Section 1 + 2

Solving a system of linear equations

1. Elementary Row operations with no additions / using backward substitution.
2. Gaussian Elimination method. (reaching the leading ones in an echelon form)
3. Jordan-Gauss Elimination method. (Making the numbers above leading ones zeros) reaching what is called reduced ~~row~~ echelon form.

* The matrix could have one solution, no solution, or infinitly many solutions.

* إلي إلها حل واحد بطلع لكل مجهول قيمة محددة.
* إلي إلها عدد لانهائي من الحلول تكون فيها leading variables و free variables

positions of leading ones → pivot positions ← بنوصفه بالصف والعمود
columns " " " → " columns.

$\boxed{0 \quad 0 \quad 0 \quad -9}$ ← إلي ما إلها أي حل تكون آخر صف فيها هيك *

$0x_3 \neq -9$

بنوضحها بآخر شابتر قبل ال Hom. sys

Homogeneous system.

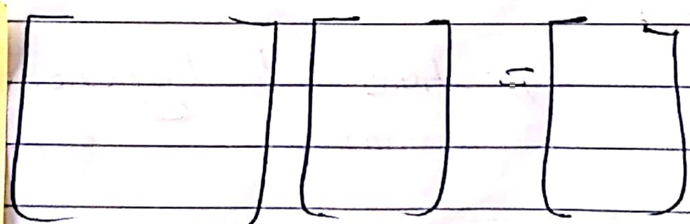
only has infinitly many solutions
besides the solution could be ① trivial
all varibles = 0
② nontrivial
with leading veribibles
and free variables

ممكن يجيلي السيستم بأكتر من طريقة أحله

① الطريقة العادية معادلات وماتركس

$$\begin{bmatrix} a+b & -- \\ c+d & — \end{bmatrix} = \begin{bmatrix} * & * \\ * & * \end{bmatrix}$$ ②

$Ax = b$ ③



A x b

خصائص ال reduced row e.f.

① أول رقم leading one

② ال 1.0 ماشيين للعمق

③ الأرقام الباقية تحتهم كلها zeros

④ ال rows الي كلها zeros مجمعين تحت

Section 3

column matrix / column vector: a matrix with only one column.

Row matrix / Row vector: a matrix with only one row.

[SQUARE MATRIX]

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix}$$

* The square matrix has a diagonal and this is it

* The square matrix has a trace and is denoted by tr(A) → when A is (nxn)

trace(A) = the sum of entries of diagonal.

* [IDENTITY MATRIX]

is a $n \times n$ matrix donated by I and $= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \vdots & \vdots \\ 0 & 0 & 1 & \vdots \\ \vdots & & & \ddots \\ 0 & 0 & 0 & 1 \end{bmatrix}$

and → $IA = AI = A$

$AA^{-1} = I$

Section 3

column matrix / colum
only one c

Row matrix / Row ve
only one row.

Properties:-

① matrices are equal if they have the same size and their corresponding entries are equal

② Transpose of A matrix is denoted by $\rightarrow A^T$
such that $\rightarrow$
$A = \begin{bmatrix} 1 & 2 & 7 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$

[SQUARE MATRIX]

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix}$$

③ for Transpose

$(AB)^T = B^T A^T$

$(A \mp B)^T = A^T \mp B^T$

$(KA)^T = KA^T$ (k is a constant)

* If A is invertible then A^T is invertible with $(A^T)^{-1} = (A^{-1})^T$
* A matrix is symmetric if $A = A^T$

* The square matrix has at...

$tr(A) \rightarrow$ when A is $(n \times n)$

trace(A) = the sum of entries of diagonal.

* [IDENTITY MATRIX]

is a $n \times n$ matrix donated by I and $= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & & \vdots \\ 0 & 0 & 1 & \vdots \\ \vdots & & & \ddots \\ 0 & 0 & 0 & 1 \end{bmatrix}$

and $\rightarrow IA = AI = A$

$AA^{-1} = I$

(4) If $P(X) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$,
then $P(A) = a_0 I + a_1 A + a_2 A^2 + \cdots + a_n A^n$

Ex: If $P(X) = 2x^2 - 3x + 4$ and $A = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$

then find $P(A)$

Solu:

$P(A) = 2A^2 - 3A + 4I$

$$= 2\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}^2 - 3\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ 0 & 13 \end{bmatrix}$$

Operations:-

① Substraction and addition:-

* to make these operation successful, matrices should be with the same size.

