CIRCUITS II

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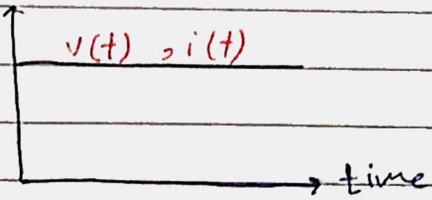
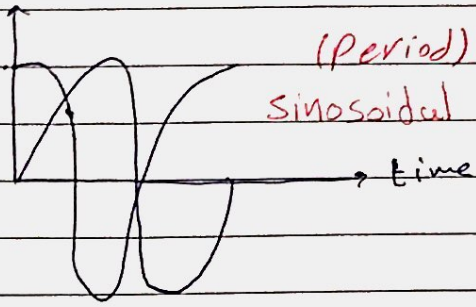
POWERUNIT

* source $\rightarrow$ Load

D.C $\rightarrow$ Direct current



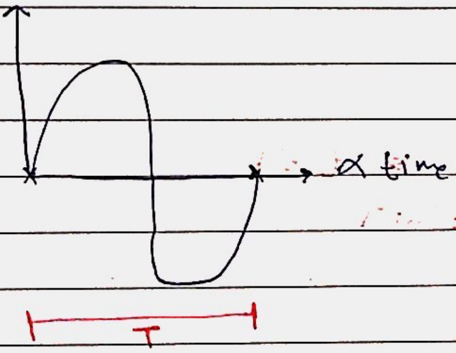
A.C $\rightarrow$ Alternating current



(Period)
sinosoidal source

* sinosoidal generation:-

1) period $\Rightarrow$ T (sec)



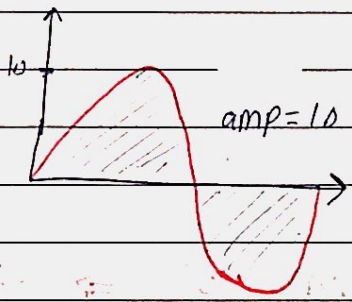
2) frequency

#of periods $\Rightarrow$ 1 sec
1 period $\Rightarrow$ T sec

$f = \frac{1}{T}$ $(sec)^{-1}$ (Hz)

3) Max $\rightarrow$ ref = 0
(Peak)

Amplitude $\rightarrow$ ref = avg $\rightarrow$ max



amp = 10 peak = 10

$y(t) = A \sin t$

$y_{avg} = \frac{1}{T} \int_0^T y(t)\, dt$
(period)

$= \frac{1}{T} \int_0^T A \sin t \, dt = -\frac{A}{T} \cos \Big]_0^t = -\frac{A}{T} [1-1] = \text{zero}$

Ex: $V(t) = 5 + 3\cos t$

① avg $= \frac{1}{T}\int_0^T 5 + \cancel{3\cos t}$ ② peak $= 5 + 3 = 8$

$= 5 = Dc$

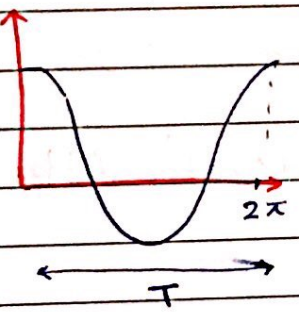
③ Amp $= 8 - 5 = 3$ ④ peak - peak min $\Rightarrow 5 + 3 * -1 = 2$

min → max $\Rightarrow 8 - 2 = 6$

angular speed $\Rightarrow$ $w = \frac{\alpha}{t}$ → angle

$\boxed{\alpha = wt}$

$5\cos x = 5\cos\alpha = 5\cos wt$



at $\alpha = 2\pi \Rightarrow t = T$

$\alpha = wt = wT$

$w = \frac{2\pi}{T} \Rightarrow w = 2\pi f$

$w \Rightarrow$ (rad/sec)

Assume a sinosoidal current

Amp = 10 A

period = 0.12 sec = T

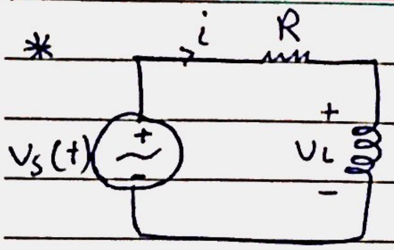
Determine the times at wich i = 5A

$i(t) = 10\sin(wt) \Rightarrow w = \frac{2\pi}{t} = \frac{2\pi}{0.12} = 52.36$ rad/sec

$5 = 10\sin(52.36)t \Rightarrow 0.5 = \sin(52.36t) \Rightarrow$

$\frac{\pi}{6} = 30^\circ \Rightarrow t_1 = 10$ m sec

$\frac{5\pi}{6}$



$V_R = IR$

$V_L = L\frac{di}{dt}$

$i_c = c\frac{dv}{dt}$

$V_s(t) = V_m \cos\omega t$

$\Rightarrow$ KVL

$$-V_m \cos\omega t + Ri(t) + L\frac{di}{dt} = 0$$

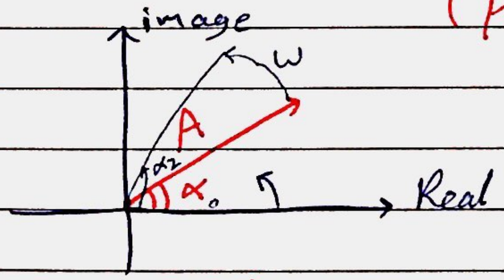
$$\boxed{Li' + Ri = V_m \cos\omega t} \rightarrow \text{Numerical} \checkmark$$

diff $\rightarrow$ x

* Introduction (phasor) :-

Vector $\rightarrow$ space

(Phasor diagram)



① at $t=0$

② rotate angular speed

$A \measuredangle \alpha \Rightarrow$ Polar

$\omega = \frac{\alpha}{t} \Rightarrow \alpha = \omega t + \alpha_0$

③ Projection x-axis

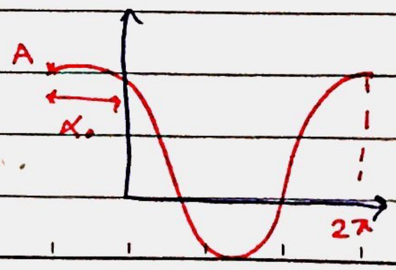
$t=0 \Rightarrow A\cos\alpha_0$

$t=1 \Rightarrow A\cos\alpha_2 \Rightarrow A\cos(\omega t + \alpha_0)$

$\hookrightarrow A \measuredangle \alpha_0$

Phasor domain

* Polar form



Five Apple

polar form $\Rightarrow$ A ∡ phase-shift $= \alpha_0$

$\Rightarrow V(t) = 5\sin(\omega t + 30^\circ)$

phasor domain

$V(t) = 5\cos(\omega t + 30^\circ)$ -90°

$\sin x = \cos(x - 90)$ $+30^\circ$

$5 \measuredangle -60$

*

$A \measuredangle \theta_1$

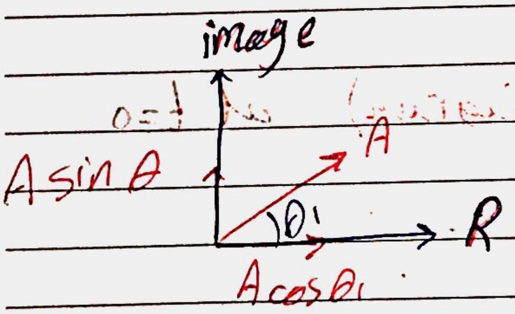
$B \measuredangle \theta_2$

* $(A \measuredangle \theta_1)(B \measuredangle \theta_2)$

$AB \measuredangle \theta_1 + \theta_2$

* $\dfrac{A \measuredangle \theta_1}{B \measuredangle \theta_2} = \dfrac{A}{B} \measuredangle \theta_1 - \theta_2$

polar $A \measuredangle \theta \rightarrow$ rec form $j = \sqrt{-1}$



$A\cos\theta_1 + jA\sin\theta_1$

$A_2 = \sqrt{R^2 + imag^2}$

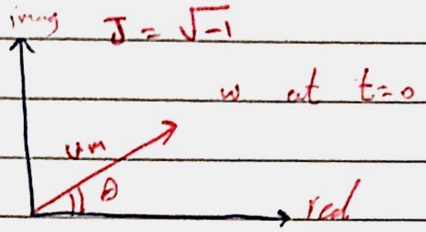
$\theta = \tan^{-1} \dfrac{imag}{real}$

time domain:

