$w \sim N(0, \sigma^2)$

$$f_w(w) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-w^2/2\sigma^2} \qquad E(w) = 0$$

$$\sigma^2 = E[w^2] - (E[w])^2$$

$y_1 = s \cdot h$

$$E[y_1 \cdot y_1^*] = \cancel{y} E[s \cdot h \cdot s \cdot h^*]$$
$$= E[s^2 \cdot |h|^2] = E[s^2] \, E[|h|^2]$$

$z = x \cdot y$

$E[z] = E[x] \, E[y]$ if both x and y are independent.

$S \in \{-1, 1\}$

$p\{S = -1\} = \frac{1}{2}$

$p\{s = 1\} = \frac{1}{2}$

Find $E[s^2]$

$$E[s] = \sum_{k=1}^{N} p\{s = k\} \, k$$

$$T.\ \beta = \sum_{i=1}^{N} |h_i|^2$$

$|h|$ Gaussian

Rally dist. / $|h_i|$ ⇒ Rarely

Outage prob. ⇒ prob. of SNR to be below a certain threshold.

Outage prob ↓ ⇒ better Comm.

To evaluate outage prob and BER

You have to evaluate SNR

$C = \log_2 (1 + SNR)$ bits/s/Hz

→ max amount of mutual info. between i/p and o/p

→ maximum Rate you can achieve

$C = \max I(x;y)$. x لـ cdf of

throughput ⇒

→ this will be max when x is Gaussian R.V

you can't transmit Gaussian

You only stick with Binary modulation

Gama Random Variable

$P_e = Q(\sqrt{9\ SNR})$

→ BER

$$\bar{P}_e = E_{SNR}\left[Q\left(\sqrt{\alpha\ SNR}\right) \right]$$

Scanned by CamScanner

Over View :-

Complex Gaussian R.V with zero mean and variance Ω

Tx (s)   Rx

h → Channel coefficient.

$y = S.h + W$ → Gaussian noise

$W \sim (0, \sigma^2)$.

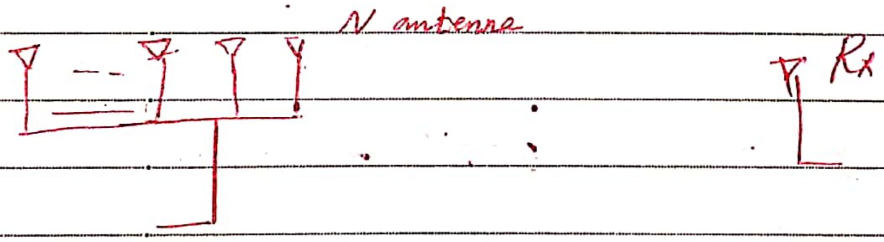
S → transmitted Signal

$$\underset{\text{(recieved)}}{SNR} \triangleq \frac{\text{signal power}}{\text{noise power}} = \frac{E[(S.h).(S.h)^*]}{E[W^2]} = \frac{E[|S|^2 . |h|^2]}{E[w^2]}$$

$E[W^2]$ ↳ = variance because the mean is zero

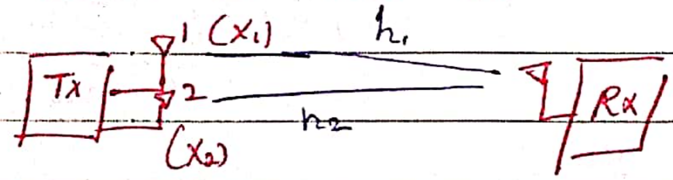
$$= \frac{|h|^2 \overbrace{E_s[|s|^2]}^{P}}{\underbrace{E[w^2]}_{\sigma^2}} = \frac{P|h|^2}{\sigma^2}$$

⊛ Transmit Diversity.



MISO system ⊘ multiple input Single Output.

Consider 2x1 MISO system



$$y = (x_1 h_1 + x_2 h_2) + w$$

Best SNR

$$SNR \propto (|h_1|^2 + |h_2|^2)$$

Let (complex values).

$$\underline{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}_{N\times 1} \qquad \underline{a}^* = [a_1^*, a_2^*, \ldots, a_N^*]_{1\times N}$$

$$\underline{a} \cdot \underline{a}_T = \sum_{i=1}^{N} |a_i|^2$$

$$a^{*T} \triangleq a^H \Rightarrow \underline{a}^H \cdot \underline{a} = \sum_{i=1}^{N} |a_i|^2$$

$$y = x_1 h_1 + x_2 h_2 + w$$

$$= \underbrace{\underline{X}^T}_{1\times 2} \cdot \underline{h} + w \Rightarrow [h_1 \ h_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w$$

$$\underline{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \underline{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}_{2\times 1}$$

$$y = \underline{h}_T \cdot X + w \Rightarrow (1)$$

$$\Rightarrow y = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} [x_1 \ x_2] + \quad (X)$$

What is the ~~appropriate~~ optimal transmission scheme

Let's assume the transmit symbol is $\underline{x}$

consider $\hat{\underline{x}}^* = \frac{1}{\|h\|}\begin{bmatrix} h_1^* \\ h_2^* \end{bmatrix} x$

