* Tragenometric Identites:-

$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$.

$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$.

$\sin(2x) = 2\sin(x)\cos(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 \Rightarrow 1 - 2\sin^2(x)$.

$\sin(x)\sin(y) = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$.

$\cos(x)\cos(y) = \frac{1}{2}\left[(\cos(x-y)) + \cos(x+y)\right]$.

$\sin(x)\cos(y) = \frac{1}{2}\left[\sin(x-y) + \sin(x+y)\right]$

$\sin^2(x) = \frac{1}{2}\left[1 - \cos(2x)\right]$

$\cos^2(x) = \frac{1}{2}\left[1 + \cos(2x)\right]$

$\sin^2(x) + \cos^2(x) = 1$

$\sin(\pi/2 \pm y) = \cos(y)$.

$\cos(\pi/2 \pm y) = \mp\sin(y)$.

$\sin(\pi \pm x) = \mp \sin(x)$.

$\cos(\pi \pm x) - \cos(x)$.

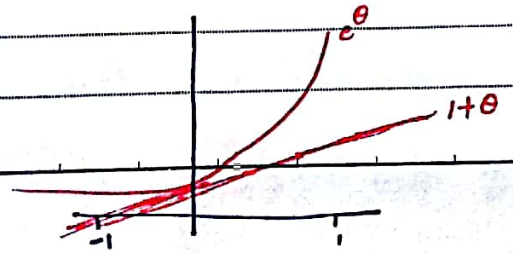
* maclaurian series expansion for:-

* $\cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots = \sum_{K=0}^{\infty} \frac{(-1)^K \theta^{2K}}{(2K)!}$

* $\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots = \sum_{K=0}^{\infty} (-1)^K \frac{\theta^{(2K+1)}}{(2K+1)!}$

* $e^\theta = 1 + \theta + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} = \sum_{K=0}^{\infty} \frac{e^K}{K!}$

* $e^\theta \approx 1 + \theta$ for $\theta \rightarrow$ goes to zero (-1 to 1)



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* complex number :-

let $j = \sqrt{-1}$, the complex number Z is given By

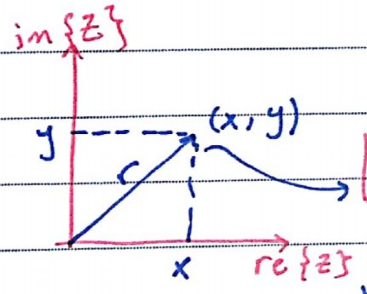
$\boxed{Z = x + jy}$, where x and y are real num.

<u>example:-</u> $3 + 5j$:

real part $= 3$

imaginary part $= 5$

* cartesian representation:-



$$\boxed{Z = x + jy} = \underbrace{r\cos\theta}_{x} + j\,(\underbrace{r\sin\theta}_{y})$$

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

using ⚠

$j = \sqrt{-1}$

$j^2 = -1$

$j^3 = -j$

$j^4 = 1$

$* \; Z = re^{j\theta}$

$\downarrow \; e^{j\theta} = \cos\theta + j\sin\theta$.

* Taylor series:-

$$e^{\theta} = 1 + \theta + \frac{\theta^2}{2!} + \cdots \xrightarrow{e^{j\theta}} 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \cdots$$

$$e^{j\theta} = \underbrace{1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots}_{\cos\theta} + j\underbrace{\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots\right)}_{\sin\theta}$$

$e^{j\theta} = \cos\theta + j\sin\theta \Rightarrow$ Multiply By r

$re^{j\theta} = r\cos\theta + jr\sin\theta$

$re^{j\theta} = r\left[\cos\theta + j\sin\theta\right) \Longrightarrow Z = re^{j\theta}$ ##

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* Cartesian to polar Transf.

let $Z = x + jy$ given, $r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}(\frac{y}{x})$

$\rightarrow Z = re^{j\theta}$

* polar to Cartesian Transf.

$re^{j\theta} = r\cos\theta + jr\sin\theta$

Ex: $Z = 3 + 3j$, Trans Z in polar form.

* Observation:

1. $e^{\pm j\pi} = -1$
2. $e^{j0} = 1$
3. $e^{j\pi/2} = j$
4. $e^{-j\pi/2} = -j$
5. $e^{j2\pi} = 1$
6. $e^{j2\pi K} = 1$, where K is integer, $K \in (-\infty, \infty)$.
7. $e^{-j\infty} = \cos(-\infty) + j\sin(-\infty) =$ undifined.
8. $e^{j\infty} = \cos(\infty) + j\sin(\infty) =$ undifined.

$e^{j\theta} = \cos\theta + j\sin\theta$ --- ①

$e^{-j\theta} = \cos(-\theta) + j\sin(-\theta)$

$= \cos\theta - j\sin(\theta)$ --- ②

① + ② $\Rightarrow \cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$ $\rightarrow \cos(j\theta) = \cosh(\theta)$

$-1 \leq \sin(x) \leq 1$

for Real x

① − ② $\Rightarrow \sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2}$

$-\infty \leq \sin(Z) \leq \infty$

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$\sin x = \frac{1}{17}$

$\sin x = 3 \rightarrow$ real x, complex y

* $\cos x = 3$, find x?

$$\frac{e^{jx} + e^{-jx}}{2} = 3 \Rightarrow \boxed{e^{jx} + e^{-jx} = 6}$$

$$e^{j2x} + 1 = 6e^{jx} \Rightarrow \underbrace{e^{2jx}}_{y^2} - 6\underbrace{e^{jx}}_{y} + 1 = 0$$

$$e^{jx} = +3 \quad , \quad e^{jx} = -2.$$

* operation on two complex numbers.

let $z_1 = x_1 + jy_1 = r_1 e^{j\theta_1}$

$z_2 = x_2 + jy_2 = r_2 e^{j\theta_2}$

1. $z_1 + z_2 = (x_1 \pm x_2) + j(y_1 \pm y_2)$
2. $z_1 \cdot z_2 = (x_1 + jy_1)\cdot(x_2 + jy_2) = \underbrace{(x_1 x_2 - y_1 y_2)}_{Re\{z_1 \cdot z_2\}} + j\underbrace{(x_1 y_2 + y_1 x_2)}_{Im\{z_1 \cdot z_2\}}$

OR $z_1 \cdot z_2 = r_1 e^{j\theta_1} \cdot r_2 e^{j\theta_2}$

$= r_1 r_2 e^{j(\theta_1 + \theta_2)}$

$= r_1 r_2 \cos(\theta_1 + \theta_2) + j r_1 r_2 \sin(\theta_1 + \theta_2).$

* definition: conjugate of a complex number is

$z = x + jy = re^{j\theta}$ is definde as

$(\bar{z}) \equiv z^* = x - jy = re^{-j\theta} \rightarrow \boxed{z \cdot z^* = (x+jy)\cdot(x-jy) = x^2 + y^2}$

3. $\frac{z_1}{z_2} = \frac{x_1 + jy_1}{x_2 + jy_2} * \frac{x_2 - jy_2}{x_2 - jy_2}$ multiplyBy

$$= \underbrace{\frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2}}_{Re\{z_1/z_2\}} - j\underbrace{\frac{(y_2 x_1 - x_2 y_1)}{x_2^2 + y_2^2}}_{Im\{z_1/z_2\}}$$

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$\Rightarrow$ OR $\frac{Z_1}{Z_2} = \frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}} = \frac{r_1}{r_2} e^{j(\theta_1-\theta_2)}$

$= \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + j\sin(\theta_1-\theta_2)\right]$

4. $Z_1 . \overline{Z_1} = x_1^2 + y_1^2 = |Z_1|^2$, $\boxed{|Z_1| = r}$

$|Z_1| = \sqrt{Z_1 . Z_1^*}$

example: $Z_1 = 1+j$ find $|Z_1|$

sol. 1- $|Z_1| = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

2- $|Z_1| = \sqrt{(1+j)(1-j)} = \sqrt{2}$.

5. $\overline{(Z_1 + Z_2)} = \overline{Z_1} + \overline{Z_2}$ (H.W proof)

6. $\overline{(Z_1 \cdot Z_2)} = \overline{Z_1} . \overline{Z_2} \Rightarrow \overline{(r_1 e^{j\theta_1} . r_2 e^{j\theta_2})}$

$= \overline{(r_1 r_2 e^{j(\theta_1+\theta_2)})} = r_1 r_2 e^{-j(\theta_1+\theta_2)}$

$\Rightarrow \underbrace{r_1 e^{-j\theta_1}}_{(\overline{Z_1})} . \underbrace{r_2 e^{-j\theta_2}}_{(\overline{Z_2})}$

7. $\overline{\left(\frac{Z_1}{Z_2}\right)} = \frac{\overline{Z_1}}{\overline{Z_2}}$

8. $|Z_1 . Z_2| = |Z_1| . |Z_2| \Rightarrow$ also $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$.

example: let $Z = \frac{1}{1+j\alpha}$ find $|Z|$

Sol:- 1. $|Z| = \frac{|1|}{|1+\alpha j|} = \frac{1}{\sqrt{1+\alpha^2}}$

2. $|Z| = \sqrt{Z . Z^*} = \sqrt{\frac{1}{1+j\alpha} * \frac{1}{1-j\alpha}} = \frac{1}{\sqrt{1+\alpha^2}}$

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* The n^{th} power of a complex power number:-

let $Z = x+jy = re^{j\theta}$

Then $Z^n = (x+jy)^n = (re^{j\theta})^n$ for integer.

$r^n e^{jn\theta} = r^n[\cos(n\theta) + j\sin(n\theta)]$

$Z^n = (re^{j\theta})^n = (r\cos\theta + jr\sin\theta)^n$

$= r^n[(\cos\theta + j\sin)^n] = \cos(n\theta) + j\sin(n\theta)$

Remember that

$(a+b)^n = \sum_{K=0}^{n} \binom{n}{K} a^K b^{n-K}$

Binomial Theorem

$\boxed{(\cos\theta + j\sin\theta)^n = \cos(n\theta) + j\sin(n\theta)}$

Demouvels format

* The n^{th} root of a complex number

$Z = x + jy = re^{j\theta}$

$w = \sqrt[n]{Z}$, where w the n^{th} root of Z

* $e^{j2\pi K} = 1$, for integer.

$Z = Z.e^{j2\pi K}$, K is integer.

$Z = re^{j(\theta + 2\pi K)}$, K is integer $\Rightarrow$ but we need w.

* $W_K = (re^{j(\theta+2\pi K)})^{1/n}$, $K = -\infty$ to ∞ (integer)

$w_K = r^{1/n} \cdot e^{j\frac{(\theta+2\pi K)}{n}}$

$w_0 = r^{1/n} \cdot e^{j\theta/n}$

$w_1 = r^{1/n} \cdot e^{j\frac{(\theta+2\pi)}{n}}$

$w_n = r^{1/n} \cdot e^{j\frac{(\theta+2\pi n)}{n}}$ (n root)

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So,

$$\boxed{\omega_K = r^{\frac{1}{n}} \, e^{j\left(\frac{\theta + 2\pi K}{n}\right)}}$$

where $Z = re^{j\theta}$ is given.

example : calculate $\sqrt[3]{1}$.

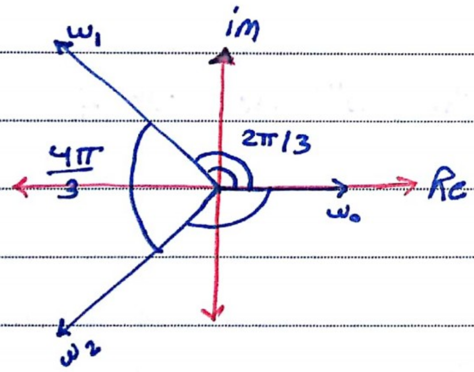
$Z = 1 + 0j$, $r = |Z| = 1$, $\theta = $ zero.

Sol. $\omega_K = e^{\frac{j2\pi K}{3}}$, $K = 0, 1, 2, \ldots$

$\omega_0 = e^{j0} = 1$

$\omega_1 = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

$\omega_2 = e^{j\frac{4\pi}{3}} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$



$\longrightarrow \omega_0 + \omega_1 + \omega_2 = 0$

So for $\sqrt[n]{1} \Rightarrow \boxed{\omega_K = e^{\frac{j2\pi K}{n}} = 0}$

$$\sum_{n=0}^{n-1} \omega_K = 0 \longrightarrow \sum_{n=0}^{n-1} e^{\frac{j2\pi K}{n}} = 0$$

\# the sum of root of unity is equal to zero.

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* The logarithem of a complex number

$$Z = x + jy = re^{j\theta}$$

$\ln(Z) = ??$

$Z = re^{j(\theta + 2\pi K)}$, K is integer

now, $\ln(Z) = \ln\left(r\, e^{j(\theta + 2\pi K)}\right)$

$= \ln r + \ln\left(e^{j(\theta + 2\pi K)}\right)$.

$= \underbrace{\ln(r) + j\theta}_{\text{principal value of } \ln(Z)} + j2\pi K$

$K = (-\infty, \infty)$

*** example ⇒** find $\ln(-1)$?

$Z = -1 + 0j \longrightarrow \ln(Z) = \ln(r) + j(\theta + 2\pi K)$

$= \ln(1) + j(\pi + 2\pi K)$

$\ln(Z{-}1) = j\pi$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\boxed{\theta = \pi}$

Note: $\cos(x) = 2x \longrightarrow \cos(Z) = 2$

where Z is a cmplx num.

*** example ⇒** find $(j)^j$?

$j^j = e^{\ln(j^j)} = e^{j\ln j} = e^{Z}$, where $Z = j\ln j$.

$\ln(j) = \underbrace{\cancel{\ln}^{0}(1) + j\frac{\pi}{2}}_{\text{principal value}} + j\,2\pi K$

$e^{j\ln(j)} = e^{j(j\pi/2 + j2\pi K)} = e^{-\frac{\pi}{2}} \cdot e^{-2\pi K}$

so $\boxed{j^j = e^{-\frac{\pi}{2}}}$

real root for complex num.

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example : (H.w)

1. is That true → $(e^{z^*}) = (e^{z})^*$ → conjugate.

2. find the magnitude of $Z = e^{jt} + e^{j3t}$

→ $|Z| = |e^{jt} + e^{j3t}|$

remember that :

$|Z_1 + Z_2| \neq |Z_1| + |Z_2|$

* geometric series (we use it in sum of a polar terms)

$$S_n = a + a.b + a.b^2 + ab^3 + \cdots + ab^{n-1} \rightarrow N \text{ terms}$$

$$* \quad S_n = \begin{cases} \dfrac{a(1-b^n)}{1-b} , & b \neq 1 \\ N.a , & b = 1 \end{cases}$$

multiply S_n By (b) :- $bS_n = a.b + a.b^2 + a.b^3 + \cdots + a.b^n$

→ now $(S_n - bS_n) = a - ab^n$

$$(1-b)\, S_n = a(1-b^n)$$

$$\rightarrow S_n = \frac{a(1-b^n)}{(1-b)}$$

example :

$$1 + 2^{-1} + 2^{-2} + 2^{-3} + \cdots + 2^{-99}$$

sol.

$$S_{100} = \frac{\left(1 - \frac{1}{2}^{100}\right)}{1 - \frac{1}{2}} = 2\left(1 - \left(\frac{1}{2}\right)^{100}\right)$$

$$S_{100} \approx \frac{1}{1 - \frac{1}{2}} = 2$$

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* The n^{th} root of unity :-

$$\omega_K = e^{\frac{j\pi K}{n}} \quad , K = 0, 1, 2, ----, n-1$$

$$\sum_{K=0}^{n-1} e^{+j\frac{2\pi K}{n}} = 0$$

$$S_N = 1 + e^{j2\pi \cdot 1/n} + e^{j2\pi \cdot 2/n} + --- \; e^{j2\pi(n-1)/n}$$

$$b = e^{j2\pi/n} \quad , a = 1$$

$$S_N = \frac{1 \cdot \left[1 - \left(e^{\frac{j2\pi}{n}}\right)^n\right]}{\left(1 - e^{j2\pi/n}\right)} \longrightarrow e^{j2\pi} = 1$$

so $S_N = 0$.

* Integrals of complex function.

ex: $$\int_0^{\pi/4} e^{j2t} \cdot dt = \left.\frac{e^{j2t}}{2j}\right|_0^{\frac{\pi}{4}} = \frac{e^{j\pi/2} - 1}{2j} = \frac{1}{2} + \frac{1}{2}j$$

ex: $$\boxed{\int_0^b e^{-\alpha t} \cdot dt = \frac{1 - e^{-\alpha b}}{\alpha}}$$ given in exam. --- ①

* integrate By part :-

$$\int_0^b t\, e^{-\alpha t} \cdot dt .$$

Starting with ①

$$\frac{d}{d\alpha}\left[\int_0^b e^{-\alpha t} \cdot dt\right] = \frac{d}{d\alpha}\left[\frac{1 - e^{-\alpha b}}{\alpha}\right]$$

$$\int_0^b t\, e^{-\alpha t} dt = \frac{-d}{d\alpha}\left[\frac{1 - e^{-\alpha t}}{\alpha}\right]$$

$$= -\left[\frac{1 - e^{-\alpha t} - \alpha e^{-\alpha b}}{\alpha^2}\right]$$ ※

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⇒ integration By parts :-

$$\int_a^b u(t)\, d[v(t)]\, dt = u(t)\cdot v(t)\Big|_a^b - \int_a^b v(t)\, d\,u(t)\, dt$$

example :- $\int_0^1 \underbrace{t}_{u(t)}\ \underbrace{e^{-j\omega t}\, dt}_{d(v(t))}$

$u(t) = t$ $\qquad$ $dv(t) = e^{-j\omega t}\, dt$

$d(u(t)) = 1$ $\qquad$ $v(t) = \frac{j e^{-j\omega t}}{\omega}$

$$\frac{j t e^{-j\omega t}}{\omega}\Big|_0^1 - \int_0^1 \frac{j e^{-j\omega t}}{\omega}\, dt$$

$$\frac{j\omega e^{-j\omega}}{\omega^2} + \left(\frac{e^{-j\omega t}}{\omega^2}\right)\Big|_0^1$$

$$\frac{e^{-j\omega} - 1}{\omega^2}$$

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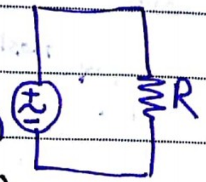
* Chapter 1 :-

→ mathmatical funct.

what is a signal ? a signal is a function of an independent variable (time) that carries some information or describe some physical phenomenones.

$i(t) = \sin(\pi t)$



if $\rightarrow v(t) = \sin(\pi t)$

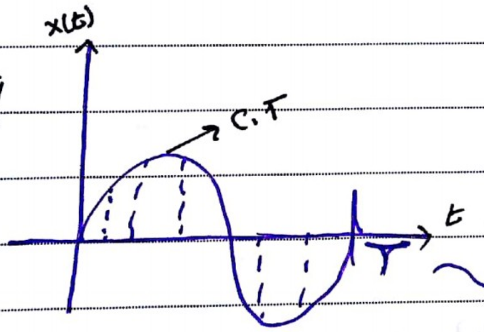
information $\begin{cases} v(0) = 0 \\ v(\pi/2) = 1 \\ v(\pi) = 0 \end{cases}$

- classification of signals :

continous time signals and Discrete time signals.

(C.T) (D.T)

ex: C.T



for any $t \in \{0, T\}$

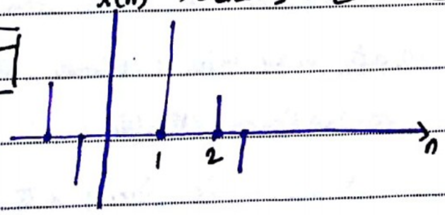
$x(t) \rightarrow$ defined

→ (there are some lost information).

↳ just when we transf from C.T to D.T

$x(n) \rightarrow$ denotes the sample

ex: D.T



$x(\frac{1}{2}) = ? \rightarrow$ undifined.

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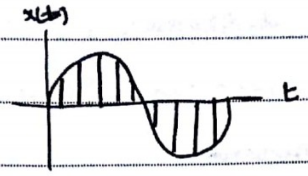
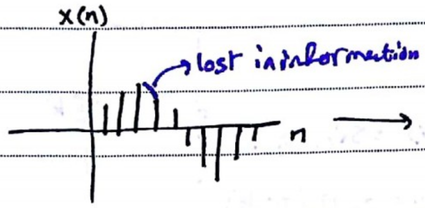
* Sampiling :- aprosses that converts C.T signals into a D.T Signals.

reducing of information.



→ C.T



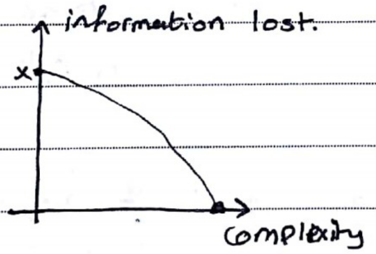
→ D.T →

* اذا بدنا نحول من D.T لـ C.T بنستخدم معادلة على كمبيوتر وتنحل المشكلة.

$x[n] = \{0 \;\; x \;\; x^2 \;\; \text{———} \}$

* نفس المشكلة لما بدنا نحول C.T لـ D.T وهون رح يكون عنا ضياع بالمعلومات

Lost in information. (reduce).



highest complexity.

* Deterministic Vs Random signals.

Deterministic: complitly spesified at any given time

$x(t) = 20 \sin(\pi t)$

$x(1) = 20$

$x(0) = 0$

Random signals: can be speified statistically (at any given time we have a random value).

$x(t) = 20 \sin(2\pi(t + \frac{\theta}{2}))$

* Real & complex signals :-

let $z(t)$ be a complex signal, then $z(t) = x(t) + j\, y(t)$

real signals

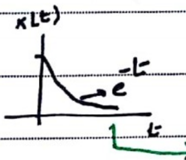
ex: $z(t) = e^{jt} \Rightarrow \cos(t) + j \sin(t)$.

* we can describe signals or / and graphically.

mathematically

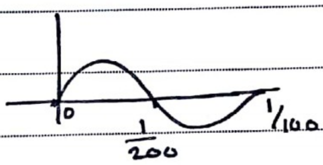
ex) • $x(t) = e^{-t}$, $t \geq 0$

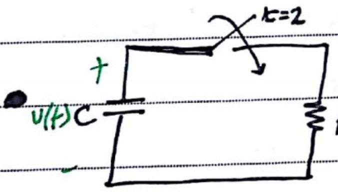
mathmetically



graphically.

• $x(t) = \sin(200\pi t)$





$\rightarrow v(t) = \begin{cases} 10 & , t < 2 \\ 10 e^{-t/RC} & , t \geq 2 \end{cases}$

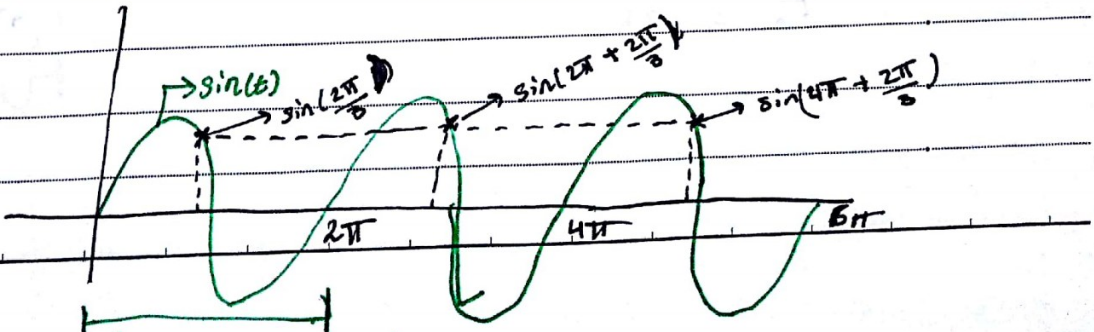
* Periodic signals :

a signal $x(t)$ is said to be periodic with period "T" if there is a positive non-zero value of T such that $\boxed{x(t+T) = x(t)}$ for all t.

$T = 2\pi$

$T = 4\pi$

$T = 6\pi$



2π = fundimental period

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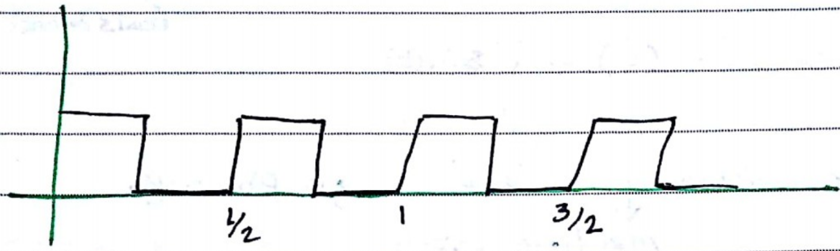
* the minimum value of the period "T" that satisfies $x(t) = x(t+T)$ is called the fundimental period "T_0"

ex:



what is the period of this signals?

$T = 1/2, 1, 3/2, 2, \ldots$

$T_0 = \frac{1}{2}$

※ Show That $x(t) = \sin(\omega_0 t)$ is periodic signal and find the period. Is there any "T" such that $x(t+T) = x(t)$ for all t.

→ Is there any "T" such that

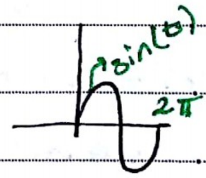
$\sin(\omega_0 (t+T)) = \sin(\omega_0 t)$

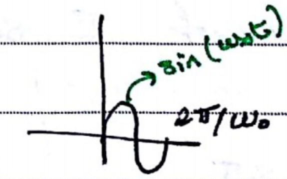
↳ $\sin(\omega_0 t + \omega_0 T) = \sin(\omega_0 t)$?

$\omega_0 T = 2\pi, 4\pi, 6\pi$

$T = \frac{2\pi}{\omega_0} + \frac{4\pi}{\omega_0} + \frac{6\pi}{\omega_0}$

$T_0 = \frac{2\pi}{\omega_0}$





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Q:- is $x(t) = e^{\sin(t)}$ periodic ??

is there any "T" such that

$$e^{\sin(t+T)} = e^{\sin(t)}$$

$T = 2\pi, 4\pi, 6\pi$

$T_0 = 2\pi$, $\omega_0 = \frac{2\pi}{T_0} = 1$ rad/s

example : $x(t) = \sin(\omega_0 t + \theta)$ find T_0 ?

↳ what is the values "T" s. that

$\sin(\omega_0 t + \theta + \omega_0 T) = \sin(\omega_0 t + \theta)$?

