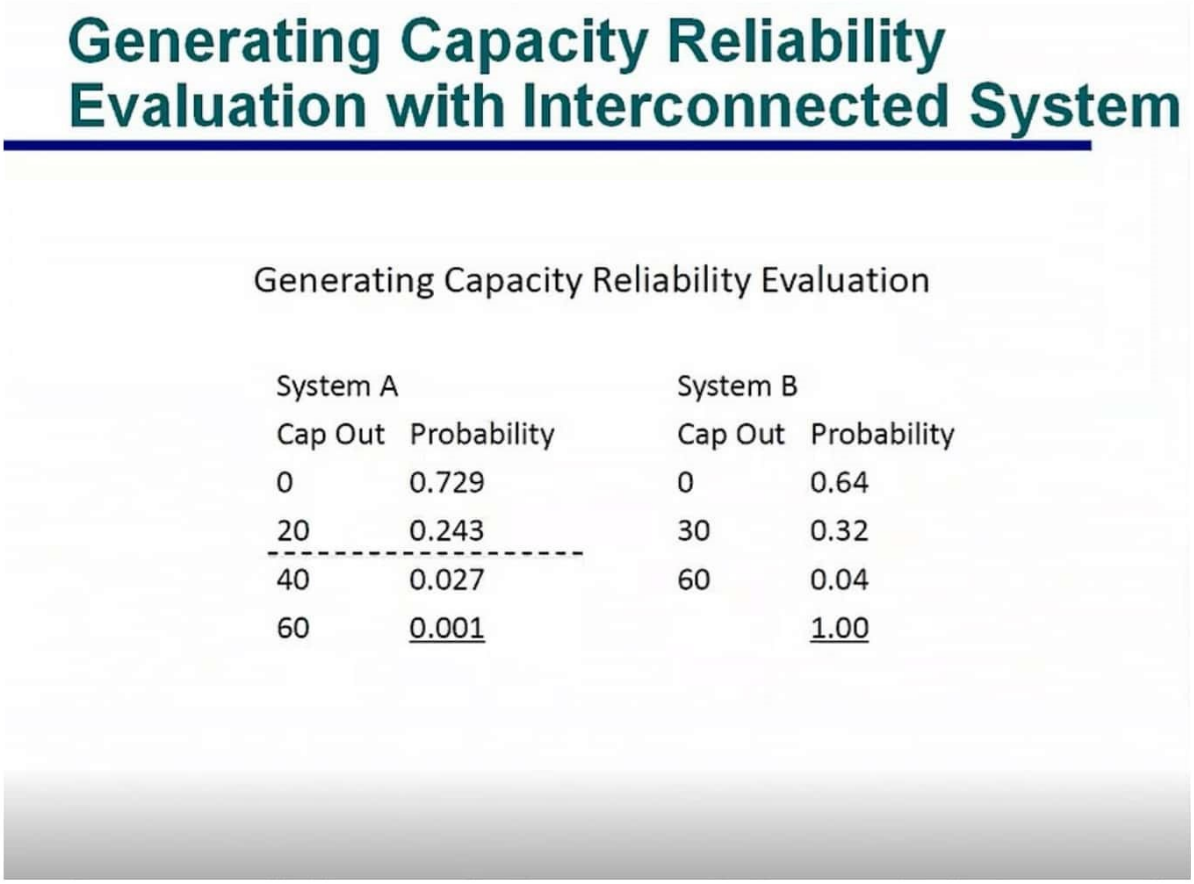


Generating Capacity Reliability Evaluation with Interconnected System

Two power systems are interconnected by a 20 MW tie line. System A has three 20 MW generating units with forced outage rate of 10%. System B has two 30 MW units with forced outage rates of 20%. Calculate the LOLE in System A for a one-day period, given that the peak load in both System A and System B is 30 MW.

