EM I

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POWERUNIT

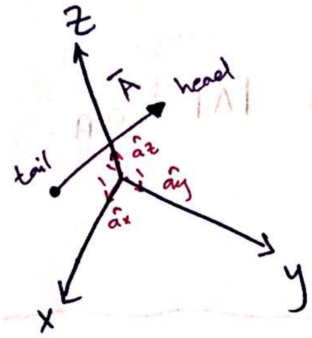
lecture (1) : 2/2/2020 :-

CH.1 : Vectors Review:-

* Vector : magnitude + direction.
* scalar : magnitude only.

$\bar{A}$: Vector A.

A : Scalar A.



① $\bar{A} : A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$. : long format.

* A_x , A_y , A_z : Vector components (scalar)معاملات/ أرقام
* $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$: unit vectors.

unit vector along (X-axis)

ie $\bar{B} : B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z$.

② $\bar{A}$: (A_x , A_y , A_z). : Short format.

Vector

بدون bar

يكون احداثيات

(x,y,z) . نقطة

* Vector magnitude (Vector Norm):-

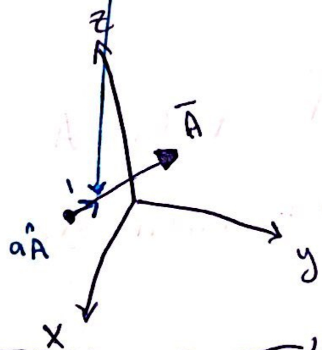
$$|\bar{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}.$$

V.imp

* $\hat{a}_A$: unit Vector along Vector 'A'.

$$\hat{a}_A = \frac{\bar{A}}{A} = \frac{\text{Vector}}{\text{magnitude}} = \frac{A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

③ $\bar{A}$: $|\bar{A}| \cdot \hat{a}_A$ ضرب عادي X مش كثير مفضّلة



lecture (2) : 3/2/2020.

* Operations on Vectors :-

1. Addition and subtraction :

$\boxed{\text{Add}}$

$$\bar{C} = \bar{A} + \bar{B}.$$

$$\bar{C} = (A_x + B_x)\hat{a}_x + (A_y + B_y)\hat{a}_y + (A_z + B_z)\hat{a}_z.$$

then

$$\bar{C} = C_x \hat{a}_x + C_y \hat{a}_y + C_z \hat{a}_z.$$

$\boxed{\text{sub}}$

$$\bar{D} = \bar{A} - \bar{B}.$$

$$\bar{D} = (A_x - B_x)\hat{a}_x + (A_y - B_y)\hat{a}_y + (A_z - B_z)\hat{a}_z.$$

$$\bar{D} = D_x \hat{a}_x + D_y \hat{a}_y + D_z \hat{a}_z.$$

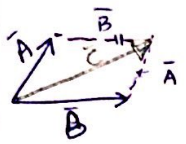
* addition & subtraction:

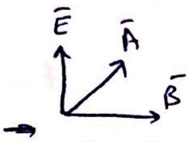
ie $\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$\bar{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$.

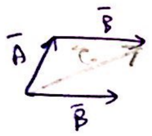
$\bar{C} = \bar{A} + \bar{B}$, $\bar{D} = \bar{A} - \bar{B}$.

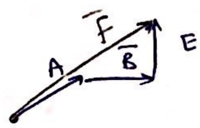
* Graphically :-





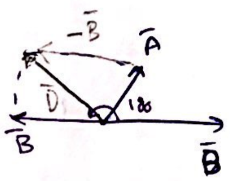
$\vec{F} = \bar{A} + \bar{B} + \bar{E}$



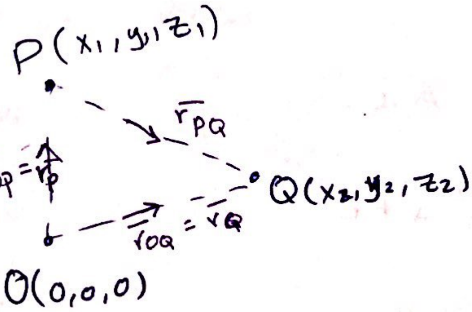


* subtraction :-

$\bar{D} = \bar{A} - \bar{B} = \bar{A} + (-\bar{B})$.



* Distance :-



$\bar{r}_{PQ} = \bar{r}_Q - \bar{r}_P$

$= (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$.

$|\bar{r}_{PQ}| = r_{PQ} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$.

$\overline{r_{QP}} = -\overline{r_{PQ}}$.

$\hat{a}_{r_{PQ}}$: unit vector بتاع $\overline{r_{PQ}}$. $= \dfrac{\overline{r_{PQ}}}{r_{PQ}} = \dfrac{\text{vector}}{\text{mag}}$

* multiplication :-

① Dot product :- $\Rightarrow$ Scalar.

$$\bar{A} \cdot \bar{B} = A\,B \cos\theta_{AB}$$

$$\theta_{AB} = \cos^{-1} \frac{\bar{A} \cdot \bar{B}}{AB}.$$

$$\hat{a}_x \cdot \hat{a}_x = (1)(1) \cos 0 = 1.$$

$$\hat{a}_x \cdot \hat{a}_y = (1)(1) \cos 90 = 0.$$

$$\hat{a}_x \cdot \hat{a}_z = (1)(1) \cos 90 = 0$$

$$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z.$$

Properties :-

* $\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$.
* $\bar{A} \cdot (\bar{B} + \bar{C}) = \bar{A} \cdot \bar{B} + \bar{A} \cdot \bar{C}$.
* $\bar{A} \cdot \bar{A} = A^2 = A_x^2 + A_y^2 + A_z^2$.
* $\hat{a}_n \cdot \hat{a}_m = \begin{cases} 1, & \text{if } n = m \\ 0, & n \neq m. \end{cases}$

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② Cross Product:- $\Rightarrow$ vector.

$$|\bar{A} \times \bar{B}| = A B \sin\theta_{AB}.$$

$$\theta_{AB} = \sin^{-1} \frac{|\bar{A} \times \bar{B}|}{AB}.$$

$$\bar{A} \times \bar{B} = \begin{array}{c|ccc|} & 1 & 2 & 3 \\ 1 & \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 2 & A_x & A_y & A_z \\ 3 & B_x & B_y & B_z \end{array}$$

طريقة الdiff

determint

الصف+العمود

$$\bar{A} \times \bar{B} = (-1)^{1+1} (A_y B_z - B_y A_z)\,\hat{a}_x + (-1)^{1+2} (A_x B_z - A_z B_x)\,\hat{a}_y + (-1)^{1+3} (A_x B_y - A_y B_x)\,\hat{a}_z.$$

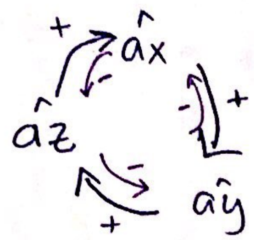
$$\bar{A} \times \bar{B} \perp \bar{A}, \bar{B}.$$

ناتجهم عمودي كلاهما

* properties:-

* $\bar{A} \times \bar{B} = -\bar{B} \times \bar{A}.$
* $\bar{A} \times (\bar{B} + \bar{C}) = \bar{A} \times \bar{B} + \bar{A} \times \bar{C}.$
* $\bar{A} \times (\bar{B} \times \bar{C}) \neq (\bar{A} \times \bar{B}) \times \bar{C}.$
* $|\hat{a}_x \times \hat{a}_x| = (1)(1)\sin 0 = 0$ متوازيين
* $|\hat{a}_x \times \hat{a}_y| = (1)(1)\sin 90 = 1$
* $|\hat{a}_x \times \hat{a}_z| = (1)(1)\sin 90 = 1$

$\Big\}$ متعامدين

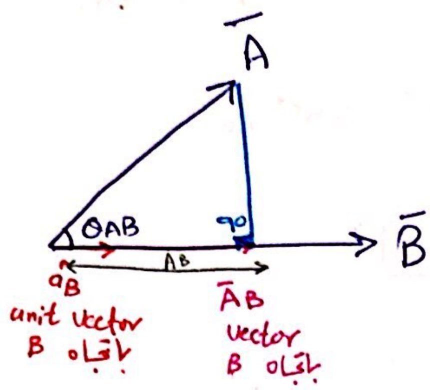


$$\hat{a}_z \times \hat{a}_y = -a_x$$

من z إلى y عكس

السهم يعني (-) الشيء الثالث. يعني y, z يعطي x

* Projection of vector along another vector:-



$A_B \equiv$ scalar projection of A along B.

$\bar{A}_B \equiv$ Vector projection of A along B.

$$\cos\theta_{AB} = \frac{A_B}{|\bar{A}|} \Rightarrow \boxed{A_B = A\cos\theta_{AB}}$$

AB : scalar

$\bar{A}_B$ vector = scalar × unit vector

$$\bar{A}_B = A_B\,\hat{a}_B \qquad \hat{a}_B = \frac{\bar{B}}{B}.$$

$$A_B = \cancel{A}\left(\frac{\bar{A}\cdot\bar{B}}{\cancel{A}B}\right) \Rightarrow \boxed{A_B = \bar{A}\cdot\hat{a}_B}\ \#$$

then

so

$$\boxed{\bar{A}_B = (A\cdot\hat{a}_B)\,\hat{a}_B}\ \#$$

***Ex:-** if $\bar{A} = 3\hat{a}_x + 4\hat{a}_y + \hat{a}_z$. Find θ_{AB}, A_B, $\bar{A}_B$.

$\bar{B} = 2\hat{a}_y - 5\hat{a}_z$. $\Rightarrow B_x = 0$.

$$\theta_{AB} = \cos^{-1}\left(\frac{\bar{A}\cdot\bar{B}}{AB}\right) \Rightarrow$$

$\bar{A}\cdot\bar{B} = 0 + 8 - 5 = 3.$

$A = \sqrt{26}$

$B = \sqrt{29}.$

$$\theta_{AB} = \cos^{-1}\left(\frac{3}{\sqrt{26}*\sqrt{29}}\right) = 83.73° = \sin^{-1}\left(\frac{|\bar{A}\times\bar{B}|}{AB}\right)$$

لازم الطريقتين يزبطو.

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$$A_B = A \cos\theta_{AB} = \sqrt{26} \cdot \frac{3}{\sqrt{26}\cdot\sqrt{29}}$$

الطرق لنفس الناتج.

Same same

$$A_B = \bar{A} \cdot \hat{a}_B = (3,4,1) \cdot \frac{(0,2,-5)}{\sqrt{29}} = \frac{8-5}{\sqrt{29}} = \frac{3}{\sqrt{29}}$$

$$\bar{A}_B = A_B \, \hat{a}_B = \frac{3}{\sqrt{29}} \cdot \frac{(0,2,-5)}{\sqrt{29}} = \frac{6}{29}\hat{a}_y - \frac{15}{29}\hat{a}_z$$

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lecture (3): 6/2/2020.

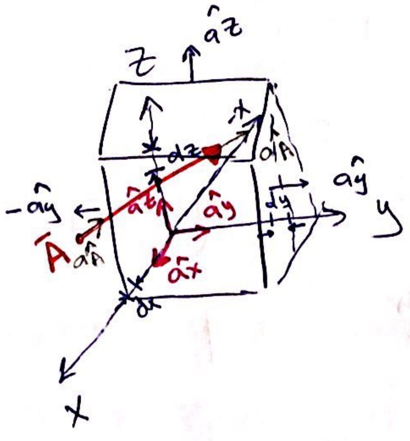
coordinate system:-

* cartesian system:-

$-\infty < x < \infty$
$-\infty < y < \infty$
$-\infty < z < \infty$

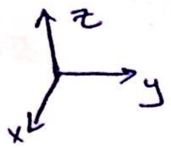
3D object.
Infinite box.

unit vectors : $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$.



* Differential elements :-

dx , dy , dz



* differential length :- ($\bar{dL}$) :- vector.

$$\bar{dL} = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}.$$

* differential Normal surface Area :- ($\bar{ds}$) :- vector.

$$\bar{ds}_{front} = dy\,dz\,\hat{a_x}.$$

$$\bar{ds}_{back} = (dy\,dz)(-\hat{a_x})$$

نأخذ دائماً العمودي على السطح الخارج منه

$$\bar{ds}_{right} = dx\,dz\,\hat{a_y}.$$

$$\bar{ds}_{left} = -dx\,dz\,\hat{a_y}$$

$$\bar{ds}_{top} = dx\,dy\,\hat{a_z}$$

$$\bar{ds}_{bottom} = -dx\,dy\,\hat{a_z}$$

* differential Volume :- (dV) :- scalar.

$dV \rightarrow$ scalar.

$$dV = dx\,dy\,dz.$$

2D - Surfaces:-

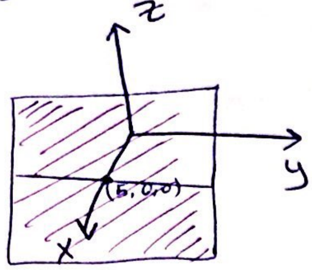
تثبيت

$\boxed{*}$ by fixing one variable

$X = \text{constant}$., $\left(-\infty < \begin{matrix} y \\ z \end{matrix} < \infty\right)$.

Ex:

* $X = 5$



inf plane parallel to y-z plane . *note: along yz plane if $\boxed{x=0}$.

منطبق عليه .

* $y = -2$. inf plane parallel to x-z plane.

* $z = 0$. inf plane along x-y plane.

// ——— // ——— // ——— // ———

1D segment:-

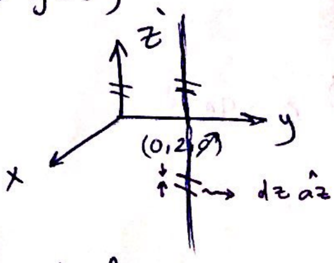
$\boxed{*}$ by fixing two Variables.

* x, y are constants: inf line parallel to z-axis.

or along z-axis if ($x=0$ and $y=0$)

* $X=0$, $y=2$. * ماحكى عن z يعني

$-\infty < z < \infty$



* x, z are constants, y variable. → inf line prallel to y-axis.

↳ inf line along y-axis if ($x=0=z$)

cylindrical coordinates:-

نق

$0 \leq \rho \leq \infty$

الدوران

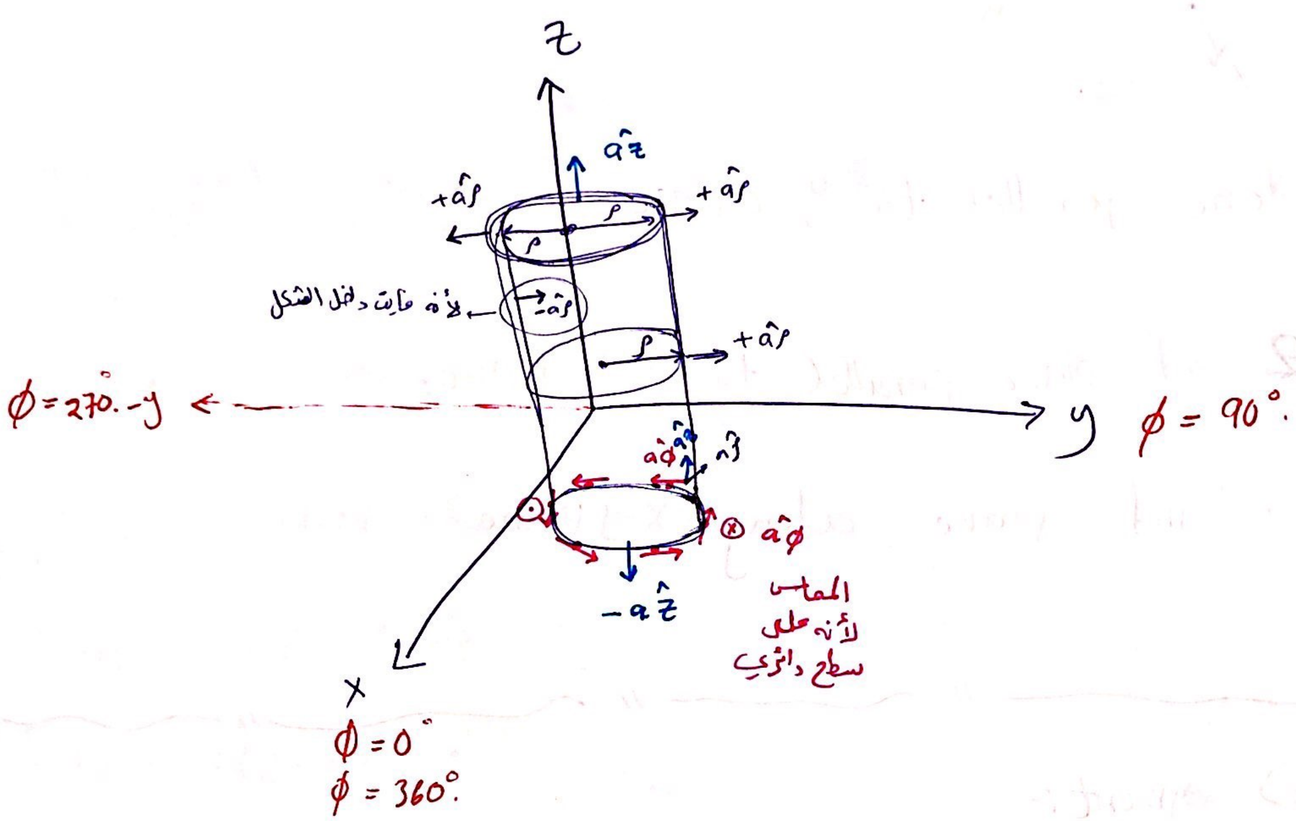
$0 \leq \phi \leq 2\pi$ (360° degrees)

الارتفاع

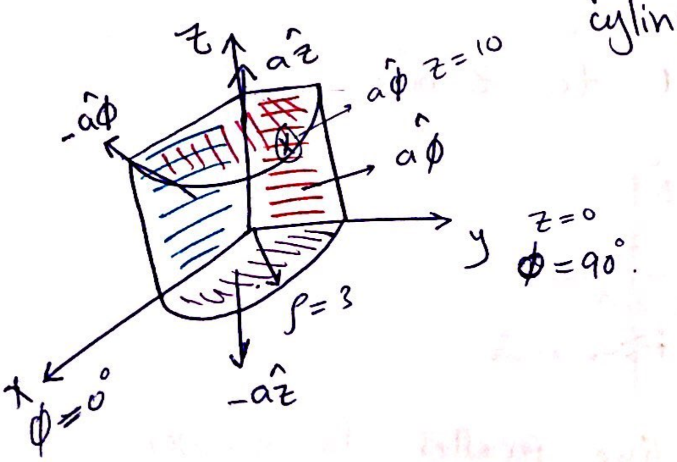
$-\infty < z < \infty$

} 3D object.
inf - solid cylinder.
مصمتة

$\boxed{\text{new}}$ unit vectors:- $\hat{a}_\rho$, $\hat{a}_\phi$, $\hat{a}_z$



quarter cylinder ربع اسطوانة



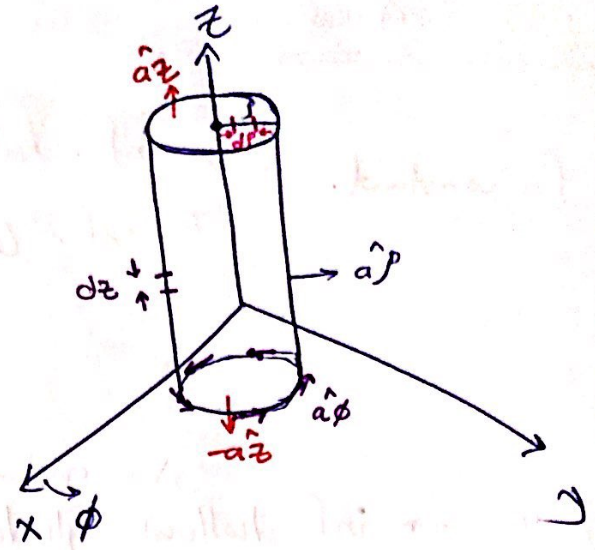
$0 \leq \rho \leq 3$ نصف قطر

$0 \leq \phi \leq 90°$. زاوية

$0 \leq z \leq 10$. height

Lecture (4):- 9/2/2020

Cylidrical cordinate:



Differential Elements:

$d\rho$, $d\phi$, dz.