↳ $T = \frac{2\pi}{\omega_0}, \frac{4\pi}{\omega_0}$

↳ $T_0 = \frac{2\pi}{\omega_0}$

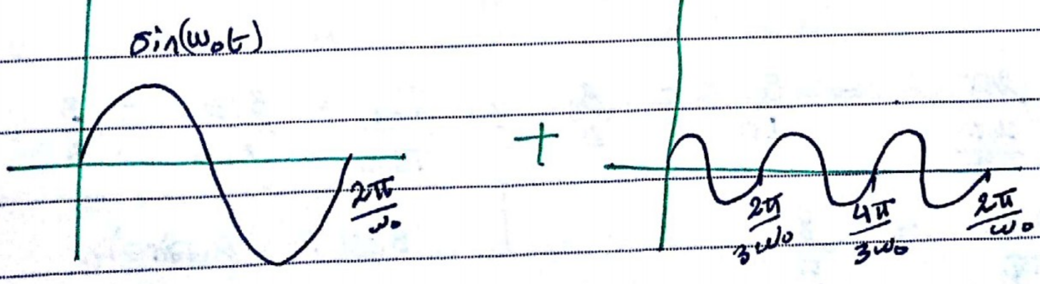
example: $x(t) = \sin(5t + \pi/4)$ find T_0.

$T_0 = \frac{2\pi}{5}$

* $s_1(t) = \sin(\omega_0 t) + \cos(\omega_0 t)$ → periodic

$s_2(t) = \sin(\omega_0 t) + \sin(3\omega_0 t)$ → periodic

بنجمعو عادي بدون اي مشكلة



we can represent the sum.

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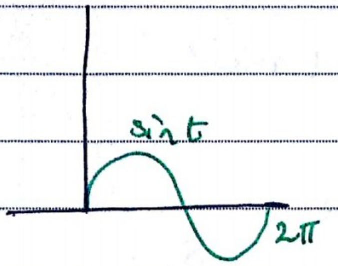
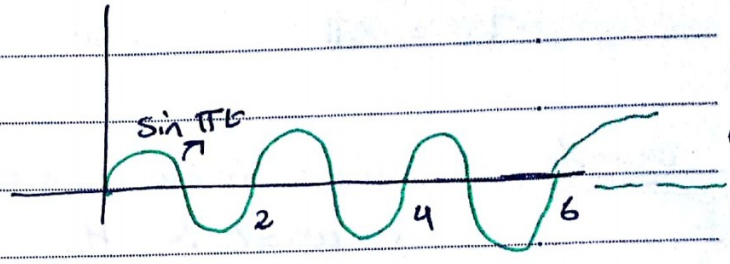
when we can't represent the sum of two sinosoidal signals ?

ex: $\sin(t) + \sin(\pi t) \longrightarrow$ non-periodic





مستحيل توصل لل 2π $\longrightarrow$

so we can't represent the sum.

* let $s(t) = \underbrace{\sin(\omega_1 t)}_{x_1(t)} + \underbrace{\cos(\omega_2 t)}_{x_2(t)}$

$s(t)$ is periodic if $\frac{T_{01}}{T_{02}}$ = ratio of integers.

where T_{01} : fundamental of period $x_1(t)$

T_{02} : " " " $x_2(t)$

* let T_0 be the fund period of $s(t)$.

Then $T_0 = LCM(T_{01}, T_{02})$

example:- $s(t) = \cos(\frac{10\pi}{3} t) + \sin(\frac{5\pi t}{4})$, is $s(t)$ periodic? find T_0.

$T_{01} = \frac{2\pi}{\frac{10\pi}{3}} = \frac{6}{10} = \frac{3}{5}$

$T_{02} = \frac{2\pi}{\frac{5\pi}{4}} = \frac{8}{5}$

$\frac{T_{01}}{T_{02}} = \frac{3/5}{8/5} = \frac{3}{8}$

$s(t)$ is periodic

$\hookrightarrow T_0 = LCM(\frac{3}{5}, 8/5) = 1/5\ LCM(3, 8) = 24/5$

$\boxed{\omega_0 = 2\pi/T_0 = 5\pi/12}$

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↳ if we have 3 signals.

* $\frac{T_{01}}{T_{02}} = \frac{I_1}{I_2}$, $\frac{T_{01}}{T_{03}} = \frac{I_3}{I_4}$

$\Rightarrow \frac{T_2}{T_3}$ is ratio of integer also.

$\boxed{\text{example}}$:

$$x(t) = \underbrace{\sin\left(\frac{5\pi t}{6}\right)}_{x_1} + \underbrace{\cos\left(\frac{3\pi t}{4}\right)}_{x_2} + \underbrace{\sin\left(\frac{\pi t}{3}\right)}_{x_3}$$

$T_{01} = 12/5$, $T_{02} = 8/3$, $T_{03} = 6$

$\frac{T_{01}}{T_{02}}$ and $\frac{T_{01}}{T_{03}}$ are ratio of integers.

$$T_0 = LCM\,(T_{01}, T_{02}, T_{03})$$

$$= LCM\left(\frac{12 \times 3}{5 \times 3}, \frac{8 \times 5}{3 \times 5}, \frac{15 \times 6}{15}\right)$$

$$= \frac{1}{15} LCM\,(36, 40, 90) = \frac{380}{15} = 24$$

$\boxed{\text{example}}$: $x(t) = \sin(5t) + \cos(3\pi t) + \sin(7t)$.

Is $x(t)$ periodic? (No

$\frac{T_{01}}{T_{02}} \neq$ ratio of integer

is (aperiodic) signal.

↳ Not periodic a.)

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* energy signals :-

$$E = \int_{0}^{\infty} p(t) \cdot dt$$

energy

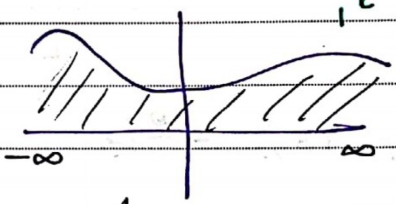
power as a function of time (inst power)

$$p = \frac{v^2(t)}{R} \quad , \quad P = i(t)^2 \cdot R$$

$$E_x = \int_{-\infty}^{\infty} x^2(t) \cdot dt$$

$x^2(t)$ → $i^2(t)$, $v^2(t)$.

x → i , v



the are of x^2 not

* السگنال اما تكون energy signal او Power signals

* اذا السگنال invalid ← You should find avg Power

$$I_{rms} = \sqrt{\frac{1}{T} \int_{0}^{T} x^2(t) \cdot dt}$$

↳ for a real signal $x(t)$: $E_x = \int_{-\infty}^{\infty} x^2(t) \cdot dt$

↳ for a cmplx signal $x(t)$: $E_x = \int_{-\infty}^{\infty} x(t) \cdot x(t) \cdot dt$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \cdot dt$$

real +ve value for energy ← لكل الـ magnitude تربيع

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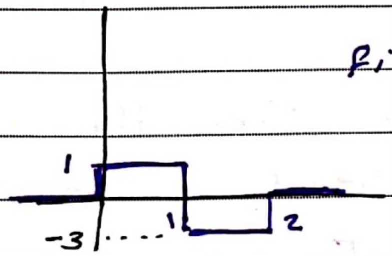
* E_x : The energy is a measure of how big is the signal $x(t)$.

* for D.T signal $x[n]$

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

example



find $E_x \int_{-\infty}^{\infty} x^2(t)\, dt$.

$$= \int_{-\infty}^{0} 0^2 \, dt + \int_{0}^{1} 1^2 \, dt + \int_{1}^{2} (-3)^2 \, dt + \int_{2}^{\infty} 0^2 \, dt.$$

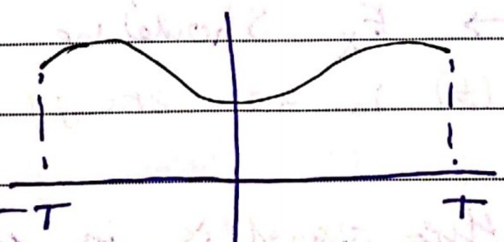
it can be cos or sin.

$$= 1 + 9 = 10$$

power in DB → $10 \log_{10} P$ = -70 DB.

$P = 10^{-7}$ → watts

* Now, The avg power of the signal $x(t)$.



$$P_x = \lim_{T\to\infty} \frac{\int_{-T}^{T} x^2(t)\, dt}{2T}$$

if $\infty > P_x > 0$ → power signal

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P_x it might be
① $E_x > 0$ it is energy E_x finite, $P_x \to 0$ not a power signal
② $P = \alpha$ must be power $\to \lim_{T \to \infty} \int_{-T}^{T} x^2(t)\,.dt$
③ $P = \infty \to$ not energy & not a power

* Show that if $P_x = 0$, then the signal $x(t)$ must be ~~an~~ energy signal.

Sol. $P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\,.dt$

$\Rightarrow \int_{-T}^{T} x^2(t)\,.dt \leq \int_{-\infty}^{\infty} x^2(t)\,.dt \to$ for any "T"

$\hookrightarrow P_x \leq \lim_{T \to \infty} \frac{\int_{-\infty}^{\infty} x^2(t)}{2T}$ E_x

if $P_x = 0 \longrightarrow E_x$ Should be finite
$\hookrightarrow x(t)$ is an energy signal.

* we say that the signal is an energy signal if
$$\infty > E_x = \int_{-\infty}^{\infty} x^2(t)\,:dt > 0.$$

$$\infty > E_x > 0.$$

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* Note: $P = 0 \Rightarrow$ Not a power signal
$\Rightarrow$ energy signal

$P = \alpha \Rightarrow$ power signal
↳ No energy (∞)

$P = \infty$

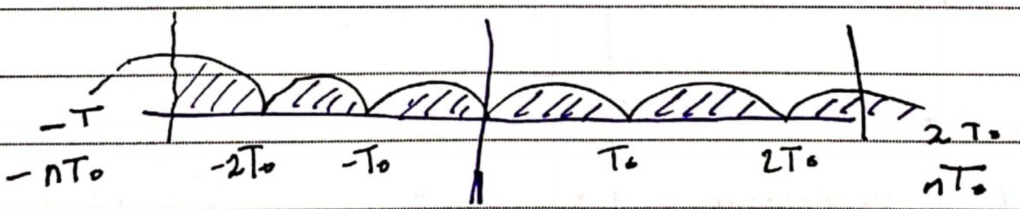
* for any signal $x(t)$

$$P_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 \, dt$$

for periodic signal $x(t)$.

$$P_x = \frac{1}{T_0}\int_{T_0} |x(t)|^2 \, dt$$

T_0 → fundamental period of $x(t)$



for periodic signal $x(t)$

$$P_x = \lim_{n\to\infty} \frac{\int_{-nT_0}^{nT_0} |x(t)|^2 \, dt}{2nT_0} = \lim_{n\to\infty} \frac{\cancel{2n}\int_{T_0} |x(t)|^2 \, dt}{\cancel{2n}}$$

$$\boxed{= \frac{1}{T_0}\int_{T_0} |x(t)|^2 \, dt}$$

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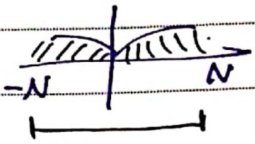
⇒ for a. D.T signal $x[n]$

$$P_x = \lim_{N \to a} \frac{\sum_{n=-N}^{N} |x[n]|^2}{2N+1}$$



$$\Rightarrow P_x = \frac{\sum_{n=0}^{N_0-1} |x[n]|^2}{N_0}$$ ⟶ for periodic signal $x(t)$.

Summary

Energy

for → C.T signal $x(t)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \cdot dt$$

for : D.T signal $x[n]$

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 .$$

If $0 < E_x < \infty$

→ $x(t)$ / $x[n]$ is an energy signal.

Power

C.T signal $x(t)$.

$$P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \cdot dt$$

if $x(t)$ is periodic

$$P_x = \frac{1}{T_0} \int_{T_0} |x(t)|^2 \cdot dt$$

for D.T signal

$$P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

for periodic signal $x[n]$

$$P_x = \frac{\sum_{n=0}^{N_0-1} |x[n]|^2}{N_0}$$

$$\infty > P_x > 0$$

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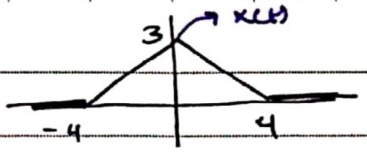
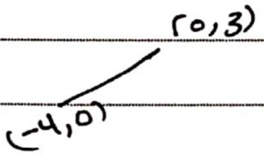
example



Find $x(t)$ and P_x.

to find equ for linear funct:-

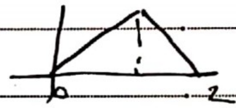


$$m = \frac{3-0}{0+4} = \frac{3}{4}$$

$$x(t) - 0 = \frac{3}{4}(t+4) = 3\left(1+\frac{t}{4}\right)$$

$$x(t) = \begin{cases} 3\left(1+\frac{t}{4}\right), & 0 \geq t \geq -4 \\ 3(1-t/4), & 4 \geq t \geq 0 \end{cases}$$

but:-



$$x(t) = \begin{cases} t, & 1 \geq t \geq 0 \\ 2-t, & 2 \geq t \geq 1 \end{cases}$$

* energy:

$$E_x = \int_{-4}^{4} x^2(t)\,dt$$

$$= \int_{-4}^{0} \left(3\cdot\left(1+\frac{t}{4}\right)\right)^2 dt + \int_{0}^{4} \left(3\left(1-\frac{t}{4}\right)\right)^2 dt$$

$$= \frac{9\left(1+\frac{t}{4}\right)^3}{3\cdot\left(\frac{1}{4}\right)}\Bigg|_{-4}^{0} - \frac{9}{3\cdot\left(\frac{1}{4}\right)}\left(1-\frac{t}{4}\right)^3\Bigg|_{0}^{4}$$

$$= 12\,[1-(-1)] = 24$$ energy signal.

* power: $P_x = 0$

(Because the energy signal is finait = value

* example: let $x(t) = A\cos(\omega_0 t+\theta)$ find P_x

$$P_x = \frac{1}{T_0}\int_{T_0} x^2(t)\,dt.$$

$$= \frac{1}{T_0}\int_{\epsilon}^{T_0+\epsilon} A^2\cos^2(\omega_0 t+\theta)\,dt.$$

$$= \frac{A^2}{2T_0}\left[\int_{\epsilon}^{T_0+\epsilon} (1+\cos(2\omega_0 t+2\theta))\,dt\right]$$

$$= \frac{A^2}{2.T_0}\left[T_0 + \int_{\epsilon}^{T_0+\epsilon} \underbrace{\cos(2\omega_0 t+2\theta)}_{I}\,dt.\right]$$

0 (proof!)

proof! → $I = \frac{\sin(2\omega_0+2\theta)}{2\omega_0}\Big|_{\epsilon}^{T_0+\epsilon}$

$$= \sin((\overset{4\pi}{\cancel{2\omega_0 T_0}}) + (2\omega_0\epsilon + \cancel{2\theta})) - \sin(2\omega_0\epsilon + 2\theta)$$

zero

$$= 0$$

$$= \frac{A^2}{2.\cancel{T_0}}.\cancel{T_0} = \frac{A^2}{2}$$

* example: $x(t) = A_1\cos(\omega t+\theta_1) + A_2\cos(\omega t+\theta_2)$

find P_x.

different fundmental (T_0) Period

* another case: different freq + different T_0

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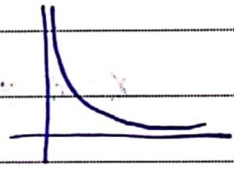
* **example** let $x[n] = \left(\frac{1}{3}\right)^n$, $n \geq 0$

Find $E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$

$$= \sum_{n=\text{zero}}^{\infty} \left(\frac{1}{3}\right)^{2n} = \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n = \frac{1}{1-1/9}$$

* **example** $x(t) = e^{-\alpha t}$, $t > 0$, $\alpha > 0$

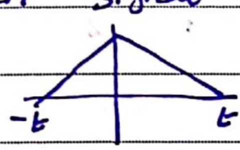
$$E_x = \int_0^{\infty} e^{-2\alpha t} \, dt = \frac{1}{2\alpha}$$

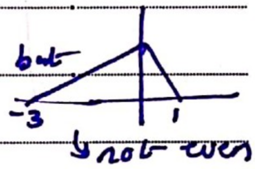


* even and odd signal.

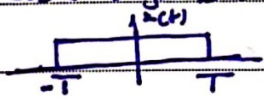
① A signal $x(t)$ is an even signal if

$$x(t) = x(-t)$$



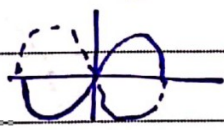


common even signals : $x(t) = \cos(t)$

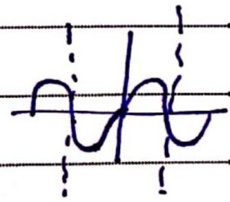


② A signal $x(t)$ is an odd signal if

$$x(-t) = -x(t)$$



common odd signal : $x(t) = \sin(t)$



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* Any real signal $x(t)$ can be expressed as a sum of even part and odd part.

$$x(t) = x_e(t) + x_o(t) \quad \text{--- (I)}$$

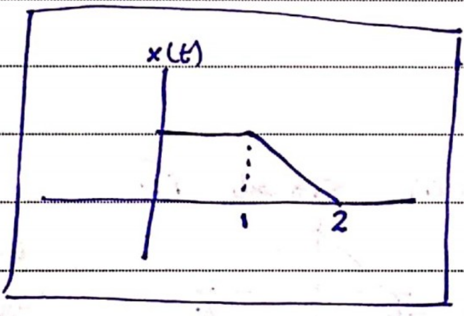
so → $x(-t) = x_e(-t) + x_o(-t)$

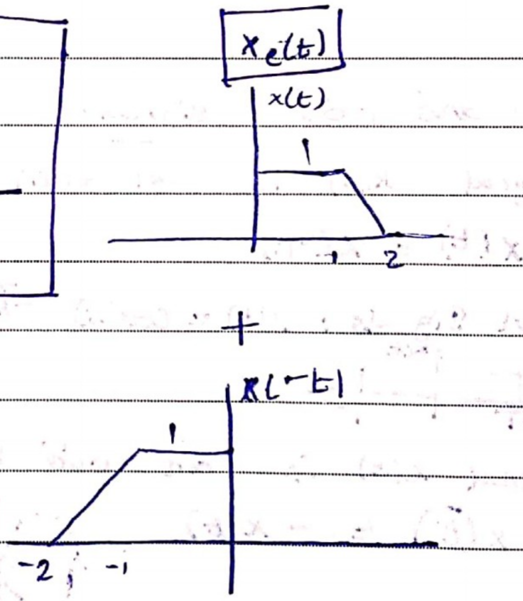
↳ $x(-t) = x_e(t) - x_o(t) \quad \text{--- (II)}$

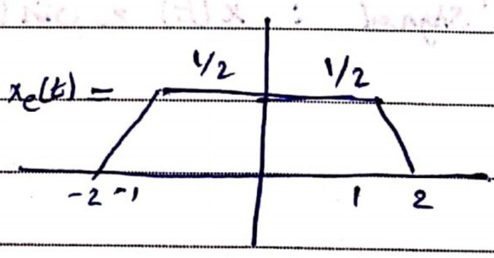
Then $x_e(t) = \dfrac{x(t) + x(-t)}{2}$

$x_o(t) = \dfrac{x(t) - x(-t)}{2}$

example :







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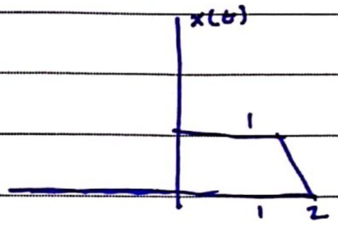
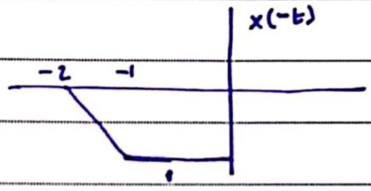
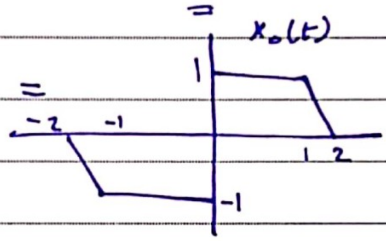
$x_o(t) \Rightarrow$



$+$



$=$



example : $x(t) = \cos(t) + \sin(t) + \cos(t)\sin(t).$

$$x(t) = \cos(t) + \sin t + \cos t \sin t$$

$$+$$

$$\frac{\cos(t) - \sin(t) - \sin(t)\cos(t)}{2}$$

$$= \cos(t)$$

$\boxed{ex}$ $x_o(t) = \sin(t) + (\sin(t) * \cos(t)).$

example :- $x(t) = (t+1)^2 = \underbrace{t^2}_{even} + \underbrace{2t}_{odd} + \underbrace{1}_{even}$

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* for a D.T signal . $x[n]$

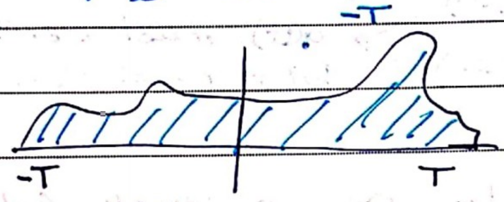
$$x_e[n] = \frac{x[n] + x[-n]}{2} \quad \text{(even)}$$

$$x_o[n] = \frac{x[n] - x[-n]}{n} \quad \text{(odd)}$$

* The average value of the signal $x(t)$

$$A_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\,dt = \begin{cases} 0, & x(t) \text{ is odd} \\ \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t)\,dt, & x(t) \text{ is even} \end{cases}$$

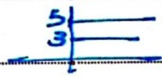
area



Notes:- 1- $E \mp E = E$

2. $O \mp O = O$ → No part have even.

↳ $x(t) = 3$, $x_2(t) = 5$] even signal.



$$\frac{x[t] - x[-t]}{2}$$

↳ let $x_1(t)$ and $x_2(t)$ are odd signals

let $s(t) = x_1(t) + x_2(t)$

↳ $s(-t) = x_1(-t) + x_2(-t)$

$= -x_1(t) - x_2(t)$

$= -1(x_1(t) + x_2(t))$

$s(-t) = -s(t)$ → odd signal.

3. $E \mp O = NE\ NO$

4. $E \times O = O$

5. $E \times E = E$

$E \times O = O$

$O \times O = E$

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* complex signals :-

let $x(t) = a(t) + j\,b(t) = V(t)\,e^{j\theta(t)}$

A signal $x(t)$ is said to be conjugate symmetric (C.S) signal if

$$\boxed{\begin{aligned} x(-t) &= x^*(t) \\ x(t) &= x^*(-t) \end{aligned}}$$

$a(t) + j\,b(t) = (a(t) + j\,b(-t))^*$

$= (a(-t) - j\,b(-t))$

$a(t) = a(-t)$
$b(t) = -b(-t)$ $\rightarrow$ $a(t)$ must be even, $b(t)$ must be odd

* A signal $x(t)$ is said to be conjugate Anti-symmetric signal (C.A.S) if $x(-t) = -x^*(t)$

or $x(t) = -x^*(-t)$

example :- $x(t) = \sin(t) + j\cos(t)$

O + E $\rightarrow$ C.A.S

* Any complex signal $x(t)$ can be represent as :

$$x(t) = x_{c.s}(t) + x_{c.A.S}(t)$$

$$x_{c.s}(t) = \frac{x(t) + x^*(-t)}{2}$$

$$x_{c.A.S}(t) = \frac{x(t) - x^*(-t)}{2}$$

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Time transformation :-

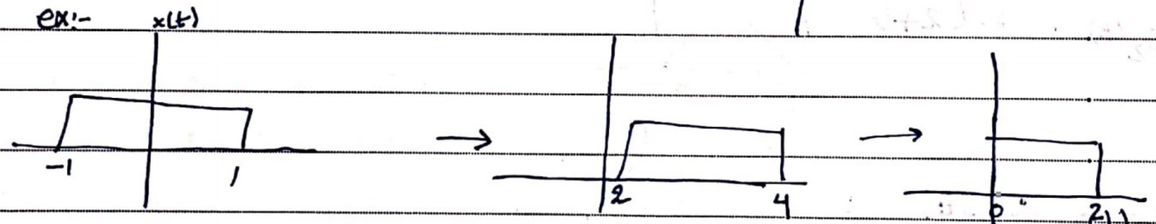
For a given $x(t)$, what is $y(t) = x(at-b)$,
where a, b are real number, $a \neq 0$.
→ const

example عدد ـ ما بتغير دال

$$x(at-b) = \begin{cases} a=1 \rightarrow y(t) = x(t-b) & \text{(Time shift) T.sh} \\ b=0 \rightarrow y(t) = x(at) & \text{(Time scaling)} \end{cases}$$

* Time shifting :

$$y(t) = x(t-b) = \begin{cases} \text{shift right by "b" unit} & b>0 \\ \text{" left " " "} & b<0 \end{cases}$$



$y(t) = x(t-1)$

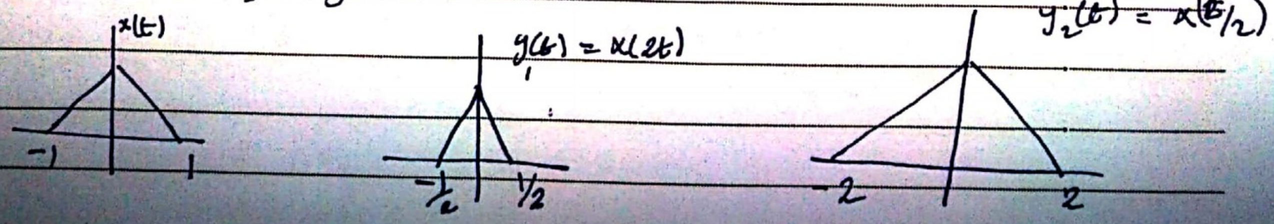
$y(1) = x(0) = 1$

$y(2) = (x(1)) = 0$

$\vdots$

$$= \begin{cases} a>1, & \text{Compression (x(t) will be compressed)} \\ a<1, & \text{expantion (x(t) will be expanded)} \end{cases}$$

* Time scaling : $y(t) = x(at)$



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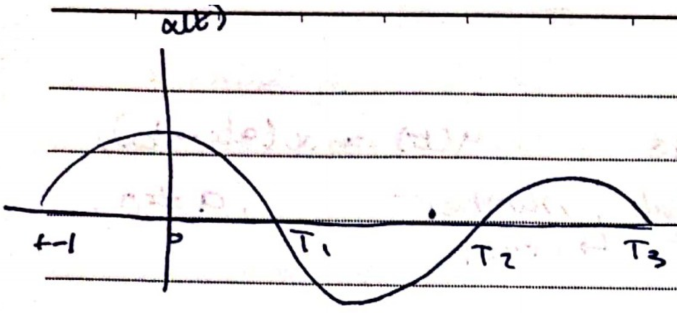
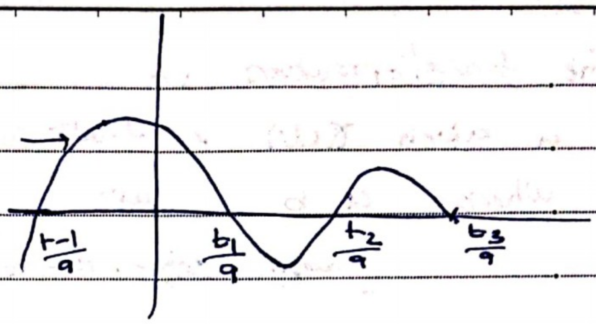
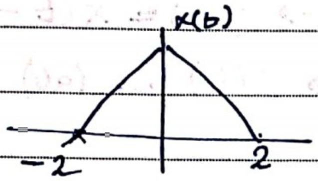


