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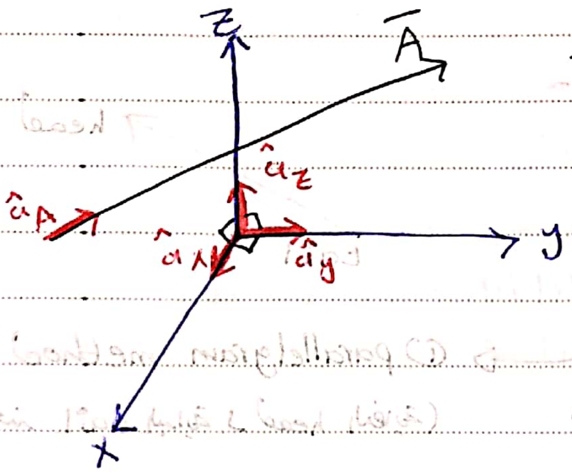
POWERUNIT

Chapter 1 :- Vectors Review

- How to write a vector ?

$\overline{A} \Rightarrow$ vector A (magnitude & direction)

$\rightarrow$ In cartesian coordinates :-



$\overline{A}$ $\rightarrow$ long format :-
$\overline{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$\rightarrow$ short format :-
$\overline{A} = (A_x, A_y, A_z)$

, where $A_x, A_y, A_z \equiv$ vector components
$\rightarrow$ (scaler) (magnitude only)

& $\underbrace{\hat{a}_x, \hat{a}_y, \hat{a}_z}$ $\equiv$ unit vectors
$\rightarrow$ magnitude $=1$ (Direction only)

$|\overline{A}| \Rightarrow$ magnitude , $|\overline{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\hat{a}_A \Rightarrow$ direction , $\hat{a}_A = \frac{\overline{A}}{A} = \frac{A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$

$\rightarrow \overline{A} = A \hat{a}_A$

* if vector B

$$\overline{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$$

ثابتين

$$\overline{B} = (B_x, B_y, B_z)$$

- ex: $\overline{B} = (4, 0, 5)$
- ex: $\overline{C} = 3\hat{a}_x + 5\hat{a}_z$

$\rightarrow \hat{a}_c = \frac{\overline{C}}{C}$, $C = \sqrt{34}$

$= \frac{3\hat{a}_x}{\sqrt{34}} + \frac{5\hat{a}_z}{\sqrt{34}}$

→ Operations on vectors :-

1) Addition & subtraction ⇒ vector ناتج

$\bar{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$

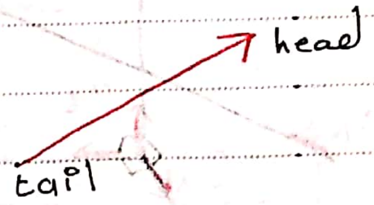
$\bar{C} = \bar{A} + \bar{B} = (A_x + B_x)\hat{a}_x + (A_y + B_y)\hat{a}_y + (A_z + B_z)\hat{a}_z$

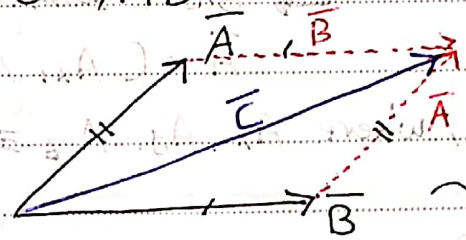
$\bar{C} = \bar{A} - \bar{B} = (A_x - B_x)\hat{a}_x + (A_y - B_y)\hat{a}_y + (A_z - B_z)\hat{a}_z$

$= C_x \hat{a}_x + C_y \hat{a}_y + C_z \hat{a}_z$

- Graphical addition :-

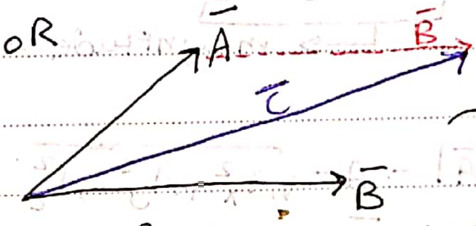
$\bar{C} = \bar{A} + \bar{B}$





→ ① parallelgram method
(عند tail البداية لـ head النهاية)

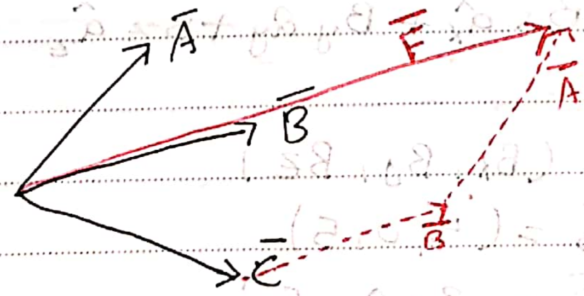
oR



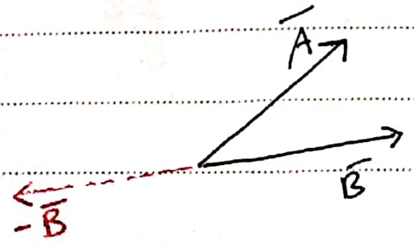
→ ② Arrow method
(عند نهاية الـ vector الأول نحط الـ vector الثاني وبعدها بوصل من tail الأول لـ head الثاني)

أفضل

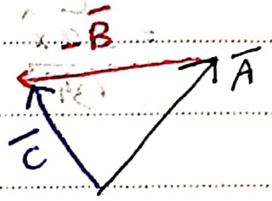
ex :- $\bar{F} = \bar{A} + \bar{B} + \bar{C}$



- Graphical subtraction :-



→



$\bar{C} = \bar{A} - \bar{B}$

$= \bar{A} + (-\bar{B})$

- **Properties of Addition & subtraction :-**

① $\bar{A}+\bar{B} = \bar{B}+\bar{A}$, $\bar{A}-\bar{B} \neq \bar{B}-\bar{A}$

② $K(\bar{A} \pm \bar{B}) = K\bar{A} \pm K\bar{B}$

③ $\bar{A}+(\bar{B}+\bar{C}) = (\bar{A}+\bar{B})+\bar{C}$, $\bar{A}-(\bar{B}-\bar{C}) \neq (\bar{A}-\bar{B})-\bar{C}$

2 Multiplication :-

1- Dot product → ناتج scaler

$$\bar{A}\cdot\bar{B} = |\bar{A}|\,|\bar{B}| \cos\theta_{AB}$$

$\bar{A}\cdot\bar{B}$ → ضرب عادي $(A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z) \times (B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z)$

2→ $\hat{a}_x\cdot\hat{a}_x = |1|\,|1| \cos(0) = 1$

$\hat{a}_x\cdot\hat{a}_y = |1|\,|1| \cos(90^\circ) = 0$

$$\Rightarrow \bar{A}\cdot\bar{B} = A_xB_x + A_yB_y + A_zB_z$$

so

$$\boxed{\begin{aligned} \cos\theta_{AB} &= \frac{\bar{A}\cdot\bar{B}}{AB} \quad \frac{\text{scaler}}{\text{scaler}} \\ \theta_{AB} &= \cos^{-1}\left(\frac{\bar{A}\cdot\bar{B}}{AB}\right) \end{aligned}}$$

- **properties :-**

① $\bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{A}$

② $\bar{A}\cdot(\bar{B}+\bar{C}) = \bar{A}\cdot\bar{B} + \bar{A}\cdot\bar{C}$

③ $\bar{A}\cdot\bar{A} = A^2 = A_x^2 + A_y^2 + A_z^2$

2- cross product → ناتج vector

$|\bar{A}\times\bar{B}| = |\bar{A}|\,|\bar{B}| \sin\theta_{AB}$, $\underbrace{\bar{A}\times\bar{B}}_{\text{new vector}} \perp \bar{A}$ & $\perp \bar{B}$

$$\sin\theta_{AB} = \frac{|\bar{A}\times\bar{B}|}{AB}$$

→

$$\bar{A}\times\bar{B} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

row ← → column

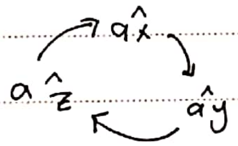
$$\bar{A}\times\bar{B} = (-1)^{1+1}(A_yB_z - A_zB_y)\,\hat{a_x}$$
$$+ (-1)^{1+2}(A_xB_z - A_zB_x)\,\hat{a_y}$$
$$+ (-1)^{1+3}(A_xB_y - A_yB_x)\,\hat{a_z}$$

* $|\hat{a_x} \times \hat{a_y}| = |1||1| \sin 90^\circ = 1$

$|\hat{a_x} \times \hat{a_x}| = 0$

$|\hat{a_x} \times \hat{a_z}| = 1$

$\hookrightarrow \hat{a_x} \times \hat{a_y} = \hat{a_z}$ (Reght hand Role - RHR)



مع السهم موجب
عكس السهم
سالب

$\hookrightarrow \hat{a_z} \times \hat{a_y} = -\hat{a_x}$

Properties :-

① $\bar{A}\times\bar{B} \neq \bar{B}\times\bar{A}$

$\bar{A}\times\bar{B} = -\bar{B}\times\bar{A}$

② $\bar{A}\times(\bar{B}+\bar{C}) = (\bar{A}\times\bar{B}) + (\bar{A}\times\bar{C})$

③ $\bar{A}\times(\bar{B}\times\bar{C}) \neq (\bar{A}\times\bar{B})\times\bar{C}$

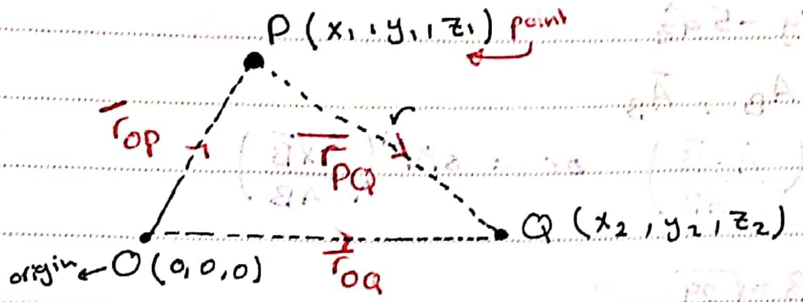
$\bar{A}\times\bar{B}\times\bar{C} \Rightarrow$ Triple cross product

ما نستخدم العلاقة الا بكتابة

بشكل 2cross product

* division $\Rightarrow \dfrac{\text{vec}}{\text{scaler}}$ or $\dfrac{\text{scaler}}{\text{scaler}}$ ولا $\dfrac{\text{vector}}{\text{vector}}$

→ Distance : [vector]



$\bar{r}_P \equiv \bar{r}_{oP}$

$\bar{r}_Q \equiv \bar{r}_{oQ}$

$\bar{r}_{PQ}$ → vector

$$\bar{r}_{PQ} = \bar{r}_Q - \bar{r}_P$$
$$= (x_2, y_2, z_2) - (x_1, y_1, z_1)$$
$$= (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$

$$|\bar{r}_{PQ}| = r_{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

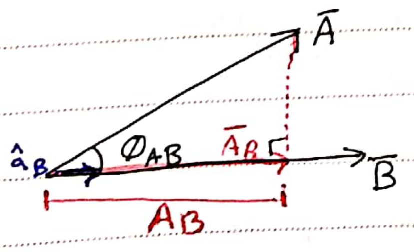
→ magnitude

→ direction

$$\hat{a}_{r_{PQ}} = \frac{\bar{r}_{PQ}}{r_{PQ}} = -\hat{a}_{r_{QP}}$$

* $|\bar{r}_{QP}| = |\bar{r}_{PQ}|$ but $\bar{r}_{PQ} = -\bar{r}_{QP}$

→ Vector projection along another vector :-



$$\cos\theta_{AB} = \frac{A_B}{A} \quad \frac{\text{مجاور}}{\text{وتر}}$$

A_B → scaler projection of $\bar{A}$ along $\bar{B}$ [$\bar{B}$ على $\bar{A}$ إسقاط]

$\bar{A}_B$ → vector " " " " "

* $\cos\theta_{AB} = \frac{A_B}{A}$ → $A_B = A\cos\theta_{AB}$ where, $\cos\theta_{AB} = \frac{\bar{A}\cdot\bar{B}}{AB}$, $\hat{a}_B = \frac{\bar{B}}{B}$

$$= \frac{\bar{A}\cdot\hat{a}_B}{A}$$

$$\boxed{A_B = \bar{A}\cdot\hat{a}_B}$$

$$\boxed{\begin{aligned}\bar{A}_B &= A_B\,\hat{a}_B \\ &= (\bar{A}\cdot\hat{a}_B)\,\hat{a}_B\end{aligned}}$$

ex :- if $\bar{A} = 3a\hat{x} + 4a\hat{y} + a\hat{z} \rightarrow = (3,4,1)$

$\bar{B} = 2\hat{a}y - 5a\hat{z}$

find θ_{AB}, A_B, $\bar{A}_B$

sol :- $\theta_{AB} = \cos^{-1}\left(\frac{\bar{A}\cdot\bar{B}}{AB}\right)$ or $= \sin^{-1}\left(\frac{\bar{A}\times\bar{B}}{AB}\right)$

$A = \sqrt{26}$, $B = \sqrt{29}$

$\bar{A}\cdot\bar{B} = 0 + 8 - 5 = 3$

$\theta_{AB} = \cos^{-1}\left(\frac{3}{\sqrt{26}\sqrt{29}}\right) = \boxed{83.73^\circ}$

2 degree

$A_B = \bar{A}\cdot\hat{a}_B$, $\hat{a}_B = \frac{\bar{B}}{|B|} = \frac{2}{\sqrt{29}} a\hat{y} - \frac{5}{\sqrt{29}} a\hat{z}$

$= (3,4,1)\cdot\frac{(0,2,-5)}{\sqrt{29}}$

$A_B = 0 + \frac{8}{\sqrt{29}} \frac{-5}{\sqrt{29}} = \boxed{\frac{3}{\sqrt{29}}}$

$\bar{A}_B = A_B\,\hat{a}_B$

$= \frac{3}{\sqrt{29}}\left(0, \frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}\right)$

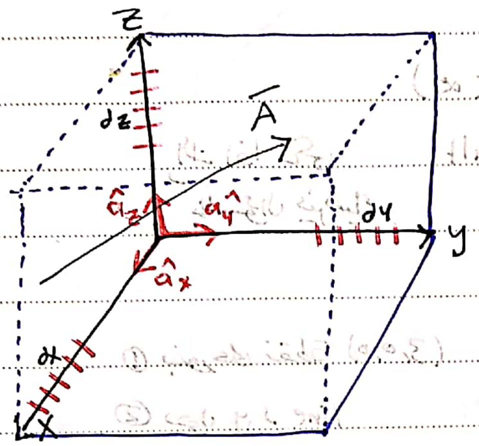
$= \frac{6}{(\sqrt{29})^2} a\hat{y} - \frac{15}{(\sqrt{29})^2} a\hat{z} = \frac{6}{29} a\hat{y} - \frac{15}{29} \hat{a}z$

CH2 : Coordinate systems

① Cartesian coordinates

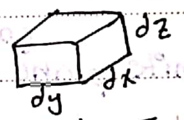
$-\infty < x < \infty$
$-\infty < y < \infty$ → 3D object
$-\infty < z < \infty$ Infinite Box

unit vectors : $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$



- Differential elements :-

dx , dy , dz



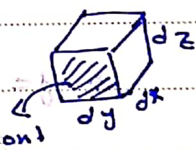
- Differential length ($\bar{dl}$) [vector]

$\bar{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

- Differential Normal surface Area ($\bar{ds}$) [vector]

* $\bar{ds}_{front} = \underbrace{dy\,dz}_{\text{scaler مساحة}}\,\hat{a}_x$ → عمودي [Normal]

لحتى يكون vector = vector



* $\bar{ds}_{back} = dy\,dz\,(-\hat{a}_x)$

* $\bar{ds}_{right} = dx\,dz\,\hat{a}_y$

* $\bar{ds}_{left} = -dx\,dz\,\hat{a}_y$

* $\bar{ds}_{top} = dx\,dy\,\hat{a}_z$

* $\bar{ds}_{bot} = -dx\,dy\,\hat{a}_z$

- Differential Volume (dv) :- [scaler]

$dv = dx\,dy\,dz$

→ 2D surfaces :-

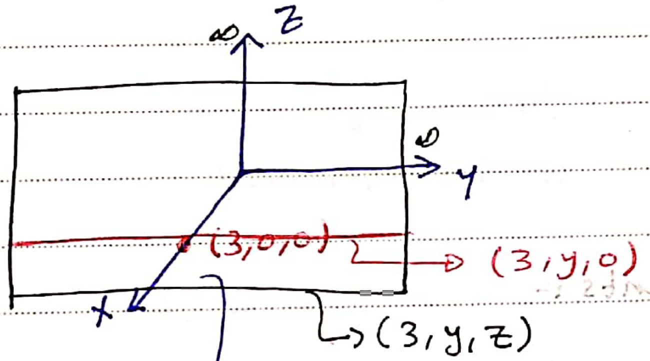
⇒ by fixing one variable

* $x =$ constant

$x = 3$ $\quad (-\infty < \frac{y}{z} < \infty)$

⇒ default

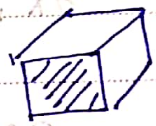
إلا إذا حكى
بالسؤال غير هيك



① بنور على نقطة (3,0,0)

② بحول y لـ line

③ بعمل variable y / z
بس x عند 3



(parallel)
infinite plane // yz plane

→ what is the normal on that surface :-

$\hat{a}_x$ / $-\hat{a}_x$, if $(6,0,0) \rightarrow \hat{a}_x$

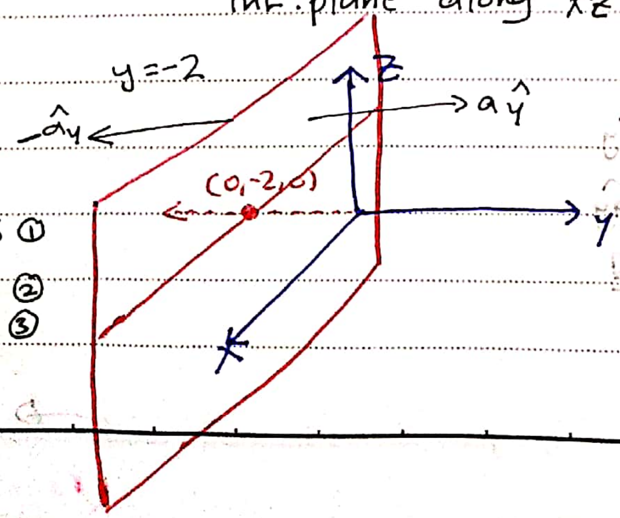
⊙ ⊗

if $x = 0$ → inf. plane along yz plane

* $y =$ constant

inf. plane // xz plane if $(y \neq 0)$

inf. plane along xz plane if $(y = 0)$



① نقطة (0,-2,0)

② line ← x

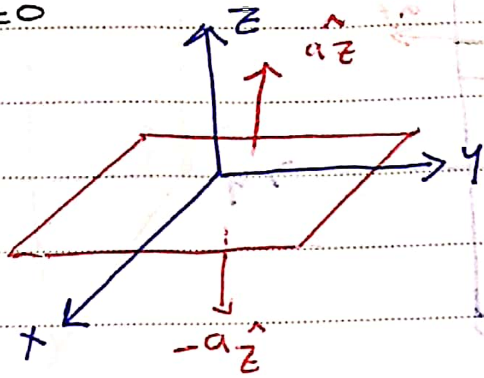
③ x/ z بنغيرهم

* $z = \text{constant}$

inf. plane // xy plane if $(z \neq 0)$

inf. plane along xy plane if $(z = 0)$

$z = 0$



→ 1D segment:

↳ by fixing two variables.

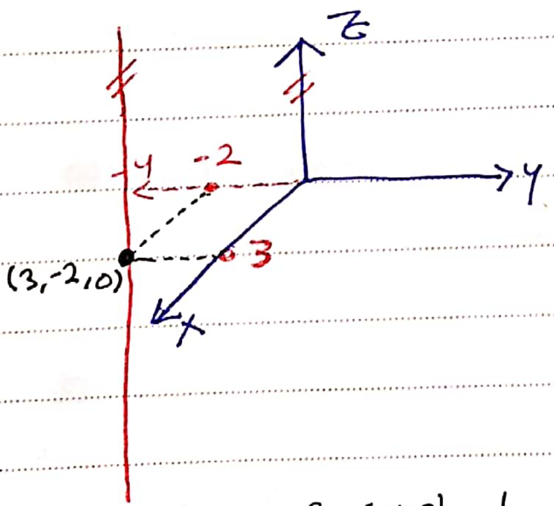
* x, y are constants $(-\infty < z < \infty)$

↳ inf. line // z axis , otherwise (ex: $x=0, y \neq 0$)

inf. line along z axis if $(x = 0$ & $y = 0)$

له لازم التنين =0

$x = 3, \; y = -2$



* x, z are constant → inf. line // y-axis

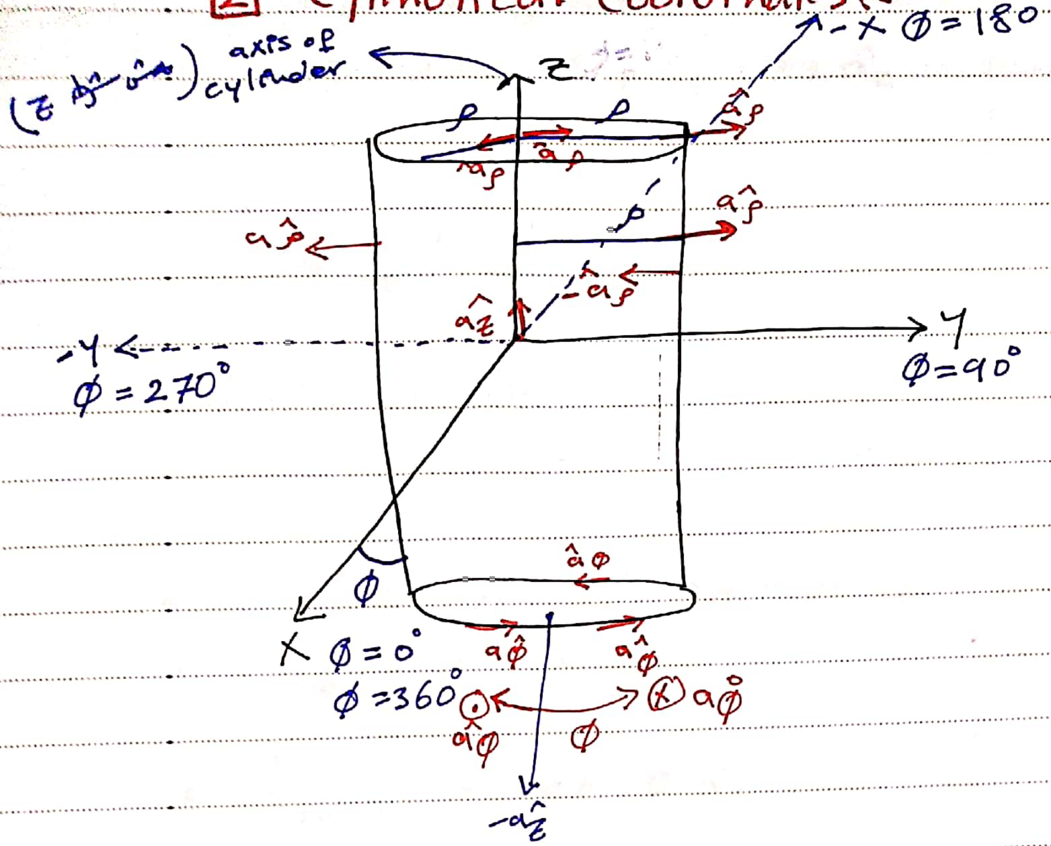
↳ inf. line along y-axis (if $x=0$ & $z=0$)

* y, z are constant → inf. line // x-axis

↳ inf. line along x-axis (if $y=0$ & $z=0$)

→ point: by fixing the three variables.

