NO.

Sunday ~~26/9~~ 29/9

*

$$\text{unit step} \rightarrow \frac{4}{s^2+4s+4} \rightarrow \text{scope}$$

⇒ For second or first order system the cofficent of the ~~domin~~ denominator are all the same then the system is stable, further more for stable system the steady state value due to unit step is $\boxed{\lim_{s\to 0} G(s) = T.F}$

⇒ $\text{unit step} \rightarrow \frac{5s+4}{s^2+4s+4} \rightarrow \text{scope}$

* الربط ما اله علاقة بال stability انما له علاقة بال transient ال steady state

⇒ Routh's stability criteria :-

Ex:- given the characteristics equ.: $s^3 + 4s^2 + 4s + K = 0$

s^3	1	4	0
s^2	4	K	0
s^1	$\frac{16-K}{4}$	0	
s^0	K		

* the system is stable when all the ~~coff~~ charges are the same

① $K > 0$

② $16 - K > 0$

$\underline{0 < K < 16}$

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NO.

* Given the following system — Tuesday 1/10/2019

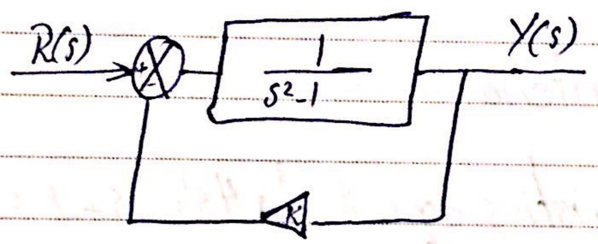
$$\underline{\underline{G(s)}} = \frac{1}{s^2-1}$$

T.F after reduction

i) Study this system :- the system will be always unstable irrespective of the nature of excitation

so, it's need stabilization

Think of a simple controller such as "negative feed back"



$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{1}{s^2+K-1}$$

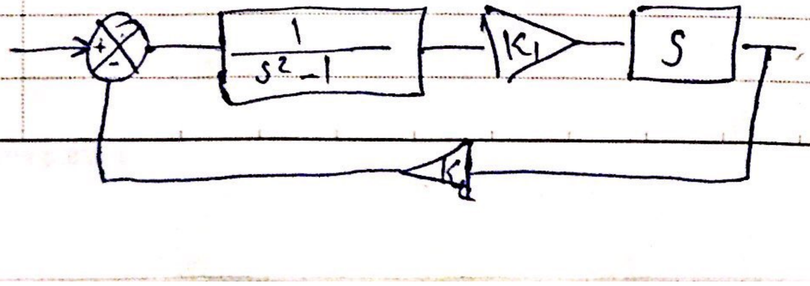
$K>1$, sustained oscilliation

still doesn't work

Conglusion :- Think of diminshing oscilliation by damping

(ال S هي اللي بتعمل damping)

$\Rightarrow$ if one tries a design like this



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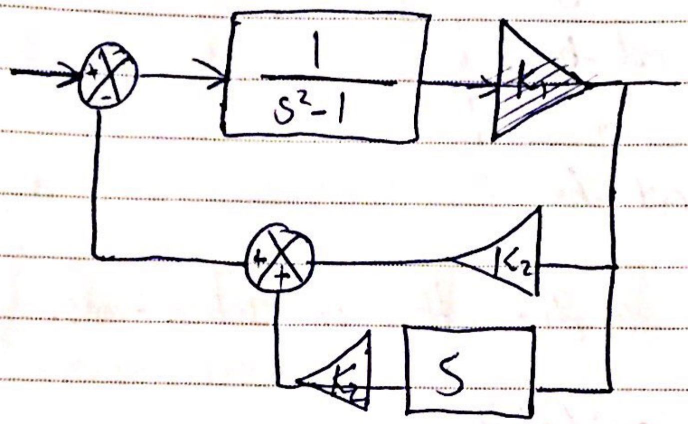
⇒ so, $\frac{Y(s)}{R(s)} = \frac{K_1 S}{S^2 - 1 + K_1 K_2 S} = \frac{K_1 S}{S^2 + K_1 K_2 S - 1}$

↳ we will cancel this sign to make it stable

⇒



⇒ If there is a change in sign or a missing power in the characteristic equation → unstable system

thursday 3/10

⇒ methods based on calculation are not welcomed.

* Symbolic matlab programming:-

```
>> syms t s a b c d
>> A= [a b;c d]
>> dA = det(A) , invA = inv(A)
```

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

```
dA = a*d - b*c
```

$invA = \begin{bmatrix} \frac{d}{a*d - b*c} , & \frac{-b}{a*d - b*c} \\ \frac{-c}{a*d - b*c} , & \frac{a}{a*d - b*c} \end{bmatrix}$

```
>> simplify (invA)   % the same answer
```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NO.

>> pretty (invA) or >> pretty (simplify (invA))

$$\begin{bmatrix} \frac{d}{ad-bc}, & \frac{-b}{ad-bc} \\ \frac{c}{ad-bc}, & \frac{a}{ad-bc} \end{bmatrix}$$

>> AT = A' % gives the conjicate transpose

[conj(a) conj(c)]

[conj(b) conj(d)]

>> transpose(A)

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

*other examples:-

>> A = [0 1; -2 -3] , B = [0; 1] ; C = [5 1]

>> G = C* inv(s*eye(2) - A) *B

>> collect (G)

ans = (s+5)/(s^2 + 3*s +2)

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Sunday 6/10

```
>> Syms s t lam1 lam2

>> A= [0 1 ; -lam1*lam2  lam1+lam2],

>> [r e] = eig (A)
```

$r \rightarrow$ eigen vector $\begin{bmatrix} 1/\lambda_1 , & 1/\lambda_2 \\ 1 , & 1 \end{bmatrix}$

$e \rightarrow$ eigen value $\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

```
>> A=[0 1; -lam1*lam2  lam1+lam2], [r e] = eig(A), trace(A)
det(A), inv (A)
```

trace(A): diagonal مجموع element

Ex:-

$$\frac{dx}{dt} = \overbrace{\begin{bmatrix} -7 & 3 & 3 \\ -6 & 1 & 4 \\ 0 & 1 & -2 \end{bmatrix}}^{A} x + \overbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}^{B} u$$

$$y = \overbrace{[1 \ 0 \ 1]}^{C} x + \overbrace{0}^{D} u$$

$$G(s) = C\left[sI_3 - A\right]^{-1} B + D$$

```
>> A = [          ] , B = [1; 0; 1], C = [1 0 1]
D = 0 , G = C*inv(s*eye(3) - A) * B + D

>> collect(G)   % the same as simplify (G)
```

FIVE APPLE

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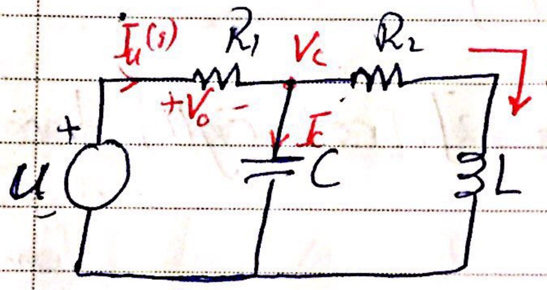
⇒ to ~~obtain~~ obtain time response

* System modeling:-

Tuesday 8/10

i) <u>Input-Output Approach</u> (s-domain): best illustrated by an electrical example



$$\Rightarrow I_u(s) = \frac{U(s) - V_c(s)}{R_1}$$

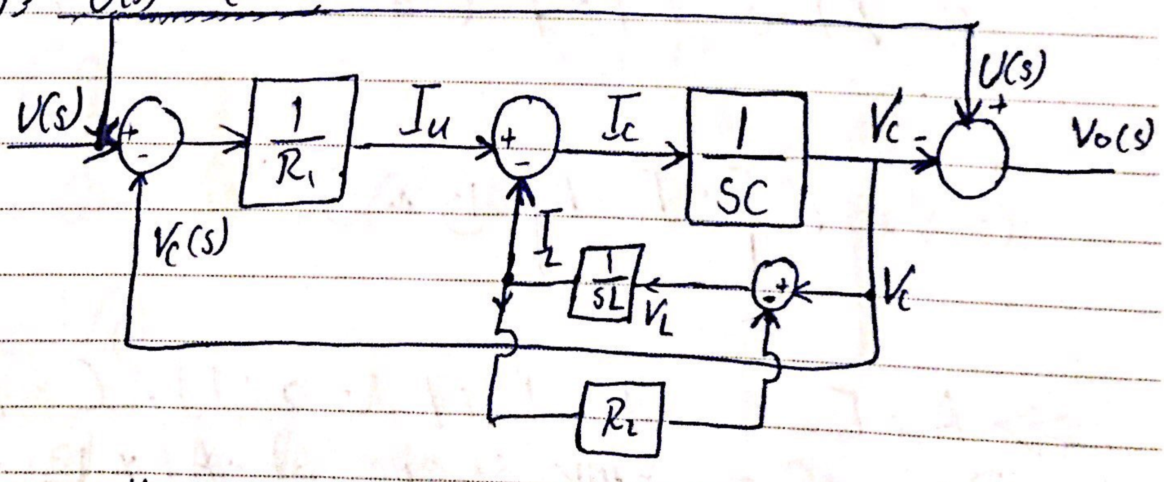
$$\Rightarrow I_c = sC\,V_c(s).$$

$$\Rightarrow \therefore V_c(s) = \frac{1}{sC} I_c(s)$$

$$\Rightarrow I_L(s) = I_u(s) - I_c(s)$$

$$\Rightarrow V_c(s) = I_L(s) R_2 + V_L(s) \qquad \Rightarrow V_L(s) = sL\,I_L(s)$$

$$\Rightarrow V_o(s) = U(s) - V_c(s)$$



Ex: reduce the above block diagram; obtain $\frac{V_o(s)}{V_i(s)}$

FIVE APPLE

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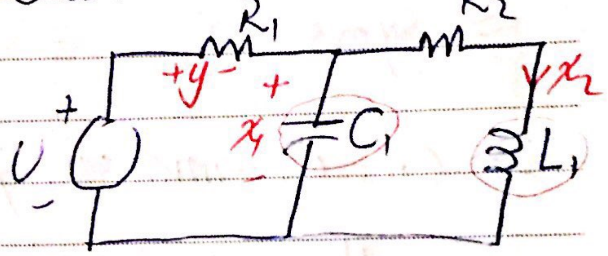
⇒ Simulate the system using $R_1 = 0.5\Omega$, $R_2 = 4\Omega$
$C = 0.2F$, $L = 0.125H$