$V(t) = V_m \cos(\omega t + \theta)$

→ phasor domain (polar form)

$V_m \angle \theta$



Ex $V(t) = -10 \sin(\omega t + 30) + 20 \cos(\omega t - 45)$ same ω *

$\underbrace{-10 \sin(\omega t + 30)}_{V_1}$, $\underbrace{20 \cos(\omega t - 45)}_{V_2}$

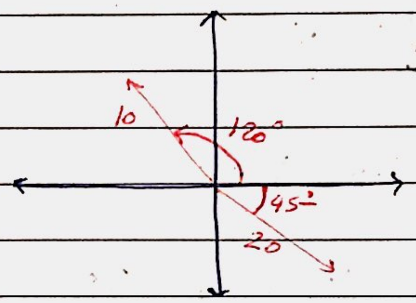
$V_1 \Rightarrow -10 \cos(\omega t + 30 - 90)$ → +180

$V_1 = 10 \cos(\omega t + 120^\circ)$

$V_2 = 20 \angle -45^\circ$

* $\sin x = \cos(x - 90^\circ)$

mode ⇒ complex



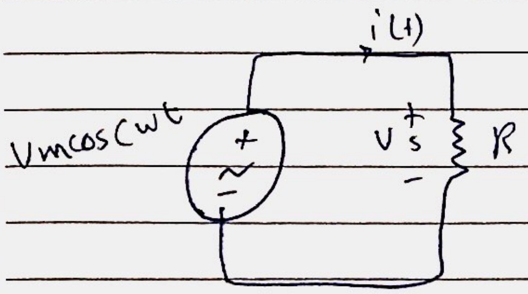
$10\angle 120^\circ + 20 \angle -45^\circ = 10.66 \angle -30.95 = 10.66 \cos(\omega t - 30.95)$

graficaly بالتحليل ←

Source ⇒ phasor load ⇒ R, L, C

$V_m \cos(\omega t + \theta)$

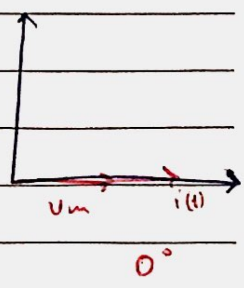
I_m

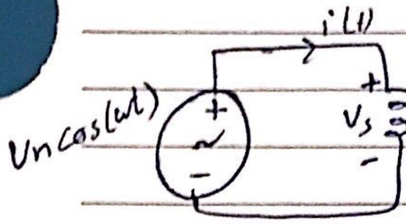


$V = IR$

$V_m \angle 0 = IR$

$I = \frac{V_m}{R} \angle 0$





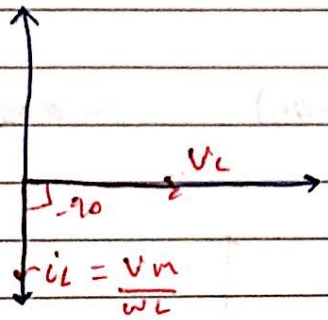
$$V_L = L\frac{di}{dt} \Rightarrow i = \frac{1}{L}\int_0^t V_L(t)\,dt$$

$$i(t) = \frac{1}{L}\int_0^t V_m$$

$$V_L = V_m \cos(\omega t)$$

$$i_L(t) = \frac{V_m}{\omega L}\sin \omega t \Rightarrow \frac{V_m}{\omega L} \measuredangle -90^\circ$$

$\cos(\omega t - 90)$



Phase shift .. 90°

V_L leads i_L by 90°

i_L lags V_L by 90°

$$V_L = V_m \measuredangle 0$$

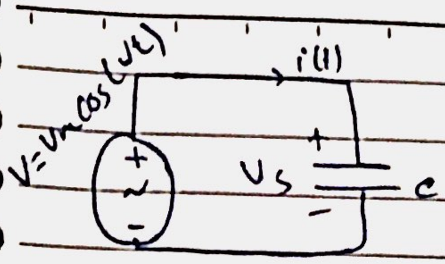
$$I_L = \frac{V_m}{\omega L} \measuredangle -90^\circ$$

$$\frac{V_L}{I_L} = \frac{V_m \measuredangle 0^\circ}{\frac{V_m}{\omega L}\measuredangle -90} = \omega L \measuredangle 90$$

$$\underline{\text{Impedense}} = \frac{V_L}{I_L} = j\omega L\ (\Omega)$$

$\hookrightarrow$ complex

$$\boxed{Z_L = j\omega L\ (\Omega)}$$



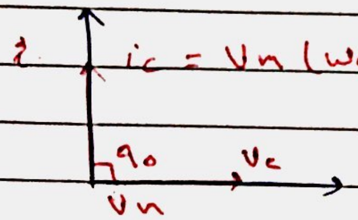
$i_c = C \frac{dv}{dt}$

$i_c = -C\,(v_m(w)\sin wt)$

$i_c = v_m\,(wc)\cos(wt - 90 + 180)$

$i_c = V_m\,(wc) \measuredangle +90$

$V_c = V_m \measuredangle 0$



i_c leads V_c by 90

$Z_c = \frac{V_c}{I_c} = \frac{V_m \measuredangle 0}{V_m W c \measuredangle 90}$

$Z_c = \frac{1}{wc} \measuredangle -90$

$Z_c = \frac{-j}{wc}\ \Omega \qquad V_c = I_c Z_c$

*in summury :-

$R \longrightarrow R\ (\Omega)$

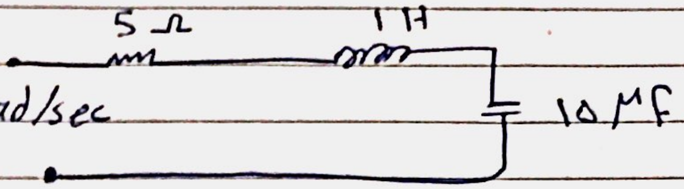
$i \longrightarrow JwL\ (\Omega)$

$c \longrightarrow \frac{-j}{w}\ (\Omega)$

(Z_1, Z_2)

- series
  $Z_{eq} = Z_1 + Z_2$
- parallel
  $Z_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2}$

Ex:-



$W = 10$ rad/sec

$Z_C = \frac{-J}{10^{-4}} = -J10^4$

$Z_L = 10J$

$Z_{eq} = 5 + j10 - j10^4 \Rightarrow 5 - j9990$

* in general :-

$Z =$ Real $\pm$ j x

$Z = R \pm jX$

+ve resistive part

↳ reactive +ve

image +Ve ⇒ inductive impedance

I_Z lags V_Z by θ_Z

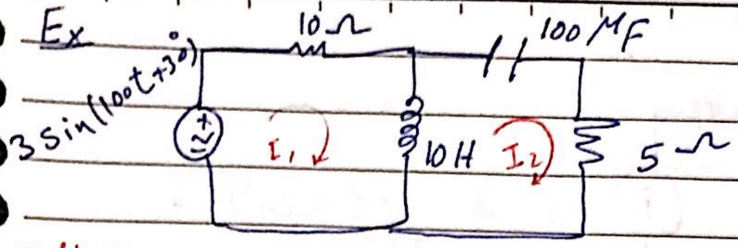
image part −Ve ⇒ capacitive impedence

i_Z lead V_Z by θ_Z

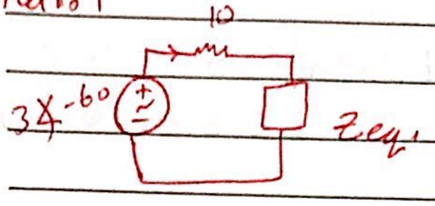
$Z = \frac{V_Z}{I_Z} = \frac{V_m}{I_m} \measuredangle \theta_V - \theta_i$ $(\theta_Z = \theta_V - \theta_i)$ phase shift

