SIGNALS

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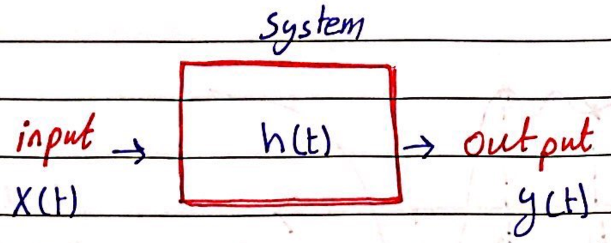
POWERUNIT

Signals & Systems.



x(t) : input signal
y(t) : output signal.
h(t) : impolse signal

* Classification of Signals

[1] Continous or discrete signal.

[2] Analog or digital signal

[3] casual or Non casual signal.

[4] Deterministic or Random signal

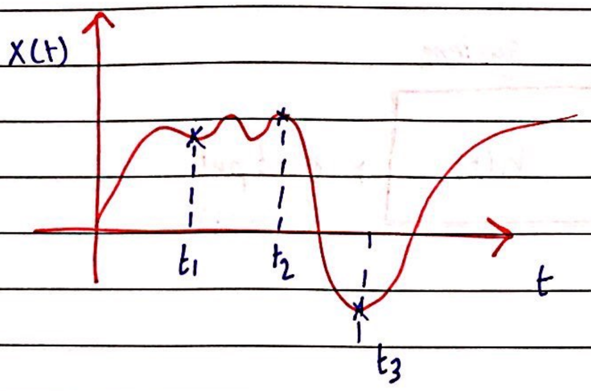
[5] odd or even signal

[6] power or energy signal

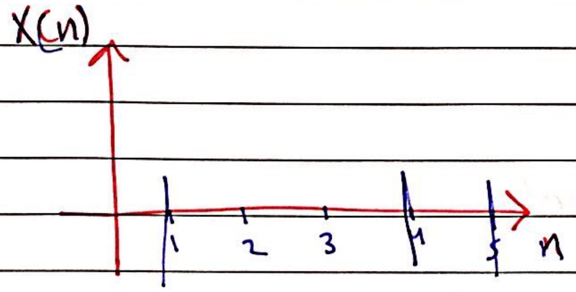
[7] stable or unstable signal

[8] periodic or aperiodic signal.

$\boxed{1}$ Cont. or dis.

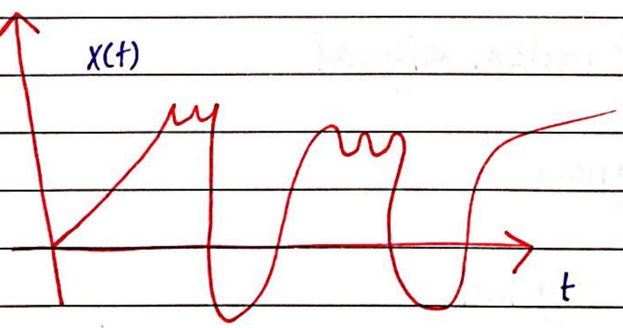


Continous signal has a value at any time.

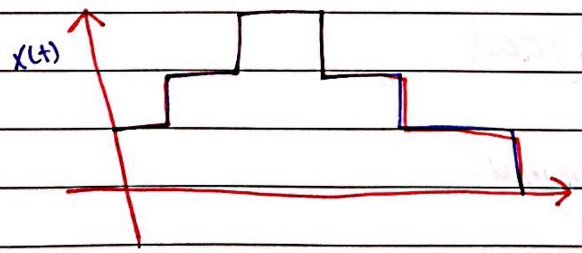


Discrete signal at certain values.

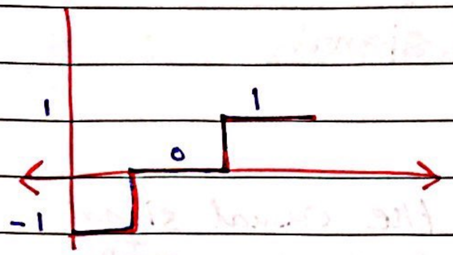
$\boxed{2}$ Analog or digital signal



analog (continous)



digital (continous)



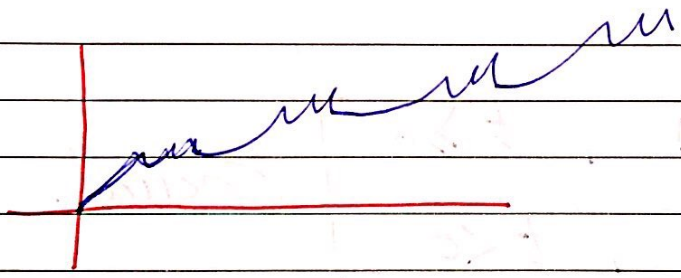
Digital signal.

* Deterministic or Random signal.

↳ we can predict the future value.

$x(t) = e^{-4t}$

$x(t) = 4\cos 10t$



Random signal.

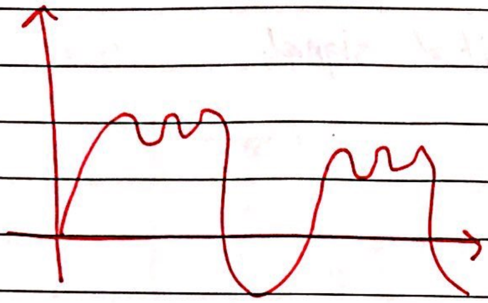
* Stable or unstable signal

$\lim_{t \to \infty} x(t) = K \Rightarrow$ stable

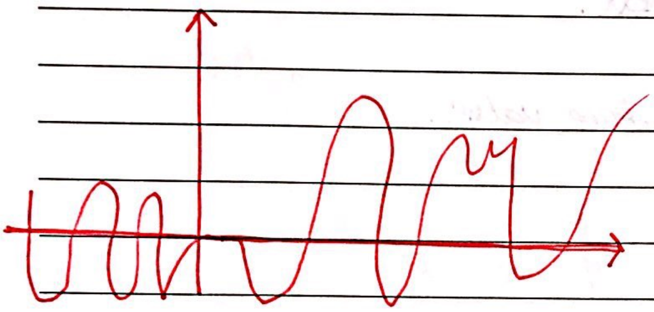
قيمة محددة

$\lim_{t \to \infty} x(t) = \infty, -\infty \Rightarrow$ unstable.

* Casual or non casual signal



the casual signal starts from ZERO.



the non-casual signal $(-\infty, \infty)$, doesn't start from zero.

$$x(t) = \begin{cases} x(t) & , \ t \geq 0 \\ 0 & , \ t < 0 \end{cases}$$ Casual.

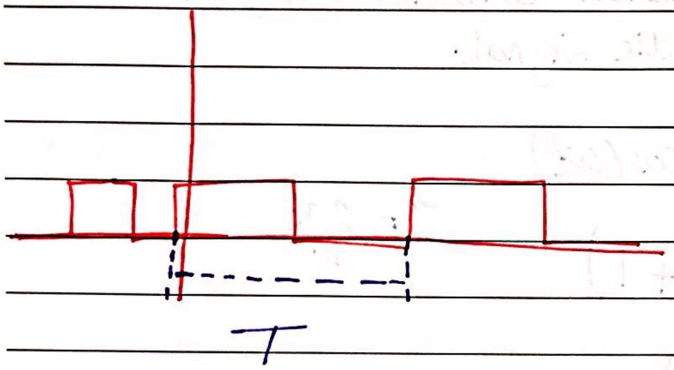
$$x(t) = \begin{cases} x_1(t) & , \ t \geq 0 \\ x_2(t) & , \ t < 0 \end{cases}$$ non casual.

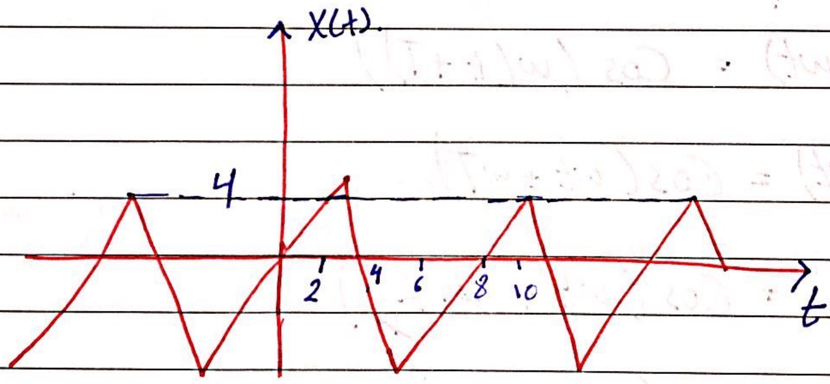
* Periodic OR non-periodic signal.

↳ It repeats it self every (T)

T: periodic time.

$$x(t) = x(t+T) \quad , -\infty < t < \infty$$

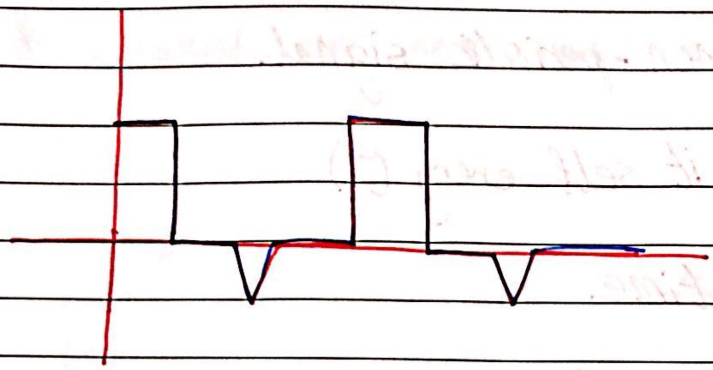




$T = 8$ sec.

$W = \frac{2\pi}{T} = \frac{\pi}{4}$ rad/sec.

$F = \frac{1}{T} = \frac{1}{8} = 0.125$ HZ



non-periodic system.
(لإنها مش من $(-\infty,\infty)$)

~~every indi~~ every individual sinsoidal signal is a periodic signal.

$$x(t) = A\cos(wt)$$

$$x(t) = x(t+T) \qquad T = \frac{2\pi}{w}$$

$$x(t) \overset{?}{=} x(t+T)$$

$$\cos(wt) = \cos(w(t+T))$$

$$\cos(wt) = \cos(wt + wT)$$

$$\cos(wt) = \cos(wt + \cancel{w}\cdot\frac{2\pi}{\cancel{w}})$$

$$\cos(wt) = \cos(wt + 2\pi).$$

→ It's periodic signal

بتعيد نفسها كل 360°.

$$= (\cos wt \cdot \underset{1}{\cos 2\pi}) + \cancel{(\sin wt \cdot \underset{0}{\sin 2\pi})}.$$

$$= \cos(wt).$$

* Ex: $x(t) = 4\cos 40\pi t$

$A = 4$

$w = 40\pi$

$F = \frac{W}{2\pi} = \frac{40w}{2\pi} = \cancel{20}\ 20\ HZ.$

$T = \frac{1}{F} = \frac{1}{20}$ sec.

* Ex: $x(t) = 3\sin 5t$

$A = 3$

$w = 5$

$T = \frac{2\pi}{w} = \frac{2\pi}{5}$ sec.

* Ex: $x(t) = e^{-3t}\, u(t)$ → يعمل عل الـ limits.

$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$ u(t): unit step function.

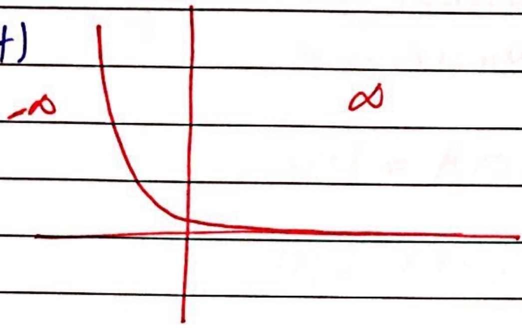
$x(t) = 5\,ut \quad x(t) = \begin{cases} 5, & t \geq 0 \\ 0, & t < 0. \end{cases}$

$$x_1(t) = e^{-3t}$$

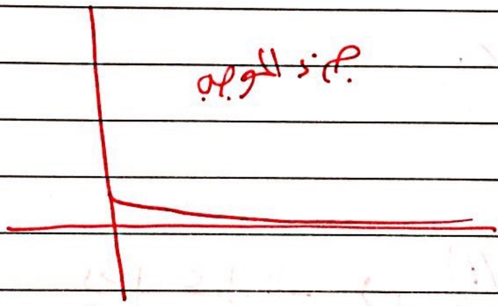
$$x_2(t) = e^{-3t}u(t)$$

$$x_3(t) = e^{-3t}u(-t).$$

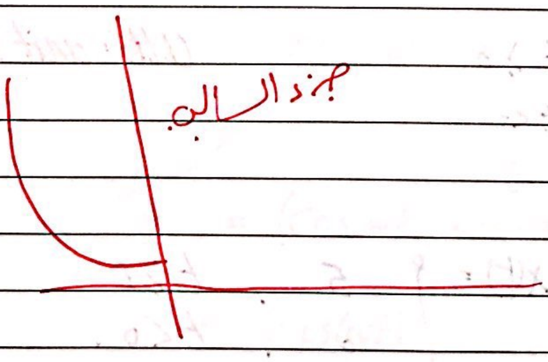
$x_1(t)$



$x_2(t)$



$x_3(t)$



$$u(-t) = \begin{cases} 0 , & t \geq 0 \\ 1 , & t < 0. \end{cases}$$

$$e^{\pm j\omega t} = e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$a + jb = r\angle\theta$$

Ex: $x(t) = 3\cos 10t + 4\sin 20t$.

is it periodic or aperiodic signal?

sol: $T_1 = \frac{2\pi}{10} = \frac{\pi}{5}$

$T_2 = \frac{2\pi}{20} = \frac{\pi}{10}$

$\therefore T_1 / T_2 = \frac{\cancel{\pi}}{\cancel{5}} \times \frac{10}{\cancel{\pi}} = 2 \rightarrow$ int.

$\therefore T_0 = T_1 = 2T_2 = \frac{\pi}{5}$

$\therefore$ periodic signal.

Ex: $y(t) = 4\cos 10t - 6\cos 20\pi t$

$T_1 = \frac{2\pi}{10} = \frac{\pi}{5}$

$T_2 = \frac{2\pi}{20\pi} = \frac{1}{10}$.

$\frac{T_1}{T_2} = \frac{\pi}{5} \times \frac{10}{1} = 2\pi \longrightarrow$ not int.

$\therefore$ non-periodic signal.

عدد منتهي

$*\ \dfrac{T_1}{T_2} = \dfrac{0.02}{0.45} = \dfrac{2}{45} \longrightarrow$ periodic.

عدد غير منتهي

$*\ \dfrac{T_1}{T_2} = \dfrac{\sqrt{2}}{6}$ non periodic.

Ex:- $x(t) = 4e^{-j12\pi t} + 4\cos 8\pi t$

sol. $T_1 = \dfrac{2\pi}{12\pi} = \dfrac{1}{6}$

$T_2 = \dfrac{2\pi}{8\pi} = \dfrac{1}{4}$

$\dfrac{T_1}{T_2} = \dfrac{1}{6} * \dfrac{4}{1} = \dfrac{2}{3} \begin{array}{l} \to \text{int} \\ \to \text{int} \end{array}$ So, periodic signal.

