

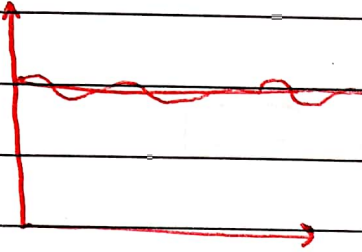
# CIRCUITS

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BY:KHALID ALNASER



$$i = \frac{\partial Q}{\partial t} \rightarrow \int_{t_0}^{t_1} i dt = Q$$

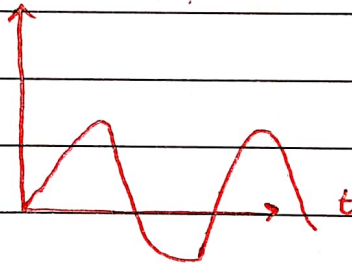
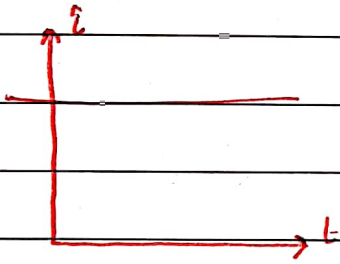


DC current

$I$

ac current

$i(t)$



يكون اتجاه التيار خارج البطارية من الموجب للسالبي (عكس اتجاه  $E$ )

Exemple 1.2.8

$$q = 5t \sin(4\pi t) \text{ m C}$$

$$i = \frac{\partial q}{\partial t} = \left[ \underset{\substack{\uparrow \\ 0.5}}{5t} \cdot \underset{\substack{\uparrow \\ 0.5}}{(4\pi)} \cos(4\pi t) + \underset{\substack{\uparrow \\ 0.5}}{\sin(4\pi t)} \cdot 5 \right] \text{ mA}$$

$$= [2.5 \times 4] \pi \times 1 + 0$$

$$= \boxed{10\pi \text{ mA}}$$

Example 1.3

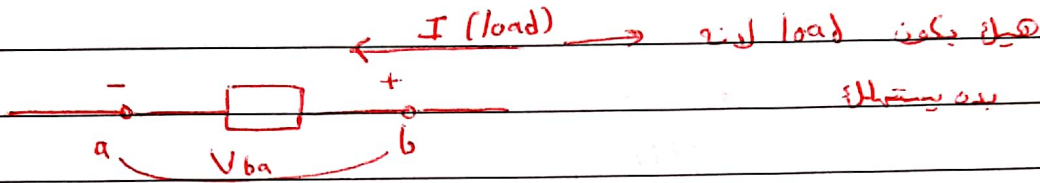
$$i = 3t^2 - t$$

$$t_2 = 2 \text{ s}$$

$$Q = \int_{t_1=1\text{s}}^{t_2=2\text{s}} (3t^2 - t) dt$$

$$= \left[ t^3 - \frac{1}{2}t^2 \right]_1^2 = [8 - 2] - [1 - \frac{1}{2}] = 6 - 0.5 = \boxed{5.5 \text{ C}}$$

\*

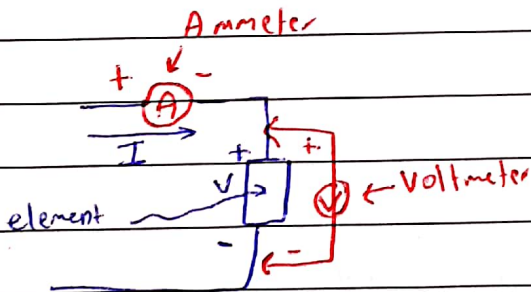


$$V_{ba} = V_b - V_a \quad \xrightarrow{I} \text{(Source)}$$

$$V_{ba} = V_b - V_a > 0$$

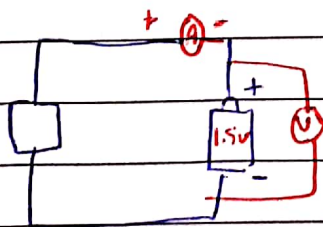
$$V_{ab} = V_a - V_b \quad \leftarrow \text{عكس جهت}$$

12/2/2020



at  $t_0, t_1$

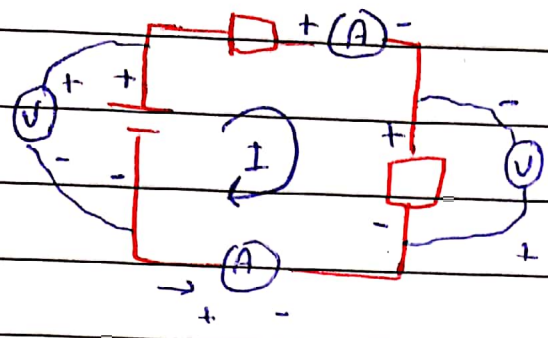
$$\left. \begin{aligned} V(t_1) &= 5 \text{ V} \\ i(t_1) &= 3 \text{ A} \end{aligned} \right\} P(t_1) = 5 \times 3 = 15 \text{ W} > 0 \quad \text{receiving} \\ \text{absorbing}$$



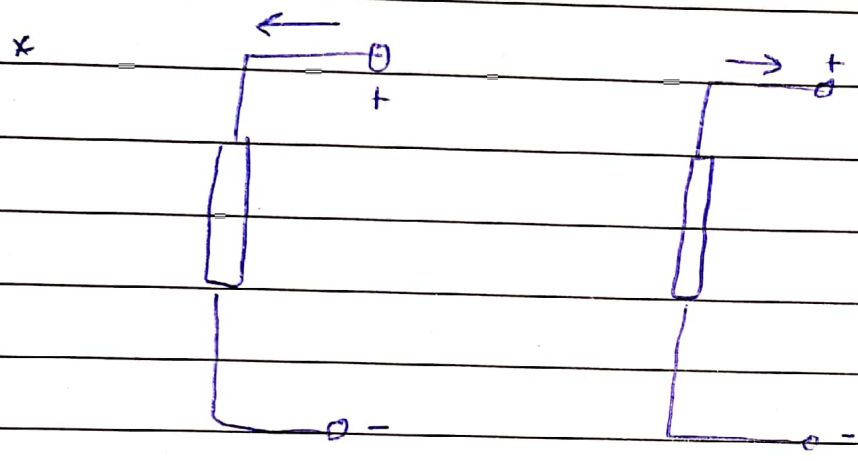
$$\left. \begin{aligned} V &= 1.5 \text{ V} \\ A &= -0.5 \text{ A} \end{aligned} \right\} P = IV = 1.5 \times -0.5 = -0.75 \text{ W} \\ \text{supplying}$$

