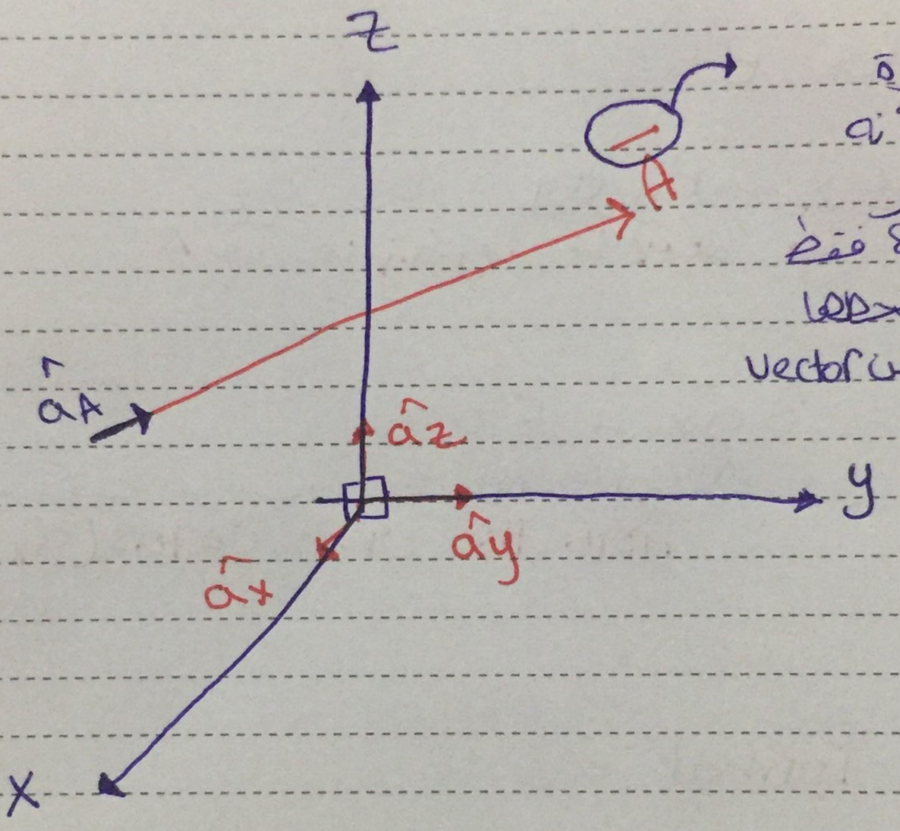
CH1 Vectors Review

- How to write a vector:-

$\bar{A}$ ≡ Vector A (Magnitude & vector)

- in cartesian coordinates



هاي الإشارة
مهمة جدًا لأنه
بدونها بيصير
قيمة فقط (Scalar)
- ما ننسى نحطها
لما نحكي عن vector

* Unit vector (Direction only) (magnitude = 1)

$\hat{a}_x$ ≡ Unit vector in x - direction

هاي الإشارة بتدل على إنه Unit vector

$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$ (Long format)

A_x, A_y, A_z = Vector components (Magnitude only)
(Scalar) ↲ يعني وحدة vector A

- if vector B

$\bar{B} = B_x \underline{\hat{a}_x} + B_y \underline{\hat{a}_y} + B_z \hat{a}_z$

هدول ما بتغيروا ثابتين ↳

- ex $\bar{C} = 3\hat{a}_x + 5\hat{a}_z$

- كيف عرفت إنه cartesian ؟

From the unit vectors ($\hat{a}_x$, $\hat{a}_z$)

② Short Format

$\bar{A} = (A_x, A_y, A_z)$

ex $\bar{C} = (3, 0, 5)$

** Note

don't mix between points & vectors.

هون مساواة

$\bar{A} = (A_x, A_y, A_z) \rightarrow$ vector

$P(x, y, z) \rightarrow$ point.

* Vector Magnitude (Scalar Quantity).

$$|\bar{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

* Unit vector along $\bar{A}$:-

$$\hat{a}_A = \frac{\bar{A}}{A} = \frac{A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

-ex $\bar{C} = 3\hat{a}_x + 5\hat{a}_z$

$$\hat{a}_c = \frac{3}{\sqrt{34}} \hat{a}_x + \frac{5}{\sqrt{34}} \hat{a}_z$$

③ $\bar{A} = A \hat{a}_A$ → كمان طريقة لكتابة الـ vector
بس مش كتير مستخدمة

* Operations on vectors :-

[1] Addition and Subtraction

الناتج لكلا لعمليتين هو vector

ex $\overline{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$\overline{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$

$\overline{C} = \overline{A} + \overline{B}$

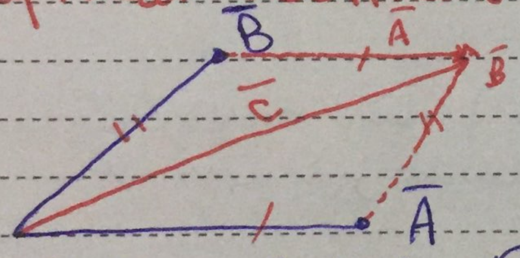
$= (A_x \oplus B_x)\hat{a}_x \oplus (A_y + B_y)\hat{a}_y \oplus (A_z + B_z)\hat{a}_z$

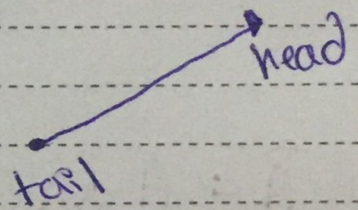
اللي هوا
بيدل على لعملية

From vector definition

$= C_x \hat{a}_x + C_y \hat{a}_y + C_z \hat{a}_z$

* Graphical addition :-

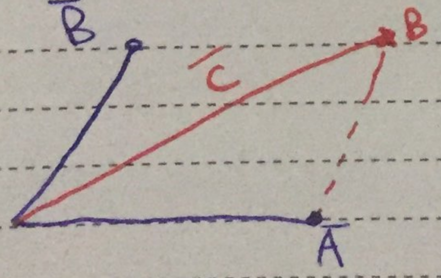




نظرية متوازي الاضلاع : كل vector برسم اله vector
يكون parallel معه بعدين من ال tail البداية لـ head النهاية.

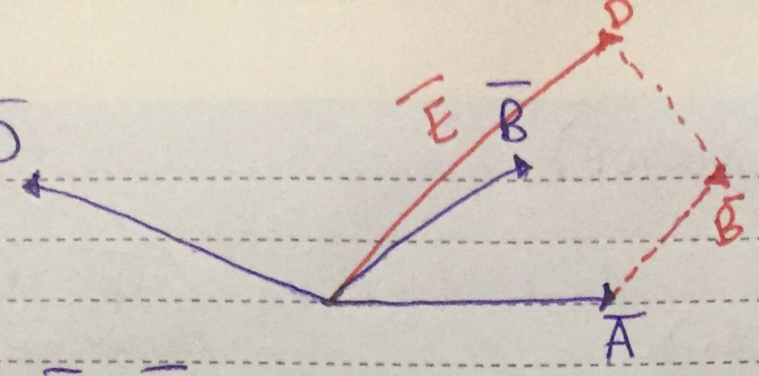
أفضل

- Arrow method :-



عند نهاية ال vector الأول
جيب ال vector الثاني ، بتوصيل
من tail الأول ل head الثاني

ex



$$\bar{E} = \bar{A} + \bar{B} + \bar{D}$$

① بجيب B عند نهاية A
② بجيب D عند نهاية B
③ بوصل بين tail A و head D

* Subtraction:-

$$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

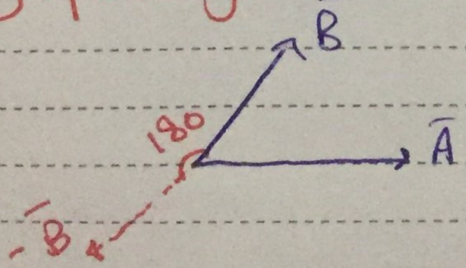
$$\bar{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$$

$$\bar{D} = \bar{A} - \bar{B}$$

$$= (A_x - B_x)\hat{a}_x + (A_y - B_y)\hat{a}_y + (A_z - B_z)\hat{a}_z$$

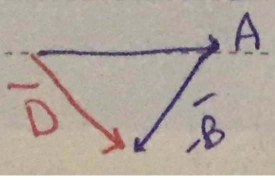
$$= D_x \hat{a}_x + D_y \hat{a}_y + D_z \hat{a}_z$$

* graphically:-



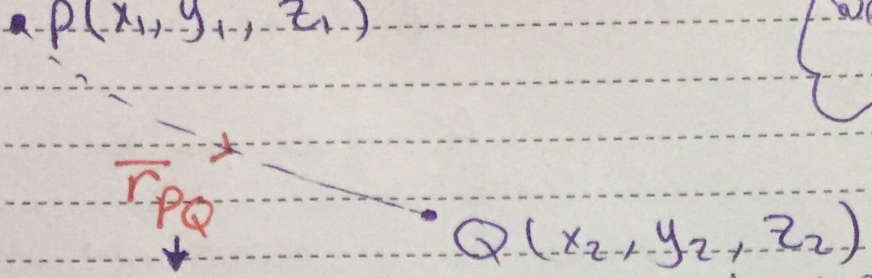
$$\bar{D} = \bar{A} - \bar{B}$$

$$= \bar{A} + (-\bar{B})$$



* Distance :- (Vector)

$\bar{r}_P = r_{\bar{OP}}$
يعنى المسافة بين النقطة
و الorigin هي نفسها
النقطة



↳ point هاي

معناها إنه المسافة
من P لQ

$r_{PQ} = \bar{r}_Q - \bar{r}_P$

$= (x_2, y_2, z_2) - (x_1, y_1, z_1)$

↳ Vector هاي

$= (x_2 - x_1)\hat{a}_x + (y_1 - y_2)\hat{a}_y + (z_2 - z_1)\hat{a}_z$ meters

* $\bar{r}_{PQ} = -\bar{r}_{QP}$

متر

$\rightarrow |\bar{r}_{PQ}| = r_{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ (m)

هيك طلعت الـ
magnitude للمسافة

كيف اطلع الdirection ؟

$a_{\hat{r}_{PQ}} = \frac{\bar{r}_{PQ}}{r_{PQ}} = (-)\hat{a}_{r_{QP}}$

السالب اجى من الdirection عكسه بس نفس
الmagnitude

② Mutliplication :-

a) Dot product $\bar{A} \cdot \bar{B}$ → Scalar quantity.

$\bar{A} = (A_x, A_y, A_z)$
$\bar{B} = (B_x, B_y, B_z)$

$$\boxed{\bar{A} \cdot \bar{B} = |\bar{A}| \, |\bar{B}| \cos \theta_{AB}}$$

له هون عندي مجهول اللي هو الزاوية
عشان هيك بضرب ال Vectors ببعض
كل term من $\bar{A}$ بضربه ب $\bar{B}$

$\hookrightarrow A_x \hat{a}_x \cdot B_x \hat{a}_x$
$= A_x B_x (\hat{a}_x \cdot \cancel{\hat{a}_x})^{1}$ → ليش هاد لقوس قيمته = 1
$= A_x B_x$ لأنه لزاوية بين ax و ay
$\hookrightarrow$ that's why it's scalar. صفر و $1 = \cos(0)$

$$\Rightarrow \boxed{\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z}$$

$$* \; \hat{a}_n \cdot \hat{a}_m = \begin{cases} 1 & , \; n = m \\ 0 & , \; n \neq m. \end{cases}$$

$$\boxed{* \; \theta_{AB} = \cos^{-1}\left(\frac{\bar{A} \cdot \bar{B}}{AB}\right)}$$

* Properties:-

1. $\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$
2. $\bar{A} \cdot \bar{A} = A^2 = A_x^2 + A_y^2 + A_z^2$
3. $\bar{A} \cdot (\bar{B} + \bar{C}) = \bar{A} \cdot \bar{B} + \bar{A} \cdot \bar{C}$

b) cross product → vector

$$|\bar{A} \times \bar{B}| = AB \sin \theta_{AB}$$

$$\theta_{AB} = \sin^{-1} \frac{|\bar{A} \times \bar{B}|}{AB}$$

$$\bar{A} \times \bar{B} \perp \bar{A}, \bar{B}$$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}_{3\times 3}$$

ودايمًا أول سطر بنحط
Unit بعديه تاني
Vector سطر بنحط
أول Vector وتالت
سطر بنحط الVector
التاني

$$\bar{A} \times \bar{B} = (-1)^{1+1} (A_y B_z - A_z B_y)\, \hat{a}_x$$
$$+ (-1)^{1+2} (A_x B_z - A_z B_x)\, \hat{a}_y$$
$$+ (-1)^{1+3} (A_x B_y - A_y B_x)\, \hat{a}_z$$

*Properties:-

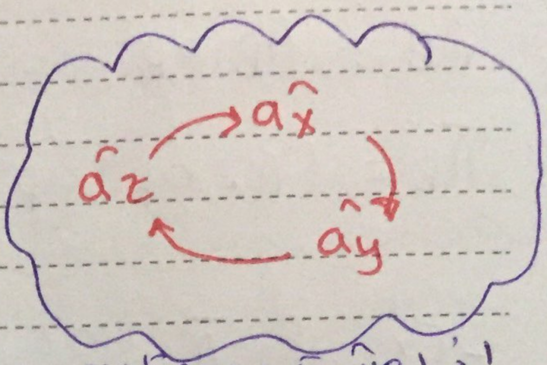
$\boxed{1}$ $\bar{A} \times \bar{B} \neq \bar{B} \times \bar{A}$ $\left(\bar{A} \times \bar{B} = -\bar{B} \times \bar{A} \right)$

$\boxed{2}$ $\bar{A} \times (\bar{B} \times \bar{C}) \neq (\bar{A} \times \bar{B}) \times \bar{C}$ (الأولوية للأقواس)

$\boxed{3}$ $|\hat{a}_x \times \hat{a}_x| = (1)(1) \sin 0 = 0$

* $|\hat{a_x} \times \hat{a_y}| = (1)(1) \sin 90° = 1$

$\hat{a_x} \times \hat{a_y} = +\hat{a_z}$

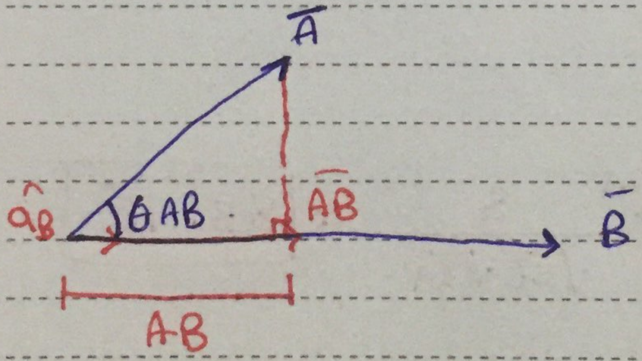


إذا مشيت مع عقارب
الساعة بكون موجب
إذا عكس عقارب الساعة
سالب

* Vector projection along another vector :-

→ A_B = Scalar projection of (A) along (B)

$\bar{A}_B$ = Vector projection of (A) along (B)



$\cos\theta AB = \frac{AB}{A}$ $\frac{\text{المجاور}}{\text{الوتر}}$

* $A_B = A \cos\theta AB$

* $\cos\theta AB = \frac{\bar{A} \cdot \bar{B}}{AB} = \frac{\bar{A} \cdot \hat{a}_B}{A(1)}$, $\hat{a}_B = \frac{\bar{B}}{B}$

$\boxed{\bar{A}_B = \bar{A} \cdot \hat{a}_B}$ → هاد بستخدمه
لما ما أعرف ال angle

$A_B = \bar{A} \cdot \hat{a}_B$

$\bar{A}_B = A_B \, \hat{a}_B$

$\bar{A}_B = (\bar{A} \cdot \hat{a}_B) \, \hat{a}_B$

- Ex: if $\bar{A} = 3\hat{a}_x + 4\hat{a}_y + \hat{a}_z$

$\bar{B} = 2\hat{a}_y - 5\hat{a}_z$

Find a) θ_{AB} b) $\bar{A}_B$.

Sol a) $\theta_{AB} = \cos^{-1} \frac{\bar{A} \cdot \bar{B}}{AB} = \sin^{-1} \frac{|\bar{A} \times \bar{B}|}{AB}$

$\bar{A} \cdot \bar{B} = 0 + 8 - 5 = 3$

$A = \sqrt{26}$

$B = \sqrt{29}$

$\theta_{AB} = \cos^{-1} \frac{3}{\sqrt{26}\sqrt{29}} = \boxed{83.73^\circ}$

b) $\bar{A}_B = (\bar{A} \cdot \hat{a}_B) \, \hat{a}_B$

$\hat{a}_B = \frac{\bar{B}}{B} = \frac{(0,2,5)}{\sqrt{29}}$

$\hat{a}_B = \frac{2}{\sqrt{29}} \hat{a}_y - \frac{5}{\sqrt{29}} \hat{a}_z$

$$\overline{A}_B = \left(0 + \frac{8}{\sqrt{29}} - \frac{5}{\sqrt{29}}\right)\left(0, \frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}\right)$$

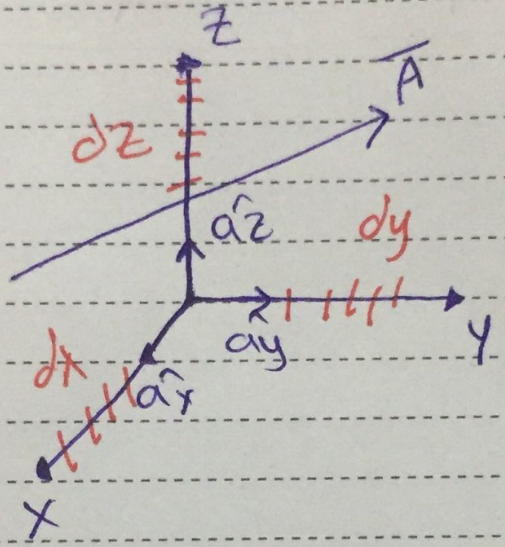
$$= \frac{3}{\sqrt{29}} \cdot \frac{1}{\sqrt{29}}(0, 2, -5)$$

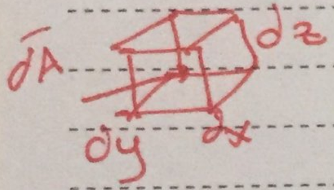
$$= \boxed{\frac{6}{29}\hat{a_y} - \frac{15}{29}\hat{a_z}}$$

CH2 Coordinate Systems and Transformations

1) Cartesian coordinates:-

$-\infty < X < \infty$
$-\infty < y < \infty$
$-\infty < z < \infty$
} → 3D object Infinite Box





- Differential elements:-
dx, dy, dz

* Differential length ($\bar{dL}$) (vector).

$$\bar{dL} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

* Differential Normal Surface Area ($\bar{ds}$) (vector).

$$\bar{ds}_{front} = dy\,dz\,\boxed{\hat{a}_x}$$

→ هاي ٨ة بدي الـNormal
يعني عامودي ومتجه
يكون outside the object.

اله ٨ة $\bar{ds}$
٦ أوجه.
(top, front, bottom, left, right, back).

$\bar{ds}_{back} = dy\,dz\,(-\hat{a_x})$.

المساحة مش سالبة
بس للاتجاه.

$\bar{ds}_{right} = dx\,dz\,\hat{a_y}$

$\bar{ds}_{left} = -dx\,dz\,\hat{a_y}$

- بحدد العامودي بالأول
بجيبه بضربه اللي
ضلو بعض.

$\bar{ds}_{top} = dx\,dy\,\hat{a_z}$

$\bar{ds}_{bottom} = -dx\,dy\,\hat{a_z}$

* Differential Volume ($\bar{dv}$) (Scalar).

$dv = dx\,dy\,dz$.

* 2D Surfaces.

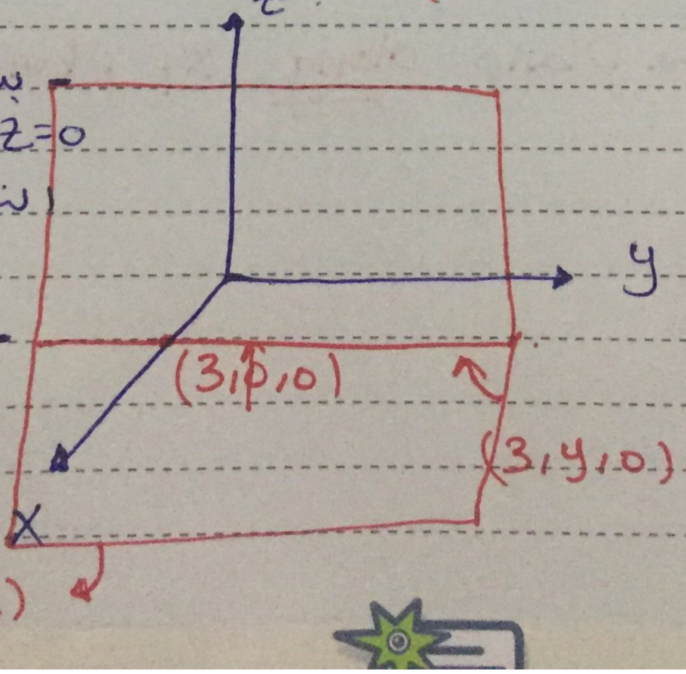
↳ by fixing one variable.

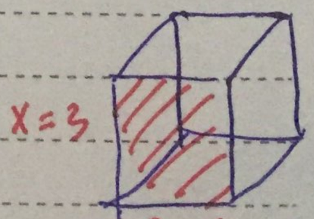
- if $x=3$

↳ we know now that $(-\infty < z, y < \infty)$

بنعتبر أول اشي إنه
$y=z=0$ يعني بندور على
نقطة $(3,0,0)$

- بعد هيك نحول y
لـ Line.





infinite plane parallel to yz plane.

- what is the normal on that surface ?
- $\hat{a}_x$ or $(-\hat{a}_x)$

ع حسب وين النقطة

if $(10,0,0) \rightarrow$ النقطة أبعد من

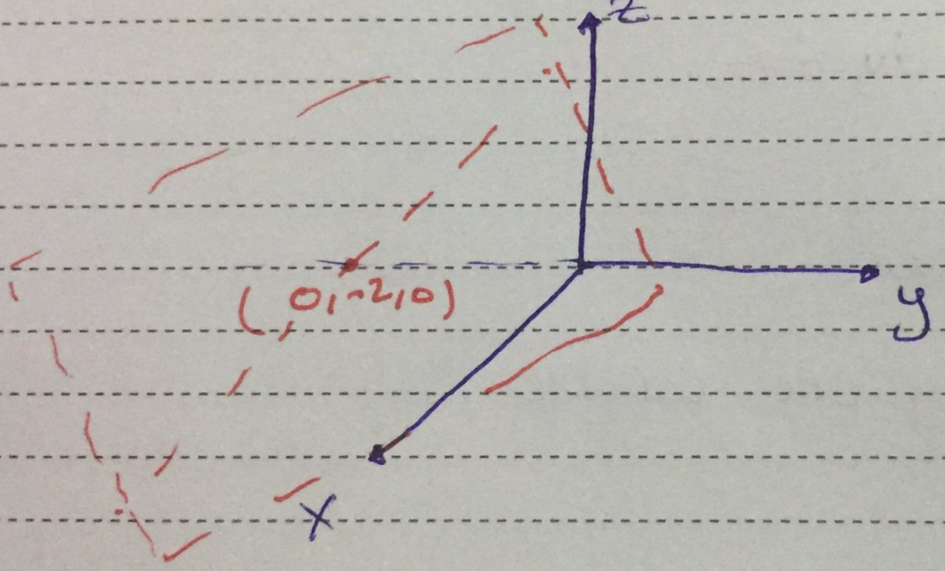
ال plane فهو $+\hat{a}_x$

- if $x=0$

↳ infinite plane along yz plane.

- if $y=-2$

↳ infinite plane parallel to xz plane.



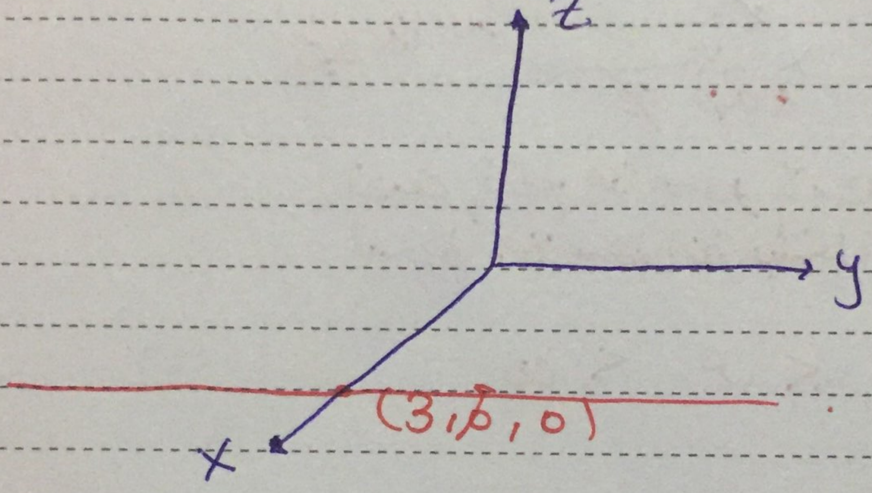
- if $z=0$

↳ infinite plane along xy plane.

