* CH #1

* How to write a vector:-

↳ Cartesian: $\bar{A} = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$ OR (A_x, A_y, A_z)

هدول اسمهم Unit vector
قيمتهم = 1 بس بدلو على الاتجاه .
ومنفهم بعرف إنو هاد ال Vector
موجود بال Cartesian coordinate.

↳ Cylindrical: $\bar{A} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}$ OR (A_ρ, A_ϕ, A_z)

↳ Spherical: $\bar{A} = A_r \hat{a_r} + A_\theta \hat{a_\theta} + A_\phi \hat{a_\phi}$ OR (A_r, A_θ, A_ϕ).

Note

don't mix between points & vectors!
$p(x, y, z)$ → point
$\bar{A} = (A_x, A_y, A_z)$ → vector

* Vector Magnitude:- (Scalar).

$$|\bar{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Scalar Quantities:-
time / mass / distance / temperature / entropy
electric potential / Population
هدول ما الهم اتجاه بس قيمة

* Unit vector along a vector:-

$$\hat{a_A} = \frac{\bar{A}}{A} = \frac{A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

لما يطلب مني هاد الشي
١) بطلع ال magnitude لل Vector
٢) بقسم ال Vector على ال magnitude

Vector Quantities:-
Velocity / force / displacement
Electric field
هدول الهم قيمة + اتجاه .

* Operations on vectors:-

[1] Addition.

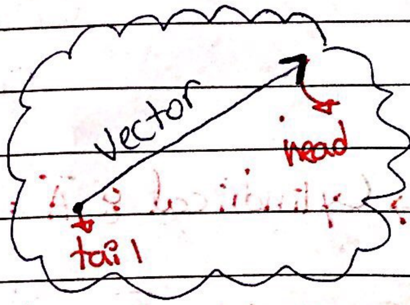
$$\bar{C} = \bar{A} + \bar{B}$$

$$= (A_x + B_x)\,\hat{a}_x + (A_y + B_y)\,\hat{a}_y + (A_z + B_z)\,\hat{a}_z$$

هاي بتدل على العملية الي عندي

from vector definition.

- Graphically:-



كيف ارسم جمع 2 vectors ؟

↳ Arrow method.

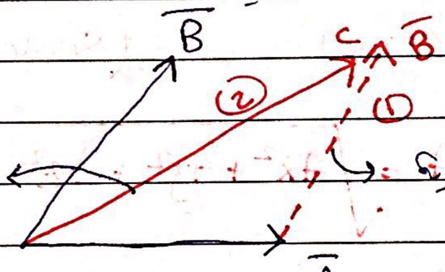
- عند نهاية ال vector الأول (يعني ال head)

بجيب ال vector الثاني و برسمه ، بعد ين

حاصل الجمع بكون توصيل بين ال tail الأول

لـ head الثاني.

مثال



وصلت بين tail $\bar{A}$ و head $\bar{B}$

رسمت $\bar{B}$ عند نهاية $\bar{A}$ (عند head $\bar{A}$)

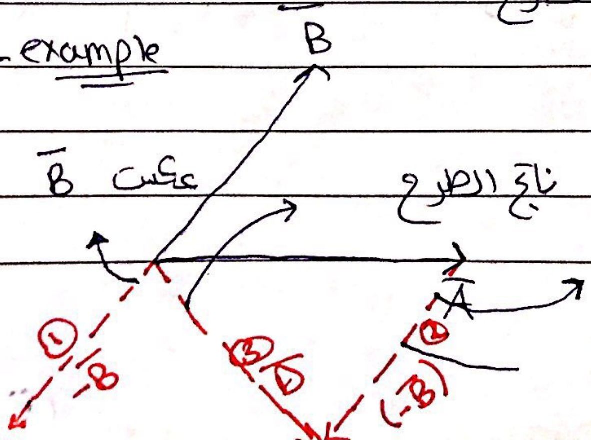
[2] Subtraction

$$\bar{C} = \bar{A} - \bar{B}$$

$$= (A_x - B_x)\,\hat{a}_x + (A_y - B_y)\,\hat{a}_y + (A_z - B_z)\,\hat{a}_z$$

- Graphically :- نفس طريقة الرسم ، لكن لازم أعكس ال vector الي بعد إشارة الطرح.

- example



$\bar{B}$ عكس

ناتج الطرح

جبت $\bar{B}$ عند نهاية $\bar{A}$

Scanned with CamScanner

- Application on Subtraction is Distance (vector)

$P(x_1, y_1, z_1)$ → Point

$Q(x_2, y_2, z_2)$ → Point

$\bar{r}_{PQ}$ → distance vector from P to Q

$\bar{r}_P = \bar{r}_{OP} = \bar{r}_P - \bar{r}_O$

المسافة لأي نقطة هي المسافة بين النقطة نفسها ونقطة الأصل

$$\bar{r}_{PQ} = \bar{r}_Q - \bar{r}_P$$

$$= (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$ → هيك طلعت ال distance vector

- إذا طلب ال distance لحال يعني قيمة فقط باخذ ال magnitude لل $\bar{r}_{PQ}$ حسب القانون. ⇒

$$|\bar{r}_{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- إذا طلب ال direction

$$\hat{a}_{r_{PQ}} = \frac{\bar{r}_{PQ}}{r_{PQ}} = -\hat{a}_{r_{QP}}$$

عكس الاتجاه

3 Multiplication

↳ a) Dot product.

$$\bar{A} \odot \bar{B} = |\bar{A}|\,|\bar{B}| \cos\theta_{(AB)}$$ → هاد القانون بستخدمه لما أكون بعرف الزاوية بين ال Vectors.

$\odot$ ← إشارة ال dot product

- إذا ما بعرف الزاوية بنفهم بستخدم هاد القانون ⟸

$$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z$$

رح يطلع يعني الجواب رقم يعني scalar

$$\theta_{AB} = \cos^{-1}\left(\frac{\bar{A} \cdot \bar{B}}{|\bar{A}|\,|\bar{B}|}\right)$$ ⟸ لو طلب مني الزاوية بستخدم هاد القانون

* ليش الـ dot product بطلع قيمته scalar ؟

لأنه القانون بعتمد على $\cos\theta$ ، ولما آجي أضرب vectors بضرب الـ terms المتشابهة ، الزاوية بين unit vector الهم = صفر ، $\cos(0) = 1$

توضيح ↲ $A_x \hat{a}_x \cdot B_x \hat{a}_x$

$= A_x B_x (\hat{a}_x \cdot \hat{a}_x)$ أخذنا طلعنا بعامل مشترك

$= A_x B_x (1)$

لأنه الزاوية بينهم صفر

$$\hat{a}_n \cdot \hat{a}_m = \begin{cases} 1, & n = m \\ 0, & n \neq m \end{cases}$$

properties of dot product :-

1) $\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$

2) $\bar{A} \cdot \bar{A} = A^2 = A_x^2 + A_y^2 + A_z^2$

3) $\bar{A} \cdot (\bar{B} + \bar{C}) = \bar{A} \cdot \bar{B} + \bar{B} \cdot \bar{C}$

↳ b) Cross product (it has magnitude + direction).

$|\bar{A} \times \bar{B}| = AB \sin\theta_{AB}$ ← هاد القانون بستخدمه لما أعرف الزاوية بين $\bar{A}$ و $\bar{B}$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

⇐ إذا ما بعرف الزاوية بينهم بحل cross product بينهم باستخدام الـ matrix

أول صف بحط الـ unit vector

ثاني صف بحط أول vector

ثالث صف بحط ثاني vector

$\bar{A} \times \bar{B} = (-1)^{1+1} (A_y B_z - A_z B_y) \hat{a}_x$

$+ (-1)^{1+2} (A_x \cancel{B_z} - \cancel{A_z} B_x) \hat{a}_y$

$+ (-1)^{1+3} (A_x B_y - A_y B_x) \hat{a}_z$

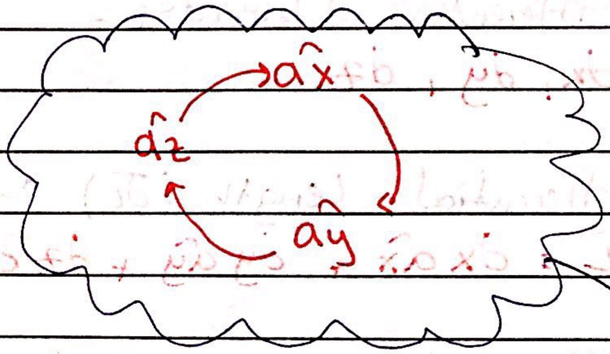
← أول مرة بخفي أول عامود و بضرب الثاني والثالث ببعض ، ثاني مرة بخفي ثاني عامود و بضرب الأول والثالث ، وآخر اشي بخفي ثالث عامود ← بضرب الأول والثاني .

$$\theta = \sin^{-1}\left(\frac{|\bar A \times \bar B|}{|\bar A|\,|\bar B|}\right)$$

* إذا طلب مني الزاوية ←
بطلع $\bar A \times \bar B$ باستخدام الـ matrix حسب القانون، بعدين باخذ الـ magnitude للـ vector الي طلع معي من الـ matrix، وباخذ الـ magnitude لـ $\bar A$ و $\bar B$ وبقسم $\frac{|\bar A \times \bar B|}{|\bar A|\,|\bar B|}$ وبعدين باخذ الـ $(\sin^{-1})$ للناتج القسمة.