Thursday 10/10

ii) A time domain approach modeling : the state space approch we write, obtain or get first order differential equation.

best illustrated consdering an electrical circuit



let $x_1 = V_c$, $x_2 = I_L$

$$C\dot{x}_1 = \frac{u - x_1}{R_1} - x_2$$

$$L\dot{x}_2 = x_1 - R_2 x_2$$

$$y = u - x_1$$

$\Rightarrow \dot{x}_1 = 10u - 10x_1 - 5x_2$; subsitute $R_1 = 0.5$, $R_2 = 4$, $C = 0.2F$, $L = 0.125H$

$$\boxed{\begin{aligned} \dot{x}_1 &= -10x_1 - 5x_2 + 10u \quad \text{---} (1) \\ \dot{x}_2 &= 8x_1 - 32x_2 \quad \text{---} (2) \\ y &= -x_1 + u \quad \text{---} (3) \end{aligned}}$$

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$$\dot{x} = \underbrace{\begin{bmatrix} -10 & -5 \\ 8 & -32 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 10 \\ 0 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} -1 & 0 \end{bmatrix}}_{C} x + \underbrace{1}_{D} u$$

```
>> A=[ ], B=[ ], C=[ ], D=1;
>> syms s
>> G=C*inv(s*eye(2)-A)*B+D
>> pretty(collect(G))
```

$$\boxed{\frac{s^2+32s+40}{s^2+42s+360} = G(s)}$$

by using matlab

* if D not equal zero so the order of numerator is the same as the order of ~~denominator numerator~~ ~~denominator~~ denominator *

* the pools are -30, -12 | >> roots([1, 42, 360])

>> poles(G)

*

* calculate the eigen values of A

>> [r, e] = eig(A)

↳ e: eigen values

FIVE APPLE

NO.

Sunday 13/10

* Obtaining state-Space models using diff. equ and T.F

⇒ N.B: The state variable are not unique. However the Outputs are.

Conclusion:- A system can have many state-space representations (in fact an infinite number).

⇒ First Case:- the DE's do not include dervation of the $u(t)$

Given the following D.E:

$$u(t) = y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \dots + a_n y \quad , \quad y^{(n)} = \frac{dy^{(n)}}{dt^{(n)}}$$

Since the system is n^{th} order a minimum of n states are needed.

تحول.

let $x_1 = y \longrightarrow \dot{x}_1 = \dot{y} = x_2$

$x_2 = \dot{y} \longrightarrow \dot{x}_2 = \ddot{y} = x_3$

$x_3 = \ddot{y}$

$\vdots$

$x_n = y^{(n-1)} \qquad \dot{x}_n = y^{(n)} = -a_1 y^{(n-1)} - a_2 y^{(n-2)} - \dots - a_n y + u(t)$

$= -a_1 x_n - a_2 x_{n-1} - \dots - a_n x_1 + u$

FIVE APPLE

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stair-case form

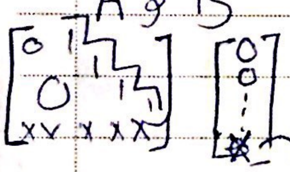
$$\Rightarrow \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 1 \\ \vdots & & & & \\ -a_n & \cdots & -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u$$

$$y = [1\ 0\ 0 \cdots 0]\, x + 0u$$

⇒ This known as the controllable ~~form~~ companian phase-variable representation

⇒ stair-Case name relates to matrix A controllable form relate to the special structure of A & B

Tuesday 15/10



* **Example:** $y^{(2)} = \frac{dy^2}{dt^2} = 2u$

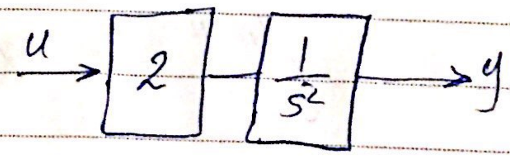
Sol:

$x_1 = y$

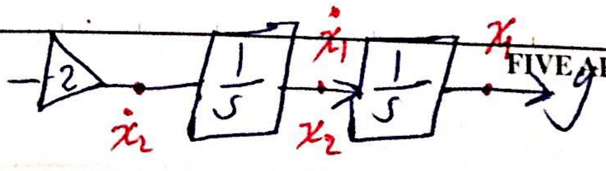
$x_2 = \dot{y} = \dot{x}_1$

$\dot{x}_2 = \ddot{y} = 2u$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$



$$y = [1\ \ 0]\, x + 0\, u$$



FIVE APPLE

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Example:- Given a system described by:-

$$y''' + 5y'' + 9y' + 5y = 4u$$

, y is the output of the system

$\Rightarrow x_1 = y$

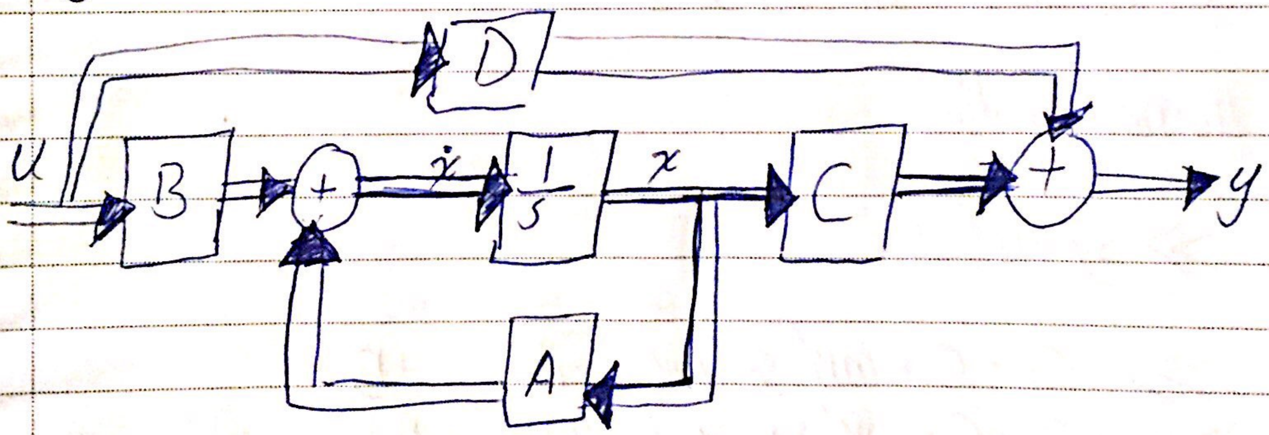
$x_2 = \dot{y} = \dot{x}_1$

$x_3 = \ddot{y} = \dot{x}_2$

$x_4 = \dddot{y} = \dot{x}_3 = -5x_1 - 9x_2 - 5 + 4u$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -9 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0]\, x + 0u$$



$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

FIVE APPLE

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NO. Thursday 17/10

Ex: Given i) $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = \begin{bmatrix} 5 & 1 \end{bmatrix} x$

obtain the differential equ. describing the system

ii) $\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & 11 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} u$

$y = \begin{bmatrix} 20 & 9 & 1 \end{bmatrix} x$

⇒ The transferfunction matrix is given by

$G(s) = C[s I_n - A]^{-1} B + D$

using matlab

```
>> syms s

>> G = C*inv(s*eye(2) - A)*B + B
or >> G = C*inv(s* eye(size(A)) - A)*B + D
```

$G(s) = \dfrac{s+5}{s^2+3s+2}$

or

```
>> [n,d] = ss2tf(A,B,C,D) , g2 = tf(n,d)
```

$g2 = \dfrac{s+5}{s^2+3s+2}$

FIVE APPLE

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>> n = [5 1]; d = [1 -3 2]; [A, B, C, D] = ~~tf2ss(n,d)~~ tf2ss(n,d)

20/10 sunday

* state space analysis :- (ch 9)

⇒ Give a system described by

$$y^{(n)} + a_1 y^{(n-1)} + \dots + a_n y = b_0 u^{(n)} + b_1 u^{(n-1)} + \dots + b_n u$$

taking the laplace transform (all intial conditions are zero)

$$\boxed{\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \dots + b_n}{s^n + a_1 s^{n-1} + \dots + a_n}}$$

⇒ Some of Convenient forms are the following :-

i) Controllable companian form

رفيقي

stair-Case

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{bmatrix} x + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [\underbrace{b_n - a_n b_0} \quad \underbrace{b_{n-1} - a_{n-1} b_0} \quad \dots \quad \underbrace{b_1 - a_1 b_0}] x + b_0 u$$

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ii) Observable Companion form

$$\dot{x} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & -a_n \\ 1 & 0 & 0 & \cdots & 0 & -a_{n-1} \\ 0 & 1 & 0 & \cdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -a_1 \end{bmatrix} x + \begin{bmatrix} b_n - a_n b_0 \\ \vdots \\ b_1 - a_1 b_0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} x + b_0 u$$

N.B: For both forms the T.F $(G(s) = C(sI_n - A)^{-1}B + D)$ is identically the same

>> A=[0 1 0 ; 0 0 1 ; -6 11 -6] , Poly (A)

>> A2=[-7 3 3 ; -6 1 4 ; 0 1 -2] , Poly (A2)