Five Apple

Ex



Method 1



$$Z_{eq} = \frac{5 - j100 \times j1000}{5 + j900}$$

$$I_1 = \frac{3\angle -60}{10 + Z_{eq}}$$

$$I_2 = I_1 \left(\frac{Z_{eq}}{5 - j100}\right)$$

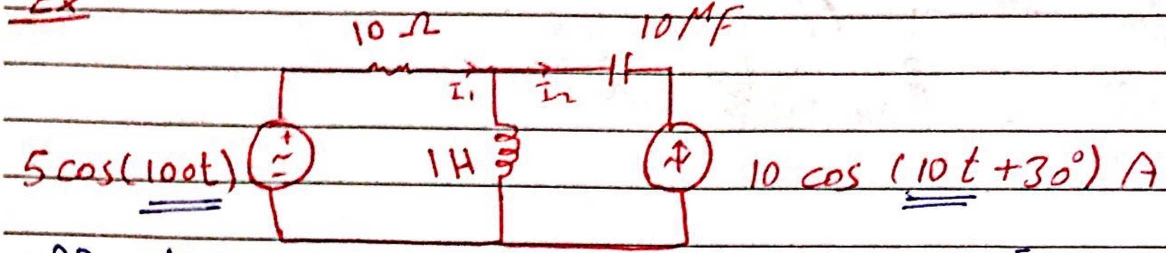
method 2 Loops

$$3\angle -60^\circ = +(10 + j1000)\, I_1 - I_2\,(j1000)$$

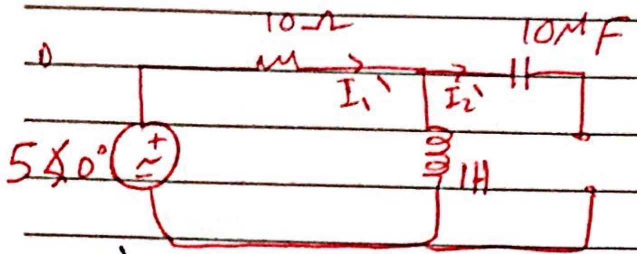
$$0 = (-j1000)\, I_1 + I_2\,(5 + j1000 - j100)$$

$$I_1 = I_2 * \frac{5 + j900}{j1000}$$

Ex



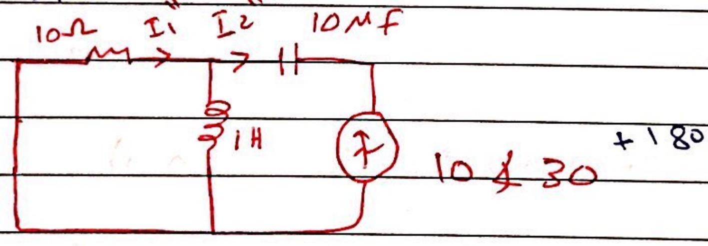
diffirent frequency ⇒ superposition ⇒ Killing $\begin{bmatrix} V.S \rightarrow S.C \\ C.S \rightarrow O.C \end{bmatrix}$



$I_2' = 0$ $\quad\omega = 100$ rad/sec

$Z_{1H} = j100\ \Omega$ $\qquad Z_{10\mu F} = \dfrac{-j}{100+10\mu F} = -j\,1000\ \Omega$

$I_1' = \dfrac{5}{10+j100}$



$W_2 = 10$ rad/sec

$Z_{1H} = j10\ \Omega$ $\qquad Z_{10\mu F} = \dfrac{-j}{(10\mu F)(10)} = -j10^4\ \Omega$

$I_2'' = -10\angle 30 \; +180$

$10 I_1'' + j10\,(I_1'' + 10\angle 30) = 0 \Rightarrow I_1'' = \dfrac{100\angle -60}{10+j10} = A_1''\angle\theta_1''$

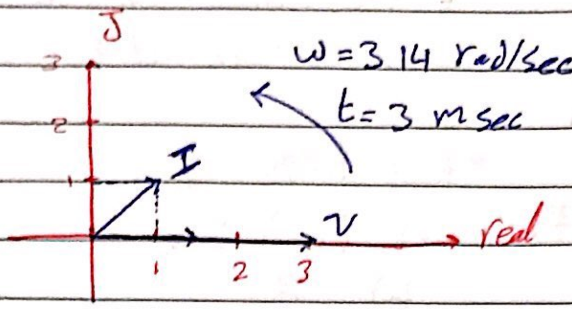
↳ w=10

$I_1' = A_1'\angle\theta_1'$

↳ w=100

$I_1 = A_1' \cos(100t + \theta_1') + A_1'' \cos(10t + \theta_1'')$ ✓

Ex.



$\omega = 314$ rad/sec

$t = 3$ msec

find $V(t)$?

$i(t)$

$V(t) = V_m \cos(314t + \theta_v)$

$i(t) = i_m \cos(314t + \theta_i)$

$V_m = 3V$ $\quad I_m = \sqrt{2}\ A$

$0 = 314 * 3m + \theta_v$

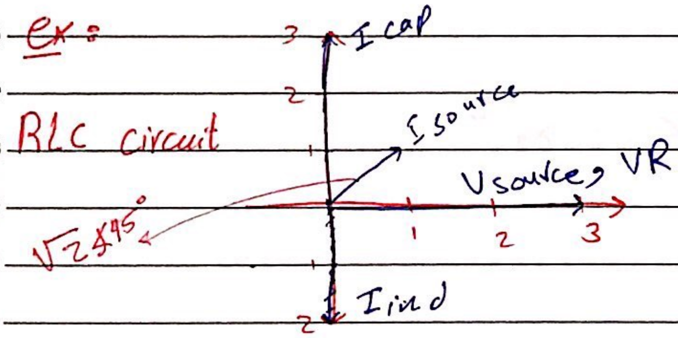
$\theta_v = -314 * 3 * 10^{-3} = -942 \times 10^{-3}$ rad $* \frac{180}{\pi} = -53.97^\circ$

$V(t) = 3\cos(314t - 53.97^\circ)$

$i(t) = \sqrt{2}\cos(314t + -53.97^\circ + 45^\circ)$ I leads V by 45°

ex:

RLC circuit



at $t = 0$

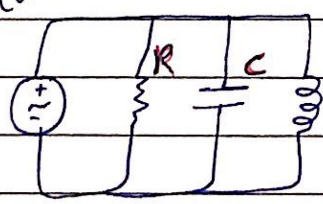
I_L lags V_L by 90°

I cap leads Vcap

Draw the current ponding circuit, show all z in Ω :-

$V_S = 3\cos(\omega t)$

$V_S = 3\angle 0^\circ$



$Z_L = \frac{V_L}{I_L} = \frac{3\angle 0}{2\angle -90} = 1.5j\ \Omega$

$Z_C = \frac{3\angle 0}{3\angle 90} = -J\ \Omega$

$R = 3\angle 0 / I_R$

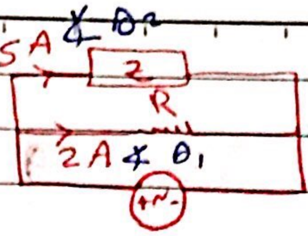
$I_R = \sqrt{2}\angle 45 - (3\angle 90 + 2\angle -90)$