* 1 D Segment :
↳ by fixing 2 Variables.

if X, Z are constants ($x \neq 0$, $z \neq 0$).
↳ infinite Line // y-axis.

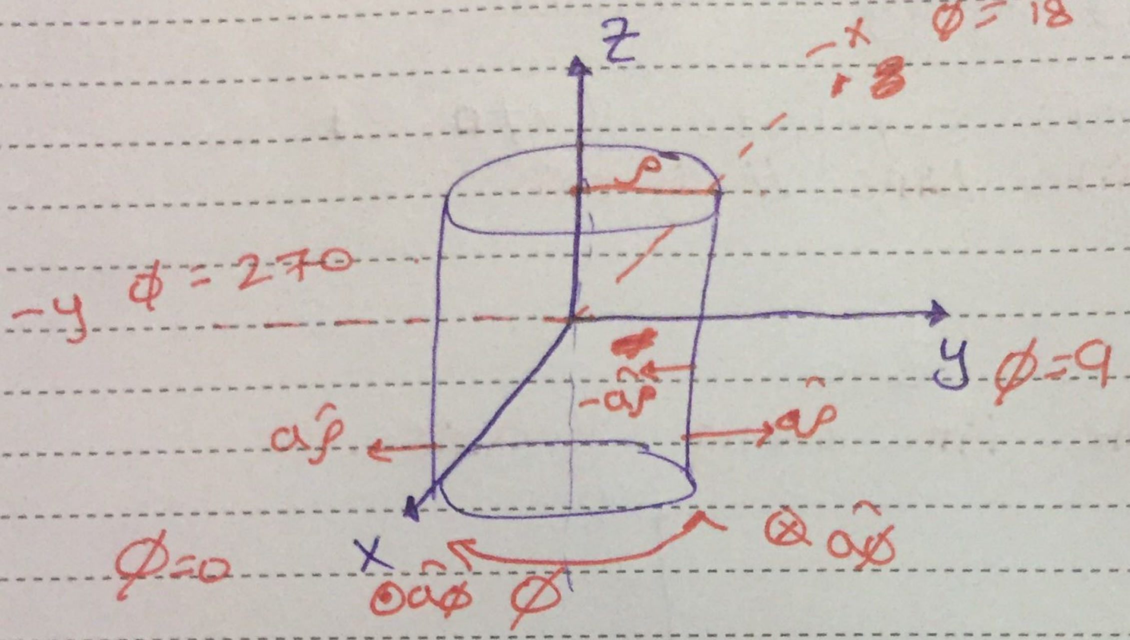
- if $x = 0$, $z = 0$.
↳ infinite line along y-axis.



- if $y = -2$, $z = -2$
↳ infinite Line // x-axis.

- infinite line along z-axis.
↳ $x = 0$, $y = 0$.

* Cylindrical coordinates:-



$\rho \rightarrow$ نصف القطر من محور الاسطوانة لسطح الاسطوانة بشكل عامودي

$0 \leq \rho < \infty$

$\emptyset \rightarrow$ rotation.

$0 \leq \emptyset \leq 2\pi$

بحددها إنه بخلي الابهام باتجاه المحور وحركة إيد بتكون حركة الاسطوانة

$-\infty < Z < \infty$

- How to write a vector in cylindrial coordinates -

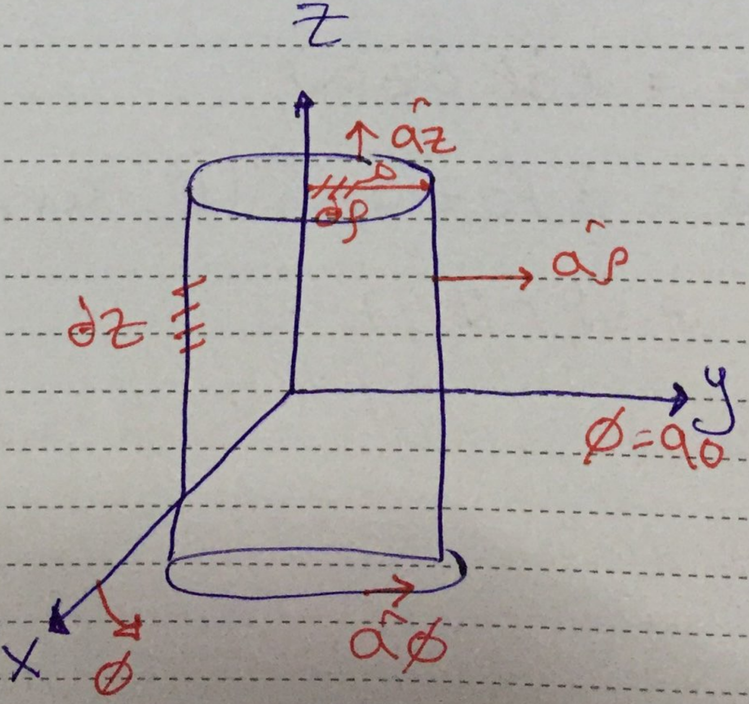
$\overline{A} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}$.

* unit vectors -

$\hat{a_\rho}$, $\hat{a_\phi}$, $\hat{a_z}$:

* $$\begin{array}{ccc} (3, & 60^\circ, & 4) \\ \uparrow & \uparrow & \uparrow \\ \rho & \phi & z \end{array}$$

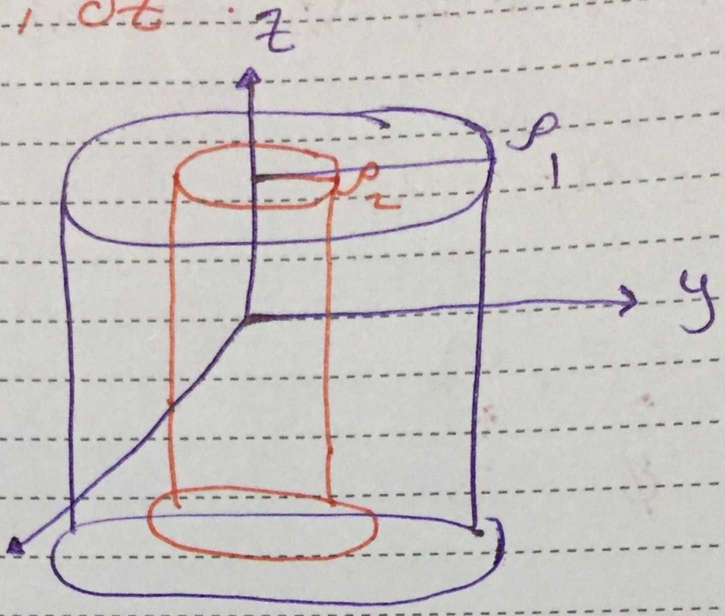
* $$\left.\begin{array}{l} 0 \leq \rho < \infty \\ 0 \leq \phi \leq 2\pi \\ -\infty < z < \infty \end{array}\right\} \rightarrow \text{infinite solid cylindrical}$$



*Differential Elements:-

$d\rho$, $\rho d\phi$, dz :

$\rho_1 > \rho_2$



$\overline{dL} = d\rho\, \hat{a}_\rho + \rho d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$

$\overline{ds}_{top} = \rho\, d\rho\, d\phi\, \hat{a}_z$

$\overline{ds}_{bottom} = -\rho\, d\rho\, d\phi\, \hat{a}_z$

$\overline{ds}_{side} = \rho\, d\phi\, dz\, \hat{a}_\rho$

$\overline{ds}_{cut} = d\rho dz\, \hat{a}_\phi$ (ϕ = constant).

$dv = \rho\, d\rho\, d\phi\, dz$.

* 2D Surfaces :-

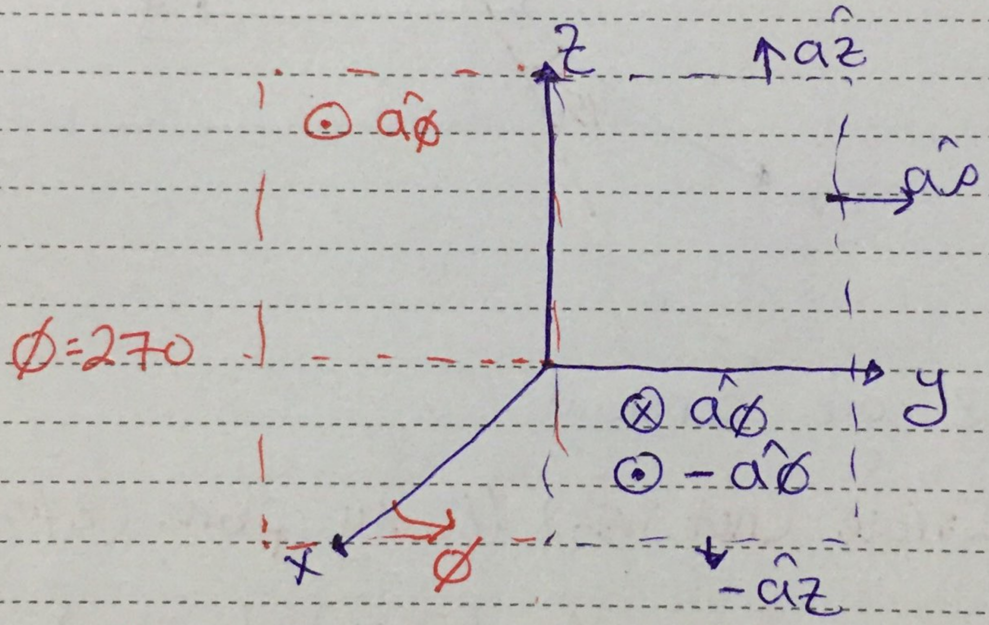
- ρ = constant → inf. hollow Cylinder

i.e. $\rho = 5$, ϕ, z → Variables.

if $\rho = 0$ → inf. line along z-axis.

- ϕ = constant → inf. plane.

i.e. $\phi = 90°$ → inf. plane along yz plane.



- z = constant → inf. Disk // xy Plane (if $z \neq 0$)

inf. Disk along xy plane (if $z=0$)

* 1D Segments -

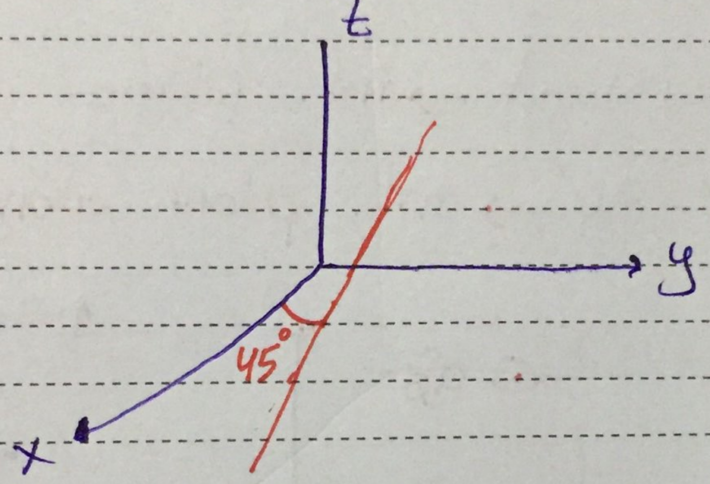
① ρ, ϕ are constants

↳ inf. Line // z-axis ($\rho \neq 0$)

↳ inf. Line along z-axis ($\rho = 0$).

i.e. $\rho = 3$, $\phi = 45°$

$\overline{\partial L} = dz\, \hat{a_z}$



② ρ, z are constants

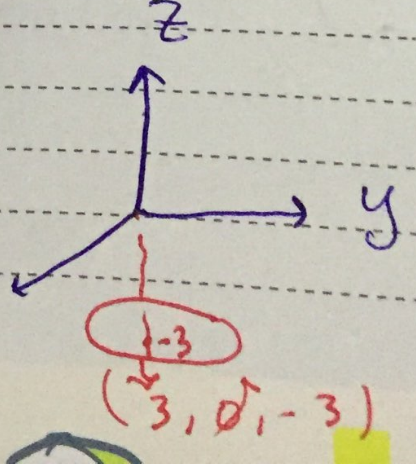
↳ Circle (not inf.) // xy plane ($z \neq 0$) ($\rho > 0$)

↳ Circle along xy ($z = 0$) ($\rho > 0$)

↳ point ($\rho = 0$).

i.e. $\rho = 3$, $z = -3$

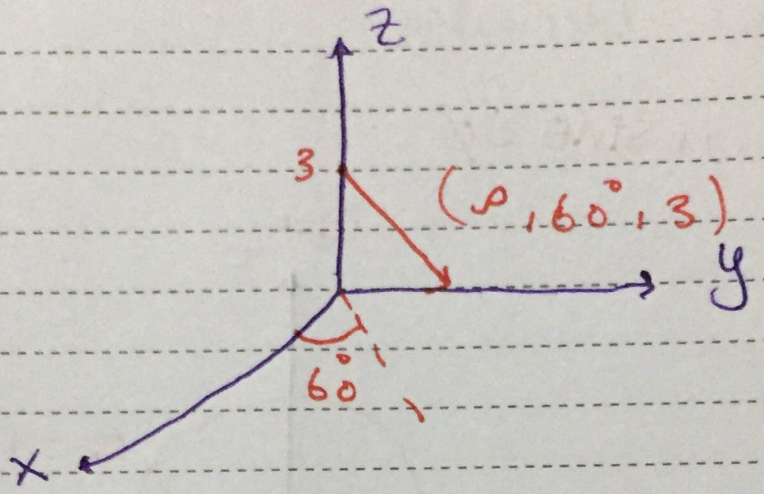
$\overline{\partial L} = 3 d\phi\, \hat{a_\phi}$



③ ϕ, z are constants.

↳ Semi-inf. Line.

i.e. $\phi = 60°$, $z = 3$

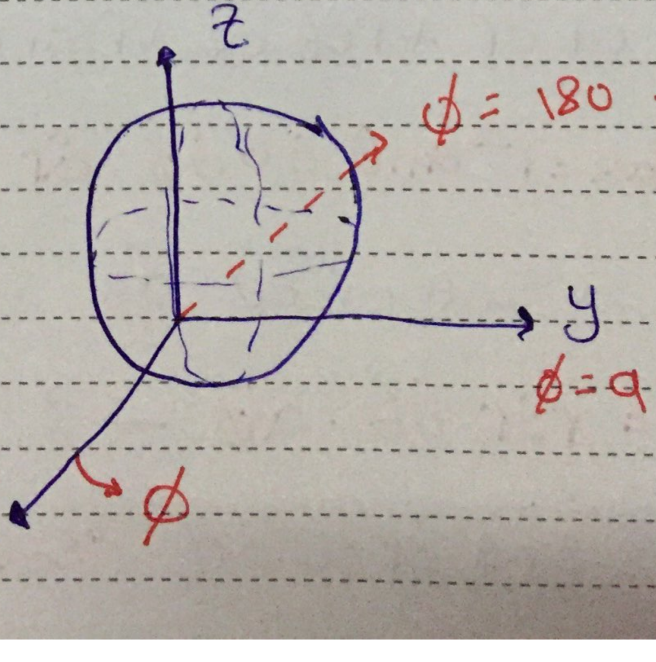


* Sperical coordinates :-

$$\left.\begin{array}{l} 0 \le r < \infty \\ 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{array}\right\} \rightarrow$$

3D object
Inf. solid
sphere.

- لازم θ و ϕ تكون وحدة منهم 180 ووحدة 360

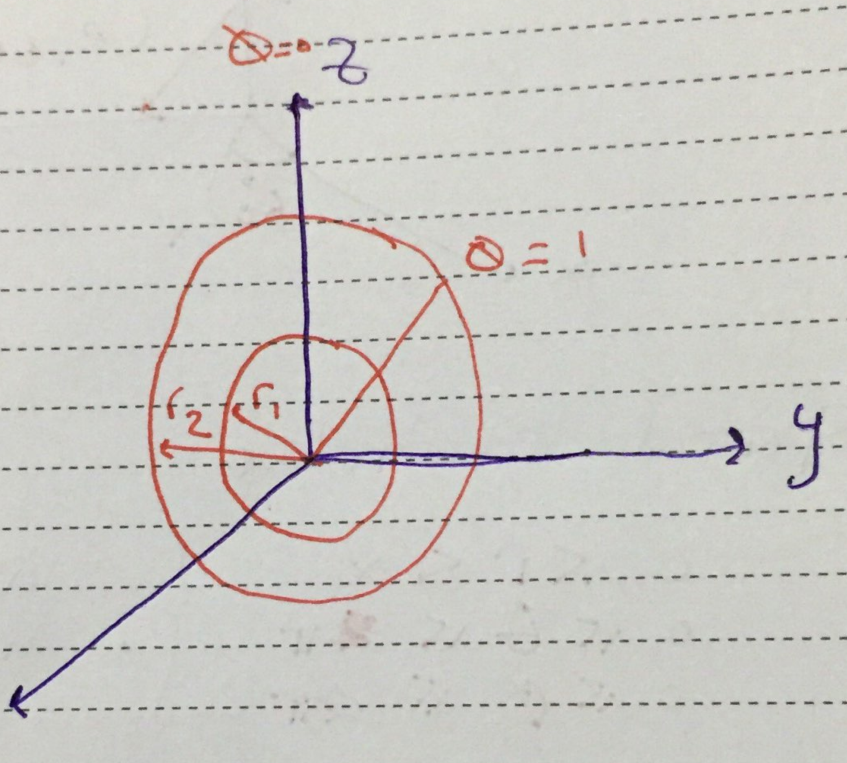


* Unit vectors:-

$\hat{a_r}$, $\hat{a_\theta}$, $\hat{a_\phi}$

-Differential Elements:-

dr, $r d\theta$, $r \sin\theta \, d\phi$.



$-\overline{dl} = dr\,\hat{a_r} + r d\theta\,\hat{a_\theta} + r\sin\theta\, d\phi\,\hat{a_\phi}$ → r const.

$-\overline{ds}_{surface} = r^2 \sin\theta \, d\theta \, d\phi \;\; \hat{a_r}$

$-\overline{ds_\theta} = r \sin\theta \, dr \, d\phi \;\; \hat{a_\theta}$

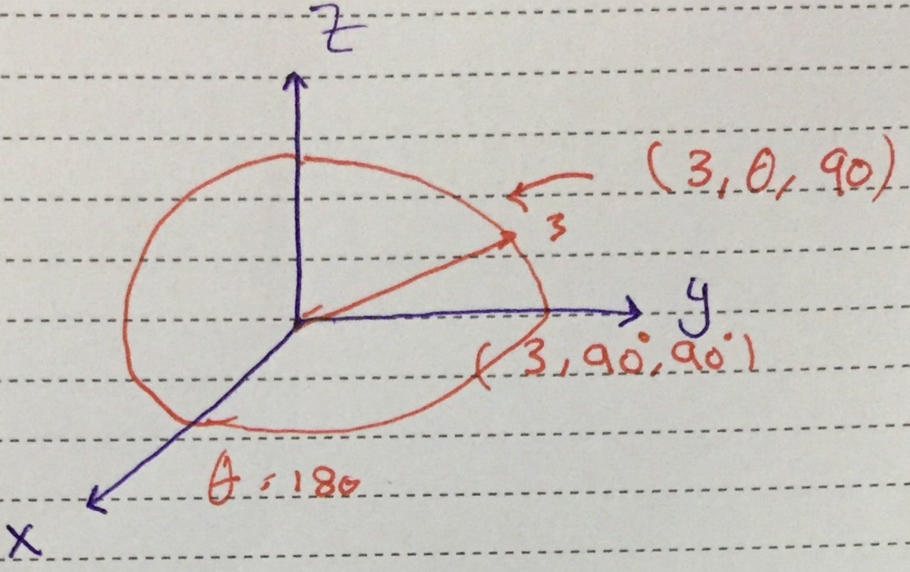
$-ds_\phi = r dr \, d\theta \cdot \hat{a_\phi}$ → ϕ const.

$-\overline{dv} = r^2 \sin\theta \, dr \, d\theta \, d\phi$ "Scalar"

* 2D Surface

- r = constant → hollow sphere ($r>0$)

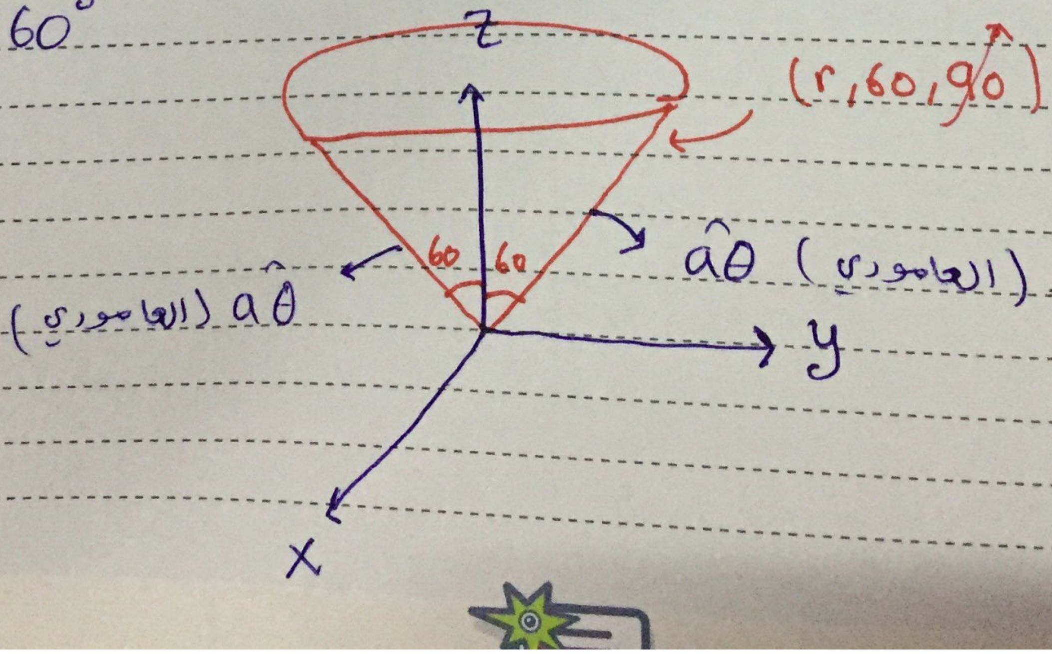
i.e $r = 3$



- if $r = 0$ → point.

- θ = constant
  - ↳ inf. hollow cone $\theta \in (0,90)$ and $(90,180)$
  - ↳ inf. disk along xy plane ($\theta = 90°$).
  - ↳ Semi-inf. Line → +ve z-axis ($\theta = 0°$)
    - ↳ -ve z-axis ($\theta = 180°$).