III Model form (Digital and/or Block diagonal form): Tuesday 22/10
⇒ Best illustrated by an example

Given the system $G(s) = \dfrac{2s^2+3s+4}{s^2+9s+20} = \dfrac{Y(s)}{U(s)}$

$\therefore G(s) = 2 + \dfrac{-15s-36}{(s+4)(s+5)}$

$$\begin{array}{r|l} & 2 \\ \hline s^2+9s+20 & 2s^2+3s+4 \\ & -(2s^2+18s+40) \\ \hline & -15s-36 \end{array}$$

$= \dfrac{24}{s+4} - \dfrac{39}{s+5} + 2$

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$$\Rightarrow Y(s) = \underbrace{\frac{24}{s+4}}_{x_1(s)} u(s) + \underbrace{\frac{-39}{s+5}}_{x_2(s)} u(s) + 2u(s)$$

① $x_1(s) = \frac{24}{s+4} u(s) \Rightarrow s x_1(s) + 4 x_1(s) = 24 u(s)$

Take lablace inverse $\mathcal{L}^{-1}(s x_1(s) + 4 x_1(s) = 24 u(s))$

$$\Rightarrow \dot{x}_1 + 4x_1 = 24u$$

$$\therefore \boxed{\dot{x}_1 = -4x_1 + 24u}$$

② $x_2(s) = \frac{-39}{s+5} u(s) \Rightarrow s x_2(s) + 5 x_2(s) = -39 u(s)$

$$\mathcal{L}^{-1}(s x_2(s) + 5 x_2(s) = -39 u(s)) = \boxed{\dot{x}_2 = -5x_2 - 39u}$$

$$\therefore Y(s) = X_1(s) + X_2(s) + 2U(s)$$

$$y(t) = x_1(t) + x_2(t) + 2u(t)$$

$$\dot{x} = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix} x + \begin{bmatrix} 24 \\ -39 \end{bmatrix} u$$

eigen values

if this is Zero uncontrollability

* To check validity of representation

$$G(s) = C[sI_n - A]^{-1} B + D$$

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12 The Jordan Form :- (needed when the poles are identical)

Suppose :- $\frac{Y(s)}{U(s)} = \frac{n_1}{(S+P_1)^3} + \frac{n_2}{(S+P_1)^2} + \frac{n_3}{S+P_1}$

$$Y(s) = \underbrace{\frac{n_1 U}{(S+P_1)^3}}_{X_1} + \underbrace{\frac{n_2 U}{(S+P_2)^2}}_{X_2} + \underbrace{\frac{n_3 U}{(S+P_1)}}_{X_3}$$

let $Y(s) = n_1 X_1 + n_2 X_2 + n_3 X_3$

① where $X_3 = \frac{U}{S+P_1} \Rightarrow \boxed{\dot{X}_3 = -P_1 X_3 + U}$ --- ①

② $x_2 = \frac{U}{(S+P_1)^2}$, $\frac{1}{(S+P_1)} * \frac{U}{(S+P_1)}$, $\frac{1}{S+P_1} x_3(s)$

$\mathcal{L}^{-1}((S+P_1)x_2(s) = x_3(s)) \Rightarrow \boxed{\dot{x}_2 = -P_1 x_2 + x_3}$ --- ②

③ $x_1 = \frac{U}{(S+P_1)^3}$, $\frac{1}{(S+P_1)} * \frac{U}{(S+P_1)^2}$, $\frac{1}{(S+P_1)} * x_2(s)$

$\Rightarrow \boxed{\dot{x}_1 = -P_1 x_1 + x_2}$ -- ③

$$\dot{x} = \begin{bmatrix} -P_1 & 1 & 0 \\ 0 & -P_2 & 1 \\ 0 & 0 & -P_3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [n_1 \ n_2 \ n_3] x + 0 u$$

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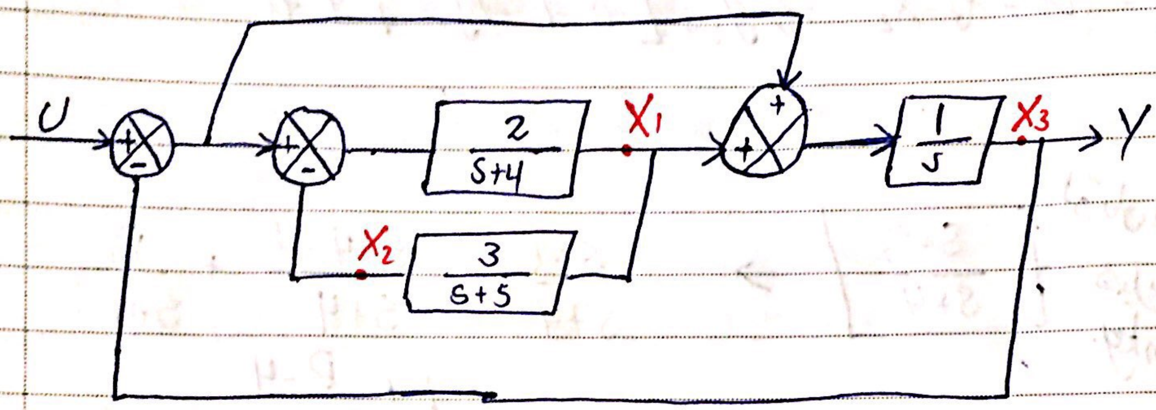
Thursday 24/10

⇒ Obtaining a state Space model from a block diagram:-

don't attempt to reduce the diagram. Best illustrated by an example:-



$\Rightarrow X_3 = \frac{1}{s}\left(X_1 + (U - X_3)\right)$

$sX_3 = X_1 + U - X_3 \quad \Rightarrow \quad \boxed{\dot{x}_3 = x_1 - x_3 + u}$

$\Rightarrow X_1 = \frac{2}{s+4}\left(U - X_3 - X_2\right)$

$sX_1 + 4X_1 = -2X_2 - 2X_3 + 2U \quad \Rightarrow \quad \boxed{\dot{x}_1 = -4x_1 - 2x_2 - 2x_3 + 2u}$

$\Rightarrow X_2 = \frac{3}{s+5}(X_1) \quad \Rightarrow \quad \boxed{\dot{x}_2 = 3x_1 - 5x_2}$

$\Rightarrow Y = X_3$

$\therefore \dot{x} = \begin{bmatrix} -4 & -2 & -2 \\ 3 & -5 & 0 \\ 1 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} u$

* مهما اختلف افرض ال s
بالرتم لازم ال diagonal
نفس الاشي ونفس ال dot

$y = [0 \quad 0 \quad 1]\, x + 0u$

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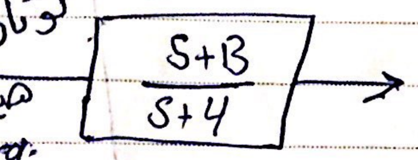
⇒ For the

$y_2 = u - x_3 - x_2 \Rightarrow y_2 = [0\ -1\ -1]x + 1 \cdot u$

⇒ لوكان ال block هيك . نحوله لصيغة $\frac{1}{s}$ Intg.

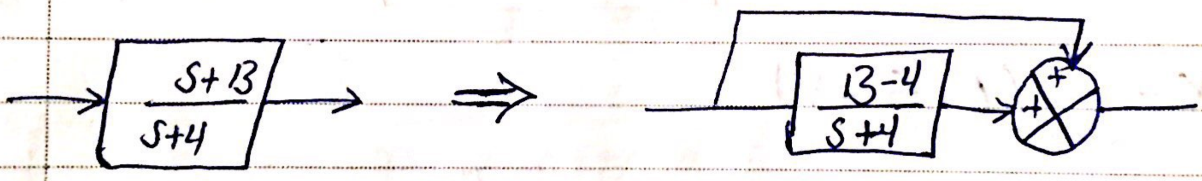


$\frac{s+13}{s+4} = \frac{s+4-4}{s+4} + \frac{13}{s+4}$

$= 1 + \frac{13-4}{s+4}$

⇒ Variation :-

1) if having $\frac{s+13}{s+4} = 1 + \frac{13-4}{s+4}$



Exercies :- det state 6 as shown. check your answer using In diagram.
trans det and T.F

NO. Sunday 27/10

⇒ Obtaining an S.S model using T.F :-

best illustrated by an example

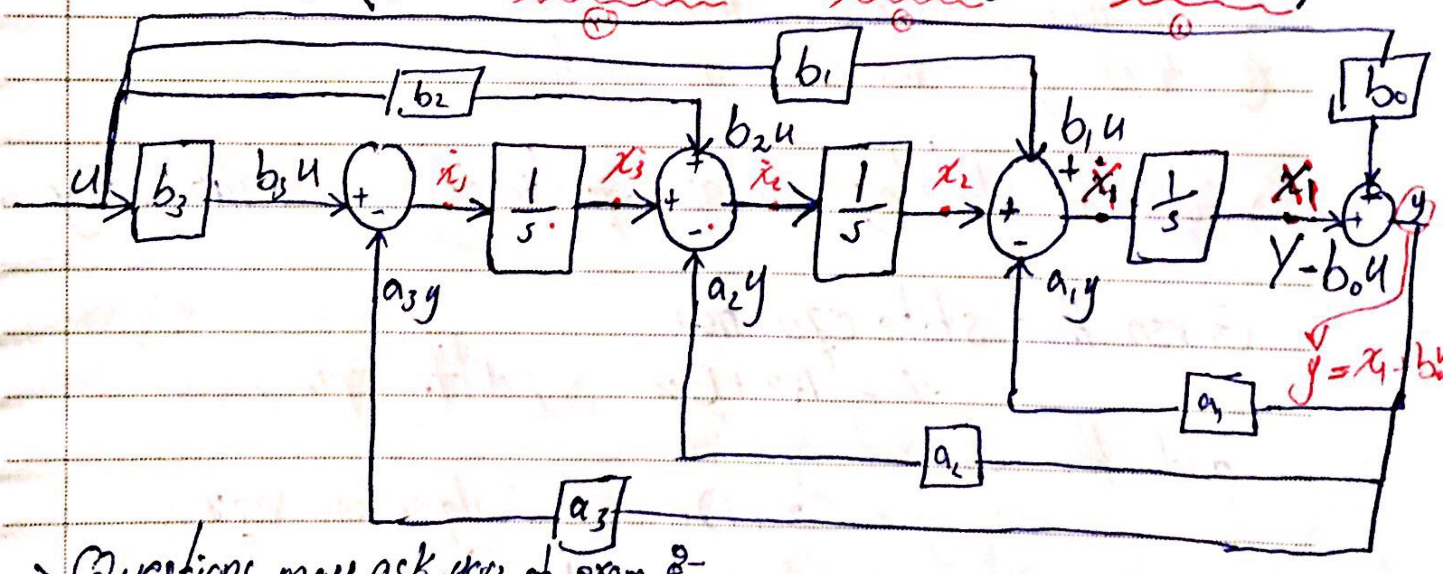
$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^3 + b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