2 Cylindrical Coordinates:-



$\hat{a}_\rho \rightarrow$ طالع من الاسطوانة

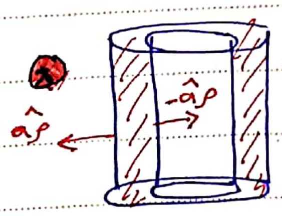
$-\hat{a}_\rho \rightarrow$ داخل للاسطوانة

- $0 \leq \rho < \infty$ [$\rho \rightarrow$ radius] [من محور الاسطوانة لـ سطح الاسطوانة]
  $0 < \phi \leq 2\pi$ [$\phi \rightarrow$ rotation]
  $-\infty \leq z < \infty$ } 3D object Inf. solid cylinder

  ارتفاع الاسطوانة

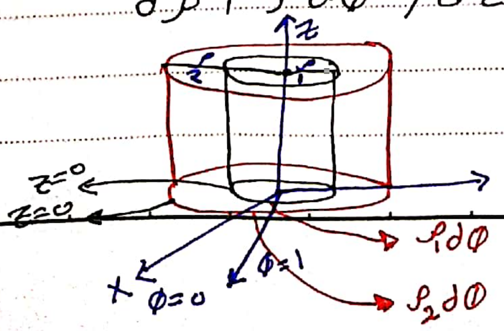
- unit vectors:-

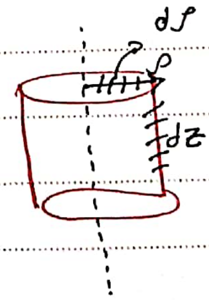
  $\hat{a}_\rho$, $\hat{a}_\phi$, $\hat{a}_z$

- 

- Differential elements &:

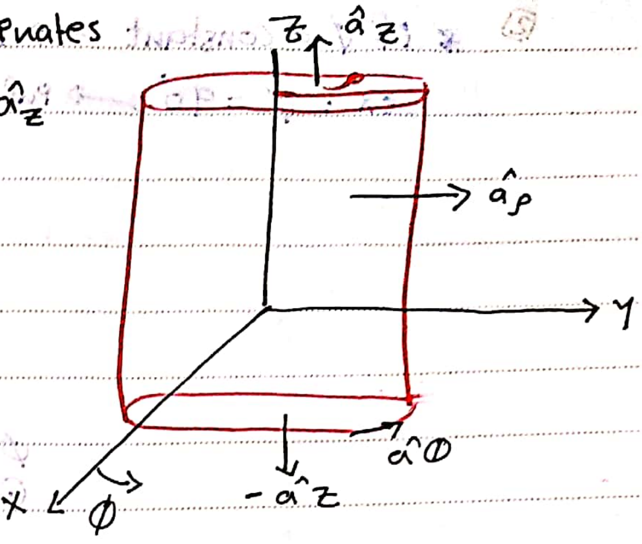
  $d\rho$, $\rho\, d\phi$, dz





→ vector in cylindrical coordinates

$\overline{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$



* $(3, 60^\circ, 4)$
  ρ ↙ ϕ ↙ z

* differential elements :
  $d\rho$, $\rho d\phi$, dz

* $\overrightarrow{dL} = d\rho\, \hat{a}_\rho + \rho d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$
* $\overrightarrow{ds}_{top} = \rho\, d\rho\, d\phi\, \hat{a}_z$
* $\overrightarrow{ds}_{bottom} = -\rho\, d\rho\, d\phi\, \hat{a}_z$
* $\overrightarrow{ds}_{side} = \rho\, d\phi\, dz\, \hat{a}_\rho$
* $\overrightarrow{ds}_{cut} = d\rho\, dz\, \hat{a}_\phi$ $(\phi = \text{constant})$

} vectors

* $dv = \rho\, d\rho\, d\phi\, dz$ } scaler

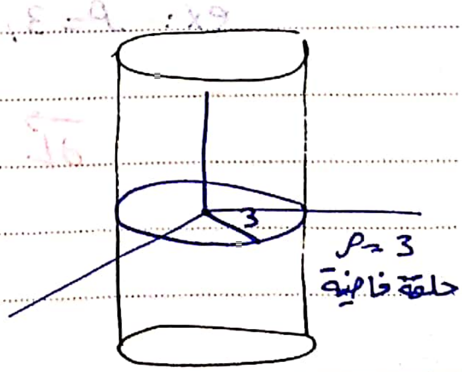
→ 2D Surface :-

① * if ρ = constant → infinite hollow Cylinder

ex : $\rho = 3$, $[\phi, z]$ → variables

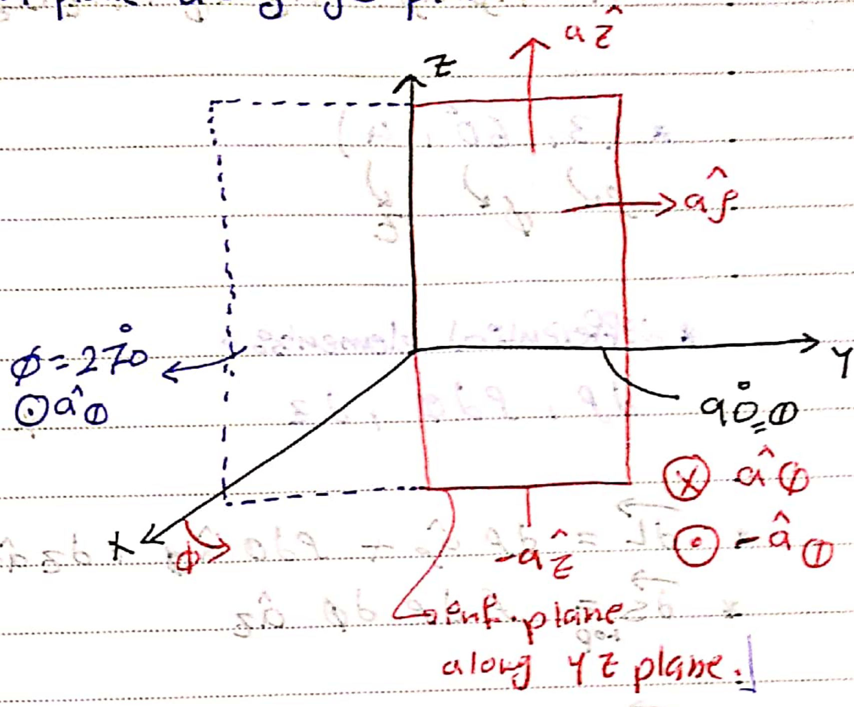
but if $\rho = 0$ → infinite line along z-axis [1D segment]

⇒ use $\overrightarrow{ds} = \rho\, d\phi\, dz\, \hat{a}_\rho$



② * if ϕ = constant → inf. plane

ex: $\phi = 90^\circ$ → inf. plane along yz plane

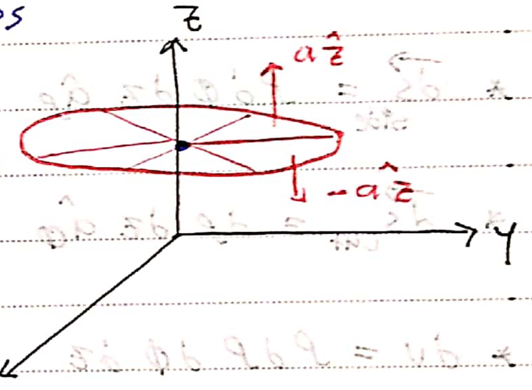


③ * if z = constant

ex: z = 2, (ϕ, ρ) → variables

→ inf. disk // xy plane (if $z \neq 0$)

inf. disk along xy plane (if $z = 0$)



→ 1D segment

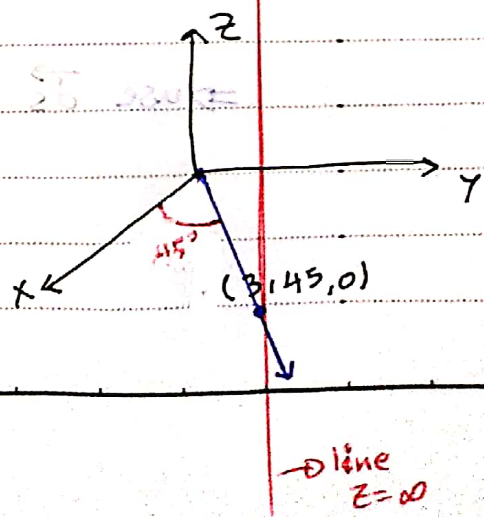
① ρ, ϕ are constants

→ inf. line // z axis ($\rho \neq 0$, $\rho > 0$)

→ inf line along z-axis ($\rho = 0$)

ex: $\rho = 3$, $\phi = 45^\circ$

$\vec{dL} = dz\, \hat{a_z}$



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[2] ρ, z are constants

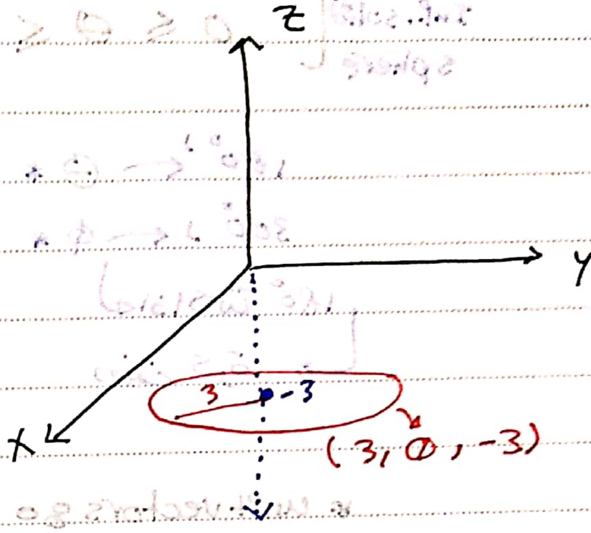
↳ circle (not inf) // xy plane [$\rho > 0$, $z \neq 0$]

↳ circle along xy ($z = 0$) ($\rho > 0$)

↳ point at z axis [$\rho = 0$]

ex: $\rho = 3$, $z = -3$

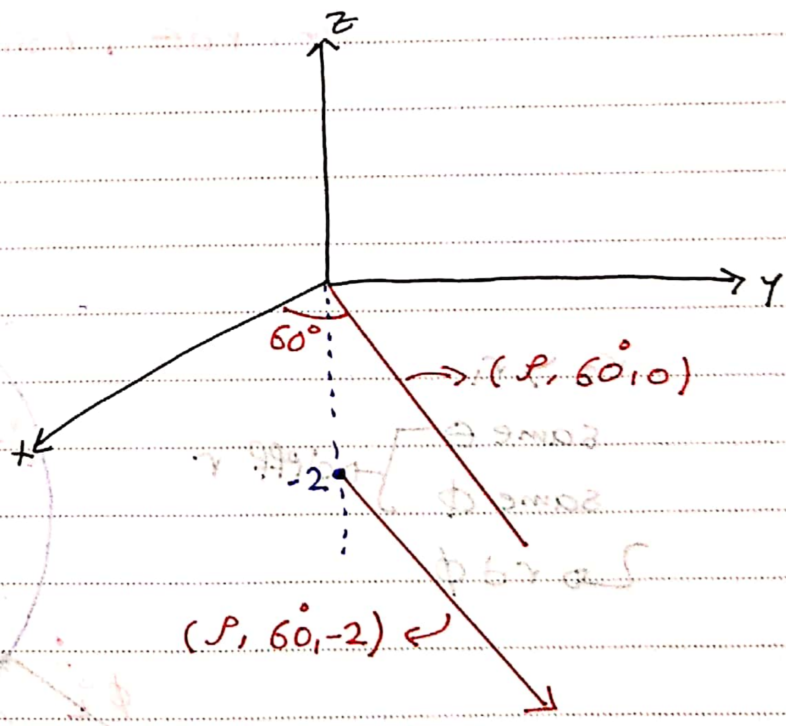
$\vec{dL} = 3\, d\phi\, \hat{a}_\phi$



[3] ϕ, z are constants $\quad 0 \leq \rho < \infty$

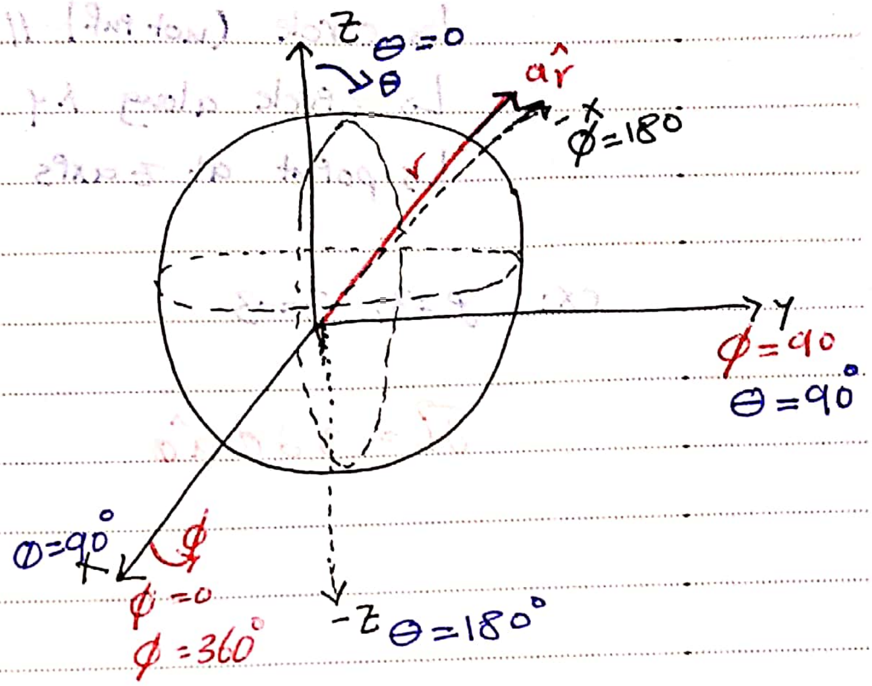
↳ semi-inf line

ex: $\phi = 60^\circ$, $z = -2$



3 Sperical coordinates :

3D object Inf. solid sphere
$$\begin{cases} 0 \leqslant r < \infty \\ 0 \leqslant \phi \leqslant 2\pi \\ 0 \leqslant \theta \leqslant \pi \end{cases}$$



θ * $\rightarrow$ 180° د

ϕ * $\rightarrow$ 360° د

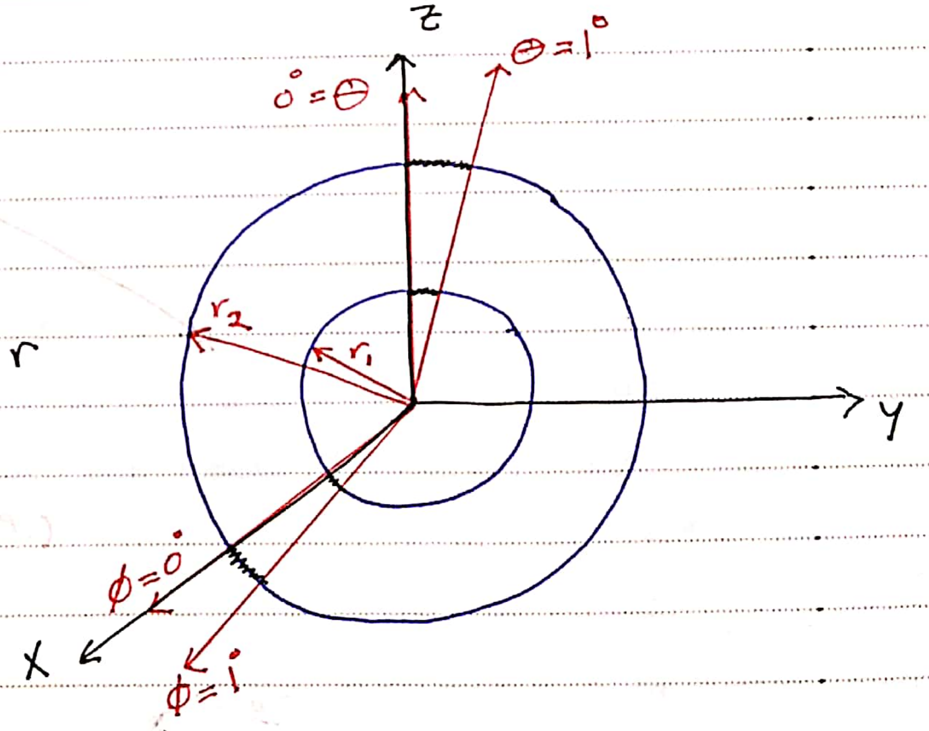
واذا كانت 180°
$\rightarrow$ نصف كرة

* unit vectors :

$\hat{a_r}$, $\hat{a_\phi}$, $\hat{a_\theta}$

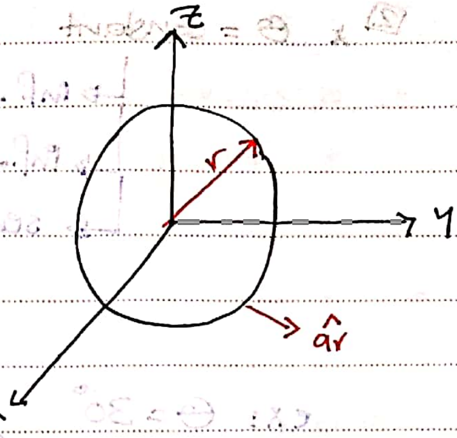
* Differential elements :

dr , $r d\theta$, $r \sin\theta \, d\phi$



$r_2 > r_1$
same θ
same ϕ } $\rightarrow$ diff r
$\rightarrow r\,d\phi$

* $\vec{dL} = dr\,\hat{a_r} + r\,d\theta\,\hat{a_\theta} + r\sin\theta\,d\phi\,\hat{a_\phi}$



* $\vec{ds}_{surface} = r^2\sin\theta\,d\theta\,d\phi\,\hat{a_r}$

$\vec{ds}_{H.cut} = r\sin\theta\,dr\,d\phi\,\hat{a_\theta}$

$\vec{ds}_{V.cut} = r\,dr\,d\theta\,\hat{a_\phi}$

$dv = r^2\sin\theta\,dr\,d\theta\,d\phi$ (scaler)

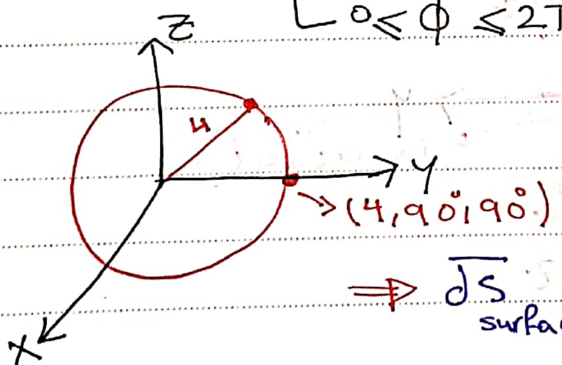
→ 2D surface

1) * $r =$ constant

→ hollow sphere $(r>0)$

→ point at the origin $(r=0)$

ex: $r=4$ $\left[\begin{array}{l} 0\le\theta\le\pi \\ 0\le\phi\le 2\pi \end{array}\right]$



$\Rightarrow \vec{ds}_{surface} = r^2\sin\theta\,d\theta\,d\phi\,\hat{a_r}$

[منأخذ القانون الي فيه المتغيرات θ / ϕ]

→

② * θ = constant

↳ inf. hollow cone [$\theta \in (0°, 90°)$ / $\theta \in (90°, 180°)$] — cone لفوق / cone لتحت

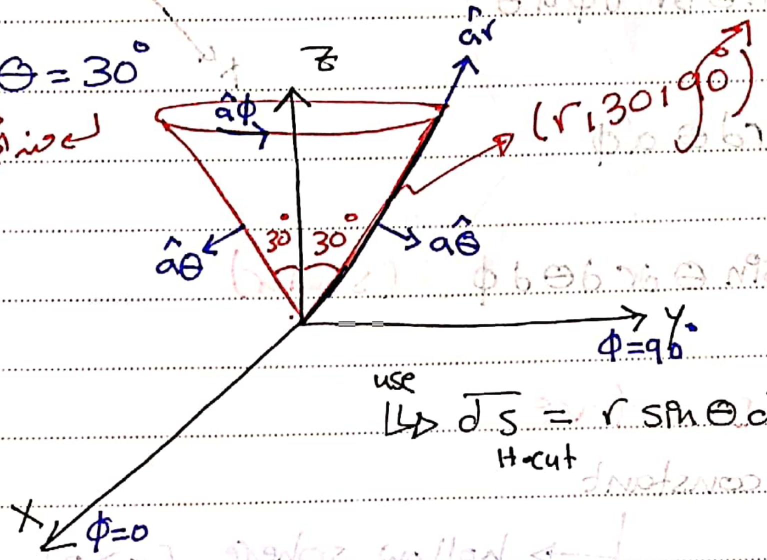
↳ inf. disk along x-y plane [$(\theta = 90°)$]

↳ semi-inf. line → +ve z axis [$(\theta = 0°)$]

or −ve z axis [$(\theta = 180°)$]

ex: $\theta = 30°$

بحسب أي جهة

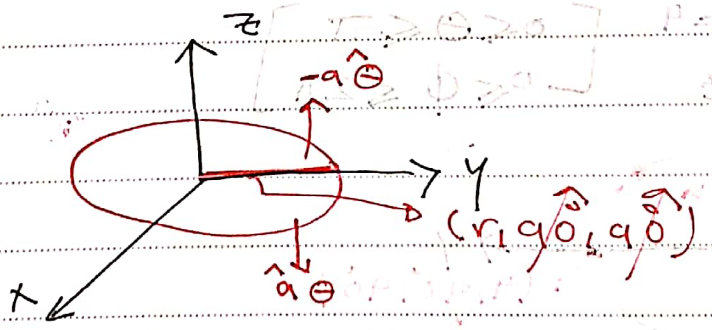


use

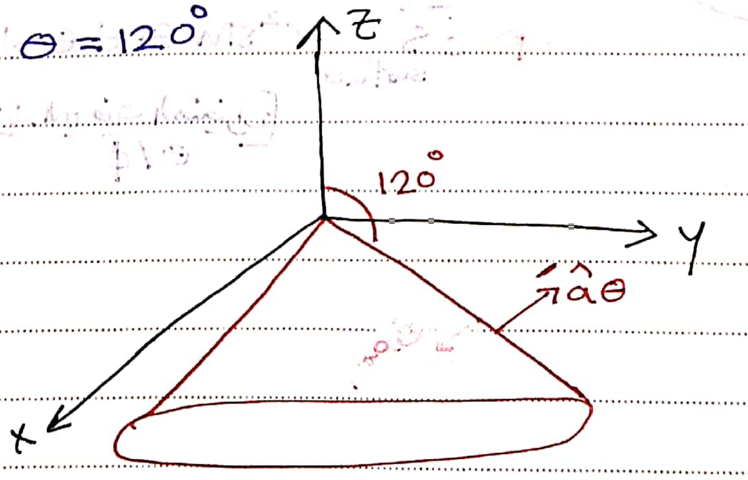
↳ $\vec{ds} = r \sin\theta \, dr \, d\phi \, \hat{a}_\theta$

H-cut

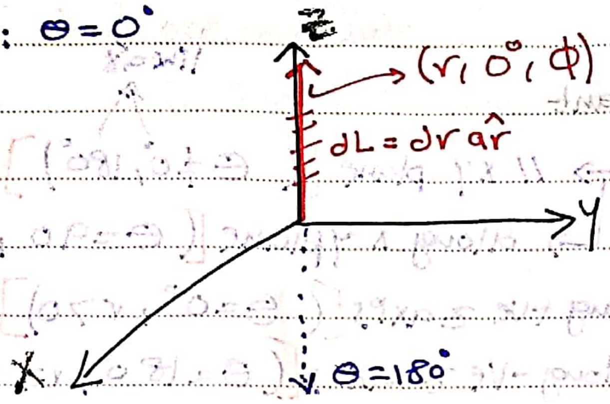
ex: $\theta = 90°$ → inf. disk along x-y plane



ex: $\theta = 120°$



ex: $\theta = 0°$



* اذا كان line بنربط ع $\vec{dL}$ مش ع $\vec{ds}$

* ال ϕ بيطل الها تأثير بالدرجة بحالتين :- $r\sin\theta\,d\phi$

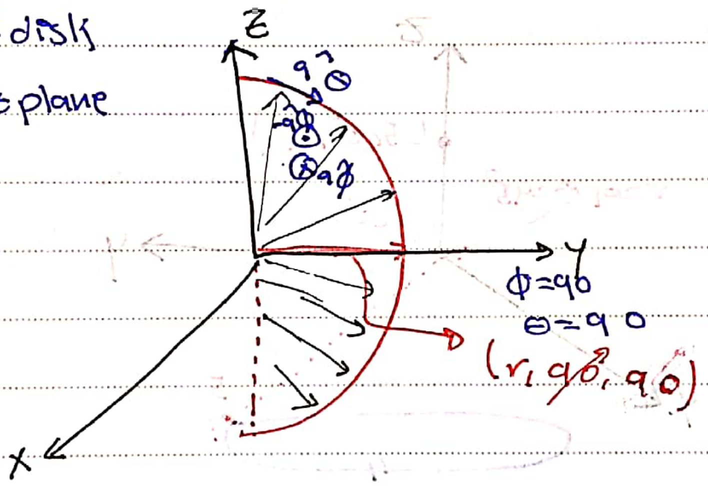
$r = 0$ or $\theta = 0, 180°$

3 * ϕ = constant

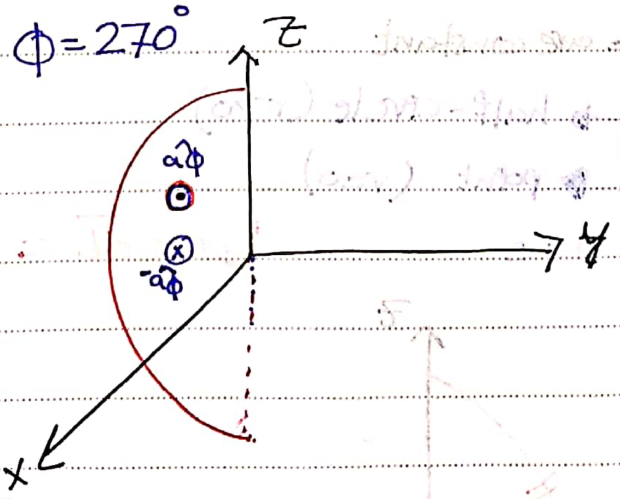
↳ semi-inf Disk

ex: $\phi = 90°$

semi-inf disk along yz plane



ex: $\phi = 270°$



→ 1D segment

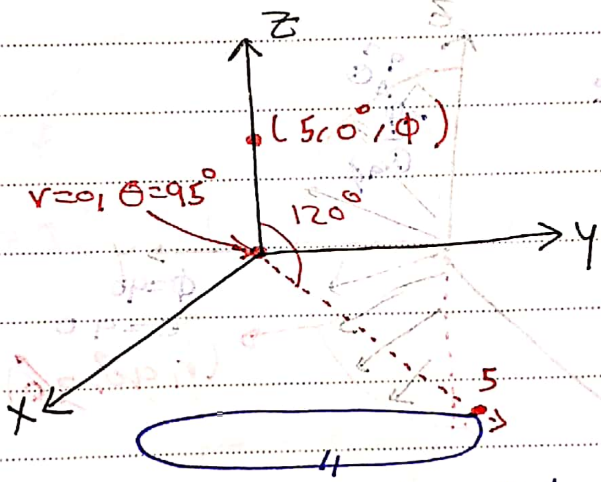
[1] * r, θ = are constant

- circle → // xy plane [$(\theta \neq 0°, 180°)$] linear
  - along x y plane [$(\theta = 90, r > 0)$]
- point along +ve z axis [$(\theta = 0°, r > 0)$]
- point along -ve z-axis [$(\theta = 180°, r > 0)$]
- point at the origin [$(r = 0)$]

↳ use $\vec{dL} = r \sin\theta \, d\phi \, \hat{a}_\phi$

المتغير هو ϕ

ex: $r = 5, \theta = 120°$



2 circle not com.

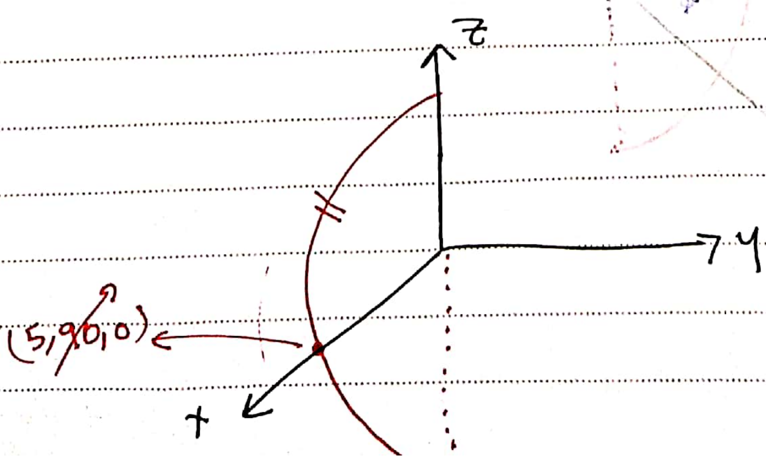
[2] * r, ϕ = are constant

- half-circle $(r > 0)$
- point $(r = 0)$

ex: $r = 5, \phi = 0$

↳ use $\vec{dL} = r \, d\theta \, \hat{a}_\theta$

المتغير θ



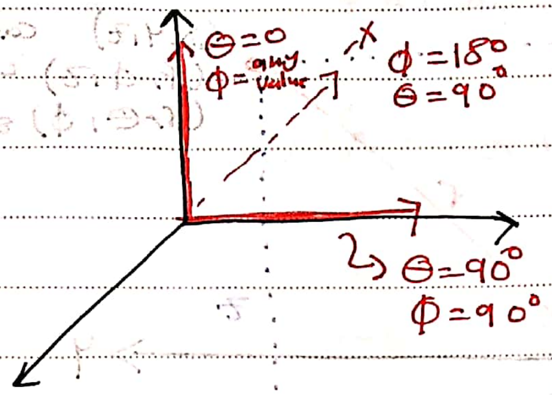
$\vec{dL} = 5 \, d\theta \, \hat{a}_\theta$

3 * θ, ϕ are constants:-

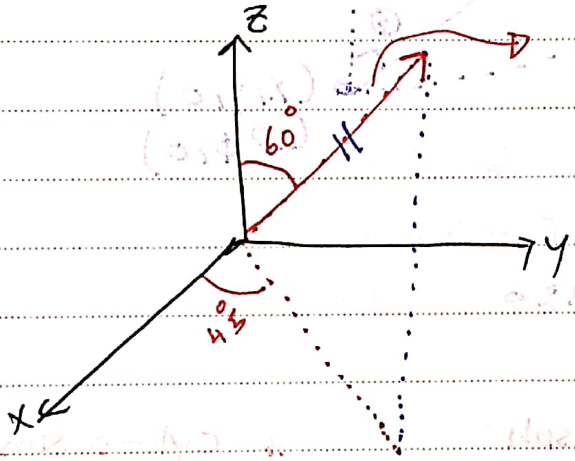
↳ semi-inf line (Ray)

↳ use $d\bar{L} = dr\hat{a}_r$

(المتغير r)



ex: $\theta = 60°$, $\phi = 45°$



★ هاد هو الـ line واسقاطه
العامودي 45° على plane y,x

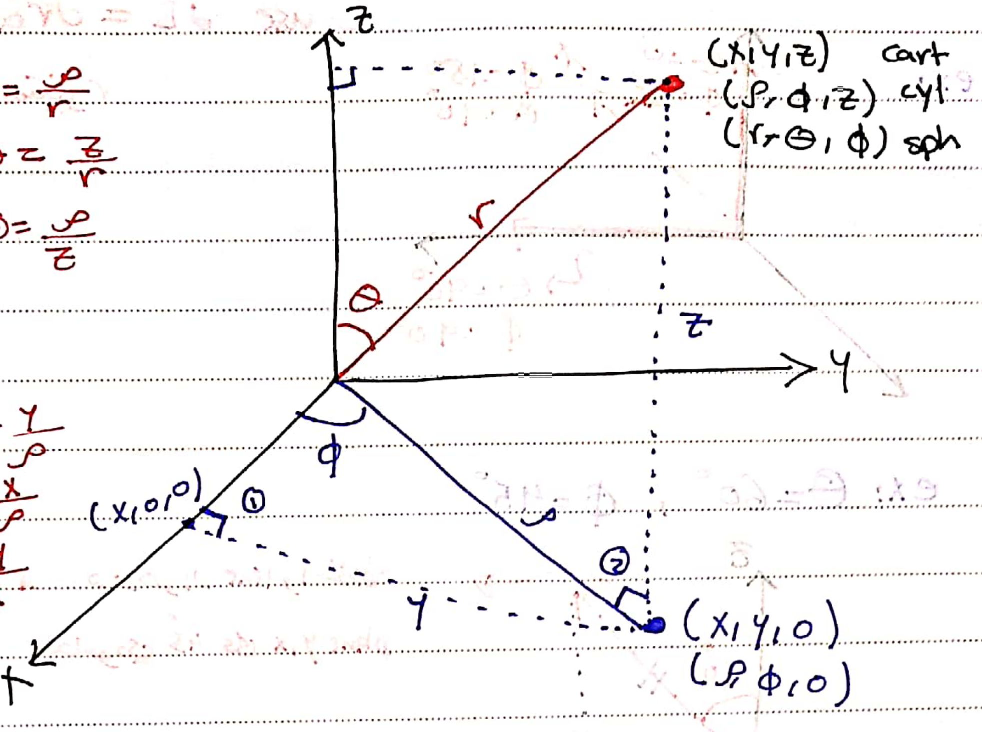
→ Conversion between coordinates :

based on the given geometry.