Unit vectors : $\hat{a}_\rho$, $\hat{a}_\phi$, $\hat{a}_z$.

$$\bar{dL} = d\rho\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + dz\, \hat{a}_z .$$

ρ: constant:

$\bar{ds}_{\text{side (جانبي)}} = \rho\, d\phi\, dz \,.\, \hat{a}_\rho$ العمود عليه

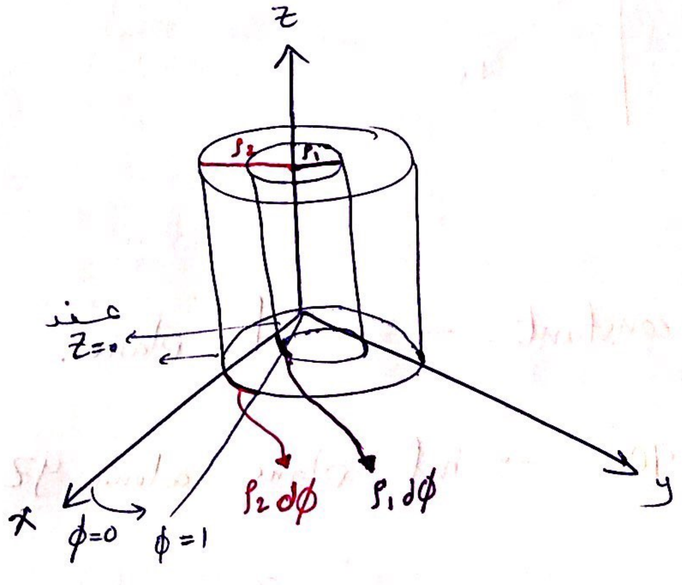
ϕ : constant:

$\bar{ds}_{\text{cut (قطع)}} = d\rho\, dz \,.\, \hat{a}_\phi$

z : constant:

$\bar{ds}_{\text{Top (فوق)}} = \rho\, d\rho\, d\phi \,.\, \hat{a}_z$

$\bar{ds}_{\text{bottom (تحت عكس)}} = -\rho\, d\rho\, d\phi \,.\, \hat{a}_z$ عكس



$$\boxed{\rho_2 > \rho_1}$$

$$dv = \rho\, d\rho\, d\phi\, dz .$$

(scalar).

2D - surfaces:

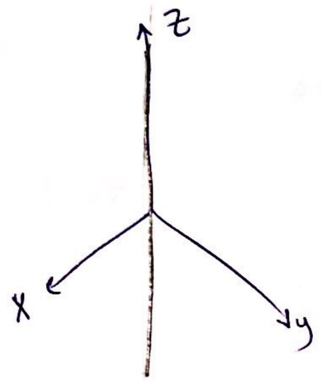
ρ = constant. → inf. hollow cylinder ($\rho \neq 0$).
→ inf. line along z-axis ($\rho = 0$).

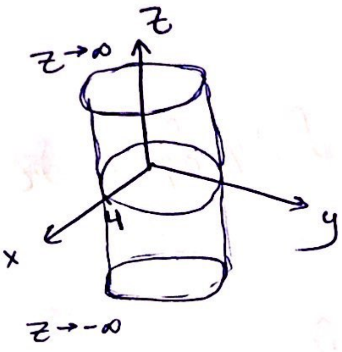
Ex:

إسطوانة فارغة.

$\rho = 4$. → inf hollow cylinder.

(4,0,0).

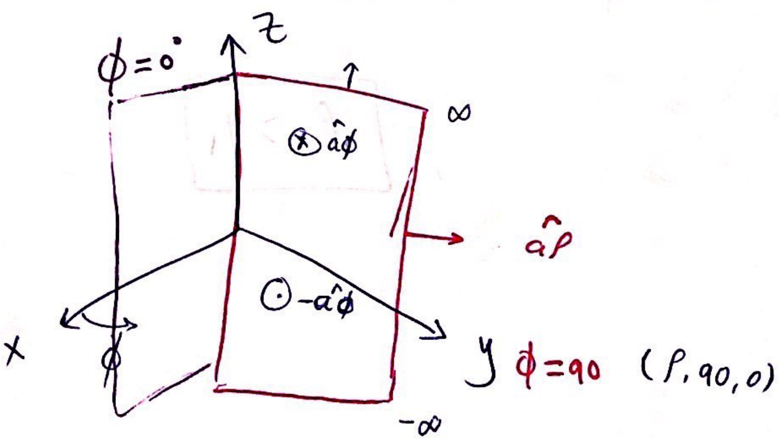




$0 < \phi < 2\pi$

ϕ = constant. → inf plane.

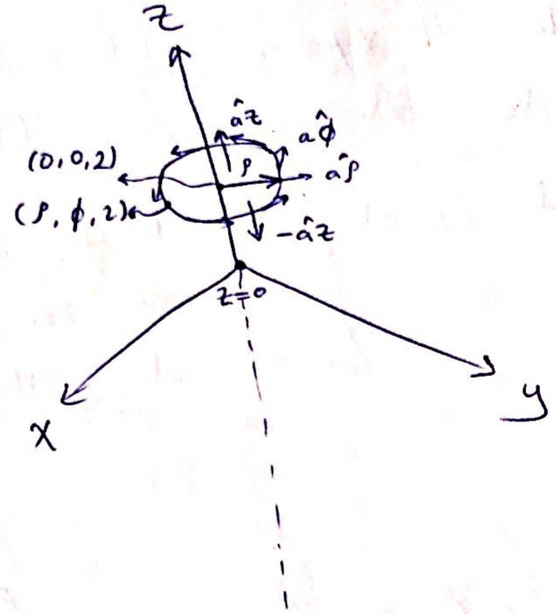
$\phi = 90°$ → inf. plane along yz - plane.



Z = constant → inf Disk parallel xy plane $(z \neq 0)$.
alone xy plane $(z = 0)$.

Ex:

$Z = 2$.



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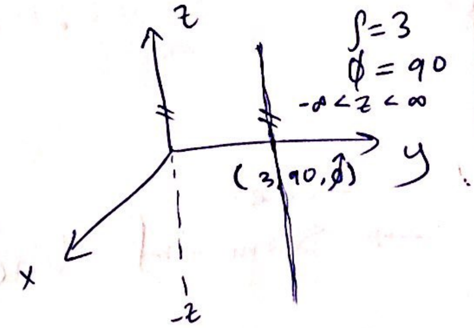
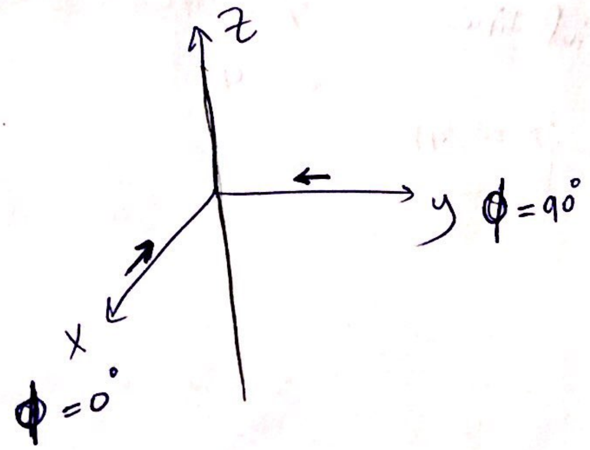
1 D segment :- ρ, ϕ are constants → inf. line ∥ z-axis $(\rho \neq 0)$.
→ inf. line along z-axis $(\rho = 0)$

Ex:

$\rho = 3$, $\phi = 90°$.



$\rho = 0$, $\phi = 0$.



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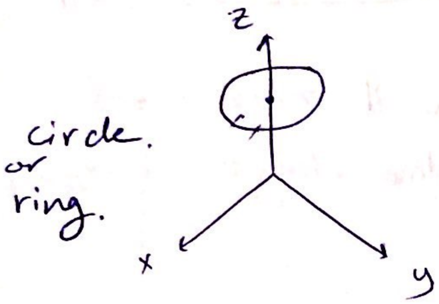
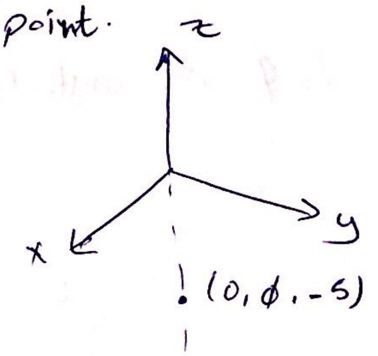
ρ, z are constants : $\rightarrow \bar{dl} = \rho \, d\phi \, \hat{a}_\phi$.

→ circle // xy plane ($\rho > 0$, $z \neq 0$).
→ circle along xy plane ($\rho > 0$, $z = 0$).
→ point ($\rho = 0$) → at the origin ($z = 0$)
→ at the +ve z axis ($z > 0$).
→ at the -ve z axis ($z < 0$).

EX: $\rho = 3$, $z = 2$.

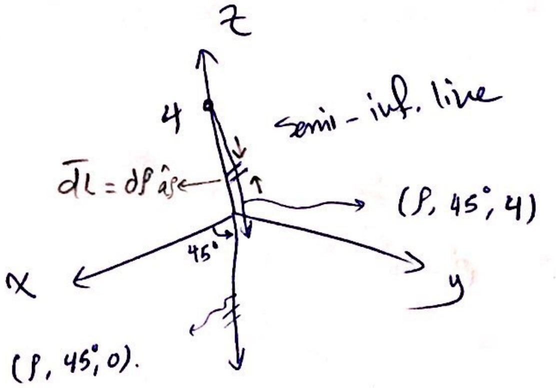


$\rho = 0$
$z = -5$.
point.



ϕ, z are constants → Semi-inf. line (ray). شعاع

نصف



$\phi = 45°$
$z = 4$.

lecture (5) : 11-2-2020

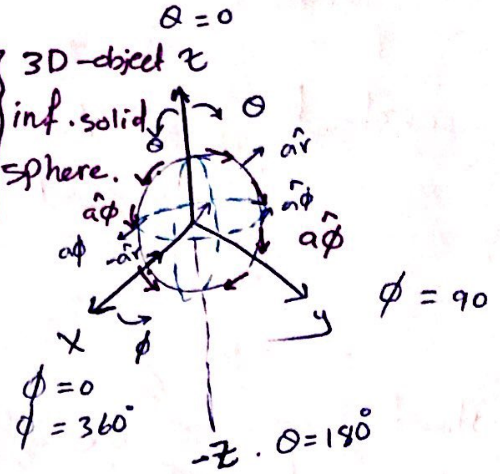
Spherical coordinates :

$0 \le r < \infty$

$0 < \theta < \pi \rightarrow -2\pi < \theta < \pi$ الحجم الفرق بينهم 180°.

$0 < \phi < 2\pi$ — الحجم الفرق بينهم 360°.

3D-object
inf. solid
sphere.



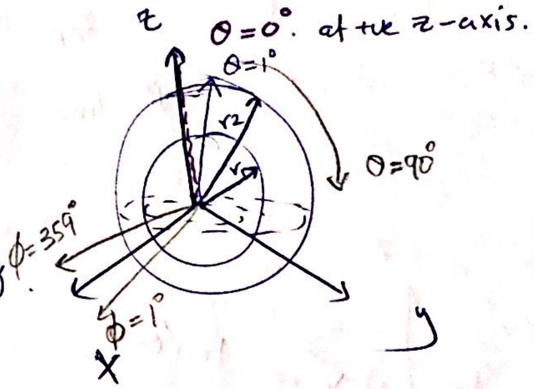
*unit vectors:

$\hat{a_r}$, $\hat{a_\theta}$, $\hat{a_\phi}$.

*Differential Elements:-

$\theta = 90^\circ$: at any point along xy-plane

dr, $r d\theta$, $r \sin\theta \, d\phi$



$r_2 > r_1$

* $\overline{dl} = dr\,\hat{a_r} + r d\theta\,\hat{a_\theta} + r\sin\theta\, d\phi\,\hat{a_\phi}$.

$\overline{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$.

shell
قشرة الكرة

نخبّي
الـ $\hat{a_r}$
ونضرب الباقي
ببعض.

$\overline{ds} = r \sin\theta \, dr \, d\phi \, \hat{a_\theta}$

horizontal
cut
قطع أفقي.

~~$\overline{ds}$ = r² sinθ dr dθ~~ .
vertical
~~cut~~

$\overline{ds} = r \, dr \, d\theta \, \hat{a_\phi}$
V. cut

volume:

$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$.

2D- Surfaces:-

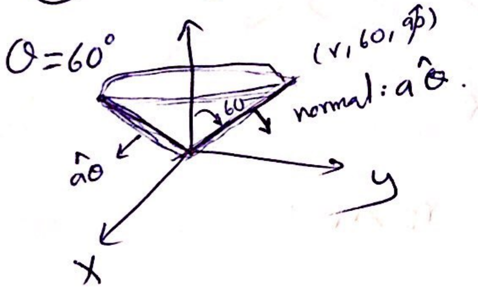
$r=$ constant → hollow sphere ($r \neq 0$).
→ point at origin ($r=0$).

$\theta =$ constant.
$\theta = 90^\circ$ special case.

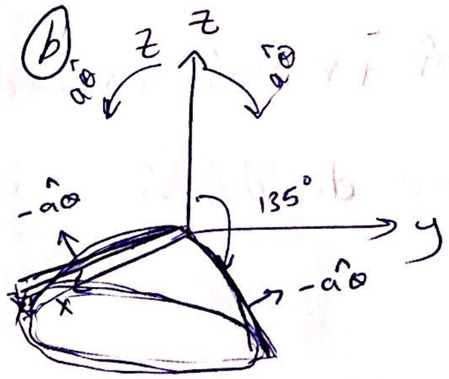
نستخدم ds تاعت
ال $\hat{a_\theta}$
لما يكون ثابت θ
دهكـ ا:

→ (a) $0 <$ any value in the range $(\theta) < 90^\circ$: inf. hollow cone around +ve z-axis.
→ (b) $90^\circ < \theta < 180^\circ$; inf hollow cone around -ve z-axis
→ inf. disk along xy-plane ($\theta = 90^\circ$).
→ line +ve z-axis ($\theta = 0^\circ$)
→ line -ve z-axis ($\theta = 180^\circ$).