It can be shown that : $(Y_{(s)} - b_0 u_{(s)})$

$$(Y_{(s)} - b_0 u_{(s)}) = \frac{1}{s^3}\left((b_1 u - a_1 y)s^2 + (b_2 u - a_2 y)s + (b_3 u - a_3 y)\right)$$

$$= \frac{1}{s}\left((b_1 u - a_1 y) + (b_2 u - a_2 y)\frac{1}{s} + (b_3 u - a_3 y)\frac{1}{s^2}\right)$$

$$= \frac{1}{s}\left(\frac{1}{s}\left[(b_3 u - a_3 y)\frac{1}{s} + (b_2 u - a_2 y)\right] + (b_1 u - a_1 y)\right)$$



⇒ Questions may ask you at exam :-

① given a Transfer function obtain a block diagram only

② and use it to find state space representation

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$$\Rightarrow \dot{x} = \begin{bmatrix} -a_1 & 1 & 0 \\ -a_2 & 0 & 1 \\ -a_3 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} b_1 - a_1 b_0 \\ b_2 - a_2 b_0 \\ b_3 - a_3 b_0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x + b_0 u$$

>> Check the answer using matlab

```
>> syms a1 a2 a3 b0 b1 b2 b3 s
>> A=[ ] , B=[ ] , C=[ ] , D=[ ]
>> G= C* inv(s*eye(size(A)) -A)*B + D
>> pretty (collect(G))
```

Ex: given $G(s) = \dfrac{2s^2 + 3s + 4}{s^2 + 5s + 6}$

① obtain a SS reperesntation

② check your answer using matlab

*Solution of state-Space Equations:-

29/10 Tuesday

Given that state equation

$\dot{x} = Ax + Bu$; diff. equ.

and the Output equation

$y = Cx + Du$; algebraic equ

it can be shown that $x(t) = e^{A(t-t_0)} x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)} B u(\tau)\, d\tau$

where e^{At} is known as the exponential matrix behind it

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NO.

$$\Rightarrow e^{At} = I + At + A^2\frac{t^2}{2!} + A^3\frac{t^3}{3!} + \dots + A^N\frac{t^n}{N!} \; ;$$

sutiable numerically

or $\boxed{e^{At} = \mathcal{L}^{-1}(sI_n - A)^{-1}}$; given a closed form solution

*Ex:- det e^{At} using as many methods as you know, given: $\dot{x} = Ax + \underline{B}u$ — 31/10 Thursday

where $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$; $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$; $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

check your answer using matlab command expm( )

```
>> A = [0 1; -2 -3]
>> g = expm(A*t)
```

*Properties of the exponetial matrix e^{At}:

1) $e^{At}\big|_{t=0} = I_n$, used to check regarding the validity of e^{At}

2) $|e^{At}| > 0$

3) $e^{A(t_1+t_2)} = e^{At_1} \cdot e^{At_2}$, t = scaler

4) $e^{(A+M)t} = e^{At} \cdot e^{Mt}$, Provided $AM = MA$ & B, A matrix

5) $[e^{At}]^{-1} = e^{A(-t)}$

6) $\frac{d}{dt}(e^{At}) = Ae^{At} = e^{At} \cdot A$

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7) $\int e^{At}\,dt = [e^{At} - I_n]\cdot A^{-1}$; inverse لازم ال A يكون الها

8) trace $e^{At} > 0$

9) $e^{(At)^n} = e^{A(tn)}$

10) $e^{A(t-t_2)} \cdot e^{A(t_2-t_1)} = e^{A(t-t_1)}$

in the book $\phi(t) = e^{At}$, $\phi(t)$ is Known as transition matrix

* Time Solution due to step input :-

3/11 Sunday

$\Rightarrow$ let $\overset{i/p}{u(t)} = K\,u(t)$

then $\boxed{x(t) = e^{At}x(0) + e^{At}\int_0^t e^{-A\tau} B\, u(\tau)\, d\tau}$ حفظ

$\Rightarrow$ to prove the above equ. :-

$\dot{x} = Ax + Bu$

$e^{-At}\dot{x} = e^{-At}Ax + e^{-At}Bu$

$e^{-At}(\dot{x}) - e^{-At}Ax = e^{-At}Bu$

$\frac{d}{dt}(e^{-At}x(t)) = e^{-At}B\cdot u$

Integrate from $0 \rightarrow t$

$e^{-A(\tau)}x(\tau)\Big|_0^t = \int_0^t e^{A(-\tau)} B\,u(\tau)\, d(\tau)$

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NO.

⇒ It can be shown that (see book)

$$\boxed{x(t) = e^{At}x(0) + [e^{At} - I_n]A^{-1}B\,K} \rightarrow \text{const.}$$

* If the system is asympototically stable and matrix A is invertible then:

$$\boxed{x_{ss} = \lim_{t\to\infty} x(t) = -A^{-1}BK}$$

For step input

⇒ It can be also shown that the respons due to ramp input of value $tu(t)\underset{\text{const.}}{V}$

$$\boxed{x(t) = e^{At}x(0) + [e^{At}A^{-2} - A^{-2} - tA^{-1}]V}$$

N.B.: For a time-invarent system (the ones we study)

if $u(t) \longrightarrow x(t)$

then $\frac{d}{dt}u(t) \longrightarrow \dot{x}(t)$

$\int u(t) \longrightarrow \int x(t)\,dt$

Ex.: given $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}x + \begin{bmatrix} 0 \\ 6 \end{bmatrix}u$, calculate $x(t)$ due to a unit step excitation using as many methods as you known. check x_{ss} ~~your~~ including matlab, Plot relevant graphs.

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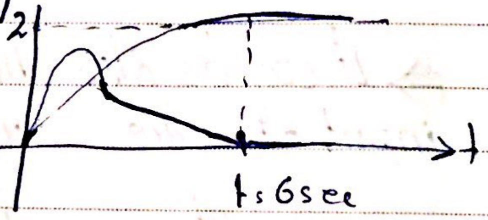
Example: $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u$, asympotically stables

since $\lambda[A] = \{-1, -2\} \Rightarrow$ a steady state value due to step input exist

Simulate using state space block with $C = I_2$ and $D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

by matlab

$\Rightarrow$ you will get something like that



by cal

$\Rightarrow x_{ss} = -A^{-1} B R = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \Rightarrow x_{ss1} = 2$

$x_{ss2} = 0$

(1) Determination of eigenvalues of a square matrix:

$\Rightarrow$ Given a transformation $T(\cdot)$ & a non-zero vector x

$\Rightarrow$ what is x such that $T(x) \propto x \overset{i.e}{\Rightarrow} T(x) = \lambda x$

$\Rightarrow$ If the transformation is linear then it can be represented by a matrix leading to $Ax = \lambda x$

$\Rightarrow \underset{n\times n}{A} \underset{n\times 1}{x} = \overset{?}{\lambda} \overset{?}{x} = \underset{n\times 1}{x} \underset{1\times 1}{\lambda}$

لازم تكون لها inverse

$[A - \lambda I_n] x = 0$

$\boxed{|A - \lambda I_n| = 0}$ $\rightarrow$ gives a monic n^{th} order polynomial solved by eigen value λ

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NO. Thursday 7/11

⇒ to determine the eigen vectors solve the singular system of equations:

$$[A-\lambda_i I_n]x_i = 0$$

Use Gauss-Elimination method

Ex:- let $A = \begin{bmatrix} -5 & 3 & 3 \\ -6 & 3 & 4 \\ 0 & 1 & 0 \end{bmatrix}$ ⇒ $|A-\lambda I_3| = \begin{vmatrix} -5-\lambda & 3 & 3 \\ -6 & 3-\lambda & 4 \\ 0 & 1 & 0-\lambda \end{vmatrix} = 0$

⇒ $\lambda^3 + 2\lambda^2 - \lambda - 2 = 0$

$(\lambda+1)(\lambda-1)(\lambda+2) = 0$

$\lambda_1 = -1$, $\lambda_2 = 1$, $\lambda_3 = -2$; eigen values

N.B:- Σ eigen values = negative cofficient of second highest order only if the cofficient of higest power is 1

$\lambda_1 + \lambda_2 + \lambda_3 = -2$ ✓

⇒ to determine the eigen vector associated with $\lambda = -2$

Solve $\begin{bmatrix} -3 & 3 & 3 \\ -6 & 5 & 4 \\ 0 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$m = \frac{-6}{-3} = 2$

Pivot = 1 $\begin{bmatrix} -3 & 3 & 3 \\ 0 & -1 & -2 \\ 0 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ⇒ $\begin{bmatrix} -3 & 3 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$m = \frac{1}{-1} = -1$

diagonal تحت الـ

stop ← zero

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$\Rightarrow$ let $\boxed{x_3 = r}$

