الأحد 2019/10/27	تفاضل وتكامل 3: الامتحان الأول	الجامعة الأردنية
مدرس المادة: د. عمار ابو اجج وقت المحاضرة: ٨ - ١٠		اسم الطالب: [redacted] الرقم الجامعي: [redacted]

يتكون الامتحان من 6 أسئلة في 3 ورقات

[1] Let $\bar{a} = \langle 2,-2,1\rangle, \bar{b} = \langle 3,-1,2\rangle$

(a) (2 marks) Find $Proj_{\bar{b}}\,\bar{a}$ $= \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\cdot\vec{b} \Rightarrow \dfrac{10}{(\sqrt{14})^2}\cdot\langle 3,-1,2\rangle$

2

$\vec{a}\cdot\vec{b} = 6+2+2 = 10$

$|\vec{b}| = \sqrt{9+1+4} = \sqrt{14}$ $\Rightarrow \left\langle \dfrac{30}{14}, \dfrac{-10}{14}, \dfrac{20}{\sqrt{14}} \right\rangle$

(b) (2 marks) Let $\bar{c}$ be a vector with magnitude 3 that makes an angle $\frac{\pi}{3}$ with x-axis, an angle $\frac{\pi}{4}$ with y-axis and an obtuse angle with z-axis. Find $\bar{c}$.

$\cos^2 a + \cos^2 B + \cos^2\alpha = 1$

$\cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{4} + \cos^2\alpha = 1$

$\frac{1}{4} + \frac{1}{2} + \cos^2\alpha = 1$

$\cos^2\alpha = \frac{4}{4} - \frac{3}{4}$

$\cos^2\alpha = \frac{1}{4}$

$\cos\alpha = \frac{1}{2}$

$\boxed{\alpha = \frac{\pi}{6}}$

1

$\frac{1}{4} + \frac{1}{2}$

$\frac{3}{4}$

$\frac{4}{4} - \frac{3}{4}$

$\frac{1}{4}$

$\vec{c} = 3\left\langle \cos\frac{\pi}{3}, \cos\frac{\pi}{4}, \cos\frac{\pi}{6} \right\rangle$

$\vec{c} = \left\langle \frac{3}{2}, \frac{3}{\sqrt{2}}, \frac{3\sqrt{3}}{2} \right\rangle$

[2] (3 marks) Write an equation of the sphere centered at the point (1,3,0) and touches the plane $2x - y + 5z = 1$.

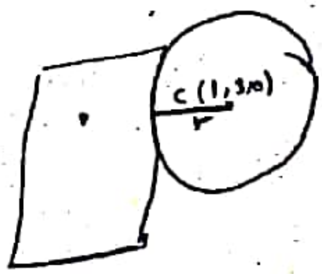
$2-3+0=1$

$\vec{P}: \langle 2,-1,5\rangle$

$2=0$

$2=1$

$2x - y + 5 = 1$



[3] (3 marks) Find parametric equations of the line through the point (2,1,−3) and parallel to the two planes $x+y-z=2$ and $2x-y+3z=1$

$P_1 : \langle 1, 1, -1 \rangle$

$P_2 : \langle 2, -1, 3 \rangle$

$$P_1 \times P_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{vmatrix} = \langle 2, -5, -3 \rangle$$

$x = 2 + 2t$

$y = 1 - 5t$

$z = -3 - 3t$

3

$t = \frac{2}{2}$

$t = 1 - y$

$y = 1 - t$

[4] (3 marks) Does the point $(1,2,-3)$ belong to the line $\frac{x-2}{3} = 1 - y = \frac{z}{2}$? If not, find an equation of the plane containing them.

$x = 2 + 3t$

$y = 1 - t$

$z = 2t$

$t = 0$

$R(2, 1, 0)$

$\vec{PC} : \langle 1, -1, 3 \rangle$

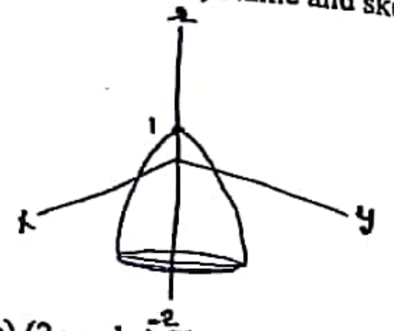
$L : \langle 3, -1, 2 \rangle$

$$\vec{PC} \times L = \begin{vmatrix} i & j & k \\ 1 & -1 & 3 \\ 3 & -1 & 2 \end{vmatrix}$$

$= \langle 1, 7, 2 \rangle$

$P \Rightarrow 1(x-1) + 7(y-2) + 2(z+3) = 0$

[5] (a) (2 marks) Name and sketch the surface $x^2 = 1 - z - y^2$



$\cancel{y^2 + y^2 + z - 1 = 0}$

$x^2 + y^2 = 1 - z$

Paraboloid by z-axis

2

(b) (2 marks) Sketch the surface described by the inequalities $\sqrt{x^2 + y^2} \leq z \leq 4$

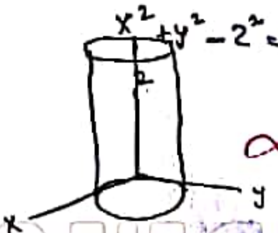
$z = \sqrt{x^2+y^2}$

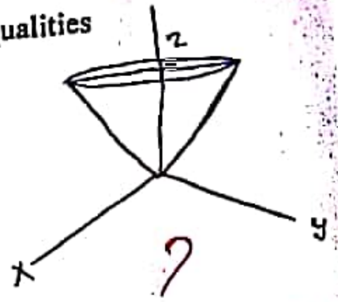
$z^2 = x^2 + y^2$

$x^2 + y^2 - z^2 = 0$

$4 = \sqrt{x^2+y^2}$

$16 = x^2 + y^2$





Cone by z-axis

?

circle cylinder by z-axis

0

[6] (3 marks) Consider the points $A(2,1,-3)$, $B(2,-2,1)$ and $C(1,1,4)$.

Are the points collinear? If yes, find the equation of the line containing them, otherwise find the area of the triangle made up of them.

$5 + \sqrt{19} \neq 5\sqrt{2}$ Not Collinear

$d(AB) = \sqrt{0^2 + 9 + 16} = \sqrt{25} = 5$

$d(BC) = \sqrt{1 + 9 + 9} = \sqrt{19}$

$d(AC) = \sqrt{1 + 0 + 49} = \sqrt{50} = \sqrt{2 \times 25} = 5\sqrt{2}$

area of traingle $= \frac{1}{2}$ | Parallelogram

$\vec{AB} = \langle 0, -3, 4 \rangle$

$\vec{BC} = \langle -1, 3, 3 \rangle$

$$\text{area} = |\vec{AB} \times \vec{BC}| = \begin{vmatrix} i & j & k \\ 0 & -3 & 4 \\ -1 & 3 & 3 \end{vmatrix} = \langle -21, -4, -3 \rangle$$

$|\vec{AB} \times \vec{BC}| = \sqrt{21^2 + 4^2 + 3^2}$

$\sqrt{9 + 16 + 441}$

$= \sqrt{466}$

3

area of tringle $= \frac{1}{2} \times \sqrt{466}$