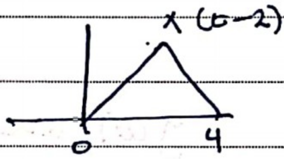


example

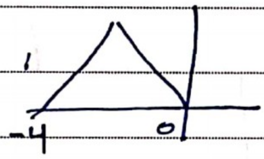


Find:

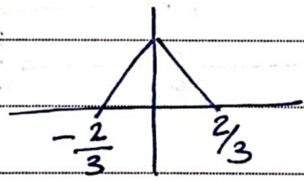
① $y_1(t) = x(t-2)$:



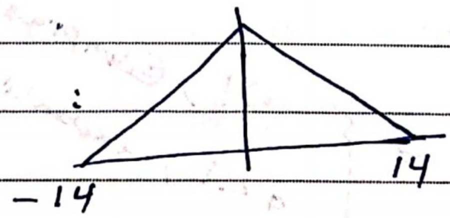
② $y_2(t) = x(2+t)$:



③ $y_3(t) = x(3t)$:



④ $y_4(t) = x(t/7)$:



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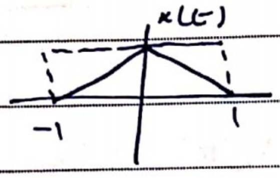
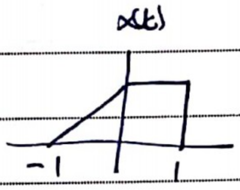
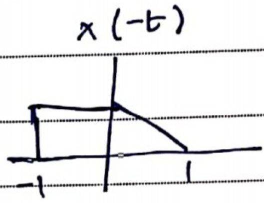
* Time Reversed :

$x(t)$ is given , $y(t) = x(-t)$



* example





mirror

method 2 :- if $x(t)$ is given $\longrightarrow$ find $y(t) = x(at-b)$

① Scaling then shifting

$$x(t) \xrightarrow{\text{T.Sca by "a"}} y_1(t) = x(at) \xrightarrow{\text{T.Sh by "b/a"}} y_2(t) = y_1(t - b/a)$$

$$= x(a(t-b/a))$$

$$= x(at-b)$$

so : $x(t) \xrightarrow{\text{T.S by 'a'}} \boxed{x(at)} \xrightarrow{\text{shift by "b/a"}} \boxed{x(at-b)}$

$$y(t) = x(at-b)$$
$$= x(a.(t-b/a))$$

T.S   time shift

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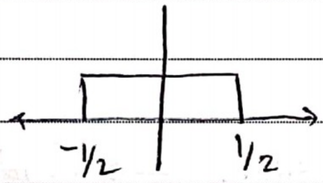
* method [2] : x(t) is given, i want $y(t) = x(at - b)$

$$x(t) \xrightarrow[\text{by 'b' unit}]{T.Sh} \boxed{x(t-b)} \xrightarrow[\text{by 'a'}]{T.\,Scaling} \boxed{x(at-b)}$$

$$x(t) \longrightarrow y_1(t) = x(t-b) \xrightarrow{\text{time scaling}} y_2(t) = y_1(at) = x(at-b)$$

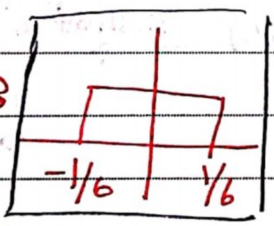
a) * example

$x(t)$, find $y(t) = x(3t+2)$

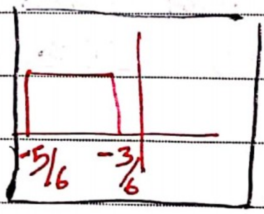


[1] method 1 : $x(3(t + 2/3))$

$x(t) \xrightarrow[\text{by '3'}]{T.\,scaling}$

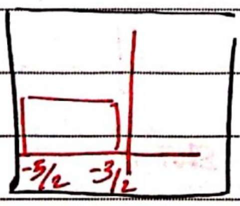


$\xrightarrow[\text{by } 2/3]{T.\,shifting}$

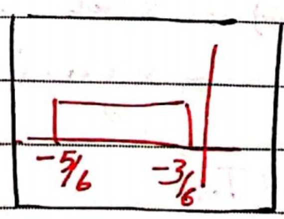


[2] method 2:

$x(t) \xrightarrow[\text{by '2'}]{T.Sh}$



$\xrightarrow[\text{by } 3]{T.\,scaling}$



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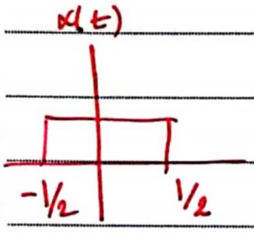
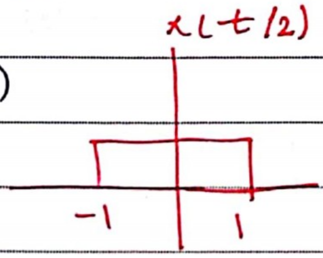
b) * example : $g(t) = x(2 - t/2) = x\left(\frac{-1}{2}(t-4)\right)$

T.s ↗ ↖ T.sh

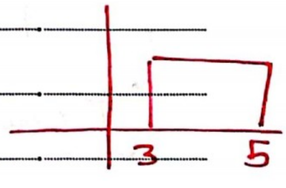
method ① :-



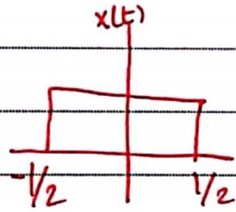
T.s by (-1/2) →



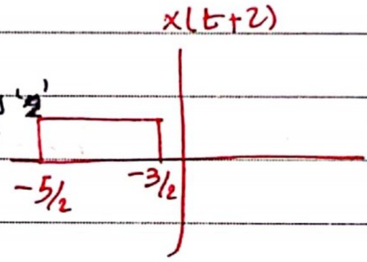
T.sh → by 4 (right t)



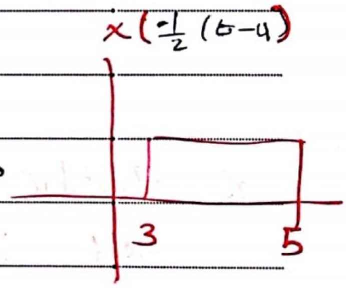
method ② :-



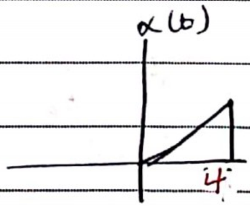
T.sh by '2' →



T.scaling → by '-1/2'

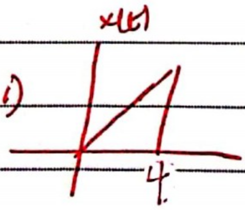


c) * example:

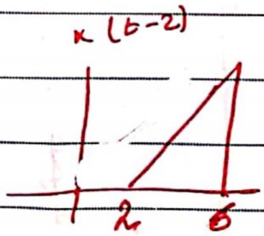


1) find $y(t) = x(3t-2)$

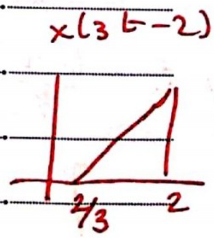
1)



T.sh by '2' → right

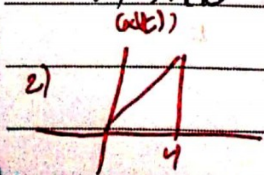


T.S by '3' →

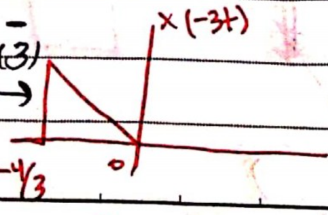


2) find $g(t) = x(2-3t) = x(-3(t-2/3))$

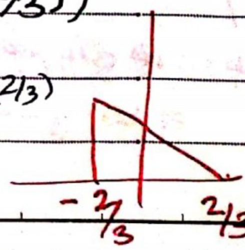
2)



T.s by (-3) →



T.sh by (2/3) →



بنتمم ع ٣ بعدين ريفلكت.

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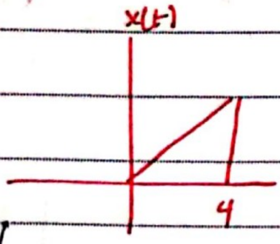
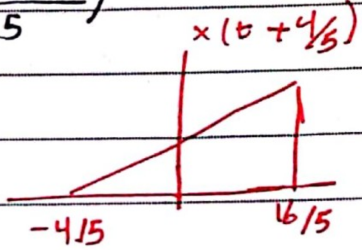
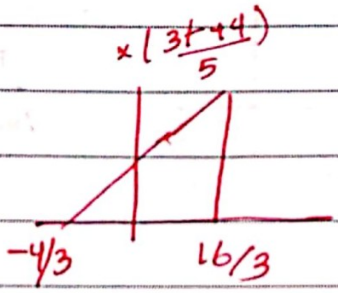
d) * example $\quad c(t) = x\left(\frac{3t+4}{5}\right)$



T.Sh by 4/5 $\longrightarrow$



T.Scaling by '3/5' $\longrightarrow$

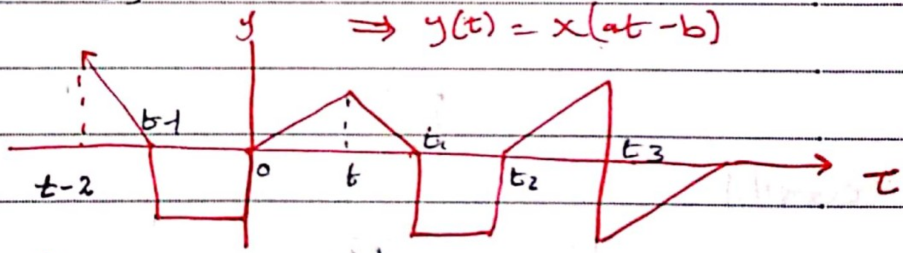


* method ③ III :- (book)

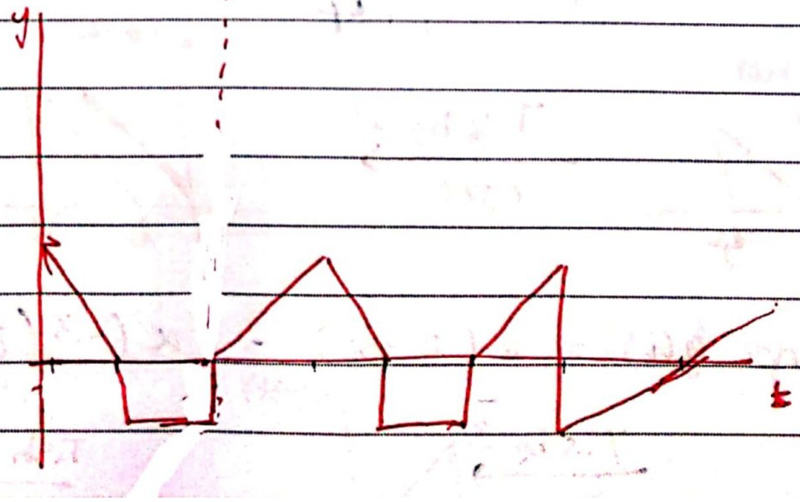
x(t) is given

$y(t) = x(at-b)$

given x(t) :



$\boxed{\begin{array}{l} \tau = at - b \\ t = \frac{\tau + b}{a} \end{array}}$



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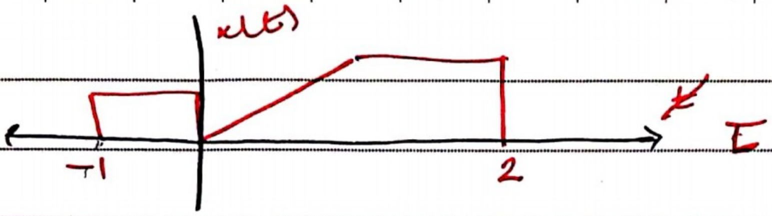
example

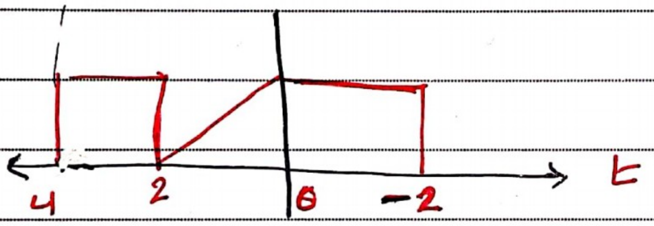


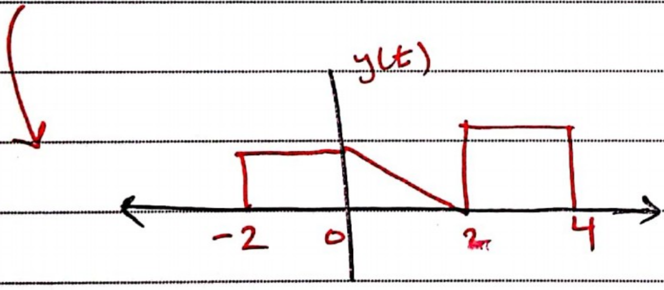
find $x(1 - t/2)$

$\tau = 1 - \frac{t}{2}$

$t = 2 - 2\tau$

τ	t
-1	4
0	2
1	0
2	-2





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* Mathematically : (very important)

$x(t)$ is given $y(t) = x(at-b)$

ex: $x(t) = t^2$

* $y(t) = x(3t+1) = (3t+1)^2$
* $g(t) = x(4t) = (4t)^2$

ex: let $x(t) = \begin{cases} 2t, & t>0 \\ -t, & t<0 \\ 0, & t=0 \end{cases}$, Find $y(t) = x(1-2t)$

Sol. $y(t) = x(1-2t) = \begin{cases} 2(1-2t), & (1-2t)>0 \\ -(1-2t), & (1-2t)<0 \\ 0, & 1-2t=0 \end{cases}$

$= \begin{cases} 2-4t, & t<\frac{1}{2} \\ 2t-1, & t>1/2 \\ 0, & t=1/2 \end{cases}$

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$\boxed{*}$ let $x(t) = \begin{cases} g_1(t), & h_1(t) > 0 \\ g_2(t), & h_2(t) > 0 \\ \vdots \\ g_N(t), & h_N(t) > 0 \end{cases}$

Then $x(f(t)) \overset{\circ}{=} x \text{ of } (t)$

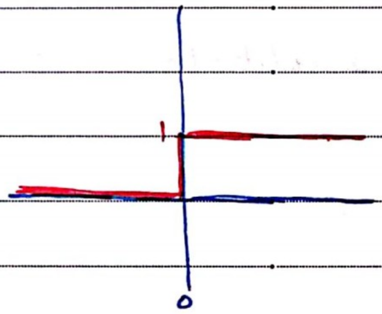
$$= \begin{cases} g_1(f(t)), & h_1(f(t)) > 0 \\ g_2(f(t)), & h_2(f(t)) > 0 \\ \vdots \\ g_N(f(t)), & h_N(f(t)) > 0 \end{cases}$$

lec (15) = 27/10

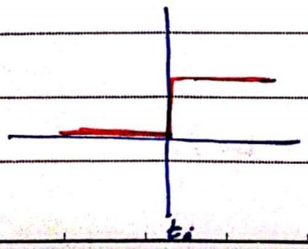
* Basic C.T signals

A) unit step function.

1. $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \\ \text{undef}, & t = 0 \end{cases}$



2. $u(t - t_0) = \begin{cases} 1, & t - t_0 > 0 \\ 0, & t - t_0 < 0 \end{cases}$



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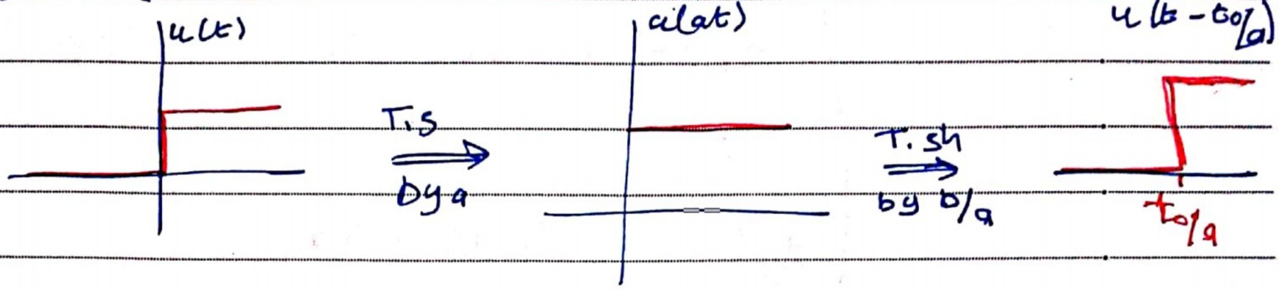
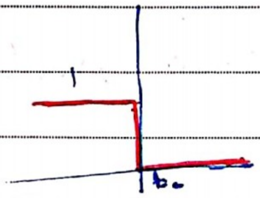
3. $u(at - t_0)$

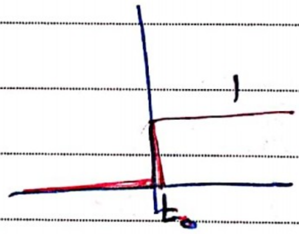


$$u(at - t_0) \begin{cases} u\left(t - \frac{t_0}{a}\right) & , a > 0 \\ u\left(\frac{t_0}{a} - t\right) & , a < 0 \end{cases}$$

* $u(t_0 - t)$ =

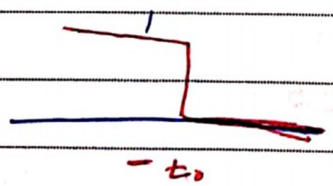


* $u(t - t_0)$ =



$$u(-t - t_0) = u(-(t + t_0)) \Rightarrow u(-t-t_0) = \begin{cases} 1 & , -t-t_0 > 0 \\ 0 & , -t-t_0 < 0 \end{cases}$$

1



4- $[u(t - t_0)]^N = u(t - t_0)$.

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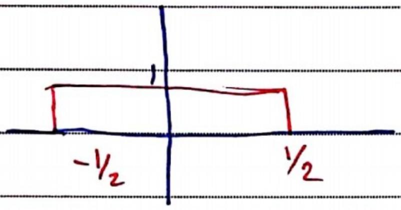
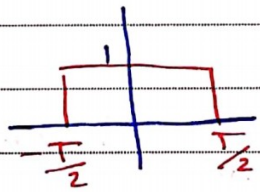
* rectangular function:-

(*) $\text{rect}(t) = \begin{cases} 1 & , -\frac{1}{2} < t < \frac{1}{2} \\ 0 & , \text{o.w} \end{cases}$

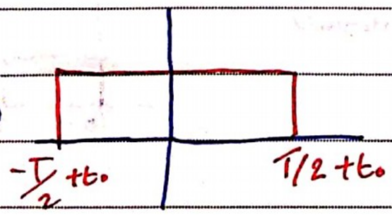


(*) $\text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & , \frac{T}{2} > t > -\frac{T}{2} \\ 0 & , \text{o.w} \end{cases}$



$1, \frac{1}{2} > \frac{t-t_0}{T} > -\frac{1}{2}$

(*) $\text{rect}\left(\frac{t-t_0}{T}\right) = \begin{cases} 1 & , \frac{T}{2}+t_0 > t > -\frac{T}{2}+t_0 \\ 0 & , \text{o.w} \end{cases}$



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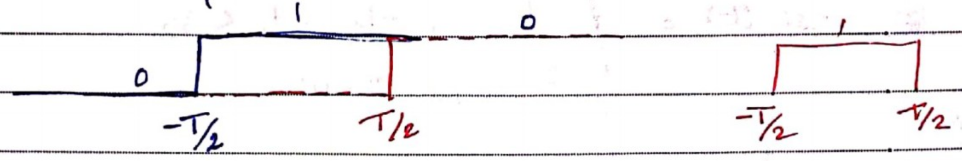
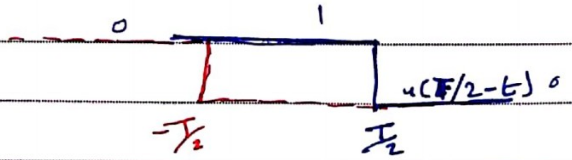
$\boxed{*}$ $\text{ret}\left(\frac{t}{T}\right) = \begin{cases} u(t+T/2)\cdot u(T/2-t) & ---③ \\ u(t+T/2) - u(t-T/2) & ---① \text{ proof!} \\ u(T/2-t) - u(-t-T/2) & ---② \end{cases}$

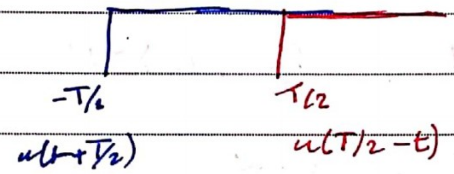
① proof!



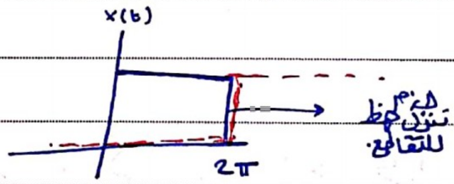
② proof!



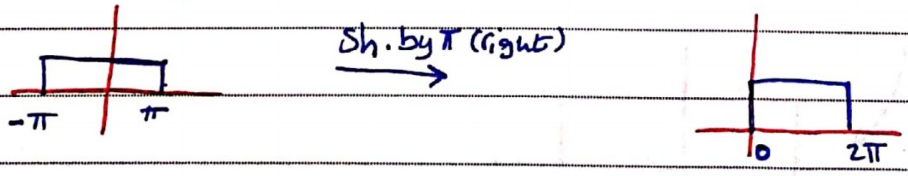
③ proof!



* example : Express $x(t) = u(t) - u(t-2\pi)$ in terms of rect funct.



$\hookrightarrow \text{rect}\left(\frac{t}{2\pi}\right)$



* power in unit step funct:

$$\lim_{T\to\infty} \int_0^T \frac{(u(t))^2}{2T}\, dt = \lim_{T\to\infty} \frac{T}{2T} = 1/2$$

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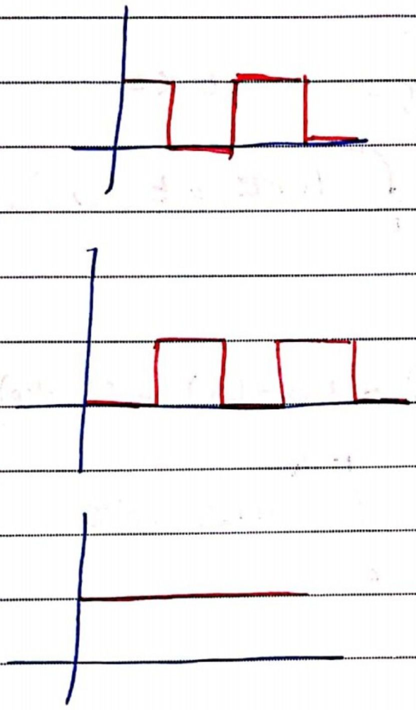
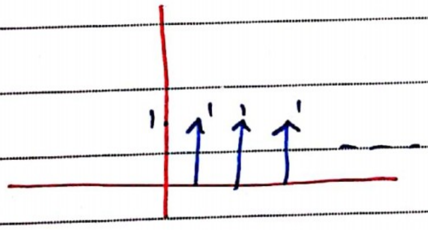
* unit step funct isn't periodic signals. (a periodic)

↳ مش شرط اذا السيجنالز مش (periodic) يكون ال power لها Infinity



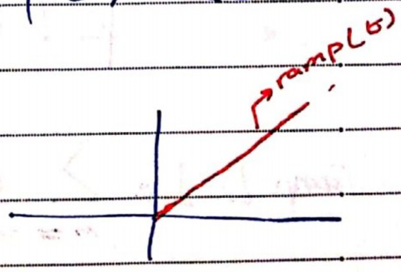
* D.T unit step function



$$u[n] = \begin{cases} 1 & , n \geq 0 \\ 0 & , n < 0 \end{cases}$$

* ramp signals

$$ramp(t) = \begin{cases} t & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$



$$\boxed{ramp(t) = t \cdot u(t)} \quad \text{- - - (I)}$$

$$t \cdot \begin{cases} 1 & , t > 0 \\ 0 & , t < 0 \\ \text{undf} & , t = 0 \end{cases}$$

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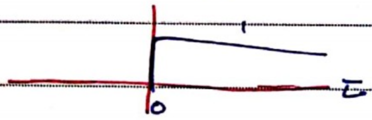
$$\boxed{ramp(t) = \int_{-\infty}^{t} u(\tau) \cdot d\tau} \quad \text{——} \quad II$$

proof :

$$\int_{-\infty}^{t} u(\tau) \cdot d\tau = \begin{cases} 0 & t < 0 \\ \int_{0}^{t} 1 \cdot d\tau = t & , \ t \geq 0 \end{cases}$$

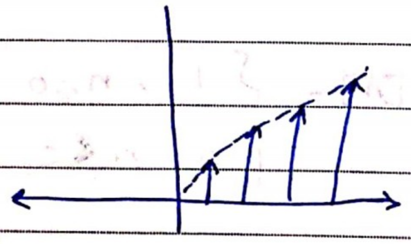


I.a $\rightarrow$ $ramp(t - t_0) = (t - t_0)\, u(t - t_0)$

II.a $ramp(t - t_0) = \int_{-\infty}^{t - t_0} u(\tau) \cdot d\tau$

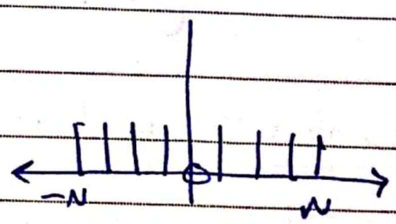
* D.T ramp signal.

$ramp[n] = n u[n]$



$$ramp[n] = \sum_{m=-\infty}^{n} u[m-1]$$

* D.T rect signal



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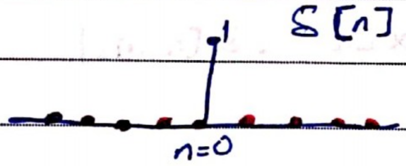
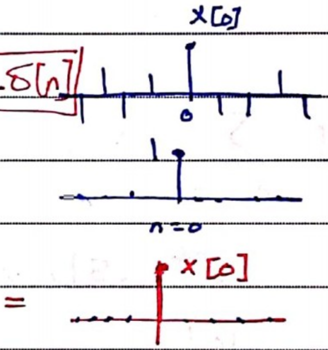
* Discrete Time Delta function (Kronecker Delta)

we define D.T delta function as $\delta[n] = \begin{cases} 1 & , n=0 \\ 0 & , n \neq 0 \end{cases}$



δ funct: it used to model events that happen at a particular time.