Voltage → Value + Polarity  
Currents → Value + Direction

موجب



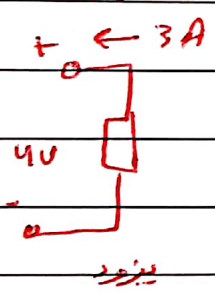
سالب



سالب

سالب

Figure 1.10



$$P = 3 \times 4 = -12$$

سالب

\*  $1 \text{ Wh} = 3600 \text{ J}$

Example 1.4

$$W = 2.3 \text{ kJ}$$

$$t = 10 \text{ s}$$

$$P = \frac{W}{t} = \frac{2.3}{10} = 0.23 \text{ kW} = 230 \text{ W}$$

$$I = 2 \text{ A}$$

$$P = VI = \frac{230}{2} = V = 115 \text{ V}$$

Example 1.5

Example 1.6

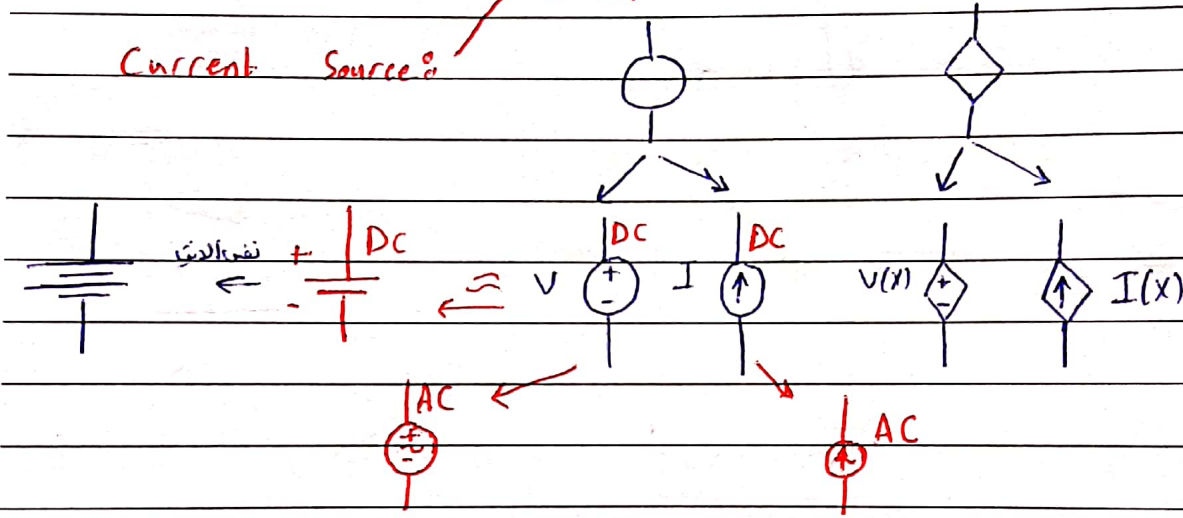
17/2/2020

### Circuit Elements

Voltage Source

Current Source

Independent or Dependent



☆

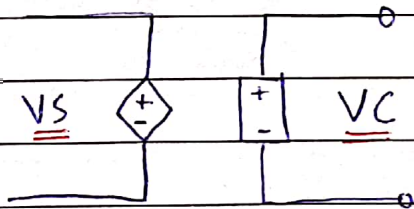
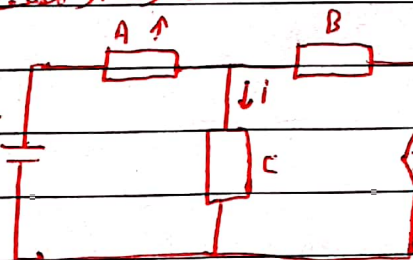


Figure 1.14

مقاومة او مكثف او حث

independent voltage source

5V

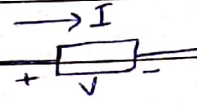
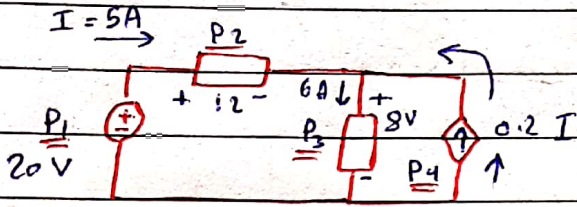


dependent voltage source

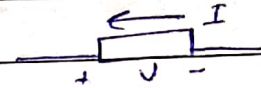
5 Elements circuit

Example 1.7

\* Note:



(absorb)  $P = IV$   
dissipator



(supply)  $P = -IV$   
Supplier

Sol:

$$P_1 = I \times V = -5 \times 20 = -100 \text{ W}$$

$$P_2 = I \times V = 5 \times 12 = 60 \text{ W}$$

$$P_3 = I \times V = 5 \times 8 = 40 \text{ W}$$

$$P_4 = I \times V = -1 \times 8 = -8 \text{ W}$$

$$\Rightarrow I = 0.2 \times 5 = 1 \text{ A}$$

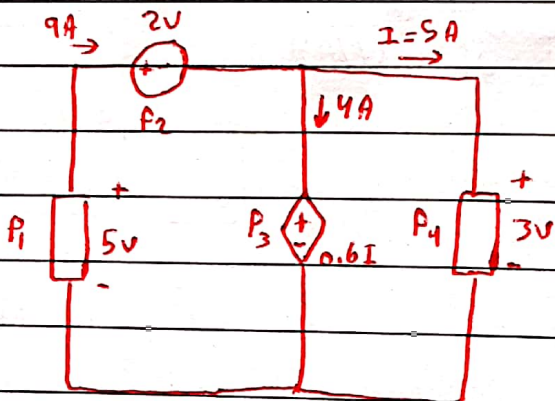
$$V = 8 \text{ V}$$

Check:

$$P_1 + P_2 + P_3 + P_4 \stackrel{?}{=} 0 \text{ W}$$

(zero) *total power*

Practice 1.7



$$P_1 = -5 \times 9 = -45 \text{ W}$$

$$P_2 = 9 \times 2 = 18 \text{ W}$$

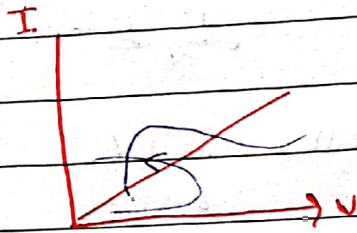
$$P_3 = 4 \times 3 = 12 \text{ W}$$

$$P_4 = 5 \times 3 = 15 \text{ W}$$

$$\Sigma = 0 \text{ W}$$

Example 1.9

19/2/2020

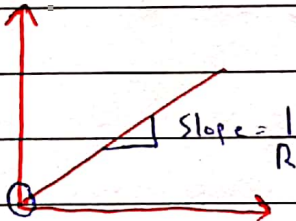
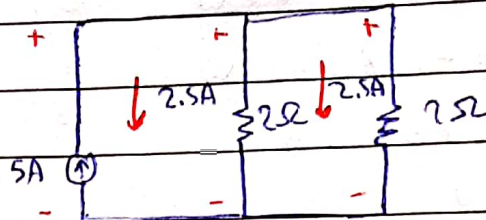
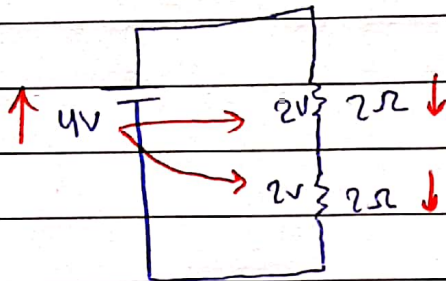