$= \frac{1}{\|h\|} \cdot \underline{h}^* \cdot x$

Transmit Beam former

$$\hat{x}^* = \begin{bmatrix} \hat{x}_1^* \\ \hat{x}_2^* \end{bmatrix} = \begin{bmatrix} \frac{1}{\|h\|} \cdot h_1^* \cdot x \\ \frac{1}{\|\underline{h}\|} \cdot h_2^* \cdot x \end{bmatrix} = \frac{1}{\|\underline{h}\|} \cdot \underbrace{\begin{bmatrix} h_1^* \\ h_2^* \end{bmatrix}}_{\underline{h}^*} x$$

the Choosen vector.

plug $\hat{x}$ above in (1) instead of x

$$y = \underline{h}^T \cdot \frac{1}{\|\underline{h}\|} \cdot \underline{h}^* \cdot x + w$$

$$y = \frac{1}{\|h\|} \left(|h_1|^2 + |h_2|^2\right) \cdot S + w$$

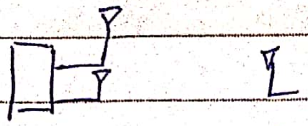
$\|h\| = \sqrt{|h_1|^2 + |h_2|^2}$

$$= \sqrt{\left(|h_1|^2 + |h_2|^2\right)} \cdot S + w$$

$$\boxed{SNR = \frac{P}{\sigma^2}\left(|h_1|^2 + |h_2|^2\right)}$$

الأربعاء ـ ٢ / ١٠

(*)



$$y = \underline{h}^T \cdot x + w$$

$$\underline{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \qquad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\hat{x} = \frac{1}{\|h\|} \begin{bmatrix} \overset{\hat{x}_1 \|h\|}{h_1^* x} \\ \underset{\hat{x}_2 \|h\|}{h_2^* x} \end{bmatrix}$$

transmit symbol

$$\hat{x} = \frac{S}{~~\|h\|~~} \begin{bmatrix} \frac{h_1^*}{\|h\|} \\ \frac{h_2^*}{\|h\|} \end{bmatrix}$$

Transmit Beam former.

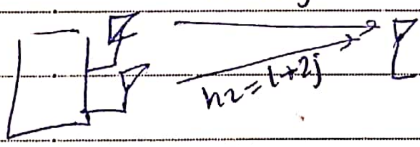
$$y = \frac{h^T}{\|h\|} \cdot h^* \cdot S + w$$

$$y = \frac{\|h\|^2}{\|h\|} S + w \qquad SNR = \|h\|^2 = \frac{|h_1|^2 + |h_2|^2 \cdot P}{\sigma^2}$$

* Transmit Beam forming is Optimal since it Maximized the SNR at the Reciever but T.B requires full channel state information (h_1, h_2)
CSI

$\boxed{\text{Example}}$

consider a 2x1 MISO system



① Find an expression for the Recieved Signal y.

$$y = \begin{bmatrix} 2+j & 1+2j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w$$

2- Find the Optimal Transmit Beamformer given the CSI is available at the Transmitter

$$\hat{x} = \frac{1}{\|h\|} \cdot \begin{bmatrix} h_1^* \\ h_2^* \end{bmatrix} s$$

$$\|h\| = \sqrt{\underline{h}^H \cdot \underline{h}} = \sqrt{[2-j \quad 1-2j]\begin{bmatrix} 2+j \\ 1+2j \end{bmatrix}} = \sqrt{|h_1|^2 + |h_2|^2}$$

$$= \sqrt{5+5} = \sqrt{10}$$

$$\boxed{\hat{x} = \frac{1}{\sqrt{10}} \begin{bmatrix} 2-j \\ 1-2j \end{bmatrix} s}$$

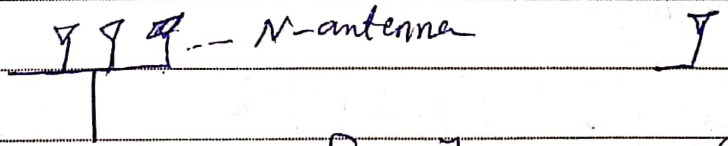
3- Find the Recieved SNR

$$y = \frac{1}{\sqrt{10}} \cdot [2+j \quad 1+2j] \begin{bmatrix} 2-j \\ 1-2j \end{bmatrix} s + w$$

$$= \sqrt{10} \cdot s + w$$

$$SNR = \frac{P\,(10)}{\sigma^2}$$

General Case



$$\underline{y} = \underline{h}^T \cdot \underline{x} + w \qquad \underline{h} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \end{bmatrix} \qquad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$$

$x_i =$ $i \in N$
transmit signal from the ith transmit antenna.

$h_i =$ the channel between the Reciever and the ~~A~~ ith antenna.

$$\hat{x} = \frac{1}{\|\underline{h}\|} \cdot \begin{bmatrix} h_1^* \\ h_2^* \\ \vdots \\ h_N^* \end{bmatrix} S = \frac{1}{\|h\|} \cdot \underline{h}^* . S$$

$$y = \frac{\underline{h}^T \cdot \underline{h}^*}{\|h\|} \cdot S + w$$

$$\underline{h}^T \cdot \underline{h}^* = \|\underline{h}\|^2$$

$$y = \|\underline{h}\| . S + w$$

$$SNR = \frac{P}{\sigma^2} \|\underline{h}\|^2$$

$$\sum_{i=1}^{N} |h_i|^2$$

- X is exponential with parameter λ (mean λ)

PDF $f_X(x) = \frac{1}{\lambda} e^{-x/\lambda}, \quad x \geq 0$

CDF $F_X(x) = 1 - e^{x/\lambda} \quad x \geq 0$

$$f_{|h_i|^2}(x) = \frac{1}{\Omega} e^{-x/\Omega}$$

only

1 antenna
SNR ⇒ exponential R.V
multiple Antenna
SNR ⇒ summation of exponential Γ-R.V.