3-20 MW
U=0.1
L=30 MW

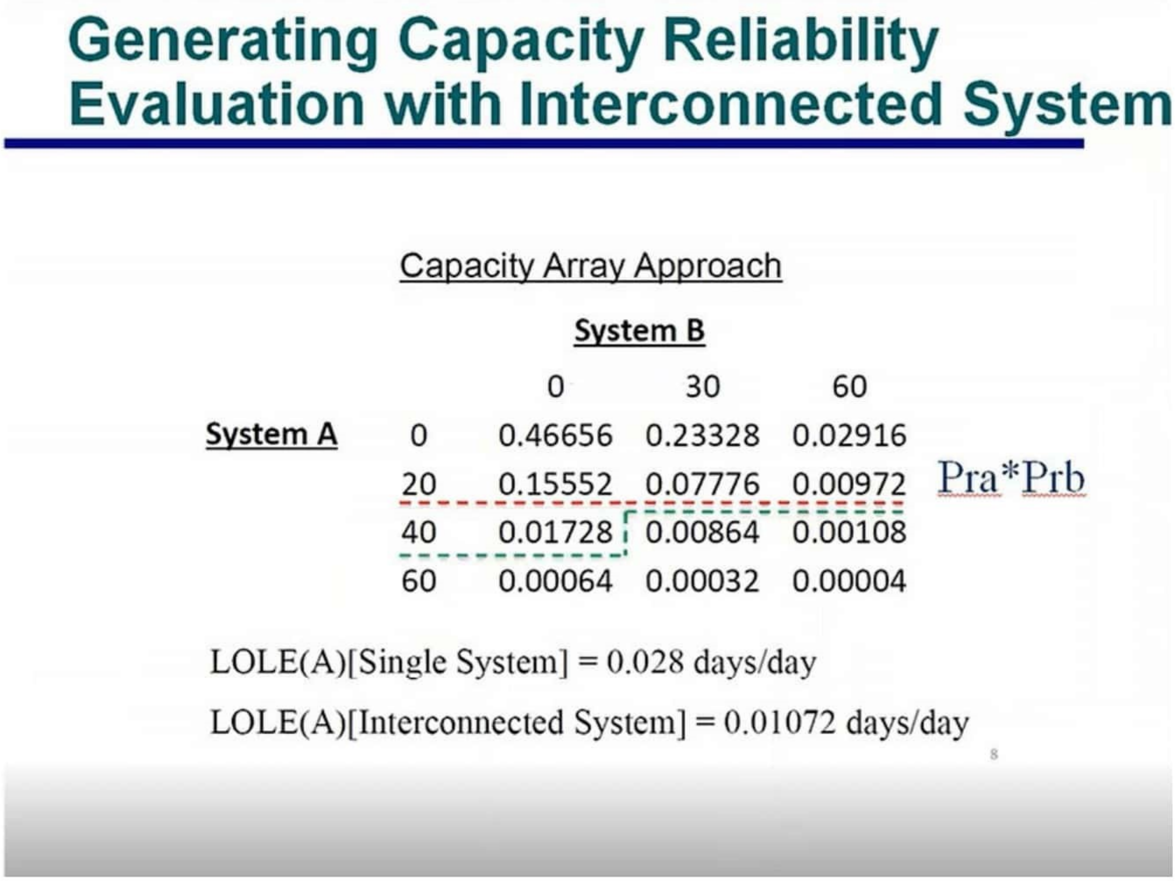
2-30 MW
U=0.2
L=30 MW



Generating Capacity Reliability Evaluation with Interconnected System

Generating Capacity Reliability Evaluation

System A		System B	
Cap Out	Probability	Cap Out	Probability
0	0.729	0	0.64
20	0.243	30	0.32
40	0.027	60	0.04
60	0.001		1.00