$\sin\theta = \frac{\rho}{r}$

$\cos\theta = \frac{z}{r}$

$\tan\theta = \frac{\rho}{z}$



$\sin\phi = \frac{y}{\rho}$

$\cos\phi = \frac{x}{\rho}$

$\tan\phi = \frac{y}{x}$

$\boxed{1}$ point conversion :

* cart → cyl
(x,y,z) (ρ,ϕ,z)

$$\rho = \sqrt{x^2+y^2}$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$
$$z = z$$

* cart → sph
(x,y,z) (r,θ,ϕ)

$$r = \sqrt{x^2+y^2+z^2}$$
$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

* cyl → sph
(ρ,ϕ,z) (r,θ,ϕ)

$$r = \sqrt{\rho^2+z^2}$$
$$\theta = \tan^{-1}\left(\frac{\rho}{z}\right)$$
$$\phi = \phi$$

* cyl → cart
(ρ,ϕ,z) (x,y,z)

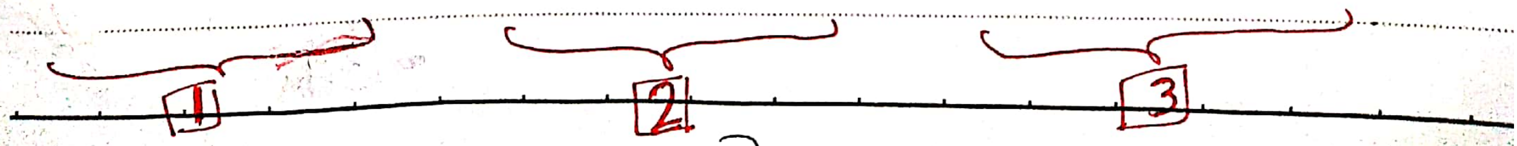
$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

* sph → cart
(r,θ,ϕ) (x,y,z)

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

* sph → cyl
(r,θ,ϕ) (ρ,ϕ,z)

$$\rho = r\sin\theta$$
$$\phi = \phi$$
$$z = r\cos\theta$$



بنجمع بين قوانين $\boxed{1}$ و $\boxed{3}$

$\boxed{2}$ vectors conversion :

* [cart $\rightarrow$ cyl]

$$\bar{A} = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$$

$$\bar{A} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}$$

$$\underbrace{\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}}_{3\times1} = \underbrace{\begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{3\times3} \underbrace{\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}}_{3\times1}$$

مطلوب ← ... ← given

How?? 2 steps

→ $\boxed{1}$ بنفك ال matrix

$A_\rho = \cos\phi\, A_x + \sin\phi\, A_y$ ضرب الصف الاول بالعامود

$A_\phi = -\sin\phi\, A_x + \cos\phi\, A_y$ الثاني بالعامود " "

$A_z = A_z$

+ بنعوض مكان $A_z / A_y / A_x$

مثلاً $A_y = xy$, $A_x = x^2$

$\boxed{2}$ x و y يتم تبديل

$x = \rho\cos\phi$ و $y = \rho\sin\phi$, $z = z$

A_ρ مثلاً $A_\rho = \cos\phi\, x^2 + \sin\phi\, xy$

$= \cos\phi\,(\rho\cos\phi)^2 + \sin\phi(\rho^2\cos\phi\sin\phi)$

....

* [cyl → cart]

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

ال transpose T^{-1}
لل matrix القبل

same steps.

+ بقدر أعمل unit vector conversion بنفس ال matrix

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a_\rho} \\ \hat{a_\phi} \\ \hat{a_z} \end{bmatrix}$$

$$\hat{a_y} = \sin\phi\, \hat{a_\rho} + \cos\phi\, \hat{a_\phi}$$

* [cart → sph]

cart $\bar{A} = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$

sph $\bar{A} = A_r \hat{a_r} + A_\theta \hat{a_\theta} + A_\phi \hat{a_\phi}$

given.

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\theta & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

or unit vector

same

$$\begin{bmatrix} \hat{a_r} \\ \hat{a_\theta} \\ \hat{a_\phi} \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} \hat{a_x} \\ \hat{a_y} \\ a_z \end{bmatrix}$$

ex: $\hat{a_\phi} = -\sin\phi\, \hat{a_x} + \cos\phi\, \hat{a_y}$

* [sph → cart]

الأصل matrix لل T^{-1} بجيب

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\theta & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

or unit vector ل .

* [cyl → sph]

cyl. $\bar{A} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}$

sph → $\bar{A} = A_r \hat{a_r} + A_\theta \hat{a_\theta} + A_\phi \hat{a_\phi}$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$A_r = \sin\theta\, A_\rho + \cos\theta\, A_z$

$A_\theta = \cos\theta\, A_\rho - \sin\theta\, A_z$

$A_\phi = A_\phi$

or unit vector ل .

* بحل بالكتاب cyl → cart of cart → sph .   ↑ نفسهم.

* [sph → cyl].

T^{-1} لل matrix

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

or unit vector.

CH3: Vector calculus :-

- Integration :-

(cart) $\overline{dL} = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$

(cyl) $\overline{dL} = d\rho\,\hat{a_\rho} + \rho\, d\phi\,\hat{a_\phi} + dz\,\hat{a_z}$

(sph) $\overline{dL} = dr\,\hat{a_r} + r\,d\theta\,\hat{a_\theta} + r\sin\theta\, d\phi\,\hat{a_\phi}$

$\boxed{1}$ line Integral :-

$$\int_l \bar{A} \cdot \overline{dL}$$

i.e : $\bar{A} = (A_x, 0, A_z) \rightarrow$ cart

$\rightarrow \overline{dL} = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$ → step1

$\bar{A} \cdot \overline{dL} = A_x\,dx + A_z\,dz$ → step2 (dot product)

$\int_l (A_x\,dx + A_z\,dz)$ → step3 (integral)

$l\,(x,y,z)$ general

$= \int_x A_x\,dx + \int_z A_z\,dz$ 2 single .

i.e : $\bar{A} = (A_\rho, A_\phi, A_z) \rightarrow$ cyl

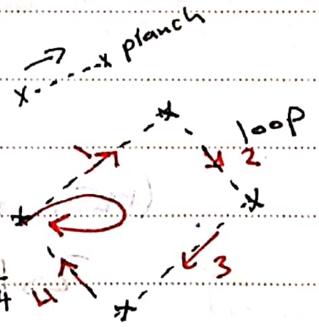
$\rightarrow \overline{dL} = d\rho\,\hat{a_\rho} + \rho\,d\phi\,\hat{a_\phi} + dz\,\hat{a_z}$ → ①

$\bar{A} \cdot \overline{dL} = A_\rho\,d\rho + \rho A_\phi\,d\phi + A_z\,dz$ → ②

$\int_l A_\rho\,d\rho + \rho A_\phi\,d\phi + A_z\,dz$ → ③

$= \int_\rho A_\rho\,d\rho + \int_\phi \rho A_\phi\,d\phi + \int_z A_z\,dz$

→ closed Line Integral



$$\oint_l \bar{A} \cdot \overline{dL}$$

$= \int_{l_1} \bar{A} \cdot \overline{dL_1} + \int_{l_2} \bar{A} \cdot \overline{dL_2} + \int_{l_3} \bar{A} \cdot \overline{dL_3} + \int_{l_4} \bar{A} \cdot \overline{dL_4}$

step1 (أخلصه منه (circulation

$dy\,\hat{a_y} + dz\,\hat{a_z}$ $dy\,\hat{a_y} + dz\,\vec{a_z}$

step بحل ال مثل فوق

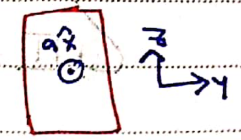
* [ما بنحط السالب ع dL بنلعب بحدود التكامل]

2) [2] surface Integral :-

$$\iint_S \bar{A} \cdot \bar{ds}$$

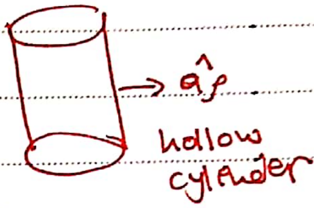
i.e: $\bar{ds} = dy\, dz\, \hat{a}_x$ →

$\bar{A} = (A_x, A_y, A_z)$



i.e: $\bar{A} = (A_\rho, A_\phi, A_z)$

$\bar{ds} = \rho\, d\phi\, dz\, \hat{a}_\rho$

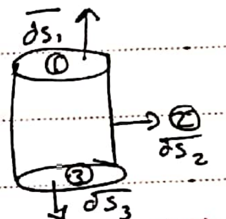


$$\int_S \bar{A} \cdot \bar{ds} = \int_z \int_\phi A_\rho\, \rho\, d\phi\, dz$$

→ closed surface Integral :

volume

$$\oint_S \bar{A} \cdot \bar{ds} \quad , \bar{A} = (A_\rho, A_\phi, A_z)$$



cyl. ل

$$\begin{cases} \bar{ds}_1 = \rho\, d\rho\, d\phi\, \hat{a}_z \\ \bar{ds}_2 = \rho\, d\phi\, dz\, \hat{a}_\rho \\ \bar{ds}_3 = -\rho\, d\rho\, d\phi\, \hat{a}_z \end{cases}$$

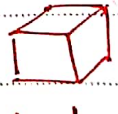
لو بوحدة hollow متكون مش closed surface Int

$$\oint_S \bar{A} \cdot \bar{ds} = \underbrace{\int_{S_1} \bar{A} \cdot \bar{ds}_1}_{\to A_z} + \underbrace{\int_{S_2} \bar{A} \cdot \bar{ds}_2}_{\to A_\rho} + \underbrace{\int_{S_3} \bar{A} \cdot \bar{ds}_3}_{\to \bar{A}_z}$$

*ال ds عادي تكون سالبة / + بس $\bar{dL}$ لازم + (نيلعب بحدود التكامل)

2) [3] Volume Integral :-

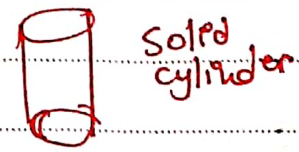
$$\int_V |\bar{A}|\, dv \quad \to \bar{A}$$



i.e: $dv = dx\, dy\, dz$

$$\int_V |\bar{A}|\, dv = \int_z \int_y \int_x A\, dx\, dy\, dz$$

i.e: $dv = \rho\, d\rho\, d\phi\, dz$



$$\int_V |\bar{A}|\, dv = \int_z \int_\phi \int_\rho A\, \rho\, d\rho\, d\phi\, dz$$

Del operator :- (vector tool)

$\nabla \rightarrow$ هو vector بس مانجط -

$$\nabla = \frac{\partial}{\partial x}\hat{a_x} + \frac{\partial}{\partial y}\hat{a_y} + \frac{\partial}{\partial z}\hat{a_z}$$

$\rightarrow$ إله 4 استخدامات : vector / scaler

✓ [1] Gradiant $\rightarrow \nabla V \rightarrow$ vector

✗ [2] Divergence $\rightarrow \nabla \cdot \bar{A} \rightarrow$ scaler

[3] curl $\rightarrow \nabla \times \bar{A} \rightarrow$ vector

[4] Laplacian $\rightarrow \nabla \cdot (\nabla V) = \nabla^2 V$

Gradiant :-

in cart :- $\nabla V = \frac{\partial V}{\partial x}\hat{a_x} + \frac{\partial V}{\partial y}\hat{a_y} + \frac{\partial V}{\partial z}\hat{a_z}$

in cyl :- $\nabla V = \frac{\partial V}{\partial \rho}\hat{a_\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a_\phi} + \frac{\partial V}{\partial z}\hat{a_z}$

in sph :- $\nabla V = \frac{\partial V}{\partial r}\hat{a_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a_\phi}$

Destance :-

(x_1, y_1, z_1)
(ρ_1, ϕ_1, z_1)
(r_1, θ_1, ϕ_1)

d

(x_2, y_2, z_2)
(ρ_2, ϕ_2, z_2)
(r_2, θ_2, ϕ_2)

* cart: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

* cyl: $d^2 = \rho_1^2 + \rho_2^2 - 2\rho_1\rho_2\cos(\phi_2 - \phi_1) + (z_2 - z_1)^2$

لا نستخدم/بنحول ل cart

OR $\boxed{\text{if } (\phi_1 = \phi_2) \rightarrow d^2 = (\rho_2 - \rho_1)^2 + (z_2 - z_1)^2}$

عالأغلب بيجو

* sph :- $d^2 =$ ---- طويلة

لا نستخدم/بنحول ل cart

OR $\boxed{\text{if } (\theta_1 = \theta_2 \text{ \& } \phi_1 = \phi_2) \rightarrow d^2 = (r_2 - r_1)^2}$

Scanned with CamScanner

EM → part1 :- mathmatics Review

CH1 → CH3.

→ part2 :- electrostatics

CH4 → CH6

→ part3 :- magnetostatic

CH7 – CH8

→ part4 :- AC fields of application

CH9 → CH 11

Scanned with CamScanner

CH4 : Electrostatic Fields (DC field)

* Electrostatic sources:

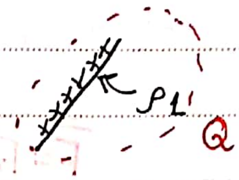
[1] point charge (Q) [C] pos

[2] continuous charge distribution

→ a) line charge (1D segment)

$(\rho_L) \rightarrow (C/m)$

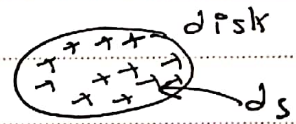
$$Q = \int_{l} \rho_L \, dL$$



→ b) surface charge (ρ_s) (2D surface)

$(\rho_s) \rightarrow (C/m^2)$

$$Q = \int_{s} \rho_s \, ds$$

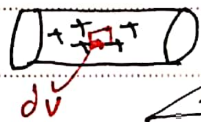


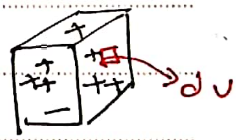
→ c) Volume charge (3D object) (ρ_v)

$(\rho_v) \rightarrow (C/m^2)$

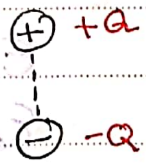
$$Q = \int_{v} \rho_v \, dv$$

$\rho_v \equiv$ density (volume charge)





[3] electric Dipole



[4] polarized Dielectric . [CH5]

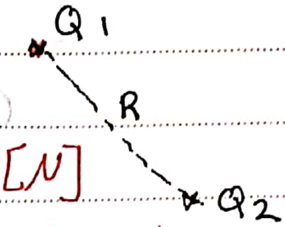
[*] [* static → not moving
→ No current → No magnetic field]

→ Major laws :

1. Colomb's law [general]
2. Gauss's law [special case of 1]
   → applied in 4 cases.

* Colomb's Law :-

→ by experiements



$\vec{F} = \vec{F_e} \equiv$ electrical Force [N]

Relation $F \propto \frac{Q_1 Q_2}{R^2}$

$$\left[F = K \frac{Q_1 Q_2}{R^2} \ [N] \quad , K = \frac{1}{4\pi \varepsilon_0} \right]$$

→ this gives magnitude only.

K: propotionality constant.
→ depends in 2 factors:-

$K = \frac{1}{4\pi \varepsilon_0}$
→ unit used $\left(\frac{1}{4\pi}\right)$
→ Media surrounded the two charge (ε_0)

ε: permitivity

ε_0: Free space permitivity (F/M)

$$\varepsilon_0 = \frac{10^{-9}}{36\pi} \ F/M = 8.85 * 10^{-12} \ F/M$$

$$K = \frac{1}{4\pi * \frac{10^{-9}}{36\pi}} = 9 \times 10^{9} \text{ in free space}$$

$\vec{F}$ → vector Quantity

$$\left[F = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R^2} \right] \rightarrow \text{magnitude only.}$$

→ the Force on (Q_2) due to (Q_1)

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R_{12}^2} \hat{a}_{R_{12}} \; [N]$$

$\overline{R}_{12}$ — Q_1 • $\hat{a}_{R_{12}}$ (unit vector) — $\vec{R}_{12}$ → • Q_2 $\qquad \overline{R}_{12} = \overline{r}_2 - \overline{r}_1$

* $|\vec{F}_{21}| = |\vec{F}_{12}|$

* $\vec{F}_{21} = -\vec{F}_{12}$, $\hat{a}_{R_{12}} = -\hat{a}_{R_{21}}$ (نفس المقدار بس عكس الاتجاه)

* $\hat{a}_{R_{12}} = \dfrac{\vec{R}_{12}}{R_{12}}$

so → $\vec{F}_{12} = \dfrac{Q_1 Q_2}{4\pi \varepsilon_0 R_{12}^2} \hat{a}_{R_{12}} \; (N)$ $\qquad \hat{a}_{R_{12}} = \dfrac{\overline{R}_{12}}{R_{12}}$

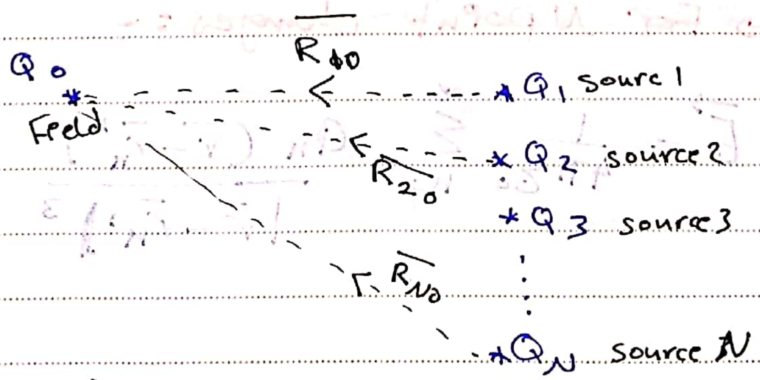
or $\vec{F}_{12} = \dfrac{Q_1 Q_2 \vec{R}_{12}}{4\pi \varepsilon_0 R_{12}^3} \; (N)$ $\qquad \vec{R}_{12} = \overline{r}_2 - \overline{r}_1$

or $\vec{F}_{12} = \dfrac{Q_1 Q_2 (\overline{r}_2 - \overline{r}_1)}{4\pi \varepsilon_0 |\overline{r}_2 - \overline{r}_1|^3} \; (N)$ → الأكثر استخدام

Source Q_1 —————— Field Q_2

Distance = Field location − source location.

→ the Force on a point charge due to N-point charges.



* $\vec{F}_0 = \vec{F}_{10} + \vec{F}_{20} + \cdots + \vec{F}_{N0}$

$$= \frac{Q_1 Q_0 (\overline{r}_0 - \overline{r}_1)}{4\pi \varepsilon_0 |\overline{r}_0 - \overline{r}_1|^3} + \cdots + \frac{Q_N Q_0 (\overline{r}_0 - \overline{r}_N)}{4\pi \varepsilon_0 |\overline{r}_0 - \overline{r}_N|^3}$$

→

→ $\vec{F_0} = \frac{Q_0}{4\pi\varepsilon_0}\sum_{k=1}^{N} \frac{Q_k(\bar{r}_0 - \bar{r}_k)}{|\bar{r}_0 - \bar{r}_k|^3}$ (N)

→ Electric Field Intensity ($\bar{E}$)

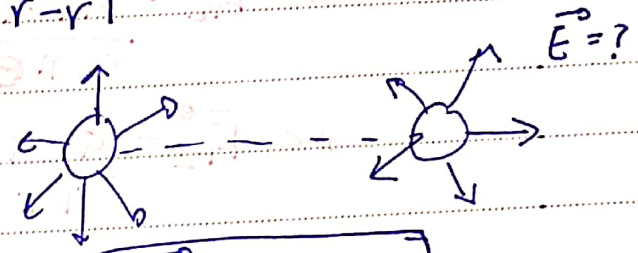
$$\vec{E} = \frac{\vec{F}}{Q} = \frac{Q_{source}}{4\pi\varepsilon_0 R^2}\hat{a}_R \quad (N/C) \text{ or } (V/m)$$

Q → field point

→ For a point charge

OR ⇒ $\vec{E} = \frac{Q\bar{R}}{4\pi\varepsilon_0 R^3}$

$$\vec{E} = \frac{Q(\bar{r} - \bar{r}')}{4\pi\varepsilon_0|\bar{r} - \bar{r}'|^3}$$



$\boxed{\vec{E} \to 0,\ r \to \infty}$

* convention :- use dashes with source and without dashes with the field.

→ For N point charges :-

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{\infty}\frac{Q_k(\bar{r} - \bar{r}_k')}{|\bar{r} - \bar{r}_k'|^3} \quad (V/m)$$

$\longrightarrow$ Electric Flux Density $(\vec{D})$

$$D = \underset{F/m}{\varepsilon_0} \underset{V/m}{\vec{E}} = \frac{\varepsilon_0 F}{Q_{\text{field}}} \qquad \frac{F \cdot V}{m^2} \equiv C/m^2$$

$\hookrightarrow$ like ρ_s

$$\vec{D} = \frac{Q}{4\pi R^2}\hat{a_r} \equiv \frac{Q\,\bar{R}}{4\pi R^3} = \frac{Q(\bar{r}-\bar{r}')}{4\pi|r-\bar{r}'|^3}$$

Ex:- Point charges $\underbrace{(1\,mC)}_{\text{Source 1}}$ and $\underbrace{(-2\,mC)}_{\text{Source 2}}$ are located at $(3,2,-1)$ and $(-1,-1,4)$, Find $\vec{F}$ and $\vec{E}$ and $\vec{D}$ at $\underbrace{(10\,nC)}_{\hookrightarrow \text{Field}}$ charge located at $(0,3,1)$

Sol:- $\boxed{1}$

$$\vec{F} = \frac{(1*10^{-3})*(10*10^{-9})*\overset{\text{vector}}{(-3,1,2)}}{4\pi*\frac{10^{-9}}{36\pi}*(14)^{3/2}} + \frac{(-2*10^{-3})*(10*10^{-9})*\overset{\text{vector}}{(1,4,-3)}}{4\pi*\frac{10^{-9}}{36\pi}*(26)^{3/2}}$$

$$= -6.507\hat{a_x} - 3.817\hat{a_y} + 7.506\hat{a_z}\ (mN)$$

$\boxed{2}$ $$\vec{E} = \frac{\vec{F}}{Q} = \frac{\vec{F}}{10*10^{-9}} = 10^8 . \vec{F}$$

$$= -650.7\hat{a_x} - 381.7\hat{a_y} + 750.6\hat{a_z}\ (kN/C)$$

or (kV/m)

$\boxed{3}$ $$\vec{D} = \varepsilon_0\vec{E} = \frac{10^{-9}}{36\pi}.\vec{E} = \dots$$

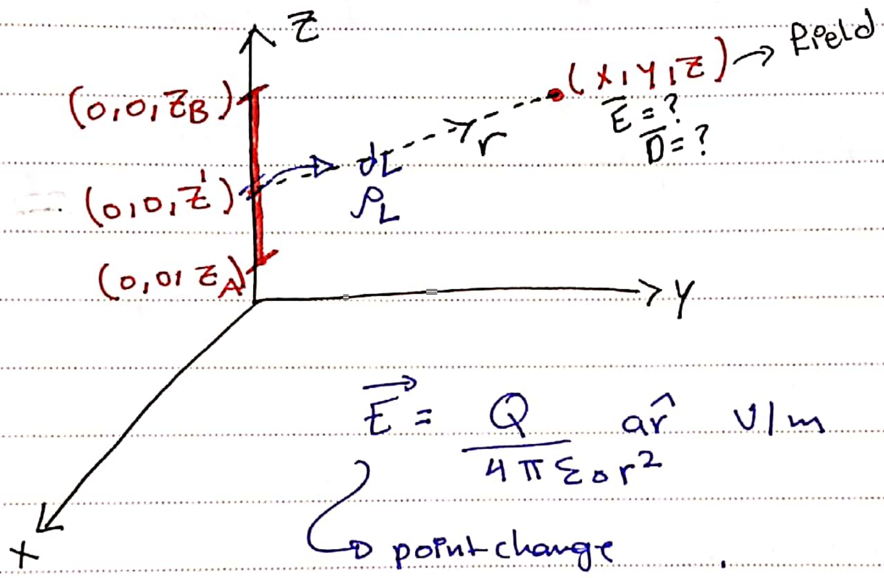
* Continuos charge distribution :-

↳ $\vec{E}$ field for a line charge

(ρ_L) in (c/m)

ex:- ∫ derivative + سحر finite line.

consider a finite line along z-axis carry charge of ρ_L (c/m). Find $\bar{E}$ and $\bar{D}$ at point (x, y, z)



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{a_r} \quad V/m$$

↳ point charge.

but in this example → line charge

$$Q = \int_L \rho_L \, dL$$

$$\hookrightarrow \vec{E} = \int_L \frac{\rho_L \, dL}{4\pi\varepsilon_0 r^2}\,\hat{a_r}$$

$$\vec{E} = \int_L \frac{\rho_L \, \bar{r} \, dL}{4\pi\varepsilon_0 r^3}$$

$\bar{dL} = dz'$ → along z-axis + ما بنبط a_z

$\bar{r} = x\hat{a_x} + y\hat{a_y} + (z-z')\hat{a_z}$

$r = \sqrt{x^2+y^2+(z-z')^2}$

$$\hookrightarrow \bar{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{z_A}^{z_B} \frac{(x, y, z-z')}{\left[(x^2+y^2+(z-z')^2)\right]^{3/2}}\, dz'$$