h_i for $i = 1, \dots, N$ are independent R.V

$$f_{\sum_{i=1}^{N} |h_i|}(x) = \frac{1}{\Omega \Gamma(n)} \left(\frac{x}{\Omega}\right)^{n-1} . e^{-\frac{x}{\Omega}}, \quad x \geq 0$$

$$F_{\sum_{i=1}^{N} |h_i|^2}(x) = 1 - e^{-\frac{x}{\Omega}} \sum_{j=0}^{N-1} \frac{\left(\frac{x}{\Omega}\right)}{j!}$$

$$\Gamma(x) = \int_0^\infty s^{x-1} e^{-s} \, ds.$$

For a Concave function $g(x)$

Jensen's Inequality $E[g(x)] \geq g[E(x)]$

$$E[\log(1+X)] = \int_0^\infty \log(1+X) \left(\frac{1}{\Omega}\right) e^{-\frac{X}{\Omega}} X^{N-1} \, dX.$$

$f_{[h_i]}(x)$

$$= \log[1+E(X)]$$

$$\boxed{\text{for } x = \sum_{i=1}^{N} |h_i|^2}$$

$$= \log\Big(1+E\big[\underbrace{\textstyle\sum |h_i|^2}_{N\Omega}\big]\Big)$$

$$= \log(1+\Omega N)$$

R $\Rightarrow$ throughput $\quad R \sim O(\log(N))$ as $N \to \infty$

which is the domenant term?

Monday. 7.10

under T.B the Signal to Noise ratio (X)

$$X = \sum_{i=1}^{N} |h_i|^2$$

X follows gamma distribution

$$f_X(x) = \frac{x^{N-1} e^{-x/\Omega}}{\Gamma(N)\,\Omega^n}$$

$$\cancel{F_X(x) = 1 - e^{-x/\Omega} \sum_{j=0}^{N-1} \frac{x}{\Omega}}$$

$$F_X(x) = 1 - \sum_{j=0}^{N} \frac{\left(\frac{x}{\Omega}\right)^j e^{-\frac{x}{\Omega}}}{j!}$$

1 Outage prop.

$$P_{out} = P_r\{X \le \bar{\gamma}\}$$

specified threshold.

$$= F_X(\bar{\gamma})$$

$$= 1 - \sum_{j=0}^{N-1} \frac{\left(\frac{\gamma}{\Omega}\right)^j e^{-\frac{\bar{\gamma}}{\Omega}}}{j!}$$

2 Throughput

Capacity Under T.B

$$X = \sum_{j=1}^{N} |h_i|^2$$

$C = \log(1+X)$ (instentanious), so we took the average.

SNR

Instentanious Capacity :-

$$C = \log_e(1+X) \quad \text{Nats/s/Hz}$$

Ergodic capacity :- $\bar{C} = \underset{X}{E}[\log_e(1+X)]$ Nats/s/Hz

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) \cdot \underbrace{f_X(x)}_{\text{pdf}}\, dx .$$

$\bar{C} =$

$$\bar{C} = \int_0^{\infty} \log(1+x) \cdot f_X(x) = \underbrace{\int_0^{\infty} \log(1+x) \frac{x^{N-1} e^{-x/\Omega}}{\Gamma(n)\,\Omega^n} dx}_{\text{intractable}}$$

$$\bar{C} \leq \int \log(1+E[X])$$

$$E[X] = \int_0^{\infty} X \cdot \frac{X^{N-1} e^{-X/\Omega}}{\Gamma(\Omega)\,\Omega^N} dx .$$

$$= N\Omega$$

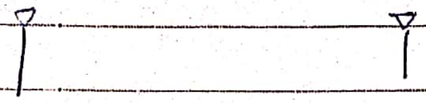
$$\bar{C} \leq \log(1+N\Omega)$$

$$\bar{C} \sim \log(N) \quad \text{as } N \to \infty$$

in T.B

SNR $\rightarrow$ linearly.

$\bar{C} \sim \log(N)$ as $N \to \infty$



$C_{target} = 100$ bits/s

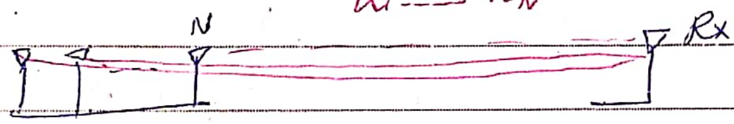
T.B. 100 antenna

$C = \log(101) \ll 100$.

الأربعاء ٩/١٠

Transmit antenna selection.

Consider a transmitter which is equiped with N-antennas



Based on the values of $h_1, h_2, \dots, h_N$ the Reciever will ~~select~~ feedback the maximum value of $(h_1, h_2 \dots h_N)$ and the index of ~~the~~ the antenna.

at the transmitter One antenna is activated, the One that leads to the Best SNR, and Other antennas will be Silent.

the Recieved Signal at Rx is

SNR =

$$y = s \cdot h^{o} + w$$

Symbol

h^{o} corresponds for $\max |h_i|^2$, $i = 1, 2, \dots N$

But it's not $|h_i|^2$ its h for the max magnitude squeare.