Properties of cross product

1) $\bar A \times \bar B = -\bar B \times \bar A$ → السالب للاتجاه.
2) $\bar A \times (\bar B \times \bar C) \neq (\bar A \times \bar B) \times \bar C$ → الأولوية للأقواس
3) $|\hat a_x \times \hat a_x| = 0$
لأنه الزاوية بينهم صفر
$(\sin 0) = 0$



إذا مشيت مع عقارب الساعة بكون الجواب موجب، وإذا عكس عقارب الساعة سالب.

* Vector projection along another vector:—

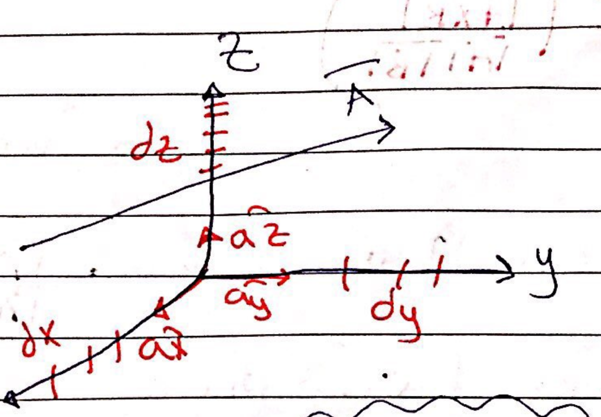
↳ 1) Scalar projection:—
$A_B = \bar A \cdot \hat a_B$

2) Vector projection:—
$\bar A_B = (\bar A \cdot \hat a_B)\,\hat a_B$, $\hat a_B = \frac{\bar B}{B}$

- لما يطلب مني Vector projection أول اشي بوجد $\hat a_B = \frac{\bar B}{B}$ الي رح تكون Vector، بعدين رح أعمل dot product بين $\bar A$ و $\hat a_B$، بطلع معي جواب Scalar بضرب الجواب هاد بالـ $\hat a_B$.

* CH #2

1) Cartesian Coordinates



$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$-\infty < x < \infty$
$-\infty < y < \infty$
$-\infty < z < \infty$

3D object
Infinite Box

- Differential Elements:-

dx, dy, dz

* differential Length ($\bar{dL}$) :-

$\bar{dL} = dx\, \hat{a}_x + dy\, \hat{a}_y + dz\, \hat{a}_z$

- Differential Normal Surface Area ($\bar{ds}$).

$\bar{ds}_{front} = dy\, dz\, (\hat{a}_x)$ → هاي اسمها العاموري

بالواجهة الأمامية بكون عندي اللي طالع من ال object هو العاموري هو $\hat{a}_x$.

$\bar{ds}_{back} = -dy\, dz\, \hat{a}_x$ — السالب للاتجاه لأنه العاموري للداخل مش للخارج.

$\bar{ds}_{right} = dx\, dz\, \hat{a}_y$

$\bar{ds}_{left} = -dx\, dz\, \hat{a}_y$

$\bar{ds}_{top} = dx\, dy\, \hat{a}_z$

$\bar{ds}_{bottom} = -dx\, dy\, \hat{a}_z$

Differential Volume (dv)

$dv = dx\, dy\, dz$ — دايماً الحجم scalar مش vector

* 2D Surface (we fix one variable only).

- if x = constant (not zero).
  ↳ infinite plane parallel to yz plane.

لو سألني شو العامودي على هاد ال plane ؟
لازم تكون ($\hat{a}_x$) أو ($-\hat{a}_x$) حسب إذا النقطة فوق ال plane أو تحته.

- if $x = 0$
  ↳ infinite plane along yz plane.

- if y = constant (not zero)
  ↳ infinite plane parallel to xz plane

- if $y = 0$:
  ↳ infinite plane along xz plane

- if z = constant (not zero)
  ↳ infinite plane parallel to xy plane

- if $z = 0$
  ↳ infinite plane along xy plane.

** 1D Segment (we fix two variables).

- if x, z are constants (not zero) ($x \neq 0$, $z \neq 0$).
  ↳ infinite line parallel to y-axis.

- if $x = 0$, $z = 0$
  ↳ infinite line along y-axis.

- if y, z constants ($y \neq 0$, $z \neq 0$).
  ↳ infinite line parallel to x-axis.

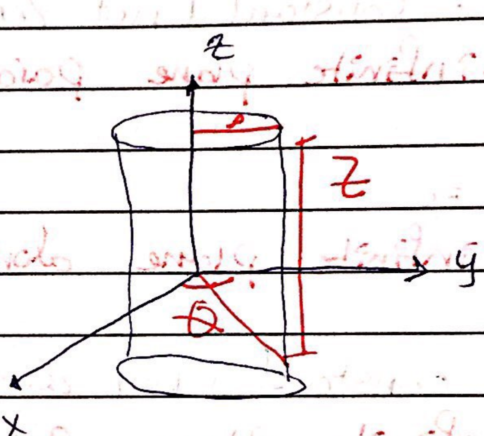
- if $y=0,\ z=0$.
  ↳ infinite Line along x-axis.

- if x, y are constants ($x \neq 0,\ y \neq 0$)
  ↳ infinite Line parallel to z-axis.

- if $x=0,\ y=0$.
  ↳ infinite Line along z-axis.

* Cylindrical coordinates.

$$0 \leq \rho < \infty$$
$$0 \leq \phi \leq 2\pi$$
$$-\infty < z < \infty$$



Infinite solid Cylinder.

$$\bar{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

* Differential Elements :-

$d\rho,\ \rho\, d\phi,\ dz$.

$$\bar{dL} = d\rho\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$$

$$\bar{dS}_{top} = \rho\, d\rho\, d\phi\, \hat{a}_z$$

$$\bar{dS}_{bottom} = -\rho\, d\rho\, d\phi\, \hat{a}_z$$

$$\bar{dS}_{side} = \rho\, d\phi\, dz\, \hat{a}_\rho$$

$$\bar{dS}_{cut}\ (\phi = \text{constant}) = d\rho\, dz\, \hat{a}_\phi$$

$$dV = \rho\, d\rho\, d\phi\, dz.$$

كيف عرفته ؟ بحدد العامودي على السطح و بضرب الباقية ببعض

لما يحكيلي اشي هو const بكون عامودي

* 2D Surface (we fix one variable).
  - ρ = constant → infinte hollow cylinder
    $\rho = 0$ → inf line along z-axis

  - ϕ = constant → semi-infinite plane
    $\phi = 90^\circ$ → semi-inf plane along yz plane.

  - z = constant → inf. Disk // xy plane
    z = 0 → inf. Disk along xy plane.

* 1 D Surface (we fix 2 variables).
  - ρ, ϕ constants & $\rho \neq 0$
    ↳ inf. Line parallel to z-axis

  - ρ, ϕ constants & $\rho = 0$
    ↳ inf. Line along ~~y~~ z-axis

  - ρ, z constants ($z \neq 0$, $\rho > 0$).
    ↳ Circle // xy plane

  - ρ, z constants ($z = 0$, $\rho > 0$)
    ↳ Circle along xy plane

  - ρ, z constants ($\rho = 0$)
    ↳ point

  - ϕ, z constants
    ↳ semi-inf Line (Ray)

Scanned with CamScanner

* Spherical Coordinates:-

$0 \leq r < \infty$

$0 \leq \theta \leq \pi$

$0 \leq \phi \leq 2\pi$

3D object
Infinite Solid
Sphere.