(a)

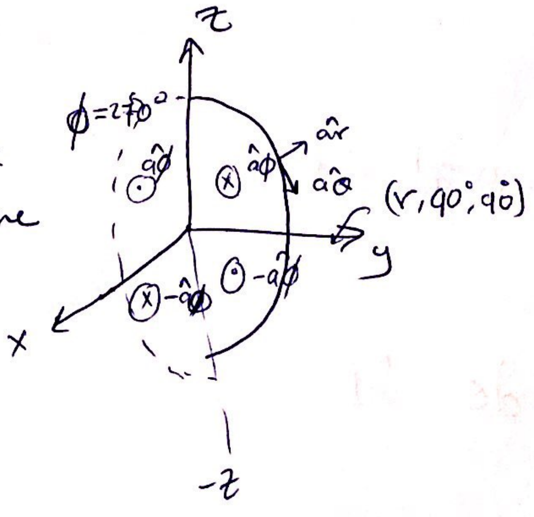


(b)



ϕ constant : → semiinf disk.
$\phi = 90^\circ$: → semi-inf disk along yz plane

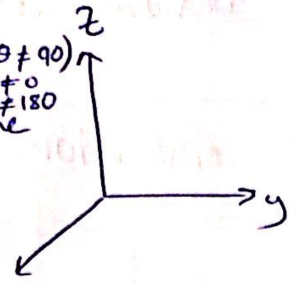
$\boxed{ds \text{ v.cut.}}$



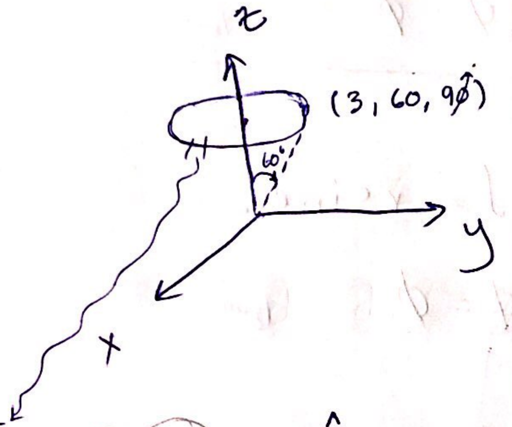
1D_segment:

⊛ r, θ are constant:
- → circle parallel to xy-plane if ($r \neq 0, \theta \neq 90$), $\theta \neq 0$, $\theta \neq 180$
- → circle along xy-plane ($r \neq 0$, $\theta = 90$)
- → point at origin ($r = 0$)
- → point at +ve z-axis ($r > 0$, $\theta = 0°$)
- → point at −ve z-axis ($r > 0$, $\theta = 180°$)

Ex:
$r = 3$, $\theta = 60°$

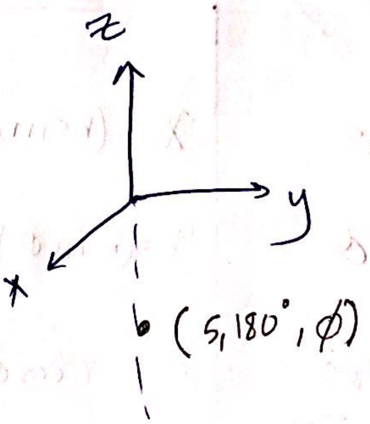


$r = 3$, $\theta = 60°$,



$$\bar{dL} = \underbrace{r}_{3}\underbrace{\sin\theta}_{\frac{\sqrt{3}}{2}}\, d\phi\, \hat{a_\phi}$$

$r = 5$, $\theta = 180°$



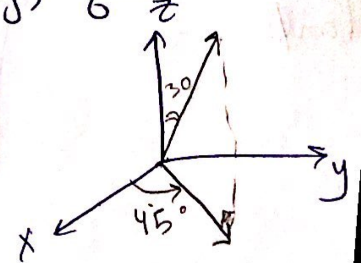
⊛ r, ϕ are constants:
- → half circle ($r \neq 0$)
- → point ($r = 0$)

⊛ θ, ϕ are constants: → semi-inf line (ray)

+ve x-axis → $\theta = 90°$
→ $\phi = 0°$

+ve z-axis → $\theta = 0°$, ϕ any value.

$\theta = 30$, $\phi = 45°$



lecture (6) :- 13 - 2 - 2020

Convert between coordinates :-

* Points conversion:

cart ⇄ cyl (x,y,z) → (ρ,ϕ,z)	cart ⇄ sph (x,y,z) → (r,θ,ϕ)	cyl ⇄ sph (ρ,ϕ,z) → (r,θ,ϕ)
$\rho = \sqrt{x^2+y^2}$	$r = \sqrt{x^2+y^2+z^2}$.	$r = \sqrt{\rho^2+z^2}$.
$\phi = \tan^{-1}\left(\frac{y}{x}\right)$	$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$	$\theta = \tan^{-1}\left(\frac{\rho}{z}\right)$.
$z = z$ نفسها	$\phi = \tan^{-1}\left(\frac{y}{x}\right)$.	$\phi = \phi$ نفسها
$x = \rho\cos\theta$	$x = (r\sin\theta)\cos\phi$	$\rho = r\sin\theta$.
$y = \rho\sin\theta$	$y = (r\sin\theta)\sin\phi$	$\phi = \phi$ نفسها
$z = z$ نفسها لا تتغير	$z = r\cos\theta$	$z = r\cos\theta$.

$$\sin\phi = \frac{y}{\rho}$$

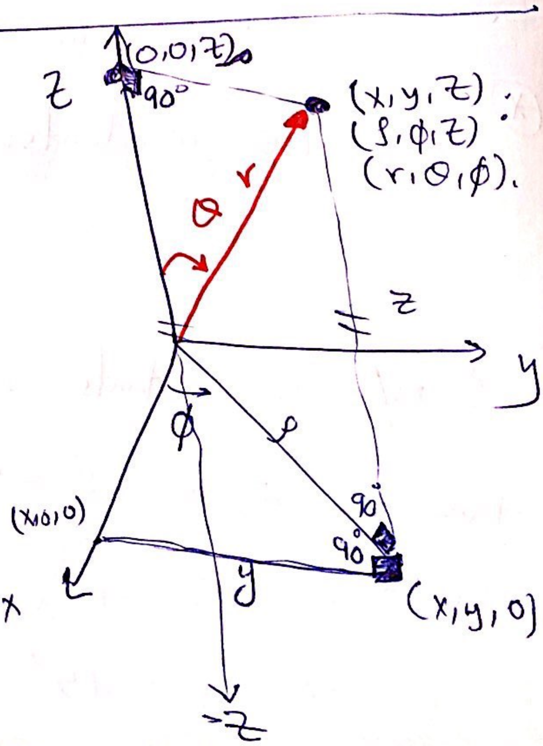
$$\cos\phi = \frac{x}{\rho}$$

$$\tan\phi = \frac{y}{x}$$

$$\sin\theta = \frac{\rho}{r}$$

$$\cos\theta = \frac{z}{r}$$

$$\tan\theta = \frac{\rho}{z}$$



* Vector conversion:-

Cart $\rightarrow$ cyl :-

$\bar{A} = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$.

$\bar{A} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}$.

وهكذا
حسب
cyl , sph

اشتق

$$\underset{3\times1}{\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}} = \underset{3\times3}{\begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}}$$

given in the exam

لو كان
بدلالة
$x^2 = \ldots$

$xy = \ldots$
بدلالة

co
cy
sp

* Two steps to convert any vector:-

(1)

$A_\rho = A_x \cos\phi + A_y \sin\phi$.

ضرب
matrices.
diff عادي

$A_\phi = -A_x \sin\phi + A_y \cos\phi$.

$A_z = A_z$. ← cyl *

(2)

we know that: $x = \rho\cos\phi$, $y = \rho\sin\phi$.

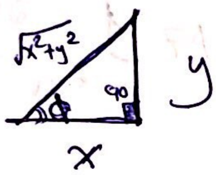
Cyl $\longrightarrow$ cart:-

$(A_\rho, A_\phi, A_z) \longrightarrow (A_x, A_y, A_z)$.

تبديل مواقع.

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\cos\phi = \cos\left(\tan^{-1}\left(\frac{y}{x}\right)\right)$$



$\tan\phi = \frac{y}{x}$.

$\cos\phi = \frac{x}{\sqrt{x^2+y^2}}$

// ———— // ———— //

Cart $\rightarrow$ sph :- $\begin{bmatrix} \hat{a}_r \\ \hat{a}_\theta \\ \hat{a}_\phi \end{bmatrix}$ and $\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}$ يمكن تبديلهم بال unit vectors

$(A_x, A_y, A_z) \rightarrow (A_r, A_\theta, A_\phi)$.

given

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

① step①

$\hat{a}_r = \sin\theta\cos\phi\,\hat{a}_x + \sin\theta\sin\phi\,\hat{a}_y + \cos\theta\,\hat{a}_z$.

We know that :-

② step ③ $x = (r\sin\theta)\cos\phi$.

$y = (r\sin\theta)\sin\phi$

cyl $\longrightarrow$ sph :- from kraus book.

$(A_\rho, A_\phi, A_z) \rightarrow (A_r, A_\theta, A_\phi)$.

given

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}.$$

we know that

$$\boxed{A_\phi = A_\phi}$$

// ——— // ——— //

lecture (7): 16-2-2020

CH.3 :- Line Integral.

تكامل على مسار . path. مسار

$$\int_l \bar{A} \cdot \bar{dl}$$

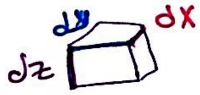
$\bar{A}$ → (A_x, A_y, A_z).

$\bar{dl}$:

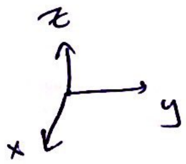
- Cart: $dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$.
- cyl: $d\rho\,\hat{a}_\rho + \rho\, d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$.
- sph: $dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\, d\phi\,\hat{a}_\phi$.

$$\oint_l \bar{A} \cdot \bar{dl}$$: closed line Integral.

تكامل على مسار مغلق. ~~[illegible]~~. loop. مسار مغلق



$$\int_x A_x\, dx + \int_y A_y\, dy + \int_z A_z\, dz.$$



*

lecture (7): 16-2-2020

CH.3 :

Line Integral. تكامل على طريق.
path. مسار

$$\int_{l} \bar{A} \cdot \bar{dl}$$

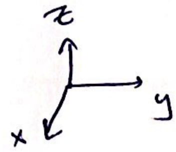
$\bar{A} \to (A_x, A_y, A_z)$.

- cart: $dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$.
- cyl: $d\rho\,\hat{a_\rho} + \rho\,d\phi\,\hat{a_\phi} + dz\,\hat{a_z}$.
- sph: $dr\,\hat{a_r} + r\,d\theta\,\hat{a_\theta} + r\sin\theta\,d\phi\,\hat{a_\phi}$.

$\oint_{l} \bar{A} \cdot \bar{dl}$: closed line Integral. تكامل على مسار مغلق.

loop. مسار مغلق

$$\int_{x} A_x\,dx + \int_{y} A_y\,dy + \int_{z} A_z\,dz.$$



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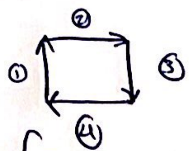
* line Integral:



$$\oint_{l} \bar{A} \cdot \bar{dl} = \int_{l_1} \bar{A} \cdot \bar{dl} + \int_{l_2} \bar{A} \cdot \bar{dl} + \int_{l_3} \bar{A} \cdot \bar{dl} + \int_{l_4} \bar{A} \cdot \bar{dl}.$$

$\bar{A} \to (\cancel{A_x}, \cancel{A_y}, A_z)$, $\bar{dl} \to dz\,\hat{a_z}$

dot product

العمودي ← cos 90 ← صفر

* Surface Integral: (double)

$\int_{s} \bar{A} \cdot \bar{ds}$. on front face . $\bar{ds}_{front} = dy\,dz\,\hat{a_x}$

عرية

$\bar{A} = (A_x, \cancel{A_y}, \cancel{A_z})$

$$\iint A_x \cdot dy\,dz$$

Surface

* ~~Closed Integral~~

* Closed surface Integral :-

$\bar{ds} = \rho \, d\rho \, d\phi \, \hat{a_z}$

$\bar{ds} = \rho \, d\phi \, dz \, \hat{a_\rho}$

$\bar{ds} = -\rho \, d\rho \, d\phi \, \hat{a_z}$

$0 \le \phi \le 2\pi$

$0 \le \rho \le 3$

$0 \le z \le 5$

$$\oint_S \bar{A} \cdot \bar{ds} = \int_{s_1} \bar{A} \cdot \bar{ds_1} + \int_{s_2} \bar{A} \cdot \bar{ds_2} + \int_{s_3} \bar{A} \cdot \bar{ds_3}$$

* Volume Integral :- (Triple) :-

$$\int_V \underbrace{|\bar{A}|}_{\text{scalar}} \cdot \underbrace{dV}_{\text{scalar}} = \text{Scalar.}$$

// ——————————— // ——————————— //

Del operator : (vector) :

$$\text{cart. } \nabla = \frac{\partial}{\partial x} \hat{a_x} + \frac{\partial}{\partial y} \hat{a_y} + \frac{\partial}{\partial z} \hat{a_z}.$$

$$\text{cyl. } \nabla = \frac{\partial}{\partial \rho} \hat{a_\rho} + \frac{\partial}{\rho \partial \phi} \hat{a_\phi} + \frac{\partial}{\partial z} \hat{a_z}.$$

$$\text{sph. } \nabla = \frac{\partial}{\partial r} \hat{a_r} + \frac{\partial}{r \partial \theta} \hat{a_\theta} + \frac{\partial}{r \sin\theta \, \partial \phi} \hat{a_\phi}.$$

* usage of del operator:-

1 Gradient (vector):-

$\nabla V \equiv$ gradient of V.

Scalar

2 Divergence (scalar):-

$\nabla \cdot \bar{A} \equiv$ divergence of $\bar{A}$.

3 carl (vector):-

$\nabla \times \bar{A} \equiv$ carl of $\bar{A}$.

4 $\nabla^2 V$ $\equiv$ laplacian of V.

$\nabla \cdot (\nabla V) = (\nabla \cdot \nabla) V$.

* in cartesian coordinates:-

$$\nabla V = \frac{\partial V}{\partial x} a\hat{x} + \frac{\partial V}{\partial y} a\hat{y} + \frac{\partial V}{\partial z} a\hat{z}.$$

* in cylidrical coordinates:-

$$\nabla V = \frac{\partial V}{\partial \rho} a\hat{\rho} + \frac{\partial V}{\rho \partial \phi} a\hat{\phi} + \frac{\partial V}{\partial z} a\hat{z}.$$

* in spherical coordinates:-

$$\nabla R = \frac{\partial R}{\partial r} a\hat{r} + \frac{\partial R}{r \partial \theta} a\hat{\theta} + \frac{\partial R}{r \sin\theta \, \partial \phi} a\hat{\phi}.$$

* Ex:-

$$V = x^2 y e^{-z}.$$

$$\nabla V = 2xye^{-z} a\hat{x} + x^2 e^{-z} a\hat{y} + -e^{-z} \cdot x^2 y \, a\hat{z}.$$

$$\nabla V \Big|_{(2,3,0)}$$

(x, y, z)

تعويض مباشر

Divergence :

$\boxed{*}$ $\nabla \cdot \bar{A} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (A_x, A_y, A_z)$.

$\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$. # in cart.

---

voh component (ρ dolos)

$\boxed{*}$ $\nabla \cdot \bar{A} = \frac{1}{\rho}\left(\frac{\partial \rho A_\rho}{\partial \rho}\right) + \frac{\partial A_\phi}{\rho \partial \phi} + \frac{\partial A_z}{\partial z}$. ← given

in cyl.

← given

$\boxed{*}$ $\nabla \cdot \bar{T} = \frac{1}{r^2}\left(\frac{\partial r^2 T_r}{\partial r}\right) + \frac{1}{r \sin\theta}\left(\frac{\partial \sin\theta T_\theta}{\partial \theta}\right) + \frac{\partial T_\phi}{r \sin\phi \partial \phi}$. # in sph.

Ex: $\bar{A} = 3x^2 y \, a\hat{x} + 25e^{-z} \, a\hat{z}$.

$\nabla \cdot \bar{A} = 6xy + 0 - 25e^{-z}$.

Curl :-

* in cart :- (not given).

$$\nabla \times \bar{A} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

* in cyl :- (given).

$$\nabla \times \bar{A} = \frac{1}{\rho} \begin{vmatrix} \hat{a_\rho} & \rho \hat{a_\phi} & \hat{a_z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

* in sph :- (given).

$$\nabla \times \bar{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a_r} & r\hat{a_\theta} & r\sin\theta\, \hat{a_\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{vmatrix}$$

Divergence Theorem:

$$\oint_S A \cdot \overline{ds} = \int_V \nabla \cdot \bar{A} \cdot dv$$

$\longrightarrow$
$\longleftarrow$

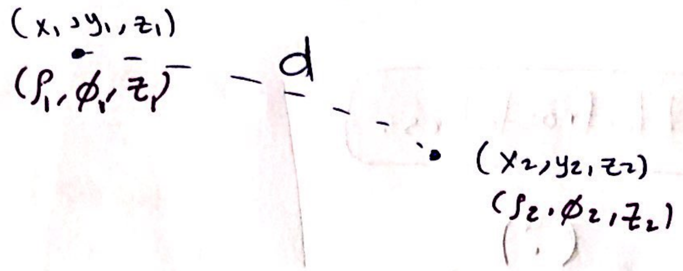
Stocke's Theorem:

$$\oint_l \bar{A} \cdot \overline{dl} = \int_S \nabla \times \bar{A} \cdot ds$$

$\longrightarrow$
$\longleftarrow$

2 - Directions

* Distance :-



in cart :

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2.$$

in cyl :

$$d^2 = \rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 \cos(\phi_2 - \phi_1) + (z_2 - z_1)^2$$

if $\phi_1 = \phi_2$

$$d^2 = (\rho_2 - \rho_1)^2 + (z_2 - z_1)^2.$$

in sph :

if $\phi_1 \neq \phi_2$ $\longrightarrow$ $d^2 = (r_2 - r_1)^2$

if $\theta_1 = \theta_2$

$\phi_1 = \phi_2$

$$d^2 = r_2^2 + r_1^2 - 2r_1r_2 \cos\theta_2 \cos\theta_1 - 2r_1r_2 \sin\theta_2 \sin\theta_1 \cos(\phi_2 - \phi_1)$$

lecture (8) : 18-2-2020

CH.4. Electrostatic Fields.

(*) Sources of Electrostatics:-

(1) point charge. ($\overset{Q}{\cdot}$)

(2) continuous charge distribution:-

1. line charge. (1D شكل segment).
2. surface charge. (2D surfaces).
3. Volume charge. (3D objects).