$-x_2 - 2x_3 = 0 \Rightarrow \boxed{x_2 = -2r}$

$-3x_1 + 3x_2 + 3x_3 = 0 \Rightarrow \boxed{x_1 = -r}$

$\therefore x = \begin{bmatrix} -r \\ -2r \\ r \end{bmatrix}, x \neq 0 \Rightarrow r \neq 0$

let $r = -1 \Rightarrow x = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

Ex: Calculate the remaining two eigen vector

* Determination of eigen vectors using the adjoint method

$\Rightarrow$ best illustrated by an example:

Suppose $\text{adj}(A - \lambda_3 I_3) = \text{adj}\begin{bmatrix} -3 & 3 & 3 \\ -6 & 5 & 4 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -3 & -3 \\ 12 & -6 & -6 \\ -6 & 3 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

N.B: Revise calculating the adjoint

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الخميس

Thursday

⇒ Calculation e^{At} using ~~an example~~ eigen vectors :

let V be an $n \times n$ matrix representing the eigen vector of A; it can be shown that:

$$e^{At} = V e^{\Lambda t} V^{-1} = V * \begin{bmatrix} e^{\lambda_1 t} & & 0 \\ & e^{\lambda_2 t} & \\ 0 & & \ddots \; e^{\lambda_n t} \end{bmatrix} * V^{-1}$$

⇒ only valid when the eigen values are distinct (different)

```
>> A = [0 1; -2, -3]
>> [r, e] = eig(A)
>> syms t
>> expA = r*expm(e*t)*inv(r)
```

Example:- let $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, it can be shown that

$$\Lambda = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \text{ or } \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \text{ in which case}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$V = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \text{ or } \begin{bmatrix} +1 & +1 \\ -2 & -1 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} * \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix} * \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$$

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* Generalized eigen vectors :-

<u>motivation:</u> let $A = \begin{bmatrix} 0 & 1 \\ -4 & 4 \end{bmatrix} \Rightarrow \lambda_1 = \lambda_2 = 2$

To check

مج λ = مج الـ diagonal

$\lambda_2 * \lambda_1 =$ الـ det

eigen vector associated with $\lambda_1 = 2$

$$V_i = adj\left(\begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix}\right) = \begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \Rightarrow V_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

For the second eigen value we calculate a generalized eigen vector as follows :-

$$[A - \lambda_1 I]\, w_2 = w_1$$

$$[A - \lambda_2 I]^2\, w_2 = 0 \qquad ; \text{remember } \left|[A - \lambda I]\right| = 0$$

يعني ملهاش inverse

so, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} w_2 = 0$

w_2 any vector independent of V_1 ; so, w_2 may be ~~any number~~ $w_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

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Example: let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 27 & -27 & 9 \end{bmatrix}$, $\lambda[A] = \{3, 3, 3\}$

$\gg [r, e] = eig(A)$ X

$r = \begin{bmatrix} 0.1048 & 0.1048 & 0.1048 \\ 0.3145 & 0.3145 & 0.3145 \\ 0.9435 & 0.9435 & 0.9435 \end{bmatrix}$ $V_1 = V_2 = V_3$

so, it doesn't work

$e = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\gg [r, e] = Jordan(A)$

generilaized eigen vector

$r = \begin{bmatrix} 9 & -3 & 1 \\ 27 & 0 & 0 \\ 81 & 27 & 0 \end{bmatrix}$

eigen vector

$\Rightarrow$ if vectors can be constitutes square matrix then they are independent only if the determine of that matrix $\neq$ zero

$e = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$, called Jordan form

ei

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NO. 12/11/2019 Tuesday

* Independant of eigen vectors. or vectors in general :-

⇒ independance of vectors easily judged algebrically as follows

1) generate a matrix contains those vectors
2) look for a square sub matrix having non-zero determinant
3) If the determinant not zero then the vectors are independent

Ex: $M_1 = [V_1\ V_2] = \begin{bmatrix} 1 & -5 \\ -2 & -10 \end{bmatrix}$

$|-5| \neq 0$ ⇒ at least One independent eigen vector (IV)

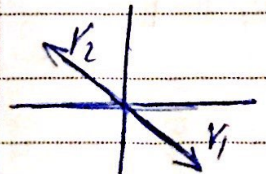
$\begin{vmatrix} +1 & -5 \\ -2 & -10 \end{vmatrix} = 0$ ⇒ no 2 IV

$M_2 = [V_1\ V_2] = \begin{bmatrix} 1 & -2 \\ -4 & 8 \\ -2 & +4 \end{bmatrix}$ ⇒ no 2x2 submatrix of non zero determinant
V_1 & V_2 are dependent

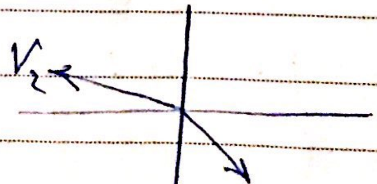
$M_3 = [V_1\ V_2] = \begin{bmatrix} +1 & -2 \\ -4 & 8 \\ 2 & 4 \end{bmatrix}$ ⇒ there exist a 2x2 sub matrix of a non-zero determinant
V_1 & V_2 are independent

* theorium for a square matrix to have an inverse the coloumbs should be independent.

⇒ Geometrically :-



"dependent"



"independent"

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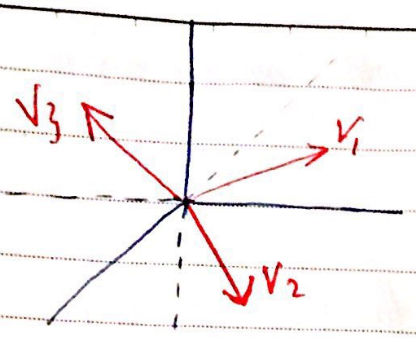

⇒ if all V_1, V_2, V_3 are in plane ⇒ dependent

⇒ if at least one of them is not on a plane containing the other ⇒ independent

*** Similarity transformation:-** a method to get a special form of matracies ($\bar{A}$) of matrix (A)

Ex:- what are the eigen values and the determinante of

$$A = \begin{bmatrix} -7 & 3 & 3 \\ -6 & 1 & 4 \\ 0 & 1 & -2 \end{bmatrix} \quad \text{and} \quad A^{-1} = P^{-1} A P \quad \text{where} \quad P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

by simulink $\bar{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

$\det(A) = \det(\bar{A}) = -1 * -4 * -3 = -12$

$\lambda[A] = \{-1, -4, -3\}$

Thursday
12/11/2019

*** Jordan forms:-**

certain matrices of repeated eigen values can have the following forms after similarity transformation

Ex:- A matrix 3X3 may have three identical eigen values say -2, -2, -2, when transformed of the following form may result:

$$\bar{A} = P^{-1} * A * P$$

; where P contain eigen vectors and generalized eigen vectors as coloumbs

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⇒ $\bar{A}$ may look like $\begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix}$ ← not diagonal matrix

or like this $\begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ or like this $\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix}$

or this extreme case $\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

* Exponential matrix of diagonal and Jordan forms:

⇒ If $A = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_3 \end{bmatrix}$ then $e^{At} = \begin{bmatrix} e^{\lambda_1 t} & 0 & 0 \\ 0 & e^{\lambda_2 t} & 0 \\ 0 & 0 & e^{\lambda_3 t} \end{bmatrix}$

⇒ If $A = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$ then $e^{At} = \begin{bmatrix} e^{\lambda t} & te^{\lambda t} \\ 0 & e^{\lambda t} \end{bmatrix}$

⇒ If $A = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}$ then $e^{At} = \begin{bmatrix} e^{\lambda t} & te^{\lambda t} & \frac{t^2}{2} e^{\lambda t} \\ 0 & e^{\lambda t} & te^{\lambda t} \\ 0 & 0 & e^{\lambda t} \end{bmatrix}$

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Ex:- Use properties of matrices to evaluate e^{At} when A:

① $A = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix}$

④ $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

② $A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$

⑤ $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$

③ $A = \begin{bmatrix} -5 & 1 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$

* The Cayley-Hamiltan Therom (CH theorem): Very Useful

⇒ given the characteristic polynomial $p(\lambda)$ of a matrix

i.e $p(\lambda) = |A - \lambda I_n| = |\lambda I_n - A| = 0$

$= \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \cdots + a_n = 0$

then $P(A) = 0$

i.e $A^n + a_1 A^{n-1} + a_2 A^{n-2} + \cdots + a_n I_n = 0$

i.e every square matrix satisfy it's characteristic ~~equation~~ polynomial

⇒ in MATLAB: $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ no error in matlab

⇒ in linear algebra:- cannot be done undefined operation

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17/11/2019 Sunday

Ex: let ① $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$

$|A - \lambda I| = \lambda^2 - 3\lambda - 10 = 0$ } ⇒ 1 , -3 , -10

```
>> poly(A)
```

Hence $A^2 - 3A - 10I_2 = O_{2\times 2}$

```
>> A^2-3*A-10*eye(2)
ans =   0   0
        0   0
```

② $A = \begin{bmatrix} -7 & 3 & 3 \\ -6 & 1 & 4 \\ 0 & 1 & -2 \end{bmatrix}$

```
>> Poly(A)
ans=
   1  8  19  12
```

∴ $CE = \lambda^3 + 8\lambda^2 + 19\lambda + 12 = 0$

```
>> A^3+8*A^2+19*A+12
ans   0  0  0
      0  0  0
      0  0  0
```

* If we want to calculate A^3 :

1) $A^3 = -8A^2 - 19A - 12$

2) by matlab to check your answer $A^3 = A^3$

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NO.