Generating Capacity Reliability Evaluation with Interconnected System

Capacity Array Approach

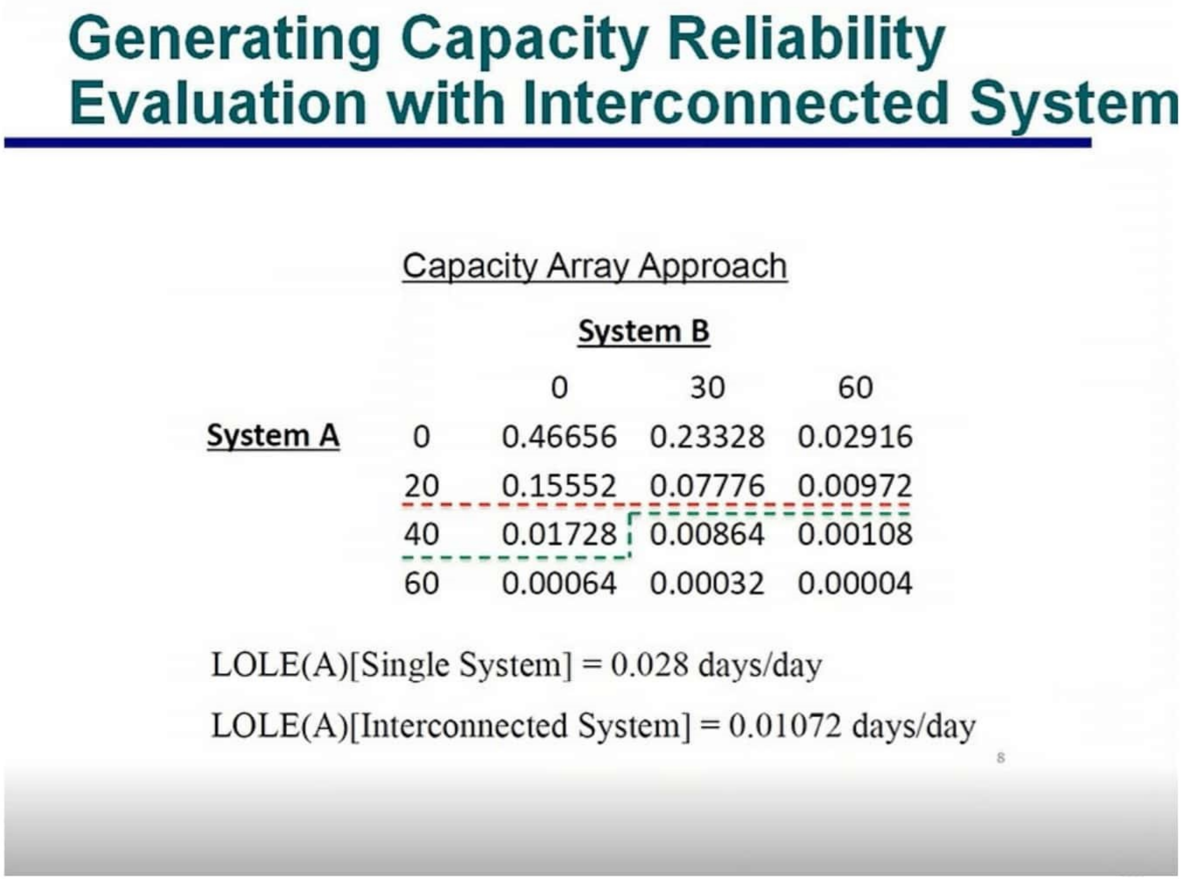
System B

		0	30	60
System A	0	0.46656	0.23328	0.02916
	20	0.15552	0.07776	0.00972
	40	0.01728	0.00864	0.00108
	60	0.00064	0.00032	0.00004

Pra*Prb

LOLE(A)[Single System] = 0.028 days/day

LOLE(A)[Interconnected System] = 0.01072 days/day



Generating Capacity Reliability Evaluation with Interconnected System

Capacity Array Approach

System B

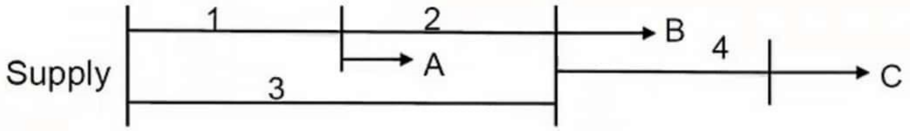
		0	30	60
System A	0	0.46656	0.23328	0.02916
	20	0.15552	0.07776	0.00972
	40	0.01728	0.00864	0.00108
	60	0.00064	0.00032	0.00004

LOLE(A)[Single System] = 0.028 days/day

LOLE(A)[Interconnected System] = 0.01072 days/day

Transmission System Reliability Evaluation

1. Consider the following system



The supply is assumed to have a failure rate of 0.5 f/yr with an average repair time of 2 hours. The line data are as follows.