* z_A, z_B طول السلك

table مع

cart

↳ cartكي ما بنحل

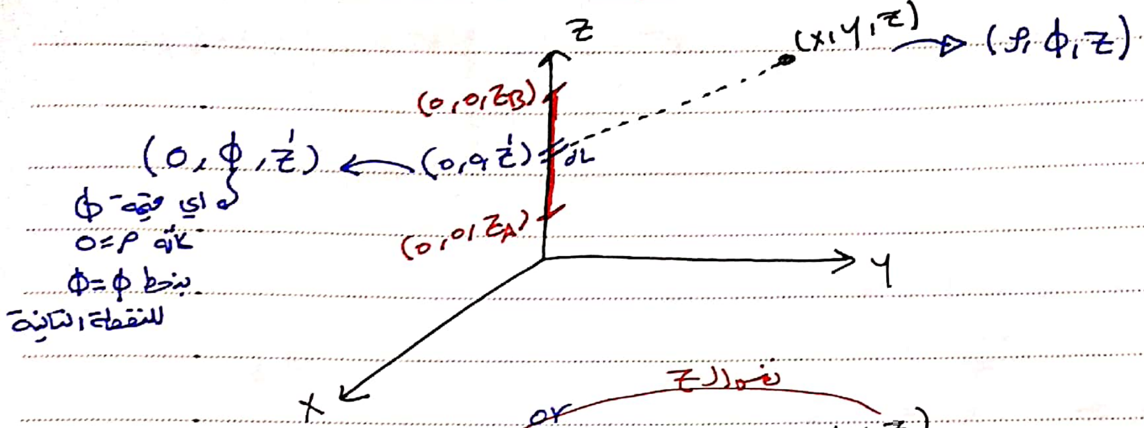
لأنه التكامل بدو table

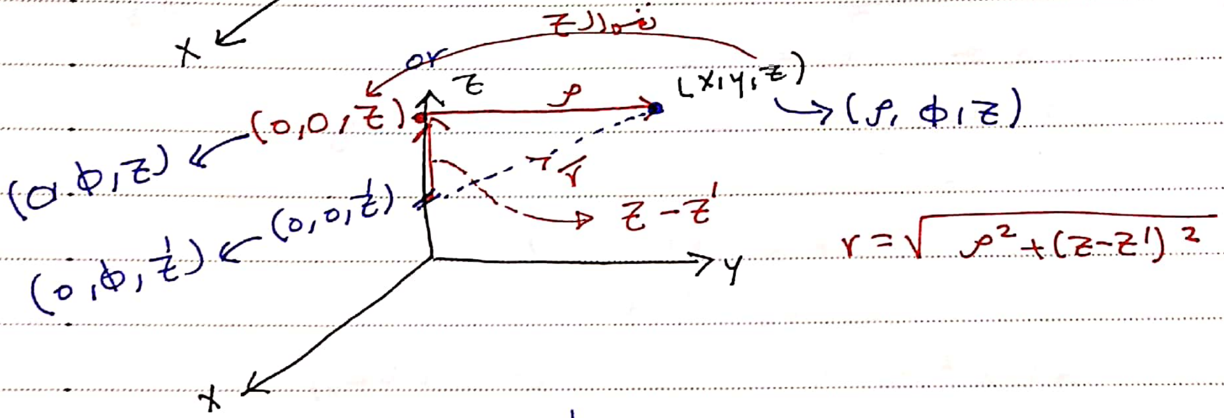
دائماً لما يكون line بنحل

كي cyl ونحول

لـ cyl

→ line is a cylinder of $\rho = 0$ so convert to cyl.





$$dL = dz'$$

$$\bar{r} = \rho\, \hat{a_\rho} + (z - z')\, \hat{a_z} \qquad (\text{Distance when } \phi = \phi)$$

$$r = \sqrt{\rho^2 + (z-z')^2}$$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_{z_A}^{z_B} \frac{\rho\, \hat{a_\rho} + (z-z')\, \hat{a_z}}{[\rho^2 + (z-z')^2]^{3/2}}\, dz'$$

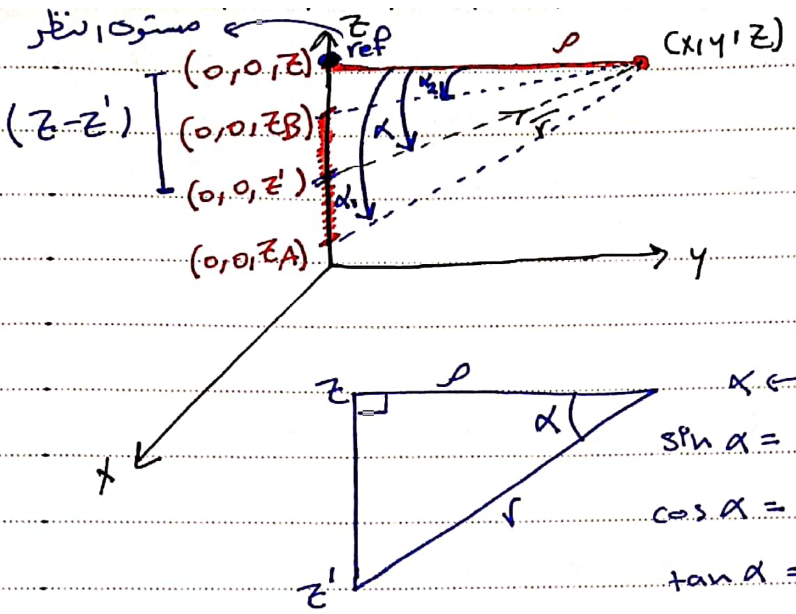
table بدو

تكاملين الأول بنحل هاد table of integral والثاني تعويض

[بالواقع ما بنروح نقيس طول برج مثلاً؟] introduce angle برضو صعب بدل أنو

instead of integration from $z_A \to z_B$
we will introduce angles α, α_1, α_2

→



بدي أحول من $z \leftarrow \alpha$

$$\sin\alpha = \frac{z-z'}{r} \rightarrow z-z' = r\sin\alpha$$

$$\cos\alpha = \frac{\rho}{r} \rightarrow \rho = r\cos\alpha$$

$$\tan\alpha = \frac{z-z'}{\rho} \rightarrow z-z' = \rho\tan\alpha$$

$$dz' \rightarrow d\alpha$$

$$z-z' = \rho\tan\alpha$$

(اشتقاق ضمني) $\quad -1(dz') = \rho\sec^2\alpha\, d\alpha$

بحطها بقانون ال $\vec{E}$ بدل dz' $\leftarrow$ $\boxed{dz' = -\rho\sec^2\alpha\, d\alpha} \rightarrow *$

$$r^2 = \rho^2 + (z-z')^2$$

$$= \rho^2 + \rho^2\tan^2\alpha$$

$$= \rho^2(1+\tan^2\alpha) \longrightarrow \text{متطابقة}$$

$$= \rho^2\left(\frac{\cos^2\alpha}{\cos^2\alpha} + \frac{\sin^2\alpha}{\cos^2\alpha}\right)$$

$$= \frac{\rho^2}{\cos^2\alpha} = \rho^2\sec^2\alpha$$

$$r = \rho\sec\alpha$$

$$\boxed{r^3 = \rho^3\sec^3\alpha} \rightarrow *$$

$\boxed{z-z' = r\sin\alpha} \rightarrow *$ نعوضها

$z-z' = \rho\tan\alpha$

$\boxed{\rho = r\cos\alpha} \rightarrow *$

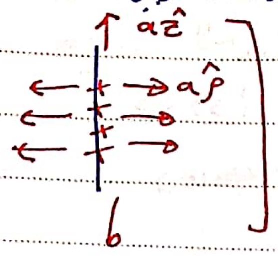
$\rho\hat{a}_\rho + (z-z')\hat{a}_z$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2} \frac{\cancel{\rho}\sec\alpha(\cos\alpha\,\hat{a}_\rho + \sin\alpha\,\hat{a}_z)\cdot -\cancel{\rho}\,\cancel{\sec^2}\alpha\, d\alpha}{\underbrace{\cancel{\rho^3}\,\cancel{\sec^3}\alpha}_{r^3}}$$

بدل dz'

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0\rho}\left[(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\rho - (\cos\alpha_2 - \cos\alpha_1)\hat{a}_z\right] \quad V/m$$

→ مركزين ليه $\hat{a_\rho}$ و $\hat{a_z}$?? بـ finite line.

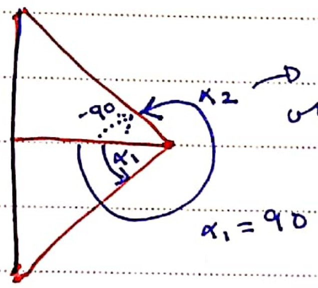


electric line

مش top & bottom & الجوانب

→ لو line infinite $\hat{a_z}$ بدون

⟹ For an infinite line.



→ $\alpha_2 = 270 = -90$

ناخذ ال angle من نفس الجهة.

حد ال $\hat{a_z}$ يختفي

$(\cos -9 - \cos -90)\, a_z$

$\sin\alpha = 1$

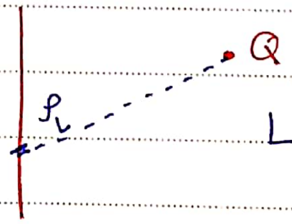
.... ⟹ $\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a_\rho}}$ V/M

always for any inf. line

ρ :- the shortest distance between the source of the field.

$\boxed{\vec{D} = \varepsilon_0\vec{E}}$

$\vec{F} = Q\vec{E}$



← شحنة ال Q

لو $\vec{F}_{LQ} = Q \cdot \vec{E}$

ال E اللي طلعناها ل inf. line

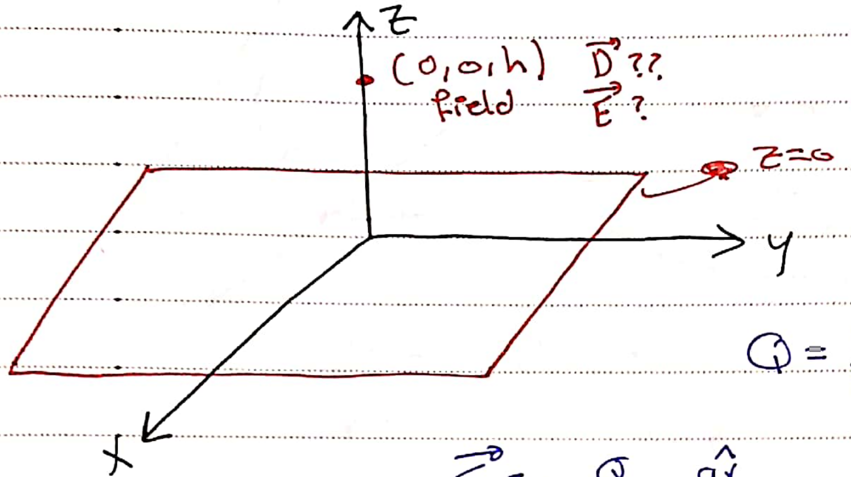
لو بدي $F_{QL} = Q\, E_Q$

$\int_l \rho_L dL$ ل line

نقطة point charge $= \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a_r}$

↳ E. field for a surface charge :-

ex.: find $\vec{E}$ and $\vec{D}$ at $(0,0,h)$ due to an infinite sheet placed along xy plane and carry a charge ρ_s (c/m^2), where $(h>0)$.



coulomb law

$$Q = \int_s \rho_s \, ds$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_o r^2}\hat{a}_r \longrightarrow \text{point charge}$$

$$\vec{E} = \int_s \frac{\rho_s \, ds \, \hat{a}_r}{4\pi\varepsilon_o r^2}$$

→ for surface charge.

$$\vec{E} = \int_s \frac{\rho_s \, \vec{r} \, ds}{4\pi\varepsilon_o r^3}$$

لو وصلنا cart رح يطلع تكامل قبل بدم 2 integral table

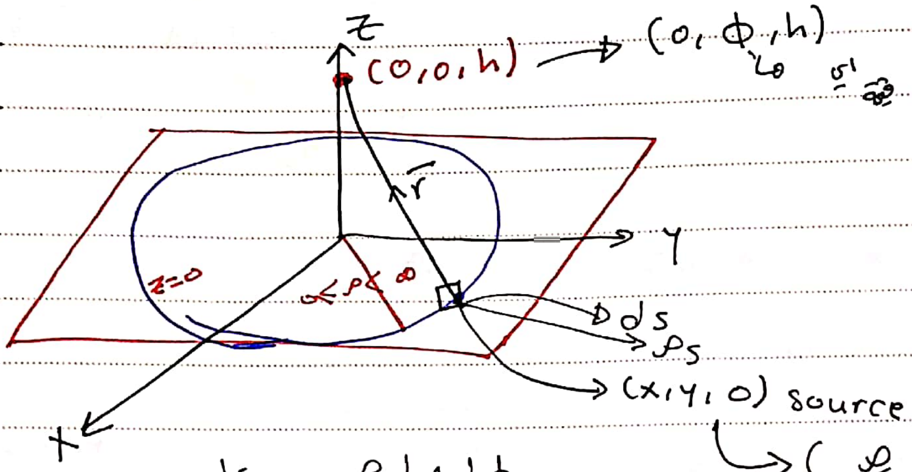
$$\vec{E} = \frac{\rho_s}{4\pi\varepsilon_o}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\ldots) \, dx' dy'$$

لأنها sheet → disk لأنها infinit لو كانت finit اجباري أحطها مربع (cart)

→

→ convert to cyl.

infinite sheet → inf. disk



$(0,0,h) \to (0,\phi,h)$ ↳ لانه فوقه $\rho=0$

$(x,y,0)$ source ↳ $(\rho, \phi, 0)$

$ds = \rho\, d\rho\, d\phi$

$\bar{r} = -\rho\, a\hat{\rho} + h\, a\hat{z}$

$r = \sqrt{\rho^2 + h^2}$

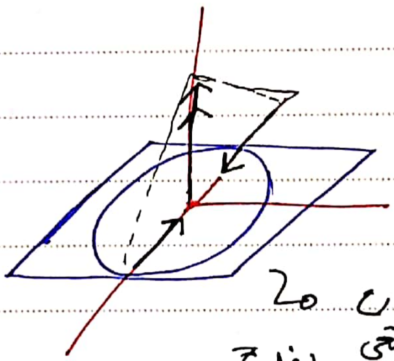
$$\vec{E} = \frac{\rho_s}{4\pi\varepsilon_0} \int_0^{2\pi}\int_0^{\infty} \frac{\cancel{-\rho\, a\hat{\rho}} + h\, a\hat{z}}{[\rho^2+h^2]^{3/2}}\, \rho\, d\rho\, d\phi$$

لو حلينا $\int_0^{2\pi}$ اول بيصير ✓

↳ due to symmetry

$(0 \to 2\pi)$ دورة كاملة ϕ

the ρ-component will be cancelled.



ص ر ↳ ملتقي بغل z

$$\Rightarrow \vec{E} = \frac{\rho_s h}{2\varepsilon_0} \int_0^{\infty} \frac{\rho\, d\rho}{[\rho^2+h^2]^{3/2}}\, a\hat{z}$$

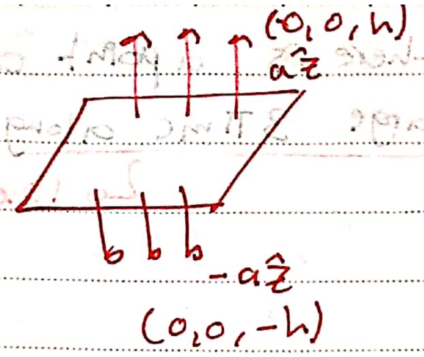
let $\rho^2 + h^2 = u \Rightarrow du = 2\rho\, d\rho \longrightarrow \rho\, d\rho = \frac{du}{2}$

$$\vec{E} = \frac{\rho_s h}{2(2\varepsilon_0)} \int \frac{du}{u^{3/2}}\, a\hat{z}$$

$$= \frac{\rho_s h}{(2)2\varepsilon_0} \cdot \frac{u^{-1/2}}{\frac{-1}{2}} \Big|\, a\hat{z} \Rightarrow \vec{E} = -\frac{\rho_s h}{2\varepsilon_0} \frac{1}{\sqrt{\rho^2+h^2}} \Big|_0^{\infty} a\hat{z}$$

$$\Rightarrow \vec{E} = \frac{-\rho_s h}{2\varepsilon_0}\left(0 - \frac{1}{h}\right) a\hat{z} \Rightarrow \boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}\, a\hat{z} \quad V/m}$$

→

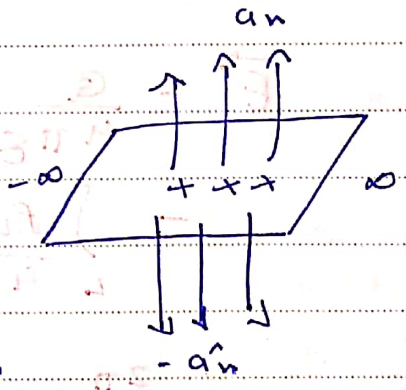


$$\Rightarrow \boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}\,\hat{a_z}}$$ for infinite sheet along xy plane

in general $\Rightarrow$ $$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}\,\hat{a_n}}$$ always for any infinite sheet.

- along x $\hat{a_x}$
- along y $\hat{a_y}$
- along z $\hat{a_z}$

$$\Rightarrow \boxed{\vec{D} = \frac{\rho_s}{2}\,\hat{a_n}}$$



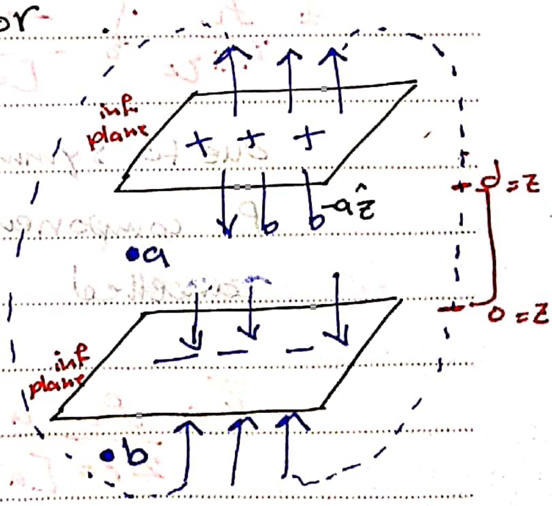
Example :- parallel plate capacitor

Find $\bar{D}$ @ point (a) & (b)

$\Rightarrow \bar{D}|_a = \bar{D}_+ + \bar{D}_-$

$= \frac{\rho_s}{2}(-\hat{a_z}) + \frac{-\rho_s}{2}(\hat{a_z})$

$\bar{D} = -\rho_s\,\hat{a_z}$ $\rightarrow$ C/m²



$\Rightarrow \bar{D}|_b = \bar{D}_+ + \bar{D}_-$

$= \frac{\rho_s}{2}\hat{a_z} + \frac{-\rho_s}{2}\hat{a_z} = 0$

$\rightarrow$ at $z > d$

Ex: Find $\bar{D}$ at $(4,0,3)$ if there is a point charge -5π mC at $(4,0,0)$ and a Line charge 3π mC along y-axis

↳ line charge ($\bar{D}_L$)

↳ point charge src1 ($\bar{D}_Q$)

Sol: $\bar{D} = \bar{D}_Q + \bar{D}_L$

$$= \frac{Q}{4\pi r^2}\hat{a}_r + \frac{\rho_L}{2\pi\rho}\hat{a}_\rho$$

$\bar{r} = 3\hat{a}_z$

$r = 3$

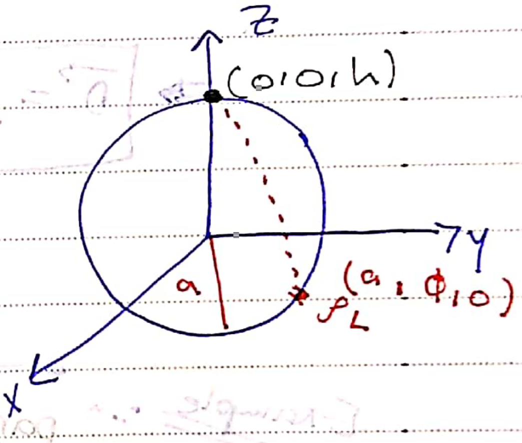
$$= \frac{Q\bar{r}}{4\pi r^3} + \frac{\rho_L\bar{\rho}}{2\pi\rho^2}$$

$\bar{\rho} = 4\hat{a}_x + 3\hat{a}_z$

$\rho = 5\ m$ ↳ shortest distance

$$\bar{D} = 240\hat{a}_x + 42\hat{a}_z \ \mu C/m^2$$

Ex: For a ring of radius (a) placed along xy plane carry ρ_L c/m Find $\bar{E}$ at $(0,0,h)$



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r$$

$$= \int_L \frac{\rho_L dL}{4\pi\varepsilon_0 r^3}\bar{r}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{-a\hat{a}_\rho + h\hat{a}_z}{[a^2+h^2]^{3/2}}\, a\, d\phi$$

① ↳ $Q = \int_L \rho_L dL$

$dL = \rho\, d\phi = a\, d\phi$

$\bar{r} = -a\hat{a}_\rho + h\hat{a}_z$

$r = \sqrt{a^2+h^2}$

due to symmetry the ρ component well be cancelled

$$\bar{E} = \frac{\rho_L\, a h}{2\varepsilon_0[a^2+h^2]^{3/2}}\hat{a}_z \quad V/m$$

if $a=0 \Rightarrow$ show $\bar{E}$ will look like $\bar{E}$ of a point charge $= \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r$

$Q = \int_L \rho_L dL = \int_0^{2\pi}\rho_L a\, d\phi \Rightarrow Q = \rho_L 2\pi a \Rightarrow \rho_L = \frac{Q}{2\pi a}$

$$\vec{E} = \frac{\frac{Q}{2\pi a}\cdot a h}{2\varepsilon_0[a^2+h^2]^{3/2}}\hat{a}_z \Big|_{a=0} \Rightarrow \boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 h^2}\hat{a}_z}$$

↳ $r = h$

$\hat{a}_r = \hat{a}_z$

Scanned with CamScanner

* $\vec{E}$ field due to volume charge dist. :-

حالة volume بنعملش Gaussian

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a_r} \quad , \quad Q = \int_v \rho_v \, dv$$

↳ point charge

$$\Rightarrow \vec{E} = \int_v \frac{\rho_v \, dv}{4\pi\varepsilon_0 r^2}\hat{a_r} \quad \text{V/m}$$

* Gauss's law

↳ special case from coulomb's Law.

$$\oint_s \bar{D} \cdot \bar{ds} = Q_{enclosed}$$

$$= Q \rightarrow \text{point charg} \quad , \bar{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a_r}$$

$$= \int_l \rho_L \, dL \rightarrow \text{infinite line} \, , \bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\hat{a_\rho}$$

$$= \int_s \rho_s \, ds \rightarrow \text{infinit sheet} \, , \bar{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a_n}$$

$$= \int_V \rho_v \, dv \rightarrow \text{uniform volume charge} \, , \bar{E} = ??$$

$$\left[\oint_s \bar{D} \cdot \bar{ds} = \int_v \rho_v \, dv \right] \Rightarrow$$ First Maxwell's eq. in integral for (Gauss's Law)

[1] $\bar{E}$ or $\bar{D}$ for a point charge

3 steps :-

[1] اختار ، شحنة Q enclosed

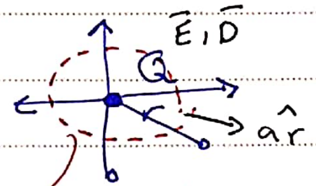
[2] نحدد $\bar{D}$ & $\bar{ds}$

[3] احل التكامل

① ⟶ $\oint_s \bar{D}\cdot\bar{ds} = Q$

② ⟶ $\bar{D} = D_r \hat{a_r} + D_\theta \hat{a_\theta} + D_\phi \hat{a_\phi}$

$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$



Gassian surface

اللي طالع منه Q ← r

كرة أحصر نقطة محيط

كرة وهمية (hollow) sph

③ ⟶ ال $\oint$ بتروح لانه سطح واحد

$$\oint_s D\,ds = \int_s \bar{D}\cdot\bar{ds} = \int_0^{2\pi}\int_0^{\pi} D_r r^2 \sin\theta \, d\theta \, d\phi = Q$$

$$D_r r^2 (2)(2\pi) = Q$$

$$\int_0^{\pi} \sin\theta \, d\theta = -\cos\theta \Big|_0^{\pi} = 2$$

حفظ

$$\underbrace{4\pi r^2}_{\text{مساحة كرة نصف قطرها } r} D_r = Q$$

$$D_r = \frac{Q}{4\pi r^2}$$

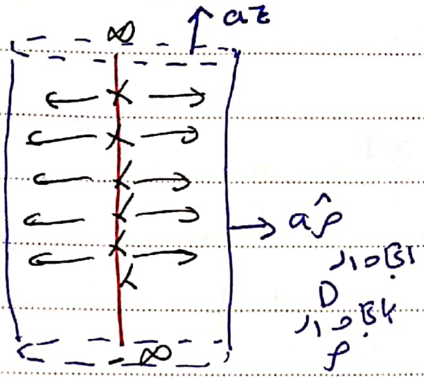
$$\boxed{\bar{D} = \frac{Q}{4\pi r^2}\hat{a_r}} = D_r \hat{a_r}$$

$$\boxed{\vec{E} = \frac{\bar{D}}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a_r}}$$

⟹ كان point charge

Scanned with CamScanner

2) $\vec{E}, \vec{D}$ for an infinite line of charge



هسا أحصر الـ line هاد بـ cylinders وهمية hollow → Gaussian surface (hollow cyl.)