$T_0 = 3T_1 = 3 * \dfrac{1}{6} = \dfrac{1}{2}.$

Ex: $4\cos 10t . \sin 6t$

sol.

$x(t) = \dfrac{1}{2}\sin(4t) + \dfrac{1}{2}\sin(6t).$

$T_1 = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

$T_2 = \dfrac{2\pi}{6} = \dfrac{\pi}{3}$

$\dfrac{T_1}{T_2} = \dfrac{\cancel{\pi}}{2} * \dfrac{3}{\cancel{\pi}} = \boxed{\dfrac{3}{2}}$ ~~periodic signal~~.

$T_0 = 3T_2$

$= \cancel{3} * \dfrac{\pi}{\cancel{3}}$

$= \boxed{\pi}$, non periodic signal.

* Power or energy signal.

Energy signal:

شرطين → $\boxed{1}$ $E = \int_{-\infty}^{\infty} |x(t)|^2 . dt = K$ k: finite number.

→ $\boxed{2}$ power = Zero.

power signal:

شرطين → $\boxed{1}$ $P = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 . dt$ ← finite.

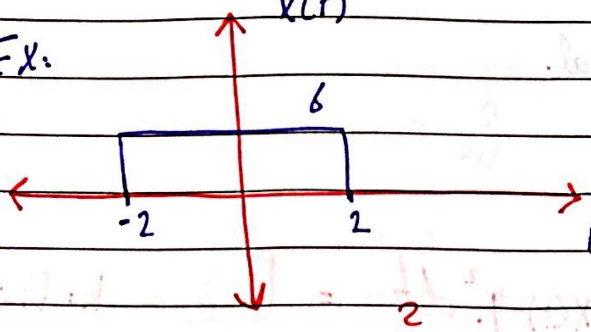
→ $\boxed{2}$ $E = \infty$, infinity.

لاحظ:

periodic → power

non-periodic → energy or non.

Ex:



non periodic

ما عادت نفسها .

بتكررها بال ∞ .

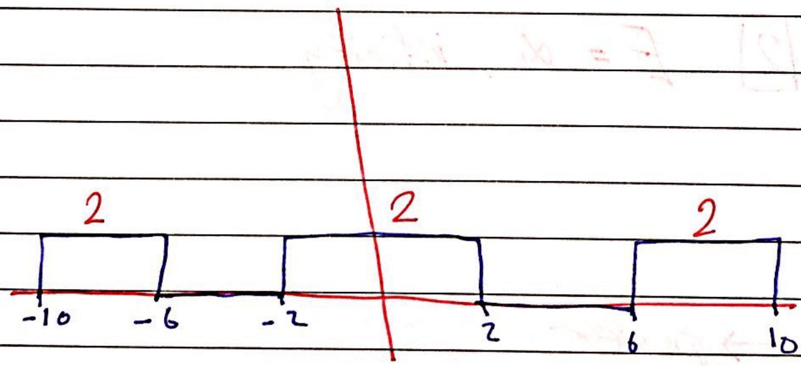
$$E = \int_{-2}^{2} 6^2\, dt = 36t \Big|_{-2}^{2} = 144\ J.$$

$$\text{power} = \frac{1}{\infty} \int \sim dt = 0.$$

$\therefore$ Energy signal. ✓

* اذا كانت non periodic بنفرض $\boxed{\infty = T}$

Ex:



periodic $\qquad \boxed{T = 8}$

$$\text{power} = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt$$

$$= \frac{1}{8} \int_{-4}^{4} 2^2\, dt = \frac{1}{8}\left[4(8)\right] = 4.$$

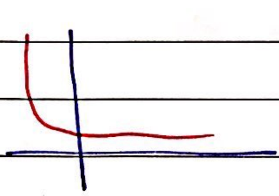
$$E = \int_{-\infty}^{\infty} \sim dt = \infty.$$

Five Apple

* Complex or Real signal.

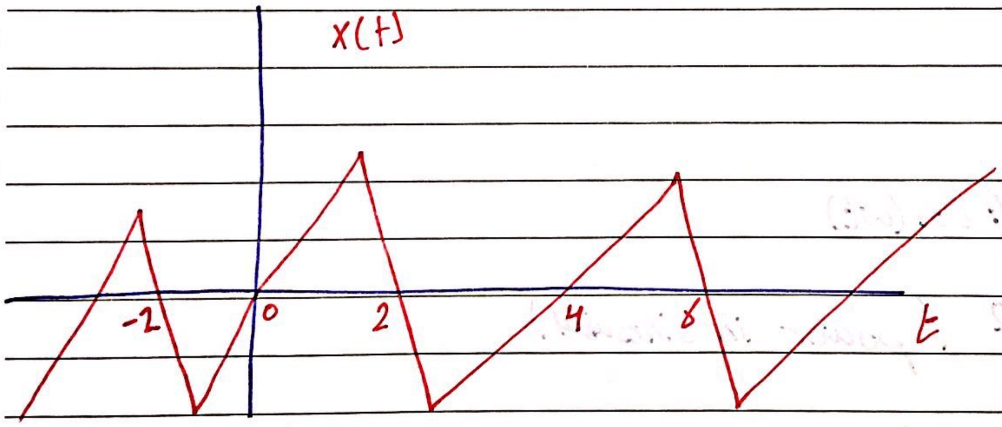
$e^{j5} = \cos 5 + j\sin 5$

$= a + jb$ culer's formula.] Complex signal.

e^{-5t} ] Real signal.

الفرق هو وجود الـ ن.

* تكملة عن power & energy.



$T = 4$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt. = \frac{1}{4} \int_{-2}^{2} \left(\tfrac{3}{2}t\right)^2 dt.$$

$$= \frac{9}{16} \frac{t^3}{3} \Big|_{-2}^{2}$$

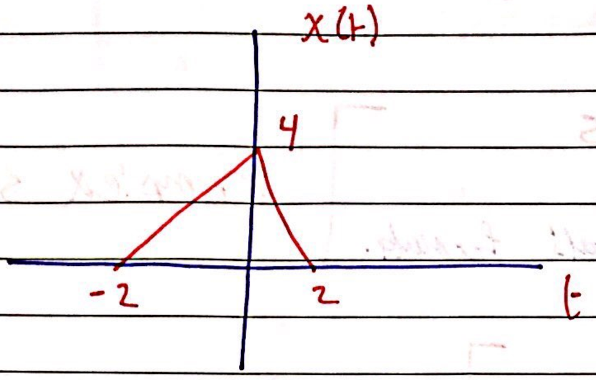
$X(t) = 6/4\,t = 3/2\,t$

$$= \frac{3}{16}\left[+2^3 - (-2)^3\right]$$

$$= \frac{3}{16}\left[8 + 8\right) = 3W.$$

Five Apple

Ex:



- aperiodic signal.

energy signal.

Sol: $\cancel{\int_{-\infty}^{-2} 0^2} + \int_{-2}^{0} 0^2 + \int_{0}^{2} 0^2 + \cancel{\int_{2}^{\infty} 0^2}$

Ex: $X(t) = A \cos(\omega t)$

$P = A^2/2$ (power in sinosuidal)

$X(t) = 3 \cos 10t$

$P = \frac{9}{2} = 4.5$ W.

Ex: $x(t) = 3\sin 10t + 4\cos 20t.$

Sol. $T_1 = \frac{2\pi}{10}$, $T_2 = \frac{2\pi}{20}$

$$\frac{T_1}{T_2} = \frac{2\cancel{\pi}}{10} \cdot \frac{20}{2\cancel{\pi}} = 2.$$

$$T_0 = \cancel{2\pi} = T_1 = 2T_2 = \frac{2\pi}{10}$$, periodic signal.

$$P = 3^2/2 + 4^2/2 \quad W.$$ (super position principle)

Ex: $x(t) = Ae^{jwt}$

$$P = \frac{1}{2T}\int_{-T}^{T} |A \cdot e^{jwt}|^2 . dt.$$

$$= \frac{1}{2T}\int_{-T}^{T} Ae^{jwt} . Ae^{-jwt} . dt.$$

$$= \frac{1}{2T} . A^2 \int e^0 dt = \frac{A^2}{2T} t \Big|_{-T}^{T} = \frac{A^2(\cancel{2T})}{\cancel{2T}}$$

$$= A^2.$$

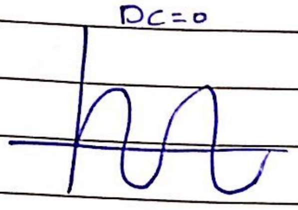
Ex: $y(t) = \cos^2 10t$

Sol. $\cancel{= \frac{1}{2}}$

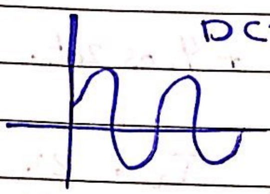
$$\cos^2 10t = \frac{1}{2} + \frac{1}{2}\cos 20t$$

$$P = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2/2 = \frac{1}{4} + \frac{1}{8} = \frac{3}{8} \; w.$$

DC=0



DC= $\frac{1}{2}$.



* Operations on signals.

1. Time shift.

2. Time scale.

3. Refliction / Reversal.

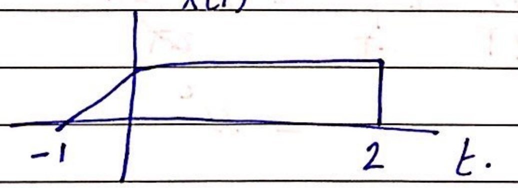
* Time - Shift.

$x(t - t_0)$ → ~~sh~~ shift to the right by t_0 units.

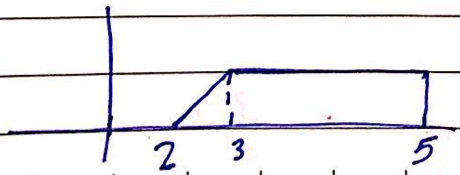
$x(t + t_0)$ → shift to the left by t_0 units.

Ex:

$x(t)$



$x(t - 3)$



Time - scale.

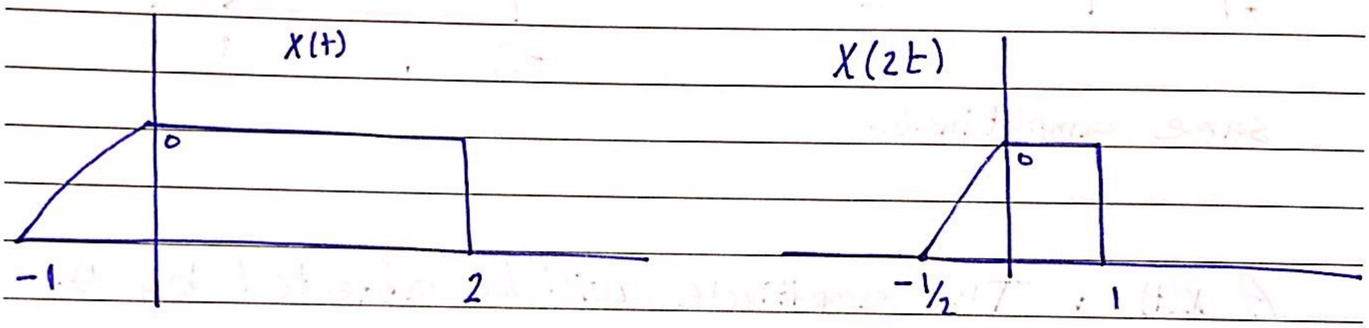
* الأرقام للفعلية.

$X(at)$. كل رقم مفصلي بقسمه على ② .

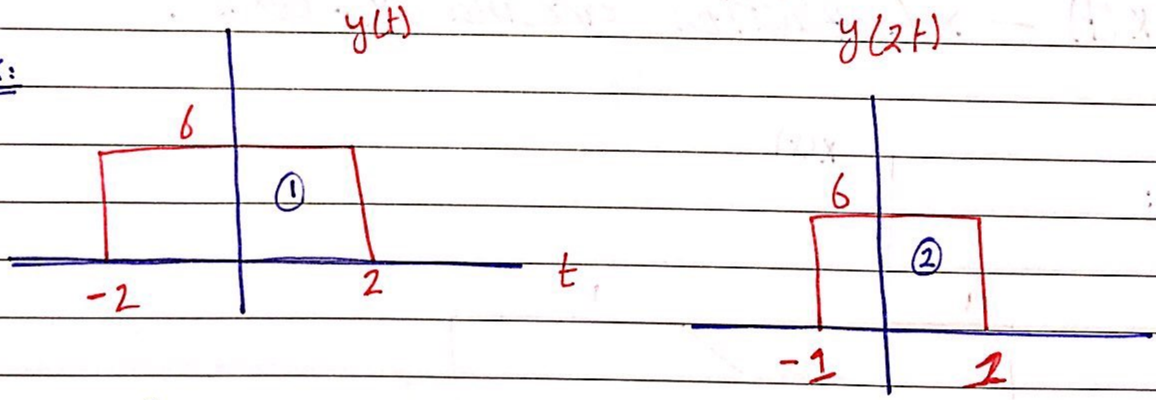
if $|a| > 1 \longrightarrow$ Compression.

if $|a| < 1 \longrightarrow$ expansion. كل رقم مفصلي بضربه بـ ②

Ex: sketch $X(2t)$.



Ex:



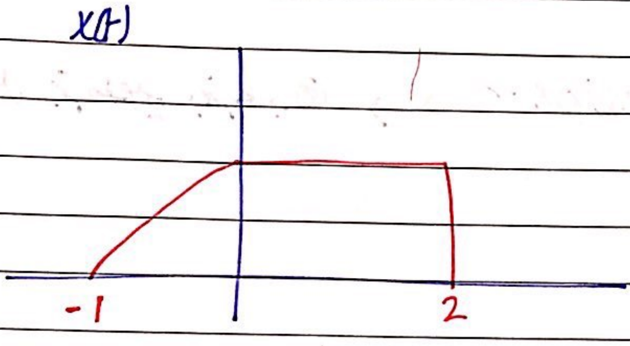
① $E = \int_{-2}^{2} 6^2 \, dt = 36t \Big|_{-2}^{2} = 144 J$.

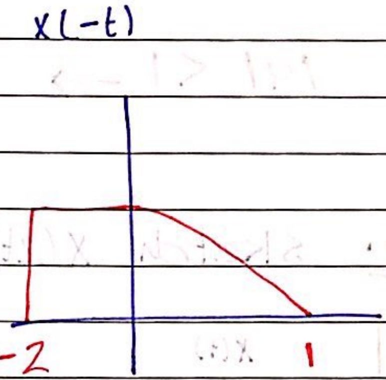
② $E = \int_{-1}^{1} 6^2 \, dt = 36t \Big|_{-1}^{1} = 72 J$.

بال compression بقسم الـ E على الـ معامل t.

- Refliction.

$x(-t) \rightarrow$ Refliction arond the y-axis.