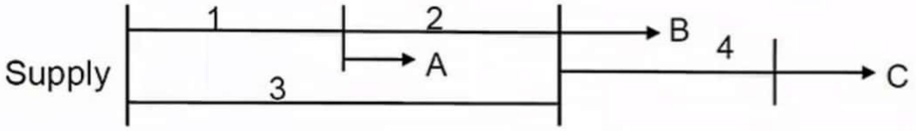
Line	Failure Rate	Average Repair Time
1	4.0 f/yr	8 hrs
2	2.0	6
3	6.0	8
4	2.0	12

Use the minimal cut set approach to calculate a suitable set of indices at each load point.

Transmission System Reliability Evaluation

Load Point A

Min Cut	λ (f/yr)	r (hrs)	U (hrs/yr)
Supply	0.5	2.0	1.0
1, 3	0.043836	4.0	0.175344
1, 2	0.012785	3.4286	0.043835
	0.556621	2.19	1.219179

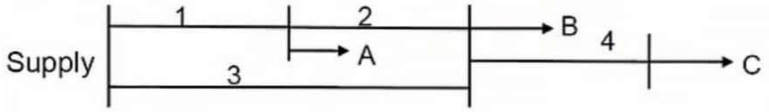


Scanned with CamScanner

Transmission System Reliability Evaluation

Load Point A

Min Cut	λ (f/yr)	r (hrs)	U (hrs/yr)
Supply	0.5	2.0	1.0
1, 3	0.043836	4.0	0.175344
1, 2	0.012785	3.4286	0.043835
	0.556621	2.19	1.219179



System failure rate, $\lambda_s = \lambda_1.\lambda_2 (r_1 + r_2)$

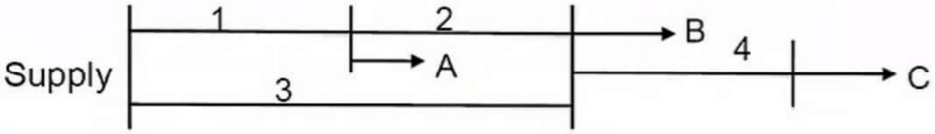
System average down time, $r_s = r_1.r_2 / (r_1 + r_2)$

System unavailability, $U_s = \lambda_s\, r_s$

Transmission System Reliability Evaluation

Load Point B

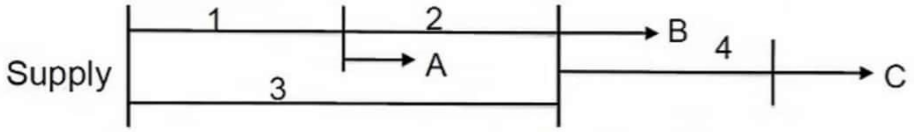
Min Cut	λ (f/yr)	r (hrs)	U(hrs/yr)
Supply	0.5	2.0	1.0
1, 3	0.043836	4.0	0.175344
2, 3	0.019178	3.4285	0.065753
	0.563014	2.2044	1.241097



Transmission System Reliability Evaluation

Load Point C

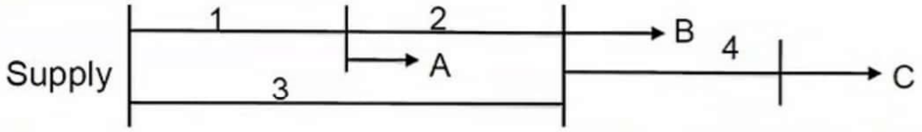
Min Cut	λ (f/yr)	r (hrs)	U(hrs/yr)
At B	0.563014	2.2044	1.241097
4	2.0	12	24
	2.563014	9.848	25.241097



Transmission System Reliability Evaluation

Summary

Min Cut	λ (f/yr)	r (hrs)	U (hrs/yr)
A	0.5566	2.19	1.219
B	0.5630	2.20	1.241
C	2.5630	9.85	25.241



Lecture 5

19/4/2020-23/4/2020

Part 1

Composite System Reliability Evaluation

2. A four unit hydro plant serves a remote load through two transmission lines. The four units are connected to a single step-up transformer which is then connected to twc transmission lines. The remote load has a daily peak load variation curve which is a straight line from the 100% to the 60% point. Calculate the annual loss of load expectation for a forecast peak of 70 MW using the following data.