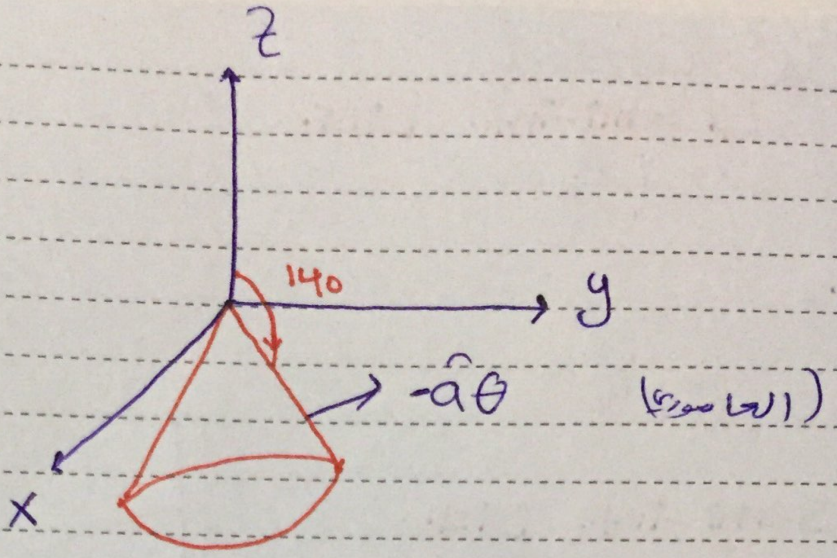
i.e. $\theta = 60°$



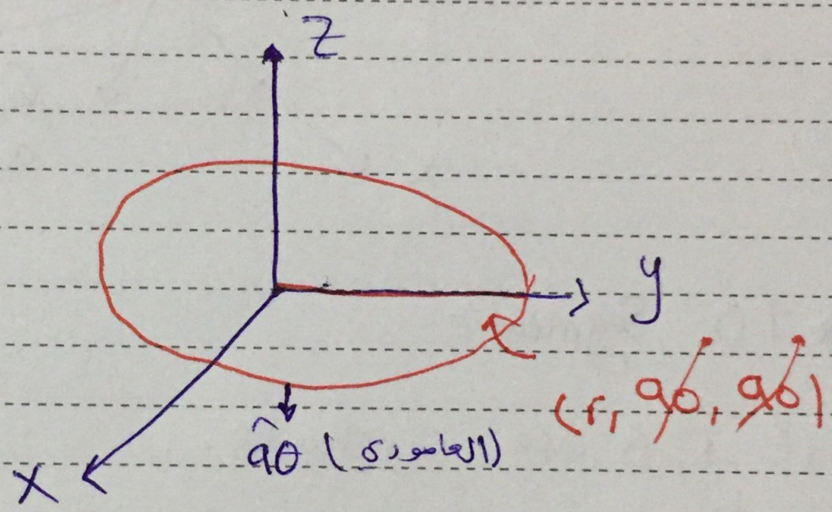
$\theta < 90$
الـ cone
لفوق

- if θ = 140°

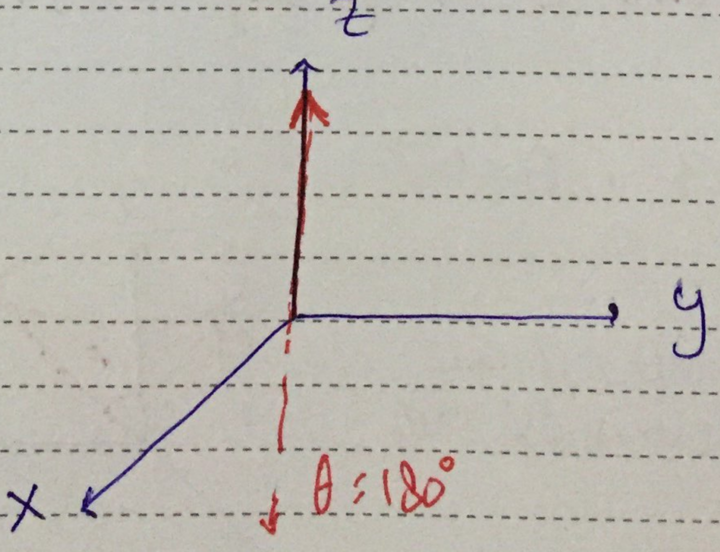
θ > 90
ال cone لتحت



- if θ = 90°



- if θ = 0°

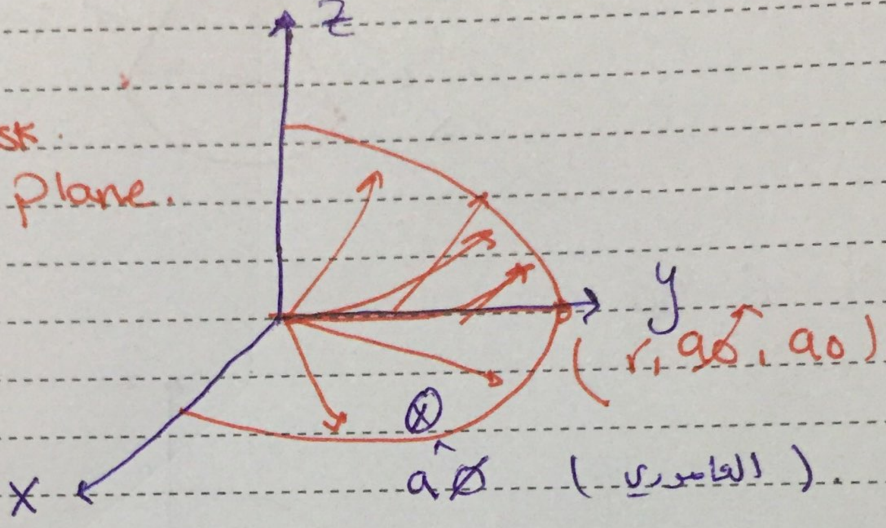


if ϕ = constant

$\hookrightarrow$ semi-inf. Disk.

i.e. $\phi = 90°$

Semi-inf. Disk. along yz plane.



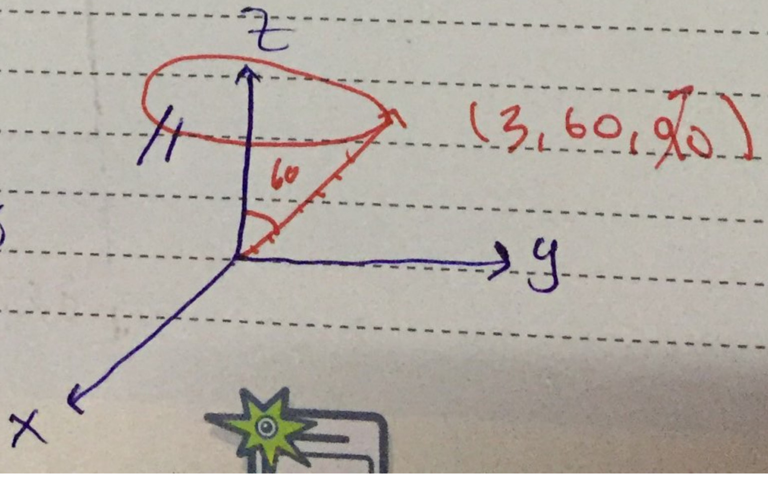
★ 1D Segments:-

if r, θ are constants.

- $\hookrightarrow$ Circle // xy plane. (r>0, θ ≠ 90)
- $\hookrightarrow$ Circle along xy plane (r>0, θ = 90)
- $\hookrightarrow$ point (r=0) (θ any value).

if r = 3, θ = 60°

$$\vec{dL} = r\sin\theta \, d\phi \, \hat{a}_\phi$$
$$= 3\sin(60)\, d\phi \, \hat{a}_\phi$$

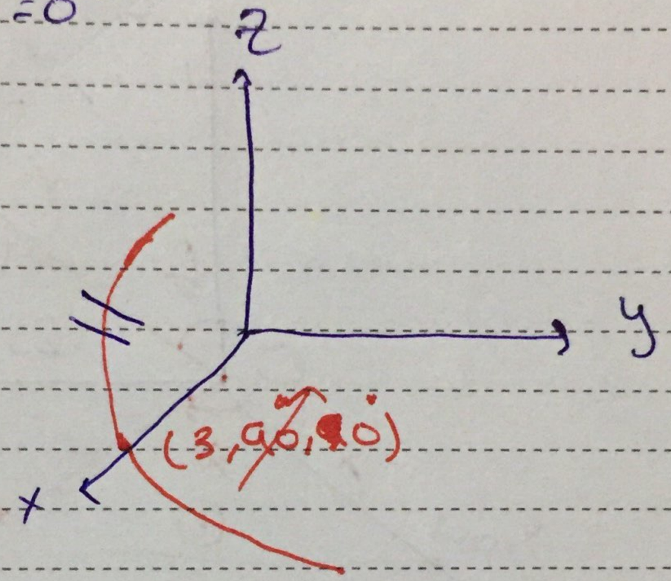


- r, ϕ are constants.
  - ↳ half-circle $(r>0)$
  - ↳ point $(r=0)$
  - ↳

i.e. $r = 3$, $\phi = 0°$

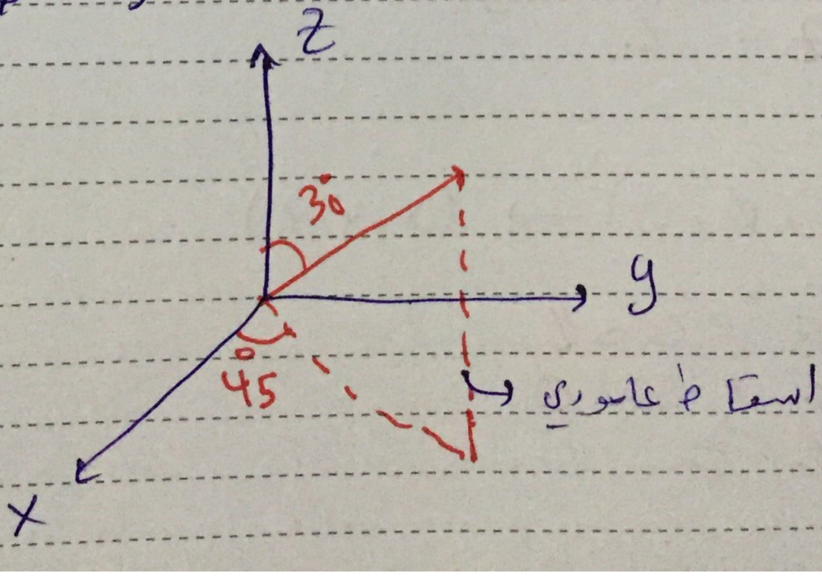
$$\bar{dL} = r\,d\theta\,\hat{a}_\theta$$

$$= 3\,d\theta\,\hat{a}_\theta$$



- θ, ϕ are constants.
  - ↳ Semi-inf. Line (Ray).

i.e. $\theta = 30°$, $\phi = 45°$

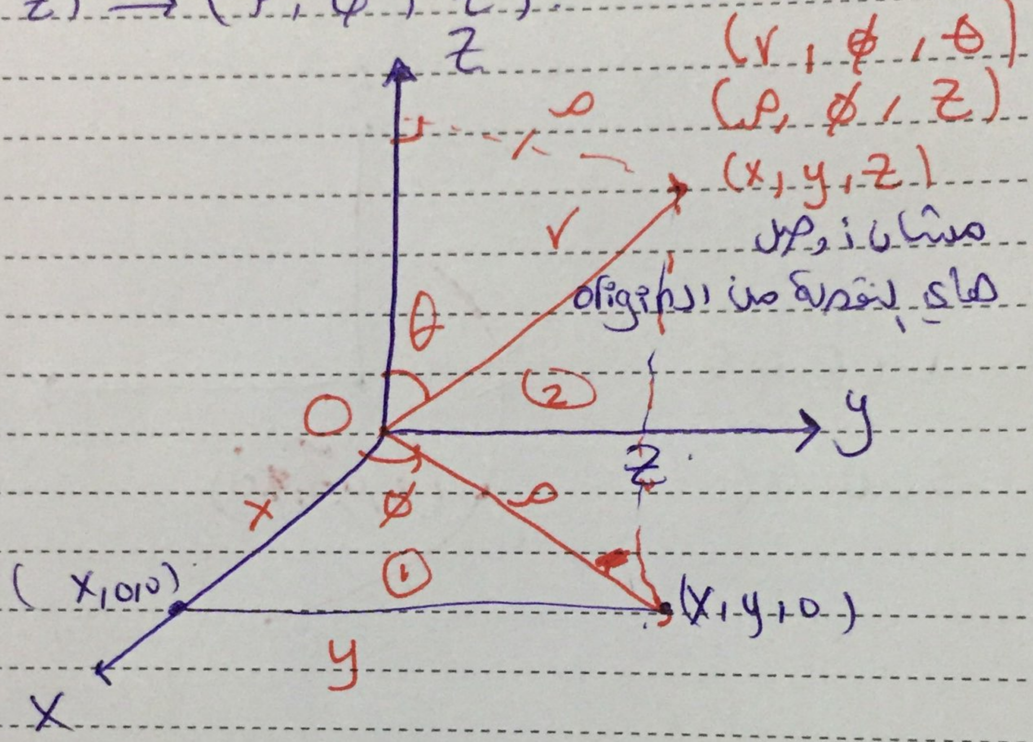


* Transformation between coordinates:

- Point conversions:-

↳ if given cartesian and we want Cylindrical:

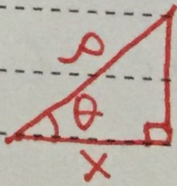
$(x, y, z) \longrightarrow (\rho, \phi, z)$



$* (x, y, z) \longrightarrow (\rho, \phi, z)$

$\rho = \sqrt{x^2 + y^2}$
$\phi = \tan^{-1}(y/x)$
$z = z$

① من مثلث



$\sin\theta = y/\rho$
$\cos\theta = x/\rho$
$\tan\theta = y/x$

$* (\rho, \phi, z) \longrightarrow (x, y, z)$

$x = \rho \cos\phi$
$y = \rho \sin\phi$
$z = z$

* Cart. → Spherical

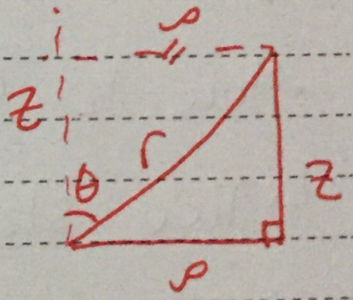
$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$$
$$\phi = \tan^{-1}(y/x)$$

* Spherical → Cart.

$$x = r\sin\theta\ \cos\phi$$
$$y = r\sin\theta\ \sin\phi$$
$$z = r\cos\theta$$

* Cylindrical → Sph.

$$r = \sqrt{\rho^2 + z^2}$$
$$\theta = \tan^{-1}(\rho/z)$$
$$\phi = \phi$$



$\sin\theta = \rho/r$
$\cos\theta = z/r$
$\tan\theta = \rho/z.$

* Sph. → Cyl.

$$\rho = r\sin\theta$$
$$\phi = \phi$$
$$z = r\cos\theta.$$

- Vectors conversion :-

- Cart. → Cyl.

i.e. $\bar{A} = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$

$\hookrightarrow \bar{A} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}$

هاد المطلوب

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

↳ This matrix is given

هاد المعطى

- How to use this matrix?

↳ Two steps.

1) $A_\rho = \cos\phi \, A_x + \sin\phi \, A_y$ ضرب الصف الأول بالعامود

$A_\phi = -\sin\phi \, \boxed{A_x} + \cos\phi \, A_y$. ضرب الصف الثاني بالعامود :

$A_z = A_z$ i.e. $A_x = xy$.

2) $x = \rho \cos\phi$
$y = \rho \sin\phi$

$xy = \rho^2 \cos\phi \, \sin\phi$

Scanned with CamScanner

* Cyl. → Cart.

$$\begin{array}{c} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{array} \leftarrow \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} \begin{array}{c} \hat{a}_\rho \\ \hat{a}_\phi \\ \hat{a}_z \end{array}$$

له ممكن يكون معطيني الماتركس اللي قبل وأنا أطلع هاي ماتركس، كيف؟ باخذ ال transpose للماتركس؛ إنه كل صف بصير عامود.

- Unit Vector Conversion

$$\hat{a}_x = \cos\phi\, \hat{a}_\rho + \sin\phi\, \hat{a}_\phi$$

* Cart. → Sph.

$$\bar{A} = A_x\, \hat{a}_x + A_y\, \hat{a}_y + A_z\, \hat{a}_z$$

$$\bar{A} = A_r\, \hat{a}_r + A_\theta\, \hat{a}_\theta + A_\phi\, \hat{a}_\phi$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\hat{a}_\phi = -\sin\phi\, \hat{a}_x + \cos\phi\, \hat{a}_y$$

* Cyl. → Sph.

$\bar{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$

↳ $\bar{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$

Sph. → Cyl.

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & \overset{0}{\cancel{\cos\theta}} \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$ Given

$A_\phi = A_\phi$

* CH3 Vector Calculus

- Integration.

↳ 1) Line Integral. , $\bar{dL}$ (differential Length).

$$\int_\ell \bar{A}\cdot\bar{dL}$$

↳ $dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

↳ $d\rho\,\hat{a}_\rho + \rho d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$

↳ $dr\,\hat{a}_r + r d\theta\,\hat{a}_\theta + r\sin\theta\, d\phi\,\hat{a}_\phi$

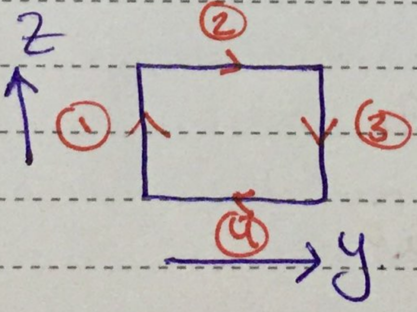
- i.e. $\bar{A} = (A_\rho, A_\phi, A_z)$.

$$\int_\ell (A_\rho, A_\phi, A_z)\cdot(d\rho, \rho d\phi, dz)$$

$$= \int_\rho A_\rho d\rho + \int_\phi \rho A_\phi d\phi + \int_z A_z dz$$

* Closed Line Integral

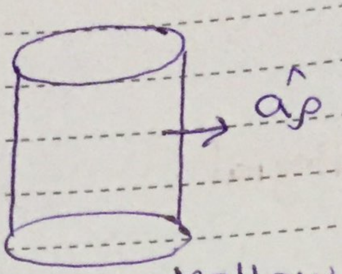
$$\int_\ell \bar{A}\cdot\bar{dL}$$



$$\int_\ell \bar{A}\cdot\bar{dL} = \int_{\ell_1} \bar{A}\cdot\bar{dL_1} + \int_{\ell_2} \bar{A}\cdot\bar{dL_2} + \int_{\ell_3} \bar{A}\cdot\bar{dL_3} + \int_{\ell_4} \bar{A}\cdot\bar{dL}$$

* Surface Integral :-

$$\int_S \bar{A}\cdot\bar{ds}$$



hollow cylinder

$\bar{A} = (A_x, A_y, A_z)$
Using matrix

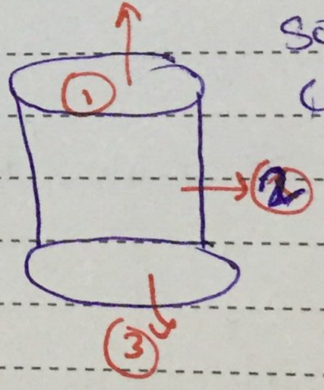
↳ $\bar{A} = (A_\rho, A_\phi, A_z)$

$\bar{ds} = \rho\, d\phi\, dz\, \hat{a}_\rho$

$$\int_S \bar{A}\cdot\bar{ds} = \int_z\int_\phi A_\rho\, \rho\, \underline{\underline{d\phi}}\, \underline{\underline{dz}}$$

* Closed Surface Integral:-

$$\oint_S \bar{A}\cdot\bar{ds}$$



solid cylinder

$\bar{ds_1} = \rho\, d\rho\, d\phi\, \hat{a}_z$

$\bar{ds_2} = \rho\, d\phi\, dz\, \hat{a}_\rho$

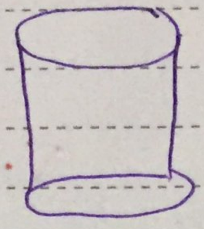
$\bar{ds_3} = -\rho\, d\rho\, d\phi\, \hat{a}_z$

$$\oint \bar{A}\cdot\bar{ds} = \int_{S_1} \bar{A}\cdot\bar{ds_1} + \int_{S_2} \bar{A}\cdot\bar{ds_2} + \int_{S_3} \bar{A}\cdot\bar{ds_3}$$

$\searrow A_z$ $\qquad$ $\searrow A_\rho$ $\qquad$ $\rightarrow A_z$

* Volume Integrals -

$$\int_V |\bar{A}|\, dv$$



Solid cylinder.

i.e. $dv = \rho\, d\rho\, d\phi\, dz$

$$\int_z \int_\phi \int_\rho A \cdot \rho\, d\rho\, d\phi\, dz$$

* Del operator (∇)

↳ vector

- in Cartesians -

$$\nabla_V = \frac{\partial v}{\partial x}\hat{a}_x + \frac{\partial v}{\partial y}\hat{a}_y + \frac{\enclose{circle}{\partial v}}{\partial z}\hat{a}_z$$

هاي اسمها partial.

- in cylindrical.

$$\nabla_u = \frac{\partial u}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial u}{\partial \phi}\hat{a}_\phi + \frac{\partial u}{\partial z}\hat{a}_z$$

- in spherical

$$\nabla_T = \frac{\partial T}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial T}{\partial \phi}\hat{a}_\phi + \frac{\partial T}{\partial \theta}\hat{a}_\theta$$

كيف بنكتبهم
فوق بنحط
(∂) بارشال
وبالمقام بنحط
differential element.

* Del Usage :-

1) Gradiant $\overset{\text{Vector}}{\nabla}\overset{\text{scalar}}{V} \equiv$ vector

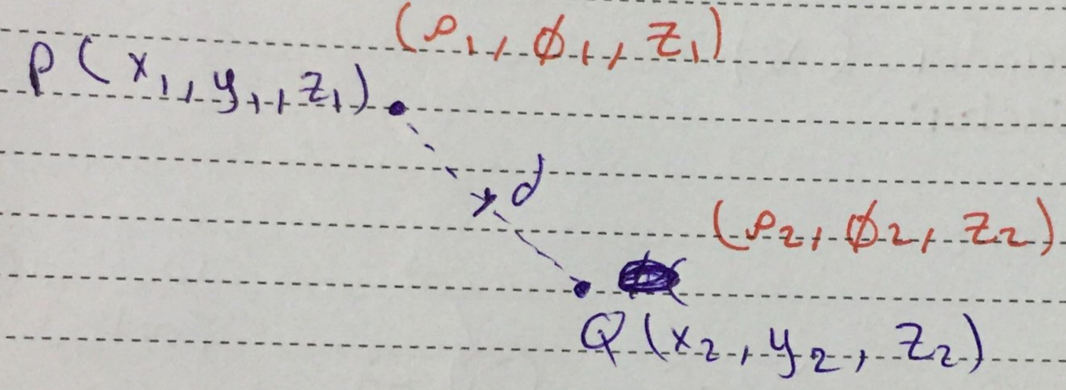
2) Divergance $\nabla \cdot \bar{A} \equiv$ scalar ✗

3) Curl $\nabla \times \bar{A} \equiv$ vector ✗

4) Laplacian $\nabla \cdot \nabla V \equiv \nabla^2 V$ ✗

مش مطلوبين

* Distance :-



$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \quad \text{cart.}$$

$$d^2 = \rho_2^2 + \rho_1^2 - 2\rho_1\rho_2\cos(\phi_2 - \phi_1) + (z_2 - z_1)^2 \quad \text{cylin.}$$

degree وحدة الزوايا ← هاد صار هيك ثابتة

ووحدة ρ, z متر

↳ if $\phi_1 = \phi_2 \rightarrow (\rho_2 - \rho_1)^2 + (z_2 - z_1)^2$

in sph. $d^2 = (r_2 - r_1)^2$ if $\phi_1 = \phi_2$

$\theta_1 = \theta_2$

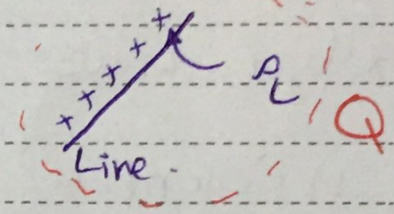
* CH4 Electrostatic Fields.

- Electrostatic Sources -

1) point charge (Q). (Unit in C)
↳ كولوم .

2) Continuos charge distribution.

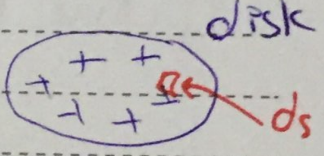
↳ a) Line charge (1D segment).
(ρ_L) → (C/m).
(كولوم/متر)



$$Q = \int_l \rho_L \, dL$$

↳ b) Surface charge (ρ_s) (2D surfaces)
(C/m²)

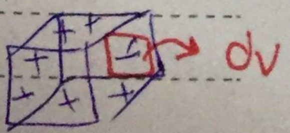
$$Q = \int_s \rho_s \, ds$$



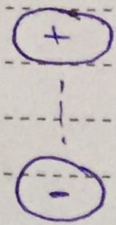
مابتعامل مع المساحة
كامل لو كان inf
بقسمه

↳ c) Volume charge (3D obj) (ρ_V) (C/m³).

$$Q = \int_V \rho_V \, dV$$



3) Electric Dipole



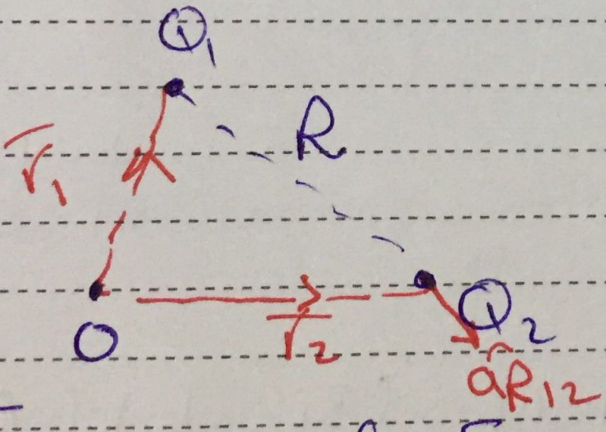
4) polarized Dielectric.

*Major Laws:-

1) Colomb's Law (General)

2) Gauss's Law (Special case).

*Colomb's Law :-
(point charge).



$F_e = F \equiv$ Electrical Force (N)

$F \propto \frac{Q_1 Q2}{R^2}$ (Relation).

$\rightarrow F = \frac{k Q_1 Q_2}{R^2}$ (N) → this gives magnitude only

k: propotionality constant

$K = \frac{1}{4\pi \varepsilon_0}$
→ Units used ($\frac{1}{4\pi}$)
→ Media surronded the two charges (ε_0).

ε : permitivity (F/m)
↳ Farad.

ε_0 : Free space permitivity (F/m).

$= \frac{10^{-9}}{36\pi}$ F/m $= 8.85 * 10^{-12}$ F/m

* F as a vector Quantity.

