$\frac{24.5}{30}$

الجامعة الأردنية	0301201 تفاضل وتكامل 3	الامتحان الثاني: 2019/11/27
اسم الطالب: [redacted] الرقم الجامعي: [redacted]		مدرس المادة: د عمار ابراهيم وقت المحاضرة: ٩ _ ١٠

يتكون الامتحان من 7 أسئلة في 3 ورقات

[1] Let $\vec{r}(t) = \langle e^t \cos t,\ e^{2t},\ e^t \sin t\rangle$.

(a) (3 marks) Sketch the graph of $\vec{r}(t)$.

$(x)^2 = (e^t \cos t)^2$

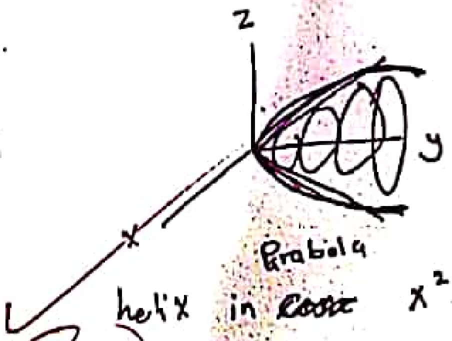
$y = e^{2t}$

$(z)^2 = (e^t \sin t)^2$

$\frac{x^2}{e^{2t}} + \frac{z^2}{e^{2t}} = 1$

$= \frac{x^2}{y^2} + \frac{z^2}{y^2} = 1$

$x^2 + z^2 = y^2$



Prabola

helix in ~~Cone~~ $x^2 + z^2 - y^2 = 0$

3

(b) (4 marks) Find the equation of the tangent line to the curve of $\vec{r}(t)$ at the point (1,1,0).

$\vec{r}'(t) = \langle -e^t \sin t + e^t \cos t,\ 2e^{2t},\ e^t \cos t + \sin t\, e^t \rangle$

$e^{2t} = 1$

$\boxed{t = 0}$

$r'(0) = \langle -e^0 \sin 0 + e^0 \cos 0,\ 2e^0,\ e^0 \cos 0 + e^0 \sin 0 \rangle$

$r'(0) = \langle 1, 2, 1 \rangle$

$x = 1 + t$

$y = 1 + 2t$

$z = t$

4

على ظهر الورقة الاخير

[2] (4 marks) Find the arc length of the curve $\vec{r}(t) = \sqrt{2t}\, i + e^t\, j + e^{-t}\, k$, $0 \le t \le 1$.

$\vec{r}'(t) = \langle \frac{2}{2\sqrt{2t}},\ e^t,\ -e^{-t} \rangle$

$\vec{r}'^2(t) = \langle \frac{1}{2t},\ e^{2t},\ e^{-2t} \rangle$

$\int_0^1 \sqrt{\frac{1}{2t} + \underbrace{e^{2t} + e^{-2t}}_{0}}\, dt = \int_0^1 \sqrt{\frac{1}{2t} + 0}\, dt$

$\int_0^1 \sqrt{\frac{1}{2t}}\, dt$

$\int_0^1 \left(\frac{1}{2t}\right)^{\frac{1}{2}} dt = \frac{2}{3}\left(\frac{1}{2t}\right)^{\frac{3}{2}}\Big]_0^1$