$\bar{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$

* Differential Elements:-

dr , $r d\theta$, $r \sin\theta \, d\phi$

- $\overline{dL} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin\theta \, d\phi \, \hat{a}_\phi$

$\overline{ds}$ surface $= r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$

$\overline{ds_\theta} = r \sin\theta \, dr \, d\phi \, \hat{a}_\theta$

$ds_\phi = r dr \, d\theta \, \hat{a}_\phi$

$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$

* 2D Surface (we fix one variable).

- if $r =$ constant $(r \neq 0)$ → hollow sphere.
- if $r = 0$ → point
- if $\theta =$ constant
  - ↳ $\theta \in (0, 90)$ and $(90, 180)$ → inf. hollow ~~sphere~~ cone.
  - ↳ $\theta = 90^\circ$ → inf. disk along xy plane
  - ↳ $\theta = 0^\circ$ → Semi-inf. Line in the +ve z-axis
  - ↳ $\theta = 180^\circ$ → semi-inf Line in the -ve z-axis.

- if $\phi =$ constant → semi-inf. Disk
  - ↳ $\phi = 90^\circ$ → semi-inf Disk along yz plane.

* 1D Segment (we fix 2 variables).

- if r, θ constants
  - $(r > 0), (\theta \neq 90°) \rightarrow$ Circle // xy plane
  - $(r > 0), (\theta = 90°) \rightarrow$ Circle along xy plane
  - $r = 0 \rightarrow$ point

- if r, ϕ constants.
  - $(r > 0) \rightarrow$ half circle
  - $(r = 0) \rightarrow$ point

- if θ, ϕ are constants
  - semi-inf. Line (Ray).

* Transformation between coordinates :-

المعطى المطلوب

- $(x, y, z) \longrightarrow (\rho, \phi, z)$

$$\rho = \sqrt{x^2 + y^2} \quad , \quad \phi = \tan^{-1}\left(\frac{y}{x}\right) \quad , \quad z = z .$$

- $(\rho, \phi, z) \longrightarrow (x, y, z)$

$$x = \rho \cos\phi \quad , \quad y = \rho \sin\phi \quad , \quad z = z$$

- $(x, y, z) \longrightarrow (r, \theta, \phi)$.

$$r = \sqrt{x^2 + y^2 + z^2} \quad , \quad \theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) \quad , \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

* $(r, \theta, \phi) \rightarrow (x, y, z)$

$x = r\sin\theta\cos\phi$, $y = r\sin\theta \sin\phi$, $z = r\cos\theta$

* ~~[illegible]~~

* $(\rho, \phi, z) \rightarrow (r, \theta, \phi)$

$r = \sqrt{\rho^2 + z^2}$, $\theta = \tan^{-1}\left(\frac{\rho}{z}\right)$, $\phi = \phi$

* $(r, \theta, \phi) \rightarrow (\rho, \phi, z)$

$\rho = r\sin\theta$, $\phi = \phi$, $z = r\cos\theta$

القوانين الي قبل كلها لتحويل نقاط فقط مش Vectors.

* Vectors Transformation

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

المطلوب   المعطى

هاي ال matrix بتكون معطاة بالامتحان.

$A_\rho = \cos\phi \, A_x + \sin\phi \, A_y$.

لو طلب مني ال cartesian وكان معطيني ال cylindrical كيف أحلها؟

باخد ال transpose للماتركس بعني كل عامود بصير صف وبكمل بنفس الطريقة.

* CH # 3

* Line Integral :-

$$\int_{l} \bar{A} \cdot \bar{dL}$$

ل ممكن تكون أي وحدة من القوانين اللي أخدناهم .

* Surface Integral :-

$$\int_{s} \bar{A} \cdot \bar{ds}$$

* Volume Integral :-

$$\int |\bar{A}| \, dv$$

* Del operator (∇)

- in Cartesian

هاي اسمها Partial

$$\nabla v = \frac{\partial v}{\partial x} \hat{a_x} + \frac{\partial v}{\partial y} \hat{a_y} + \frac{\partial}{\partial z} \hat{a_z}$$

تطبيق على ∇

Gradient

$\nabla * V$ = vector

∇ ↓ vector, V ↓ scalar

- in cylindrical

$$\nabla u = \frac{\partial u}{\partial \rho} \hat{a_\rho} + \frac{1}{\rho} \frac{\partial u}{\partial \phi} \hat{a_\phi} + \frac{\partial u}{\partial z} \hat{a_z}$$

- in spherical

$$\nabla T = \frac{\partial T}{\partial r} \hat{a_r} + \frac{1 T}{r} \frac{\partial}{\partial \theta} \hat{a_\theta} + \frac{1}{r \sin\theta} \frac{\partial T}{\partial \phi} \hat{a_\phi}$$

* CH 4

- coulomb's Law
  - the Force between two point charges is along the line joining them, directly propotional to Q_1, Q_2 and inversely propotional to the square of the distance between them.

$$F = k\frac{Q_1 Q_2}{R^2} \ (N), \quad k = \frac{1}{4\pi\epsilon_0}, \quad \epsilon_0 = \frac{10^{-9}}{36\pi} \ (F/m).$$

ده القانون magnitude only

* F as a vector quantity:-
  - ① $\overline{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}\hat{a}_{R_{12}}$ — Force on Q_2 due to Q_1 — $\hat{a}_{R_{12}} = \frac{\overline{R}_{12}}{|\overline{R}_{12}|}$
  - ② $\overline{F}_{12} = \frac{Q_1 Q_2 \cdot \overline{R}_{12}}{4\pi\epsilon_0 R^3}$
  - ③ $\overline{F}_{12} = \frac{Q_1 Q_2 (\overline{r}_2 - \overline{r}_1)}{4\pi\epsilon_0 |r_2 - r_1|^3}$

* ملاحظة: قانون ① و ② و ③ نفس الاشي ولكن بأشكال مختلفة.

$\overline{F}_{12} = -\overline{F}_{21}$

نفس المقدار, عكس الاتجاه.

* Force due to N-point charges:-

$$\overline{F} = \frac{Q}{4\pi\epsilon_0}\sum_{k=1}^{N}\frac{Q_k(\overline{r} - \overline{r}_k)}{|r - r_k|^3}$$

- Using superposition principle.

Field → اللي بدي أحسب (عنصر اللي بأثروا عليها)

Note:
1. the distance between Q_1 & Q_2 must be large compared to their bodies (point charges)
2. Q_1 & Q_2 must be static (at rest)

Source → اللي بأثروا عالنقطة اللي بدي أحسب عندها.

+ve

* Electric field intensity: the force that a +ve charge experiences when placed in an Electric field.

$$\bar{E} = \frac{\bar{F}}{Q} \quad (N/C) \text{ or } (V/m)$$

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r = \frac{Q(\bar{r}-\bar{r}')}{4\pi\epsilon_0 |r-r'|^3}$$

$r \to$ field
$r' \to$ source

* For N-point charges:-

$$\bar{E} = \frac{1}{4\pi\epsilon_0}\sum_{k=1}^{N} \frac{Q_k(\bar{r}-\bar{r}_k)}{|r-r_k|^3}$$

Electric fields due to continuous charge distributions:-

$\rho_L \to$ Line charge density (C/m)
$\rho_s \to$ Surface charge density (C/m²)
$\rho_v \to$ Volume charge density (C/m³)

don't mix between ρ for cylindrical radial and ρ_L, ρ_s, ρ_v.

* Total charge:-

1) Line charge.

$$dQ = \rho_L\, dL \to Q = \int_L \rho_L\, dL$$

2) Surface charge

$$dQ = \rho_s\, ds \to Q = \int_S \rho_s\, ds.$$

3) Volume charge

$$dQ = \rho_v\, dv \to Q = \int_V \rho_v\, dv.$$

- Electric field intensity

1) Line charge.

$$\bar{E} = \int_L \frac{\rho_L\, dL}{4\pi\epsilon_0 R^2}\,\hat{a}_r$$

2) Surface charge

$$\bar{E} = \int_S \frac{\rho_s\, ds}{4\pi\epsilon_0 R^2}\,\hat{a}_r$$

3) Volume charge.

$$\bar{E} = \int_V \frac{\rho_v\, dv}{4\pi\epsilon_0 R^2}\,\hat{a}_r$$

* Line Charge

- finite line charge

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0 \rho}\left[-(\sin\alpha_2 - \sin\alpha_1)\,\hat{a_\rho} + (\cos\alpha_2 - \cos\alpha_1)\,\hat{a_z}\right] \; (V/m)$$

- infinite line charge.