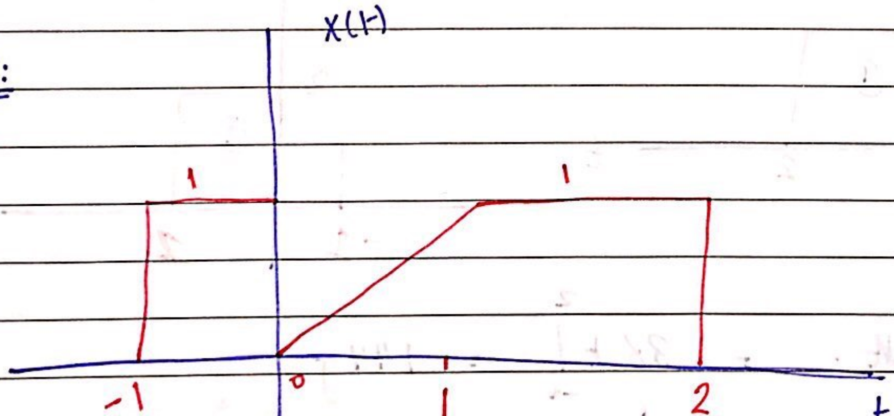


same amplitinde.

$A\,x(t)$: The amplitinde will be affected by A.

$-x(t) \rightarrow$ Refliction around x-axis.

Ex:



sketch

$x(1 - t/2)$.

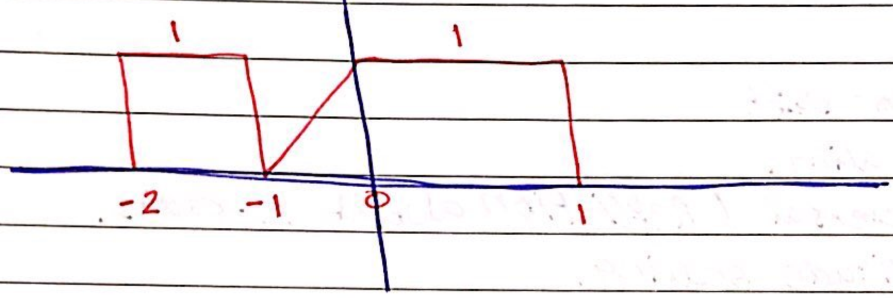
① find time shift

② find time scale.

③ find Refliction.

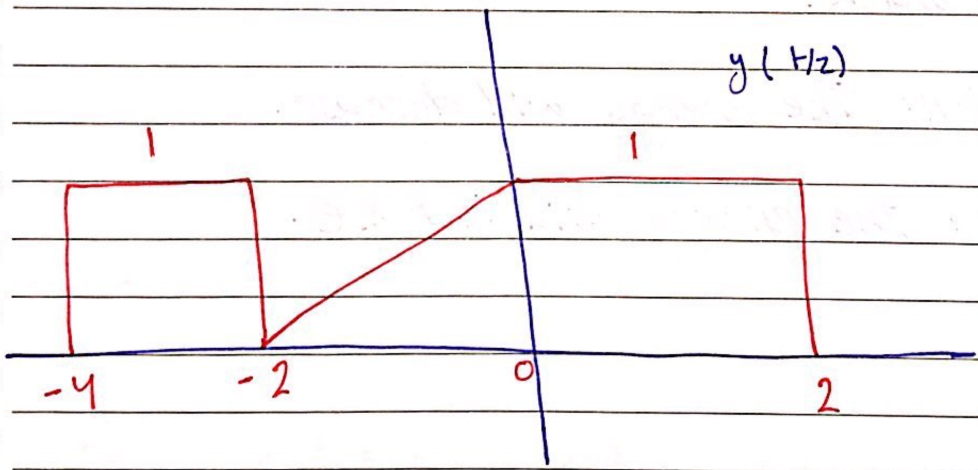
① time shift.

$x(t+1) = y(t)$

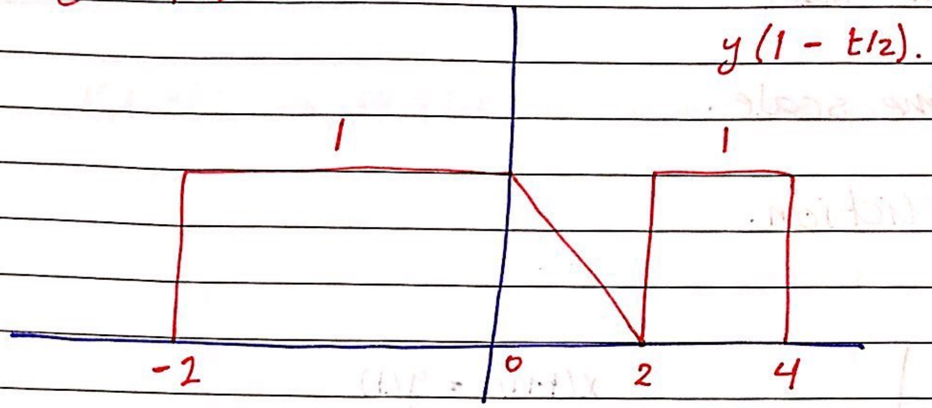


② time scale. (expansion)

$y(t/2)$



③ Refliction.



final plot.

- $x(t \pm t_0)$: time shift
- $x(at)$: time scaling.
- $x(-t)$: time reversal / Refliction about x-axis.
- $a x(t)$: Amplitude scaling.
- $-x(t)$: Amplitude reversal / Refliction about y-axis

* Energy signal : the signal that has the ability to do work.

* in Comprision : The energy will decrease.

* in expansion : The energy will increase.

* odd & Even signals.

* Even Function :

$$x(t) = x(-t).$$

Ex:

$$\cos(\theta) = \cos(-\theta).$$

* odd Function:

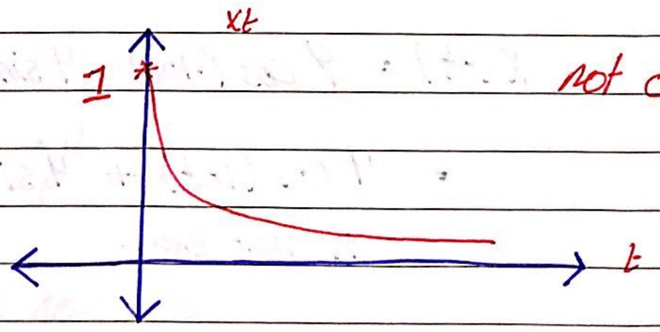
$$x(t) = -x(-t).$$

* for any signal $x(t)$, $x(t) = x_{even}(t) + x_{odd}(t)$

if even signal, the odd component $=0$. (pure even).
if odd signal, the even component $=0$. (pure odd).

- $x_{odd}(t) = \frac{1}{2} x(t) - \frac{1}{2} x(-t)$.] to find the odd comp.
- $x_{even}(t) = \frac{1}{2} x(t) + \frac{1}{2} x(-t)$] " " " even comp.

Ex. $x(t) = e^{-t}$ not odd nor even.



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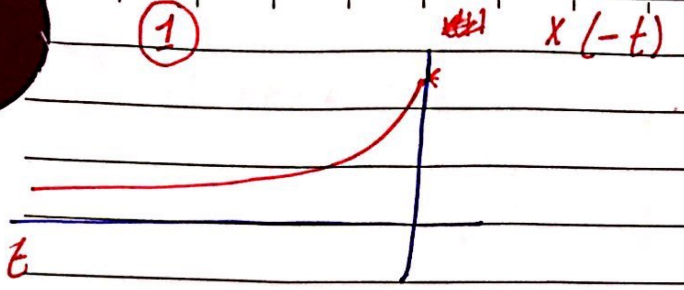
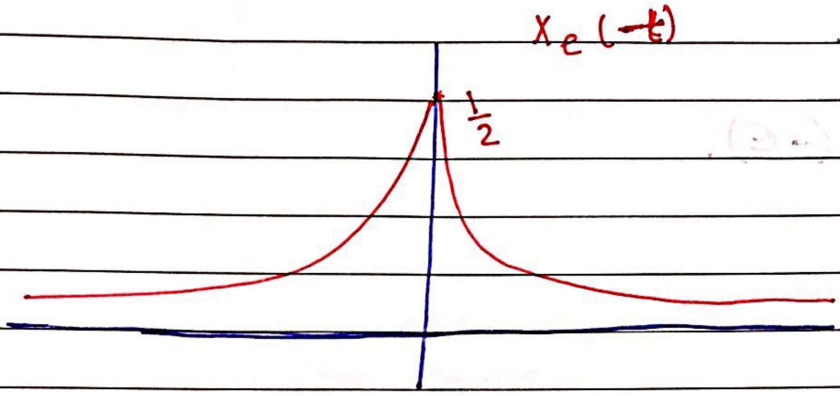
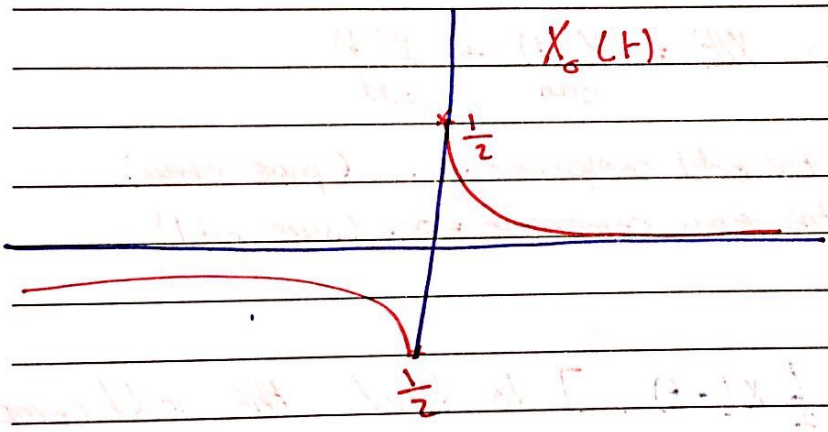






Ex: Check if $x(t) = 4\cos(10t) - 4\sin(6t)$. is even?

$$x(-t) = 4\cos(-10t) - 4\sin(-6t)$$

$$= 4\cos(10t) + 4\sin(6t) \neq x(t)$$

is not even.

Check if odd?

$-x(-t) = -4\cos(-10t) - (-4\sin(-6t))$.

$= -4\cos(10t) - 4\sin(6t) \neq x(t)$.

so, not even nor odd.

- Find the even component:

$x_e = \frac{1}{2}x(t) + \frac{1}{2}x(-t)$.

$= \frac{1}{2}[4\cos 10t - 4\sin 6t] + \frac{1}{2}[4\cos 10t + 4\sin 6t]$

$\boxed{x_e = 4\cos 10t.}$

- Find the odd component:

$x_o = -4\sin(6t)$.

Ex: $x(t) = e^{jt}$, is it even or odd?

$x(t) \stackrel{?}{=} x(-t)$

$x(-t) = e^{-jt} \neq x(t)$.

$-x(-t) = -e^{-jt} \neq x(t)$

not even nor odd.

Find X_e:

$$X_e = \frac{1}{2} x(t) + \frac{1}{2} x(-t)$$

$$= \frac{1}{2} e^{jt} + \frac{1}{2} e^{-jt} = \cos t.$$

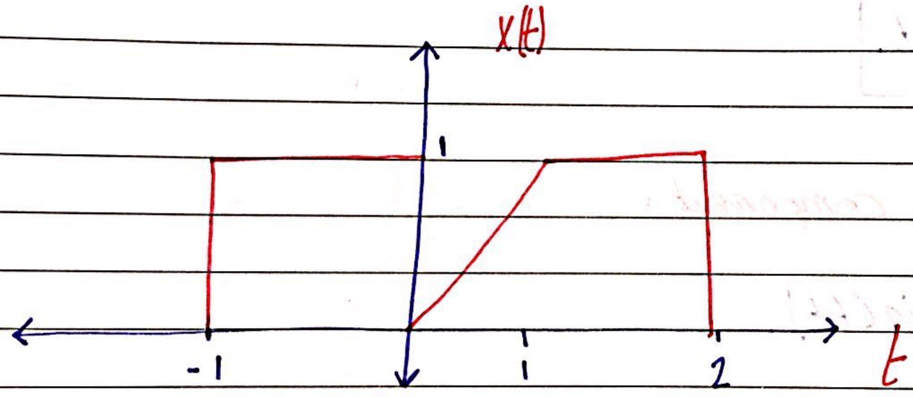
$$\left[\begin{array}{l} \cos x = \frac{1}{2} e^{jx} + \frac{1}{2} e^{-jx} \\ \sin x = \frac{1}{2j} e^{jx} - \frac{1}{2j} e^{-jx} \end{array} \right.$$

$$X_o = \frac{1}{2} x(t) - \frac{1}{2} x(-t)$$

$$= \frac{1}{2} e^{jt} - \frac{1}{2} e^{-jt} = j \sin t.$$

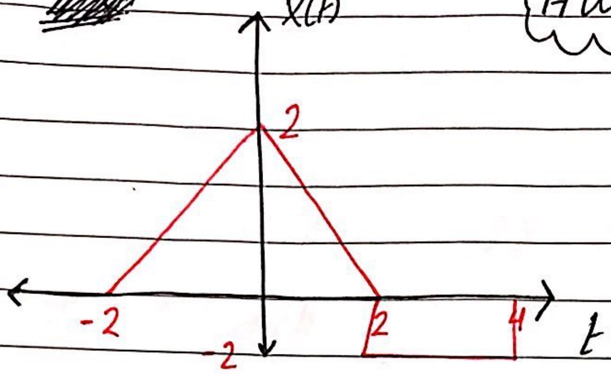
$$\left[e^{\mp jx} = \cos x \mp j \sin x. \right.$$

* H.W # 1



Check if even or odd & find the componets & sketch them.

HW. 2



a) find $E_{x(t)}$

b) let $y(t) = 2x(-2-4t)$

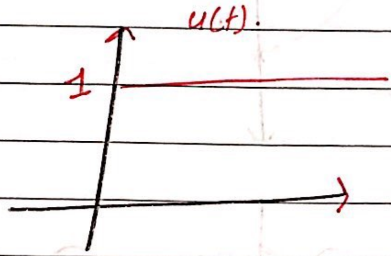
sketch $y(t)$ & find $E_y(t)$

c) Write your comments.

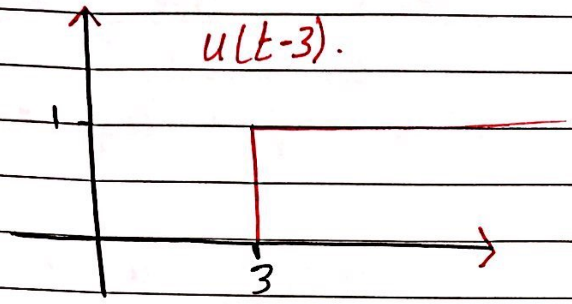
* Singularity, Elementary, Basic signals

* Unit step function.

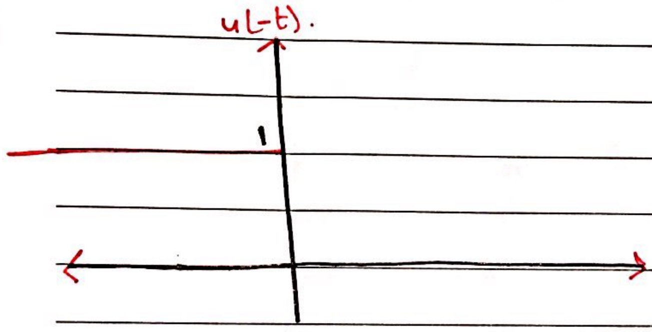
$$u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0. \end{cases}$$



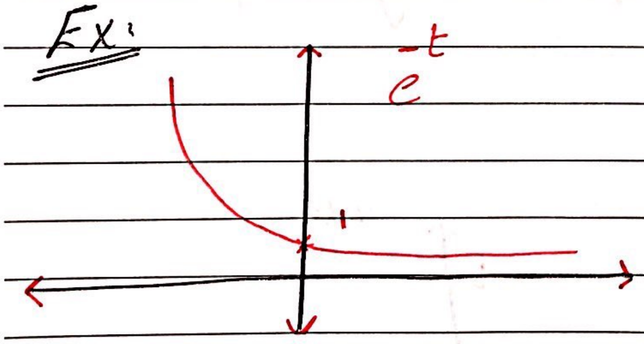
Ex: $u(t-3)$.