$SNR = \frac{P}{\sigma^2} \cdot |h^{a}|^2$ where $|h^{i}|^2 = \max_{i=1,\dots,N} |h_i|^2$

$\underline{SNR} = \frac{P}{\sigma^2} X$ where $X = \max_{i=1,\dots,N} |h_i|^2$

exponential distribution

PDF

always positive

$f_{|h_i|^2}(x) = \left(\frac{1}{\Omega} e^{-x/\Omega}\right)$ as $|h_i|^2$ are IID

$F_{|h_i|} = 1 - e^{-x/\Omega}$

$\boxed{\text{CDF}}$

when $X = \max_{i=1,\dots,N} |h_i|^2$

$F_X(x) = \left[F_{|h_i|}(x)\right]^N$

$$\boxed{F_X(x) = \left[1 - e^{-x/\Omega}\right]^N}$$

$p\left\{\frac{X \cdot P}{\sigma^2} \le x\right\} = p\left\{X \le \frac{\sigma^2}{P} x\right\}$

$= \left[1 - e^{-x/N\frac{P}{\sigma^2}}\right]^N$

x, y independent

$p\{\max(x,y) \le z$

$p\{X \le z \mid x > y, \; y \le z \mid x < y\}$

$\underline{p\{x \le z\}} \cdot p\{y \le z\}$

$F_x(z) \cdot F_y(z)$.

$Z = \max(x,y)$

then $\Rightarrow F_Z(z) = F_x(z) \cdot F_y(z)$

x, y IID

$= F_x(z)^2$

1- Outage propability

$$P\{X \le \bar{\gamma}\} = \left(1 - e^{-\bar{\gamma}/\Omega}\right)^N$$

2- Ergodic Capacity

$$\bar{C} = E[\log(1+X)]$$

$$= \int_0^\infty \log(1+x)\, d[cdf)$$

$$= \int_0^\infty \log(1+x) \frac{d\left[1-e^{-x/\Omega}\right]^N}{dx} dx$$

$$f_X(x) = \cancel{\ } N \left(1 - e^{-x\Omega}\right)^{N-1} \cdot \frac{e^{-x/\Omega}}{\Omega}$$

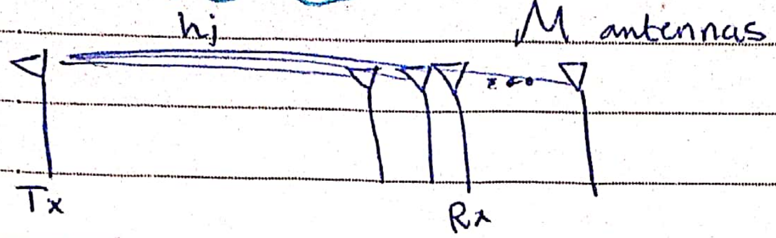
$\hookrightarrow$ $E(X) = \psi(N) + E_0$

$$\psi(N) = \log(N) \qquad N \to \infty$$

So $\bar{C} = \log[\log(N)] \qquad N \to \infty$

30/10 الأربعاء .

Recieved Diversity.



h_i : the channel coeff. from the transmitter to the Recieve antenna.

- Maximal ratio combining $\xrightarrow{\text{مكافئ}}$ T.B @ Tx Diversity

S.D @ Tx $\xrightarrow{\text{تكافئ}}$ selection combining @ Rx

Def: y_i denote the Recieved ~~ant~~ Signal at the Recieve antenna.

$$y_1 = \underbrace{S}_{\text{transmitted symbol}} h_1 + Z_1$$ where Z_1 · the noise and it can be modeled as complex Gaussian R.V with Zero mean & variance (σ^2)

$$y_2 = S . h_2 + Z_2$$

$\vdots$

$$y_m = S h_m + Z_m$$

IN vector form

$$\underline{y} = \underline{S} . \underline{h} + \underline{Z} \quad \ldots (1)$$

$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_M \end{bmatrix} = S \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_M \end{bmatrix}$$

Maximal Ratio Combining @ Rx better than T.B @ Tx
Because 1- don't need feedback
2- can acheive same SNR of T.B
CSI. بدون

Recieved Signal Combining:
at the Rx we will combine the Recieved Signals

$$u = w_1 y_1 + w_2 y_2 + w_3 y_3 - - - + w_m y_m$$

u ← combiner Output

w_1, w_2, w_3, w_m ← combining coeff.

in Vector form
$$u \triangleq \underline{w}^T \cdot \underline{y} \quad ---- (2) \qquad \underline{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix}$$

- Optimization coeff.
W in the Same direction of h give maximal Ratio Combining

① in T.B @ Tx ⟹ the Best $\underline{s}$ when $\underline{s}$ in Same direction of $\underline{h}$ ⟶ maximum SNR

- Making use of ① and ②

$$\underline{u} = \underline{w}^T \cdot (s \cdot \underline{h} + \underline{z}) = \underbrace{\underline{w}^T \cdot s \cdot \underline{h}}_{\text{Signal component}} + \underbrace{\underline{w}^T \underline{z}}_{\text{Noise Component}}$$

$$SNR = \frac{|\underline{w}^T \cdot \underline{h}|^2 \; E[|s|^2]}{E_z[|w^T \cdot z|^2]}$$

to find ②

$E\left[|w^T . z|^2\right]$

$w^T . z = w_1 z_1 + w_2 z_2 + \cdots + w_m z_m$

$$E\left[|w^T . z|^2\right] = E\left[\left(\sum_{i=1}^{M} w_i z_i\right)\left(\sum_{i=1}^{M} w_i^* z_i^*\right)\right]$$

$$= E\left[\sum_{i=1}^{M} |w_i|^2 |z_i|^2\right] + E\left[\sum_{i=1}^{M} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_i w_j^* z_i z_j^*\right]$$

$$= \underbrace{\sum_{i=1}^{M} |w_i|^2}_{\|w\|^2} . \underbrace{E\left[|z_i|^2\right]}_{\sigma^2} + \sum_{i=1}^{M} \sum_{\substack{j=1 \\ j \neq i}} w_i w_j^* \; E\left[z_i z_j^*\right]$$