$\frac{2}{3}\sqrt{\left(\frac{1}{2t}\right)^3}\Big]_0^1$

$\frac{2}{3}\left(\frac{1}{8} - \infty\right.$

$\frac{1}{2t} + \underbrace{e^{2t} + e^{-2t}}_{0}$

$\frac{1}{2t} + \frac{2t}{2t}$

$\frac{1 + 2t}{2t}$

$e^{2t} + \frac{1}{2t} + e^{-2t}$

$(e^t \quad 1 \quad (e^t$

$3\frac{1}{2}$

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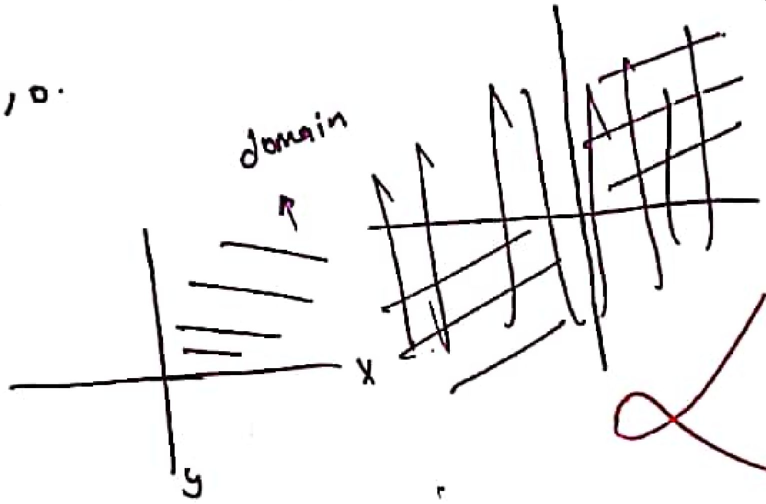
[3] (3 marks) Sketch the domain of the function $f(x,y) = \frac{1}{\sqrt{xy-1}}$

$xy - 1 \geq 0$

$xy \geq 1$



[4] (4 marks) Find $\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$ if it exists. $= \frac{0}{0}$!!

along $y \to x=0$

$\lim_{(x,y)(0,0)} \frac{(0)\,y^4}{0+y^2} = \frac{0}{y^2} = 0$

along $x \to y=0$

$\lim_{(x,y)(0,0)} \frac{x\cdot(0)}{x^2+0} = \frac{0}{x^2} = 0$

$\lim_{(x,y)(0,0)} \frac{(y^4)^2 (y^4)}{y^8 + y^2} = \frac{y^8}{2y^8} = \frac{1}{2}$

$\lim_{(x,y)} f(x,y)$ d.N.E

1 : 4
2 : 8
1 : 4

$x = y^4$

[illegible]

4

[5] (4 marks) Let $f(x,y) = \sqrt[3]{x^3+y^3}$. Use the definition of the derivative to find $f_x(0,0)$.

$f_x(0,0) = \lim_{h\to 0} \frac{f(h+0,0) - f(0,0)}{h}$

$\lim \frac{f(h,0) - f(0,0)}{h}$

$f(h,0) = \sqrt[3]{h^3 + 0}$

$f(h,0) = h$

$\lim \frac{h-0}{h} = \frac{\cancel{h}}{\cancel{h}} = 1$

3

POWERUNIT

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[6] (4 marks) Find the tangent plane to the surface $z = \frac{2x+3}{4y+1}$ at the point (0,0).

$Z = Z_0 + f_x(x-a) + f_y(y-b)$

$Z_0 = \frac{2(0)+3}{4(0)+1} = \frac{3}{1} = 3$

$Z = 3 + 2(x-0) + -12(y-0)$

$f_x = \frac{(4y+1)2 - (2x+3)(0)}{(4y+1)^2}$

$2x - 12y - 2 + 3 = 0$

4

$f_x(0,0) = \frac{2}{1} = 2$

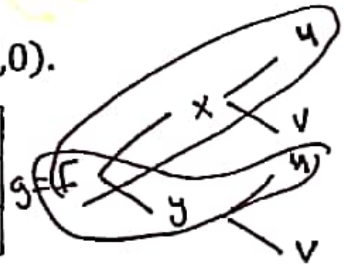
$f_y = \frac{(4y+1)(0) - (2x+3)4}{(4y+1)^2} =$

$f_y(0,0) = \frac{-12}{1} = -12$

POWERUNIT

[7] (4 marks) Suppose that f is a differentiable function and $g(u,v) = f(e^u + \sin v, e^u + \cos v)$. Use the table to find $g_u(0,0)$.

	f	g	f_x	f_y
(0,0)	3	6	4	8
(1,2)	6	3	2	5



$x = e^u + \sin v$

$\frac{dg}{du} = \frac{dg}{dx} \cdot \frac{dx}{du} + \frac{dg}{dy} \cdot \frac{dy}{du}$

$\frac{dx}{du} = e^u + 0 \qquad \frac{dx}{du}(0,0) = e^0 = 1$

$= f_x \cdot \frac{dx}{du} + f_y \cdot \frac{dy}{du}$

$y = e^u + \cos v$

$\frac{dy}{du} = e^u + 0 \qquad \frac{dy}{du}(0,0) = e^0 = 1$

$= 4 \cdot 1 + 8 \cdot 1$

$4 + 8 = 12$

3

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