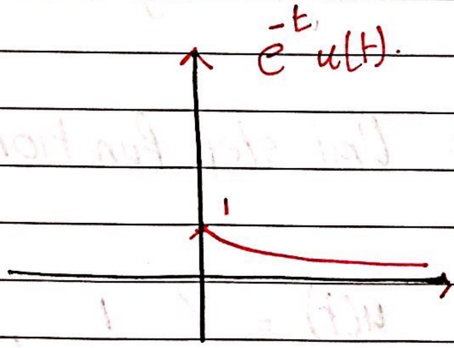
$u(-t)$

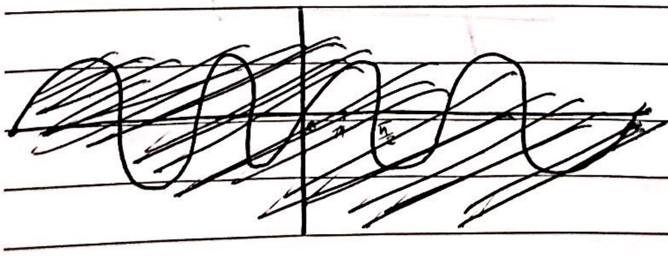


Ex:



$\Rightarrow$



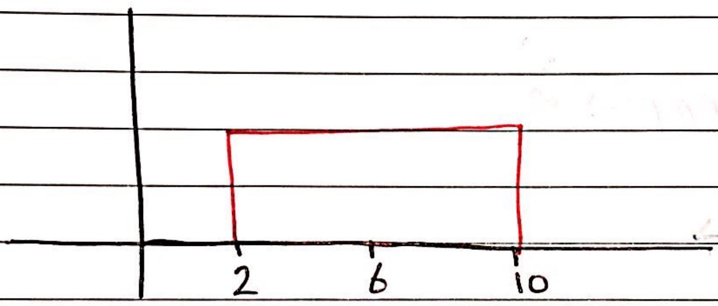


* $\text{rect}\,A\left(\frac{t-a}{\tau}\right)$

a: Center A: Amplitiude.

τ: Width.

sketch: $\text{rect}\left(\frac{t-6}{8}\right)$



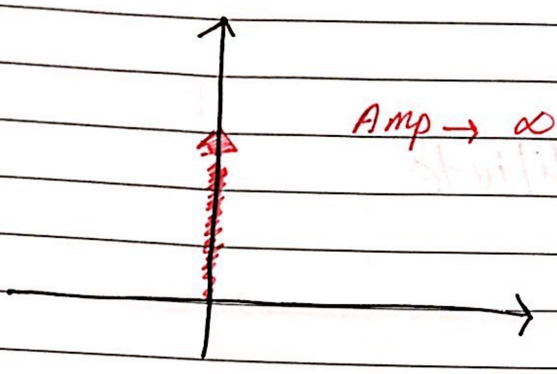
$x(t) = u(t-2) - u(t-10)$.

* Dirac, Delta, impulse function.

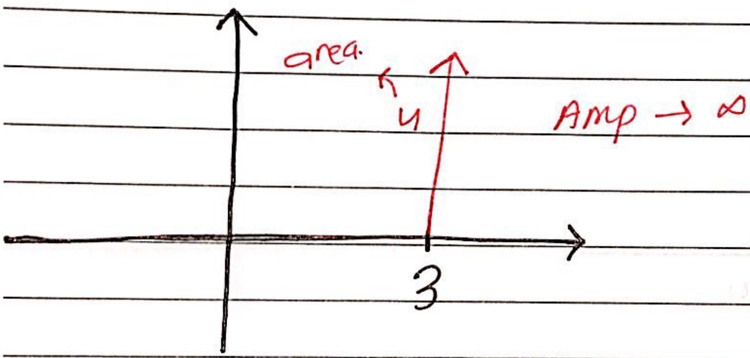
$$\delta(t) = \begin{cases} \delta(t) & , t=0 \\ 0 & , \text{O.W.} \end{cases}$$

Area of rect = 1

sketch $\delta(t)$:



sketch $\delta(t-3)$

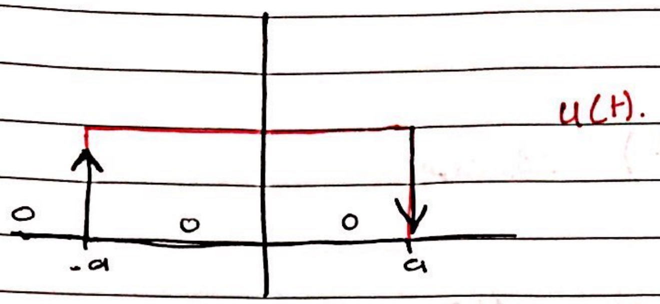


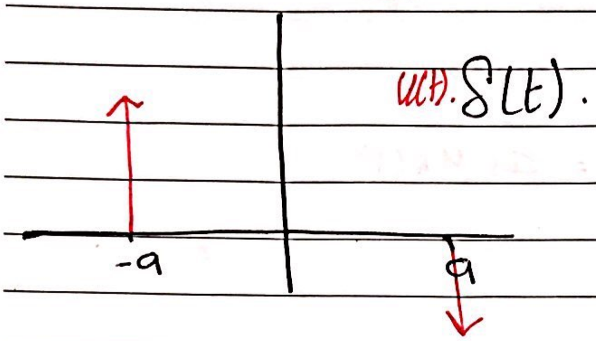
* properties of ~~δ(t)~~. $\delta(t)$.

1) $\int_{-\infty}^{\infty} \delta(t).dt = 1$

✓ مشتقة ال $u(t) \longrightarrow \delta(t)$.

2) $\int_{-\infty}^{t} \delta(\tau).d\tau = u(t). \Rightarrow \frac{d\,u(t)}{dt} = \delta(t).$

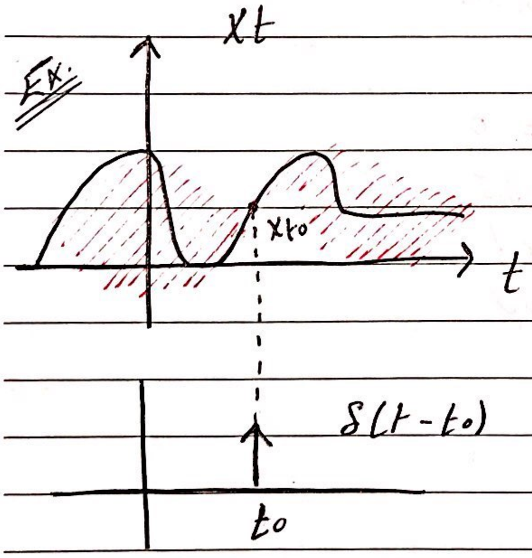




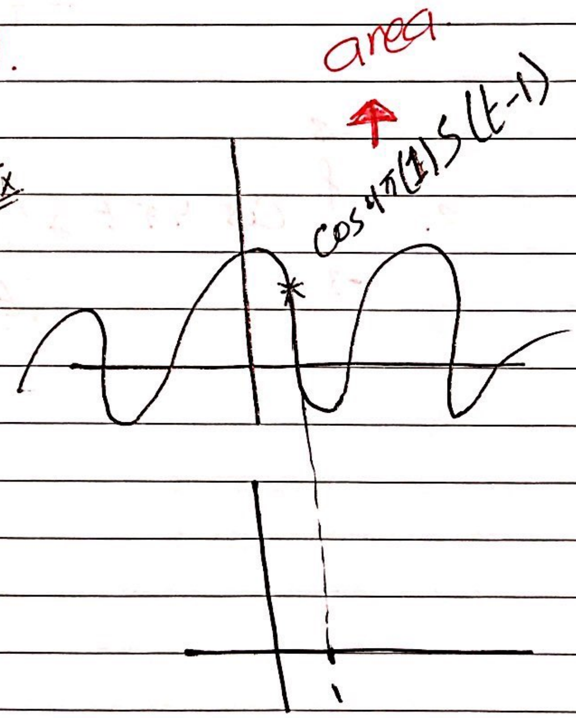
3) Sampling property

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0).$$

Ex.



Ex.



4) Sifting property.

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$$ بعدين دلتا.

$$\int_{-\infty}^{\infty} \underbrace{x(t_0)}_{\text{Const.}}\ \underbrace{\delta(t-t_0)}_{\text{كاملها } 1}\,dt = x(t_0)$$ بدون دلتا.

Ex. 1. $$\int_{-2}^{2} \cos 4\pi t\ \delta(t-1)\,dt = \cos 4\pi(1).$$

2. $$\int_{-2}^{3} \cos 4\pi t\ \delta(t-4)\,dt = 0.$$

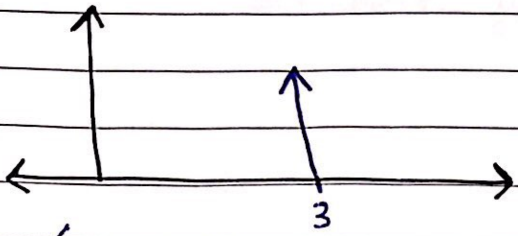
$\boxed{t=4}$ → برا حدود التكامل.

3. $$\int_{-3}^{3} \cos 4\pi t\ \delta(t-3)\,dt = 0$$

$\boxed{t=3}$ → كان الحد بارز بط.

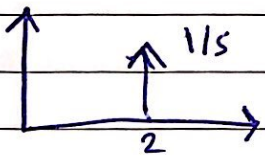
Sunday 23/Feb (1)

*



* Scalling property :-

- $\delta(at-b) = \left|\frac{1}{a}\right| \delta\left(t - \frac{b}{a}\right)$
- $\delta(t-3)$
- $\delta(5t-10) = \frac{1}{5}\delta(t-2) \rightarrow$



Ex:-

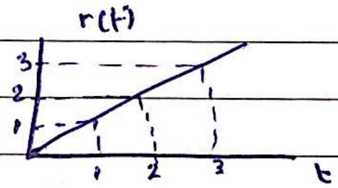
* $e^{-4t}\,\delta(t/3 - 4) \rightsquigarrow$ Sampling.

* $e^{-4t}\,|3|\,\delta(t-12)$ $\quad t = 12$

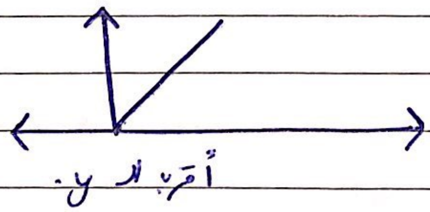
$= 3e^{-4(12)}\,\delta(t-12) \rightsquigarrow$ Scaling.

* Ramp $f^{n.}$

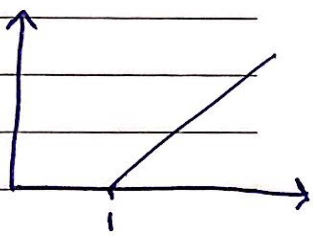
- $r(t) = \begin{cases} t \;, & t \geq 0 \\ 0 \;, & t < 0 \end{cases}$



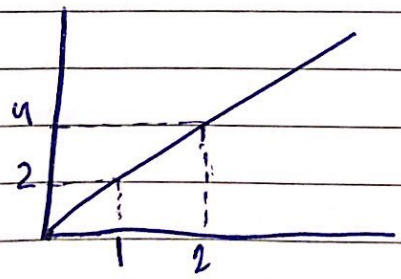
- at



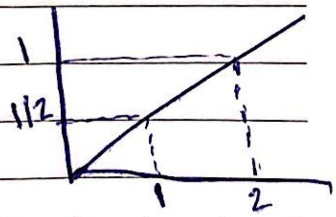
- $(t-1)$



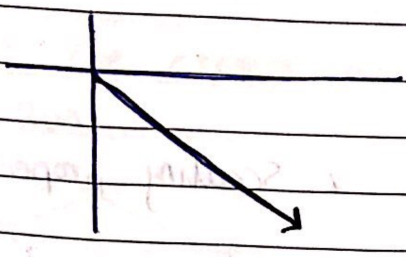
- 2t



- 1/2t



$\cdot(-t)$



Five Apple

Sunday 23/Feb ②

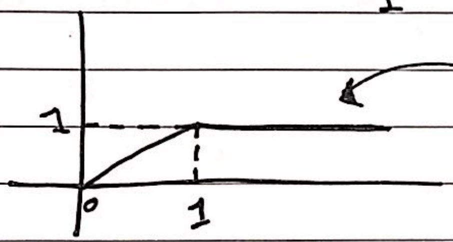
* $u(t) = \dfrac{dr(t)}{dt}$

$$\int_0^t u(\tau)\,d\tau = r(t)$$

* Sketch

$r(t) - r(t-1)$

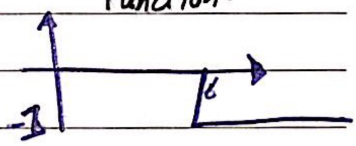
0 1



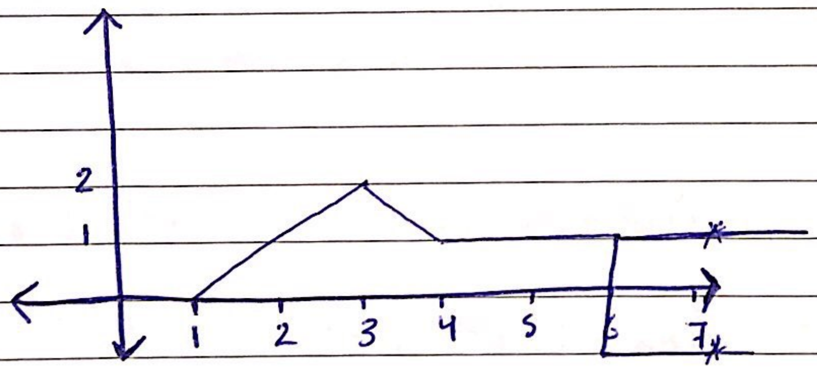
Const. = ② قوة + ① قوة مجهلة

* Sketch $f(t) = r(t-1) - 2r(t-3) + r(t-4) - \underbrace{u(t-6)}_{\text{unit step function.}} - r(t-3)$

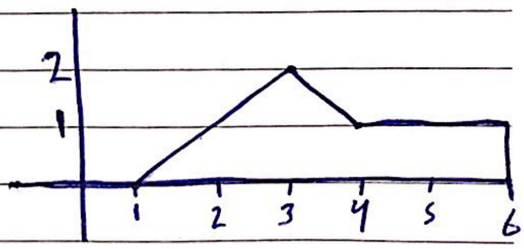
$- r(t-3)$.