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho}\,\hat{a_\rho} \quad (V/m)$$

* Surface Charge.

- infinite sheet of charge

$$\bar{E} = \frac{\rho_s}{2\epsilon_0}\,\hat{a_n} \quad (V/m)$$, $\hat{a_n}$: unit vector normal to the sheet.

- for parallel plate capacitor.

$$\bar{E} = \frac{\rho_s}{\epsilon_0}\,\hat{a_n} \quad (V/m)$$

Volume Charge

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a_r} \quad (V/m)$$

Electric Flux Density :-

$$\bar{D} = \epsilon_0 \bar{E} \quad (c/m^2)$$

infinite sheet of charge

$$\bar{D} = \frac{\rho_s}{2}\,\hat{a_n} \quad (c/m^2)$$

نفس قانون E بس بنشيل ϵ_0

infinite line charge

$$\bar{D} = \frac{\rho_L}{2\pi\rho}\,\hat{a_\rho} \quad (c/m^2)$$

Volume charge distribution

$$\bar{D} = \int_v \frac{\rho_v\, dv}{4\pi R^2}\,\hat{a_r}$$

The flux $(\bar{D})$ is independent of the meduim يعتمد فقط على الشحنة وموقعها

* Gauss's Law:-
↳ total electric flux through closed surface is equal to the total charge enclosed by that surface.

$$Q = \oint_S \overline{D} \cdot ds = \int_V \rho_v \, dv$$

يعني
[illegible]
enclosed surface.

- First Maxwell's equation in integral form:

$$\oint \overline{D} \cdot ds = \int_V \rho_v \, dv$$

- First Maxwell's equation in point form

$$\rho_v = \nabla \cdot \overline{D}$$

* Electric Potential:-

$$W = -Q \int_A^B \overline{E} \cdot \overline{dL} \quad (J)$$

W: Work (potential energy) required to move Q from point A to point B.

negative sign means
the work is done by an external agent.

$$V_{AB} = \frac{W}{Q} = -\int_A^B \overline{E} \cdot \overline{dL} \quad (J/C) \; (V)$$

V_{AB}: potential difference between A & B
A is the initial point
B is the final point

if V_{AB} is positive
↳ gain in potential
↳ external agent performs the work

if V_{AB} is negative
↳ loss in potential
↳ work is being done by the field.

$$V_{AB} = V_B - V_A$$

* For a point charge:-

$$V_{AB} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$V(r) = \frac{Q}{4\pi\epsilon_0 |r-r'|}$$

إذا ما كان معطيني ref. point بالسؤال بعتبر ∞ و $V_\infty = 0$

* Line charge

$$V(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho_L \, dL'}{|r-r'|}$$

- Surface charge

$$V(r) = \frac{1}{4\pi\epsilon_0}\int_s \frac{\rho_s \, ds'}{|r-r'|}$$

- Volume charge

$$V(r) = \frac{1}{4\pi\epsilon_0}\int_v \frac{\rho_v \, dv'}{|r-r'|}$$

- $\boxed{E = -\nabla V}$

$\bar{E}$ is opposite to the direction in which V increases

Electric dipole:-

$$V = \frac{Q\,(r_2 - r_1)}{4\pi\epsilon_0 r_1 r_2}$$ when locations are given.

d

or

$$V = \frac{Q\,d\cos\theta}{4\pi\epsilon_0 r^2}$$

- define dipole moment $\bar{P} = Q\bar{d}$

$$V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2} = \frac{P\cdot(\bar{r}-\bar{r}')}{4\pi\epsilon_0 |r-r'|^3}$$

or

$$V = \frac{\bar{P}\cdot\hat{a}_r}{4\pi\epsilon_0 r^2} \qquad \bar{E} = \frac{P}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right)$$

* Energy Density of Electrostatic fields:-

$$W_E = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k \quad (J)$$ for point charges.

- for line charge

$$W_E = \frac{1}{2}\int_L \rho_L V dL \quad (J)$$

- for surface charge

$$W_E = \frac{1}{2}\int_S \rho_s V ds \quad (J)$$

- for volume charge

$$W_E = \frac{1}{2}\int_V \rho_v V dv \quad (J)$$

* To find energy from $\bar{E}$

$$W_E = \frac{1}{2}\int_V \bar{D}.\bar{E}\, dv \quad (J)$$

or

$$W_E = \frac{1}{2}\int_V \epsilon_0 E^2 dv \quad (J)$$ $\bar{D} = \bar{E}\epsilon_0$

or

$$W_E = \frac{1}{2}\int_V \frac{D^2}{\epsilon_0} dv \quad (J)$$ $\bar{E} = \frac{\bar{D}}{\epsilon_0}$

* Energy density (w_E)

$$w_E = \frac{dW_E}{dv} = \frac{1}{2}\bar{E}.\bar{D} = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\frac{D^2}{\epsilon_0} \quad (J/m^3)$$

$$W_E = \int_V w_E\, dv \quad (J/m^3)$$

CH5 - Materials based on it's electrical properties

↳ Conductors ($\sigma \ggg 1$)

i.e. (Cu, Al, Ag, Lead)

↳ Semi-conductors ($\sigma > 1$)

i.e. (Si, GaAs, Ge)

↳ Dielectrics (Insulators) ($\sigma \lll 1$)

i.e (Li, Mica, polythelene, polyst-)

σ: conductivity (S/m)

↳ depends on:-

1) Temperature ($T\uparrow, \sigma\downarrow$)

2) Frequency.

* $\boxed{I = \frac{\Delta Q}{\Delta t} \quad (A)}$

Current: the electric charge passing through an area per unit time.

- Current Density ($\bar{J}$)

$$\boxed{\bar{J} = \frac{\Delta I}{\Delta S}} \quad (A/m^2)$$

- Types of current:-

1] Convention Current

$I = \rho_v u_y \Delta S \quad (A)$

$\bar{J} = \rho_v \bar{u} \quad (A/m^2)$

2] Conduction Current

$\bar{J} = \sigma \bar{E}$

$\sigma = \frac{n^2 e \tau}{m}$

$\rho_v = ne$

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* Conductors:
↳ if the conductor is isolated.
↳ $\bar{E}$ inside $=0$.
$\bar{D}$ inside $=0$
ρ_v inside $=0$.
V_{ab} inside $=0$.
- The conductor is called equipotential body.

↳ if the conductor is not isolated

$$R = \frac{V}{I} = \frac{L}{\sigma S} = \frac{\rho_c L}{S} \qquad \left(\rho = \frac{1}{\sigma}\right)$$

⇒ cross section uniform لو كانت الـ conductor لها ثابت ، هاد القانون لا

$$R = \frac{V}{I} = \frac{\int_L \bar{E}\cdot d\bar{L}}{\int_S \sigma \bar{E}\cdot d\bar{s}} \rightarrow \text{in general.}$$

* power: -

$$P = \int_V \bar{J}\cdot \bar{E}\, dV$$

or

$$P = \sigma \int_V E^2 dV \ (W) \left(\bar{J} = \sigma \bar{E}\right)$$

or

$$P = \int_V \frac{J^2}{\sigma} dV \ (W) \left(\bar{E} = \frac{\bar{J}}{\sigma}\right)$$

* Power Density.

$$w_p = \frac{dP}{dV} = \bar{J}\cdot\bar{E} = \sigma E^2 = \frac{J^2}{\sigma} \ (W/m^3)$$

$$P = \int_V w_p\, dV$$

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- Non-polarized dielectrics:-

$$\bar{P} = \lim_{\Delta V \to 0} \frac{\sum_{k=1}^{N} Q_k \bar{d}_k}{\Delta V}$$

- Polarized dielectric:-

$$V = \int_S \frac{\rho_{ps}\, ds}{4\pi\epsilon_o r} \qquad V = \int_V \frac{\rho_{pv}\, dv}{4\pi\epsilon_o r}$$