$\Rightarrow$ if you want $\underline{A^4} = -8A^3 - 19A^2 - 12A$

$$A^4 = -8A^3 - 19A^2 - 12A$$
$$= -8(-8A^2 - 19A - 12I_3) - 19A^2 - 12A$$
$$A^4 = 45A^2 + 140A + 96I_3$$

Exercies: Use the CH theorem to calculate A^{-1}

sol:

A^{-1} * $\quad A^3 + 8A^2 + 19A + 12I_3 = 0$

$$A^2 + 8A + 19I_3 + 12A^{-1} = 0$$
$$\therefore A^{-1} = -\frac{(A^2 + 8A + 19A)}{12}$$

Tuesday 19/11/2019

***Eigen values properties and characterstics: (See notes)**

① $\lambda[M^T] = \lambda[M] = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$; IF $M = M^T$ eigen values are real numbers

② $\lambda[\alpha M] = \alpha\lambda[M]$

③ $\lambda[M\cancel{N}] = \lambda[NM]$

④ $\lambda[P^{-1}MP] = \lambda[M]$; Similarity transformation

⑤ $\lambda[M^{-1}] = \dfrac{1}{\lambda[M]}$

⑥ $\lambda[M + \alpha I] = \lambda[M] + \alpha$.

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NO.

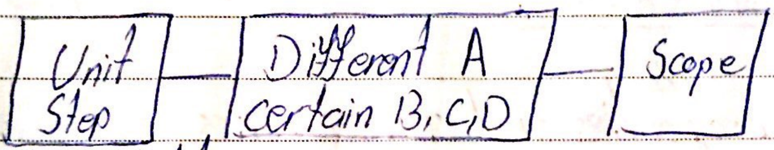
* Structural properties of System:

Stability

⇒ a stable system doesn't grow without bond due to a bounded excitiation

⇒ Stability of linear systems is determined by the eigen values of A

⇒ Demeonstration



check stability visually and using eig(A)

Sunday 24/11

⇒ -ve real part of $\lambda\{A\}$ then the system is stable

Ex: $\dot{x} = \begin{bmatrix} -5 & 3 & 3 \\ -6 & 3 & 4 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} u$

trace(A) = -5 + 3 + 0 = -2 ; not conclusive to determine stability

Suppose we change -5 to -2

trace(A) = -2 + 3 + 0 = 1 ; unstable because there at least a positive real part
conclosive decision

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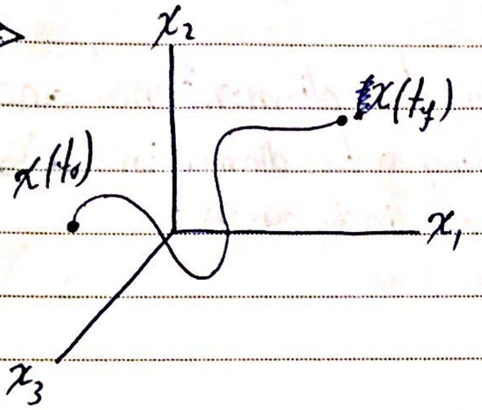
* Controllability:-

⇒ A system is called controllable if it is possible to more the states of the system $x(t_0)$ at $t = t_0$ to any arbitrary final state $x(t_f)$ in finite time using an unconstrained control law

⇒



$x(t_0)$ & $x(t_f)$ are not contained within a plane

⇒ controllable in $\mathbb{R}^3$

Sunday 1/12/2019

⇒ Test for controllability (cc)

Def: n vectors $v_1, v_2, \ldots, v_n$ are ~~id~~ independent

if $c_1 v_1 + c_2 v_2 + \cdots + c_3 v_3 = 0$

implies $c_1 = c_2 = \cdots = c_n = 0$

Rank: The maximum number of independent coloumbs of an matrix $n \times m$

Ex: $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $v_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

sol: mathematically $\begin{bmatrix} 1 \\ 2 \end{bmatrix} c_1 + \begin{bmatrix} 2 \\ 1 \end{bmatrix} c_2 = 0 \Rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0$

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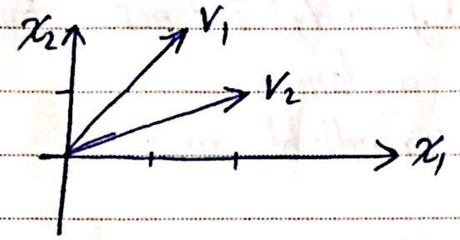
NO.

$$\Rightarrow \overbrace{\cancel{\frac{-1}{3}\begin{bmatrix}1 & -2\\ -2 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}}}^{I_2}\begin{bmatrix}c_1\\ c_2\end{bmatrix} = \frac{-1}{3}\begin{bmatrix}1 & -2\\ -2 & 1\end{bmatrix}\begin{bmatrix}0\\ 0\end{bmatrix}$$

* نضرب بال inv

$$I_2\begin{bmatrix}c_1\\ c_2\end{bmatrix} = \begin{bmatrix}c_2\\ c_1\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$; then they are independent

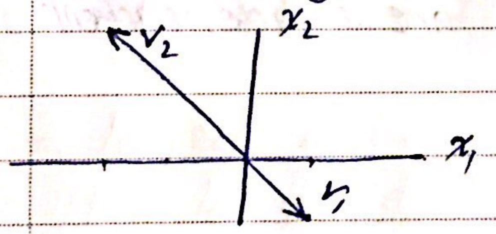
⇒ geometrically :-



being two diomentional and Spaning a two diomentional plane hence: independent.

Ex:- $V_1 = \begin{bmatrix}1\\ -2\end{bmatrix}$, $V_2 = \begin{bmatrix}-2\\ 4\end{bmatrix}$

⇒ geometrically :-



dependent

⇒ mathmetically :-

$$\begin{bmatrix}1 & -2\\ -2 & 4\end{bmatrix}\begin{bmatrix}c_1\\ c_2\end{bmatrix} = 0$$

satisfied For all $c_1 = 2c_2 \neq 0$ ⇒ dependent

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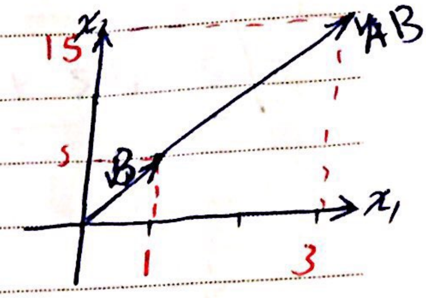
NO. Controllability test

[1] The Rank test: A system is completely (CC) iff

$$\operatorname{rank}(M) = \operatorname{rank}([B \ \ AB \ \ A^2B \ \ \cdots \ \ A^{n-1}B]) = n$$

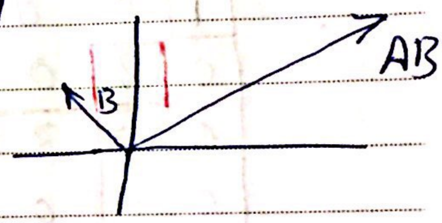
Ex: $\dot{x} = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}_{n\times n} x + \begin{bmatrix} 1 \\ 3 \end{bmatrix} u$



solution: $M = [B \ \ AB] = \begin{bmatrix} 1 & 5 \\ 3 & 15 \end{bmatrix}$

$\operatorname{rank}(M) = 1 \Rightarrow$ uncontrollable

If $B = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \Rightarrow \operatorname{rank}(M) = \operatorname{rank}\begin{bmatrix} -1 & 7 \\ 3 & 9 \end{bmatrix} = 2$ controllable



Ex: ① $\operatorname{rank}\left(\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}\right) = 2$

② $\operatorname{rank}\left(\begin{bmatrix} -7 & 21 \\ 8 & -24 \end{bmatrix}\right) = 1$ ③ $\operatorname{rank}\left(\begin{bmatrix} 4 & -8 & 4 \\ 2 & -4 & 2 \\ -3 & 6 & -3 \end{bmatrix}\right) = 1$

N.B: For cases where $M_{n\times n} = [B \ \ AB \ \cdots \ A^{n-1}B]$ Tuesday 3/12/2019
like single-input system then the system is cc iff $|M| \neq 0$

[2] Diagonalization test: cc can be judged visually (by inspection) after diagonalizing the system through a similarity transformation

$\Rightarrow x = Sz$, $|S| \neq 0$

$z = S^{-1}x$

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$\Rightarrow \quad \dot{x} = Ax + Bu$

$S\dot{z} = ASz + Bu$

$S^{-1}S\dot{z} = S^{-1}ASz + S^{-1}Bu$

$\dot{z} = S^{-1}AS\,z + S^{-1}Bu$

$\therefore \boxed{\dot{z} = Jz + B_z u}$ where $J = S^{-1}AS$

$B_z = S^{-1}B$

S : contains the eigen vector of A as columbs

$\Rightarrow J$ is a block diagonal (i.e of the form).

$$J = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

$\Rightarrow$ To judge CC using J & B_z :

1) Know to Jordan blocks should be associated the same eigen value

2) the last row in B_z should be non-zero

3) For distinct eigen value the row of z associated with it should be non-zero

Ex: $J = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$, $B_z = \begin{bmatrix} -1 \\ 0 \\ -7 \\ 0 \end{bmatrix}$

sol: -2 is CC

-3 is $\overline{CC}$

4 is $\overline{CC}$

$\therefore$ the system is uncontrollable

Ex: 9.12 (P 650)

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Tuesday 3/12/2017

*Judging controbility through the (T.F):-

$\Rightarrow$ Given $\dot{x} = Ax + Bu \quad x \in R^n$

$y = Cx + Du \quad u \in R^m$

$$G(s) = C[sI_n - A]^{-1}B + D$$

assuming the system is observable; then the system is completlly controllable iff there is no poles-zeros cancallation in the T.F

Ex: $G(s) = \dfrac{s+3}{s^2-3-10} = \dfrac{s+3}{(s+2)(s-5)}$; no cancellation $\Rightarrow$ CC