$I_R = \sqrt{2}\angle 45^\circ - j$

$= 1$

$\therefore R = 3\Omega$

Ex



if the current in the impedence Z lags voltage at Z

*I_Z lags V_Z

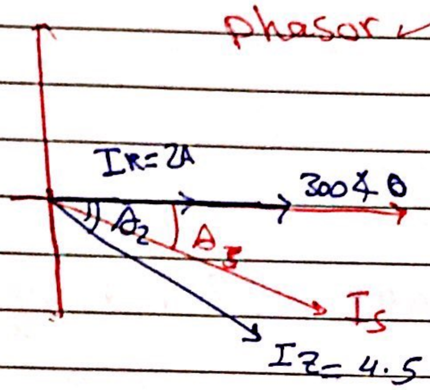
$v(t) = 300\cos(wt)$

$V_Z = 300\angle 0$

a) find $I_Z(t)$

b) Z

phasor



$I_s \cos\theta_s = 2 + 4.5 + \cos\theta_2$

$I_s \sin\theta_s = 4.5 \sin\theta_2$

$I_s^2 = (2 + 4.5 + \cos\theta_2)^2 + (4.5 \sin\theta_2)^2$

$\theta_2 = \#$

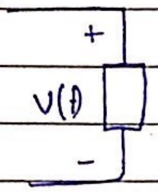
* Ac power

power = sign V I

→ ⊕ absorbed

→ ⊖ deliver

$p(t) = \text{sign } I(t)\, V(t)$



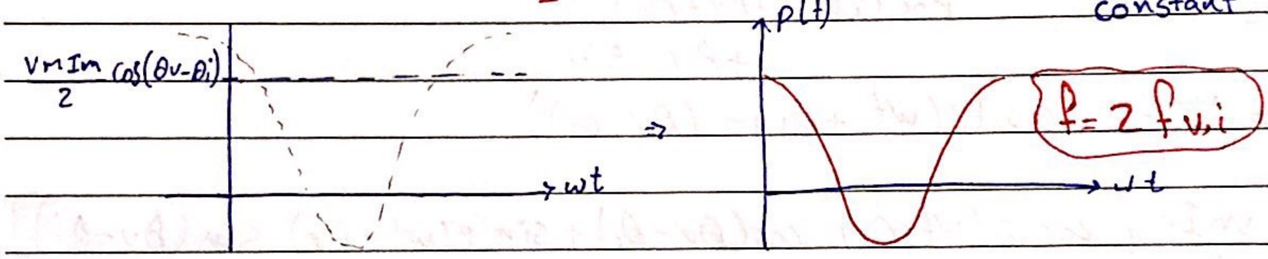
Ac circuit

$v(t) = V_m \cos(\omega t + \theta_v)$

$i(t) = I_m \cos(\omega t + \theta_I)$

$p(t) = V_m I_m \cos(\omega t + \theta v) \cos(\omega t + \theta_I)$

$= \frac{V_m I_m}{2} \left[\cos(2\omega t + \theta_v + \theta_i) + \underbrace{\cos(\theta_v - \theta_i)}_{\text{constant}} \right]$



$\frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = \text{Power avg}$

by $\quad P_{avg} = \frac{1}{T} \int_0^T p(t)\,.dt$

$p(t) = \frac{1}{T} \int_0^T \frac{V_m I_m}{2} \left[\cos(2\omega t + \theta_v + \theta_i) \cos(\theta_v - \theta_i) \right]$

$P_{avg} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$ → phase shift if $\theta_v - \theta_i = 0 \Rightarrow$ in phase

$$P = \frac{V_m I_m}{2}$$

if $\theta_v - \theta_i = \pm 90$ $\qquad P_{avg_{c,L}} = \text{zero}$

$$P = 0$$

$$p(t) = \underbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i)}_{P_2(t)} + \underbrace{\frac{V_m I_m}{2} (\theta_v - \theta_i)}_{P_{avg}}$$

$$P_2(t) = \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i + \theta_v - \theta_v)$$

$$= \frac{V_m I_m}{2} \cos(2(\omega t + \theta_v) - (\theta_v - \theta_i))$$

$$= \frac{V_m I_m}{2} \left[\cos 2(\omega t + \theta_v) \cos(\theta_v - \theta_i) + \sin 2(\omega t + \theta_v) \sin(\theta_v - \theta_i)\right]$$

$$p(t) = P_{avg} + P_2(t)$$

$$= \boxed{\frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \left[1 + \cos(2(\omega t + \theta_v))\right]} \rightarrow \text{Real power}$$

$$+ \boxed{\frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \sin 2(\omega t + \theta_v)} \rightarrow \text{reactive power}$$

Amp real (active) power $= P(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$

Amp reactive power $= Q(t) = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$

Complex power $= \underbrace{\frac{V_m I_m}{2}\cos(\theta_v-\theta_i)}_{\text{Real}} + j\underbrace{\frac{V_m I_m}{2}\sin(\theta_v-\theta_i)}_{Q}$

$\boxed{S = P_{avg} + jQ}$ * S unit → V.A

$V I^* = \frac{V_m I_m}{2} \angle \theta_v - \theta_i \implies \boxed{S = \frac{1}{2}\vec{V}\vec{I}^{*}}$ → conj

$S = P_{avg} + jQ \implies |S| \angle \theta_s = |S| \angle \theta_v - \theta_i$

$-90^\circ < \theta_v - \theta_i < +90^\circ$

$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \geq 0$

$Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)$

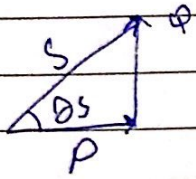
$\theta_v - \theta_i > 0 \Rightarrow Q > 0$ V leads i (inductor)

$\theta_v - \theta_i < 0 \Rightarrow Q < 0$ i lead V (capasitor)

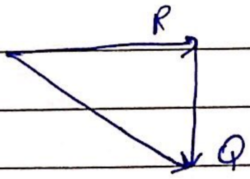
$\theta_v = \theta_i \Rightarrow Q = 0$ pure resistance

$|S| = \sqrt{P^2 + Q^2}$

$\theta_s = \theta_v - \theta_i = \tan^{-1}\frac{Q}{P}$

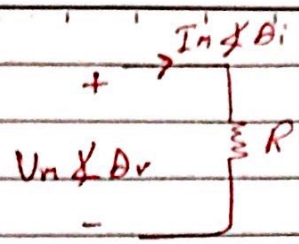


inductor



capasitor

Ex:



ωt

plot instantanous power $(P(t))$
show all max, min, zero crossing

$$P(t) = \frac{1}{2} V_m I_m \cos(\underbrace{\theta_v - \theta_i}_{zero}) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

V_R and I_R are in phase $\rightarrow (\theta_v = \theta_i)$

$$P(t) = \frac{1}{2} V_m I_m [1 + \cos(2\omega t + 2\theta)]^{*}$$

at $\omega t = 0$

$$P(0) = \frac{1}{2} V_m I_m [1 + \cos(2\theta)]$$

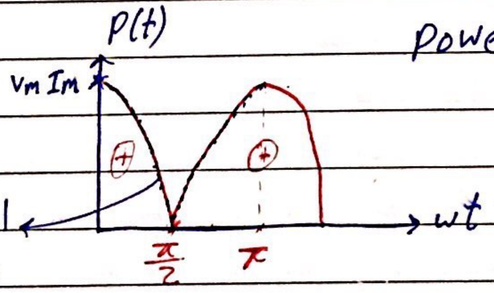
$P(t) \geq 0$

$P_{max} \Rightarrow$ at $2\omega t + 2\theta = 0$ $(\omega t = -\theta)$

$$P_{max} = \frac{1}{2} V_m I_m (1+1) = V_m I_m$$

$$P_{min} = zero \Rightarrow 2\omega t + 2\theta = 180^\circ = \pi \Rightarrow \omega t = \frac{\pi - 2\theta}{2}^{*}$$

if $\theta = 0$



انعكست الدورة
عند π

Powe always +ve in R (absorbed)

$\omega_P = 2\omega_{V,I}$

Peak-peak $\Rightarrow P\text{-}P = V_m I_m$
(max $\rightarrow$ min)

$$P_{avg} = \frac{V_m I_m}{2}$$

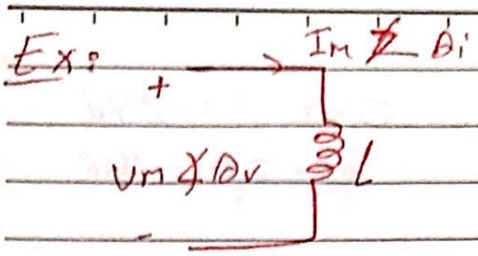
$$P(t) = \frac{1}{2} V_m I_m [\cos(2\omega t) + 1]$$

$\cos(2\omega t) = 0 \Rightarrow P(t) = P_{avg}$

$2\omega t = \frac{\pi}{2}$

$\boxed{\omega t = \frac{\pi}{4}} \rightarrow P_{avg}$

Ex:



$p(t)$ . . -

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$\theta_v - \theta_i =$ +90°

ind $\Rightarrow$ i leads

$\Rightarrow$ if $\theta_v = 0 \Rightarrow \theta_i = -90°$

$$p(t) = \frac{1}{2} V_m I_m \cos(2\omega t - 90°)$$

* $p(\omega t = 0) = 0$

$p(\omega t = ?) = p_{max}$

at $2\omega t - 90° = 0 \Rightarrow \omega t = \frac{90°}{2} = 45° = \frac{\pi}{4}$

