EM I

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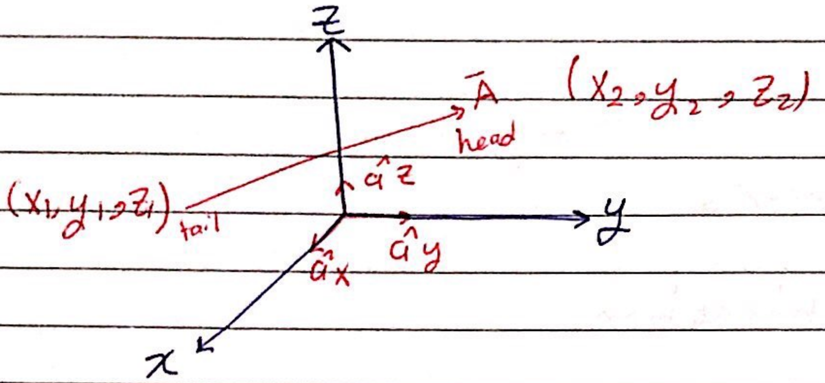
POWER UNIT

CH1 - vectors

Vector → mag + dir　　　Vector A → $\bar{A}$ → Bold in the book

scaler → mag only　　　scaler A → A

$*$ in cartesian coordinate :-



$*$ unit vector : only direction (mag = 1)

$\hat{a}x$ → unit vector in x-dir

$$\bar{A} = \underbrace{A_x \hat{a}x + A_y \hat{a}y + A_z \hat{a}_z}_{\text{vector componant (scaler)}} \quad \text{---- } \boxed{1} \text{ Long formal}$$

$$\bar{A} = (A_x, A_y, A_z) \quad \text{---- } \boxed{2} \text{ short formal}$$

Point → $P(x, y, z)$

ex: $\bar{C} = \overset{\to cx}{3} \hat{a}x + \overset{\to cz}{4} \hat{a}_z$

$\bar{C} = (3, 0, 4)$

$*$ magnitude of the vector

$$|\bar{A}| = A = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$$

unit vector in $\bar{A}$ dir

$$\hat{a}_A = \frac{\bar{A}}{A} = \frac{A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z}{\sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}}$$

$\boxed{3}$ $\bar{A} = A\,\hat{a}_A$

* operation on vector :-

-Addition and subtraction

$$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

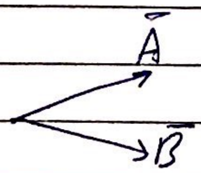
$$\bar{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$$

$$\bar{C} = \bar{A} \oplus \bar{B}$$

$$\bar{C} = (A_x + B_x)\hat{a}_x \oplus (A_y + B_y)\hat{a}_y \oplus (A_z + B_z)\hat{a}_z$$

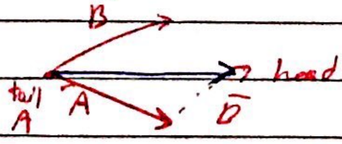
$$\bar{C} = C_x \hat{a}_x + C_y \hat{a}_y + C_z \hat{a}_z$$

*graficaly :-



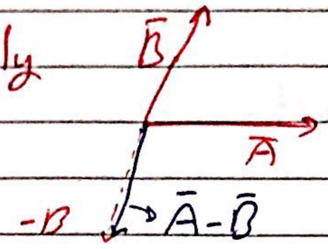
$$\bar{C} = \bar{A} + \bar{B}$$

Arrow method



$\bar{D} = \bar{A} - \bar{B} \longrightarrow \bar{D} = (A_x - B_x)\,\hat{a}_x + (A_y - B_y)\,\hat{a}_y + (A_z - B_z)\,\hat{a}_z$

grafically



* $\bar{A} + \bar{B} = \bar{B} + \bar{A}$, $\bar{A} + (\bar{B} + \bar{C}) = (\bar{A} + \bar{B}) + \bar{C}$, $K(\bar{A} + \bar{B}) = K\bar{A} + K\bar{B}$

*multiplication:

a) Dot product $\longrightarrow$ scaler

$\bar{A} \cdot \bar{B} = |\bar{A}|\,|\bar{B}| \cos\theta_{AB}$

$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z$

$\hat{a}_x \cdot \hat{a}_x = (1)(1)\cos 0 = 1$

$\hat{a}_x \cdot \hat{a}_y = (1)(1)\cos 90 = \text{zero}$

$\hat{a}_n \cdot \hat{a}_m = 0$ if $n \neq m$

$\hat{a}_n \cdot \hat{a}_m = 1$ if $n = m$

$\cos\theta_{AB} = \dfrac{\bar{A} \cdot \bar{B}}{A\,B}$, $\bar{A} \cdot \bar{A} = |\bar{A}|^2 = A^2 = A_x^2 + A_y^2 + A_z^2$

$\bar{A} \cdot \bar{B} = \bar{B} \cdot A$, $\bar{A} \cdot (\bar{B} + \bar{C}) = \bar{A} \cdot \bar{B} + \bar{A} \cdot \bar{C}$

X

b) cross product : (answer = vector)

$|\bar{A} \times \bar{B}| = AB \sin\theta$

↳ magnitude for cross product

$\sin\theta_{AB} = \dfrac{|\bar{A} \times \bar{B}|}{AB}$

$$\bar{A} \times \bar{B} = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{ccc} ① & ② & ③ \\ \left|\begin{array}{ccc} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{array}\right| \end{array} \begin{array}{c} ① \\ ② \\ ③ \end{array}$$

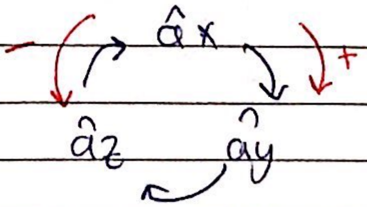
$\bar{A} \times \bar{B} = -\bar{B} \times \bar{A}$

$\bar{A} \times \bar{B} = (-1)^{1+1} (A_y B_z - A_z B_y)\hat{a}_x + (-1)^{1+2} (A_x B_z - A_z B_x)\hat{a}_y$

$+ (-1)^{1+3} (A_x B_y - A_y B_x)\hat{a}_z$

$\bar{A} \times \bar{B} \perp \bar{A}$ and $\perp \bar{B}$

- $\bar{A} \times (\bar{B} + \bar{C}) = \bar{A} \times \bar{B} + \bar{A} \times \bar{C}$
- $\bar{A} \times (\bar{B} \times \bar{C}) \neq (\bar{A} \times \bar{B}) \times \bar{C}$
- $\bar{A} \times \bar{B} \times \bar{C}$ = eq. sll B



- $\hat{a}_x \times \hat{a}_x$ = zero

$|\hat{a}_x \times \hat{a}_y| = 1$ → but $\hat{a}_x \times \hat{a}_y = \hat{a}_z$

$\hat{a}_z \times \hat{a}_y = -\hat{a}_x$

* Vector projection along another vector :-

A_B : projection of A along B .
↳ scaler

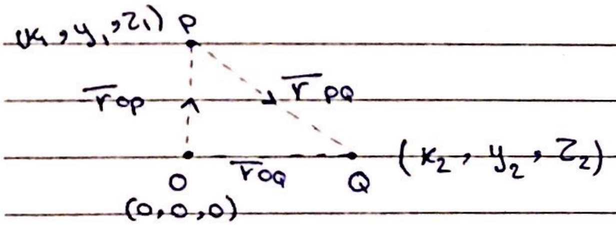
$\bar{A}_B$: " " " " " .
↳ vector

- $\cos\theta_{AB} = \frac{A_B}{A} \longrightarrow A_B = A\cos\theta_{AB}$

- $A_B = \bar{A} \cdot \hat{a}_B \longrightarrow \hat{a}_B = \frac{\bar{B}}{B}$, $\cos\theta_{AB} = \frac{\bar{A}\cdot\bar{B}}{AB}$

- $\bar{A}_B = (\bar{A} \cdot \hat{a}_B)\,\hat{a}_B$

Distrance : → vector



$\bar{r}_{PQ} = \bar{r}_Q - \bar{r}_P$
$= (x_2, y_2, z_2) - (x_1, y_1, z_1)$
$= (x_2 - x_1)\,\hat{a}_x + (y_2 - y_1)\,\hat{a}_y + (z_2 - z_1)\,\hat{a}_z$

$|\bar{r}_{PQ}| = |r_{PQ}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$\hat{a}_{rPQ} = \frac{\bar{r}_{PQ}}{r_{PQ}} = \hat{a}_{rQP}$

$\left[\ \bar{r}_{QP} = -\bar{r}_{PQ}\ \right]$

Ex : Given $\bar{A} = 3\hat{a}_x + 4\hat{a}_y + \hat{a}_z$
$\vec{B} = 2\hat{a}_y - 5\hat{a}_z$

Find : (a) θ_{AB} (b) $\bar{A}_B$

(a) $\theta = \cos^{-1} \frac{\bar{A}\cdot\bar{B}}{AB}$ or $\theta = \sin^{-1} \frac{|\bar{A}\times\bar{B}|}{AB}$ $A = \sqrt{26}$, $B = \sqrt{29}$

$\bar{A}\cdot\bar{B} = 0 + 8 - 5 = 3$ $\theta = \cos^{-1}\left(\frac{3}{\sqrt{29}\sqrt{24}}\right) = 83.73^\circ$

b) $\bar{A}_B = (\bar{A} \cdot \hat{a}_B) \; \hat{a}_B$

$\hat{a}_B = \frac{\bar{B}}{B} = \frac{(0, 2, -5)}{\sqrt{29}}$

$= \frac{2}{\sqrt{29}} \hat{a}_y \quad \frac{5}{\sqrt{29}} \hat{a}_z$

$(\bar{A} \cdot \hat{a}_B) = \left(0 + \frac{8}{\sqrt{29}} - \frac{5}{\sqrt{29}}\right) \cdot \underbrace{\left(0, \frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}\right)}_{\hat{a}_B}$

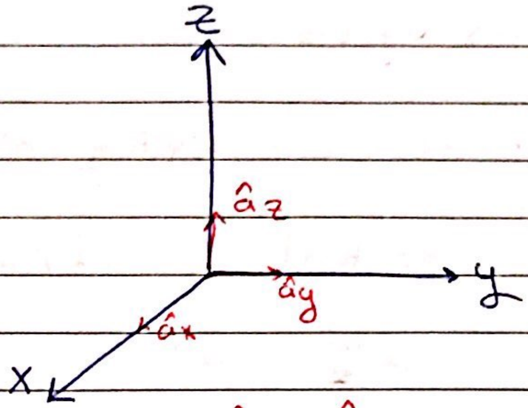
$= \frac{3}{29} (0, 2, -5)$

$\boxed{\bar{A}_B = \frac{6}{29} \hat{a}_y - \frac{15}{29} \hat{a}_z}$

CH-2 coordinate systems:-

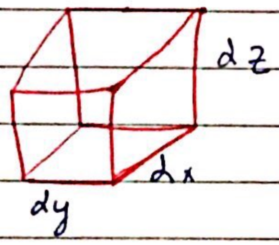
1) cartesian coordinate

$-\infty < x < \infty$
$-\infty < y < \infty$
$-\infty < z < \infty$



3D object → infinite Box

$\hat{a}_x, \hat{a}_y, \hat{a}_z$ → unit vector



→ differential Box

* Differential elements → (dx, dy, dz)

* differential lengh ($\bar{d}L$) → vector

$$\bar{d}L = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

* diffential Normal surface Area ($\bar{d}s$) → vector

$\bar{d}s$ front $= dy\,dz\,(\hat{a}_x)$
$\bar{d}s$ back $= dy\,dz\,(-\hat{a}_x)$
$\bar{d}s$ left $= dx\,dz\,(-\hat{a}_y)$
$\bar{d}s$ right $= dx\,dz\,(\hat{a}_y)$
$\bar{d}s$ top $= dx\,dy\,(\hat{a}_z)$
$\bar{d}s$ bot $= dx\,dy\,(-\hat{a}_z)$

* differential volume (dv) → scaler

$dv = dx\, dy\, dz$

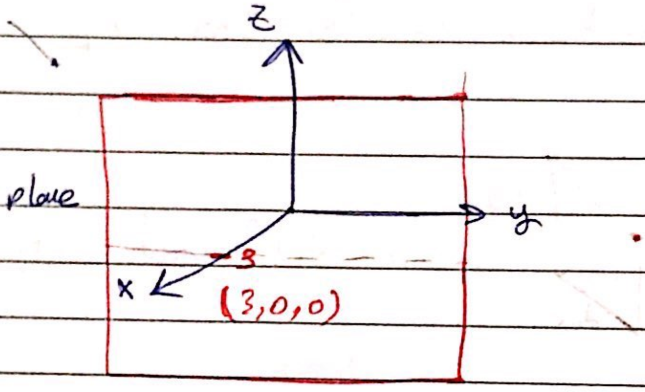
- 2D surface : by fixing one variable

• if x = constant → inf plane // yz plane

ex:

$x = 3$ $\quad (-\infty < {}^{y}_{z} < \infty)$

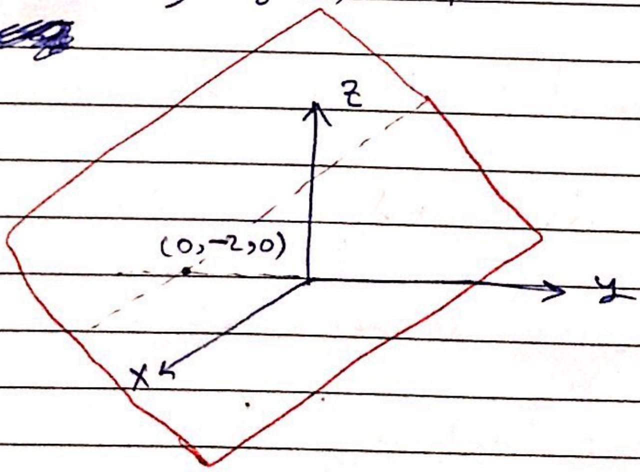
if $x=0$ → inf plane along yz plane



• if y = constant

↳ inf plane // or along ($y=0$) xz plane

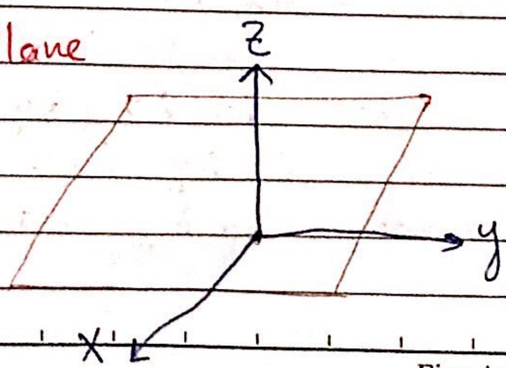
ex: $y = -2$



• if z = constant

↳ inf plan along xy Plane

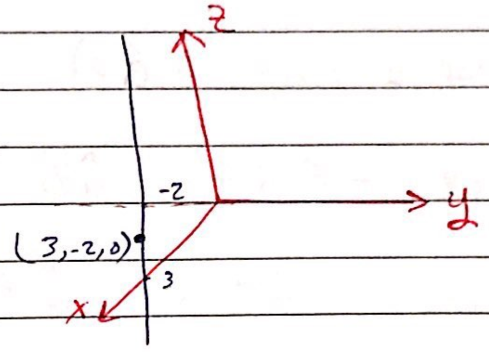
ex $z = 0$



* 1D Segmet : by fixing tow Variables

• if x,y are constant

$x = -3 , y = -2 \rightarrow$ inf line // z-axis



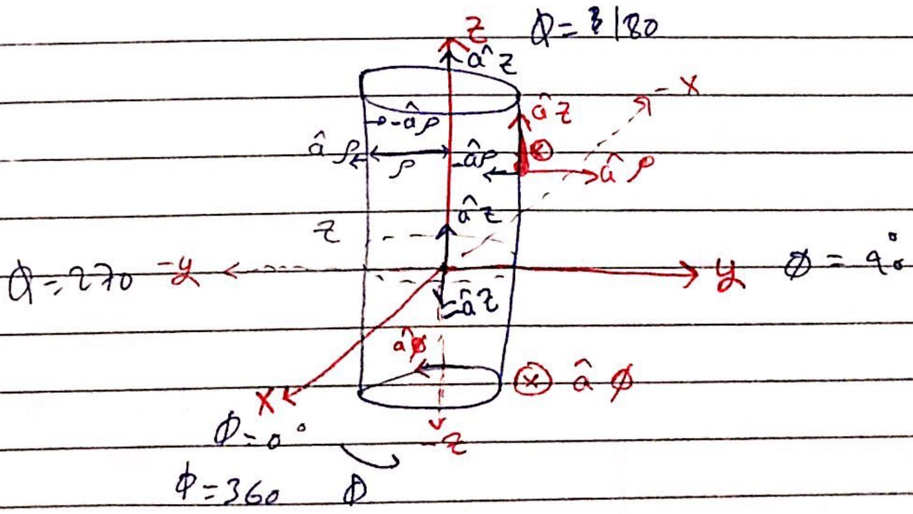
* Sames

- x, z are constant
  - ↳ inf line // or along y axis
- y,z are constant
  - ↳ inf line // or along x-axis

* 2) Cylindrical coordinate

$$\left.\begin{array}{l} 0 < \rho < \infty \\ 0 \leq \phi \leq 2\pi \\ -\infty < z < \infty \end{array}\right\} \text{3D object inf solid Cylindrical}$$

unit vector $\rightarrow \hat{a}_\rho , \hat{a}_\phi , \hat{a}_z$



*Differential elements:-

$(d\rho, \rho d\phi, dz)$

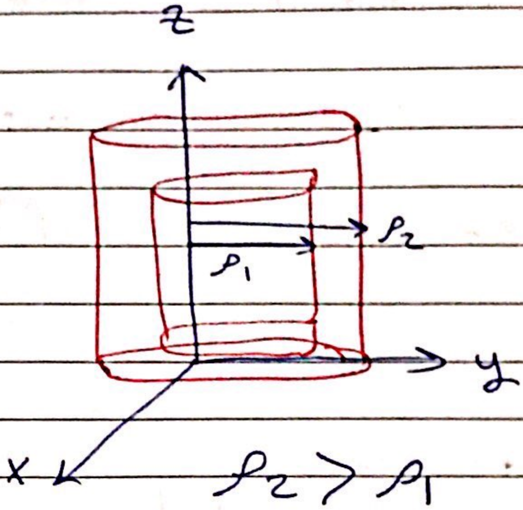
$\overline{dl} = d\rho\, \hat{a}_\rho + \rho d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$

$\overline{ds}_{top} = \rho d\rho d\phi (\hat{a}_z)$

$\overline{ds}_{bot} = -\rho d\rho d\phi (\hat{a}_z)$

$\overline{ds}_{side} = \rho d\phi\, dz\, (\hat{a}_\rho)$

$ds_\phi = d\rho\, dz\, \hat{a}_\phi$



$\rho_2 > \rho_1$

$z = 0$

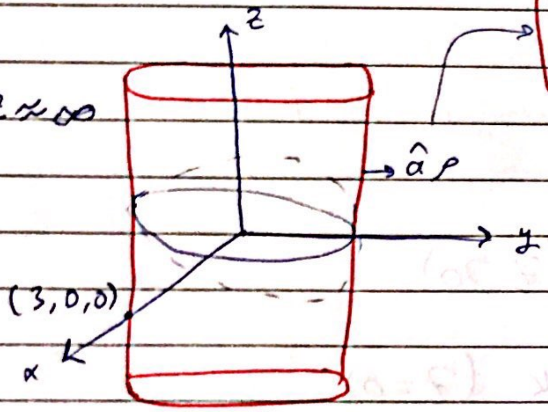
• $dv = \rho\, d\rho\, d\phi\, dz$

* 2D surface

- ρ = constant
  - ↳ inf hollo cylinder
  - ↳ inf line ($\rho = 0$) along z-axis

$\rho = 3$

$(3, \phi, 0)$



$$\boxed{\bar{ds} = \rho\, d\phi\, dz\, \hat{a}_\rho}$$

$d\rho$

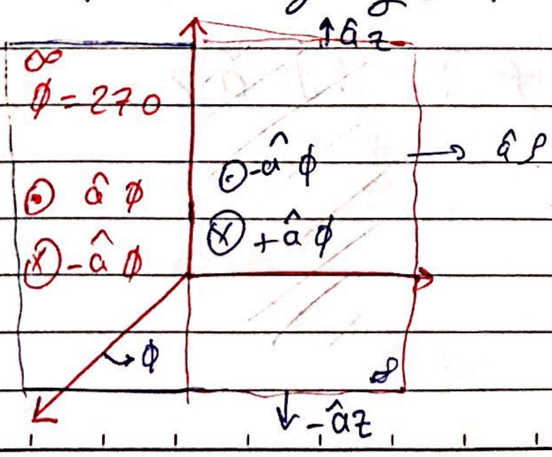
$\rho\, d\phi$

dz

$0 \leq \rho \leq 3$

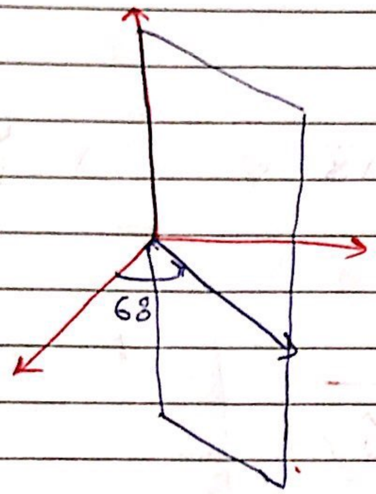
- ϕ = constant
  - ↳ inf plane

$\phi = 90$
↳ inf plane along yz-plane



$\bar{ds} = d\rho\, dz\, \hat{a}_\phi$

$\phi = 60$

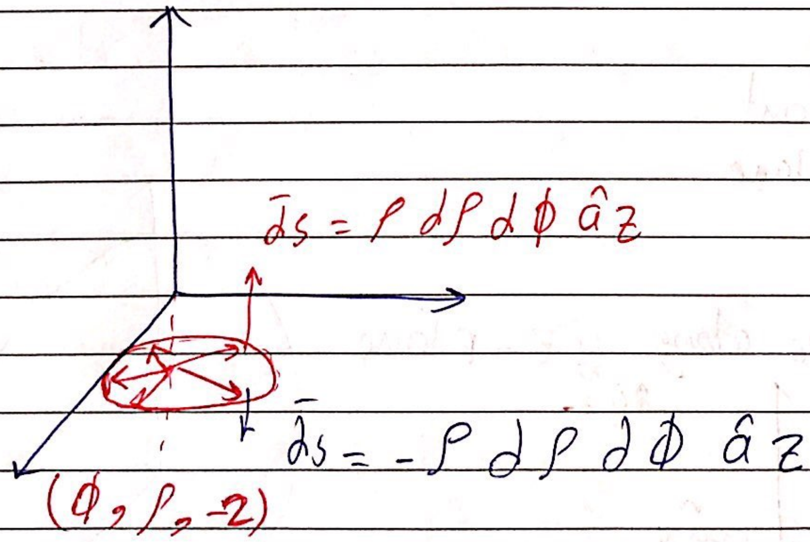


• $z =$ constant

$\hookrightarrow$ inf disk // xy plane ($z > 0$)

along xy plane ($z = 0$)

$z = -2$



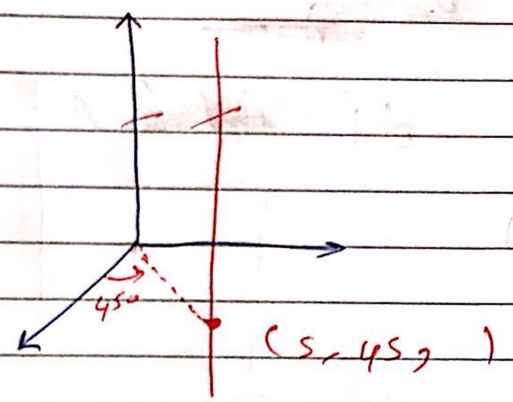
* 1D Segment

• ρ, ϕ are constant

↳ inf line // z axis ($\rho > 0$)

along z axis ($z = 0$)

$\rho = 5 \quad \phi = 45^\circ$



$\rho = 0 \quad \phi = 0^\circ$

$\rho = 0 \quad \phi = 90^\circ$

$dL = dz\,\hat{a}_z + \cancel{d\rho\,\hat{a}_\rho} + \cancel{\rho\,d\phi\,\hat{a}_\phi}$

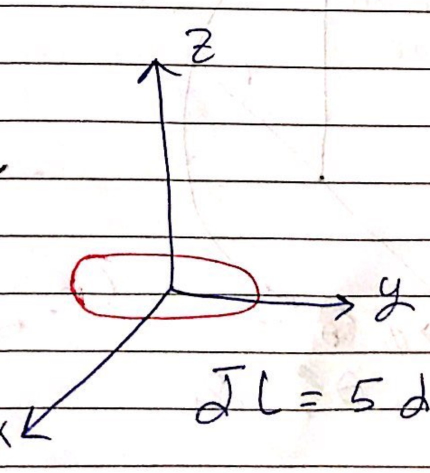
• ρ, z are constant:

↳ circle // xy plane ($\rho \neq 0$, $z \neq 0$)

↳ circle along xy plane ($\rho \neq 0$, $z = 0$)

↳ point → ($\rho = 0$)

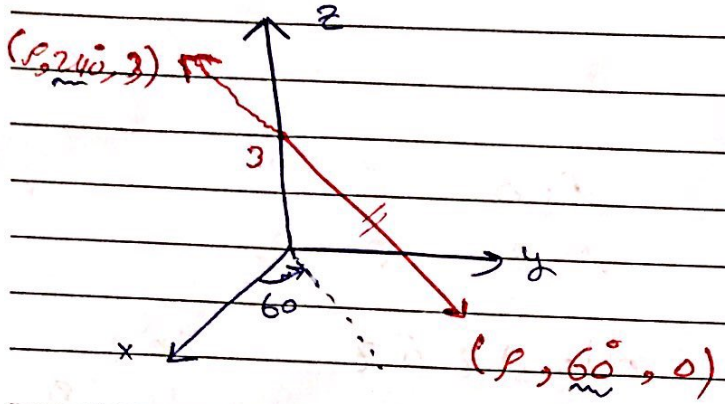
$\rho = 5 \quad z = 0$



$dL = 5\,d\phi\,\hat{a}_\phi$

- ϕ, z are constant
  ↳ semi-inf-line (ray)

ex $\phi = 60$ $z = 3$



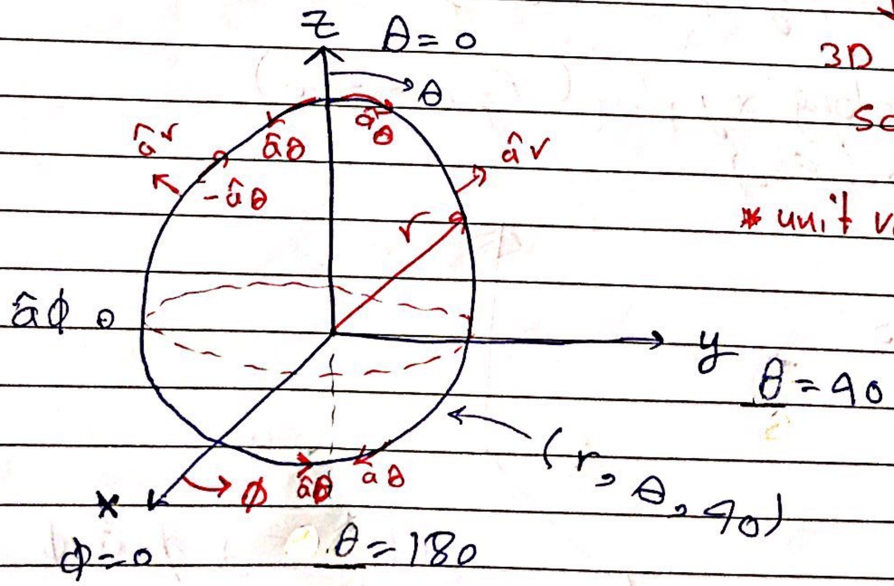
$d\bar{l} = d\rho\ \hat{a}_\rho$

* 3) spherical coordinate.

$0 \le r < \infty$, $0 \le \theta \le \pi$, $0 \le \phi \le 2\pi$

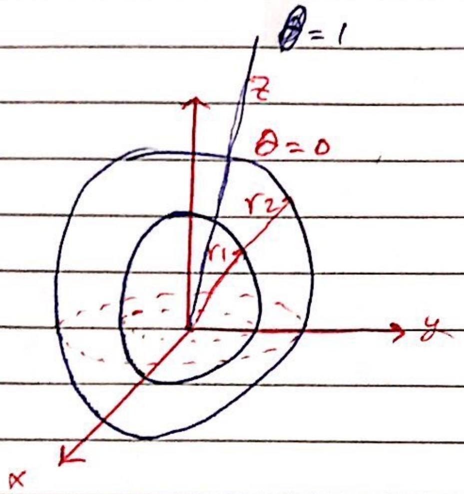
↓ 3D object inf solid sphere.

* unit vector → $\hat{a}_r, \hat{a}_\theta, \hat{a}_\phi$

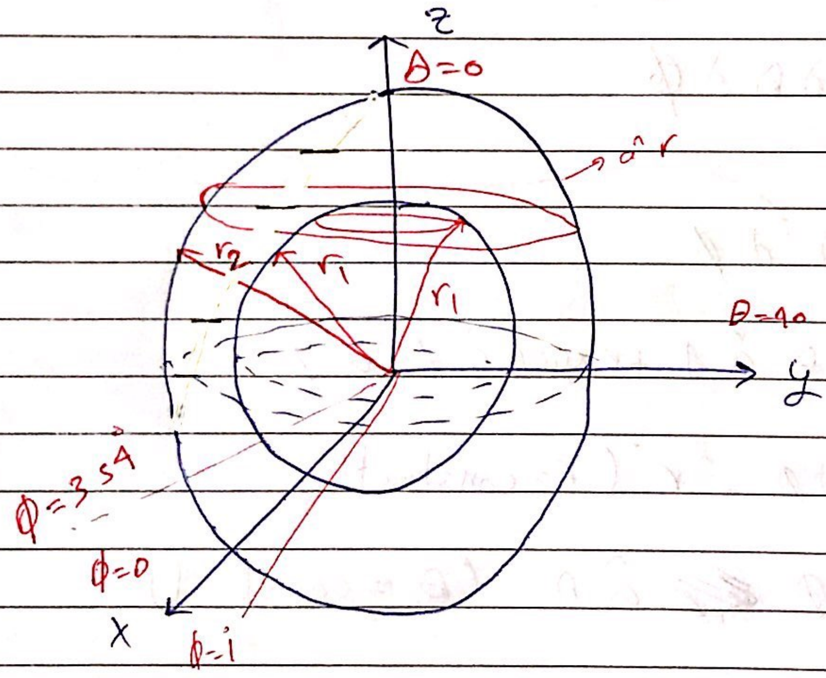


Differential elements

$dr,\ r\,d\theta,\ r\sin\theta\,d\phi$



$r_2 > r_1$



θ, ϕ const $\quad r, \phi$ cont $\quad r, \theta$ const

$\bar{dL} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$

$\bar{ds} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$

surface shell

$\bar{ds}_\theta = r\sin\theta\, dr\, d\phi\, \hat{a}_\theta$

θ = constant

$\bar{ds}_\phi = r\, dr\, d\theta\, \hat{a}_\phi$

ϕ constant

$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$dr,\ r\,d\theta,\ r\sin\theta\, d\phi$

$\bar{dL} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\phi + r\sin\theta\, d\phi\, \hat{a}_\phi$

$\bar{ds} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$ (r = constant)

$\bar{ds} = r\sin\theta\, dr\, d\phi\, \hat{a}_\theta$ (θ = constant)

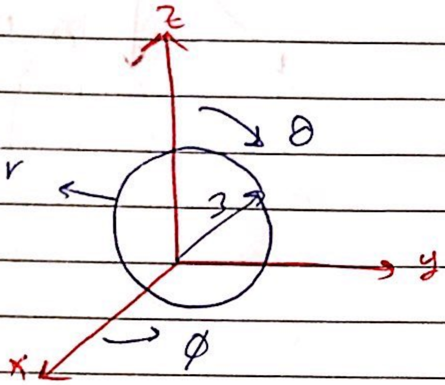
$\bar{ds} = r\, dr\, d\theta\, \hat{a}_\phi$ (ϕ = constant)

$dr = r^2 \sin\theta\, dr\, d\theta\, d\phi$

2D - surface

- r = constant
  - → hollow sphere $(r>0)$
  - → point $(r=0)$

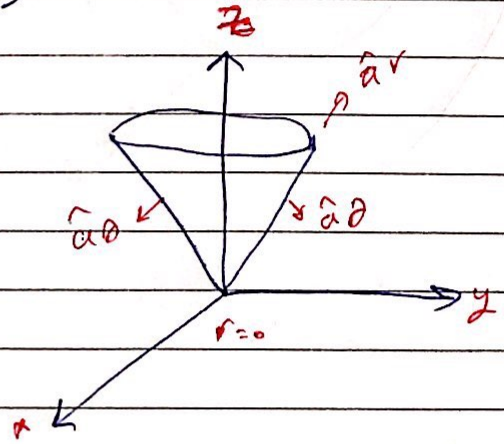
$r=3$



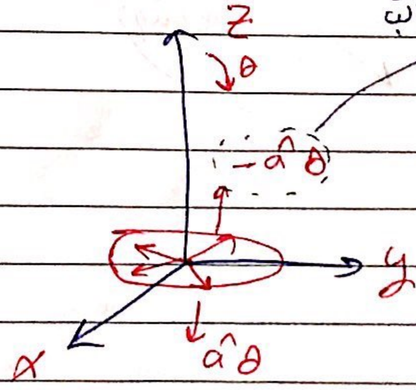
$$\text{Area}=\int ds=\int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta \, d\theta \, d\phi$$

- θ = constant
  - → inf hollow cone $\theta \in (\overset{\text{not include}}{0}, 90)$
  - → inf Disk along xy plane $(\theta=90^\circ)$
  - → semi-inf-line along (+ve z axis $\theta=0^\circ$) OR (−ve z axis $\theta=180$)

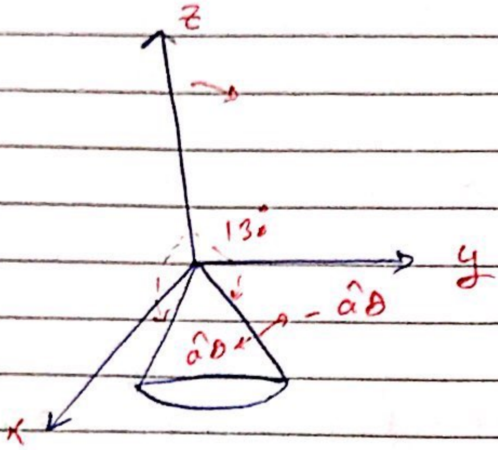
$\theta=45^\circ$



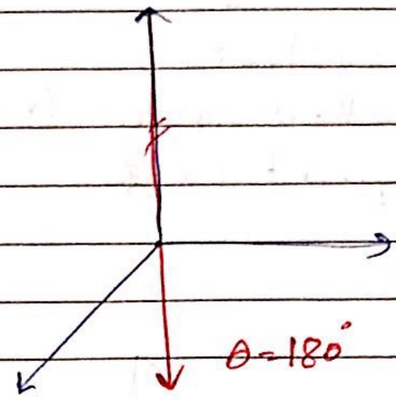
$\theta=90^\circ$. طالما أمشي عكس θ
بحط سالب.



$\theta = 130^\circ$



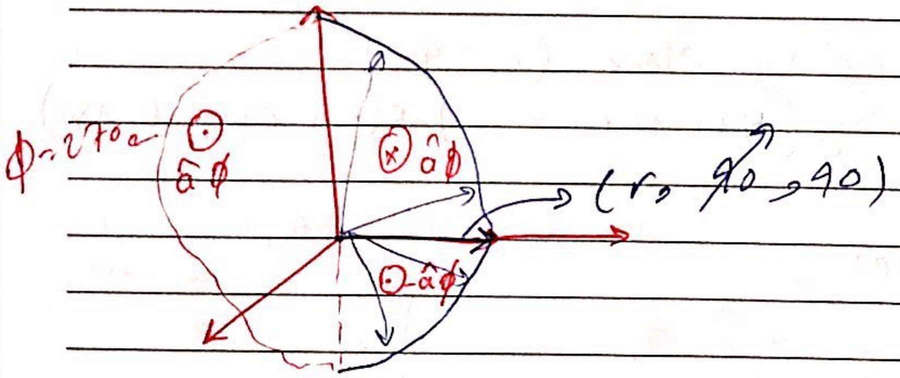
$\theta = 0$



- ϕ = constant

↳ semi inf Disk

$\phi = 90$

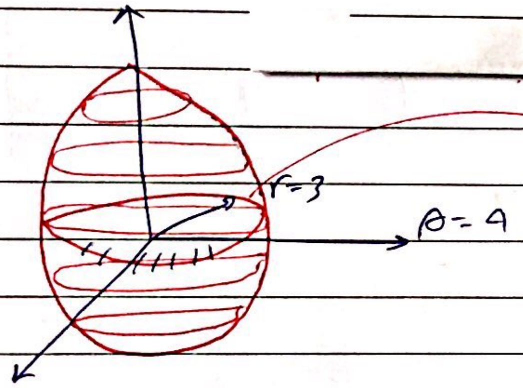


*1D segment

r, θ are constant

→ circule // xy plane ($r \neq 0$, $\theta = 90°$)
along xy plane ($r \neq 0$, $\theta = 90°$)

→ point ($r = 0$)

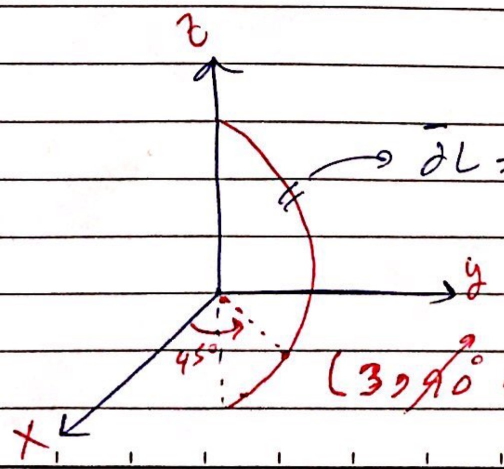


$\bar{\partial L} = r \sin\theta \, \partial\phi \, \boxed{\hat{a}\phi}$
$= 3 \sin 90 \, \partial\phi \, \hat{a}\phi$

• r, ϕ constant

→ half circle ($r \neq 0$)
→ point ($r = 0$)

$r = 3$ $\theta = 45°$



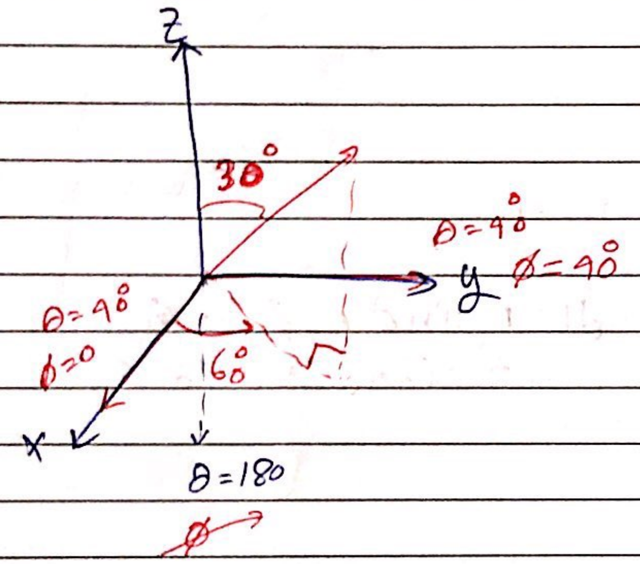
$\bar{\partial L} = r \, \partial\theta \, \boxed{\hat{a}\theta}$

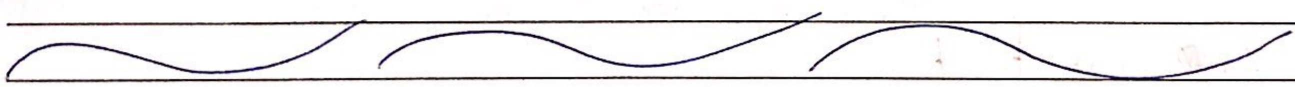
$(3, \cancel{90°}, 45°)$

• θ, ϕ are constant

↳ semi - inf - Line (ray)

$\theta = 30°$ $\phi = 60°$





1

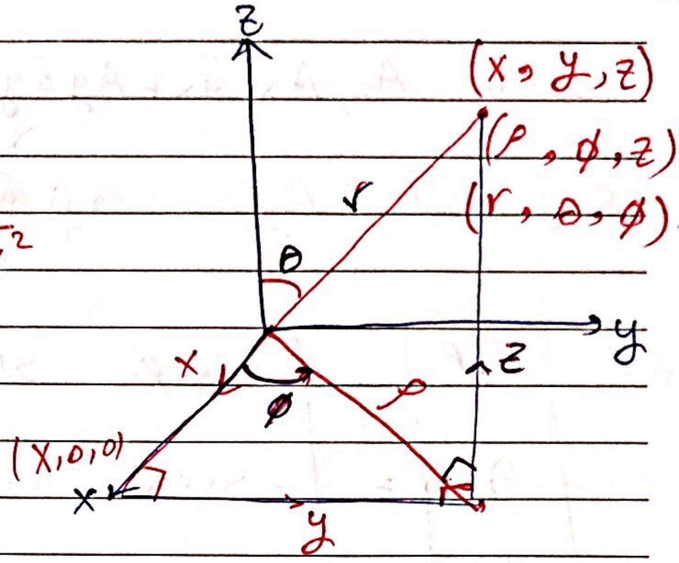
* point and vector transformation

↳ point conversion

$\sin\phi = \frac{y}{\rho} = \frac{y}{\sqrt{x^2+y^2}}$

$\cos\phi = \frac{x}{\rho}$

$\tan\phi = \frac{y}{x}$



* convert from:

• cart → cyl
$(x, y, z) \rightarrow (\rho, \phi, z)$?