\- The Force on (Q_2) due to (Q_1)

$$\overline{F}_{12} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R_{12}} \hat{a}_{R_{12}} \quad (N).$$

$$\overline{R}_{12} = \overline{r}_2 - \overline{r}_1$$

→ $\overline{F}_{12} = \frac{Q_1 \, Q_2}{4\pi \epsilon_0 R_{12}} \boxed{a\hat{R}_{12}} \rightarrow \frac{\overline{R}_{12}}{R_{12}}$

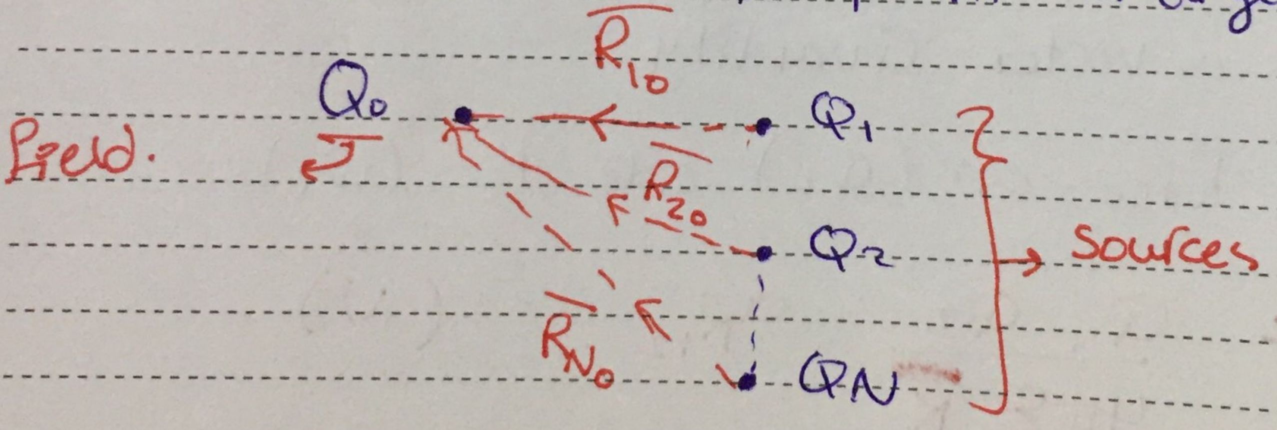
② $\overline{F}_{12} = \frac{Q_1 \, Q_2 \, \overline{R}_{12}}{4\pi \epsilon_0 \, R_{12}^3} \quad (N)$

كلهم نفس القانون
ولكن بأشكال
مختلفة.

③ $\overline{F}_{12} = \frac{Q_1 Q_2 \, (\overline{r}_2 - \overline{r}_1)}{4\pi \epsilon_0 \, |\overline{r}_2 - \overline{r}_1|^3} \quad (N)$

$\overline{F}_{21} = \boxed{-} \, \overline{F}_{12}$ نفس المقدار ولكن عكس الاتجاه.
↳ للاتجاه.

* The Force due to N-point charges:-



- The Force on Q_0 :-

$$\bar{F} = \overbrace{\frac{Q_1 Q_0 (\bar{r}_0 - \bar{r}_1)}{4\pi\epsilon_0 |\bar{r}_0 - \bar{r}_1|^3}}^{F_{10}} + \overbrace{\frac{Q_2 Q_0 (\bar{r}_0 - \bar{r}_2)}{4\pi\epsilon_0 |\bar{r}_0 - \bar{r}_2|^3}}^{F_{20}}$$

$$+ \;----\; + \underbrace{\frac{Q_0 Q_N (\bar{r}_0 - \bar{r}_N)}{4\pi\epsilon_0 |\bar{r}_0 - \bar{r}_N|^3}}_{F_{N0}} \qquad (N)$$

$$\rightarrow \boxed{F = \frac{Q_0}{4\pi\epsilon_0} \sum_{k=1}^{N} \frac{Q_k (\bar{r}_0 - \bar{r}_k)}{|\bar{r}_0 - \bar{r}_k|^3} \qquad (N)}$$

* Electric Field Intensity ($\bar{E}$)

$$\bar{E} = \frac{\bar{F}}{\underset{\text{field point}}{Q}} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r \qquad \left(\frac{N}{C}\right) \text{ or } \left(\frac{V}{m}\right)$$

↳ For a point charge.

$$\hookrightarrow \bar{E} = \frac{Q(\bar{r} - \bar{r}')}{4\pi\epsilon_0 |\bar{r} - \bar{r}'|^3}$$

Use dashes with the source
and without dashes with the field.

- For N point charges :-

$$\bar{F} = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^{N} \frac{Q_k (\bar{r} - \bar{r}_k')}{|\bar{r} - \bar{r}_k'|^3}$$

Source 1 Source 2.

- **Ex** point charges (1mc) and (-2mc) are located at (3, 2, -1), (-1, -1, 4) Find $\overline{F}$ and $\overline{E}$ at (10nc) charge located at (0, 3, 1)?

↳ Field.

Sol

Vector

$$\overline{F} = \frac{(1*10^{-3})*(10*10^{-9})*(-3,1,2)}{4\pi * \frac{10^{-9}}{36\pi} * (14)^{3/2}}$$

هاد للشحنة الأولى

vector

$$+\frac{(-2*10^{-3})(10*10^{-9})*(1,4,-3)}{4\pi * \frac{10^{-9}}{36\pi} * (26)^{3/2}} =$$

هاد للشحنة الثانية

$$= -6.507\,\hat{a}_x - 3.817\,\hat{a}_y + 7.506\,\hat{a}_z \ (mN)$$

$$\overline{E} = \frac{\overline{F}}{Q} = \frac{\overline{F}}{10*10^{-9}}$$

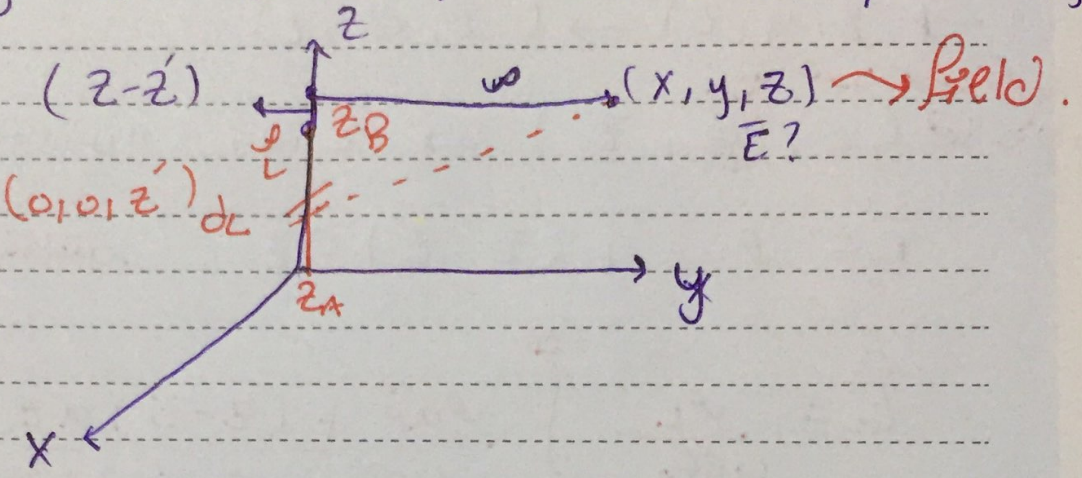
$$= -650.7\,\hat{a}_x - 381.7\,\hat{a}_y + 750.6\,\hat{a}_z \ \left(\frac{KN}{C}\right)$$

or (KV/m

* Continuos charge distribution :-

→ Line charge (ρ_L) in (C/m)

- ex Consider a finite Line along z-axis carry a charge of ρ_L (C/m) . Find $\bar{E}$ at point (x, y, z)



Sol $\vec{E} = \dfrac{Q}{4\pi\epsilon_0 r^2}\,\hat{a_r}$ → point charge.

$\vec{E} = \int\limits_{L} \dfrac{\rho_L\, dL}{4\pi\epsilon_0 r^2}\,\hat{a_r}$ → for Line charge.

$dL = dz'$

$\bar{r} = x\,\hat{a_x} + y\,\hat{a_y} + (z-z')\,\hat{a_z}$

$r = \sqrt{x^2 + y^2 + (z-z')^2}$

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon_0}\int_{z_A}^{z_B} \frac{(x, y, (z-z')}{[x^2+y^2+(z-z')^2]^{3/2}}\, dz'$$

ZA, ZB
لو طول السلك

Note

التكامل اللي طلع عنا صعب حله إلا بال table عشان

هيك لازم أحل ال Line باستخدام cylindrical coordinates

- Line is a cylinder of $\rho=0$ So convert to cyl.

$\rightarrow (x, y, z) \rightarrow (\rho, \phi, z)$

هون ممكن تكون أي قيمة لأنه $\rho = 0$

$(0, \phi', z') \rightarrow$ Line: هاي من ال

$\bar{r} = \rho \hat{a_\rho} + (z-z')\hat{a_z}$

فرضنا ال ϕ بالنقطتين يساووا بعض

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0}\int_{z_A}^{z_B} \frac{\rho \hat{a_\rho} + (z-z')\hat{a_z}}{(\rho^2 + (z-z')^2)^{3/2}}\, dz$$

هاد بنقسم لتكاملين الأول بنحل عن table of integral

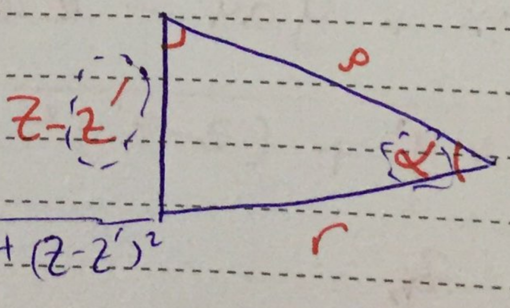
والثاني تحويلها

له بوخذه صعب!

Solution is introduce Angles!

- Introduce angles : $\alpha_1, \alpha_2, \alpha$.

بدي أحوّل من z' ل α



$\sin\alpha = z-z'/r \quad \sqrt{\rho^2+(z-z')^2}$

$\cos\alpha = \rho/r$

$\tan\alpha = z-z'/\rho \rightarrow z-z' = \rho\tan\alpha$

$+dz' = \cancel{-}\rho\sec^2\alpha\, d\alpha$

$$r^2 = \rho^2 + (z - z')^2$$

$$= \rho^2 + \rho^2 \tan^2\alpha$$

$$= \rho^2 (1 + \tan^2\alpha)$$

$$= \rho^2 \left(1 + \frac{\sin^2\alpha}{\cos^2\alpha}\right)$$

$$= \rho^2 \left(\frac{\cos^2\alpha}{\cos^2\alpha} + \frac{\sin^2\alpha}{\cos^2\alpha}\right)$$

$$r^2 = \rho^2 \sec^2\alpha$$

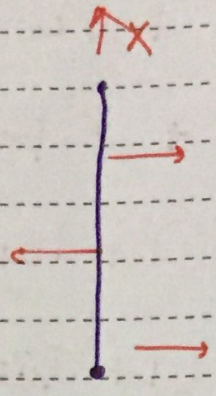
$$r^3 = \rho^3 \sec^3\alpha .$$

$$\hookrightarrow \vec{E} = \frac{\rho_L}{4\pi\epsilon_0} \int_{\alpha_1}^{\alpha_2} \frac{\rho \cancel{\sec\alpha} (\cos\alpha\, \hat{a}_\rho + \sin\alpha\, \hat{a}_z)}{\underbrace{\rho^{\cancel{3}} \cancel{\sec^3\alpha}}_{\text{هاي مقام}}} \underbrace{(-\rho \sec^2\alpha\, d\alpha)}_{\text{هاي بدل } dz'}$$

الحدود صارت بـ
α بدل z .

$$\vec{E} = \frac{-\rho_L}{4\pi\epsilon_0 \rho} (\sin\alpha_2 - \overset{\sin\alpha_1}{[illegible]})\, \hat{a}_\rho - ((\cos\alpha_2 - \cos\alpha_1)\, \hat{a}_z \quad (V/m)$$

*For an infinite Line:



ما عندي عامودي من فوق ومن تحت
[illegible] ← جـ ل ح لازم
يختفي.

$\alpha_1 = 90°$

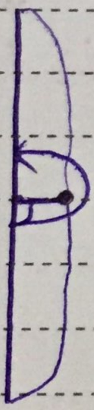
$\alpha_2 = 270°$ or $-90°$

$$\rightarrow \boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho}$$

حفظ

↳ Always for any infinite Line.

- كيف اطلع α_1, α_2 ؟

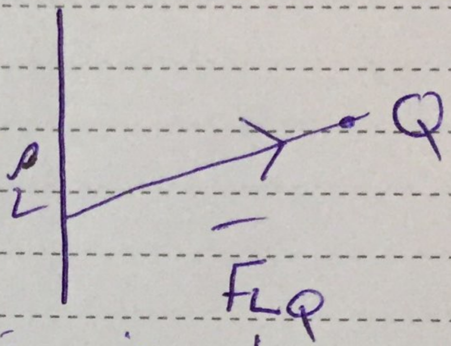


بمسك النقطة الي عندي وبطلع
من عندها لنهاية ال Line
إذاً الزاوية الأولى $\alpha_1 = 90°$
و ثاني مرة بطلع من النقطة
لبداية الخط (من نفس المرجع)
و بشوف الزاوية

* $\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \boxed{\rho}} \hat{a}_\rho$

ρ ↓ the shortest distance between the source & the field.

$\vec{F} = Q\vec{E}$



↲ بضرب قيمة $\bar{E}$ الي طلعتها بقيمة لشحنة Q

if we want F_{QL}

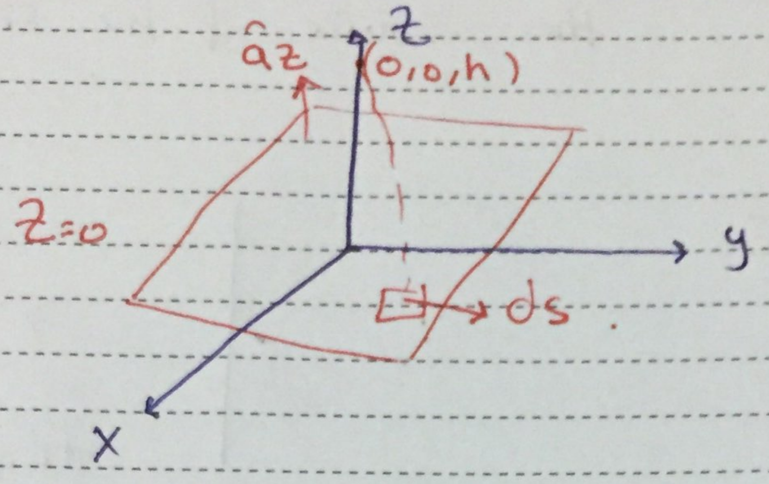
$$F_{QL} = Q \quad E_Q$$

$Q \rightarrow \int \rho_L dL$

$E_Q \rightarrow \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$

* E-field due to a surface charge:-

- Ex: Consider a $z = 0$ plane carry a charge ρ_s c/m². Find $\vec{E}$ at $(0,0,h)$ where $h > 0$



Sol: $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r$ → point charge

$$Q = \int_S \rho_s \, ds .$$

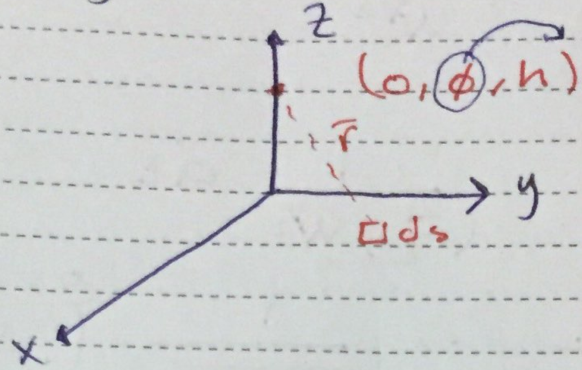
$$\vec{E} = \int_S \boxed{\frac{\rho_s \, ds}{4\pi\epsilon_0 r^2}} \hat{a}_r$$

$ds = dx' dy'$,
$\vec{r} = -x'\hat{a}_x - y'\hat{a}_y + h\hat{a}_z$

$r = \sqrt{x'^2 + y'^2 + h^2}$

$$\vec{E} = \frac{\rho_s}{4\pi\epsilon_0} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{(-x', -y', h)}{\underbrace{[x'^2 + y'^2 + h^2]^{3/2}}_{\text{table of integral.}}} \, dx' \, dy'$$

→ Convert to cylindrical because $z=0$ can be represented in cyl.



ϕ عشان تحتوي
قيمة ركز $\rho = 0$.

$ds = \rho\, d\rho\, d\phi$
$\bar{r} = -\rho \hat{a}_\rho + h \hat{a}_z$
$r = \sqrt{\rho^2 + h^2}$

$\phi \leftarrow 2\pi \quad \infty \rightarrow \rho$

$$\bar{E} = \frac{\rho_s}{4\pi\epsilon_0} \int_0^{2\pi}\int_0^{\infty} \frac{-\rho \cancel{\hat{a}_\rho} + h\hat{a}_z}{[\rho^2 + h^2]^{3/2}} \, \rho\, d\rho\, d\phi$$

-due to symmetry the ρ-component will be cancelled

$$\bar{E} = \frac{\rho_s}{4\pi\epsilon_0} \int_0^{2\pi}\int_0^{\infty} \frac{\rho h \hat{a}_z}{[\rho^2+h^2]^{3/2}} \, d\rho\, d\phi$$

$$= \frac{\rho_s h}{2\epsilon_0} \int_0^{\infty} \frac{\rho \hat{a}_z}{[\rho^2+h^2]^{3/2}} \, d\rho$$

Let $u = \rho^2 + h^2$
$du = 2\rho\, d\rho \rightarrow \rho\, d\rho = \frac{du}{2}$

$$\overline{E} = \frac{\rho_s h}{2\epsilon_0(2)} \int \frac{du}{u^{3/2}} \hat{a_z}$$

$$= \frac{\rho_s h}{4\epsilon_0} \cdot \frac{1}{\left(\frac{-1}{2}\right) u^{1/2}} \hat{a_z}$$

$$= \frac{-\rho_s h}{2\epsilon_0} \cdot \frac{1}{\sqrt{\rho^2+h^2}} \Bigg]_0^\infty \hat{a_z}$$

$$= \frac{-\rho_s h}{2\epsilon_0} \left(0 - \frac{1}{h}\right) \hat{a_z}$$

$$= \boxed{\frac{\rho_s}{2\epsilon_0} \hat{a_z}}$$ For infinite sheet along xy plane.

$$\boxed{\overline{E} = \frac{\rho_s}{2\epsilon_0} \hat{a_n}}$$ Always for any infinite sheet.

* $\bar{E}$-field due to Volume charge.

$$\boxed{\bar{E} = \int_{r} \frac{\rho_v \, dv}{4\pi\epsilon_0 r^2} \hat{a_r}} \quad V/m$$

* Electrical Flux Density ($\bar{D}$)

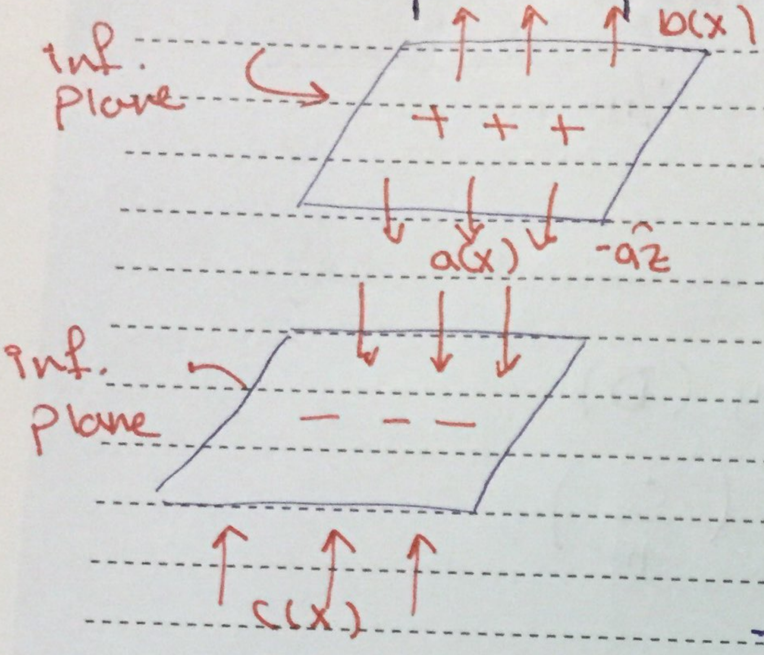
$$\bar{D} = \underset{F/m}{\underset{\downarrow}{\epsilon_0}} \; \underset{V/m}{\underset{\downarrow}{\bar{E}}} \qquad \left(\frac{FV}{m^2}\right)\left(\frac{C}{m^2}\right).$$

- For a point charge.

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a_r} \qquad \qquad \bar{D} = \frac{Q}{4\pi r^2} \hat{a_r}$$

For point charge	For inf. line	For inf. sheet
$\bar{D} = \frac{Q}{4\pi r^2} \hat{a_r}$	$\bar{D} = \frac{\rho_L}{2\pi\rho} \hat{a_\rho}$	$\bar{D} = \frac{\rho_s}{2} \hat{a_n}$ $\bar{E} = \frac{\rho_s}{2\pi\epsilon_0} \hat{a_n}$

* for a parallel plate capacitor



at point (a)

$$\overline{E} = \overline{E}_+ + \overline{E}_-$$

$$= \frac{\rho_s}{2\epsilon_0}(-\hat{a}_z) + \frac{-\rho_s}{2\epsilon_0}(\hat{a}_z)$$

$$= \left(\frac{-\rho s}{2\epsilon_0} - \frac{\rho_s}{2\epsilon_0}\right)\hat{a}_z$$

$$= \frac{-\rho_s}{G_0}\hat{a}_z \quad V/m$$

-at point b (z>d)

$$\overline{E} = \overline{E}_+ + \overline{E}_-$$

$$= \frac{\rho_s}{2\epsilon_0}\hat{a}_z + \frac{-\rho_s}{2\epsilon_0}\hat{a}_z = 0.$$

- Ex Find $\overline{D}$ at (4, 0, 3) if there is a point charge [redacted] -5π mC at $\boxed{(4,0,0)}$ and a line charge 3π mC along $\underline{y\text{-axis}}$

point charge src 1

↳ line charge

Sol ⇒ $\overline{D} = \overline{D}_Q + \overline{D}_L$

$$= \frac{Q}{4\pi r^2}\,\hat{a}_r + \frac{\rho_L}{2\pi\rho}\,\hat{a}_\rho$$

field → اللي
بدي أحسب المجال
عنه.

$$= \frac{Q\overline{r}}{4\pi r^3} + \frac{\rho_L\,\overline{\rho}}{2\pi\rho^2}$$

$\overline{r} = 3\hat{a}_z$

$r = 3$

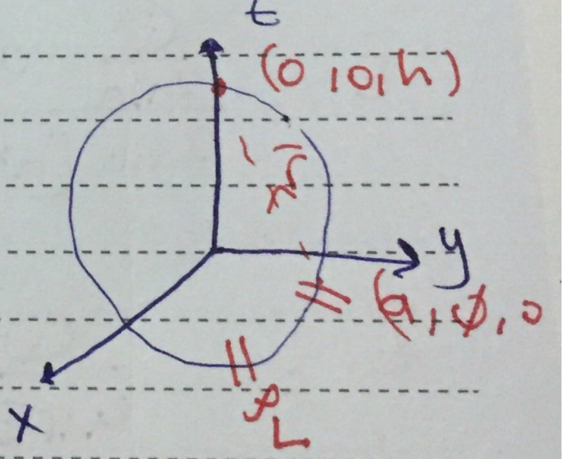
$\overline{\rho} = 4\hat{a}_x + 3\hat{a}_z$

$\rho = 5$ m

↳ shortest distance.

$\overline{D} = 240\,\hat{a}_x + 42\hat{a}_z$ μc/m²

- Ex for a ring of radius (a) placed along xy plane carry ρ_L c/m ① find $\overline{E}$ at (0, 0, h).



$$\overline{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r$$

$$= \int_L \frac{\rho_L\,dL}{4\pi\epsilon_0 r^3}\,\overline{r}$$

$$= \frac{\rho_L}{4\pi\epsilon_0}\int_0^{2\pi} \frac{-a\hat{a}_\rho + h\hat{a}_z}{\left[a^2+h^2\right]^{3/2}}\,a\,d\phi$$

$Q = \rho_L\,dL$.

