SIGNALS

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POWERUNIT

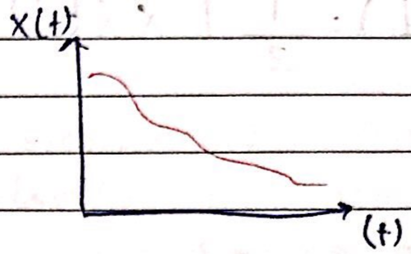
Signals :-

$\theta(t)$: Temprature

$\theta(t, x, y, z)$

$x(t)$

dependent Variable — independent Variabls



According to the nature of the independent variable

1) continuos - time signals
2) Discrete - time signals

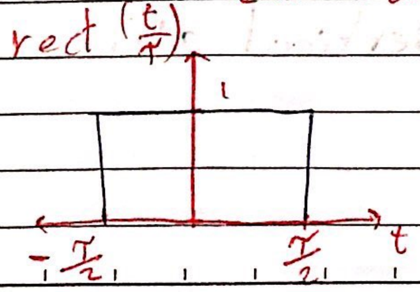
→ a continuoas - time signal $x(t)$ is said to be discontinoas in amplitude at $t = t_1$ if $x(t_1^-) \neq x(t_1^+)$ where $(t_1 - t_1^-)$ and $(t_1^+ - t_1)$ infinitesimal positive numbers.

* Signal $x(t)$ is continuoas at $t = t_1$ if

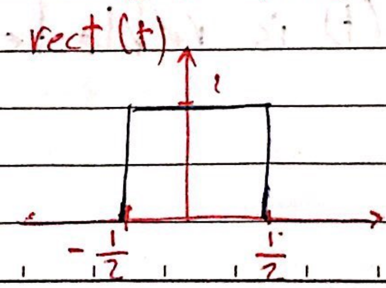
$$x(t_1^-) = x(t_1^+) = x(t)$$

there are many continuoas - time signals that are not continuoas at all points of t. an example is the rectangular polar function defined by.

$$\text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & |t| < \frac{T}{2} \\ 0 & |t| > \frac{T}{2} \end{cases} \qquad \text{rect}(t) = \begin{cases} 1 & |t| < \frac{1}{2} \\ 0 & |t| > \frac{1}{2} \end{cases}$$



⟹ for $T = 1$



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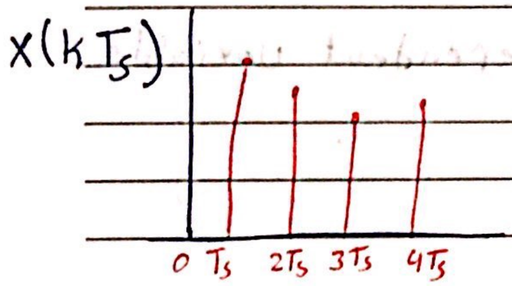
we define the value of the signal at a point of discontinuity t_1 as

$$x(t_1) = \frac{1}{2}\left[x(t_1^+) - x(t_1^-)\right]$$

if the independent variable taken only discrete-values $t = KT_s$, where T_s is a fixed positive real number. and $K = 0, \pm 1, \pm 2, \dots$



$$x(KT_s) = x[K] = \{ - - - - \}$$

$x[1.5] \neq 0 \rightarrow$ undifind

not integer

Transformations of continuous-time signals:-

1 Time Transformations and 2 Amplitude Transformations

Independent variable — dependent variable

$\boxed{1}$ Time transformations

1) Time reversal

$$y(t) = x(-t)$$

$x(t)$ is reflected about the vertical axis

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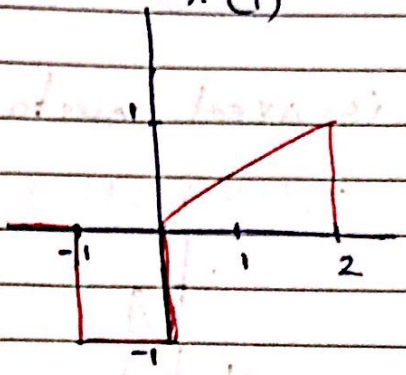
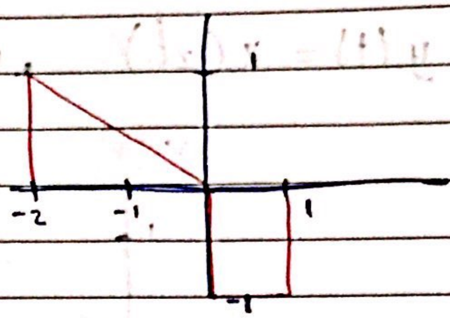
Ex: $x(t)$ $\Rightarrow$ $y(t) = x(-t)$

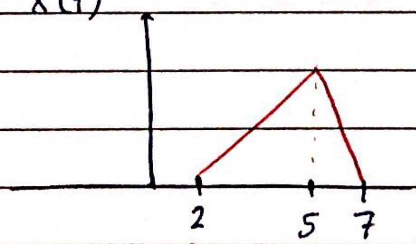




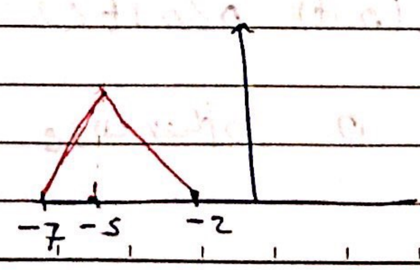
$$x(t) = \begin{cases} -1 & -1 < t < 0 \\ \frac{1}{2}t & 0 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow y(t) = x(-t) = \begin{cases} -1 & -1 < -t < 0 \\ \frac{1}{2}(-t) & 0 < -t < 2 \\ 0 & \text{otherwise} \end{cases} = \begin{cases} -\frac{1}{2}t & -2 < t < 0 \\ -1 & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

Ex: $x(t)$



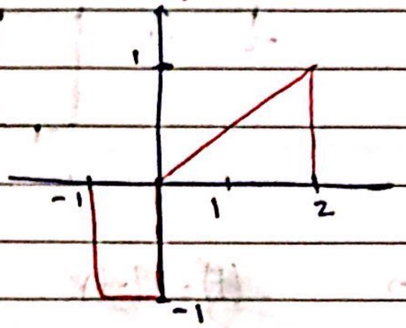
$y(t) = x(-t)$

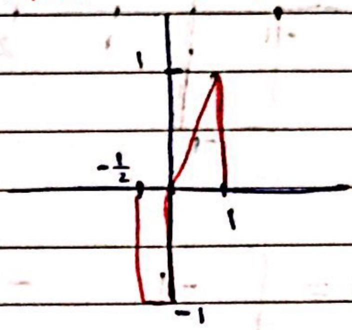


2) time scaling:

$y(t) = x(at)$, wher a is areal constant

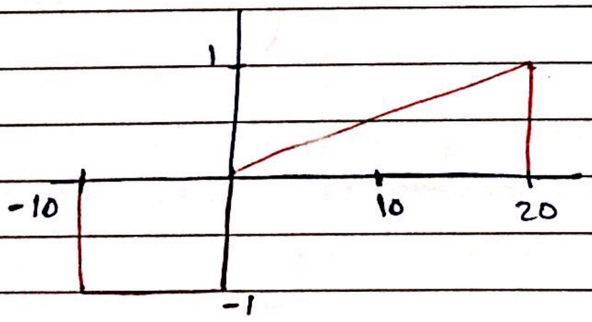
$x(t)$ $\quad\quad$ $x_1(t) = x(2t)$





$$y_1(t) = x(2t) = \begin{cases} -1 & -1 < 2t < 0 \\ \frac{1}{2}(2t) & 0 < 2t < 2 \\ 0 & \text{o.w} \end{cases} = \begin{cases} -1 & -\frac{1}{2} < t < 0 \\ t & 0 < t < 1 \\ 0 & \text{o.w} \end{cases}$$

$y_2(t) = x(0.1t)$



$$y_2(t) = x(0.1t) = \begin{cases} -1 & -1 < 0.1t < 0 \\ \frac{1}{2}(0.1t) & 0 < 0.1t < 2 \\ 0 & \text{otherwise} \end{cases} = \begin{cases} -1 & -10 < t < 0 \\ \frac{t}{20} & 0 < t < 20 \\ 0 & \text{o.w} \end{cases}$$

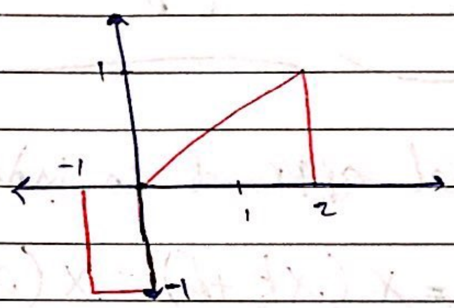
3) time shifting

$y(t) = (x(t - t_0))$, wher t_0 is a constant

* Not that: $y(t_0) = x(t_0 - t_0) = x(0)$, Hence if t_0 is positive, the shifted signal $y(t)$ is delayed in time (shifted to the right relative to $x(t)$),
if t_0 is negative $y(t)$ is advanced in time (shifted to the left)

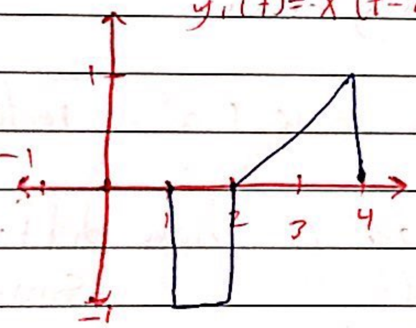
Ex:

$$x(t) = \begin{cases} -1 & -1 < t < 0 \\ \frac{1}{2}t & 0 < t < 2 \\ 0 & o.w \end{cases}$$



find $y_1(t) = x(t-2)$
$y_2(t) = x(t+1)$

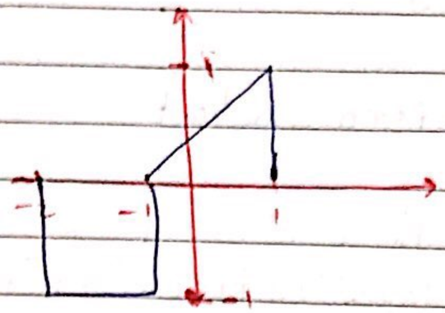
$y_1(t) = x(t-2)$



$$y_1(t) = x(t-2) = \begin{cases} -1 & -1 < t-2 < 0 \\ \frac{1}{2}(t-2) & 0 < t-2 < 2 \\ 0 & o.w \end{cases}$$

$$= \begin{cases} -1 & 1 < t < 2 \\ \frac{1}{2}t - 1 & 2 < t < 4 \\ 0 & o.w \end{cases}$$