[1] $x[n] \cdot \delta[n] = \boxed{x[0] \cdot \delta[n]}$



→ multiplied

extract information (specify value & location)

example

$$Yazan = \begin{array}{ccccc} prob & signal & c_1 & c_2 & c_m \\ A & B & B^+ & B^- & C \end{array}$$

$$\left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \end{array}\right]$$

يزن جاب A في مادة ال prob $\quad [A \;\; 0 \;\; 0 \;\; 0 \;\; 0]$

[2] $\underbrace{\sum_{n=-\infty}^{\infty} x[n]\, \delta[n] = x[0]}_{\text{sifting property}}$

← غربلة

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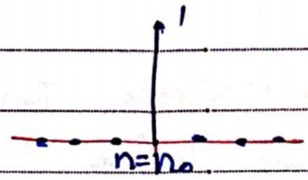
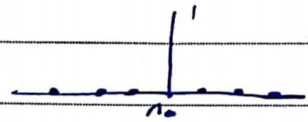
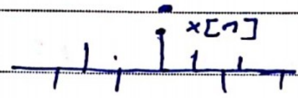
$* \quad \delta[n-n_0] = \begin{cases} 1 & , n = n_0 \\ 0 & , n \neq n_0 \end{cases}$

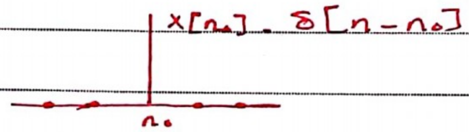


$\boxed{1}$ $x[n] \cdot \delta[n-n_0] = x[n_0] \cdot \delta[n-n_0]$





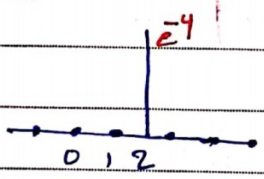
$=$



$\boxed{2}$ $\sum_{n=-\infty}^{\infty} x[n]\,\delta[n-n_0] = x[n_0]$

example: $e^{-2n} \cdot \delta[n-2] = e^{-2\cdot 2}\,\delta[n-2]$

$= e^{-4} \cdot \delta[n-2]$

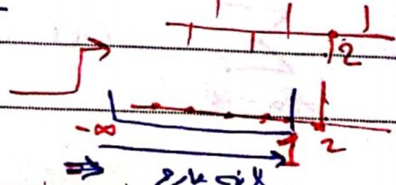


example: $\cos\left(\frac{\pi n}{8}\right) \cdot \delta[n-2] = \cos\left(\frac{2\pi}{8}\right) \cdot \delta[n-2]$

$= \frac{1}{\sqrt{2}}\,\delta[n-2]$

example: $\sum_{n=-\infty}^{\infty} \cos\left(\frac{n\pi}{8}\right) \delta[n-2] = \frac{1}{\sqrt{2}}$

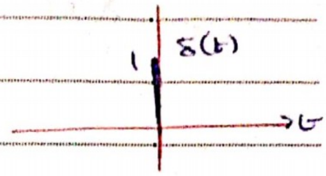
example: $\sum_{n=-\infty}^{1} \cos\left(\frac{n\pi}{8}\right) \delta[n-2] = 0$



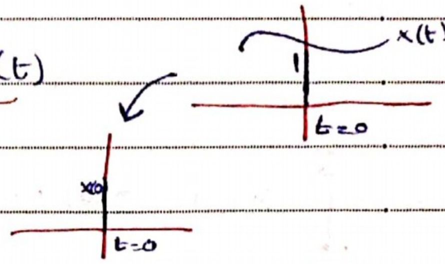
$\Rightarrow$ لانه ما رح
يوصل ال ٢
من $-\infty$ لـ 1

C.T Delta function (Dirac Delta)

let $\delta = \begin{cases} 1, & t=0 \\ 0, & t \neq 0 \end{cases}$



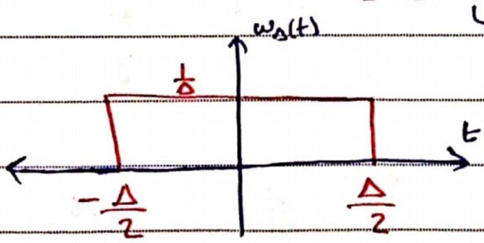
1) $x(t) \cdot \delta(t) = \underbrace{x(0) \cdot \delta(t)}$ $\rightarrow =$



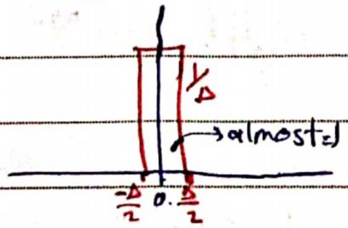
2) $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = 0 \neq x(0)$

so Delta function above is not correct (is not good definition)

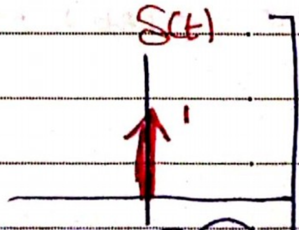
So :- let $\delta(t) = \lim_{\Delta \to 0} \underbrace{\frac{1}{\Delta}\,\text{rect}\left(\frac{t}{\Delta}\right)}_{w_\Delta(t)}$



as $\Delta \longrightarrow 0$



$\Rightarrow$



$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

it used to model event that happen over a short time interval but transfer non negligible amount of physical quantity!

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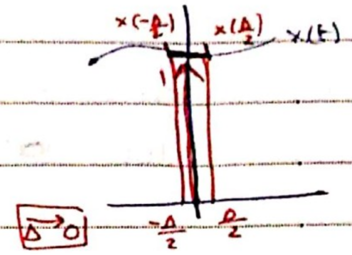
① $x(t) \cdot \delta(t) = x(0) \cdot \delta(t)$



$x(-\frac{\Delta}{2}) = x(-\frac{\Delta}{2}) = x(0)$

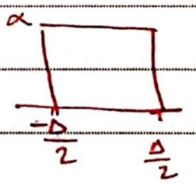
② $\int_{-\infty}^{\infty} x(t) \cdot \delta(t) \cdot dt = x(0)$

$\int_{-\infty}^{\infty} x(t) \cdot \delta(t) \cdot dt = \int_{-\infty}^{\infty} x(t) \left(\lim_{\Delta \to 0} \left(\frac{1}{\Delta} \operatorname{rect}\left(\frac{t}{\Delta}\right)\right)\right) dt$

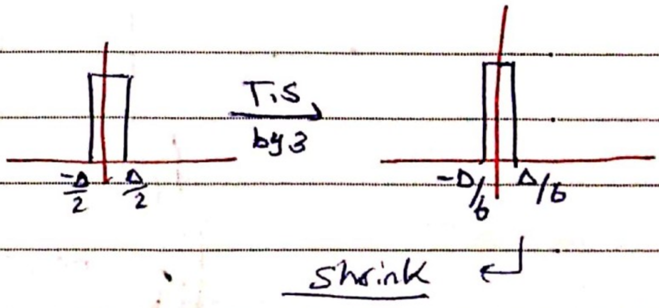
$= \lim_{\Delta \to 0} \left(\frac{1}{\Delta} \int_{-\infty}^{\infty} x(t) \cdot \operatorname{rect}\left(\frac{t}{\Delta}\right) \cdot dt \right)$

$= \lim_{\Delta \to 0} \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} x(t) \cdot dt = x(0)$

$= (\Delta \cdot x(0))$



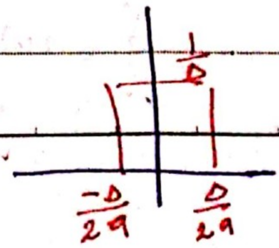
example: find $\delta(3t) \rightarrow ?$



example: show that $\delta(at) = \frac{1}{|a|} \delta(t)$, $a \neq 0$

$\delta(t) = \lim_{\Delta \to 0} \frac{1}{\Delta} \operatorname{rect}\left(\frac{t}{\Delta}\right)$

$\delta(at) = \lim_{\Delta \to 0} \frac{1}{\Delta} \operatorname{rect}\left(\frac{t}{\Delta/a}\right)$



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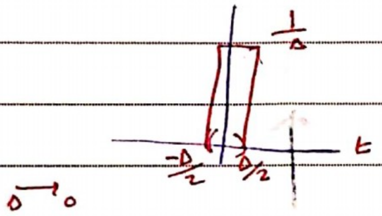
* unit step function and Dirac Delta funct.

let us try to evaluate

$$\int_{-\infty}^{t} \delta(t)\,.dt = \begin{cases} 1 & , t>0 \\ 0 & , t<0 \\ \text{und} & , t=0 \end{cases}$$



$= u(t)$

$$\text{so,} \quad u(t) = \int_{-\infty}^{t} \delta(t)\, d\tau \quad \text{—— I}$$

from I, we have:

$$\frac{d\,u(t)}{dt} = \delta(t) \quad \text{—— II}$$

$$\underbrace{\int_{-\infty}^{\infty} x(t).\,\delta(t)\,.dt}_{L.H.S} = \underbrace{x(0)}_{R.H.S} \quad \text{—— III}$$

let us verify that $\delta(t) = \frac{du(t)}{dt}$ in III

$$L.H.S = \int_{-\infty}^{\infty} x(t).\left(\frac{du(t)}{dt}\right) dt = \underbrace{x(t)\,u(t)\Big|_{-\infty}^{\infty}}_{x(\infty)} - \underbrace{\int_{-\infty}^{\infty} u(t).\frac{dx(t)}{dt}\,dt}_{x(t)\big|_0^{\infty} = x(\infty) - x(0)}$$

$$\Rightarrow \cancel{x(\infty)} - \cancel{x(\infty)} + x(0) = x(0)$$

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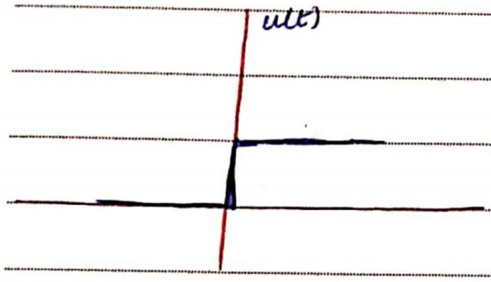
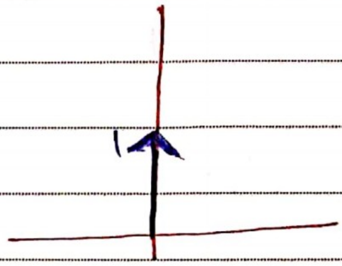
$\boxed{1-}$ $\displaystyle\int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$

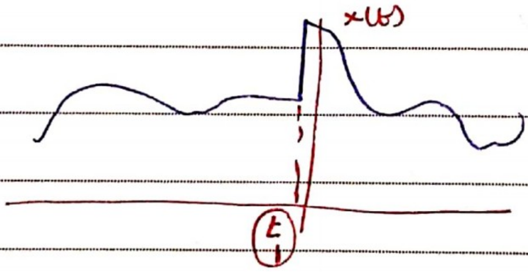
$\boxed{2-}$ $\displaystyle\frac{d\,u(t)}{dt} = \delta(t)$



$\longrightarrow \dfrac{du(t)}{dt}$



let $x(t)$ be discontinous at $t = t_1$



Then:

$$\frac{d\,x(t)}{d(t)} = \begin{cases} x(t) & , \boxed{t \neq 0} \\ x(t) + -x(t_1^-)\,\delta(t-t_1) & \boxed{t = t_1} \end{cases}$$

$$\frac{dx(t)}{dt} = x'(t) + (x(t_1^+) - x(t_1^-))\,\delta(t-t_1)$$

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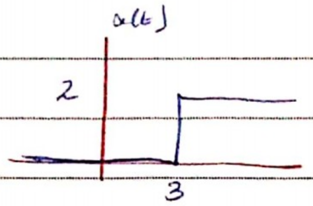
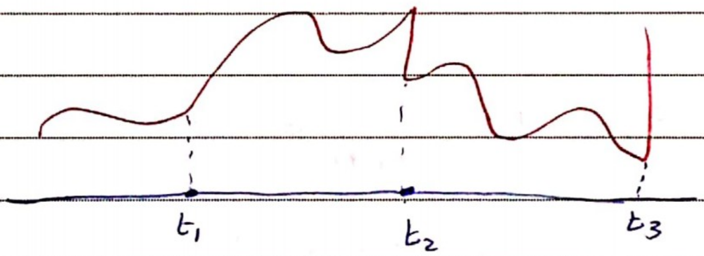
* example



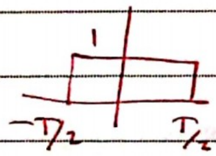
sol. $$\frac{dx(t)}{dt} = 2\delta(t-3)$$

* let $t_1, t_2, t_3, \ldots t_N$ be the discontinuity points :

$$\frac{dx(t)}{dt} = x'(t) + \sum_{i=1}^{N} \left(x(t_i^+) - x(t_i^-)\right)\delta(t-t_i)$$



* example let $x(t) = rect\left(\frac{t}{T}\right)$ find $x'(t)$



$$\frac{dx(t)}{dt} = (1-0)\,\delta(t+T/2) + (0-1)\,\delta(t+T/2)$$
$$= \delta(t+T/2) - \delta(t-T/2)$$

* rect : $u(t+T/2) - u(t-T/2)$

$\delta(t+T/2) - \delta(t-T/2)$.

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Quick Summary:

let $x(t)$ be a cont. function at $t = t_o$

1- $x(t) \cdot \delta(t - t_o) = x(t_o)\, \delta(t - t_o)$

2- $\int_{-\infty}^{\infty} x(t)\, \delta(t - t_o)\, dt = x(t_o)$

3- $\int_{-\infty}^{\infty} x(t - t_o)\, \delta(t) \cdot dt = x(t - t_o)\Big|_{t=0} = x(-t_o)$

4- a) $u(t) = \int_{-\infty}^{t} \delta(\tau) \cdot d\tau$

b) $u(t - t_o) = \int_{-\infty}^{t-t_o} \delta(\tau) \triangleq \int_{-\infty}^{t} \delta(\tau - t_o)\, d\tau$

5- a) $\delta(at - t_o) = \frac{1}{|a|}\, \delta\left(t - \frac{t_o}{a}\right)$

Proof: $\int_{-\infty}^{\infty} x(t)\, \delta(\underbrace{at - t_o}_{v})\, dt$

let $v = at - t_o \Rightarrow \frac{v + t_o}{a} = t$

$dv = a \cdot dt$

$\int_{-\infty}^{\infty} x\left(\frac{v + t_o}{a}\right) \cdot \delta(v) \cdot \frac{dv}{|a|} = \frac{1}{|a|} * x\left(\frac{t_o}{a}\right)$

$= \int_{-\infty}^{\infty} x(t) \cdot \underbrace{\delta\left(t - \frac{t_o}{a}\right)}_{|a|}\, dt \Rightarrow$ Note $\left(\delta(at - t_o) = \frac{1}{|a|} \delta\left(t - \frac{t_o}{a}\right)\right)$

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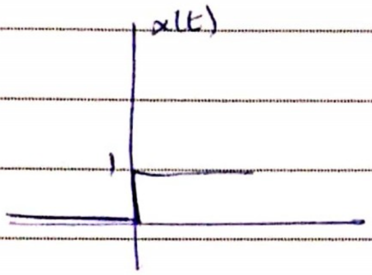
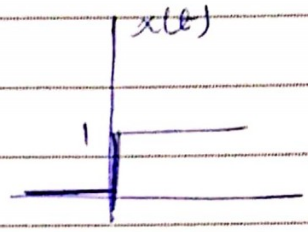
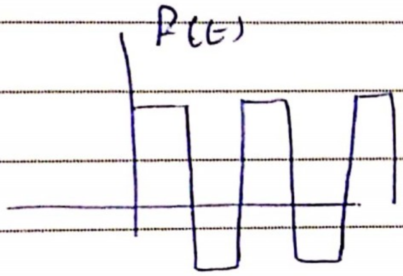
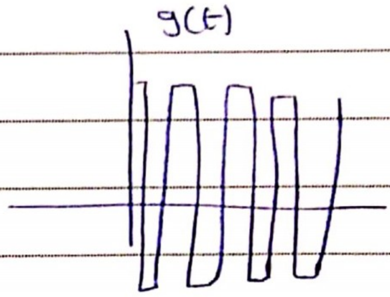
let $\int_{-\infty}^{\infty} x(t)\, f(t)\, dt = \int_{-\infty}^{\infty} x(t)\, g(t)\,.dt$

is not nessesary that $g(t) = f(t)$









So. 5- a) $\delta(at - t_0) = \frac{1}{|a|}\,\delta\left(t - \frac{t_0}{a}\right)$

b) $\int_{-\infty}^{\infty} x(t)\, \delta(at - t_0)\, dt = \frac{1}{|a|}\, x\left(\frac{t_0}{a}\right)$

c) $\delta(t) = -\delta(t)$.

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examples: ① $\int_{-1}^{1} (3t^2+1)\ \delta(t)\, dt = (3(0)^2+1) = 1$

② $\int_{-\infty}^{\infty} e^{-t}\ \delta(2t-2)\ dt = \frac{1}{2}\int_{-\infty}^{\infty} e^{-t}\ \delta(t-1)\, dt$

$= \frac{1}{2}\ e^{-1}$

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* Systems

$x(t)$ ——— [System] ——— $y(t)$

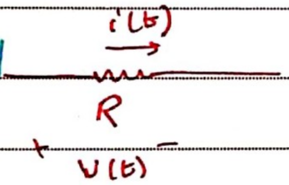
input signal — output signal

A system is a process that transforms the input signal into an output signal.

$x(t)$ ——— [T] ——— $y(t)$

$$y(t) = T\{x(t)\}$$

example



$$v(t) = R \cdot i(t)$$

output — input

$$\boxed{v(t) = T\{i(t)\} = R \cdot i(t)}$$

* شو تأثير R على الـ system ؟

هو عبارة عن $\boxed{\text{system parameter}}$ (بغير الـ output بالسيجنال)

$i(t)$ ——— [T] ——— $v(t)$

systems parameter

~~example~~

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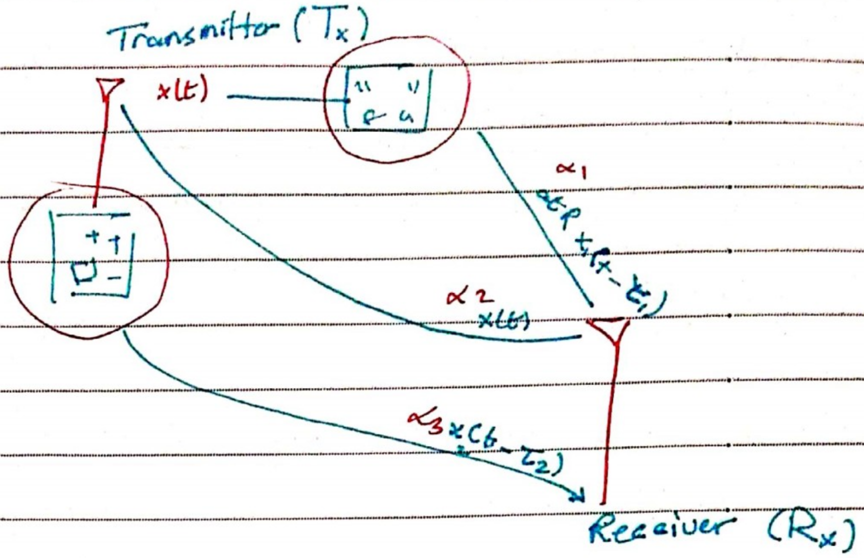
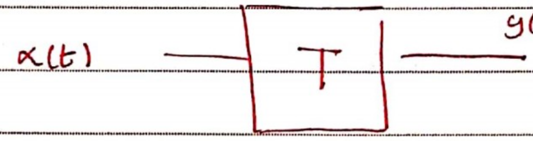
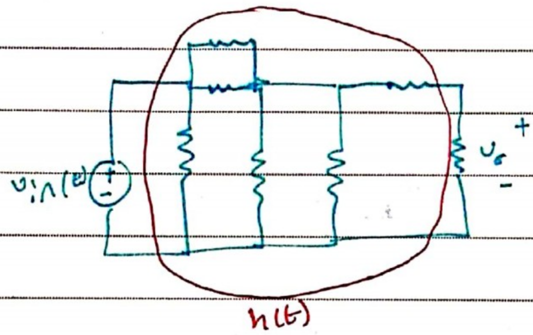
* Example 2

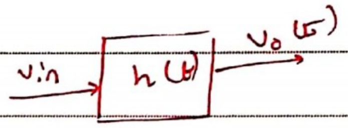




$y(t) = \alpha_1 + \alpha_2 + \alpha_3$

$= x_1(t) + x(t-\tau_1) + x(t-\tau_2)$





* let $y_1(t) = T\{x_1(t)\}$

$y_2(t) = T\{x_2(t)\}$

if $T\{x_1(t) + x_2(t)\} = T\{x_1(t)\} + T\{x_2(t)\}$

↳ system is linear.

$v(t) = i(t) \cdot R + c$; جهاز ohm's low لو كان ال *

$v_1(t) = i_1(t) R + c$

$v_2(t) = i_2(t) R + c$

$V_\Sigma = (i_1 + i_2) R + c \neq i_1 R + c + i_2 R + c$ (non-linear)

$\boxed{\text{if } c = 0}$ ؟ متى يكون الآلة تكون الـ system linear

* Memoryless Systems & System with memory

a system is said to ~~have~~ have memory if the output signal at $t = t_0$ depends on the ~~xxx~~ input signal at $t = t_1$, where $t_1 \neq t_0$. otherwise the system is memoryless

$y(t_0)$ = depends on $x(t_0)$ ⟶ memoryless (static)

$y(t_0)$ = depends on $x(t_1)$ ⟶ $t_1 \neq t_0$ $\xrightarrow{\text{has}}$ Memory (Dynamic)

examples: ① $y(t) = x(t) + x(t-1)$ ⟶ has memory

② $y(t) = \sqrt{x(t)}$ ⟶ memoryless system

③ $y(t) = \cos(t+1)\, x(t)$ ⟶ memoryless system.

لانو ال cos هو عبارة عن function
وهي بس العلاقة بين y و x
يعني اعتماد y هو بس على x

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Ex:- $y(t) = \cos(t+1)\, x(t)$. $\longrightarrow$ memoryless system.

$$y[n] = y[n-1] + x[n]$$

let $y[0] = 0$

$x[0] = 0$

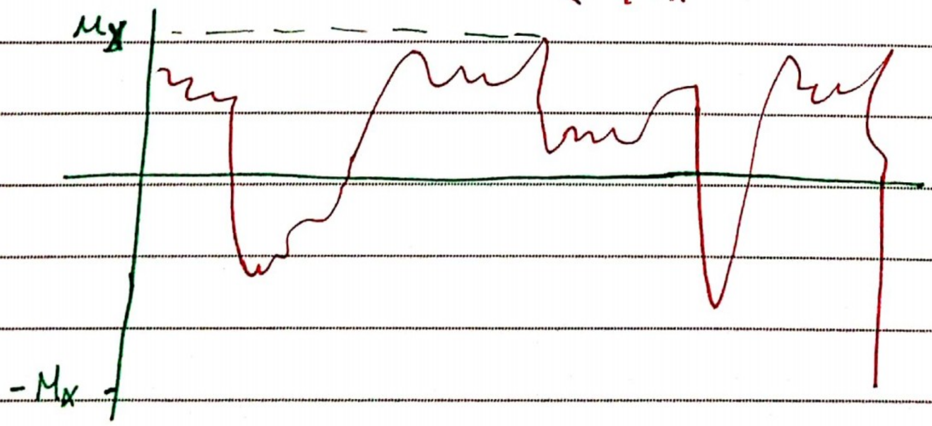
$$y[1] = \cancel{y[0]}^{\,0} + x[1]$$

$y[2] = y[1] + x[2]$ $\quad$ $y[1]$ [illegible] $y[2]$

$y[2] = x[1] + x[2]$ $\longrightarrow$ so, the system ~~is~~ with memory

* Bounded input, Bounded output (BIBO) stability.

A signal $x(t)$ is said to be bounded if there is M_x such that $|x(t)| \leq M_x < \infty$



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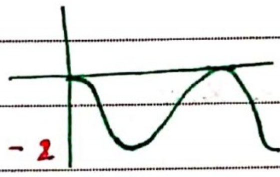
ex: $x(t) = \cos(t)$.

$|x(t)| \leq 1 < \infty$ (Bounded)

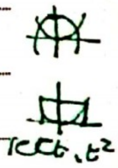
ex: $x(t) = \cos(t) - 1$



$|\cos(t) - 1| \leq 2$ (Bounded signal)

ex: $x(t) = e^{-t}$, $x(t) = t$ $\longrightarrow$ unbounded

ex: $e^{-t}u(t)$, $e^{t}u(-t)$ $\longrightarrow$ bounded

* a system is BIBO ~~system~~ stable if every bounded input signal produces a bounded output signal. 

ex:- IS $y(t) = t^2 x(t)$ BIBO STABLE ??

if $x(t) = 1$ $\longrightarrow$ bounded

$y(t) = t^2 . 1$ $\longrightarrow$ unbounded

] so, NOT BIBO Stable

* عشان نقرر اذا - system تكون دايماً stable اولاً

~~[illegible]~~ لازم نجرب لنلاقي ع الأقل وحدة بتعطي

NOT BIBO Stable

يعني احنا بنهتم لما تكون عنا Bounded input بتعطي unbounded

* سهل تعرف انه الـ system ← not BIBO stable بس صعب تعرف انه stable

(show that stable دايما لما لقيت احكي لازم نفكر بالـ Bounde (القاعدة العامة)

ex: if $y(t) = e^{x(t)}$. the system is BIBO stable.

Suppose that $|x(t)| \leq M_x < \infty$

$y(t) = e^{x(t)} \leq e^{M_x} < \infty$

↳ so it's BIBO stable.

ex: $y(t) = e^{t}\, x(t)$

let $x(t) = 1$ ⟶ bounde

$y(t) = e^{t}$ ⟶ unbounde

↳ Not BIBO stable.

* linear systems :-

1) Additivity property:

$$x(t) \longrightarrow \boxed{T} \longrightarrow y(t) \qquad y(t) = T\{x(t)\}$$

let $T\{x_1(t)\} = y_1(t)$
$T\{x_2(t)\} = y_2(t)$

Then the system is additive
if $T\{x_1(t) + x_2(t)\} = T\{x_1(t)\} + T\{x_2(t)\}$

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$y(t) = t\ x(t)$. Is the system additive?

$T\{x_1(t)\} = t\ x_1(x) = y_1(t)$

$T\{x_2(t)\} = t\ x_2(x) = y_2(2)$

$\hookrightarrow T\{x_1(t) + x_2(t)\} = t \cdot (x_1(t) + x_2(t))$

$= t\ x_1(t) + t x_2(t)$

$= y_1(t) + y_2(t)$ $\longrightarrow$ so it's additive.