$d_L = \rho\,d\phi = a\,d\phi$

$\overline{r} = -a\hat{a}_\rho + h\hat{a}_z$

$r = \sqrt{a^2+h^2}$

due to symmetry → the ρ component will be cancelled.

$$\boxed{\bar{E} = \frac{\rho_L\, a h}{2\epsilon_0 [a^2+h^2]^{3/2}}\ \hat{a_z} \quad V/m}$$

② if $a \to 0$, show that the $\bar{E}$ will look like the $\bar{E}$ of a point charge

Sol $\quad \bar{E} = \left.\frac{\rho_L\, a h}{2\epsilon_0[a^2+h^2]^{3/2}}\right|_{a=0} \overset{?}{=} \frac{Q}{4\pi\epsilon_0 r^2}\ \hat{a_r}$

$$Q = \int_L \rho_L\, dL$$

$$= \int_0^{2\pi} \rho_L\, a d\phi \Rightarrow Q = \rho_L\, 2\pi\, a$$

$$\rho_L = \frac{Q}{2\pi a}$$

$$\bar{E} = \left.\frac{\frac{Q}{2\pi a}\, a h}{2\epsilon_0 [a^2+h^2]^{3/2}}\ \hat{a_z}\ \right|_{a\to 0}$$

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 h^2}\ \hat{a_z}$$

$r = h$

$\hat{a_r} = \hat{a_z}$

* Gauss's Law :-
↳ special case from coulomb's law

$$\Psi_e = \oint_S \underset{\underset{m^2}{\downarrow}}{\bar{D}} \cdot \underset{\underset{m^2}{\downarrow}}{\bar{ds}} = \underset{\underset{C}{\downarrow}}{Q}$$

- Electric Flux (Ψ_e) in (c)

$$\Psi_e = \int_S \bar{D} \cdot \bar{ds} \quad \text{Scalar.}$$

* Applications of Gauss's Law :-

$$\oint_S \bar{D} \cdot \bar{ds} = Q = \int_L \rho_L \, dL \quad \text{Point / Line .}$$

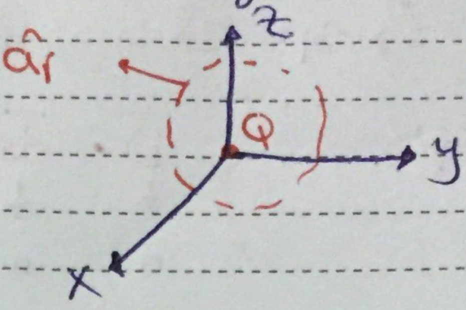
$$= \int_S \rho_s \, ds \quad \text{surface .}$$

$$= \int_V \rho_v \, dv \quad \text{Volume .}$$

$$* \boxed{\oint_S \bar{D} \cdot \bar{ds} = \int_V \rho_v \, dv}$$

First Maxwell's eq. in integral form (Gauss's Law

① $\bar{E}$ or $\bar{D}$ due to a point charge



$\bar{D} = D_r \hat{a_r}$

$\overline{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$

$$\oint_S \bar{D} \cdot \overline{ds} = \int_S \bar{D} \cdot \overline{ds} = \int_0^{2\pi} \int_0^{\pi} D_r \hat{a_r} \cdot r^2 \sin\theta \, d\phi \, d\theta \, \hat{a_r} = Q$$

$$= r^2 D_r (2)(2\pi) = Q$$

$$= 4\pi r^2 D_r = Q$$

$$D_r = \frac{Q}{4\pi r^2}$$

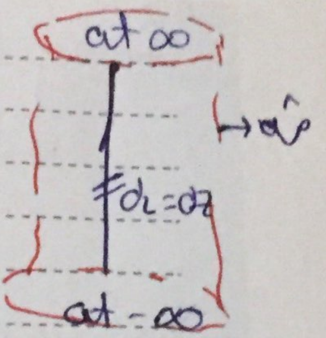
$$\bar{D} = D_r \hat{a_r}$$

$$\bar{D} = \frac{Q}{4\pi\epsilon_0} \hat{a_r}$$

$$\bar{E} = \frac{\bar{D}}{\epsilon_0} = \frac{Q}{4\pi r^2 \epsilon_0} \hat{a_r}$$

$\bar{D} = D_\rho \, \hat{a}_\rho$

$\bar{ds} = \rho \, d\phi \, dz \, \hat{a}_\rho$



$$\int_L \bar{D} \cdot \bar{ds}$$

$$= \int_{-L/2}^{L/2} \int_0^{2\pi} D_\rho \, \hat{a}_\rho \cdot \rho \, d\phi \, dz \, \hat{a}_\rho$$

$$= \int_{-L/2}^{L/2} \rho_L \, dz.$$

$L \to \infty$

$D_\rho (\rho \, 2\pi L) = \rho_L L.$

$$\boxed{\bar{D} = \frac{\rho_L}{2\pi\rho} \hat{a}_\rho} \quad , \quad \boxed{\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho}$$

3) $\bar{E}$ or $\bar{D}$ due to an infinite sheet of charge

$$\oint_S \bar{D} \cdot \bar{ds} = Q_{en} = \int_S \rho_s \, ds$$

$$\int_{S_{top}} \bar{D} \cdot \bar{ds}_{top} + \int_{S_{bot}} \bar{D} \cdot \bar{ds}_{bot} = \int_S \rho_s \, ds.$$

$$\bar{D} = \begin{cases} D_z \hat{a}_z & , z > 0 \\ D_z(-\hat{a}_z) & , z < 0 \end{cases}$$

$$\begin{cases} \bar{ds}_{top} = dxdy\,\hat{a}_z \\ \bar{ds}_{bot} = -dxdy\,\hat{a}_z \end{cases}$$

$$D_z A + D_z A = \rho_s A$$

$$D_z = \frac{\rho_s}{2}$$

$$\boxed{\bar{D} = \frac{\rho_s}{2}\,\hat{a}_n} \qquad \boxed{\bar{E} = \frac{\rho_s}{2\epsilon_0}\,\hat{a}_n}$$

4) $\bar{E}$ or $\bar{D}$ due to uniform volume charge

- Ex: consider a sphere of radius (a) has:

$$\rho_r = \begin{cases} \rho_0 & , r < a \\ 0 & , r > a \end{cases}$$

Find $\bar{E}$ & $\bar{D}$ everywhere.

↳ Gauss's Law.

→

Sol $\oint_S \bar{D}\cdot\bar{ds} = Q_{enc} = \int \rho_v dv$

For $r < a$

$\underline{D} = D_r \hat{a_r}$

$\underline{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$

$$\int_0^{2\pi}\int_0^{\pi} D_r \cancel{\hat{a_r}} \cdot r^2 \sin\theta \, d\theta \, d\phi \, \cancel{\hat{a_r}}$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho_0 r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$D_r (r^2)(2)(2\pi) = \rho_0 \frac{r^3}{3} (2)(2\pi)$$

$$D_r = \frac{\rho_0 r}{3}$$

$$\overline{D_r} = \frac{\rho_0 r}{3} \hat{a_r}$$

$$\bar{E} = \frac{\rho_0 r}{3\epsilon_0} \hat{a_r}$$

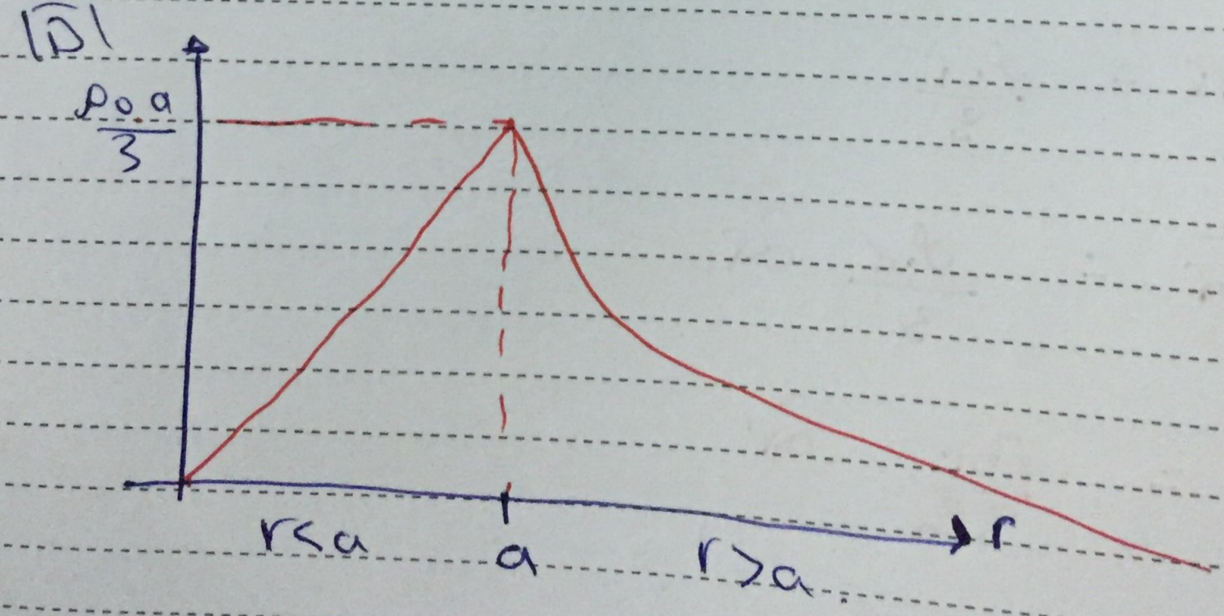
For $r > a$

$$4\pi r^2 D_r = \int\int\int_0^a \rho_0 r^2 \sin\theta \, dr \, d\theta \, d\phi$$

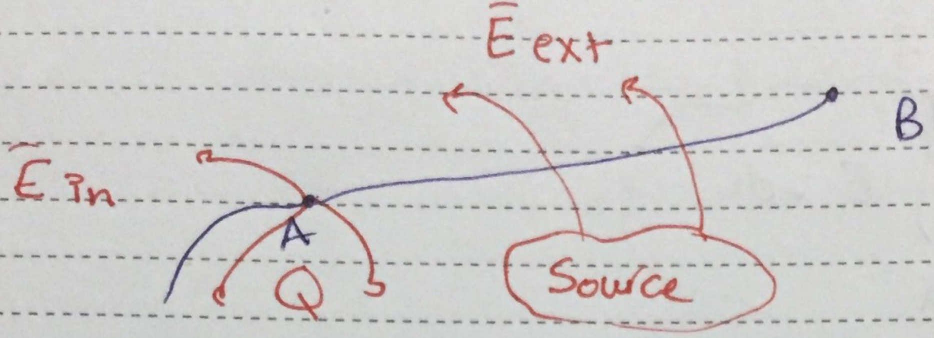
$$+ \int_0\int\int_a^r \text{zero}$$

$$\cancel{4\pi} r^2 D_r = \rho_0 \frac{a^3}{3} \cancel{(2)(2\pi)}$$

$$\left.\begin{aligned} \bar{D} &= \frac{\rho_0 a^3}{3 r^2} \hat{a}_r \\ \bar{E} &= \frac{\rho_0 a^3}{3 \epsilon_0 r^2} \hat{a}_r \end{aligned}\right\} \rightarrow \text{for } r \geq a$$



*Electric Potential in (V) Scalar quantity.



To move Q from A to B

$V_{AB} = V_B - V_A$

$$\text{Voltage} = \frac{\text{work}}{\text{charge}}$$

$$1V = \frac{1J}{1C}$$

work = Voltage * charge.

$$\boxed{W = Q\,V_{AB}}$$

↳ potential difference between two points.

$W = \bar{F} \cdot \bar{L}$ (N.m).

$= \textcircled{-} Q \, \bar{E}_{ext} \cdot \int$

↳ the work is done by the external field.

$$\frac{W}{Q} = - \int_{f} \bar{E} \cdot dL$$

$$\boxed{V_{AB} = - \int_{r_A}^{r_B} \bar{E} \cdot \bar{dL}}$$

- For a point charge :-

$$V_{AB} = - \int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon_0 r^2} \hat{a_r} \cdot dr \, \hat{a_r}$$

$$V_{AB} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_B} - \frac{1}{r_A} \right) \quad V$$

$$= \boxed{\frac{Q}{4\pi\epsilon_0 r_B}} - \boxed{\frac{Q}{4\pi\epsilon_0 r_A}}$$

$$V_{AB} = V_B - V_A$$

*if point A is at ∞

$$\bar{r_A} = \infty$$

$$V_{\infty B} = \frac{Q}{4\pi\epsilon_0 r_B} - \cancel{\frac{Q}{4\pi\epsilon_0 \infty}} \rightarrow 0$$

$$= V_B - V_\infty$$

$$= V_B - 0$$

$$= \boxed{V_B}$$

* **Notes**

$V_\infty = 0$ Always, Since $r = \infty$

- لو حكالي فرق الجهد عن نقطة معناها فرق الجهد بين النقطة والـ ∞ .

- إذا طلب مني potential بستخدم

$$\boxed{V = -\int_{f} \bar{E}\cdot d\bar{L}}$$

- To find (V):-

↳ From $\bar{E}$

$$V = -\int_{f} \bar{E}\cdot d\bar{L}$$

↳ From the Source

↳ point charge $V = \dfrac{Q}{4\pi\epsilon_0 r}$

↳ Line charge $V = \displaystyle\int_{l} \frac{\rho_L \, dL}{4\pi\epsilon_0 r}$

↳ Surface charge $V = \displaystyle\int_{s} \frac{\rho_s \, ds}{4\pi\epsilon_0 r}$

↳ Volume charge $\underset{\text{Voltage}}{\textcircled{V}} = \displaystyle\int_{\underset{\text{Volume}}{\textcircled{v}}} \frac{\rho_v \, \overset{\text{Volume}}{\textcircled{dv}}}{4\pi\epsilon_0 r}$

-Ex Two point charges $(-4\mu C)$ and $(5\mu C)$ are located at $(2,-1,3)$ and $(0,4,-2)$ find the potential at $p(1,0,1)$.

Sol refrence is at ∞

$$V_{\infty p} = V_p - V_\infty$$
$$= V_p - 0$$
$$= \boxed{V_p} \rightarrow$$ المطلوب

$$V_p = \boxed{\sum_{k=1}^{N}} \frac{Q_k}{4\pi\epsilon_0 r_k} = \frac{-4 * 10^{-6}}{4\pi \frac{* 10^{-9}}{36\pi} * \sqrt{6}} + \frac{5 * 10^{-6}}{4\pi * \frac{10^{-9}}{36\pi} *}$$

هاد لأنه عندي أكتر من نقطة

اخترت هاد القانون لأنه عندي point charge.

$$\rightarrow \bar{r}_1 = (1,0,1) - (2,-1,3)$$
$$= (-1,1,-2).$$

$$\bar{r}_2 = (1,0,1) - (0,4,-2)$$
$$= (1,-4,3).$$

$$V = -5.872\ kV$$

Scanned with CamScanner

- What is the meaning of the minus sign?

if V is -ve

↳ drop in Potential.

↳ the work is done by the field it self.

↳ $\int_{f} E \cdot \partial l$ is +ve.

- if V is +ve

↳ gain in Potential.

↳ the work is done by an external field.

- Ex At point charge of (5nC) loacted at (-3,4,0) and a line y=1, z=1 carries $\rho_L = 2$ nC/m if V=100V at B(1,2,1) find the potential at C(-2,5,3)??

Sol $V = V_Q + V_L$

refrence is (B)

$V_{BC} = V_{BCQ} + V_{BCL}$.

$= V_C - V_B$

$= V_C - 100$.

$$\boxed{V_C = V_{BC} + 100}$$

$$V_{BCQ} = \frac{\cancel{\ }Q}{4\pi\epsilon_0}\left(\frac{1}{r_C} - \frac{1}{r_B}\right)$$

or

بستخدم هاي إذا ماكنت حافظة القانون اللي فوق.

$$V_{BCQ} = -\int_{f}^{} \bar{E}\cdot\partial\bar{L} = -\int_{r_B}^{r_C} \frac{Q}{4\pi\epsilon_0 r^2}\,\cancel{\hat{a}_r}\cdot dr\,\cancel{\hat{a}_r}$$

$$= \frac{Q}{4\pi\epsilon_0 r^2}\left(\frac{1}{r_C} - \frac{1}{r_B}\right).$$

$$r_C = |(1,1,3)| = \sqrt{11}$$

$$r_B = |(4,-2,1)| = \sqrt{21}$$

$$V_{BC\cancel{l}} = \int_{-\infty}^{\infty} \frac{\rho_L\,\partial L}{4\pi\epsilon_0 r} \rightarrow \text{complicated}$$

$$\text{So}\quad V_{BCl} = -\int_{f}^{} \bar{E}\cdot\partial\bar{L}$$

$$= -\int_{\rho_B}^{\rho_C} \frac{\rho_L}{2\pi\epsilon_0\rho}\,\cancel{\hat{a}_\rho}\cdot d\rho\,\cancel{\hat{a}_\rho}$$

$$= \frac{-\rho_L}{2\pi\epsilon_0}\ln(\rho_C) - \ln(\rho_B) \longrightarrow$$

$$\frac{-\rho_L}{2\pi\epsilon_0} \cdot \ln\left(\frac{\rho_C}{\rho_B}\right)$$

$$= \frac{\rho_L}{2\pi\epsilon_0} \ln\left(\frac{\rho_B}{\rho_C}\right)$$

$\rho_C = \left|\left(\overset{0}{\cancel{2}}, 4, 2\right)\right| = \sqrt{20}$ أقصر مسافة

$\rho_B = \left|(0, 1, 0)\right| = \sqrt{1}$

$V_{BC} = -50.175$ V

$V_C = 49.825$ w

★ Electric potential :-

$$V = -\int_L \bar{E} \cdot \bar{dL}$$

otential $dV = -\bar{E} \cdot \bar{dL}$

-if in cart. coordinates:-

$\bar{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z$

$\bar{dL} = dx\, \hat{a}_x + dy\, \hat{a}_y + dz\, \hat{a}_z$

$\rightarrow \bar{E}\,\bar{dl} = E_x\,\partial x + E_y\,dy + E_z\,\partial z.$

$$\partial v = \frac{\partial v}{\partial x}\,dx + \frac{dv}{\partial y}\,dy + \frac{\partial v}{\partial z}\,\partial z$$

Equating Similar components:-

$$\frac{\partial v}{\partial x}\,\cancel{dx} = -E_x\,\cancel{\partial x}$$

$$E_x = -\frac{\partial v}{\partial x}$$

$$E_y = -\frac{\partial v}{\partial y}$$

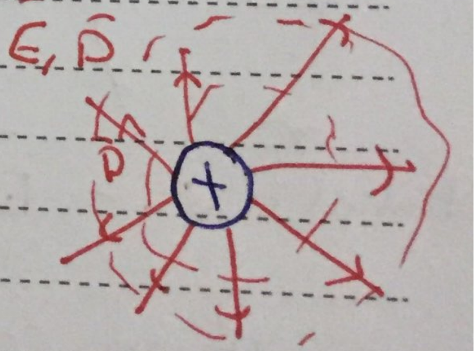
$$E_z = -\frac{\partial v}{\partial z}.$$

$$\rightarrow \bar{E} = -\left(\frac{\partial v}{\partial x}\,\hat{a}_x + \frac{\partial v}{\partial y}\,\hat{a}_y + \frac{\partial v}{\partial z}\,\hat{a}_z\right)$$

$\boxed{\bar{E} = -\nabla V}$ → to find $\bar{E}$ if potential is given.

* Point charge.

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r$$

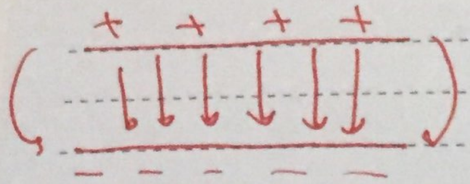


flux lines : $V = \dfrac{Q}{4\pi\epsilon_0 r}$

$$\bar{D} = \epsilon_0\,\bar{E}$$

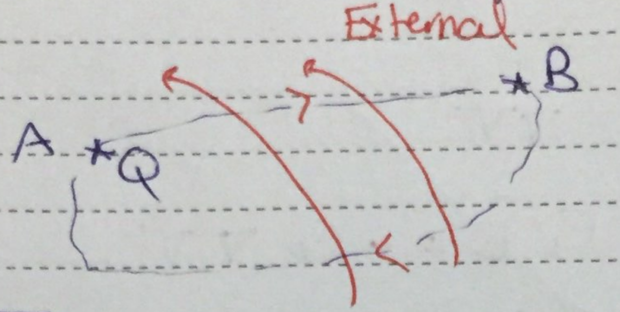
$\bar{E}, \bar{D}$

الهم نفس الاتجاه



capacitor

* Potential around a closed paths -



بدي أنقل Q
من A لـ B

$$\boxed{W = Q\,V_{AB}}$$

if Q is moved from A to B.

$$V_{AB} = \bigcirc\!\!\!\!-\,\int_{A}^{B} \bar{E}\cdot\bar{\partial L}$$

[illegible] لشغل
خارجي

2nd Maxwell's equation in integral form

$$V = \oint_{L} \bar{E}\cdot\bar{\partial L} = 0$$

1st Maxwell's equ.

$$\oint_{L} \bar{D}\cdot\bar{\partial s} = \int_{V} \rho_v\, dv$$

$$V = \int_{L} \bar{E}\cdot\bar{\partial L}$$

$$\underset{V_B}{V_{AA}} = -\int_{r_A}^{r_B} \bar{E}\,\bar{\partial L} + -\int_{r_B}^{r_A} \bar{E}\cdot\bar{\partial L}$$

$$= -\int_{r_A}^{r_B} \bar{E}\cdot\bar{\partial L} + \int_{r_A}^{r_B} \bar{E}\cdot\bar{\partial L} = 0.$$

$$V_{AA} = V_A - V_A = 0$$

- Ex Given $V = \frac{10}{r^2} \sin\theta \cos\phi$ V

a) Find $\overline{D}$ at $(2, \frac{\pi}{2}, 0)$

b) calculate the work in moving a (10 μC) charge from A(1, 30°, 120°) to B(4, 90°, 60°)

Sol a) $\overline{E} = -\nabla V$

$$\overline{D} = \epsilon_0 \overline{E} = -\epsilon_0 \nabla V$$

$$\overline{E} = -\left(\frac{\partial v}{\partial r} \hat{a_r} + \frac{\partial v}{r \partial\theta} \hat{a_\theta} + \frac{\partial v}{r \sin\theta \partial\phi} \hat{a_\phi} \right)$$

$$\overline{E} = -\left(\frac{-20}{r^3} \sin\theta \cos\phi \hat{a_r} + \frac{10}{r^3} \cos\theta \cos\phi \hat{a_\theta} - \frac{10}{r^3} \sin\phi \hat{a_\phi} \right)$$

$$\rightarrow \overline{D}\Big|_{(2, \frac{\pi}{2}, 0)} = \epsilon_0 \overline{E}\Big|_{(2, \frac{\pi}{2}, 0)} = \frac{10^{-9}}{36\pi} \left(\frac{20}{8} \hat{a_r} \right)$$

$$= 22 \hat{a_r} \text{ pC/m}^2$$

b.) Method (1)

$$W = Q\,V_{AB} = Q\,(V_B - V_A) \rightarrow V_B$$

كيف أجيب VB
بعوض لنقطة B
جوا معادلة ال V

$$= 10 * 10^{-6}\left(\frac{10}{16}\cdot\frac{1}{2} - 10\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\right)$$

$$= 10 * 10^{-6}\left(\frac{5}{16} + \frac{40}{16}\right)$$

$$= 10 * 10^{-6}\left(\frac{45}{16}\right)$$

$$= \boxed{28.125\ \mu J}$$

- Method (2)

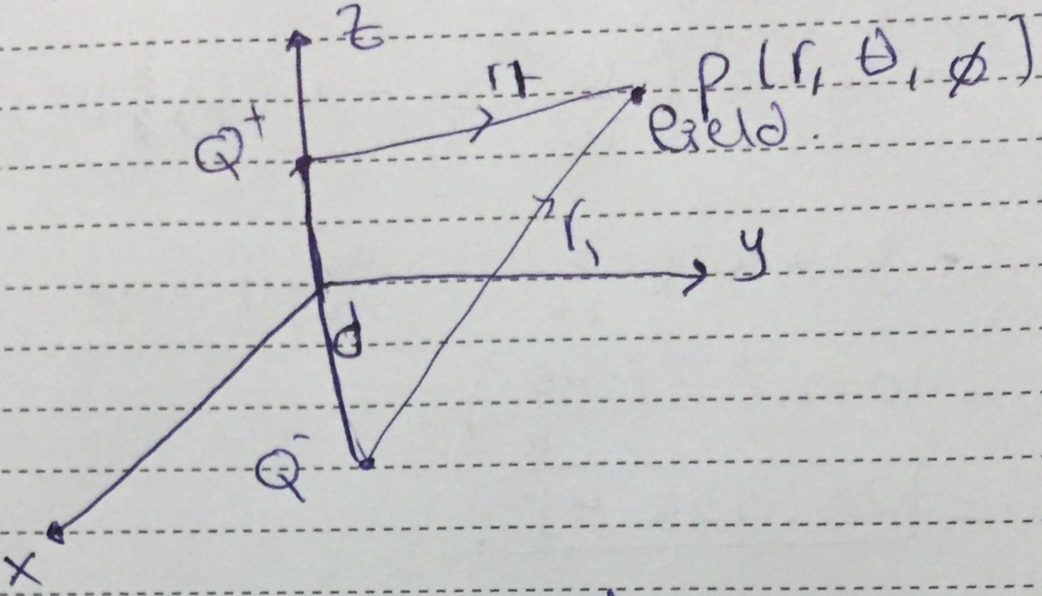
$$W = -Q\int_{rA}^{rB} \bar{E}\cdot \bar{dl} \qquad V = -\int \bar{E}\cdot\bar{dL}.$$

$$= -10 * 10^{-6}\left[\left.\int_{1}^{4} E_r\,dr\right|_{\substack{\theta=30\\ \phi=120}} + \left.\int_{30^\circ}^{90^\circ} E_\theta\, r\,d\theta\right|_{\substack{r=4\\ \phi=120^\circ}} + \left.\int_{120^\circ}^{60^\circ} E_\phi\, r \sin\theta\, d\phi\right|_{\substack{r=4\\ \theta=90^\circ}}\right].$$

* Electric Dipole8 -

- How to find $\bar{E}$, $\bar{D}$ and V

r>>>d
must
for a
dipole



- if $r \geq d$ then it's two point charges.