$G(s) = \dfrac{s+4}{s^2+7s+12} = \dfrac{\cancel{s+4}}{\cancel{(s+4)}(s+3)} = \dfrac{1}{s+3}$; cancellation $\Rightarrow \overline{CC}$

Ex:- $\dot{x} = \begin{bmatrix} -8 & 5 & 3 \\ -6 & 0 & 4 \\ 0 & 1 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u$

$y = [1 \ 0 \ 0]x + u$

>> A=[ ]; B=[ ]; C=[ ];
D=[ ];
>> G = C*inv(s*eye(2)-A)*B+D
>> pretty(collect(G))

$\dfrac{s+6}{s+3}$; it must be third order to be CC

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Sunday 8/12

*** Observability:** A system is observable if it's possible to obtain the states at (t_0) from observation of the output over a finite period of time

*** Test for Observability: (OO)**

[1] The Rank test: A system is completly OO iff

$$\text{rank}(N) = \text{rank}\left(\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}\right) = n$$

Ex:
$$\dot{x} = \begin{bmatrix} -20 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 1 & -2 \end{bmatrix} x + \begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix} x$$

$$\text{rank}(N) = \left(\begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}\right) = \text{rank}\left(\begin{bmatrix} 0 & 1 & -2 \\ 0 & -2 & 2 \\ 0 & 2 & 0 \end{bmatrix}\right) = 2 < n \Rightarrow \overline{OO}$$

[2] Diagonalization Test:

let $x = Sz \Rightarrow z = S^{-1}x$

and

$$\dot{z} = S^{-1}ASz + S^{-1}Bu$$
$$y = CSz + Du$$

where columbs of S are the eigen vectors of A

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⇒ The same criteria used for controllability applies except now columns of C_T are used.

Ex: Consider the previous example. $S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1+j & 1-j \\ 0 & 1 & 1 \end{bmatrix}$ = [eigen vector of A]

then

$$\Rightarrow \dot{z} = \begin{bmatrix} -20 & 0 & 0 \\ 0 & -1+j & 0 \\ 0 & 0 & -1-j \end{bmatrix} x + \begin{bmatrix} 5 \\ -j \\ j \end{bmatrix} u$$

$$y = [0 \quad -1+j \quad -1-j] z + D u \quad \Rightarrow -20 \text{ is } \overline{OO}$$

$\boxed{3}$ Cancellation test: examine the transfer function after cancelling poles with zeros

⇒ If the order of $G(s)$ is less than n then the system is $\overline{OO}$

Ex: Consider the previous example.

$$G(s) = C[sI_n - A]^{-1} B + D = \frac{2s}{s^2+2s+2} \Rightarrow \text{order } 2 < n$$

hence $\overline{OO}$

N.B: Stability, Controllability and Observability don't implicate each other; a system can be stable, controlable and obervable ... etc where the total combination is $\underline{8}$ up to unstable, uncontrollable and unobservable.

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N.13:- Cancellation may be due to uncontrollable or unobservable or both, if you have cancellation cheek one of them in order to make right conclousion.

Exercise:- Consider the following system and determine cc & oo using the three methods.

$$\dot{x} = \begin{bmatrix} -5 & 3 & 3 \\ -6 & 3 & 4 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} x$$

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Tuesday 10/12

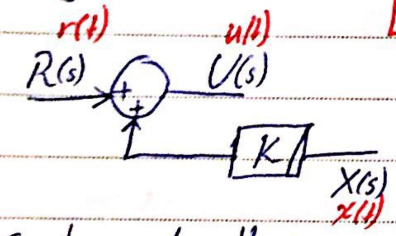
#ch 10: Designe of control System (state Feed Back)

-use Feed Back the states not the output

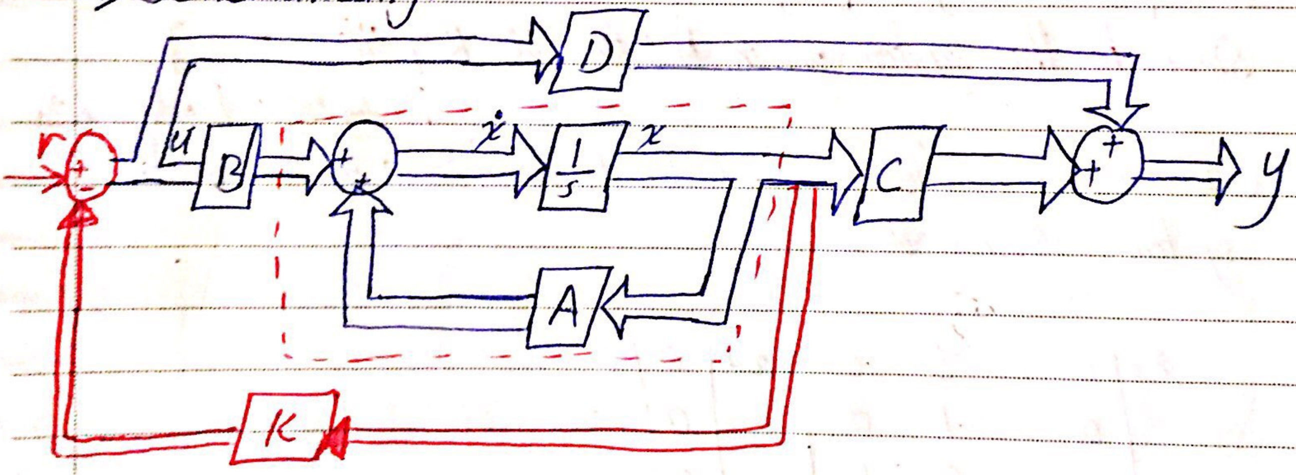
⇒ Consider a system given by: $\dot{x} = Ax + Bu$; $x \in R^n$, $u \in R^n$

In state feed back method the states are fed-back through the input signal u (i.e $u = r - Kx$)

↳ refrence value



⇒ Schematically:-



before introduction of $\underline{K}$ the System is without Feed back

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12/12 Thursday

⇒ stability of linear system is not dicated by the exciting input

⇒ $u = r - Kx$

$$\dot{x} = Ax + B(r - Kx) = \underline{(A - BK)x} + B\,r$$

هي اللي بتحدد ال stability

State feed back seeks a K which results in acceptable system dynamics

motoriational example:-

$$\dot{x} = \begin{bmatrix} -5 & 3 & 3 \\ -6 & 3 & 4 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

Simulate this system when i) $K = 0$ ii) $K = [-4/3 \quad 3 \quad 5]$

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Choosing K Frobenous (controllable form):- in this form the system is described as:-

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4 & -a_3 & -a_2 & -a_1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \quad 1 \rightarrow \neq 0$$

Pro. of this matrix:

① $C.E = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$

② eigen vector $= \begin{bmatrix} 1 \\ \lambda_i \\ \lambda_i^2 \\ \lambda_i^3 \end{bmatrix}$ ⇒ For distinct eigen value

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$\Rightarrow$ in which case

$$A_K = A - BK = A - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} [K_4 \; K_3 \; K_2 \; K_1]$$

$$A_K = A - \begin{bmatrix} & 0 & & \\ K_4 & K_3 & K_2 & K_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4-K_4 & -a_3-K_3 & -a_2-K_2 & -a_1-K_1 \end{bmatrix}$$

$$\dot{x} = (A - BK)x + Br$$
$$\dot{x} = A_K x + Br$$

A_K is required to have desirable eigen value say ($\lambda_1, \lambda_2, \lambda_3$ and λ_4) giving a closed loop C.E

$$\phi(\lambda) = (\lambda-\lambda_1)(\lambda-\lambda_2)(\lambda-\lambda_3)(\lambda-\lambda_4)$$
$$\phi(\lambda) = \lambda^4 + \delta_1\lambda^3 + \delta_2\lambda^2 + \delta_3\lambda + \delta_4$$

$$A_K = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\delta_4 & -\delta_3 & -\delta_2 & -\delta_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4-K_4 & -a_3-K_3 & -a_2-K_2 & -a_1-K_1 \end{bmatrix}$$

there for $\boxed{-a_i - K_i = -\delta_i}$ $\quad i = 1,2,3,4$; solve for $K_i = \delta_i - a_i$

Ex: Consider the following

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \underset{=-a_3}{4} & \underset{=-a_2}{3} & \underset{=-a_1}{2} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

trace(A) = 2 unstable

$\lambda^3 - 2\lambda^2 - 3\lambda - 4 = 0$ unstable

$\lambda^3 - 2\lambda^2 + 3\lambda + 4 = 0$ unstable

$\lambda^3 + 2\lambda^2 + 3\lambda + 4 = 0$ idk

assign $-1, -2, -3$

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$\phi(\lambda) = (\lambda+1)(\lambda+2)(\lambda+3) = \lambda^3 + \underbrace{6}_{\delta_1}\lambda^2 + \underbrace{11}_{\delta_2}\lambda + \underbrace{6}_{\delta_3}$

$K_1 = 6+2 = 8$, $K_2 = 11+3 = 14$, $K_3 = 6+4 = 10$

$\therefore K = [10 \quad 14 \quad 8]$

$\gg eA = eig(A)$, $e_K = eig(A - BK)$

Tuesday 17/12

*Eigen values assigment for general A and B of controllable:-

$\Rightarrow$ Given $\dot{x} = Ax + Bu$ of no special form, $x \in R^n$
$u \in R^1$ (single input)

then to calculate k:-

i) Evalute the C.E., put it in the form of
$\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_n$; a_i given by
$\gg$poly(A)

ii) let $M_{n\times n} = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B]$; $\gg$ctrb(A,B)

iii) let $T = MW = M\begin{bmatrix} a_3 & a_2 & a_1 & 1 \\ a_2 & a_1 & 1 & 0 \\ a_1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

deconv
conv

iv) Calculate $\bar{K}$ as for Frobeniors form

v) $K = \bar{K}T^{-1}$

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Exercise:- Given $\dot{x} = \begin{bmatrix} -5 & 3 & 3 \\ -6 & 3 & 4 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$