$z_i . z_j^*$ (independent)

$E\left[z_i . z_j^*\right]$

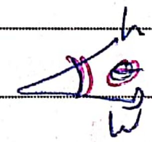
$= E[z_i] . E[z_j^*]$

$= 0 . 0 = 0$

$$= \boxed{\sigma^2 \|w\|^2}$$

① $|w^T . h|^2 = |w . h|^2 = \|w\| . \|h\|^2 , \cos\theta^2$

$$SNR = \frac{P . \|w\|^2 . \|h\|^2 \cos^2\theta}{\sigma^2 \; \|w\|^2}$$



→ (Maximum Ratio Combining)

SNR is Maximum at $\theta = 0$

w must be in the direction h

$$SNR = \frac{P\|h\|^2}{\sigma^2} \rightarrow \theta = 0°$$

wireless → maximal Ratio Combining
more difficult than equal Gain combining

optical channel → Equal gain combining.
↳ very simple أسهل طريقة

→ for M Rx antenna → the worst scheme
* كأني بختار أي واحدة وأعمل system
SISO Randomly

→ X in the Same direction of Y means equal of multiply by constant.

$\hat{w} = \frac{h}{\|h\|}$ (maximal ratio combining) $\hat{\ } \triangleq$ optimal value

① in Cellular I can Use T.B-Tx or selectivity.

② if I have single antenna user
أستخدم ولا T.B Tx
M.R.C

③ in Uplink we use Rx antenna selection.

* Same analysis for T.B and M.R.C
$\bar{\varepsilon}$, BER, Pout, Rx antenna selection
$\bar{C} = \log(M)$
for M.RC = $\max_{i=1,\dots,M} |h_i|^2$

QPSK:

- Monday. 4/11.

BER Bit error rate

Assume Binary PSK (BPSK)

$$P_e = Q\left(\sqrt{\alpha\, SNR}\right)$$

SISO channel

$$T_x \xleftarrow{\quad h \quad} R_x$$

$$P_{e|h} = Q\left(\sqrt{\frac{\alpha P |h|^2}{\sigma^2}}\right)$$

conditioning on

$$\bar{P}_e = \int_{-\infty}^{\infty} Q\left(\sqrt{\frac{\alpha P |h|^2}{\sigma^2}}\right) f_{|h|^2}(x)\, dx \quad \text{(Expected value).}$$

Transmit Beam forming / Maximal Ratio Combining.

$$= \frac{P \sum_{i=1}^{N} |h_i|^2}{\sigma^2} = X \qquad \text{assume } \frac{P}{\sigma^2} = 1$$

$$P_e = E_X\left[Q\left(\sqrt{\frac{\alpha P}{\sigma^2} X}\right)\right] \qquad f_X(x) = \frac{x^{N-1} e^{-x/\Omega}}{\Omega^N \Gamma(N)}$$

$$= \int_0^{\infty} Q\sqrt{\frac{\alpha P x}{\sigma^2}}\, f_X(x)\, dx$$

$$= \int_0^{\infty} Q\left(\sqrt{\frac{\alpha P x}{\sigma^2}}\right) \cdot \frac{x^{N-1} e^{-x/\Omega}}{\Omega^N \Gamma(N)}\, dx$$

$Q(x) \le e^{-x^2/2}$ $\quad Q(\sqrt{\alpha x}) \le e^{-\alpha x/2}$

$$E\left[\sqrt{\alpha x}\right] \le E\left[e^{-\alpha x/2}\right]$$

- for T.B & M.R.C X is Γ R.V
- SISO , X is exponential
- Selection : X is max of exponentials.

But $Q(\sqrt{x}) \le e^{-x/2}$

So $\bar{p}_e = E_x\left[Q\left(\sqrt{\alpha x}\right)\right] \le E_x\left[e^{-\alpha x/2}\right]$

The moment generating function of a R.V X is

$$M_x(t) = E_x\left[e^{-t.x}\right] = \int_0^\infty e^{-tx} f_x(x)$$

$\Rightarrow \bar{p}_e \le M_x\left(\frac{\alpha}{2}\right)$

↳ for SISO → $M_x\left(\frac{\alpha}{2}\right)$ → for exponential function

for T.B or M.R.C = gama function.

for selection → for max of exponential.