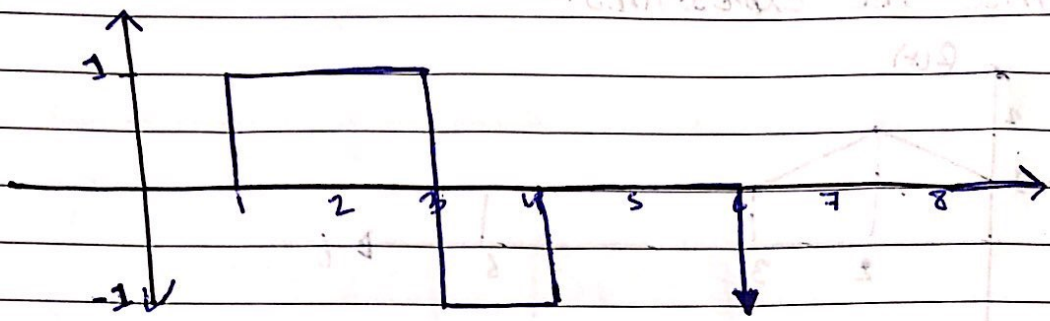
* slope = 1



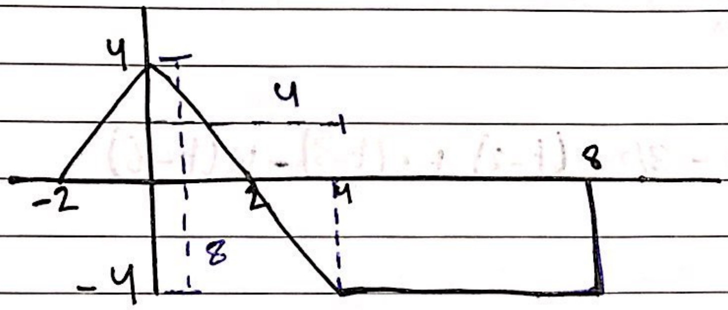
* derive $f(t)$ and sketch it



* $f(t) = u(t-1) - 2u(t-3) + u(t-4) - \delta(t-6)$ ③



* Describe the following functions :-



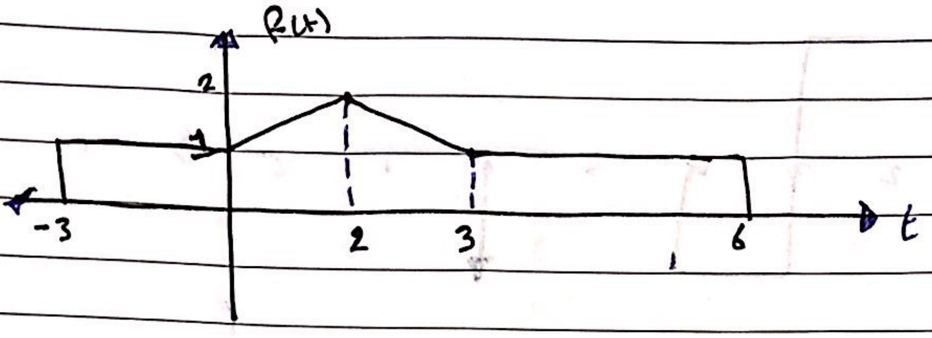
Slope $= \frac{8}{4} = 2$

* $\underline{\underline{2}}r(t+2) - 2(r(t)) - 2r(t) + 2r(t-4) + 4u(t-8)$
  ↳ 4 طالع, 2 مشيت

* $2r(t+2) - 4r(t) + 2r(t-4) + 4u(t-8)$

. $f(t) = 2u(t+2) - 4u(t) + 2u(t-4) + 4\delta(t-8)$.

* Write the expressions:-

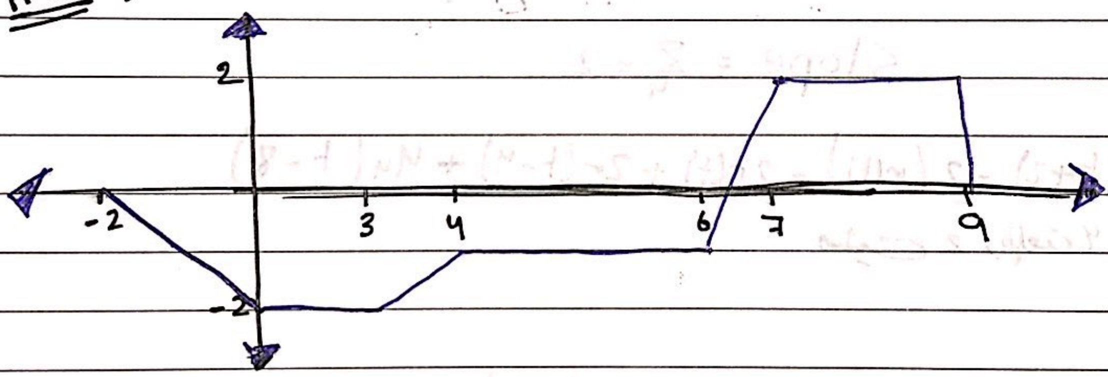


$* f(t) = u(t+3) + \underbrace{1/2}_{\frac{\Delta y}{\Delta x}} r(t) - 1/2 r(t-2) - r(t-2) + r(t-3)$

$+ u(t-6)$

$= u(t+3) + 1/2 r(t) - 3/2 r(t-2) + r(t-3) - u(t-6)$

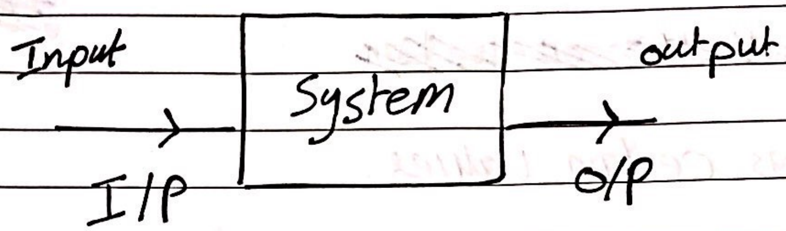
H.W 3



Tuesday 25/Feb.

The system is processing the signals.

We describe the system using mathmatical eq's.



* Classification of systems:

[1] Linear or non linear system.

[2] Time Varying or time invermiant system

[3] Casual or non - casual system

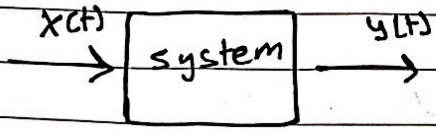
[4] Memory (dynamic) or memoryless (static) system.

[5] Continous or discrete system.

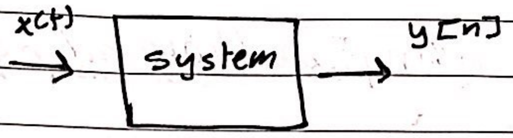
[6] Invertable or non-invertable system.

[7] stable or unstable system.

5



Continous system (input x(t) / output y(t))

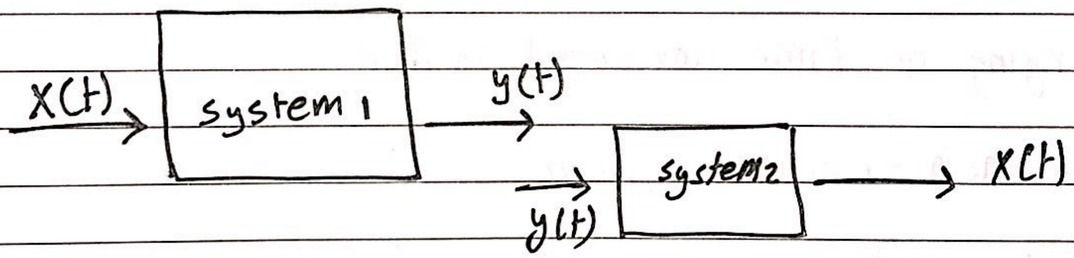


Discrete system (input x(t) / output y[n])

~~y[n] has values on certain~~

y[n]: has certain values.

6 * اذا ال System الأول دخلت x(t) واعطاني y(t) ورجعت على system ثاني دخلت y(t) واعطاني x(t) بكون ال System is invertable.

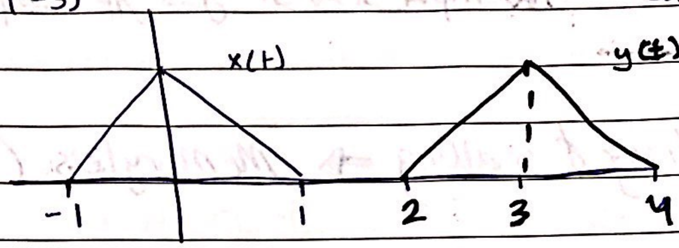


this is called invertable system, which is a single system devide to multiple stages.

$\boxed{3}$

— Causal system: If the input proceeds the out-put or at the same time.

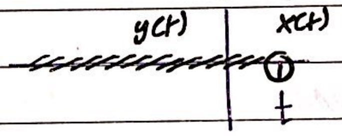
$y(t) = x(t-3)$



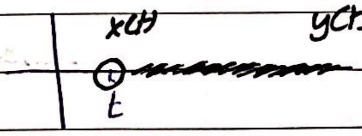
$\hookrightarrow$ $x(t) = y(t)$.

Ex:

- $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \Rightarrow$ non-causal system.



- $y(t) = \int_{t}^{\infty} x(\tau)\, d\tau \Rightarrow$ causal system



$\boxed{7}$ To check the stability :-

$\lim_{t \to \infty} x(t) = k \Rightarrow$ Bouned Input (BI).

$\lim_{t \to \infty} y(t) = M \Rightarrow$ Bounded output (BO)

k, M: constant values. $\boxed{BIBO} \rightarrow$ stabile system.

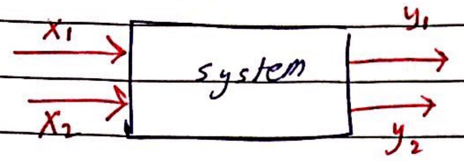
$\boxed{4}$ Static system: بكون (memoryless) بـظهروا بنفس اللحظة input & output * إذا ال

* إذا ال output ظهر قبل أو بعد ال input بكون: dynamic system (memory).

* $x(t)$ without shiftting & scalling ⇒ Memoryless (static)

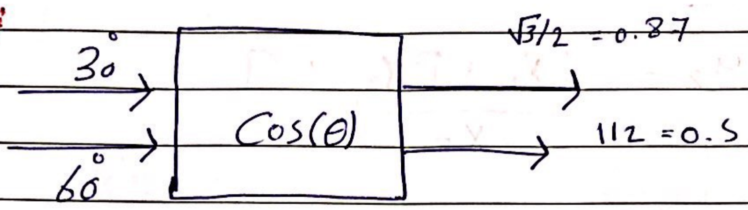
- $y(t) = x(t/4)$ → dynamic
- $y(t) = 4 + x(t)$ → static
- $y(t) = \sqrt{x(t)}$ → static
- $y(t) = x^2(t)$ → static
- $y(t) = x(t-2)$ → dynamic
- $y(t) = e^{-x(t)}$ → static.

1 Linear or non-linear system:-



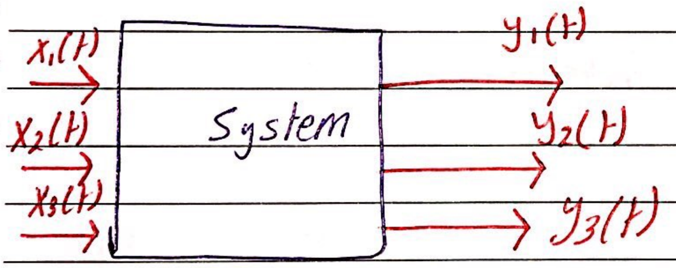
if $\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha y_1(t) + \beta y_2(t)$.

Ex:



$(30+60) = 90 \longrightarrow 0 \neq 0.87 + 0.5$.

$\therefore$ Non-linear system



$$\boxed{\begin{array}{l} X_3 = \alpha X_1 + \beta X_2 \\ y_3 = \alpha y_1 + \beta y_2 \end{array}}$$

β, α : are constants.

$X_1 \longrightarrow y_1$

$X_2 \longrightarrow y_2$

$X_3 \longrightarrow y_3$

$$\alpha X_1 + \beta X_2 \xrightarrow{??} \alpha Y_1 + \beta y_2$$

if yes, system is linear

else, system is non-linear.

Ex:-

$$y(t) = x(t) + a \quad , \quad a : \text{const.}$$

$$y_1 = x_1 + a$$
$$y_2 = x_2 + a$$
$$y_3 = x_3 + a$$

$$\alpha y_1 + \beta y_2 = \underbrace{\alpha x_1 + \beta x_2}_{x_3} + a$$

$\therefore$ This system is non-linear.

Ex:-

$$y(t) = a\,x(t)$$
$$y_1 = a x_1$$
$$y_2 = a x_2$$
$$y_3 = a x_3$$

$$\cancel{\alpha y_1 + \beta y_2 = \alpha}$$

$$\alpha y_1 + \beta y_2 = a(\alpha x_1 + \beta x_2)$$

$$\Rightarrow \quad \alpha y_1 + \beta y_2 = a\alpha x_1 + a\beta x_2$$

$\therefore$ This system is linear.

Ex:-

$$y(t) = \frac{dx(t)}{dt} + x(t)$$

$$y_1 = \frac{dx_1}{dt} + x_1$$

$$y_2 = \frac{dx_2}{dt} + x_2$$

$$y_3 = \frac{dx_3}{dt} + x_3$$

$$\alpha y_1 + \beta y_2 = \frac{d}{dt}\left[\alpha x_1 + \beta x_2\right] + \left[\alpha x_1 + \beta x_2\right]$$

$$\Rightarrow \alpha y_1 + \beta y_2 = \frac{d\,\alpha x_1}{dt} + \frac{d}{dt}\beta x_2 + \alpha x_1 + \beta x_2$$

This system is linear.

* لازم بس أجمع ال input يجمع ال output.

Ex:

$$y(t) = e^{x(t)}$$

$$y_1 = e^{x_1}$$

$$y_2 = e^{x_2}$$

$$y_3 = e^{x_3}$$

$$\alpha y_1 + \beta y_2 = e^{\alpha x_1 + \beta x_2} = e^{\alpha x_1} \boxed{\cdot} e^{\beta x_2}$$

multiplication is not allowed.

$\therefore$ This system is not linear.

Ex:

$$y(t) = \sqrt{x(t)}$$

$$y_1 = \sqrt{x_1}$$

$$y_2 = \sqrt{x_2}$$

$$y_3 = \sqrt{x_3}$$

$$\alpha y_1 + \beta y_2 = \sqrt{\alpha x_1 + \beta x_2} \neq \sqrt{\alpha x_1} + \sqrt{\beta x_2}$$

Non-linear.

Ex:

$$y(t) = \int x(t)\, dt.$$

Ex: $y(t) = x^2(t)$.

Ex:

$$y(t) = x(t) \cos t.$$

Ex: $y(t) = \cos(x(t))$.

Ex:

$$\frac{dy}{dt} + t^2 y(t) = (2t+3)\,x(t)$$

Sol:

$$\frac{dy_1}{dt} + t^2 y_1 = (2t+3)\,x_1$$

$$\frac{dy_2}{dt} + t^2 y_2 = (2t+3)\,x_2$$

$$\frac{dy_3}{dt} + t^2 y_3 = (2t+3)\,x_3$$

$$\alpha \frac{dy_1}{dt} + \beta \frac{dy_2}{dt} + t^2 \alpha y_1 + t^2 \beta y_2 = (2t+3)\alpha x_1 + (2t+3)\beta x_2$$

This system is linear.