(3) Electric dipole. ($\oplus$ — $\ominus$)

(4) polarised dielectric.

(*) Fields:-

- Eletric Field Intensity ($\bar{E}$). قوة المجال الكهربائي
- Eletric Flux Density ($\bar{D}$). كثافة المجال الكهربائي

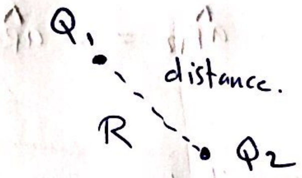
* Major laws:-

* [1] coulomb's law. (general).
* [2] Gauss's law. (special case).

---

[*] coulomb's law:-

$F \propto \frac{Q_1 Q_2}{R^2}$ (relation).



$$F = \frac{k Q_1 Q_2}{R^2} \quad (N).$$

* $k = \frac{1}{4\pi\epsilon_0}$.
  - → unit used
  - → Type of media surrounding the charges. → الوسط المحيط (ϵ_0)

$k = 9*10^9$.

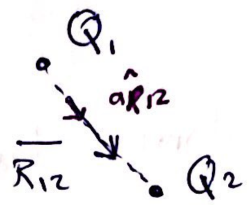
* ϵ_0 : free space.

$\epsilon_0 = \frac{10^{-9}}{36\pi}$ F/m $= 8.858 * 10^{-12}$ F/m.

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R^2} \quad (N)$$ Electric Force. (vector quantity).

F ↓ magnitude only.

The direction of forces:-



* $F_{12} \equiv$ The force on Q_2 due to Q_1

1) $$\bar{F}_{12} = \frac{\overset{\text{scalar}}{Q_1}\ \overset{\text{scalar.}}{Q_2}}{4\pi\epsilon_\circ (\bar{R}_{12})^2}\ \hat{a}_{R_{12}}.$$

vector

scalar يصير

وتنتج unit vector

$\bar{F}_{21} = -\bar{F}_{12} \rightarrow$ because $\hat{a}_{R_{12}} = -\hat{a}_{R_{21}}$ $\hat{a}_{R_{12}} = \dfrac{\bar{R}_{12}}{|\bar{R}_{12}|}.$

$|\bar{F}_{21}| = |\bar{F}_{12}| \rightarrow$ as a magnitude.

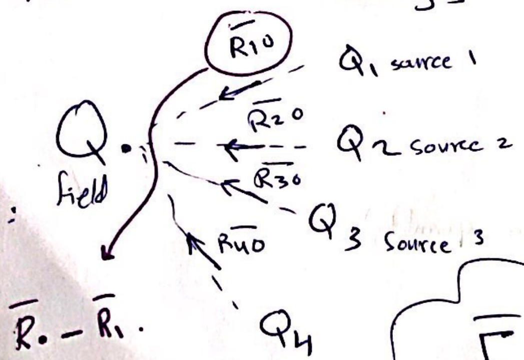
2) $$\bar{F} = \frac{Q_1 Q_2 \overbrace{\bar{R}_{12}}^{\hat{a}_{R_{12}}}}{4\pi\epsilon_\circ |R_{12}|^3}\ (N) \qquad \bar{R}_{12} = \bar{r}_2 - \bar{r}_1 .$$

location of Q_2 ; location of Q_1

3) $$\bar{F} = \frac{Q_1 Q_2 (\bar{r}_2 - \bar{r}_1)}{4\pi\epsilon_\circ |\bar{r}_2 - \bar{r}_1|^3}\ (N).$$

(magnitude)^3.

For N-point charges:-



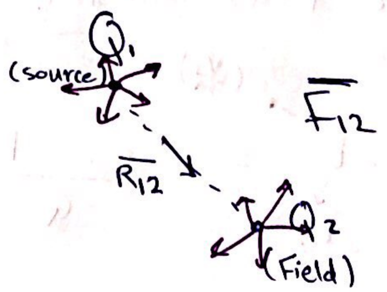
$$\bar{F}_\circ = \frac{Q_1 Q_\circ (\bar{r}_\circ - \bar{r}_1)}{4\pi\epsilon_\circ |\bar{r}_\circ - \bar{r}_1|^3} + \frac{Q_2 Q_\circ (\bar{r}_\circ - \bar{r}_2)}{4\pi\epsilon_\circ |\bar{r}_\circ - \bar{r}_2|^3} + \text{———} \frac{Q_N Q_\circ (\bar{r}_\circ - \bar{r}_N)}{4\pi\epsilon_\circ |\bar{r}_\circ - \bar{r}_N|^3}$$

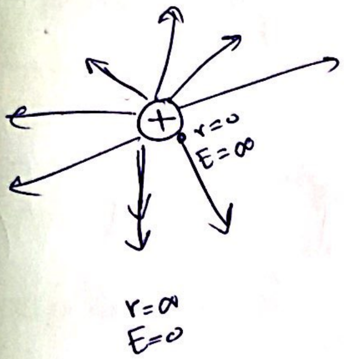
$$\boxed{\bar{F}_\circ = \frac{Q_\circ}{4\pi\epsilon_\circ} \sum_{K=1}^{N} \frac{Q_K (\bar{r}_\circ - \bar{r}_K)}{|\bar{r}_\circ - \bar{r}_K|^3}\ (N).}$$

* Electric Field Intinsity : ($\bar{E}$)

$$\bar{E} = \frac{\bar{F}}{Q_{(field)}} = \left(\frac{N}{C}\right)$$

المتأثرة





دائماً المسافة R هي ال $Q_{field} - Q_{source}$

يعني دائماً ال $R_{field} - R_{source}$

وذلك لأنه $R_{field} - R_{source}$ يعني أن Q_{source} هي

التي تؤثر بقوة المجال ($\bar{E}$) على ال Q_{field}.

وصورة اشتقاقه

$$① \quad \bar{E} = \frac{Q_{source}}{4\pi\epsilon_\circ R^2}\,\hat{a_r}$$

$$② \quad \bar{E} = \frac{Q\,\bar{r}_{(vector)}}{4\pi\epsilon_\circ r^3_{(magnitude)^3}}$$

$$③ \quad \bar{E} = \frac{Q(\bar{r} - \bar{r}')}{4\pi\epsilon_\circ |\bar{r} - \bar{r}'|^3}$$

magnitude

Convention : . اصطلاحاً

شحطة عليه مش مشتقة.

r : for field charge

r' : for source charge.

Use dashes with a source and without dashes with a field charge

For N- charges :-

$$\bar{E} = \frac{1}{4\pi\epsilon_\circ}\sum_{k=1}^{N}\frac{Q_k(\bar{r} - r_k')}{|\bar{r} - r_k'|^3} \; (N/C) \text{ or } (V/m).$$

---

* Electric Flux density:- ($\bar{D}$)

⊛ $\bar{D}$ unit : C/m^2 $\boxed{\text{coulomb}/(\text{meter})^2}$.

$$\bar{D} = \underset{(\frac{F}{m})}{\epsilon_o} \underset{(\frac{N}{C})}{\overset{(\frac{V}{m})}{\bar{E}}} = \frac{\epsilon_o \bar{F}}{Q}$$

⊛ CH.6 : $C = \frac{Q}{V}$, $F = \frac{C}{V}$

* for point charge:-

$$\bar{F}_{12} = \frac{Q_1 Q_2 (\bar{r}_2 - \bar{r}_1)}{4\pi\epsilon_o |\bar{r}_2 - \bar{r}_1|^3}.$$

$$\bar{E} = \frac{Q}{4\pi\epsilon_o r^2} \hat{a_r}.$$

$$\bar{D} = \frac{Q}{4\pi r^2} \hat{a_r}.$$ ⟹ will not affect by media.
الوسط المحيط.

Ex:- Two point charges (1mC and -2mC) located in $\underset{\text{source 1}}{(3,2,-1)}$ and $\underset{\text{source 2}}{(-1,-1,4)}$. Find $\bar{F}, \bar{E}, \bar{D}$ at a (10nC) charge located $\overset{\text{field.}}{(0,3,1)}$. all in Free space. ← by default.

$$\bar{F} = \frac{1*10^{-3} * 10*\cancel{10^{-9}} \overset{r_2 - r_1}{(-3,1,2)}}{\cancel{4\pi} * \frac{\cancel{10^{-9}}}{9\,\cancel{36\pi}} * \underset{\text{magnitude}}{(14)^{\frac{3}{2}}}} + \frac{-2*10^{-3} * 10*\cancel{10^{-9}} (1,4,-3)}{\cancel{4\pi} * \frac{\cancel{10^{-9}}}{9\,\cancel{36\pi}} * (26)^{\frac{3}{2}}}$$

$$*\bar{F} = -6.507\,\hat{a_x} - 3.817\,\hat{a_y} + 7.566\,\hat{a_z} \quad mN.$$

* $\bar{E} = \frac{\bar{F}}{Q_{field}} = \frac{\bar{F}}{10\times10^{-9}} = 650.7\,\hat{a_x} - 381.7\,\hat{a_y} + 750.6\,\hat{a_z}$ KN

* $\bar{D} = \epsilon \cdot \bar{E} = \frac{10^{-9}}{36\pi} \cdot \bar{E} = \ldots$

" ——————— " ——————— "

lecture (9) :- 20-2-2020

توزيع شحنة خطي.

Ex:- Line charge dist. ⇒ Exist on 1-D segment. Q ++++ ρL (c/m) توزيع الشحنات خطي

شحنة

consider a finite line along Z-axis carry a charge

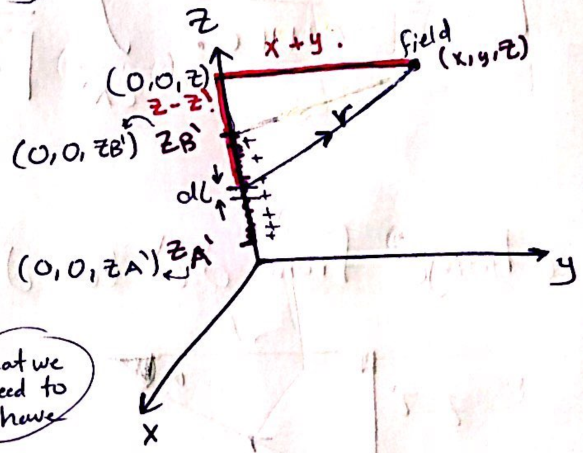
ρL (c/m) find $\bar{E}$ and $\bar{D}$ at point (x, y, z) :- ρL: الشحنة لكل مقطع

الشحنة الكلية = عدد الشحنات * الطول

$$\int dQ = \int \rho L\, dl \Rightarrow Q = \int_l \rho L . dl$$

from the unit we know it's charge (Q) ← $\frac{\text{columb}}{\cancel{\text{meter}}} * \cancel{\text{meter}}$

(0, 0, z') source ال



$$\bar{E} = \frac{Q}{4\pi\epsilon_\circ r^2}\hat{a_r} = \int \frac{\rho L\, dl}{4\pi\epsilon_\circ r^2}\hat{a_r}$$

point charge

what we need to have

→ convert to cylindrical (أسهل)

$dl = dz'$ → along السلك أو Z-axis only

$\bar{r}$ as a vector $= x\hat{a_x} + y\hat{a_y} + (z - z')\hat{a_z}$

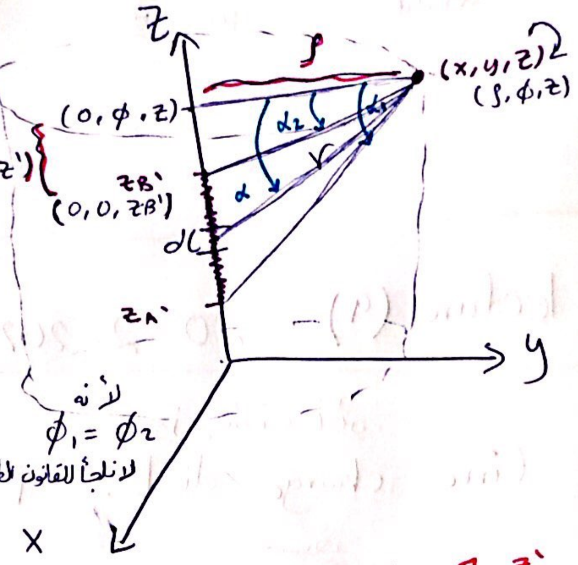
$|r|$ as a magnitude $= \sqrt{x^2 + y^2 + (z-z')^2}$

⇒ $$\bar{E} = \frac{\rho L}{4\pi\epsilon_\circ} \int_{Z_{A'}}^{Z_{B'}} \frac{(x, y, z - z')\, . dz'}{\left[x^2 + y^2 + (z-z')^2\right]^{\frac{3}{2}}}$$

constants

* Line segment ⇒ cylindrical coordinates
أفضل للتكامل!

* convert to cylindrical coordinates:-



in cyl.

$$\boxed{\begin{aligned} dl &= dz' \\ \bar{r} &= \rho \hat{a}_\rho + (z - z')\hat{a}_z \\ |r| &= \sqrt{\rho^2 + (z-z')^2} \end{aligned}}$$

because $\underset{\text{cart}}{z} = \underset{\text{cyl}}{z}$

$\longrightarrow$ لأنه $\phi_1 = \phi_2$ لا نلجأ للقانون الطويل

$\sin\alpha = \frac{z - z'}{r}$

$\cos\alpha = \frac{\rho}{r}$

$\tan\alpha = \frac{z - z'}{\rho}$

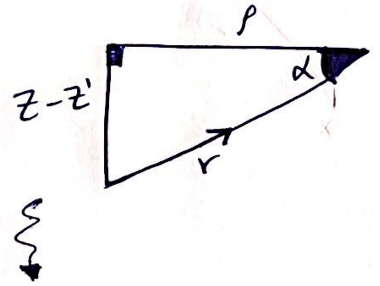
$\Downarrow$

$\boxed{z - z' = \rho \tan\alpha}$

$dz' = -\rho \sec^2\alpha \, d\alpha$

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0} \int_{z_A'}^{z_B'} \frac{\rho \hat{a}_\rho + (z - z')\hat{a}_z}{\left[\rho^2 + (z-z')^2\right]^{\frac{3}{2}}} \cdot dz'$$

Change from dz' to $d\alpha$.



$r^2 = \rho^2 + (z - z')^2$

$r^2 = \rho^2 (1 + \tan^2\alpha)$

$r^2 = \rho^2 \sec^2\alpha$.

$r^3 = \rho^3 \sec^3\alpha$.

* $z - z' = r \sin\alpha$.
* $\rho = r\cos\alpha$

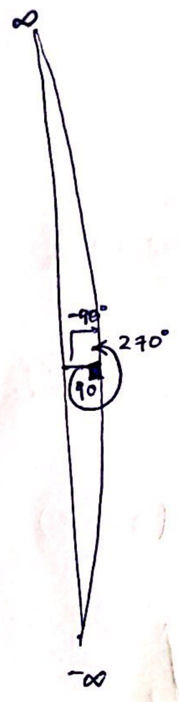
$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0} \int_{\alpha_1}^{\alpha_2} \frac{\rho \sec\alpha \,(\cos\alpha \, \hat{a}_\rho + \sin\alpha \, \hat{a}_z)}{\rho^3 \sec^3\alpha} (-\rho \sec^2\alpha \, d\alpha).$$

$$\bar{E} = \frac{-\rho_L}{4\pi\epsilon_0 \rho} \left[(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\rho - (\cos\alpha_2 - \cos\alpha_1)\hat{a}_z \right]$$

for an infinite-line :-

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon_\circ \rho} \hat{a}_\rho .$$ # always for any inf. line

ρ ≡ shortest distance between the source and the field.



$\alpha_1 = 90°$

$\alpha_2 = -90°$ or $270°$

نعوّض بالقانون
السابق بطلع
قانون جديد
ثابت وهو .

// ——— // ——— //

lecture (10) :- 23-2-2020

lecture (10) 23-2-2020

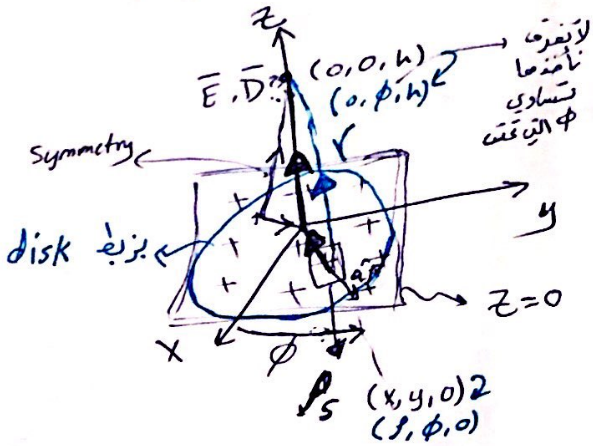
$\bar{E}$ - field for a surface charge dist :- توزيع الشحنة على سطح (مساحة)

* consider a plane $z = 0$ (infinite) carry charge ρ_s (c/m^2) .

Find $\bar{E}$ and $\bar{D}$ at $(0,0,h)$.

$$\bar{E} = \frac{Q}{4\pi\epsilon_\circ r^2}\,\hat{a}_r$$

$$Q = \int \rho_s \cdot ds$$



$$\bar{E} = \int \frac{\rho_s \cdot ds}{4\pi\epsilon_\circ \cdot r^2}\cdot\hat{a}_r$$

$$\bar{E} = \int \frac{\rho_s \cdot ds\,\bar{r}}{4\pi\epsilon_\circ \cdot r^3}$$

* $ds = \rho\, d\rho\, d\phi$

* $\bar{r}$ (vector) $=$ field $-$ source

$$\bar{r} = -\rho\hat{a}_\rho + h\hat{a}_z$$

* r (magnitude) $= \sqrt{\rho^2 + h^2}$.

$$\bar{E} = \frac{\rho_s}{4\pi\epsilon_\circ}\int_0^{2\pi}\int_0^{\infty} \frac{-\rho\hat{a}_\rho + h\hat{a}_z}{[\rho^2+h^2]^{\frac{3}{2}}}\cdot\rho\, d\rho\, d\phi$$

due to symmetry , the ρ component will be cancelled.

single Int $\leftarrow$ double Int

$$\bar{E} = \frac{\rho_s \cdot h}{2\epsilon_\circ}\int_0^{\infty}\frac{\hat{a}_z}{[\rho^2+h^2]^{\frac{3}{2}}}\,\rho\, d\rho$$

let $u = \rho^2 + h^2$

* تكامل بالتعويض .