- find V: (ref is at ∞).

$$V = V^+ + V^-$$

$$= \frac{Q}{4\pi\varepsilon_0 r_1} + \frac{-Q}{4\pi\varepsilon_0 r_2}$$

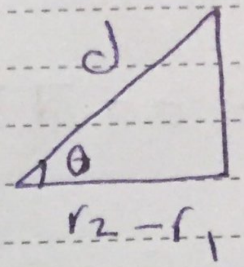
$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\boxed{V = \frac{Q(r_2 - r_1)}{4\pi\varepsilon_0 r_1 r_2}}$$

Q and it's location
are not given

- Approximations:

$r_1 r_2 \simeq r^2$



$$\cos\theta = \frac{r_2 - r_1}{d}$$

$$r_2 - r_1 = d\cos\theta .$$

$$\rightarrow \boxed{V = \frac{Q\, d\cos\theta}{4\pi \epsilon_0 r^2}}$$

- define a dipole moment as $\bar{p} = Q\bar{d}$ in (C.m)

$$\rightarrow \boxed{V = \frac{p\cos\theta}{4\pi \epsilon_0 r^2}}$$

($\bar{p}$) and the center of the dipole are usually given.

$r = |$ field − center of the dipole $|$.

θ is the angel between ($\bar{p}$) and ($\bar{r}$).

$$\boxed{V = \frac{\bar{p}\cdot \hat{a}_r}{4\pi \epsilon_0 r^2}} \rightarrow \bar{p}\cdot\hat{a}_r = p\,(1)\,\cos\theta .$$

$$= p\cos\theta .$$

$$* \ \bar{E} = \bar{E}^{+} + \bar{E}^{-}$$

$$\text{or} \quad \bar{E} = -\nabla V$$

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \ (V).$$

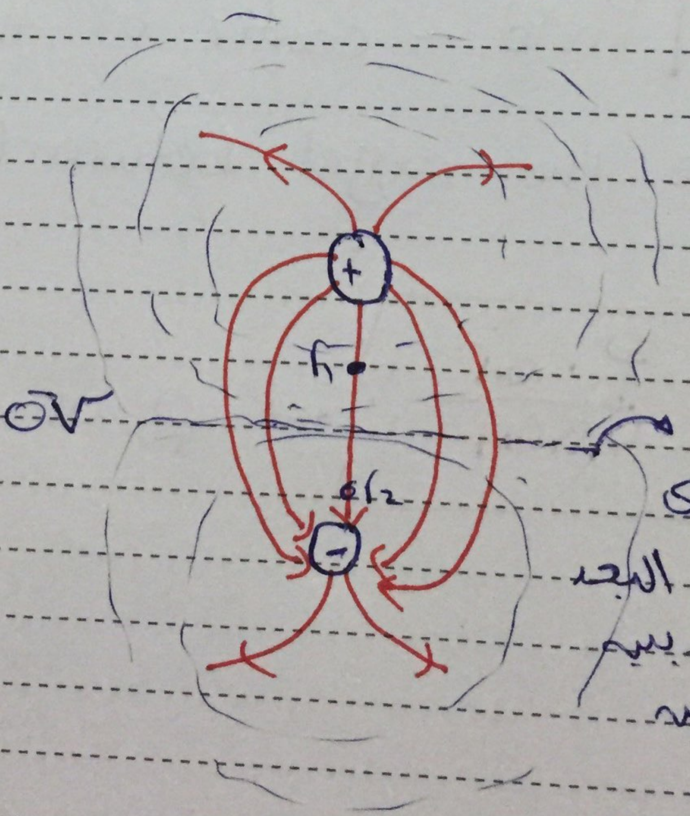
$$\rightarrow \bar{E} = -\nabla V = -\left(\frac{\partial V}{\partial r}\hat{a}_r + \frac{\partial V}{r\partial\theta}\hat{a}_\theta + \frac{\partial V}{r\sin\theta\partial\phi}\hat{a}_\phi \right)$$

$$= \frac{-p}{4\pi\varepsilon_0}\left(\frac{-2\cos\theta}{r^3}\hat{a}_r - \frac{\sin\theta}{r^3}\hat{a}_\theta \right).$$

$$\rightarrow \boxed{\bar{E} = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta + \sin\theta \, \hat{a}_\theta \right)}$$

$$\rightarrow \bar{D} = \varepsilon_0 \bar{E}$$

Flux lines
for the
dipole.



0 نه أي
مكان على
هاد خط البعد
بينه وبين
الشحنتين
متساوي

- for N-dipoles :-

$$V = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^{N} \frac{\overline{P_k} \cdot (\overset{\text{field}}{\overline{r}} - \overset{\text{source}}{\overline{r_k'}})}{(r - r_k')^3} \quad (u)$$

- Ex Two dipoles with dipole moments $(-5\hat{a_z})$ ncm and $(9\hat{a_z})$ nc.m located at (0,0,-2) and (0,0,3). Find the potential at the origin.

Sol

$$V_{\infty o} = \frac{-5\hat{a_z} * 10^{-9} \cdot 2\hat{a_z}}{4\pi \frac{* 10^{-9}}{36\pi} * 8}$$

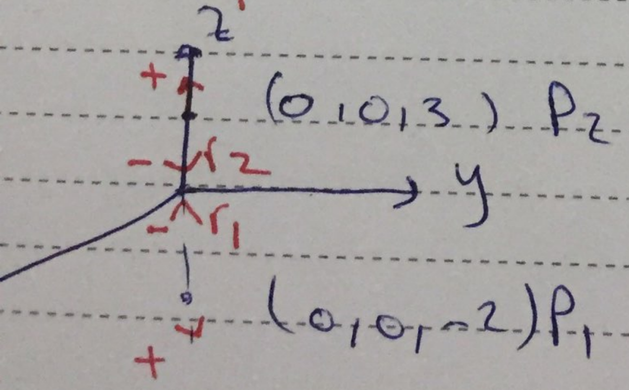
$$+ \frac{9\hat{a_z} * 10^{-9} \cdot (-3\hat{a_z})}{4\pi \frac{* 10^{-9}}{36\pi} * 27} = \boxed{-} 20.25 \text{ v}$$

drop, work done by field itself

- Find $\overline{E}$ at origin?

$$\overline{E} = \overline{E_1} + \overline{E_2}$$

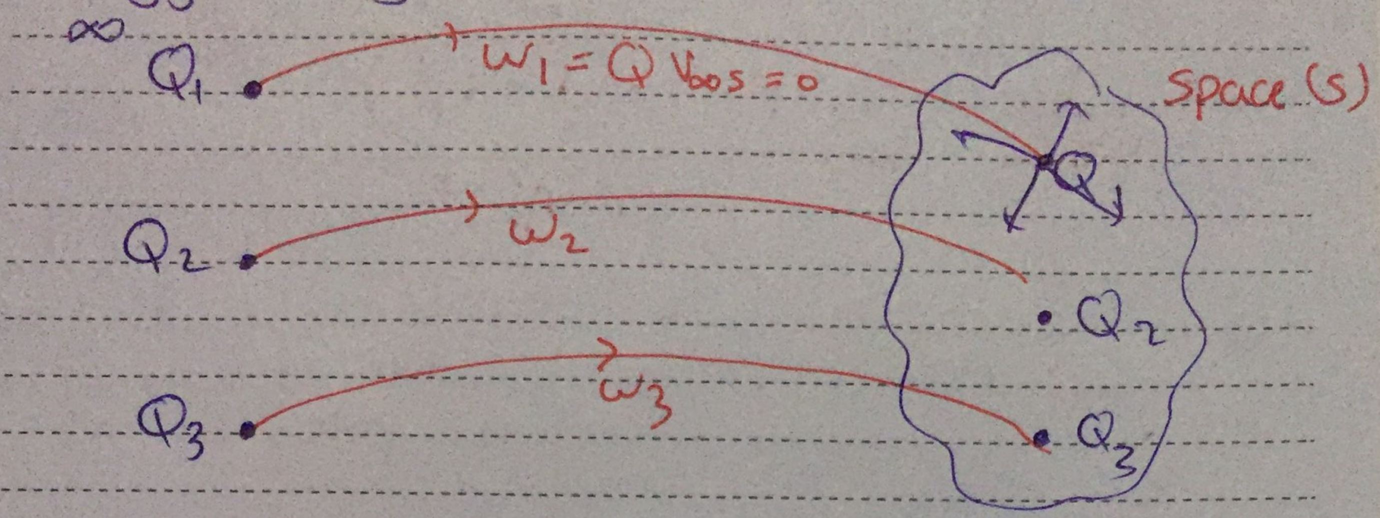
$$= \frac{P}{4\pi\epsilon_0 r_1^3}(2\cos\theta_1 \hat{a_r} + \sin\theta_1 a_\theta)$$



$P_1 = 5$ncm
$P_2 = 9$ncm
$r_1 = 2$
$r_2 = 3$

الزاوية بين r و P
$\theta_1 = 180$
$\theta_2 = 180$

*Energy Density of Electrostatic Fields:-



$\underrightarrow{Q_1, Q_2, Q_3}$

W_E : Electrical Energy (J).

$$W_E = W_1 + W_2 + W_3$$
$$0 + Q_2(V_{12}) + Q_3(V_{13} + V_{23}) \text{ --- } ①$$

* $Q_3 \longrightarrow Q_2 \longrightarrow Q_1$

$$W_E = W_1 + W_2 + W_3$$

$$Q_1(V_{21} + V_{31}) + Q_2(V_{32}) + 0 \text{ --- } ②$$

① + ②

$$2W_E = Q_1(\overset{V_1}{\underline{V_{21} + V_{31}}}) + Q_2(\overset{V_2}{\underline{V_{12} + V_{32}}}) + Q_3(\overset{V_3}{\underline{V_{13} + V_{23}}})$$

$$W_E = \frac{1}{2}(Q_1 V_1 + Q_2 V_2 + Q_3 V_3)$$

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$$\boxed{W_E = \frac{1}{2} \sum_{k=1}^{N} Q_k V_k}$$ (J) for N-point charges.

- for Line Charges -

$$W_E = \frac{1}{2} \int_L \rho_L V \, dL$$ potential

- for surface charges -

$$W_E = \frac{1}{2} \int_S \rho_s V \, ds$$

- for volume charges -

$$W_E = \frac{1}{2} \int_V \rho_v V \, dv$$

* How to relate energy to $\bar{E}$?

$$W_E = \frac{1}{2} \int_V \rho_v V \, dv$$

Using Gauss Law.

$$\oint_S \bar{D} \cdot ds = \int_V \rho_v \, dv$$

⋮

$$\boxed{W_E = \frac{1}{2}\int_V \bar{E} \cdot \bar{D} \, dv}$$

or

$$\boxed{W_E = \frac{1}{2}\int_V \epsilon_0 E^2 \, dv} \qquad (\bar{D} = \epsilon_0 \bar{E})$$

or

$$\boxed{W_E = \frac{1}{2}\int_V \frac{D^2}{\epsilon_0} \, dv} \qquad \left(\bar{E} = \frac{\bar{D}}{\epsilon_0}\right)$$

* $\bar{E} \cdot \bar{E} = |\bar{E}|^2$

* Energy Density (w_E)

$$w_E = \frac{W_E}{\text{Volume}} \qquad (J/m^3)$$

or

$$w_E = \frac{1}{2} \bar{E} \cdot \bar{D}$$

$$= \frac{1}{2} \epsilon_0 E^2$$

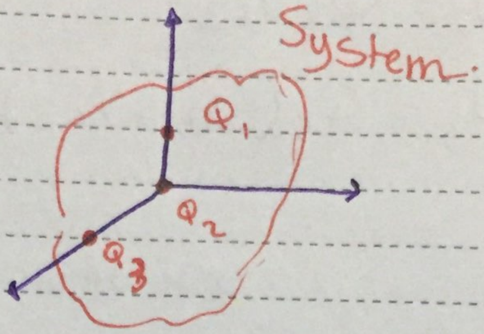
$$= \frac{1}{2} \frac{D^2}{\epsilon_0}$$

$$W_E = \int_V w_E \, dv$$

لو طلب W_E وكان معطيني w_E

- Ex: The point charges (-1nC) ; (4nC) and (3nC) are located at (0,0,0) , (0,0,1) and (1, 0, 0) . Find the energy in the System?

Sol:



$$W_E = \frac{1}{2} \sum_{k=1}^{3} Q_k V_k .$$

$$= \frac{1}{2} \left[Q_1 V_1 + Q_2 V_2 + Q_3 V_3 \right]$$

$$= \frac{1}{2} \left[Q_1 (V_{21} + V_{31}) + Q_2 (V_{12} + V_{32}) + Q_3 (V_{13} + V_{23}) \right] .$$

$$= \frac{1}{2} \left[Q_1 \left(\frac{Q_2}{4\pi\epsilon_0 (1)} + \frac{Q_3}{4\pi\epsilon_0 (1)} \right) + Q_2 \left(\frac{Q_1}{4\pi\epsilon_0 (1)} + \frac{Q_3}{4\pi\epsilon_0 (\sqrt{2})} \right) \right.$$

$$\left. + Q_3 \left(\frac{Q_1}{4\pi\epsilon_0 (1)} + \frac{Q_2}{4\pi\epsilon_0 (\sqrt{2})} \right) \right]$$

$$= \frac{1}{2 (4\pi\epsilon_0)} \left[Q_1 Q_2 + Q_1 Q_3 + Q_2 Q_1 + \frac{Q_2 Q_3}{\sqrt{2}} + Q_3 Q_1 + \frac{Q_3 Q_2}{\sqrt{2}} \right.$$

$$= \boxed{13.37 \text{ nJ}}$$

$\hookrightarrow W_1 + W_2 + W_3 .$

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لو طلب مني احسب ال work لكل شحنة

Q_1 $\downarrow$ Q_2 $\downarrow$ Q_3

$$W_1 = 0$$

$$W_2 = Q_2 V_{12}$$

$$W_3 = Q_3 (V_{13} + V_{23}).$$

- لو كان السؤال ال System بشكل معين

$$W_E = \frac{1}{2} \int_V \epsilon_0 \boxed{E^2} \boxed{dv} \rightarrow r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\int_0^2 \int_0^\pi \int_0^{2\pi}$$

$$E^2 \rightarrow \left(\frac{Q}{4\pi\epsilon_0 r^2} \right)^2$$

r_{12}

r_{13}

r_{23}

* CH5 Electric field in Materials:-

- Classification of materials based on it's electrical properties

↳ Conductors (Metals) ($\sigma >>> 1$)
Cu, AL, Ag , lead

↳ Semi - conductors ($\sigma > 1$)
Si, GaAs , Ge

↳ Dielectric (Insulators) $0 < \sigma <<< 1$
Li, Mica, polythelene , polysthren

σ = conductivity (s/m) or $(\Omega m)^{-1}$

- $\sigma = 0$ → free space

σ depends on:-

↳ Temperature (°K)
↳ Frequency (Hz)

i.e lead at (20°C) (293°K)
$\sigma = 10^6$ S/m

at (4°K)
$\sigma = 10^{20}$ (s/m) (super conductor)
perfect "
good "

* $T \uparrow \longrightarrow \sigma \downarrow$

$\sigma = \frac{1}{\rho}$

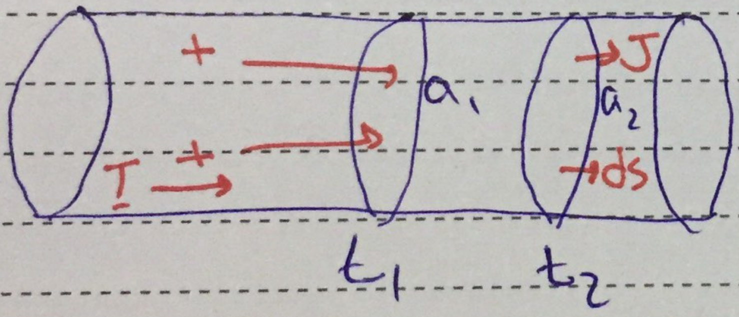
$\rho \longrightarrow$ resistivity (Ωm)

* Types of currents :-

1) Conduction current (DC or AC)
2) convection current (DC) يمر بالمادة العازلة -
3) Displacement current (AC)

①

* $\boxed{I = \frac{\Delta q}{\Delta t}} = \frac{a_2 - a_1}{t_2 - t_1}$



Unit $\rightarrow \frac{C}{s} = \boxed{A}$

*define J = current density

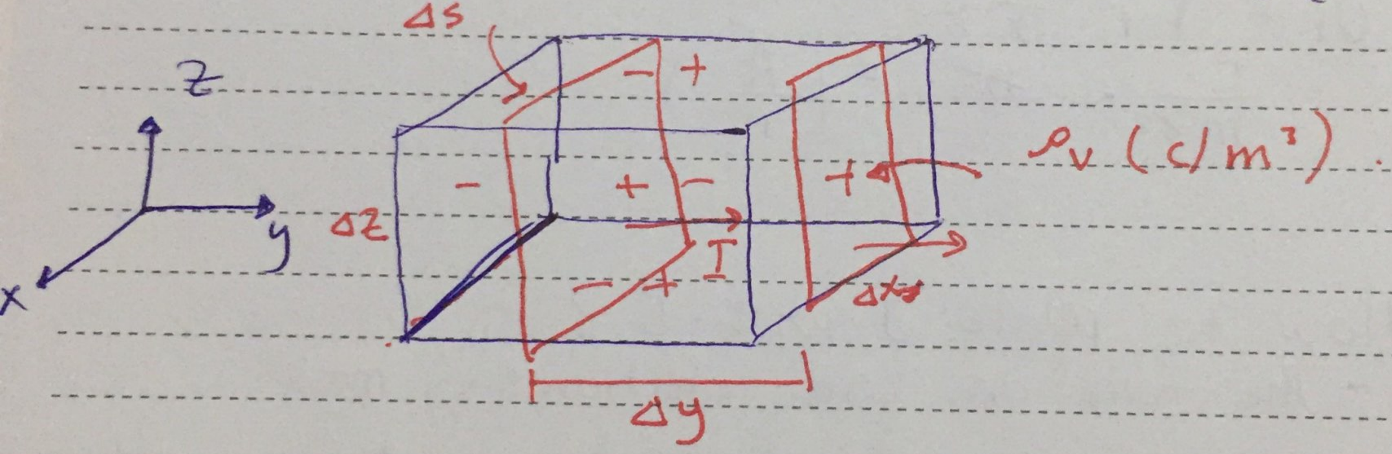
$J = \frac{I}{S} = (A/m^2)$

$I = JS$ (if uniform)

②
$$\boxed{\underbrace{I}_{\text{Scalar}} = \int_S \bar{J} \cdot \bar{dS}}$$

* Convection Current (Density) 8-

-A Portion of a dielectric materical



$= \Delta S = \Delta x \, \Delta z$

$= \Delta V = \Delta x \, \Delta y \, \Delta z = \Delta y \, \Delta S$

$Q = \int_v \rho_v \, dv$

$dq = \rho_v \, dv$.

$I = \frac{\Delta q}{\Delta t} = \frac{dq}{dt} = \frac{\rho_v \, dv}{dt}$

السرعة

$= \frac{\rho_v \, \Delta y \, \Delta s}{dt}$

$\frac{dy}{dt}$: drift velocity (m/s).

$\frac{dy}{dt} = u_y$.

$\rightarrow \boxed{I = \rho_v u_y \Delta S}$ I: convection current.

$J = \frac{I}{\Delta S} = \rho_v u_y$.

$\boxed{\bar{J} = \rho_v \bar{u}}$, $\bar{u} = u_y \hat{a_y}$

↳ Convection current density.

Unit for $\bar{J}$:-

$\frac{C}{m\,32} \cdot \frac{\cancel{m}}{s} = \left(\frac{A}{m^2}\right)$

* How to relate $\bar{J}$ with $\bar{E}$?

- The force on one electron :- mass

$Q = -e$, $\bar{F} = Q\bar{E} = -e\bar{E} = \overset{\downarrow}{m}\bar{a}$ → acceleration

$\bar{a} = \frac{du}{dt} = \frac{\Delta u}{\Delta t} = \frac{u}{\tau}$

τ : time between Collisions (s) → affected by Temperature

t : الزمن الي بتحتاجه الشحنة حتى
تتحرك مسافة معينة

$$\rightarrow -e\bar{E} = m\frac{\bar{u}}{\tau}$$

$$\boxed{\bar{u} = \frac{-e\tau}{m}\bar{E}}$$

at a certain Temperature.

$\frac{-e\tau}{m}$ is constant $= \mu =$ mobility $\left(\frac{C\cdot s}{kg}\right)$ Unit.

$$\bar{v}_d = \boxed{\bar{u} = \mu\bar{E}}$$

- For $\rho_v = -ne$, $n =$ # of charges per volume (#/m

n depends on the portion volume.

$$\bar{u} = \mu\bar{E}$$

$$\bar{J} = \rho_v\bar{u}$$

$$\bar{J} = -ne\mu\bar{E} = -ne\left(-\frac{e\tau}{m}\right)\bar{E}$$

$$\boxed{\bar{J} = \left(\frac{ne^2\tau}{m}\right)\bar{E}}$$

$\downarrow$

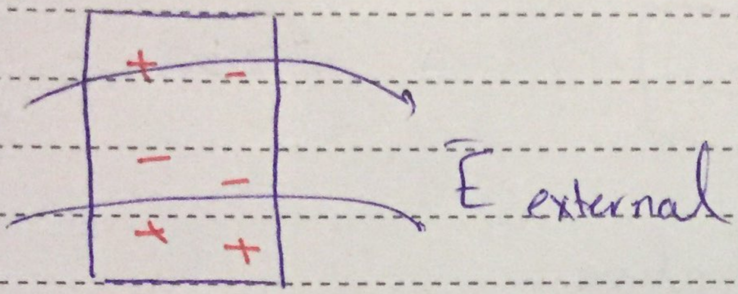
هاد كله ثابت واسمه σ

$$\boxed{\bar{J} = \sigma\bar{E}}$$

$\downarrow$

Conduction current density.

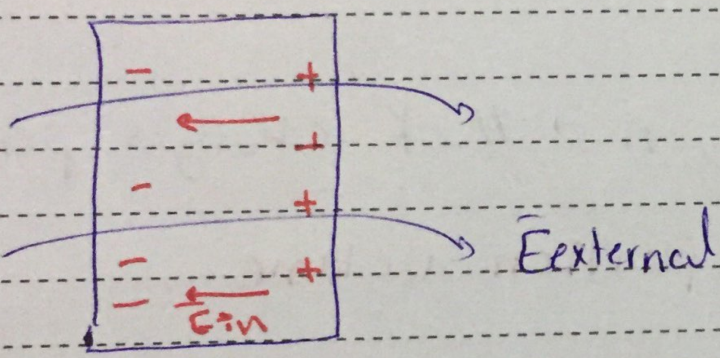
Conductors:

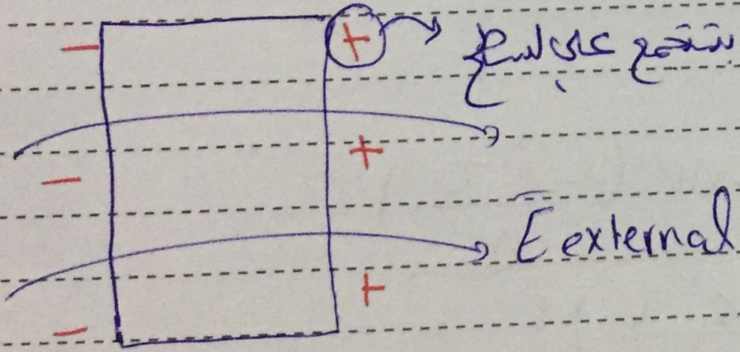


$\bar{E} = \nabla V$

$Q_{total} = 0$

$\bar{F} = Q \, \bar{E}_{external}$





الشحنات بتتجمع على السطح

inside:-

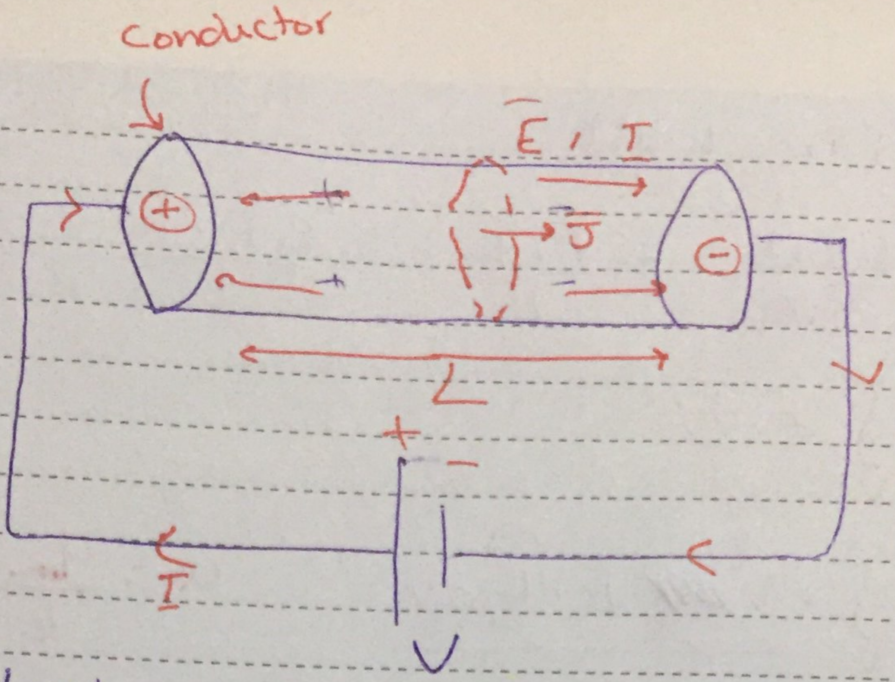
$\bar{E} = 0$

$\bar{D} = 0$

$a = 0$

$V_{diff} = 0$.