i.v characteristics إحداثيات

$$R = \frac{P \cdot L}{A} \quad (\Omega \cdot m)$$

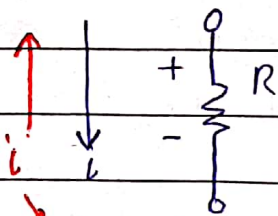
Voltage Division

Current division



$$V = R \cdot I$$

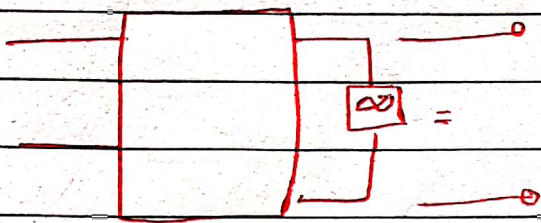
$$R = \frac{V}{I} = \frac{V/A}{I}$$



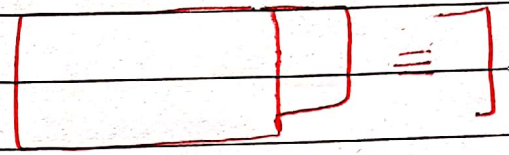
لنظم إشارة سالبة لحظتها

$$R = \frac{P \cdot L}{A} \longrightarrow G = \frac{1}{R} = \frac{1}{\frac{P \cdot L}{A}} = \frac{A}{P \cdot L} = \frac{i}{V}$$

Conductance



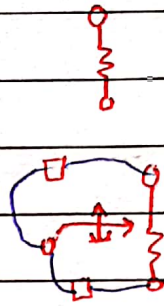
Open circuit  $\equiv$  No load



Short circuit

Resistors

- Fixed
- Variable



$$\begin{aligned}
 P &= i \cdot V \\
 &= i \cdot (i \cdot R) \\
 &= i^2 \cdot R
 \end{aligned}$$

$$\begin{aligned}
 P &= i \cdot V \\
 &= \frac{V}{R} \cdot V \\
 &= \frac{V^2}{R}
 \end{aligned}$$

always positive = passive element

Example 2.1 ✓

Practice Problem 2.1 ✓

Example 2.2 ✓

Practice Problem 2.2 ✓



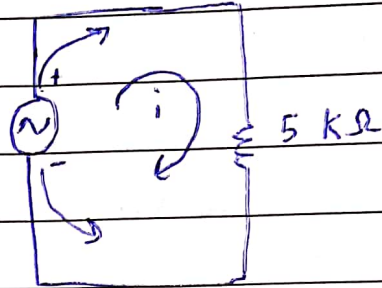
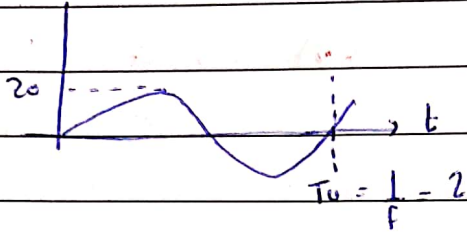
24/2/2020

Example 2.3

$$V_s = 20 \sin \pi t$$

$$\omega = 2\pi f = \pi$$

$$f = \frac{1}{T} = 0.5 \text{ Hz}$$



$$i = \frac{V}{R} = \frac{20 \sin \pi t \text{ V}}{5 \text{ k}\Omega} = 4 \sin \pi t \text{ mA}$$

$$P = i \times V$$

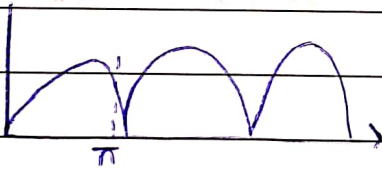
$$= \frac{V^2}{R}$$

$$= i^2 \cdot R$$

$$= 4 \text{ mA} \times 20 \text{ V} (\sin \pi t)^2$$
  
$$= 80 \sin^2 \pi t \text{ mW}$$

Note: Hz = 1/s

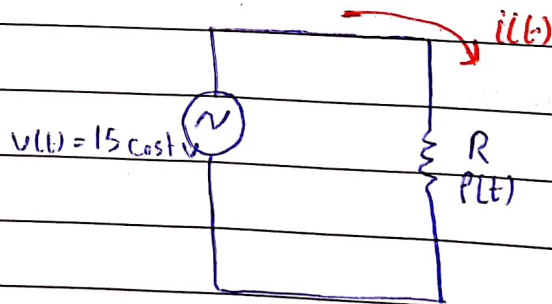
Power



Practice Problem 2.3

$$P(t) = 30 \cos^2 t \text{ (mW)}$$

$$V = 15 \cos t \text{ (V)}$$



$$i(t) = \frac{P}{V} = \frac{30 \cos^2 t \text{ mW}}{15 \cos t \text{ V}} = 2 \cos t \text{ (mA)}$$

$$R = \frac{V}{i} = \frac{V^2}{P} = \frac{(15)^2 (\cos t)^2 \text{ V}^2}{30 (\cos t)^2 \text{ mW}} = 7.5 \text{ k}\Omega$$

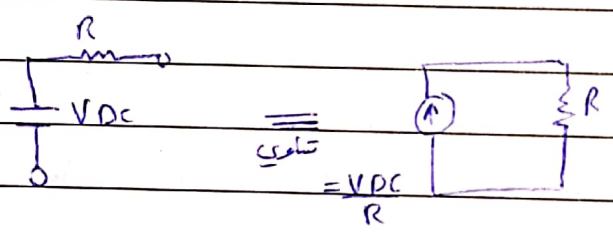
ال network يمكن يكون فيها اكثر من circuit  
 و ال circuit لوزم تكون بتلقه بين ال network من شرط  
 Element dia branch JS \*