Hydro Units – 25 MW	Transformer – 110 MVA
FOR = 2%	U = 0.2%

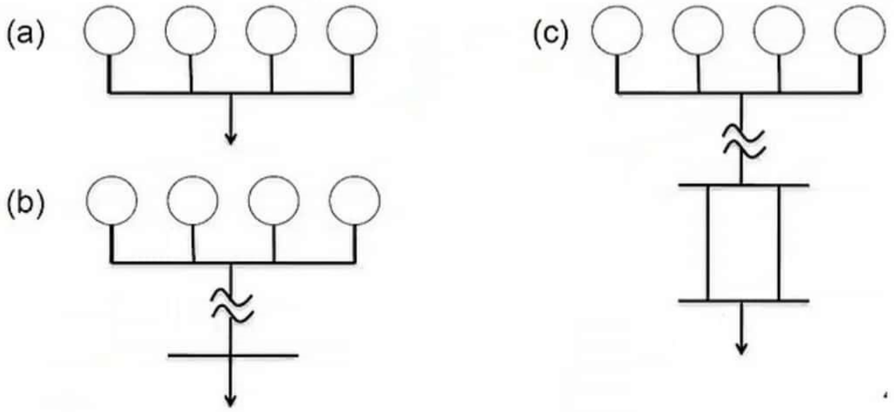
Transmission lines – Carrying capability 50 MW per line

– Failure rate = 2 f/yr

– Average repair time = 24 hrs

Composite System Reliability Evaluation

Calculate the LOLE in three stages using the following configurations.

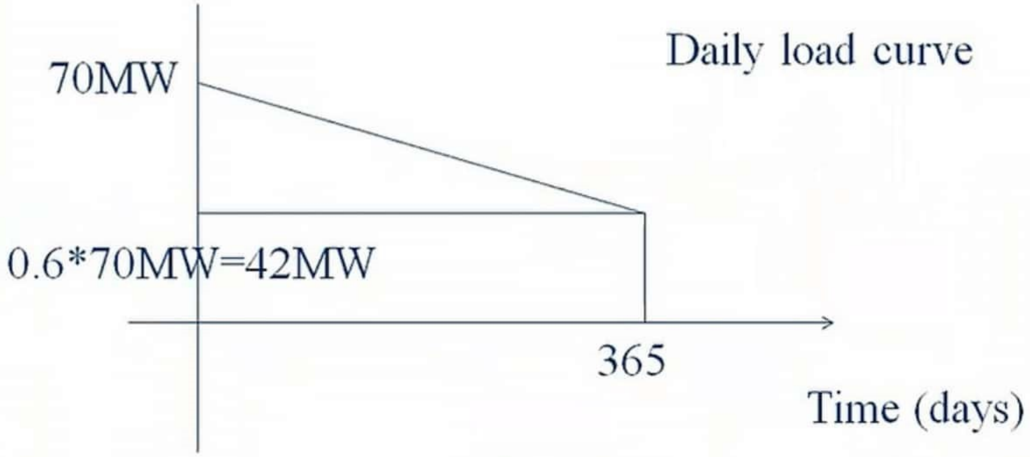


Composite System Reliability Evaluation

Configuration (a)

Capacity in	Capacity Out	Probability	Time	Expectation
100	0 MW	0.922368	0.0	
75MW	25	0.075295	0.0	
50	50	0.002305	260.71	0.600937
25	75	0.000032	365.0	0.011680
0	100	-	365.0	-
		1.000000		0.612617

LOLE = 0.613 days/yr



Composite System Reliability Evaluation

Configuration (b)

Capacity Out	Probability	Time	Expectation
0 MW	0.920524	0.0	
25	0.075144	0.0	
50	0.002300	260.71	0.599633
75	0.000032	365.0	0.011680
100	0.002000	365.0	0.730000
	1.000000		1.341313

LOLE = 1.341 days/yr

Composite System Reliability Evaluation

Configuration (c)

Transmission lines $\lambda = 2 \quad f/yr$

$$\mu = \frac{1}{r} = \frac{8760}{24} = 365 \quad r/yr$$

$$Unavailability = \frac{\lambda}{\lambda+\mu} = \frac{2}{2+365} = 0.005450$$

$Availability = 0.994550$

Cap. Out	Probability
0 MW	0.989130
50	0.010840
100	0.000030
	1.000000

Composite System Reliability Evaluation

Generation – In (MW)