$y_2(t)$



$$y_2(t) = x(t+1) = \begin{cases} -1 & -1 < t+1 < 0 \\ \frac{1}{2}(t+1) & 0 < t+1 < 2 \\ 0 & \text{O.W} \end{cases} = \begin{cases} -1 & -2 < t < -1 \\ \frac{1}{2}t + \frac{1}{2} & -1 < t < 1 \\ 0 & \text{O.W} \end{cases}$$

General approach to independent-variable transformation:

$$y(t) = x(\alpha t + \beta) = x\left(\alpha\left(t + \frac{\beta}{\alpha}\right)\right)$$

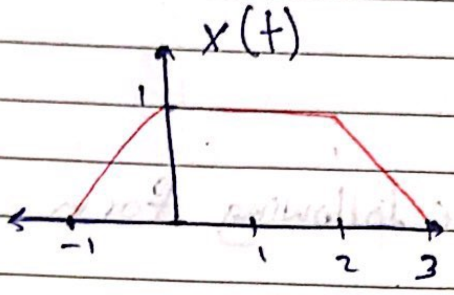
where α, β are assume to be real numbers. the operation should be performed in the following order:

1) scale by α. if α is negative, reflect about vertical axis

2) Shift to the right by $\frac{\beta}{\alpha}$ if β and α have different signs and to the left by $\frac{\beta}{\alpha}$ if α and β have the sam sing

*Not that: the operation of reflecting and time scaling are commutitive where as the operation of shifting and reflecting or shifting and time scaling are not.

Ex: $x(t) = \begin{cases} t+1 & -1 \le t \le 0 \\ 1 & 0 < t \le 2 \\ -t+3 & 2 < t \le 3 \\ 0 & o.w \end{cases}$



find $y(t) = x(3t-6)$

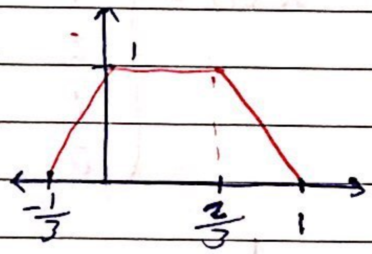
$$y(t) = x(3t-6) = \begin{cases} (3t-6)+1 & -1 < 3t-6 \le 0 \\ 1 & 0 < 3t-6 < 2 \\ -(3t-6)+3 & 2 < 3t-6 \le 3 \\ 0 & o.w \end{cases}$$

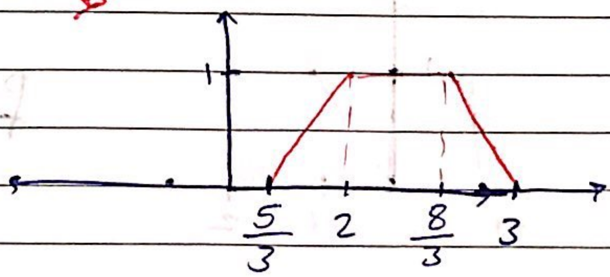
$$y(t) = \begin{cases} 3t-5 & \frac{5}{3} < t \le 2 \\ 1 & 2 < t < \frac{8}{3} \\ -3t+9 & \frac{8}{3} < t < 3 \\ 0 & o.w \end{cases}$$

$y(t) = x(3t-6) = x(3(t-2))$

$y_1(t) = x(3t)$

$\rightarrow y(t) = y_1(t-2)$





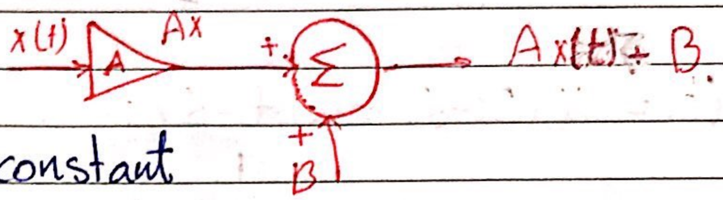
* Amplitude transformation:

the three transfomation in amplitude are.

1) amplitude Reversal
2) " scaling
3) " shifting

it can be written in the following form

$y(t) = A\,x(t) + B$

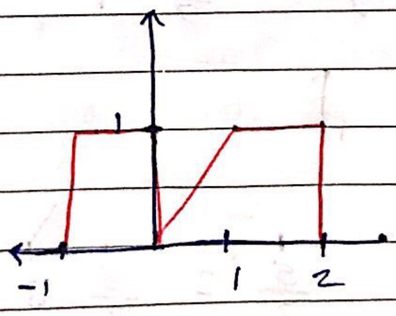


where A and B are constant

Ex: $y(t) = -3x(t) - 5$

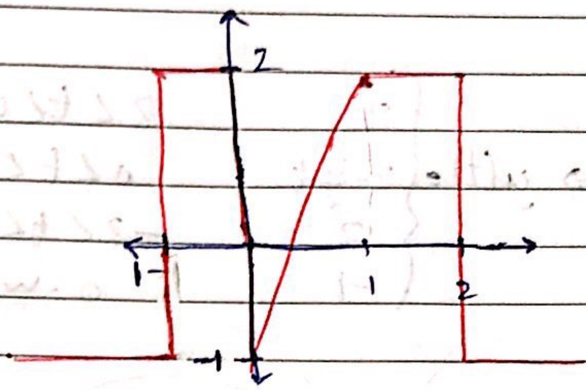
the Value $A=-3$ yields amplitude reversal (the minus sing) and amplitude scaling by $(|A|=3)$ and the value $B=-5$ shifts the amplitude of the signal down by 5 units

Ex: let $x(t) = \begin{cases} 1 & -1 < t < 0 \\ t & 0 < t < 1 \\ 1 & 1 \le t < 2 \\ 0 & \text{o.w} \end{cases}$



find $y(t) = 3x(t) - 1$

$$y(t) = 3x(t) - 1 = \begin{cases} 2 & -1<t<0 \\ 3t-2 & 0<t<1 \\ 2 & 1<t<2 \\ -1 & o.w \end{cases}$$



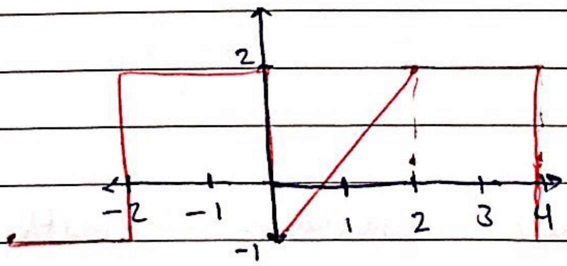
• Time and amplitude transformation of a signal

Ex: consider $y(t) = 3x(1-\frac{t}{2}) - 1$

① let $y_1(t) = 3x(t) - 1$

$$y_1(t) = \begin{cases} 2 & -1<t<0 \\ 3t-1 & 0<t<1 \\ 2 & 1<t<2 \\ -1 & o.w \end{cases}$$

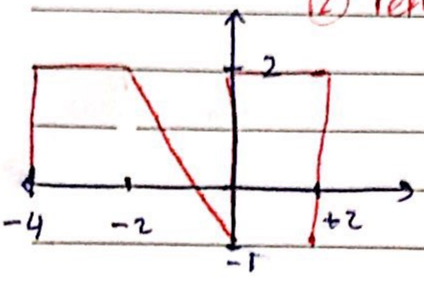
$$y(t) = y_1(1-\frac{t}{2}) = y_1(-\frac{1}{2}(t-2))$$



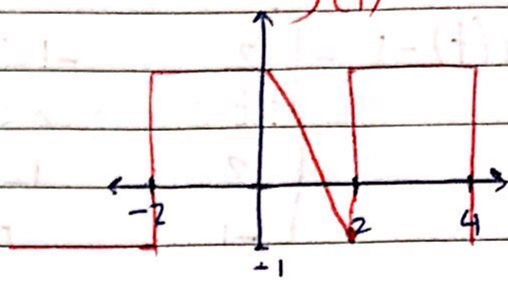
① scale by $\frac{1}{2}$

⇨

(2) reflect



$g(t)$

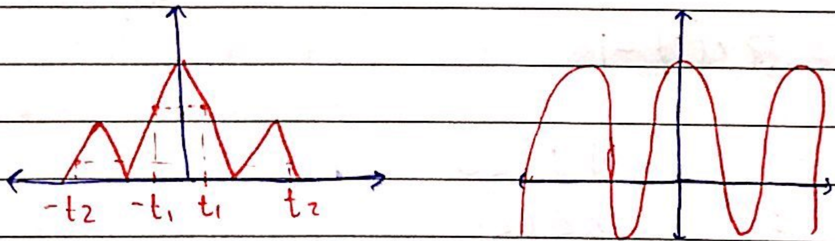


$$y(t)=\begin{cases}2 & -1<1-\frac{t}{2}<0\\ 3(1-\frac{t}{2})-1 & 0<1-\frac{t}{2}<1\\ 2 & 1<1-\frac{t}{2}<2\\ -1 & o.w\end{cases} \Rightarrow y(t)=\begin{cases}2 & 2<t<4\\ 2-\frac{3}{2}t & 0<t<2\\ 2 & -2<t<0\\ -1 & o.w\end{cases}$$

* Signal charastaristics :-

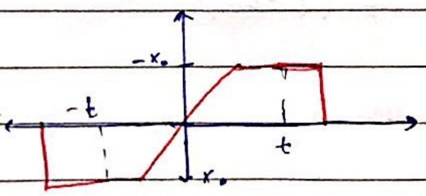
1) even and odd signal

- a signal is even if $x_e(t)=x_e(-t)$



ex: $x(t)=\cos(wt)$ is even because $\cos(wt)=\cos(-wt)$

- asignal is odd if $x_o(t)=-x_o(-t)$



ex: $x(t)=\sin(wt)$ is odd because $\sin(wt)=-\sin(-wt)$.