$p(0) = 0 \Rightarrow 2\omega t - 90° = 90°$

$\omega t = \frac{180°}{2} = 90°$

$p_{min} \Rightarrow 2\omega t - 90° = 180°$

$\omega t = \frac{270°}{2} = 135° \Rightarrow \frac{3\pi}{4}$

$p_{max} \Rightarrow \frac{1}{2} V_m I_m$

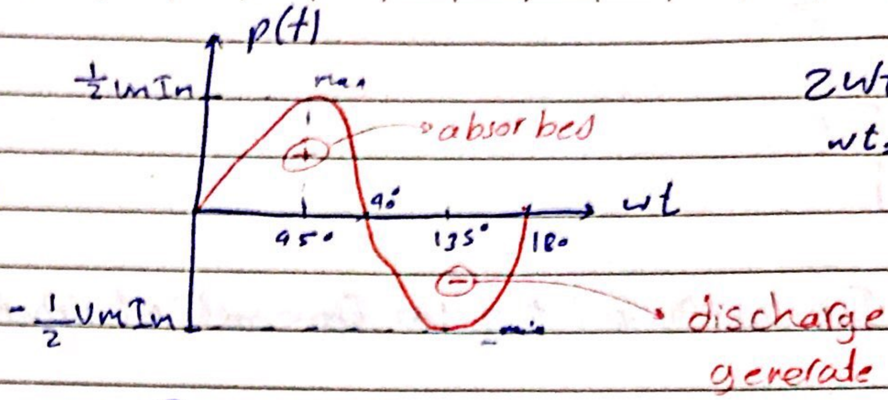
$p_{min} \Rightarrow -\frac{1}{2} V_m I_m$

$\Rightarrow$

$2\omega t - 90 = 270°$

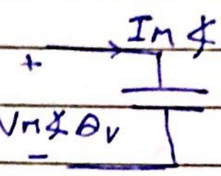
$\omega t = \frac{360}{2} = 180°$

$P_{avg} = 0$
ind



$P\text{-}P = V_m I_m = 2 |S|$

Ex capacitor



plot

i leads v by 90°

$\theta_v - \theta_i = -90$

if $\theta_v = 0 \Rightarrow \theta_i = 90°$

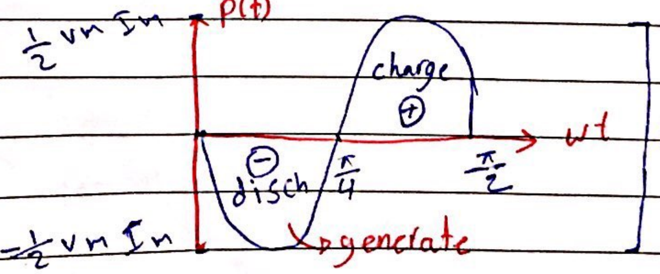
$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$

$\theta_v - \theta_i = -90°$

$\frac{1}{2} V_m I_m \cos(2\omega t + 90°)$

$\frac{1}{2} V_m I_m [\cos 2\omega t \cos 90 - \sin \omega t \sin 90]$

$Q(t) = -\frac{1}{2} V_m I_m \sin 2\omega t$



$P\text{-}P = I_m V_m = 2 S$

$P_{avg} = 0 \Rightarrow$ capacitor
inductor

Ex $v(t) = 100\sqrt{2}\cos(\omega t)$

$I(t) = \sqrt{2}\cos(\omega t - \theta)$

Find a- P-P inst power $(P(t))$ → a) $\theta = 0$

Avg power → b) $\theta = 60^\circ$

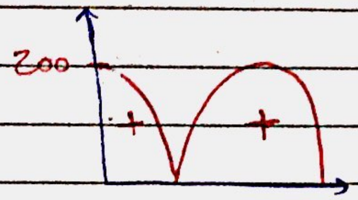
Reactive power → c) $\theta = 80^\circ$

$$p(t) = \frac{1}{2}(100)(\sqrt{2})(\sqrt{2})\cos(-\theta i) + \frac{1}{2}(100\sqrt{2})(\sqrt{2})\cos(2\omega t + \theta i)$$

a) $\theta = 0 \Rightarrow p(t) = 100 + 100\cos(2\omega t) = 100(1 + \cos 2\omega t)$

$\therefore$ $\theta v = \theta i = 0$

↳ in phase ⇒ resistance



$P\text{-}P = 200 \Rightarrow V_m I_m = 100\sqrt{2}\sqrt{2} = 200$

$P_{avg} = 100$ watt

Reactive $= 0$

$$|S| = \text{constant} = \frac{V_m I_m}{2} = \frac{P\text{-}P}{2} = \sqrt{P_{avg}^2 + Q^2}$$

b) $\theta = 60^\circ \Rightarrow 100\underbrace{\cos(-60^\circ)}_{-\frac{1}{2}\times 100} + 100\underbrace{\cos(2\omega t + 60)}_{-1}$

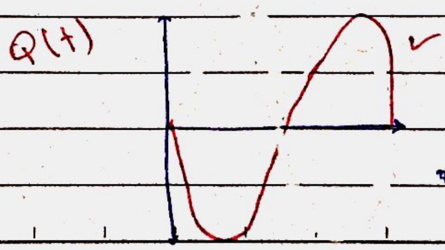
$= 50 + 100\cos(2\omega t + 60^\circ) \rightsquigarrow p(t)$

$\overline{I}$ $P_{avg} = 50$, $P_{max} = 150$

$P\text{-}P = \underline{\underline{200}} \Rightarrow P_{min} = 50 - 100 = -50$

$$Q = \frac{1}{2} V_m I_m \sin(0 - 60)\sin(2\omega t) = 100\sin(-60)\sin 2\omega t$$

$Q = -100\frac{\sqrt{3}}{2} = -50\sqrt{3} \Rightarrow$ capacitive $Q(t) = -50\sqrt{3}(\sin 2\omega t)$

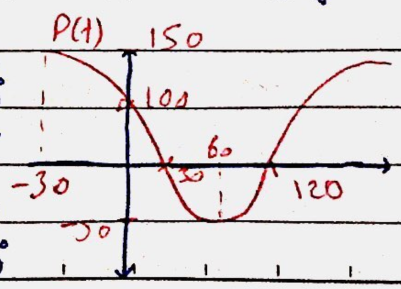


$* \; 2\omega t + 60 = 120^\circ$

$\omega t = \frac{60}{2} = 30^\circ$

$* \; 2\omega t + 60 = 180$

$\omega t = \frac{120}{2} = 60^\circ$



Five Apple

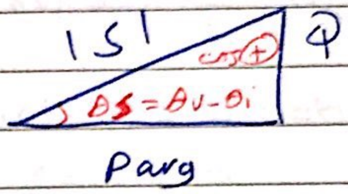
* Ac power

$S = P_{avg} + jQ$

* power factor = PF = pf

$$PF = \frac{P_{avg}}{|S|} = \frac{P_{avg}}{\sqrt{P_{avg}^2 + Q^2}} \leq 1$$

$$|S| = \sqrt{P^2 + Q^2}$$



$$\cos \theta_S = \frac{P_{avg}}{|S|} = pf$$

$\cos(\theta v - \theta i)$

$0 \leq Pf = \cos(\theta v - \theta i) \leq 1$

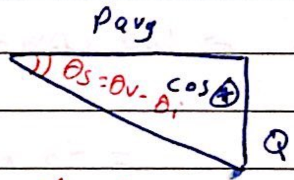
$Q \geq 0 \Rightarrow$ inductive

$Pf = \cos(\theta v - \theta i)$

$\theta v - \theta i \geq 0$

$Q = 0 \Rightarrow pf = 1 \Rightarrow$ unity

Resistive $\Rightarrow P_{avg}$



$pf = 0.8 \rightarrow$ i lags v

$\rightarrow$ i leads V

load capacitive

$pf = \cos(\theta v - \theta i)$

$\theta v - \theta i \leq 0$

cos (+)

pf $\Rightarrow$ lag $\Rightarrow$ i lags v

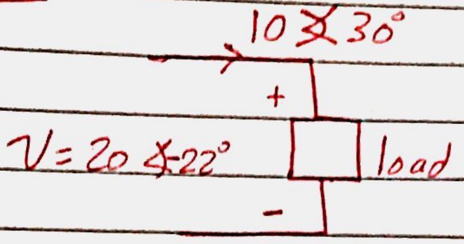
$\therefore$ inductive $\Rightarrow Q \geq 0$