$\rho = \sqrt{x^2+y^2}$ $\quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$

$z = z$

• cyl → cart
$(\rho, \phi, z) \rightarrow (x, y, z)$

$x = \rho \cos\phi$ $\quad y = \rho \sin\phi$

$z = z$

• car → spher
$x, y, z \rightarrow (r, \theta, \phi)$

$r = \sqrt{x^2+y^2+z^2}$ $\quad \theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$\phi = \tan^{-1} \frac{y}{x}$

cyl → sph
$(\rho, \phi, z) \rightarrow (r, \theta, \phi)$

$r = \sqrt{\rho^2+z^2}$ $\quad \theta = \tan^{-1}\left(\frac{\rho}{z}\right)$

$\phi = \phi$

sph → cart
$(r, \theta, \phi) \rightarrow (x, y, z)$

$x = r \sin\theta \cos\phi$ $\quad y = r \sin\theta \sin\phi$

$z = r\cos\theta$

sph → cyl $\quad \rho = r\sin\theta \quad z = r\cos\theta \quad \phi = \phi$

* Vector conversion :-

• car → cyl

cart : $\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

cyl : $\bar{A} = \underset{?}{A_\rho} \hat{a}_\rho + \underset{?}{A_\phi} \hat{a}_\phi + \underset{?}{A_z} \hat{a}_z$

$$\begin{array}{cc} \hat{a}_\rho \leftarrow & \\ \hat{a}_\phi \leftarrow & \\ \hat{a}_z \leftarrow & \end{array} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \begin{array}{c} \rightarrow \hat{a}_x \\ \rightarrow \hat{a}_y \\ \rightarrow \hat{a}_z \end{array}$$

unit vector لازم نستخدمها بقدر

Given

Tow step to use

① $A_\rho = \cos\phi \cdot A_x + \sin\phi \, A_y$

$A_\phi = -\sin\phi \, A_x + \cos\phi \, A_y$

$A_z = A_z$

② $x = \rho\cos\phi$
$y = \rho\sin\phi$

ex $A_x = x^2 y \Rightarrow (\rho\cos\phi)^2 \, \rho\sin\phi$

Cyl $\to$ cart

$(A_\rho, A_\phi, A_z) \to (A_x, A_y, A_z)$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

cart $\to$ sph

$\bar{B} = (B_x, B_y, B_z)$

$\to \bar{B} = (B_r, B_\theta, B_\phi)$

$$\begin{bmatrix} B_r \\ B_\theta \\ B_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

* بنربط ؟ بستخدم ال unit vector

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}^{-1} \begin{bmatrix} B_r \\ B_\theta \\ B_\phi \end{bmatrix}$$

• Cyl $\rightleftarrows$ Sph

$(A_\rho, A_\phi, A_z) \rightarrow (A_r, A_\theta, A_\phi)$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

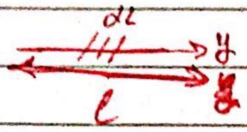
$$A_\phi = A_\phi$$

CH3 Vector calculas

Integration:-

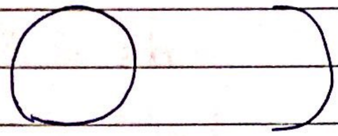
* Line integral $\longrightarrow \bar{dL}$

$\int_l f_l \, dL$ scaler or $\int_l \bar{A} \cdot \bar{dL}$ Vector



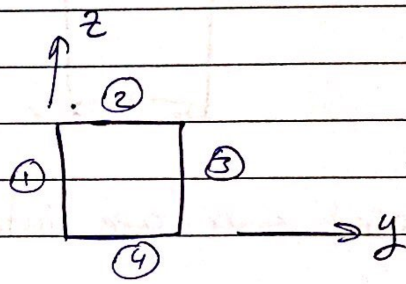
$dL = dy$ $\bar{dL} = dy \, \hat{a}_y$

1D segments:



• closed line integral

$\oint_l \bar{A} \cdot \bar{dL}$



$$\oint_l \bar{A} \cdot \bar{dL} = \int_{l_1} \bar{A} \cdot \bar{dL_1} + \int_{l_2} \bar{A} \cdot \bar{dL_2} + \int_{l_3} \bar{A} \cdot \bar{dL_3} + \int_{l_4} \bar{A} \cdot \bar{dL_4}$$

$\bar{dL_1} = dz \, \hat{a}_z$ $\bar{dL_2} = dy \, \hat{a}_y$

$\bar{dL_3} = dz \, \hat{a}_z$, $\bar{dL_4} = dy \, \hat{a}_y$

* surface integral

$$\int_S \rho_s \, ds \quad \text{or} \quad \int_S \bar{A} \cdot \bar{ds}$$

ex

$\bar{A} = (A_\rho, 0, A_z)$

$\rho = 3$

$\bar{ds} = \rho \, d\phi \, dz \, \hat{a}_\rho$

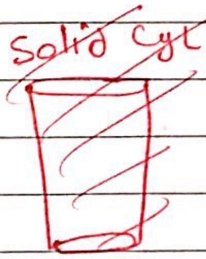
$$\int_{-4}^{3} \int_{0}^{2\pi} \rho \, A_\rho \, d\phi \, dz$$

• closed surface integral :-

$$\oint_S \bar{A} \cdot \bar{ds}$$

ex $\bar{A} = (A_x, A_y, A_z)$

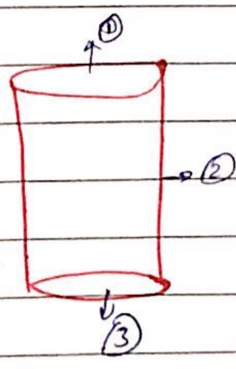
use matrix $\rightarrow \bar{A} = (A_\rho, A_\phi, A_z)$ solid cyL



$\bar{ds_1} = \rho \, d\rho \, d\phi \, \hat{a}_z$

$\bar{ds_2} = \rho \, d\phi \, dz \, \hat{a}_\rho$

$\bar{ds_3} = -\rho \, d\rho \, d\phi \, \hat{a}_z$



$0 \leq \rho \leq 3$

$0 \leq \phi \leq 2\pi$

$-2 \leq z \leq 2$

$$\oint_S \bar{A} \cdot \bar{ds} = \int_{S_1} \bar{A} \cdot \bar{ds_1} + \int_{S_2} \bar{A} \cdot \bar{ds_2} + \int_{S_3} \bar{A} \cdot \bar{ds_3}$$

$\downarrow A_z$ $\quad \downarrow A_\rho$ $\quad \rightarrow A_z$

* Volume integral

$$\int_v \rho_v \, dv \qquad dv = \rho \, d\rho \, d\phi \, dz$$

$$\int_{-2}^{2} \int_{0}^{2\pi} \int_{0}^{3} \rho_v \, \rho \, d\rho \, d\phi \, dz$$

* Del operator = (Vector) (∇) $\vec{\nabla} \rightarrow \times$

in cart.

$$\nabla = \frac{\partial}{\partial x} \hat{a}_x + \frac{\partial}{\partial y} \hat{a}_y + \frac{\partial}{\partial z} \hat{a}_z \quad \text{.....} (1)$$

in cyl.

$$\nabla = \frac{\partial}{\partial \rho} \hat{a}_\rho + \frac{1}{\rho} \frac{\partial}{\partial \phi} \hat{a}_\phi + \frac{\partial}{\partial z} \hat{a}_z \quad \text{.....} (2)$$

in sph.

$$\nabla = \frac{\partial}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi} \hat{a}_\phi \quad \text{.....} (3)$$

- Del has 4 usage

1) Gradiant → Vector

∇V = gradiant of V ← scaler

بحط جنب (1), (2), (3) V

$$\nabla V = \frac{\partial \boxed{V}}{\partial x} \hat{a}_x + \dots \dots$$ مثلاً

2) Divergence $\longrightarrow$ scaler

$\nabla . \bar{A}$ = Divergence of (A)

- in cart $\nabla . \bar{A} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (A_x, A_y, A_z)$

$$\nabla . \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

- in cyL (given)

$$\nabla . \bar{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\bar{A} = \underbrace{\rho^2}_{A\rho} \hat{a}_\rho + \underbrace{\cos \phi}_{A\phi} \hat{a}_\phi \quad , A_z = 0$$

- in sph (given)

$$\nabla . \bar{A} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}(\sin \theta \, A_\theta)$$
$$+ \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

3) curl $\rightarrow$ vector

$\nabla \times \bar{A}$ = curl of (A)

- in cart

$$\nabla \times \bar{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

- in cyl (given)

$$\nabla \times \bar{A} = \frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho \hat{a}_\phi & \hat{a}_z \\ \partial/\partial \rho & \partial/\partial \phi & \partial/\partial z \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

- in sph (given)

$$\nabla \times \bar{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta \, \hat{a}_\phi \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial \phi \\ A_r & rA_\theta & r\sin\theta \, A_\phi \end{vmatrix}$$

4) Laplacian $\rightarrow$ scaler $\rightarrow$ CH6

$\nabla \cdot \nabla V = \nabla^2 V$ = Laplacian of (v)

* Divergence theorem:-

$$\oint_s \bar{A} \cdot \partial s = \int_v \nabla \cdot \bar{A} \, \partial V$$

حسب المعطيات

* Stock's theorem:-

$$\oint_l \bar{A} \cdot \partial \bar{L} = \int_s \nabla \times \bar{A} \cdot \partial \bar{s} \quad \text{curl}$$

حسب المعطيات والإجبار على استخدام الطريقة

* Distance:-

⊙ (r_1, θ_1, ϕ_1)

⊛ (ρ_1, ϕ_1, z_1)

$P(x_1, y_1, z_1)$

⊙ (r_2, θ_2, ϕ_2)

⊛ (ρ_2, ϕ_2, z_2)

$Q(x_2, y_2, z_2)$

d

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

⊛ $$d^2 = \rho_1^2 + \rho_2^2 - 2\rho_1 \rho_2 \cos(\phi_2 - \phi_1) + (z_2 - z_1)^2$$

if $\phi_1 = \phi_2 \rightarrow d^2 = (\rho_2 - \rho_1)^2 + (z_2 - z_1)^2$

⊙ $d^2 = (r_2 - r_1)^2$ if $\theta_1 = \theta_2$, $\phi_1 = \phi_2$

أو عن طريق التحويل بين Cart, Cyl, sph

CH-4 Electrostatic fields:-

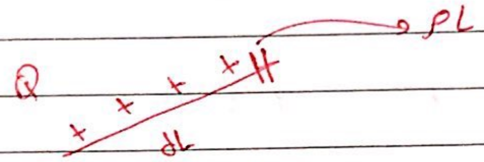
Sources:-

1) point charge (Q) in (c) → colounb

2) continuous charge distribution

→ a) Line charge (ρ_L) in (c/m)

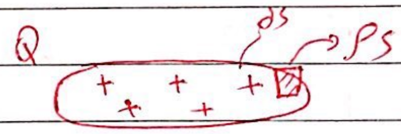
(1D segment)



$$Q = \int_{l} \rho_L \, dL$$

b) surface charge (ρ_s) → (c/m²)

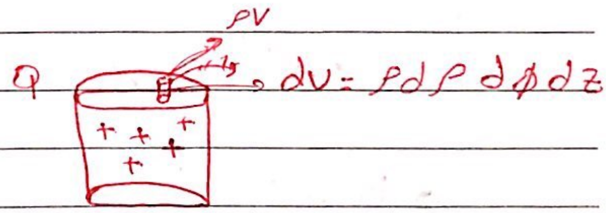
(2D surface)



$$Q = \int_{s} \rho_s \, ds$$

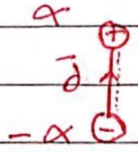
c) Volume charge dist (ρ_v) → (c/m³)

(3D object)



$$Q = \int_{v} \rho v \, dv$$

3) Electric Dipol

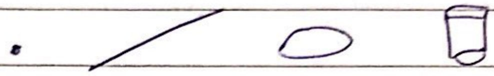


4) Polarized Dielectric → CH5

*Major Laws :-

1) Coloumb's Law
↳ general

2) Gauss's law (special case)



• Coloumb's Law (by Exp)

$F_e \equiv F \equiv$ electrical force (N)

$$F \propto \frac{Q_1 Q_2}{r^2}$$ relation

$$F = \frac{K Q_1 Q_2}{r^2}$$, K = proporitionality constant

$$K = \frac{1}{4\pi\epsilon_o}$$ → CH4 → free space

→ unit used $\frac{1}{4\pi}$

→ media ϵ

ϵ = permittivity (F/m), ϵ_o = free space permittivity

$$\epsilon_o = \frac{10^{-9}}{36\pi} \text{ F/m} = 8.85 \times 10^{-12} \text{ F/m}$$

Five Apple

$$F = \frac{Q_1 Q_2}{4 \pi \epsilon_0 r^2} \quad (N) \longrightarrow \text{magnitude only}$$

* Force as avector :-

- the force on (Q_2) due to (Q_1)

① $$\bar{F}_{12} = \frac{Q_1 Q_2}{4 \pi \epsilon_0 r_{12}^2} \hat{a}_{r_{12}} \longrightarrow \hat{a}_{r_{12}} = \frac{\bar{r}_{12}}{r_{12}}$$

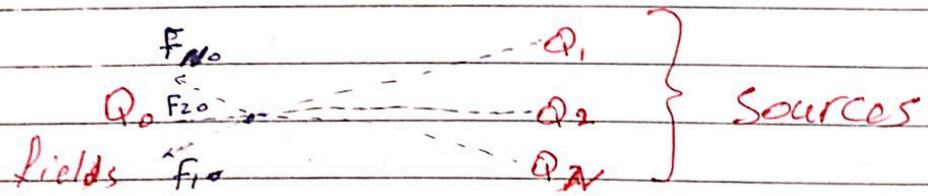
② $$\bar{F}_{12} = \frac{Q_1 Q_2 \bar{r}_{12}}{4 \pi \epsilon_0 (r_{12})^3}$$

vector

③ $$\bar{F}_{12} = \frac{Q_1 Q_2 (\bar{r}_2 - \bar{r}_1)}{4 \pi \epsilon_0 |\bar{r}_2 - \bar{r}_1|^3} \quad (N)$$ الصورة النهائية للقانون

distance = field - source

- the Force due to N-Point charges

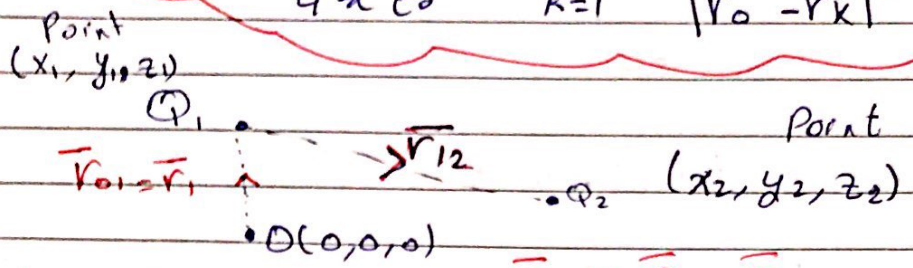


the force on Q_0

$$\bar{F} = \frac{Q_1 Q_0 (\bar{r}_0 - \bar{r}_1)}{4 \pi \epsilon_0 |\bar{r}_0 - \bar{r}_1|^3} + \frac{Q_2 Q_0 (\bar{r}_2 - \bar{r}_0)}{4 \pi \epsilon_0 |\bar{r}_2 - \bar{r}_0|^3} + \cdots + \frac{Q_0 Q_N (\bar{r}_0 - \bar{r}_N)}{4 \pi \epsilon_0 |\bar{r}_0 - \bar{r}_N|^3}$$

Five Apple

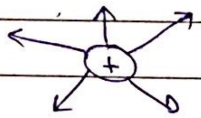
$$\boxed{\bar{F} = \frac{Q_0}{4\pi\epsilon_0} \sum_{k=1}^{N} \frac{Q_K(\bar{r}_0 - \bar{r}_K)}{|\bar{r}_0 - \bar{r}_K|^3} \quad (N)}$$



$$\bar{r}_{12} = \bar{r}_1 - \bar{r}_2$$

* Electric feild intensity :- $(\bar{E})$

$$\bar{E} = \frac{\bar{F}}{Q_{\text{feild point}}} \quad \text{in } (N/C) \text{ or } (V/m)$$ "from coulomb law"



① $\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$ (Q → source)

② $\bar{E} = \frac{Q\bar{r}}{4\pi\epsilon_0 r^3}$ V/m

③ $\bar{E} = \frac{Q(\bar{r} - \bar{r}')}{4\pi\epsilon_0 |\bar{r} - \bar{r}'|^3}$

[illegible]

convention: use dashs with the sources and without dashes with feilds

$$\bar{F}_{21} = -\bar{F}_{12}$$

$\overrightarrow{\hat{a}_{r12}}$ $\overleftarrow{\hat{a}_{r21}}$

Ex: Tow point charges 1mc and -3nc are located $(3,2,-1)$ and $(-1,-1,4)$. Find $\bar{F}$ and $\bar{E}$ at a 10mc charge located $(0,3,1)$.

S_1 S_2

feild

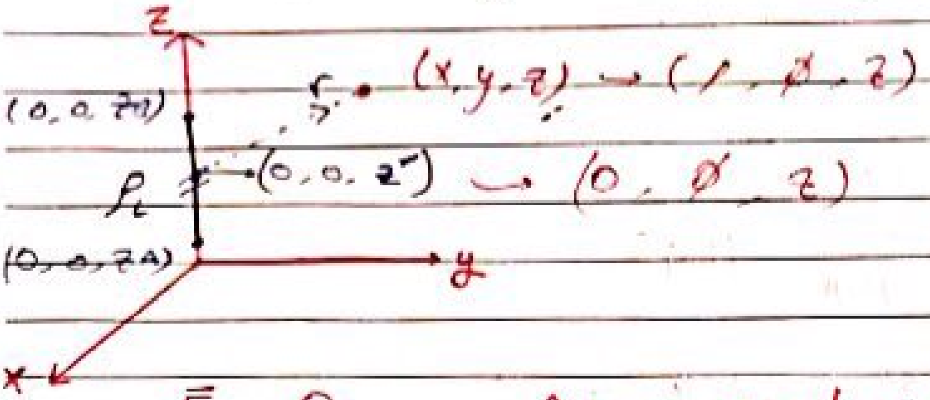
$$\vec{F} = \frac{1\times10^{-3} \times 10\times10^{-9} \; (-3,1,2)}{4\pi \times \frac{10^{-9}}{36\pi} \times (14)^{\frac{3}{2}}} + \frac{-3\times10^{-6}\times10\times10^{-9} \; (1,4,-3)}{4\pi\varepsilon \times \frac{10^{-9}}{36\pi} \; (26)^{\frac{3}{2}}}$$

$$\bar{F} = -6.507\,\hat{a}_x - 3.817\,\hat{a}_y + 7.506\,\hat{a}_z \quad mN$$

$$\bar{E} = \frac{\bar{F}}{10\times10^{-9}} \longrightarrow \bar{E} = -650.7\,\hat{a}_x - 381.7\,\hat{a}_y + 750.6\,\hat{a}_z \quad KV/m$$

* $\bar{E}$ - feild due to line charge :-

Ex: consider a finite line along z axis carry charge ρ_L C/m Find $\bar{E}$ at (x, y, z).



$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r \longrightarrow \text{point charge}$$

$$Q = \int_L \rho_L \, dL \longrightarrow \bar{E} = \int_L \frac{\rho_L \, dL}{4\pi\epsilon_0 r^3}\,\bar{r}$$

scaler

1 طريقة

$dL = dz'$ $\qquad \bar{r} = x\hat{a}_x + y\hat{a}_y + (z - z')\hat{a}_z$

$$r = \sqrt{x^2 + y^2 + (z - z')^2}$$

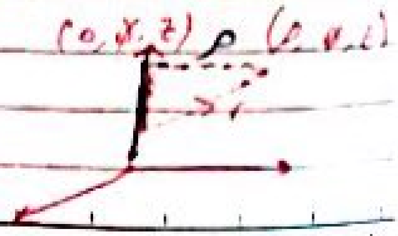
$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0} \int_{z_A}^{z_B} \frac{(x, y, (z - z'))}{[x^2 + y^2 + (z - z')]^{\frac{3}{2}}}\, dz'$$

Table of integral

2 طريقة convert to cylindrical:

$dL = dz'$ $\qquad \bar{r} = \rho\,\hat{a}_\rho + (z - z')\,\hat{a}_z$

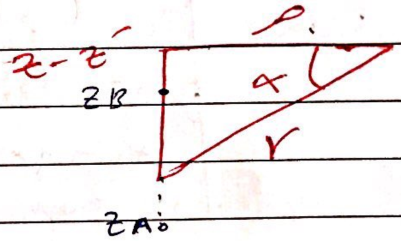
$r = \sqrt{\rho^2 + (z - z')^2}$



$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0} \int_{z_A}^{z_B} \frac{\rho\, \hat{a}_\rho + (z - z')\, \hat{a}_z}{[\rho^2 + (z - z')^2]^{3/2}}\, dz'$$

* introduce angles α, α_1, α_2 and replace dz' by $d\alpha$

$$\sin\alpha = \frac{z - z'}{r} \longrightarrow \checkmark$$



$$\cos\alpha = \frac{\rho}{r}$$

$$\tan\alpha = \frac{z - z'}{\rho}$$

$$z - z' = r \sin\alpha \qquad (-1)\, dz' = (\rho \sec^2\alpha)\, d\alpha$$

$$z - z' = \rho \tan\alpha$$

$$dz' = -\rho \sec^2\alpha\, d\alpha$$

$$r^2 = \rho^2 + (z - z')^2 \longrightarrow \rho^2 + \rho^2 \tan^2\alpha$$

$$= \rho^2 (1 + \tan^2\alpha) = \rho^2 \sec^2\alpha$$

$$r = \rho \sec\alpha \qquad r^3 = \rho^3 \sec^3\alpha$$

$$z - z' = r \sin\alpha = \rho \sec\alpha \sin\alpha$$

$$\rho = r \cos\alpha = \rho \sec\alpha \cos\alpha$$

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0} \int_{\alpha_1}^{\alpha_2} \frac{\cancel{\rho} \sec\alpha (\cos\alpha\, \hat{a}_\rho + \sin\alpha\, \hat{a}_z)}{\rho\, \cancel{\rho^3}\, \cancel{\sec^3}\alpha} - \cancel{\rho} \sec^2\alpha\, d\alpha$$

$$\bar{E} = \frac{-\rho_l}{4\pi\epsilon_0 \rho}\left[(\sin\alpha_2 - \sin\alpha_1)\,\hat{a}_\rho - (\cos\alpha_2 - \cos\alpha_1)\,\hat{a}_z\right] \quad V/m$$

for finit line along z-axis

• for inf. line

$\alpha_2 = 270^\circ = -90$

$\alpha_1 = 90^\circ$

$$\bar{E} = \frac{\rho_l}{2\pi\epsilon_0 \rho}\,\hat{a}_\rho$$

c/m

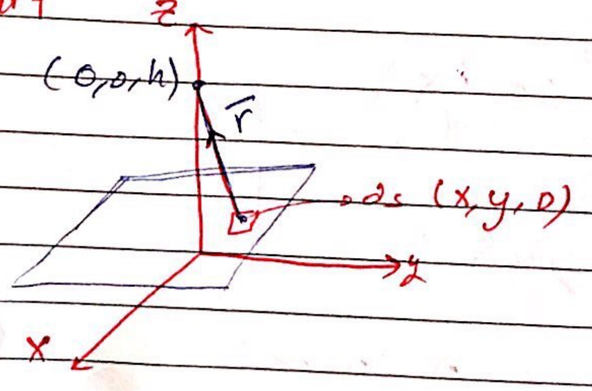
always for any line

ρ: is the shortest distance between the source and the feild

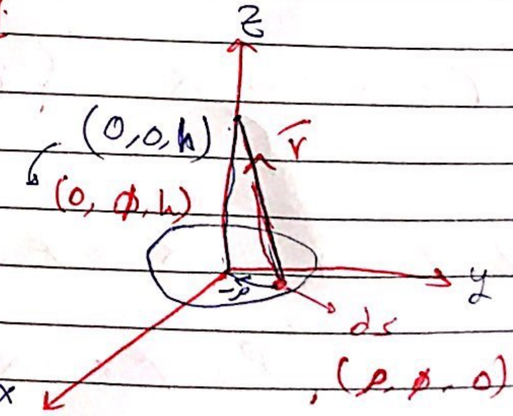
* $\bar{E}$. Feild due to a surface charge

ex: consider a sheet on $z=0$ carry charge ρ_s c/m^2. find $\bar{E}$ at $(0, 0, h)$.

cart



cyl



$ds = dx\,dy$

$\bar{r} = -x\hat{a}_x - y\,\hat{a}_y + h\hat{a}_z$

$r = \sqrt{x^2 + y^2 + h^2}$

$ds = \rho\,d\rho\,d\phi$

$\bar{r} = -\rho\,\hat{a}_\rho + h\,\hat{a}_z$

$r = \sqrt{\rho^2 + h^2}$

→ for Poin charg only

$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r \quad , \quad Q = \int_s \rho_s\, ds$

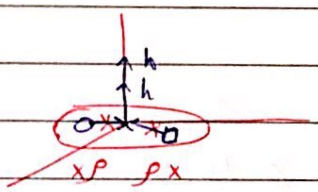
scaler

$$\bar{E} = \int_s \frac{\rho_s\, ds}{4\pi\epsilon_0 r^2}\,\hat{a}_r \longrightarrow \bar{E} = \int_s \frac{\rho_s\, ds\, \bar{r}}{4\pi\epsilon_0 r^3}$$

$$\bar{E} = \frac{\rho_s}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{\infty} \frac{-\rho\,\hat{a}_\rho + h\hat{a}_z}{[\rho^2+h^2]^{\frac{3}{2}}}\,\rho\, d\rho\, d\phi$$

• Due to symmetry, the ρ-component will be cancelled

طلعنا $\hat{a}_\rho$



$$\bar{E} = \frac{\rho_s h}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{\infty} \frac{\rho}{[\rho^2+h^2]^{\frac{3}{2}}}\, d\rho\, d\phi\ \hat{a}_z$$

$$\bar{E} = \frac{\rho_s h}{2\epsilon_0}\int_0^{\infty} \frac{\rho}{[\rho^2+h^2]^{\frac{3}{2}}}\, d\rho\ \hat{a}_z$$ كاملة بالنسبة لـϕ أول

let $u = \rho^2 + h^2 \longrightarrow du = 2\rho\, d\rho$

$$\bar{E} = \frac{\rho_s h}{2\epsilon_0 (2)}\int \frac{du}{u^{\frac{3}{2}}} \longrightarrow \bar{E} = \frac{\rho_s h}{4\epsilon_0(-\frac{1}{2})}\frac{1}{\sqrt{\rho^2+h^2}}\Bigg|_0^{\infty}$$

→

Five Apple

$$\bar{E} = \frac{-\rho_s h}{2\epsilon_0}\left(0-\frac{1}{h}\right)\hat{a}_z \longrightarrow \bar{E} = \frac{\rho_s}{2\epsilon_0}\hat{a}_z \ \text{V/m}$$

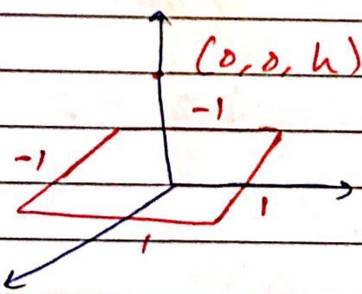
Note

if $(0,0,-h)$ $\rightarrow$ $\bar{E} = \frac{\rho_s}{2\epsilon_0}(-\hat{a}_z)$ in cart

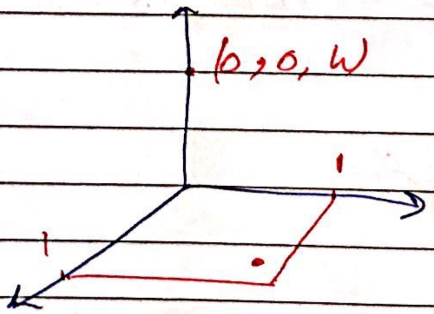
$(0, \phi, -h)$ $\rightarrow$ $\bar{r} = -\rho\hat{a}_\rho - h\hat{a}_z$ in cyl

in general $\bar{E} = \frac{\rho_s}{2\epsilon_0}\hat{a}_n$ $\rightarrow$ always

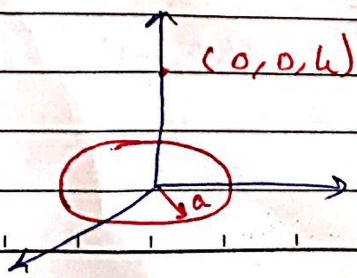
Ex. for infite surface



cart
sym → x, y

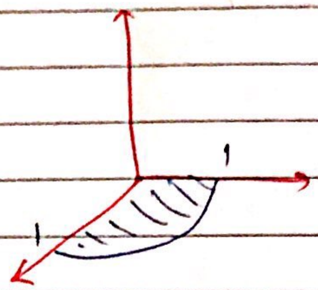


cart
No sym



cyl
sym → ρ
$\hat{a}_z$

Ex.



Cyl No Sym

disk $ds = \rho\, d\rho\, d\phi$

ring $dL = \rho\, d\phi$

* $\bar{E}$. feild for a volume:
charge disk.

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r \longrightarrow \text{Point}$$

$$Q = \int_v \rho_v \, dv \qquad \bar{E} = \int_v \frac{\rho_v \, dv}{4\pi\epsilon_0 r^2}\hat{a}_r$$

So charge disk $\longrightarrow$ Gauss's law only

* Electric feild flux density ($\bar{D}$)

$$\bar{D} = \epsilon_0 \bar{E}$$

F U = c $\bar{F} \downarrow \bar{E} \downarrow \bar{D}$

. for point charge

$$\bar{D} = \frac{Q}{4\pi r^2}\hat{a}_r$$

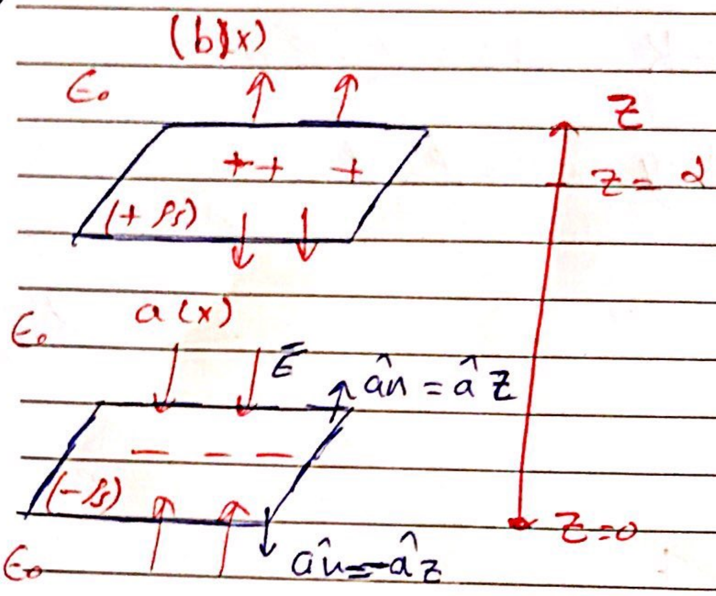
. for inf line

$$\bar{D} = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho$$

for inf sheet

$$\bar{D} = \frac{\rho_s}{2}\hat{a}_n$$

Ex. for a parallel plate capacitor



(c) x

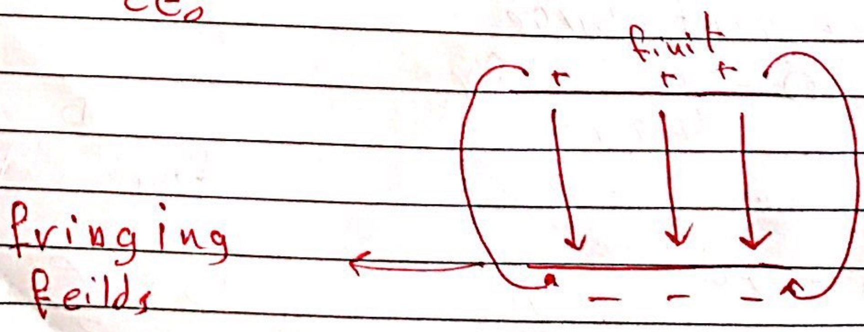
$\bar{E}$ at point (a)

$$E = \bar{E}_+ + \bar{E}_-$$

$$= \frac{\rho_s}{2\epsilon_0}(-\hat{a}_z) + \frac{-\rho_s}{2\epsilon_0}(\hat{a}_z) = \frac{\rho_s}{\epsilon_0}(-\hat{a}_z)$$

$$\bar{E}_b = \bar{E}_+ + \bar{E}_-$$

$$= \frac{\rho_s}{2\epsilon_0}\hat{a}_z + \frac{-\rho_s}{2\epsilon_0}\hat{a}_z = 0$$



feild

Ex Find $\bar{D}$ at $(4,0,3)$ if there is a point charge -5π mc at $(4,0,0)$ and a line charge 3π m/c along y-axis

$$\bar{D} = \bar{D}_Q + \bar{D}_L = \frac{Q\bar{r}}{4\pi r^3} + \frac{\rho_L \, \bar{\rho}}{2\pi\rho^2}$$

$\bar{r} = 3\hat{a}_z$, $r = 3m$

$\bar{\rho} = 4\hat{a}_x + 3\hat{a}_z$, $\rho = 5m$

feild - source

$(0,4,0)$

$(0,0,0)$

نقطة عشوائية

$\bar{D} = 240\hat{a}_x + 42\hat{a}_z \ \mu c/m^2$ final answer

Note: capacitor

at point (a) = $\bar{E} = \frac{\rho_s}{\epsilon_0}(-a_z)$

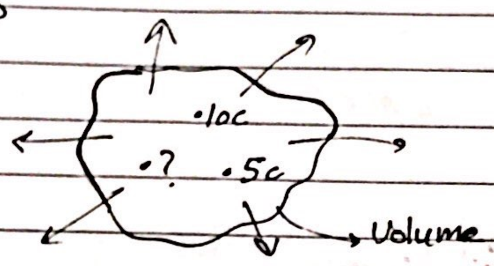
$$\bar{E} = \frac{\rho_s}{\epsilon_0}\hat{a}_n$$

$$\bar{D} = \rho_s \ \hat{a}_n$$

$\boxed{\bar{D}\cdot\hat{a}_n = \rho_s}$ always

*Gauss's law:-

$$\oint_S \bar{D} \cdot \bar{ds} = Q_{enclosed}$$

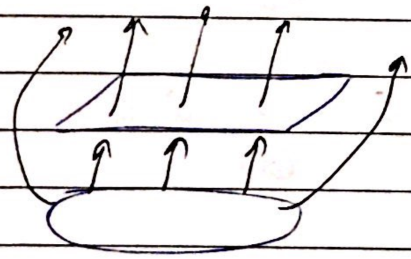


$\equiv 7c \rightarrow Q_{enc}$

$$\oint_S \bar{A} \cdot \bar{ds} = \int_V \nabla \cdot \bar{A} \, dv$$

$\Psi_e \equiv$ Electric flux   Scaler in (c)

$\Psi_e = \int_S \bar{D} \cdot \bar{ds} \longrightarrow \Psi_e = \oint_S \bar{D} \cdot \bar{ds} \rightarrow$ Gauss's



• Application of Gauss's law

$\oint_S \bar{D} \cdot \bar{ds} = Q_{enc} = Q \longrightarrow$ point charge

$\int_L \rho_L \, dL \rightarrow$ Line "

$\int_S \rho_s \, ds \longrightarrow$ surface "

$\int_v \rho_v \, dv \longrightarrow$ Volume charge

gauss's law :

$$\boxed{\oint_s \bar{D} \cdot \bar{ds} = \int_v \rho_v \, dv}$$ → 1st maxwell's equ in integral form

- Apply Divergence theroem

$$\oint_s \bar{D} \cdot \bar{ds} = \int_v \nabla \cdot \bar{D} \, \cancel{dv} = \int_v \rho_v \, \cancel{dv}$$

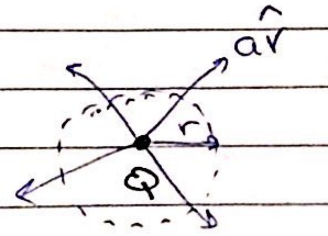
$$\boxed{\nabla \cdot \bar{D} = \rho_v}$$ → 1st maxwell's equ Differential form

$\rho_s = \bar{D} \cdot \hat{a_n}$, $\bar{D} = \rho_s \hat{a_n}$

$\rho_v = \nabla \cdot \bar{D}$ $\quad \bar{D} = \epsilon_0 \bar{E}$ $\quad D <<< E$

1) $\bar{E}$ or $\bar{D}$ for apoint charge

$$\oint_s \bar{D} \cdot \bar{ds} = Q_{enc} = Q$$



* $\bar{D} = D_r \hat{a_r}$

* Gauss's ian surface

$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$

$$\int_0^{2\pi}\int_0^{\pi} D_r\, r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r = Q$$

$$\int_0^{\pi} \sin\theta\, d\theta = -\cos\theta \Big|_0^{\pi}$$

$$4\pi r^2 D_r = Q \qquad D_r = \frac{Q}{4\pi r^2}$$

$$D = \frac{Q}{4\pi r^2}\hat{a}_r$$

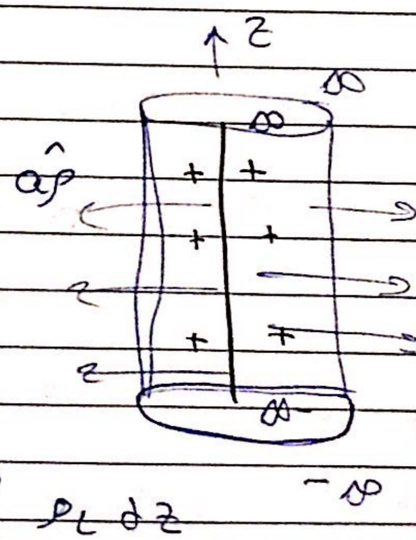
$$\bar{E} = \frac{\bar{D}}{\epsilon_0} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r$$ → same coulmb law

2) $\bar{E}$ or D for an infinit line

$$\oint_s \bar{D}\cdot \bar{ds} = \int_L \rho_L\, dL$$

$\bar{D} = D_\rho\, \hat{a}_\rho$ $\qquad \boxed{dL = dz}$

$\bar{ds} = \rho\, d\phi\, dz\, \hat{a}_\rho$



$$\int_{-L/2}^{L/2}\int_0^{2\pi} D_\rho\, \hat{a}_\rho\cdot \rho\, d\phi\, dz\, \hat{a}_\rho = \int_{-L/2}^{L/2} \rho_L\, dz$$

$l \to \infty$

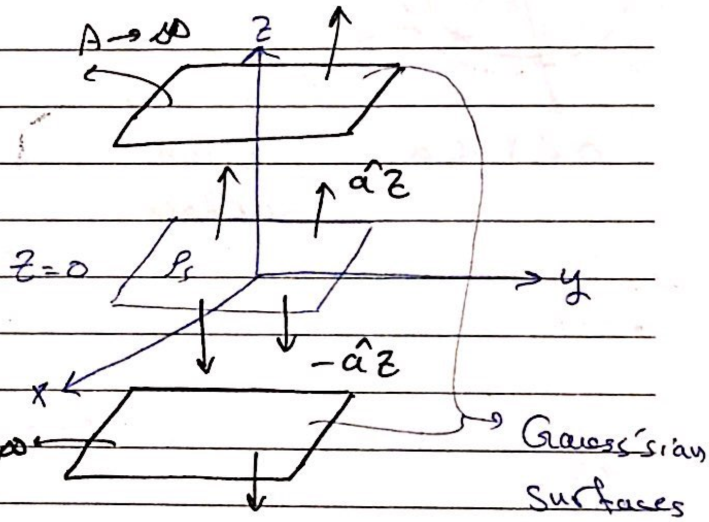
$$2\pi\rho L\, D_\rho = \rho_L L$$

$$D_\rho = \frac{\rho_L}{2\pi\rho} \rightarrow \bar{D} = \frac{\rho_L}{2\pi\rho}\,\hat{a}_\rho \rightarrow \bar{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\,\hat{a}_\rho$$