① $\oint_S \bar{D} \cdot \bar{ds} = Q_{enc} = \int_L \rho_L \, dL$

② $\bar{D} = D_\rho \, \hat{a_\rho}$

$\bar{ds} = \rho \, d\phi \, dz \, \hat{a_\rho}$

③ $\oint_S \bar{D} \cdot \bar{ds} \Rightarrow \int_L \rho_L \, dL$

$\rightarrow = \int_{-L}^{L} \int_0^{2\pi} D_\rho \, \rho \, d\phi \, dz = \int_{-L}^{L} \rho_L \, dz$

$L \to \infty$

∮ جزوجناه ع واحد

الأصل $\int_{-\infty}^{\infty}$ بس حطينا $\infty \to L$ هسا نقدر نشطب

$D_\rho \underbrace{\rho \, 2\pi (\cancel{2L})}_{} = \rho_L (\cancel{2L})$

محيط القاعدة × الارتفاع

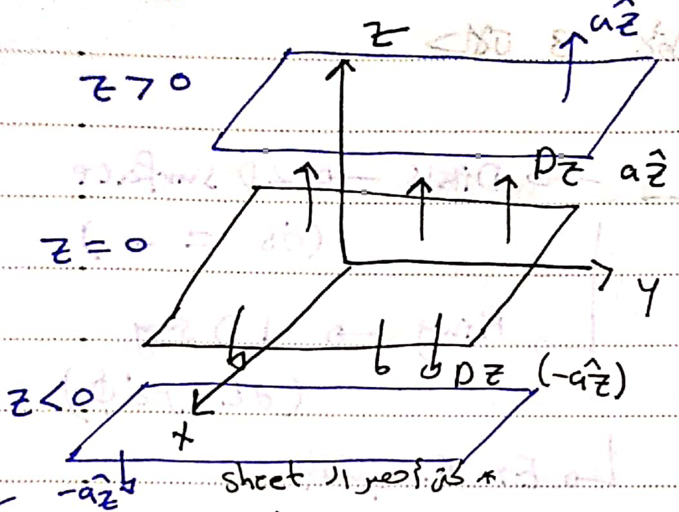
$$D_\rho = \frac{\rho_L}{2\pi\rho}$$

$$\rightarrow \boxed{\bar{D} = \frac{\rho_L}{2\pi\rho} \hat{a_\rho}}$$

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\epsilon_0\rho} \hat{a_\rho}}$$

Scanned with CamScanner

[3] $\bar{E}$ or $\bar{D}$ for an inf. sheet of charge



* كأن أحصر ال sheet

بدي box فارغ بس حافة دايرة الجوانب

لأنه حافي D بهاي الاتجاهات ! اعمم

Gaussian surfaces

① $\oint_s \bar{D}\cdot\bar{ds} = Q_{enc} = \int_s \rho_s\, ds$

② $\bar{D} = \begin{cases} D_z \hat{a}_z & , z>0 \\ D_z(-\hat{a}_z) & , z<0 \end{cases}$

$\bar{ds}_{top} = dx\,dy\,\hat{a}_z \quad , z>0$

$\bar{ds}_{bot} = -dx\,dy\,\hat{a}_z \quad , z<0$

③ عنّا هيك 2 surface [$\oint$ غير مغلق] $\oint \neq S$

$$\oint_s \bar{D}\cdot\bar{ds} = \int_{s_{top}} \bar{D}\cdot\bar{ds}_{top} + \int_{s_{bot}} \bar{D}\cdot\bar{ds}_{bot} = \int_s \rho_s\, ds$$

$$\hookrightarrow o = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_z\, dx\,dy + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_z\, dx\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho_s\, dx\,dy$$

put $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\,dy = A$

$$= D_z \cancel{A} + D_z \cancel{A} = \rho_s \cancel{A}$$

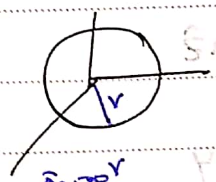
$$2 D_z = \rho_s$$

$$\boxed{D_z = \frac{\rho_s}{2}} \quad , \bar{D} = \begin{cases} \frac{\rho_s}{2}\hat{a}_z & , z>0 \\ \frac{\rho_s}{2}(-\hat{a}_z) & , z<0 \end{cases}$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a}_n} \qquad \hookrightarrow \boxed{\bar{D} = \frac{\rho_s}{2}\hat{a}_n}$$

↳ كنا نحل in general لأول 3 حالات

Note :-



r محددة

→ Disk → 2D surface
(ds = ...)

↳ Ring → 1D seg
← فاضي من جوا
($dL = \rho d\phi$)

↳ Ex Ex بالكتاب

← ما بقدر أحلها على
Gauss's لأنو

ال r محددة نحلها على columb's law

[4] $\vec{E}$ or $\bar{D}$ for a uniform volume charge .

ده يتحل فقط Gauss's يا بتحل ب colomb's ds .

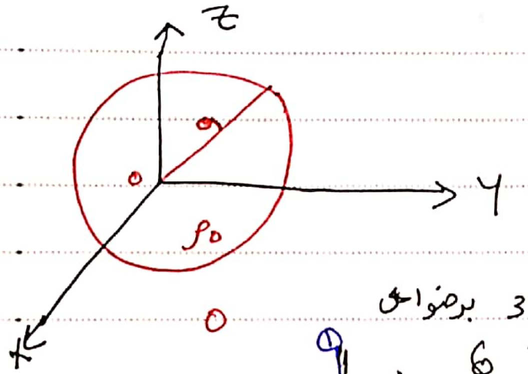
ex :- Consider a sphere of radius (a) has

$$\rho_v = \begin{cases} \rho_0 , & r < a \\ 0 , & r > a \end{cases} \quad \text{in } C/m^3$$

→ volume charge dosity .

Find $\bar{E}$ and $\bar{D}$ every where

[every wher uniform] → indicate انو نحلها ب Gauss's law



برضواله 3 steps :

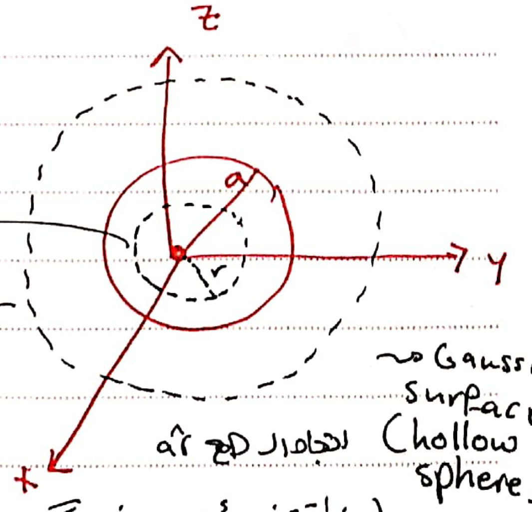
① → $\oint_s \bar{D} \cdot \bar{ds} = Q_{enc} = \int_v \rho_v \, dv$

but for ① $\boxed{0 < r < a}$

② → $\bar{D} = D_r \hat{a_r}$

$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$

③ → $\oint_s \bar{D} \cdot \bar{ds} =$



هنه ري أقل من a

هنه a بكون

→ Gaussian surface (hollow sphere)

$\hat{a_r}$ اتجاهه مع

$$\int_0^{2\pi}\int_0^{\pi} D_r \, r^2 \sin\theta \, d\theta \, d\phi = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho_0 \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\cancel{4\pi r^2} D_r = \rho_0 \frac{r^3}{3} \cancel{(2)(2\pi)}$$

$$D_r = \frac{\rho_0 r}{3}$$

$$\boxed{\bar{D} = \frac{\rho_0 r}{3} \hat{a_r}}$$

$$\boxed{\vec{E} = \frac{\rho_0 r}{3 \varepsilon_0} \hat{a_r} \Rightarrow 0 \leq r < a}$$

(منطقتين رح أحسب فيها $\bar{E}$

كل وحدة لحال ودائماً ه بنبلش من

origin لبرا يعني باكوا بعدين لبرا .

→

For $a < r < \infty$

$\hookrightarrow \; 4\pi r^2 D_r = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho_o \, r^2 \sin\theta \, dr \, d\theta \, d\phi$ الداخلية ρ_v

$+ \int_0^{2\pi}\int_0^{\pi}\int_a^{r} \cancel{0 \, r^2 \sin\theta \, dr \, d\theta \, d\phi}$

Zero ρ_v الخارجية

$$4\pi r^2 D_r = \frac{\rho_o \, 4\pi a^3}{3}$$

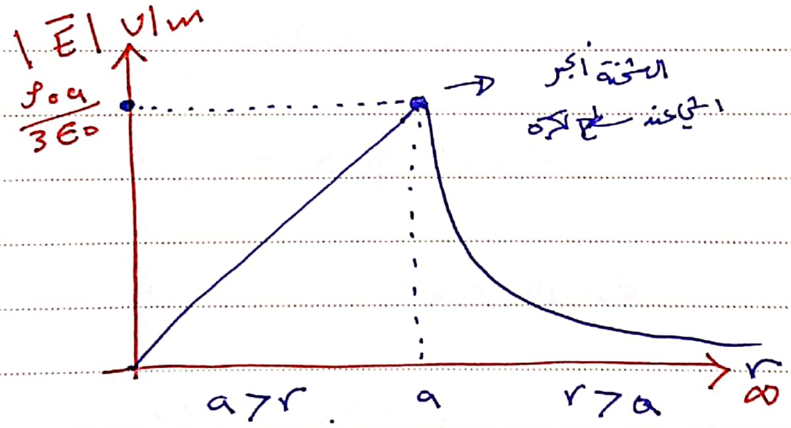
$$D_r = \frac{\rho_o a^3 \, \hat{a}_r}{3 r^2}$$

$$\boxed{\bar{D} = \frac{\rho_o a^3}{3 r^2} \hat{a}_r}$$

$$\boxed{\bar{E} = \frac{\rho_o a^3}{3 \epsilon_o r^2} \hat{a}_r} \rightarrow a \leq r < \infty$$

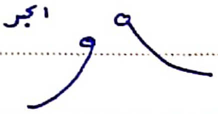
V/m

فتصل عند ال a



→ الشحنة أكبر اللي عند سطح الكرة

اذا كانت الشحنة خارج الكرة أكبر .



⇒ * Electric Flux (Ψ_e)

$$\Psi_e = \int_s \bar{D} \cdot ds = (C)$$

C/m^2 ، m^2

- Electric Flux (Ψ_e) in (C)

* Gauss's law

$$\Psi_e = \oint_s \bar{D} \cdot \bar{ds} = Q_{enc}$$

$\left[\oint_s \bar{D} \cdot \bar{ds} = \int \rho_v \, dv \right] \rightarrow$ 1st max well's eq. in integral Form.

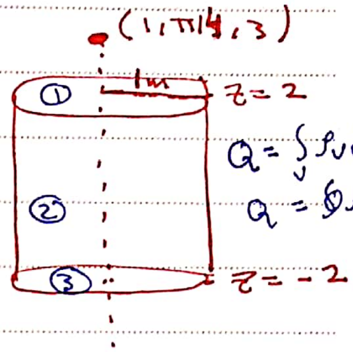
Ex:- Given $\overline{D} = z\rho\cos^2\phi\, \hat{a}_z$ C/m² .

calculate : a) The charge density at $(1, \pi/4, 3)$

b) The total charge enclosed by the cylender of radius 1m with $-2 \leq z \leq 2$m

Q

$\rho_s = \overline{D} \cdot \hat{a}_n$



$Q = \int \rho_v dv$

$Q = \oint \rho_s ds$

ⓑ $Q = \oint_s \rho_s ds$

$= \oint_s \overline{D} \cdot \underbrace{\hat{a}_n ds}_{\overline{ds}}$

$= \oint_s \overline{D} \cdot \overline{ds}$

$= \int_{s_{top}} \overline{D} \cdot \overline{ds}_{top} + \cancel{\int_{s_{side}} \overline{D} \cdot \overline{ds}_{side}} + \int_{s_{bot}} \overline{D} \cdot \overline{ds}_{bot}$

$\overline{ds}_{top} = \rho\, d\rho\, d\phi\, \hat{a}_z$

$\overline{ds}_{side} = \rho\, d\phi\, dz\, \hat{a}_\rho$

$\overline{ds}_{bot} = -\rho\, d\rho\, d\phi\, \hat{a}_z$

$Q = \int_0^{2\pi}\int_0^1 z\rho\cos^2\phi\, \rho\, d\rho\, d\phi \Big|_{z=2} + \int_0^{2\pi}\int_0^1 z\rho\cos^2\phi\, \rho\, d\rho\, d\phi \Big|_{z=-2}$

$= 2\left(\frac{1}{3}\right)\pi + 2\left(\frac{1}{3}\right)\pi$

$Q = \left(\frac{4\pi}{3}\right)$ C

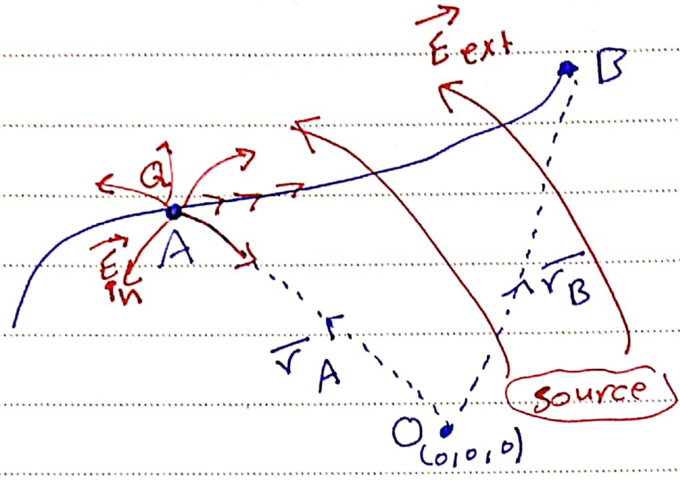
ⓐ $\rho_s = \overline{D} \cdot \hat{a}_n <$ $\hat{a}_z$, $\hat{a}_\rho$, $-\hat{a}_\rho$

$\rho_{s\,top} = z\rho\cos^2\phi$

$\rho_{s\,top}\Big|_{(1, \pi/4, 3)} = (3)(1)\left(\frac{1}{2}\right) = 1.5$ C/m²

$\rho_{bot} = -z\rho\cos^2\phi = -1.5$ C/m²

* Electric Potential :- in (v) [Scalar Quantity]



Work = (w)

To move Q from A to B

$$V_{AB} = V_B - V_A$$

$$\text{voltage} = \frac{\text{work}}{\text{charge}}$$

$$1V = \frac{1J}{1C}$$

work = voltage * charge

$$W = Q\,V_{AB}$$

$$W = \overline{F}\,\overline{L} \quad (N.m)$$

$$W = -Q\,\overline{E}_{ext}$$ → The work is done by the Eext

$$\Rightarrow \underbrace{\int_L dw}_{\text{scalar}} = \int_L -Q\,\overline{E}_{ext}\cdot\overline{dL}$$

$$\boxed{W = -Q\int_L \overline{E}\cdot\overline{dL}}$$

potential diff between 2 point.

$$V_{AB} = \frac{W}{Q} = -\int_L \overline{E}\cdot\overline{dL} = V_B - V_A = -\int_{r_A}^{r_B} \overline{E}\cdot\overline{dL}$$

$$\boxed{V = -\int_L \overline{E}\cdot\overline{dL}}$$ ⟹ potential diff. (v)

→ for moving a point charge :-

$$\bar{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r$$

$\bar{dL} = dr\hat{a}_r$

$$V_{AB} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon_0 r^2}\cancel{\hat{a}_r}\cdot dr\,\cancel{\hat{a}_r}$$

$$= \frac{Q}{4\pi\varepsilon_0 r}\Big]_{r_A}^{r_B}$$

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) V$$

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0 r_B} - \frac{Q}{4\pi\varepsilon_0 r_A}$$

$$V_{AB} = V_B - V_A$$

if point A is at ∞

$r_A \to \infty$

$$V_{\infty B} = \frac{Q}{4\pi\varepsilon_0 r_B} - 0$$

$$= V_B - V_\infty \quad , V_\infty = 0 \text{ since } r_\infty \to 0$$

* ∞ defual ref if it is not mentioned

→ ref → dest and voltage

لو حكى فرق الجهد عند نقطة معناها فرق الجهد بين النقطة وال ∞

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 r}}$$ → if ref is at ∞

* Electric Potential $\Rightarrow$

$V_{AB} = \frac{W}{Q} = -\int_{\ell} \bar{E} \cdot \bar{dL}$

$\hookrightarrow$ From $\bar{E}$ (field)

$\rightarrow$ From the source

$\hookrightarrow$ For a point charge from $A \rightarrow B$

* $V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$ (V)

* if ref is at ∞ $\Rightarrow$ $V = \frac{Q}{4\pi\varepsilon_0 r}$, $V_{\infty} = 0$

point

$\hookrightarrow$ For line charge :

* if ref is at ∞ $\Rightarrow$ $V = \int_{\ell} \frac{\rho_L \, dL}{4\pi\varepsilon_0 r}$

$\hookrightarrow$ For surface charge :

* if ref is at ∞ , $V_{\infty} = 0 \Rightarrow V = \int_{S} \frac{\rho_s \, ds}{4\pi\varepsilon_0 r}$

$\hookrightarrow$ For volume charge :

* ref is at ∞, $V_{\infty} = 0 \Rightarrow V = \int_{v'} \frac{\rho_v \, dv'}{4\pi\varepsilon_0 r}$

volume

Potential

$\hookrightarrow$ volume

$\Leftarrow$ بستخدم هاي القوانين لما ما أقدر أطلع V من أول قانون [أوجد $\bar{E}$ بالأول]

Ex : Two point charges $-4\mu C$ & $5\mu C$ are located at $\underbrace{(2,-1,3)}_{}$ (S_1) and point $\underbrace{(0,4,-2)}_{}$ (S_2), Find the potential at $\boxed{p(1,0,1)}$ → Field

Sol :- ref is at ∞ ($V_\infty = 0$) → اذا ماذكر بالسؤال اي ref عند potential

مثلا لو كان $C\,[10V]$ ← تكون هوا ال ref

اذا ماذكر بنحط $ref \to \infty$

⇒ مطلوب V at p ⇒ قانون V ل pointcharge & $ref \to \infty$

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

→ we can write it as for N-charges

$$V = \frac{1}{4\pi\epsilon_0} \sum_{K=1}^{N} \frac{Q_K}{r_K}$$

oR

$$V = \frac{-4 * 10^{-6}}{\frac{4\pi * 10^{-9}}{36\pi} \underbrace{\left|(-1,1,-2)\right|}_{|r_1| = \sqrt{6}}} + \frac{5 * 10^{-6}}{\frac{4\pi * 10^{-9}}{36\pi} * \underbrace{\sqrt{26}}_{|r_2|}}$$

$\vec{r_2}$ = field $- S_2 = (1,0,1) - (0,4,-2)$

$|\vec{r_2}| = |(1,-4,3)|$

$\vec{r_1}$ = field $- S_1 = (1,0,1) - (2,-1,3)$

$|\vec{r_1}| = |(-1,1,-2)|$

$$\Rightarrow V\Big|_{(1,0,1)} = V_p = -5.872\ KV$$

$[V_{\infty p} = V_p - V_\infty = V_p - 0 = V_p]$ أو $[\infty = ref]$ لازم اكتب انو قبل ←

* $V_p = (-)5.872\ KV$, what is the meaning of the minus sign?

→ drop in potential (the work is done by the field it self) $\left[\int_L \vec{E}\cdot d\vec{L} \text{ is +ve}\right]$

if V is (+ve)

→ gain in Potential (the work is done by the external field) $\left[\int_L \vec{E}\cdot d\vec{L} \text{ is -ve}\right]$

Ex :- A point charge of 5 nC located at $\underbrace{(-3, 4, 0)}_{S_1}$, and a line $\underbrace{y=1, z=1}_{S_2}$ carries uniform charge (ρ_L) (infinite line). of 2 nC/m.

بالكتاب →

a) ref = ∞ ....

b) if $\underbrace{V = 100\,V}_{\text{ref}}$ at $\underbrace{B(1,2,1)}_{\text{field }(B)}$, Find V at $\underbrace{C(-2,5,3)}_{\text{field }(C)}$

sol :- المطلوب V_C → كاما حابقدر ، فنجد V_{CB} OR V_{BC}

لأنو $ref \neq \infty$

ref at B

$$\Rightarrow V_{BC} = V_C - V_B = V_C - 100$$

$$\boxed{V_C = V_{BC} + 100}$$

V_{BC} ??

$$\Rightarrow V_{BC} = V_{BC_Q} + V_{BC_L}$$

V_{BC_Q} ← من point charge

V_{BC_L} ← من Line

$$* \; V_{BC_Q} = -\int_L \bar{E} \cdot d\bar{L} \; , \; \bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r$$

ـــ بحل هيك إذا ما كنت حافظة القانون

$$= -\int_{r_B}^{r_C} \frac{Q}{4\pi\epsilon_0 r^2} \cancel{\hat{a}_r}\, dr\, \cancel{\hat{a}_r}$$

لأنه $\bar{E}$ sphere

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_C} - \frac{1}{r_B}\right)$$

[المسافة من Q لـ C] $r_C = \underset{C \;\; - \;\; Q}{\text{field} - \text{source}} = |(1,1,3)| = \sqrt{11}$

[المسافة من Q لـ B] $r_B = \underset{B \;\; - \;\; Q}{\text{field} - \text{source}} = |(4,-2,1)| = \sqrt{21}$

$$\text{so } V_{BC_Q} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{11}} - \frac{1}{\sqrt{21}}\right)$$

→

$$* \; V_{BCL} = -\int_L \bar{E} \cdot \bar{dL} \quad , \quad \bar{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a_\rho}$$

$$= -\int_{\rho_B}^{\rho_C} \frac{\rho_L}{2\pi\epsilon_0 \rho} \cancel{\hat{a_\rho}} \, d\rho \, \cancel{\hat{a_\rho}}$$

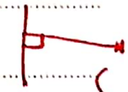
ال $\bar{E}$ بتغير بال ρ

$$= \frac{-\rho_L}{2\pi\epsilon_0} \left(\ln \rho_C - \ln \rho_B \right)$$

$$= \frac{-\rho_L}{2\pi\epsilon_0} \ln \left(\frac{\rho_C}{\rho_B} \right)$$

بعد نقطة عن نقطة $\int_{r_A}^{r_B}$

بعد نقطة عن Lin $\int_{\rho_B}^{\rho_C}$



بناخذ اقصر مسافة عشان ميلها ρ

$$\rho_C = \left| \underbrace{(-2,5,3) - (-2,1,1)}_{\text{field - source}} \right|$$

اقصر مسافة z=1 , y=1 & ال x مثل

$$= \sqrt{20}$$

$$\rho_B = \left| \underbrace{(1,2,1) - (1,1,1)}_{\text{field - Source}} \right|$$

z=1 , y=1 وال x مثل

$$= 1$$

$$\ln \left(\frac{\rho_C}{\rho_B} \right) > 1 \Rightarrow +ve \Rightarrow V_{BC_L} (-ve)$$

ملاحظة : ممكن استخدم بس صعب $\left[V_{BC_L} = \int_L \frac{\rho_L \, dL}{4\pi\epsilon_0 r} \right. \Leftarrow$ بالعادة لل finite

وال infinite بنحل على أول قانون الي فيه $\vec{E}$.

$$\Rightarrow V_{BC} = V_{BCQ} + V_{BC_L} = -50.175 \, (V)$$

drop in potential .

$$V_B > V_C$$

$$V_C = V_{BC} + V_B = -50.175 + 100 = \boxed{49.825 \, (V)}$$

⇒ For the (4th) case in Gauss's [volume]

↳ in sphere example

$$\Rightarrow \rho_v = \begin{cases} \rho_o & , \ r < a \\ 0 & , \ r > a \end{cases}$$

$$\Rightarrow \bar{E} = \begin{cases} \dfrac{\rho_o r}{3\epsilon_o} & , \ 0 \le r \le a \\[2ex] \dfrac{\rho_o a^3}{3\epsilon_o r^2} & , \ a \le r < \infty \end{cases}$$

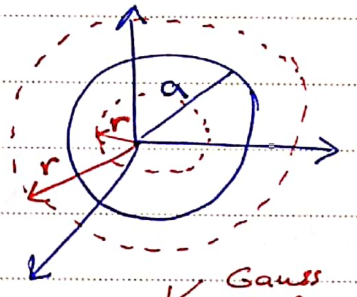
زيادة على السؤال ↳ Find (V) every where .

*بال $\vec{E}$ اجباري نبدأ من جوا لبرا من $r=0$ لبرا $r=\infty$

*بال $\bar{V}$ دائماً بنبدأ من ref (ممكن) ال ref

يكون عند 0 يعني بنبدأ من جوا بس صعب

ال ref ← ∞ (من برا لجوا)



+ بدنا ref

← ref is at ∞

⇒ ref is at ∞

→ ① for $a < r < \infty$

$$V = -\int_L \bar{E} \cdot \bar{dL} = -\int_{\infty}^{r} \frac{\rho_o a^3}{3\epsilon_o r^2} \cancel{\hat{a}_r} \, dr \, \cancel{\hat{a}_r}$$

$$= +\frac{\rho_o a^3}{3\epsilon_o}\left(\frac{1}{r} - \cancel{\frac{1}{\infty}}^{\,0}\right) = \frac{\rho_o a^3}{3\epsilon_o r} \quad (V)$$

→ ② for $0 < r < a$

$$V = -\int_L \bar{E} \cdot \bar{dL} = -\int_{\infty}^{a} \bar{E} \cdot \bar{dL} - \int_{a}^{r} \bar{E} \cdot \bar{dL}$$

بقسم لأني راح نحسب من ∞ لل r الي جوا

$$= \frac{\rho_o a^3}{3\epsilon_o a} - \int_a^r \frac{\rho_o r}{3\epsilon_o} \hat{a}_r \, dr \, \hat{a}_r$$

[ال $\bar{E}$ continuous أو ال V]

$$V = \frac{\rho_o a^2}{3\epsilon_o} - \frac{\rho_o}{3\epsilon_o}\left(\frac{r^2 - a^2}{2}\right)$$

عند $\uparrow r = 0 \Rightarrow V \uparrow$ لو طلب V منتصف الكرة

Scanned with CamScanner

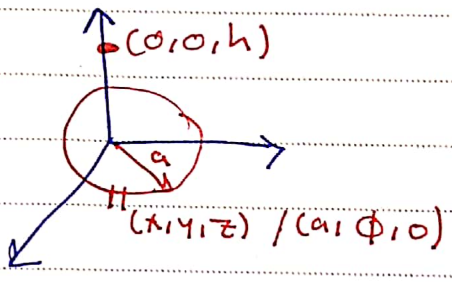
Note :

$$V = -\int_{l} \bar{E}\cdot\bar{dL} \quad , \quad V = \int_{l} \frac{\rho_L \, dL}{4\pi\varepsilon_0 r}$$

Finding (V) from the field . — Finding (V) from the source .

Ex : Ring

V ↑ كل القوانين تكون حلول بس اتنين أحول .



a) E ?   b) V ?

[لو طلب بس V بنقدر نستخدم] $V = -\int_{l} \bar{E}\cdot\bar{dL}$ أو $V = \int_{l} \frac{\rho_L \, dL}{4\pi\varepsilon_0 r}$

طول ال L حسب (z)
$$\bar{dL} = dz\,\hat{a_z}$$
$$\int_{\infty}^{h} = \int_{L} \text{ يعني}$$

حسب ϕ
$$\bar{dL} = a\,d\phi$$
$$\int_{0}^{2\pi} = \int_{L} \text{ يعني}$$

لو استخدمنا $V = -\int_{L} \bar{E}\cdot\bar{dL} \rightarrow \bar{dL} = dz\,\hat{a_z}$

$$\bar{E} = \int_{l} \frac{\rho_L \, dL}{4\pi\varepsilon_0 r^2} \hat{a_r} = \int_{l} \frac{\rho_L \, \bar{r} \, dL}{4\pi\varepsilon_0 r^3}, \quad \hat{a_r} = \frac{\bar{r}}{r}$$

$\bar{E}$ in cyl. so $dL = a\,d\phi$ in $\bar{E}$

$$\Rightarrow \bar{r} = (0, \phi, h) - (a, \phi, 0)$$

$$\bar{r} = -a\,\hat{a_\rho} + h\,\hat{a_z} \quad , \quad r = \sqrt{a^2+h^2} \quad , \quad dL \text{ in } \bar{E} = a\,d\phi$$

$$\bar{E} = \frac{\rho_L a}{4\pi\varepsilon_0} \int_{0}^{2\pi} \frac{\cancel{-a\,\hat{a_\rho}}^{\text{symmetry}} + h\,\hat{a_z}}{[a^2+h^2]^{3/2}} \, d\phi$$

حسب $2\pi \leftarrow 0$

$$\Rightarrow = \frac{\rho_L a h}{2\varepsilon_0 [a^2+h^2]^{3/2}} \hat{a_z}$$

$$\Rightarrow V = -\int_{L} \bar{E}\cdot\bar{dL} \rightarrow \bar{dL} = dz\,\hat{a_z}$$

$$= -\int_{-\infty}^{h} \frac{\rho_L a h \, \cancel{\hat{a_z}} \cdot dz \, \cancel{\hat{a_z}}}{2\varepsilon_0 [a^2+h^2]^{3/2}} \quad , \quad \text{replace}(h) \text{ by } (z)$$

$$= -\int_{-\infty}^{h} \frac{\rho_L a z}{2\varepsilon_0 [a^2+z^2]^{3/2}} \, dz \quad \ldots \rightarrow$$

لو اتخذنا $\rightarrow$ (الأسهل)

$$V = \int_L \frac{\rho_L \, dL}{4\pi \epsilon_o r} \rightarrow dL = a\,d\phi \;,\; \int_L = \int_0^{2\pi}$$

$$V = \int_0^{2\pi} \frac{\rho_L \, a \, d\phi}{4\pi \epsilon_o \sqrt{a^2+h^2}}$$

$$\boxed{V = \frac{\rho_L \, a}{2\epsilon_o \sqrt{a^2+h^2}} \quad (V)}$$

⇒ How to Find $\bar{E}$ From potential :-

$$V = -\int_L \bar{E}\cdot\bar{dL}$$

in cart :-

$$\bar{E} = E_x \hat{a_x} + E_y \hat{a_y} + E_z \hat{a_z}$$

$$\bar{dL} = dx \hat{a_x} + dy \hat{a_y} + dz \hat{a_z}$$

$$\bar{E}\cdot\bar{dL} = E_x dx + E_y dy + E_z dz$$

$$\Rightarrow dV = -\bar{E}\cdot\bar{dL}$$

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

by equating similar components

$$\frac{\partial V}{\partial x} \cancel{dx} = -E_x \cancel{dx} \Rightarrow E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

$$\bar{E} = -\left[\frac{\partial V}{\partial x} \hat{a_x} + \frac{\partial V}{\partial y} \hat{a_y} + \frac{\partial V}{\partial z} \hat{a_z}\right]$$

$$\boxed{\bar{E} = -\nabla V}$$ ← cart / cyl / sph

⇒ to Find $\bar{E}$ if potential is given.