Transmission-In (MW)

T/G	100	75	50	25	0
100	100	75	50	25	0
50	50	50	50	25	0
0	0	0	0	0	0

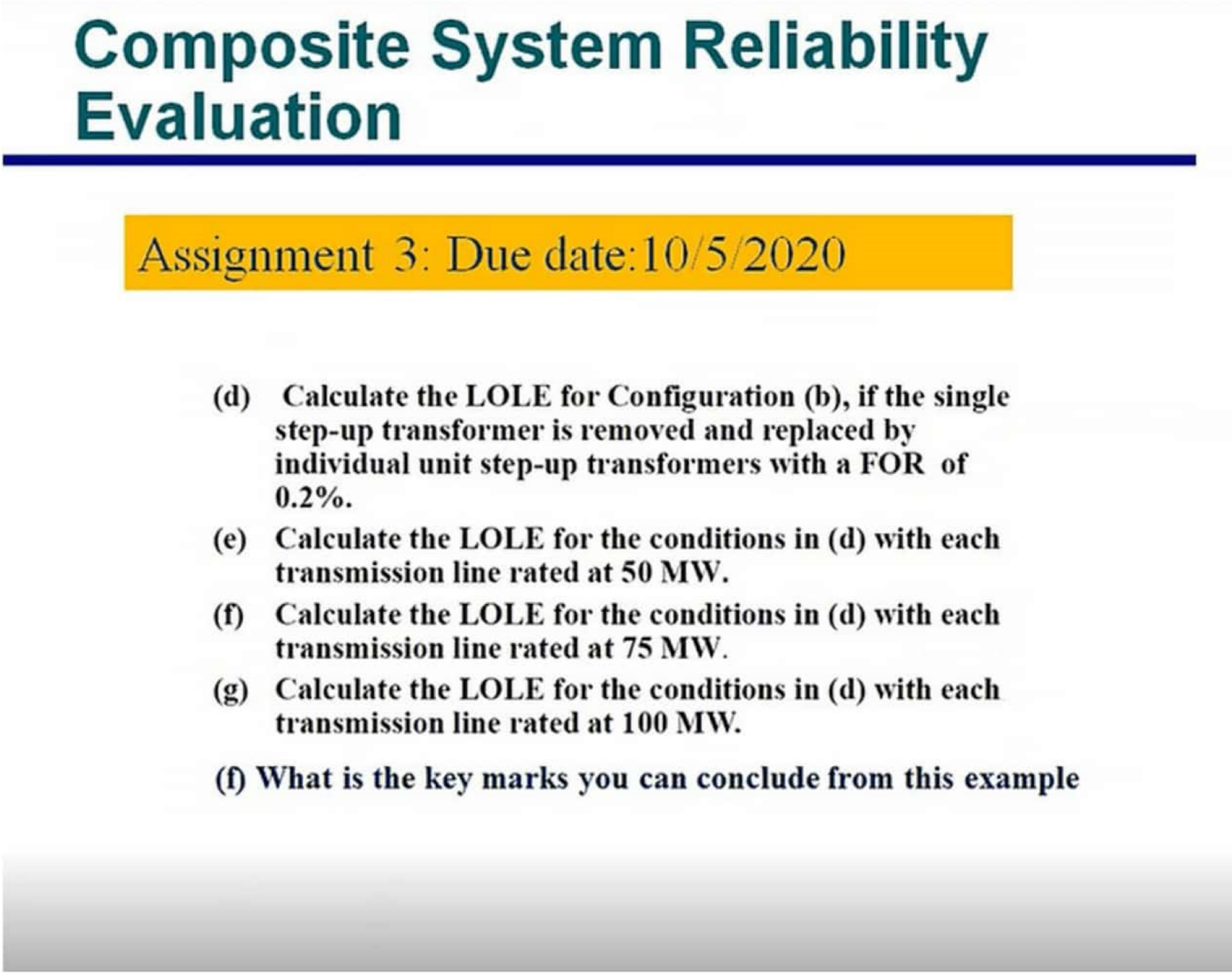
System Capacity States

Composite System Reliability Evaluation

Configuration (c)