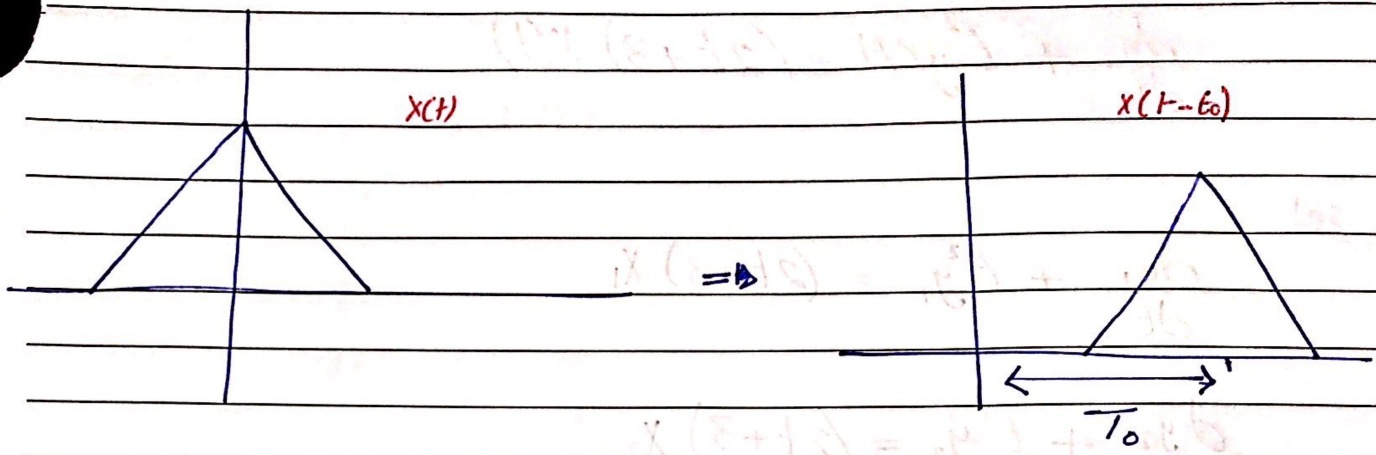
Time Varying system & Time invariant system.

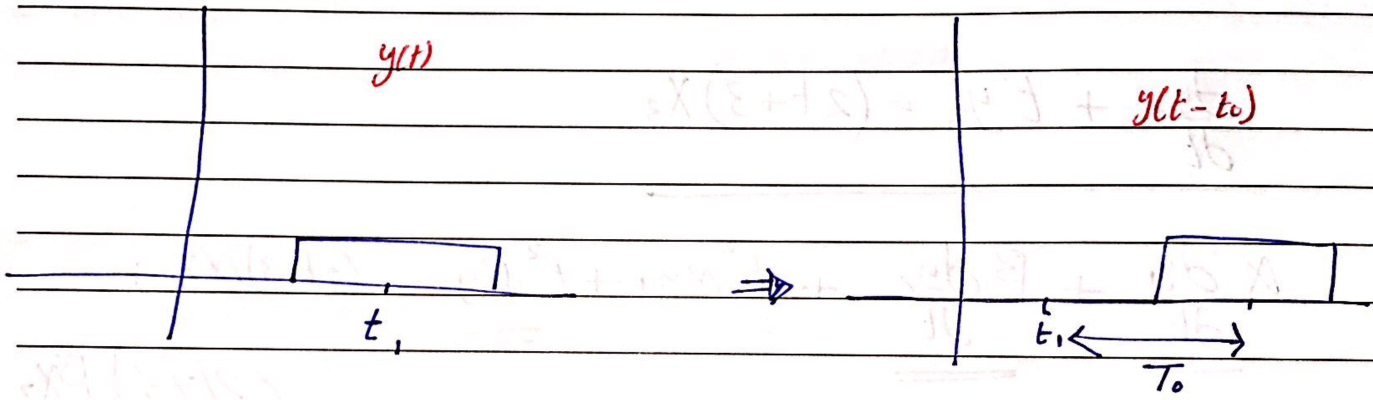
(T.V) (TIS)

1. $x(t) \rightarrow \boxed{\text{sys}} \xrightarrow{y(t)} \boxed{\text{delay } t_0} \rightarrow y(t-t_0)$

2. $x(t) \rightarrow \boxed{\text{delay } t_0} \xrightarrow{x(t-t_0)} \boxed{\text{sys}} \rightarrow y_2$

if $y_1 = y_2$ The system said to b (T.IS)





Ex:

$y(t) = x(t) + 2$

Sol.

1. $x(t) \rightarrow \boxed{Sys} \xrightarrow{x(t)+2} \boxed{t_0} \rightarrow x(t-t_0)+2 = y_1$

2. $x(t) \rightarrow \boxed{t_0} \xrightarrow{x(t-t_0)} \boxed{Sys} \rightarrow x(t-t_0)+2 = y_2$

$\therefore \boxed{y_1 = y_2}$, TIS.

Ex: $y(t) = \sin(t) \cdot x(t)$

Sol.

Delay.

1. $x(t) \rightarrow \boxed{Sys} \xrightarrow{\sin(t) \cdot x(t)} \boxed{t_0} \rightarrow \sin(t-t_0) \cdot x(t-t_0) = y_1$

2. $x(t) \rightarrow \boxed{t_0} \xrightarrow{x(t-t_0)} \boxed{Sys} \rightarrow \sin(t) \cdot x(t-t_0) = y_2$

$\therefore \boxed{y_1 \neq y_2}$, T.V

Ex:

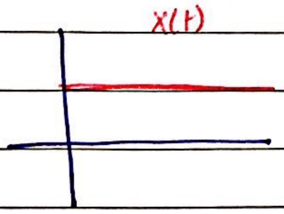
$x(t) \rightarrow \boxed{\text{Time reversal}} \rightarrow x(-t)$

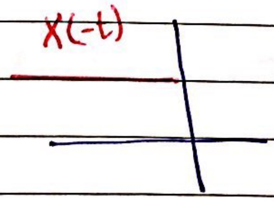
Let $x(t) = u(t)$

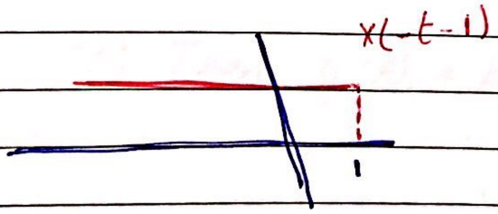
let $t_0 = 1$

1.

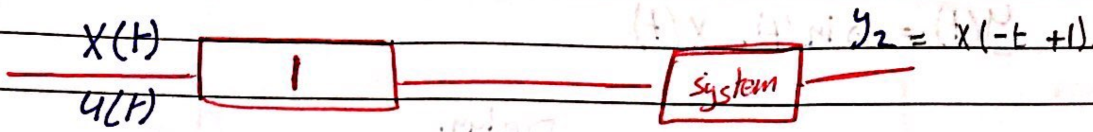
$\underset{x(t)}{u(t)} \rightarrow \boxed{Sys} \xrightarrow[x(-t)]{u(-t)} \boxed{1} \rightarrow u(-(t+1)) = u(-t-1)$

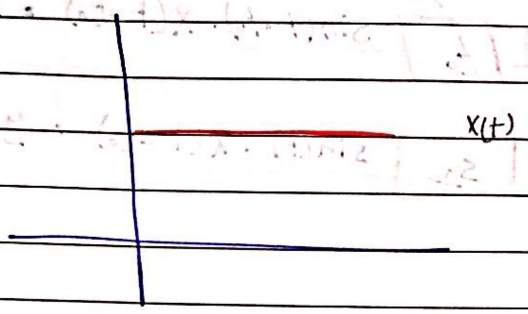


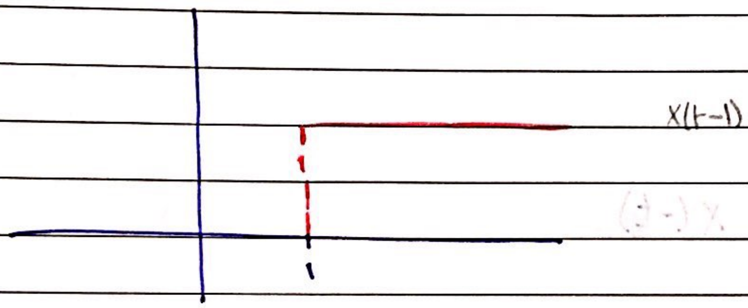


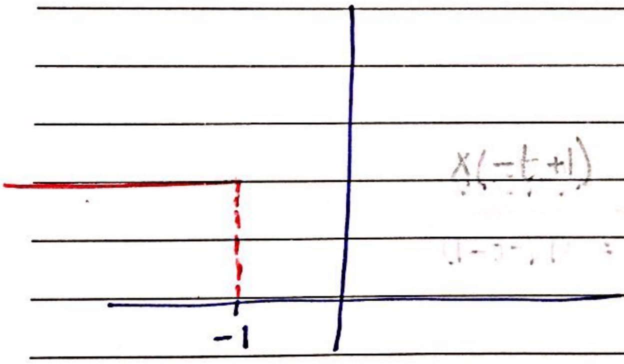


2.







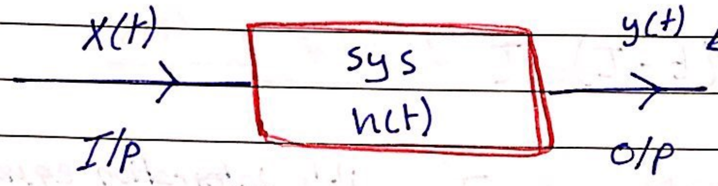


T. V system.

* Continous Linear Time Invarient Sys. (LTI).

CLTIS

Let The system CLTI



Response

response → output when the input is Delta of (t).

h(t): Impolse response.

* Convolution : shows the relation between the I/P & o/p and the Impolse response.

$$y(t) = x(t) * h(t)$$

$*$ → convolution sign.

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} x(t-\tau)\, h(\tau)\, d\tau.$$

If $x(t) = \delta(t)$ Then $y(t) = h(t)$.

proof..

$$y(t) = x(t) \cdot h(t)$$

$$= \delta(t) \cdot h(t)$$

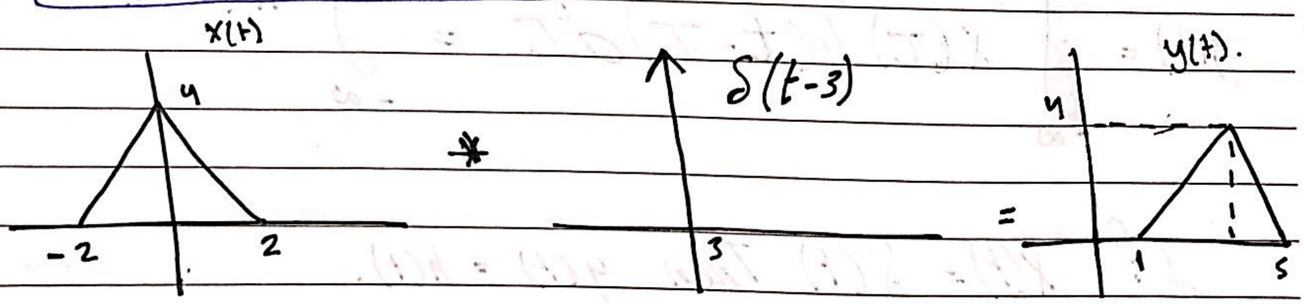
$$y(t) = \int_{-\infty}^{\infty} \delta(t-\tau) \cdot h(\tau) d\tau$$

$$= \int_{-\infty}^{\infty} h(t) \cdot \delta(t-\tau) d\tau$$

$$= h(t) \left[\int_{-\infty}^{\infty} \delta(t-\tau) d\tau \right] \rightarrow$$ it's integration equal to 1.

$\boxed{y(t) = h(t)}$

$\boxed{\begin{aligned} x(t) * \delta(t) &= x(t) \\ x(t) * \delta(t-t_0) &= x(t-t_0) \end{aligned}}$



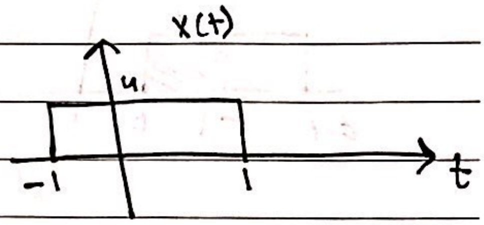
* Convolution procedure:

بقلب أي واحد بس

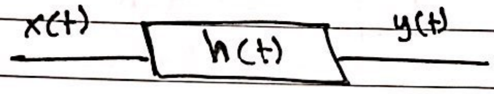
① واحد بقلبه و بعمل shift بمقدار (t). الأحسن بقلب الي عرضه أصغر.

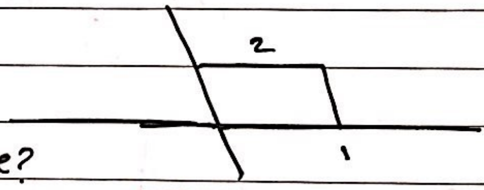
② بمشي أجزاء فيه لحد ما يوصل الاقتران الثاني.

③ بشوف مناطق التداخل وبتكامل عليها.

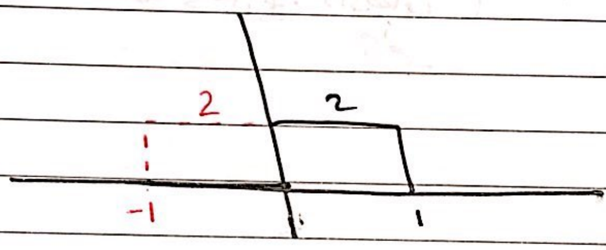


Ex: answer LTIS.



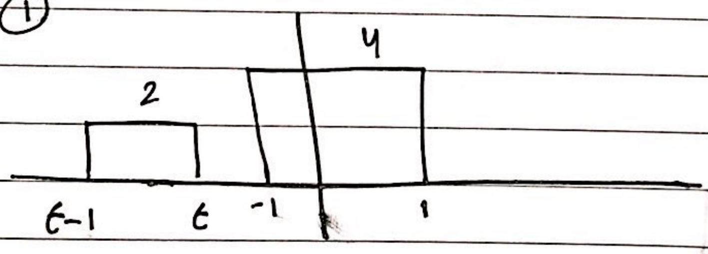


Sol: Find y(t), response?