Figure 2.10

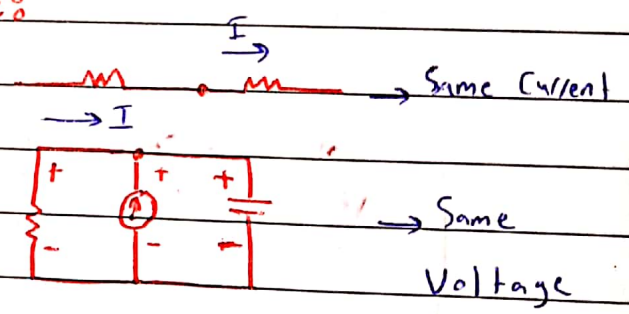
- 5 branches
- 3 nodes
- 3 loops

$$b = I + n - 1$$

\*Notes:



Notes:



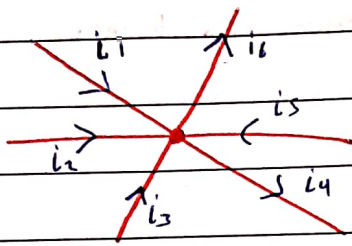
Example 2.4

- 4 branches
- 3 nodes

Practise Problem 2.4

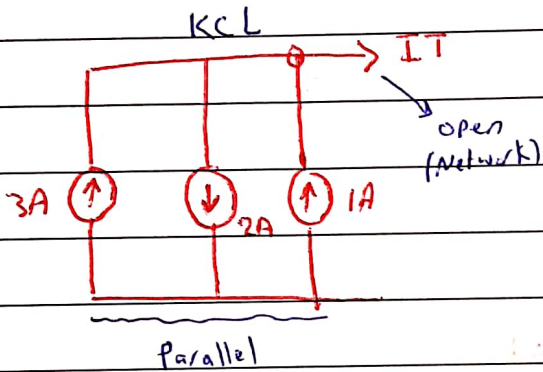
- 5 branches
- 3 nodes

26/2/2020



inward : +ve  $\equiv$  entering  
 outward : -ve  $\equiv$  leaving

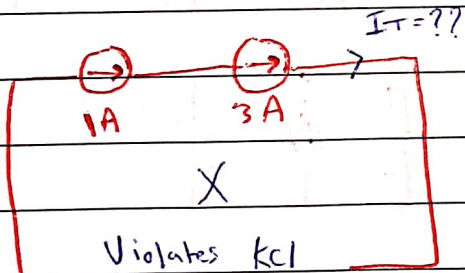
$$+i_1 + i_2 + i_3 - i_4 - i_5 - i_6 = 0$$



$$Out = I_N$$

$$I_T + 2 = 3A + 1A$$

$$I_T = 3 + 1 - 2 = \boxed{2A}$$

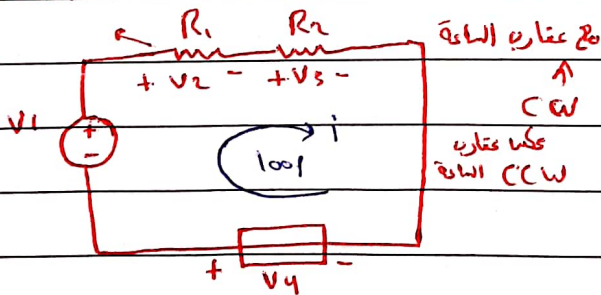


still as two different currents

Parallel is 1st basic rule to apply in circuit \*

terminal

KVL

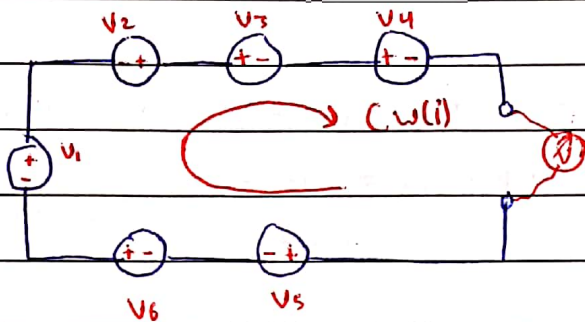


$$KVL: -V_1 + V_2 + V_3 - V_4 = 0$$

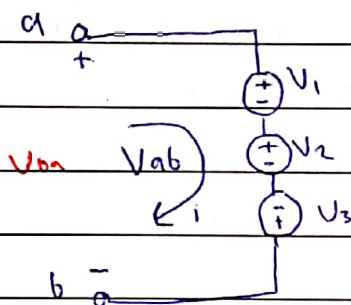
$$KVL: V_1 - V_2 - V_3 + V_4 = 0$$

$$\sum_{m=1}^M V_m = 0$$

2 terminal all elements

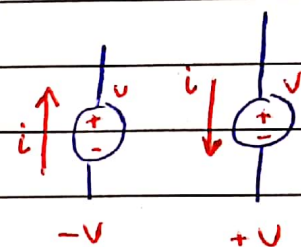


$$-V_1 - V_2 + V_3 + V_4 + V_5 - V_6 = 0 \Rightarrow V_1 + V_2 - V_3 - V_4 - V_5 + V_6$$

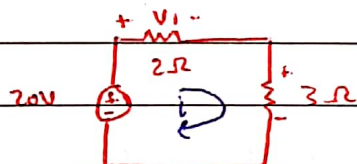


$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

$$V_{ab} = V_1 + V_2 - V_3$$



Example 2.5:



$$\text{KVL: } -20 + V_1 + V_2 = 0$$

$$\text{ohm's law: } V_1 = 2 \times i$$

$$V_2 = 3 \times i$$

$$= -20 + 2i + 3i = 0$$

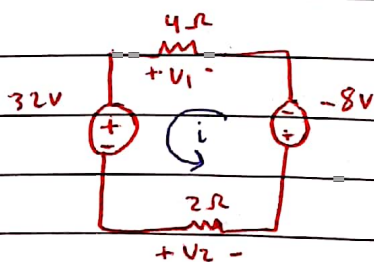
$$5i = 20$$

$$i = 4A$$

$$V_1 = 2 \times 4 = 8V$$

$$V_2 = 3 \times 4 = 12V$$

Practice Problem 2.5:



$$\text{KVL: } -8 - V_1 + 32 + V_2 = 0$$

$$\text{ohm's law: } V_1 = -4i = -4 \times -4 = 16V$$

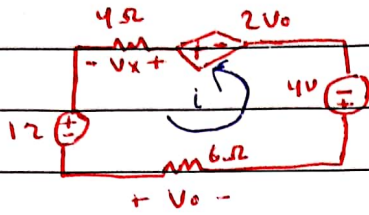
$$V_2 = +2i = 2 \times -4 = -8V$$

$$= -8 + 4i + 32 + 2i = 0$$

$$+6i = -24$$

$$i = -4A$$

Example 2.6:



$$\textcircled{1} - \text{KVL: } +12 + V_o + 4i - 2V_o + V_x = 0$$

$$\text{But } V_o = i \times 6 \text{ --- } \textcircled{2}$$

$$V_x = i \times 4 \text{ --- } \textcircled{3}$$

$$\text{Let } \textcircled{3} \text{ \& } \textcircled{2} \text{ in } \textcircled{1} \rightarrow 16 + i \times 6 - 2 \times i \times 6 + i \times 4 = 0$$

$$i(6 - 12 + 4) = -16$$

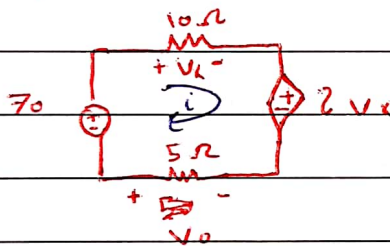
$$i(-2) = -16 \rightarrow \boxed{i = 8 \text{ A}}$$

$$V_o = 8 \times 6 = 48 \text{ V}$$

$$V_x = 8 \times 4 = 32 \text{ V}$$

$$\underline{\underline{\text{KVL}}} \quad 12 + 48 + 4 - 96 + 32 = 0 \quad \checkmark$$

Practice Problem 2.6:



$$\text{KVL: } -70 + V_x + 2V_x - V_o = 0$$

$$V_x = 10i = 20 \text{ V}$$

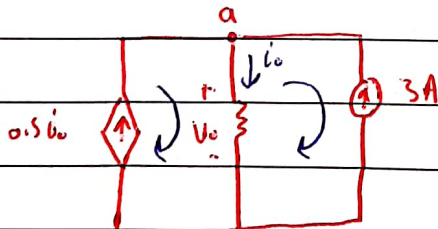
$$V_o = -5i = -10 \text{ V}$$

$$-70 + 10i + 2 \times 10i - (-5i) = 0$$

$$i(10 + 20 + 5) = 70$$

$$i = \frac{70}{35} = 2$$

Example 2.7:



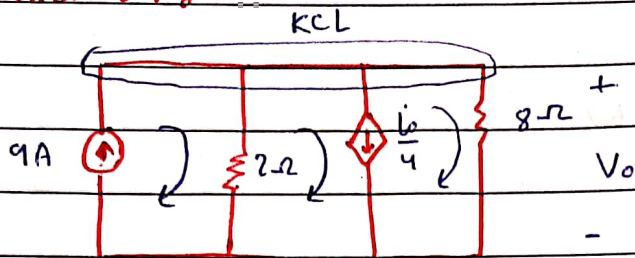
$$\text{KCL: } +0.5i_o - i_o + 3 = 0$$

$$i_o(0.5 - 1) = -3$$

$$i_o = \frac{-3}{-0.5} = 6 \text{ A}$$

$$V_o = 4 + i_o = 4 \times 6 = 24 \text{ V}$$

## Practice Problem 2.7



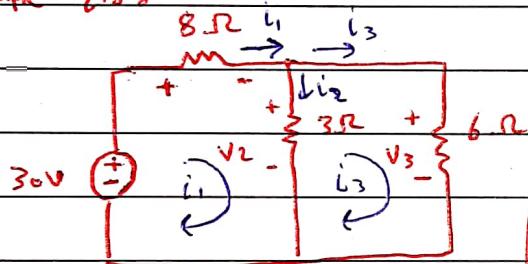
$$9 - i_o - \frac{i_o}{4} - \frac{V_o}{8} = 0 \rightarrow i_o \left( -1 - \frac{1}{4} - \frac{1}{4} \right) = -9 \rightarrow i_o (-1.5) = -9$$

$$i_o = \frac{9}{1.5} = \boxed{6A}$$

But:  $V_o = 2 * i_o = 2 * 6 = 12V$

2/3/2020

## Example 2.8



① KVL 1:  $-30 + 8i_1 + 3i_2 = 0$

② KVL 2:  $6i_3 - 3i_2 = 0$

③ KCL:  $i_1 - i_2 - i_3 = 0 \rightarrow i_2 = i_1 - i_3$

$$\rightarrow 8i_1 + 3i_2 + 0i_3 = 30 \rightarrow 8i_1 + 3(i_1 - i_3) = 30$$

$$0i_1 - 3i_2 + 6i_3 = 0 \rightarrow 11i_1 - 3i_3 = 30$$

$$= -3(i_1 - i_3) + 6i_3 = 0$$

$$= -3i_1 + 9i_3 = 0$$

$$\boxed{i_1 = 3i_3}$$

$$\parallel (3i_3) - 3i_3 = 30 \rightarrow i_3 = \frac{30}{30} = 1A$$

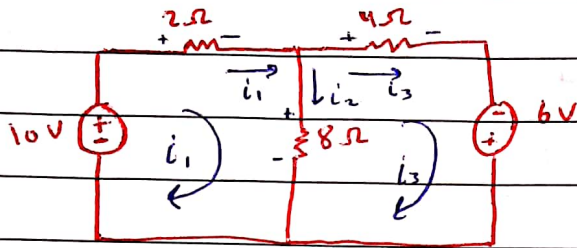
$$\therefore i_1 = 3i_3 = 3A$$

$$\therefore i_2 = i_1 - 3i_3 = 3 - 1 = 2A$$

Voltagess:  $V_1 = 8i_1 = 8 * 3 = 24V$

$$V_2 = 3i_2 = 3 * 2 = 6V$$

### Practice Problem 2.8



$$\text{KVL 1: } -10 + 2i_1 + 8(i_1 - i_3) = 0 \rightarrow i_1(2+8) - i_3(8) = 10 \rightarrow (10i_1 - 8i_3 = 10) \quad \text{①}$$

$$-8(i_1 - i_3) + 4i_3 - 6 = 0 \rightarrow -i_1(8) + i_3(4+8) = 6 \rightarrow -8i_1 + 12i_3 = 6 \quad \text{②}$$

$$8i_1 - 6.4i_3 = 8$$

$$8.6i_3 = 14$$

$$i_3 = \frac{14}{8.6} = 2.5 \text{ A}$$

$$\text{From ①} \rightarrow 10i_1 = 8i_3 + 10 = 8 \times 2.5 + 10 = 30 \rightarrow i_1 = \frac{30}{10} = 3 \text{ A}$$

$$\text{KCL: } i_1 = i_3 + i_2$$

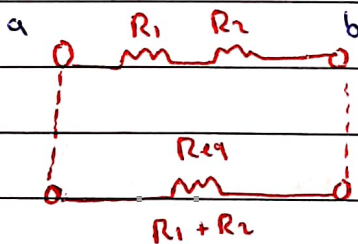
$$\therefore i_2 = i_1 - i_3 = 3 - 2.5 = 0.5 \text{ A}$$