$\alpha = 2$ in BPSK

7/11 الخميس

Convergence in distribution

A sequence of R.Vs $X_1, X_2, \ldots X_N$ converges in distribution to a Random Variable (Z) if the limit

$$\lim_{N \to \infty} F_{X(N)}(X) = F_Z(X)$$

for all X and F in Continuous.

$$\Rightarrow X(N) \xrightarrow{D} Z \qquad N \to \infty$$

* Central limit theorem: (CLT)

Let $X_1, X_2, \ldots X_N$ be i.i.d Rvs with $E[x_i] = \mu$ and varience $(x_i) = \sigma^2 < \infty$

then and define

$$X(N) = \sum_{i=1}^{N} \frac{X_i - N\mu}{\sigma\sqrt{N}}$$

then $\lim_{N \to \infty} P\{X(N) \leq u\} = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t/2} \, dt.$

$$X(N) \xrightarrow{D} N(0,1)$$

M.R.C

Ex: consider the T.B SNR

$$SNR \triangleq X = \sum_{i=1}^{N} |h_i|^2 \; ; \text{where } |h_i|^2 = \Omega$$

$$X = \sum_{i=1}^{N} |h_i|^2 \sim N(N\Omega, \Omega^2 N)$$

$$\text{or} \quad \frac{X - N\Omega}{\sqrt{N}\,\Omega} \sim N(0,1)$$

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$G(x) = e^{-e^{-x}} \quad , -\infty < x < +\infty$

Moment generation function implies in the CLT,

11/11 الخميس

Extreme value Theory.

Let $X_1(N)$

Let $X(N) = \max(X_i)$ $\quad X \quad i = 1, 2, \dots \infty$

$X \Rightarrow$ expon.

$$EVT = \frac{X(N) - \Omega \ln(N)}{\Omega} \xrightarrow{D} e^{-e^{-t}}$$

a_N ... b_N

Gumbell.

$$P_e \leq E\left[e^{-(\alpha/2)x} \right] = M_x\left(\frac{\alpha}{2}\right)$$

From EVT let $X(N) = \max(X_i)$

Then $\dfrac{X(N) - a_N}{b_N} \xrightarrow{D}$ gumbel

X is Gumbel R.V

$$\lim_{N \to \infty} E\left[e^{+t \frac{(X(N) - a_n)}{b_n}} \right] = E\left(e^{t X} \right)$$

$X(N) = \max(X_i)$

$P_e \le E\left[e^{-tX(N)}\right]\Big|_{t=-\alpha/2} = E\left[e^{t\left(\frac{(X(N)-a_N)}{b_N}b_N + a_N\right)}\right]$

$\underset{X[n]}{E}\left[e^{t\left(\frac{X[n]-a(N)}{b_n}\right)b_N}\right]e^{a_N}$

$= \underset{\text{gumbelle }X}{E}\left[e^{tb_nX} \cdot e^{a_N}\right]$

$= e^{a_N} E_X\left[e^{tb_nX}\right]$

$N \to \infty$

13/11

الأربعاء

$$\lim_{N\to\infty} E\left[e^{\frac{t(x(N)-a_N)}{b_N}}\right] = E\left[e^{tx}\right] = M_x(t), \text{ where } t<0 \rightarrow ①$$

but $F_X(x) = e^{-e^{-x}}$, $\infty > x > -\infty$

$$M_x(t) = \int_{-\infty}^{\infty} e^{tx}\, e^{-x} \cdot e^{-e^{-x}}\, dx = \frac{\Gamma(1-t)}{0!} = \Gamma(1-t),\ t<1$$

Using ①

$$E\left[e^{t.x(N)}\right] = E\left[e^{\frac{t(x(N)-a_N)}{b_N} b_N + a_N}\right]$$

$$= E\left[e^{t.b_N.x}\right] e^{t.a_N} = -e^{t a_N}\,\Gamma(1-t b_N),\ t<0$$

selection diversity

$$\bar{P}_e \le M_{X(N)}\left(\frac{-\alpha}{2}\right)$$

BPSK → upper bound [Q function]

non coherent - FSK → equality.

$X(N) = \max |h_i|^2 \quad i=1,2,\dots N$

as $N \to \infty$

$$\bar{P}_e = e^{-\alpha/2 .\, a_N}\, \Gamma\left(1+\frac{\alpha}{2} . b_N\right) \rightarrow$$

How does BER scale with N ??

$X(N) = \max |h_i|^2$

$C(N) = \log(1 + X(N))$

↳ throughput of the max SNR link

$C(N) = \log(1 + X(N))$

if $\dfrac{X(N) - a(N)}{b_N} \xrightarrow{D} X$ ← Gumbel

then $\dfrac{C(N) - a^*(N)}{b(N^*)} \xrightarrow{D} X$ ← gumbel

$a^*(N) = \log(1 + Pa_N)$

$b^*(N) = \log\left(1 + \dfrac{Pb_N}{1 + Pa_n}\right)$

$\bar{C}(N) = E[C(N)] = a^*_N + \gamma b^*_N$

let $|h_i|^2$ be an exponential with mean of Ω

Selection ↓

$X(N) = \max\limits_{i=1,\ldots,N} (|h_i|^2$ $\quad a_N = \Omega \log(N)$

$\quad b_N = \Omega$

$C(N) = \log(1 + \rho X(N))$

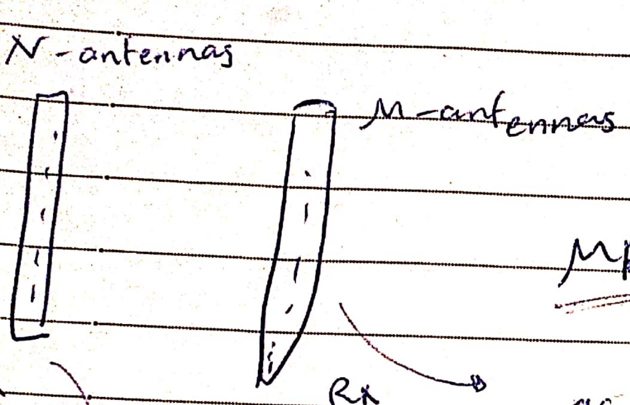
→ Ergodic capacity

$\bar{C}(N) = a^*_N + \gamma b^*_N$; $\gamma = 0.577$ ← Euler Constant

$a^*_N = \log(1 + \rho\Omega\log(N))$

$b^*_N = \log\left(1 + \dfrac{\rho\Omega}{1 + \rho\Omega\log N}\right)$

18/11 Monday



MRC & Antenna Selection.