هدول المعادلات للشحنات جوا ال dielectric

ρ_{ps}: polarized (bound) surface charge density

ρ_{pv}: polarized (bound) volume charge density

$$V = \int_S \frac{\rho_s\, ds}{4\pi\epsilon_o r} \qquad V = \int_V \frac{\rho_v\, dv}{4\pi\epsilon_o r}$$

هدول معادلات ال free charge

$$* \ \boxed{\rho_{ps} = \bar{P} \cdot \hat{a}_n} \qquad \boxed{\rho_s = \bar{D} \cdot \hat{a}_n}$$

$$* \ \boxed{\bar{D} = \epsilon_o \bar{E} + \bar{P}}$$

$$\boxed{\bar{P} = X_e \epsilon_o \bar{E}}$$

$$\boxed{\bar{D} = \epsilon \bar{E}}, \quad \boxed{\epsilon = \epsilon_r \epsilon_o}$$

$$\downarrow$$

$$\boxed{\epsilon_r = 1 + X_e}$$

↳ Relative permativity

$$* \ \boxed{\bar{P} = (\epsilon_r - 1)\epsilon_o \bar{E}}$$

إذا مش معطيني X_e بالسؤال

$$Q_{b^+} = \int_S \rho_{ps}\, ds$$

$$Q_{b^-} = \int_V \rho_{pv}\, dv$$

$Q_{btot} = 0 \rightarrow$ material is electrically neutralized.

* Continuity Equation

$$\oint_S \overline{J} \cdot \overline{ds} = -\int_V \frac{d\rho v}{dt} \, dV$$

* Relaxation Time

$$T_r = \frac{\epsilon_0 \epsilon_r}{\sigma} \quad (s)$$

$$\rho v = \rho v_0 \, e^{-t/T_r} \quad (c/m^3)$$

ρv_0 : initial charge density

* Boundry conditions :-

A) Dielectric - to - dielectric.

$$\overline{E_1} = \overline{E_{1t}} + \overline{E_{1n}}$$

$$\overline{E_{1n}} = (\overline{E_1} \cdot \hat{a_n}) \cdot \hat{a_n}$$

$$\overline{E_{1t}} = \overline{E_1} - \overline{E_{1n}}$$

$$\overline{E_{2t}} = \overline{E_{1t}}$$

$$\overline{D_{2n}} - \overline{D_{1n}} = \rho_s \quad (\text{if } \rho_s = 0 \rightarrow \overline{D_{1n}} = \overline{D_{2n}})$$

$$\overline{D_2} = \epsilon_0 \epsilon_r \overline{E_2}$$

$$\overline{E_{2n}} = \frac{\epsilon_{r1}}{\epsilon_{r2}} \overline{E_{1n}}$$

$$\theta_2 = \sin^{-1}\left(\frac{E_{2t}}{E_2}\right) = \cos^{-1}\left(\frac{E_{2n}}{E_2}\right) = \tan^{-1}\left(\frac{E_{2t}}{E_{2n}}\right)$$

$$\alpha_2 = 90 - \theta_2$$

B) Dielectric - to - Conductor

$$\overline{D_{1n}} = \rho_s \hat{a_n}$$

$$\overline{E_{1n}} = \frac{\rho_s}{\epsilon_0 \epsilon_{r1}} (-\hat{a_z})$$

CH6 Resistance and Capacitance :-

$$R = \frac{V}{I} = \frac{-\int_L \bar{E}\cdot\bar{dL}}{\int_S \sigma \bar{E}\cdot ds}$$

$$C = \frac{Q}{V} = \frac{\int_S \epsilon \bar{E}\, \bar{ds}}{-\int_L \bar{E}\cdot\bar{dL}}$$

- Steps to find R&C :-

1) Choose suitable coordinates
2) Assume (+Q) and (-Q)
3) $Q = \oint_S \epsilon \bar{E}\, \bar{ds}$ (find $\bar{E}$)
4) Find $V = -\int_L \bar{E}\, \bar{dL}$ / $I = \int_S \sigma \bar{E}\cdot\bar{ds}$

<u>Notes:</u> R&C must be +ve values

CH7 Magnetostatic Fields :-

- Biot - Savart's Law :

$dH \propto \frac{I \sin(\alpha)\, dL}{r^2}$, α : angle between dL & r

- for line current

$\bar{H} = \int_L \frac{I\, \bar{dL} \times \bar{r}}{4\pi r^3}$, I : Line Current (A/m)

- for surface current

$\bar{H} = \int_S \frac{\bar{K} \times \bar{r}}{4\pi r^3}\, ds$, k : Surface Current (A/m)

for volume Current

$\bar{H} = \int_V \frac{\bar{J} \times \bar{r}}{4\pi r^3}\, dV$, J : Volume Current (A/m)

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- $\overline{H}$ for Straight finite line :

$$\overline{H} = \frac{I}{4\pi\rho} (\cos\alpha_2 - \cos\alpha_1)\, \hat{a_\phi}$$

$$\hat{a_\phi} = \hat{a_L} \times \hat{a_\rho}$$

- $\overline{H}$ for semi-infinite line :-

$$\overline{H} = \frac{I}{4\pi\rho}\, \hat{a_\phi}$$

- $\overline{H}$ for infinite line :-

$$\overline{H} = \frac{I}{2\pi\rho}\, \hat{a_\phi}$$

* Ampere's Laws :-

$$\oint_L \overline{H} \cdot \overline{dL} = I_{enc}.$$

* $\overline{H}$ for infinite Sheets :-

$$\overline{H} = \frac{1}{2} \overline{K} \times \hat{a_n}$$

$\hat{a_n}$: unit vector directed from the Sheet to the point.

* Magnetic flux density $(\overline{B})$:-

$$\overline{B} = \mu_0 \overline{H}$$

μ_0 : free space permeability $(4\pi * 10^{-7})$

- $\overline{B}$ for infinite line

$$\overline{B} = \frac{\mu_0 I}{4\pi\rho} (\cos\alpha_2 - \cos\alpha_1)\, \hat{a_\phi}$$

نفس قانون $\overline{H}$ بس مضروب عليه μ_0

- $\overline{B}$ for infinite sheet

$$\overline{B} = \frac{\mu_0}{2} \overline{K} \times \hat{a_n}$$

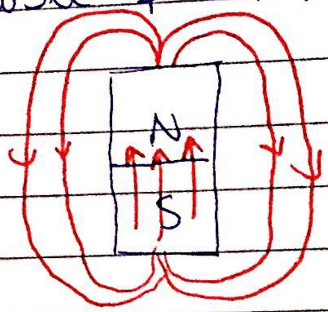
* Magnetic Flux (Ψ_m)

$$\Psi_m = \int_S \bar{B} \cdot \bar{ds} \quad (wb)$$

* Flux Lines :-

[1] Continuos

[2] From N→S outside & S→N inside



* Magnetic Gauss Law:-

$$\Psi_m = \int_S \bar{B} \cdot \bar{ds} = 0$$

* Magnetic Potentials:-

$$\bar{H} = -\nabla V_m$$

(V_m : Scalar potential)

($\bar{A}$: Vector potential).

- Vector Potential for line currents

$$\bar{A} = \int_L \frac{\mu_0 I}{4\pi r} \bar{dL}$$

- Vector Potential for Surface Currents

$$\bar{A} = \int_S \frac{\bar{K} \mu_0}{4\pi r} ds$$

- Vector Potential for Volume Current:-

$$\bar{A} = \int_V \frac{\bar{J} \mu_0}{4\pi r} dv$$

* $\boxed{\Psi_m = \oint_L \bar{A} \cdot \bar{dL}}$

$\boxed{\Psi_m = \int_S \bar{B} \cdot \bar{ds}}$

CH.8 Magnetization:-

$$\overline{M} = \lim_{\Delta V \to 0} \sum_{k=1}^{N} \frac{\overline{m}_k}{\Delta V}$$, $\overline{m}$: magnetic moment

- Free current

$\overline{K} = \overline{H} \cdot \hat{a_n}$

- Bound Current

$\overline{K_b} = \overline{M} \times \hat{a_n}$

* $\boxed{\overline{M} = \chi_m \overline{H}}$, $\chi_m = \mu_r - 1$ (Magnetization)

* Classification of magnetic materials:-
  - ↳ Diamagnetic ($\mu_r \le 1$) (i.e. Cu, lead, AL) } Linear
  - ↳ paramagnetic ($\mu_r \geq 1$) (i.e. Air) } Linear
  - ↳ Ferromagnetic ($\mu_r \ggg 1$) (i.e. iron, Nickle, rebolt) ⟶ Nonlinear.