$$V_1 = 2i_1 = 6 \text{ V}$$

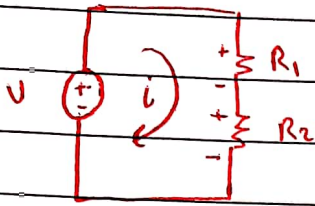
$$V_2 = 8i_2 = 4 \text{ V}$$

$$V_3 = 4i_3 = 10 \text{ V}$$

### Resistors in Series :



# Voltage Divider Networks:

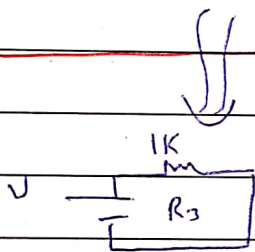
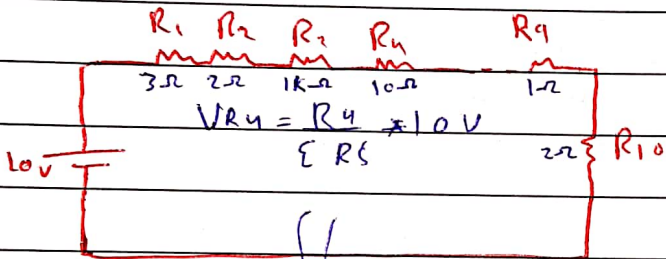


$$\text{KVL: } -V + iR_1 + iR_2 = 0$$

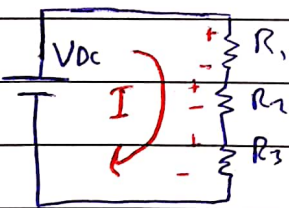
$$i = \frac{V}{R_1 + R_2}$$

$$VR_1 = i \times R_1 = \frac{R_1}{R_1 + R_2} V$$

$$VR_2 = i \times R_2 = \frac{R_2}{R_1 + R_2} V$$



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$$VR_1 = VDC \frac{R_1}{R_1 + R_2 + R_3}$$

$$VR_{23} = VDC \frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

$$VR_2 = VDC \frac{R_2}{R_1 + R_2 + R_3}$$

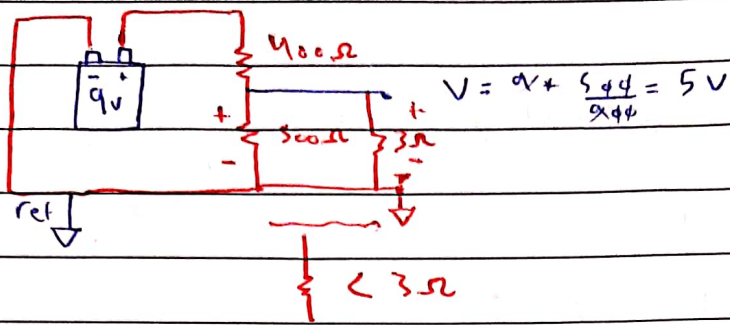
$$= VR_2 + VR_3$$

$$VR_3 = VDC \frac{R_3}{R_1 + R_2 + R_3}$$

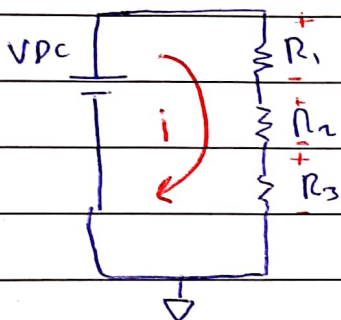
$$I = \frac{VDC}{R_1 + R_2 + R_3}$$

$R_{eq}$



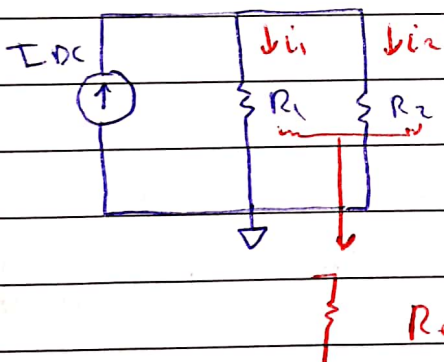


### Voltage Division :



$$V_{R\#} = V_{DC} \frac{R_{\#}}{R_1 + R_2 + R_3}$$

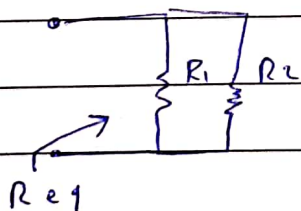
### Current Division :



$$i_{R1} = I_{DC} \frac{R_2}{R_1 + R_2}$$

$$i_{R2} = I_{DC} \frac{R_1}{R_1 + R_2}$$

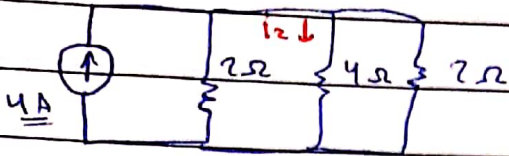
$$R_{eq} = R_{11} = \frac{R_1 R_2}{R_1 + R_2}$$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

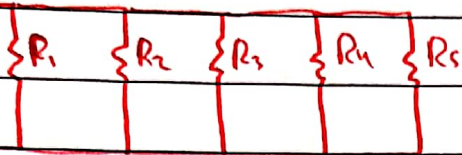
$$G_{eq} = G_1 + G_2$$

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2} \rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$



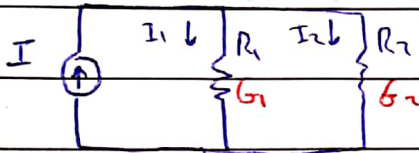
$$i_2 = 4 \cdot \frac{1}{1+4} = \frac{4}{5} \text{ A}$$

$$i_2 = I \frac{R_1 // R_3}{R_1 // R_3 + R_2}$$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$$