Assign $-2, -3$, and -4, for more exerciess chang $\underline{B}$ $\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$

* Ackermann's method:

Given $\dot{x} = Ax + Bu$; $x \in R^n$, $u \in R^1$, system is CC

$\Rightarrow$ to assign $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ as eigen values. Generate $\phi(\lambda)$

$$\phi(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3) \dots (\lambda - \lambda_n) \quad \text{; Closed loop C.E}$$
$$= \lambda^n + \alpha_1 \lambda^{n-1} + \alpha_2 \lambda^{n-2} + \dots + \alpha_n \quad \text{; using conv in matlab}$$

$\Rightarrow K = [0\ 0\ \dots\ 0\ 1][B\ \ AB\ \dots\ A^{n-1}B]^{-1} \phi(A)$

$= [0\ 0\ \dots\ 0\ 1][B\ \ AB\ \dots\ A^{n-1}B](A^n + \alpha_1 A^{n-1} + \alpha_2 A^{n-2} + \dots + \alpha_n I_n)$

Example:- Given $\dot{x} = \begin{bmatrix} -7 & 3 & 3 \\ -6 & 1 & 4 \\ 0 & 1 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} u$ Thursday 20/12

assign $-2 \pm j3$, -5

sol:- $K = [0\ 0\ 1][B\ AB\ A^2B]^{-1} \phi(A)$

$$\phi(\lambda) = (\lambda + 2 - j3)(\lambda + 2 + j3)(\lambda + 5) = \lambda^3 + 9\lambda^2 + 3\lambda + 65$$

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```
>>syms lam

>>(lam+2-j3)*(lam+2+3*j)*(lam+5)
ans
    lam^3 + 9*lam^2 + 33*lam + 65

>> K=[0 0 1]*inv(ctrb(A,B))*(A^3+9*A^2+33*A+65*eye(3))
```

$K = \left[\frac{97}{45} \quad \frac{-61}{45} \quad \frac{14}{45}\right]$

To check the correctness of K :- >>eig(A-B*K)

Sunday 22/12

*State Observers :- Used to estimate the states from the system Outputs(s) in cases, where the state are available due to inaccessiblity or differentes in measurment.

Consider the system $\dot{x} = Ax + Bu$

$y = Cx$

and an observer the following form :-

$$\dot{\tilde{x}} = A\tilde{x} + Bu + K_e(y - \tilde{y})$$
$$\tilde{y} = C\tilde{x}$$

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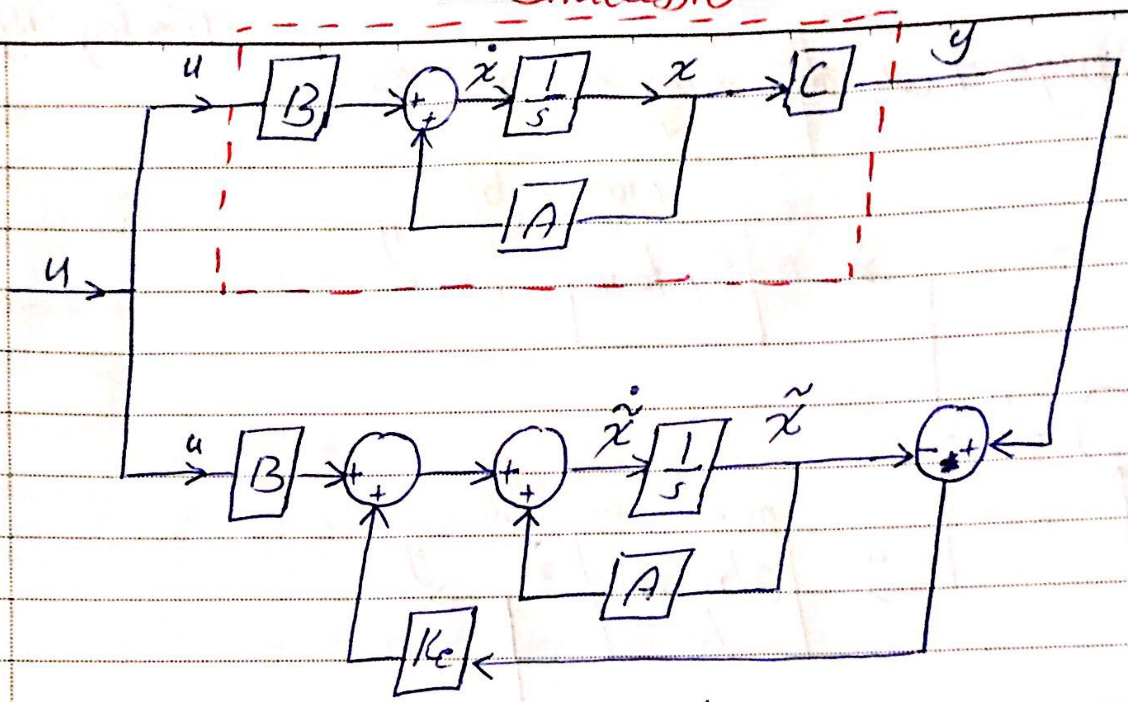
NO. Inaccessible



⇒ The design in observers center on a proper selection of $\underline{Ke}$ such that the estimated state $\underline{\underline{\tilde{x}}}$ approch those inaccessible system state.

⇒ For the seleetion of $\underline{Ke}$ the ackermann's method can be adoppted to have desirable eigen value

⇒ Ackermann's method: $Ke = \phi(A) \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$

Ex: Consider the system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u \qquad \Rightarrow Ke = acker(A', C', [-4, -5])'$$

$$y = [1 \quad 0] x$$

Choose Ke for (A-KeC) to have eigen value -4, -5

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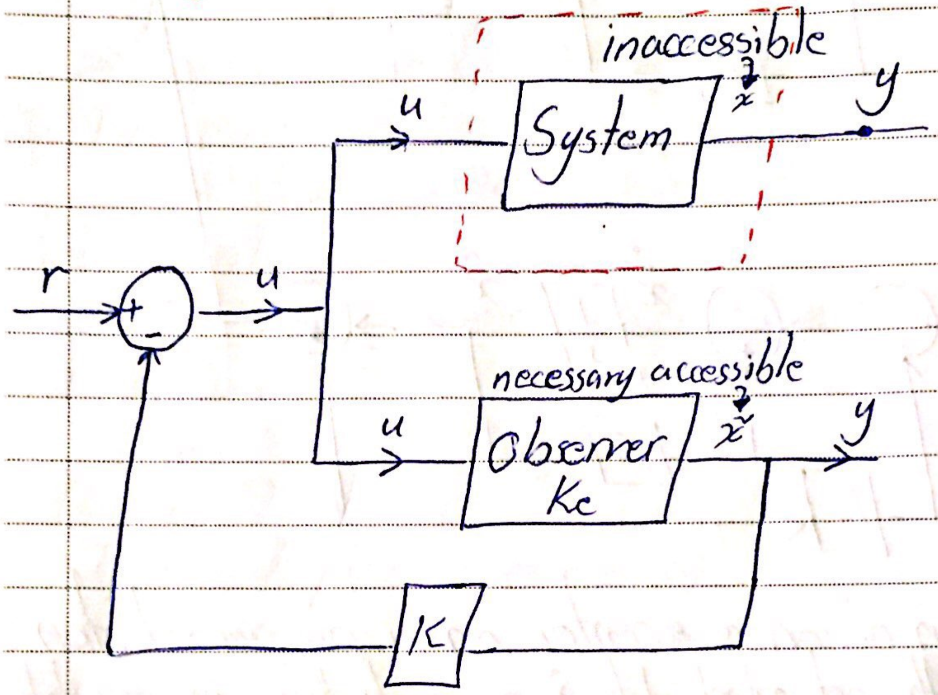
NO. Tuesday 24/12

* Design of controller using observers :



⇒ to calculate $\underline{K}$ use ackermann's method as usuall ending with desirble eigen value for $\underline{\underline{A-BK}}$

⇒ to calculate Ke use ackermann's method ~~as usuall~~ either ~~after~~ after adaptiation or using matlabe ending with desirble eigen values for $\underline{\underline{A-K_eC}}$

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NO. Sunday 29/12

* Quadratic optimal control:

⇒ Given $\dot{x} = Ax + Bu$, optimal control seeks a controller $u = -Kx$ to minimize a performance index.

$$J = \int_0^{\infty} (\underbrace{x^T Q x}_{\text{state}} + \underbrace{u^T R u}_{\text{control law}})\, dt$$

⇒ It can be shown that: $\boxed{K = R^{-1} B^T P}$

where P is positive definite matrix satisfying

$$A^T P + PA - PBR^{-1}B^T P + Q = 0$$

⇒ It can be shown that $J = \underset{1\times n}{x^T(0)}\ \underset{n\times n}{P}\ \underset{n\times 1}{x(0)} = \text{scaler}_{1\times 1}$

Example: See book

using matlab [K, P, E] = Lqr(A, B, Q, R)

Ex: $\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$, $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $R = 1$

```
>> [K,P,E] = lqr(A, B, Q, R)
```

$K = [1 \quad 1.7321]$

$P = \begin{bmatrix} 1.7321 & 1 \\ 1 & 1.7321 \end{bmatrix} \Rightarrow \begin{bmatrix} \sqrt{3} & 1 \\ 1 & \sqrt{3} \end{bmatrix} = P^T$

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$$e = \begin{bmatrix} -0.866 + j0.5 \\ -0.866 - j0.5 \end{bmatrix}$$

⇒ For positive definitess :-

① Symtric

② $P_{11} > 0$

③ $\begin{vmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{vmatrix}$ ✓ det.

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