$du = 2\rho \, d\rho$

$\rho \, d\rho = \frac{du}{2}$

$$\bar{E} = \frac{\rho_s h}{2(2\epsilon_0)} \int \frac{du}{u^{3/2}} \cdot \hat{a}_z$$

$$\bar{E} = \frac{\rho_s h}{2 \cdot 4\epsilon_0} \cdot \frac{u^{-\frac{1}{2}}}{-\frac{1}{2}} \Big|_{?}^{?} \hat{a}_z$$

عوضنا $\frac{1}{\infty}$, $\frac{1}{h}$

$$\bar{E} = \frac{-\rho_s h}{2\epsilon_0} \cdot \frac{1}{\sqrt{\rho^2+h^2}} \Big|_{0}^{\infty} \hat{a}_z \longrightarrow \frac{-\rho_s h}{2\epsilon_0}\left(0 - \frac{1}{h}\right)\hat{a}_z$$

$$\boxed{\Rightarrow \bar{E} = \frac{\rho_s}{2\epsilon_0} \underbrace{\hat{a}_z}_{\hat{a}_n} \; V/m.} \; \#$$

$\hat{a}_z$ $\hat{a}_\rho$

unit vector العمودي على ال sheet

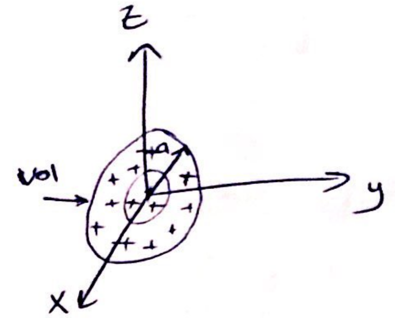
* In general for any infinite sheet the law is

If the point is $(0,0,-h) \rightarrow$ بجيب العمودي $-\hat{a}_z$

$-\hat{a}_z$

$$* \; \bar{D} = \frac{\rho_s \,(c/m^2)}{2} \hat{a}_n \quad c/m^2$$

* for ρ Volume charge distribution:-



$$Q = \int_V \rho_v \, dv.$$

* coloumb's law ✗,
* Gauss's law ✓.

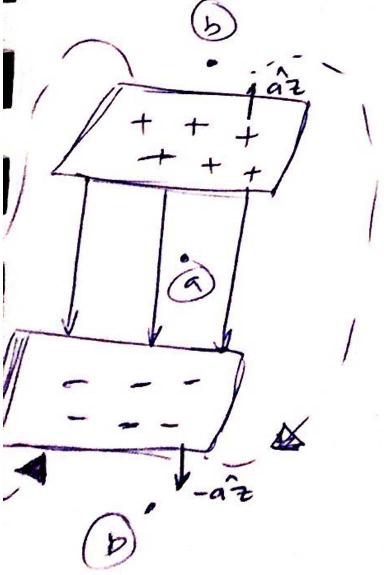
$$\bar{E} = \int \frac{\rho_v \, . \, dv}{4\pi\epsilon \, . \, r^2} \, \hat{a_r}$$

---

* Ex: parallel plate capacitor. ⊛ infinite sheets:-

find $\bar{D}$ at points : ⓐ between the plates.

ⓑ outside the plates.



at point ⓐ $\bar{D} = \bar{D}_+ + \bar{D}_-$

$$\bar{D} = \frac{+\rho_s}{2}(-\hat{a_z}) + \frac{-\rho_s}{2}(\hat{a_z})$$

$$\bar{D} = \rho_s(-\hat{a_z})$$

$$\bar{D} = \rho_s \, \hat{a_n}$$

$$\boxed{\rho_s = \bar{D} \, \hat{a_n}}$$

لكي نحصل على المصدر ،

(المسبب) (الشحنة) من

الـ $\bar{D}$

lecture (11) :- 25-2-2020 :-

Gauss's law:-

$$\Psi = \oint_s \bar{D} \cdot \bar{ds} = Q_{enclosed}.$$

→ Q →: point charge

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 \cdot r^2} \hat{a_r}$$

→ $\int_l \rho_L \cdot dl$ →: infinite line.

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon \cdot \rho} \hat{a_\rho}$$

→ $\int_s \rho_s \, ds$ →: infinite sheet.

$$\bar{E} = \frac{\rho_s}{2\epsilon} \hat{a_r}$$

→ $\int_v \rho_v \cdot dv$ →: uniform volume.

$\bar{E} = ??$ Gauss's law.

Electric flux (Ψ)

$$\Psi = \int_s \bar{D} \cdot \bar{ds}$$

we know that

$$\boxed{\rho_s = \bar{D} \cdot \hat{a_n}}$$

$$\boxed{\oint_s \bar{D} \cdot ds = \int_V \rho_v \cdot dV}$$

Gauss's law

1st Maxwell equation integrated from.

$$\oint_s \bar{D} \cdot \bar{ds} = \int_V \nabla \cdot \bar{D} \, dv = \int_V \rho_v \cdot dv$$

$$\boxed{\nabla \cdot \bar{D} = \rho_v}$$

1st maxwell's equation in differential form.

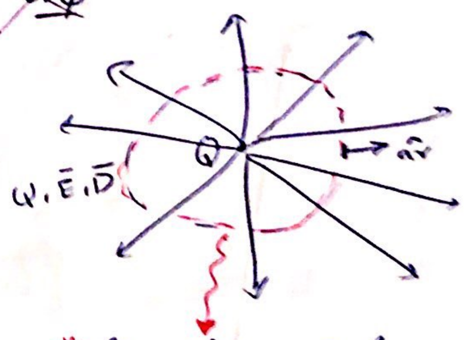
* Applications on Gauss's law:-

① $\bar{E}$ or $\bar{D}$ for a point charge.

$$\oint_s \bar{D} \cdot \bar{ds} = Q_{enclosed} = Q.$$

$$\bar{D} = D_r \hat{a_r} + \cancel{D_\theta \hat{a_\theta}} + \cancel{D_\phi \hat{a_\phi}}.$$

$$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}.$$



* Gaussian surface hollow sphere. *

$$\int_0^{2\pi}\int_0^{\pi} D_r . r^2 \sin\theta \, d\theta \, d\phi = Q$$

$$D_r \, r^2 (2)(2\pi) = Q$$

$$4\pi r^2 D_r = Q$$

$$D_r = \frac{Q}{4\pi r^2}$$

$$\bar{D} = \frac{Q}{4\pi r^2} \hat{a_r}$$

$$\bar{E} = \frac{\bar{D}}{\epsilon_0} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a_r}$$

$$\int_0^{\pi} \sin\theta \, d\theta = -\cos\theta \Big]_0^{\pi} = -(-1-1) = 2$$

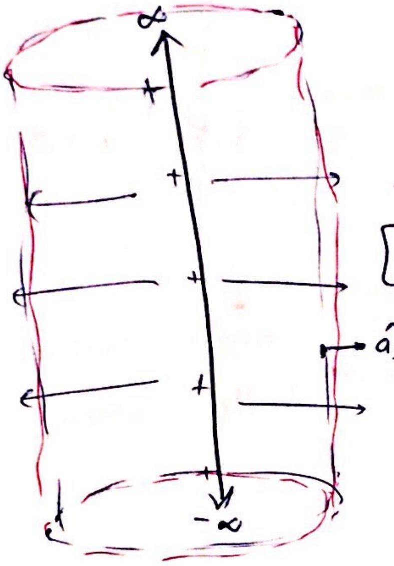
② $\bar{E}$ or $\bar{D}$ for an infinite line of charge:-

$$\oint_S \bar{D} \cdot \bar{ds} = Q_{enclosed} = \int_l \rho_L \, dl$$

$$\bar{D} = \bar{D}_\rho \hat{a_\rho} \quad , \quad dl = dz.$$

$$\bar{ds} = \rho \, d\phi \, dz \, \hat{a_\rho}.$$

$$\int_{-l \to -\infty}^{l \to \infty} \int_0^{2\pi} D_\rho \, \rho \, d\phi \, dz = \int_{-l}^{l} \rho_L \, dz.$$



$\boxed{l \to \infty}$

$\boxed{\text{area}}$

$$D_\rho \cdot \rho \, (2\pi)(\cancel{2l}) = \rho_L (\cancel{2l})$$

$$D_\rho = \frac{\rho_L}{2\pi\rho}$$

hollow cylinder.

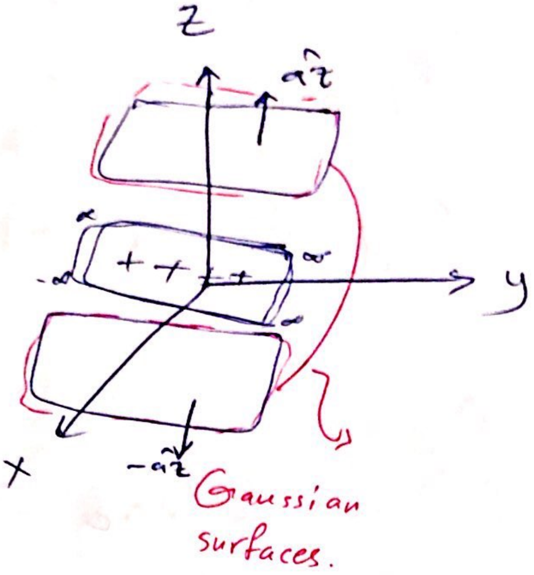
Gaussian surface.

$$\bar{D} = \frac{\rho_L}{2\pi\rho} \hat{a_\rho} \quad c/m^2.$$

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a_\rho} \quad V/m.$$

③ $\bar{E}$ or $\bar{D}$ for an infinite sheet:-

$$\oint_S \bar{D} \cdot \bar{ds} = Q_{enclosed} = \int_S \rho_s . ds$$



$$\bar{D} = \begin{cases} D_z \, \hat{az} , & z > 0 \\ D_z \, (-\hat{az}) , & z < 0 \end{cases}$$

$\bar{ds}_{top}$: $dx \, dy \, \hat{az}$, $z > 0$

$\bar{ds}_{bot}$: $-dx \, dy \, \hat{az}$, $z < 0$

$$\int_{s_{top}} \bar{D} \cdot \bar{ds}_{top} + \int_{s_{bot}} \bar{D} \cdot \bar{ds}_{Bot} = \int_S \rho_s . ds$$

$$\iint_{-\infty}^{\infty} D_z \, dx \, dy + \iint_{-\infty}^{\infty} D_z \, dx dy = \iint_{-\infty}^{\infty} \rho_s \, dx \, dy .$$

let $\iint_{-\infty}^{\infty} dx \, dy = A$.

$$\bar{E} = \frac{\rho_s}{2\epsilon .} \hat{an}$$

Then :- $D_z \cancel{A} + D_z \cancel{A} = \rho_s \cancel{A}$.

$2 D_z = \rho_s$

so, $\boxed{D_z = \frac{\rho_s}{2}}$ #

$$\bar{D} = \begin{cases} \frac{\rho_s}{2} \, \hat{az} , & z > 0 \\ \frac{\rho_s}{2} (-\hat{az}) , & z < 0 \end{cases} \qquad \bar{D} = \frac{\rho_s}{2} \hat{an} \begin{cases} \hat{az} \\ -\hat{az} \end{cases}$$

Scanned with CamScanner

④ $\bar{E}$ or $\bar{D}$ for a uniform volume charge:-

* consider a sphere of radius (a) has:-

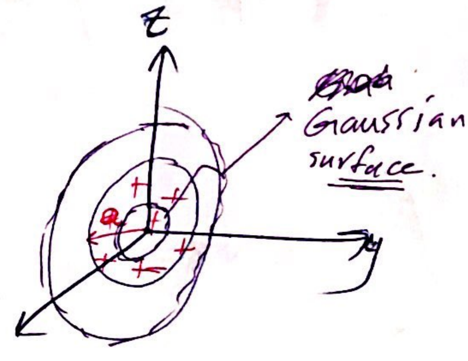
$$\rho_v = \begin{cases} \rho_o & , r<a \\ 0 & , r>a \end{cases}$$

find $\bar{E}$ and $\bar{D}$ $\boxed{\text{every where.}}$

hint:
لوحدو
استخدم
Gauss's law.

$$\oint_S \bar{D} \cdot \bar{ds} = Q_{enc} = \int_V \rho_v dv$$

$$\bar{D} = D_r \cdot \hat{a_r}$$
$$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}.$$
$$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$$



* for $0<r<a$:

$$\int_0^{2\pi}\int_0^{\pi} D_r r^2 \sin\theta \, d\theta \, d\phi = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho_o \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$D_r \, 4\pi r^2 = \rho_o \frac{r^3}{3} 4\pi$$

$$D_r = \frac{\rho_o r}{3}$$

$$\boxed{\bar{D} = \frac{\rho_o r}{3} \hat{a_r}} \quad 0\leq r<a \qquad \boxed{\bar{E} = \frac{\rho_o r}{3\epsilon_o} \hat{a_r}}$$

lecture (12): 27-2-2020

Ex:- Given $\bar{D} = z \rho \cos^2 \phi \, a_z$ c/m^2

calculate :- (a) The charge density at $(1, \frac{\pi}{4}, 3)$.

(b) The total charge enclosed by the surface $\rho = 1$, with $-2 \leq z \leq 2$ m.

* Method (1)

* $\rho_L = X$ no equ.
$\rho_s = \bar{D} \cdot \hat{a_n}$
$\rho_v = \nabla \cdot \bar{D}$

Not used, because we don't know the surface.

* $\rho_v = \frac{\partial D_z}{\partial z}$ $\xrightarrow{\text{بما أن}}$ $\nabla \cdot \bar{A} = \cancel{\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho)} + \cancel{\frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi}} + \frac{\partial A_z}{\partial z}$

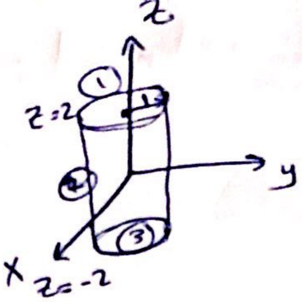
(a)

$$\rho_v = \frac{\partial D_z}{\partial z} = \rho \cos^2 \phi \ \text{c/m}^3$$

$$\rho_v \Big|_{(1, \frac{\pi}{4}, 3)} = 1 \cdot \frac{1}{2} = 0.5 \text{c/m}^3.$$

(b)

(b) $Q = \Psi = \oint_s \bar{D} \cdot \bar{ds}$.



$$Q = \int_{S1} \bar{D} \cdot \bar{ds}_1 + \cancel{\int_{S2} \bar{D} \cdot \bar{ds}_2} + \int_{S3} \bar{D} \cdot \bar{ds}_3 .$$

$$Q = \int_v \rho_v \cdot dv$$

عدد التكاملات لعدد الأسطح. (*)

$$\bar{ds}_1 = \rho d\rho d\phi \hat{a_z}$$

$$\bar{ds}_2 = \rho d\phi dz \hat{a_\rho}$$

$$\bar{ds}_3 = -\rho d\rho d\phi \hat{a_z}$$

(*) عمودي ← ρ و z يعني تكامل zero

$$\bar{D} = \int_0^{2\pi}\int_0^1 z \rho \cos^2\phi \, \rho d\rho \, d\phi \Big|_{\substack{z=2 \\ \text{constant}}} - \int_0^{2\pi}\int_0^1 z \rho \cos^2\phi \, \rho \, d\rho d\phi \Big|_{z=-2}$$

$$= 2 \cdot \frac{1}{3} \cdot \pi + 2 \cdot \frac{1}{3} \cdot \pi = \frac{4}{3} \pi$$

$$\int_0^{2\pi} \cos^2\phi = \int_0^{2\pi} \frac{1}{2} (1 + \cos 2\phi) \, d\phi = \pi + \underbrace{\cancel{\frac{\sin \phi}{4}}}_{\text{Zero}} \Big]_0^{2\pi}$$

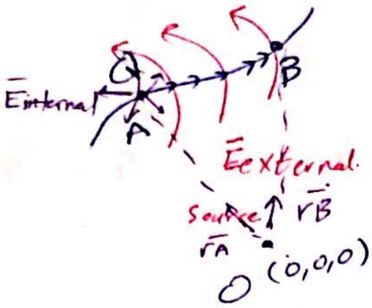
* Method (2)

$$Q = \int_v \rho_v \, dv = \int_{-2}^{2}\int_0^{2\pi}\int_0^{\pi} 1 \cos^2\phi \, \rho d\rho \, d\phi \, dz . =$$

$$\left(\frac{1}{3}\right)(\pi)(4) = \frac{4}{3} \pi \, C .$$

4.7

* Electric potential :-



$$\underset{\text{scalar}}{\overset{\text{work}}{W}} = \underset{\text{vectors}}{\bar{F}\cdot\bar{\ell}}$$ unit (N.m) or (Joul.)

$$\underset{Q\text{source}}{\bar{E}} = \frac{\bar{F}}{Q_{\text{field}}}$$

then $w = Q\,\underset{ext}{\bar{E}} \cdot \bar{dl}$

$W = (-)Q\,\bar{E} \cdot \bar{dl}$

↳ The work is done by the external field.