$$G_{eq} = G_1 + G_2 + G_3 + G_4 + G_5$$



$$I_1 = \frac{R_2}{R_1 + R_2} I$$

$$I_1 = \frac{G_1}{G_1 + G_2} I$$

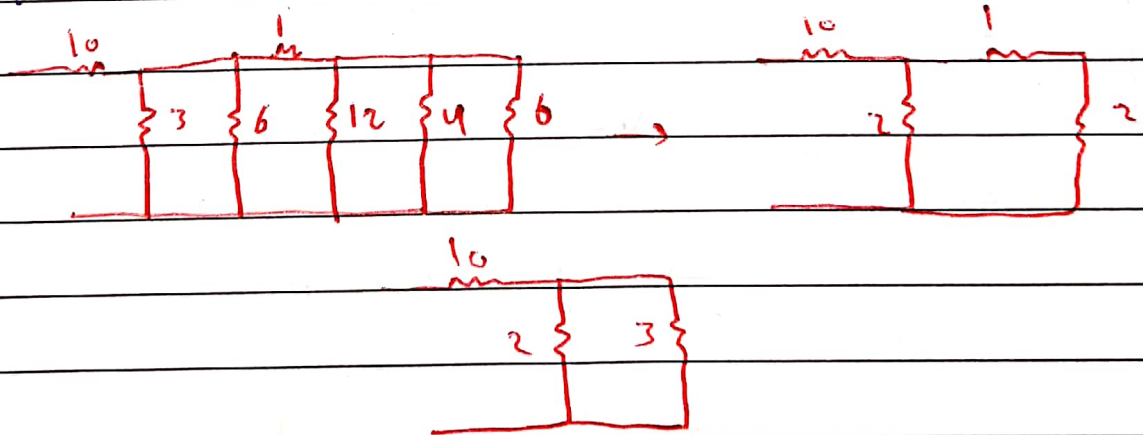
$$I_2 = \frac{R_1}{R_1 + R_2} I$$

$$I_2 = \frac{G_2}{G_1 + G_2} I$$

Example 2.9

Practice Problem 2.9

Example 2.10



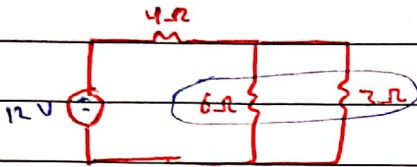
Practice ~~Problem~~ <sup>Problem</sup> 2.10

Example 2.11 :

$$S = \frac{1}{R} = \frac{1}{\Omega^{-1}} = \text{mho}$$

Practice Problem 2.11 :

Example 2.12 :



$$6\Omega \parallel 3\Omega = 2\Omega$$

$$V_o = 12 \frac{2}{2+4} = 4V$$

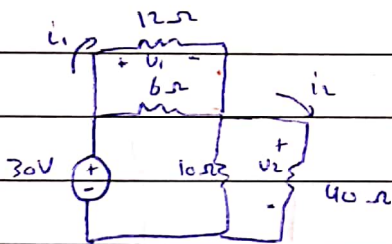
$$i_o = \frac{4V}{3\Omega}$$

$$P_{3\Omega} = i_o \times V_o = \frac{4}{3} \times 4 = \frac{16}{3} W$$

$$= i_o^2 \times R$$

$$= \frac{V_o^2}{R}$$

Practice Problem 2.12 :



$$V_1 = 30 \times \frac{4}{4+8} = 30 \times \frac{4}{12} = 10V$$

$$V_2 = 30 - V_1 = 20V$$

$$i_1 = \frac{V_1}{12} = \frac{10}{12} = \frac{5}{6} A$$

$$i_2 = \frac{V_2}{40} = \frac{20}{40} = \frac{1}{2} A$$

Example 2.13 :

$$\rightarrow R_{eq} = 6k$$

$$V_2 = 6k + 30m = 180V$$

$$V_1 = V_2 \frac{9k}{9k+3k}$$

$$= 180 \times \frac{9}{12} = \boxed{135V}$$

$$\rightarrow i_o = 30mA$$

$$i_1 = i_o \frac{18k}{27k} = 30m \times \frac{6}{9} = 20mA$$

$$i_2 = i_o - i_1 = 30m - 20m = 10mA$$

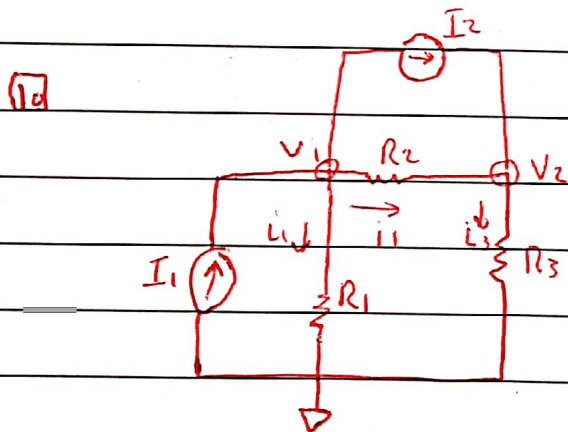
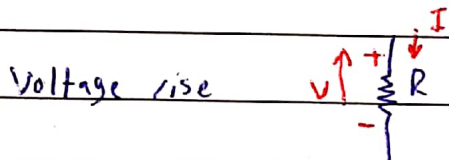
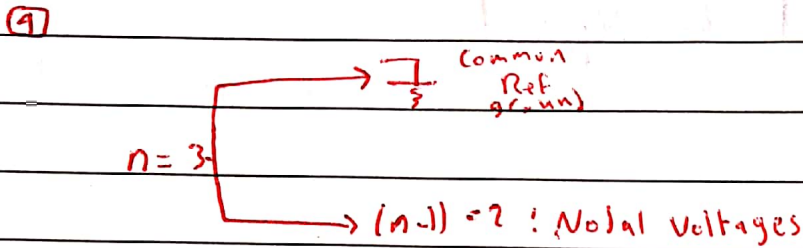
Practice Problem

# Chapter 30

Voltage Nodes KCL  $\rightarrow$  Nodal equations

Current Loops KVL  $\rightarrow$  Mesh equations

Nodal analysis:



KCL (1) @  $V_1$ :  $I_1 - I_2 - i_1 - i_2 = 0$

$\therefore I_1 = I_2 + i_1 + i_2$

KCL (2) @  $V_2$ :  $I_2 + i_2 - i_3 = 0$

$\therefore i_3 = I_2 + i_2$

$I_1 = I_2 + \frac{V_1 - 0}{R_1} + \frac{V_1 - V_2}{R_2} = 0$

$I_1 = \frac{V_2 - 0}{R_3} = I_2 + \frac{V_1 - V_2}{R_2} = 0$

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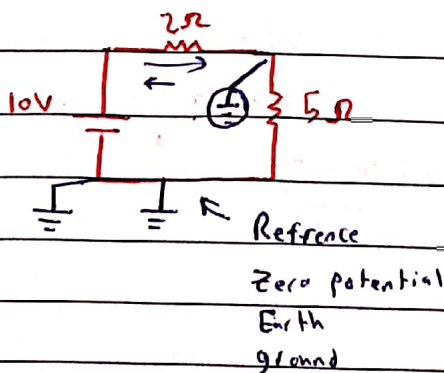


Figure 3.2

Kcl @  $n_1$ :  $I_1 - i_2 - i_1 = 0$   
 $I_n = \text{out}$

$I_1 = i_2 + i_1 + I_2$

Kcl @  $n_2$ :  $I_2 + i_2 = i_3$

$i_1 = \frac{V_1 - 0}{R_1}$

$i_2 = \frac{V_1 - V_2}{R_2}$

$i_3 = \frac{V_2 - 0}{R_3}$

Ohm's law

①  $\rightarrow I_1 = \frac{V_1 - V_2}{R_2} + \frac{V_1}{R_1} + I_2$

$\left[ \frac{1}{R_1} + \frac{1}{R_2} \right] V_1 + \left[ \frac{-1}{R_2} \right] V_2 = I_1 - I_2$