as one antenna

$\gamma_j = h = \sum_{i=} |h_i|^2$

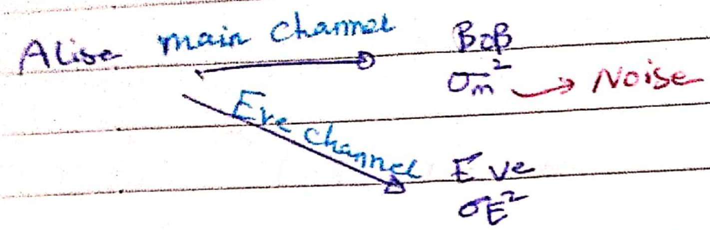
$\downarrow$

MRC

selection

$$\max \sum_{i=1}^{N} (\gamma_j)$$

Secrecy Capacity



p :- transmit power at alice

$$C_M^{\text{main channel}} = \log_2\left(1+\frac{P}{\sigma_m^2}\right)$$

$$C_E = \log_2\left(1+\frac{P}{\sigma_E^2}\right) \qquad \text{Eve channel}$$

Secrecy capacity :-

$$C_s \begin{cases} \log\left(1+\frac{P}{\sigma_m^2}\right) - \log\left(1+\frac{P}{\sigma_E^2}\right) ; & \sigma_E^2 > \sigma_m^2 \\ \text{if } \sigma_E^2 < \sigma_m^2 , & C_s = 0 \end{cases}$$

secrecy capacity under fading effects.

$$C(\gamma_m, \gamma_E) = \begin{cases} \log_2(1+\gamma_m) - \log_2(1+\gamma_e) & \gamma_m \leq \gamma_E \\ 0 & \gamma_E \leq \gamma_m \end{cases}$$

R.V

$$\gamma_E = \frac{P|h_m|^2}{\sigma_m^2} \quad ; \quad \gamma_m = \frac{P|h_e|^2}{\sigma_e^2}$$

* perfect secrecy is guaranteed only when (Active Eve the Tx know full CSI of Bob and Eavesdropping).

*** passive Eavesdropping

Eve's CSI is not available at alice.
Let us assume that alice transmits confidential data at constant Rate ("Rs")

$R_s \leq C_s \rightarrow$ perfect secrecy capacity.
$R_s > C_s \rightarrow$ security is compromized (No perfect secrecy).

We are interested

$$P_{out}(R_s) = P_r\{C_s \leq R_s\}$$

→ secrecy outage probability.

$$P_{out} = P_e\{C_s \le R_s\}$$

$$P_{out} = 1 - P_e\{R_s < C_s\}$$

$$P_{out} = 1 - P_e\left\{\log_2\left(\frac{1+\gamma_m}{1+\gamma_e}\right) > R_s\right\}$$

$$P_{out} = 1 - P_e\left\{\gamma_m > 2^{R_s}(1+\gamma_e) - 1\right\}$$

$$1 - \int_0^\infty P_r\{\gamma_m > 2^{R_s}(1+\overset{X}{\gamma_e}) - 1\} f_{\gamma_e}(X)$$

$$1 - \int_0^\infty \left(\int_{2^{R_s}(1+X)-1}^{\infty} f_{\gamma_m}(y)\,dy\right) f_X(x)$$

$$= 1 - \int_0^\infty f_{\gamma_e}(x)\left[F_{\gamma_{em}}(\infty) - F_{\gamma_e}\left(2^{R_s}(1+x)-1\right)\right]$$

$$= \left(\cancel{1} - \cancel{1} + \int f_{\gamma_e}(x) F_{\gamma_m}\left(2^{R_s}(1+X)-1\right)dx\right.$$

$$= \int_0^\infty f_{\gamma_e}(x)\, F_{\gamma_m}\left(2^{R_s}(1+X)-1\right) dx$$

$$\boxed{P\{C_s \le 0\} = \int_0^\infty f_{\gamma_e}(x)\, F_{\gamma_m}(x)\, dx.}$$

Ergodic secrecy capacity. Monday 9-12-2019

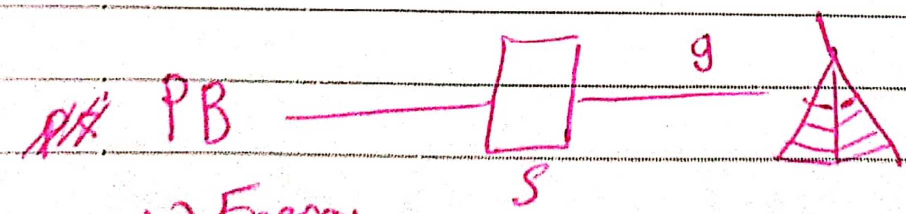
$$\bar{C}_s = \int_0^\infty \int_0^\infty \underbrace{C_s(\gamma_m, \gamma_e)}_{\log\left(\frac{1+\gamma_m}{1+\gamma_e}\right) \; ; \; \gamma_m > \gamma_e} f_{\gamma_m}(x) \, f_{\gamma_e}(x) \, dx \, dy$$

$\therefore$ ** $\bar{C}_s = \frac{1}{\ln(2)} \int \frac{F_{\gamma_e}(x)}{1+x} \left(1 - F_{\gamma_m}(x)\right) dx$

Backscatter communications.