3) $\bar{E}$ or $\bar{D}$ for an infinit sheet

$$\oint_s \bar{D}\cdot \bar{ds} = Q_{enc} = \int_s \rho_s\, ds$$

$$\bar{D} = \begin{cases} D_z\,\hat{a}_z, & z>0 \\ D_z\,(-\hat{a}_z), & z<0 \end{cases}$$



$$\bar{ds}_{top} = dx\,dy\,\hat{a}_z$$

$$\bar{ds}_{bot} = -dx\,dy\,\hat{a}_z$$

$$\int_{s_{top}} \bar{D}\cdot\bar{ds}_{top} + \int_{s_{bot}} \bar{D}\cdot\bar{ds}_{bot} = \int_s \rho_s\, ds$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_z\, dx\, dy + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_z\, dx\, dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho_s\, dx\, dy$$

$$D_z A + D_z A = \rho_s A$$

$$D_z = \frac{\rho_s}{2} \rightarrow \boxed{\bar{D} = \frac{\rho_s}{2}\,\hat{a}_n} \begin{cases} \hat{a}_z & (z>0) \\ -\hat{a}_z & (z<0) \end{cases}$$

$$E = \frac{\rho_s}{2\epsilon_0}\,\hat{a}_n \begin{cases} \hat{a}_z \\ -\hat{a}_z \end{cases}$$

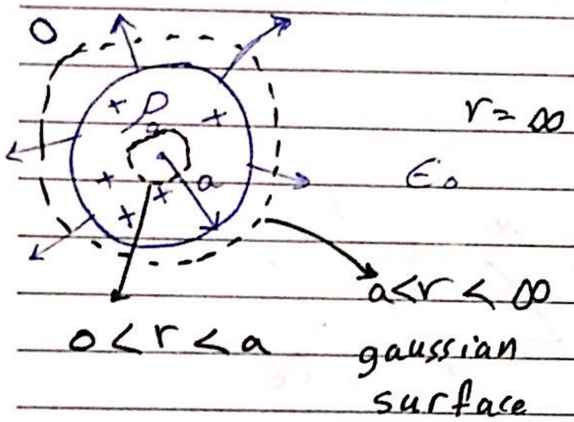
* أيضا أنه كولوم نفس غاوس للإنفنت

4) $\bar{E}$ or $\bar{D}$ for volume charge dist

Ex: consider a sphere of radius (a) has a

$$\rho_v = \begin{cases} \rho_0 , & r < a \\ 0 , & r > a \end{cases} \quad C/m^3 \text{, Find } \bar{E} \text{ and } \bar{D}$$

every where



• For $r < a$:

$$\oint_s \bar{D} \cdot \bar{ds} = Q_{enc} = \int \rho_v \, dv$$

$$\bar{D} = D_r \hat{a}_r \qquad \bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\int_0^{2\pi} \int_0^{\pi} D_r \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r = \int_0^{2\pi} \int_0^{\pi} \int_0^{r} \rho_0 r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\cancel{4\pi} \cancel{r^2} D_r = \rho_0 \frac{\cancel{4\pi} r^{\cancel{3}}}{3}$$

$$\bar{D} = \frac{\rho_0 r}{3} \hat{a}_r \qquad \bar{E} = \frac{\rho_0 r}{3 \epsilon_0} \hat{a}_r \qquad 0 < r < a$$

• for $\infty > r > a$

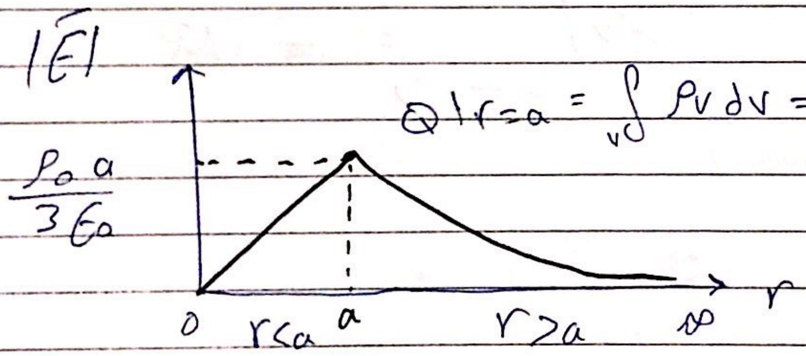
$$4\pi r^2 D_r = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho_o \sin\theta \, dr \, d\theta \, d\phi$$

$$+ \int\int\int_a^r \cancel{0} \dots$$

$$4\pi r^2 D_r = \rho_o \, 4\pi \frac{a^3}{3}$$

$$\bar{D} = \frac{\rho_o a^3}{3r^2} \hat{a_r} \qquad \bar{E} = \frac{\rho_o a^3}{3\epsilon_o r^2} \hat{a_r}$$

$$a \le r < \infty$$



$$Q|_{r=a} = \int_v \rho_v \, dv = \frac{\rho_o 4\pi a^3}{3} \quad C$$

إذا مش صفر

$$4\pi r^2 D_r = Q = \rho_o \frac{4\pi a^3}{3} + \int\int\int_a^r \rho_v \, dv$$

$$\bar{D} = \frac{\rho_o a^3}{3r^2} \hat{a_r}$$

Ex: Given $\bar{D} = z\rho\cos^2\phi\,\hat{a}_z$ C/m²

a) calculate the charge density at $(1, \frac{\pi}{4}, 3)$

ρ_L, ρ_s, ρ_v

b) the total charge enclosed by the cylinder of radius 1 m with $-2 \le z \le 2$

$\rho_s = \bar{D} \cdot \hat{a}_n \longrightarrow$ what is the surface

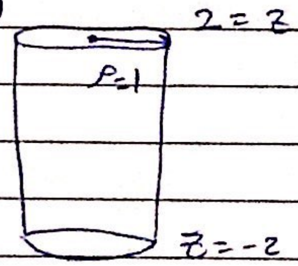
$\rho_v = \nabla \cdot \bar{D}$

$$\rho_v = \nabla\cdot\bar{D} = \underbrace{\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho)}_{\times} + \underbrace{\frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi}}_{\times} + \underbrace{\frac{\partial D_z}{\partial z}}_{\checkmark}$$

$$= \frac{\partial D_z}{\partial z}$$

$$\rho_v = \rho\cos^2\phi \qquad \rho_v\Big|_{(1,\frac{\pi}{4},3)} = \frac{1}{2}\ \text{C/m}^3$$

b)



$$Q = \int_v \rho_v\, dv$$

$$= \int_{-2}^{2}\int_{0}^{2\pi}\int_{0}^{1} \rho\cos^2\phi\,\rho\,d\rho\,d\phi\,dz$$

$$Q = \frac{1}{3}\int_0^{2\pi}\frac{1}{2}(1+\cos 2\phi)\,d\phi \cdot 4$$

$$= \frac{4}{3}\left(\pi + \frac{\sin 2\phi}{2}\Big|_0^{2\pi}\right) = \frac{4\pi}{3}\ \text{C}$$

Method (2) : Gauss

$$Q_{enc} = \oint_S \bar{D} \cdot \bar{ds} = Q = \int \rho_L \, dL$$

$$\rightarrow = \int \rho_v \, dv$$

$$Q = \int_{s_{top}} \bar{D} \cdot \bar{ds}_{top} + \int_{s_{bot}} \bar{D} \cdot \bar{ds}_{bot} + \cancel{\int_{side} \bar{D} \cdot \bar{ds}_{side}}$$

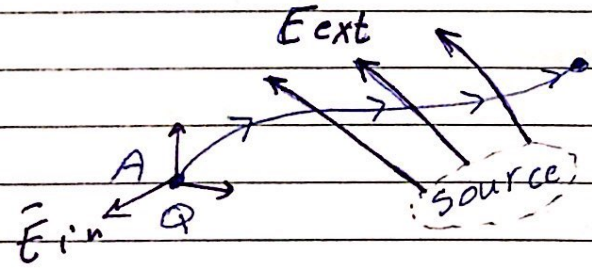
$\hat{a}_z$, $-\hat{a}_z$, $\rightarrow \hat{a}_\rho$

$$Q = \int_0^{2\pi}\int_0^1 z \rho \cos^2\phi \, \rho \, d\rho \, d\phi \Big|_{z=2} - \int_0^{2\pi}\int_0^1 z \rho \cos^2\phi \, \rho \, d\rho \, d\phi \Big|_{z=-2}$$

$$Q = \frac{2\pi}{3} + \frac{2\pi}{3} = \frac{4\pi}{3} \; C$$

* Electric potential:

Scaler quantity $\longrightarrow$ in (V) volt



work (scaler) $\rightarrow$ $w = \bar{F} \cdot \bar{L}$ (N.m) or (J) Joule

$w = Q \bar{E}_{ext} \cdot \bar{L}$, $\bar{F} = Q\bar{E}$ $\longrightarrow$ $w = - Q \bar{E} \cdot \bar{L}$ $\rightarrow$ work done by $\bar{E}_{ext}$

$w = -Q \bar{E} \cdot \bar{L}$ $\rightarrow$ $w = -\int_L Q \, \bar{E} \cdot d\bar{L}$ $\rightarrow$ general $\longrightarrow$

for uniform length

$V_{AB} = \frac{w}{Q} = -\int_L \bar{E} \cdot \bar{dL}$, $\frac{1J}{1C} = 1V$

coulmb's Gauss's

$V_{AB} = V_B - V_A$

- based on $V_{AB} = -\int_L \bar{E} \cdot \bar{dL}$

- if the voltage is $-ve$:

-drop in potential (higher pot → lower pot)

- the work is done by the feild it self.

- if the voltage is $+ve$

- gain in potintial (lower pot → higher pot)

- the work is done by the ext feild.

- for a point charge :-

$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$ → coulmbs → Gauss's

$V = -\int_L \bar{E} \cdot \bar{dL} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon_0 r^2} \hat{a_r}\, dr\, \hat{a_r}$

$= \frac{+Q}{4\pi\epsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) V = \frac{Q}{4\pi\epsilon_0 r_B} - \frac{Q}{4\pi\epsilon_0 r_A} = V_B - V_A$

- if point A is at $\infty \rightarrow r_A \simeq \infty$

$V_{AB} = V_B - V_A = V_B - 0 = V_B$

$$= \frac{Q}{4\pi\epsilon_0 r_B} - \cancel{\frac{Q}{4\pi\epsilon_0 r_A}}^{\,0} \longrightarrow V_B = \frac{Q}{4\pi\epsilon_0 r_B}$$ if ref is at ∞

$V_\infty = 0 \rightarrow$ ref

$$V = \frac{Q}{4\pi\epsilon_0 r} \quad \text{(if ref is at } \infty)$$

- for N-point charges

$$V = \sum_{k=1}^{N} \frac{Q_k}{4\pi\epsilon_0 |r - r'_k|} \quad \text{if } V_\infty = 0$$

$r' \rightarrow$ source

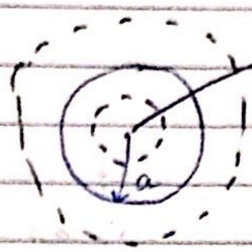
$$V = \int_L \frac{\rho_L \, dL}{4\pi\epsilon_0 r} \longrightarrow \text{Line charge}$$

$$V = \int_S \frac{\rho_s \, ds}{4\pi\epsilon_0 r} \longrightarrow \text{surface charge}$$

$$V = \int_v \frac{\rho_v \, dv}{4\pi\epsilon_0 r} \longrightarrow \text{Volume charge}$$

$$V(\bar{r}) = \int_{v'} \frac{\rho_v(\bar{r}')\, dv'}{4\pi\epsilon_0 |r - r'|}$$

from past example



$$\rho_v = \begin{cases} \rho_0, & \cancel{r \le a}\ r < a \\ 0, & r > a \end{cases}$$

$$\bar{E} = \begin{cases} \dfrac{\rho_0 r}{3\epsilon_0}\hat{a}_r, & 0 < r \le a \\[2ex] \dfrac{\rho_0 a^3}{3\epsilon_0 r^2}\hat{a}_r, & a \le r < \infty \end{cases}$$

+ find V everywhere:

for $r > a$

$$V = -\int_L \bar{E}\cdot\bar{dl} \longrightarrow V = -\int_{\infty}^{r} \frac{\rho_0 a^3}{3\epsilon_0 r^2}\,\hat{a}_r\cdot dr\,\hat{a}_r$$

$$= +\frac{\rho_0 a^3}{3\epsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{\rho_0 a^3}{3\epsilon_0 r}, \quad a \le r < \infty$$

for $r < a$

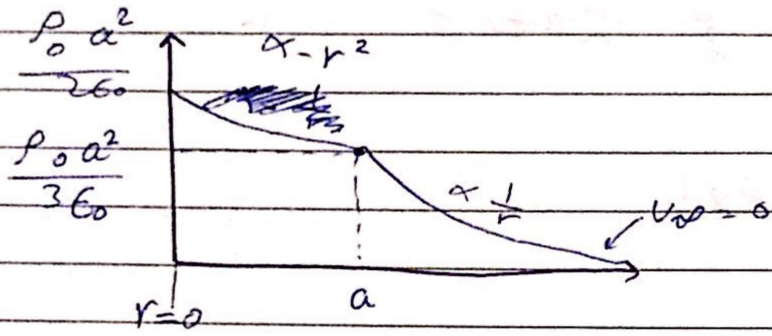
$$V = -\int_L \bar{E}\cdot\bar{dl} \longrightarrow -\int_{\infty}^{r} \bar{E}\cdot\bar{dl}$$

$$= -\int_{\infty}^{a} \bar{E}\cdot\bar{dl} - \int_{a}^{r} \bar{E}\cdot\bar{dl}$$

$$= \frac{\rho a^2}{3\epsilon_0} - \int_{a}^{r} \frac{\rho_0 r}{3\epsilon_0}\,\hat{a}_r\cdot dr\,\hat{a}_r = \frac{\rho_0 a^2}{3\epsilon_0} - \frac{\rho_0 (r^2 - a^2)}{6\epsilon_0}\ V$$

$$0 < r \le a$$

$$V(r=0) = \frac{\rho_0 a^2}{3\epsilon_0} + \frac{\rho_0 a^2}{6\epsilon_0} = \frac{\rho_0 r^2}{2\epsilon_0}$$



Ex. A point charge of 5 nc located at $(-3, 4, 0)$ and line $y = 1$, $z = 1$ carries $\rho_L = 2$ nc/m $\rightarrow (x, 1, 1)$

سؤال كبير بس موضع

a) ....   b) if $V = 100$ V at $B(1, 2, 1)$ find V at $C(-2, 5, 3)$

c)...

ref is point B

V_C?

$$V_{BC} = V_C - V_B \Rightarrow V_C = V_{BC} + 100 \qquad (V_{BC} = V_{BCQ} + V_{BCL})$$

$$V_{BCQ} = -\int_L \vec{E} \cdot d\vec{L} = \int_{r_B}^{r_C} \frac{Q}{4\pi\epsilon_0 r^2} \cancel{\hat{a}_r} \cdot dr \cdot \hat{a}_r$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_C} - \frac{1}{r_B}\right)$$

$\longrightarrow$

$r_c = |(0,1,3)| = \sqrt{11}$, $r_B = |(4,1,3)| = \sqrt{26}$

$$V_{BC} = -\int_{C}^{\ } \bar{E} \cdot \bar{dL} = -\int_{\rho_B}^{\rho_c} \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho \, d\rho \, \hat{a}_\rho$$

$$= \frac{-\rho_L}{2\pi\epsilon_0} (\ln \rho_c - \ln \rho_B)$$

$$= \frac{\rho_L}{2\pi\epsilon_0} \ln\left(\frac{\rho_B}{\rho_c}\right)$$

$\rho_B = |(0,1,0)| = 1$

$\rho_c = |(0,4,2)| = \sqrt{20}$

$V_{BC} = -50.175$

$V_c = 49.825$ V

• How to find $\bar{E}$ from (V) :-

$$V = -\int_L \bar{E} \cdot \bar{dl}$$

Potential $dV = -\bar{E} \cdot \bar{dl}$

in cartisians:-

$$\bar{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z$$

$$\bar{dL} = dx \, \hat{a}_x + dy \, \hat{a}_y + dz \, \hat{a}_z$$

$$\bar{E} \cdot \bar{dl} = E_x dx + E_y dy + E_z \hat{a}_z$$

$$\overset{\text{Diff}}{dV} = \overset{\text{partial}}{\frac{\partial V}{\partial x}} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

by Equating similar terms:-

$$\frac{\partial V}{dx} \cancel{dx} = -E_x \cancel{dx} \longrightarrow \boxed{E_x = -\frac{\partial V}{\partial x}}$$

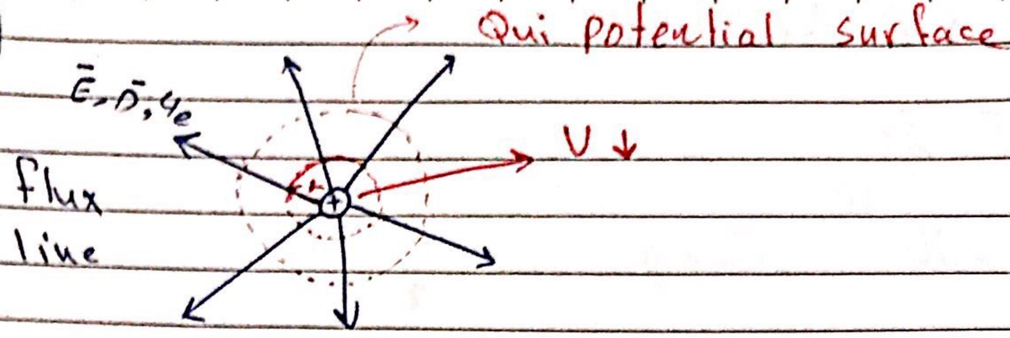
$$\boxed{E_y = \frac{-\partial V}{\partial y}} \qquad \boxed{E_z = -\frac{\partial V}{\partial z}}$$

$$\bar{E} = -\left(\frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z\right)$$

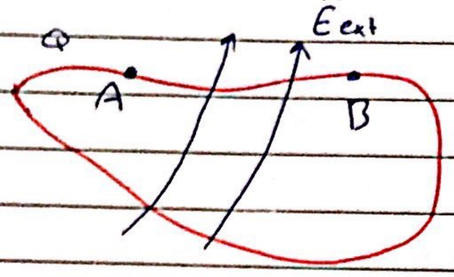
$$\boxed{\bar{E} = -\nabla V} \equiv \frac{V}{m}$$

→ 3^{rd} way to find $\bar{E}$. feild (if (V) is given or easy to find)

Five Apple



* potential over a closed path:-



Q : A → B then B → A

$$V = -\int_{r_A}^{r_B} \bar{E} \cdot \bar{dl} + -\int_{r_B}^{r_A} \bar{E} \cdot \bar{dl} = 0$$

$V_{AA} = V_A - V_A = $ zero

$$\boxed{V = \oint_{l} \bar{E} \cdot \bar{dl} = 0}$$ always

→ 2nd maxwell equation in integral form.

* by applying stocke's theorem:-

$$\oint_{c} \bar{E} \cdot \bar{dl} = \int_{s} \nabla \times \bar{E} \cdot \bar{ds} = \int 0 \, ds$$

$$\boxed{\nabla \times \bar{E} = 0}$$ → 2nd maxwell's equation in Differential form

Ex: Given $V = \frac{10}{r^2} \sin\theta \cos\phi$

a) find $\bar{D}$ at $(2, \frac{\pi}{2}, 0)$

b) calculate the work done in moving $10\,\mu C$ charge from $A(1, 30°, 120°)$ to $B(4, 90°, 60°)$.

$\bar{E} = -\nabla V$, $\bar{D} = \epsilon_0 \bar{E}$

$\bar{D} = -\epsilon_0 \nabla V$

$$\bar{E} = -\left(\frac{\partial v}{\partial r}\hat{a}r + \frac{\partial v}{r\partial\theta}\hat{a}\theta + \frac{\partial v}{r\sin\theta\,\partial\phi}\hat{a}\phi \right)$$

$$\bar{E} = \left(\frac{+20}{r^3}\sin\theta\cos\phi\,\hat{a}r - \frac{10}{r^3}\cos\theta\cos\phi\,\hat{a}\theta + \frac{10}{r^3}\cos\phi\,\hat{a}\phi \right)$$

a) $\bar{D} = \epsilon_0 \bar{E}$

$$\bar{D}\Big|_{(2, \frac{\pi}{2}, 0)} = \epsilon_0 \left(\frac{20}{8}\hat{a}r \right) \, c/m^2$$

r θ ϕ

$$\bar{D} = 22.1\,\hat{a}r \; pc/m^2$$

b) method (I)

$W = Q\,V_{AB} \rightarrow Q(V_B - V_A)$

$$10\times10^{-6}\left(\frac{10}{16}\cdot\frac{1}{2} - \frac{10}{1}\cdot\frac{1}{2}\cdot\frac{-1}{2} \right) = 10\times10^{-6}\left(\frac{5}{16} + \frac{10}{4} \right)$$

$$= 10\times10^{-6}\underbrace{\left(\frac{45}{16} \right)}_{V_{AB}} = 28.125 \; \mu J$$

method (II)

$$W = -Q \int_L \bar{E} \cdot \bar{dl}$$

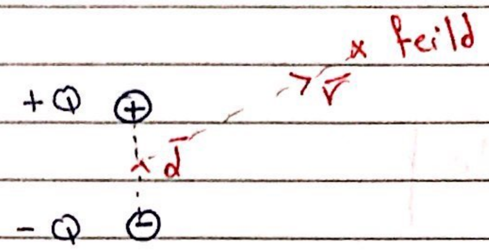
$$W = -10 \times 10^{-6} \int (E_r\, dr + E_\theta\, r\, d\theta + E_\phi\, r \sin\theta\, d\phi)$$

$$W = -10 \times 10^{-6} \left[\int_1^4 E_r\, dr \Big|_{\substack{\theta = 30 \\ \phi = 120}} + \int_{30^\circ}^{90^\circ} E_\theta\, r\, d\theta \Big|_{\substack{r=4 \\ \phi = 120^\circ}} + \int_{120^\circ}^{60^\circ} r \sin\theta\, E_\phi\, d\phi \Big|_{\substack{r=4 \\ \theta = 90}} \right]$$

$A(1, 30^\circ, 120^\circ$ $\quad\quad$ $B(4, 90^\circ, 60^\circ) \xrightarrow{} \phi$

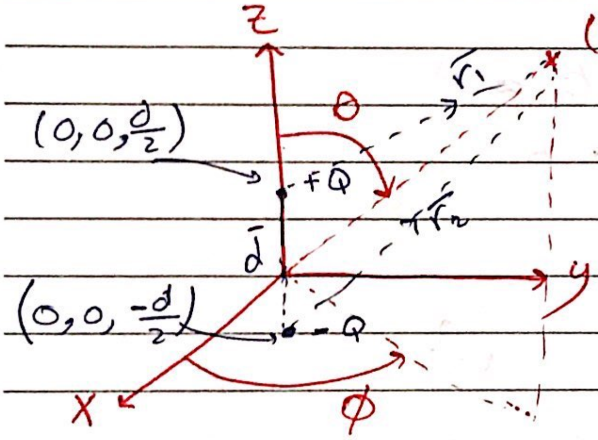
$\xrightarrow{r} (4, 30^\circ, 120^\circ) \xrightarrow{\theta} (4, 90^\circ, 120^\circ)$

* Electric dipoles :-



$d <<< r$

Find $\bar{E}, \bar{D}, V$?



$\bar{E}$, $\bar{D}$, V ?

$d <<< r$

$V = V_+ + V_-$ (ref is at ∞)

$$= \frac{Q}{4\pi\epsilon_0 r_1} + \frac{-Q}{4\pi\epsilon_0 r_2} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\boxed{1} \quad V = \frac{Q(r_2 - r_1)}{4\pi\epsilon_0 r_1 r_2}$$

Q?

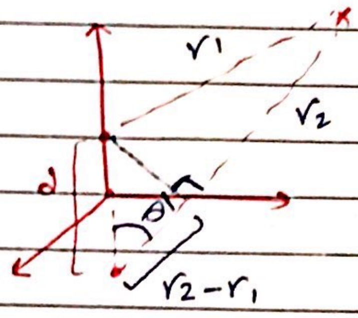
location $\rightarrow (0,0,\frac{d}{2})$
$\rightarrow (0,0,-\frac{d}{2})$?

*Approximations:- ①,②

① $r_1 r_2 \approx r^2$

$r = |\text{field} - \text{center of the dipole}|$

②



$$r_2 - r_1 = d\cos\theta$$

$$\theta' \approx \theta$$

$$\cos\theta = \frac{r_2 - r_1}{d}$$

$\boxed{2}$ $$V = \frac{Qd\cos\theta}{4\pi\epsilon_0 r^2} \quad (V)$$

Q? d?

define: dipole moment

$$P = Q\bar{d} \quad \text{in } (C.m)$$

$\boxed{3}$ $$V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2} \quad (V)$$ → P center of the dipole

θ is the angle between $(\bar{p})$ and $(\bar{r})$

$\boxed{3'}$ $$V = \frac{\bar{P}\cdot\hat{a}_r}{4\pi\epsilon_0 r^2}$$ → P(1) cos θ

$\boxed{3''}$ $$V = \frac{\bar{P}\cdot\bar{r}}{4\pi\epsilon_0 r^3}$$

$$\bar{E} = -\nabla V$$

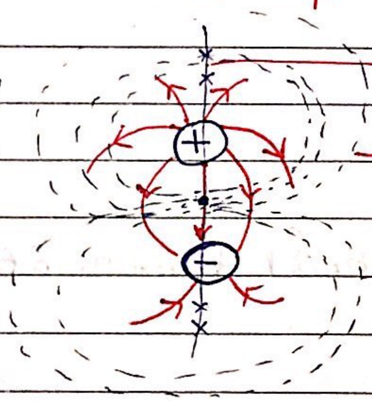
$$= \frac{-P}{4\pi\epsilon_0}\left(\frac{-2\cos\theta}{r^3}\,\hat{a}r - \frac{\sin\theta}{r^3}\,\hat{a}\theta\right.$$

$$\boxed{\bar{E} = \frac{P}{4\pi\epsilon_0 r^3}\left(2\cos\theta\ \hat{a}r + \sin\theta\ \hat{a}\theta\right)\ V/m}$$

* for N-dipoles

$$V = \frac{1}{4\pi\epsilon_0}\sum_{K=1}^{N}\frac{\bar{P}_K\cdot(\bar{r}-\bar{r}_K')}{|\bar{r}-\bar{r}_K'|^3}\quad (V)$$

flux line



logical of all equi potential surface radii

equi potential surface

Ex: tow dipoles with dipole moments $-5\,\hat{a}_z$ nC.m and $9\,\hat{a}_z$ nC.m are located at $(0,0,-2)$ and $(0,0,3)$ find the potential at the oregion.

ref is at ∞

$$V = \frac{\bar{P}_1 \cdot \bar{r}_1}{4\pi\epsilon_0 r_1^3} + \frac{\bar{P} \cdot \bar{r}_2}{4\pi\epsilon_0 r_2^3}$$

$\bar{r}_1 = 2\hat{a}_z \rightarrow r_1 = 2m$

$\bar{r}_2 = -3\hat{a}_z \rightarrow r_2 = 3m$

$$V = \frac{-5\hat{a}_z \times 10^{-9} \cdot 2\hat{a}_z}{\frac{4\pi 10^{-9}}{36\pi} \cdot 8} + \frac{9\hat{a}_z \times 10^{-9} \cdot -3\hat{a}_z}{\frac{4\pi\, 10^{-9}}{36\pi} \times 27}$$

$V = -20.25\,V$

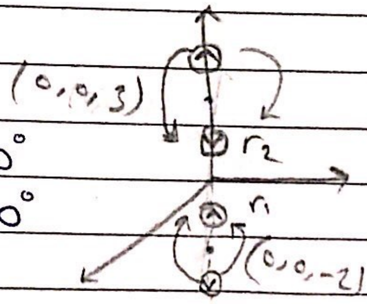
$\rightarrow$ find $\bar{E}$ at the origion

$$\bar{E}_1 = -\nabla V = \frac{P_1}{4\pi\epsilon_0 r_1^3} (2\cos\theta_1 \hat{a}_r + \cancel{\sin\theta_1 \hat{a}_\theta})$$

$\bar{E}_2 = \ldots\ldots$

$P_1 = -5 \times 10^{-9}$ C.m $\qquad \theta_1 = 180^\circ$

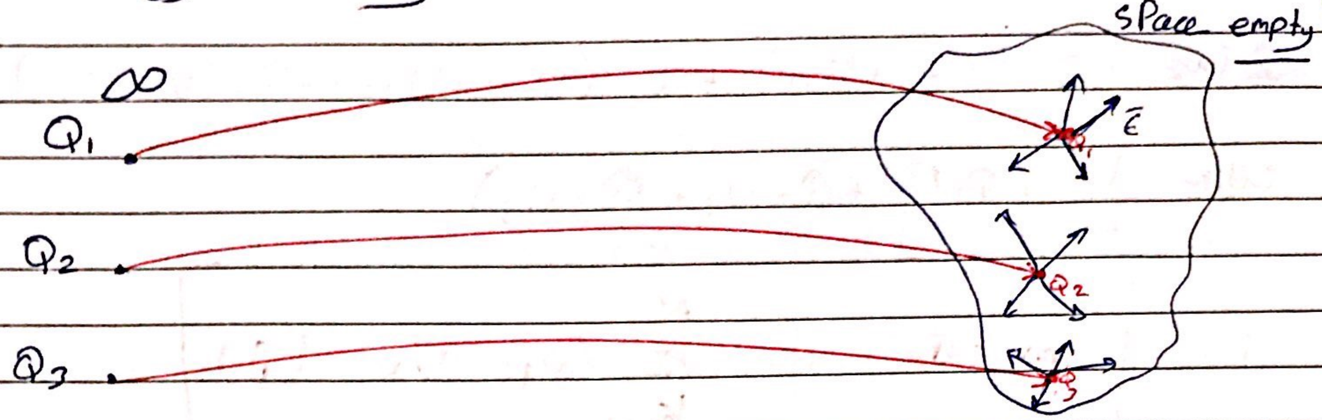
$P_2 = 9 \times 10^{-9}$ C.m $\qquad \theta_2 = 180^\circ$



$$\bar{E} = \frac{10^{-9}}{\frac{4\pi \times 10^{-9}}{36\pi}} \left(\frac{+5}{8}(-2)\hat{a}_r + \frac{9}{27}(-2)\hat{a}_r \right)$$

$\theta(\qquad)\,\hat{a}_r$ V/m

* Energy Density in electrostatic Fields :-



$Q_1 \longrightarrow Q_2 \longrightarrow Q_3$

$W_E \equiv$ electrical energy

$W_E = W_1 + W_2 + W_3$

$W_1 = Q_1 V_{\infty s} = 0$, $Q_1 , Q_2 , Q_3 \neq 0$

$V_\infty = 0$, $V_{\substack{\text{space}\\\text{empty}}} = 0$

$W_2 = Q_2 (V_{12})$, $W_3 = Q_3 (V_{13} + V_{23})$

$W_E = 0 + Q_2 V_{12} + Q_3 (V_{13} + V_{23})$ --- $\boxed{1}$

$Q_3 \longrightarrow Q_2 \longrightarrow Q_1$:

$W_E = W_1 + W_2 + W_3$

$= Q_1 (V_{21} + V_{31}) + Q_2 V_{32} + 0$ ---- $\boxed{2}$

$\boxed{1} + \boxed{2}$

$$2W_E = Q_1(V_{21} + V_{31}) + Q_2(V_{12} + V_{32}) + Q_3(V_{13} + V_{23})$$

$$W_E = \frac{1}{2}(Q_1 V_1 + Q_2 V_2 + Q_3 V_3)$$

$$\boxed{* \quad W_E = \frac{1}{2} \sum_{K=1}^{N} Q_K V_K \quad (J)}$$

For N-Point charges :-

$$W_E = \frac{1}{2} \int_L \rho_L V \, dL \longrightarrow \text{for line charge}$$

$$W_E = \frac{1}{2} \int_S \rho_S V \, ds \longrightarrow \text{for surface charge}$$

$$W_E = \frac{1}{2} \int_{\underset{\text{Volume}}{V'}} \rho_V' \underset{\text{Potential}}{V} \, dV' \longrightarrow \text{for volume charge}$$

* How to finde W_E from $\bar{E}$?

$$\bar{D} = \epsilon_o \bar{E}$$

$$\rho_V = \nabla \cdot \bar{D}$$

Five Apple

Scanned with CamScanner

$$W_E = \frac{1}{2}\int_V \rho_v V \, dv = \frac{1}{2}\int_V \nabla\cdot\bar{D} \; V \, dv$$

• Scaler Identity :- (given) (general)

$$\nabla\cdot V\bar{A} = \bar{A}\cdot\nabla V + V(\nabla\cdot\bar{A}) \quad , \bar{D} \equiv \bar{A}$$

$$W_E = \frac{1}{2}\left[\underbrace{\int_V \nabla\cdot(V\bar{D})\,dv}_{1} - \underbrace{\int_V \bar{D}\cdot\nabla V \, dv}_{2}\right]$$

Apply divergence theorem on integral 1 :

$$W_E = \frac{1}{2}\left[\oint_s V\bar{D}\cdot\bar{ds} - \int_V \bar{D}\cdot\nabla V \, dv\right]$$

$-\bar{E} = +\nabla V$

$$\left.\begin{aligned} V &\propto \frac{1}{r} \\ \bar{D} &\propto \frac{1}{r^2} \\ ds &\propto r^2 \end{aligned}\right]$$

$\frac{1}{r}$

as $(r \to \infty) \to \oint_s V\bar{D}\cdot\bar{ds} \simeq 0$

$$W_E = \frac{1}{2}\int_V \bar{D}\cdot\bar{E} \, dv$$

$\bar{D} = \epsilon_0 \bar{E}$

$$W_E = \frac{1}{2}\int_V \epsilon_0 E^2 \, dv \quad \text{or} \quad W_E = \frac{1}{2}\int_V \frac{D^2}{\epsilon_0} \, dv$$

Energy density :- (w_E)

$$w_E = \frac{W_E}{\text{Volume}} \quad \left(\frac{J}{m^3}\right)$$

$$w_E = \frac{1}{2}\,\bar{E}\cdot\bar{D} = \frac{1}{2}\,\epsilon_0 \bar{E}^2 = \frac{1}{2}\,\frac{D^2}{\epsilon_0}$$

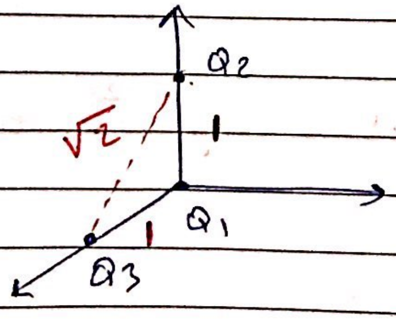
$$W_E = \int_v w_E \, dv$$

Ex:- the point charges $-1nc$, $4nc$ and $3nc$ are located at $(0,0,0)$, $(0,0,1)$ and $(1,0,0)$ find the energy in the system. لازم أختار القانون صح حسب المعطيات

$$W_E = \frac{1}{2} \sum_{K=1}^{3} Q_K V_K$$

$$= \frac{1}{2}\left(Q_1 V_1 + Q_2 V_2 + Q_3 V_3\right)$$

$$= \frac{1}{2}\left[Q_1 (V_{21} + V_{31}) + Q_2 (V_{12} + V_{32}) + Q_3 (V_{13} + V_{23})\right]$$



$$W_E = \frac{1}{2}\left[Q_1 \left(\frac{Q_2}{4\pi\epsilon_0 (1)} + \frac{Q_3}{4\pi\epsilon_0 (1)} \right) \right.$$

$$+ Q_2 \left(\frac{Q_1}{4\pi\epsilon_0 (1)} + \frac{Q_3}{4\pi\epsilon_0 \sqrt{2}} \right)$$

$$\left. + Q_3 \left(\frac{Q_1}{4\pi\epsilon_0 (1)} + \frac{Q_2}{4\pi\epsilon_0 \sqrt{2}} \right) \right]$$

$$W_E = \frac{1}{8\epsilon_0\pi}\left[2Q_1Q_2 + 2Q_1Q_3 + \frac{2Q_2Q_3}{\sqrt{2}} \right]$$

$$= 13.37 \text{ nJ}$$

"END OF CHAPTER (4)"

CH5 :- Electrostatic in materials :-

classification of materials based on it's elc. properties:-

→ conductors (Metals) $\sigma >>> 1$
(Cu, Al, lead, Au, Ag, brass ....

→ semi - conductors $\sigma > 1$.
Si, Ge, gaAS, InP, ...