pf $\Rightarrow$ leads $\Rightarrow$ i leads v

$\therefore$ load capacitive $\Rightarrow Q < 0$

Five Apple

Ex



find [1] P_{avg}

[2] pf

[1] $P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_V - \theta_i)$

$= \frac{1}{2}\, 20\ 10\ \cos(-22-30) = 927$ watt

[2] $pf = \frac{427}{S}$, $|S| = \sqrt{P_{avg}^2 + Q^2}$

$Q = \frac{1}{2} V_m I_m \sin(\theta_V - \theta_i)$

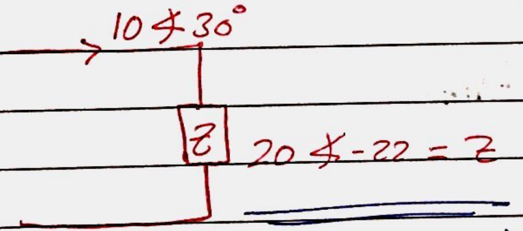
$= \frac{1}{2}(20)(10) \sin(-22-30)$

$= \quad\quad < 0$

$pf = \cos(\theta_V - \theta_i)$

$= \cos(-30+22) = \cos(-55)$ lead

Ex



find [1] P_{avg}

[2] pf

↳ capacitive load

[1] $P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_V - \theta_i)$

$V_Z = I_Z = (10 \angle 50)(20 \angle -22) = 200 \angle 8^\circ$

$P_{avg} = \frac{1}{2} * 200 \times 10 \cos(8 - 30^\circ) =$

$\theta_Z = \theta_V - \theta_i = \theta_S$

OR

$p_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$V = (I_m \measuredangle \theta_i) * |Z| \measuredangle \theta_v - \theta_i$

$V = \underline{\underline{I_m |Z|}} \measuredangle \theta_v$
V_m

$P_{avg} = \frac{1}{2} (I_m) |Z| I_m \cos(\theta_v - \theta_i)$

$= \frac{1}{2} I_m^2 \boxed{|Z| \cos(\underbrace{\theta_v - \theta_i}_{\theta_Z})} \Rightarrow R$

$P_{avg} = \frac{1}{2} I_m^2 R$

$Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)$

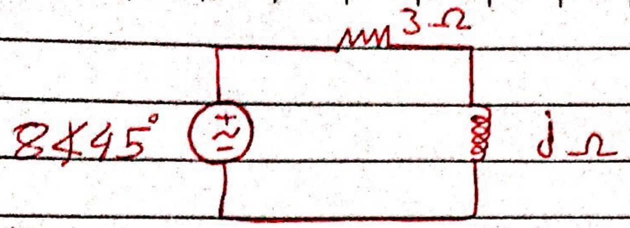
$= \frac{1}{2} I_m^2 |Z| \sin(\theta_Z)$

$Q = \pm \frac{1}{2} I_m^2 \underline{\underline{X}} \rightarrow$ reactince

$|S| = \sqrt{P_{avg}^2 + Q^2}$

$\theta_S = \theta_v - \theta_i = \theta_Z$

Ex



find

[1] Pavg absorbed by R and L.

[2] Pavg supply by the source.

[3] Q supply / consume by the source.

$P_{avg} = \frac{1}{2} I_m^2 R$

$I = \frac{8\angle 45}{3+j} = \underline{2.53}_{I_m} \angle 26.58°$

$P_{avg,R} = \frac{1}{2} (2.53)^2 * 3 = 9.6$ watt consume.

$P_{avg,L} = \frac{1}{2} V_m I_m \cos(\theta v - \theta i)$

$= \frac{1}{2} (8)(2.53) \cos(45 - 26.58) = -9.6$ watt supply

$Q_{source} = -\frac{1}{2} \times 8 \times 2.53 \sin(45 - 26.58)$

≥ 0

$Q < 0 \Rightarrow$ supply

$L \Rightarrow$ consume $Q \geq 0$

$Q_{Load} = +\frac{1}{2} I_m^2 X_L$

$20\angle -22 = Z$

capacitive $\Rightarrow$ i leads v

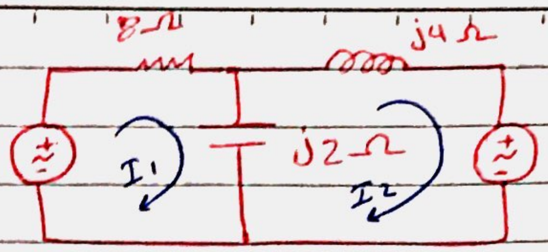
$P_{avg} = \frac{1}{2} (200)(10) \cos(8-30) =$

$\theta z = \theta s = \theta v - \theta i$

$V_Z = I Z = (10\angle 30)(20\angle -22) = 200\angle 8°$

Five Apple

Ex



a) pavg in each element

b) find pr for the voltage sources

$p_{avg} = \frac{1}{2} V_m I_m \cos(\theta v - \theta i)$, $p_{avg} = \frac{1}{2} I_m^2 * R = \frac{1}{2} \frac{V_m^2}{R}$

mesh $\quad 40\angle 0 = (8 - j2) I_1 - I_2 (-j2) \quad --- \boxed{1}$

$-j20 = -I_1(-j2) + I_1 (j4 - j2) \quad --- \boxed{2}$

cramers

$$\underset{20\angle -90}{\begin{bmatrix} 40\angle 0 \\ -j20 \end{bmatrix}} = \begin{bmatrix} 8-j2 & j2 \\ j2 & j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$B = AX \Longrightarrow X = A^{-1} B$

$I_1 = 5 \angle 53.14^\circ \qquad I_2 = 13.6 \angle -17.11$

sings (+, -)

$P_{avg\,40\angle 0} = -\frac{1}{2}(40)(5)\cos(0 - 53.14)$

$= -100 \cos(-53.14) = -60$ watt (generate Power)

(deliver)

$P_{avg\,j2} = +\frac{1}{2}(20)(13.6)\cos(90 - (-17.6)) = -40$ watt generate

$Q_{40\angle 0} = -\frac{1}{2}(40)(5)\sin(0 - 53)$

$Q_{40\angle 0} > 0$ # consume

$Q_{j20} = + \frac{1}{2} (20)(13.6) \sin(90 - 17.11)$

$Q_{j20} > 0$ # consume

$Q_{j4} = + \frac{1}{2} I^2 * X = + \frac{1}{2} * (13.6)^2 * 4$

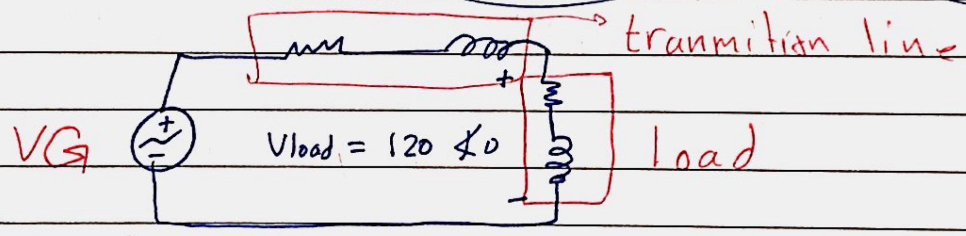
$Q_{-j2} = - \frac{1}{2} (|I_1 - I_2|)^2 * 2$ generate

$pf = \cos(\theta_v - \theta_i)$

$Pf_{S_1} = 40 = \cos(0 - 53)$ lead

$Pf_{S_2} = j20 = \cos(90 - (-17.11)) = -$ lag

Ex



$Z_{T.L} = 1 + j2 \ \Omega$ $\quad Z_{load} = 8 + j6 \ \Omega$

find real and reactive power supplies by the generator?