*Curie temperature: The temperature that if a ferromagnetic is heated above it loses its nonlinear properties and becomes a linear paramagnetic material. Thus if a permenant magnet is headed above the curie temp. it loses its magnetization.

*Magnetic Boundary Conditions:-

$\mu = \mu_0 \mu_r$ (of free space, $\mu_r = 1$)

$\overline{H_{1n}} = (\overline{H_1} \cdot \hat{a_n}) \cdot \hat{a_n}$

$\overline{H_{1t}} = \overline{H_1} - \overline{H_{1n}}$, $\sin\theta_1 = \dfrac{H_{1t}}{H_1}$

$\overline{B_{1n}} = (\overline{B_1} \cdot \hat{a_n}) \cdot \hat{a_n}$

$\overline{B_{1t}} = \overline{B_1} - \overline{B_{1n}}$, $\cos\theta_1 = \dfrac{H_{1n}}{H_1}$

$\overline{B_{1n}} = \overline{B_{2n}}$

$(\overline{H_1} - \overline{H_2}) \times \hat{a_n} = \overline{K}$ (if $\overline{K} = 0 \rightarrow \overline{H_{1t}} = \overline{H_{2t}}$) $\tan\theta_1 = \dfrac{H_{1t}}{H_{1n}}$

$\alpha_1 = 90 - \theta_1$

* Inductors:

$$L = \frac{\lambda}{I} \ (H), \quad \lambda = N\Psi, \quad \Psi = \int_S \overline{B} \cdot \overline{ds}$$

* Magnetic energy:-

$$W_m = \frac{1}{2} \int_V \overline{H} \cdot \overline{B} \, dV \quad (J)$$

$$= \frac{1}{2} \int_V \mu H^2 \, dV \quad (J)$$

$$= \frac{1}{2} \int_V \frac{B^2}{\mu} \, dV \quad (J)$$

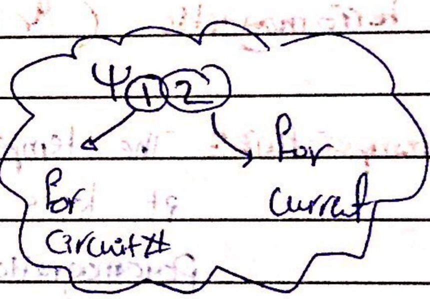
- if uniform:-

$$W_m = \frac{1}{2} L I^2 \Rightarrow \boxed{L = \frac{2W_m}{I^2}}$$

* Self inductance:-

$$L_{11} = \frac{\lambda_{11}}{I_1} = \frac{N_1 \Psi_{11}}{I_1}$$

$$L_{22} = \frac{\lambda_{22}}{I_2} = \frac{N_2 \Psi_{22}}{I_2}$$



* Mutual inductance:-

$$M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \Psi_{12}}{I_2}$$

$$M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2 \Psi_{21}}{I_1}$$

- total flux in circuit 1:- (Ψ_1)

$$\Psi_1 = \Psi_{11} \pm \Psi_{12}$$

+ → same direction

− → opposite direction.

* Total inductance of circuit I:

$$L_1 = L_{11} \pm M_{12}$$

+ if flux +

− if flux −

* Total energy:

$$W_m = \frac{1}{2} L_{11} I_1^2 + \frac{1}{2} L_{22} I_2^2 + \left(\frac{1}{2} M_{12} I_2^2 + \frac{1}{2} M_{21} I_1^2\right)$$

* $\boxed{\frac{I_2}{I_1} = \frac{N_1}{N_2}}$

* Procedure to find (L):

1) Choose suitable coordinates
2) Assume current
3) Find B
4) $\Psi = \int \bar{B} \cdot \bar{ds} \longrightarrow L = \frac{N\Psi}{I}$

or

4) $W_m = \frac{1}{2} \int_V \frac{B^2}{\mu} dV \rightarrow L = \frac{2W_m}{I^2}$

* Magnetic Circuits:

Electrical Circuit	Magnetic circuit
V_{em}	$V_{emf} = F$
I	Ψ
$R = \frac{V}{I} = \frac{L}{G}$	$R_m = \frac{F}{\Psi}$
$G = 1/R$	$P = 1/R_m$
$I = \int_S \bar{J} \cdot \bar{ds}$	$\Psi = \int_S \bar{R_s} \cdot \bar{ds}$
$W_E = \frac{1}{2} \int_V \epsilon E^2 dV$	$W_m = \frac{1}{2} \int_V \mu H^2 dV$
ϵ	μ

principle of duality

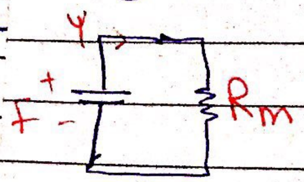
* Magnetic Resistance (Reluctance):-

$$R_m = \frac{L_c}{\mu_0 \mu_r A_c}$$

$L_c = 2\pi \rho_0$ (toroid)

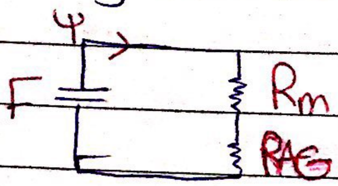
$A_c = \pi t^2$ (toroid)

* Magnetic circuit:-



$$\Psi = \frac{F}{R_m} = \frac{NI}{\frac{L_c}{\mu_0 \mu_r A_c}}$$

* Magnetic circuit with air gap:-



$$R_m = \frac{L_c}{\mu_0 \mu_r A_c}$$

$$R_{AG} = \frac{L_{CA}}{\mu_0 A_c} \quad (\mu_r = 1)$$

$R_{eq} = R_m + R_{AG}$

$$\Psi = \frac{F}{R_{eq}}$$

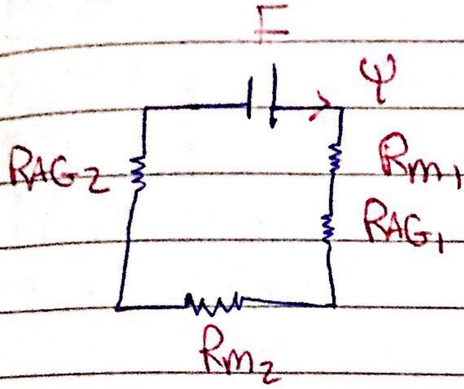
Force in the air gap:-

(Magnetic Tractive Force) (F_{MT})

$$F_{MT} = \frac{B^2 A_c}{2\mu_0} \qquad B = \frac{\Psi}{A_c}$$

$$\text{Pressure} = \frac{F_{MT}}{A_c}$$

*Magnetic Limitations:-



$Rm_1 = \frac{Lc_1}{\mu_0 \mu_{r_1} Ac}$ $Rm_2 = \frac{Lc_2}{\mu_0 \mu_{r_2} Ac}$

$RAG_1 = \frac{LAG}{\mu_0 Ac}$ $RAG_2 = RAG_1$

$Req = 2RAG + Rm_1 + Rm_2$

$FM_T = mg = \frac{B^2 Ac}{2\mu_0}$

*CH9 Time Varying fields

- Sources for time varying fields:-
1) Charge moving with accelaration
2) Ac current flowing in wire.