*



- Ohm's Law

$$R = \frac{V}{I} = \frac{L}{\sigma S} = \frac{\rho L}{S} \quad (\Omega) \text{ if uniform.}$$

$$\boxed{R = \frac{-\int_L \bar{E}\cdot\bar{dl}}{\int_S \sigma \bar{E}\cdot\bar{ds}}}$$

in general

↳ if uniform

↳ $R = \frac{E \cdot L}{\sigma E S} = \frac{\rho L}{S}$

*power is in watt

$$P = \frac{\text{Work}}{\text{Time}} = \frac{\bar{F} \cdot \bar{L}}{t} = \frac{Q\bar{E} \cdot \bar{L}}{t}$$

$$Q = \int_v \rho v \, dv$$

$$\rightarrow P = \frac{\int_v \rho v \, \bar{E} \cdot \bar{L} \, dv}{t}$$

$$\bar{u} = \frac{\bar{L}}{t}$$

$$P = \int_v \underline{\rho v} \, \bar{E} \cdot \underline{\bar{u}} \, dv = \int_v \underline{\bar{J}} \cdot \bar{E} \, dv$$

$$\boxed{P = \int_v \bar{J} \cdot \bar{E} \, dv}$$

or

$$\boxed{P = \sigma \int_v E^2 \, dv} \quad , \quad J = \sigma \bar{E}$$

or

$$\boxed{P = \int_v \frac{J^2}{\sigma} \, dv} \quad , \quad E = \frac{J}{\sigma}$$

- in circuit, $P = V \cdot i = \frac{V^2}{R} = i^2 R = G V^2 = \frac{i^2}{G}$

* $P = \int_{V} \bar{J} \cdot \bar{E} \, dv$ in general

- if uniform

$V = \ell s$

$$P = \int_{S}\int_{\ell} \bar{J} \cdot E \, d\ell \, ds$$

$$P = \int_{S} \bar{J} \cdot ds \int_{\ell} \bar{E} \cdot dL$$

$$\boxed{P = I V}$$

*Power Density:-

$$w_p = \frac{\text{Power}}{\text{Volume}} \quad (w/m^3)$$

$$w_p = \bar{J} \cdot E$$

$$= \sigma E^2$$

$$= \frac{J^2}{\sigma}$$

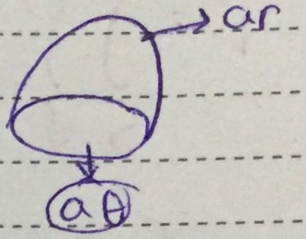
$$P = \int_{V} w_p \, dv$$

***Ex** if $\overline{J} = \frac{1}{r^3}(2\cos\theta\, \hat{a_r} + \sin\theta\, \hat{a_\theta})$ A/m²

Find the current passing through:

a) A hemi spherical shell of radius 20 cm
$0 < \theta < \frac{\pi}{2}$, $0 < \phi \leq 2\pi$

b) A spherical shell of radius 10 cm.



هاد السطح مش
موجود لأنه حاكي
عن shell يعني
مفرّغة من جوا

$\overline{ds} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{a_r}$

Sol a) $I = \int_S \overline{J} \cdot \overline{ds}$

$$= \int_0^{2\pi}\int_0^{\pi/2} \frac{1}{r} \underbrace{2\cos\theta\sin\theta}_{\sin 2\theta}\, d\theta\, d\phi \Big|_{r=0.2m}$$

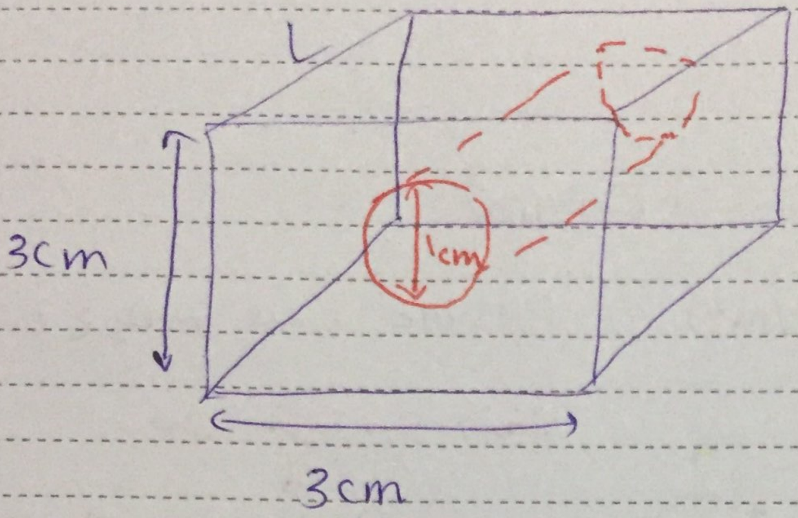
$= 10\pi$ A $= 31.4$ A

b) $I = \int_S \overline{J} \cdot \overline{ds}$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{1}{r} \sin 2\theta\, d\theta\, d\phi \Big|_{r=0.1m}$$

$= 0$ A

*Ex 3 A lead $\sigma = 5 * 10^6$ S/m , $L = 4m$, Find R?



Sol $R = \frac{L}{\sigma A}$

$A = (3 * 10^{-2})^2 - \pi (\frac{1}{2} * 10^{-2})^2$

$R = 974 \mu\Omega$.

*Polarization in Dielectrics -

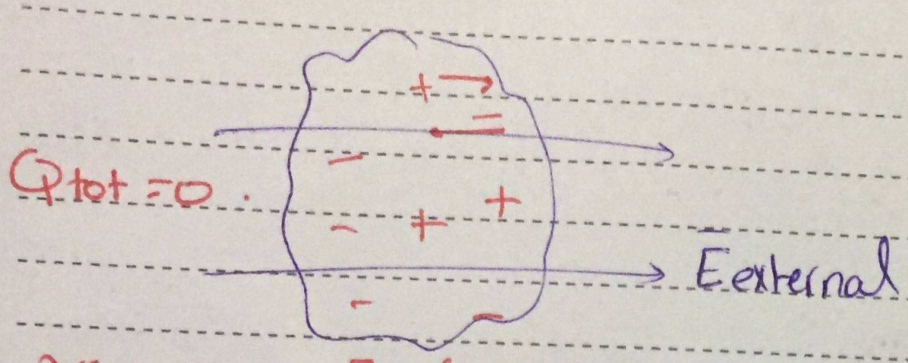
① Non-polar Dielectrics

O_2 , H_2 , N_2 , Rare gases (Kr, Xe, Ne)

② Polar Dielectrics (permenant dipoles

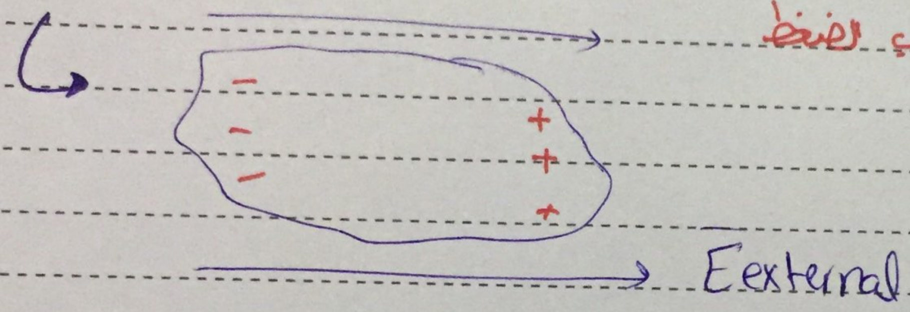
H_2O , NH_3 , HCL

- Non-Polar dielectrics



pressure = $\frac{F}{S}$ (N/m²)

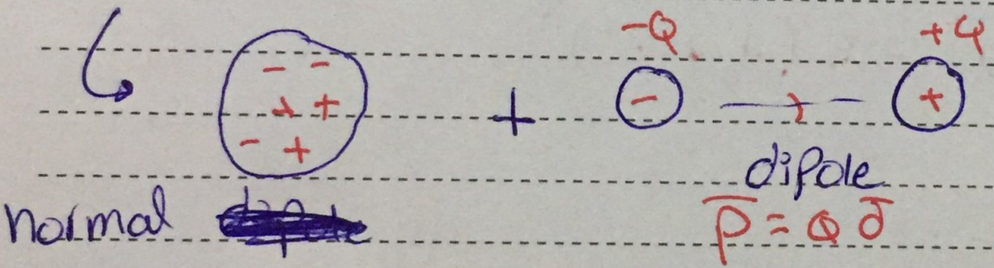
رح يصير عندي Pressure



صارت مشوهة بسبب الضغط

↓

Distorted charge distribution.



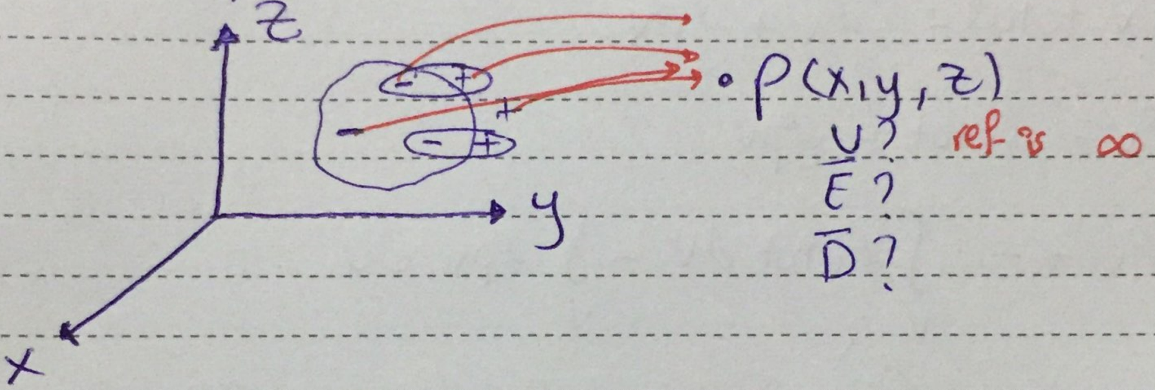
$\bar{p} = Q\,\partial$

لما أشيل $\bar{E}$ بترجع المادة طبيعية و بتبطل dipole.

ρ_s : Free ~~[illegible]~~ Surface charge ~~[illegible]~~ distribution (C/m^2)
ρ_v : free volume charge distribution (C/m^3)
ρ_{ps} : polarized (bound) surface charge dist. (C/m^2)
ρ_{pv} : polarized ~~[illegible]~~ (bound) volume charge dist. (C/m^3).

$$* \underset{\underset{(C/m^2)}{\downarrow}}{\bar{P}} = \lim_{\Delta V \to 0} \frac{\sum_{k=1}^{N} \bar{P}_K}{\Delta V} \rightarrow \text{dipole moment.}$$

* For a polarized Dielectric :-



$$- V = \int_S \frac{\rho_{ps}\, ds}{4\pi\epsilon_0 r} \qquad V = \int_V \frac{\rho_{pv}\, dv}{4\pi\epsilon_0 r}$$

هدول طلعوا نتيجة للشحنات جوا ال dielectric

$$V = \int_S \frac{\rho_s\, ds}{4\pi\epsilon_0 r} \qquad V = \int_V \frac{\rho_v\, dv}{4\pi\epsilon_0 r}$$

هدول لل free charges.

$$\bar{E} = \int_S \frac{\rho_s\, ds}{4\pi\epsilon_0 r^2}\, \hat{a}_r \qquad \bar{E} = \int_S \frac{\rho_{ps}\, ds}{4\pi\epsilon_0 r^2}\, \hat{a}_r$$

or

$$\bar{E} = -\nabla V \qquad \bar{D} = \epsilon_0 \bar{E}$$

$\rho_s = \bar{D} \cdot \hat{a_n}$

$\rho_v = \oint_S \bar{D} \cdot ds = \int_V \rho_v dv$

- Duality :-

$\rho_{ps} = \bar{P} \cdot \hat{a_n}$

$\rho_{pv} = -\nabla \cdot \bar{P}$ القانون هاد مش مطلوب

- ρ_v total = $\rho_v + \rho_{pv}$

$\rho_v = \rho_{v\,tot} - \rho_{pv}$

$\int_V \rho_v dv = \int_V \rho_{v\,tot}\, dv - \int_V \rho_{pv}\, dv$

$\oint_S \bar{D} \cdot \bar{ds} = \oint_S \underset{E\epsilon_0}{\cancel{\bar{D}}} \cdot \bar{ds} - - \oint_S \bar{P} \cdot \bar{ds}$

$\boxed{\bar{D} = \epsilon_0 \bar{E} + \bar{P}}$

- For some dielectrics

↳ linear , isotropic

$\boxed{\bar{P} = \chi_e \epsilon_0 \bar{E}}$

χ_e : electric susceptability (constant for each material)

$$\bar{D} = \epsilon_0 \bar{E} + \chi_e \epsilon_0 \bar{E}$$

$$= \epsilon_0 \bar{E} \underbrace{(1+\chi_e)}_{\epsilon_r \equiv \text{relative permitivity.}}$$

$$\bar{D} = \epsilon_0 \epsilon_r \bar{E}$$

$$\boxed{\bar{D} = \epsilon \bar{E}} \quad , \underset{\text{permitivity}}{\underset{\downarrow}{\overset{\overset{F/m}{\downarrow}}{\epsilon}}} = \overset{\overset{F/m}{\downarrow}}{\epsilon_0} \, \underset{\hookrightarrow \text{ Unitless}}{\epsilon_r}$$

in the air → $\epsilon_r = 1$

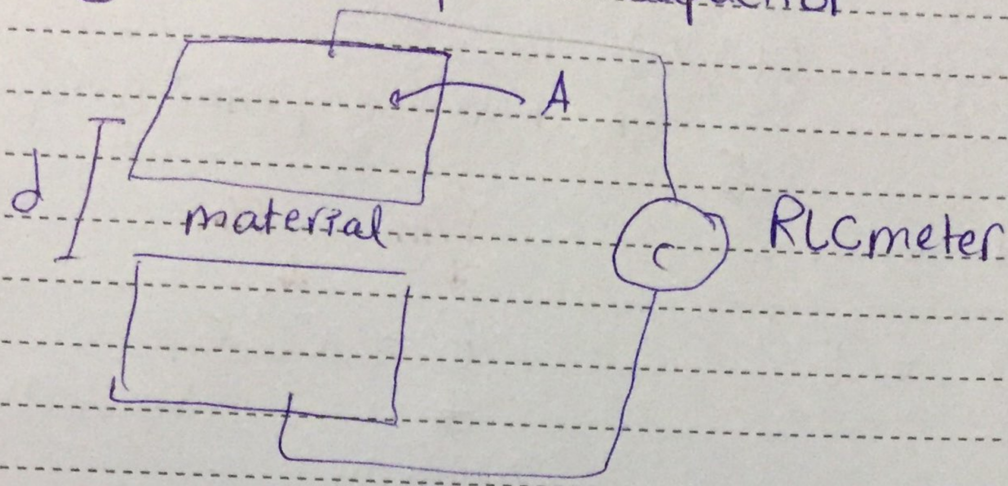
$$\boxed{\begin{aligned} \epsilon_r &= 1 + \chi_e \\ \epsilon_r &= \frac{\epsilon}{\epsilon_0} \end{aligned}}$$

* ϵ_r : Dieletric constant (Dk)

i.e: Teflon: 2.15
water: 81
FR-4: 4.4
Free space: 1
air : 1.0006 ≃ 1
Conductors : 0.999 ≃ 1

$$\boxed{\bar{P} = (\epsilon_r - 1)\, \epsilon_0 \bar{E}}$$ إذا مش معطيني χ_e بالسؤال

- How to measure ϵ_r :-
  - by using parallel plate capacitor



- Fix d = 1 cm
  material : Air → $\epsilon_r = 1$
  رح يقرأ قراءة اسمها C_o
  C_o : Capacitance of free space.

- بغير ال material لخشب / زجاج / :-
  ويقرأ قراءة اسمها Cmat.

$$\epsilon_r = \frac{C_{mat.}}{C_o}$$

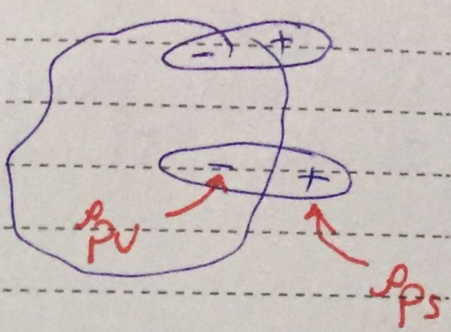
* $C = \frac{\epsilon A}{d}$ $\quad C_o = \frac{\epsilon_o A}{d}$ $\quad C_{mat} = \frac{\epsilon_o \epsilon_r A}{d}$

* DI >>> DI
Dielectric ($\epsilon_r > 1$) — air Cond. $(\bar{D} = \epsilon_o \bar{E} + 0)$

$\bar{D} = \epsilon_o \bar{E} + \bar{P}$

* Bound Charges- (Q_b) in (C)

$Q_{b+} = \int_S \rho_{ps}\, ds$



$Q_{b-} = \int_V \rho_{pv}\, dv$

$Q_{btot} = Q_{b+} + Q_{b^-} = 0$

material is electrically neutarilzed.

* Dielectric Breakdowns -

1) Nature of the material

2) Tempreture ($T\uparrow$, $\sigma\downarrow$)

3) Humidity ($H\uparrow$, $\sigma\uparrow$)

4) Applied $\bar{E}$-field.

5) Time of applied field.

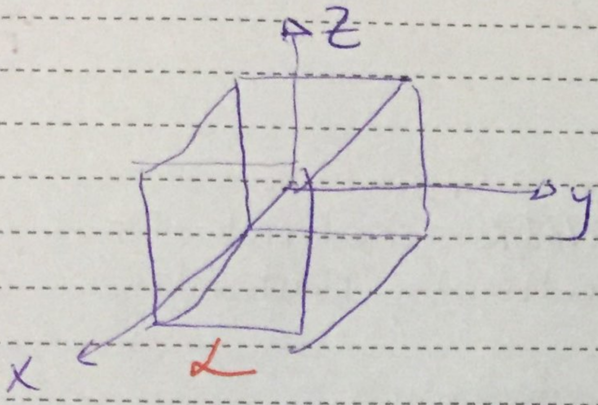
* Dielectric Strengths -

Maximum value of $\bar{E}$-field that the dielectric can tolerate without being breaked down.

-Ex A dielectric cube of length (L) centered at the origin has $\bar{P} = a\bar{r}$, a is constant

Find all bond charge densities & the total charge if $\rho_r = -3a$ C/m³

Sol



$\rho_s = \underline{P} \cdot \hat{a}_n$

$\bar{P} = -ax\,\hat{a}_x + ay\,\hat{a}_y + az\,\hat{a}_z$

$\rho_s\Big|_{front} = ax\Big|_{x=\frac{L}{2}} = \frac{aL}{2}$ C/m²

$\rho_s\Big|_{back} = -ax\Big|_{x=\frac{-L}{2}} = \frac{aL}{2}$ C/m² $(a_n = -\hat{a}_x)$

-All Surfaces have $\rho_s = \frac{aL}{2}$ C/m²

$Q_b(total) = Q_b^+ + Q_b^- = 0.$

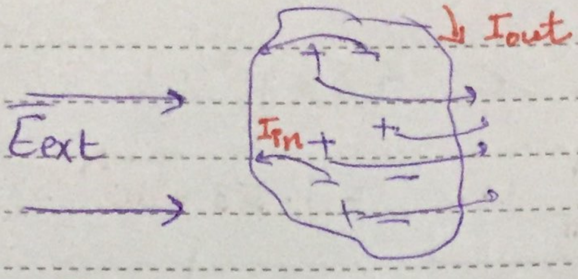
$Q_b^+ = 6\int \rho_s ds$

$\boxed{Q_b^+ = 3aL^3 \text{ C}}$

$ds = L \cdot L$

$\boxed{Q_b^- = \int \rho_{pv} dv = -3aL^3}$

*Continuity Equation and Relaxation time (T_r)



KCL: $I_{out} = I_{in}$

$$\oint_S \overline{J} \cdot \overline{ds} = \boxed{-} \frac{dq}{dt}$$

الشحنات الى طلعت هوا سالبة
~~[illegible]~~ الفكرة ع تناقص [illegible]
شحنات بتطلع على السطح

$$\boxed{\oint_S \overline{J} \cdot \overline{ds} = -\int_V \frac{d\rho v}{dt} \, dv}$$

$Q = \int_V \rho v \, dv$

Continuity Equation in integral form.

$\nabla \cdot \overline{J} = -\frac{d\rho v}{dt}$ X

*Relaxation Time (T_r).

-till ~~[illegible]~~ of charges are on the outer surface
62.8% Start conduction
or 36.8% are still inside the volume.

inside

$e^{-1} = 0.368 \rightarrow 1 \, T_r$

$e^{-2} = 0.14 \rightarrow 2 \, T_r$

$e^{-3} \simeq 0.05 \rightarrow 3 \, T_r$

$e^{-4} \simeq 0.018 \rightarrow 4 \, T_r$

$e^{-5} < 0.01 \simeq 0.006 \rightarrow 5 \, T_r$.