Generally :- Any signal can be expressed as the sum of an even part and an odd part that is

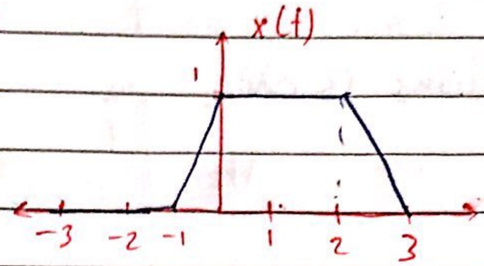
$$x(t) = x_e(t) + x_o(t) \quad --① $$

$$x(-t) = x_e(-t) + x_o(-t)$$

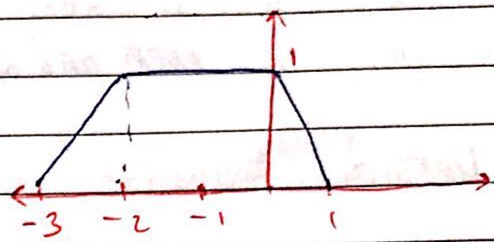
$$x(-t) = x_e(t) - x_o(t) \quad --②$$

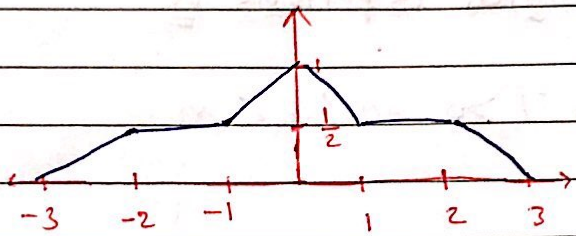
$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right] \qquad x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

ex:



⇒





$$x(t) = \begin{cases} t+1 & -1 < t < 0 \\ 1 & 0 < t < 2 \\ 3-t & 2 < t < 3 \\ 0 & o.w \end{cases} \Rightarrow x(-t) = \begin{cases} -t+1 & -1 < -t < 0 \\ 1 & 0 < -t < 2 \\ 3-(-t) & 2 < -t < 3 \\ 0 & o.w \end{cases}$$

$$x(-t) = \begin{cases} 1-t & 0 < t < 1 \\ 1 & -2 < t < 0 \\ t+3 & -3 < t < -2 \\ 0 & o.w \end{cases} \Rightarrow x_e(t) = \begin{cases} \frac{t+3}{2} & -3 < t < -2 \\ \end{cases}$$

even and odd function have the following properefies:-

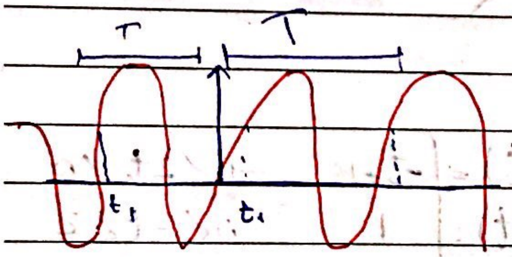
1) the sum of tow even function is even.
2) " " " " odd " " odd.
3) " " " even function and odd function is niether even nor odd.

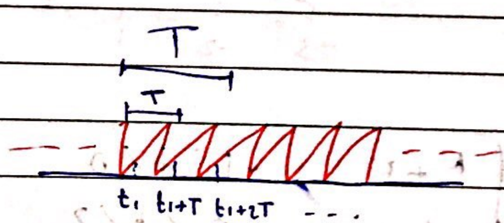
4) the product of tow even function is even.
5) " " " " odd " ".
6) " " " even and odd functions is odd.

2) Periodic signals:-

a continuoas time signal is periodic if

$x(t) = x(t+T)$, $T>0$ $-\infty < t < \infty$





$x(t) = x(t+T) = x(t+2T)$

$x(t+2T) = x(t+3T)$

$x(t) = x(t+nT)$; n is an integer

* the minimum value of the period $T>0$ that satisfies the definition $x(t) = x(t+T)$ is called the fundemental period of the signal, and is denoted by (T_0).

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wish T_0 in second, the fundemental frequency in hertz (Hz)

$f_0 = \frac{1}{T_0}$ Hz the number of periods per second

$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$ rad/sec

$x(t) = A \cos(\omega_0 t)$
$= A \cos(2\pi f_0 t)$

* the sum of continuous-time periodics signals is periodic if and only if the ratios op the periodics of the individual signals are ratios of integers. (rational number)

* if a sum of n periodic signals is periodic the fundemental period can be found as follows:-

1- convert each period ratio $\boxed{\frac{T_{01}}{T_{0i}}}$, $2 \leq i \leq N$ to a ratio of integers, where T_{01} is the fundemental peiod of the first signal considered and T_{0i} is the period of one of the other $(N-1)$ signals. IF one or more of there ratios is not periodic
↳ the sum of signal is not periodic

2- eliminate common factors from each numerater and dononinater of ratio of integers.

3- The fundemental period of the sum of signals is $\boxed{T_0 = K_0 T_{01}}$ where K_0 is the least common multiple (LCM) of the denominaters of the individual ratios of integers.

Ex: There are three periodics signal:-

$$x_1(t) = \cos(3.5t)$$
$$x_2(t) = \sin(2t)$$
$$x_3(t) = 2\cos\left(\frac{7}{6}t\right)$$
$$v(t) = x_1(t) + x_2(t) + x_3(t)$$

is $v(t)$ periodic? if yes, what is it's fundemental period?

$$T_{01} = \frac{2\pi}{\omega_1} = \frac{2\pi}{3.5}$$

$$T_{02} = \frac{2\pi}{\omega_2} = \frac{2\pi}{2}$$

$$T_{03} = \frac{2\pi}{\omega_3} = \frac{2\pi}{\frac{7}{6}}$$

$$\frac{T_{01}}{T_{02}} = \frac{\cancel{\frac{2\pi}{3.5}}}{\cancel{\frac{2\pi}{2}}} = \frac{2}{3.5} = \frac{4}{7}$$

$$\frac{T_{01}}{T_{03}} = \frac{\cancel{\frac{2\pi}{3.5}}}{\cancel{\frac{2\pi}{\frac{7}{6}}}} = \frac{\cancel{7}^{2}}{6} \times \frac{1}{\cancel{3.5}} = \frac{1}{3}$$

$\therefore v(t)$ is periodic

$$K_0 = LCM(7, 3) = 21$$

$$T_0 = K_0 T_{01} = 21\left(\frac{2\pi}{3.5}\right) = 12\pi \text{ sec}$$

last example

Ex: $w(t) = \underset{\sim}{z}(t) + x_4(t)$

where $x_4(t) = 3\sin(5\pi t)$

$\frac{T_{01}}{T_{04}} = \frac{\frac{2\pi}{3.5}}{\frac{2\pi}{5\pi}} = \frac{10\pi}{7} \Rightarrow$ irratinal $\therefore w(t)$ is a periodic

* Energy and power signal:-

let $x(t)$ be a real value signal

. if $(x(t)$ is a voltage $\rightarrow$ The instantanuoas power $p(t) = \frac{x^2(t)}{R}$

. if $x(t)$ is a current $\rightarrow$ The instantanuoas power $p(t) = x^2(t) R$

in general we don't necessarly know wether $x(t)$ is a voltage or current, so in order to normalise power we assume $R = 1\Omega$ $\underline{p(t) = x^2(t)}$.

. The signal energy of time interval $2L$ is defind by:-

$$E_{2L} = \int_{-L}^{L} |x(t)|^2 . dt$$

. and the total energy in the signal over the rang $t \in (-\infty, \infty)$

can be defind as:-

$$E = \lim_{L \to \infty} \int_{-L}^{L} |x(t)|^2 . dt$$

. and the avarge power can be defined as:-

$$P = \lim_{L \to \infty} \left[\frac{1}{2L} \int_{-L}^{L} |x(t)|^2 dt \right]$$

. if E exists , i.e $0 < E < \infty \Rightarrow$ the signal is said to be an energy signal.

Energy signals have zero average power

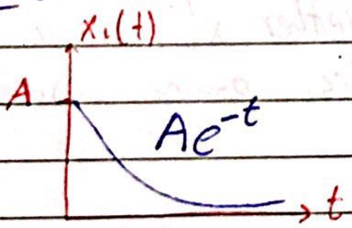
on the other hands if $0 < p < \infty$, then $x(t)$ is power signal

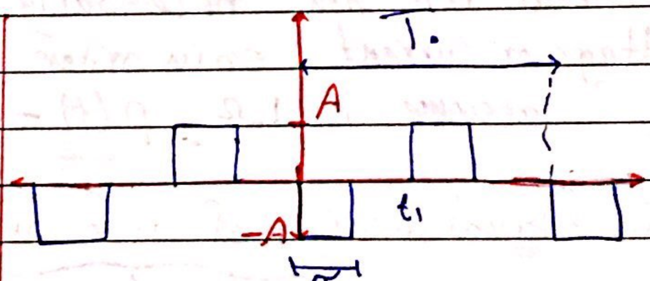
power signals have infinit energy.

. if the signal is periodic $\Rightarrow$ Power Signal

$$p = \frac{1}{T} \int_0^T |x(t)|^2 \cdot dt$$

Ex:





$$E x_1 = \int_0^\infty A^2 e^{-2t} \cdot dt$$

$$= A^2 \frac{e^{-2t}}{-2} \Big|_0^\infty = \frac{A^2}{2}$$

$$\frac{A^2 [0-1]}{-2} = \frac{A^2}{2}$$

$$\boxed{P x_1 = 0}$$

$$P x_2 = \frac{1}{T_0} \left[\int_0^{T_0} |x(t)|^2 \cdot dt \right]$$

$$= \frac{1}{T_0} \left[A^2 t \Big|_0^\tau + A^2 t \Big|_{t_1}^{t_1+\tau} \right.$$

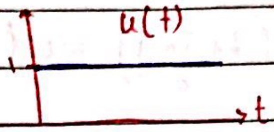
$$= \frac{1}{T_0} (A^2 \tau + A^2 \tau = \frac{2A^2 \tau}{T_0}$$

$$\boxed{E x_2 = \infty}$$

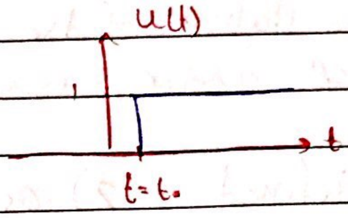
* Elementary signals:-

1) The unit step function (Signal).

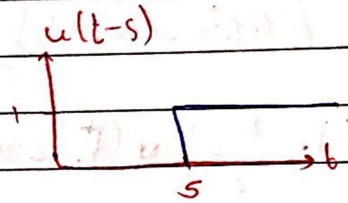
$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



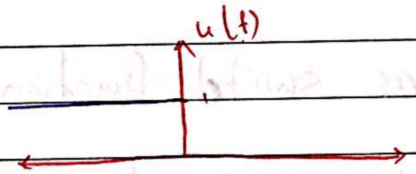
$$u(t-t_0) = \begin{cases} 1 & t-t_0 > 0 \Rightarrow t > t_0 \\ 0 & t-t_0 < 0 \Rightarrow t < t_0 \end{cases}$$



$$u(t-5) = \begin{cases} 1 & t > 5 \\ 0 & t < 5 \end{cases}$$



$$u(-t) = \begin{cases} 1 & t < 0 \\ 0 & t > 0 \end{cases}$$



- the unit step function has the property

$$u(t-t_0) = [u(t-t_0)]^2 = [u(t-t_0)]^K$$

wish K is any ~~~~ positive integer

thies property is based on the relations.