*Faraday's Laws:-

$V_{emf} = -\frac{d\Psi}{dt}$ (if $N=1$)

($\Psi = \int_S \bar{B} \cdot \bar{ds}$)

$V_{emf} = \oint_L \bar{E} \cdot \bar{dl} = -\frac{d}{dt} \int_S \bar{B} \cdot \bar{ds}$

[1] Stationary loop in time-varying $\bar{B}$ field (Transformer emf)

$V_{emf} = \oint_L \bar{E} \cdot \bar{dl} = -\int_S \frac{d\bar{B}}{dt} \cdot \bar{ds}$

[2] Moving Loop in static $\bar{B}$ field (Motional emf)

$V_{emf} = \oint_L \bar{E} \cdot \bar{dl} = \oint_L (\bar{u} \times \bar{B}) \cdot \bar{dl}$

u: velocity

3) Time varying field with moving loop

$$V_{emf} = \oint_L \bar{E} \cdot \bar{dL} = -\int_S \frac{d\bar{B}}{dt} \cdot \bar{ds} + \oint_L (\bar{u} \times \bar{B}) \cdot \bar{dL}$$

* Displacement Current:- (I_d)

$$\oint_L \bar{H} \cdot \bar{dL} = \int_S (\bar{J} + \bar{J_d}) \cdot \bar{ds}$$

$\bar{J_d}$: Displacement current density (A/m^2)

* Continuity equation:-

$$\oint_L \bar{H} \cdot \bar{dL} = \int_S \left(\bar{J} + \frac{d\bar{D}}{dt}\right) \cdot \bar{ds}$$

$$\bar{J_d} = \frac{d\bar{D}}{dt}, \quad \bar{D} = \epsilon \bar{E}$$

$\bar{J}_{conduction} = \sigma \bar{E}$

$\bar{J}_{convention} = \rho_v \bar{u}$

$\bar{J}_{displacement} = \frac{d\bar{D}}{dt}$

* Time harmonic fields:-

→ Complex Numbers

↳ Euler identity: $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

- Rectangular form of complex numbers:-

$Z = (x) + j\,(y)$

Real ↲ ↳ imaginary

- Polar form

$Z = r\angle\theta$

$r = \sqrt{x^2+y^2}$ $\quad \theta = \tan^{-1}\left(\frac{\text{imaginary}}{\text{real}}\right)$

- exponential form

$r\,e^{j\theta}$

Complex conjugate (*)

$Z = x + jy = r\angle\theta = r\,e^{j\theta}$

$Z^* = x - jy = r\angle -\theta = r\,e^{-j\theta}$

* Phasor domain:-

$I_o\,e^{j\theta} = I_s$ (phasor)

$$\frac{d}{dt}\left(e^{j\omega t}\right) = (j\omega)\, e^{j\omega t}$$

$$\int_t e^{j\omega t}\, dt = \frac{1}{j\omega}\, e^{j\omega t}$$

CH 10 Electromagnetic wave Propagation:-

λ: wavelength
u: speed
T: period
ω: radian frequency

$\lambda = uT$ (m)

$u = f\lambda$ $\left(T = \frac{1}{f}\right)$

$\omega = 2\pi f$ (rad/s)

$\beta = \frac{\omega}{u}$ (rad/m)

$T = \frac{2\pi}{\omega}$ (s)

$\beta = \frac{2\pi}{\lambda}$

إشا. موجبة عكس إشا. β

مثال: $\bar{E} = 50\cos(10^8 t + \beta x)$

$\hat{a}_k = -\hat{a}_x$

$\sin\left(\psi \pm \frac{\pi}{2}\right) = \pm\cos\psi$

$\sin(\psi \pm \pi) = -\sin\psi$

$\cos\left(\psi \pm \frac{\pi}{2}\right) = \mp\sin\psi$

$\cos(\psi \pm \pi) = -\cos\psi$

*wave propagation in lossless media:-

$(\varepsilon = \varepsilon_0\varepsilon_r \;,\; \mu = \mu_0\mu_r \;,\; \sigma = 0)$

$\gamma = \alpha + j\beta$

γ: Propagation constant; α: attenuation constant; β: phase constant

$\alpha = 0$ (since $\sigma = 0$)

$\beta = \omega\sqrt{\mu\varepsilon}$

$u = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$

$\lambda = \frac{2\pi}{\beta}$

$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ speed of light

$\eta = \sqrt{\frac{\mu}{\varepsilon}} \angle 0°$ → $\bar{E}$ & $\bar{H}$ are in phase

η: intrinsic impedance.

$\bar{E} = E_0\cos(\omega t - \beta z)\,\hat{a}_x$

$\bar{H} = \frac{E_0}{\eta}\cos(\omega t - \beta z)\,\hat{a}_y$ → (بنحول) $\hat{a}_z \times \hat{a}_x$

$H_0 = \frac{E_0}{\eta}$

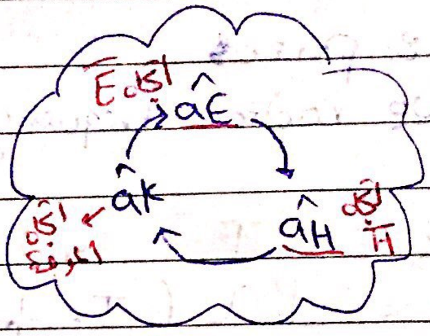
*Wave propogation in free space:-

$(\epsilon = \epsilon_0 \quad , \quad \mu = \mu_0 \quad , \quad \sigma = 0)$

$\alpha = 0$ (since $\sigma = 0$)

$\beta = \omega\sqrt{\mu_0 \epsilon_0} = \dfrac{\omega}{C}$

$u = \dfrac{1}{\sqrt{\mu_0 \epsilon_0}} = C$ Speed of light



$\eta = \sqrt{\dfrac{\mu_0}{\epsilon_0}} = 120\pi \ \Omega$

$\bar{E}$ & $\bar{H}$ are in phase

$\lambda = \dfrac{2\pi}{\beta}$

*Skindepth (δ)

$\delta = \dfrac{1}{\alpha} = \infty$

$1 \ NP = 20 \log_{10} e$

$1 NP = 8.686 \ dB$

*Wave propogation in a good conductor:-

$(\epsilon = \epsilon_0 \quad , \quad \mu = \mu_0 \mu_r \quad , \quad \sigma \simeq \infty)$

$\alpha = \sqrt{\dfrac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu \sigma}$

$\beta = \alpha = \sqrt{\pi f \mu \sigma}$

$\eta = \sqrt{\dfrac{j\omega\mu}{\sigma}} = \sqrt{\dfrac{\omega\mu}{\sigma}} \angle 45^\circ$

$u = \sqrt{\dfrac{2\omega}{\mu\sigma}}$

$\bar{E}$ leads $\bar{H}$ by 45°

$\lambda = \dfrac{2\pi}{\beta}$

$\bar{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{a}_x$

$\hat{a}_z \times \hat{a}_x$

$\delta = \dfrac{1}{\alpha} = \dfrac{1}{\sqrt{\pi f \mu \sigma}}$

$H = \dfrac{E_0}{\eta} e^{-\alpha z}$ ~~[illegible]~~

$\eta = \sqrt{\dfrac{\omega\mu}{\sigma}}$; $\cos(\omega t - \beta z - 45^\circ) \hat{a}_y$

* Application of skin depth (Coaxial cable) :-

at DC ($f = 0$)

$$R = \frac{L}{\sigma_c A}, \quad A = \pi a^2$$

$f\uparrow \rightarrow \delta\downarrow$

at AC ($f > 0$)

$$R = \frac{L}{\sigma_c A}, \quad A = 2\pi a$$

$R_{AC} \gg R_{DC}$ at higher freq.

* Power :-

$$\overline{P} = \overline{E} \times \overline{H} \quad (W/m^2)$$

$\overline{P}$ → Poynting vector

$$\overline{P}_{avg} = \frac{1}{T} \int_t \overline{E} \times \overline{H} \, dt$$

$$\overline{P}_{avg} = \frac{1}{2} \, Re\{ \overline{E}_s \times \overline{H}_s^* \} \quad (W/m^2)$$

* total power :-

$$P_{avg} = \int_S \overline{P}_{avg} \cdot \overline{ds}$$

* CH 11 Transmission Lines (T.L.)