$y(t) = K \cdot x(t) + c$, where K, c are const.

Is the system additive?

$T\{x_1(t)\} = K\ x_1(t) + c = y_1(t)$

$T\{x_2(t)\} = K x_2(t) + c = y_2(t)$

$\hookrightarrow T\{x_1(t) + x_2(t)\} = K x_1(t) + K x_2(t) + c$

$\neq T\{x_1(t)\} + T\{x_2(t)\}$

The system is additive just if $c=0$.

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* Additivity: $x(t) \longrightarrow \boxed{T} \longrightarrow y(t)$

let $T\{x_1(t)\} = y_1(t)$

$T\{x_2(t)\} = y_2(t)$

Then the system is additive if

$$T\{x_1(t) + x_2(t)\} = T\{x_1(t)\} + T\{x_2(t)\}$$

* example: $y(t) = t \cdot x(t)$

$T\{x_1(t)\} = y_1(t) = t\, x_1(t)$

$T\{x_2(t)\} = y_2(t) = t\, x_2(t)$

$T\{x_1(t) + x_2(t)\} = t\,(x_1(t) + x_2(t)) = t\, x_1(t) + t\, x_2(t)$

$\Longrightarrow$ The system is additive

an electrical Aplication on Additivity: Super postion

	1	2	
2 indep. src	on	off	$\longrightarrow v'$
	off	on	$\longrightarrow v''$
	on	on	$\longrightarrow v = v' + v''$

H.W: ① $y(t) = x^2(t)$

② $y(t) = x(t) * (t-1)$.

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* Homogenity:

$$x(t) \longrightarrow \boxed{T} \longrightarrow y(t)$$

let $T\{x(t)\} = y(t)$

then The system is Homogenues if

$$T\{a x(t)\} = a\, T\{x(t)\} = a \cdot y(t)$$

* example $\quad y(t) = t \cdot x(t)$

$$T\{a\, x(t)\} = t \cdot a\, x(t)$$
$$= a \cdot t\, x(t)$$
$$= a \cdot y(t)$$

$\Longrightarrow$ Homo.

* example $\quad y(t) = K\, x(t) + C$

$$T\{a\, x(t)\} = K a\, x(t) + C \neq a\, y(t)$$

$\Longrightarrow$ non homo.

* example $\quad y(t) = x^2(t) \Rightarrow$ NON Homo.

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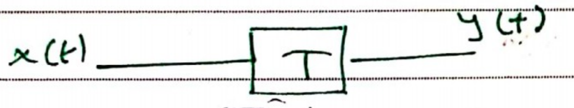
* linearity :- A system is linear if it is both additive and homogenuous.

consider a system:



let $T\{x_1(t)\} = y_1(t)$
and $T\{x_2(t)\} = y_2(t)$

Then the system is liner if:

$$T\{a_1 x_1(t) + a_2 x_2(t)\} = T\{a_1 x_1(t)\} + T\{a_2 x_2(t)\}$$ by additivity

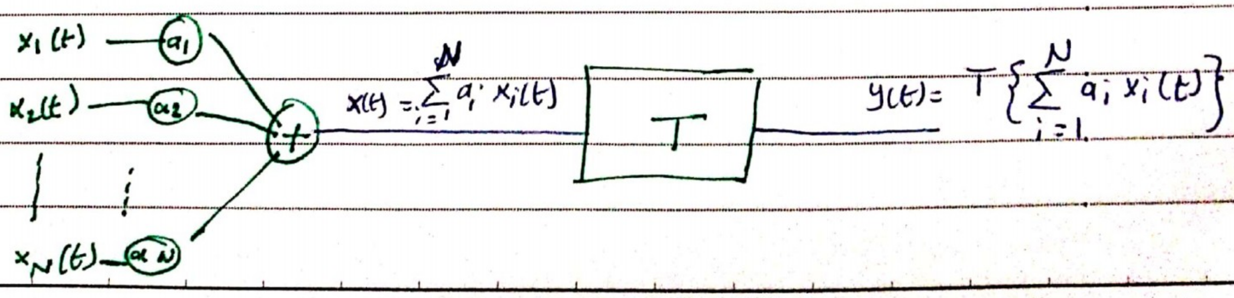
$$= a_1 T\{x_1(t)\} + a_2 T\{x_2(t)\}$$ by homo.

$$= a_1 y_1(t) + a_2 y_2(t).$$

* In general : let $T\{x_i(t)\} = y_i(t)$ for $i = 1, 2, \ldots N$
then the system is Liner if :

$$T\left\{\sum_{i=1}^{N} a_i x_i(t)\right\} = \sum_{i=1}^{N} T\{a_i x_i(t)\}$$ by additivity.

$$= \sum_{i=1}^{N} a_i T\{x_i(t)\}.$$



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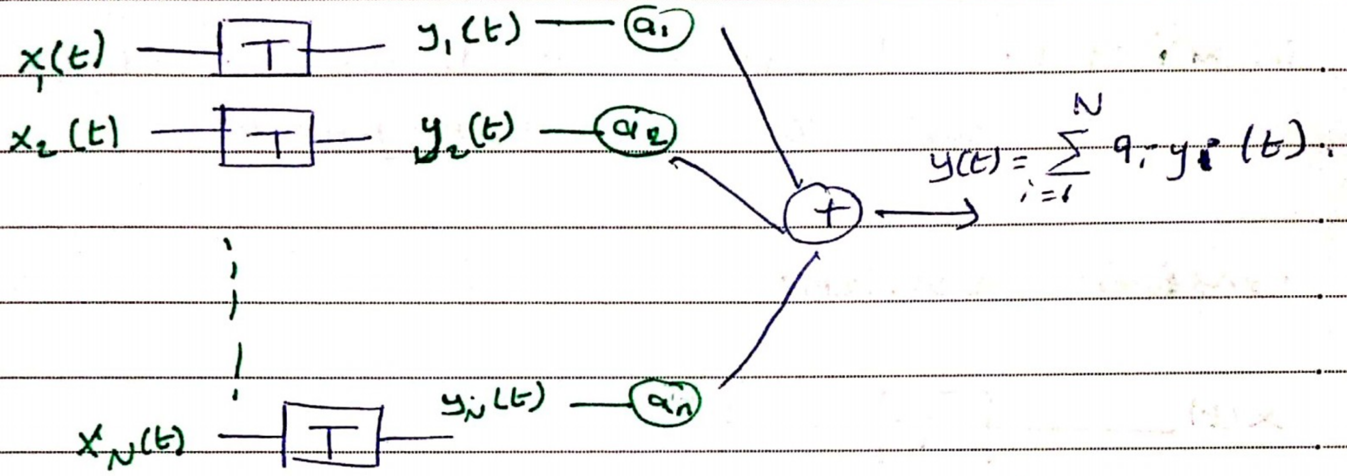

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ex:

Consider a linear system.

$$x(t) \rightarrow \boxed{T} \rightarrow y(t).$$

with $T\{x_1(t) = e^{j2\pi t}\} = e^{j3\pi t}$

and $T\{x_2(t) = e^{-j2\pi t}\} = e^{-j3\pi t}$

Find $y(t) = T\{\sin 2\pi t)\}$.

using the fact that.

$$\sin(2\pi t) = \frac{e^{j2\pi t}}{2j} + \frac{-1}{2j} e^{-j2\pi t}$$

$$T\{\sin(2\pi t)\} = T\left\{\frac{1}{2j} e^{j2\pi t} + \frac{-1}{2j} e^{-j2\pi t}\right\}$$

$$= \frac{1}{2j} T\{e^{j2\pi t}\} - \frac{1}{2j} T\{e^{-j2\pi t}\}.$$

$$= \frac{1}{2j} e^{j3\pi t} - \frac{1}{2j} e^{-j3\pi t} \qquad = \sin(3\pi t)$$

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/ /

Time invariace:

the output corresponding to a delayed version of the input signal is the same as delaying the output.

$x(t) \rightarrow \boxed{\text{parameters}} \rightarrow y(t)$

A system is T.I if

$T\{x(t-t_0)\} = y(t-t_0).$

T.I : system parameter don't change with time

note:

$x(t) \longrightarrow \boxed{s^{t_0}} \longrightarrow x_1(t) = x(t-t_0)$

s^{t_0}: delay

$x(t) \longrightarrow \boxed{s^{t_0}} \xrightarrow{x_1(t) = x(t-t_0)} \boxed{T} \longrightarrow y_1(t) = T\{x_1(t)\}$

$x(t) \longrightarrow \boxed{T} \xrightarrow{y(t)} \boxed{s^{t_0}} \longrightarrow y_2(t) = y(t-t_0)$

$y_1(t) = y_2(t).$

the system is T.I

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Ex: $y(t) = \cos(x(t))$, is the system T.I :

$x(t) \rightarrow \boxed{S^{t_0}} \rightarrow x_1(t) = x(t-t_0) \rightarrow \boxed{T} \rightarrow y_1(t) = \cos(x_1(t))$

$= \cos(x(t-t_0))$.

$x(t) \rightarrow \boxed{T} \rightarrow y(t) = \cos(x(t)) \rightarrow \boxed{S^{t_0}} \rightarrow y_2(t) = y(t-t_0)$

$= \cos(x(t-t_0))$.

$y_1(t) = y_2(t)$.

T.I

ex: $y(t) = x(2t)$ is the system T.I :

$x(t) \rightarrow \boxed{S^{t_0}} \rightarrow x_1(t) = x(t-t_0) \rightarrow \boxed{T} \rightarrow y_1(t) = T\{x_1(t)\}$

$x_1(2t)$

$x(2t-t_0)$

$x(t) \rightarrow \boxed{T} \rightarrow y(t) = x(2t) \rightarrow \boxed{S^{t_0}} \rightarrow y_2(t) = y(t-t_0)$.

$= x(2(t-t_0))$

$= x(2t-2t_0)$

$y_1(t) \neq y_2(t)$

T.V

ex: $y(t) = x(t\cos(t))$

$x(t) \rightarrow \boxed{S^{t_0}} \rightarrow x_1(t) = x(t-t_0) \rightarrow \boxed{T} \rightarrow y_1(t) = x_1(t\cos(t))$

$= x(t\cos(t) + t_0)$

$x(t) \rightarrow \boxed{T} \rightarrow y(t) = x(t\cos(t)) \rightarrow \boxed{S^{t_0}} \rightarrow y_2(t) = y(t-t_0)$

$= x((t-t_0)\cos(t-t_0))$.

T.V

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Causality:

A system is non causal if the output at t_0 $y(t_0)$ depends on the input at t_1 $x(t_1)$ for $t_1 > t_0$. otherwise the system causal.

Ex: $y(t) = x(t+1)$ non causal

$y(t) = x(t-1)$ causal.

$y(t) = x(t+1) + x(t-1)$

$y[n] = \dfrac{x[n] + x[n+1] + x[n-1]}{3}$ non causal.

$\tilde{y}[n] = y[n-1]$

$= \dfrac{x[n-1] + x[n] + x[n-2]}{3}$ causal

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* Impulse response:- is the output when the input signal $x(t) = \delta(t)$.

We use $h(t)$ to denote the impulse response

* consider a system

$$x(t) \longrightarrow \boxed{T} \longrightarrow y(t)$$

$$y(t) = T\{x(t)\}$$

impulse response $\leftarrow$ $h(t) = T\{\delta(t)\}$

for discrete signal D.T:-

$$x[n] \longrightarrow \boxed{T} \longrightarrow y[n]$$

impulse response $\leftarrow$ $h[n] = T\{\delta[n]\}$

example:- $y(t) = x(t) + x(t-1)$

Find $h(t)$?

sol. $h(t) = \delta(t) + \delta(t-1)$

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→ * The impulse response completely characterizes the LTI system, we can find the output "y(t)" for any input signal "x(t)", provided that "h(t)" is given.

Consider a D.T LTI system.

$$x[n] \longrightarrow \boxed{LTI} \longrightarrow y[n]$$

$$y[n] = T\{x[n]\}$$

$$h[n] = T\{\delta[n]\}$$

For time Invariance,

$$h[n-K] = T\{\delta[n-K]\}, \text{ for all } K \text{ integer}$$

The system's Output

$$y[n] = T\{x[n]\} \quad \text{———} \quad \mathrm{I}$$

using sifting property, we have

$$x[n] = \sum_{K=-\infty}^{\infty} x[K]\,\delta[n-K] \quad \text{———} \quad \mathrm{II}$$

making use of II in I

$$y[n] = T\left\{\sum_{K=-\infty}^{\infty} x[K]\,\delta[n-K]\right\}$$

$$= \sum_{K=-\infty}^{\infty} T\{x[K]\,\delta[n-K]\} \longrightarrow \text{Additivity}$$

$$= \sum_{K=-\infty}^{\infty} x[K]\cdot T\{\delta[n-K]\} \longrightarrow \text{Homogenity}$$

$$= \sum_{K=-\infty}^{\infty} x[K]\,h[n-K] \longrightarrow \text{by Time Invariance}$$

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$\Rightarrow$ $y[k] = x[n] * h[n]$. $\longrightarrow$ D.T convolution

* Consider a C.T LTI System.

$x(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$

$T\{x(t)\} = y(t)$

$h(t) = T\{\delta(t)\}$

impulse response

Time Invariance $\longrightarrow h(t-\tau) = T\{\delta(t-\tau)\}$

$y(t) = T\{x(t)\}$ —— ①

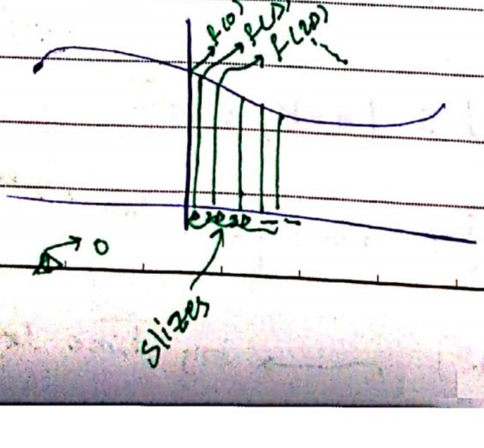
using sifting, we can write $x(t)$ as

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau \quad \text{—— ②}$$

making use ② in ①, we have

$$y(t) = T\left\{\int_{-\infty}^{\infty} x(t)\,\delta(t-\tau)\,d\tau\right\}$$

using $\int_{-\infty}^{\infty} f(x)\,.dx = \sum_{K=-\infty}^{\infty} f(K.\Delta).\Delta$ ④



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let $f(\tau) = x(\tau)\,\delta(t-\tau)$ ⑤

$f(K \cdot D) = \delta(t - K \cdot D)$

making use of 4 and 5 in 3

$$y(t) = T\left\{\sum_{K=-\infty}^{\infty} \Delta \cdot x(K \cdot D)\,\delta(t - K \cdot D)\right\}$$

$$= \sum_{K=-\infty}^{\infty} T\left\{\Delta \cdot x(K \cdot \Delta)\,\delta(t - K \cdot \Delta)\right\}$$ $\Rightarrow$ additivity.

$$= \sum_{K=-\infty}^{\infty} \Delta \cdot x(K \cdot \Delta)\, T\left\{\delta(t - K \cdot D)\right\}$$ Homo.

$$= \sum_{K=-\infty}^{\infty} \Delta \cdot x(K \cdot D)\, h(t - K \cdot D)$$ time Invariance

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$ "convolution Integral"

$$= x(t) * h(t)$$

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* LTI System.

for C.T $\quad x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

for D.T $\quad x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$

$$y[n] = x[n] * h[n]$$
$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

* properties of Convelution :-

1. Commutativity $\qquad$ (الاول ضرب الثاني = الثاني ضرب الاول.)

$$x(t) * h(t) = h(t) * x(t)$$

proof /

L.H.S $\quad = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$$= \int_{-\infty}^{\infty} x(t-v) \cdot h(v)\, dv \quad , \quad v = t - \tau \quad dv = -d\tau$$

$$= \int_{-\infty}^{\infty} h(v)\, x(t-v)\, dv \quad = h(t) * x(t).$$

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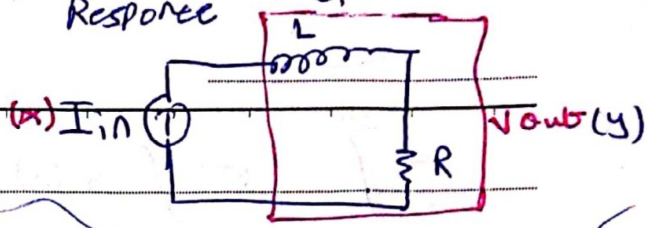
سؤال امتحان ع ال impulse Response (لازم نلاقي علاقة بين x و y)
(Liner or non Liner → فالـ لي الـ liner system دخل.)



2. associativity:-

$$[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$$

$$x(t) \longrightarrow \boxed{LTI} \xrightarrow{[x(t) * h_1(t)]} \boxed{h_2(t)} \longrightarrow y(t) = (x(t) * h_1(t)) * h_2(t).$$

$$x(t) \longrightarrow \boxed{h_1(t) * h_2(t)} \longrightarrow y(t) = x(t) * (h_1(t) * h_2(t))$$

$$h_{eq} = h_1(t) * h_2(t).$$

proof!

$$L.H.S \triangleq \underbrace{(x(t) * h_1(t))}_{f_1(t)}$$

$$= \int_{-\infty}^{\infty} f_1(t)\ h_2(t-\tau)\ d\tau \quad \longrightarrow (1)$$

but $$f_1(t) = \int_{-\infty}^{\infty} X(\delta)\ h_1(t-\delta)\ d\delta \quad \longrightarrow (2)$$

making use (2) in (1), yeilds,

$$L.H.S = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} x(\delta)\ h_1(t-\delta)\ d\delta \right) h_2(t-\tau)\ d\tau$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\delta)\ h_1(t-\delta)\ h_2(t-\tau)\ d\tau\ d\delta$$

let $\lambda = \tau - \delta$

$d\lambda = d\tau$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\delta)\ h_1(\lambda)\ h_2(t-\lambda-\delta)\ d\lambda\ d\delta$$

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$$= \int_{-\infty}^{\infty} x(\delta) \underbrace{\int_{-\infty}^{\infty} h_1(\lambda)\, h_2(t-\lambda-\delta)\, d\lambda)}_{f_2(t-\delta)}\, d\delta$$

$$\begin{cases} \text{let } f_2(t) = h_1(t) * h_2(t) \\ \qquad = \int_{-\infty}^{\infty} h_1(\lambda)\, h_2(t-\lambda)\, d\lambda \\ f_2(t-\delta) = \int_{-\infty}^{\infty} h_1(\lambda)\, h_2(t-\lambda-\delta)\, d\lambda \end{cases}$$

$$= \int_{-\infty}^{\infty} x(\delta)\, f_2(t-\delta) = x(t) * f_2(t)$$

$$= x(t) * (h_1(t) * h_2(t)).$$

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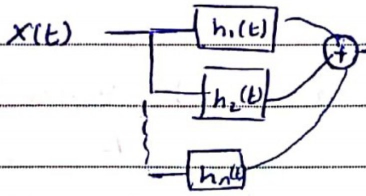
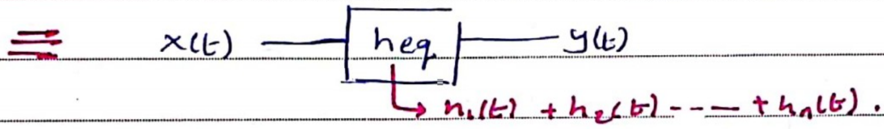
3. Distributivity:

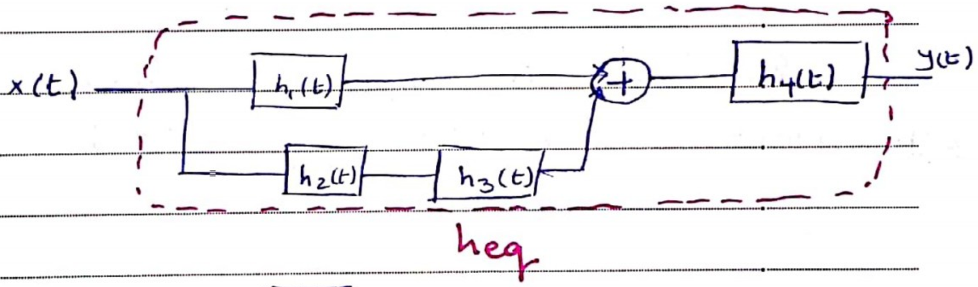
$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$

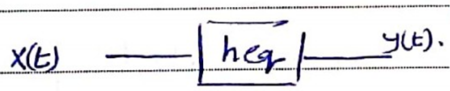


$$y(t) = x(t) * h_1(t) + x(t) h_2(t)$$
$$= x(t) * \underbrace{[h_1(t) + h_2(t)]}_{heq}$$



* example





4. Homogenity of Convolution

$$\boxed{x(t) * (a \cdot h(t)) = a\,(x(t) * h(t))}$$

5. convolution with Impulse delta funct.

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau = x(t)$$

عنصر محايد

ex:- $x(t) * \delta(t - t_0) = x(t - t_0)$

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Ex:- Consider a system,

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

1- Find the output when $x(t) = t\,u(t)$

Sol. $y(t) = \int_{-\infty}^{t} \tau \cdot u(\tau)\, d\tau = \int_{0}^{t} \tau \cdot d\tau$

$= \frac{\tau^2}{2}\Big|_0^t = \frac{t^2}{2}\, u(t)$ ↳ $0 \le t < \infty$

2- Find the impulse response of the system.

Sol. $h(t) = T\{\delta(t)\} = \int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$.

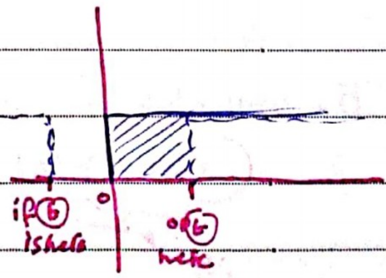
3- It can be shown that the system is LTI system.

Hence, $y(t) = x(t) + h(t)$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \tau\, (u(\tau) \cdot u(t-\tau))\, d\tau .$$

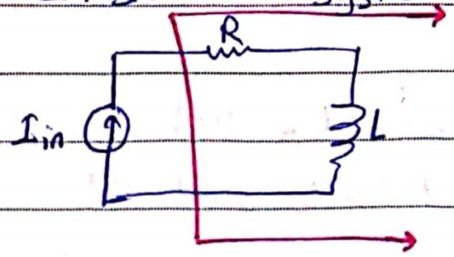
$$u(\tau)\,u(t-\tau) = \begin{cases} 1, & t > \tau > 0 \\ 0, & \text{o.w} \end{cases}$$



$$= \int_{0}^{t} \tau\, d\tau = \frac{t^2}{2}\, u(t)$$

سؤال ال Apet →

consider a system for ckt



لو اعطاني ال L مشحونة من قبل (IC) ما نقدر نستخدم ال convolution

initial condition بكون مو linear

$$V_{out}(t) = R\,I_{in}(t) + L\,\frac{d\,I_{in}(t)}{d(t)}$$

نحل ع ال convolution

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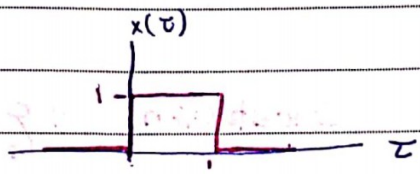
C.T convolution:

ex:- let $x(t) = rect(t - 1/2)$

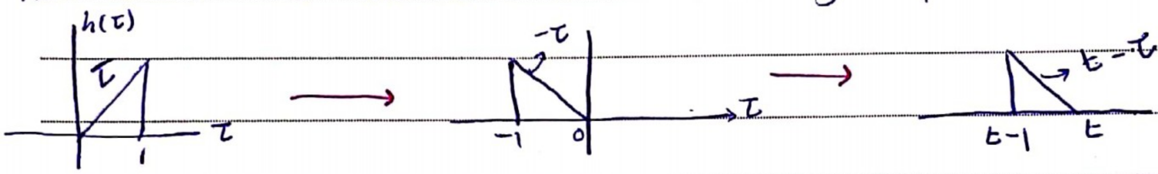
$h(t) = t[u(t) - u(t-1)]$

find $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

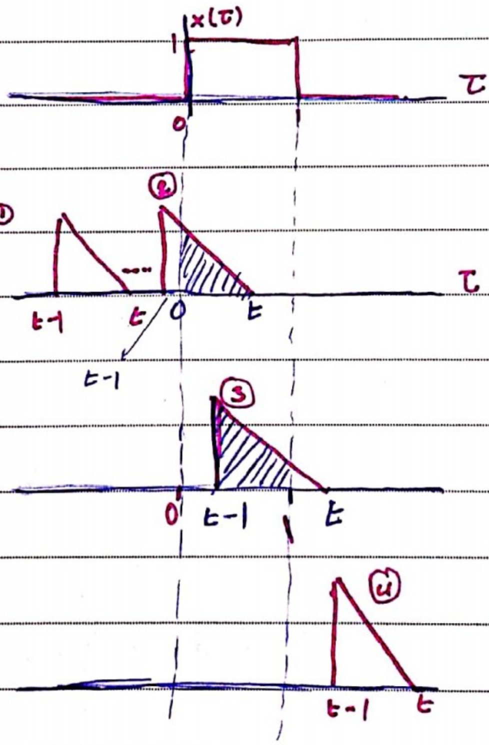
$x(\tau) =$



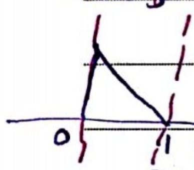
find $h(\tau) \longrightarrow h(-\tau) \longrightarrow$ shift to right by "t" $\longrightarrow h(t-\tau)$



sol



we neglect this case because it's already included (continues)



$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)$$

$$= \begin{cases} 0 , & t<0 \\ \int_0^t 1.(t-\tau).d\tau = \frac{(t-2)^2}{2(-1)}\Big|_0^t = \frac{t^2}{2} , & 1 > t \geq 0 \\ \int_{t-1}^{t} 1.(t-\tau)d\tau = t - t^2/2 , & 2 > t \geq 1 \\ 0 , & t > 2. \end{cases}$$

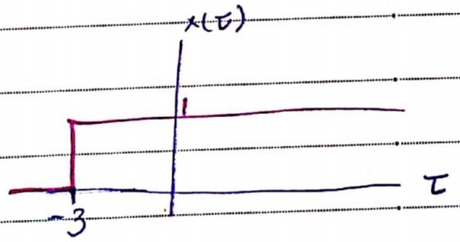
example:- let $x(t) = u(t+3)$

$h(t) = e^{-t} u(t)$

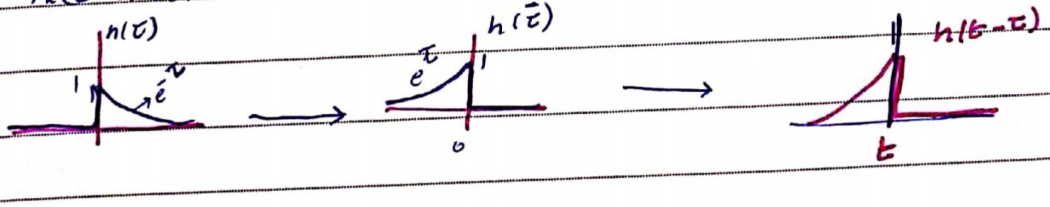
Find $y(t) = x(t) * h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\; h(t-\tau)\, .d\tau$$

$x(t) = u(t+3) \Rightarrow x(\tau) = u(\tau+3)$



Find $h(\tau) \longrightarrow h(-\tau) \longrightarrow$ shift right by "t" $\longrightarrow h(t-\tau)$



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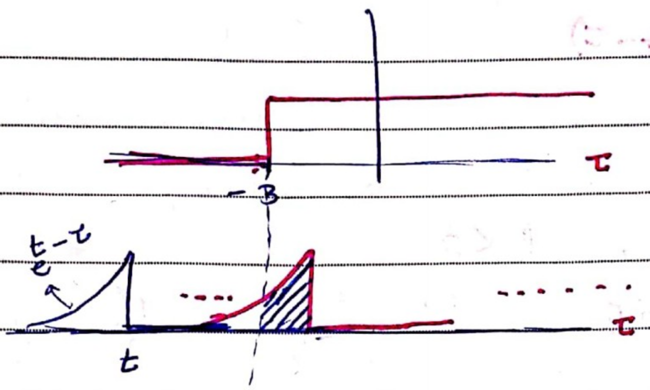
sol.