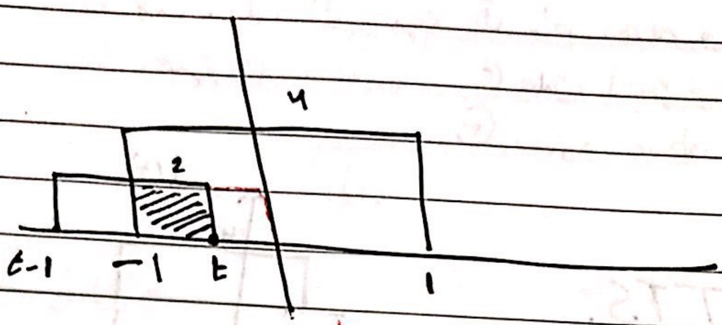
$y(t) = x(t) * h(t)$

①



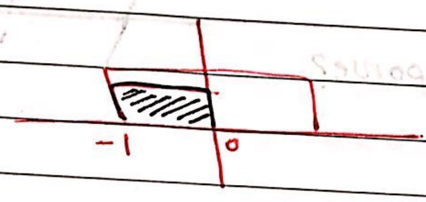
$y(t) = 0 \quad , \quad t < -1$

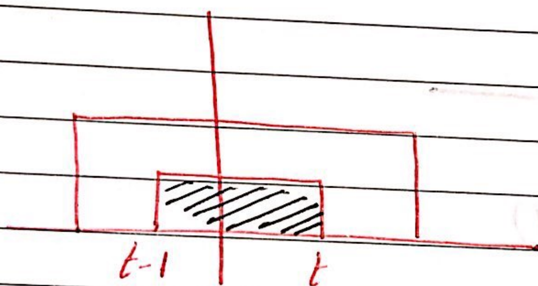
② مرحلة بدء الدخول.

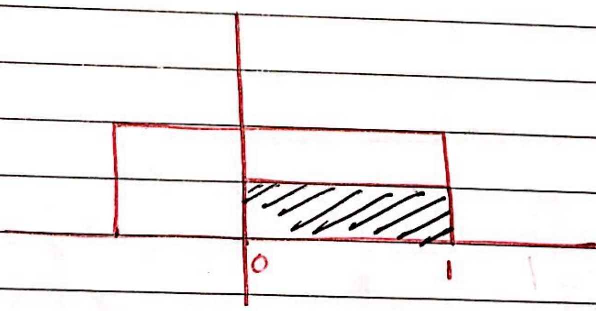


$$y(t) = \int_{-1}^{t} (4)(2)\, d\tau = 8\tau \Big]_{-1}^{t} = 8t + 8.$$

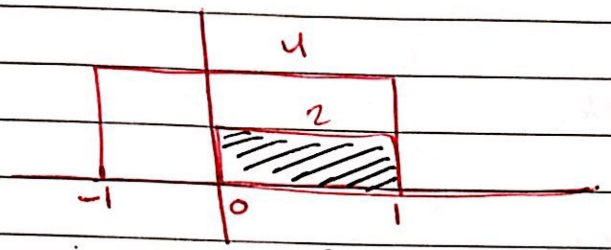
$-1 \le t \le 0.$ ← عند الصفر يكون الاقتران كله دخل بوا (انطباق تام.





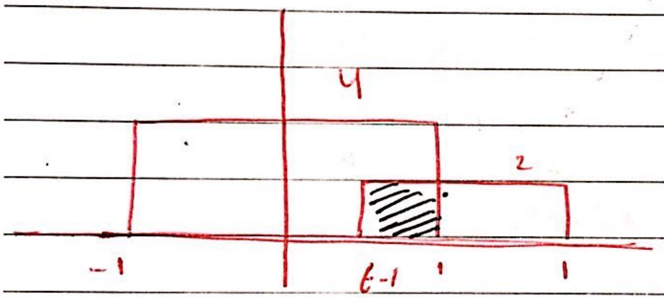


③ باخر أي حالة من حالات
الأنطباق التام



$$y(t) = \int_{-1}^{0} 4(2)\, d\tau = 8\tau \Big]_{-1}^{0} = \boxed{8} \quad 0 \leq t \leq 1$$

④ مرحلة بد الخروج



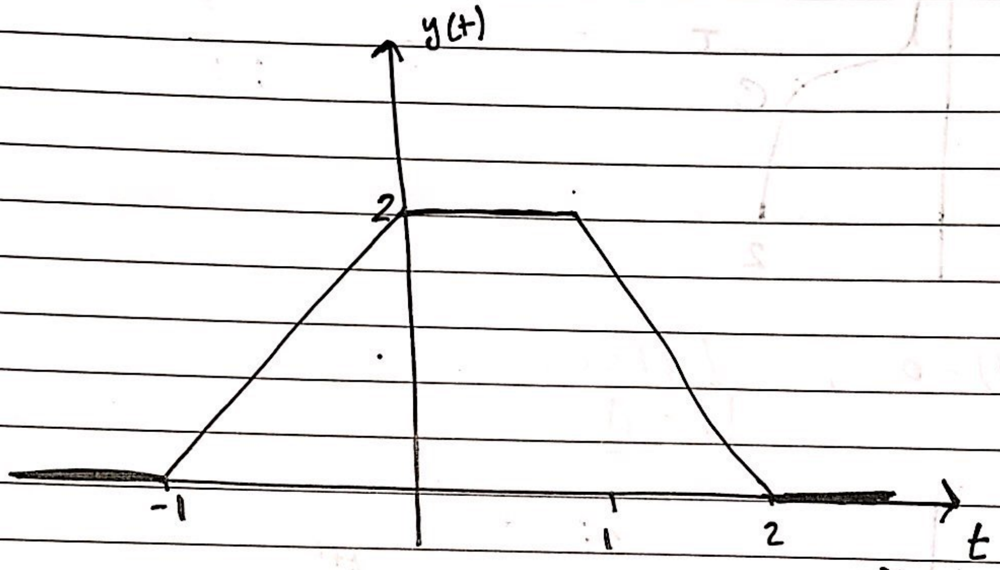
$$y(t) = \int_{t-1}^{1} 4(2)\, d\tau = 8\tau \Big]_{t-1}^{1}$$

$$= 8[1 - t + 1]$$

$$= 16 - 8t \quad 1 < t \leq 2.$$

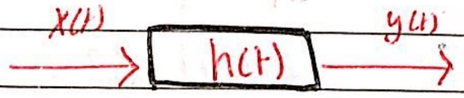
⑤ $y(t) = 0 \quad t \geq 2.$

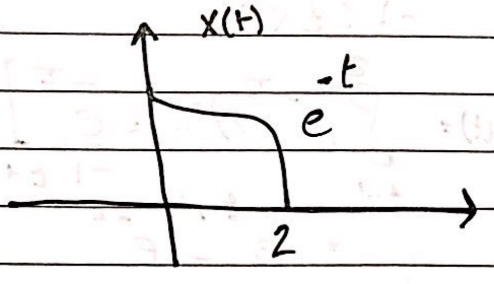
$$y(t) = \begin{cases} 0 & , \quad t < -1 \\ 8t+8 & , \quad -1 < t \leq 0 \\ 8 & , \quad 0 < t \leq 1 \\ 16 - 8t & , \quad 1 < t \leq 2 \\ 0 & , \quad t > 2 \end{cases}$$

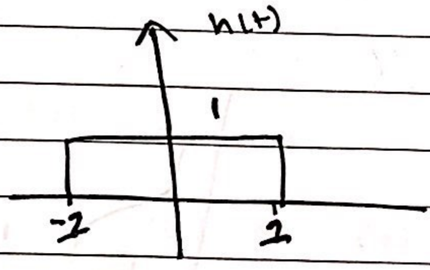


Ex 2:

Assume LTIS

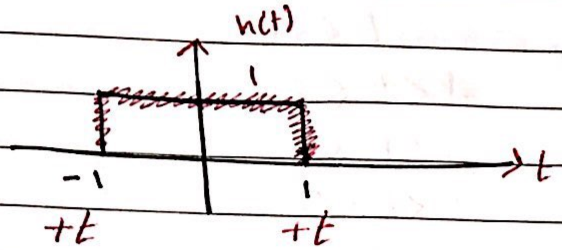




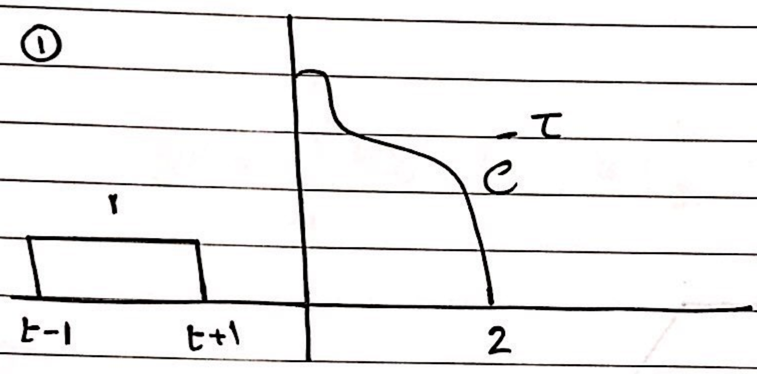


* When the two functions have the same width, No need to do the complete overlaping.

Sol.



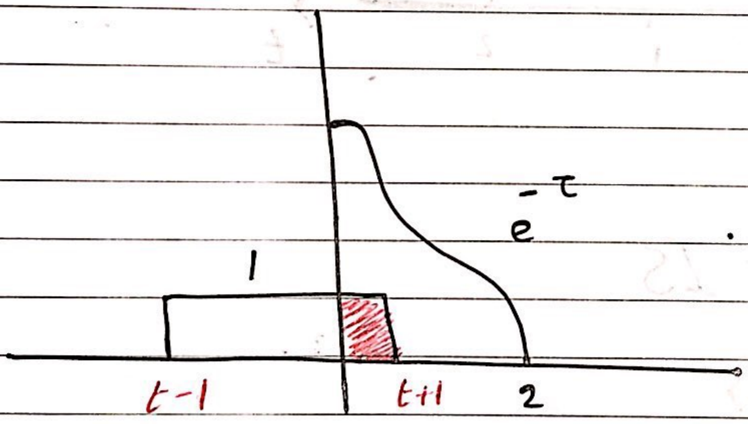
①



$y(t) = 0 \quad , \quad t+1<0.$

$\boxed{t<-1}$

②



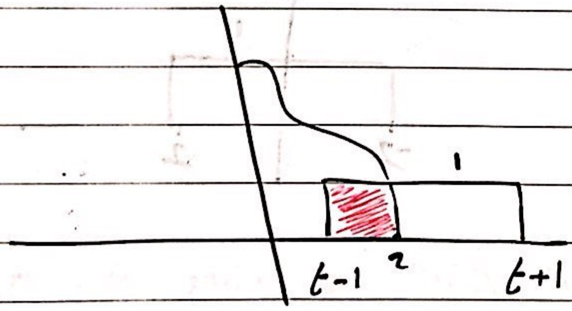
$$y(t) = \int_0^{t+1} e^{-\tau} (1)\, d\tau$$

$$= \frac{e^{-\tau}}{-1}\Big]_0^{t+1} = 1 - e^{-t-1}$$

$0 < t+1 < 2.$

$-1 < t < 1$

③



$$y(t) = \int_{t-1}^{2} e^{-\tau} d\tau = \frac{e^{-\tau}}{-1}\Big]_{t-1}^{2}$$

$$= e^{-t+1} - e^{-2}$$

$2 < t+1 < 4$

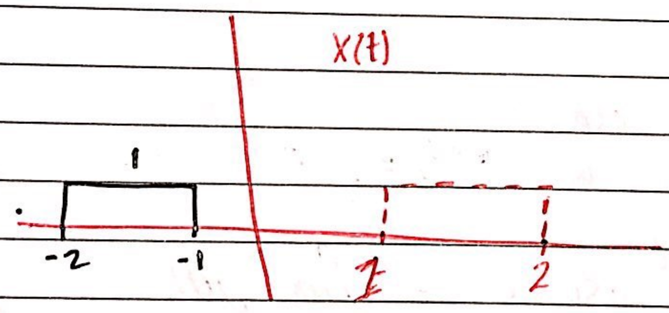
$1 < t < 3$

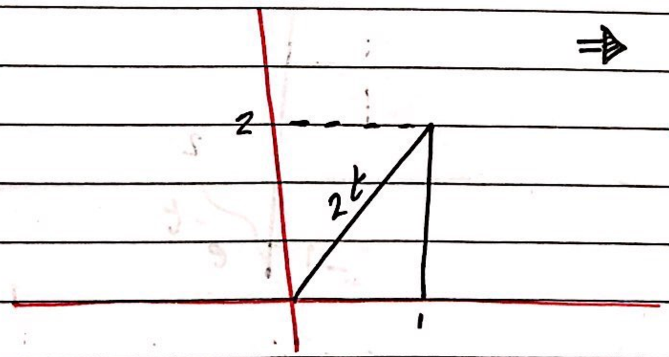
④ $y(t) = 0 \quad t > 3.$

$y(t) = \begin{cases} \sim \\ \sim \\ \sim \\ \sim \end{cases}$

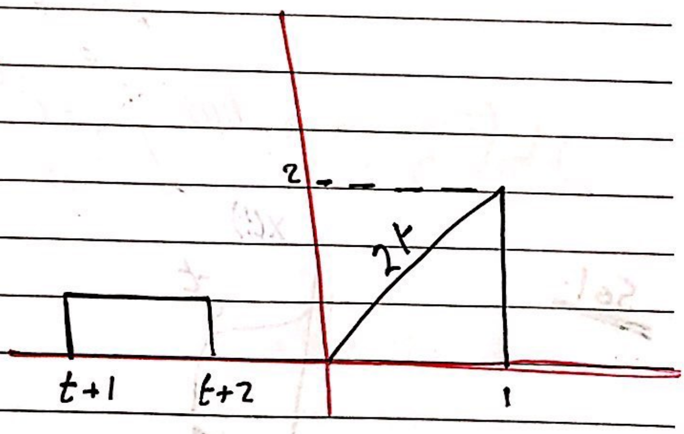
* اذا بدي استبدل x(t) بقلبه بشفته وبدل كل (τ) بحط $(t-\tau)$

Ex LTIS, find $y(t)$.