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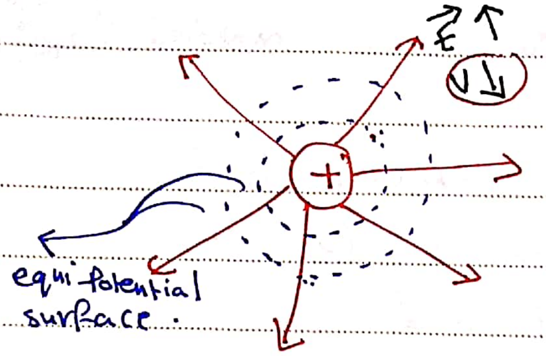


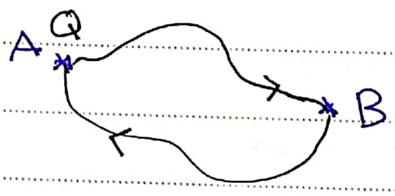
$$V = \frac{Q}{4\pi\epsilon_0 r}$$

$$\bar{D} = \epsilon_0 \bar{E}$$ $\bar{E}$ & $\bar{D}$ لهم نفس الاتجاه

⟹ Potential over a closed loop :

Q from A to A

$$V_{AA} = -\int_A^B \bar{E}\cdot\overline{dL} - \int_B^A \bar{E}\cdot\overline{dL} = 0$$



$$= V_A - V_A \quad , \quad V_A \neq 0$$

$$\boxed{W = Q \cdot V_{AA} = 0}$$

محصلة الشغل = صفر

$$\boxed{V = \oint_L \bar{E}\cdot\overline{dL} = 0}$$ ⟹ second Maxwell's eq. in integral form.

closed path / مسار ثابت

↳ (equi-potential surface).

$$V = -\int_L \bar{E}\cdot dL$$

↳ no closed path.

Ex :- Given $V = \frac{10}{r^2} \sin\theta \cos\phi$ (v)

Find a) $\bar{D}$ at $(2, \frac{\pi}{2}, 0) \rightarrow$ cyl or sph

b) The work done in moving a 10 μC charge from $A(1, 30°, 120°)$ to $B(4, 90°, 60°)$.

$\rightarrow$ sol :-

(a) $\bar{D} = \epsilon_0 \bar{E}$, $\bar{D} = -\epsilon_0 \nabla V$ ($\bar{E} = -\nabla V$)

$$\bar{E} = -\left(\frac{\partial v}{\partial r}\hat{a}_r + \frac{\partial v}{r\partial\theta}\hat{a}_\theta + \frac{\partial v}{r\sin\theta\,\partial\phi}\hat{a}_\phi\right)$$

$$\bar{E} = -\left(\left(\underbrace{\frac{-20}{r^3}\sin\theta\cos\phi}_{E_r}\,\hat{a}_r + \underbrace{\frac{10}{r^3}\cos\theta\cos\phi}_{E_\theta}\,\hat{a}_\theta + \underbrace{\frac{-10}{r^3\cancel{\sin\theta}}\cancel{\sin\theta}\,\sin\phi}_{E_\phi}\,\hat{a}_\phi\right)\right)$$

$$\Rightarrow \bar{D}\Big|_{(2,\frac{\pi}{2},0)} = \epsilon_0 \bar{E}\Big|_{(2,\frac{\pi}{2},0)} = \frac{10^{-9}}{36\pi}\left(\frac{20}{8}\hat{a}_r\right) \quad c/m^2$$

(b) work.

Method (1) :-

$$W = Q V_{AB} = Q(V_B - V_A)$$

$$= 10 * 10^{-6}\left(\frac{10}{16}\cdot\left(\frac{1}{2}\right)(1) - 10\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\right)$$

$$= 10 * 10^{-6}\left(\frac{5}{16} - \frac{-40}{16}\right)$$

$$= 10 * 10^{-6}\left(\underbrace{\frac{45}{16}}_{V_{AB}}\right) J = \boxed{28.125\,\mu J}$$

Method (2) :-

$$W = Q V_{AB} = -Q\int_{r_A}^{r_B} \bar{E}\cdot\overline{dL}$$

$$= -10 * 10^{-6}\left[\int_{r_A}^{r_B}\left[E_r\,dr + E_\theta\, r\,d\theta + E_\phi\, r\sin\theta\, d\phi\right]\right]$$

$$= -10\times10^{-6}\left[\int_{1}^{4} E_r\,dr\Big|_{\theta=30^\circ,\ \phi=120} + \int_{30}^{90} E_\theta\, r\,d\theta\Big|_{r=4,\ \phi=120} + \int_{120}^{60} E_\phi\, r\sin\theta\, d\phi\Big|_{r=4,\ \theta=90}\right]$$

بنقل r بس θ / ϕ ما بنقلهم

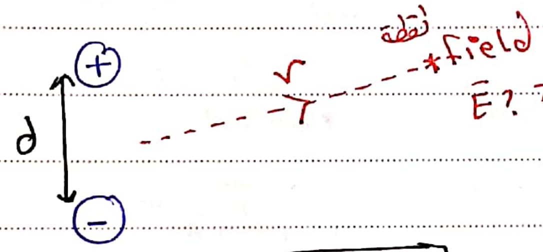
بنقل θ والـ r نقلتها بس ϕ ما بنقلها

بنقل الـ ϕ والـ r / θ نقلتهم .

معكن أغير ترتيب النقل
لو نقلت θ أول اشي
$\int_{30}^{90} E_\theta\, d\theta\Big|_{r=1,\ \phi=120}$ ← أول تكامل
< ما نقلتهم
+ ---- + ----

$$= \boxed{28.125\,\mu J}$$

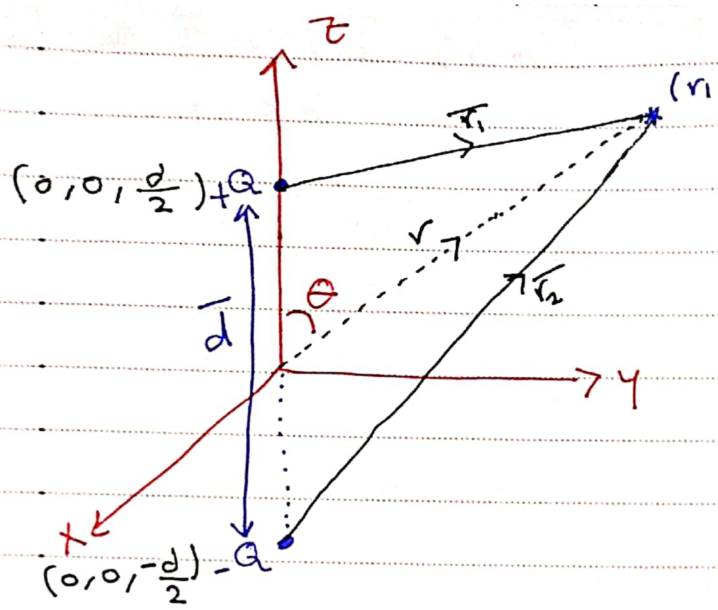
Electric Dipole :-



$\bar{E}$? $\bar{D}$? V?

$\boxed{d \ll r}$ → dipole condition .

كأنو الناظر من بعيد
بشوفهم كأنهم شحنة وحدة.



$\bar{E}$? $\bar{D}$? $\bar{V}$? if $d <<< r$

Sol: (*) ref is at ∞, $V_\infty = 0$

$$V = V_+ + V_-$$

$$= \frac{Q}{4\pi\epsilon_o r_1} - \frac{Q}{4\pi\epsilon_o r_2}$$

$$= \frac{Q}{4\pi\epsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\boxed{V = \frac{Q\,(r_2 - r_1)}{4\pi\epsilon_o r_1 r_2}} \quad (1) \ (V)$$

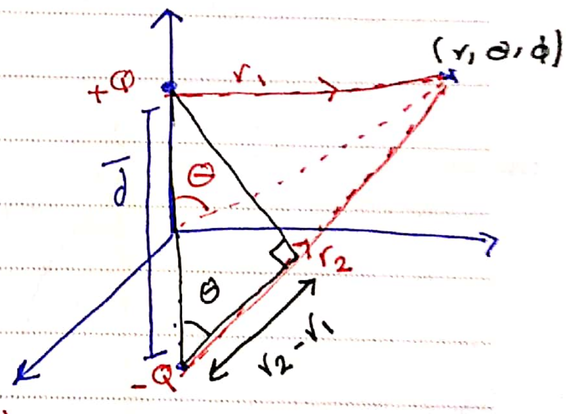
بالواقع صعب تحسب الوضع صح هما

يعني Q/ r_2/r_1 صعب نجيبها الا اذا أعطاه.

(*) $\cos\theta = \frac{r_2 - r_1}{d}$

$r_1 r_2 \approx r^2$

$$\Rightarrow \boxed{V = \frac{Q\,d\cos\theta}{4\pi\epsilon_o r^2}} \ (V) \quad (2)$$



لو كان الـ d/ Q صعب نجيبهم

فالكل define dipole moment ($\bar{P}$)

$\Rightarrow \bar{P} = Q\,\bar{d}$ (C.m) — من السالب للموجب (الاتجاه الرابطة مش اتجاه ال $\bar{E}$)

$p = Q\,d$

$$\boxed{V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2}} \quad (V) \quad (3)$$

p → scaler

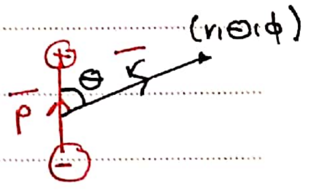
r = field − center of the dipole (source)

لو عطينا $\bar{p}$ vector كحاجة ال

$$\boxed{V = \frac{\bar{p}\cdot\hat{a_r}}{4\pi\epsilon_0 r^2}} \quad (3)'$$

$\bar{p}\cdot\bar{a_r} = $ ؟ $p(1)\cos\theta$ (بين ال $\bar{p}$ / $\bar{r}$)

$$\equiv \; V = \frac{\bar{p}\cdot\bar{r}}{4\pi\epsilon_0 r^3}$$



$\bar{E} = \bar{E}_+ + \bar{E}_-$

OR $\bar{E} = -\nabla V$ ⇒ أسهل / $\nabla V \to$ sph(θ, r لأنه فيه)

from (3)

$$\Rightarrow \bar{E} = \frac{-p}{4\pi\epsilon_0}\left(\frac{-2\cos\theta}{r^3}\hat{a_r} - \frac{\sin\theta}{r^3}\hat{a_\theta}\right)$$

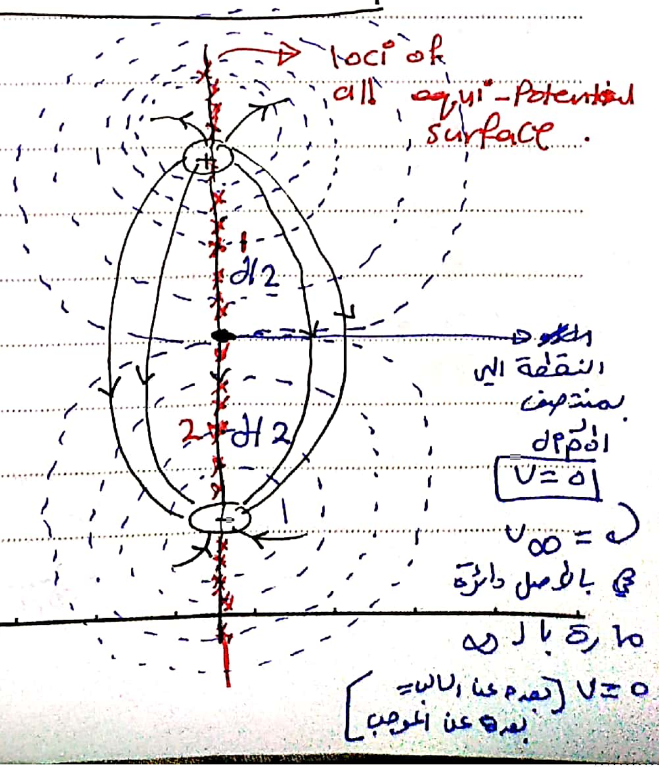
φ مش موجودة

$$\boxed{\bar{E} = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{a_r} + \sin\theta\,\hat{a_\theta}\right)} \quad V/m$$

p → scaler

$$\boxed{\bar{D} = \epsilon_0\bar{E} \quad (c/m^2)}$$

[هي بالأصل دائرة مش قطع]



⇒ For N Dipoles :

Field — النقطة عن center ال dipole (source).

Field ال نقطة

$$V = \frac{\bar{p}\cdot\hat{a}_r}{4\pi\varepsilon_0 r^2} = \frac{\bar{p}\cdot(\bar{r}-\bar{r}_1')}{4\pi\varepsilon_0|\bar{r}-\bar{r}_1'|^3}$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{K=1}^{N}\frac{\bar{P}_K\cdot(\bar{r}-\bar{r}_K')}{|\bar{r}-r_K'|^3} \quad (V)$$

Ex : Two dipoles with dipole moments $-5\hat{a}_z$ nC·m and $9\hat{a}_z$ nC·m are located at $(0,0,-2)$ and $(0,0,3)$. Find the potential and $\bar{E}$ at the origin.

sol :- ref is at ∞

بنقدر نحل بدون رسم .

$$\Rightarrow V = \frac{\bar{P}_1\cdot\hat{a}_{r_1}}{4\pi\varepsilon_0 r_1^2} + \frac{\bar{P}_2\cdot\hat{a}_{r_2}}{4\pi\varepsilon_0 r_2^2}$$

$$= \frac{-5\hat{a}_z\times10^{-9}\cdot 2\hat{a}_z}{4\pi\times\frac{10^{-9}}{36\pi}\times 8} + \frac{9\hat{a}_z\times10^{-9}\times -3\hat{a}_z}{4\pi\times\frac{10^{-9}}{36\pi}\times 27}$$

$$\boxed{V = -20.25\ \text{V}}$$

↳ drop , work done by field itself .

$$\Rightarrow \bar{E} = \bar{E}_1 + \bar{E}_2$$

بنعوض بقانون ال $\bar{E}$

$-5a_z$ direction.

(-5 بنحط 5 مش -5 scalar) $P_1 = 5$ nC·m

(-2 بنحط 2 مش -2 scalar) $r_1 = 2$ m

$\theta_1 = 180°$

angle between $\bar{P}_1$ & $\bar{r}_1$

$\theta_1 \measuredangle\ \bar{P}_1$ & $\bar{r}_1$

$-\hat{a}_z \quad \hat{a}_z$

$\theta_1 = 180°$

$P_2 = 9$ nC·m

$r_2 = 3$ m

$\theta_2 = 180°$

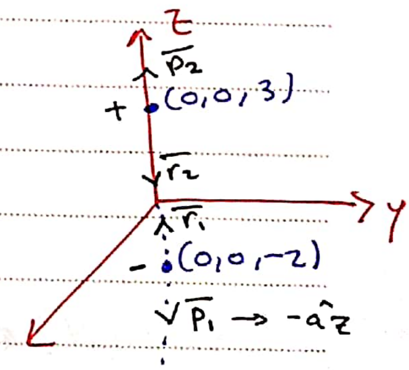
angle between $\bar{P}_2$ & $\bar{r}_2$

$\theta_2 \measuredangle\ \bar{P}_2$ & $\bar{r}_2$

$\hat{a}_z \quad -\hat{a}_z$

$\theta_2 = 180°$

لو رسمة

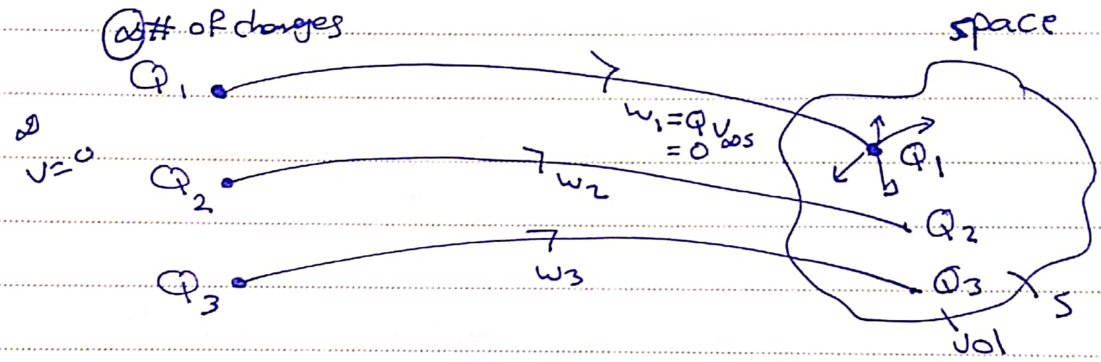


* $V = \dfrac{\overline{P} \cdot \hat{a_r}}{4\pi \varepsilon_0 r^2}$ (V)

Point charge	Dipole
$V \propto \frac{1}{r}$	$V \propto \frac{1}{r^2}$
$E \propto \frac{1}{r^2}$	$E \propto \frac{1}{r^3}$

Energy Density in Electrostatic Fields :-

W_E = Electrical Energy in (J).



بدنا ننقلهم للـ space بهاد الترتيب * $Q_1 \to Q_2 \to Q_3$

لما نقلنا Q_1 ← بالبداية موجود فاضي ← $V=0$ و $V_{\infty}^{ref}=0$ ⇐ بالتالي $\boxed{W=0}$

بعدها لما نقلنا Q_2 كانت Q_1 موجودة فأثرت وهكذا وعند نقل Q_3 اثرت عليها Q_1 / Q_2 .

$W_E = W_1 + W_2 + W_3$

$= 0 + Q_2 V_{12} + Q_3 (V_{13} + V_{23}) \longrightarrow (1)$

الشغل المبذول لنقل الشحنة الأولى.

* $Q_3 \to Q_2 \to Q_1$

$W_E = W_1 + W_2 + W_3$

$= Q_1 (V_{21} + V_{31}) + Q_2 V_{32} + 0 \longrightarrow (2)$

$\Rightarrow$ Add (1) & (2)

$2W_E = Q_1 \underbrace{(V_{21} + V_{31})}_{} + Q_2 (V_{12} + V_{32}) + Q_3 (V_{13} + V_{23})$

$V_1 \rightarrow$ total potential on Q_1; V_2; V_3

$W_E = \frac{1}{2} (Q_1 \cdot V_1 + Q_2 \cdot V_2 + Q_3 \cdot V_3)$

* $\boxed{W_E = \frac{1}{2} \sum_{K=1}^{N} Q_K V_K}$ in (J) for N-point charges.

ملاحظة : بنستخدم هاد القانون إذا طلب E. Energy ، بس لو حكى
verify your answer by calculate work
بنستخدم $W_E = W_1 + W_2 + W_3 + W_4 \dots$

⇒ For line charge :

$$W_E = \frac{1}{2}\int_L \rho_L V \, dL \quad (J)$$

$V \rightarrow$ potential.

⇒ For surface charge :

$$W_E = \frac{1}{2}\int_S \rho_s V \, ds \quad (J)$$

كلام Soura.

⇒ For volume charge :

$$W_E = \frac{1}{2}\int_{V'} \rho_v V' \, dv' \quad (J)$$

⇒ How to relate W_E with $\bar{E}$??

* $W_E = \frac{1}{2}\int_V \rho_v V \, dv$

$\rho_v = \nabla \cdot \bar{D}$

$\bar{E} = -\nabla V$

* $W_E = \frac{1}{2}\int_V \bar{E} \cdot \bar{D} \, dv$

$= \frac{1}{2}\int_V \epsilon_0 E^2 \, dv$ عادي ✓

$= \frac{1}{2}\int_V \frac{D^2}{\epsilon_0} \, dv$

بنختار أبعاد الـ Volume.

$, \bar{D} = \epsilon_0 \bar{E}$

$\bar{E} = \frac{\bar{D}}{\epsilon_0}$

$\bar{E} \cdot \bar{E} = |\bar{E}|^2$

⇒ Energy Density = $\frac{\text{Energy } (W_E)}{\text{Volume}}$ (J/m^3)

$$w_E = \frac{W_E}{\text{Volume}}$$

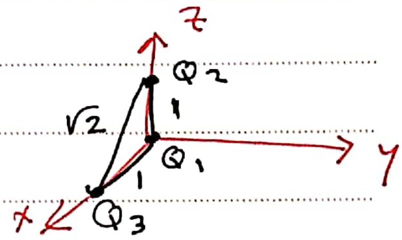
$$\boxed{w_E = \frac{W_E}{\text{Vol}} \ (J/m^3)}$$

OR

$$\boxed{\begin{aligned} w_E &= \frac{1}{2}\bar{E} \cdot \bar{D} \\ &= \frac{1}{2}\epsilon_0 E^2 \\ &= \frac{1}{2}\frac{D^2}{\epsilon_0} \end{aligned}}$$

* $W_E = \int_V w_E \cdot dv$ → لو طلب W_E وكان معطى w_E

Ex :- The point charges $-1\,nC$, $4\,nC$ and $3\,nC$ are located at $(0,0,0)$, $(0,0,1)$, $(1,0,0)$. Find the energy in the system ??



$$W_E = \frac{1}{2}\sum_{K=1}^{N} Q_K V_K$$

$$= \frac{1}{2}\left(Q_1 V_1 + Q_2 V_2 + Q_3 V_3\right)$$

$$= \frac{1}{2}\Big[Q_1\big(\underbrace{V_{21}}_{\frac{Q_2}{4\pi\epsilon_0(1)}} + \underbrace{V_{31}}_{\frac{Q_3}{4\pi\epsilon_0(1)}}\big) + Q_2\big(\underbrace{V_{12}}_{\frac{Q_1}{4\pi\epsilon_0(1)}} + \underbrace{V_{32}}_{\frac{Q_3}{4\pi\epsilon_0\sqrt{2}}}\big)$$

$$+ Q_3\big(\underbrace{V_{13}}_{\frac{Q_1}{4\pi\epsilon_0(1)}} + \underbrace{V_{23}}_{\frac{Q_2}{4\pi\epsilon_0(\sqrt{2})}}\big)\Big]$$

$$\boxed{W_E = 13.137\,nJ}$$

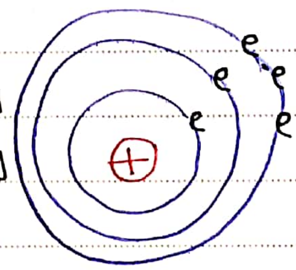
* لما طلب work ل system وما طلب energy density طلب energy فبنستخدم هاد القانون
↳ $W = W_1 + W_2 + W_2$

CH5 : Electric Field in Materials :

* Classification of materials based on it's electrical properties :
  - → conductors (Metals) , ($\sigma >>> 1$)
    Cu , AL , Ag , lead
  - → semi - conductors , ($\sigma > 1$)
    Si , GaAs , Ge
  - → Dielectrics (Insulators) , ($0 < \sigma <<< 1$)
    Li , Mica , Teflon , glass , poly stren , polythene

$\sigma \equiv$ Conductivity (S/m) , $S = \Omega^{-1}$, $\sigma \ (\Omega \cdot m)^{-1}$
$\rho \equiv$ Resistivity (Ω-m) $= \frac{1}{\sigma}$

$T \uparrow \longrightarrow \sigma \downarrow$ [اذا زادت الحرارة يتزيد حركة e بشكل عشوائي σ بتقل]
$T \downarrow \longrightarrow \sigma \uparrow$ [اذا قلت الحرارة كل e في مكانه خال σ بتزيد]



* $\sigma = 0 \Rightarrow$ Free space
* So σ depends on :
  - → Temperature (°K)
  - → Frequency (Hz)

* اي مادة ال $T > 0°K$ بتكون إلها طاقة حركية ← طاقة إشعاعية
  ↲ if $T = 0°K$ ← ما بتكون إلها حركة

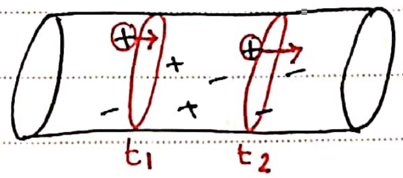
* lead at $T = 20°C$ (293°K)
  → $\sigma \approx 10^{6}$ (S/m)
  if T is reduced to (4°K)
  → $\sigma \approx 10^{20}$ (S/m)   (super conductor)
  (perfect conductor)
  (good conductor)

* Types of current :

1. conduction current , (conductors , DC or AC)

2. convection current , (dielectrics , DC)

3. Displacement current , (dielectrics , AC)
   ↳ CH : 9



$$I = \frac{\Delta Q}{\Delta t} = \frac{Q_2 - Q_1}{t_2 - t_1} \equiv (C/S) \text{ or } (A)$$ ← بستخدمه لما التغير يكون منتظم وبالواقع ما يكون منتظم فا بنستخدمه .

$$I = \frac{dq}{dt} \quad \text{in general}$$ ← للتغير غير منتظم

$$\Rightarrow \frac{I}{\Delta S} = J \quad , \quad J \equiv \text{current density } (A/m^2)$$

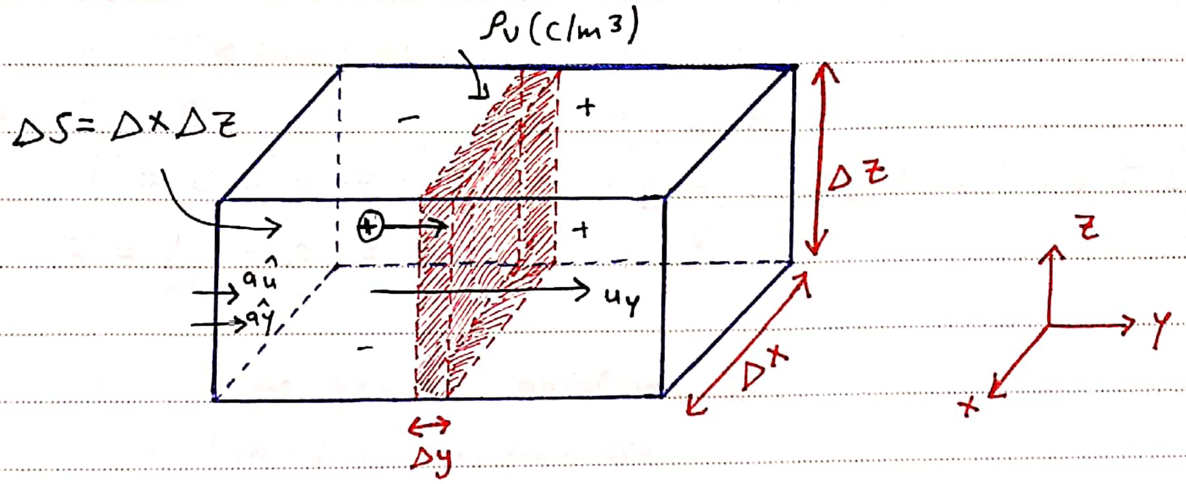
scaler ← $I = \oint_S \overline{J} \cdot \overline{ds}$

$\overline{J}$ ← بمشي current وإله اتجاه

$\oint$ ← حتى لو ما حطيت $\oint$ لازم أعرف إنه السلك لازم يكون مغلق لحتى يصير I فيه .

$$\boxed{I = \int_S \overline{J} \cdot \overline{ds}}$$

* Consider a filament of a dielectric material :-



$$I = \frac{\Delta Q}{\Delta t} \qquad , \; Q = \int_v \rho_v \, dv$$

$$dQ = \rho_v \, dv$$

$$\boxed{\Delta Q = \rho_v \, \Delta v}$$

$$= \frac{\rho_v \, \Delta v}{\Delta t}$$

$$= \frac{\rho_v \, \Delta s \, \Delta y}{\Delta t} \qquad , \; \Delta s = \Delta x \, \Delta z$$

$$, \; \Delta v = \Delta s * \Delta y = \Delta x \, \Delta y \, \Delta z$$

convection current

$$\boxed{I = \rho_v \, \Delta s \, u_y} \qquad , \; u_y = \frac{\Delta y}{\Delta t} \qquad , \; \boxed{\bar{u} = u_y \hat{a_y}}$$

كم السرعة

$$J = \frac{I}{\Delta s} = \rho_v \, u_y$$

$$\boxed{\bar{J} = \rho_v \, \bar{u}} \equiv \frac{c}{m^3} \cdot \frac{m}{s} = \left(\frac{A}{m^2}\right)$$

convection current density .