- Energy harvesting

- Harvest then transmit protocol



S is Energy constrained node

S has to harvest energy from PB

↪ Energy harvesting phase/stage "τ"

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assuming that the Total time is $\underline{T}$ then

E.h time $= \tau \cdot T$

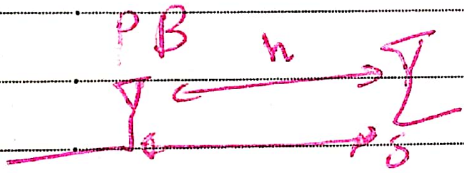
For harvesting and transmit

transmit: source uses the harvested energy to communicate with $\underline{D}$

information transfere

$(1-\tau)\,T$ sec

- assuming $T = 1s$



τs

$$\boxed{\vec{y}_s = \sqrt{p}\;\; s_1 \cdot h + \sigma_s^2}$$

transmit power at the P.B

$P_{received} = P \cdot |h|^2$

$E = \eta\, \tau_0\; P|h|^2$

η :- energy harvesting efficiency

E → harvested Energy.

$0 \le \eta \le 1$

information transfere

$$y_D = \sqrt{P_s} \cdot g \cdot S_2 + w_D^2$$

transmit power at the source

$$SNR = \frac{P_s \cdot |g|^2}{\sigma_D^2}$$

where $P_s = \frac{P_r}{(1-\tau)} = \frac{\eta \tau P_{in} |h|^2}{(1-\tau)}$

$$\underset{(R.V)}{SNR} = \frac{\eta \tau |h|^2 |g|^2 / P}{(1-\tau)\sigma^2}$$

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16/92

(1 - Pout)
throughput

Pout =

① Delay limited (DL)
(intolerant)

$R_{DL}(\tau) = R(1 - P_{out}(\tau))(1-\tau)$

E.H time

$p_{out} = Pr\{\log_2(1+SNR) \leq R\}$

$= Pr\{SNR = 2^R - 1\}$

$$= \boxed{Pr\left\{\frac{\cancel{\eta}\,\tau |g|^2 |h|^2}{(1-\tau)\,\sigma^2} < \gamma_{th}\right\}}$$

$$Pr\{|g|^2|h|^2\} \leq \frac{(1-\tau)\,\sigma^2\gamma_{th}}{\eta\,T}$$

$F_X(x) = (1 - e^{x/\lambda_g})\,u(x)$

$F_Y(y) = (1 - e^{-y/\lambda_H})\,u(y)$

$Z = X \cdot Y$

$$F_Z(z) = \int_0^\infty f_{Z/X}(x)\, f_X(x)\, dx$$

$$F_Z(z) = P\{X \cdot Y \le z\}$$

$$P\left\{Y \le \frac{z}{x}\right\} = \int_0^\infty F_Y\left(\frac{z}{x}\right) f_X(x)\, dx$$

Bessel function

$$F_Z(z) = \frac{\sim 2\sqrt{z}\ I\left(\frac{2\sqrt{z}}{\sqrt{\beta\lambda}}\right)}{\sqrt{\beta\lambda}}$$

$$Pr\left\{z \le \frac{(1-\tau)\sigma^2 \gamma_{th}}{\eta\tau}\right\}$$

$$= 1 - F\left(\frac{(1-\tau)\sigma^2 \gamma_{th}}{\eta\tau}\right)$$

Delay Tolerent

$$R_{DT}(\tau) = (1-\tau)\tau$$

$$(1-\tau)\, E\left(\log\left(1 + \frac{\eta(1+\tau)}{\tau |g|^2 |h|^2}\right)\right)$$

$$= (1-\tau)\int_0^\infty \log(\quad)\, F_Z(z)$$

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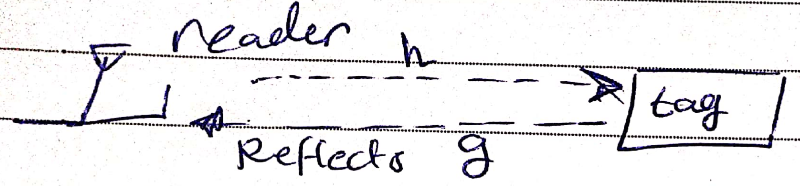
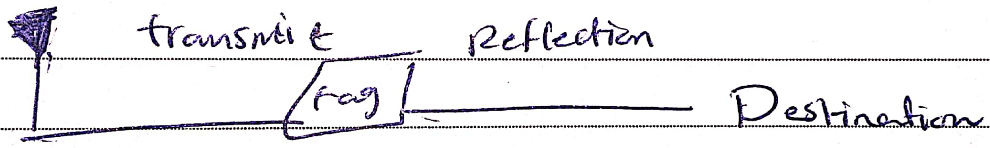


$g = h$ in Monostatic (Fully correlated channels).

I transmit and Recieve using the same antenna

Bistatic



$$SNR = \frac{|h|^2 |g|^2 P}{\sigma^2}$$