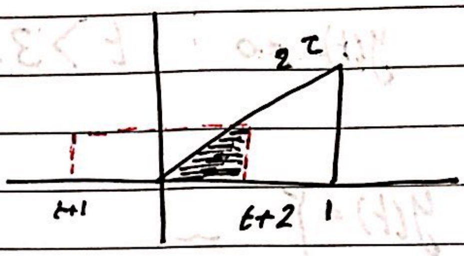
$\Rightarrow$



1. $y(t) = 0 \quad , \quad t+2 < 0$

$t < -2$

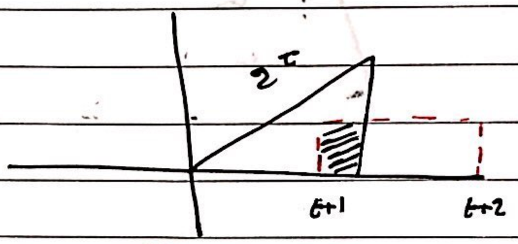
2. $y(t) = \int_{0}^{t+2} (1)(2\tau)\,d\tau$



$0 < t+2 \leq 1$

$-2 < t < -1$

3. $y(t) = \int_{t+1}^{1} (1)(2\tau)\,d\tau$



$1 < t+2 \leq 2$

$-1 < t \leq 0$

4. $y(t) = 0$ $\quad t+2 > 2$

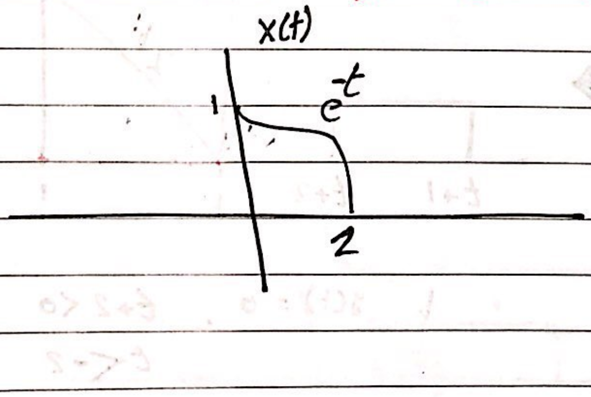
$t > 0.$

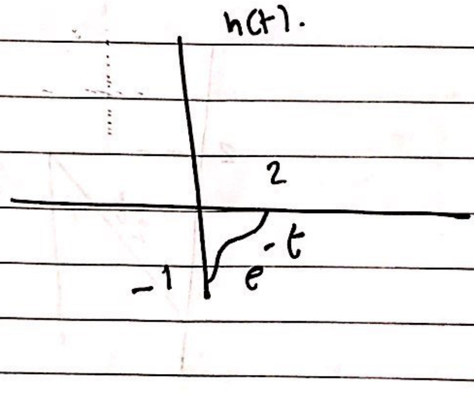
Ex: ~~Let~~ LTIS

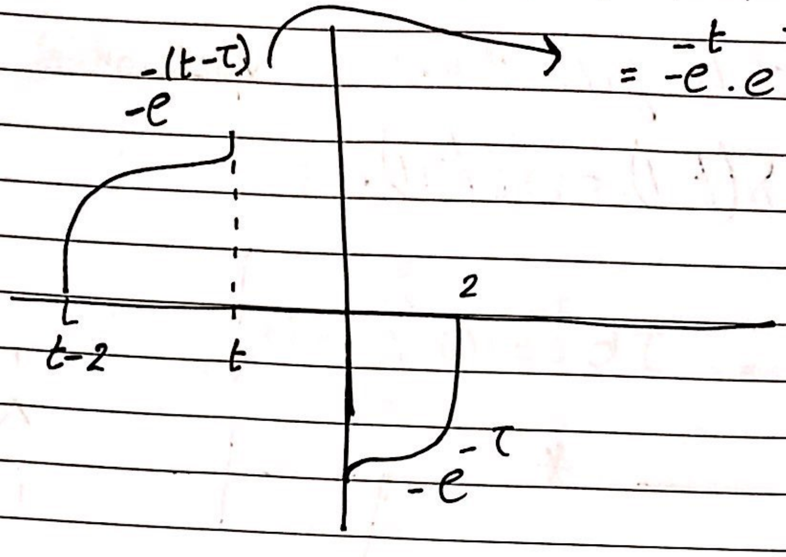
Let $x(t) = \begin{cases} e^{-t} & 0 < t \leq 2 \\ 0 & o.w \end{cases}$

$h(t) = \begin{cases} -e^{-t} & 0 < t \leq 2 \\ 0 & o.w \end{cases}$ $\quad$ Find $y(t)$.

Sol.



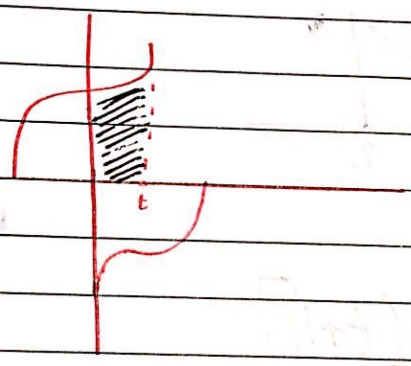




$= -e^{-t} \cdot e^{\tau}$

① $y(t) = 0 \; , \quad t<0$

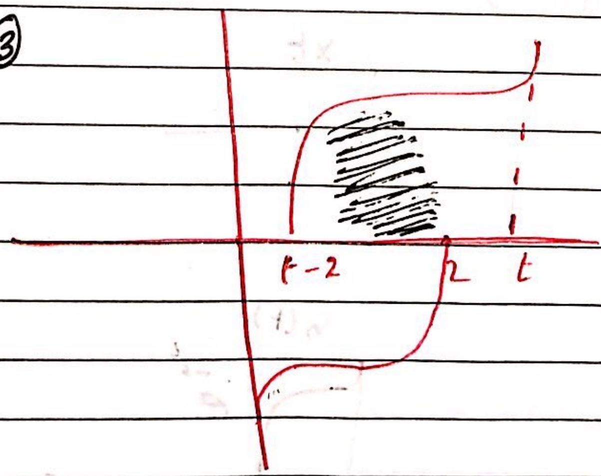
②



$$y(t) = \int_0^t e^{-t} \cdot e^{\tau} \cdot (-e^{-\tau}) \cdot d\tau = -\tau e^{-t} \Big]_0^t$$

$$= -te^{-t} \; , \quad 0<t\leq 2.$$

③



$$y(t) = \int_{t-2}^{2} e^{-t} e^{\tau} (-e^{-\tau}) d\tau$$

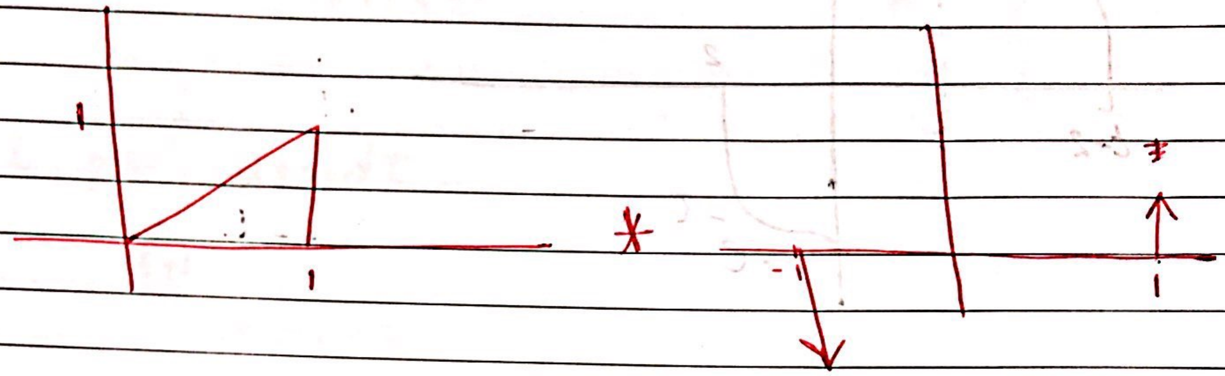
$$= -\tau e^{-t}$$

$2 \leq t \leq 4$.

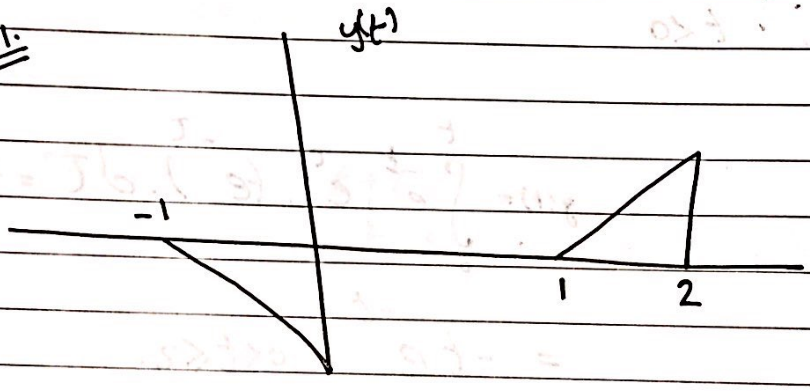
Ex:

Find & sketch $y(t)$ & find it's expression.

$$h(t) = \delta(t-1) - \delta(t+1).$$



Sol:



$$y(t) = x(t) * [\delta(t-1) - \delta(t+1)]$$

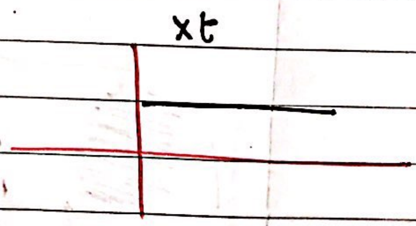
$$= x(t-1) - x(t+1).$$

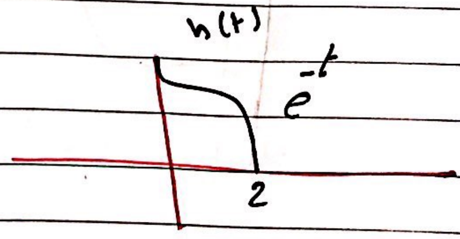
ex:

For LTIS

~~X(t)~~ $X(t) = u(t)$

$h(t) = e^{-t} \quad 0 < t < 2.$



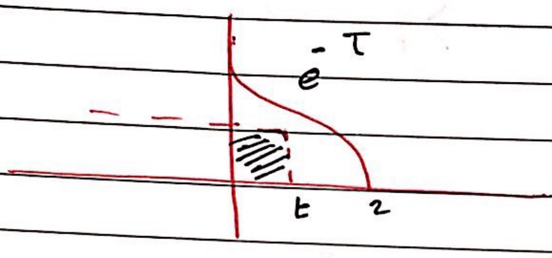


sol.

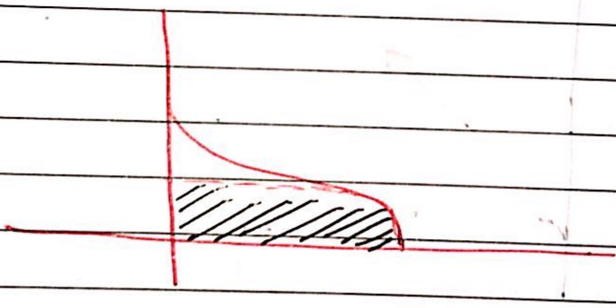
1. $y(t) = 0 \quad , \quad t < 0.$

2. $y(t) = \int_0^t (1)(e^{-\tau}) d\tau = -e^{-\tau}\Big|_0^t = 1 - e^{-t}$

$0 < t \leq 2$



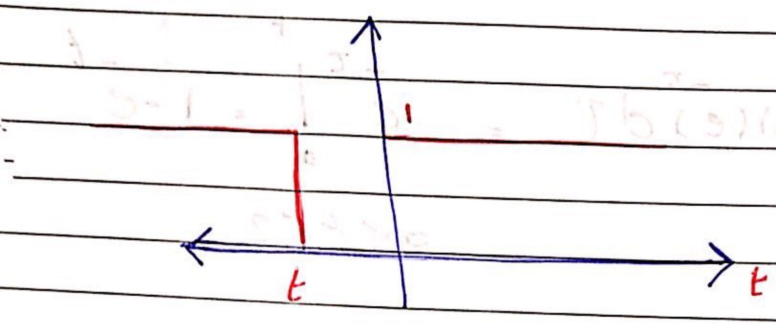
3. $y(t) = \int_0^2 e^{-\tau} (1) d\tau \qquad t > 2$



This case we have one function infinte & the other is finite.

* Assume LTIS.

$x(t) = u(t)$ and $h(t) = u(t)$

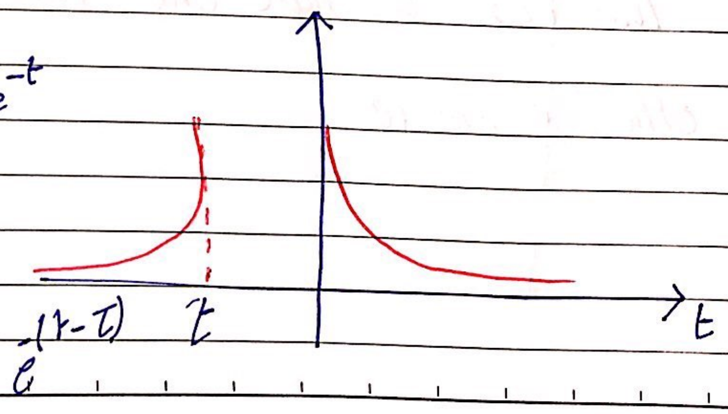


1. $y(t) = 0$, $t < 0$

2. $y(t) = \int_0^t 1 * 1 \, dt = t$ $t > 0$

* Assume LTIS :-

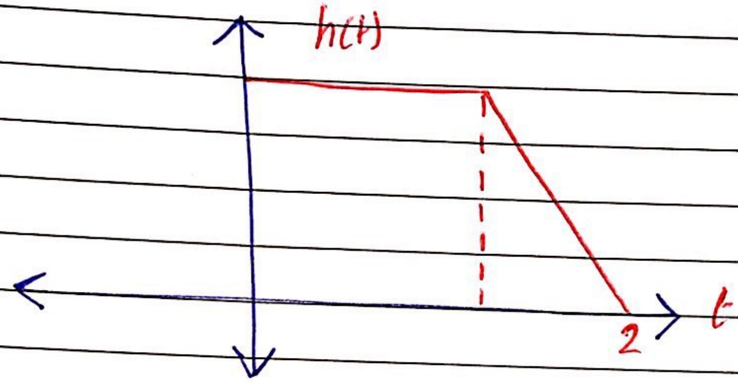
$x(t) = e^{-t}$, $h(t) = e^{-t}$

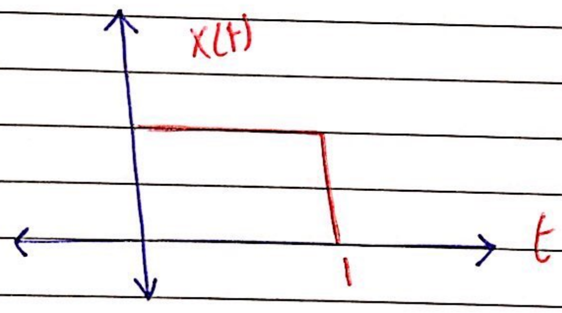


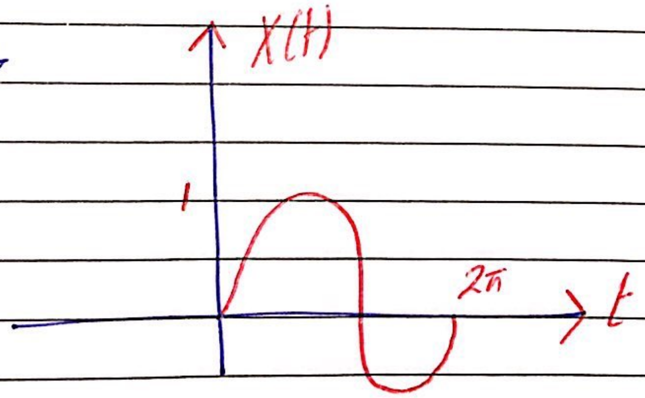
1. $y(t) = 0, \quad t < 0$

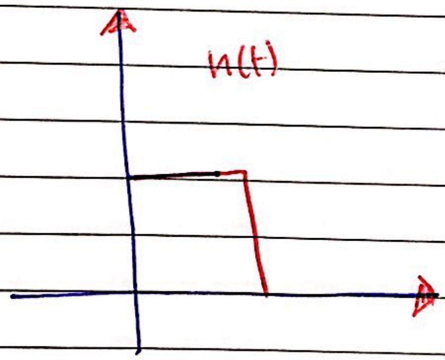
2. $y(t) = \int_0^t (e^{\tau} \cdot e^{-t}) * e^{-\tau} \, d\tau = \tau e^{-t} \Big|_0^t = te^{-t}, \quad t \geq 0.$

Homework.









$X(t) = \sin(t), \quad 0 \leq t \leq 2\pi$

$= \sin t \, (u(t) - u(t - 2\pi)).$