→ Dielectrics (insulators) $0 < \sigma <<< 1$
$\sigma = 0$ → free space

σ = conductivity (S/m) or $(\Omega \cdot m)^{-1}$

ρ = Resistivity $(\Omega \cdot m)$, $\rho = \frac{1}{\sigma}$

σ depends mainly on
→ Tempreture (o K°)
→ Frequency (Hz) → CH.10

i.e
lead
at $20°C$ → $293°K$

$\sigma \approx 10^6$ S/m

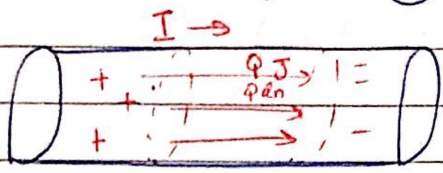
at $4°K$
$\sigma \approx 10^{20}$ S/m → super, Perfect, good

* Types of currents:-

1) conduction current (DC or AC)
2) convection current (DC)
3) Displacement current (AC)

$I = \frac{\Delta Q}{\Delta t}$ uniform sections

→ C/s or (A)



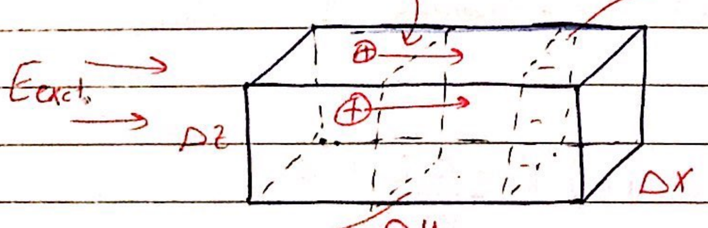
$I = \frac{Q_2 - Q_1}{t_2 - t_1}$

$I = \frac{dq}{dt}$ in general

$\boxed{I = \int_s \bar{J} \cdot \bar{ds}}$ $\bar{J}$ = current density $J = \frac{I}{S}$

* Portion of dielectric material:-

$0 < \sigma << 1$ ρ_v (C/m^3) Area $= \Delta S = \Delta X \Delta Z$



$\Delta V = \Delta X \Delta y \Delta Z = \Delta S \Delta y$

$I = \frac{\Delta Q}{\Delta t} = \frac{\rho_v \Delta V}{\Delta t} \longrightarrow I = \frac{\rho_v \Delta S \Delta y}{\Delta t} \rightarrow \frac{\Delta y}{\Delta t} = u_y$

$I = \rho_v \Delta S\, u_y$, $J = \frac{I}{\Delta S} = \rho_v u_y$

$\boxed{\bar{J} = \rho_v \bar{u}}$, $\bar{u} = u_y \hat{a}_y$ $\quad \frac{C}{m^3} \cdot \frac{m}{s} = \frac{A}{m^2}$

convection current density $\left(\frac{A}{m^2}\right)$

$\bar{u} = \bar{v}_d \equiv$ drift velocity (m/s)

*The force on one electron :-

$Q = -e \longrightarrow \bar{F} = Q\bar{E} = -e\bar{E} = m\bar{a} = \frac{m\bar{u}}{\tau}$

$\bar{v}_d = \bar{u} = \left(\frac{-e\tau}{m}\right)\bar{E} = \mu \bar{E}$

τ = time between collisions (s) (depend on temprature)

$\mu \equiv$ mobility $\left(\frac{C.s}{kg}\right) \longrightarrow \boxed{\bar{u} = \mu \bar{E}}$

+for $\rho_v = -ne$, $n \equiv$ # of charges for volume ($1/m^3$)

$\bar{J} = \rho_v \bar{u} = (-ne)\left(\frac{-e\tau}{m}\right)\bar{E}$

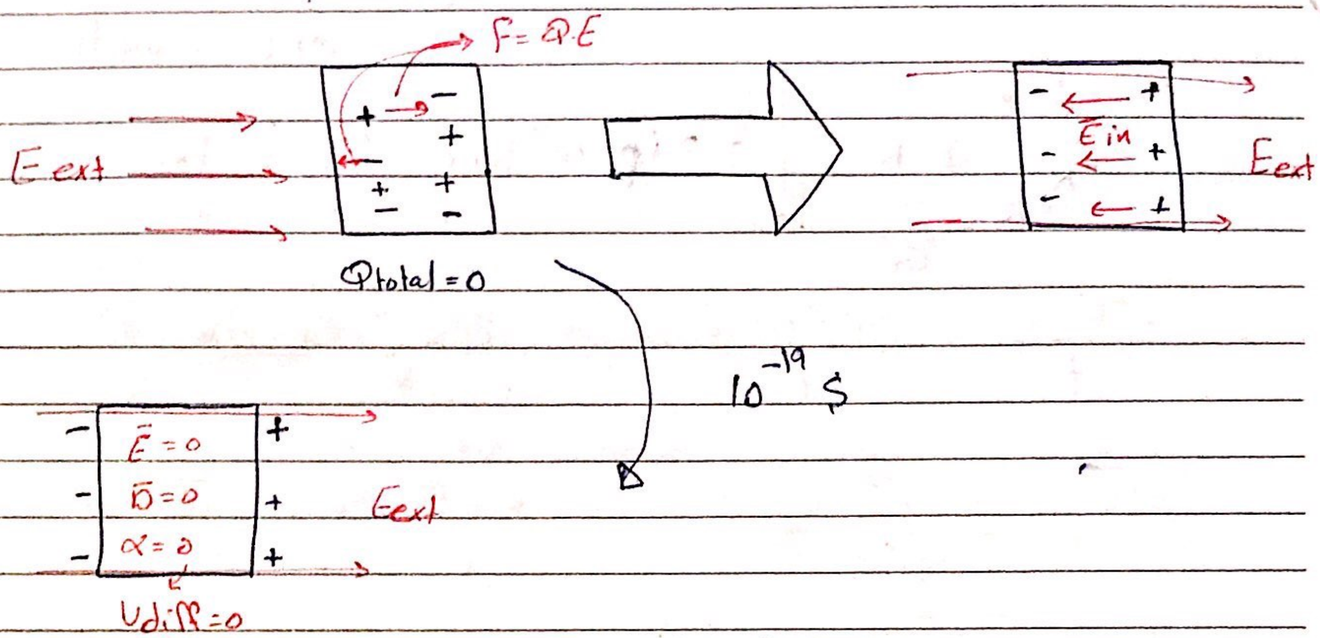
$= \left(\frac{ne^2\tau}{m}\right)\bar{E}$

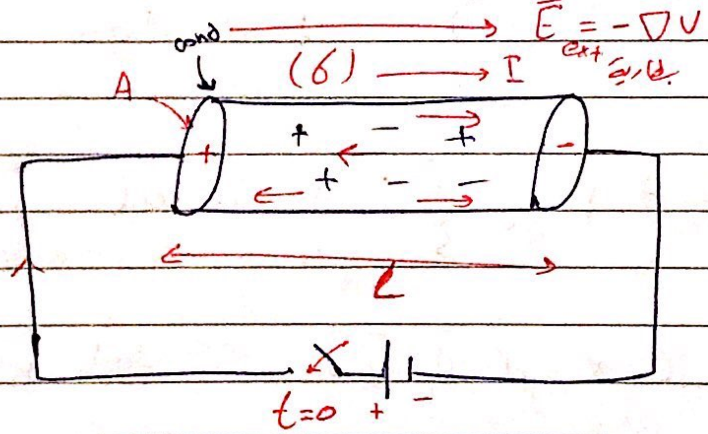
$\boxed{\bar{J} = \sigma \bar{E}} \longrightarrow$ ohm's law in point form

conduction current density $\frac{1}{\Omega m} \cdot \frac{V}{m} = \left(\frac{A}{m^2}\right)$

$\boxed{\frac{\sigma}{\mu} = \rho_v}$

* conductors :-





$$R = \frac{V}{I} = \frac{-\int_L \bar{E} \cdot \bar{dL}}{\int_S \sigma \bar{E} \cdot \bar{ds}}$$

if uniform section $\ell = \frac{1}{\sigma}$, A

$$R = \frac{E\ell}{\sigma E A} = \frac{\ell}{\sigma A} = \frac{\rho L}{A}$$

$$power = \frac{work}{time} \Rightarrow P = \frac{W}{t} \quad in\ (J/s)$$

or (W) watt

$$P = IV = \frac{V^2}{R} = I^2 R = GV^2 = \frac{I^2}{G}$$ → if uniform (Jouls law)

$$P = \frac{-\bar{F} \cdot \bar{l}}{t} = \frac{Q \bar{E}_{ext} \cdot \bar{l}}{t}, \quad Q = \int_v \rho_v \, dv$$

$$P = \int_v \frac{\rho_v \bar{E} \cdot \bar{l}}{t} dv \rightarrow P = \int_v \rho_v \bar{E} \cdot \bar{u} \, dv$$

$$\boxed{P = \int_v \bar{E} \cdot \bar{J} \, dv}$$ → Jouls law

if uniform → $P = \int_l \int_s \bar{E} \cdot \bar{J} \, ds \, dl$

$$P = \int_l \bar{E} \cdot \bar{dl} \times \int_s \bar{J} \cdot \bar{ds}$$ → Jouls تمثل قوانين

$$P = \begin{cases} = \int_v \bar{E} \cdot \bar{J} \, dv \\ \int_v \sigma E^2 \cdot dv \\ \int_v \frac{J^2}{\sigma} dv \end{cases}$$

Power density (wp) in (W/m³), $w_p = \begin{cases} = \bar{E} \cdot \bar{J} \\ = \sigma E^2 \\ = \frac{J^2}{\sigma} \end{cases}$

$$P = \int_v W_p \, dv \quad , \quad I = \oint_s \bar{J} \cdot \bar{J}_s = \int_v \nabla \cdot \bar{J} \, dv$$

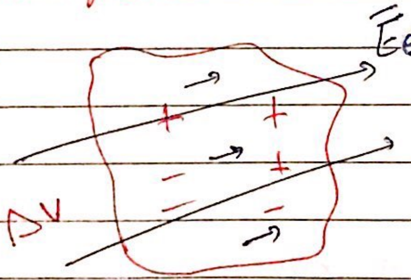
* polarization in dialectrics:-

→ Types of dielectrics:-

→ non-polar
O_2, H_2, N_2 + Rage gases
(Kr, Ne, Xe)

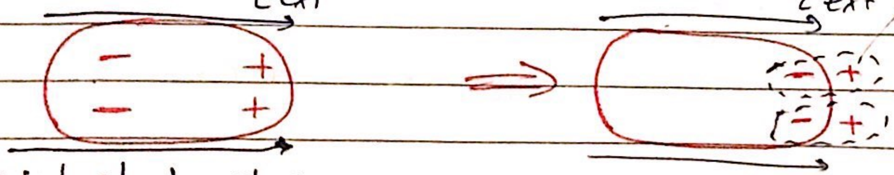
→ polar
(permenant DiPols)
Alloys H_2O, NH_3, HCl, ...

* Non-Polar



P = polarization
$P = 0$

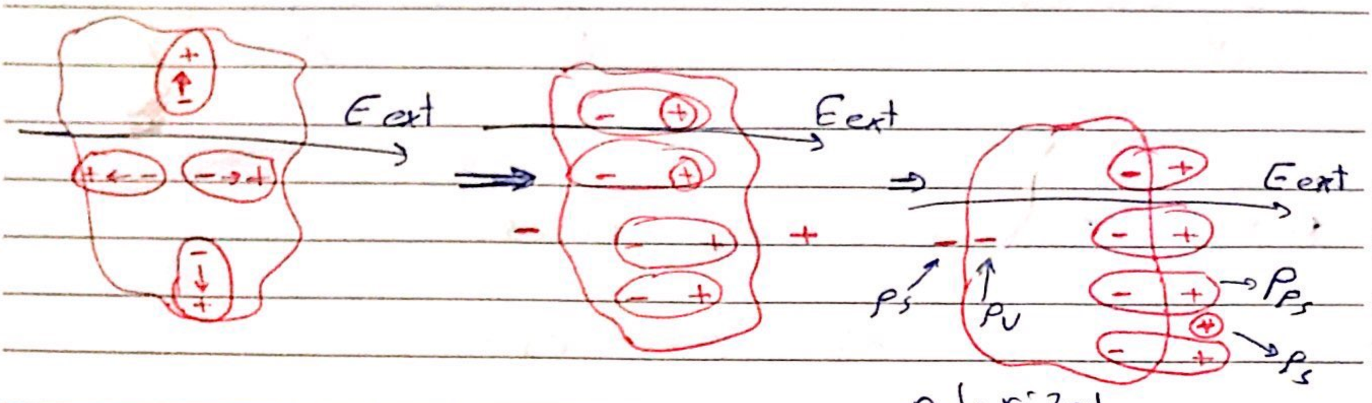
Eext ↑ , F ↑ → pressure $= \frac{F}{S}$ N/m^2



شحنات غير حرة الحركة
(حرة الحركة تلقائي راحت عالسطح)

Distorted charge distribution

* for polar Dielectrics :-



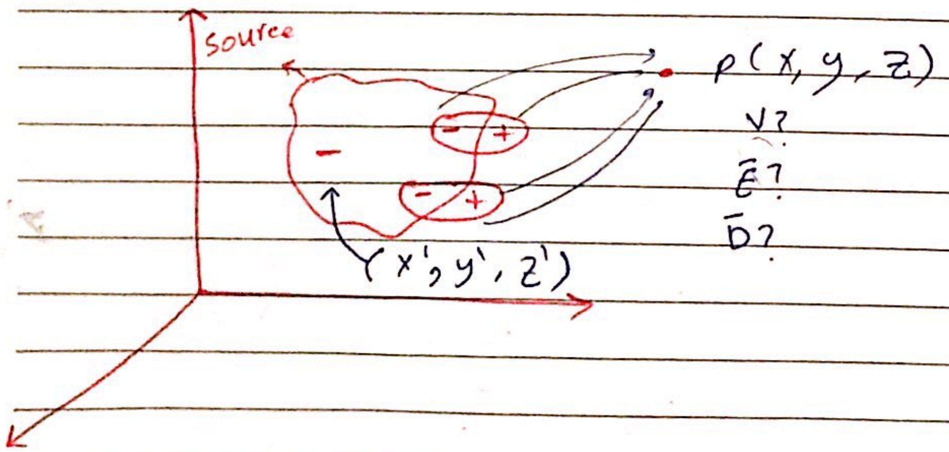
$\rho_s \equiv$ free surface charge dist (c/m²)

$\rho_{ps} \equiv$ polarized (bond) surface charge dist (c/m²)

ρ_v = free volume charge dist (c/m³)
ρ_{pv} = polarized " " " (c/m³)

$$\bar{P} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} P_k}{\Delta v} = \frac{c \cdot m}{m^3} = \frac{c}{m^2}$$

* for polarized Dielectric :-



$\rightarrow$ start $\quad V = \dfrac{\bar{P} \cdot \hat{a}_r}{4\pi\epsilon_0 r^2}$ $\quad$ * r = field - source , r' = source - feild

$$\frac{\hat{a}_r}{r^2} = \frac{\bar{r}}{r^3} = -\nabla\left(\frac{1}{r}\right)\hat{a}_r = -\left(-\frac{1}{r^2}\right)\hat{a}_r = \nabla'\left(\frac{1}{r}\right)$$

derivation w.R.T (x', y', z')

$$V = \frac{\bar{P} \cdot \nabla'\left(\frac{1}{r}\right)}{4\pi\epsilon_0}$$

$$\int dv = \int \frac{\bar{P} \cdot \nabla'\left(\frac{1}{r}\right)}{4\pi\epsilon_0} dv' \rightarrow V = \int_{v'} \frac{\bar{P} \cdot \nabla'\left(\frac{1}{r}\right)}{4\pi\epsilon_0} dv'$$

identity:- $\quad \nabla \cdot V\bar{A} = \bar{A} \cdot \nabla V + V(\nabla \cdot \bar{A})$

(given) $\quad V = \dfrac{1}{R}$, $\bar{A} = \bar{P}$, $\nabla = \nabla'$

$$V = \frac{1}{4\pi\epsilon_0}\left[\int_{v'} \nabla' \cdot \frac{\bar{P}}{R} dv' - \int_{v'} \frac{1}{R} \nabla' \cdot \bar{P} \, dv'\right]$$

* by divergence theorem on integral (1)

$$V = \oint_s \underbrace{\frac{\bar{P} \cdot ds}{4\pi\epsilon_0 R}}_{\oplus} - \int_{v'} \underbrace{\frac{\nabla' \cdot \bar{P}}{4\pi\epsilon_0 R} dv'}_{\ominus}$$

$\oplus, \ominus$ chargedipole

* from CH4 $\quad V = \displaystyle\int_{s'} \frac{\rho_s \, ds'}{4\pi\epsilon_0 r} \rightarrow \rho_s$, $V = \displaystyle\int \frac{\rho_v \, dv'}{4\pi\epsilon_0 r} \rightarrow \rho_v$

$$\rho_{\rho s} = \bar{P} \cdot \hat{a}_n \Rightarrow \rho_s = \bar{D} \cdot \hat{a}_n$$

$$\rho_{\rho v} = -\nabla \cdot \bar{P} \Rightarrow \rho_v = \nabla \cdot \bar{P}$$

* $\bar{E} = -\nabla V \rightarrow (V \neq \text{number})$

or $\bar{E} = \int_S \frac{\rho_s\, ds\, \hat{a}_r}{4\pi\epsilon_0 r^2} + \int_V \frac{\rho_v\, dv\, \hat{a}_r}{4\pi\epsilon_0 r^2} + \int_S \frac{\rho_{ps}\, ds\, \hat{a}_r}{4\pi\epsilon_0 r^2} + \int_V \frac{\rho_{pv}\, dv\, \hat{a}_r}{4\pi\epsilon_0 r^2}$

V = number

* $\bar{D} = \epsilon_0 \bar{E}$ (without ϵ_0 term, same as $\bar{E}$)

* For some dielectrics: → linear
→ isotropic

$\bar{P} = \chi_e \epsilon_0 \bar{E}$

↳ electrical susceptability (constant)

free space → $\epsilon_r = 1$
air → $\epsilon_r = 1.00006 \simeq 1$
conductor → $\epsilon_r = 0.9999 \simeq 1$
→ $\chi_e = \text{zero}$

ρ_v total $= \rho_v + \rho_{pv}$ → dipole charge

charge free inside

$\rho_v = \rho_v \text{ total} - \rho_{pv} = \nabla \cdot \bar{D}$

$\rho_v = \nabla \cdot \bar{D} = \nabla \cdot \epsilon_0 \bar{E} - (-\nabla \cdot \bar{P}$

$\cancel{\nabla} \cdot \bar{D} = \cancel{\nabla} \cdot (\epsilon_0 \bar{E} + \bar{P})$

$\bar{D} = \epsilon_0 \bar{E} + \chi_e \epsilon_0 \bar{E}$

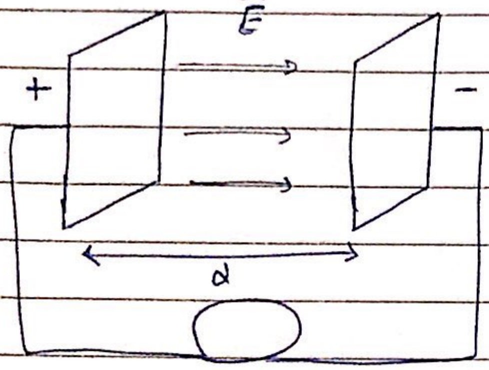
$= (1 + \chi_e)\epsilon_0 \bar{E}$

$\bar{D} = \epsilon_r \epsilon_0 \bar{E}$, ϵ_r = relative permitivity (Dimentionless)

$\boxed{\bar{D} = \epsilon \bar{E}}$ ↳ dielectric constant (Dk)

$\bar{D}|_{\text{dielectric}} \gg \bar{D}|_{\text{air or cond}}$

* $\epsilon_r = 1 + \chi_e \rightarrow \epsilon_r = \frac{\epsilon}{\epsilon_0}$ (by exp)



parallel plates capacitor
+RLC meter
+material sheet

fix $d = 1\,cm$

$$C_o = \frac{\epsilon_o A}{d} \longrightarrow \underset{(material)}{C_{wood}} = \frac{\epsilon_o \epsilon_r A}{d}$$

$$\epsilon_r = \frac{C}{C_o}$$

$$\epsilon = \epsilon_o \epsilon_r$$

$\epsilon \to F/m$, $\epsilon_o \to F/m$, $\epsilon_r \to$ unitless

total charge $\to Q = \int \rho_s \, ds$
$\to Q = \int \rho_v \, dv$

* total bond charge (Q_b)

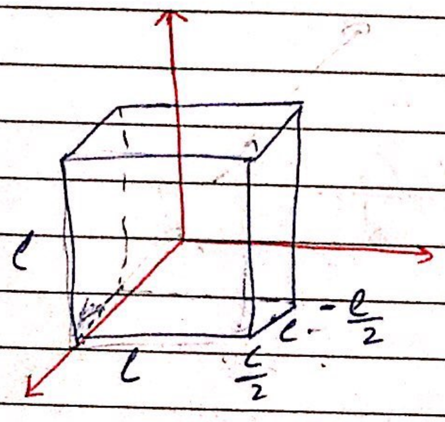
$$Q_{b+} = \int \rho_{ps} \, ds$$

$$Q_{b-} = \int \rho_{pv} \, dv$$

$$Q_{b^+} + Q_{b^-} = Q_{total} = 0$$

(electrically neuteralized)

Ex a dielectric cube of lengh (ℓ) centered at the origin has $P = a\bar{r}$ where $\bar{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ and (a) is constant Find all bond charge densities and the total charge:-



$\rho_{ps}\Big|_{\substack{\text{front} \\ (x=\frac{\ell}{2})}} = \bar{P}\cdot\hat{a}_n \Big|_{\hat{a}_n = \hat{a}_x} = ax\Big|_{x=\frac{L}{2}}$

$\rho_{ps}\Big|_{\text{front}} = \frac{a\ell}{2}$ C/m^2

$\rho_{ps}\Big|_{\text{back}} = -ax\Big|_{x=-\frac{\ell}{2}} = \frac{a\ell}{2}$ C/m^2

$x = -\frac{\ell}{2}$

$\hat{a}_n = -\hat{a}_x$

$\rho_{ps}\Big|_{\text{righ}}$ / All $\rho_{ps} = \frac{a\ell}{2}$ C/m^2

$\rho_{pv} = -\nabla\cdot\bar{P} = -a(1+1+1) = -3a$ C/m^3

Five Apple

$$Q_{b\,total} = Q_{b+} + Q_{b-}$$

$$Q_{b+} = \oint_S \rho_{ps}\, ds = 6 \int_S \rho_{ps}\Big|_{front} ds$$

$$= 6\,\frac{al}{2}\cdot l \cdot l = 3al^3\ c$$

$$Q_{b-} = \int_V \rho_{pv}\, dv = -3al^3\ c$$

$$Q_{b\,total} = 0$$

Dielectric Break down:
depends mainly on:-

1) NAture of the material
2) temprefure $\downarrow$ $\sigma\uparrow$
3) Humidity $\uparrow$ $\sigma\uparrow$
4) Applied E-field
5) time of the applied field.

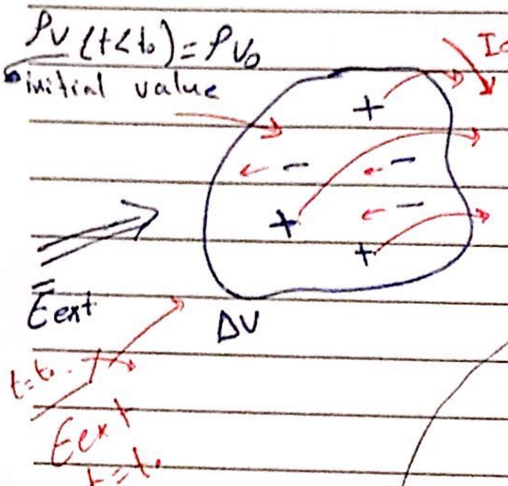
Dielectric strength

the max value of E_{ext} the

* continuity equation and relaxation time (τ_r)

$\rho_v (t<t_0) = \rho_{v_0}$

initial value



KCL : $I_{out} = I_{in}$

$$\oint_s \bar{J} \cdot \bar{ds} = -\frac{\partial q}{dt}$$

$$q = \int_v \rho_v \, dv$$

$$\frac{dq}{dt} = \int_v \frac{d\rho_v}{dt} \, dv$$

$$\int_v \nabla \cdot \bar{J} \, \partial v = \int_v -\frac{\partial \rho_v}{\partial t} \, \partial v$$

$$\boxed{\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}}$$ → in diff form

$$\boxed{\int_v \nabla \cdot \bar{J} \, dv = \int_v -\frac{\partial \rho_v}{\partial t} \, \partial v}$$ → in integral form

Duality from CKt

$$\rho_v = \rho_{v_0} \, e^{-t/\tau_r}$$

$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}$ * , $\nabla \cdot \bar{D} = \rho_v$, $\bar{D} = \epsilon \bar{E}$, $\bar{J} = \sigma \bar{E}$

if ϵ is homogeneous

$$\sigma \, \nabla \cdot \bar{E} = \frac{\sigma \rho_v}{\epsilon} \rightarrow \nabla \cdot \bar{J} = \frac{\sigma \rho_v}{\epsilon_0}$$

$$\frac{\sigma \rho_v}{\epsilon} = -\frac{\partial \rho_v}{\partial t} \rightarrow \int_{\rho_{v_0}}^{\rho_v} \frac{\partial \rho_v}{\rho_v} = \int_{t_0}^{t} -\frac{\sigma}{\epsilon} \, \partial t$$

$$\ln \rho_v \Big|_{\rho_{v_0}}^{\rho_v} = -\frac{\sigma}{t}(t-t_0)$$

$$e^{\ln\left(\frac{\rho_v}{\rho_{v_0}}\right)} = e^{-\frac{\sigma}{t}(t-t_0)}$$

$$\frac{\rho_v}{\rho_{v_0}} = e^{-\frac{\sigma}{\epsilon}(t-t_0)}$$

$$\rho_v = \rho_{v_0} e^{-\frac{\sigma}{\epsilon}(t-t_0)}$$ if $t_0 = 0$ and let $\frac{\epsilon}{\sigma} = T_r$

T_r = relaxation time in (s)

$$\rho_v = \rho_{v_0} \, e^{-t/T_r}$$

$$V = V_0 e^{-t/\tau}$$

$$I = I_0 e^{-t/\tau}$$

$t = \tau$

$V = V_0 \, e^{-1}$

0.368

$1 \, T_r = e^{-1}$ or 36.8

of charge are left inside the material

$\hookrightarrow$ 63.2% are on the surface

- for copper , $\epsilon_r = 1$, $\sigma = 5.8 \times 10^7$ S/m

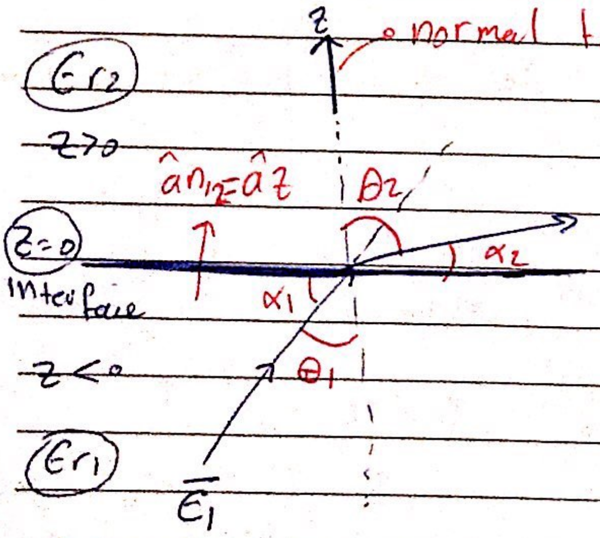
$$T_r = \frac{\epsilon}{\sigma} = \frac{\epsilon_0 \epsilon_r}{\sigma} = \frac{\frac{10^{-9}}{36\pi}}{5.8 \times 10^7} = 1.53 \times 10^{-19} \text{ s}$$

- for fused Quartz , $\epsilon_r = 5$, $\sigma \simeq 10^{-17}$ S/m

$$T_r = \frac{\epsilon}{\sigma} = \frac{\frac{10^{-9}}{36\pi} \times 5}{10^{-17}} = 51.2 \text{ days}$$

* Boundary conditions:-

A) Dielectric - to - Dielectric interface



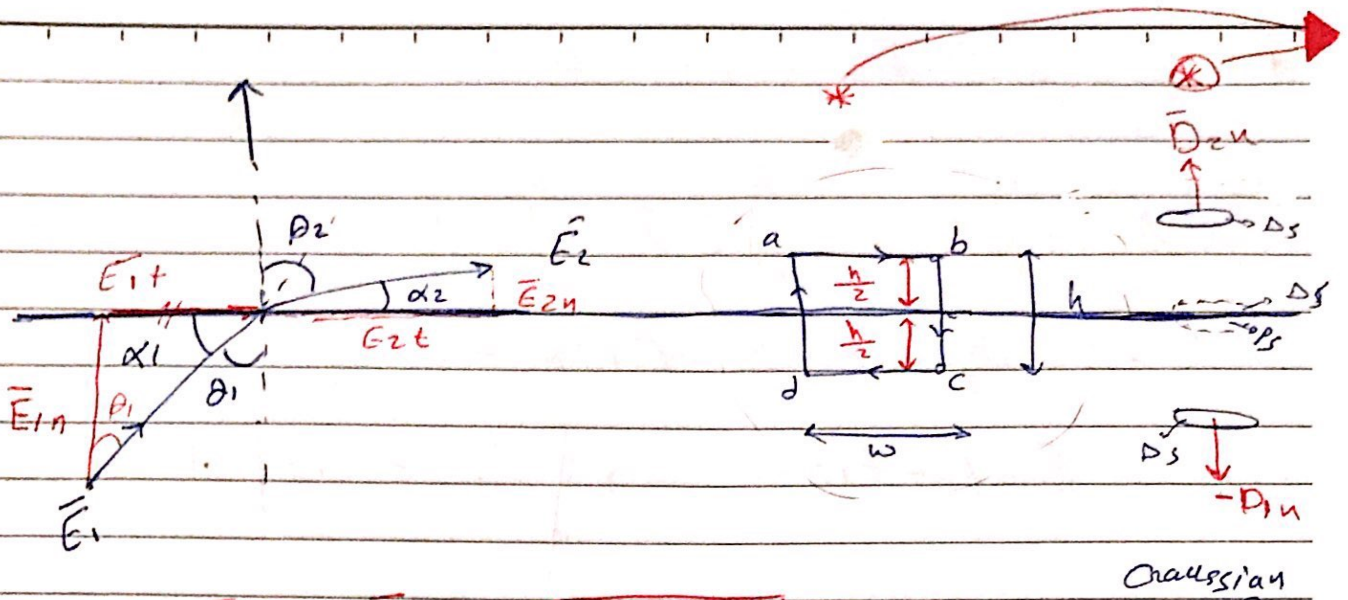
$\bar{E}_2$

$\bar{D}_2 = \epsilon_0 \epsilon_{r2} \bar{E}_2$

$\bar{P}_2 = (\epsilon_{r2} - 1) \epsilon_0 \bar{E}_2$

$\bar{D}_1 = \epsilon_0 \epsilon_{r1} \bar{E}_1$

$\bar{P}_1 = (\epsilon_{r1} - 1) \epsilon_0 \bar{E}_1$



$$E_1 = \bar{E}_{1n} + \bar{E}_{1t} \quad , \quad \boxed{\hat{a}_n = \hat{a}_z}$$

$$\bar{E}_{1n} = (\bar{E}_1 \cdot \hat{a}_n)\,\hat{a}_n \quad , \quad E_{1t} = \bar{E}_1 - \bar{E}_{1n}$$

$$\theta_1 = \sin^{-1}\frac{E_{1t}}{E_1} = \cos^{-1}\frac{E_{1n}}{E_1} = \tan^{-1}\frac{E_{1t}}{E_{1n}}$$

$$\alpha_1 = 90 - \theta_1$$

to find $\bar{E}_2 \rightarrow \bar{E}_2 = \underset{?}{\bar{E}_{2n}} + \underset{?}{\bar{E}_{2t}}$

use maxwell's

① $\oint_s \bar{D}\cdot d\bar{s} = Q_{enc} = \int_s \rho_s\, ds$

② $\oint_l \bar{E}\, d\bar{l} = 0$

Apply on the interface $z=0$

* Apply $\oint_l \bar{E}\cdot\bar{dl} = 0$ at $z=0$

$$\int_a^b \bar{E}\cdot\bar{dl} + \int_b^c \bar{E}\cdot\bar{dl} + \int_c^d \bar{E}\cdot\bar{dl} + \int_d^a \bar{E}\cdot\bar{dl} = 0$$

$$E_{2t} W - E_{2n}\frac{h}{2} - E_{1n}\frac{h}{2} - E_{1t} W + E_{1n}\frac{h}{2} + E_{2n}\frac{h}{2} = 0$$

$$E_{2t} - E_{1t} = 0 \longrightarrow \boxed{\bar{E}_{2t} = \bar{E}_{1t}} \text{ always}$$

* Apply $\oint_s \bar{D}\cdot\bar{ds} = \int_s \rho_s\, ds$

$$\int_{S_{top}} \bar{D}\cdot\bar{ds}_{top} + \int_{S_{bot}} \bar{D}\cdot\bar{ds}_{bot} = \int_s \rho_s\, ds$$

$$D_{2n}\,\Delta S - D_{1n}\,\Delta S = \rho_s\,\Delta S$$

$$\boxed{D_{2n} - D_{1n} = \rho_s} \text{ Not always}$$

if $\rho_s = 0$

$\bar{D}_{2n} = \bar{D}_{1n}$

$$\bar{E}_{2n} = \frac{\epsilon_{r1}\,\bar{E}_{1n}}{\epsilon_{r2}}$$

$$|\bar{D}_{1n}| = \epsilon_0\,\epsilon_{r1}\,|\bar{E}_{1n}|$$

$$D_{2n} = \epsilon_0\,\epsilon_{r2}\,E_{2n}$$

$$E_{2n} = \left(\frac{D_{2n}}{\epsilon_0\,\epsilon_{r2}}\right)\hat{a}_n$$

$$\bar{D}_2 = \epsilon_0\,\epsilon_{r2}\,\bar{E}_2$$

$$\bar{P}_2 = (\epsilon_{r2}-1)\,\epsilon_0\,\bar{E}_2$$

$$\theta_2 = \sin^{-1}\frac{E_{2t}}{E_2}$$

$$\cos^{-1}\frac{E_{2n}}{E_2} = \tan^{-1}\frac{E_{2t}}{E_{2n}}$$

$\alpha_2 = 90 - \theta_2$

$$E_{1t} = E_{2t}$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2 \longrightarrow ①$$

if $\rho_s = 0 \longrightarrow D_{1n} = D_{2n}$

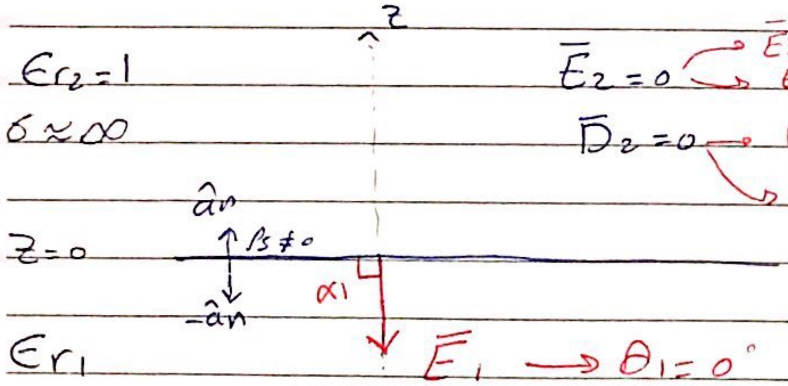
$$\epsilon_0 \epsilon_{r1} E_{1n} = \epsilon_0 \epsilon_{r2} E_{2n}$$

$$\epsilon_{r1} E_1 \cos\theta_1 = \epsilon_{r2} E_2 \cos\theta_2 \longrightarrow ②$$

$$\frac{①}{②} \longrightarrow \frac{\tan\theta_1}{\epsilon_{r1}} = \frac{\tan\theta_2}{\epsilon_{r2}}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}} \longrightarrow \text{if } \rho_s = 0$$

B) conductor-to-Dielectric



$\bar{E}_2 = 0 \rightarrow \bar{E}_{2t} = 0 \rightarrow \bar{E}_{1t} = 0$, $E_{2n} = 0$

$\bar{D}_2 = 0 \rightarrow \bar{D}_{2t} = 0$, $\bar{D}_{2n} = 0$

$\bar{E}_1 \longrightarrow \theta_1 = 0°$, $\alpha_1 = 90°$

$\left.\begin{matrix}\theta_2 \\ \alpha_2\end{matrix}\right] \rightarrow$ does not exist

$$\bar{E}_1 = \cancel{\bar{E}_{1t}}^{\,0} + \bar{E}_{1n}$$

$$\cancel{\bar{D}_{2n}}^{\,0} - \bar{D}_{1n} = \rho_s \rightarrow D_{1n} = -\rho_s \longrightarrow E_{1n} = \frac{-\rho_s}{\epsilon_0 \epsilon_{r}} \hat{a}_n$$

Five Apple

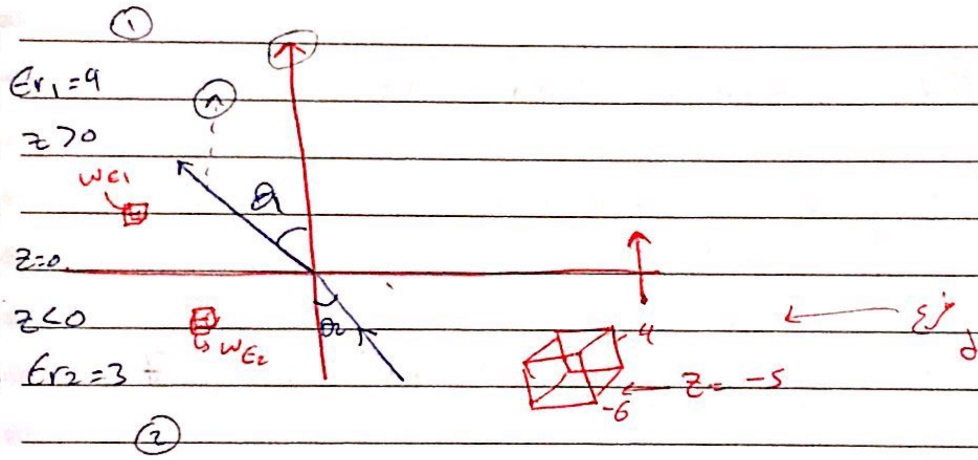
Ex: tow homogeneous dieledrics meet on Plane $z=0$ for $z<0$ $\rightarrow \epsilon_{r2}=3$ and for $z>0 \rightarrow \epsilon_{r1}=4$ and $\bar{E}_1 = 5\hat{a}x - 2\hat{a}y + 3\hat{a}z$ $\frac{KV}{m}$

Find: a) E_2 for $z \leq 0$

b) $\theta_1, \theta_2, \alpha_1, \alpha_2$

c) energy density in both region

d) the energy in a cube of side (2m) centerted at (3, 4, -5)



a) $\hat{a}n = +\hat{a}z$

$\bar{E}_{1n} = 3\,\hat{a}z$ KV/m

$\bar{E}_{1t} = 5\hat{a}x - 2\hat{a}y$ KV/m $= \bar{E}_{2t}$

$D_{1n} - D_{2n} = \rho_s \rightarrow 0$

$\bar{D}_{2n} = \bar{D}_{1n}$

$\epsilon_{r2}\bar{E}_{2n} = \epsilon_{r1}\bar{E}_{1n}$

$\bar{E}_{2n} = \frac{4}{3}\, 3\times 10^3\ \hat{a}z$

$\bar{E}_{2n} = 4\,\hat{a}z$ KV/m

$\bar{E}_2 = 5\hat{a}z - 2\hat{a}y + 4\hat{a}z$ KV/m $\rightarrow D_2 \rightarrow \rho_{s2}, \rho_{v2}$; $\rightarrow \bar{P}_2$

Five Apple

b) $\theta_1 = \sin^{-1} \frac{E_{1t}}{E_1} = \sin^{-1}\left(\frac{\sqrt{29}}{\sqrt{38}}\right) = \cancel{\cos^{-1}}\left(\frac{3}{\sqrt{35}}\right)$

$= 60.9^\circ \longrightarrow \alpha_1 = 90 - \theta_1 = 29.1^\circ$

$\theta_2 = ?$ since $\rho_s = 0$

$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}} = \frac{4}{3} \longrightarrow \theta_2 = 53.4^\circ$

$\alpha_2 = 36.6^\circ$

$\theta_1 > \theta_2$ since $\epsilon_{r1} > \epsilon_{r2}$

c) $w_{E1} = \frac{1}{2}\epsilon_1 E_1^2 = \frac{1}{2} * \frac{10^{-9}}{36\pi} * 4 * 38 * 10^6 = 672\ \mu J/m^3$

$w_{E2} = \frac{1}{2}\epsilon_0 \epsilon_{r2} E_2^2 = \frac{1}{2} * \frac{10^{-9}}{36\pi} * 3 * 45 * 10^6 = 597\ \mu J/m^3$

d) the cube is in region (2) $(3, 4, -5)$

$W_E = \frac{1}{2}\int_v \epsilon_2 E_2^2 = \int_v w_{E2}\, dv$

$= 597 \times 10^{-6} \underbrace{\int_{-6}^{-4}\int_{3}^{5}\int_{2}^{4} dx\, dy\, dz}_{2 \times 2 \times 2 = 8\ m^2}$

$= 4.776\ mJ$

END OF CH 5

CH6 : Boundary value problems

↳ if the charge or potential is known for some part of object

$\nabla \cdot \bar{D} = \rho_v$, $E = -\nabla V$, $\bar{D} = \epsilon \bar{E}$

- $\nabla \cdot \epsilon(-\nabla V) = \rho_v \longrightarrow \boxed{-\nabla \epsilon \nabla \cdot V = \rho_v}$

→ poison's eq for non-homogeneous media ; ϵ_r is function

- if $\rho_v = 0$ (source free region)

$\rho_v = 0$, $\bar{J} = 0$

$\boxed{\nabla \cdot \epsilon \nabla V = 0}$ → Laplace's eq for non homogeneous media

- if ϵ is homogeneous

$\nabla \cdot \nabla = \nabla^2$

$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$ → poisson's eq for homo media

- if ϵ_r is homo $\rho_v = 0$

$\boxed{\nabla^2 V = 0}$ → Laplace's eq for homo media

CH 6 - Boundary value problems

$\nabla \cdot -\epsilon \nabla V = \rho_v$

$\nabla \cdot \epsilon \nabla V = 0$

$$\left.\begin{array}{l} \nabla^2 V = -\dfrac{\rho_v}{\epsilon} \\ \nabla^2 V = 0 \end{array}\right\} \text{Er is homogeneus}$$

$\nabla^2 = \nabla \cdot \nabla$ = laplacian operator

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z \rightarrow \text{cart}$$

cart

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

→ laplacian of (V) → scaler

in cyl

↓

(given)

↑

in shp

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right)$$

$$+ \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

*procedure to solve B.V.P :-

1) chose a suitube coordinates then chose the equation to solve (poisson or laplace).

2) Solve laplace or poisson's equation either:

a) by direct integration if (v) is function of one var.

b) by separation of variables if (v) is function of tow or three var.

3) Apply the tow initial corditions to find the unique potential.