Capacity		Probability	Time	Expectation
In	Out			
100	0	0.910518	0.0	
75	25	0.074327	0.0	
50	50	0.013093	260.71	3.413476
25	75	0.000032	365.0	0.011680
0	100	0.002030	365.0	0.740950
		1.000000		4.166106

LOLE = 4.166 days/yr



Composite System Reliability Evaluation

Assignment 3: Due date:10/5/2020

(d) **Calculate the LOLE for Configuration (b), if the single step-up transformer is removed and replaced by individual unit step-up transformers with a FOR of 0.2%.**

(e) **Calculate the LOLE for the conditions in (d) with each transmission line rated at 50 MW.**

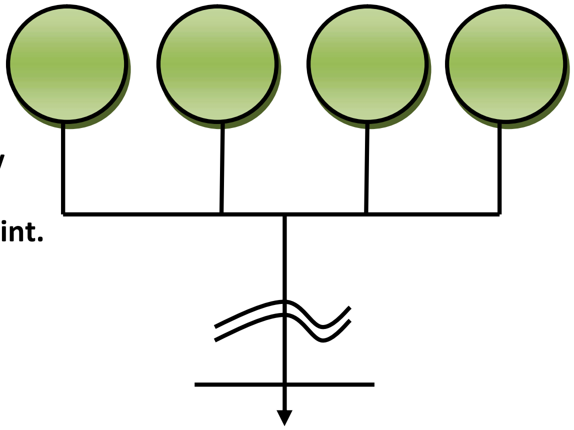
(f) **Calculate the LOLE for the conditions in (d) with each transmission line rated at 75 MW.**

(g) **Calculate the LOLE for the conditions in (d) with each transmission line rated at 100 MW.**

(f) What is the key marks you can conclude from this example

Problem: Four units, connected with 2 Transmission lines and a unit step-up transformers with a FOR of 0.2%. The load is 70 MW that has a straight line daily peak load variation curve from the 100% to the 60% point.

***Calculate the LOLE:**



Solution:

*25MW Failure rate=0.02
*110MVA Failure rate = 0.002
Transmission lines with 50 MW per line :
*Failure rate = 2 f/yr
*Average repair time = 24 hrs

➢ FOR =(0.02) + (0.002) – (0.02)*(0.002)
= 0.0219
So the unavailability =0.0219
Availability = 1-0.0219
= 0.978040

IN	OUT	Probability	Time	Expectation
100	0	0.915012	0.0	
75	25	0.082179	0.0	
50	50	0.002768	260.7	0.7216
25	75	0.000041	365	0.0149
0	100		365	

➢ LOLE =0.734 days/year

Problem: calculate the LOLE for the conditions in the previous part with each transmission line rated at 50 MW:

IN	OUT	Probability	Time	Expectation
100	0	0.905066	0.0	
75	25	0.081286	0.0	
50	50	0.013577	260.7	3.53966
25	75	0.000041	365	0.01496
0	100	0.000030	365	0.01095

➢ LOLE= 3.566 days/year

Problem: calculate the LOLE for the conditions in the previous part with each transmission line rated at 75 MW:

IN	OUT	Probability	Time	Expectation
100	0	0.905066	0.0	
75	25	0.092095	0.0	
50	50	0.002768	260.7	0.721645
25	75	0.000041	365	0.014965
0	100	0.000030	365	0.010950

- LOLE=0.748 days/year

Problem: calculate the LOLE for the conditions in the previous part with each transmission line rated at 100 MW:

IN	OUT	Probability	Time	Expectation
100	0	0.914985	0.0	
75	25	0.082177	0.0	
50	50	0.002768	260.7	0.721645
25	75	0.000041	365	0.014965
0	100	0.000030	365	0.010950

- LOLE=0.748 days/year

Problem: What are the key marks you can conclude from this example?

- In this problem we realize that the Loss of load expectation does increase when changing the single step-up transformer to individual units of step up transformers.
- We also realize that the more rated power the transmission lines are at, the lower the Loss of load expectation becomes.
- We realize that after a cretin rated power for the transmission lines, the Loss of load expectation becomes almost constant and does not change.
- Another note is that the time for the system or units to fail is not affected by changing the rated power for the transmission lines and stays the same through the entire calculation process.