$\int dW = \int -Q\,\bar{E} \cdot \bar{dl}$ ← الwork عكس المجال

Qsource

$W = -Q \int_{\ell} \bar{E} \cdot \bar{dl}$

field ↳ من colonmb's law and Gauss's law

$W \rightarrow + \rightarrow$ internal work.

$W \rightarrow - \rightarrow$ External work.

$$\frac{W}{Q} = V_{AB} = \int_{A\to B} = -\int_{V_A}^{V_B} \bar{E} \cdot \bar{dl}$$

$$1V = \frac{1J}{1C}$$

* if : VAB is (+ve) : Gain in potential.

↳ The work is done by the external field.

move from lower to higher potential بواسطة.

* if : VAB is (-ve): Drop in potential.

→ The work is done by the field itself.

move from high to low potential

* To move Q from A to B :

$$V_{AB} = -\int_{r_A}^{r_B} \bar{E} \cdot d\bar{l}$$

$$V_{AB} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon \cdot r^2} \hat{a}_r \cdot dr\, \hat{a}_r$$

(1)

معطى من السؤال

$$V_{AB} = \frac{Q}{4\pi\epsilon \cdot r} \Big]_{r_A}^{r_B} = \frac{Q}{4\pi\epsilon} \left(\frac{1}{r_B} - \frac{1}{r_A}\right) V$$

$$= \frac{Q}{4\pi\epsilon} \left(\frac{1}{r_B} - \frac{1}{r_A}\right) V = \frac{Q}{4\pi\epsilon \cdot r_B} - \frac{Q}{4\pi\epsilon \cdot r_A} V$$

$$\boxed{V_{AB} = V_B - V_A.}$$

lecture (13) : 1/3/2020

طرق ايجاد (V)

$$V_{AB} = \frac{W}{Q} = -\int_{l} \bar{E} \cdot d\bar{L}$$

(1) (2)

for a point charge.

$$V_{AB} = \frac{Q}{4\pi\epsilon_\circ}\left(\frac{1}{r_B} - \frac{1}{r_A}\right).$$

if refrence at ∞ $(V_\infty = 0)$

$$V = \frac{Q}{4\pi\epsilon_\circ r}$$

// ———————— // ———————— //

* if refrence at (∞) for line charge:- $\boxed{\int \rho_L \, dL} = Q$

(3) from source:-

$$V = \int_{l} \frac{\rho_L \cdot dL}{4\pi\epsilon |r|}$$

magnitude

* اذا شي finite بنفع
أرجع للsource

for surface charge:-

$$V = \int \frac{\rho_s \, ds}{4\pi\epsilon_\circ r}$$

for Volume charge :-

$$V = \int_{V'} \frac{\rho_v \, dV'}{4\pi\epsilon_\circ r}$$

Volume

Voltage (potential)

* for N point charges:-

$$V = \frac{1}{4\pi\epsilon_\circ} \sum_{k=1}^{N} \frac{Q_k}{r_k}$$

*Ex:- Two point charges ($-4\mu c$) and ($5\mu c$) are located at $(2,-1,3)$ and $(0,4,-2)$. Find the potential at $P(1,0,1)$:-

refrence is at (∞)

$\boxed{V_\infty = 0}$

لأ نومش محدد
نفترضه (∞)
من عنا .

$$V_{\infty P} = V_P - V_\infty = V_P - 0 = V_P.$$

field - source.

$$V_P = \frac{-4*10^{-6}}{4\pi * \frac{10^{-9}}{36\pi} * \sqrt{6}} \rightarrow |-1, 1, -2| = \sqrt{4+1+1} = \sqrt{6}$$

$$+ \frac{5*10^{-6}}{4\pi * \frac{10^{-9}}{36\pi} * \sqrt{26}} \rightarrow |1, -4, 3| = \sqrt{9+16+1} = \sqrt{26}.$$

$$V_P = -5.872 \text{ KV}$$

Drop in potential (The work is done by the field itself).

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* EX:- A point charge of (5nc) located at $(-3,4,0)$ and a line $y=1$, $z=1$ carries a uniform charge (2nc/m):. ⓐ ~~find the potential~~. / ⓒ potential between two point

إلا لو عطوهو ∞

لالو جد مرجع

* معطي نقطة ومعلوم جهدها معناه هي المرجع

ⓑ if $V=100V$ at $B(1,2,1)$. Find V at $C(-2,5,3)$

$\boxed{\text{ref is at } B}$

$$V_C? \rightarrow V_{BC} = V_C - V_B. \qquad \rightarrow V_{CB} = V_B - V_C.$$

نفس القيمة
بس سالبة

$$V_{BC} = V_C - 100 \longrightarrow \boxed{V_C = V_{BC} + 100} \text{ --- } (*)$$

$$V_{BC} = V_{BC\,(point)} + V_{BC\,(line)}$$

$$V_{BC}\text{ (point)} = -\int_{l} \bar{E}\cdot\bar{dl} = -\int \frac{Q}{4\pi\epsilon_\circ r}\,\hat{a_r}\cdot \bar{dl}$$

$\bar{dl} \rightarrow dr\,\hat{a_r}$

$$= \frac{Q}{4\pi\epsilon_\circ}\left(\frac{1}{r_c} - \frac{1}{r_B}\right)$$

r_c: بعد C عن source r_B: بعد B عن source

$$r_c = \left|\underbrace{(-2,6,3)}_{\text{field (c)}} - \underbrace{(-3,4,0)}_{\text{source (Q)}}\right| = \sqrt{11}$$

$$r_B = \left|\underbrace{(1,2,1)}_{\text{field (B)}} - \underbrace{(-3,4,0)}_{\text{source (Q)}}\right| = \sqrt{21}.$$

V_{BC} (point) = تعويض بالقانون . V

---

from the electric field.

$$V_{BC}\text{ (line)} :- \; -\int_{l} \bar{E}\cdot\bar{dl} \quad , \quad \bar{E} = \frac{\rho_L}{2\pi\epsilon_\circ r}\,\hat{a_r}$$

or from the source

$$V_{BC} = \int_{l} \frac{\rho_L\, dl}{4\pi\epsilon_\circ r} \quad \times$$

infinite لأنه line

أسهل

$$V_{BC} = -\int_{\rho_B}^{\rho_C} \frac{\rho_L}{2\pi\epsilon_\circ \rho}\,\hat{a_\rho}\cdot d\rho\,\hat{a_\rho}$$

ρ أقصر مسافة.

$$V_{BC} = \frac{-\rho_L}{2\pi\epsilon_\circ}\left(\ln\rho_c - \ln\rho_B\right)$$

to find   field − source (line)

$\rho_C = |(-2,5,3) - (-2,1,1)| = \sqrt{20}$

$\rho_B = |(1,2,1) - (1,1,1)| = \sqrt{1} = 1$

$$V_{BC} = \frac{-\rho_L}{2\pi\epsilon_\circ}\ln\left(\frac{\rho_c}{\rho_B}\right).$$

Drop in potential ← $V_{BC} = -50.175$ V

$V_c = 49.825$

صعود جهد .

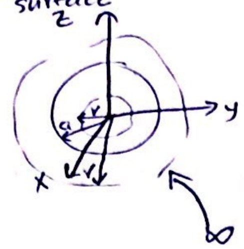
جهد نقطة B = 100

جهد نقطة C = 49.4

* Gauss's Example on a sphere :-

$$\rho_r = \begin{cases} \rho_0 , & 0 < r < a \\ 0 , & r > a \end{cases}$$

* نشتغل من ال ref لل gaussian surface



من برا لجوا عكس من $\bar{E}$ من جوا لبرا.

$$\bar{E} = \begin{cases} \dfrac{\rho_0 r}{3\epsilon_0} , & 0 \leq r \leq a. \\ \dfrac{\rho_0 a^3 \hat{a}_r}{3\epsilon_0 r^2} , & a \leq r \leq \infty \end{cases}$$

find (V) everywhere. $\boxed{V_\infty = 0}$ refrence.

$\boxed{\text{for } a < r < \infty}$

$$V = -\int_l \bar{E} \cdot \bar{dl} = -\int_\infty^r \frac{\rho_0 a^3}{3\epsilon_0 r^2} \cancel{\hat{a}_r} \cdot dr \cancel{\hat{a}_r} = \frac{\rho_0 a^3}{3\epsilon_0} \left(\frac{1}{r} - \overset{(0)}{\cancel{\frac{1}{\infty}}}\right) =$$

$$\frac{\rho_0 a^3}{3\epsilon_0 r} \ (V).$$

$\boxed{\text{for } 0 < r < a}$ ⊛ again we take from ∞ to the new radius.

⊛ مثل $\bar{E}$ بداية من $0 \leftarrow r$
وكذلك من $0 \leftarrow a$

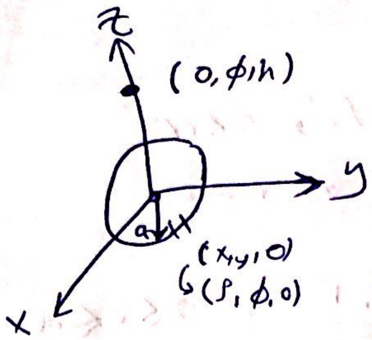
جوا نجزّأ التكامل :-

$$V = -\int_l \bar{E} \cdot \bar{dl} = -\int_\infty^a \frac{\rho_0 a^3}{3\epsilon_0 r^2} \cancel{\hat{a}_r} \, dr \cancel{\hat{a}_r} - \int_a^r \frac{\rho_0 r}{3\epsilon_0} dr \cdot \cancel{\hat{a}_r} \cdot \cancel{\hat{a}_r}$$

$$= \frac{\rho_0 a^2}{3\epsilon_0} - \frac{\rho_0}{3\epsilon_0} \frac{r^2 - a^2}{2} \ (V)$$

* في المنتصف للكرة (V) أكبر ما يمكن لأنه طول المسار منه يزداد

* Ex:- ring:



(a) $\bar{E}$.

(b) find V in ⇒ ref $V_\infty = 0$
(0,ϕ,h).

$dl = a\,d\phi$

$\bar{r} = -a\hat{a}_\rho + h\hat{a}_z$

$r = \sqrt{a^2+h^2}$

$$\bar{E} = \int_l \frac{\rho_L\, dl}{4\pi\epsilon_\circ r}\, \hat{a}_r .$$

find (V) using :- (1) $V = -\int_l \bar{E} \cdot \overline{dl}$ → $dz\hat{a}_z$

(2) $V = \int_l \frac{\rho_L\, dl}{4\pi\epsilon_\circ r}$

lecture (14) 3-3-2020

perfect day ♡

How to find $\bar{E}$ from potential : → when the source is missing.

$$V = -\int_{l} \bar{E} \cdot \bar{dl}$$

$$dV = -\bar{E} \cdot \bar{dl}$$

اشتقاق للتخلص التكامل.

in cartesian:-

$$\bar{E} = E\hat{ax} + E\hat{ay} + E\hat{az}$$

$$\bar{dl} = dx\,\hat{ax} + dy\,\hat{ay} + dz\,\hat{az}$$

$$\rightarrow \bar{E} \cdot \bar{dl} = E_x\,dx + E_y\,dy + E_z\,dz.$$

or can be written as:-

$$dV = \frac{\partial V}{\partial x} dx + \frac{dV}{dy} dy + \frac{dV}{dz} dz$$

* by Equating components:-

$$\frac{dV}{dx} \cancel{dx} = -E_x \cancel{dx} \rightarrow E_x = \frac{-dV}{dx}$$

$$E_y = \frac{-dV}{dy}$$

$$E_z = -\frac{dV}{dz}.$$

تكامل

$E \rightarrow V$

اشتقاق

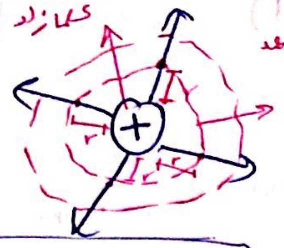
$$\bar{E} = -\left(\frac{\partial V}{\partial x} \hat{ax} + \frac{\partial V}{\partial y} \hat{ay} + \frac{\partial V}{\partial z} \hat{az} \right).$$

$$\boxed{\bar{E} = -\nabla V}$$

→ إيجاد المجال والpotential بطريقة ثالثة عندما يكون المصدر غير موجود no source.

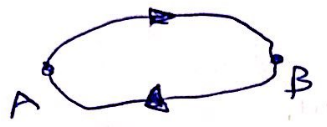
$$* \; V = \frac{Q}{4\pi\epsilon_0 r}$$

* اتجاه المجال هو اتجاه تناقص الجهد.



$\boxed{\text{refrence } \infty.}$ $V_\infty = 0$

* potential around a closed path:-



$$V_{AA} = V_A - V_A = 0$$

$$= -\int_A^B \bar{E}\cdot\bar{dL} + \int_{(B)}^{(A)} \bar{E}\cdot\bar{dL} = 0$$

$$\boxed{V = \oint_l \bar{E}\cdot\bar{dL} = 0}$$

always for static field.

(*) مسار مغلق
إشارة السالب
غير مهمة.

* Apply stockes Theorem:- ← 2^{nd} maxwell's equation in integration form (equi_ potential surface).

$$\oint_l \bar{E}\cdot\bar{dL} = \int_s \nabla\times\bar{E}\cdot\bar{ds} = 0$$

→ curl = 0
دوران
irrotational or conservative.

2^{nd} _maxwell's equation in Differential form.

*but : Solenoidal ملازوني . $\nabla\cdot\bar{E} = 0$

*** Ex:-** Given the potential $V = \frac{10}{r^2} \sin\theta \cos\phi$.

sph

* لا يوجد معلومات عن ال source

Find

نعتبرها sph أسهل لأن المسألة sph

a) $\bar{D}$ at $(2, \frac{\pi}{2}, 0)$.

b) The work done in moving a $(10\mu c)$ charge from $A(1, 30°, 120°)$ to $B(4, 90°, 60°)$.

---

a) $\bar{D} = \epsilon_0 \bar{E}$, $\bar{E} = -\nabla V$

then, $\bar{D} = -\nabla V \epsilon_0$

$$\bar{E} = -\nabla V = -\left(\frac{\partial V}{\partial r} \hat{a_r} + \frac{\partial V}{r\partial\theta} \hat{a_\theta} + \frac{\partial V}{r\sin\theta\,\partial\phi} \hat{a_\phi}\right).$$

E_r

$$\bar{E} = -\left(\left(\frac{20}{r^3} \sin\theta \cos\phi \hat{a_r} + \frac{10}{r^3} \cos\theta \cos\phi \hat{a_\theta} + \frac{-10}{r^3} \sin\phi \hat{a_\phi}\right)\right) \text{v/m}$$

E_θ E_ϕ

$$\bar{D}\Big|_{(2, \frac{\pi}{2}, 0)} = \frac{10^{-9}}{36\pi} * \bar{E} \Rightarrow \frac{10^{-9}}{36\pi} * \frac{20}{8} \hat{a_r} \ c/m^2 \longrightarrow$$

الحدود الباقية بتصفر

b) * Method (1):-

work = ??

من $A \leftarrow B$

$$W = Q V_{AB}$$

$$W = 10*10^{-6} * (V_B - V_A) \longrightarrow$$

* نعوض من احداثيات B, A بال expression لنطلع V_A, V_B

$$W = 10*10^{-6} * \left(\frac{5}{16} + \frac{40}{16}\right) =$$

V_{AB} : gain in potential , the work is done by the ext field.

$$W = 10*10^{-6} * \boxed{\frac{45}{16}}$$

$$\boxed{W = 28.125 \mu J.}$$

نستخدم عند :
* إذا $\bar{E}$ معي و
$\bar{V}$ مش معي.

* b) Method (2):-

$$W = -Q\int_{r_A}^{r_B} \bar{E}\cdot d\bar{l}$$

$$W = -10\times10^{-6} * \int_{r_A}^{r_B}[E_r dr + E_\theta\, r d\theta + E_\phi\, r\sin\theta\, d\phi].$$

$$W = -10\times10^{-6}\left(\int_{1}^{4} E_r dr + \int_{30^\circ}^{90^\circ} E_\theta\, r d\theta + \int_{120^\circ}^{60^\circ} E_\phi\, r\sin\theta\, d\phi\right).$$

مهم

r غيرنا
بس $\theta = 30^\circ$ ثوابت هون
$\phi = 120^\circ$
نحنا واقفين عند A

$r=4$
$\phi = 120^\circ$

$r=4$
$\theta = 90^\circ$

* كل مرة تكامل متغير واحد فقط والاثنين الباقيين ثوابت #.

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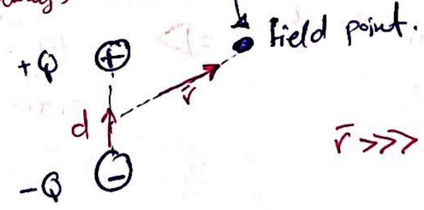
lecture (15): 5-3-2020

* Electric Dipole :-

always find $\bar{E}, \bar{D}, V$???



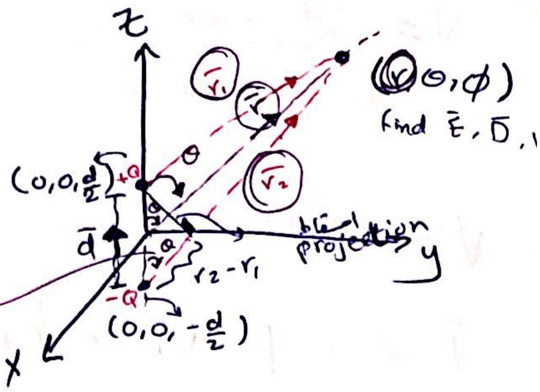
$\bar{r} >>> d$: نعتبره dipole
لم نعتبره 2 point charges.