ما كملت الحل لأنه خلصت المحاضرة

سامحوني ! 😢

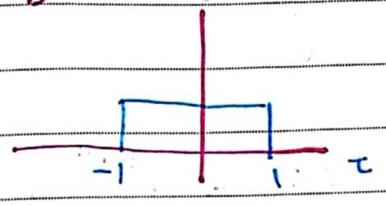
حل بس المشاكس نتج y(t)

Ex:- let $x(t) = u(t+1) - u(t-1)$

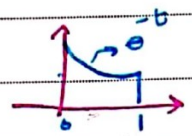
$h(t) = e^{-t}[u(t) - u(t-1)]$

Find $y(t) = x(t) * h(t)$

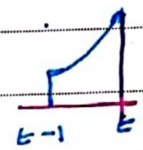
$x(t) = u(t+1) - u(t-1) \rightarrow x(\tau)$



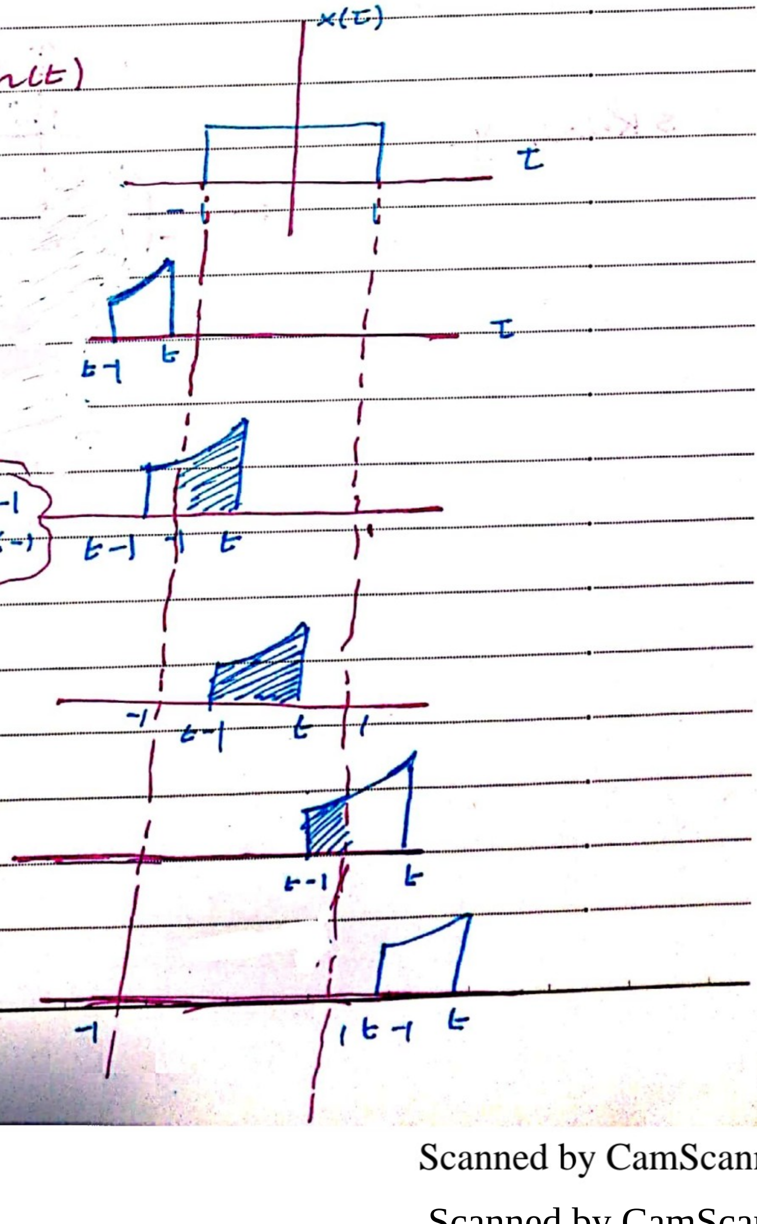
$h(\tau) \longrightarrow$



$\longrightarrow h(t-\tau) = e^{\tau - t}$



sol. $y(t)$:- $x(t) * h(t)$

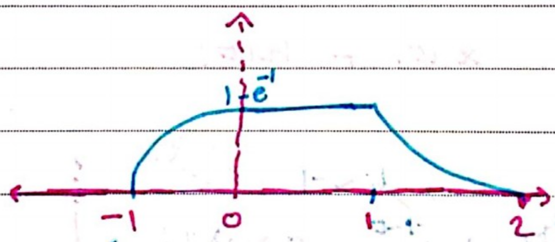


$t > -1$

$t-1 < -1$

$$
y(t) = \begin{cases}
0 & , \quad t < -1 \\
\displaystyle\int_{-1}^{t} (1).\, e^{\tau - t}\, d\tau = 1 - e^{-1-t} & , \quad 0 > t \geq -1 \\
\displaystyle\int_{t-1}^{t} 1.\, e^{\tau - t}\, d\tau = 1 - e^{-1} & , \quad 1 > t \geq 0 \\
\displaystyle\int_{t-1}^{1} 1.\, e^{\tau - t}\, d\tau = e^{1-t} - e^{-1} & , \quad 2 > t \geq 1 \\
0 & ; \quad t \geq 2
\end{cases}
$$

Sketch y :-



* properties of LTI system :-

1- An LTI system is said to be memoryless iff its impulse response is $\boxed{h(t) = K\,\delta(t)}$

So since this is LTI System

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t_0) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t_0) = \int_{-\infty}^{\infty} x(\tau)\, h(t_0-\tau)\, d\tau.$$

* consider a system $y(t) = f(x(t))$.

such the system is memoryless & LTI

it can shown that

$y(t) = K\, x(t)$.

is the only memoryless LTI system.

recall that,

$$y(t_0) = K\, x(t_0) = \int_{-\infty}^{\infty} x(\tau) \cdot K\, \delta(t_0-\tau)\, d\tau$$

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2- BIBO stability of LTI system.

An LTI system is BIBO stable if the impulse response satisfies.

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$

$$\Downarrow$$

$h(t)$ is absolutely integraible

proof: consider an LTI system

$$y(t) = T\{x(t)\}$$

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau. \quad --- ①$$

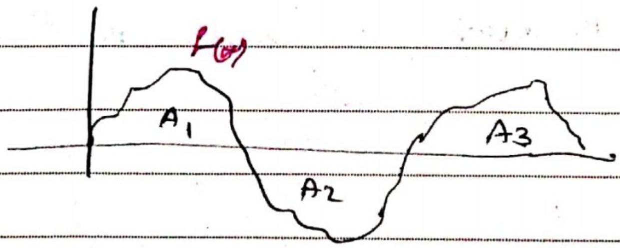
↳ let us assume that $|x(t)| \leq M_x < \infty$ (Bounded)

but from ①:

$$|y(t)| = \left| \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau \right|$$

$\boxed{\text{note:}}$ $\left| \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau \right| \leq \int_{-\infty}^{\infty} |x(t-\tau)|\, |h(\tau)|\, d\tau$

to proof this↑:



$$\left| \int_{-\infty}^{\infty} f(x)\, dx \right| = |A_1 - A_2 + A_3|$$

$$\int_{-\infty}^{\infty} |f(x)\, dx| = |A_1| + |A_2| + |A_3|$$

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how to proof note Mathmitically :-

$$\left| \int_{-\infty}^{\infty} f(x)\, dx \right| \le \int_{-\infty}^{\infty} |f(x)|\, dx.$$

$$|f(x)| \geq f(x) \geq -|f(x)|$$

$$\underbrace{\int_{-\infty}^{\infty} |f|\, dx}_{a} \geq \int_{-\infty}^{\infty} f(x) \cdot dx \geq \underbrace{\int_{-\infty}^{\infty} -|f(x)|\, dx}_{-a}$$

$$\Rightarrow \left| \int_{-\infty}^{\infty} f(x) \cdot dx \right| \leq a$$

$$= \left| \int_{-\infty}^{\infty} f(x) \cdot dx \right| \leq \int_{-\infty}^{\infty} |f(x)|\, dx.$$

استبدلنا $x(t-\tau)$ بـ M_x $\boxed{|x(t)| \le M_x < \infty}$

Back to ① :-

$$|y(t)| = \left| \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau \right| \leq \int_{-\infty}^{\infty} M_x \cdot |h(\tau)|\, d\tau.$$

for $|y(t)| \leq M_y < \infty$

$$= \int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty.$$

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Ex: Is the LTI System whose $h(t) = e^{-3t} u(t)$ BIBO stable ??

$$\int_{-\infty}^{\infty} |h(t)| \cdot dt < \infty$$

$$\int_{-\infty}^{\infty} e^{-3t} \, dt = 1/3 < \infty \Rightarrow \text{BIBO stable.}$$

$$h(t) = e^{+3t} u(t) \qquad \int_{-\infty}^{\infty} h(t) \cdot dt = \infty$$

↳ not BIBO stable.

3- Causality :-

An LTI System is causal if $h(t) = 0$, $t < 0$.

$$y(t) = \int_{-\infty}^{\infty} \underbrace{x(t-\tau)}_{\tau \geq 0} \, h(\tau) \, d\tau .$$

$\Rightarrow ((h(\tau) = 0), \tau < 0)$.

↳ $h(t) = 0$, $t < 0$) → so the output $y(t)$ is causal.

For causal LTI System:

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau) h(\tau) \cdot d\tau = \int_{0}^{\infty} x(t-\tau) h(\tau) d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{t} x(\tau) h(t-\tau) d\tau .$$

↳ so $h(t-\tau) = 0$, $t - \tau < 0$

$\boxed{\tau > t}$

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LTI System. $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

* Unit step Response:

$$S(t) = T\{u(t)\} \triangleq u(t) \underset{\text{convolu.}}{*} h(t)$$

$$= \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau) \cdot d\tau$$

$$S(t) = \int_{0}^{\infty} h(t-\tau) \cdot d\tau$$

or

$$S(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau = \int_{-\infty}^{t} h(\tau)\, d\tau$$

$$S(t) = \int_{-\infty}^{\infty} h(\tau)\, d\tau$$

$$\boxed{\frac{d\,s(t)}{dt} = h(t)}$$

$$T\{u(t)\} = \int_{-\infty}^{\infty} T\{\delta(t)\} \cdot dt$$

but $u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau.$

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* $s(t) = \int_{-\infty}^{t} h(\tau) \cdot d\tau$

* $h(t) = \frac{d\, s(t)}{dt}$

* **Ex:** let $s(t) = (1 - e^{-3t})\, u(t)$

zero (من خصائص $\delta(t)$)

Find $h(t) = \frac{d}{dt} s(t) = (1-e^{-3t})\,\delta(t) + 3e^{-3t}\, u(t)$

$\delta(t) = (2 - e^{-t})\, u(t)$

---

* Fourier Series :-

① orthognality

let $x(t)$ and $y(t)$ be defined over $(0 \leq t \leq T)$

Then if $\int_{0}^{T} x(t) \cdot g^{*}(t)\, dt = \text{zero}$

$\Rightarrow$ $x(t)$ and $g(t)$ are orthognal

$\Rightarrow$ A set of "N" signals $\{ g_1(t), \quad , \text{———} \, g_N(t) \}$ defined over $0 \leq t \leq T$. Then the set is said to be orthognal if

$$\int_{0}^{T} g_i(t) * g_j^{*}(t) \cdot dt = \begin{cases} v_i & , i = j \\ 0 & , i \neq j \end{cases}$$

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Ex: $\cos(\omega_1 t)$ and $\cos(\omega_2 t)$

are orthognal on $0 < t < T_0$

⇒ linear combination:- A linear combination of "N" orthognal signals is the form:-

C^{T} → transpos (بدل العمود ← لصف)

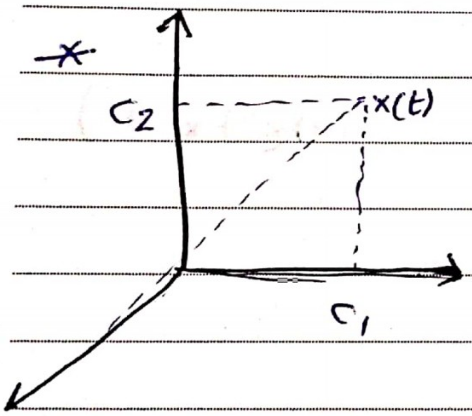
$$x(t) = \sum_{i=1}^{N} C_i g_i(t) = C^{T} \cdot g(t)$$

$$x(t) = \sum_{K=1}^{\infty} C_K \cos(K\omega_0 t + \theta) = C^{T} \cdot g(t)$$

$\frac{1}{T_0}$

$$= \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} \cdot \begin{bmatrix} g_1(t) \\ g_2(t) \\ g_3(t) \\ g_4(t) \end{bmatrix}$$

*



← orthogonal set

* F.S for real signal

$$x(t) = \sum_{K=1}^{\infty} \alpha_K \cos(K\omega_0 t + t_0)$$

* F.S for complex signal

$$x(t) = \sum_{K=-\infty}^{\infty} C_K e^{jK\omega_0 t}$$ } complx exponetial F.S

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* F.S :

Given periodic signal $x(t)$, the F.S for this signal is given

$$x(t) = \sum_{K=-\infty}^{\infty} C_K \, e^{jK\omega_0 t} \quad \text{——— (I)}$$

$\omega_0 \rightarrow$ Fund. Freq $= \frac{2\pi}{T_0}$

* $K\omega_0$ = The Kith harmonic

$$x(t)\, e^{-jn\omega_0 t} = \sum_{K=-\infty}^{\infty} C_K \, e^{jK\omega_0 t} \cdot e^{-jn\omega_0 t} \quad \text{——— (II)}$$

$$\int_0^{T_0} x(t) \cdot e^{-jn\omega_0 t}\, dt = \sum_{K=-\infty}^{\infty} C_K \underbrace{\int_0^{T_0} e^{j\omega_0 t(K-n)} \cdot dt}_{I_{n,K}}$$

$$I_{n,K} = \begin{cases} T_0 , & K = n \\ \left. \dfrac{e^{j\omega_0 (K-n) t}}{(K-n)} \right|_0^{T_0} = 0 , & K \neq n. \end{cases}$$

$$\sum_{K=-\infty}^{\infty} C_K \begin{cases} T_0 , & n = K \\ 0 , & n \neq K \end{cases} = C_n T_0$$

$$C_n = \frac{1}{T_0} \int_0^{T_0} x(t)\, e^{-jn\omega_0 t} \, dt$$

$$\boxed{\therefore C_K = \frac{1}{T_0} \int_0^{T_0} x(t) \cdot e^{-jK\omega_0 t} \, dt}$$

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10/12

$$C_0 = \frac{1}{T_0}\int_{T_0} x(t) \equiv \frac{\text{Area under 1 period}}{T_0}$$

* For Real $x(t)$, we have:

$$C_{-K} = C_K^*$$

$$x(t) = \sum_{K=-\infty}^{\infty} C_K e^{jK\omega_0 t}$$

$$= C_0 + \sum_{K=-\infty}^{\infty} C_K e^{jK\omega_0 t} + \sum_{K=1}^{\infty} C_K e^{jK\omega_0 t}$$

$$= C_0 + \sum_{K=1}^{\infty} C_{-K} e^{-jK\omega_0 t} + \sum_{K=1}^{\infty} C_K e^{jK\omega_0 t}$$

⇒ Recall that C_K is a complex coff.

$C_K = |C_K| e^{j\theta_K}$, where $\theta_K = \tan^{-1}\left[\frac{Im\{C_K\}}{Re\{C_K\}}\right]$

$$= C_0 + \sum_{K=1}^{\infty} \left[\left(C_K e^{j\omega_0 Kt}\right)^* + C_K e^{jK\omega_0 t}\right]$$

$$\Rightarrow x(t) = C_0 + \sum_{K=1}^{\infty} |C_K| \left[\left(e^{jK\omega_0 t + \theta_K}\right)^* + e^{jK\omega_0 t + \theta_K}\right]$$

$$\boxed{x(t) = C_0 + \sum_{K=1}^{\infty} 2|C_K| \cos(K\omega_0 t + \theta_K)}$$ ⇒ combined trigonometric form.

Trigonometric form ←

$$= C_0 + \sum_{K=1}^{\infty} \underbrace{2|C_K|\cos(\theta_K)}_{A_K} \cdot \cos(K\omega_0 t) + \underbrace{-2|K|\sin(\theta_K)}_{B_K}\sin(K\omega_0 t)$$

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$$\Rightarrow A_K = 2|C_K|\cos(\theta_K)$$
$$B_K = 2|C_K|\sin(\theta_K)$$

$$A_K^2 + B_K^2 = 4|C_K|^2$$

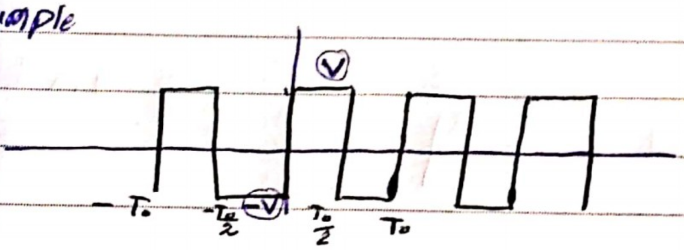
$$|C_K| = \frac{\sqrt{A_K^2 + B_K^2}}{2}$$

$$\theta_K = \tan^{-1}\left(\frac{B_K}{A_K}\right)$$

$$C_K = |C_K| e^{j\theta_K}$$

---

example



, Find the F.S expansion of $x(t)$.

1) Find C_K.

- $C_0 = 0$
- $C_K = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t) \cdot e^{-jK\omega_0 t}$.

$$= \frac{1}{T_0}\left[\int_{-T_0/2}^{0} (-V) \cdot e^{-jK\omega_0 t} + \int_{0}^{T_0/2} (V) \cdot e^{-jK\omega_0 t}\right]$$

$$C_K = \frac{V}{jK\omega_0 T_0}\left[2 - e^{jK\omega_0 \frac{T_0}{2}} - e^{-jK\omega_0 \frac{T_0}{2}}\right]$$

$$= \frac{V}{j2\pi K}\left[2 - e^{j\pi K} - e^{-j\pi K}\right]$$

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$\Rightarrow$

$$C_K = \begin{cases} 0 & , K \text{ is even and } K \neq 0 \\ \frac{2V}{2\pi K} & , K \text{ is odd} \end{cases}$$

$$C_K = \begin{cases} 2V/2\pi K & , K \text{ is odd} \\ 0 & , \text{other wise} \end{cases}$$

$$x(t) = \sum_{\substack{K=-\infty \\ \text{odd}}}^{\infty} C_K \, e^{jK\omega_0 t} = \sum_{K=-\infty}^{\infty} \frac{2V}{j\pi K} \, e^{jK\omega_0 t}$$

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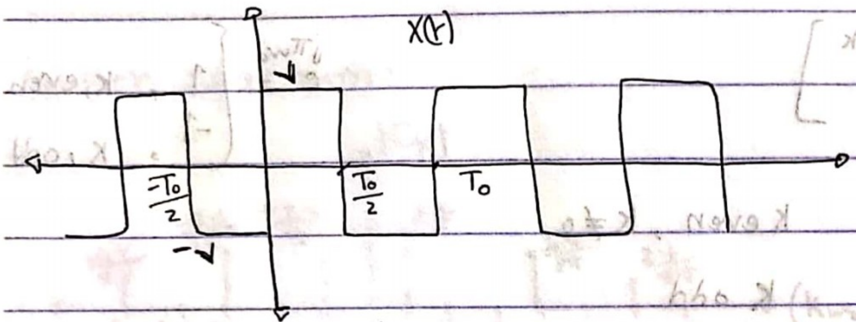
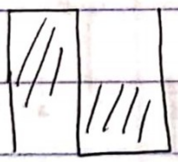
Fourier series of a square wave: 12/12/2019

V : amplitude.



$$C_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-j\omega_0 t k}.$$

$$C_0 = \frac{1}{T_0}\int_{T_0} x(t)\, dt = \frac{\text{area under the curve } x(t) \text{ in one period.}}{T_0}$$

$C_0 = 0$  equal and opposite.

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$$C_K = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\, e^{-jKw_0t}\, dt.$$

$$= \frac{1}{T_0}\left[\int_{-\frac{T_0}{2}}^{0} (-V)\, e^{-jKw_0t}\, dt + \int_{0}^{\frac{T_0}{2}} (V)\, e^{-jKw_0t}\, dt \right].$$

$$= \frac{1}{T_0}\left[\left.\frac{V e^{-jKw_0t}}{jKw_0}\right|_{-\frac{T_0}{2}}^{0} - \left.\frac{V e^{-jKw_0t}}{jKw_0}\right|_{0}^{\frac{T_0}{2}} \right]$$

$$= \frac{1}{T_0}\left[\frac{V}{jKw_0} - \frac{V e^{+jKw_0\frac{T_0}{2}}}{jKw_0} - \frac{V e^{-jKw_0\frac{T_0}{2}}}{jKw_0} + \frac{V}{jKw_0} \right].$$

$$= \frac{V}{T_0}\left[\frac{2 - e^{jw_0K\frac{T_0}{2}} - e^{-jKw_0\frac{T_0}{2}}}{jKw_0} \right]$$

$w_0T_0 = 2\pi$

$$= \frac{V}{jK2\pi}\left[2 - e^{j\pi K} - e^{-j\pi K} \right]$$

$$e^{j\pi w_0} = \begin{cases} 1, & K,\text{even} \\ -1, & K,\text{odd} \end{cases}$$

$$= \begin{cases} 0, & K\text{ even},\ K \neq 0 \\ \frac{2V}{j\pi K}, & K\text{ odd} \end{cases}$$

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Fourier series expansion.

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{2V}{j\pi k} e^{jk\omega_0 t}$$

complex exponential form.

✳ For the combined trignonometric form.

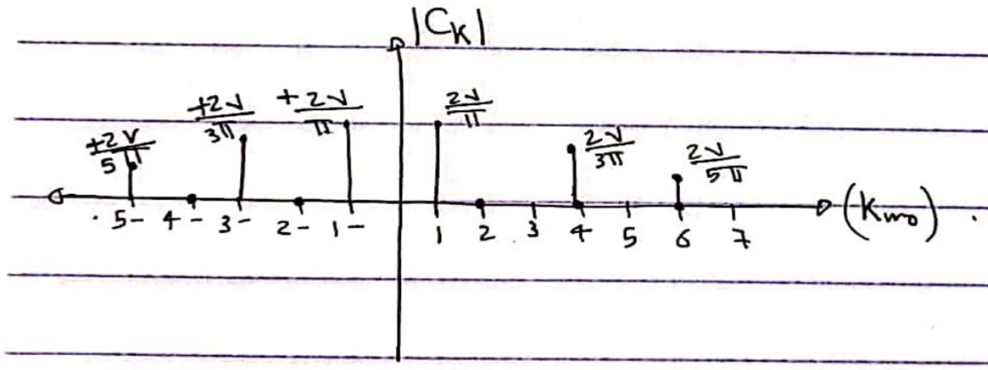
$$x(t) = C_0 + \sum_{k=1}^{\infty} |C_k| \cos(k\omega_0 t + \theta)$$

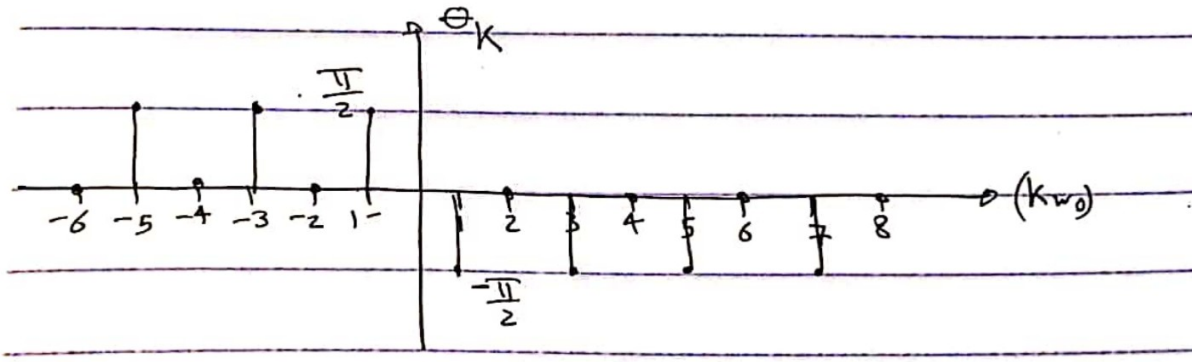
$|C_k| = \frac{2V}{\pi k}$ odd k. , $\theta = -\frac{\pi}{2}$ (Positive k) , so :-

(negative k) $\theta = \frac{\pi}{2}$.

$$x(t) = 0 + \sum_{k=1}^{\infty} \frac{4V}{\pi k} \cos\left(k\omega_0 t - \frac{\pi}{2}\right)$$

Fourier spectrum:





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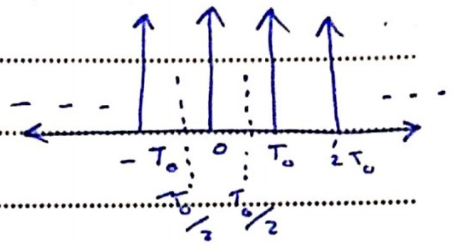
Fouri. S For impulse train



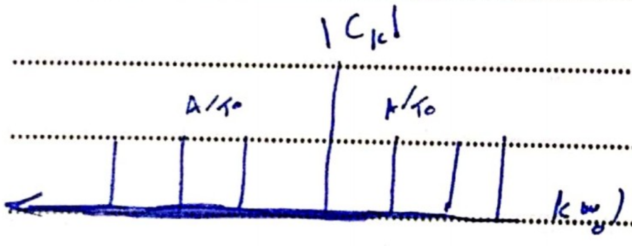
$$C_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A\, \delta(t)\, dt$$

$$= \frac{A}{T_0}\int_{-T_0/2}^{T_0/2} \underbrace{e^{-jk\omega_0 t}\, \delta(t)\, dt}_{1} = \frac{A}{T_0}$$

$$x(t) = \sum_{-\infty}^{\infty} \frac{A}{T_0}\, e^{-j\omega_0 kt}$$ ... complex exponential Form

$$x(t) = \frac{A}{T_0} + \sum_{k=1}^{\infty} \frac{2A}{T_0}\cos(k\omega_0 t)$$ ... combined trigometric Form.