4) find $\bar{E}$ using $\bar{E} = -\nabla V$

5) Find what ever is required

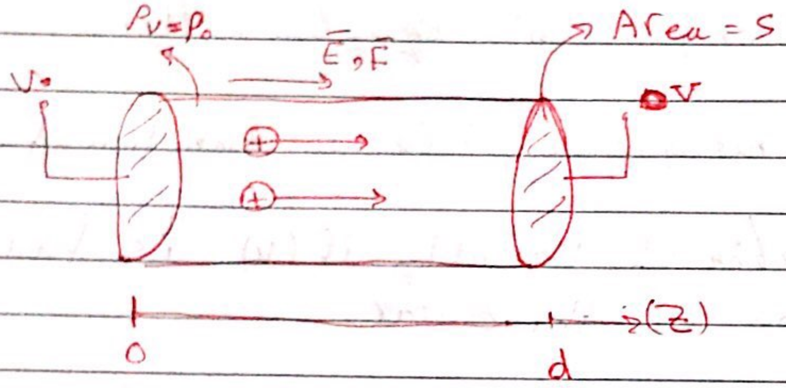
$$Q = \int_s \epsilon \bar{E} \cdot \bar{ds}$$

$$I = \int_s \sigma \bar{E} \cdot \bar{ds}$$

$$\text{pressure} = \frac{\text{Force}}{\text{Area}} \quad , \quad F = Q\bar{E}$$

Five Apple

Ex: an electrohydrodynamic pumping has a uniform ρ_o exists between the tow electrode calculate the pressure of the pump if $\rho_o = 25\ mc/m^3$ and $V_o = 22\ KV$



$$\nabla^2 V = \frac{-\rho_v}{\epsilon} = \frac{-\rho_o}{\epsilon}$$

$\rho_v \neq 0$

ϵ_r is homo (Not mention in quis)

$$\frac{d^2 V}{d z^2} = \frac{-\rho_o}{\epsilon} \longrightarrow \int \frac{d^2 V}{d z^2} = \int \frac{-\rho_o}{\epsilon}$$

$$\int \frac{dv}{dz} = \int \left(-\frac{\rho_o}{\epsilon} z + A \right)$$

$$\boxed{V = \frac{-\rho_o z^2}{2\epsilon} + Az + B}$$ general

I.C.S

$$V(z=0) = V_0 = B$$
$$V(z=d) = 0$$

$$V(z=0) = \boxed{V_0 = B}$$

$$V(z=d) = 0 = -\frac{\rho_0 d^2}{2\epsilon} + Ad + V_0 \longrightarrow \boxed{A = \frac{1}{d}\left(\frac{\rho_0 d^2}{2\epsilon} - V_0\right)}$$

$$\boxed{V = \frac{-\rho_0 z^2}{2\epsilon} + \frac{z}{d}\left(\frac{\rho_0 d^2}{2\epsilon} - V_0\right) + V_0} \longrightarrow \text{unique potential}$$

$$\bar{E} = -\nabla V = -\frac{dV}{dz}\hat{a}_z$$

$$\bar{E} = \left[+\frac{\rho_0 z}{\epsilon} - \frac{1}{d}\left(\frac{\rho_0 d^2}{2\epsilon} - V_0\right)\right]\hat{a}_z \quad V/m$$

$$\bar{F} = Q\bar{E}, \quad Q = \int_v \rho_v \, dv$$

$$\bar{F} = \int_v \rho_0 \bar{E} \, dv = \rho_0 S \int \bar{E} \, dz$$

$$\bar{F} = \rho_0 S \int_0^d \left[\frac{\rho_0 z}{\epsilon} - \frac{1}{d}\left(\frac{\rho_0 d^2}{2\epsilon} - V_0\right)\right] dz \, \hat{a}_z$$

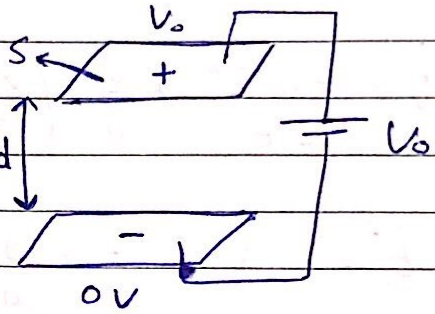
$$= \rho_0 S \left(\frac{\rho_0 d^2}{2\epsilon} - \frac{\rho_0 d^2}{2\epsilon} + V_0\right)\hat{a}_z \longrightarrow \bar{F} = \rho_0 S V_0 \hat{a}_z \; N$$

$$\text{Pressure} = \frac{|F|}{S} = \rho_0 V_0 \; (N/m^2) = 550 \; N/m^2$$

* Resistance and capacitance :-

$$R = \frac{V}{I} = \frac{-\int_{l} \bar{E} \cdot \bar{dl}}{\int_{s} \sigma \bar{E} \cdot \bar{ds}} \quad , \quad C = \frac{Q}{V} = \frac{\int_{s} \epsilon \bar{E} \cdot \bar{ds}}{-\int_{l} \bar{E} \cdot \bar{dl}}$$

• procedure to find (R) and (C) :-



1) choose a suitable coordinates :-

↓ B.V.P

2) Assume V_o difference on the tow plates.

3) solve Laplace equation and substitute the tow initial condition to find the unique potential

4) $\bar{E} = -\nabla V$

5) Find $Q = ?$ using gauss's law

$Q = \oint_{s} \epsilon \bar{E} \cdot \bar{ds}$ and find $I = \int_{s} \sigma \bar{E} \cdot \bar{ds}$

6) $\left[C = \frac{Q}{V} \quad , R = \frac{V}{I} \right]$ where (V) is assumed (V_o) in step (2)

$\longrightarrow$ Gauss

2) assume $\pm Q$

3) find $\bar{E}$ using gauss

$$Q = \oint_S \epsilon \bar{E} \cdot \bar{ds} \longrightarrow CH4$$

4) Find $V = -\int_L \bar{E} \cdot \bar{dl}$, $I = \int_S \sigma \bar{E} \cdot \bar{ds}$

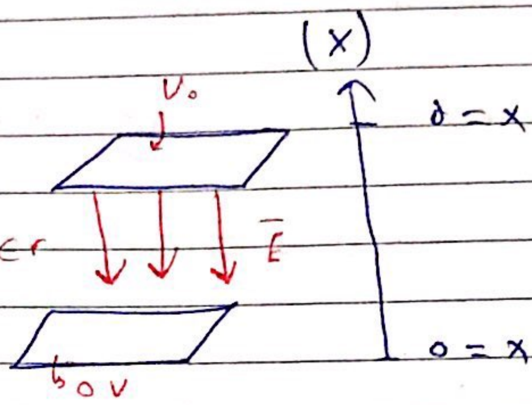
5) $C = \dfrac{Q}{V}$ $\longrightarrow$ step (3) $\longrightarrow$ step (4)

$R = \dfrac{V}{I}$ $\longrightarrow$ step (4)

* Types of capacitor :-

1) parallel plate capacitor

method (1) $\rightarrow$ B.V.P cart



$\nabla^2 V = 0$

$$\int \frac{d^2 V}{d x^2} = \int 0$$

$$\int \frac{dV}{dx} = \int A \longrightarrow \boxed{V = Ax + B}$$

$\rightarrow$ I.C
$V(x=0) = 0$
$V(x=d) = V_0$

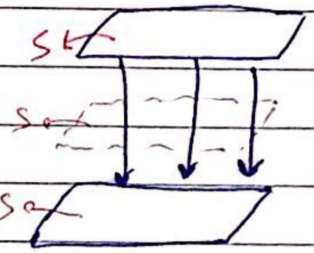
$\boxed{B = 0}$, $Ad = V_0 \longrightarrow \boxed{A = \frac{V_0}{d}}$

$$V = \frac{V_0}{d} x$$

$$\bar{E} = -\nabla V = -\frac{\partial V}{\partial x} \hat{a}_x = \frac{-V_0}{d} \hat{a}_x \quad V/m$$

نفس اتجاه $\bar{E}$ اي فرضته بالصح

$$Q = \int_s \epsilon \bar{E} \cdot \bar{ds} = \int_{-\infty}^{\infty}\int \epsilon \left(\frac{+V_0}{d} \cancel{\hat{a}_x}\right) . dy\, dz \left(\cancel{-\hat{a}_x}\right)$$



$$Q = \frac{\epsilon V_0}{d} S \longrightarrow C = \frac{Q}{V}$$

$$= \frac{\frac{\epsilon \cancel{V_0} S}{d}}{\cancel{V_0}} \longrightarrow \boxed{C = \frac{\epsilon S}{d} \ (F)}$$

farad

$$\int_{-\infty}^{\infty}\int dy\, dz = S$$

Five Apple

$R_d = \frac{V}{I}$, $I = \int_S \sigma_d \bar{E} \cdot \bar{ds}$

$I = \int\int_{-\infty}^{\infty} \left(-\frac{V_0}{d} \hat{a}x \cdot dy\, dz (-\hat{a}z)\right) \longrightarrow \boxed{I = \frac{\sigma_d V_0}{d} S \ (A)}$

$R_d = \frac{V_0}{\frac{\sigma_d V_0 S}{d}} = \frac{d}{\sigma_d S} = \boxed{\frac{d}{\sigma_d A}}$

(Ω)

method (2) → Gauss :

$Q = \oint_S \epsilon \bar{E} \cdot \bar{ds}$

$\bar{E} = E_x (-\hat{a}x)$ $\quad \bar{ds} = dy\, dz (-\hat{a}x)$

$Q = \int\int_{-\infty}^{\infty} \epsilon E_x (-\hat{a}x)\, dy\, dz (-\hat{a}x)$, $\int\int_{-\infty}^{\infty} dy\, dz = S$

نفس CH4

$Q = \epsilon E_x S \ (c)$ $\quad \bar{E} = \frac{Q}{\epsilon S} - \hat{a}x$, $E = \frac{\rho_s}{\epsilon_0} (-\hat{a}x)$

$V = -\int_L \bar{E} \cdot dL = \cancel{-}\int_{0}^{d} \frac{Q}{\epsilon S} \cancel{+\hat{a}x} \cdot dx \cancel{\hat{a}x}$ ($+Ve$ Q, $-Ve$ Q)

$V = \frac{Qd}{\epsilon S}$ (V) constant

$C = \frac{Q}{V} = \frac{\cancel{Q}}{\frac{\cancel{Q} d}{\epsilon S}} \longrightarrow \boxed{C = \frac{\epsilon S}{d} \ (F)}$

$$I = \int_S \sigma_d \, \bar{E} \cdot \bar{ds} = \iint_{-\infty}^{\infty} \sigma_d \frac{Q}{\epsilon s} (-\hat{a}_x) \, dy \, dz (-\hat{a}_x)$$

$$\boxed{I = \frac{\sigma_d Q}{\epsilon} \ (A)} \Rightarrow \frac{C}{\Omega \cdot m \cdot \frac{F}{m}} = \frac{C}{\Omega \cdot F} = A$$

$$R_d = \frac{V}{I} = \frac{\frac{Qd}{\epsilon s}}{\frac{\sigma_d Q}{\epsilon}} = \boxed{\frac{d}{\sigma_d s} \ (\Omega) = R_d}$$

$$\rightarrow \boxed{G = \frac{1}{R_d} = \frac{\sigma_d s}{d} \ (S)}$$

$$* \text{ always } R_{dc} = \tau_r = \left(\frac{d}{\sigma_d s}\right) \cdot \left(\frac{\epsilon s}{d}\right) = \frac{\epsilon}{\sigma_d}$$

$$W_E = \frac{1}{2} \int_V \epsilon E^2 \, dv \quad dv \to s\,dx, \quad s\,d \ m^3$$

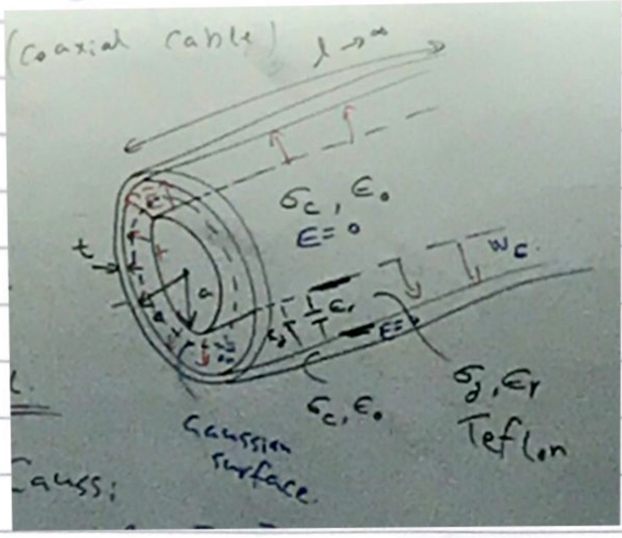
$$= \frac{\epsilon}{2} s \int_0^d \frac{Q^2}{\epsilon^2 s^2} dx = \frac{Q^2 d}{2 \epsilon s}$$

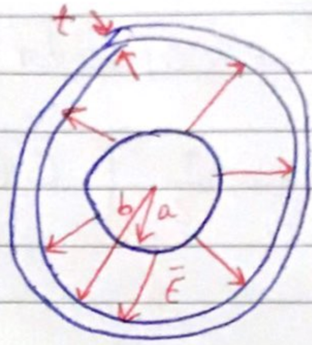
$$= \frac{Q^2}{2C} \quad , \ Q = CV$$

$$= \frac{C^2 V^2}{2C} = \frac{1}{2} C V^2$$

$$\boxed{W_E = \frac{1}{2} C V^2}$$

2) Cylindrical capacitor





Gauss

$$Q = \oint_s \epsilon \bar{E} \cdot \bar{ds} \quad , \quad \bar{E} = E_\rho \hat{a}_\rho$$

$$\bar{ds} = \rho \, d\phi \, dz \, \hat{a}_\rho$$

$$Q = \int_0^l \int_0^{2\pi} \epsilon E_\rho \hat{a}_\rho \cdot \rho \, d\phi \, dz \, \hat{a}_\rho$$

$$Q = \epsilon E_\rho \rho 2\pi l \Rightarrow \boxed{\bar{E} = \frac{Q}{2\pi\rho\epsilon l} \hat{a}_\rho}$$

in CH4

$$* \quad \bar{E} = \frac{\rho_L}{2\pi\epsilon\rho} \hat{a}_\rho \quad , \quad \rho_L = \frac{Q}{l}$$

$$V = -\int_l \bar{E} \cdot \bar{dl} = -\int_b^a \frac{\epsilon}{2\pi\epsilon\rho l} \cancel{\hat{a}_\rho} \, d\rho \, \cancel{\hat{a}_\rho}$$

$$V = \frac{+Q}{2\pi\epsilon l} \ln\left(\frac{b}{a}\right) \quad (V)$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q \ln\frac{b}{a}}{2\pi\epsilon l}} \longrightarrow \boxed{C = \frac{2\pi\epsilon l}{\ln(b/a)} \, (F)}$$

$$\frac{C}{\ell} = \frac{2\pi\epsilon}{\ln(b/a)} \quad (F/m)$$

↳ capacitor per unit lengh

$$W_E = \frac{1}{2}\int_v \epsilon E^2 \, dv \longrightarrow \frac{1}{2CV^2}$$

$$R_d = \frac{V}{I} \qquad , \quad I = \int_s \sigma_d \bar{E} \cdot \bar{d}_s$$

$$= \int_0^{\ell}\int_0^{2\pi} \sigma_d \frac{Q}{2\pi\epsilon\cancel{\rho_d}} \hat{a}_\rho \cdot \cancel{\rho} \, d\phi \, dz \, \hat{a}_\rho \longrightarrow \boxed{I = \frac{\sigma_d Q}{\epsilon} (A)}$$

$$R_d = \frac{\frac{Q}{2\pi\epsilon\ell} \ln\left(\frac{b}{a}\right)}{\frac{\sigma_d Q}{\epsilon}} \longrightarrow \boxed{R_d = \frac{\ln(b/a)}{2\pi\sigma_d \ell} (\Omega)}$$

$$G = \frac{2\pi\sigma_d \ell}{\ln(b/a)} \quad (S)$$

$$G/\ell = \frac{2\pi\sigma_d}{\ln(b/a)} \quad (S/m)$$

$$CR_d = \frac{\epsilon}{\sigma_d} = T_r$$

using B.V.P

$\nabla^2 V = 0$, $\rho_v = 0$ Er is homo

$$\frac{1}{\rho} \int \frac{\partial}{\partial \rho}\left(\rho \frac{dV}{\partial \rho}\right) = \int 0$$

general

$$\rho \int \frac{\partial V}{\partial \rho} = \int \frac{A}{\rho} \longrightarrow \boxed{V = A \ln \rho + B}$$

I.C $V(\rho = a) = V_0$
$V(\rho = b) = 0$

$A \ln a + B = V_0$ --- ①
$A \ln b + B = 0$ --- ②

$B = -A \ln b$

$A \ln a - A \ln b = V_0 \longrightarrow A \ln \frac{a}{b} = V_0$

$$\boxed{A = \frac{V_0}{\ln\left(\frac{a}{b}\right)}} \qquad \boxed{B = \frac{-V_0}{\ln\left(\frac{a}{b}\right)} \ln(b)}$$

$$V = \frac{V_0}{\ln(a/b)}\left(\ln \rho - \ln b\right)$$

$$\boxed{V = V_0 \frac{\ln \frac{\rho}{b}}{\ln \frac{a}{b}}} \longrightarrow \text{unique}$$

continue

$$\bar{E} = -\nabla V$$

$$= \frac{-\partial V}{\partial \rho} \hat{a}_\rho \longrightarrow \bar{E} = \frac{-V_0}{\ln(\frac{a}{b})\,\rho} \hat{a}_\rho$$

$$Q = \oint_s \epsilon \bar{E} \cdot \bar{ds} = \int_0^l \int_0^{2\pi} \epsilon \frac{-V_0}{\ln(\frac{a}{b})\,\rho} \hat{a}_\rho \cdot \rho\, d\phi\, dz\, \hat{a}_\rho$$

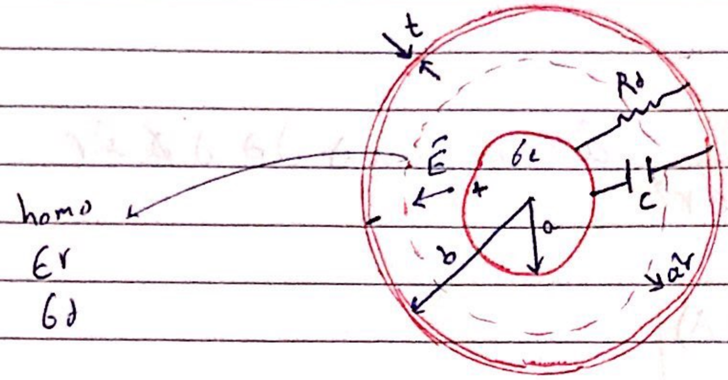
$$= \frac{V_0 \epsilon}{\ln(\frac{b}{a})} 2\pi L \; (C)$$

$$C = \frac{Q}{V} = \frac{\frac{V_0 \epsilon 2\pi l}{\ln(b/a)}}{V_0}$$

$$C/l = \frac{2\pi \epsilon}{\ln(\frac{b}{a})} \; F/m$$

$$I = \frac{\sigma_d Q}{\epsilon}, \quad R_d = \frac{\ln(b/a)}{2\pi \sigma_d l} \; (\Omega)$$

3) spherical capacitor



Gauss

$$Q = \oint_S \epsilon \bar{E} \cdot \bar{ds} \quad , \quad \bar{E} = E_r \hat{a}_r$$

$$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$Q = \int_0^{2\pi}\int_0^{\pi} \epsilon E_r r^2 \underbrace{\sin\theta \, d\theta}_{2} \underbrace{d\phi}_{2\pi} .$$

$$Q = \epsilon E_r \, 4\pi r^2 \longrightarrow \boxed{\bar{E} = \frac{Q}{4\pi \epsilon r^2} \hat{a}_r} \rightarrow \epsilon_0 \epsilon_r$$

$$V = -\int_l \bar{E} \cdot \bar{dl} = -\int_b^a \frac{Q}{4\pi\epsilon r^2} \hat{a}_r \cdot dr \, \hat{a}_r$$

$$V = \frac{Q}{4\pi\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right) \ (V)$$

$$C = \frac{Q}{V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\epsilon}\left(\frac{1}{a}-\frac{1}{b}\right)} \longrightarrow \boxed{C = \frac{4\pi\epsilon}{\frac{1}{a}-\frac{1}{b}} \ (F)}$$

$\longrightarrow$

$$I = \int_s \sigma_d \bar{E} \cdot \bar{ds}$$

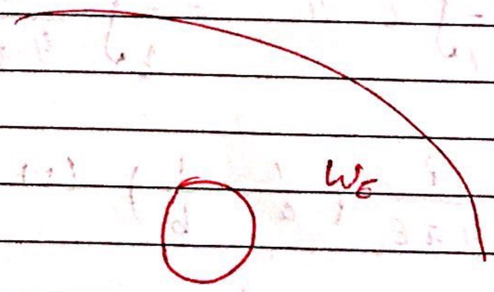
$$= \iint \sigma_d \frac{Q}{4\pi\epsilon r^2} \hat{a}_r \, r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$\boxed{I = \frac{\sigma_d Q}{\epsilon} \ (A)}$$

$$R_d = \frac{V}{I} = \frac{\frac{Q}{4\pi\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right)}{\frac{\sigma_d Q}{\epsilon}} \longrightarrow \boxed{R_d = \frac{\frac{1}{a} - \frac{1}{b}}{4\pi\sigma_d} \ (\Omega)}$$

$$W_E = \frac{1}{2}\int_v \epsilon E^2 \, dv \longrightarrow \boxed{W_E = \frac{1}{2} C V^2} \qquad \boxed{G = \frac{4\pi\sigma_d}{\frac{1}{a} - \frac{1}{b}} \ (S)}$$

4) Isolated capacitor
Isolated sphere



if $b \rightarrow \infty$

$C = 4\pi\epsilon a \ (F)$

$R_d = \dfrac{1}{4\pi\sigma_d a} \ (\Omega)$

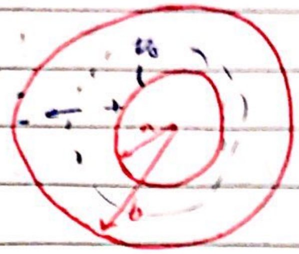
$R_{dc} = \dfrac{\epsilon}{\sigma}$

$G = 4\pi\sigma_d a \ (S)$

Spherical capacitor:-

Gauss

$$C = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}} \; (F)$$



using B.V.P

$$\nabla^2 . V = 0$$

$$\frac{1}{r^2} \int \frac{\partial}{\partial r} \left(r^2 \frac{dV}{dr} \right) = \int 0$$

$$r^2 \int \frac{dV}{dr} = \int \frac{A}{r^2} \longrightarrow V = -\frac{A}{r} + B$$

I.C.S

$V(r=a) = V_0 \longrightarrow V_0 = \frac{-A}{a} + B \quad \cdots ①$

$V(r=b) = 0$

$$0 = -\frac{A}{b} + B \quad \cdots ②$$

$$B = \frac{A}{B} \dashrightarrow \text{sub in } 1$$

$$V_0 = \frac{-A}{a} + \frac{A}{b} \longrightarrow V_0 = -A\left(\frac{1}{a} - \frac{1}{b}\right) \longrightarrow \boxed{A = \frac{-V_0}{\frac{1}{a} - \frac{1}{b}}}$$

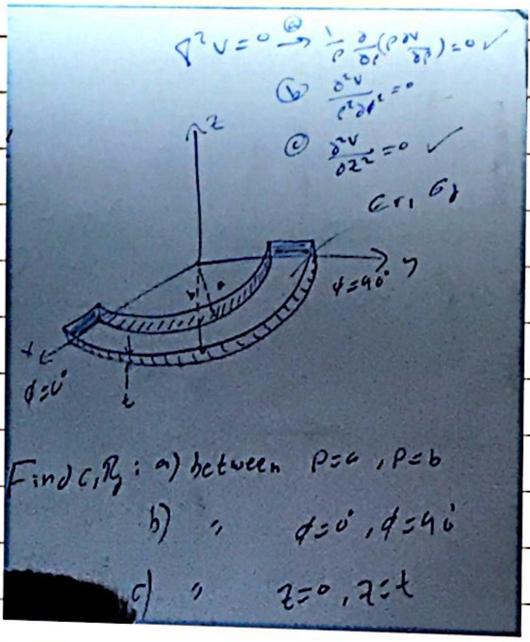
$$V = \frac{V_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} \left[\frac{1}{r} - \frac{1}{b} \right] \longrightarrow \text{unique}$$

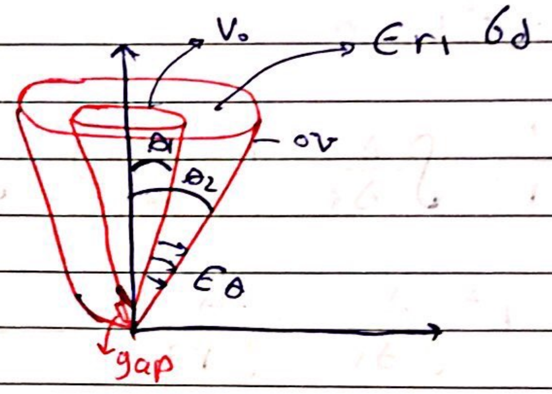
$$\boxed{\bar{E}} = -\nabla V = -\frac{dV}{dr} \hat{a}_r = \boxed{\frac{+V_0}{\left(\frac{1}{a} - \frac{1}{b}\right) r^2} \hat{a}_r \quad V/m}$$

$$Q = \oint_s \epsilon \bar{E} \cdot \bar{ds} = \int_0^{2\pi}\int_0^{\pi} \frac{\epsilon V_0}{\frac{1}{a}-\frac{1}{b}} \frac{1}{r^2}\, \hat{a}_r \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

4π

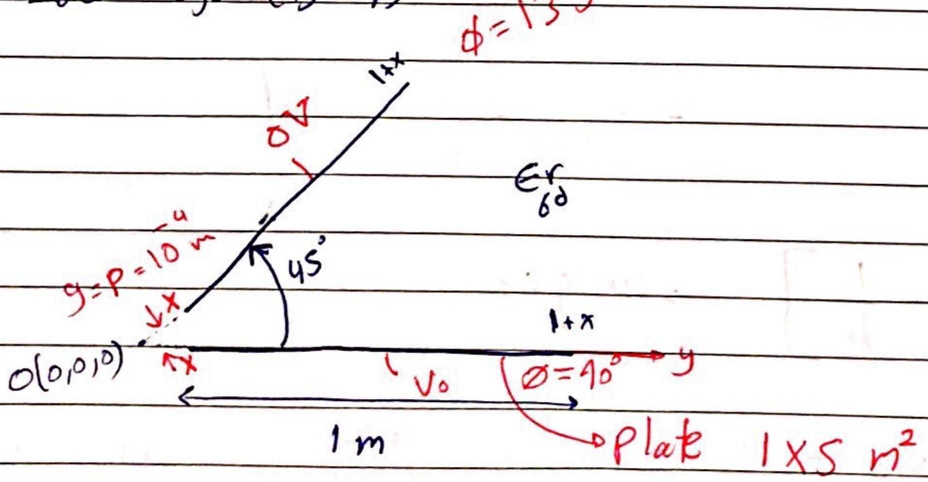
$$Q = \frac{4\pi\epsilon V_0}{\frac{1}{a}-\frac{1}{b}} \ (C) \longrightarrow C = \frac{Q}{V_0} = \frac{4\pi\epsilon}{\frac{1}{a}-\frac{1}{b}} \ (F)$$





$$\frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial V}{\partial\theta}\right) = 0$$

Ex Idea cyl (B.V.p)



$$\nabla^2 V = 0$$

$$\int \frac{\partial^2 V}{\rho^2 \partial \phi^2} = \int 0$$

$$\int \frac{\partial V}{\partial \phi} = \int A \longrightarrow V = A\phi + B$$

$$V\left(\phi = \frac{\pi}{2}\right) = V_0$$

$$V\left(\phi = \frac{3\pi}{4}\right) = 0$$

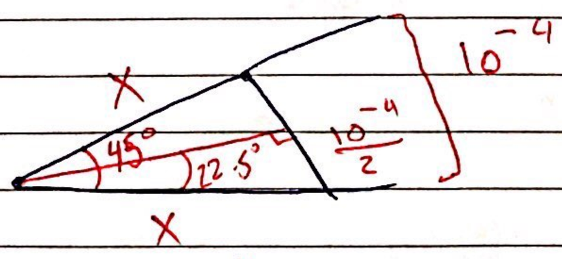
$$V_0 = A\frac{\pi}{2} + B \quad \text{---} (1)$$ solve for A, B

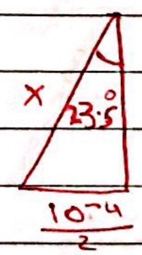
$$0 = A\frac{3\pi}{4} + B \quad \text{---} (2)$$

$$\bar{E} = -\nabla V = -\frac{1}{\rho}\frac{\partial V}{\partial \phi}\,\hat{a}\phi$$

$$Q = \oint_S \epsilon \bar{E} \cdot \bar{ds}$$

$$Q = \int_0^{S}\int_x^{H x} \epsilon E_\phi \,\cancel{\hat{a}\phi} \cdot d\rho\, dz\, \cancel{\hat{a}\phi}$$

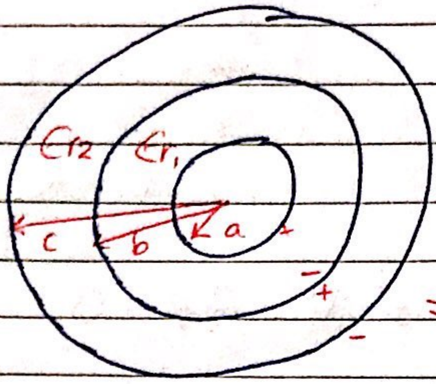




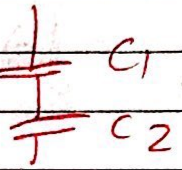
$$\sin 22.5 = \frac{\frac{10^{-4}}{2}}{x}$$

Ex spherical

Ceq ?



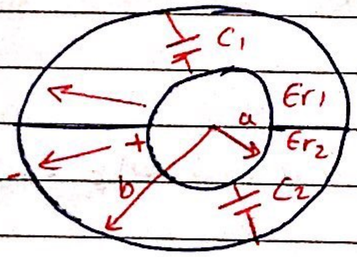
$\Rightarrow$

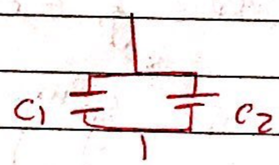


$C = \dfrac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}}$

$C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$ $\Rightarrow$ $C_1 = \dfrac{4\pi\epsilon_0\epsilon_{r1}}{\frac{1}{c} - \frac{1}{b}}$, $C_2 = \dfrac{4\pi\epsilon_0\epsilon_{r2}}{\frac{1}{b} - \frac{1}{c}}$

Ex spherical



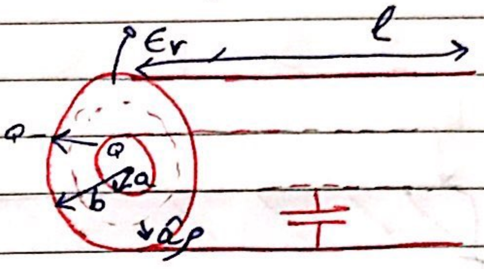


$C_1 = \dfrac{4\pi\epsilon_0\epsilon_{r1}}{\frac{1}{a} - \frac{1}{b}} \cdot \dfrac{1}{2}$

$C_2 = \dfrac{4\pi\epsilon_0\epsilon_{r2}}{\frac{1}{a} - \frac{1}{b}} \cdot \dfrac{1}{2}$

Five Apple

Ex: a coaxial cabel has $a = 1cm$ $b = 2.5$ cm is filled with a dielectric matirial has $\epsilon_r = \frac{10+\rho}{\rho}$ where ρ is in cm find C/ℓ ?



Gauss

$$Q = \oint_s \epsilon \bar{E} \cdot \bar{ds}$$

$$Q = \int_0^{\ell} \int_0^{2\pi} \underbrace{\epsilon}_{\epsilon_0 \epsilon_r} E_\rho \hat{a}_\rho \cdot \rho \, d\phi \, dz \, \hat{a}_\rho$$

$$Q = \epsilon E_\rho \rho 2\pi \ell \longrightarrow \boxed{\bar{E} = \frac{Q}{2\pi \epsilon \rho \ell} \hat{a}_\rho}$$

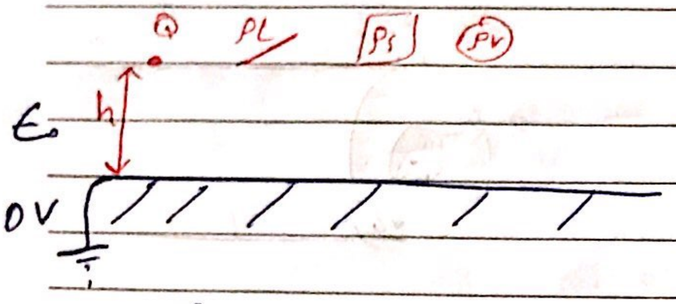
$$V = -\int_{\ell} \bar{E} \cdot \bar{dl} = -\int_b^a \frac{Q}{2\pi \epsilon_0 \left(\frac{10+\rho}{\rho}\right) \rho \ell} \hat{a}_\rho \cdot d\rho \, \hat{a}_\rho$$

$$V = \frac{+Q}{2\pi \epsilon_0 \ell} \ln\left(\frac{10+b}{10+a}\right)$$

$$\frac{C}{\ell} = \frac{Q}{V} = \frac{2\pi \epsilon_0 L}{\ln\left(\frac{10+b}{10+a}\right)}$$

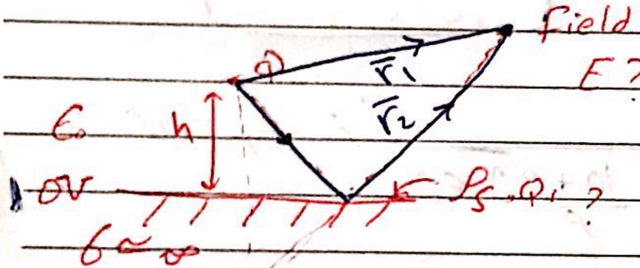
$$\frac{C}{\ell} = \frac{2\pi \frac{10^{-9}}{36\pi}}{\ln\left(\frac{12.5}{11}\right)} = 434.6 \text{ pf/m}$$

* method of Images

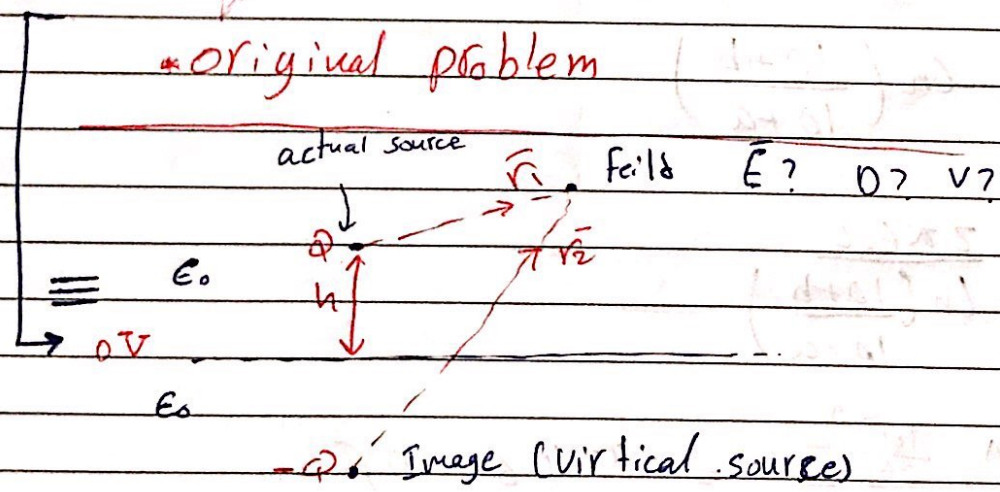


$\sigma \simeq \infty$

A) point charge above a perfect ground conducting plane:



* original problem



* equivalent problem

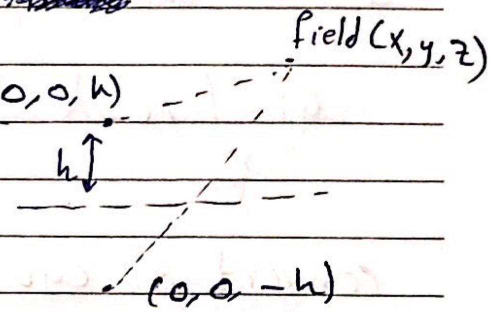
$\therefore$ prof $Q_i = -Q$

$\bar{E}=0, \bar{D}=0, \bar{V}=0 \,]$, $z<0$ ← هاى الخطوة عشان أوجد $\bar{E}$ فوق السطح، بعد ما أحسبه

eq Problem

$$\bar{E} = \bar{E}_+ + \bar{E}_- = \frac{Q\,\bar{r}_1}{4\pi\epsilon_0 r_1^3} - \frac{Q\,\bar{r}_2}{4\pi\epsilon_0 r_2^3}$$

$$\bar{E} = \frac{Q}{4\pi\epsilon_0}\left[\frac{\bar{r}_1}{r_1^3} - \frac{\bar{r}_2}{r_2^3}\right] \quad V/m$$



$\bar{r}_1 = x\hat{a}_x + y\,\hat{a}_y + (z-h)\,\hat{a}_z$

$r_1 = \sqrt{x^2+y^2+(z-h)^2}$

$\bar{r}_2 = x\hat{a}_x + y\,\hat{a}_y + (z+h)\,\hat{a}_z$

$r_2 = \sqrt{x^2+y^2+(z+h)^2}$

$$\bar{E}_{original} = \begin{cases} \dfrac{Q}{4\pi\epsilon_0}\left(\dfrac{\bar{r}_1}{r_1^3} - \dfrac{\bar{r}_2}{r_2^3}\right), & z>0 \\ \dfrac{-Q\,2h\,\hat{a}_z}{4\pi\epsilon_0\left[x^2+y^2+h^2\right]^{3/2}}, & z=0 \quad \Rightarrow \bar{D} = \epsilon_0\bar{E} \\ 0, & z<0 \end{cases}$$

$V = -\int_l \bar{E}\cdot d\bar{l}$ or $V = V_+ + V_-$

$$V = \frac{Q}{4\pi\epsilon_0 r_1} - \frac{Q}{4\pi\epsilon_0 r_2}$$

$$V_{original} = \begin{cases} \dfrac{Q}{4\pi\epsilon_0}\left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right], & z>0 \\ 0, & z\le 0 \end{cases}$$

$\rho_s = \bar{D}\big|_{z=0} \cdot \hat{a}_n \quad , \hat{a}_n = \hat{a}_z$

$$\boxed{\rho_s = \frac{-Qh}{2\pi\left[x^2+y^2+h^2\right]^{3/2}} \quad C/m^2}$$

$Q_i = ?$ and proof $Q_i = -Q$

$$Q_i = \int_s \rho_s \, ds = \int_{-\infty}^{\infty}\int \frac{-Qh \quad dx\,dy}{2\pi\left[x^2+y^2+h^2\right]^{3/2}}$$

convert to cyl

$Q_i = \int_s \rho_s \, ds \quad , ds = \rho \, d\rho \, d\phi$

$$Q_i = \frac{-Qh}{2\pi} \int_0^{2\pi}\int_0^{\infty} \frac{\rho \, d\rho \, d\phi}{\left[\rho^2+h^2\right]^{3/2}}$$

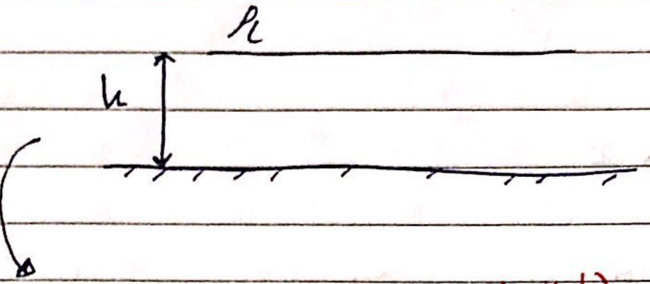
let $u = \rho^2 + h^2$

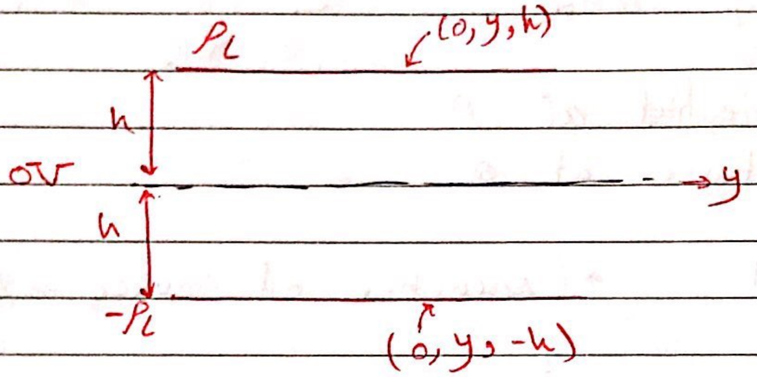
$du = 2\rho \, d\rho \longrightarrow \rho \, d\rho = \frac{du}{2}$

$$Q_i = +\frac{Qh}{\cancel{x}} \cdot \frac{1}{\frac{1}{\cancel{x}} u^{\frac{1}{2}}}$$

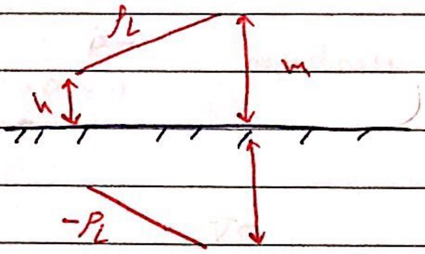
$$Q_i = Qh \frac{1}{\sqrt{\rho^2+h^2}}\Big|_0^{\infty} = Qh\left(0 - \frac{1}{h}\right) = -Q \ \#$$

B) Line charge above a perfect ground conducting plane





$\bar{E} = \bar{E}_+ + \bar{E}_-$



$$= \begin{cases} \dfrac{\rho_L \bar{\rho}_1}{2\pi\epsilon_0 \rho_1^2} - \dfrac{\rho_L \bar{\rho}_2}{2\pi\epsilon_0 \rho_2^2} & , z > 0 \\[2ex] \dfrac{-2h\rho_L}{2\pi\epsilon_0 [x^2+h^2]}\hat{a}z & , z = 0 \\[2ex] 0 & , z < 0 \end{cases}$$

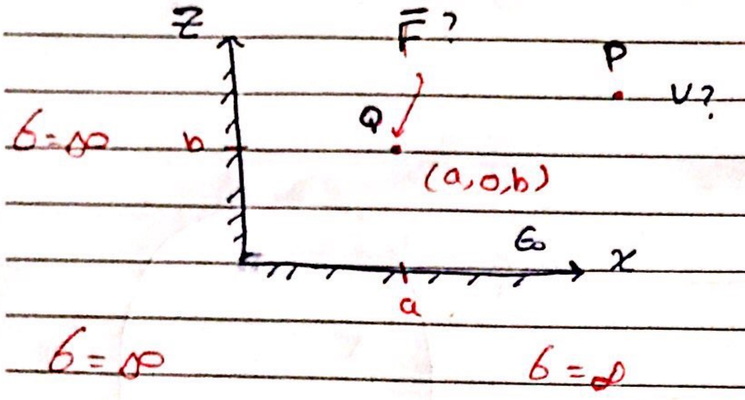
$z = 0 \Rightarrow$
$\bar{D} = \epsilon_0 \bar{E} \rightarrow \rho_s = \bar{D}\cdot\hat{a}_n \quad (z = 0)$
$V = V_+ + V_-$
$Q_i\,? \longrightarrow \rho_i\,?$