- Gause X → صعب محاولة الشكل.
- coloumb ✓
- potential ✓

*Ex:-

$d <<< r$

$\bar{d}$: من سالب للموجب



$$E = \frac{Q\bar{r}}{4\pi\epsilon_0 r^3}$$

2nd assumption
r/d بين θ
* هي نفسها θ بين r2/d

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

نبدأ منه
أسهل ومنه
نحل $\bar{E} = -\nabla V$
ونجد E أسهل .
أسهل لأنه scalar
بينما $\bar{D}/\bar{E}$ vector *

$$V = V_+ + V_- = \frac{Q}{4\pi\epsilon_0 r_1} + \frac{Q}{4\pi\epsilon_0 r_2} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

$$V = \frac{Q(r_2 - r_1)}{4\pi\epsilon_0 r_1 r_2} \text{ V} \quad ---- (1)$$

الصورة الأولى
لكن Q r_2/r_1
غير معطى
للـ dipole

للتخلص من المشكلة :

فرض .

assumption:- $r_1 r_2 \approx r^2$

$r \equiv$ field point — centre of the dipole

$$V = \frac{Qd\cos\theta}{4\pi\epsilon_0 r^2} \text{ V} \quad ---- (2)$$

we define dipole moment:-

$\bar{P} = Q\bar{d}$ in (c.m) لكون

$|\bar{P}| = a|\bar{d}| \longrightarrow$ معطى بالسؤال عادة $P = Qd$ then $V = \dfrac{P\cos\theta}{4\pi\epsilon_0 r^2}$. ———— (3)

momentum
عزم

$\cos\theta \rightarrow$ dot product.

$\sin\theta \rightarrow$ cross product.

$$V = \frac{\bar{P} \cdot \hat{a_r}}{4\pi\epsilon_0 r^2} \quad \text{------} \quad (3')$$

$\hat{a_r} = \frac{\bar{r}}{r}$

~~[illegible]~~

$\bar{P} \cdot \hat{a_r} = |\bar{P}| (1) \cos(\bar{P}, 1) =$ $\longrightarrow$ $\theta p = \theta d$

$$\boxed{\bar{P} \cdot \hat{a_r} = \bar{P} (1) \cos\theta .}$$

field source (center of dipole).

$$\boxed{V = \frac{\bar{P} \cdot (\bar{r} - \bar{r}')}{4\pi\epsilon_0 |\bar{r} - \bar{r}'|^3} \quad \text{------} \quad (3'')}$$

#حفظ

لل dipole

[illegible]

$$\bar{E} = E_+ + E_-$$

or $\bar{E} = -\nabla V$

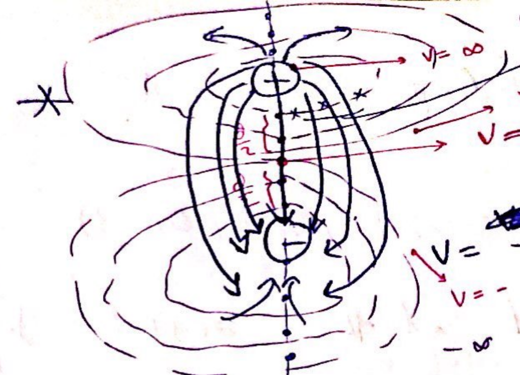
$$\bar{E} = -\left(\frac{\partial V}{\partial r} \hat{a_r} + \frac{\partial V}{r \partial \theta} \hat{a_\theta} + \frac{\partial V}{r \sin\theta \, \partial \phi} \hat{a_\phi} \right).$$

$\rightarrow$ scalar.

$$\bar{E} = \frac{-P}{4\pi\epsilon_0} \left(\frac{-2\cos\theta}{r^3} \hat{a_r} - \frac{\sin\theta}{r^3} \hat{a_\theta} + \text{zero} \right).$$

$$\boxed{\bar{E} = \frac{P}{4\pi\epsilon_0 r^3} \left(2\cos\theta \, \hat{a_r} + \sin\theta \, \hat{a_\theta} \right) \text{ N/m.}}$$ # حفظ

$$\boxed{\bar{D} = \epsilon_0 \bar{E} .}$$ # حفظ



سهم نفس الجهد
موشرط متعامد
لكن يبذل جهد

لأن

$$V = \frac{Q\,(r_2 - r_1)}{4\pi\epsilon_0\, r_1 r_2}$$

* سطح تساوي الجهد
$\Delta V = 0$ بس V عليه مش
شرط تساوي صفر

دائرة تساوي الجهد!

loci of all equi-potential surfaces.

---

* for N-dipoles :-

$$V = \frac{1}{4\pi\epsilon_0} \sum_{K=1}^{N} \frac{\bar{P}_K \cdot (\bar{r} - \bar{r}_K')}{|\bar{r} - \bar{r}_K'|^3} \; V$$

*Point charge :	*Dipole :
$V \propto \frac{1}{r}$	$V \propto \frac{1}{r^2}$
$E \propto \frac{1}{r^2}$	$E \propto \frac{1}{r^3}$

*** EX:** Two dipole moments ($-5\hat{a}_z$ nc.m) and

معناه r_1 center لا يعني احداثي واحد فقط . r_2

($9\hat{a}_z$ nc.m) are located at (0,0,-2) and (0,0,3)

Find the potential and $\bar{E}$-field at the origin (0,0,0).

* refrence at (∞)

$$V = \frac{P_1 \cdot (\bar{r} - \bar{r}_1)}{4\pi\epsilon_0 |\bar{r} - \bar{r}_1|^3} + \frac{P_2 \cdot (\bar{r} - \bar{r}_2)}{4\pi\epsilon_0 |\bar{r} - \bar{r}_2|^3}$$

($\bar{r}$ = origin)

قانون sumation (Σ)

$$V_{\infty} = \frac{-5\hat{a}_z \times 10^{-9} \cdot 2\hat{a}_z}{4\pi\epsilon_0 \cdot (8)} + \frac{9\hat{a}_z \times 10^{-9} \cdot (-3\hat{a}_z)}{4\pi\epsilon_0 \cdot (27)}$$

$V = -20.25\,V.$

drop in potential.

* $\bar{E} = -\nabla V$ X . خطأ → فقط إذا كان (V) معطى لـ expression لأنه هنا مشتقة أي رقم ثابت = Zero

الحل الصحيح*

$$\bar{E}_1 = \frac{P_1}{4\pi\epsilon_0 r_1^3}\left(2\cos\theta_1 \hat{a}_r + \sin\theta_1 \hat{a}_\theta\right)$$

* لو كانت الزاوية مركبة : نعمل

$$\cos\theta_1 = \frac{\bar{P}_1 \cdot \bar{r}_1}{|\bar{P}_1||\bar{r}_1|}$$

وناخذ $\cos^{-1}$

حفظ القانون

we need:-

$P_1 = -5a_z$ → vector ⇝ we need it as scalar 5 nc.m فقط

$r_1 = 2$ m

$\theta_1 = 180^\circ$.

* we know that

$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z}$ X ← لا ينفع!

* θ is $\measuredangle$ between P_1, r_1 → $\theta = 180$

(P_1 → $-a_z$, r_1 → a_z)

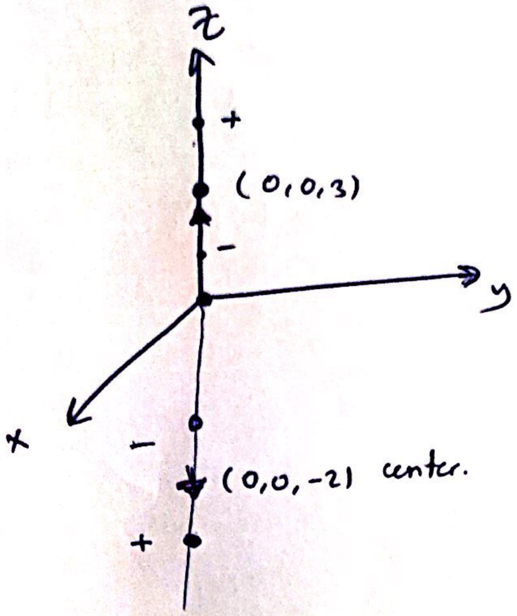
$P_2 = 9$ nc.m

$r_2 = 3$ m

$\theta_2 = 180^\circ$.

* نعلم اتجاه ميلان الـ dipole من الاتجاه
للـ (P)

نجد θ_1 , θ_2 وهكذا
من الرسمة :-

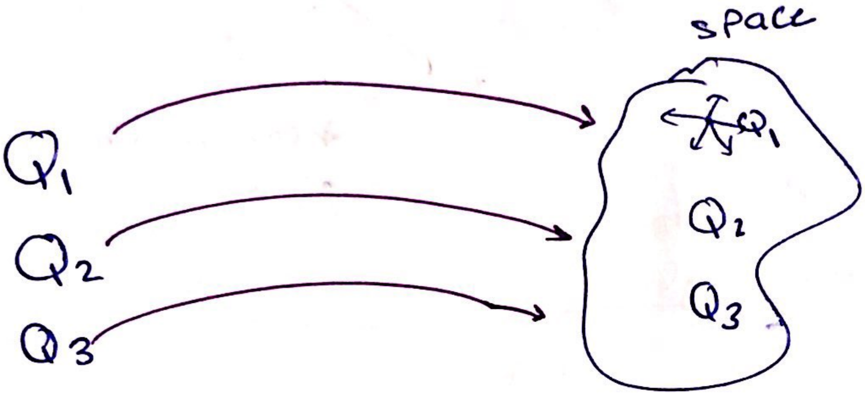


lecture (16) :- 8-3-2020

Energy Density in electrosatic Fields:

W_E = Electrical Energy (J) (joul).

W = work. (in joul).



نقلت شحنات :

$Q_1 \xrightarrow{\text{then}} Q_2 \xrightarrow{\text{ثم}} Q_3$, Work = $Q \cdot V$ (difference).

$$W_E = W_1 + W_2 + W_3.$$

$$W_E = 0 + Q_2 V_{12} + \boxed{Q_3 V_{13} + Q_3 V_{23}} \rightarrow Q_3 (V_{13} + V_{23}). \quad \text{--- \#1}$$

لما كان في فرق جهد لأنه ما في شحنات غير Q_1 التي نقلتها أنا.

$V_{12} \rightarrow V_1 - V_2$

تأثير Q_1 على Q_2

نقلت شحناتي بالترتيب التالي :

$Q_3 \xrightarrow{\text{ثم}} Q_2 \xrightarrow{\text{ثم}} Q_1$

$$W_E = W_1 + W_2 + W_3.$$

$$W_E = Q_1 (V_{31} + V_{21}) + Q_2 V_{32} + 0 \quad \text{--- \#2}$$

* جمع معادلتين
1/2

$$2W_E = Q_1(V_{21}+V_{31}) + Q_2(V_{12}+V_{32}) + Q_3(V_{13}+V_{23})$$

$$W_E = \frac{1}{2}\left(Q_1 V_1 + Q_2 V_2 + Q_3 V_3\right).$$

$$\boxed{W_E = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k .}$$

العبوة ①, for N charges ---
W_E " $\boxed{Q}$ point charge.
#1

" ——————— " ——————— "

* $W_E = \frac{1}{2}\int_{l} \rho_L V \, dL$ → for Line charge.

* $W_E = \frac{1}{2}\int_{s} \rho_s V \, ds$ → for surface charge.

* $W_E = \frac{1}{2}\int_{v'} \boxed{\rho_{v'}} V \, dv'$ → for Volume (v') charge.

$$W_E = \frac{1}{2}\int (\nabla \cdot \bar{D}) V \, dv.$$

$V = -\bar{E}\, d\bar{L}$
$\bar{E} = -\nabla V$
$\boxed{\rho_V = \nabla \cdot \bar{D}}$
$\bar{D} = \epsilon_0 \bar{E}.$

* Identity :-

$$\nabla \cdot V\bar{A} = \bar{A} \cdot \nabla V + V(\nabla \cdot \bar{A}). \longrightarrow V(\nabla \cdot \bar{D}) = \nabla \cdot V\bar{D} - \bar{D} \cdot \nabla V$$

$\bar{A} \equiv \bar{D}$ ← put it.

توزع التكامل على الطرح.

$$W_E = \frac{1}{2} \int_V \left[\nabla \cdot V\bar{D} - \bar{D} \cdot \nabla V \right] dv.$$

$$W_E = \underbrace{\frac{1}{2} \int_V \nabla \cdot V\bar{D} \, dv}_{1} - \underbrace{\frac{1}{2} \int_V \bar{D} \cdot \nabla V \, dv}_{2}$$

two Integrals → apply divergence theorem :-

* The divergence theorem is:-

$$\oint_S A \cdot \bar{ds} = \int_V \nabla \cdot \bar{A} \cdot dv$$

← this way!

$$W_E = \overbrace{\frac{1}{2} \oint_S \cancel{V\bar{D} \cdot \bar{ds}}}^{1} - \overbrace{\frac{1}{2} \int_V \bar{D} \cdot \nabla V dv}^{2}, \quad \nabla V = -\bar{E}.$$

Zero

(goes to $\frac{1}{\infty}$) so (Zero)

$$W_E = \frac{1}{2} \int \bar{E} \cdot \bar{D} \, dv \quad , \quad \bar{D} = \epsilon_0 \bar{E}.$$

$$= \boxed{\frac{1}{2} \int \bar{E}^2 \cdot \epsilon_0 \, dv} \Rightarrow \text{scalar.}$$

صارت magnitude (مقدار)^2 = قيمة فقط scalar

$$= \boxed{\frac{1}{2} \int \frac{D^2}{\epsilon_0} dv}$$

نستخدم القانون الصورة ② لل W_E هذا عندما نكون لا نعلم المصدر المسبب في السؤال

#2

no Source charge

⊛ Energy Density :- ω_E : small letter (ω) not (W)

$$\omega_E = \frac{W_E}{\text{Volume}} \text{ in } (\text{Joul}/m^3).$$ كثافة الطاقة.

$$\omega_E = \frac{1}{2}\bar{E}.\bar{D} = \frac{1}{2}\epsilon_o E^2 = \frac{D^2}{2\epsilon_o}$$

$$\boxed{W_E = \int \omega_E \, dv}$$ → ⓒ نفس الصورة #2* . الكتابة ال WE .

---

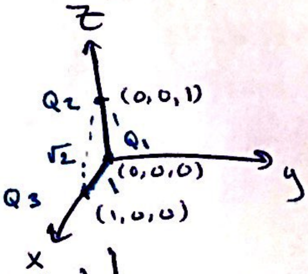
*Ex:- The point charges (-1nc, 4nc) and (3nc) are located at (0,0,0), (0,0,1) and (1,0,0), Find the energy in the system:-

لما يطلب energy

يعني (WE) عادي

Soln:

$$W_E = \frac{1}{2}\int E^2\epsilon_o \quad \text{or} \quad \frac{1}{2}\int \frac{D^2}{2\epsilon_o}$$ نعمل مرتين التكامل لكل شحنة.



or

$$W_E = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k.$$

$$= \frac{1}{2}\left(Q_1V_1 + Q_2V_2 + Q_3V_3\right).$$

$$= \frac{1}{2}\left(Q_1(V_{21}+V_{31}) + Q_2(V_{12}+V_{32}) + Q_3(V_{13}+V_{23})\right).$$

$$= \frac{1}{2}\left[Q_1\left(\frac{Q_2}{4\pi\epsilon_o(1)} + \frac{Q_3}{4\pi\epsilon_o(1)}\right) + Q_2\left(\frac{Q_1}{4\pi\epsilon_o(1)} + \frac{Q_3}{4\pi\epsilon_o(\sqrt{2})}\right) + Q_3\left(\frac{Q_1}{4\pi\epsilon_o(1)} + \frac{Q_2}{4\pi\epsilon_o(\sqrt{2})}\right)\right]$$

$$W_E = \boxed{13.37\,nJ}.$$

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lecture (17) :- 10 - 3 - 2020

CH.5 :- Electric Fields in Materials:-

The classification of materials based on it's electrical properties:-

* conductors:

  $\sigma >>> 1$

  الموصلية كثير كبيرة

  (موصلات)

  cu, AL, Au, Ag, lead.

* semi - conductors:

  $\sigma > 1$

  (أشباه موصلات)

* Dielectrics:

  $0 < \sigma <<< 1$

  الموصلية قليلة كثير

  (عازلات)

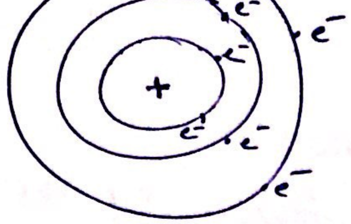
  Teflon, Mica, lithium.

$\sigma \equiv$ conductivity . (S/m) الموصلية
semin

$\sigma = \frac{1}{\rho}$. while ρ: Resistivity المقاومية (Ω.m).
ohm

When Temperature ↑
the conductivity ↓

$\boxed{T\uparrow, \sigma\downarrow}$



lead at 20°c (293K)

$\sigma \approx 10^{6}$ s/m.

at (4K)

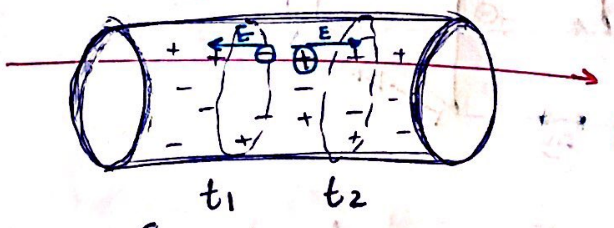
$\sigma \approx 10^{20}$ s/m.

T قلت , σ زادت

---

* Types of currents:- ⊗ أنواع التيارات حسب المادة.

1. conduction current. (DC or AC)
2. convection current. (Dielectric DC).
3. Displacement current. (Dielectric AC) → CH.9.



∵ Eexternal : مجال خارجي

* $\bar{E}_{ext} = \dfrac{\bar{F}}{Q}$.