$P_{avg,source} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)$

$I = \frac{V_{load}}{Z_{load}} = \frac{120 \angle 0}{8 + j6} = 12 \angle -36.9 \text{ A}$

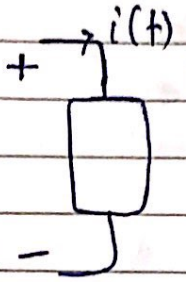
KVL

$V_G = (12 \angle -36.9)(1 + j2) + 120 \angle 0 = 144.6 \angle 4.8° \text{ V}$

$P_{avg} = -\frac{1}{2}(144.5)(12)\cos(4.8-(-36.9)) = 1295$ watt

$Q = -\frac{1}{2}(144.5)(12)\sin(4.8+36.9) = 1154$ VAr

Ex8



$v(t) = 110\cos(\omega t + 60^\circ)$

$i(t) = 15\sin(\omega t - 20^\circ)$

$\hookrightarrow \cos(\omega t - 110^\circ)$

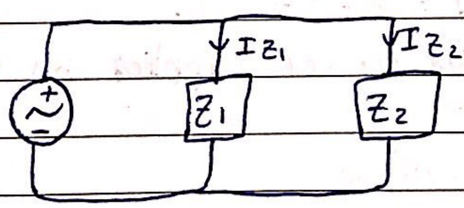
a) $p(t) = \frac{1}{2}(110)(15)\cos(170^\circ) + \frac{1}{2}(110)(15)\cos(2\omega t - 50)$

b) $P_{avg} = -821.9$ W (generate)

c) $Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i) = 825\sin(170) = \underline{143.26}$ VAr
consume

$p.f = \cos(170) = -0.984$ lagging (ind + Resistive)

Ex8



$Z_1 = 50\angle 45^\circ$, $Z_2 = 25\angle 30^\circ$

a) I_{Z_1}, I_{Z_2} c) I_{tot}

b) S absorbed d) S_{source}

Five Apple

Scanned with CamScanner

Soll:- $I_{Z_1} = 1 \measuredangle -45^\circ$ A $\quad I_{Z_2} = 2 \measuredangle -30$ A

$I = 3 \measuredangle -35^\circ$

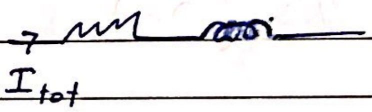
$S_{Z_1} = \frac{1}{2}(50 \measuredangle 0)(1 \measuredangle 45) \Rightarrow \frac{1}{2}\vec{I}\vec{U}$ vectors

$= 25 \measuredangle 45^\circ$ ind

$S_{Z_2} = 50 \measuredangle 30^\circ$

$S_{source} = -(S_1 + S_2) = -74.86 \measuredangle 42.5^\circ$

Ex different source with diff frequencies in the circuit

$P_{avg} = \frac{1}{2}\left[I^2_{1,\omega_1} + I^2_{2,\omega_2} \right] R \Rightarrow$ 

Ex $i(t) = 2\cos 10t - 3\cos 100t$, $R = 4\,\Omega$

$P_{avg} = \frac{1}{2}[2^2 + 3^2]\,4 = 26$ watt

$i(t) = 2\cos(10t) - 3\cos(10t)$

$P_{avg} = \frac{1}{2}(1)^2 \times 4 = 2$ watt

$i(t) = 2\cos(10t) - 3\sin(10t)$

$\hookrightarrow i(t) = 2\cos(10t) + 3\cos(10t - 90 + 180)$

$\Rightarrow 2 \measuredangle 0 + 3 \measuredangle 90 = \sqrt{13} \measuredangle 56.3$

$P_{avg} = \frac{1}{2}(\sqrt{13})^2 \times 4 = 26$ watt

Ex: $v(t) = 8\sin(200t) - 6\cos(200t - 45°) - 5\sin(100t) + 4$

$\Rightarrow \underbrace{8\cos(200t - 90) + 6\cos(200t - 45 + 180)}_{\text{same } f} - 5\sin(100t) + 4$

$8 \angle -90 + 6 \angle 135° = 5.67 \angle -48°$

$v(t)$ has three components:

1) $5.67 \angle -48°$ $(\omega = 100)$

2) $5 \angle -90°$ $(\omega = 200)$

3) 4 (DC)

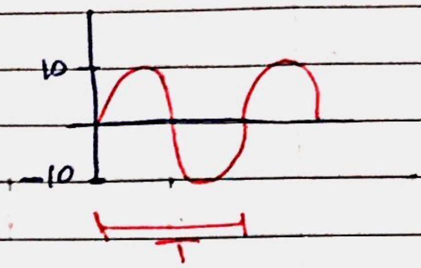
$$P_{avg} = \left(\frac{1}{2}[5.67^2 + 5^2] + 4^2 \right) \times \frac{1}{4}$$

$$\Rightarrow \frac{1}{2}\frac{V_{AC}^2}{R} + \frac{V_{DC}^2}{R}$$

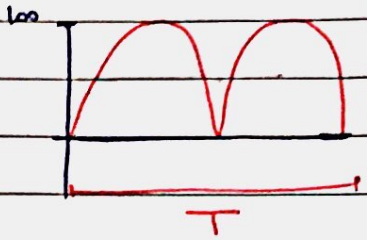
*Effictive value of voltage/current (RMS)

$Ac \Longrightarrow v(t)$

$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt = 0$



if $[v(t)]^2 \Rightarrow$

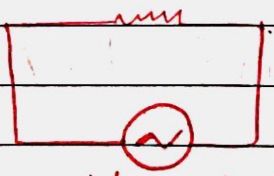


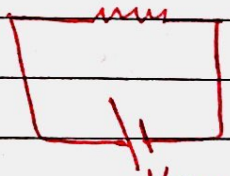
$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$ root mean square (also applyable for current)

$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T (V_m\cos(\omega t))^2\,dt} \Longrightarrow V_{rms} = \frac{V_m}{\sqrt{2}}$ only sinosoidal source

$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$

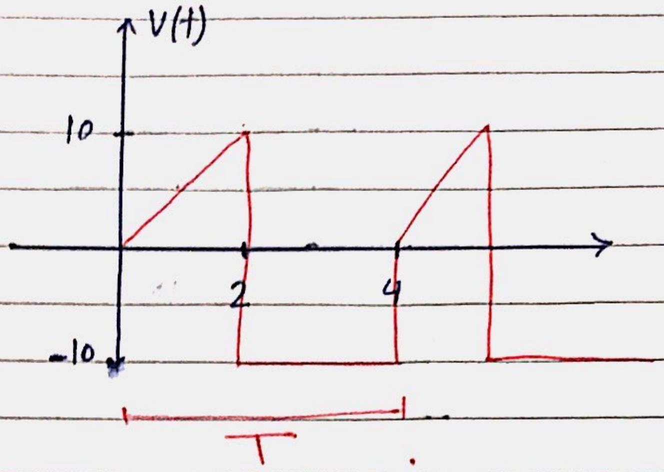
$\vec{S} = \frac{1}{2}\vec{V} I^* \Longrightarrow |S| = V_{rms} I_{rms}$





$P_{avg} = \left(\frac{V_m}{\sqrt{2}}\right)^2 / R$ * if we want to replace Ac source with Dc giving same power.

Ex if $v(t)$:- find V_{rms}



$$V_{rms} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2 dt + \int_2^4 (-10)^2 dt\right]} = \sqrt{\frac{1}{4}\left[\frac{25t^3}{3}\Big|_0^2 + 100t\Big|_2^4\right]}$$

$$= \sqrt{\frac{1}{4}\left[25 \times \frac{8}{3} + 200\right]} = 8.165 \; V_{rms}$$

* different sources $\omega_1 \neq \omega_2$

if V, I are sinosoidal (cos, sin) $\Rightarrow V_{rms\,DC} = V_{DC}$

$V_{rms} = \sqrt{V_{rms_1}^2 + V_{rms_2}^2 + \dots}$, $I_{rms} = \sqrt{I_{rms_1}^2 + I_{rms_2}^2 + \dots}$

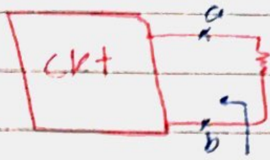
Ex: $v(t) = 6\cos(25t) + 5\sin(30t) + 4$

$$V_{rms} = \sqrt{\left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2 + 4^2} = 6.82 \; V_{rms}$$

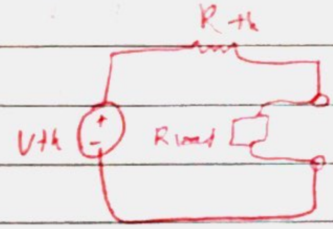
$v(t) = 6\cos(25t) + 4\sin(25t + 30^\circ)$ لا تطبق العلاقة السابقة.