$\bar{\rho}_1 = x\hat{a}x + (z-h)\hat{a}z \qquad \rho_1 = \sqrt{x^2+(z-h)^2}$

$\bar{\rho}_2 = x\hat{a}x + (z+h)\hat{a}z \qquad \rho_2 = \sqrt{x^2+(z+h)^2}$

$Q_i = \int_l \rho_L\,dl = \int_s \rho_s\,ds \rightarrow (dx\,dy) \Rightarrow \rho_i = \int_l \rho_s\,dx$

Ex



a) find the potential at P

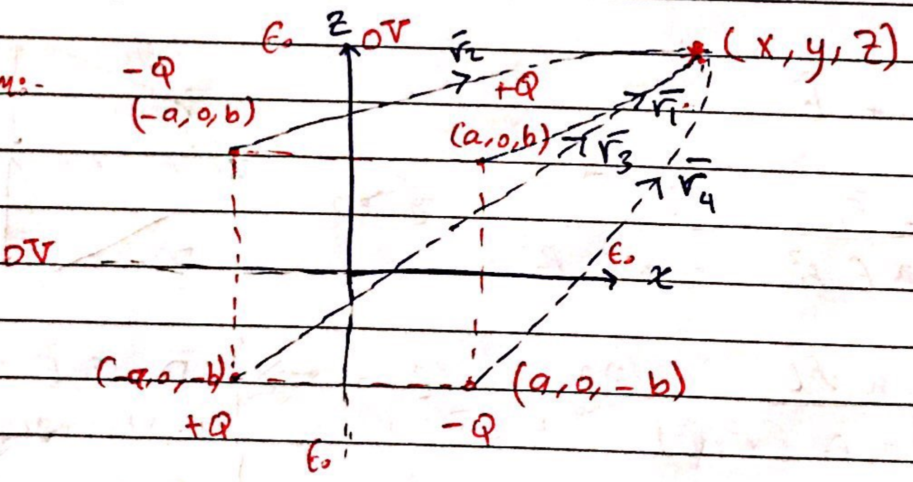
b) " " force at Q

a)

$N = \frac{360}{\phi} - 1$ * (Number of source $\rightarrow N = \frac{360}{\phi}$)

number of Images

eq problem:-



$$V\Big|_P = V_1 + V_2 + V_3 + V_4$$

$$= \begin{cases} \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_3} - \frac{1}{r_4}\right], & x > 0,\ z > 0 \\ 0, & \text{otherwise} \end{cases}$$

$\bar{r}_1 = (x-a)\hat{a}_x + y\,\hat{a}_y + (z-b)\hat{a}_z$

$\bar{r}_2 = (x+a)\hat{a}_x + y\,\hat{a}_y + (z-b)\hat{a}_z$

$\bar{r}_3 = (x+a)\hat{a}_x + y\,\hat{a}_y + (z+b)\hat{a}_z$

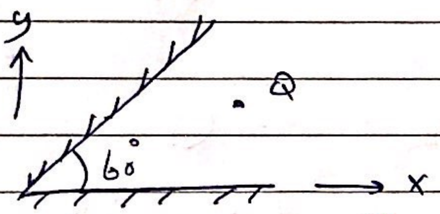
$\bar{r}_4 = (x-a)\hat{a}_x + y\,\hat{a}_y + (z+b)\hat{a}_z$

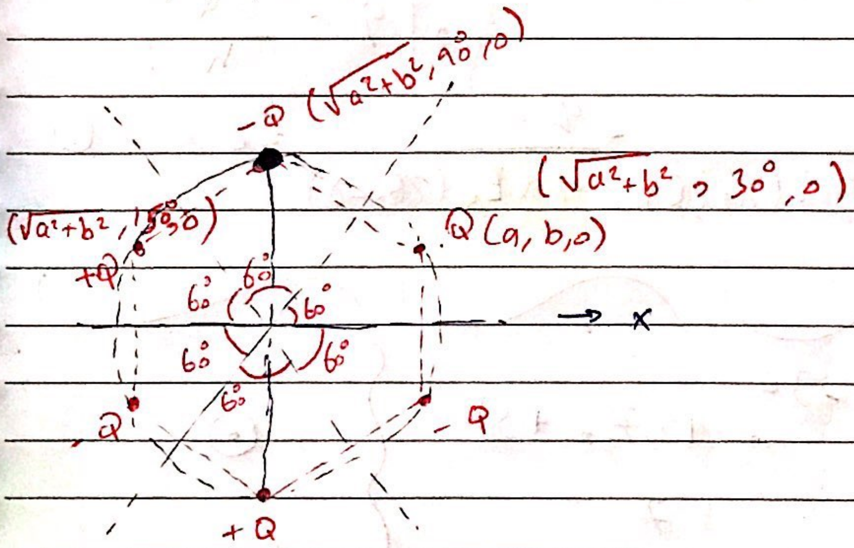
b) $\bar{F} = Q\bar{E}$

$$\bar{E} = \frac{Q}{4\pi\epsilon_0}\left[-\frac{\bar{d}_2}{d_2^3} + \frac{\bar{d}_3}{d_3^3} - \frac{\bar{d}_4}{d_4^3}\right]$$

- $\bar{d}_2 = 2a\,(+\hat{a}x) \rightarrow d_2 = 2a$
  $\bar{d}_3 = 2a\,(\hat{a}x) + 2b\,(\hat{a}z) \rightarrow d_3 = 2\sqrt{a^2+b^2}$
  $\bar{d}_4 = 2b\,(\hat{a}z) \rightarrow d_4 = 2b$

Ex :

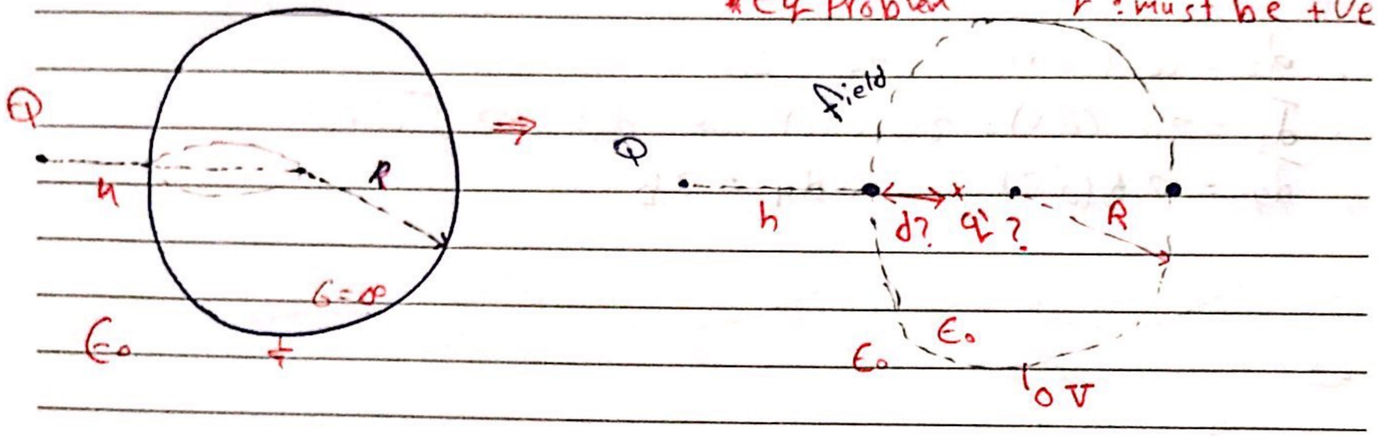




Ex :- a point charge Near a perfect ground conducting sphere :-

- find location and value of the image.

* eq Problem q' : must be +Ve

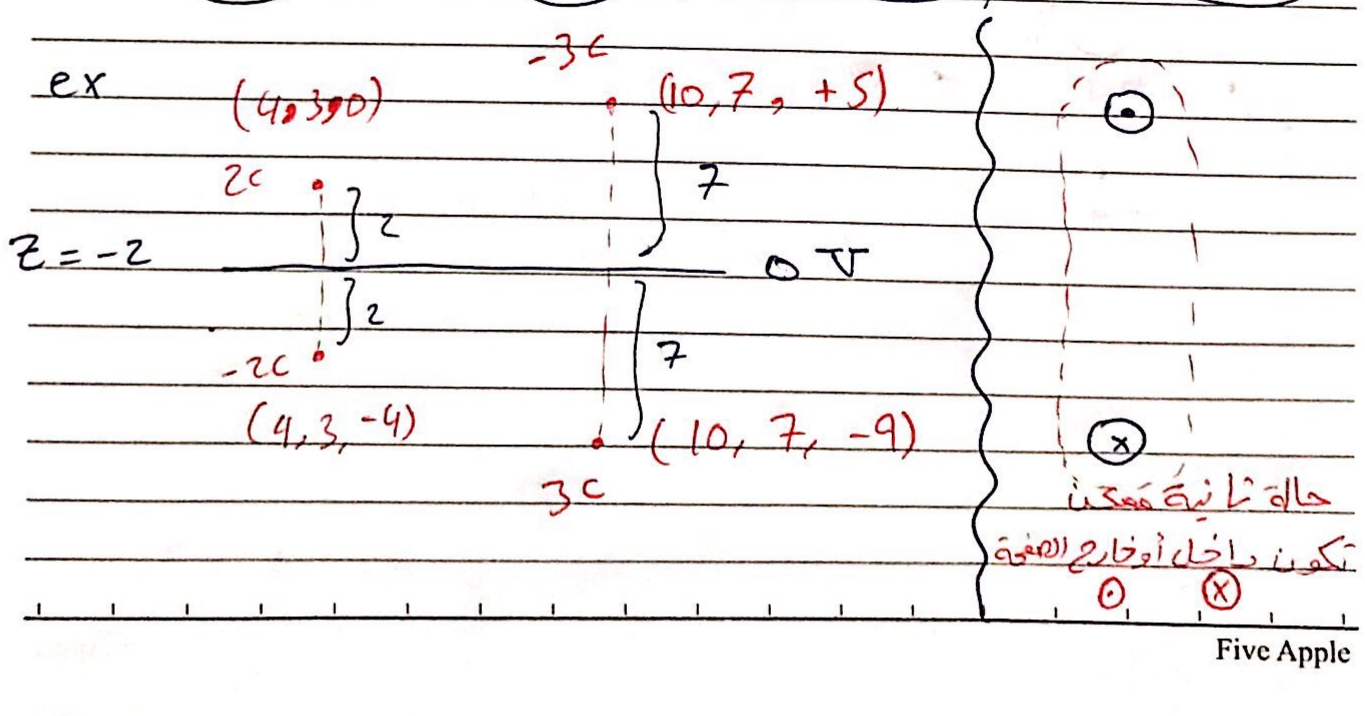


$$V\Big|_{r=h} = 0 \longrightarrow \frac{Q}{4\pi\epsilon_0 r} - \frac{q'}{4\pi\epsilon_0 d} = 0 \quad \text{--- (1)}$$

$$\boxed{q' = \frac{d\,Q}{h}}$$

$$V\Big|_{r=h+2R} = 0 \longrightarrow \frac{Q}{4\pi\epsilon_0 (h+2R)} - \frac{q'}{4\pi E_0 (2R-d)} = 0 \quad \text{--- (2)}$$

ex



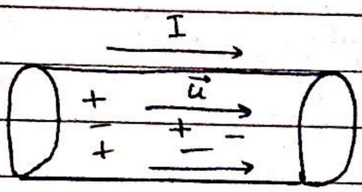
CH7: Magnetostatic Fields

*source for magnetostatic:-

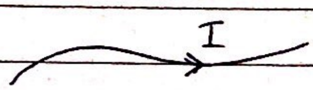
1) permenent magnet

2) charge moving with uniform velocity (constant) → Zero acceleration

3) DC current flowing in the wire



CH5

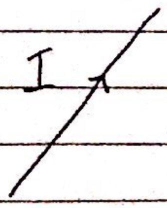


*Major laws:-

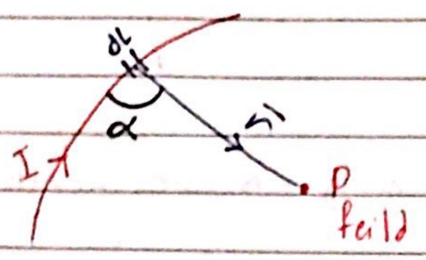
1) Biot-savart's law (general)

2) Ampere's law (special case)

oristead (Motor principle)



* Biot savart's Law
by exp



$\bar{H} \equiv$ Magnetic field intensity. (A/m)

$$dH \propto \frac{I\, dL \sin\alpha}{r^2}$$ (relation) ($\times \to$ cross poduct)

$$dH = \frac{K\, I\, dL \sin\alpha}{r^2} \quad , \quad K = \frac{1}{4\pi}$$

↳ proportimality constant

$$\int dH = \int \frac{I\, dL \sin\alpha}{4\pi r^2}$$

$$H = \int_L \frac{I\, dL \sin\alpha}{4\pi r^2}$$ ⇒ magnitude only

$\bar{H}$?

$$\bar{H} = \int_L \frac{I\, \bar{dL} \times \hat{a}_r}{4\pi r^2}$$

$|\bar{A} \times \bar{B}| = AB \sin\theta_{AB}$

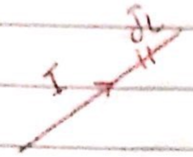
$|\bar{dL} \times \hat{a}_r| = dL\, (1) \sin\alpha$

→ $\perp \bar{dL}$ $\perp \bar{r}$, $\bar{dL} = dL\, \hat{a}_L$

$$\boxed{\bar{H} = \int_L \frac{I\, \bar{dL} \times \bar{r}}{4\pi r^3} = \int_L \frac{I\, \hat{a}_L \times \bar{r}}{4\pi r^3}\, dL \quad \text{for line current (1D segment)}}$$

Types of current distribution

1) line current $\longrightarrow$ 1D segment

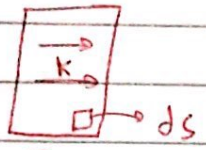


$I\,\bar{dL} \equiv A.m$

A/m^2

$$\bar{H} = \int_L \frac{I\,\bar{dL} \times \bar{r}}{4\pi r^3}$$

2) Surface current $\longrightarrow$ 2D surfaces

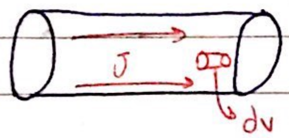


• $\bar{K}\,ds \equiv A.m$

K = surface current density (A/m)

$$\bar{H} = \int_S \frac{\bar{K}\,ds \times \bar{r}}{4\pi r^3} \Rightarrow \bar{H} = \int_S \frac{\bar{K} \times \bar{r}}{4\pi r^3}\,ds$$

3) Volume current $\rightarrow$ 3D object



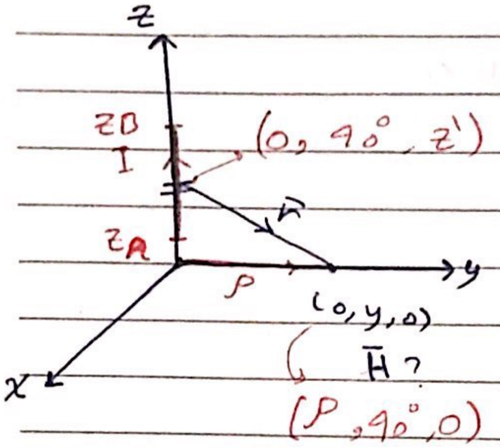
$\bar{J}\,dv = A.m$

$\bar{J}$ = Volume current density (A/m^2)

$$\bar{H} = \int_V \frac{\bar{J}\,dv \times \bar{r}}{4\pi r^3}$$

X Volume $\longrightarrow$ Ampere's Law

Ex: find $\bar{H}$ at $(0, y, 0)$ due to a finite straight wire along z-axis carry current 'I'



$$\bar{H} = \int_L \frac{I \, \bar{dL} \times \bar{r}}{4\pi r^3}$$

$\bar{dL} = dz' \, \hat{a}_z$

$\bar{r} = \rho \hat{a}_\rho - z' \hat{a}_z$

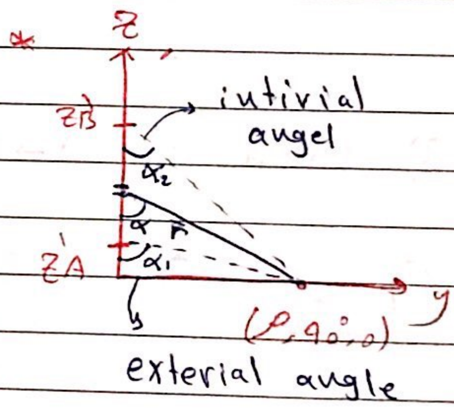
$r = \sqrt{\rho^2 + z'^2}$

$$\bar{H} = \frac{I}{4\pi} \int_{z'_A}^{z'_B} \frac{\hat{a}_z \times (\rho \hat{a}_\rho - z' \hat{a}_z)}{[\rho^2 + z'^2]^{3/2}} \, dz'$$

$\hat{a}_z \times \hat{a}_\rho = \hat{a}_\phi$

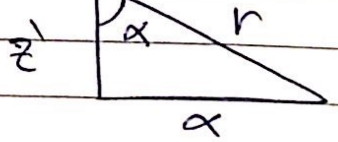
$\hat{a}_z \to \hat{a}_\rho \to \hat{a}_\phi \to \hat{a}_z$

$$\bar{H} = \frac{I}{4\pi} \int_{z'_A}^{z'_B} \frac{\rho}{[\rho^2 + z'^2]^{3/2}} \, dz' \, \hat{a}_\phi$$



introduce angle $\alpha, \alpha_1, \alpha_2$
relate z' with α.

$\sin\alpha = \frac{\rho}{r}$



$\cos\alpha = \frac{z'}{r}$

$\tan\alpha = \frac{\rho}{z'} \longrightarrow z' = \rho \cot\alpha$

$z' = \rho \cot\alpha \longrightarrow dz' = -\rho \csc^2\alpha \, d\alpha$

$\rho^2 + z'^2 = \rho^2 + \rho^2 \cot^2\alpha = \rho^2 (1 + \cos^2\alpha)$

$r^2 = \rho^2 \csc^2\alpha \quad , \; r^3 = \rho^3 \csc^3\alpha$

$$\bar{H} = \frac{I\rho}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{-\rho \csc^2\alpha}{\rho^3 \csc^3\alpha} \, d\alpha \; \hat{a}_\phi$$

$$\bar{H} = \frac{I}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \sin\alpha \, d\alpha \; \hat{a}_\phi$$

$$\boxed{\bar{H} = \frac{I}{4\pi\rho} (\cos\alpha_2 - \cos\alpha_1) \; \hat{a}_\phi}$$

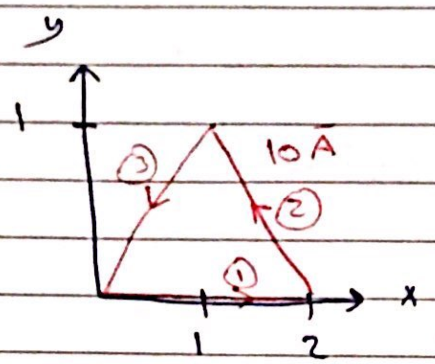
(interior - exterior) angle

always for finit / straight wire

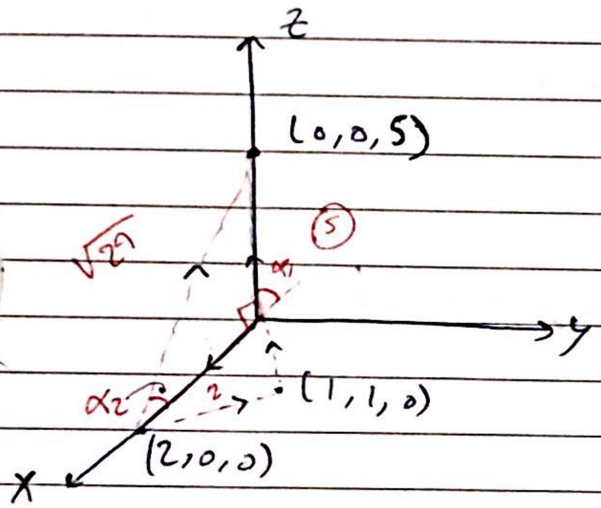
$$\boxed{\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho}$$

Ex. for a triangular loop

2D



find $\bar{H}$ at $(0,0,5)$ due to side (1) of the loop :-



$$\bar{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\hat{a}_\phi$$

$\alpha_1 = 90^\circ \rightarrow \cos\alpha_1 = 0$

$\cos\alpha_2 = \frac{2}{\sqrt{29}}$

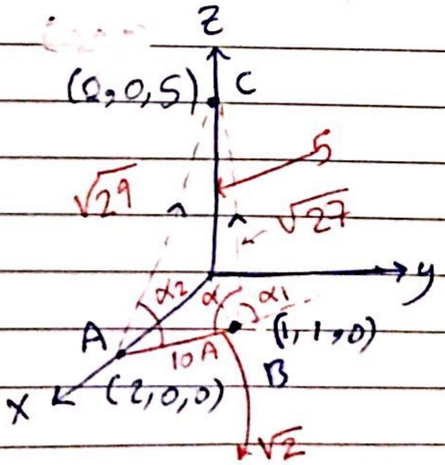
$\bar{\rho} = 5\hat{a}_z \rightarrow \rho = 5\ m$

$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho$

$= \hat{a}_x \times \hat{a}_z = -\hat{a}_y$

$\bar{H} = -54.1\ \hat{a}_y\ mA/m$

Side (2)



$\bar{AB} \cdot \bar{AC} = |\bar{AB}|\,|AC| \cos\alpha_2$

$\bar{BA} \cdot \bar{BC} = |\bar{BA}|\,|BC| \cos\theta_1$

$\cos\alpha_1 = -\cos\theta_1$

$\alpha_1 = 90^\circ \rightarrow \rho = \sqrt{27}$

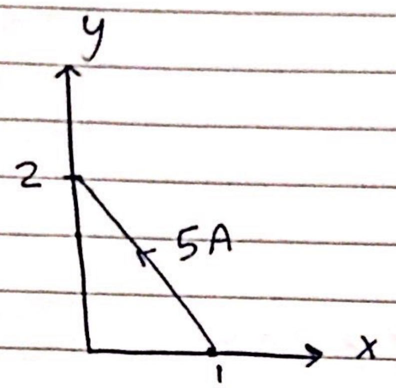
$(1,-1,0)\cdot(-1,-1,5) = \sqrt{2}\,\sqrt{27}\,\cos\theta_1$

$-1+1+0$

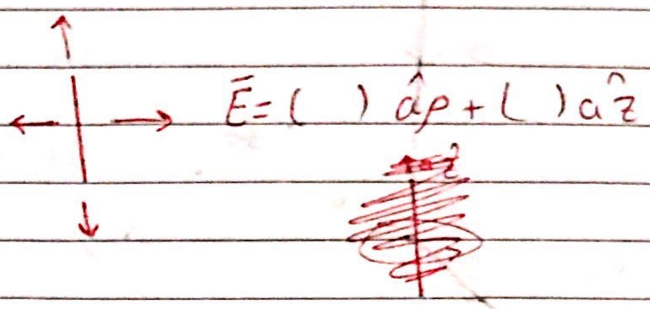
$\cos\theta_1 = 0$

$\bar{AB} = (-1,1,0) \rightarrow AB = \sqrt{2}$

ex:



Find $\bar{H}$ at $(0,0,5)$?

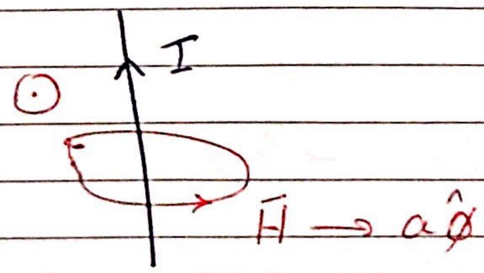


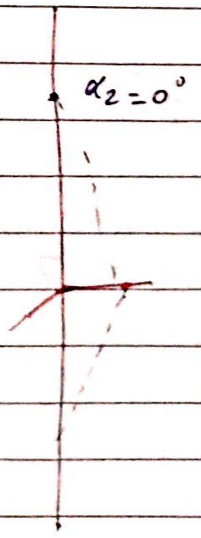
$$\bar{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\hat{a\phi}$$

for an infinit line

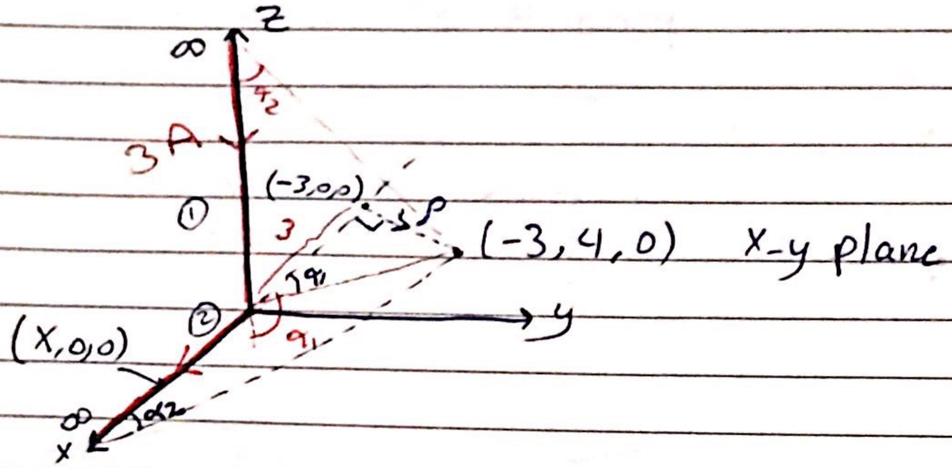
$$\boxed{\bar{H} = \frac{I}{2\pi\rho}\,\hat{a\phi}}$$

always for any inf line





Ex: find $\bar{H}$ at $(-3,4,0)$



$\bar{H} = \bar{H}_1 + \bar{H}_2$

- $H_1 = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\, a\hat{\phi}$

$\alpha_1 = 90^\circ$

$\alpha_2 = 0^\circ$

$\bar{\rho} = (-3,4,0)$, $\rho = 5\ m$

$a\hat{\phi} = a\hat{l} \times a\hat{\rho}$

$= -a\hat{z} \times \frac{(-3,4,0)}{5} = \frac{4}{5} a\hat{x} + \frac{3}{5} a\hat{y}$

$\bar{H}_1 = \frac{I}{4\pi\rho} a\hat{\phi}$

$\bar{H}_1 = -47.75\ a\hat{\phi}\ mA/m \rightarrow cyl$

$\bar{H}_1 = 38.2\ a\hat{x} + 28.65\ a\hat{y}\ mA/m \rightarrow cart$

$$\bar{H}_2 = \frac{I}{4\pi\rho}\left(1+\frac{3}{5}\right)\hat{a\phi}$$

$\cos\alpha_1 = -\frac{3}{5}$

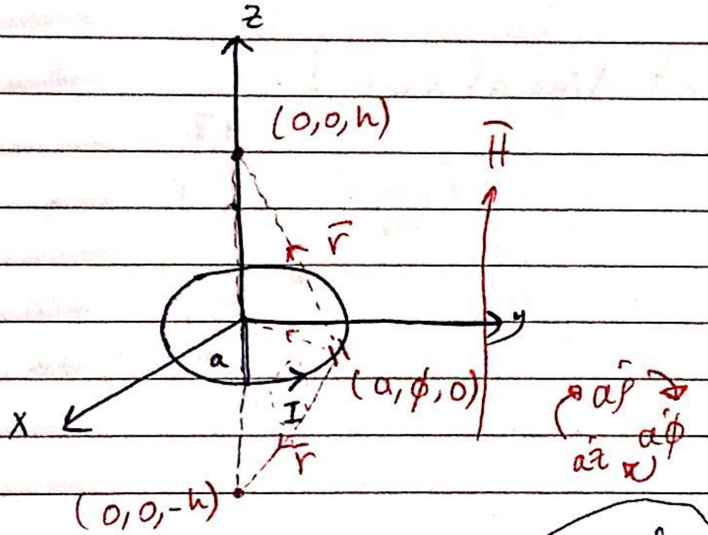
$\alpha_2 = 0°$

$\bar{\rho} = 4\hat{ay}$, $\rho = 4\,m$

$\hat{a\phi} = \hat{al} \times \hat{a\rho}$
$= \hat{ax} \times \hat{ay}$
$= \hat{az}$

$\bar{H}_2 = 23.88\hat{az}\ mA/m$

<u>ex:</u> find $\bar{H}$ at $(0,0,h)$ and $(0,0,-h)$



$$\bar{H} = \int_l \frac{I\,\bar{dl} \times \bar{r}}{4\pi r^3} \quad \text{at } (0,0,h)$$

$\bar{dl} = a\,d\phi\,\hat{a\phi}$

$\bar{r} = -a\,\hat{a\rho} + h\hat{az}$ ($-$)

$r = \sqrt{a^2+h^2}$

$$\bar{H} = \frac{I}{4\pi}\int_0^{2\pi} \frac{a\,d\phi\,\hat{a\phi} \times (-a\hat{\rho} + h\hat{az})}{[a^2+h^2]^{3/2}} \rightarrow \text{due to symmetry}$$

$\hat{az}$ $\hat{a\rho}$ ($-$)

$$\boxed{\bar{H} = \frac{Ia^2}{2(a^2+h^2)^{3/2}}\hat{az}\ A/m}$$

at $(0,0,-h)$
Same Answ

* Ampere's law :-

$$\boxed{\oint_L \bar{H}\cdot\bar{dL} = I_{enq}}$$ → 3rd maxwell's eq in integral form

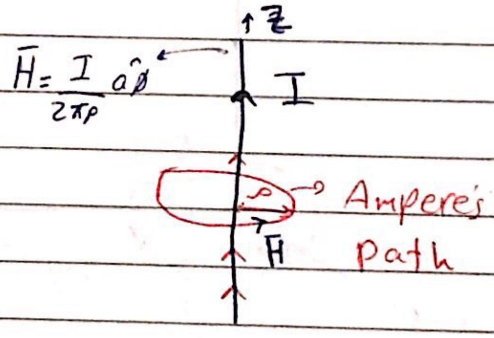
Apply Stocke's theorem

$$\oint_L \bar{H}\cdot\bar{dL} = \int_S \nabla\times\bar{H}\cdot\bar{ds} = I_{enc} = \int_S \bar{J}\cdot\bar{ds}$$

$$\boxed{\nabla\times\bar{H} = \bar{J}}$$ → 3rd maxwell's eq in diff form

* applications on Ampere's law :-

1) find $\bar{H}$ for an infinit line of current.



$$\oint_L \bar{H}\cdot\bar{dL} = I_{enc} = I$$

from R.H.R

$\bar{H} = H\phi \, \hat{a\phi}$

$\bar{dL} = \rho \, d\phi \, \hat{a\phi}$

$$\int_0^{2\pi} H\phi \, \hat{a\phi} \, \rho \, d\phi \, \hat{a\phi} = I$$

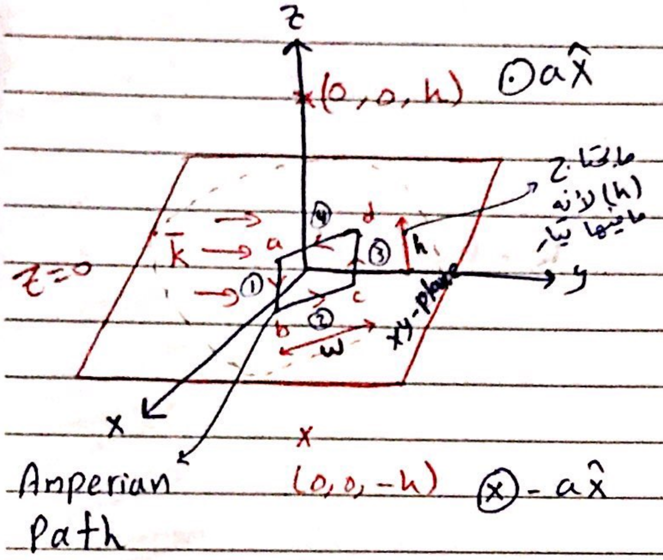
$$2\pi\rho \, H\phi = I$$

$$\bar{H} = \frac{I}{2\pi\rho} \, \hat{a\phi}$$

2) find $\bar{H}$ for an infinit sheet of current

i.e for $z=0$ plane carry current $\bar{K} = K_y \hat{a_y}$.

find $\bar{H}$ every where. $\rightarrow$ $z>0$, $z<0$



$$\oint_L \bar{H} \cdot \bar{dl} = I_{enc} = K_y W$$

(R,H,R)

$$\bar{H} = \begin{cases} H_o \hat{a_x}, & z>0 \\ H_o(-\hat{a_x}), & z<0 \end{cases}$$

$$\oint_L \bar{H} \cdot \bar{dl} = \int_a^b \cancel{\bar{H} \cdot \bar{dl_1}}^{\,0} + \int_b^c \bar{H} \cdot \bar{dl_2} + \int_c^d \cancel{\bar{H} \cdot \bar{dl_3}}^{\,0} + \int_d^a \bar{H} \cdot \bar{dl_4} = K_y W$$

$$\int_b^c H_o\, \cancel{(-\hat{a_x})} \cdot dx\, \cancel{(-\hat{a_x})} + \int_d^a H_o\, \cancel{\hat{a_x}} \cdot dx\, \cancel{\hat{a_x}} = K_y W$$

$$H_o W + H_o W = K_y W$$

$$H_o = \frac{K_y}{2}$$

* Recall CH4

$$E = \frac{\rho_s}{2\epsilon_o}$$

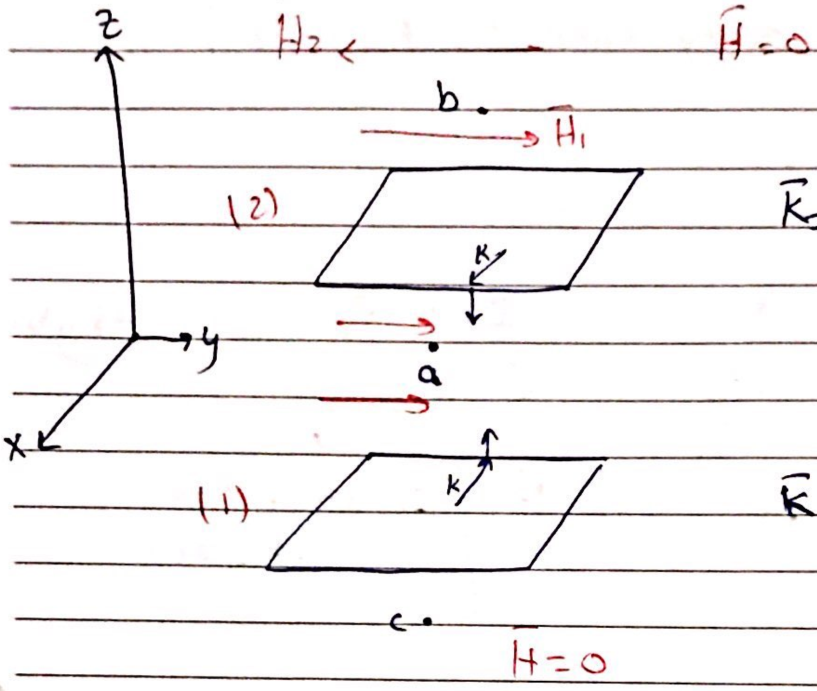
$$\bar{H} = \begin{cases} \frac{K_y}{2} \hat{a_x}, & z>0 \\ \frac{K_y}{2}(-\hat{a_x}), & z<0 \end{cases}$$

$\Rightarrow$ $\boxed{\bar{H} = \frac{1}{2} K \times \hat{a_n}}$ always for any inf sheet $(\hat{a_n})$

$\hat{a_x} \leftarrow \hat{a_y}$, $\hat{a_z}$

$-\hat{a_x} \leftarrow$, $-\hat{a_z}$

i.e for a parallel plate capacitor, find $\bar{H}$ every where



$\bar{K} = K_x \hat{a}_x$

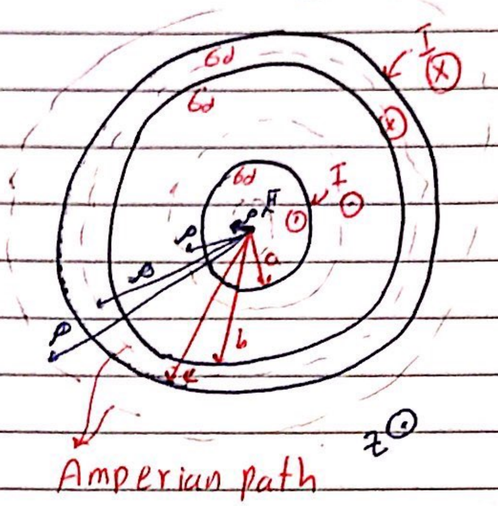
$\bar{K} = K_x(-\hat{a}_x)$

$$H\Big|_a = \bar{H_1} + \bar{H_2}$$

$$= \frac{1}{2} K_x(-a\hat{x}) \times a\hat{z} + \frac{1}{2} K_x \hat{a}x \times -a\hat{z}$$

$$= \frac{K_x}{2} a\hat{y} + \frac{K_x}{2} a\hat{y} = K_x a\hat{y}$$

3) Find $\bar{H}$ for an infinit coaxial cable :-



$$\oint_l \bar{H} \cdot \bar{dl} = I_{enc}$$

- for $0 < \rho < a$

$$\bar{H} = H_\phi \, \hat{a\phi}$$
$$\bar{dl} = \rho \, d\phi \, \hat{a\phi}$$

$$\int_0^{2\pi} H\phi \, \hat{a\phi} \, \rho \, d\phi \, \hat{a\phi} = I_{enc} = \int_s \bar{J} \cdot \bar{ds}$$

$$\underset{?}{H\phi} \; 2\pi\rho = \int_s \underset{?}{\bar{J}} \cdot \bar{ds} \quad \text{--- (1)}$$

at $\rho = a \longrightarrow I_{enc} = I \Rightarrow I = \int_s \bar{J} \cdot \bar{ds}$

$$I = \int_0^{2\pi} \int_0^{a} J_z \, \hat{az} \cdot \rho \, d\rho \, d\phi \, \hat{az}$$

$$I = J_z \frac{a^2}{2} 2\pi \Rightarrow \boxed{\bar{J} = \frac{I}{\pi a^2} \hat{az}}$$

$$H\phi \; 2\pi\rho = \int_0^{2\pi} \int_0^{\rho} \frac{I}{\pi a^2} \hat{az} \cdot \rho \, \partial\rho \, \partial\phi \, \hat{az} \Rightarrow H\phi \, 2\pi\rho = \frac{I}{\pi a^2} \frac{\rho^2}{2} 2\pi$$

$$2\pi\rho H\phi = I \frac{\pi\rho^2}{\pi a^2} \Rightarrow \boxed{\bar{H} = \frac{I\rho}{2\pi a^2} \hat{a\phi}} \quad 0 < \rho \leq a$$

• for $a<\rho<b$

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$\rightarrow 2\pi\rho H_\phi = I_{enc} = I$

*1 $\boxed{\bar{H} = \frac{I}{2\pi\rho} a\hat{\phi}}$ $a \leq \rho < b$

• for $b<\rho<c$

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$\rightarrow 2\pi\rho H_\phi = I_{enc} = I - \int_s \bar{J} \cdot ds$ --- (2)

? ?

at $\rho = c \rightarrow I_{enc} = I$ $b<\rho<c$

$$I = \int_0^{2\pi}\int_b^c J_z (-\cancel{a\hat{z}}) \cdot \rho\, d\rho\, d\phi (\cancel{-a\hat{z}})$$

$$I = J_z \frac{(c^2-b^2)}{2} 2\pi$$

$$\boxed{\bar{J} = \frac{I}{\pi(c^2-b^2)} a\hat{z}}$$ sub in (2)

$$2\pi\rho H_\phi = I - \int_0^{2\pi}\int_b^\rho \frac{I}{\pi(c^2-b^2)} a\hat{z}\, \rho\, d\rho\, d\phi\, a\hat{z}$$

$$2\pi\rho H_\phi = I - \frac{I}{\pi(c^2-b^2)} \frac{\rho^2-b^2}{2} 2\pi$$

$$2\pi\rho H_\phi = I\left(\frac{1-\pi(\rho^2-b^2)}{\pi(c^2-b^2)}\right)$$

*2 $$\boxed{\bar{H} = \frac{I}{2\pi\rho}\left(1-\frac{\rho^2-b^2}{c^2-b^2}\right)\hat{a_\phi}}$$ $b \le \rho \le c$