$(0)^K = 0$ and $(1)^K = 1$, $K = 1, 2, \ldots$

• A second property is related to scaling

$u(at-t_0) = u(t-\frac{t_0}{a})$, $a \neq 0$

$u(at+t_0) = u(a(t+\frac{t_0}{a})) = u(t+\frac{t_0}{a})$

the value of the unit step function at the point that the step accures is not:

1) defined 2) as zero 3) as one

4) $u(0) = \frac{1}{2}[u(0^+) - u(0^-)] = \frac{1}{2}$

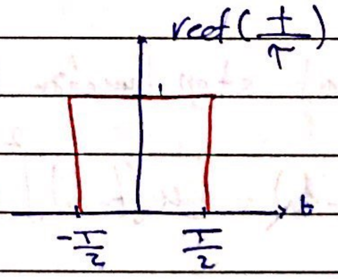
$x(t_1) = \frac{1}{2}[x(t_1) - x(t_1^-)]$

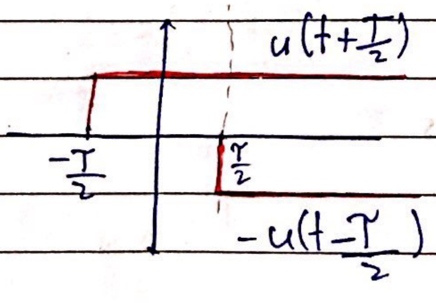
u(t) is used as switch function.

• $\cos(\omega t)\,u(t) = \begin{cases} \cos(\omega t) & t > 0 \\ 0 & t < 0 \end{cases}$

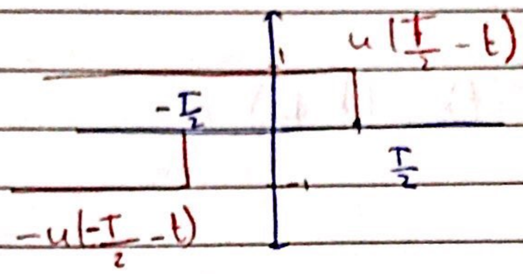
using unit step function to find rect

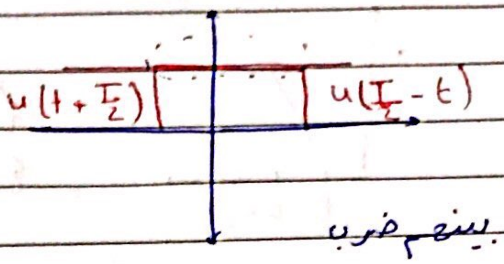
• $rect\left(\frac{t}{\tau}\right) = \begin{cases} 1 & -\frac{T}{2} < t < \frac{T}{2} \\ 0 & o.w \end{cases}$





$rect(\frac{t}{\tau}) = u(t+\frac{T}{2}) - u(t+\frac{T}{2})$





Ex: consider the signum function (written as sgn(t)).

$$Sgn(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

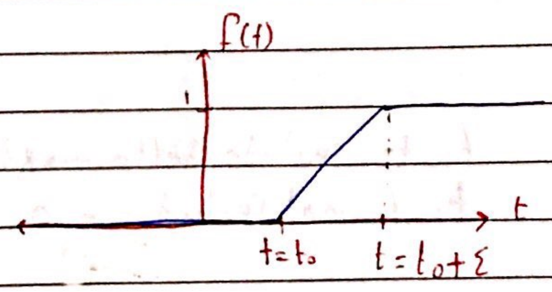
it can be expressed in terms of the unit step function as:

$$Sgn(t) = -1 + 2u(t)$$

2) unit impulse function:

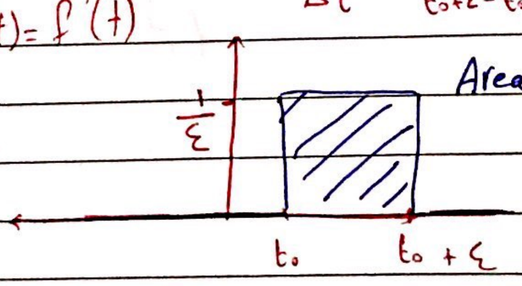
engineers have found great use for unit impluse function $\delta(t)$ even though this function cannot appear in nature.

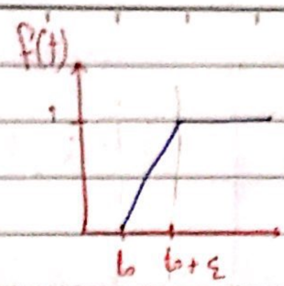
it is also called Dirac Delta function.



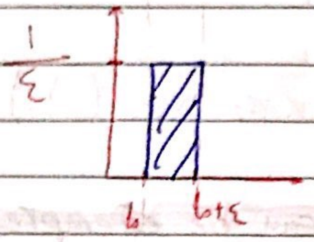
$\Rightarrow$

$\frac{\Delta f(t)}{\Delta t} = \frac{1-0}{t_0+\varepsilon-t_0} = \frac{1}{\varepsilon}$

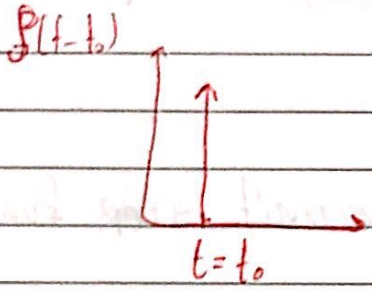




$$\lim_{\varepsilon \to 0} f(t) = u(t-t_0)$$



$$A = \frac{1}{\varepsilon} \cdot \varepsilon = 1$$



$$\lim_{\varepsilon \to 0} g(t) = \delta(t-t_0)$$

we define the unit impulse function $\delta(t-t_0)$ by relations :

$\delta(t-t_0) \quad t \neq t_0$

$$\int_{-\infty}^{\infty} \delta(t-t_0)\, dt = 1 \qquad \int_{t_0^-}^{t_0^+} \delta(t-t_0)\, dt = 1$$

$$\int_{t_1}^{t_2} x(t)\ \delta(t)\, dt = x(0) \quad t_1 < 0 < t_2$$

t_1, t_2 include Delta $\rightarrow x(0)$

t_1, t_2 not includ $\rightarrow 0$

1) $\delta(0) \longrightarrow \infty$

2) $\delta(t) = 0 \quad , t \neq 0$

3) $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

4) $\delta(t)$ is even function , $\delta(t) = \delta(-t)$

Three important property :-

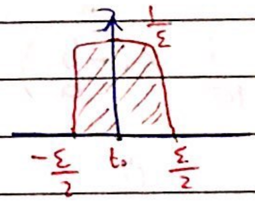
1) The sifting property

$$\int_{t_1}^{t_2} x(t)\, \delta(t-t_0)\, dt = \begin{cases} x(t_0) & t_1 < t_0 < t_2 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{t_1}^{t_2} x(t)\, \delta(t-t_0)\, dt = \frac{1}{2} x(t_0) \quad \text{if } t_0 = t_1 \text{ or } t_0 = t_2$$

$$\int_{t_0}^{t_2} x(t)\, \delta(t-t_0)\, dt = \frac{1}{2} x(t_0)$$

$$\int_{t_1}^{t_0} x(t)\, \delta(t-t_0)\, dt = \frac{1}{2} x(t_0)$$

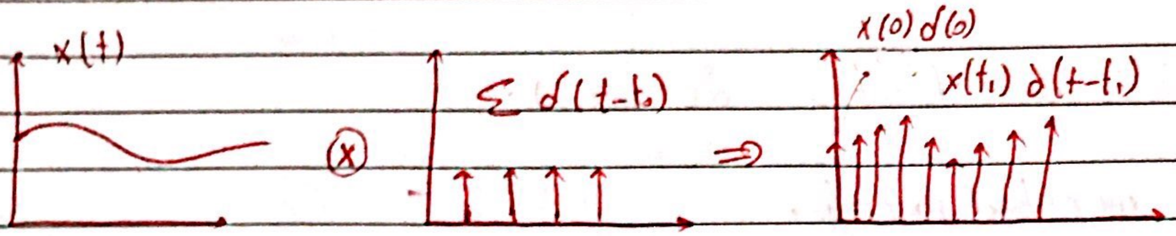


$$\int_{t_0^-}^{t_0^+} x(t)\, \delta(t-t_0)\, dt = x(t_0)$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau = x(\tau)\Big|_{\tau = t} = x(t)$$

2) The Sampling property.-

If $x(t)$ is continuoas at $t=t_0$ then $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$



3) The Scaling property

$$x(at+b) \Rightarrow x\left(a\left(t+\frac{b}{a}\right)\right)$$

$$\delta(at+b) = \frac{1}{|a|}\,\delta\left(t+\frac{b}{a}\right)$$

$$\int_{t_1}^{t_2} x(t)\,\delta(at+b)\,dt = \int_{t_1}^{t_2} \frac{1}{|a|}\,x(t)\,\delta\left(t+\frac{b}{a}\right)dt$$

$$= \begin{cases} \frac{1}{|a|}\,x\left(\frac{-b}{a}\right) & t_1 < \frac{-b}{a} < t_2 \\ 0 & \text{o.w} \end{cases}$$

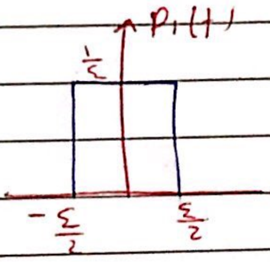
Ex: evaluate

a) $\int_{-2}^{1} (t+t^2)\,\delta(t-3)\,dt = 0$ $3 \rightarrow$ not encloude

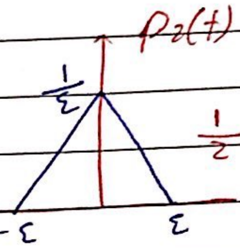
b) $\int_{-2}^{4} (t+t^2)\,\delta(t-3)\,dt = 3+3^2 = 12$

c) $\int_0^3 \exp(t-2)\, \delta(2t-4)\, dt$

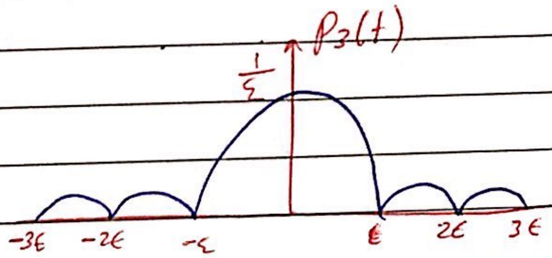
$= \int_0^3 \frac{1}{2} \exp(t-2)\, \delta(t-2)\, dt = \frac{1}{2} \exp(2-2) = \frac{1}{2} \exp(0) = \frac{1}{2}$