Note
في سؤال عالي ٣
الباميات
Table

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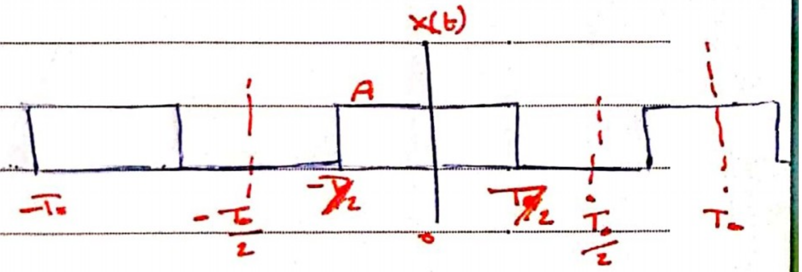
$\text{sinc}(x) = \frac{\sin x}{x}$

F.S for rect signal



$$C_0 = \frac{AT}{T_0}$$

$$C_K = \frac{1}{T_0}\int_{-T/2}^{T/2} A\, e^{-jK\omega_0 t}\, dt = \frac{A}{T_0}\, \frac{e^{-jK\omega_0 t}}{jK\omega_0}\Bigg|_{T/2}^{-T/2}$$

$$\Rightarrow C_K = \frac{A}{j2\pi K}\left[e^{jK\omega_0 T/2} - e^{-jK\omega_0 T/2}\right]$$

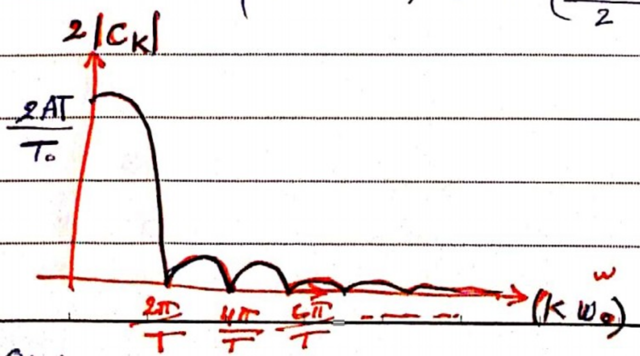
$$\Rightarrow C_K = \frac{A}{\pi K}\sin\left(\frac{K\omega_0 T}{2}\right)$$

$$= \frac{A}{\pi K}\,\text{sinc}\left(\frac{\omega_0 K T}{2}\right)\cdot\frac{K\cdot 2\pi\cdot T}{2T_0}$$

$$C_K = \underbrace{\frac{AT}{T_0}}_{C_0}\,\text{sinc}\left(\frac{\omega_0 K T}{2}\right)$$

$$\rightarrow x(t) = \sum_{-\infty}^{\infty}\frac{AT}{T_0}\,\text{sinc}\left(\frac{K\omega_0 T}{2}\right)e^{j\omega_0 K t}\quad\Bigg\}\ \text{complex exponential form}$$

where $\theta K = \begin{cases} 0, & \text{sinc}\left(\frac{K\omega_0 T}{2}\right) > 0 \\ \pi, & \text{sinc}\left(\frac{K\omega_0 T}{2}\right) < 0 \end{cases}$



* find intersection (with zero)

$$\text{sinc}\left(\frac{\overset{\omega}{K\omega_0} T}{2}\right) = 0$$

$$\Rightarrow \frac{\omega_0 T}{2} = n\cdot\pi$$

$$\omega = \frac{2n\pi}{T}$$

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15/12

parsaval's power theorem:-

let $x(t)$ be periodic with a F.S expansion,

$$x(t) = \sum_{K=-\infty}^{\infty} c_K \, e^{jK\omega_0 t} \quad \text{then.}$$

The power P_x (of $x(t)$ is)

$$P_x = \frac{1}{T_0} \int_{T_0} x^*(t) \, . \, x(t) \, . \, dt \triangleq \sum_{-\infty}^{\infty} |c_K|^2$$

Proof!

$$x(t) = \sum_{K=-\infty}^{\infty} c_K \, e^{jK\omega_0 t} \quad , \quad x^*(t) = \sum_{K=-\infty}^{\infty} c_K^* \, e^{-jK\omega_0 t} \quad \text{---} (1)$$

using (1) : $$P_x = \frac{1}{T_0} \int_{T_0} x(t) \left(\sum_{K=-\infty}^{\infty} c_K^* \, e^{-jK\omega_0 t} \right) dt$$

$$= \left(\sum_{K=-\infty}^{\infty} c_K^* \underbrace{\frac{1}{T_0} \int_{T_0} x(t) \, e^{-jK\omega_0 t} \, dt}_{c_K} \right)$$

$$= \sum_{K=-\infty}^{\infty} c_K^* \, c_K = \sum_{K=-\infty}^{\infty} |c_K|^2$$

example $x(t) = A \sin(2\pi t)$

Find c_K and F.S expansion.

$$x(t) = \frac{A}{2j} e^{j2\pi t} + \frac{-A}{2j} e^{j2\pi t(-1)} \quad \to K=-1$$

$$= \sum_{K=-\infty}^{\infty} c_K \, e^{jK\omega_0 t}$$

$$c_K = \begin{cases} A/2j & , K=1 \\ -A/2j & , K=-1 \\ 0 & , \text{otherwise.} \end{cases}$$

$$P_x = |c_{-1}|^2 + |c_1|^2$$
$$= \frac{A^2}{4} + \frac{A^2}{4} = \frac{A^2}{2} .$$

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* let $x(t)$ be periodic, then

$$x(t) = \sum_{K=-\infty}^{\infty} C_{K_x} e^{jK\omega_0 t}$$

* let $y(t) = x(at+b)$, then what is C_{Ky} (F.S coffesiont for $y(t)$).

$$\hookrightarrow y(t) = x(at+b) = \sum_{K=-\infty}^{\infty} C_{K_x} e^{jK\omega_0 (at+b)}$$

$$= \sum_{K=-\infty}^{\infty} C_{K_x} e^{j\omega_0 K b} \cdot e^{jK(a\omega_0)t}$$

For $a>0$

$$C_{Ky} = C_{K_x} e^{jK\omega_0 b}$$

but if $a<0$.

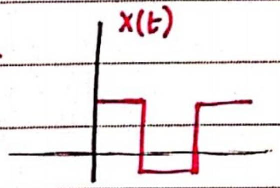
$$y(t) = \sum_{K=-\infty}^{\infty} C_{-Kx} e^{-jK\omega_0 b} \cdot e^{-jK(\omega_0 a)t}$$

so $C_{Ky} = \left(C_{Kx} e^{jK\omega_0 b}\right)^*$

$y(t) = x(at+b)$

$$C_{Ky} = \begin{cases} C_{K_x} e^{jK\omega_0 b} & , a>0 \\ \left(C_{K_x} e^{jK\omega_0 b}\right)^* & , a<0 \end{cases}$$

example:-



Find C_{Ky} ?

$$C_{K_x} = \begin{cases} \dfrac{A}{j2\pi K} & , K \text{ is odd} \\ 0 & , \text{otherwise.} \end{cases}$$

$y(t) = x(1-t)$ Find C_{Ky}

$b=1$ $a=-1$

$$C_{Ky} = \left(C_{Kx} e^{jK\omega_0 b}\right)^* = \frac{jA}{2j\pi K} e^{(-jK\omega_0 1)}$$

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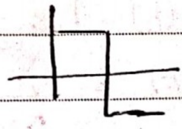
* let $x(t)$ be periodic, then



$$x(t) = \sum_{k=-\infty}^{\infty} c_{k_x} e^{jk\omega_0 t}$$

* let $y(t) = A \cdot x(t) + B$

$$= A \cdot \sum_{k=-\infty}^{\infty} c_{k_x} e^{jk\omega_0 t} + B$$

$$= \underbrace{(A \cdot C_{0x} + B)}_{C_{0y}} + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \underbrace{A\, c_{kx}}_{\substack{c_{ky} \\ k \neq 0}} e^{jk\omega_0 t}$$

$$c_{ky} = \begin{cases} A\, C_{0x} + B, & \text{if } k = 0 \\ A\, c_{k_x}, & \text{if } k \neq 0. \end{cases}$$

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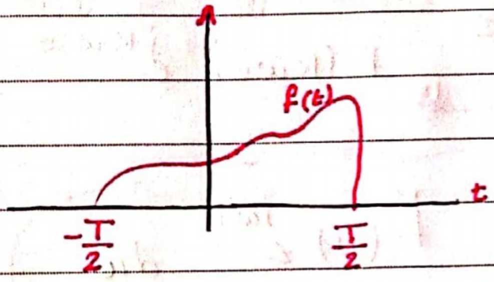
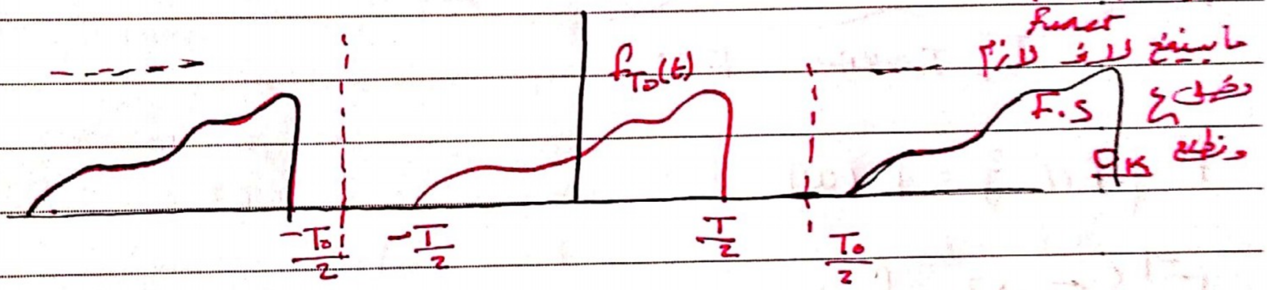
17/12.

Fourier Transform (F.T)

* Def of F.T

let $f(t)$ be aperiodic (not periodic) signal





$$f_{T_0}(t) = \sum_{K=-\infty}^{\infty} f(t - K \cdot T_0)$$

$$f(t) = \lim_{T_0 \to \infty} f_{T_0}(t).$$

$f_{T_0}(t)$ is periodic

$$f_{T_0}(t) = \sum_{K=-\infty}^{\infty} C_K e^{jK\omega_0 t} \quad . \quad \omega_0 = \frac{2\pi}{T_0}$$

$$C_K = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} f_{T_0}(t)\, e^{-jK\omega_0 t} \, . dt = \frac{1}{T_0} \int_{-\infty}^{\infty} f(t)\, e^{-jK\omega_0 t} \, . dt$$

let us define $\boxed{F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} \, dt}$ F.T

$$C_K = \frac{1}{T_0} F(K\omega_0) = \frac{\omega_0 F(K\omega_0)}{2\pi}$$

$\Rightarrow$

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$$F_{T_0}(t) = \sum_{K=-\infty}^{\infty} \frac{F(K\omega_0)}{2\pi} e^{jK\omega_0 t} \cdot \omega_0$$

$$F(t) = \lim_{T_0 \to \infty} F_{T_0}(t) \stackrel{\Delta}{=} \lim_{\omega_0 \to \infty} F_{T_0}(t)$$

$$= \lim_{\omega_0 \to \infty} \frac{1}{2\pi} \sum_{K=-\infty}^{\infty} F(K\omega_0) e^{jK\omega_0 t} \cdot \omega_0$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega_0 t} d\omega$$

Inverse F.T.

$$F\{f(t)\} = F(\omega)$$

$$F^{-1}\{F(\omega)\} = f(t).$$

$$f(t) \longleftrightarrow F(\omega)$$

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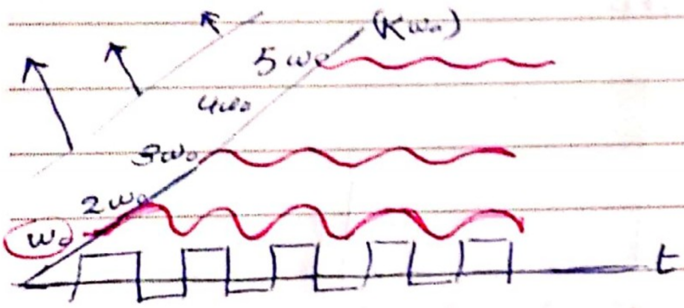
F.S.T 19/12

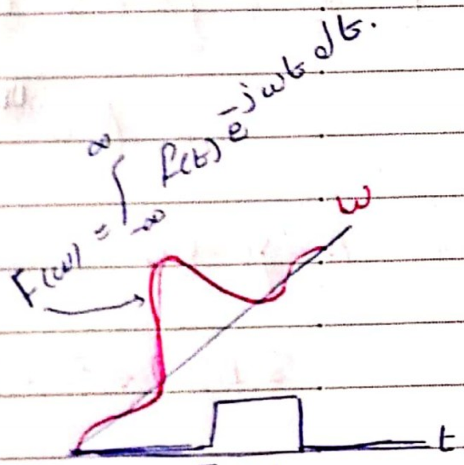
$f(t) \xleftrightarrow{\mathcal{F}} F(\omega)$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$



F.S



F(t).

* Significance of F.T

ex:

$x(t) = e^{j\omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= e^{j\omega_0 t} \int_{-\infty}^{\infty} h(\tau)\, e^{-j\omega_0 \tau}\, d\tau$$

$$= e^{j\omega_0 t} \cdot \mathcal{F}\{h(t)\}\Big|_{\omega=\omega_0}$$

$$= e^{j\omega_0 t} \cdot H(\omega_0).$$

$\boxed{f(t)}$
- bounde,
- finite, ...
- absolutly integrable ?

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Ex:- let $f(t) = A\,\delta(t-t_0)$, find $F(\omega)$

sol $F(\omega) = \int_{-\infty}^{\infty} A\,\delta(t-t_0)\, e^{-j\omega t}\, dt$.

$= A\, e^{-j\omega t_0}$

$$A\,\delta(t-t_0) \xleftrightarrow{\mathcal{F}} A e^{-j\omega t_0}$$

$$\delta(t) \xleftrightarrow{\mathcal{F}} 1$$

Ex:- let $f(t) = e^{j\omega_0 t}$, Find $F(\omega)$

$$F(\omega) = \int_{-\infty}^{\infty} e^{j\omega_0 t} \cdot e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-j(\omega-\omega_0)t}\, dt = \frac{e^{-j(\omega-\omega_0)t}}{(\omega-\omega_0)} \Bigg|_{\infty}^{-\infty} \quad \times \text{ Def. fails.}$$

$$\left\{ \begin{array}{l} e^{-j\infty} = \text{undifine} \neq 0 \\ e^{j\infty} = \text{"} \\ e^{j0} = 1 \\ e^{-\infty} = 0 \\ e^{\infty} = \infty \end{array} \right.$$

$\Longrightarrow$

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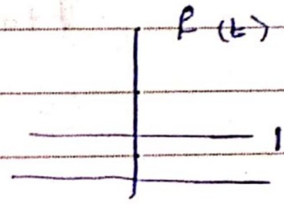
$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \cdot d\omega \longrightarrow \text{(inverse)}$$

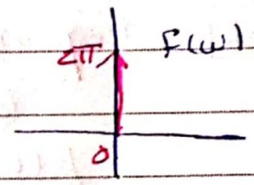
$f(t) = e^{j\omega_0 t}$ شو قيمة $F(\omega)$ الي بعطيني

$$F(\omega) = 2\pi\, \delta(\omega - \omega_0).$$



$$e^{j\omega_0 t} \overset{\mathcal{F}}{\longleftrightarrow} 2\pi\, \delta(\omega - \omega_0)$$

$$1 \longleftrightarrow 2\pi\, \delta(\omega)$$



* proparities of F.T

1. Linearity propority:

let $f_1(t) \overset{\mathcal{F}}{\longleftrightarrow} F_1(\omega)$

$f_2(t) \overset{\mathcal{F}}{\longleftrightarrow} F_2(\omega)$

then $[a_1 f_1(t) + a_2 f_2(t)] \overset{\mathcal{F}}{\longleftrightarrow} a_1 F_1(\omega) + a_2 F_2(\omega).$

2. Time Shift:

let $f(t) \overset{\mathcal{F}}{\longleftrightarrow} F(\omega)$

then $f(t - t_0) \overset{\mathcal{F}}{\longleftrightarrow} F(\omega) e^{-j\omega t_0}.$

$$F\{f(t - t_0)\} = \int_{-\infty}^{\infty} f(t - t_0) e^{-j\omega t}\, dt$$

$$= \Big(\underbrace{\int_{-\infty}^{\infty} f(t) e^{-j\omega t} \cdot dt}_{F(\omega).}\Big) e^{-j\omega t_0}$$

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3. Time transformation

let $f_1(t) \xleftrightarrow{\mathcal{F}} F(\omega)$

$$f(at - t_0) \xleftrightarrow{\mathcal{F}} \frac{1}{|a|} F\left(\frac{\omega}{a}\right) e^{\frac{-j\omega t_0}{a}}$$

4. frequency shift

let $f(t) \xleftrightarrow{\mathcal{F}} F(\omega)$

$$f(t)\, e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} F(\omega - \omega_0)$$

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/ /

[5] Convolution property.

$f_1(t) \longleftrightarrow F_1(\omega)$

$f_2(t) \longleftrightarrow F_2(\omega)$ then $f_1(t) * f_2(t) \xleftrightarrow{F} F_1(\omega) . F_2(\omega)$

proof:-

let $f(t) = f_1(t) * f_2(t)$

$$= \int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau) \, d\tau$$

let $F(\omega) = F\{f(t)\}$

$$= \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} f_1(\tau) \underbrace{\left(\int_{-\infty}^{\infty} f_2(t-\tau) e^{-j\omega t} dt \right)}_{F_2(\omega) e^{-j\omega\tau}} d\tau .$$

$$F_2(\omega) \int_{-\infty}^{\infty} f_1(\tau) e^{-j\omega\tau} d\tau$$

$$F_2(\omega) . F_1(\omega)$$

[6] let $f_1(t) \longleftrightarrow F_1(\omega)$

$f_2(t) \xleftrightarrow{S} F_2(\omega)$

then $f_1(t) . f_2(t) \xleftrightarrow{F} \frac{1}{2\pi} F_1(\omega) * F_2(\omega)$

[7] Duality property

let $f(t) \xleftrightarrow{F} F(\omega)$

then $F(t) \xleftrightarrow{F} 2\pi f(-\omega)$

$f(t) \qquad F(\omega)$

$\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi \delta(+\omega)$.

$F(t)$

8 Differentiation

let $f(t) \xleftrightarrow{F} F(w)$.

then $f'(t) \xleftrightarrow{F} jw\, F(w)$

$$\frac{d\, f(t)}{dt} \xleftrightarrow{F} (jw)^n\, F(w).$$

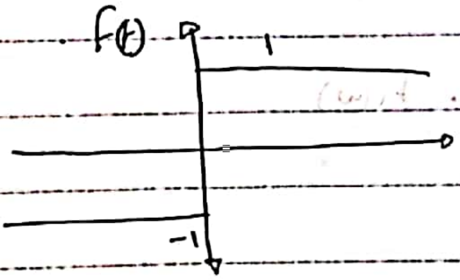
proof:-

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w)\, e^{jwt}\, dw.$$

$$\frac{d\, f(t)}{dt} = \frac{d}{dt}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)\, e^{jwt}\, dw\right).$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \underline{jw\, F(w)}\, e^{jwt}\, dt.$$

ex: $f(t) = \operatorname{sgn}(t)$: find $F(w)$



$\operatorname{sgn}(t) = 2u(t) - 1.$

$u(t) = \dfrac{1 + \operatorname{sgn}(t)}{2}$

$F\{u(t)\} = F\{\tfrac{1}{2}\} + \tfrac{1}{2} F\{\operatorname{sgn}(t)\}$

$\pi\, \delta(t) + \dfrac{1}{jw}.$

$f'(t) = 2\,\delta(t)$

take F.T for both sides.

$jw\, F(w) = 2$

$F(w) = \dfrac{2}{jw}$

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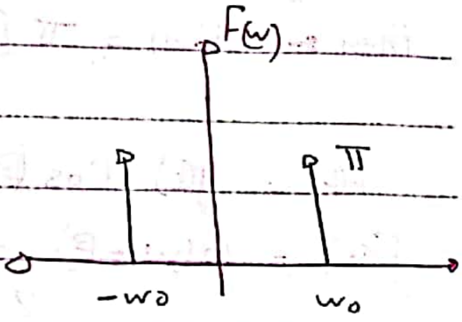
let $f(t) = \cos(w_0 t)$, find $F(w)$

$$f(t) = \frac{e^{jw_0t} + e^{-jw_0t}}{2}$$

$$F(w) = \frac{1}{2} F\{e^{jw_0t}\} + \frac{1}{2} F\{e^{-jw_0t}\}$$

$$F(w) = \pi\{\delta(w-w_0) + \delta(w+w_0)\}$$



let $f(t) = \sin(w_0t)$, find $F(w)$

$$F(w) = \frac{\pi}{j}[\delta[w-w_0] - \delta(w+w_0)].$$

* $f(t) = \sin(w_0t) = \frac{-1}{w_0} \frac{d}{dt}[\cos(w_0t)]$

$$F\{\sin(w_0t)\} = \frac{-1}{w_0} jw \, F\{\cos(w_0t)\}$$

$$= \frac{-jw\pi}{w_0}[\delta(w-w_0) + \delta(w+w_0)].$$

$$= \frac{\pi}{j}\left[\frac{w_0}{w_0}\delta(w-w_0) - \frac{w_0}{w_0}\delta(w+w_0)\right]$$

let $f(t) = g(t) \cdot \cos(w_0t)$, let $g(t) \xleftrightarrow{F} G(t)$, find $F(w)$

$$g(t) \cdot \cos(w_0t) \xleftrightarrow{F} \frac{1}{2\pi} G(t) * F\{\cos(w_0t)\}..$$

$$\frac{1}{2\cancel{\pi}} \cancel{\pi} [G(t) * (\delta(w-w_0) + \delta(w_0+w_0))].$$

$$= \frac{1}{2}(G(w-w_0) + G(w+w_0))$$

ex :- $f(t) = \cos(\alpha t) \cdot \cos(\beta t)$, find $F(\omega)$.

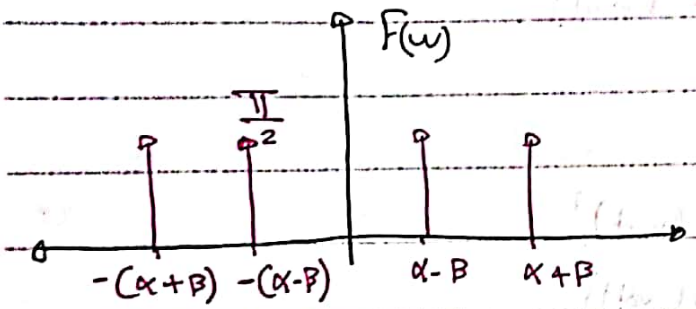
let $g(t) = \cos(\alpha t)$.

then $G(\omega) = \pi [\delta(\omega - \omega_0) + \delta(\omega - \omega_0)]$.

$f(t) = g(t) \cdot \cos(\beta t)$

$$F(\omega) = \frac{G(\omega - \beta) + G(\omega + \beta)}{2}$$

$$F(\omega) = \frac{\pi}{2} [\delta(\omega - \alpha - \beta) + \delta(\omega - \alpha + \beta) + \delta(\omega + \alpha - \beta) + \delta(\omega + \alpha + \beta)]$$

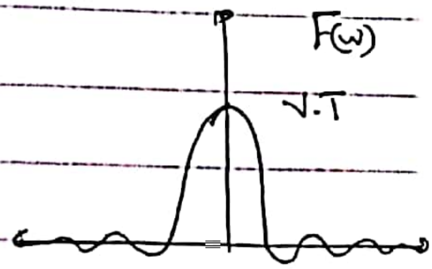


let $f(t) = V \, \text{rect}\left(\frac{t}{T}\right)$, find $F(w)$:-

$$F(w) = \int_{-\infty}^{\infty} f(t) \, e^{-jwt} \, dt.$$

$$= \int_{-\frac{T}{2}}^{\frac{T}{2}} V \, e^{-jwt} \, dt = \frac{V}{-jw} \left[e^{jw\frac{T}{2}} - e^{-jw\frac{T}{2}} \right] \cdot \frac{\cancel{2j}\, w \frac{T}{2}}{\cancel{2j}\, w \frac{T}{2}} .$$

$$= V.T \ \text{sinc}\left(w\frac{T}{2}\right).$$



• $V \, \text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{F} VT \ \text{sinc}\left(\frac{wT}{2}\right)$

ex:- let $f(t) = A \, \text{sinc}(\beta t)$, find $F(w)$.