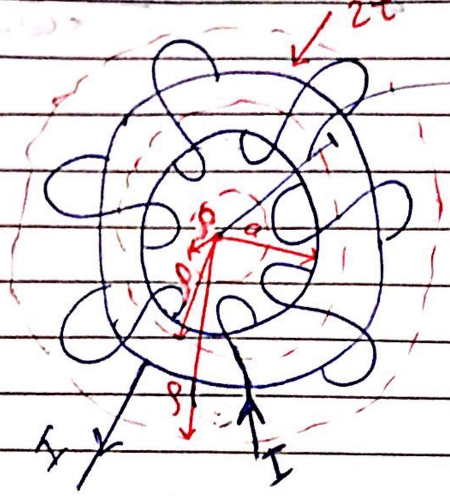
• for $\rho \ge c$

$$2\pi\rho H_\phi = I_{enc} = I - I = 0$$

*3 $\boxed{\bar{H}=0}$ $\rho \geq c$

$$\bar{H} = \begin{cases} {}^{*1}\ \dfrac{I}{2\pi\rho}\hat{a_\phi}, & a \le \rho < b \\ {}^{*2}\ \dfrac{I}{2\pi\rho}\left(1-\dfrac{\rho^2-b^2}{c^2-b^2}\right)\hat{a_\phi}, & b \le \rho \le c \\ {}^{*3}\ 0, & \rho \geq c \end{cases}$$

4) Toroid $\longrightarrow$ Find H every where



$0<\rho<a \rightarrow I_{enc}=0 \rightarrow \bar{H}=0$

$\rho>a+2t \rightarrow I_{enc}=I-I=0$

$\bar{H}=0$

or $\rho>\rho_0+t$

$\rho_0=a+t \qquad \rho_0-t<\rho<\rho_0+t$

$$\oint_l \bar{H}\cdot\bar{dl}=I_{enc}$$

$\bar{H}=H_\phi \hat{a_\phi} \qquad \bar{dl}=\rho d\phi \hat{a_\phi}$

$$2\pi\rho H_\phi=I_{enc}=NI$$

$$\boxed{\bar{H}=\frac{NI}{2\pi\rho}\hat{a_\phi}} \qquad \bar{H}=\frac{NI}{l}\hat{a_\phi}$$

$$\bar{H}=\frac{NI}{l} \rightarrow \bar{H}\cdot\bar{l}=NI$$

$$\boxed{\int_l \bar{H}\cdot\bar{dl}=NI}$$ $\rightarrow$ Ampere's Law for N-turn

$\bar{H}=\frac{NI}{l}\hat{a_\phi}=nI\hat{a_\phi}$, $\frac{N}{l}=n\equiv$ turns density

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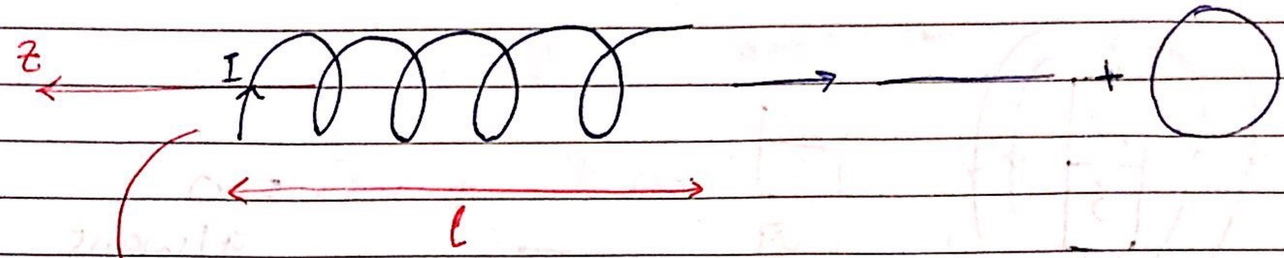
Ex in your book (solenoid)

finitline ring



finit solenoid → Biot - savart's

Infinit solenoid → Biot - savart's → Ampere's

$\bar{H} = nI\hat{a_z}$

* Magnetic flux density :-

$\bar{B} = \mu_0 \bar{H}$, $\mu_0 \equiv$ free space permeability (H/m) $= 4\pi * 10^{-7}$ (H/m)

unit of $\bar{B}$ → $\frac{H}{m}\frac{A}{m} = \frac{H.A}{m^2} = \frac{Wb}{m^2}$, Wb = weber

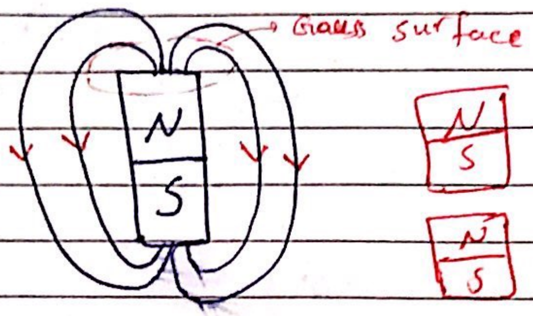
old : G ≡ Gauss $1G = 10^{-5}T$ T = Tesla

magnetic flux :-

$\Psi_m = \int_s \bar{B} . \bar{ds}$ in Wb

$\Psi_m = \oint_s \bar{B} . \bar{ds}$ → magnetic Gauss Law

Flux line :-



$\Rightarrow \Psi_m = \oint_S \bar{B} \cdot \bar{ds} = 0$ always

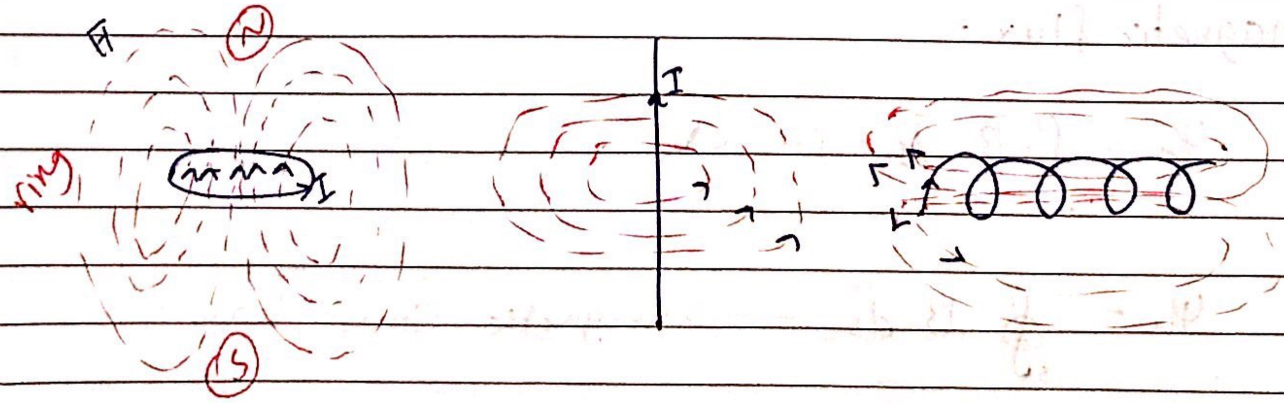
Since there is no single pole magnet

4th maxwell's eq in integral form

in Diff form
Apply divergence theorem

$$\oint_S \bar{B} \cdot \bar{ds} = \int_V \nabla \cdot \bar{B} \, dv = 0$$

$\boxed{\nabla \cdot \bar{B} = 0}$ 4th maxwell's eq in diff form



maxwell's eq for static $\overset{\text{Dc}}{\text{fields}}$

integral	diff
$\oint_S \bar{D}\cdot\bar{ds} = \int_V \rho_v\, dv$	$\nabla\cdot\bar{D} = \rho_v$
$\oint_L \bar{E}\cdot\bar{dl} = 0$	$\nabla\times\bar{E} = 0$ *
$\oint_L \bar{H}\cdot\bar{dl} = I_{enc} = \int_S \bar{J}\cdot\bar{dl}$	$\nabla\times\bar{H} = \bar{J}$ *
$\oint_S \bar{B}\cdot\bar{ds} = 0$	$\nabla\cdot\bar{B} = 0$

*magnetic potential :-

Scaler mag potential (V_m)
Vector mag potential ($\bar{A}$)

using maxwell
③ $\nabla\times\bar{H} = \bar{J}$ ... $\boxed{1}$
④ $\nabla\cdot\bar{B} = 0$ ... $\boxed{2}$

and math identities
$\nabla\cdot(\nabla\times\bar{A}) = 0$ --- ⓐ
$\nabla\times(-\nabla V) = 0$ --- ⓑ

from (1) and (b)
if $\bar{J} = 0 \Rightarrow \boxed{\bar{H} = -\nabla V_m}$ by duality $\bar{E} = -\nabla V$

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from (2) and (a)

$\bar{B} = \nabla \times \bar{A}$, $\bar{H} = \dfrac{\bar{B}}{\mu_0} \Longrightarrow \bar{B} = \mu_0 \bar{H}$

How to find $\bar{A} = ?$

by duality:

line charg $\longrightarrow$ line current

$V = \displaystyle\int_L \frac{\rho_L \, dL}{4\pi \epsilon_0 r}$ $\qquad$ $\bar{A} = \displaystyle\int_L \frac{\mu_0 I \, \bar{dL}}{4\pi r}$

surface charge $\longrightarrow$ surface current

$V = \displaystyle\int_s \frac{\rho_s \, ds}{4\pi \epsilon_0 r}$ $\qquad$ $\bar{A} = \displaystyle\int_s \frac{\mu_0 \bar{K} \, ds}{4\pi r}$

Volume charg $\longrightarrow$ $V = \displaystyle\int_v \frac{\rho v \, dv}{4\pi \epsilon_0 r}$

Volume current $\longrightarrow$ $\bar{A} = \displaystyle\int_v \frac{\mu_0 \bar{J} \, dv}{4\pi r}$, $\bar{B} = \displaystyle\int_L \frac{\mu_0 I \, \bar{dL} \times \bar{r}}{4\pi r^3}$

• $\dfrac{\bar{r}}{r^3} = -\nabla\left(\dfrac{1}{r}\right)$

magnetic flux

① $\Psi_m = \displaystyle\int_s \bar{B} \cdot \bar{ds}$

$\Psi_m = \displaystyle\int_s \nabla \times \bar{A} \cdot ds$

Applying stocke's Theorem:

② $\Psi_m = \displaystyle\oint_L \bar{A} \cdot \bar{dL}$

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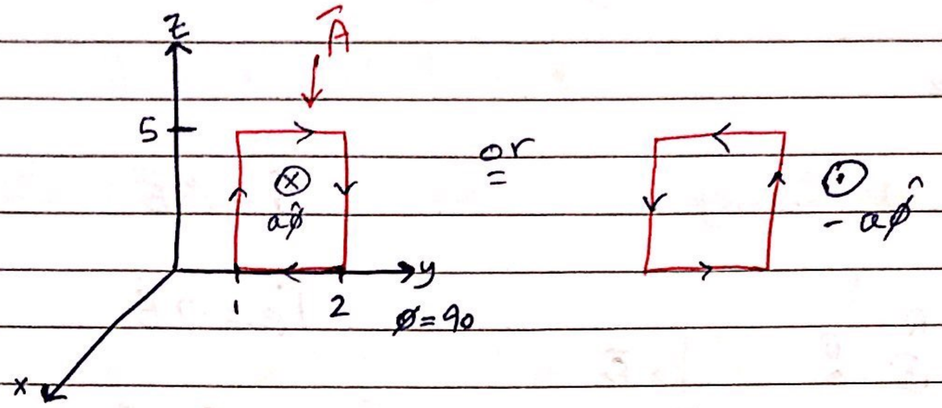
Ex Given $\bar{A} = \frac{-\rho^2}{4}\hat{a_z}$ wb/m calculate the magnetic flux crossing the surface $\phi = \frac{\pi}{2}$ $1 \le \rho \le 2$, $0 \le z \le 5$



+ve

$$\Psi_m = \oint_L \bar{A}\cdot d\bar{l} = \int_{L_1} \bar{A}\cdot d\bar{l}_1 + \cancel{\overset{0}{\int_{L_2} \bar{A}\cdot d\bar{l}_2}} + \int_{L_3} \bar{A}\cdot d\bar{l}_3 + \cancel{\overset{0}{\int_{L_4} \bar{A}\cdot d\bar{l}_4}}$$

$d\bar{l}_1 = dz\,\hat{a_z}$ $\qquad$ $d\bar{l}_2 = d\rho\,\hat{a_\rho}$

$d\bar{l}_3 = (-)\,dz\,\hat{a_z}$ $\qquad$ $d\bar{l}_4 = (-)\,d\rho\,\hat{a_\rho}$

$\int$

$$\Psi_m = \int_0^5 \underbrace{\frac{-\rho^2}{4}}_{\rho=1} \cancel{\hat{a_z}}\cdot dz\,\cancel{\hat{a_z}} + \int_5^0 \underbrace{\frac{-\rho^2}{4}}_{\rho=2} \cancel{\hat{a_z}}\cdot dz\,\cancel{\hat{a_z}}$$

$$= -\frac{1}{4}(5) - 1(-5) = -\frac{5}{4} + 5 = \frac{15}{4} \text{ wb}$$

method(2)

$\Psi_M = \int_S \bar{B}\cdot d\bar{s}$ $\qquad$ $\bar{B} = \nabla \times \bar{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a_\rho} & \rho\hat{a_\phi} & \hat{a_z} \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ \overset{0}{A_\rho} & \overset{0}{\rho A_\phi} & A_z \end{vmatrix}$ $\qquad$ $A_z = -\rho^2/4$

$\bar{B} = \frac{1}{\rho}\left[\frac{\rho}{2}\,\rho\,\hat{a_\phi}\right] \Rightarrow \Psi_m = \int_0^5\int_1^2 \frac{\rho}{2}\cancel{\hat{a_\phi}}\,d\rho\,dz\,\cancel{\hat{a_\phi}}$

$$= \frac{4-1}{4}(5) = \frac{15}{4} \text{ wb}$$

*CH8 : mgnetic Force and magnatic matirials :-

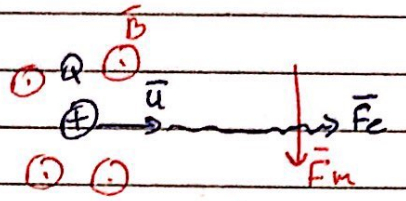
There are 3 cases for magnetic forces.

1) The force in a moving charged particle

charge

static ⊕ $\bar{F}_e$ only $\bar{u} = \mu \bar{E}$

$\bar{F}_e = Q \bar{E}$



moving
→ $\bar{F}_e$
→ F_m

$\bar{F}_m = Q \bar{u} \times \bar{B}$ $u \neq 0$
R·H·R $\bar{B} \neq 0$

$\bar{F} = \bar{F}_e + \bar{F}_m$ if $\bar{E} \neq 0$, $B \neq 0$, $u \neq 0$

$\boxed{\bar{F} = Q(\bar{E} + \bar{u} \times \bar{B}) = m\bar{a}}$ Lorentz's force of law

special case if $\bar{E} \neq 0$ $\bar{B} \neq 0$ but u is uniform $a = 0$

$\bar{F}_e + \bar{F}_m = 0$

$\therefore \bar{F}_e = -\bar{F}_m$

$Q\bar{E} = -Q \, \bar{u} \times \bar{B}$

$\bar{E} = \bar{B} \times \bar{u}$.

magnetude

$\bar{E} = Bu$

$\boxed{u = \frac{E}{B}}$ if $a = 0$

$K.E = \frac{1}{2} m |\bar{u}|^2 \ (J)$

$\bar{F}_e$ will affect K.E

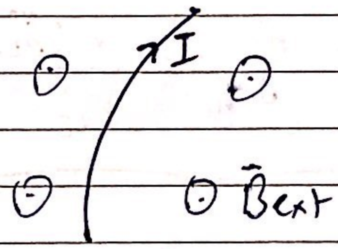
$\bar{F}_m$ will not affect K.E

$\bar{F}_e >> \bar{F}_m$ unless $\bar{u}$ is very large

2) the force on a current element $\left(\begin{array}{ll} 1D & \text{segment} \\ 2D & \text{surface} \\ 3D & \text{object} \end{array} \right)$

assume $\bar{E}_{ext} = 0$

$\bar{F}_m = ?$



* $\bar{F}_m = Q\,\bar{u} \times \bar{B}$ → moving point charge

* $I\,d\bar{l} = \bar{K}\,ds = \bar{J}\,dv$

* $\bar{J} = \rho_v\,\bar{u}$ * $Q = \int_v \rho_v\,dv$

$d\bar{F}_m = dQ\,\bar{u} \times \bar{B}$

$dQ = \rho_v\,dv$

$d\bar{F}_m = \rho_v\,dv\,\bar{u} \times \bar{B}$

$d\bar{F}_m = \bar{J} \times \bar{B}\,dv$

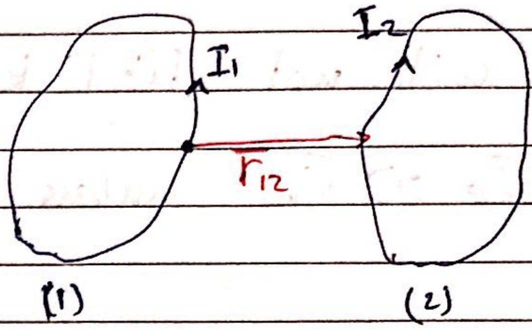
$\boxed{d\bar{F}_m = I\,d\bar{l} \times \bar{B}} = \bar{K}\,ds \times \bar{B}$

$\bar{F}_m = \oint_L I\,d\bar{l} \times \bar{B}$ $\bar{F}_m = \oint_s \bar{K}\,ds \times \bar{B}$

$\bar{F}_m = \int_v \bar{J}\,dv \times \bar{B}$

3) The force between tow current elements :-

$\overline{F}_{12}$ = The force on (2) due to (1)



$$\overline{F}_{m_{12}} = \int_{L_2} I_2 \, \overline{dL_2} \times \overline{B}_1$$

$$\overline{B}_1 = \int_{l_1} \frac{\mu_0 I_1 \overline{dL_1} \times \overline{r}_{12}}{4\pi r^3_{12}} \quad , \overline{r}_{12} = (2) - (1) \text{ coordinates}$$

$$\boxed{F_{m_{12}} = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{l_2} \oint_{l_1} \frac{\overline{dL_2} \times (\overline{dL_1} \times \overline{r}_{12})}{r_{12}^3} \quad (N)}$$

for force between tow surface current :-

$$\overline{F}_{m_{12}} = \int_{S_2} \int_{S_1} \frac{\mu_0 \ \overline{K}_2 \times (\overline{K}_1 \times \overline{r}_{12})}{4\pi \ r_{12}^3} ds_1 \, ds_2$$

$$\overline{B}_1 = \oint_{S_1} \frac{\mu_0 \ \overline{K}_1 \times \overline{r}_{12}}{4\pi \ r_{12}^3} ds_1$$

$$\overline{F}_{m_{12}} = \oint_{S_2} K_2 \, ds_2 \times \overline{B}_1$$

Ex A charged particle of mass 2kg and charge 3C starts at point (1, -2, 0) with velocity ($4\hat{a_x} + 3\hat{a_z}$ m/s) in $\bar{E} = 12\hat{a_x} + 10 a_y$ V/m At time t = 1 s, find:-

a) $\bar{a}$, b) $\bar{u}$, c) K.E , d) position

$$\bar{F} = Q\bar{E}_{ext} + \cancel{Q\bar{u} \times \bar{B}_{ext}} = m\bar{a}$$

$$Q\bar{E} = m\bar{a} \rightarrow \bar{a} = \frac{Q}{m}\bar{E} = \frac{3}{2}(12, 10, 0)$$

$$\bar{a} = 18\hat{a_x} + 15\hat{a_y} \quad m/s^2$$

$$\bar{a} = \frac{d\bar{u}}{dt}$$ vector diff equ

$$a_x = \frac{du_x}{dt} \longrightarrow \int du_x = \int a_x \, dt$$

$$u_x = 18t + c_1$$

$$a_y = \frac{du_y}{dt} \longrightarrow u_y = 15t + c_2$$

$$a_z = \frac{du_z}{dt} \longrightarrow u_z = c_3$$

find c_1, c_2, c_3

I.C $\bar{u}(t=0) = (4, 0, 3)$

$u_x(t=0) = \boxed{4 = c_1}$ $\boxed{c_2 = 0}$ $\boxed{c_3 = 3}$ $\Rightarrow$

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$\bar{u}(t) = (18t + 4, 15t, 3)$ m/s

$u(15) = 22a\hat{x} + 15a\hat{y} + 3a\hat{z}$ m/s

c) $K.E\Big|_{t=1} = \frac{1}{2}(2)(22^2 + 15^2 + 3^2)$ J $= 7.16$ J

$K.E\Big|_{t=0} = \frac{1}{2}(2)(25) = 25$ J, $\because$ $\bar{F}_c$ affect K.E

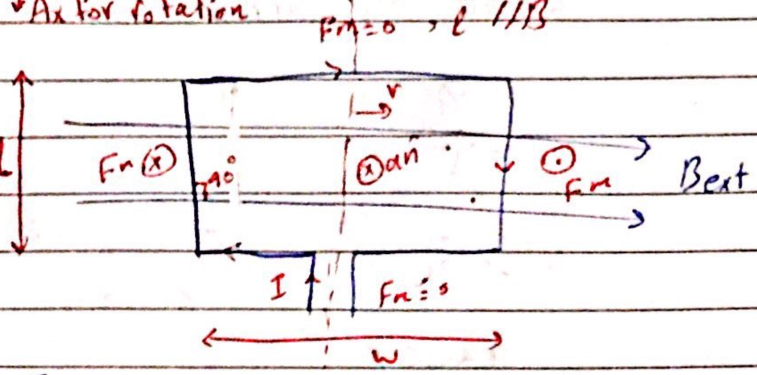
d) $\bar{l}$?

$\bar{u} = \frac{d\bar{l}}{dt}$ or $\bar{a} = \frac{d^2 l}{dt^2}$

*Magnetic Torque ($\bar{T}$) and moment ($\bar{m}$) :-

$\bar{T} = \bar{r} \times \bar{F}$ in (N.m)

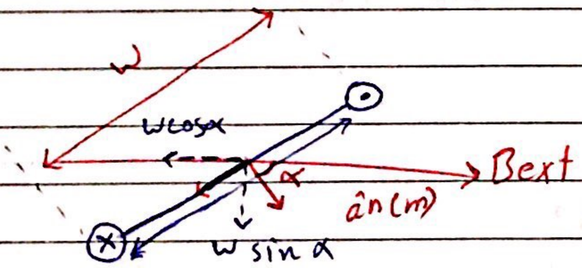
Ax for rotation



$\bar{F}_m = \oint_C I \, d\bar{l} \times \bar{B}$

if uniform $F = I \bar{l} \times \bar{B}$

$\bar{F}_{\substack{total \\ (Net)}} = 0$



$\bar{T} = \bar{r} \times \bar{F}$

$T = rF$

$= w \, I \, \ell B \sin\alpha \Rightarrow T = I B S \sin\alpha$

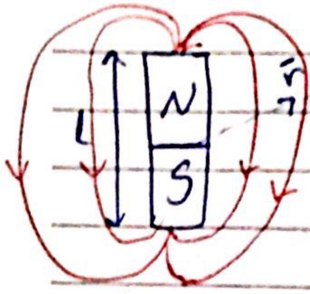
$\boxed{\bar{T} = \bar{m} \times \bar{B}} = m B \sin\alpha$

define $\bar{m} \equiv$ mag dipole moment (duality $\underline{p} \rightarrow$ elc dipole moment)

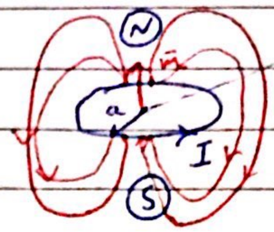
$\bar{m} = I S \hat{a}_n$

* magnatic dipole :-

small bar magnet | small loop carrying current



$L <<< r$



$\bar{m} = I_s \, a\hat{n}$

$S = \pi a^2$

$a <<< r$

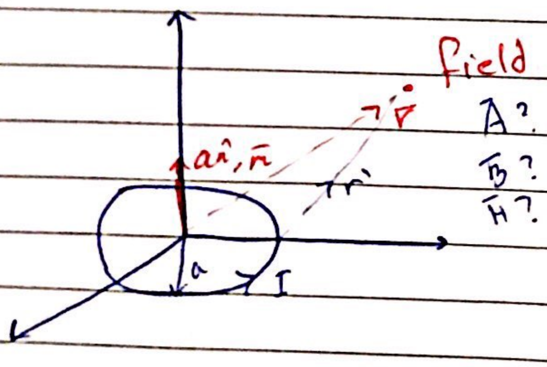
$\bar{T} = Q_m \, l$

$\bar{T} = \bar{m} \times \bar{B}_{ext}$

$$\boxed{Q_m = \frac{|\bar{m} \times \bar{B}|}{|\bar{l}|}}$$

• for a small loop of current where $a <<< r$

μ_0



$$\bar{A} = \int_L \frac{\mu_0 \, I \, d\bar{L}}{4\pi r} \Rightarrow \text{Bionomial expantion}$$

$r \rightarrow r'$

Duality from electric dipole

$a\phi \to a\hat{r} \to a\hat{\theta} \to a\phi$

$\bar{A} = \frac{\mu_0 \bar{m} \times a\hat{r}}{4\pi r^2}$ (V in elc dipole)

$$\boxed{\bar{A} = \frac{\mu_0 I \pi a^2 \sin\theta}{4\pi r^2} a\hat{\phi}}$$

$\bar{m} = I s a\hat{n}$

$= I\pi a^2 a\hat{z}$ / $\hat{a}z = \cos\theta\, \hat{a}r - \sin\theta\, \hat{a}\theta + 0\, \hat{a}\phi$

$$\bar{B} = \nabla \times \bar{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a\hat{r} & r a\hat{\theta} & r\sin\theta\, a\hat{\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ \cancel{Ar}^{0} & \cancel{rA\theta}^{0} & r\sin\theta A\phi \end{vmatrix}$$

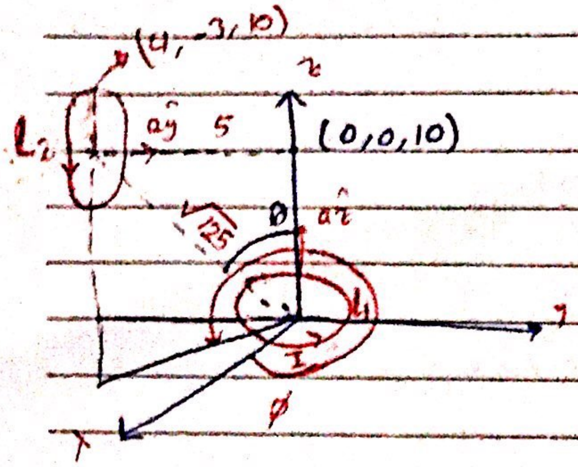
$$\bar{B} = \frac{1}{r^2 \sin\theta} \cdot \frac{\mu_0 I a^2}{4\pi} \left(\frac{2\sin\theta\cos\theta}{r^3} a\hat{r} - \frac{(-1)\sin^2\theta}{r^2} r a\hat{\theta} \right)$$

$$\bar{B} = \frac{\mu_0 I \pi a^2}{4\pi r^3} \left[2\cos\theta\, a\hat{r} + \sin\theta\, a\hat{\theta} \right]$$

$$\bar{B} = \frac{\mu_0 m}{4\pi r^3} \left(2\cos\theta\, a\hat{r} + \sin\theta\, a\hat{\theta} \right) \quad Wb/m^2$$

$$\bar{H} = \frac{\bar{B}}{\mu_0}$$

Ex: a small loop (L_1) with $\bar{m} = 5\hat{a}_z$ $A \cdot m^2$ Located at the origion while another small loop (L_2) located at $(4,-3,10)$ with $\bar{m} = 3\hat{a}_y$ $A \cdot m^2$ find the Torque on L_2 ?

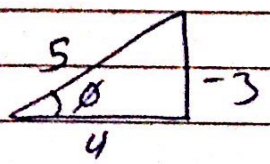


$$\bar{T}_2 = \bar{m}_2 \times \bar{B}_1$$

$$\bar{B}_1 = \frac{\mu_0 (5)}{4\pi(\sqrt{125})^3}\left(\frac{2 \cdot 10}{\sqrt{125}}\hat{a}_r + \frac{5}{\sqrt{125}}\hat{a}_\theta\right)$$

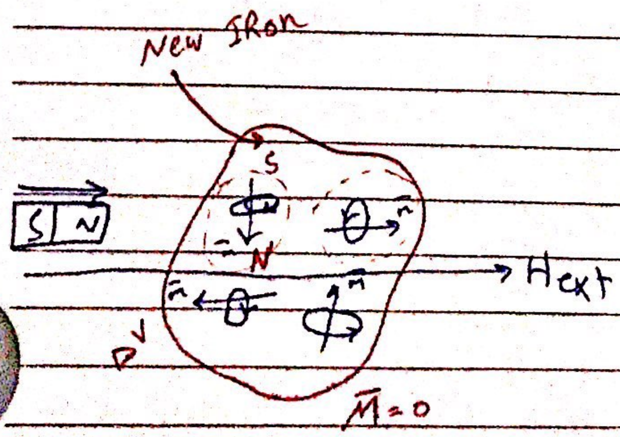
$$\bar{m}_2 = 3a_y = 3(\underbrace{\sin\theta}_{\frac{5}{\sqrt{125}}}\sin\phi\,\hat{a}_r + \underbrace{\cos\theta}_{\frac{10}{\sqrt{125}}}\sin\phi\,\hat{a}_\theta + \cos\phi\,\hat{a}_\phi)$$

$$\tan\phi = \frac{y}{x} = \frac{-3}{4}$$



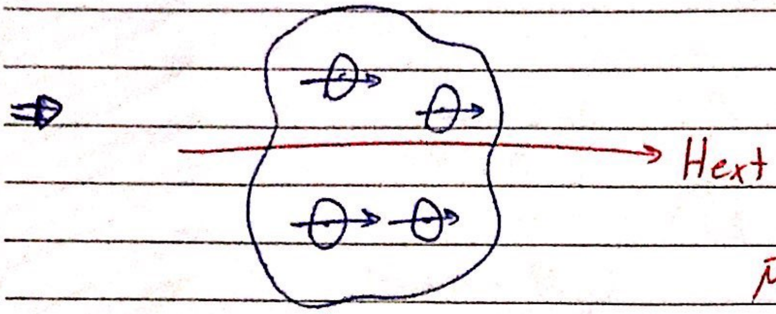
$$\sin\phi = \frac{-3}{5} \qquad \cos\phi = \frac{4}{5}$$

* magnetization in materials



$$\bar{M} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} \bar{m}_k}{\Delta v}$$

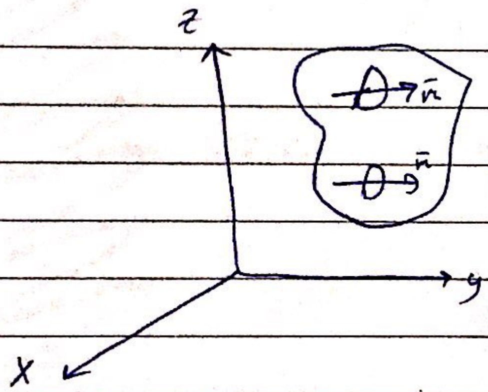
$\bar{F}_m$ wont affect K.E



$\bar{M}_s \equiv$ saturation magnetization

$\bar{M} \neq 0$

• for a magnetized material :-



• field

$\bar{A}$?

B? $\quad \bar{B} = \nabla \times \bar{A}$

H? $\quad \bar{H} = \frac{\bar{B}}{\mu_0}$

Duality

$$\boxed{\begin{aligned} \rho_{ps} &= \bar{P} \cdot \hat{a}_n \\ \rho_{pv} &= -\nabla \cdot \bar{P} \end{aligned}} \qquad \boxed{\begin{aligned} \bar{K}_b &= \bar{M} \times \hat{a}_n \\ \bar{J}_b &= \nabla \times \bar{M} \end{aligned}}$$

free: ρ_s, ρ_v ; $\bar{K}, \bar{J}$, bound: ρ_{ps}, ρ_{pv} ; $\bar{K}_b, \bar{J}_b$

$\bar{K}_b$ = bond (magnetized) surface current density (A/m)

$\bar{J}_b$ = " " volume " " (A/m²)

for some material [• linear • isotropic

$\bar{M} = \chi_m \bar{H}$, $\bar{P} = \chi_e \varepsilon_0 \bar{E}$

Types of magnetic materials:-

- → Dia magnetic ($\mu_r \lesssim 1$)
  Cu, AL, Au, ...
- → para magnetic ($\mu_r \gtrsim 1$)
  Air, ...

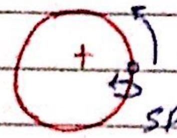
→ Linear

$\bar{B} = \mu \bar{H}$

$\bar{B} = \mu_0 \mu_r \bar{H}$

$\bar{M} = \chi_m \bar{H}$

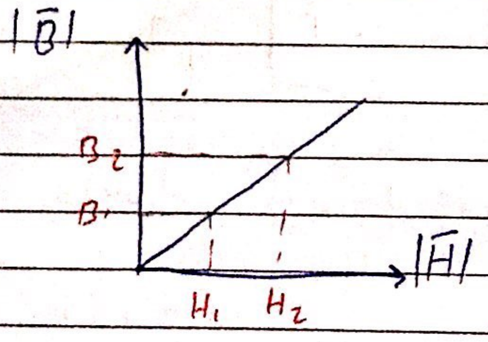
- → Ferromagnetic ($\mu_r >>>> 1$) → Non linear $\bar{B} = \mu \bar{H}$ only at point
  Iron, Nickle, cobelt...



Diamagnetic
→ B_o cancel B_s

Ferromagnetic
→ B_o in-phase with B_s

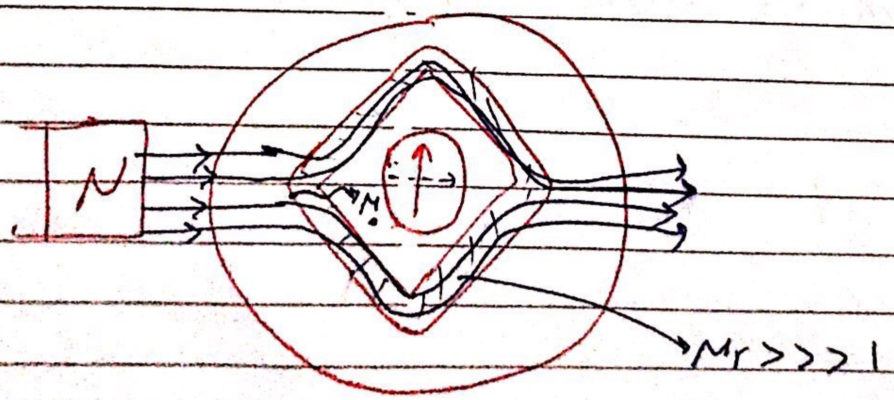
• B/H curve (For linear material)

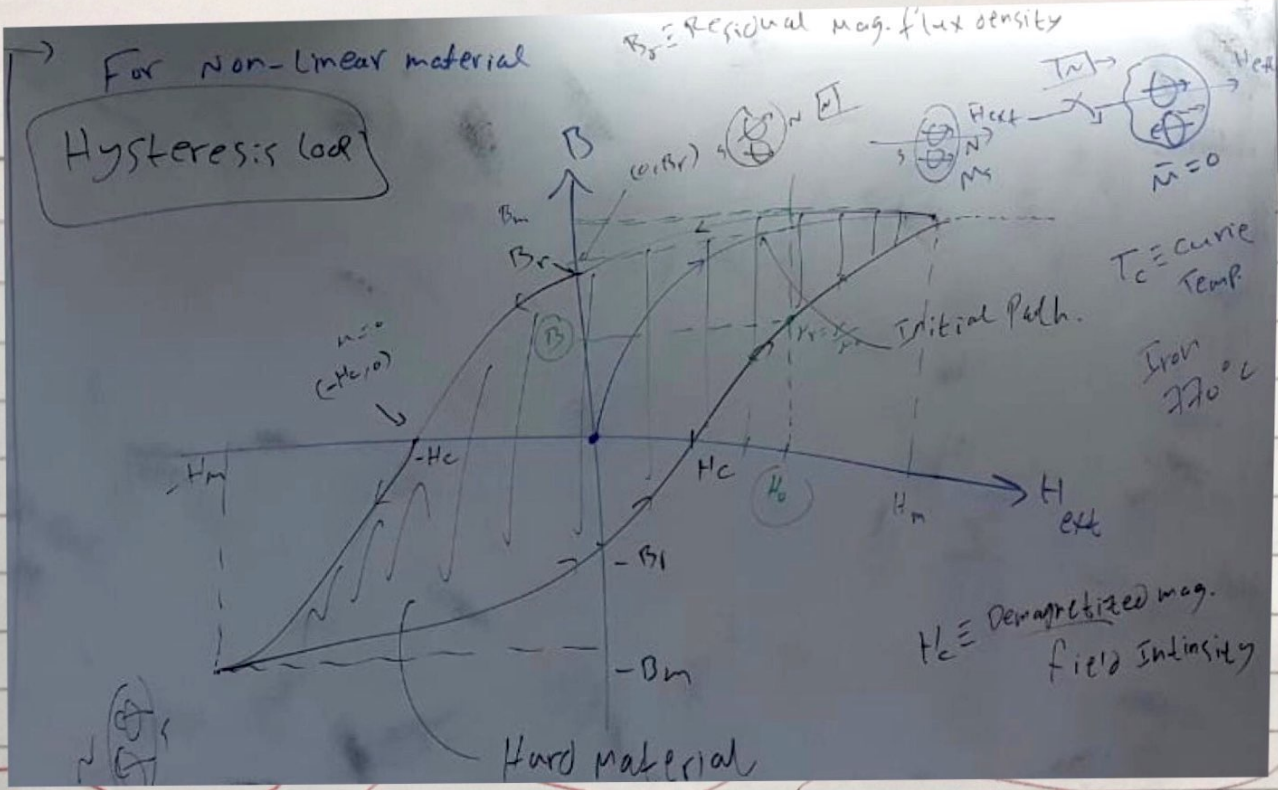


$$\mu = \frac{B_2 - B_1}{H_2 - H_1} = \mu_0 \mu_r$$

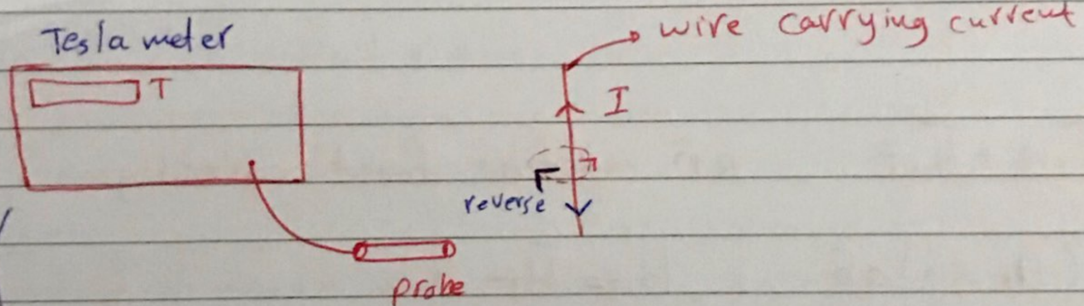
$$\mu_0 = 4\pi \times 10^{-7}$$

(for non-linear material) → Next Page





To measure B_{earth}?