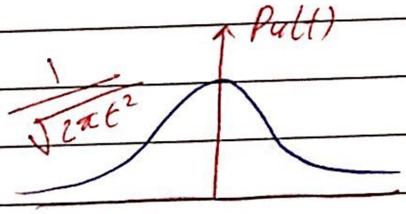
$\lim_{\epsilon \to 0} p_1(t) = \delta(t)$



$\frac{1}{2}(2\epsilon)(\frac{1}{\varepsilon}) = \boxed{1}$



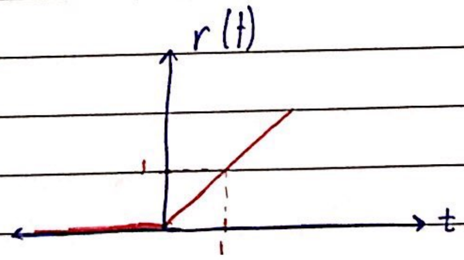
$p_3(t) = \varepsilon \left(\frac{1}{\pi t} \sin\left(\frac{\pi t}{\varepsilon} \right) \right)^2$



Gaussian pulse $p_4(t) = \frac{1}{\sqrt{2\pi\epsilon^2}} e^{-\frac{t^2}{2\epsilon^2}}$

The ramp function

$x(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$



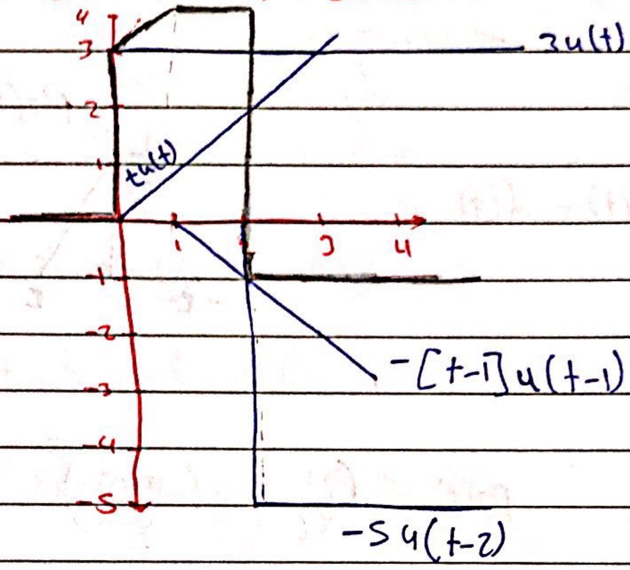
it can be obtained by integrating the unit step function

$\int_{-\infty}^{t} u(\tau)\, dt = r(t)$, $\int_0^t d\tau = \tau|_{\tau=t} = t$

Mathmatical functions for signals:-

*plotting signal waveform

$f(t) = 3u(t) + t\,u(t) - [t-1]u(t-1) - 5u(t-2)$



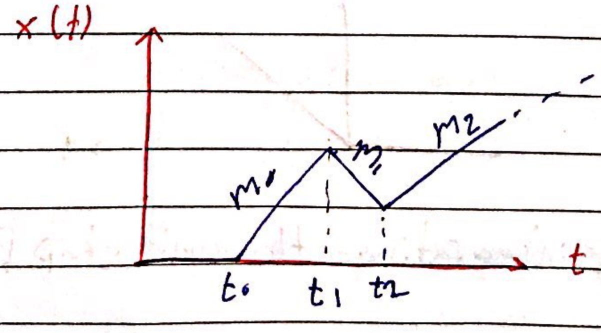
$t<0 \quad , \quad f(t) = 0+0-0-0=0$

$0<t<1, \quad f(t) = 3+t-0-0 = 3+t$

$1<t<2, \quad f(t) = 3+t-(t-1)-0 = 4$

$2<t \quad , \quad f(t) = 3+t-(t-1)-5 = -1$

*A technique for writing the equations for functions compared of straight line segments:-



$m_i = \frac{\Delta x_i}{\Delta t_i}$

$X - X_0 = m_0[t - t_0]$

$\boxed{1}$ the signal is zero for $t<t_0$

For $t<t_1$

$$x_0(t) = m_0[t-t_0]u(t-t_0) \quad , \text{for } t<t_1$$

$\boxed{2}$ to write the equation for the signal for $t<t_2$ we first set the slope to zero by subtracting the slop m_0

$$x_1(t) = x_0(t) - m_0[t-t_1]u(t-t_1)$$

$$x_2(t) = m_0[t-t_0]u(t-t_0) - m_0[t-t_1]u(t-t_1) + m_1[t-t_1]u(t-t_1)$$

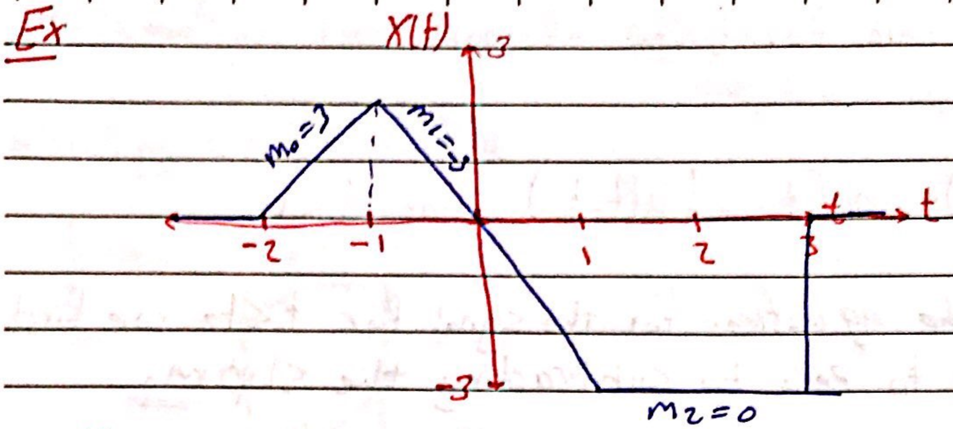
$t<t_2 \Rightarrow x_2(t) = m_0[t-t_0]u(t-t_0) + (m_1-m_0)[t-t_1]u(t-t_1)$

Generally : when

when the slope of a signal changes a ramp function is added at the point wish the slop of this ramp ~~function~~ equals to the new slop minus the previous slope (m_1-m_0).

At any point that a step occure in the signals a step function is odded.

Ex



$x(t) = \overbrace{3}^{m_0}(t+2)u(t+2) + \overbrace{[-3-3]}^{m_1-m_0}(t+1)u(t+1) + \overbrace{[0--3]}^{m_2-m_1}(t-1)u(t-1) + 3u(t-3)$

$x(t) = 3(t+2)u(t+2) - 6(t+1)u(t+1) + 3(t-1)u(t-1) + 3u(t-3)$

$t < -2$, $f(t) = 0 - 0 + 0 + 0 = 0$

$-2 < t < -1$, $f(t) = 3(t+2) - 0 + 0 + 0 = 3(t+2)$

$-1 < t < 1$, $f(t) = 3(t+2) - 6(t+1) + 0 + 0 = -3t$

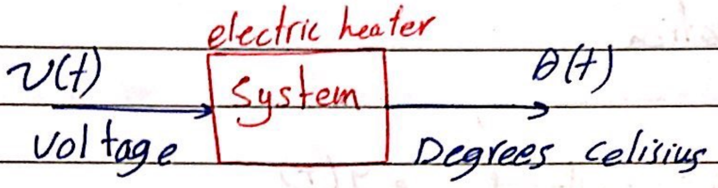
$1 < t < 3$, $f(t) = -3t + 3(t-1) = -3$

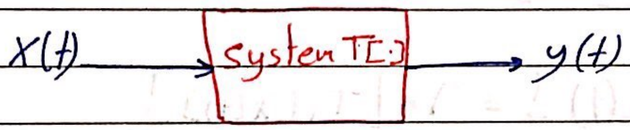
$t > 3$, $f(t) = -3 + 3 = 0$

* Continuoas time systems:-

A system is the proces for wich cause-and-effect realation eaist.

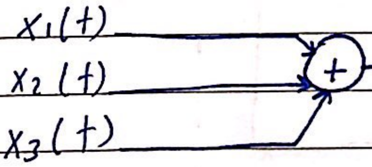
The cause is the input signal to the system and the effect is the output signal of the system.



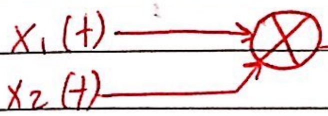


$y(t) = T[x(t)]$, $T[.]$ = Transformation

$x(t) \longrightarrow y(t)$

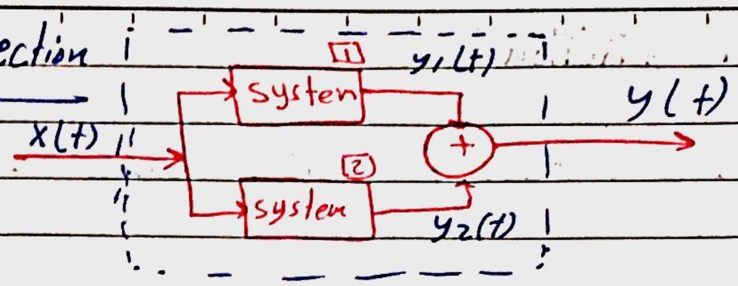


$\rightarrow y(t) = x_1(t) + x_2(t) + x_3(t)$



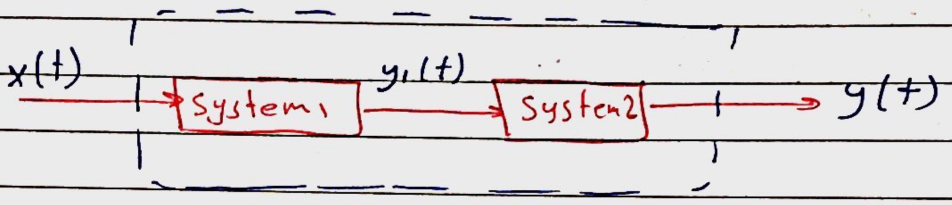
$\rightarrow y(t) = x_1(t) \times x_2(t) = x_1(t)\ x_2(t)$