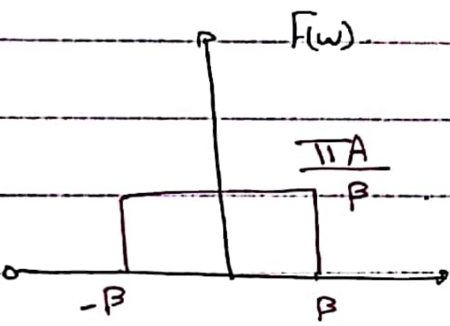
• duality

$$\text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{F} T \ \text{sinc}\left(\frac{wT}{2}\right).$$

$$T \, \text{sinc}\left(t\frac{T}{2}\right) \xleftrightarrow{F} 2\pi \ \text{rect}\left(\frac{w}{2\frac{T}{2}}\right)$$

Copy. $\frac{T}{2} = \beta$, T , $\frac{1}{\beta}$

$$A \, \text{sinc}(\beta t) \xleftrightarrow{F} \frac{\pi A}{\beta} \ \text{rect}\left(\frac{w}{2\beta}\right).$$



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Time integration :-

let $f(t) \xleftrightarrow{F} F(w)$

Then

$$g(t) = \int_{-\infty}^{t} f(\tau) \cdot d\tau \longleftrightarrow \frac{F(w)}{jw} + \pi F(0) \cdot \delta(w)$$

where $F(0)$ its $F(w)$ when $w = 0 \Rightarrow \int_{-\infty}^{\infty} f(t) \cdot e^{-j0t} \, dt$

$$\int_{-\infty}^{t} f(\tau) \cdot d\tau = \int_{-\infty}^{\infty} f(\tau) \cdot u(t-\tau) \cdot d\tau$$

$$= f(t) * u(t)$$

$$G(w) = F(w) \cdot \left[\frac{1}{jw} + \pi \delta(w) \right]$$

$$= \frac{F(w)}{jw} + \pi F(0) \delta(w) \quad \#$$

example #

$$g(t) = \int_{-\infty}^{t} V \operatorname{rect}\left(\frac{\tau}{T}\right) d\tau$$

time integration

$F(0) = V \cdot T \, \cancel{\text{sinc}\, 0}$

$G(w)$

$= V \cdot T$

So

$f(t) = V \operatorname{rect}\left(\frac{\tau}{T}\right)$

$G(w) = \dfrac{VT \operatorname{sinc}\left(\frac{wT}{2}\right)}{jw} + \pi \cdot V \cdot T \, \delta(w)$

$F(w) = VT \operatorname{sinc}\left(\frac{wT}{2}\right)$

SUBJECT: ٢٠١٩ / ١٢ / ٢٦

example:- evaluate

$$\int_{-\infty}^{\infty} \underbrace{\text{sinc}(\beta t)}_{g(t)}\, dt$$

you can use F.T

$$F(w) = \int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt$$

$$F(0) = \int_{-\infty}^{\infty} f(t)\, dt \Rightarrow F(w) = F\{\text{sinc}(\beta t)\}$$

$$F(0) = \frac{\pi}{\beta} \text{ rect } 0$$

$$F(0) = \frac{\pi}{\beta}$$

example:- let $f(t) = e^{-at} u(t)$, $Re\{a\} > 0$ Find $F(w)$

$$F(w) = \int_{-\infty}^{\infty} e^{-at} \cdot e^{-jwt} u(t)\, dt \Rightarrow \int_{0}^{\infty} e^{-(a+jw)t}\, dt$$

$$= \frac{1}{a + jw}, \quad Re\{a\} > 0$$

$$e^{-at} u(t) \xleftrightarrow{F} \frac{1}{a+jw}$$

SUBJECT: Signals & System

example :- let $F(\omega) = \frac{1}{(a+j\omega)^2}$, $R\{a\} > 0$

$$F(\omega) = \underbrace{\frac{1}{a+j\omega}}_{F_1(\omega)} * \underbrace{\frac{1}{a+j\omega}}_{F_2(\omega)}$$

using

$$f_1(t) \circledast f_2(t) = e^{-at} u(t) \circledast e^{-at} u(t)$$

$$e^{-\alpha t} \int_{-\infty}^{\infty} 1 \, d\tau$$

$$= e^{-\alpha t} \, t \, u(t)$$

$\therefore$ $t e^{-\alpha t} u(t) \xleftrightarrow{F} \frac{1}{(a+j\omega)^2}$

Another method to solve it :-

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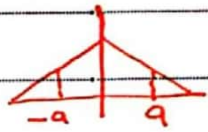
29/12/2019

* $e^{-\alpha t} \, u(t) \xleftrightarrow{\mathcal{F}} \dfrac{1}{a+j\omega}$

* $t \, e^{-\alpha t} \, u(t) \longleftrightarrow \dfrac{1}{(a+j\omega)^2}$

$|F(\omega)| = \dfrac{1}{\sqrt{a^2+\omega^2}}$



* let $f(t) \xleftrightarrow{\mathcal{F}} F(\omega)$

then $t^n f(t) \xleftrightarrow{\mathcal{F}} j^n \dfrac{d^n F(\omega)}{d\omega^n}$

$t f(t) \xleftrightarrow{\mathcal{F}} j F'(\omega)$

* proof !

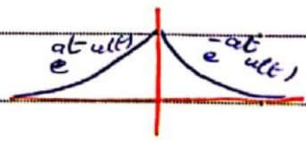
$$F(\omega) = \int_{-\infty}^{\infty} f(t) \, e^{-j\omega t} \, . dt$$

$$\frac{dF(\omega)}{d\omega} = \int_{-\infty}^{\infty} -j t f(t) \, e^{-j\omega t} \, dt$$

$$jF'(\omega) = \int_{-\infty}^{\infty} t f(t) \, e^{-j\omega t} \, dt .$$

example : let $f(t) = e^{-a \cdot |t|}$, $a > 0$. Find $F(\omega)$.

$$f(t) = \underbrace{e^{-at} u(t)}_{g(t)} + \underbrace{e^{at} u(-t)}_{g(-t)}$$



$$F(\omega) = G(\omega) + G(-\omega)$$

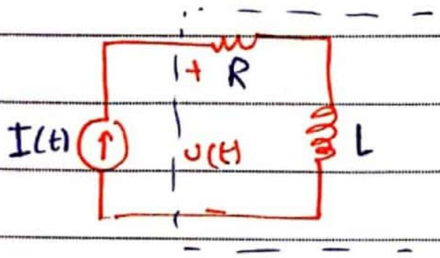
$$= \frac{1}{a+j\omega} + \frac{1}{a-j\omega} = \frac{2a}{a^2+\omega^2}$$

$$\boxed{e^{-a|t|} \xleftrightarrow{\mathcal{F}} \frac{2a}{a^2+\omega^2}}$$

$$\frac{1}{a^2+t^2} \xleftrightarrow{\mathcal{F}} \frac{\pi \ast e^{-a|\omega|}}{2a}$$

* Application of F.T

(1)



impedance $\Rightarrow R + j\omega L$

$$v(t) = R\,I(t) + L\frac{dI(t)}{dt}$$

let $v(t) \xleftrightarrow{\mathcal{F}} V(\omega)$

$I(t) \xleftrightarrow{\mathcal{F}} I(\omega)$

$\Rightarrow V(\omega) = R.I(\omega) + L \cdot j\omega I(\omega)$

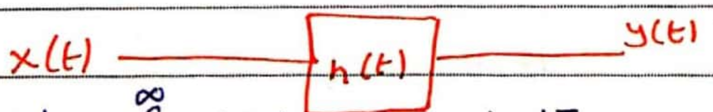
$V(\omega) = (R + j\omega L)\, I(\omega)$

$I(\omega) = \dfrac{V(\omega)}{R + j\omega L}$

if $V(\omega)$ is given

find $I(\omega)$ $\xrightarrow{\text{then}}$ find $I(t)$

(2) LTI system



$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

* $y(t) \xleftrightarrow{\mathcal{F}} Y(\omega)$
* $x(t) \xleftrightarrow{\mathcal{F}} X(\omega)$
* $h(t) \xleftrightarrow{\mathcal{F}} H(\omega)$

$Y(\omega) = X(\omega) \cdot H(\omega)$

$H(\omega) = \dfrac{Y(\omega)}{X(\omega)}$

$h(t) = \mathcal{F}^{-1}\{H(\omega)\}$

* F.T of periodic signal.

let $x(t)$ be periodic signal.

$$x(t) = \sum_{k=-\infty}^{\infty} c_k \, e^{jk\omega_0 t}$$

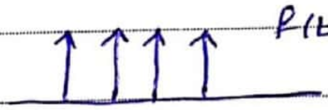
then the F.T of $x(t)$, $X(\omega)$ is

$$X(\omega) = \mathcal{F}\left\{\sum_{k=-\infty}^{\infty} c_k \, e^{jk\omega_0 t}\right\}$$

$$= \sum_{k=-\infty}^{\infty} c_k \, \mathcal{F}\left\{e^{jk\omega_0 t}\right\}$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi \, c_k \, \delta(\omega - k\omega_0).$$

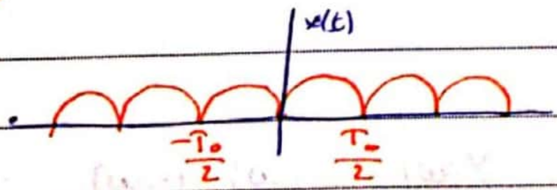
example:-



$f(t) \longrightarrow c_k = \frac{1}{T_0}$

$$F(\omega) = \sum_{k=-\infty}^{\infty} \underbrace{\left(\frac{2\pi}{T_0}\right)}_{\omega_0} \cdot \delta(\omega - k\omega_0)$$

example:- let $x(t)$ be periodic, then



$g(t) \rightarrow$ pulse shaping signal

$$x(t) = \sum_{x=-\infty}^{\infty} g(t - kT_0)$$

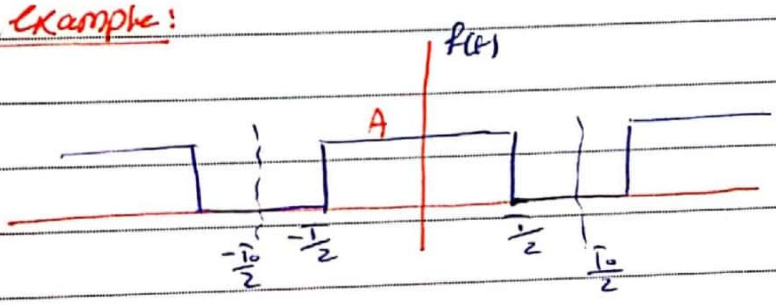
$$x(t) = g(t) \underset{\text{convol.}}{*} \left(\sum_{k=-\infty}^{\infty} \delta(t - kT_0)\right)$$

$$X(\omega) = G(\omega) \cdot \mathcal{F}\left\{ \sum_{K=-\infty}^{\infty} \delta(t - KT_0) \right\}$$, $G(\omega) = \mathcal{F}\{g(t)\}$

$$= G(\omega) \cdot \sum_{K=-\infty}^{\infty} \omega_0\, \delta(\omega - K\omega_0)$$

$C_K = \frac{G(K\omega_0)}{T_0}$

* example:



Find $F(\omega)$

$g(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$, $\frac{T_0}{2} \geq t \geq \frac{-T_0}{2}$

$G(\omega) = T.A \ \mathrm{sinc}\left(\frac{\omega T}{2}\right)$

$$F(\omega) = \sum_{K=-\infty}^{\infty} \omega_0 T.A\ \mathrm{sinc}\left(\frac{\omega_0 K T}{2}\right) \delta(\omega - K\omega_0).$$

$$C_K = \frac{T.A\ \mathrm{sinc}\left(\frac{K\omega_0 T}{2}\right)}{T_0}$$

* let $f(t) \xleftrightarrow{\mathcal{F}} F(\omega)$

$f^*(t) \xleftrightarrow{\mathcal{F}}$??

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$\mathcal{F}$

$$f^*(t) = \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t}\, dt\right)^*$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F^*(\omega)\, e^{-j\omega t}\, d\omega$$

new $\omega = -\omega$

$$\Rightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} F^*(-\omega)\, e^{j\omega t}\, d\omega$$

$$\Rightarrow f^*(t) \xleftrightarrow{\mathcal{F}} F^*(-\omega)$$

if $f(t)$ is real signal then

$$\mathcal{F}(\omega) = F^*(\omega)$$

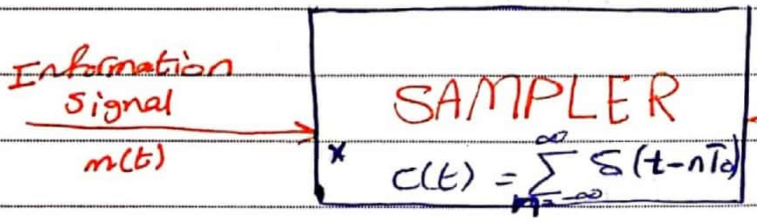
$$\mathcal{F}(-\omega) = F^*(\omega).$$

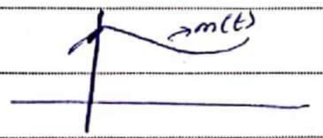
بنحول ال C.T signal الى Analog Digital (D.T) وبعدها نوصلها خلال مسافات

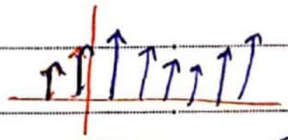
طرق الإرسال ١٢/٢٩

نرجع نرجعها لـ C.T Analog من Digital

* **sampling:** reduction of a C.T signal into a D.T signal.



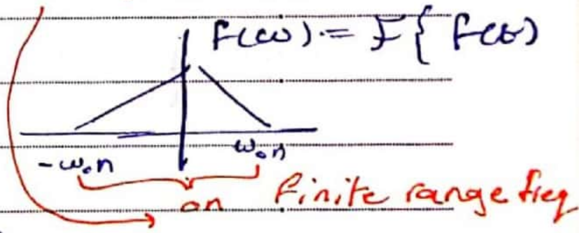




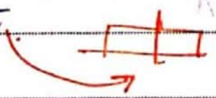
$$\boxed{m(t) \underbrace{*}_{\text{multiply}} c(t) = S(t)}$$

f(t) is Band limited signal iff F(ω) is limited

$F(\omega) = \mathcal{F}\{f(t)\}$



* example: sinc is Band limited because F.T for it = rect is limited



* ليه لازم m(t) تكون Band limited ?

← عشان نقدر نرجعها لـ cont. لما نحول Digital to Analog بصورة كاملة

* m(t) is band limited: its F.T is non zero over small finite range of frequency

* x(t) is band limited to $[-\omega_m, \omega_m]$ if $X(\omega) = \mathcal{F}\{x(t)\}$ is zero for $\omega \notin [-\omega_m, \omega_m]$

$\left\{ X(\omega) = 0 \text{ for } |\omega| > \omega_0 \right.$

$$C(t) = \sum_{K=-\infty}^{\infty} \delta(t - KT_0)$$

$$m(t) \longrightarrow \otimes \longrightarrow s(t) = C(t) \cdot m(t)$$

$$\boxed{*}\ s(t) = m(t) \sum_{K=-\infty}^{\infty} \delta(t - KT_0).$$

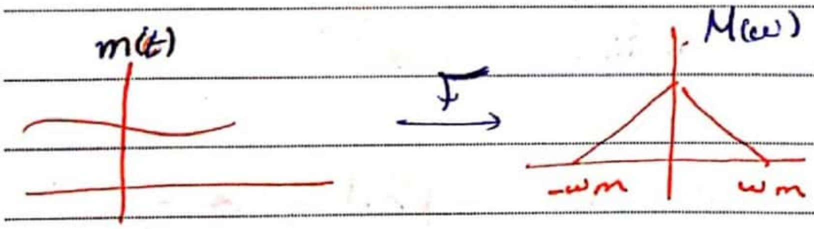
let $s(t) \xleftrightarrow{\mathcal{F}} S(\omega)$

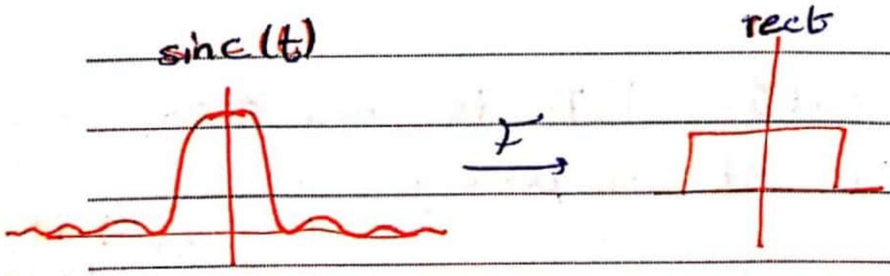
$m(t) \xleftrightarrow{\mathcal{F}} M(\omega)$

$\boxed{\omega_s = \dfrac{2\pi}{T_s}}$ sampling freq.

$$S(\omega) = \frac{1}{2\pi} M(\omega) * \sum_{K=-\infty}^{\infty} \omega_s\, \delta(\omega - K\omega_s)$$

$$S(\omega) = \frac{\omega_s}{2\pi} \sum_{K=-\infty}^{\infty} M(\omega - K\omega_s)$$

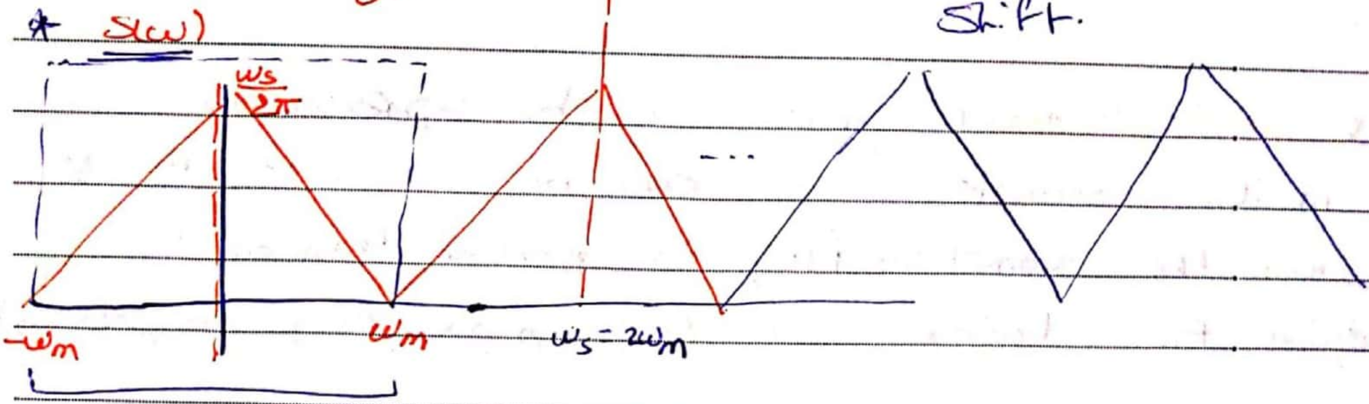




$$S(\omega) = \frac{\omega_s}{2\pi} \left[M(\omega) + M(\omega - \omega_s) + M(\omega + \omega_s) + \cdots \right]$$

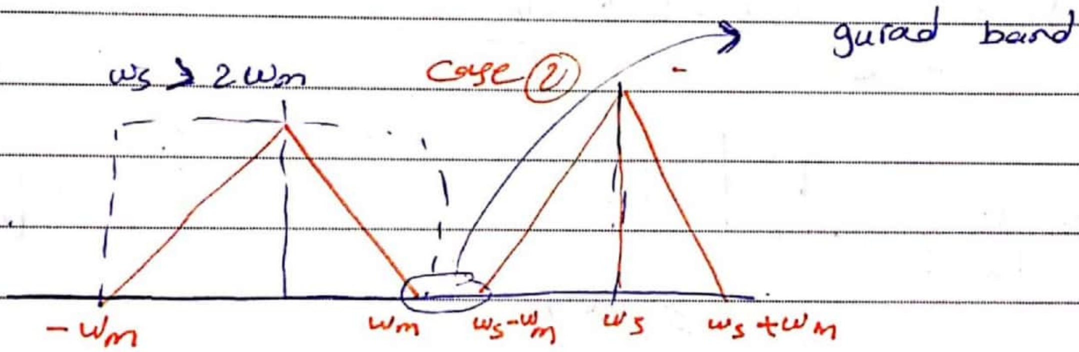
case ① $\omega_s = 2\omega_m$

* $S(\omega)$

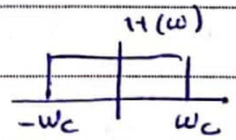


to reconstruct the signal : $S(\omega) \cdot H(\omega)$
$H(\omega) \Rightarrow$ rect from $[-\omega_c$ to $\omega_c]$ } $\omega_c = \omega_m$

* $\omega_s > 2\omega_m$ case ②



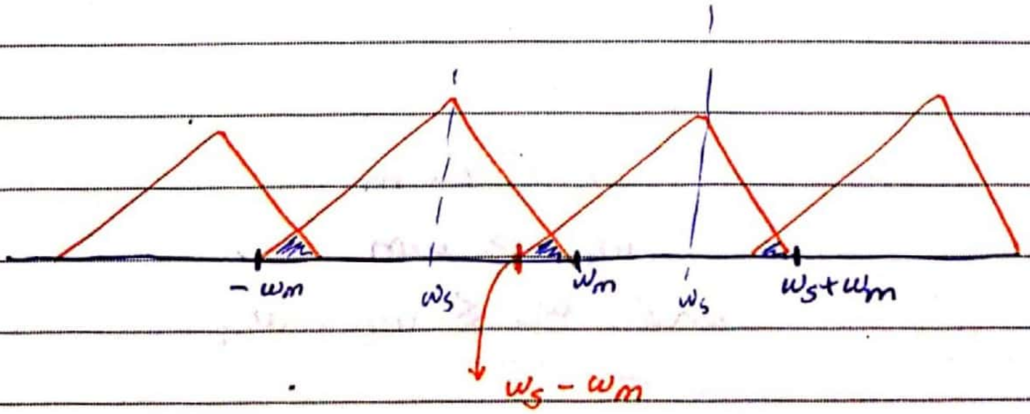
to reconstruct the signal
$S(\omega) \cdot H(\omega)$, $H(\omega)$ = rect



$\omega_s - \omega_m \geq \omega_c \geq \omega_m$

* case ③ $\omega_s < 2\omega_m$ (not interested in this case)

~~No~~ reconstructed.



note } * اذا اعطانا وحدة من هدول ال cases وطلب m(t)
لازم نضرب ب rect $(-\omega_m, \omega_m)$ وبعدها بعمل inverse وبطلع m(t).
عشان اطلع $M(\omega)$

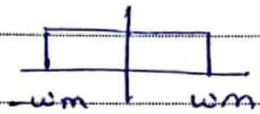
* **Sampling theorem**: a signal can be represented in its samples and can be recovered back when the sampling freq is greater than or equal to twice of the max freq ($\omega_s \geq 2\omega_m$)

سؤال امتحان

*ex: $m(t) = \text{sinc}\,(---)$

$$C(t) = \sum_{K=-\infty}^{\infty} \delta(t - K \underbrace{50 \times 10^{-3}}_{T_s})$$

$$\boxed{\omega_s = \frac{2\pi}{T_s}}$$



violated / اخترقت / not valid

$$\underline{\omega_s \geq 2\omega_m}$$

لازم نحقق الشرط عشان اطبق ال sampling theorem.

$70 \geq 100$?

Yes it's violated

if $\omega_s = 200$ $\omega_s > 2\omega_m$

$\omega_m = 70$ $\omega_c \geq \omega_m$

and $\omega_c \leq \omega_s - \omega_m$

if $\omega_s = 200$ should be

$\omega_m = 100 \longrightarrow \omega_c = 100.$

Sketch of spectrum يعني ارسم ال

وتحدد النقاط ونكتب ال guard band هي ال.

31/12/2019

parsaval's Theorem.

the energy of the signal $f(t)$:

$$E_f = \int_{-\infty}^{\infty} |f(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega .$$

area under $f(t)^2$ in time domain equals to the area under $F(\omega)$ in ω-domain $\div 2\pi$

$$f^*(t) \xleftrightarrow{F} F^*(\omega)$$

Proof:-

$$E_f = \int_{-\infty}^{\infty} f(t) \cdot f^*(t) = \int_{-\infty}^{\infty} \underbrace{\int_{-\infty}^{\infty} \frac{F(\omega)}{2\pi} e^{j\omega t} d\omega}_{f(t)} . \, f^*(t) \, dt$$

$$= \int_{-\infty}^{\infty} \frac{F(\omega)}{2\pi} \left(\int_{-\infty}^{\infty} f^*(t) e^{j\omega t} dt \right) d\omega .$$

$$= \int_{-\infty}^{\infty} \frac{F(\omega) \cdot F^*(\omega)}{2\pi} \, d\omega .$$

$$= \int_{-\infty}^{\infty} \frac{|F(\omega)|^2}{2\pi} \, d\omega .$$

$|F(\omega)|^2 = \epsilon_f(\omega)$. energy spectral density. (ESD)

$f \rightarrow$ related to $f(t)$.

$$E_f = \int_{-\infty}^{\infty} \frac{1}{2\pi} \epsilon_f(\omega) \, d\omega . = \int_{0}^{\infty} \frac{1}{\pi} \epsilon_f(\omega) \, d\omega .$$

$\hookrightarrow$ even function

$|F(\omega)|^2$:- measure of how the energy is being destriputive in. Frequency domain.

$$\int_{0}^{\infty} \_ \, d\omega = \int_{-\infty}^{0} \_ \, d\omega$$ because it's even

Ex:

Consider an LTI system.

$$x(t) = e^{-3t}u(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$h(t) = e^{-2t}u(t).$$

Find ESD of $y(t)$ $\left(\mathcal{E}_y(\omega)\right)$:

$x(t) \longleftrightarrow X(\omega)$

$y(t) \longleftrightarrow Y(\omega)$

$h(t) \longleftrightarrow H(\omega).$

$$\mathcal{E}_y(\omega) = |Y(\omega)|^2$$

but $Y(\omega) = X(\omega) \cdot H(\omega).$

$$\mathcal{E}_y(\omega) = \underbrace{|X(\omega)|^2}_{\mathcal{E}_X(\omega)} \cdot |H(\omega)|^2.$$

$$X(\omega) = F\{e^{-3t}u(t)\} = \frac{1}{3+j\omega}.$$

$$H(\omega) = F\{e^{-2t}u(t)\} = \frac{1}{2+j\omega}.$$

$$|X(\omega)|^2 = X(\omega) \cdot \overset{*}{X}(\omega) = \frac{1}{3+j\omega} \cdot \frac{1}{3-j\omega} = \frac{1}{9+\omega^2} = \mathcal{E}_X(\omega).$$

$$|H(\omega)|^2 = \frac{1}{4+\omega^2}$$

$$\mathcal{E}_y(\omega) = \frac{1}{(4+\omega^2)(9+\omega^2)}, \quad E_y = \frac{1}{\pi}\int_0^\infty \frac{1}{(9+\omega^2)(4+\omega^2)}\,d\omega = \frac{1}{12}$$

Average power and power spectral density. (PSD)

$$P_f = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} |\underbrace{f(t) \cdot rect\left(\frac{t}{T}\right)}_{f_T(t)}|^2 \, dt.$$

$$F_T(\omega) = F\{f(t) \cdot rect\left(\frac{t}{T}\right)\}$$

$$= \lim_{T \to \infty} \frac{1}{T} \frac{1}{2\pi} \int_{-\infty}^{\infty} |F_T(\omega)|^2 \, d\omega.$$

$$= \int_{-\infty}^{\infty} \frac{1}{2\pi} \underbrace{\lim_{T \to \infty} \frac{|F_T(\omega)|^2}{T}}_{PSD.} \, d\omega.$$

$$P_f(\omega) = \lim_{T \to \infty} \frac{|F_T(\omega)|^2}{T}$$

$$P_f = \frac{1}{2\pi} \int_{-\infty}^{\infty} P_f(\omega) \, d\omega.$$

هاد حكي لازم
نفهمه
بس مو داخل ما
رح يجيب عليه
سؤال