- ↳ Coaxial cable
- ↳ Twin wire cable
- ↳ planar lines → Strip line
  - ↳ micro strip line
  - ↳ Slot line

* Coaxial cable parameters :-

$$\frac{R}{l} = \frac{1}{2\pi \sigma_c S}\left(\frac{1}{a} + \frac{1}{b}\right) \quad (\Omega/m)$$

Conductor

R → conductor

R_d → Dielectric

$$\frac{G}{l} = \frac{2\pi \sigma_d}{\ln\left(\frac{b}{a}\right)} \quad (S/m)$$

Dielectric

$$\frac{C}{l} = \frac{2\pi \epsilon_o \epsilon_r}{\ln\left(\frac{b}{a}\right)} \quad (F/m)$$

Dielectric

$$\frac{L}{l} = \frac{\mu_o \mu_r}{2\pi} \ln\left(\frac{b}{a}\right) \quad (H/m)$$

Dielectric

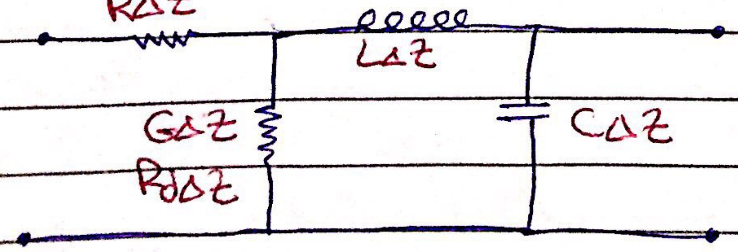
* Relaxation Time (T_r)

$$T_r = \frac{\epsilon_o \epsilon_r}{\sigma_d}$$

$$\frac{C}{G} = \frac{\epsilon}{\sigma_d}$$

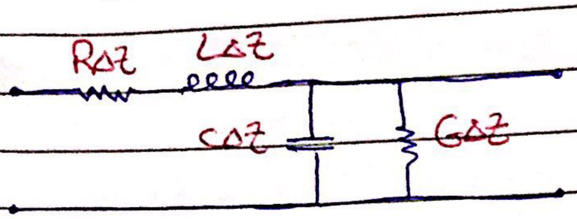
$$\frac{L_{ext}}{l} * \frac{C}{l} = \mu\epsilon$$

* Exact circuit for T.L.



* T.L. equivalent circuits (Approximate)

L - Type circuit (95% accurate)



T. L. equations:- (For L-Type)

$$\frac{d^2 V_s(z)}{dz^2} - \gamma^2 V_s(z) = 0 \quad , \quad \frac{d^2 I_s(z)}{dz^2} - \gamma^2 I_s(z) = 0$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

-Solution to the T.L. equ. (For L-Type)

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$

$$I_s(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z}$$

* Z_o: Characteristic impedance (Ω)

$$Z_o = \frac{V_o^+}{I_o^+}$$

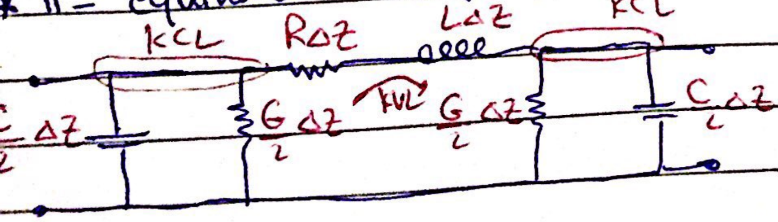
$$Z_o = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

$$u = \frac{\omega}{\beta} \quad , \quad \lambda = \frac{2\pi}{\beta}$$

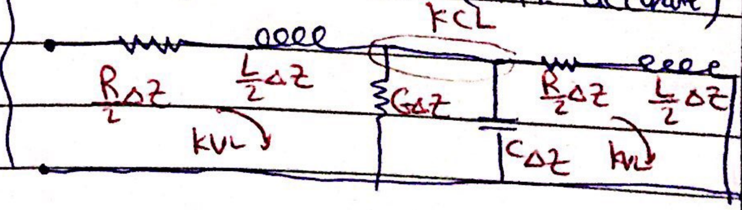
* Γ: reflection coefficient

$$\Gamma = \frac{V_o^-}{V_o^+}$$

* π- equivalent ckt (99% accurate)



* T- equivalent ckt (99% accurate)



* Lossless T.L.

$(\sigma_c = \infty \rightarrow R = 0)$

$(\sigma_d = 0 \rightarrow G = 0)$

$\gamma = j\omega\sqrt{LC}$ $\qquad Z_0 = \sqrt{\frac{L}{C}}$

$\beta = \omega\sqrt{LC}$, $\alpha = 0$ $\qquad u = \frac{\omega}{\beta}$

$\lambda = \frac{2\pi}{\beta}$

$V(z,t) = V_0^+ \cos(\omega t - \beta z) + V_0^- \cos(\omega t + \beta z)$

$I(z,t) = \underbrace{\frac{V_0^+}{Z_0}}_{I_0^+} \cos(\omega t - \beta z) + \underbrace{\frac{V_0^-}{Z_0}}_{I_0^-} \cos(\omega t + \beta z)$

* Distortionless T.L.:

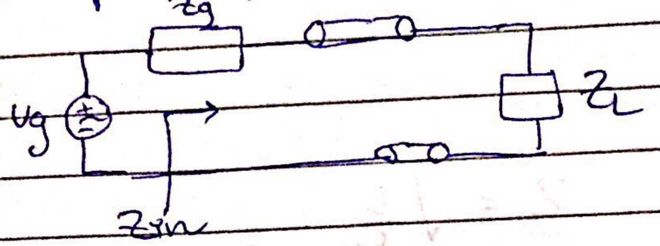
$\boxed{\frac{R}{L} = \frac{G}{C}}$ شرط

$\gamma = \underbrace{\sqrt{RG}}_{\alpha} + j \underbrace{\omega \frac{L}{R} \sqrt{RG}}_{\beta}$ $\qquad \beta = \omega\sqrt{LC}$

$Z_0 = \sqrt{\frac{L}{C}}$ $\qquad u = \frac{\omega}{\beta}$ $\qquad \lambda = \frac{2\pi}{\beta}$

$V_s(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z}$

* Input impedance :-

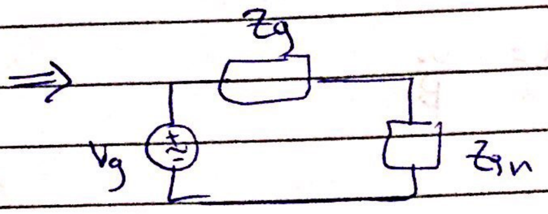


$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma L}{Z_0 + Z_L \tanh \gamma L}$$

$\downarrow$

for lossy & distortionless T.L.

$\Rightarrow$



$V(z=0) = V_0 \;,\; V(z=L) = V_L$

$I(z=0) = I_0 \;,\; I(z=L) = I_L$

$$V_0 = V_g \frac{Z_{in}}{Z_{in}+Z_g} \quad \text{(voltage division)}$$

$$I_0 = \frac{V_g}{Z_{in}+Z_g} \quad \text{(ohm's Law)}$$

* Standing wave Ratio (SWR) :-

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$|\Gamma| = \frac{|V_0^-|}{|V_0^+|}$$

* Reflection coefficient at the load :-

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (\text{if } Z_L \neq Z_0)$$

$$\text{Power} = \frac{1}{2} \operatorname{Re}\{ \underset{\text{Peak}}{V_s} * \underset{\text{Peak}}{I_s^*} \}$$

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

* Max power transfer :-

$$P = \underbrace{\frac{|V_0^+|^2}{2Z_0}}_{P_1} (1-|\Gamma|^2) \quad (\text{when } Z_L = Z_0)$$

$Z_{in}\ s.c = jZ_0 \tan(\beta l)$

$Z_{in}\ o.c = -jZ_0 \cot(\beta l)$

Lossless

Max power transfer :-

$$P = \frac{|V_0^+|^2}{2Z_0} \left(1 - |\Gamma|^2\right)$$

P_r

P_i

$P_t = P_i - P_r$

$$\boxed{P_t = \frac{|V_0^+|^2}{2Z_0}}$$ max power transfer ($Z_L = Z_0$) (if $P_r = 0$)

Scanned with CamScanner

* CH 6

Parallel plate capacitor :-

$C = \frac{\epsilon S}{d}$ $R = \frac{d}{\sigma S}$ $W_E = \frac{1}{2} C V^2$

Cylindrical capacitor (coaxial capacitor)

$\frac{C}{L} = \frac{2\pi\epsilon L}{\ln(\frac{b}{a})}$ $R_d = \frac{\ln(b/a)}{2\pi\sigma_d L}$ $W_E = \frac{1}{2} C V^2$

Spherical capacitor

$C = \frac{4\pi\epsilon}{(\frac{1}{a} - \frac{1}{b})}$ $R_d = \frac{(\frac{1}{a} - \frac{1}{b})}{4\pi\sigma_d}$

Isolated capacitor

$C = 4\pi\epsilon a$ $R_d = \frac{1}{4\pi\sigma_d a}$