parallel conection



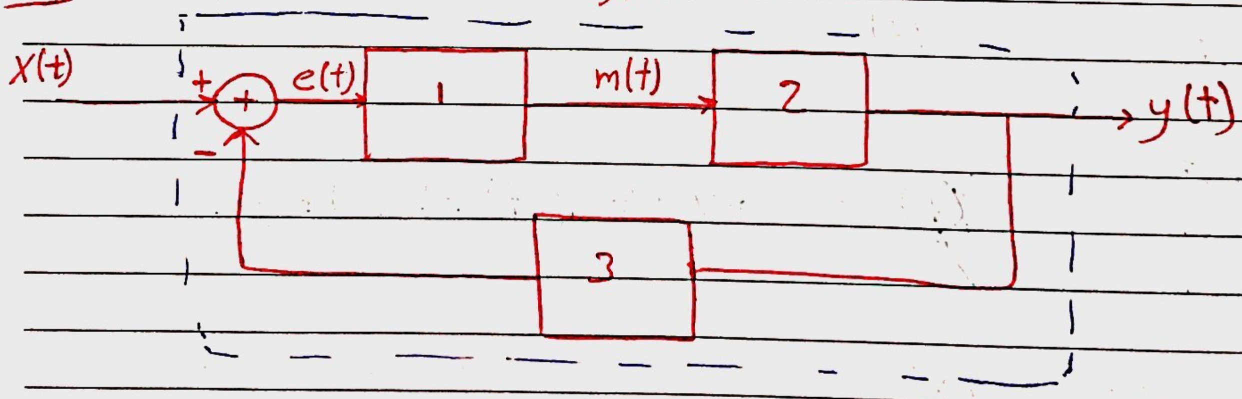
$$y(t) = T[x(t)] = y_1(t) + y_2(t) = T_1[x_1(t)] + T_2[x_2(t)]$$

Series or cascade conection



$$y(t) = T[x(t)] = T_2[y_1(t)] = T_2[T_1[x(t)]]$$

Ex. Interconection of a system :-



Feedback system

1 ⟶ controller
2 ⟶ plant
3 ⟶ sensor

Solution ⟶

$e(t) = x(t) - T_3[y(t)]$ $\quad$ $y(t) = T_2[m(t)]$

$m(t) = T_1[e(t)]$ $\quad$ $y(t) = T_2[T_1[x(t) - T_3[y(t)]]]$

properties of continuous - Time - system :-

1) A linear and nonlinear system

A system is linear if it meets the following criteria :-

1) Additivity : IF $x_1(t) \longrightarrow y_1(t)$ and $x_2(t) \longrightarrow y_2(t)$
then $x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$

2) Homogeneity :- IF $x_1(t) \longrightarrow y_1(t)$
then $a x_1(t) \longrightarrow a y_1(t)$
where a is a constant

• these tow criteria can be conbined to yields the principle of superposition

$$a x_1(t) + a_2 \cdot x_2(t) \longrightarrow a y_1(t) + a_2 y_2(t)$$

• A system is linear if it satisfies the principle of SP.

Ex: 1 $y(t) = K\, x(t)$ (ideal Amplifire)

where K is a constant

is the system linear or not?

$x_1(t) \longrightarrow K\, x_1(t) = y_1(t)$
$x_2(t) \longrightarrow K\, x_2(t) = y_2(t)$

$a_1 x_1(t) + a_2 x_2(t) \longrightarrow K\left[a_1 x_1(t) + a_2 x_2(t)\right]$

$= a_1 K x_1(t) + a_2 K x_2(t)$
$= a_1 y_1(t) + a_1 y_2(t)$

The system is linear ✓

$\boxed{2}$ $y(t) = x^2(t)$ (The squaring circuit)

$x_1(t) \longrightarrow y_1(t) = x_1^2(t)$
$x_2(t) \longrightarrow y_2(t) = x_2^2(t)$

$a_1 x_1(t) + a_2 x_2(t) \longrightarrow \left(a_1 x_1(t) + a_2 x_2(t)\right)^2$
$= a_1^2 x_1^2 + 2 a_1 a_2 x_1(t) x_2(t) + a_2^2 x_2^2(t)$
$\neq a_1 x_1^2(t) + a_2 x_2^2(t)$

Non linear system.

$\boxed{3}$ $y(t) = A\, x(t) + B$

$x_1(t) \longrightarrow y_1(t) = (A x_1(t) + B)\, a_1$
$x_2(t) \longrightarrow y_2(t) = (A x_2(t) + B)\, a_2$

$a_1 x_1(t) + a_2 x_2(t) \longrightarrow A(a_1 x_1(t) + a_2 x_2(t)) + B$
$= a_1 A\, x_1(t) + a_2 A\, x_2(t) + B$

Non linear system $\neq a_1[A x_1(t) + B] + a_2[A x_2(t) + B]$
$a_1 A x_1(t) + a_2 A x_2(t) + (a_1 + a_2) B$

$\boxed{4}$ $y(t) = B$ Try it # non linear system.

2) Time-varying and Time-Invariant Systems.

A system is said to be Time-invariant if a time shift in the input signal results only in the same time shift in the output signal.

$$x(t) \longrightarrow y(t)$$
$$x(t-t_0) \longrightarrow y(t-t_0)$$

$$x(t) \longrightarrow \boxed{\text{System}} \xrightarrow{y(t)} \boxed{\text{Delay } t_0} \longrightarrow y(t-t_0)$$

$$x[t] \longrightarrow \boxed{\text{Delay } t_0} \xrightarrow{x(t-t_0)} \boxed{\text{systen}} \longrightarrow y_d(t)$$

$$y_d(t) = T[x(t-t_0)] = y(t-t_0) \Longrightarrow \text{Time-invariant system}$$

Ex: 1) $y(t) = e^{x(t)}$ 2) $y(t) = e^{-t}\, x(t)$

1) $y(t)\big|_{t-t_0} = e^{x(t)}\big|_{t-t_0} = e^{x(t-t_0)}$

$y_d(t) = y(t)\big|_{x(t-t_0)} = e^{x(t-t_0)} = y(t-t_0)$: Time-invariant system

2) $y(t)\big|_{t-t_0} = e^{-t}\, x(t)\big|_{t-t_0} = y(t-t_0) = e^{-(t-t_0)}\, x(t-t_0)$

$y_d(t) = y(t)\big|_{x(t-t_0)} = e^{-t}\, x(t-t_0) \neq e^{-(t-t_0)}\, x(t-t_0)$: Time-varying system

3) system with and without memory :-

a system has memory if it's output at time t_0, $y(t_0)$, depends on input values other than $X(t_0)$. otherwise the system is memory less.

A system with memory $\Rightarrow$ dynamic system
A memory less System $\Rightarrow$ Static system

Ex: $y(t) = K\,X(t)$ memory less (static)

$y(t) = x(t) + x(t-5)$ system with memory

$y(t) = K \int_{-\infty}^{t} x(\tau)\, d\tau$ " " "

$y(t) = x(-t)$ " " "

4) cousal and Noncousal system :-

A system is cousal if for the output at any time t_0 is dependent on the input only for $t \leq t_0$.

All Physical system are cousal.

Ex: 1) $y(t) = x(t-2)$ causal

2) $y(t) = x(t+2)$ Non cousal

3) $y(t) = x(-t)$ Non cousal

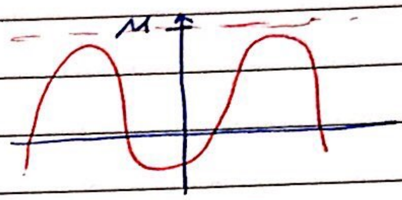
Five Apple

5) stability :-

BIBO stability criterion :-

A system is stable if the output remains bounded for any bounded input.

by definition, a signal $x(t)$ is bounded if there exist a number M such that:-



$$|x(t)| \le M \quad , \text{for all } t$$
$$M < \infty$$

A system is Bounded input bounded output stable if for a number R

$$|y(t)| \le R \quad , \text{for all } t$$

For all $x(t)$ such that $x(t)$ is bounded.

Ex:-

1) $y(t) = e^{x(t)}$ 2) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

1) $|y(t)| = |e^{x(t)}| \le e^{|x(t)|} \le e^{M}$

$|y(t)| \le e^{M} = R$ $\therefore$ BIBO system

2) $|y(t)| = \left| \int_{-\infty}^{t} x(\tau)\, d\tau \right| \le \int_{-\infty}^{t} |x(\tau)|\, d\tau \le M \int_{-\infty}^{t} d\tau$

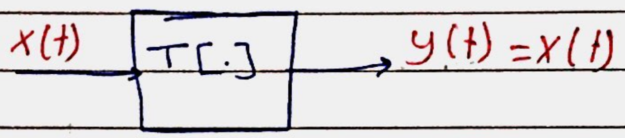
$\therefore$ unstable system.

6) Invertibility and inverse system:-

A system is said to be invertible if distinct input result in distinct outputs.

$$y(t) = x^2(t) \Longrightarrow x(t) = \pm\sqrt{y(t)} \Longrightarrow \text{not invertible system.}$$

* Identity system:- A system for wich the output is equal to its input.

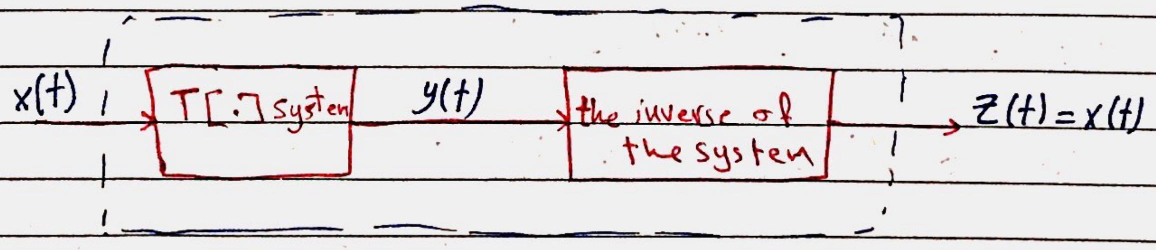


Ex: Ideal Amplifiar with unity gain

$$y(t) = k\,x(t) \quad \text{wich } k=1$$

* inverse of a system:-

The inverse of a system (denoted by T) is a second system (denoted by T_1) that when cascated with the system T yields the ideals system.



Ex Determine if each of the following system is invertible, If it is, then, construct the inverse system.