* الاتجاه العامودي على المساحة هو $\hat{ay}$ وفيفل يكون باتجاه حركة الشحنة $\hat{ay}$.

* How to relate $\overline{J}$ with $\overline{E}$?

- The force on one electron :-

$\Rightarrow$ $Q = -e$

$$\overline{F} = Q\overline{E} = -e\overline{E} = \underset{\text{mass}}{m}\,\underset{\text{acceleration}}{\overline{a}} = \frac{m\overline{u}}{\tau} \qquad [\text{if } \tau\uparrow \rightarrow \overline{F}\downarrow]$$

$$\overline{a} = \frac{du}{dt} = \Delta u = \frac{\overline{u}}{\tau}$$

$\tau \equiv$ Time between collisions (s)

$\hookrightarrow$ affected by temperature

$T\downarrow \rightarrow \tau\uparrow$

$$\rightarrow -e\overline{E} = \frac{m\overline{u}}{\tau}$$

$$\overline{u} = \frac{-e\tau}{m}\overline{E}$$, τ is fixed if Temp. is fixed

$$\boxed{\overline{u} = \mu\overline{E}}$$

(drift velocity) $\overline{v}_d$ = $\overline{u}$; μ $\rightarrow$ mobility $\left(\frac{C \cdot s}{kg}\right)$

$\Rightarrow$ For n-electrons per volume

$$\rho_v = \underset{/m^3}{n}\,(\underset{C}{-e}) \Rightarrow (C/m^3)$$, n = # of charges per volume (# /m³)

$$\overline{J} = \rho_v\overline{u}$$

$$= -ne\left(\frac{-e\tau}{m}\right)\overline{E}$$

$$\overline{J} = \underbrace{\frac{ne^2\tau}{m}}_{\sigma}\overline{E}$$

$$\boxed{\overline{J} = \sigma\overline{E}} \quad , \quad \boxed{\frac{\sigma}{\mu} = \rho_v}$$

$$\sigma \rightarrow \frac{1}{\Omega \cdot m} \cdot \frac{V}{m} = \frac{A}{m^2}$$

$\rightarrow$ conduction current density / ohm's law

(بنقدر نستخدمه مع ال dielectric بس ال σ بتكون صغيرة)

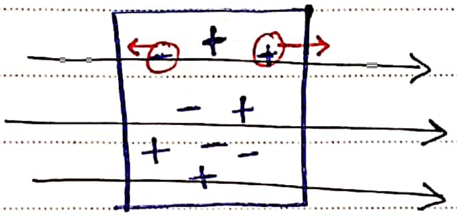
$* \; R = \frac{L}{\sigma A}$, $V = IR$ (ohm's law) $R = \frac{1}{G}$

$V = \frac{I}{G} \rightarrow I = GV$

$\bar{J} = \sigma \cdot \bar{E}$

$$\boxed{\bar{J} = \sigma . \bar{E}}$$

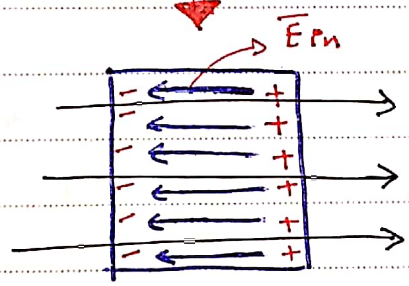
* Conductors :



$\rightarrow \bar{F} = Q \bar{E}_{ext}$

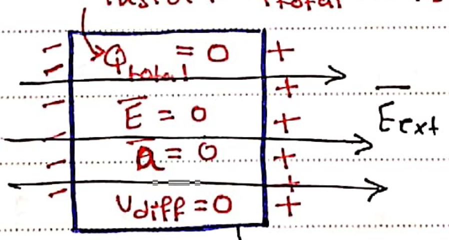
$Q_{total} = 0$

$\Downarrow$

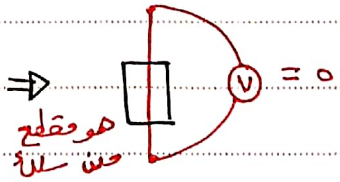


$\Downarrow$

inside :- $Q_{total} = 0$, $\bar{E} = 0$, $\bar{a} = 0$, $V_{diff} = 0$, $\bar{D} = 0$



$\Rightarrow$



$\rightarrow$ equi - Potential surface

الشحنات بتتجمع على السطح

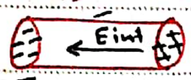
* لو شلت الـ $\bar{E}_{ext}$ يعني مثل

اني أشيل البطارية رح يرجع مثل أول مرة

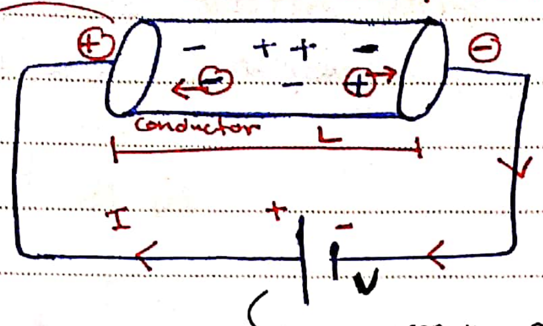
$\rightarrow$

$\longrightarrow [\bar{E}, t, I, \bar{J}, \bar{u}]$ كلم باتجاه ال $\bar{E}$

بعدها صار بصير



$\bar{E}_{int}$ ليصير عنه



هي ال source

لل $\bar{E}_{ext}$

$\bar{E} = -\nabla V$

$R = \dfrac{V}{I}$

$$\boxed{R = \frac{V}{I} = \frac{-\int_L \bar{E}\cdot\bar{dL}}{\int_S \sigma\,\bar{E}\cdot\bar{ds}}} \rightarrow \text{in general}$$

$I = \int_S \bar{J}\cdot\bar{ds}$, $\bar{J} \rightarrow \sigma E$

$$\boxed{R = \frac{L}{\sigma s} = \frac{\rho L}{s}\,(\Omega)}$$

ohm's law (if uniform)

$$\hookrightarrow R = \frac{\cancel{E}\cdot L}{\sigma \cancel{E} S} = \frac{\rho L}{S}$$

---

* Power :-

$$\left[P = V\cdot i = \frac{V^2}{R} = I^2 R = G V^2 = \frac{I^2}{G}\right] \quad \text{Joule's law}$$

$P = \dfrac{\text{work}}{\text{Time}}$ (J/s) or $(W) \rightarrow$ watt

$= \dfrac{\bar{F}\cdot\bar{L}}{t} = \dfrac{Q\,\bar{E}\cdot\bar{L}}{t}$, $Q = \int_V \rho_v\,dv$

$P = \int_V \dfrac{\rho_v\,\bar{E}\cdot\bar{L}}{t}\,dv$, $\bar{u} = \dfrac{\bar{L}}{t}$

$P = \int_V \rho_v\,\bar{E}\cdot\bar{u}\,dv$, $\bar{J} = \rho_v\,\bar{u}$

Joule's law

$$\boxed{P = \int_V \bar{E}\cdot\bar{J}\,dv}$$, $\bar{J} = \sigma\bar{E}$, $\bar{E} = \dfrac{\bar{J}}{\sigma}$

$$\boxed{P = \int_V \sigma E^2\,dv}$$

$$\boxed{P = \int_V \frac{J^2}{\sigma}\,dv} \Rightarrow \boxed{P = \int_V \rho J^2\,dv}$$

ρ: $\Omega\cdot m$ (Resistivity)

$\Rightarrow$ if uniform cross section wire

$$P = \int_V \bar{E} \cdot \bar{J} \, dv \quad \rightarrow \text{in general}$$

$$P = \int_S \int_L \bar{E} \cdot \bar{J} \, dL \, ds$$

$$P = \underline{\int_L \bar{E} \cdot \bar{dL}} \ * \ \underline{\int_S \bar{J} \cdot \bar{ds}}$$

$$\boxed{P = V \cdot i}$$

* Power density :-

$$W_p = \frac{\text{Power}}{\text{Volume}} \quad (W/m^3)$$

$$\boxed{W_p = \frac{P}{Vol} = \bar{E} \cdot \bar{J} = \sigma E^2 = \frac{J^2}{\sigma}}$$

* $P = \int_V W_p \, dv$

Ex :- if $\bar{J} = \frac{1}{r^3} (2 \cos\theta \, \hat{a_r} + \sin\theta \, \hat{a_\theta}) \ (A/m^2)$

find the current passing through

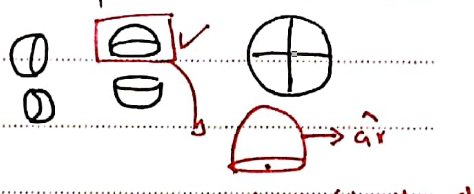
a) A hemi spherical shell of radius 20 cm with

$0 < \theta < \frac{\pi}{2}$, $0 < \phi < 2\pi$

b) A spherical hollow shell of radius 10 cm

sol :- a) $I = \int_S \bar{J} \cdot \bar{ds}$, $\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$

$$\Rightarrow I = \int_0^{2\pi} \int_0^{\pi/2} \frac{1}{r} \underbrace{2 \sin\theta \cos\theta}_{\sin 2\theta} \, d\theta \, d\phi \Big|_{r=0.2\,m}$$



Shell (hollow)

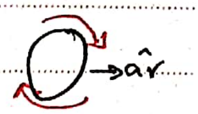
الـ I بيمشي على القشرة موصلة

$$= 10\pi \, A = 31.4 \, A$$

b)

$$I = \int_0^{2\pi} \int_0^{\pi} \frac{1}{r} \sin 2\theta \, d\theta \, d\phi \Big|_{r=0.1\,m}$$

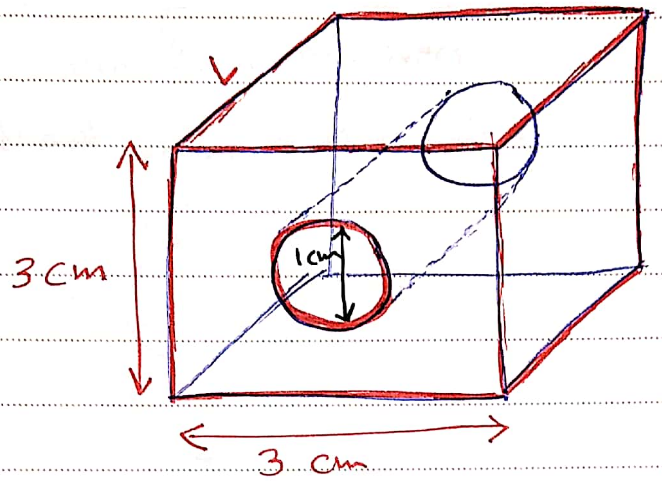
$$= 0 \, A$$



مجموع الـ current = 0

من الجزء العلوي + الجزء السفلي

Ex :- A lead $\sigma = 5 * 10^{6}$ S/m , $L = 4$ m , Find R ?



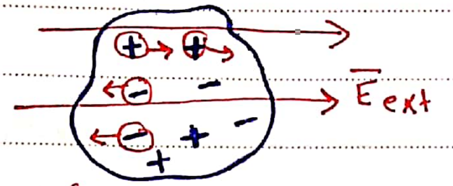
Sol :- $R = \frac{L}{\sigma A}$, $A = (3*10^{-2})^{2} - \pi(\frac{1}{2}*10^{-2})^{2}$

$\Rightarrow \boxed{R = 974 \, \mu\Omega}$

* Polarization in Dielectrics

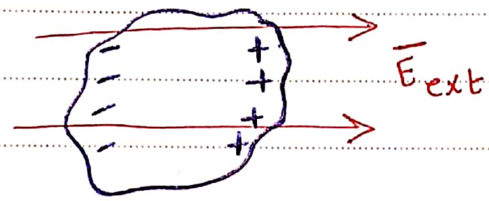
Non-Polar Dielectrics

ملاحظة [$\vec{E}_{ext}$ طبع المجال الخارجي non permenant ⇐ dipoles فيها الـ]



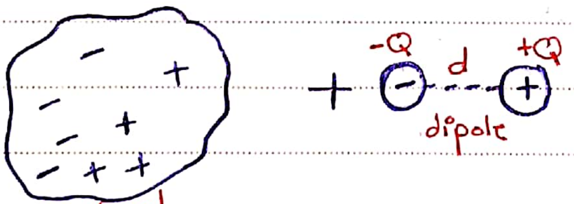
$Q_{total} = 0$

بصير فيه pressure وبتصير مشوهة



↳ called Distorted charge distribution.

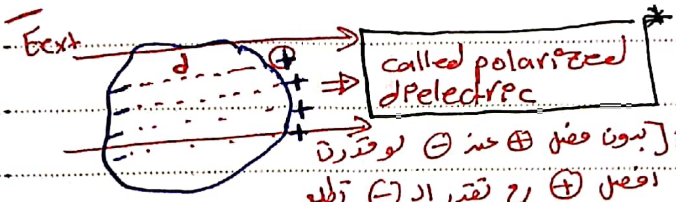
عبارة عن



normal

* لما أشيل الـ $\vec{E}$ بترجع المادة طبيعية وبتبطل dipole

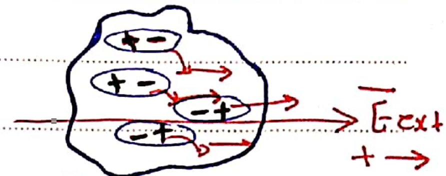
↳ نهاية انو الشحنات الموجبة بتحاول تطلع للسطح بس السالب بضل جوا المادة لأنها خفيفة وبس بضل في رابطة بين ⊕ و ⊖ [dipole]



*[بدون فصل ⊕ عن ⊖ لو قدرت افصل ⊕ رح تقدر الـ ⊖ تطلع وبالتالي بتصير conductor / semi conductor [بس ما بنفصلهم

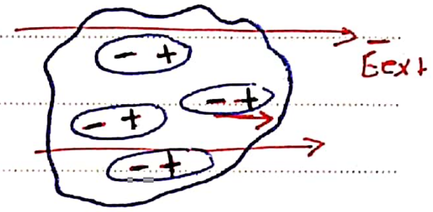
Polar Dielectric

[permenant dipoles] فيها dipole's]

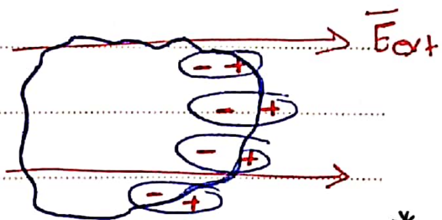


$Q_{total} = 0$

بالبداية الشحنات بصير الها إعادة ترتيب حتى تصير ع اتجاه الـ $\vec{E}$ (بتوافقه)



بعدها شحنة الموجبة بتتحرك لتصير ع الـ $\vec{E}$ وتوصل للسطح الخارجي



↳ called polarized dielectric *

[لو صار break down صعب نرجعه للوضع الطبيعي [في عوامل للـ break down]]

$\vec{P} \neq 0$ [$\vec{P}$ (polarized Dielectric)] $\vec{P} \neq 0$

← باكاليش المادة صارت قطبية [polarized] طرف موجب وطرف سالب.

$$\boxed{\bar{P} = \lim_{\Delta V \to 0} \frac{\sum_{k=1}^{N} \bar{P}_k}{\Delta V}}$$

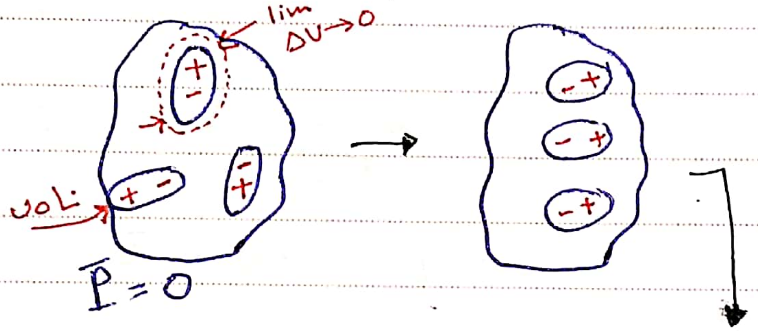
$\bar{P}_k$ → dipole moment (C·m)

ΔV → m^3

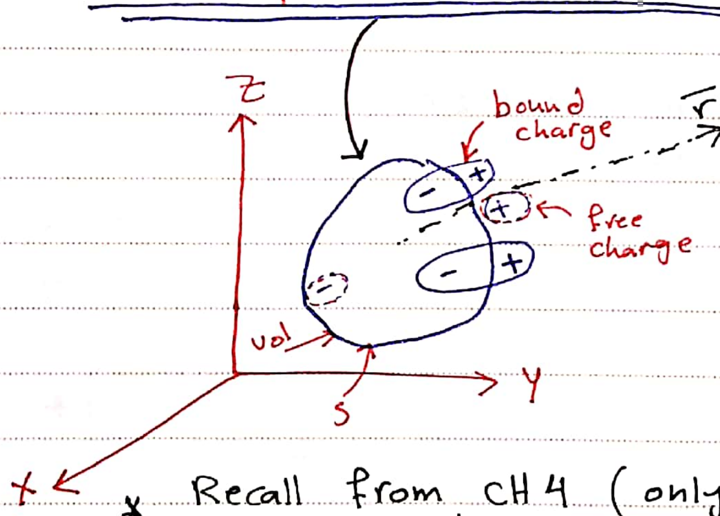
$\bar{P}$ → (C/m^2)

* Non-Polar : H_2 , O_2 , N_2 , rare gases (Kr, Xe, Ne)

* Polar : (HCL , H_2O , NH_3) (Has $\bar{P}$)



* For a polarized Dielectric :



field

V ?

$\bar{E} = -\nabla V$?

$\bar{D} = \epsilon \bar{E}$?

* Recall from CH 4 (only for $\boxed{\text{free}}$ charges)

$$V = \int_S \frac{\rho_s \, ds}{4\pi \epsilon_0 r} \quad , \quad V = \int_V \frac{\rho_v \, dv}{4\pi \epsilon_0 r}$$

→ For surface Charge

→ for volume Charge

Scanned with CamScanner

* For bound or Polarized charges :

$$V = \int_S \frac{\rho_{ps}\, ds}{4\pi \epsilon_o r} \quad , \quad V = \int_V \frac{\rho_{pv}\, dv}{4\pi \epsilon_o r}$$

field – location on the surface

ρ_{ps} : Polarized (bound) surface charge density (C/m^2)
ρ_{pv} : " " Volume " " (C/m^3)

$$\bar{E} = \int_S \frac{\rho_s\, ds}{4\pi \epsilon_o r^2}\, \hat{a_r} \qquad \bar{E} = \int_V \frac{\rho_v\, dv}{4\pi \epsilon_o r^2}\, \hat{a_r}$$

$$\bar{E} = \int_S \frac{\rho_{ps}\, ds}{4\pi \epsilon_o r^2}\, \hat{a_r} \qquad \bar{E} = \int_V \frac{\rho_{pv}\, dv}{4\pi \epsilon_o r^2}\, \hat{a_r}$$

or $\bar{E} = -\nabla V$, $\bar{D} = \epsilon_o \bar{E}$

For some dielectrics (linear) :

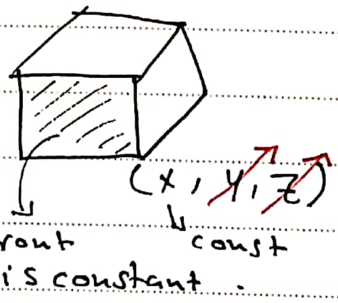
$$\bar{P} = \chi_e \epsilon_o \bar{E} \quad , \quad \chi_e \Big|_{\text{Air cond.}} = 0$$

χ_e → Electrical suscepatability (constant for each material)

* $\rho_s = \bar{D} \cdot \hat{a_n}$
* $\bar{D} = \rho_s \hat{a_n}$

* $\boxed{\rho_{ps} = \bar{P} \cdot \hat{a_n}}$

$\bar{P} = ?$



Electric Flux Density

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P} \longrightarrow \text{only exists on Dielectrics}$$

$$\bar{D}\Big|_{\substack{\text{dielectric} \\ \boxed{\epsilon_r > 1}}} > \bar{D}\Big|_{\substack{\text{Air cond.} \\ \boxed{\epsilon_r = 1}}}$$

$$\bar{D} = \epsilon_0 \bar{E} + \chi_e \epsilon_0 \bar{E}$$

$$= (1 + \chi_e)\, \epsilon_0 \bar{E}$$

$$= \epsilon_r \epsilon_0 \vec{E}$$

$\Rightarrow \bar{D} = \epsilon \vec{E}$, $\epsilon = \epsilon_0 \epsilon_r$

ϵ: F/m, ϵ_0: F/m, $\epsilon_r \rightarrow$ unitless

ϵ_r : relative Permittivity

so $\Rightarrow$ $\boxed{\bar{D} = \epsilon \bar{E}}$, $\boxed{\epsilon_r = \frac{\epsilon}{\epsilon_0}}$, $\boxed{\epsilon_r = 1 + \chi_e}$

$\boxed{P_K = (\epsilon_r - 1)\, \epsilon_0 \bar{E}}$ $\Rightarrow$ إذا أعطى χ_e بالسؤال

---

* Bound charge : Q_b in (C)

$$Q_{b+} = \int_s \rho_{ps}\, ds$$

$$Q_{b-} = \int_v \rho_{pv}\, dv$$

$$Q_b = Q_{b+} + Q_{b-} \; \begin{cases} \text{if} \\ = 0 \\ \Rightarrow \text{electrically Neutralized} \end{cases}$$

* Dielectric Breakdown :

→ Factors affects the ~~dis~~ dielectrics :

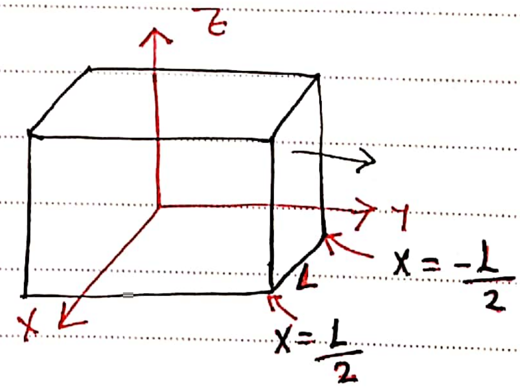
1. Nature of the material .
2. Temperature $(T\downarrow, \sigma\uparrow)$
3. Humidity $(H\uparrow, \sigma\uparrow)$
4. The applied E-field $(\rightarrow \uparrow, \sigma\uparrow)$
5. Time of the field is applied $(\rightarrow \uparrow, \sigma\uparrow)$

* Dielectric Strength :- The maximum value of E-field the dielectrics can tolerate without being breaking down .

Ex :- A dielectric cube of length (L) centered at the origin has $\bar{P} = a\bar{r}$ where (a) is constant and $\bar{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$

Find all bound charge densities and the total charge if $\rho_{PV} = -3a$ c/m^3

Sol :-



$$\rho_{PS} = \bar{P} \cdot \hat{a}_n$$

$$\rho_{PS\,\text{front}\,x=\frac{L}{2}} = \cancel{a x \hat{a}_x \cdot \hat{a}_x}$$

$$\hookrightarrow a\,x\;\hat{a}_x \cdot \hat{a}_x$$

$$= \boxed{\frac{aL}{2} \; c/m^2}$$

$$\rho_{PS\,\text{back}\,x=-\frac{L}{2}} = a\,x\;\hat{a}_x \cdot -\hat{a}_x = \boxed{\frac{aL}{2} \; c/m^2}$$

→ All surfaces have $\rho_{PS} = \frac{aL}{2}$ c/m^2

$$Q_{b(\text{total})} = Q_{b+} + Q_{b-} \quad , \; Q_{b+} = \oint_s \rho_{PS}\, ds$$

$$\hookrightarrow \; = 6\int_s \rho_{PS}\, ds \; , \; \text{Area} = L^2 \; (\text{integration})$$

$$= 6\,a\,\frac{L}{2}\,L^2 = \underline{3\,a\,L^3 \; C}$$

→

$$Q_b = \int_V \rho_{pv}\, dv$$

$$= -3aL^3 \quad C$$

$\Rightarrow Q_{b\,total} = 0$ # Neutarilized

Continuity Equation :

معادلة الاستمرارية

تتحدث عن استمرارية حركة الشحنات داخل المادة والتيار المتكون عن حركة هذه الشحنات

جميع أنواع المواد [freespace , good conductor , dielectric] جميعها

تحتوي على شحنات موجبة وسالبة متساوية و total charge=0 وهذه الشحنات اما يتكون

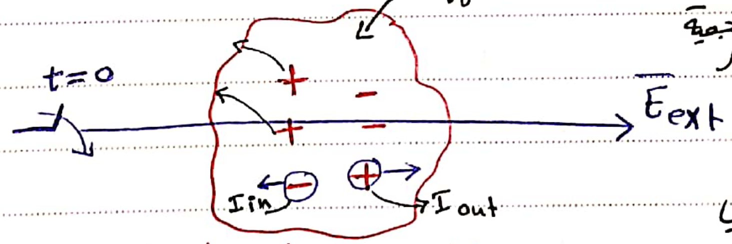
حركة الحركة أو غير قادرة على الحركة بحرية بسبب تأثير المجال الخارجي أو أنها بتحتوي على

dipoles بالبداية أو احتوت على dipoles بعدين [المهم انها مادة متعادلة كهربائياً Qtotal=0]

ρ_{v_0} at $t=0^-$ (قبل تسليط $\bar{E}$)

كان عدد الشحنات الحجمية في المادة قيمتها ρ_{v_0} وقيمة ρ_{v_0} ستقل بعد $t=0$ لأن عدد من هاي الشحنات ستظهر على السطح الخارجي

before $t=0$
no $\bar{E}$
& after $t=0$
$\bar{E}$ exist



الشحنات الموجبة بتطلع للسطح الخارجي وحركتها بتشكل Iout

والشحنات السالبة بتضل داخل المادة وحركتها بتشكل I in

$$\left\{\begin{array}{l} \boxed{t<t_0 \ (t=0^-)} \rightarrow \rho_v = \rho_{v_0} \\ \boxed{t=t_0^+ \ (t=0^+)} \rightarrow \rho_v < \rho_{v_0} \end{array}\right.$$

KCL :

حركة التيار على السطح الخارجي ← $I_{out} = I_{in}$ → حركة التيار داخل الـ material

closed surface كل يعبر فيه تيار ←

$$\oint_S \bar{J}\cdot\bar{ds} = -\frac{dq_{in}}{dt} = -\frac{d}{dt}\int_V \rho_v\, dv = -\int_V \frac{d\rho_v}{dt}\, dv$$

تناقص ← [the charges inside the volume are decreasing]

الشحنات التي بتثبت جوا المادة هي الشحنات السالبة والموجبة طلعت للسطح

$$\Rightarrow \boxed{\oint_S \bar{J}\cdot\bar{ds} = -\int_V \frac{d\rho_v}{dt}\, dv}$$

→ continuity Equation in integral form .