② Multiplying matrices.

ith row of the matrix AB = B x [ith row of A]

jth column of the matrix AB = A x [jth column of B]

and for an example:-

$2x_1 + 3x_2 - x = 0$

$x_1 + x_2 - x_3 = 1$

$2x_1 + 0 + x_3 = 5$ it is represented by : $Ax = b$

$$\underbrace{\begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\underline{\underline{x}}} = \underbrace{\begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}}_{\underline{\underline{b}}}$$

by multiplying:-

$$\underbrace{\begin{bmatrix} 2x_1 + 3x_2 - 1x_3 \\ 1x_1 + 1x_2 - 1x_3 \\ 2x_1 + 0x_2 + 1x_3 \end{bmatrix}}_{Ax} = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$$

③ Matrices with powers.

① $A^n = AA \cdots A \rightarrow$ n times

② If A is invertible, then

$$A^{-n} = \underbrace{A^{-1}A^{-1} \cdots A^{-1}}_{n\ \text{times}}$$

③ $A^0 = I \rightarrow$ I in matrice acts like (1) in numerical operations.

④ $(A^n)^m = (A^m)^n$

⑤ $(A^n)(A^m) = A^{n+m}$

⑥ If A is invertible, then A^{-1} is invertible with $(A^{-1})^{-1} = A$

* When point 6. is correct, then A^n is invertible with $(A^n)^{-1} = (A^{-1})^n$, then $\rightarrow (A^n(A^{-1})^n = (AA^{-1})^n = I^n = I)$

* When point 6. is correct and $k \neq 0$, then: kA is invertible with $(kA)^{-1} = \frac{1}{k} A^{-1}$

Section 4

Some properties:-

* for the matrix $O = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \\ 0 & \cdots & \cdots & 0 \end{bmatrix}$ $\rightarrow$ $O + A = A$

$A + (-A) = O$

$-A + A = O$

A is the matrix and O is the zero matrix

* Cancelation law does not hold for matrices

explanation,

$A = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix}$

$AB = AC = \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$ but $B \neq C$

[INVERSE]

* The matrix (A) is invertible if:-

① If A is a nxn matrix and there exists B of the same size with, $AB = BA = I$ then we say A is invertible and B is the inverse of A, and can be donated by (A^{-1})

* The matrix A is singular if:-

A has a zero column or zero row $\rightarrow$ like $\begin{bmatrix} 1 & 4 & 0 \\ 2 & 5 & 0 \\ 3 & 6 & 0 \end{bmatrix}$

Sec 6

① Theorem (3) index

② fundamenta - - -

③ Theorem (4) index

** Conditions of [b] in the system $Ax=b$

هستخدم الـ elementary operations عادية، مسأله
اطلع الكوندشنز

Sec 7

① Diagonal Matrix
all entries off the main diagonal are zeroes.

① to be invertible $d_i \neq 0$

$$D^{-1} = \begin{bmatrix} \frac{1}{d_1} & 0 & 0 \\ 0 & \frac{1}{d_2} & 0 \\ 0 & 0 & \frac{1}{d_3} \end{bmatrix}$$

② $$D^k = \begin{bmatrix} d_1^k & 0 & 0 \\ 0 & d_2^k & 0 \\ 0 & 0 & d_3^k \end{bmatrix}$$

A t خاصية الضرب بنفسه

② Triangular Matrices.

① Diagonal matrices are both upper and lower triangular matrices.

② T of an upper → lower and vice versa.

③ Triangular are invertible when entries of main diagonal are all nonzeros

④ inv of upper is upper and vice versa

⑤ product of upper's is upper and vice versa.

+ Symmetric matrices.

* find the inverse for $\underline{\underline{2\times 2}}$ matrix

to test if the matrix is invertible or not use:
[the matrix is invertible when $(ad-bc\neq 0)$]

such that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

And the inverse can be found by →

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

* If A and B are invertible, then it is not necessarly that $A \mp B$ invertible.