If it is not, then, find tow input signals to the system that have same outputs

a) $y(t) = 2x(t)$ b) $y(t) = \cos(x(t))$

c) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$; $y(-\infty)=0$ d) $y(t) = x(t+1)$

Solu.-

a) The system is invertable

$$z(t) = \frac{1}{2} y(t)$$

b) The system is noninvertible since $x(t)$ and $x(t)+2\pi$ give the same output.

c) The system is invertible and the inverse system is the differentiator:

$$z(t) = \frac{d}{dt}(y(t))$$

d) The system is invertible and the inverse is the one unit delay

$$z(t) = y(t-1)$$

Five Apple

Linear Time-invariant system:- (LTI)

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau).d\tau$$

• The sifting property

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$$

$$x(t) \longrightarrow \boxed{T[\cdot]} \longrightarrow y(t) = T\left[\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau\right]$$

For linear system $\quad y(t) = \int_{-\infty}^{\infty} T[x(\tau)\,\delta(t-\tau)].d\tau$

$$= \int_{-\infty}^{\infty} x(\tau).T[\delta(t-\tau)]\,d\tau$$

$$x(t) \longrightarrow \boxed{T[\cdot]} \longrightarrow y(t)$$

* $x(t) = \delta(t)$ $\qquad$ $y(t) = h(t)$: The impulse response
* $x(t) = u(t) \Longrightarrow y(t) = s(t)$: The step response

$T[s(t)] \longrightarrow h(t)$

$T[S(t-\tau)] \longrightarrow h(t,\tau)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau).h(t,\tau).d\tau \qquad \int_{-\infty}^{\infty} x(\tau).h(t-\tau).d\tau$$

• if the system also time-invariant

$$T[\delta(t-\tau)] \longrightarrow h(t-\tau)$$

• for a linear time-invariant system (LTI)

$$y(t) = \int_{-\infty}^{\infty} x(\tau) . h(t-\tau) . d\tau$$ convolution integral.

$$= x(t) \underbrace{*}_{\text{convolution}} h(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau) . d\tau$$ also convolution integral.

$$= h(t) \underbrace{*}_{\text{convolution}} x(t)$$

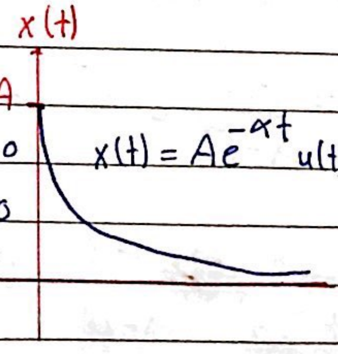
* LTI system :-

$$x(t) \longrightarrow \boxed{T[\cdot]} \longrightarrow y(t) = h(t)$$ impulse response.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) . h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

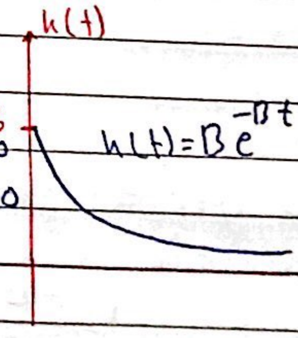
Ex :

$$x(t) = \begin{cases} Ae^{-\alpha t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$



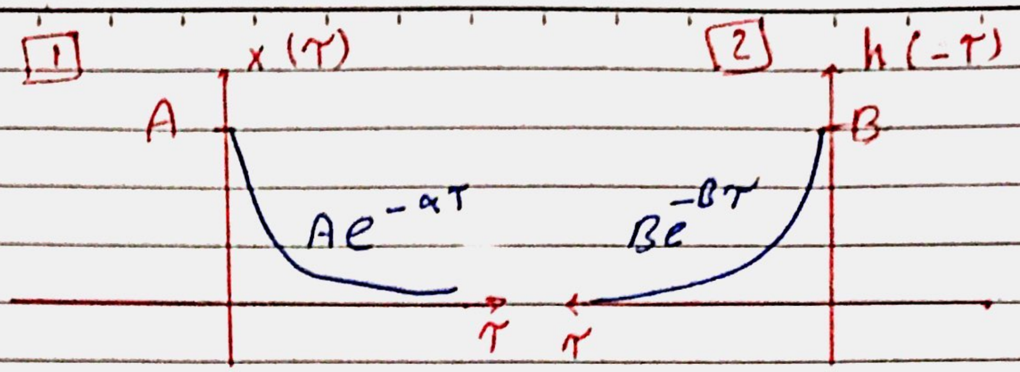
$$x(t) = Ae^{-\alpha t} u(t)$$

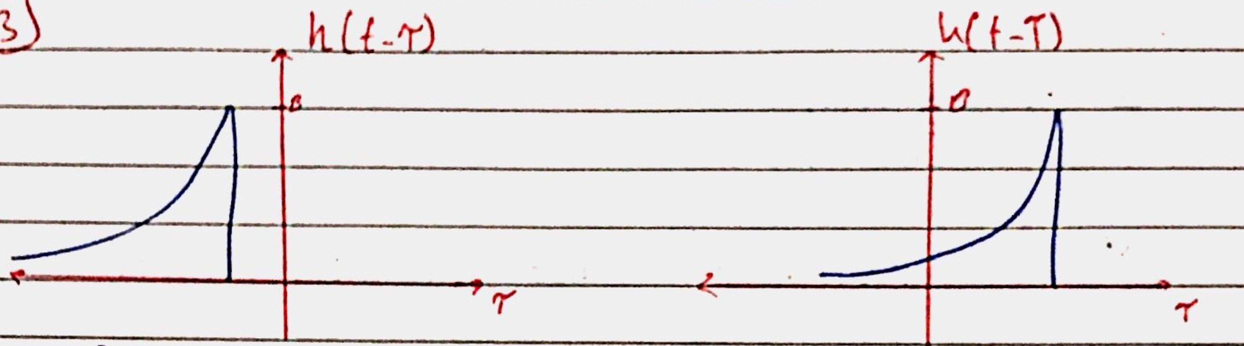
$$h(t) = \begin{cases} Be^{-\beta t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$



$$h(t) = Be^{-\beta t} u(t)$$

find $y(t) = x(t) * h(t)$ $\Longrightarrow$

1 

3 

for $t < 0$

$y(t) = \text{zero}$

for $t > 0$

$$y(t) = \int_0^t Ae^{-\alpha\tau} \cdot Be^{-\beta(t-\tau)}\, d\tau$$

$$= AB e^{-\beta t} \int_0^t e^{-\tau(\alpha-\beta)}\, d\tau$$

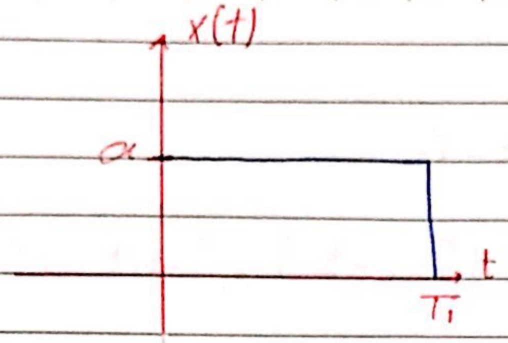
$$= AB e^{-\beta t} \left. \frac{e^{-\tau(\alpha-\beta)}}{-(\alpha-\beta)} \right]_0^t$$

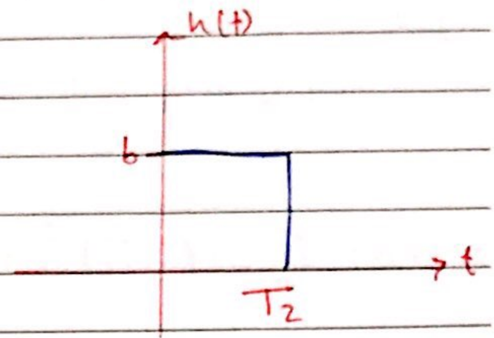
$$= \frac{AB}{\alpha-\beta}\left[e^{-\beta t} - e^{-\alpha t}\right]$$

So $y(t) = \begin{cases} \text{Zero} & , t < 0 \\ \frac{AB}{\alpha-\beta}\left[e^{-\beta t} - e^{-\alpha t}\right] & , t \geq 0 \end{cases}$

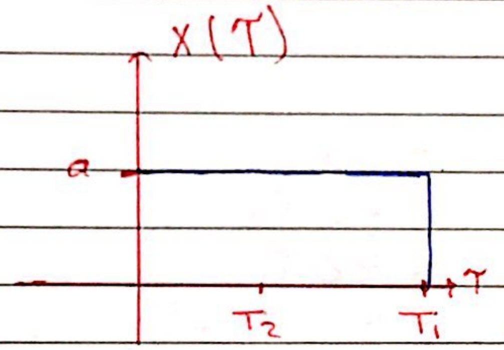
$$\Longrightarrow y(t) = \frac{AB}{\alpha-\beta}\left[e^{-\beta t} - e^{-\alpha t}\right] u(t)$$

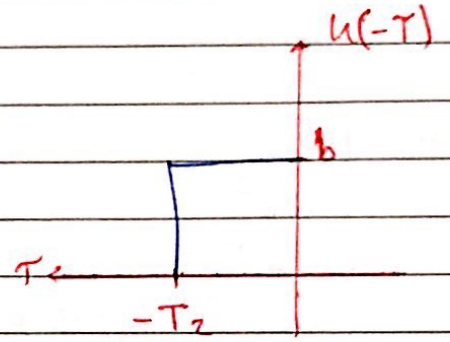
Ex:

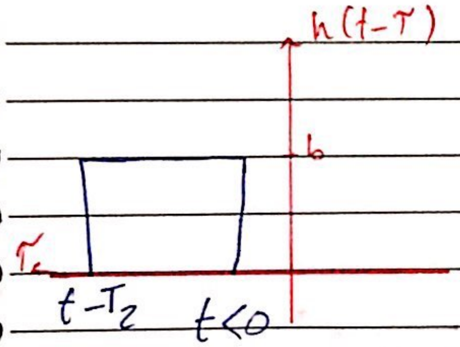


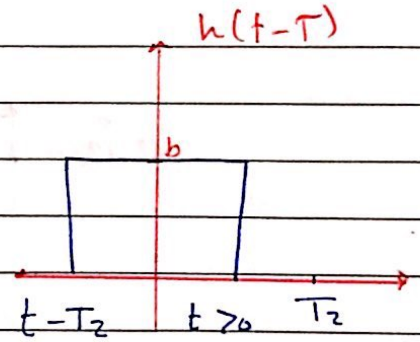


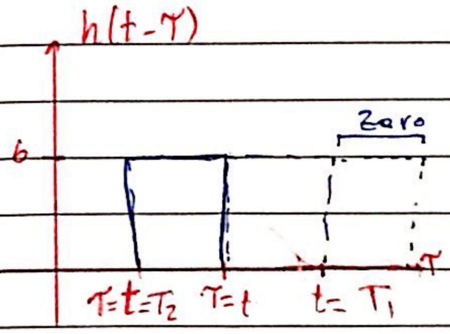
Solu











$y(t) = 0 \quad , \quad t<0$

$y(t) = \int_0^t a.b.d\tau = abt \quad , \quad 0 \le t < T_2$

$y(t) = \int_{t-T_2}^{t} a.b.d\tau = ab\,\tau \Big|_{t-T_2}^{t}$

$= abT_2 \quad , \quad T_2 \le t < T_1$

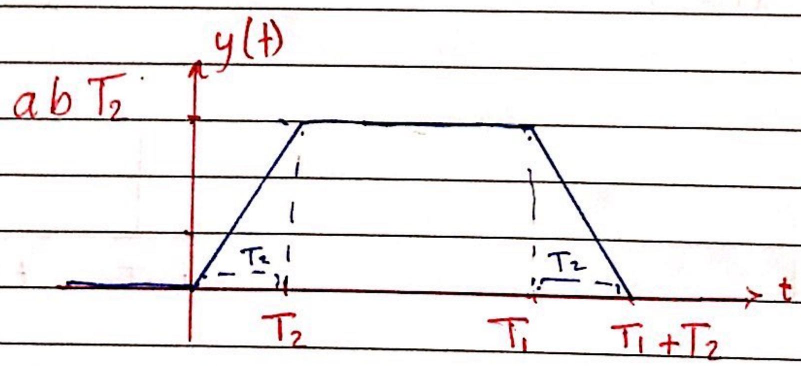
$\Rightarrow$

$$y(t) = \int_{t-T_2}^{T_1} a.b.d\tau = a.b.\tau \Big|_{t-T_2}^{T_2}$$

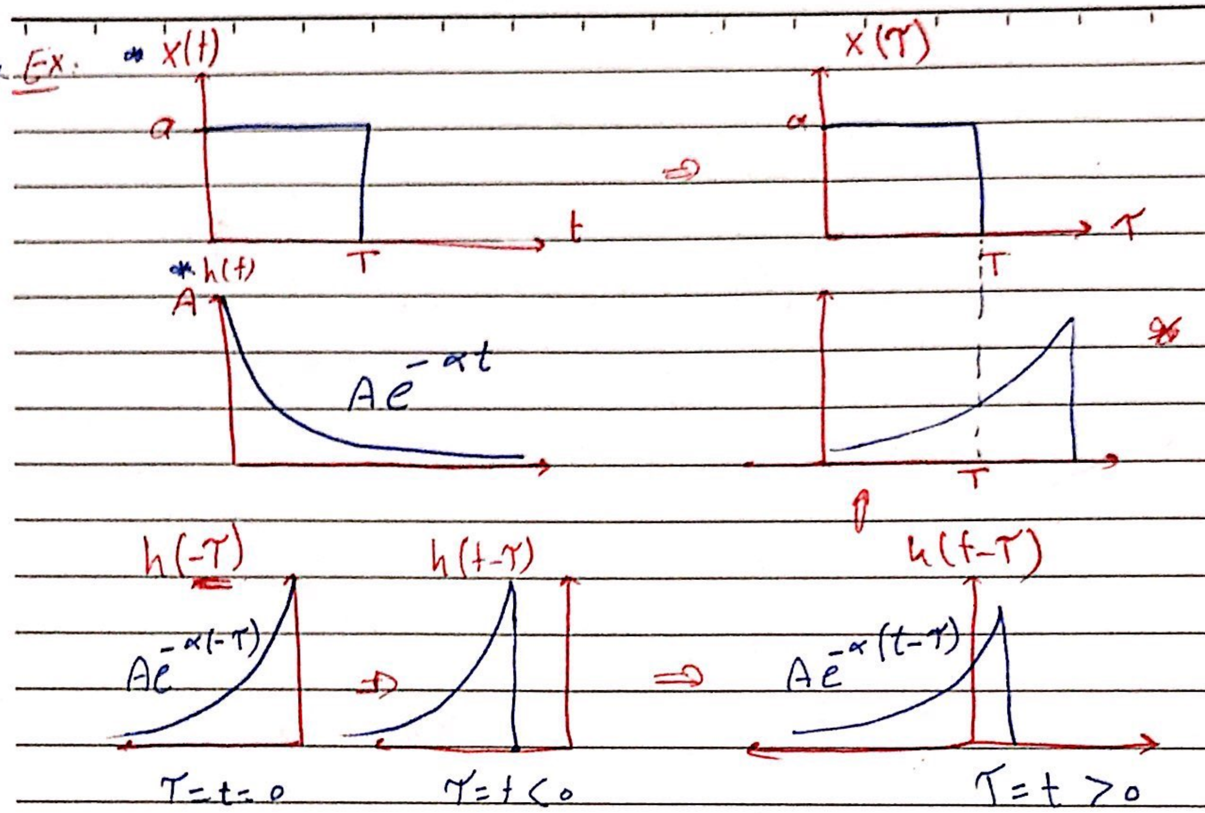
$$= ab\left[T_2 - (t-T_2)\right] = ab\left[(T_1+T_2)-t\right], \quad T_1 \leq t \leq (T_1+T_2)$$

$$y(t) = 0, \quad t > (T_1+T_2)$$

$$y(t) = \begin{cases} 0 & t<0 \\ abt & 0 \leq t < T_2 \\ abT_2 & T_2 \leq t < T_1 \\ ab\left[(T_1+T_2)-t\right] & T_1 \leq t < T_1+T_2 \\ 0 & t > T_1+T_2 \end{cases}$$



Ex:



y(t) for all t

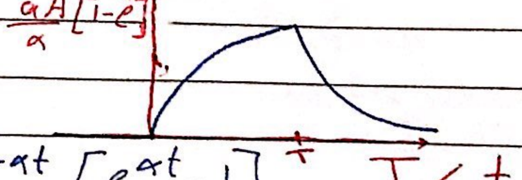
$y(t)=0 \qquad t<0$

$$y(t)=\int_0^t a.A\,e^{-\alpha(t-\tau)}\,d\tau \qquad 0<t<T$$

$$= a.A\,e^{-\alpha t}\int_0^t e^{\alpha\tau}\,d\tau = a.A\,e^{-\alpha t}\,\frac{e^{\alpha t}}{\alpha}\Big|_0^t$$

$$=\frac{a\cdot A\,e^{-\alpha t}}{\alpha}\left[e^{\alpha t}-1\right]=\frac{aA}{\alpha}\left[1-e^{-\alpha t}\right]$$

$$y(t)=\int_0^T a.\,A\,e^{-\alpha(t-\tau)}\,d\tau$$



$$=\frac{a\cdot A\,e^{-\alpha t}}{\alpha}\cdot e^{\alpha t}\Big|_0^T=\frac{aA\,e^{-\alpha t}}{\alpha}\left[e^{\alpha T}-1\right] \qquad T<t$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$\int_{-\infty}^{\infty} y(t)\, dt = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \right] dt$$

$$= \int_{-\infty}^{\infty} x(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau)\, dt \right] d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) \left[\text{area under } h(t) \right] d\tau$$

$$= \left[\text{area under } x(t) \right] \left[\text{area under } h(t) \right]$$

properties of continuos-Time LTI Systems:-

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

* memoryless system :-

$$y(t_0) = \int_{-\infty}^{\infty} x(\tau)\, h(t_0-\tau)\, d\tau$$

compare with

$$\int_{-\infty}^{\infty} x(t)\, \underbrace{\delta(t-t_0)}_{\delta(t_0-t)}\, dt = x(t_0)$$

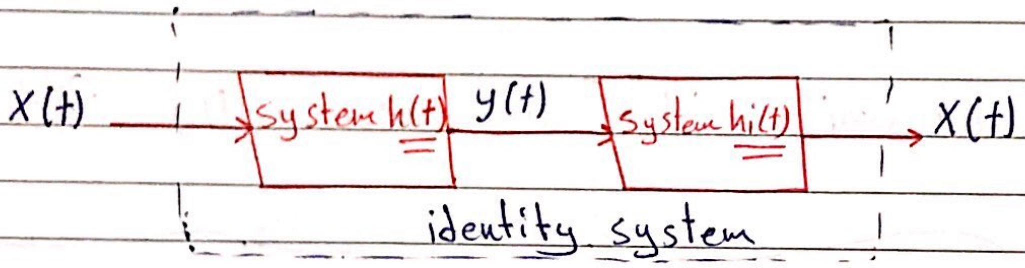
$$y(t_0) = \int_{-\infty}^{\infty} x(t)\, K\delta(t_0 - t)\, dt = K\, x(t_0)$$

$h(t) = K\delta(t)$ (memoryless system).

for $K=1 \Longrightarrow h(t) = \delta(t)$ (The identity system)

- $x(t) * \delta(t) = x(t)$

*** invertibility :-**



$$x(t) * \underbrace{h(t) * h_i(t)} = x(t)$$

$$h(t) * h_i(t) = \delta(t)$$

*** Causality :-**

$\delta(t)$ accurs at $t=0$

The impulse response $h(t)$ must be zero for $t<0$
$h(t)=0$ for $t<0$.

$$y(t) = \int_0^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_0^{t} x(\tau)\, h(t-\tau)\, d\tau$$

* stability : BIBO stable critorion :-

$$|y(t)| = \left| \int_{-\infty}^{\infty} x(t-T)\, h(T) \, dT \right| \leq \int_{-\infty}^{\infty} |x(t-T)| \, |h(T)| \, dT$$

$$|y(t)| \leq \underbrace{B_x \int_{-\infty}^{\infty} |h(T)| \, dT}_{B_y}$$

$$\int_{-\infty}^{\infty} |h(T)| \, dT = S_h < \infty$$ (Absolutaly integrable).

Ex :- 1) $h(t) = e^{-3t} u(t)$ 2) $h(t) = e^{3t} u(t)$ check stability?

1) $\int_{-\infty}^{\infty} |h(t)| \, dt = \int_{0}^{\infty} e^{-3t} \, dt = \left. \frac{e^{-3t}}{-3} \right|_0^{\infty} = \frac{1}{3} < \infty$

absolutaly integrable and the system is BIBO stable.

2) $\int_{-\infty}^{\infty} |h(t)| \, dt = \int_{0}^{\infty} e^{3t} \, dt = \left. \frac{e^{3t}}{3} \right|_0^{\infty} \rightarrow \infty$ unstable system.