⇒ to calculate the $\boxed{\text{Relaxation time}}$ from the continuity Equation :-

$$\oint_S \bar{J} \cdot \bar{ds} = -\int_V \frac{d\rho_v}{dt} dv = -\frac{d Q}{dt} = -\frac{d}{dt} \oint_S \bar{D} \cdot \bar{ds}$$

$\int_V \rho_v dv$

Q according to Gauss's law

$$\boxed{\bar{J} = -\frac{d\bar{D}}{dt}}$$

→ in time varying fields (CH 9).

→ for relaxation time:-

$$\boxed{T_r = \frac{\epsilon}{\sigma} = \frac{\epsilon_o \epsilon_r}{\sigma}}$$

→ comes from:- $\bar{J} = -\frac{d\bar{D}}{dt}$

$$\rightarrow \sigma \bar{E} = -\frac{d \epsilon \bar{E}}{dt}$$

$$\rightarrow \int \frac{d\bar{E}}{E} = -\int \frac{\sigma}{\epsilon} dt$$

$$\ldots\ldots \rightarrow T_r = \frac{\epsilon}{\sigma}$$

ex!:- For copper $\epsilon_r = 1$, $\sigma = 5.8 * 10^7$ S/m

$$\rightarrow T_r = \frac{\epsilon}{\sigma} = \frac{\epsilon_o \epsilon_r}{\sigma} = \frac{\frac{10^{-9}}{36\pi} * 1}{5.8 * 10^7} = \boxed{1.53 * 10^{-19} \text{ S}}$$

→ within this time that means:-

after 1 T_r → 36.8% of charges will remain inside the material & 63.2% will go to the outer surface.

1 T_r 36.8% → 63.2%

the lower level of a material to be conducting.

* if we take 5 time constant ($5T_r$)

→ the percent inside the material will be less than 1% and more than 99% on the outer surface so the material will be a good conductor.
(copper)

Scanned with CamScanner

ex2 : For Fused Quartz $\epsilon_r = 5$, $\sigma = 10^{-17}$ S/m

↳ $T_r = \dfrac{\frac{10^{-9}}{36\pi} * 5}{10^{-17}} = \boxed{51.2 \text{ days}}$

→ means : to have 63.2% of charges on the outer surface of a material (fused Quartz) needs 51.2 days .

↳ this material (fused Quartz) is super dielectric material and most likely this material will be breaking down before this # of charges appear on the outer surface

⇒ So Tr (relaxation time) is important to know how the material is good conductor or good dielectric .

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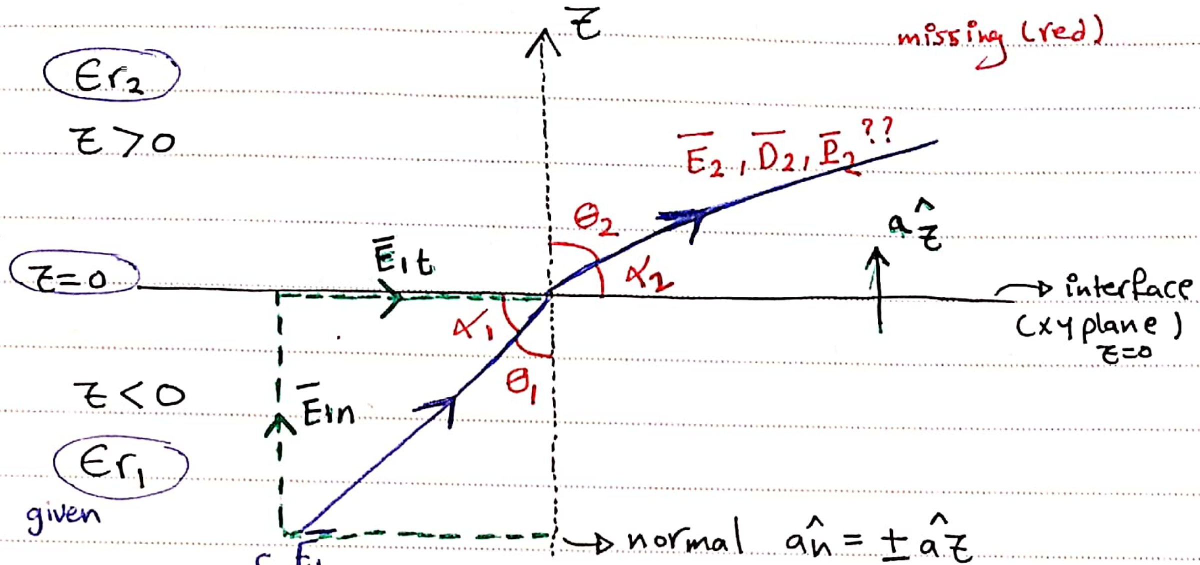
Boundary Conditions (B.C)

نظرية الحدود

شو بصير لل $\bar{E}$ لما ينتقل من وسط لوسط مختلف بإخصائص

A) Dielectric – to – Dielectric



one of these given ← $\begin{cases} \bar{E}_1 \\ \bar{D}_1 = \epsilon_0 \epsilon_{r_1} \bar{E}_1 \\ \bar{P}_1 = \chi_e \epsilon_0 \bar{E}_1 = (\epsilon_{r_1} - 1)\epsilon_0 \bar{E}_1 \end{cases}$

ϵ_{r_1} & $\epsilon_{r_2} > 1$ [so non of these material is free space]

⇒ Region 1 components $\bar{E}_1$:– → given

→ $\bar{E}_1 = \bar{E}_{1n} + \bar{E}_{1t}$

$\bar{E}_1$ normal parallel to normal ←   → $\bar{E}_1$ tangent parallel to surface.

$\hat{a_n} = \hat{a_z}$

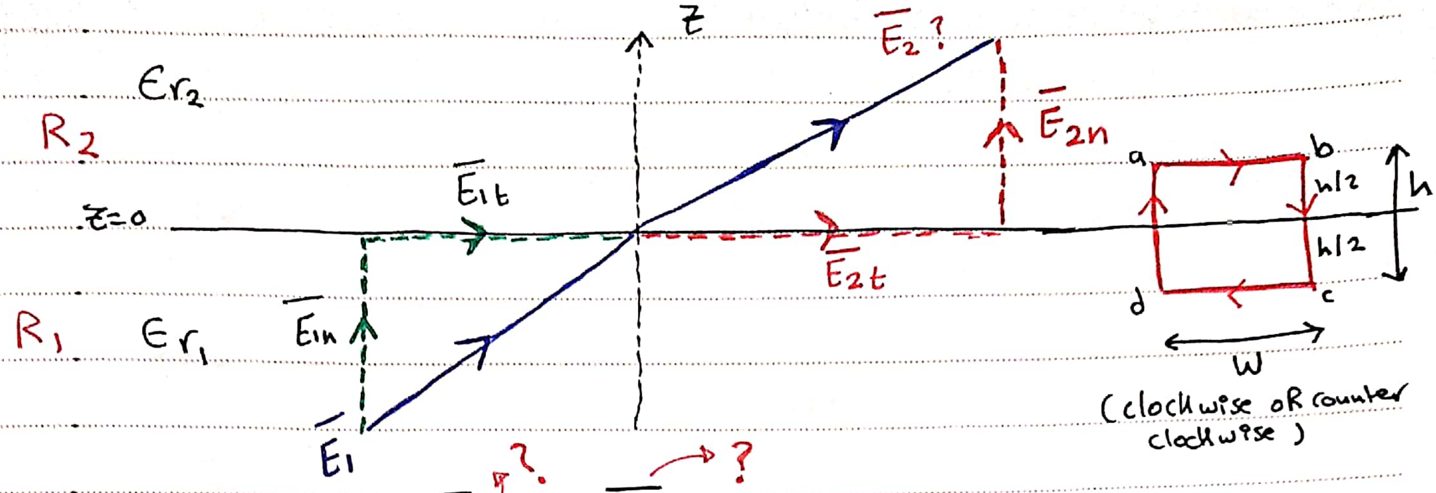
→ $\bar{E}_{1n} = \underbrace{(\bar{E}_1 \cdot \hat{a_n})}_{\text{scaler}} \hat{a_n}$] → vector

→ $\bar{E}_{1t} = \bar{E}_1 - \bar{E}_{1n}$ E_{1t}

→ $\theta_1 = \sin^{-1}\left(\frac{|\bar{E}_{1t}|}{E_1}\right) = \cos^{-1}\left(\frac{E_{1n}}{E_1}\right) = \tan^{-1}\left(\frac{E_{1t}}{E_{1n}}\right)$

→ $\alpha_1 = 90° - \theta_1$

⇒ Region 2 $\bar{E}_2$ field :-



$$\bar{E}_2 = \overset{?}{\bar{E}_{2n}} + \overset{?}{\bar{E}_{2t}}$$

[1'st & 2'nd] Maxwell's eq. at $z=0$ (~~interface~~), this eq. applied on interface $z=0$.

first maxwell equ. ← $\oint_s \bar{D}\cdot\bar{ds} = Q_{enc.} = \int_s \rho_s \, ds$

↳ point / line / surface / volume

↳ since we need apply this eq in 2D surface

second maxwell equ. ← $\oint_L \bar{E}\cdot\bar{dL} = 0$

↳ apply 2'nd maxwell equ first [make closed path around the interface].

$$\oint_L \bar{E}\cdot\bar{dL} = \int_a^b \bar{E}\cdot\bar{dL}_1 + \int_b^c \bar{E}\cdot\bar{dL}_2 + \int_c^d \bar{E}\cdot\bar{dL}_3 + \int_d^a \bar{E}\cdot\bar{dL}_4 = 0$$

R2   R2   R1   R1   R1   R2

$$= E_{2t}W - E_{2n}\cancel{\frac{h}{2}} - E_{1n}\cancel{\frac{h}{2}} - E_{1t}W + E_{1n}\cancel{\frac{h}{2}} + E_{2n}\cancel{\frac{h}{2}} = 0$$

always $\boxed{\bar{E}_{2t} = \bar{E}_{1t}}$ → the tangential component of E-field must be continuous

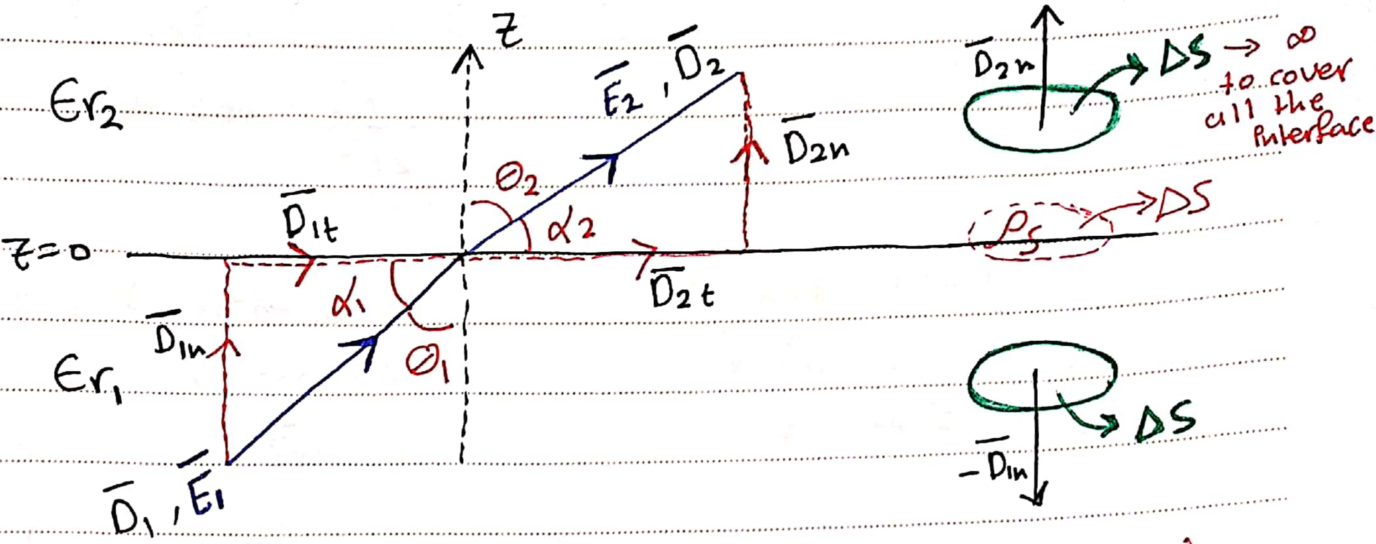
$\bar{E}_{2t}$: $z=0^+$ (above the interface)

$\bar{E}_{1t}$: $z=0^-$ (below the interface)

[below and above the interface $\bar{E}$ will not change but the change will be appeared in the normal component]

⇒ to find E_{2n} :

$\oint_s \bar{D} \cdot \bar{ds} = \int_s \rho_s \, ds$] applied 1'st maxwell eq. (Gauss's law) in interface $z=0$



2 Gaussian surface should be located above and below the interface, because if there ρ_s a charge this charge should be radiated upward or downward the interface, why? this charge (ρ_s) should be located on the whole interface (infinit in x & y direction), we don't need the other side faces because all of these faces are located at ∞.

$$\oint_s \bar{D} \cdot \bar{ds} = \int_s \rho_s \, ds$$

$$\int_{S_{top}} \bar{D} \cdot \bar{ds}_{top} + \int_{S_{bot}} \bar{D} \cdot \bar{ds}_{bot} = \int_s \rho_s \, ds$$

$$D_{2n} \cancel{\Delta S} - D_{1n} \cancel{\Delta S} = \rho_s \cancel{\Delta S}$$

$\boxed{D_{2n} - D_{1n} = \rho_s}$ → if there is a charge (ρ_s) at the surface.

field in new Region − field in the original region

$D_{2n} - D_{1n}$

(راجعين من منطقة 1 لـ منطقة 2)

$\Rightarrow$ if $\rho_s = 0$ $\rightarrow$

$$\boxed{\overline{D_{2n}} = \overline{D_{1n}}}$$ $\Rightarrow$ if there is no charge on surface.

$\Rightarrow \epsilon_0 \epsilon_{r_2} \overline{E_{2n}} = \epsilon_0 \epsilon_{r_1} \overline{E_{1n}}$

$$\Rightarrow \boxed{\overline{E_{2n}} = \frac{\epsilon_{r_1} \overline{E_{1n}}}{\epsilon_{r_2}}}$$

if diff. region, diff. ϵ_r only normal component of $\overline{E}$ will be diff & $\overline{E_{2t}} = \overline{E_{1t}}$

if same Region $\epsilon_{r_1} = \epsilon_{r_2}$

$\hookrightarrow \overline{E_{2n}} = \overline{E_{1n}}$ & $\overline{E_{2t}} = \overline{E_{1t}}$

$\hookrightarrow$ so the $\overline{E}$ will be same

$\hookrightarrow$ So $$\boxed{\overline{E_2} = \overline{E_{2n}} + \overline{E_{2t}}}$$

$$\boxed{\overline{D_2} = \epsilon_0 \epsilon_{r_2} \overline{E_2}}$$

$$\boxed{\overline{P_2} = (\epsilon_{r_2} - 1) \epsilon_0 \overline{E_2}}$$

$$\boxed{\theta_2 = \sin^{-1}\left(\frac{E_{2t}}{E_2}\right)}, \quad \boxed{\alpha_2 = 90^\circ - \theta_2}$$

$\hookrightarrow$ OR $\overline{E_{1t}} = \overline{E_{2t}}$ $\rightarrow$ from $\theta_2 = \sin^{-1}\left(\frac{E_{2t}}{E_2}\right)$

$E_1 \sin\theta_1 = E_2 \sin\theta_2$ --- (1)

if $\rho_s = 0$

$D_{2n} - D_{1n} = 0 \rightarrow \overline{D_{1n}} = \overline{D_{2n}}$

$\epsilon_0 \epsilon_{r_1} \overline{E_{1n}} = \epsilon_0 \epsilon_{r_2} \overline{E_{2n}}$

$\epsilon_{r_1} E_1 \cos\theta_1 = \epsilon_{r_2} E_2 \cos\theta_2$ ---- (2)

divide eq (1) by eq (2)

$$\frac{\tan\theta_1}{\epsilon_{r_1}} = \frac{\tan\theta_2}{\epsilon_{r_2}}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r_1}}{\epsilon_{r_2}}}$$

$\rightarrow$ only if $\rho_s = 0$

$\rightarrow$ we can find θ_2 directly without finding E_2 component (E_{2n} OR E_{2t})

B) conductor – to – Dielectric

→ very large conductivity so this region is perfect conductor.

$\sigma_2 \approx \infty$

(conductor) $\epsilon_{r_2} = 1$.

$\bar{E}_2 = 0$ → inside the conductor

↳ $\bar{E}_{2t} = 0$ & $\bar{E}_{2n} = 0$ [no field in region 2]

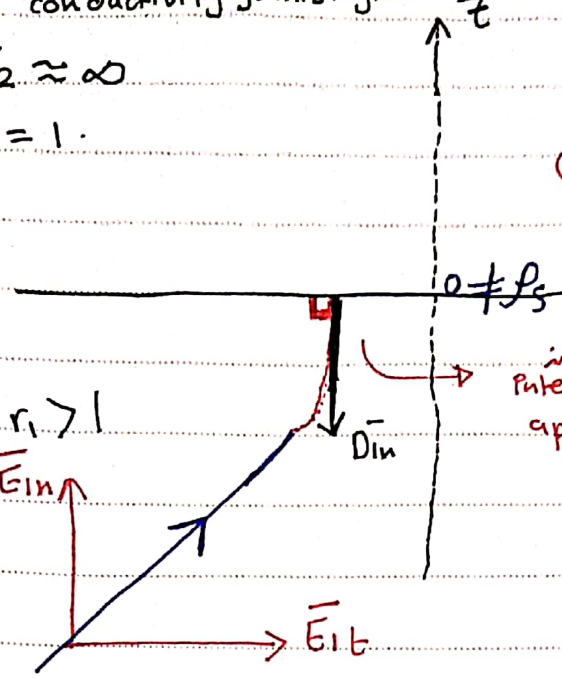
↳ $\bar{D}_2 = 0$

↳ $\bar{D}_{2t} = 0$ & $\bar{D}_{2n} = 0$

↳ so θ_2 / α_2 doesn't exist.

$z = 0$ $\quad 0 \neq \rho_s$

(dielectric) $\epsilon_{r_1} > 1$



لما يقترب من interface only normal component appear but tangent component = 0.

$\bar{E}_{1t} = \bar{E}_{2t} \Rightarrow \boxed{\bar{E}_{1t} = 0}$ at interface $z = 0$

(befor reach interface $\bar{E}_{1t}$ exist)

$\boxed{\rho_s \text{ has value } \neq 0}$

$\overset{0}{D_{2n}} - \overset{\neq 0}{D_{1n}} = \overset{\neq 0}{\rho_s}$

$D_{1n} = -\rho_s$

$\bar{D}_{1n} = \rho_s(-\hat{a_n})$

$\boxed{\bar{E}_{1n} = \dfrac{-\rho_s \hat{a_n}}{\epsilon_0 \epsilon_{r_1}}}$

↳ E_1 has only normal component without any tangent component

$\bar{D}_{1n}$ → (لما يقترب من ال interface يسقط بشكل عامودي وينعكس بشكل عامودي مؤقتاً لبعدما يرجع عادي)

↳ $\boxed{\begin{array}{l} \theta_1 = 0 \\ \alpha_1 = 90^\circ \end{array}}$

→ if Region 1 is free space (conductor to free space)

↳ $\epsilon_{r_1} = 1$ → $\bar{E}_{1t} = 0$

→ $\bar{E}_{1n} = \dfrac{-\rho_s \hat{a_n}}{\epsilon_0}$

Ex 5.9 in text book

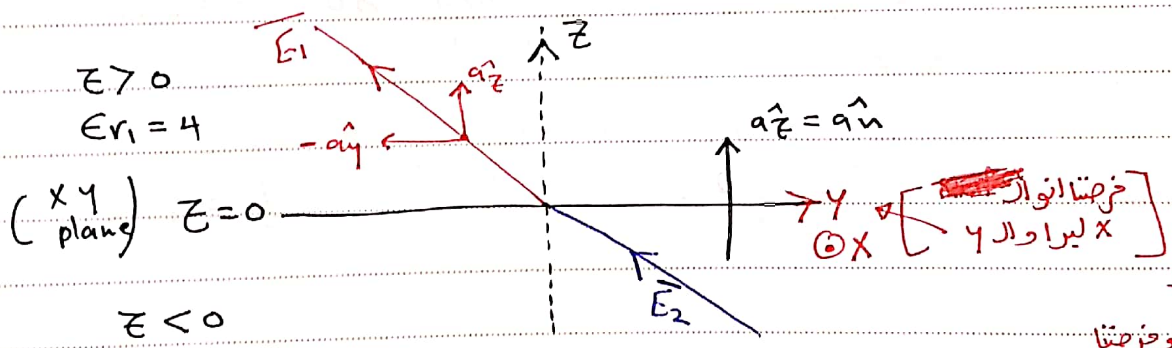
Two homogeneous isotropic dielectrics meet on plane $z=0$. for $z>0$, $\varepsilon_{r_1}=4$ and for $z<0$, $\varepsilon_{r_2}=3$, If $\bar{E}_1 = 5\hat{a_x} - 2\hat{a_y} + 3\hat{a_z}$ K V/m exists for $z \geq 0$, find:

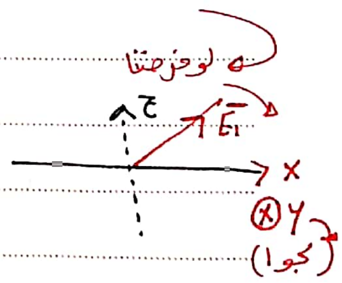
(a) $\bar{E}_2$ for $z \leq 0$?

* homogeneous $\rightarrow$ ε_r is constant foreach point (any where) in Region.
* isotropic $\rightarrow$ the permittivity is not change with direction
  $\hookrightarrow$ ε is not functional direction.

sometimes linear $\rightarrow$ ε is not a function of electric field.

ans :- $\hat{a_n} = \hat{a_z}$ [$\bar{E}$ field going from R2 to R1]





بترع تكون $\bar{E}_2$ بالمنطقة المقابلة لل $\bar{E}_1$
مش تحتها لانه هذا شكل انكاسرش اشعاع

$\rightarrow$ $\bar{E}_{1n} = (\bar{E}_1 \cdot \hat{a_n})\hat{a_n} = 3\hat{a_z}$ K V/m

$\bar{E}_{1t} = \bar{E}_1 - \bar{E}_{1n} = 5\hat{a_x} - 2\hat{a_y}$ K V/m $= \bar{E}_{2t}$

$\rho_s = 0$, why? $\boxed{1}$ not mentioned in question

$\boxed{2}$ said dielectric (the charge inside dielectrics will need large relaxation time to reach the surface so between 2 dielectrics no charge in the interface.

$D_{2n} - D_{1n} = \rho_s = 0$

$\hookrightarrow \bar{E}_{2n} = \frac{\epsilon_{r1}}{\epsilon_{r2}} \bar{E}_{1n} = \frac{4}{3} 3\hat{a}_z = 4\hat{a}_z$ KV/m

$\Rightarrow \bar{E}_2 = \bar{E}_{2n} + \bar{E}_{2t} = \boxed{5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z}$ KV/m

$\hookrightarrow$ difference only in the normal component.

(b) The angles E_1 & E_2 makes with the $\boxed{\text{interface}}$?

$\hookrightarrow \alpha_1$ & α_2 ??

$\hookrightarrow \theta_1 = \sin^{-1} \frac{E_{1t}}{E_1}$

* (angles makes with the $\boxed{\text{normal}}$ $\rightarrow \theta_1$ & θ_2)

$= \sin^{-1}\left(\frac{\sqrt{29}}{\sqrt{38}}\right) = 60.9°$

$\alpha_1 = 90° - \theta_1 = \boxed{29.1°}$

$\theta_2 = \cdots ?$ OR $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}} = \frac{4}{3} \Rightarrow \theta_2 = 53.4°$

$\Rightarrow \alpha_2 = \boxed{36.6°}$

(c) The energy densities in both dielectrics ?

$W_{E1} = \frac{1}{2}\epsilon_1 E_1^2 = \frac{1}{2}\epsilon_0 \epsilon_{r1} E_1^2 = \frac{1}{2} \frac{10^{-9}}{36\pi}(4)(\sqrt{38}\,k)^2$ $\rightarrow 38 * 10^6$

$= 672\,\mu$ J/m³

$W_{E2} = \frac{1}{2}\epsilon_0 \epsilon_{r2} E_2^2 = \frac{1}{2}\frac{10^{-9}}{36\pi}(3)(\sqrt{45}\,k)^2$ $\rightarrow 45 * 10^{-6}$

$= 597\,\mu$ J/m³

$|E_2| = \sqrt{45}$ KV/m & $|E_1| = \sqrt{38}$ KV/m .

$W_{E1} > W_{E2}$ since $\epsilon_{r1} > \epsilon_{r2}$.

(d) The energy within a cube of side 2m centered at (3, 4, −5) → total energy (not energy density)

→ we need to know at what region the cube is exist.

$$W_E = \int_V w_{E_2} \, dv$$

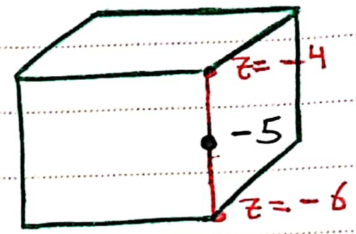
R(1) $z > 0$

$$= \underbrace{597 \times 10^{-6}}_{w_{E2} * V} \times \underbrace{8}_{\text{volum}}$$

$z = 0$

$$= 4.776 \text{ mJ}$$

R(2) $z < 0$



so cube in Region 2

if the cube is centered at the origin

$$\hookrightarrow W_E = \underbrace{W_{E1} * 4}_{\text{half cube in } R_1} + \underbrace{W_{E2} * 4}_{\text{half cube in } R_2}$$