Q total without Eext = 0
← شحنات عشوائية

* $\bar{F} = Q\bar{E}$.

* $I = \dfrac{\Delta Q}{\Delta T}$ C/s or Ampere. (A).

in general:-

$$I = \frac{dq}{dt},$$

$$\frac{I}{Area} = \frac{I}{\Delta s} = \frac{I}{ds} = J$$

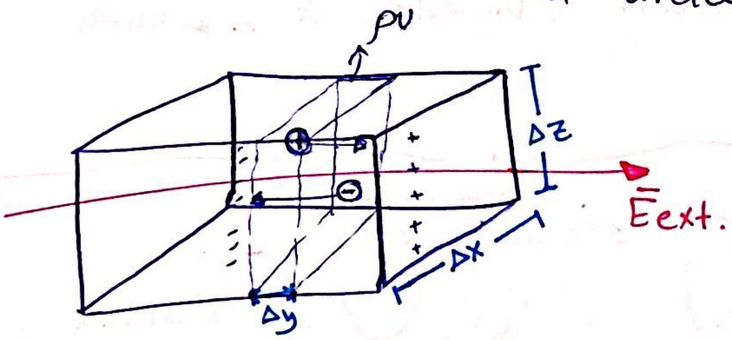
$J \equiv$ current density. كثافة التيار (A/m^2).

$$\boxed{I = \int_s \bar{J} \cdot \bar{ds}}$$

" ———————————— " ———————————— "

قطعة

(*) consider a filament of a dielectric material:-



* $\Delta S = \Delta X \cdot \Delta Z$. المساحة (Area).

* $I = \frac{\Delta Q}{\Delta t}$. * $Q = \int_v \rho v \, dv$.

* $dQ = \rho v \, dv$.

$\boxed{* \Delta Q = \rho v \, \Delta V.}$

ΔV $\Delta z, \Delta y, \Delta x$ = الحجم

$\Delta s . \Delta y$

$$* \; I = \frac{\rho_v \; \Delta s \; \Delta y}{\Delta t}$$

we know $\frac{\Delta y}{\Delta t}$ = u_y سرعة باتجاه (y)

مسافة / زمن

$$* \; \frac{I}{\cancel{\Delta s}} = \frac{\rho_v \cancel{\Delta s} \, u_y}{\cancel{\Delta s}}$$

(scalar).

نقسم على (ΔS)

$$\frac{I}{Area} = J \text{ (scalar).}$$

* $\bar{u} = u_y \, \hat{a}_y$ (vector).

* $J = \rho_v u_y$. as scalar.

$$* \; \boxed{\bar{J} = \rho_v \bar{u}}$$

as vector. V.imp

convection current density.

* The force on one electron :-

$Q = -e$. شحنة الالكترون

التسارع × الكتلة

التسارع = سرعة / زمن

$$\bar{F} = Q\bar{E} = -e\bar{E} = m\bar{a} = m\frac{\bar{u}}{\tau}$$

التصادمات.

* τ ≡ Time between collisions in (sec).

$$* \; -e\bar{E} = \frac{m\bar{u}}{\tau} \rightarrow$$

$$* \; \bar{v}_d = \bar{u} = \frac{-e\tau}{m}\bar{E}$$

(drift velocity).

Fixed, if the temperature is fixed.

$$* \; \boxed{\bar{u} = \mu \bar{E}}$$

. * μ = mobility $\left(\frac{c.s}{kg}\right)$.

* for n-electrons per volume.

$$\rho_v = -ne. \xrightarrow{\text{عددهم}} \text{in } m^3.$$

$$\bar{J} = \rho_v \bar{u} = -ne\left(\frac{-e\tau}{m}\right)\bar{E}$$

$$\bar{J} = \frac{ne^2\tau}{m}\bar{E}$$

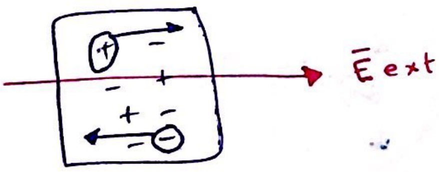
* $\bar{J} = \sigma \bar{E}$. → conduction current density.
→ oh'ms law.

conductance. $(\frac{1}{\Omega})$

$V = IR.$, $I = GV$, $\frac{\sigma}{\mu} = \rho_v$

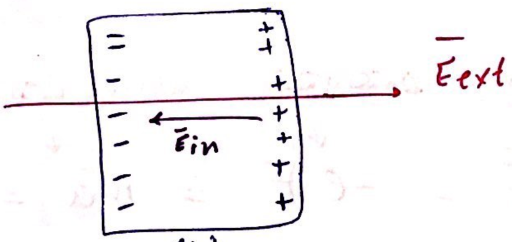
" ______ " " ______ "

* conductors:-



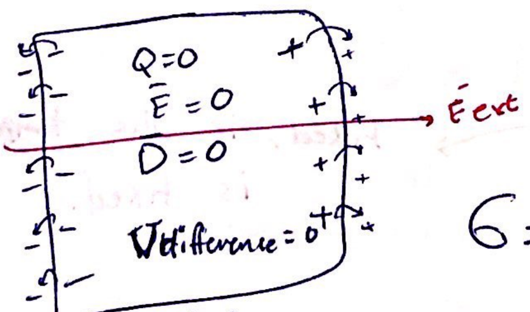
(a)

$Q_{total} = 0.$



(b)

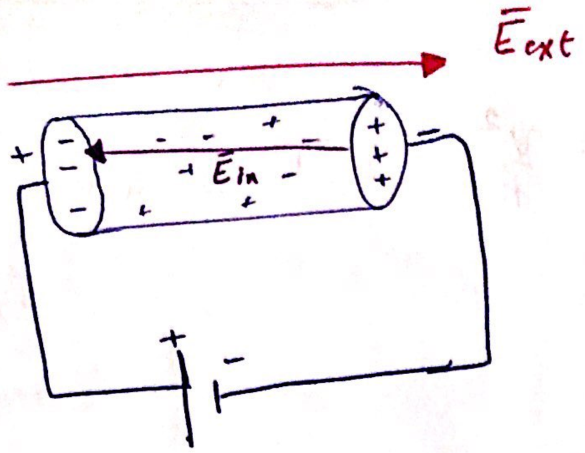
$\bar{E} = -\nabla V$



(c)

$\sigma = \infty$

* تطلع الشحنات على السطح وتكوّن
$\bar{E}$ext عكس $\bar{E}$in.



$$R = \frac{V}{I} = \frac{-\int_l \bar{E}\cdot \bar{dl}}{\int_s \bar{J}\cdot\bar{ds}} = \frac{-\int_l \bar{E}\cdot\bar{dl}}{\int_s \sigma \bar{E}\cdot\bar{ds}} = \frac{\cancel{E} l}{\sigma \cancel{E} s} =$$

$$\boxed{R = \frac{l}{\sigma A}}$$

المقاومة = الطول / الموصلية * المساحة *

* Power :-

$P = \frac{W}{t}$ ← work, ← time in J/s or (watt).

الشغل = قوة . المسافة

$$P = \frac{\bar{F}\cdot\bar{l}}{t} = \frac{Q\bar{E}\cdot\bar{l}}{t} \Rightarrow Q = \int_V \rho_v \, dv .$$

السرعة .

$$P = \int_V \rho_v \frac{\bar{E}\cdot\bar{l}}{t} dv = P = \int_V \rho_v \cdot\bar{E}\cdot\bar{u}\, dv , \quad \bar{J} = \rho_v \bar{u} .$$

$$\boxed{P = \int_V \bar{E}\cdot\bar{J}\, dv}$$ joulés law.

$$= \int_V \sigma E^2 dv = \int_V \frac{J^2}{\sigma} dv = \int_V \frac{E^2}{\rho} dv .$$

$$P = v.i = \frac{v^2}{R} = I^2 R = Q V^2 = \frac{I^2}{Q}.$$

joule's law.

$$P = \int_V \bar{E} \cdot \bar{J}\, dv \rightarrow \text{in general.}$$

if the wire ~~...~~ is uniform.

$$P = \int_S \int_l \bar{E} \cdot \bar{J}\, dl\, ds.$$

$$P = \underbrace{\int_l \bar{E} \cdot \bar{dl}}_{v} \underbrace{\int_s \bar{J} \cdot \bar{ds}}_{i}$$

lecture (18) :- 12-3-2020

Ex.:- If $\bar{J} = \frac{1}{r^3}\left(2\cos\theta\ \hat{a}_r + \sin\theta\ \hat{a}_\theta\right)$ A/m².

Find the current passing through :-

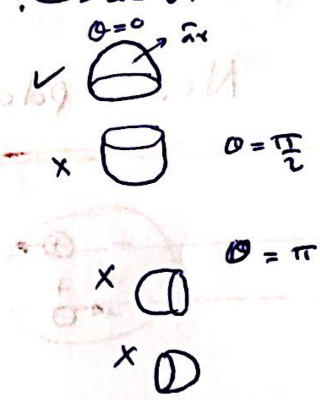
a. hemi-spherical shell (قشرة) of radius (20 cm) with

$0 < \theta < \frac{\pi}{2}$, $0 < \phi < 2\pi$

b. a spherical shell of radius (10cm).

Solu.:

الاحتمالات :



a.

$$I = \int_S \bar{J} \cdot \bar{ds}$$

$$\bar{ds} = r^2 \sin\theta\ d\theta\ d\phi\ \underline{\hat{a}_r}$$

العمودي على ال hemi sphere shell

$$I = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \frac{1}{r} \underbrace{2\sin\theta\cos\theta}_{\sin 2\theta}\ d\theta\ d\phi \Big|_{r = 0.2\,m.}$$

$$= 10\pi\ A = \cancel{31.4} \boxed{31.4\ A}$$

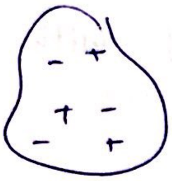
b. $= I = \int_S \bar{J} \cdot \bar{ds}$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{1}{r} \sin 2\theta\ d\theta\ d\phi \Big|_{r = 0.1\,m} = 0\ A.$$

* Polurization in Dielectric:-

Non - polar.

i.e: O_2, H_2, N_2, rare gases, Ne, Xe, Kr.



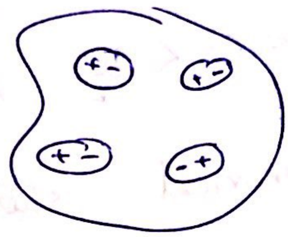
Q total $= 0$

$\bar{P} = 0$

polar.

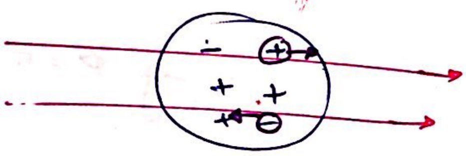
permenant dipole.

HCl, NH_3, H_2O.



$\bar{P} = 0$

* for Non - polar Dielectrics:-

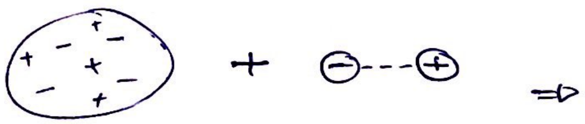


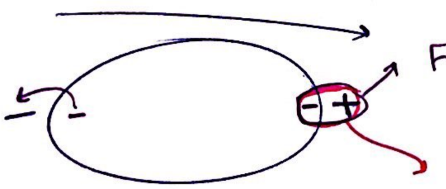
$\bar{E}$ ext

$\bar{F} = Q\bar{E}$

pressure $= \frac{F}{S(\text{Area})}$.

$\bar{E}$ ext





Free charge.

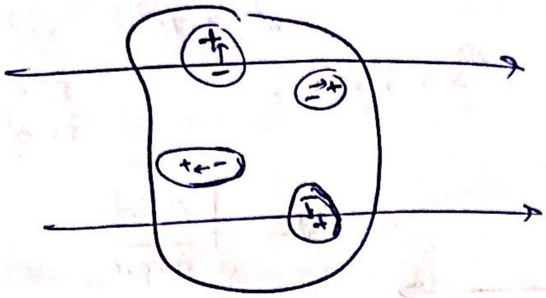
polarized dielectric.

bound (polarized). charge.

$\bar{E}$ ext

$\bar{P} \neq 0$

* For polar Dielectrics:-

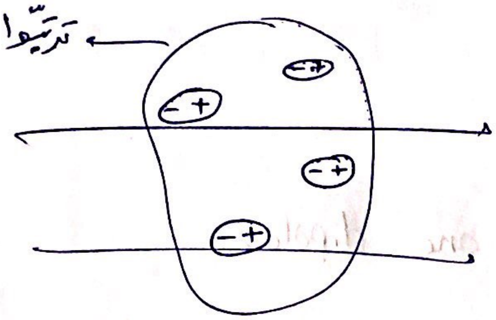


$\bar{E}_{ext}$

$\Delta V = 0$

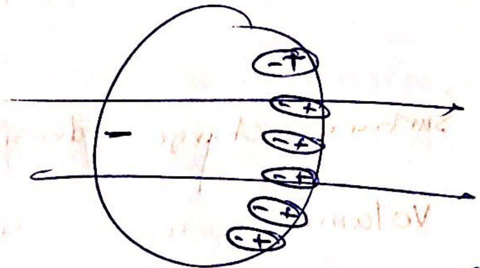
$\bar{P} = 0$

ترتيبوا جوا



$\bar{E}_{ext}$

Polarized Dielectric.



$\bar{E}_{ext}$

polar moment (c.m).

$$\bar{P} = \lim_{\Delta V \to 0} \frac{\sum_{k=1}^{N} \bar{P}_k}{\Delta V}$$

$\bar{P}$: (c/m^2)

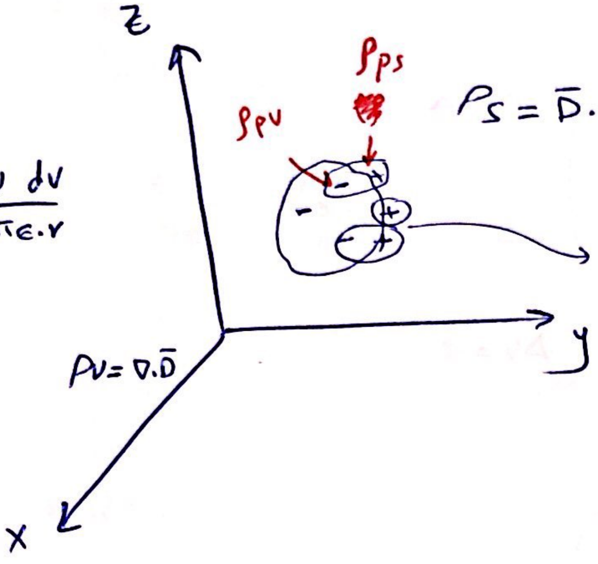
ΔV: m^3 (volume)

$\bar{P} = Q\bar{d}$

العزم

القطبي

* for a polarized Dielectric:-



$V = \int_v \frac{\rho_v \, dv}{4\pi\epsilon_o r}$

$\cdot (x, y, z)$. V?

$\bar{E}$? $= -\nabla V$

$\bar{D}$? $= \bar{E}\epsilon_o$

$\rho_s = \bar{D} \cdot \hat{a}_n$.

$V = \int_s \frac{\rho_s \, ds}{4\pi\epsilon_o r}$

$$V = \frac{\bar{P} \cdot \hat{a}_r}{4\pi\epsilon_o r^2} \text{ (V)}$$ for one dipole.

ρ_{ps} : polarized (bound) surface charge density.

ρ_{pv}: " " volume " "

$$V = \frac{\bar{P} \cdot \hat{a}_r}{4\pi\epsilon_o r^2} \quad -\nabla\left(\frac{1}{r}\right)$$

$$\nabla\left(\frac{1}{r}\right) = \frac{-1}{r^2}\hat{a}_r$$

$$\frac{a_r}{r^2} = -\nabla \frac{1}{r} = \nabla'\left(\frac{1}{r}\right)$$

$$V = \frac{\bar{P} \cdot \nabla' \frac{1}{r}}{4\pi\epsilon_o}$$ one dipole.

* **Identity:-** $\nabla \cdot V\bar{A} = \bar{A} \cdot \nabla V + V(\nabla \cdot \bar{A})$.

$\nabla \Rightarrow \nabla'$ $\quad$ $V \Rightarrow \frac{1}{r}$

$\bar{A} \Rightarrow \bar{P}$

* for polarized dielectric:-

$$V = \int_{v'} \frac{\bar{P} \cdot \nabla'(\frac{1}{r})}{4\pi\epsilon_0} dv'$$

$$\oint_s \frac{\bar{P}}{r} \cdot \bar{ds}$$

$\bar{ds} = ds\,\hat{a}_n$.

$$V = \frac{1}{4\pi\epsilon_0}\left[\underbrace{\int_{v'} \nabla' \cdot \left(\frac{\bar{P}}{r}\right) dv'}_{①} - \int_{v'} \frac{\nabla' \cdot \bar{P}}{r} dv' \right].$$

Apply divergence Theorem:- onto ①

$$V = \oint_{s'} \frac{\bar{P} \cdot \hat{a}_n}{4\pi\epsilon_0 r} ds' - \int \frac{\nabla' \cdot \bar{P}}{4\pi\epsilon_0 r} dv'$$

$$V = \oint_{s'} \frac{\rho_{ps}}{4\pi\epsilon_0 r} ds' + \int_{v'} \frac{-\rho_{pv}}{4\pi\epsilon_0 r} dv'$$

Bound.

$\rho_{ps} = \bar{P} \cdot \hat{a}_n$. $\Longleftrightarrow$ $\rho_s = \bar{D} \cdot \hat{a}_n$

(c/m^2)

Free.

$\rho_{pv} = -\nabla \cdot \bar{P}$. $\Longleftrightarrow$ $\rho_v = \nabla \cdot \bar{D}$

(c/m^3)