But → (AB) is invertible when Both A and B are invertible.

$$(A+B)^{-1} \neq A^{-1} \mp B^{-1}$$

$$\text{But } (AB)^{-1} = A^{-1}B^{-1}$$

Types of Matrices

* Square matrix

* Diagonal matrix

  $\underset{n\times n}{A}$ is diagonal iff: $a_{ij} = 0$ for $i \neq j$

* Upper triangular matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 5 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

* lower triangular matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 7 & 2 & 0 \end{bmatrix}$$

* Symmetric matrix. $(A = A^T)$

* ضروري ارجع لدفتر دكتور حسين في ال Elementary matrices.

بنقدر نستخدمها لنطلع ال inverse لماتريكسات أكبر من ($\underline{2\times 2}$)

$$\left[\begin{array}{cc|cc} \cdots & \cdots & 1 & 0 \\ \cdots & \cdots & 0 & 1 \end{array}\right]$$

method

Determinants

خط الزمان على ألواح مرتعش
إن السبيل إلى أحلامنا جلد

إن السبيل إلى جلد أحلامنا

§ 1 * Cofactor expansion *

*بنستخدم ال det(A) عشان نحدد ال INVERTIBILITY

اللهم بلغنا مناك اللهم يا عظيم

إن السبيل إلى أحلامنا جلد
خط الزمان على ألواح مرتعش
تحت الجفون بريق لو تقا يلتمع
فر للجدار أني تحري الحكاية ترتعد.

* SQURE MATRICES

① 2*2 matrix

$(ad-bc) \neq 0 \rightarrow$ A is invertible.

② square matrices of our size.
we want to extend the det to square matrices of higher size.

*det → Expansion *

$$\det(A) = 1\begin{vmatrix} 4 & 2 \\ 2 & -1 \end{vmatrix} + 3\begin{vmatrix} -1 & 2 \\ 4 & -1 \end{vmatrix} + 2\begin{vmatrix} -1 & 4 \\ 4 & 2 \end{vmatrix}$$

if $A = \begin{bmatrix} \overset{+}{1} & \overset{-}{3} & \overset{+}{2} \\ \overset{-}{-1} & \overset{+}{4} & \overset{-}{2} \\ \overset{+}{4} & \overset{-}{2} & \overset{+}{-1} \end{bmatrix}$

3*3

---

* special for 3*3 matrix
First and second columns method.

* If A a matrix has a two proportional rows or columns $\rightarrow \det(A) = 0$

* The det of triangular matrices equals the product of the entries of the main diagonal.

Minor of the square matrix

- $M_{ij} \rightarrow$ The minor matrix of entry a_{ij} is the determinant of the submatrix that remains after deleting ith row and jth column, denoted by M_{ij}

$$C_{ij} \rightarrow = (-1)^{i+j} M_{ij} \quad (\text{Cofactor of entry } a_{ij})$$

ال M_{ij} وال C_{ij} لازم يتساووا في العدد بس ممكن يختلفوا في الإشارة.

§2 Evaluating det by row reduction [Elementary operations]

* [Elementary operation effect on det]

Working on I

① Multiplying a row by k:

$\boxed{\det(E) = k}$

② Changing two rows:

$\boxed{\det(E) = -1}$

③ Adding a multiple of rows to another row:

column " " column.

$\boxed{\det(E) = 1}$

Properties:

① if A has a column of zeros or a row of zeros then $\det(A) = 0$

② $\det(A) = \det(A^T)$

③ Relating to effects above:-

* If B is the matrix that results when two ~~...~~ rows or two columns of A are interchanged, then $\det(B) = -\det(A)$

* If B is the matrix resulting when a multiple of one row of A is added to another row or when a multiple of one column is added to another column then $\det(B) = \det(A)$

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* If B is a matrix results when a single row or a single ~~col~~ column of A is multiplyed by k then $\det(B) = k\det(A)$.

كيف بدي استخدم الي بسموهم Row Reduction OPERATIONS مشان اطلع الـ det

بنستخدم الـ row operations وبستغل تأثيرهم على الـ det مشان اوصل للـ Triangular form واستغل الـ det تبعه

داماً حلي بال expansion إلا اذا طلب بالذات حلي بال Row Reduction حلي فيها

A