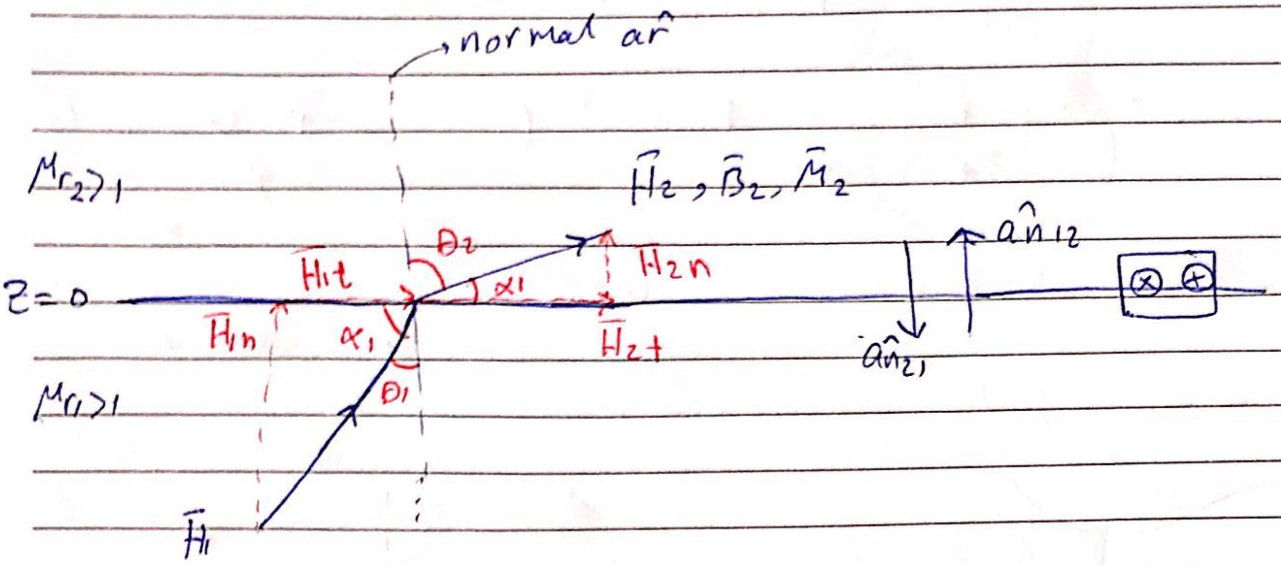
$B_1 = B_{wire} + B_{earth}$

reverse the current

$B_2 = (-) B_{wire} + B_{earth}$

$$\frac{B_1 + B_2}{2} = B_{earth}$$

* Mag Boundary condition :-



$\bar{B}_1 = \mu_0 \mu_{r_1} \bar{H}_1$

$\bar{M}_1 = (\mu_{r_1} - 1) \bar{H}_1$

$\bar{H}_1 = \bar{H}_{1n} + \bar{H}_{1t}$ $\hat{an}$ must be fount correctly

$\bar{H}_{1n} = (H_1 \cdot \hat{an}) \hat{an}$, $H_{1t} = \bar{H}_1 - \bar{H}_{1n}$

or $\bar{H}_{1t} = H_1 \times \hat{an}_{12}$ $H_{1n} = \bar{H}_1 - \bar{H}_{1t}$

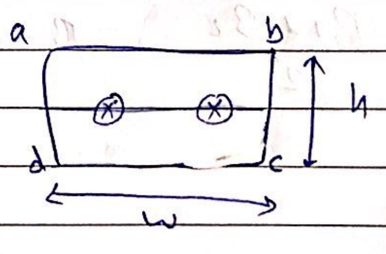
$\theta_1 = \sin^{-1} \dfrac{H_{1t}}{H_1}$ $\alpha_1 = 90^\circ - \theta_1$

---

to find $\bar{H}_2 \longrightarrow$ B.C

$\bar{H}_2 = \bar{H}_{2n} + \bar{H}_{2t} \longrightarrow 2$ B.C

$\oint_L \bar{H} \cdot \bar{dl} = I_{enc}$, $\oint_S \bar{B} \cdot \bar{ds} = 0$



$$\int_a^b \bar{H}\cdot d\bar{l}_1 + \int_b^c \bar{H}\cdot d\bar{l}_2 + \int_c^d \bar{H}\cdot d\bar{l}_3 + \int_d^a \bar{H}\cdot d\bar{l}_4 = K\,w$$

$$H_{2t}\,\cancel{w} - \cancel{H_{2n}\frac{h}{2}} - \cancel{H_{1n}\frac{h}{2}} - H_{1t}\,\cancel{w} + \cancel{H_{1n}\frac{h}{2}} + \cancel{H_{2n}\frac{h}{2}} = K\cancel{w}$$

$$\boxed{H_{2t} - H_{1t} = K} \longrightarrow \text{scaler}$$

K → given

$$\boxed{\begin{array}{l}\text{if } K = 0 \\ H_{2t} = H_{1t}\end{array}}$$

$$H_{2t} - H_{1t} = k$$

$$* \; (\bar{H}_2 - \bar{H}_1) \times \hat{a}_{n_2} = \bar{K}$$

or $$* \; (\bar{H}_1 - \bar{H}_2) \times \hat{a}_{n_{12}} = \bar{K}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r1}}{\mu_{r2}}$$

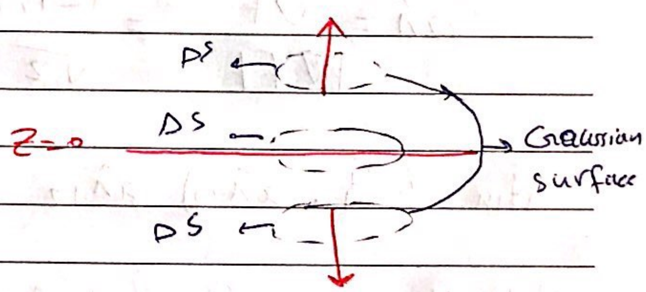
if $\bar{K} = 0$

to find $\bar{H}_{2n}$?

use $\oint_s \bar{B}\cdot d\bar{s} = 0$

$$\int_{s\,top} \bar{B}\cdot d\bar{s}_{top} + \int_{s\,bot} \bar{B}\cdot d\bar{s}_{bot} = 0$$

$$B_{2n}\,\cancel{\Delta S} - B_{1n}\,\cancel{\Delta S} = 0$$



$$\boxed{\bar{B}_{2n} = \bar{B}_{1n}}$$ always

$$\bar{H}_{2n} = \frac{\mu_{r1}}{\mu_{r2}}\,\bar{H}_{1n}$$

Given that $\bar{H}_1 = -2a\hat{x} + 6a\hat{y} + 4a\hat{z}$ A/m in region $y - x - 2 \leq 0$ where $\mu_1 = 5\mu_0$ calculate:

μ_{r_1}

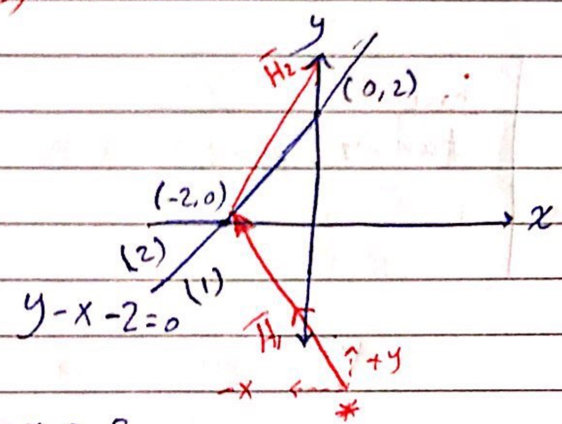
a) $\bar{B}_1$ and $\bar{M}_1$

b) $\bar{H}_2$ and $\bar{B}_2$ in region $y - x - 2 \geq 0$ if $\mu_2 = 2\mu_0$

a) $\bar{B}_1 = \mu_0 \mu_{r_1} \bar{H}_1 = (-10, 30, 20)\ \mu_0 T$

$\bar{M}_1 = (\mu_{r_1} - 1)\bar{H} = 4\bar{H} = (-8, 24, 16)$ A/m

b)



$y - x - 2 = f$

$$a\hat{n} = \frac{\nabla f}{|\nabla f|} = \frac{(-1, 1, 0)}{\sqrt{2}} \Rightarrow a\hat{n}_{12} = \frac{-1}{\sqrt{2}} a\hat{x} + \frac{1}{\sqrt{2}} a\hat{y}$$

$\bar{H}_{1n} = (\bar{H}_1 \cdot a\hat{n}_{12})\ a\hat{n}_{12} = (-4, 4, 0)$ A/m

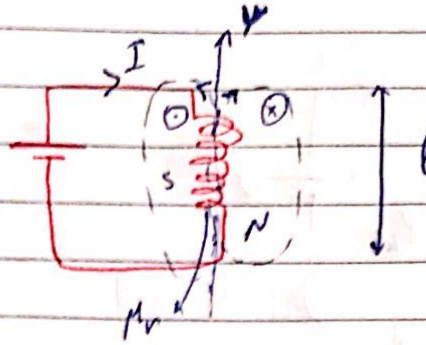
$\bar{H}_{1t} = \bar{H}_1 - \bar{H}_{1n} = (2, 2, 4)$ A/m $\Rightarrow = H_{2t}$ (since $\bar{K} = 0$)

$\bar{H}_{2n} = \frac{\mu_{r_1}}{\mu_{r_2}} \bar{H}_{1n}$ $(\bar{B}_{1n} = \bar{B}_{2n})$ $= \frac{5}{2}(-4, 4, 0) = (-10, 10, 0)$

$\bar{H}_2 = \bar{H}_{2n} + \bar{H}_{2t} = -8a\hat{x} + 12a\hat{y} + 4a\hat{z}$ A/m

$\bar{B}_2 = 2\mu_0 \bar{H}_2$, $\bar{M}_2 = \bar{H}_2$

* Inductors (L)



$I \propto B \propto \Psi \propto \lambda$, λ= flux linkage $\lambda = N\Psi$ in (Wb)

$\lambda \propto I$ = relation

$\lambda = L I$

$L \rightarrow$ proportinality constant

(Inductance)

① $L = \frac{\lambda}{I} = \frac{N\Psi}{I}$ (H) henry

• magnetic energy (W_m) in (J)

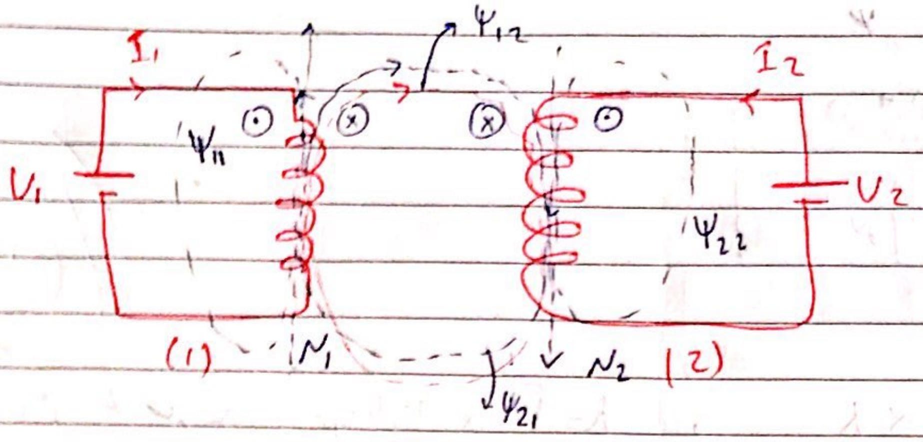
$$W_m = \frac{1}{2}\int_v \bar{H}\cdot\bar{B}\, dv = \frac{1}{2}\int \mu H^2\, dv = \frac{1}{2}\int_v \frac{B^2}{\mu}\, dv$$

$\boxed{W_m = \frac{1}{2} L I^2}$ in uniform ② $L = \frac{2W_m}{I^2}$

$w_m = \frac{W_m}{Vol}$ (J/m^3)

$\hookrightarrow$ mass energy density

* Self Inductance and mutal Inductance



Ψ_{11} , $\Psi_{22} \equiv$ Self flux

ckt No → current No

Ψ_{12}, $\Psi_{21} \equiv$ mutal flux

Self Inductance

$$L_{11} = \frac{\lambda_{11}}{I_1} = \frac{N_1 \Psi_{11}}{I_1}$$

$$L_{22} = \frac{\lambda_{22}}{I_2} = \frac{N_2 \Psi_{22}}{I_2}$$

mutal Inductance

$$M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \Psi_{12}}{I_2}$$

$$M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2 \Psi_{21}}{I_1}$$

total Inductance

$L_1 = L_{11} + M_{12} \Rightarrow L_1 = \frac{N_1 \psi_1}{I_1}$

total flux

$\psi_1 = \psi_{11} \overset{\oplus}{\ominus} \psi_{12}$ → same / opposit

CKt No

$W_m = \frac{1}{2} L_{11} I_1^2 + \frac{1}{2} L_{22} I_2^2$

$\pm \frac{1}{2} (M_{12} I_2^2 + M_{21} I_1^2)$ (J)

if $M_{12} = M_{21} \Rightarrow \frac{I_1}{I_2} = \frac{N_2}{N_1}$

procedure to find (L) or (M)

1) choose a suitable coordinates

2) Assume currents → I

3) find $\vec{B}$ (Biot savart's law, Amperes law, mag Pot) CH7

4) $\psi = \int_s \vec{B} \cdot \vec{ds}$ $\qquad W_m = \frac{1}{2} \int_v \frac{B^2}{\mu} dv$

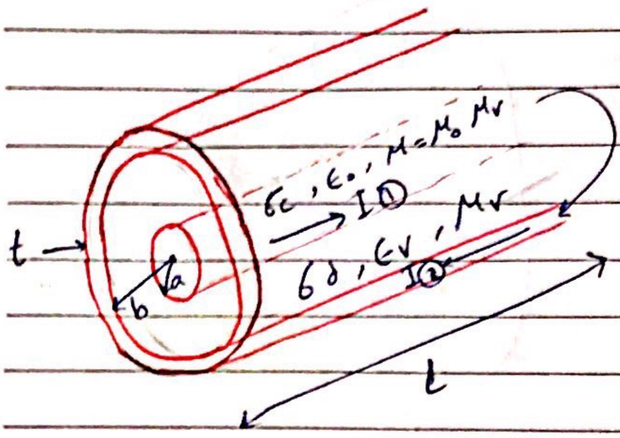
5) $\lambda = N \psi$

6) $L = \frac{N\psi}{I}$ $\qquad L = \frac{2 W_m}{I^2}$

Five Apple

Scanned with CamScanner

Ex: find the self Inductance for a coaxil cable shown:



$$\vec{B} = \begin{cases} \frac{\mu I \rho}{2\pi a^2}\hat{a}_\phi & , 0 \le \rho \le a \\ \frac{\mu I}{2\pi\rho}\hat{a}_\phi & , a \le \rho \le b \\ \times\ \frac{I}{2\pi\rho}(\text{- - -})\hat{a}_\phi & , b \le \rho \le c \\ \times\ 0 & , \rho > c \end{cases}$$

self Inductance

$L = L_{int} + L_{ext}$, for L_{int} (use $L = \frac{2W_2}{I^2}$)

$0 < \rho \le a \qquad a \le \rho < b$

$$W_n = \frac{1}{2}\int_0^l\int_0^{2\pi}\int_0^a \frac{\mu^2 I^2 \rho^2}{\mu 4\pi^2 a^4}\rho\, d\rho\, d\phi\, dz$$

$$\frac{\mu I^2}{4\cancel{8}\pi^2 a^4}\left(\frac{a^4}{4}\right)(2\pi)(l) = \frac{\mu I^2 l}{16\pi}\ J$$

$L_{int} = \frac{\mu l}{8\pi}\ H \Rightarrow L_{int}/l = \frac{\mu}{8\pi}\ (H/m)$

- L_{ext} (use $L = \frac{N\psi}{I}$)

$a \le \rho \le b$

$$\psi = \int_s \vec{B}\cdot\vec{ds} \Rightarrow \psi = \int_0^l\int_a^b \frac{\mu I}{2\pi\rho}\hat{a}_\phi\, d\rho\, dz\, \hat{a}_\phi$$

$$= \frac{\mu I l}{2\pi}\ln\left(\frac{b}{a}\right)\ wb\ (N = 1\ \text{turn}) \Rightarrow L_{ext}/l = \frac{\mu \ln\left(\frac{b}{a}\right)}{2\pi}(H/m)$$

$$\boxed{L = \frac{\mu}{2\pi}\left[\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right]\ H/m}$$

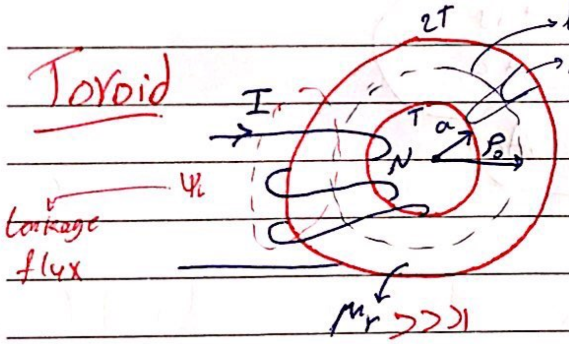
* Magnetic Circuits :-

elec ckt	mag ckt
$V_{emf} \equiv V$ $(\frac{+}{T})$	$V_{mmf} \equiv F$ $(\frac{+}{T})$
I	Ψ
$R = \frac{V}{I} = \frac{l}{\sigma A}$ (ohm's)	$R_m = \frac{F}{\Psi} = \frac{l}{\mu A_c}$ (ohm's)
σ	μ
$G = \frac{1}{R}$	$P = \frac{1}{R_m}$ Permance

* magnetic structure :- (Gapless circuit

- → mag core (l_c, A_r, μ_r)
- → Excitation coil (I, N)

Toroid



l_c = Avarage lengh of the core

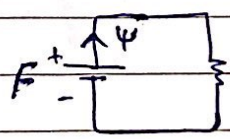
$l_c = 2\pi\rho_o$ → $a + t$

A_c = cross section Area of the core

$A_c = \pi l^2$

$R_m = \frac{l_c}{\mu_o \mu_r A_c} = \frac{m}{\frac{H}{m} \cdot m^2} = H^{-1}$

: (mag resistange) reluctance



$\Psi = \frac{F}{R_m}$

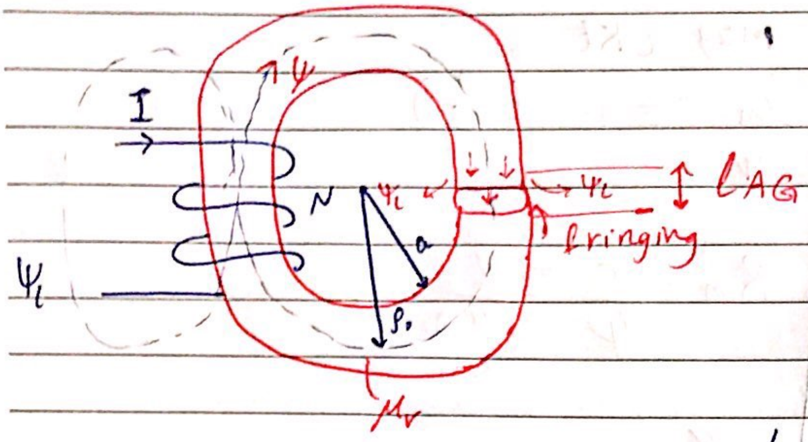
$V_{mmf} = F = NI \equiv A \cdot t$ → Amperes turns

$= \Psi R_m$ → $\oint_c \bar{H} \cdot \bar{dl}$, $NI = Hl$ if uniform

$= Hl$

2

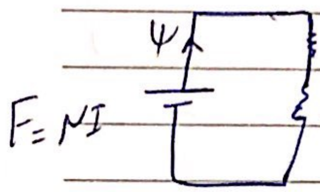
*magnetic structure with Air gap



$l = 2\pi\rho_o$

$l_{AG} = 0.5\ cm \rightarrow 1.5\ cm$

$l_c = 2\pi\rho_o - l_{AG}$



$R_m = \dfrac{l_c}{\mu_o \mu_r A_c}$

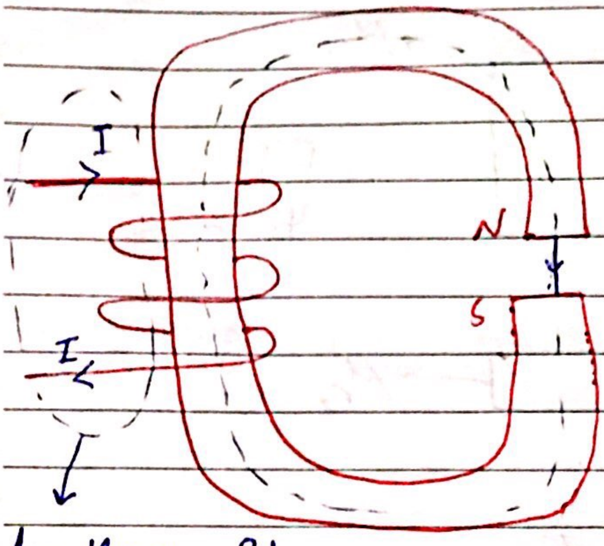
$R_{AG} = \dfrac{l_{AG}}{\mu_o (1) A_{Ag}}$

$R_{eq} = R_m + R_{AG}$

$\psi = \dfrac{F}{R_{eq}}$

FF = Fringing fActor ≥ 1 $\quad FF = \dfrac{A_{Ag}}{A_C}$

$FF = 1.5 \quad A_{Ag} = 1.5\ A_C$



from $I \to B \to \underline{\Psi}$ → leakage
→ core

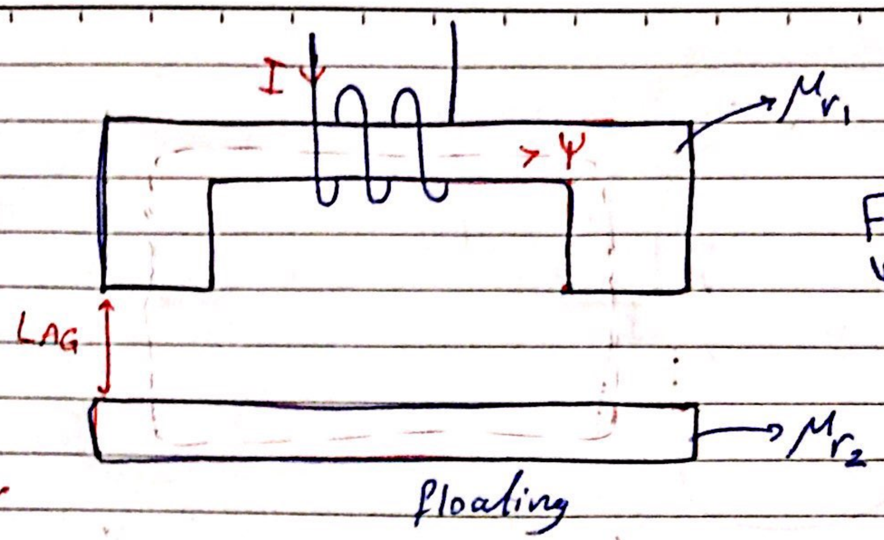
force in the airgap

F_{mT} ≡ magnetic fraction force

$$= \frac{(-)\ B^2 A_c}{2\mu_0}$$ in one airgap

air

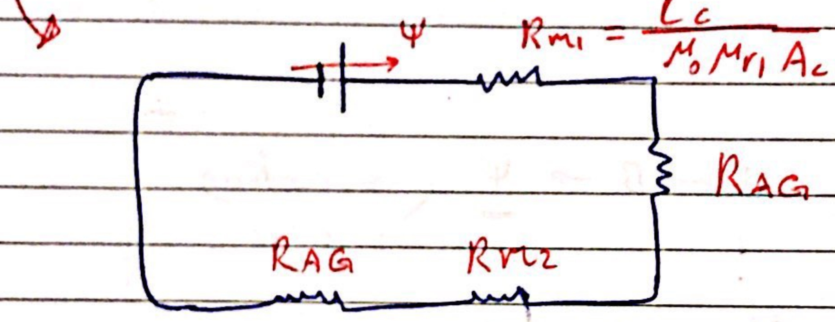
$$\text{pressure} = \frac{F_{mt}}{A_c}$$



FF=1

$\Psi_L = 0$

$g = 9.8 \ m/s^2$

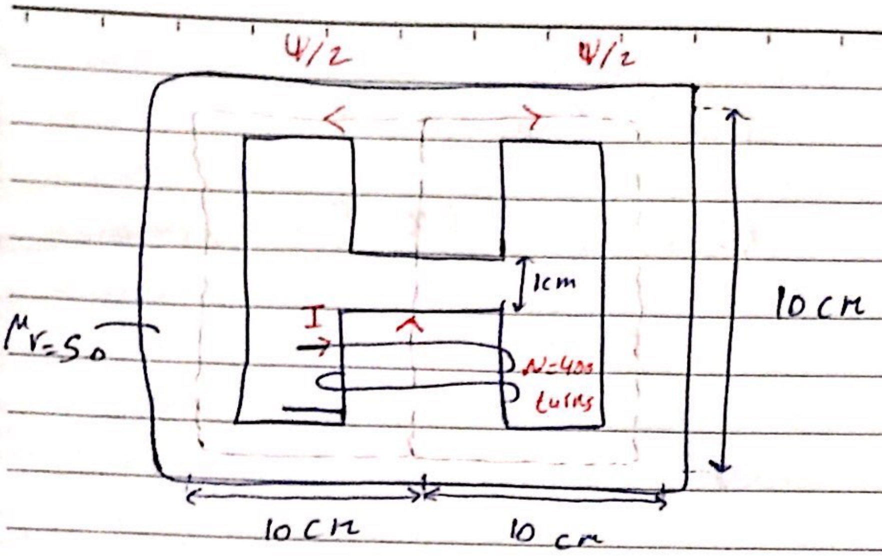


$Weight = mg = F_{MT}$

$F_{MT} = \dfrac{B^2 A_c}{\mu_o}$ for tow airgap

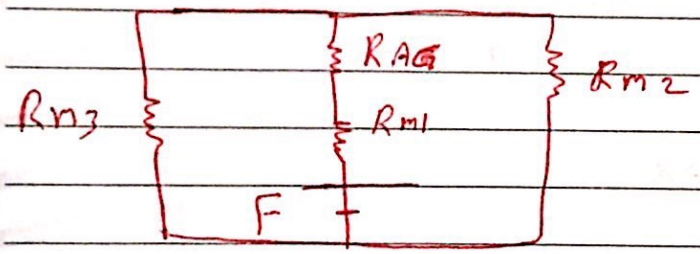
$B = \dfrac{\Psi}{A_c} \longrightarrow \Psi = \dfrac{F}{R_{eq}}$

$\boxed{F = NI}$



all $A_c = 10 \text{ cm}^2$, find I to produce 1.5 T in the airgap

FF=1, $\Psi_\ell = 0$, μ_r is given area the same



$$R_{m1} = \frac{9\times10^{-12}}{(4\pi\times10^{-7})(50)(10\times10^{-4})} \quad , \quad R_{AG} = \frac{1\times10^{-2}}{4\pi\times10^{-7}\times10\times10^{-4}}$$

$$R_{m2} = \frac{30\times10^{-2}}{(4\pi\times10^{-7})(50)(10\times10^{-4})} = R_{m3}$$

$$R_{m_{eq}} = (R_{m2} \parallel R_{m3}) + R_{m1} + R_{AG}$$

→

$F = \Psi R_{meq} = NI$

$\Psi = B A_c \quad (\Psi = \int_s \bar{B} \cdot \bar{ds}$

$I = 44.16 \ A$

$F_{MT} = \frac{B^2 A_c}{2\mu}$ $\rightarrow A_{ag}$

$L = \frac{N\Psi}{I}$

$B_c < B_{AG}$ if $\Psi_i > 0$

$W_m = \frac{1}{2} L I^2$

$A_{AG} = FF(A_c)$

$M_{core} = (\mu_r - 1) H \rightarrow H_c = \frac{B_c}{\mu_0 \mu_r}$ $H_{AG} = \frac{B_{AG}}{\mu_0 (1)}$

* if μ_r is not given

B	H
0.5	--
0.75	--
1	--
1.25	--

$\mu = \frac{B}{H}$

$FF = 1$

$\Psi_i = 0$

more application duality to CH6

$\nabla^2 V = 0$ scaler Laplace

$\nabla^2 V = \frac{-\rho v}{\epsilon}$ scaler Poisson

duality

$\nabla^2 \bar{A} = 0$ vector Laplace if $\bar{J} = 0$

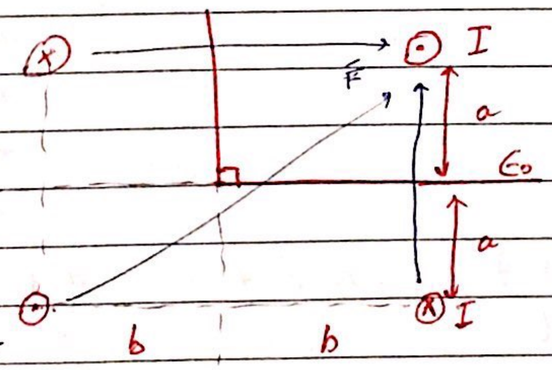
$\nabla^2 \bar{A} = -\mu \bar{J}$ vector poisson if $\bar{J} \neq 0$

$\nabla^2 A_x = 0$

$\nabla^2 A_y = 0$

$\nabla^2 A_z = 0$

* method of images



$\bar{H} = \frac{I}{2\pi \rho} a_{\hat{\phi}}$

CH 4 : maxwell's eq (time varying field)

Ac Source $\longrightarrow$ Ac fields

$\rho_v , J \longrightarrow \bar{E}, \bar{D}, \bar{H}, \bar{B}$

$(x, y, z : t) \longrightarrow (x, y, z : t)$

· Previe :-

① $\nabla \cdot \bar{D} = \rho_v \longrightarrow \oint_s \bar{D} \cdot \bar{ds} = \int_v \rho_v \, dv$

② $\nabla \times \bar{E} = 0 \longrightarrow \oint_L \bar{E} \cdot \bar{dL} = 0$

③ $\nabla \times \bar{H} = \bar{J} \longrightarrow \oint_L \bar{H} \cdot \bar{dL} = I_{enc} = \int_s \bar{J} \cdot \bar{ds}$

④ $\nabla \cdot \bar{B} = 0 \longrightarrow \oint \bar{B} \cdot \bar{ds} = 0$

Sources for Ac source

1) charg moving with accelaration

2) Ac current flowing in the wire

* Faraday's law

(generator Principle)

$$\boxed{V_{emf} = -\frac{\partial \lambda}{\partial t}} = \frac{-\partial N\psi}{dt} = -N\frac{d\psi}{dt} = \frac{-d\psi}{dt} \quad N=1$$

Lenz's law , $\psi = \bar{B} \cdot \bar{ds}$

in CH 4

$V_{emf} = \oint_L \bar{E} \cdot \bar{dL}$

$$\boxed{V_{emf} = -\frac{\partial \psi}{\partial t} = \frac{-\partial}{\partial t} \int_s \bar{B} \cdot \bar{ds}} = \oint_L \bar{E} \cdot \bar{dL}$$

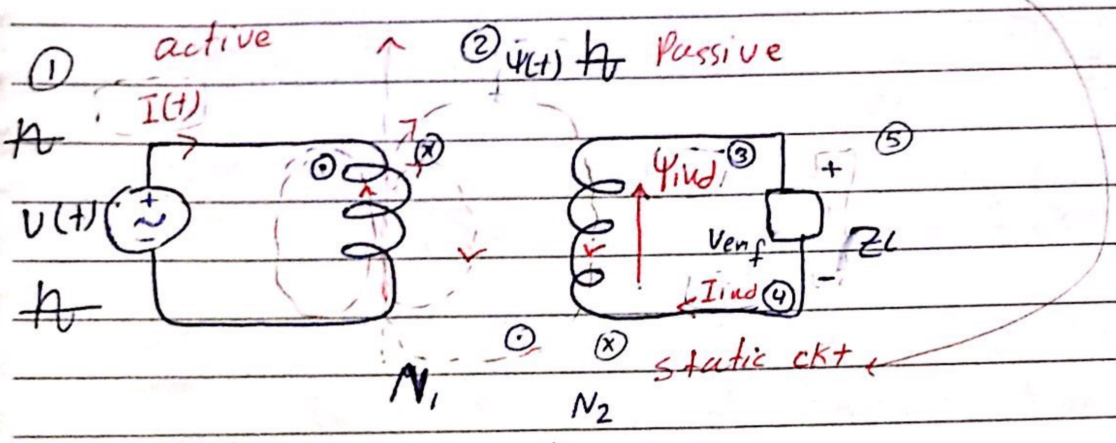
$$\boxed{\oint_L \bar{E} \cdot \bar{dL} = \frac{-\partial}{\partial t} \int_s \bar{B} \cdot \bar{ds}}$$

→ faraday's law

→ 2nd maxwell eq in time varying fields

There are 3 cases for faraday's Law:

1) time varying fields on a stationary loop

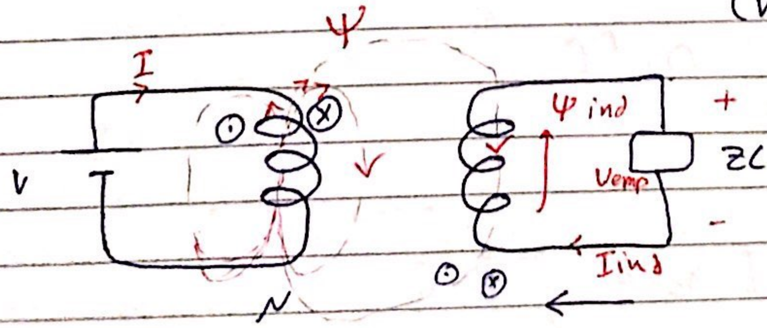


(Transformer EMF)

$$V_{emf} = \oint_L \bar{E} \cdot \bar{dl} = \int_s \frac{-\partial \bar{B}}{\partial t} \cdot \bar{ds} \Rightarrow \nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t}$$

$$\int_s \nabla \times \bar{E} \cdot \bar{ds} = \int_s - \frac{\partial \bar{B}}{\partial t} \cdot \bar{ds}$$

2) static field on amoving Loop (Dc generator)
(motional EMF)



$$V_{emp} = \oint_L \bar{E} \cdot \bar{dL} = \oint_L \bar{u} \times \bar{B} \cdot \bar{dL}$$

$$\bar{F}_m = Q \bar{u} \times \bar{B}$$

$$\frac{\bar{F}_m}{Q} = \bar{E}_m = \bar{u} \times \bar{B}$$

$$\nabla \times \bar{E} = \nabla \times (\bar{u} \times \bar{B})$$

3) time varying field on a moving loop
(induction and synchronous generators)
(transformer + motional) Emf

$$V_{emf} = \oint_l \bar{E}\,\bar{dl} = -\int_s \frac{\partial \bar{B}}{\partial t} \cdot \bar{ds} + \oint_l \bar{u} \times \bar{B} \cdot \bar{dl}$$

$$\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t} \cdot \quad \nabla \times (\bar{u} \times \bar{B})$$

* Displacement current Ac current (I_d)

$$I_d = \int_s \bar{J}_d \cdot \bar{ds}$$, $\bar{J}_d \equiv$ displacement current Density (A/m^2)

$\bar{J} = \sigma \bar{E}$ $\quad \bar{J} = \rho v \bar{u}$

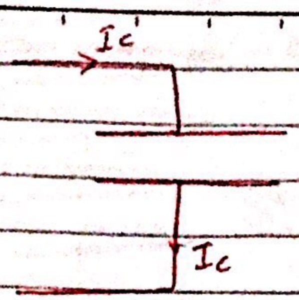
from continuity eq

$$\oint_s J \cdot \bar{ds} = -\int_v \frac{\partial \rho v}{\partial t}\, dv$$

$$\nabla \cdot \bar{J} = \frac{-\partial \rho v}{\partial t}$$ Diff

$$\oint_l \bar{H} \cdot \bar{dl} = I_{enc} = \int_s \bar{J} \cdot \bar{ds}$$ → conduction → Amperes Law

Five Apple



$\oint \bar{H}\cdot \bar{dL} = I_{enc} = I_c + I_d = \int_s \bar{J}\cdot \bar{ds} + \int_s \bar{J}_d \cdot ds$

$\hookrightarrow$ modifield Amperes law

(3d Maxwell eq) $\qquad \int_s (\bar{J} + \bar{J}_d)\, \bar{ds}$

$$\nabla \times \bar{H} = \bar{J} + \bar{J}_d$$

$\nabla \times \bar{H} = \bar{J} + \bar{J}_d$

$\cancel{\nabla \cdot (\nabla \times \bar{H})} = \nabla \bar{J} + \nabla \cdot \bar{J}_d$

$\nabla \cdot \bar{J}_d = -\nabla \cdot \bar{J} \Rightarrow \nabla \cdot \bar{J}_d = +\frac{\partial \rho v}{\partial t}, \quad \rho_V = \nabla \cdot \bar{D}$

$\nabla \cdot \bar{J}_d = +\frac{\partial}{\partial t} \nabla \cdot \bar{D}$ $\;\to xyz$

$\nabla \cdot \bar{J}_d = +\nabla \frac{\partial \bar{D}}{\partial t}$

$$\boxed{\bar{J}_d = \frac{\partial \bar{D}}{\partial t}}$$

maxwell's eq in time varying fields (in time domain)

$\nabla \cdot \bar{D} = \rho_V \; (\nabla \cdot \bar{D}(x, y, z; t) = \rho v (x, y, z; t)$

$\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t} = -\frac{\mu \partial \bar{H}}{\partial t} \Rightarrow \nabla \times \bar{H} = \sigma \bar{E} + \frac{\epsilon \partial E}{\partial t}$

$$\boxed{\nabla \cdot \bar{B} = 0}$$

* time harmonic fields :- 9.7 9.6 x

Periodic (sinusoidal)

$j = \sqrt{-1}$ → imaginary operator

$z = r \angle \theta$

$\sqrt{z} = \sqrt{r} \angle \theta/2$

Euler's Identity :-

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\left(e^{j\theta}\right)^* = e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}, \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

i.ex $I = I_o \cos(\omega t + \phi)$ A

(instantaneous form)

$$I(t) = Re\left[I_o \, e^{j(\omega t + \phi)}\right]$$

$$I(t) = Re\left[I_o \, e^{j\phi} \, e^{j\omega t}\right]$$

$\Downarrow$

$$I_s = I_o e^{j\phi} = I_o \cos\phi + j I_o \sin\phi$$

phasor

$$= I_o \angle\phi = I_o \cos\phi + j I_o \sin\phi$$

$$\frac{d}{dt} e^{j\omega t} = j\omega e^{j\omega t} \quad / \quad \int e^{j\omega t} \, dt = \frac{1}{j\omega} e^{j\omega t}$$

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maxwell's equation in phasor form

$$\nabla \cdot \bar{D}(x,y,z;t) = \rho_v(x,y,z;t)$$

$$\cancel{Re}\{\nabla \cdot D_s(x,y,z)\,\cancel{e^{j\omega t}}\} = \cancel{Re}\{\rho_{vs}(x,y,z)\,\cancel{e^{j\omega t}}\}$$

$$\nabla \cdot \bar{D}_s = \rho_{v_s} \quad (1)$$

$$\nabla \times \bar{E}(x,y,z;t) = -\frac{\partial B}{\partial t}(x,y,z;t)$$

$$\nabla \times Re\{E_s\, e^{j\omega t}\} = -Re\left\{\frac{\partial B_s}{\partial t}\, e^{j\omega t}\right\}$$

$$\nabla \times \cancel{Re}\{E_s\,\cancel{e^{j\omega t}}\} = -\cancel{Re}\{j\omega B_s\,\cancel{e^{j\omega t}}\}$$

$$\nabla \times \bar{E}_s = j\omega \bar{B}_s \quad (2) \Rightarrow \nabla \times \bar{E}_s = -j\omega \mu \bar{H}_s$$

$$\nabla \times \bar{H}_s = \sigma \bar{E}_s + j\omega \epsilon E_s \quad (3)$$

$$\nabla \cdot \bar{B}_s = 0 \quad (4)$$

Ex: given $\quad \omega \rightarrow (rad/s) \quad$ phase const $\rightarrow (rad/m)$

$$\bar{E} = \frac{50}{\rho} \cos(10^6 t + \beta z)\, \hat{a_\phi} \quad V/m$$

$$\bar{H} = \frac{H_o}{\rho} \cos(10^6 t + \beta z)\, \hat{a_\rho} \quad A/m$$

in free space and sourse free region find H_o and β

$\hookrightarrow \sigma = 0$, $c = 3 \times 10^8\ m/s$ $\hookrightarrow \rho_v = 0$, $\bar{J} = 0$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}, \quad \nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$$

*دائماً ابدأ بال[illegible] والعلاقة

$$\bar{E}_s = \frac{50}{\rho} e^{j\beta z}\, \hat{a_\phi} \quad V/m$$

$$\bar{H}_s = \frac{H_o}{\rho} e^{j\beta z}\, \hat{a_\rho} \quad A/m$$

$$\nabla \times \bar{E}_s = -j\omega \underbrace{(\mu \bar{H}_s)}_{\bar{B}_s}, \quad \nabla \times \bar{H}_s = j\omega \underbrace{(\epsilon \bar{E}_s)}_{\bar{D}_s}$$

$$\bar{H}_s = \frac{-1}{j\omega\mu} \nabla \times \bar{E}_s$$

$$\begin{bmatrix} H_{\rho s} \\ \cancel{H_{\phi s}} \rightarrow 0 \\ \cancel{H_{zs}} \rightarrow 0 \end{bmatrix} = \frac{-1}{j\omega\mu} \frac{1}{\rho} \begin{vmatrix} \hat{a_\rho} & \rho \hat{a_\phi} & a_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ \cancel{E_{\rho s}}\ 0 & \rho E_{\phi s} & \cancel{E_{zs}}\ 0 \end{vmatrix}$$

$$H_{\rho s} = \frac{1}{j\omega\mu\rho} \left(\frac{-\cancel{\rho}\, 50}{\cancel{\rho}} (j\beta)\, e^{j\beta z} \right) \hat{a_\rho}$$

$$\frac{H_o}{\rho} \cancel{e^{j\beta z}}\, \cancel{\hat{a_\rho}} = \frac{50}{\omega\mu\rho} \cancel{e^{j\beta z}}\, \cancel{\hat{a_\rho}}$$

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$$H_o = \frac{50\beta}{\omega\mu} \quad \text{--- ①}$$

$$\bar{E}_s = \frac{1}{j\omega\epsilon} \nabla \times \bar{H}_s$$

$$E_{\phi s} = \frac{1}{j\omega\epsilon\rho}\left(0 + \frac{H_o}{\rho} j\beta e^{j\beta z}\right)\rho \hat{a_\phi}$$

$$\frac{50}{\rho} e^{j\beta z} \hat{a_\phi} = \frac{H_o\beta}{\omega\epsilon\rho} e^{j\beta z} \hat{a_\phi}$$

$$50 = \frac{H_o\beta}{\omega\epsilon} \quad \text{-- ②}$$

$$50 = \frac{50\beta^2}{\omega^2\mu\epsilon} \Rightarrow \beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c}$$

$$= 10^6\sqrt{4\pi\times10^{-7} * \frac{10^{-4}}{36\pi}} = \pm\frac{1}{3}$$

$$H_o = \pm 0.1326 \ A/m$$

The END

بالتوفيق