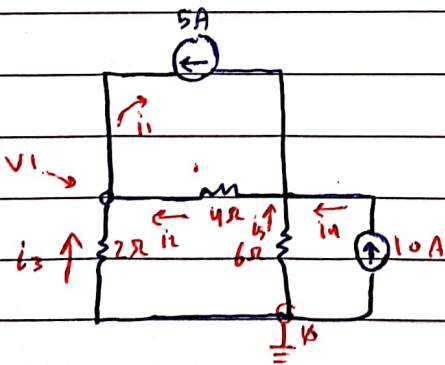
②  $\rightarrow I_2 + \frac{V_1 - V_2}{R_2} = \frac{V_2}{R_3}$

$\left[ \frac{1}{R_2} \right] V_1 + \left[ \frac{-1}{R_2} - \frac{1}{R_3} \right] V_2 = -I_2$

Matrix

$\frac{1}{R_1} + \frac{1}{R_2}$	$\frac{-1}{R_2}$	$I_1 - I_2$
$\frac{1}{R_2}$	$\frac{-1}{R_2} - \frac{1}{R_3}$	$-I_2$
$V_1$	$V_2$	$=$

Example 3.1:



① Kcl @ node 1:  $i_2 + i_3 - i_1 = 0$  — ①

Ohm's law

$i_1 = -5$

$i_2 = \frac{V_2 - V_1}{4} = \frac{20 - 40}{4} = -1.6668A$

$i_3 = \frac{0 - V_1}{2}$

$i_2 = 10$

$$\text{in } \textcircled{1} \quad \frac{V_2 - V_1}{4} + \frac{-V_1}{2} - (-5) = 0$$

$$\left[ \left[ \frac{-1}{4} \quad -\frac{1}{2} \right] V_1 + \left[ \frac{1}{4} \right] V_2 = -5 \right] \times 4$$

$$-3V_1 + V_2 = -20 \quad \dots \textcircled{1}$$

$$\text{Kcl @ node } \textcircled{2} \quad i_1 + i_4 + i_5 - i_2 = 0$$

$$-5 + 10 - \frac{V_2}{6} - \frac{(V_2 - V_1)}{4} = 0$$

$$\left[ \left[ \frac{1}{4} \right] V_1 + \left[ -\frac{1}{6} \quad -\frac{1}{4} \right] V_2 = -5 \right] \times 12$$

$$3V_1 + [-5] V_2 = -60$$

$$3V_1 - 5V_2 = -60 \quad \dots \textcircled{2}$$

$$\text{let } \textcircled{1} + \textcircled{2} \rightarrow -4V_2 = -80$$

$$\boxed{V_2 = 20 \text{ V}}$$

$$\text{In } \textcircled{1} \rightarrow \cancel{-60 + V_2} = -20$$

~~V<sub>2</sub>~~

$$-3V_1 + 20 = -20 \rightarrow \boxed{V_1 = \frac{40}{3}}$$

$$* \quad \begin{bmatrix} -3 & 1 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -20 \\ -60 \end{bmatrix}$$

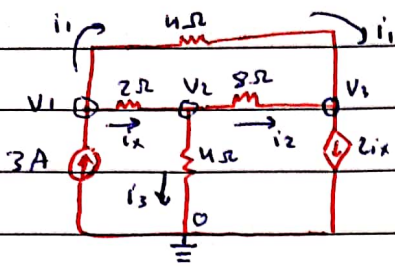
$$* \quad V_1 = \frac{\Delta_1}{\Delta} = \frac{160}{12} = \frac{40}{3} \quad V_2 = \frac{\Delta_2}{\Delta} = \frac{240}{12} = 20$$

$$\Delta = \begin{vmatrix} -3 & 1 \\ 3 & -5 \end{vmatrix} = 15 - 3 = 12$$

$$\Delta_1 = \begin{vmatrix} -20 & 1 \\ -60 & -5 \end{vmatrix} = 100 - (-60) = 160$$

$$\Delta_2 = \begin{vmatrix} -3 & -20 \\ 3 & -60 \end{vmatrix} = 180 - (-60) = 240$$

Example 3.28



By Ohm's law:

$$i_1 = \frac{V_1 - V_3}{4}$$

$$i_2 = \frac{V_2 - V_3}{8}$$

$$i_3 = \frac{V_2}{4}$$

$$i_x = \frac{V_1 - V_2}{2}$$

By KCL

① n1:  $I_n = out$

$$3 = i_1 + i_x$$

$$3 = \frac{V_1 - V_3}{4} + \frac{V_1 - V_2}{2}$$

$$4 \times \left[ \left[ \frac{1}{4} + \frac{1}{2} \right] V_1 + \left[ -\frac{1}{2} \right] V_2 + \left[ -\frac{1}{4} \right] V_3 = 3 \right]$$

$$3V_1 - 2V_2 - 1V_3 = 12 \quad \text{--- (1)}$$

② n2:  $I_n = out$

$$i_x = i_3 + i_2$$

$$\frac{V_1 - V_2}{2} = \frac{V_2}{4} + \frac{V_2 - V_3}{8}$$

$$8 \times \left[ \left[ \frac{1}{2} \right] V_1 + \left[ -\frac{1}{2} - \frac{1}{4} - \frac{1}{8} \right] V_2 + \left[ \frac{1}{8} \right] V_3 = 0 \right]$$

$$4V_1 - 7V_2 + V_3 = 0 \quad \text{--- (2)}$$

③ n3:  $I_n = out$

$$i_1 + i_2 - 2i_x = 0$$

$$\frac{V_1 - V_3}{4} + \frac{V_2 - V_3}{8} - 2 \left( \frac{V_1 - V_2}{2} \right) = 0$$

$$8 \times \left[ \left[ \frac{1}{4} - 1 \right] + \left[ \frac{1}{8} + 1 \right] V_2 + \left[ -\frac{1}{4} - \frac{1}{8} \right] V_3 = 0 \right]$$

$$-\frac{6}{3}V_1 + \frac{9}{3}V_2 - \frac{3}{3}V_3 = 0$$

$$-2V_1 + 3V_2 - V_3 = 0$$

$$\begin{bmatrix} 3 & -2 & -1 \\ 4 & -7 & 1 \\ -2 & 3 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$\underbrace{\hspace{10em}}_A \quad \underbrace{\hspace{10em}}_{\vec{v}} = b$

$$\vec{v} = \text{Inv}(A) * b$$