$\Rightarrow V_{rms} = 6.16 \; V_{rms}$ $\frac{V_m}{\sqrt{2}}$ بعد جمعهم وتطبيق

* max power transfore



$R = ?$ $\quad P_{load} \Rightarrow Max \Rightarrow$ min losses



$$P = I^2 R_{load}$$

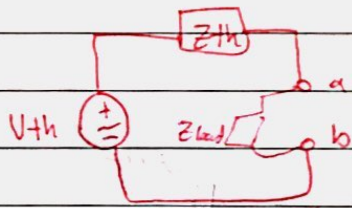
$$= \left(\frac{V_{th}}{R_{th} + R_{load}}\right)^2 * R_{load}$$

$$P_{max\,DC} = \frac{V_{th}}{4R_{th}}$$

$\hookrightarrow R_{th} = R_{load}$

$\Rightarrow$ 

$Z_{load} = R \pm jX$

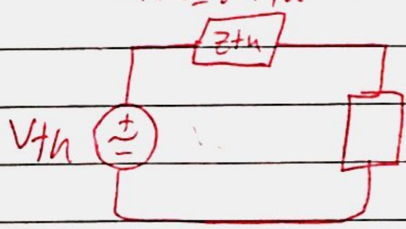


$$P = \frac{1}{2} I^2 R_{load}$$

$$I = \left| \frac{V_{th} \angle \theta}{Z_{th} + Z_{load}} \right|$$

$$\boxed{Z_{load} = Z_{th}^{*}}$$

$R_{th} \pm jX_{th}$

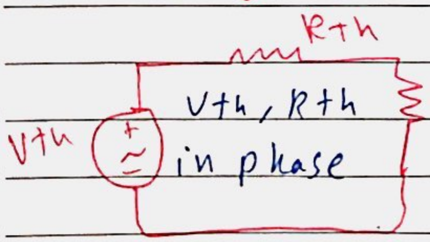


$R_{th} \pm jX_{th}$

$$P_{R_{th}} = \frac{1}{2} I^2 R_{th}$$

$\Downarrow$

$$P = \frac{1}{2}\left(\frac{V_{th}^{*}}{2R_{th}}\right)^2 R_{th}$$



R_{th}

$$= \frac{V_{th}^2}{8R_{th}} = P_{max}$$

under max condition

$$\eta = \frac{P_{out}}{P_{in}}$$

$$P_{out} = P_{load} = \frac{|V_{th}^2|}{8R_{th}}$$

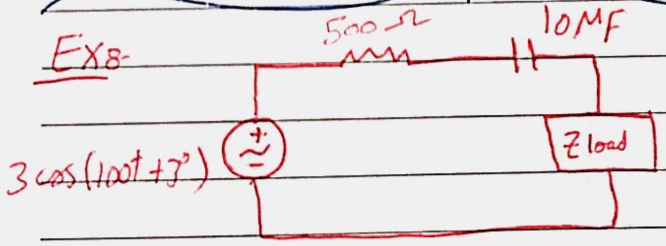
$$P_{sourse} = P_{in} = \frac{1}{2} V_{th}\, I \cos(\theta_v - \theta_i)$$ 1

$$I = \frac{V_{th}}{2R_{th}}$$

$$P_{in} = \frac{1}{2} V_{th}\left(\frac{V_{th}}{2R_{th}}\right) = \frac{V_{th}^2}{4R_{th}}$$

$$\frac{P_{out}}{P_{in}} = 0.5 = 50\%$$

Ex8-



a) what is the value of Z_{load} to allow absorbed max power ⟹ Th

b) " " " " " inductive component . . .

a)

$$10M \Rightarrow \frac{-J}{10M \times 100} = -J1000\ \Omega$$

$$Z_{th} = 500 - J1000$$

$$Z_{load} = 500 + J1000$$

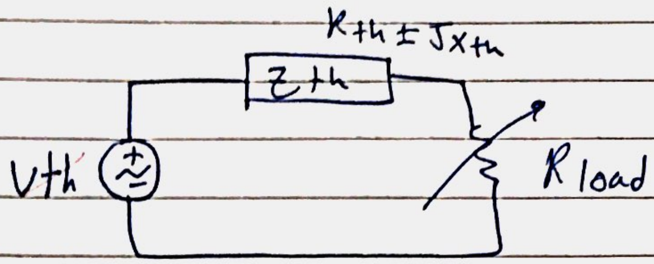
$$P_{max\ load} = \frac{V_{th}^2}{8R_{th}} = \frac{3^2}{8 \times 500}$$

b) $X = \omega L$

$$\frac{1000}{100} = 10\ H = L$$

Case II

load $\Rightarrow$ pure Resistive

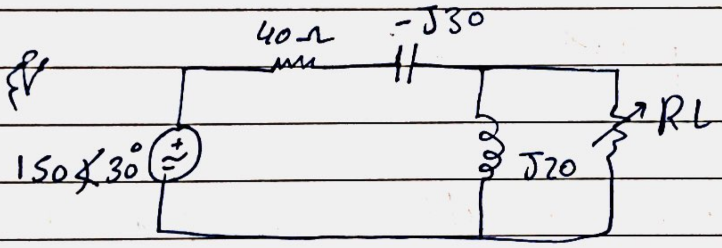


* $R_{load} = |Z_{th}|$

$P_{max} = \frac{1}{2}|I|^2 R_{load}$

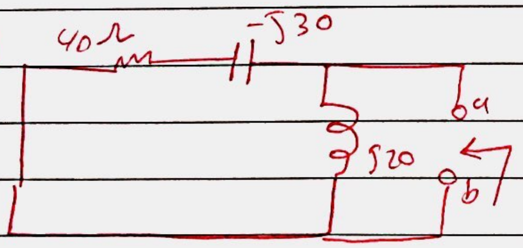
$I = \frac{V_{th}}{Z_{th} + R_{load}} = |I| \angle 0$

Ex



1) find RL that will absorbed max power from ckt
2) Pmax load

1) $R_L = |Z_{th}|$



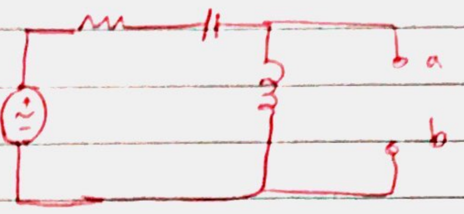
$40 - J30 \; // \; J20$

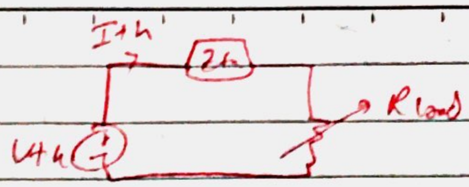
$Z_{th} = \frac{(40 - J30)(J20)}{40 - J10}$

$= 9.412 + J22.35$

$|Z_{th}| = 24.25 \, \Omega = R_{load}$

2)



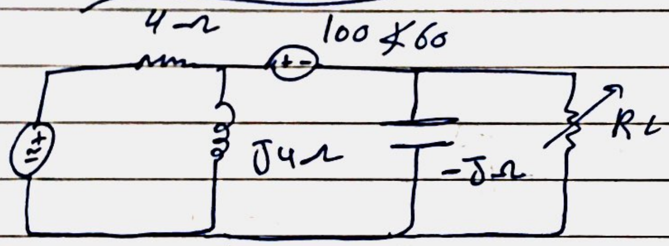


$V_{o.c} = \left(\frac{150 \angle 30}{40 - j30 + j20}\right) \times j20 = 72.76 \angle 134^\circ$

$I_{th} = \frac{72.76 \angle 134^\circ}{9.412 + j22.35 + 24.25} = 1.8 \angle 100.42$

$P_{max} = \frac{1}{2}(1.8)^2 \times 24.25 = 39.29$ watt

Ex:-



a) find RL wich would resalt in max power transfere.
b) power max, load.

a) V.S → short ckt   RL → O.C

$4 \parallel j4 \parallel -j$

$\left(\frac{4(j4)}{4+j4}\right) \parallel -j$

$Z_{th} = 1.26 \angle -71.6^\circ$ Rload $= 1.26\,\Omega$

b) $R_{load} \longrightarrow V_{o.c}$

Z_{th}

by mesh

$$75\angle 0 = (4+J4)I_1 - J4\,I_2$$

$$-100\angle 60 = -J4\,I_1 + (J4-J)I_2$$

$\vdots \quad \vdots$

$V_{o.c} = I_2(-J) \Rightarrow P_{max}$ - -