$(1 - e^{-1}) * 100\%$ outside

$$\boxed{* \quad T_r = \frac{\epsilon_0 \epsilon_r}{\sigma}}$$

- i.e. copper $\epsilon_r = 1$, $\sigma = 5.8 * 10^7$ S/m

$$T_r = \frac{\epsilon}{\sigma} = \frac{\frac{10^{-9}}{36\pi}(1)}{5.8 * 10^7} = 1.53 * 10^{-19} \text{ S}.$$

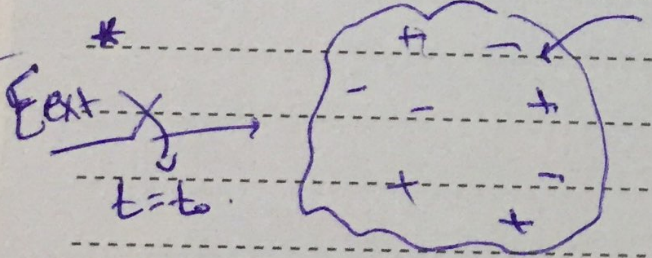
$$5 T_r \simeq 7.5 * 10^{-19}$$

- Fused Quartz

$\epsilon_r = 5$, $\sigma = 10^{-17}$ S/m

$$T_r = \frac{\epsilon}{\sigma} = \frac{\frac{10^{-9}}{36\pi}(5)}{10^{-17}} = 51.2 \text{ days}$$

*



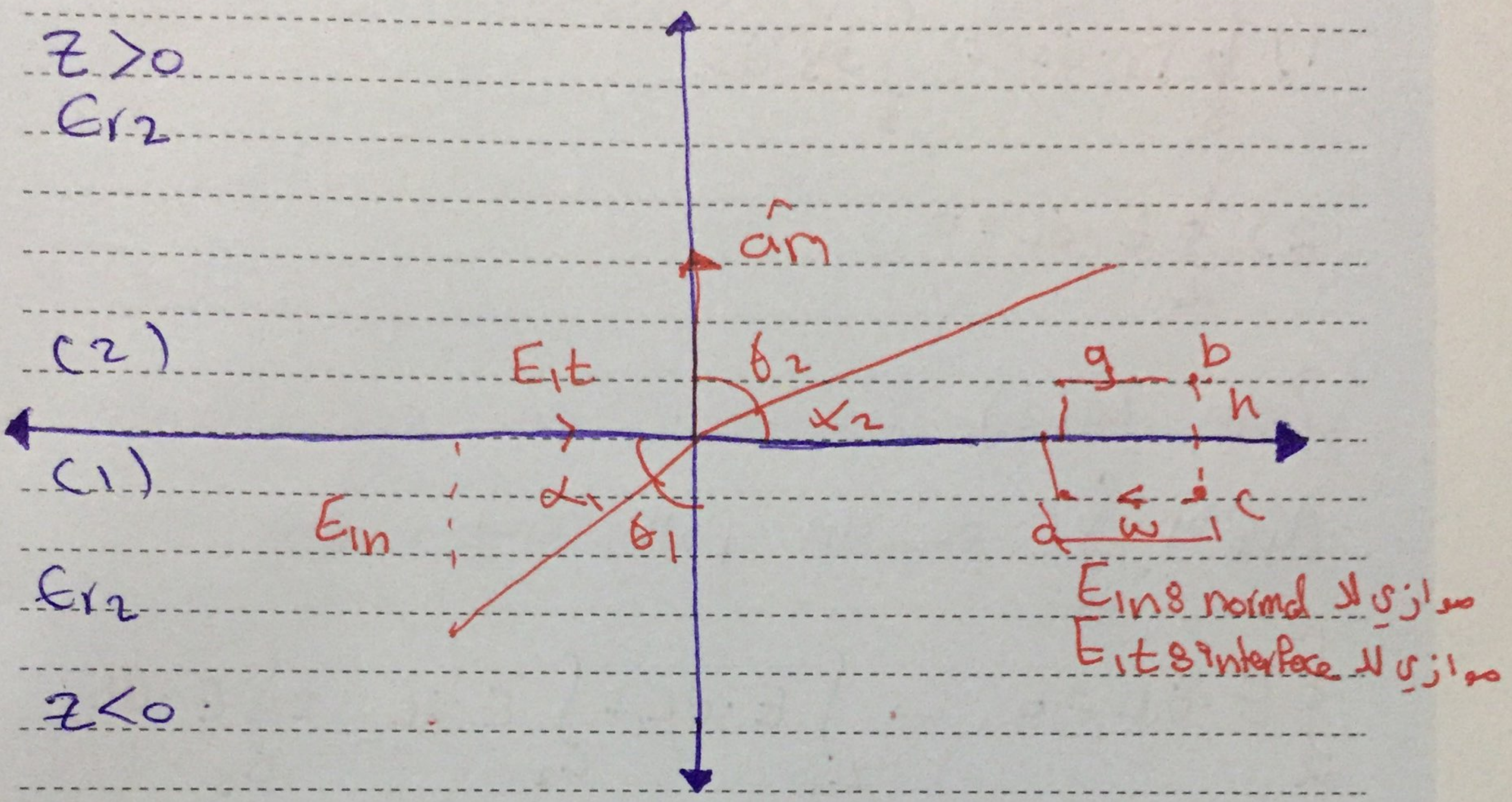
$t < t_o \rightarrow \rho_v = \rho_{vo} \rightarrow$ initial condition.

$t = t_o^+ \rightarrow \rho_v < \rho_{vo}$

$$\boxed{\rho_v = \rho_{vo} e^{-t/T_r}} \quad C/m^3$$

*Boundary Conditions.

A) Dielectric-to-Dielectric Interface.



- To find $\overline{E_2}$, $\overline{D_2}$, $\overline{P_2}$ and angles:-

Start with $\overline{E_1}$

$$\overline{E_1} = \overline{E_{1n}} + \overline{E_{1t}}$$

$$\hat{a}_n = \hat{a}_z$$

$$\overline{E_{1n}} = (\overline{E_1} \cdot \hat{a}_n)\,\hat{a}_n$$

$$\overline{E_{1t}} = \overline{E_1} - \overline{E_{1n}}$$

$\sin\theta_1 = E_1t / E_1$

$\cos\theta_1 = E_{1n} / E_1$

$\tan\theta_1 = E_{1t} / E_{1n}$

$\alpha_1 = 90° - \theta_1$

- To find $\bar{E}_t$?

Apply :-

1) $\oint_S \bar{D}\cdot d\bar{s} = \int_S \rho_s \, ds$

2) $\oint_L \bar{E}\cdot d\bar{L} = 0$

From Eq(2)

Apply $\oint_L \bar{E} \, \bar{dL}$ on path a-b-c-d-a .

$$\oint_L \bar{E}\cdot \bar{dL} = 0 \longrightarrow \int_{L_1} \bar{E}\cdot dL_1 + \int_{L_2} \bar{E}\cdot dL_2 + \int_{L_3} \bar{E}\cdot dL_3$$

$$+ \int_L \bar{E}\cdot dL_4 = 0$$

$$E_{2t} w - \cancel{E_{2n} \frac{h}{2}} + \cancel{E_{1n} \frac{h}{2}} - E_{1t} W + \cancel{E_{1n} \frac{h}{2}} + \cancel{E_{1n} \frac{h}{2}} = 0$$

$$E_{2t} w - E_{1t} w = 0$$

$\boxed{\bar{E}_{2t} = \bar{E}_{1t}}$ Always.

To find $\overline{E_{2n}}$

-Apply $\oint_S \overline{D}\cdot\overline{ds} = \int_S \rho_s \, ds$ at $z=0$.

$D_{2n} \cancel{\nabla S} + - D_{1n} \cancel{\nabla S} = \rho_s \cancel{\nabla S}$

$$\boxed{D_{2n} - D_{1n} = \rho_s}$$

$D_{1n} = \epsilon_0 \epsilon_{r1} E_{1n}$

$D_{2n} = \epsilon_0 \epsilon_{r2} E_{2n}$

$$\overline{E_{2n}} = \frac{D_{2n}}{\epsilon_0 \epsilon_{r2}} \hat{a}_n$$

$$\boxed{\overline{E_2} = \overline{E_{2t}} + \overline{E_{2n}}}$$

$$\boxed{\overline{D_2} = \epsilon_0 \epsilon_{r2} \overline{E_2}}$$

$$\boxed{\overline{P} = (\epsilon_{r2} - 1) \epsilon_0 \overline{E_2}}$$

-If $\rho_s = 0$

$\overline{D_{2n}} = \overline{D_{1n}}$

$\cancel{\epsilon_0} \epsilon_{r1} \overline{E_{1n}} = \cancel{\epsilon_0} \epsilon_{r2} \overline{E_{2n}}$

$$\boxed{\overline{E_{2n}} = \frac{\epsilon_{r1}}{\epsilon_{r2}} \overline{E_{1n}}}$$

$$\theta_2 = \sin^{-1}\left(\frac{E_{2t}}{E_2}\right)$$

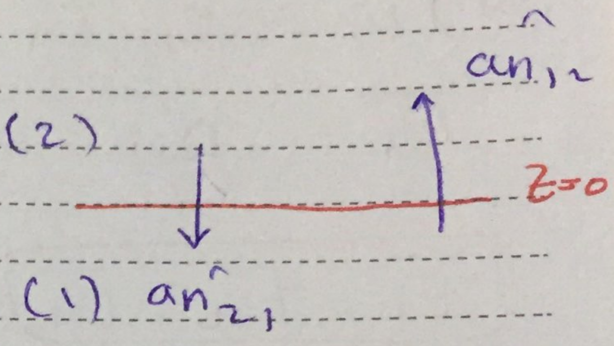
$$= \cos^{-1}\left(\frac{E_{2n}}{E_2}\right)$$

$$= \tan^{-1}\left(\frac{E_{2t}}{E_{2n}}\right)$$

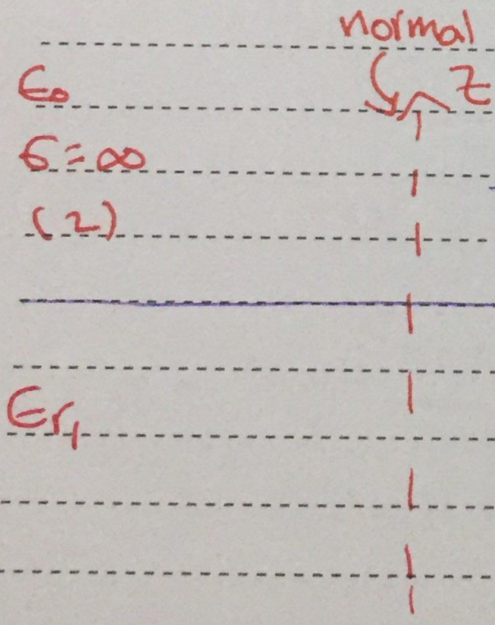
$$\alpha_2 = 90^\circ - \theta_2$$

* $\rho_{ps2} = \bar{P}_2 \cdot \hat{a}_{n21}$

$\rho_{ps1} = \bar{P}_1 \cdot \hat{a}_{n12}$



*Dielectric - to - conductor



$\bar{E}_2 = 0 \rightarrow E_{2n} = 0$, $E_{2t} = 0$

$\bar{D}_2 = 0 \rightarrow \bar{D}_{2n} = 0$, $\bar{D}_{2t} = 0$

$\rho_s \neq 0$ z=0 interface.

Air → $\epsilon = \epsilon_0$, $\sigma = 0$

Cond. → $\epsilon = \epsilon_0$, $\sigma \to \infty$.

$\bar{E}_1$?

$\bar{E}_{1t} = \bar{E}_{2t} = 0$

$\bar{E}_{2n}$?

$\cancel{\bar{D}_{2n}}^{\,0} - \bar{D}_{1n} = \rho_s$

$\boxed{\bar{D}_{1n} = -\rho_s \, \hat{a_n}}$

$$\bar{E}_{1n} = \frac{\bar{D}_{1n}}{\epsilon_0 \epsilon_{r_1}}$$

$$\boxed{\bar{E}_{1n} = \frac{+\rho_s}{\epsilon_0 \epsilon_{r_1}} (-\hat{a_z})} \leftarrow \epsilon \simeq \infty$$

- **Ex** Two homogeneous isotropic Dielectrics meet on plane $z=0$ for $z>0$, $\epsilon_{r_1} = 4$ and for $z<0$ $\epsilon_{r_2} = 3$. A uniform $\bar{E}_1 = 5\hat{a_x} - 2\hat{a_y} + 3\hat{a_z}$ kV/m exists for $z \geq 0$. Finde -

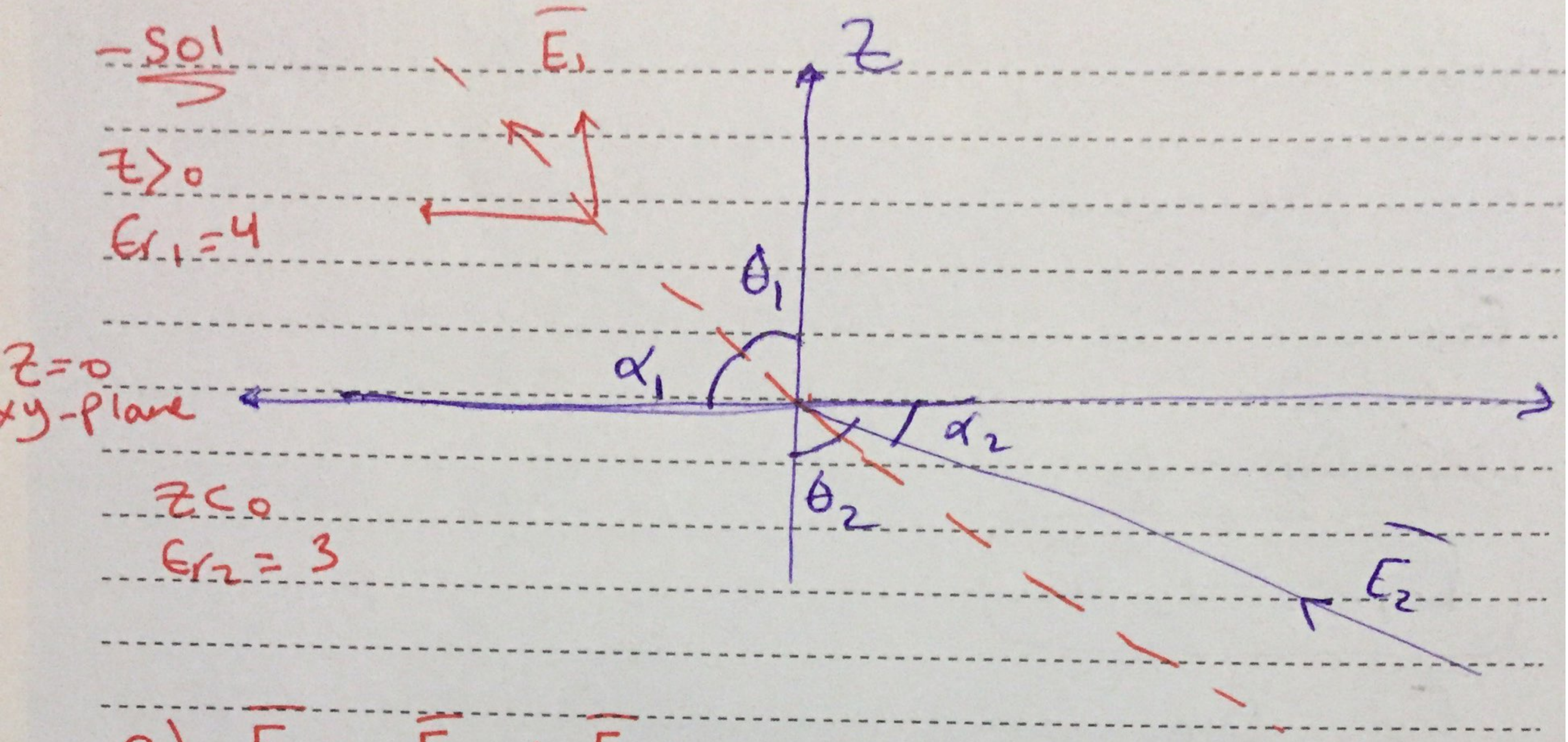
a) E_2 for $z \leq 0$

b) $\theta_1, \theta_2, \alpha_1, \alpha_2$

c) Energy density in both regions.

d) Energy in a cube of side (2m) centered at (3, 4, −5).

-Sol



a) $\bar{E}_1 = \bar{E}_{1n} + \bar{E}_{1t}$

$\hat{a_n} = \hat{a_z}$

لأن z=0 plane ، عمودي $\bar{E}$ [illegible]
$\hat{a_n} = +\hat{a_z}$ or $+3\hat{a_z}$

$\bar{E}_{1n} = (\bar{E}_1 \cdot \hat{a_n})\hat{a_n}$

$= 3\hat{a_z}$ kV/m .

$\bar{E}_{1t} = \bar{E}_1 - \bar{E}_{1n}$

$= 5\hat{a_x} - 2\hat{a_y}$ kV/m

$= \boxed{\bar{E}_{2t}}$

b) $\theta_1 = \sin^{-1}\frac{E_{1t}}{E_1} = \sin^{-1}\left(\frac{\sqrt{29}}{\sqrt{38}}\right) = \boxed{60.9^\circ}$

$\alpha_1 = 90^\circ - \theta_1 = \boxed{29.1^\circ}$

$\overline{E_{2n}} \longrightarrow \quad D_{1n} - D_{2n} = \overset{0}{\cancel{\rho_s}}$

لأنو ال field انتقل
من ٢ لـ ١

$\overline{D_{1n}} = \overline{D_{2n}}$

$\epsilon_{r1} \overline{E_{1n}} = \epsilon_{r2} \overline{E_{2n}}$

$\overline{E_{2n}} = \frac{\epsilon_{r1}}{\epsilon_{r2}} \overline{E_{1n}}$

$= \frac{4}{3} (3\, kV/m)\, \hat{a_z}$

$\overline{E_{2n}} = \cancel{4k}\ 4\hat{a_z}\ kV/m$

$\overline{E_2} = \overline{E_{2n}} + \overline{E_{2t}}$

$$\boxed{\overline{E_2} = 5\hat{a_x} - 2\hat{a_y} + 4\hat{a_z} \quad kV/m}$$

$\theta_2 = \sin^{-1}\left(\frac{E_{2t}}{E_2}\right)$

$= \cos^{-1}\left(\frac{E_{2n}}{E_2}\right)$

$= \tan^{-1}\left(\frac{E_{2t}}{E_{2n}}\right)$

$= \tan\theta_2 = \frac{\epsilon_{r2}}{\epsilon_{r1}} \tan(\theta_1)$

اشتقاق هاد لقانون موجود
بالصفحة التالية.

$= \frac{3}{4} \tan(60.9°)$

$\boxed{\theta_2 = 53.4°} \qquad \boxed{\alpha_2 = 36.6°}$

Scanned with CamScanner

To find θ_2?

$\bar{E}_{1t} = \bar{E}_{2t}$ - - - ①

$D_{2n} - D_{1n} = \rho_s = 0$ - - - ② if $\rho_s = 0$

$E_1 \sin\theta_1 = E_2 \sin\theta_2$ - - - - ⓐ

$D_{2n} = D_{1n}$

$\epsilon_0 \epsilon_{r2} E_{2n} = \epsilon_0 \epsilon_{r1} E_{1n}$

$\epsilon_{r2} E_2 \cos\theta_2 = \epsilon_{r1} E_1 \cos\theta_1$ - - - ⓑ

divide $\frac{ⓐ}{ⓑ}$

$$\frac{\cancel{E_1} \sin\theta_1}{\epsilon_{r1} \cancel{E_1} \cos\theta_1} = \frac{\cancel{E_2} \sin\theta_1}{\epsilon_{r2} \cancel{E_2} \cos\theta_2}$$

$$\frac{\tan(\theta_1)}{\epsilon_{r1}} = \frac{\tan(\theta_2)}{\epsilon_{r2}}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}}$$

بستخدمها مشان أوجد
θ_2 لما $\rho_s=0$.

c) $w_{E_1} = \frac{1}{2} \epsilon_1 E_1^2$, $\epsilon_1 = \epsilon_0 \epsilon_{r_1}$

$= \frac{1}{2} * \frac{10^{-9}}{36\pi} * 4 * 38 * 10^{6}$

$= 672 \ \mu J/m^3$ كثافة energy أعلى

$w_{E_2} = \frac{1}{2} \epsilon_2 E_2^2$

$= \frac{1}{2} * \frac{10^{-9}}{36\pi} * 3 * 45 * 10^{6}$

$= 597 \ \mu J/m^3$

d) $W_E = \frac{1}{2} \int_V \epsilon E^2 \, dv \longrightarrow W_E = \int_V w_E \, dv$.

$W_E = \int_V w_{E_2} \, dv$ المكعب موجود بالمنطقة الثانية

= [illegible]

$= \int_{z=-6}^{-4} \int_{y=3}^{5} \int_{x=2}^{4} 597 * 10^{-6} \, dx \, dy \, dz = \boxed{4.776 \ mJ}$

- if the cube is centered at $(\overset{3}{\cancel{\ }}, \overset{4}{\cancel{\ }}, 0)$.

$W_E = \int_{0}^{1} \int_{y=3}^{5} \int_{x=2}^{4} w_{E_1} dx dy dz + \int_{z=-1}^{0} \int_{y=3}^{5} \int_{x=2}^{4} w_{E_2} dx dy dz$

$= w_{E_1} * 4 + w_{E_2} * 4$

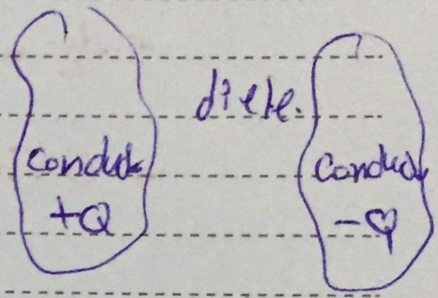
Scanned with CamScanner

CH6 Boundry Value Problems

Section 6.5 Resistance & capacitance.

$$R = \frac{V}{I} = \frac{-\int_{L} \bar{E} \cdot \bar{\partial L}}{\int_{S} \sigma \bar{E} \cdot \bar{\partial s}}$$

$$C = \frac{Q}{V} = \frac{\int_{S} \epsilon \bar{E} \cdot \bar{\partial s}}{-\int_{L} \bar{E} \cdot \bar{\partial L}}$$



* Procedure to find (C) and (R)

1) Choose a suitable coordinate

2) Assume $\begin{smallmatrix}(+)\\(-)\end{smallmatrix}$Q on each plate.

3) Find $\bar{E}$ from $\oint_{S} \epsilon \bar{E} \cdot \bar{\partial s} = Q$ (Gauss Law)

4) Find $V = -\int_{L} \bar{E} \cdot \bar{\partial L}$

$C = \frac{Q}{V}$ $\begin{smallmatrix}\text{(Step 2)}\\\text{(Step 4)}\end{smallmatrix}$

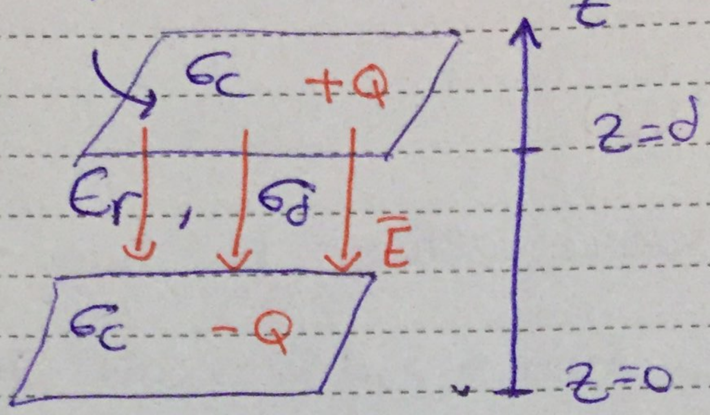
5) $I = \int_{S} \sigma \bar{E} \cdot \bar{\partial s}$

$\boxed{R} = \frac{V \rightarrow \text{(Step 4)}}{I \rightarrow \text{(Step 5)}}$

Dielectric resistance
Leakage resistance.

* Types of capacitors :-

1) Parallel plate capacitor



Find C? Rd?

Sol

Cartesian coordinate

بنشوف شو ال coordinate المناسب
ونفرض +Q و -Q و يكون
اتجاه Ē من موجب للسالب

$$\bar{E} = \bar{E}_+ + \bar{E}_-$$

$$= \frac{\rho_s}{2\epsilon_0}(-\hat{a}_z) + \frac{-\rho_s}{2\epsilon_0}(\hat{a}_z)$$

$$= \frac{-\rho_s}{\epsilon}\hat{a}_z$$

$$Q = \int_S \rho_s \, ds$$

$$\rho_s = \frac{Q}{S}$$

$$\bar{E} = \frac{Q}{S\epsilon}(-\hat{a}_z) \text{ V/m}$$

$$V = -\int_L \bar{E}\cdot \bar{dL} \quad , \quad \bar{dL} = dz\,\hat{a}_z$$

Higher potential (+ve) ⓓ

$$V = + \int_{0}^{d} \frac{Q}{S\epsilon} (+\cancel{\bar{a}_z}) \cdot dz\, \cancel{\bar{a}_z}$$

Lower ⓞ
potential (-ve)

ولما حدود التكامل
من الأقل للأعلى

$$V = \frac{Qd}{S\epsilon} \ (V)$$

↓
potential difference between the two plates.

* لو فرضت +Q لتحت و -Q لفوق لازم أعكس حدود التكامل
عشان ما يضل عندي سالب

$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{S\epsilon}} = \boxed{\frac{\epsilon S}{d}} \ (F)$$

↓
if ϵ_r is homogenous
(يعني تكون ϵ_r رقم)

- if ϵ_r is non homogenous (function)
Then apply the Procedure.
(لازم نشتق القانون).

$$I = \int_S \sigma \bar{E} \cdot \bar{ds}$$

$$I = \iint \sigma \frac{Q}{S\epsilon} \cancel{-\bar{a}_z} \cdot dx\,dy \cancel{-\bar{a}_z}$$

$$\iint dx\,dy = S$$

$$I = \frac{\sigma Q \cancel{S}}{\cancel{S}\epsilon} \rightarrow \boxed{I = \frac{\sigma Q}{\epsilon}} \ (A)$$

$$R = \frac{V}{I} = \frac{\frac{\cancel{Q} d}{S \cancel{\epsilon}}}{\frac{\sigma_d \cancel{Q}}{\cancel{\epsilon}}} \rightarrow \boxed{R = \frac{d}{\sigma_d S}} = \rho \frac{L}{A}$$

σ_d is homogenous.

$d = L$

$S = A$

$\frac{1}{\sigma_d} = \rho$

$* \; T_r = \frac{\epsilon}{\sigma_d} = R_d C$

$$R_d * C = \frac{\cancel{d}}{\sigma_d S} * \frac{\epsilon S}{\cancel{d}} = \frac{\epsilon}{\sigma_d}.$$

* Energy in capacitor.

$$W_E = \frac{1}{2} \int_V \epsilon E^2 dV \overset{?}{=} \frac{1}{2} C V^2$$

. (اثبت)

$$W_E = \frac{1}{2} \cancel{\epsilon} \frac{Q^2}{S^{\cancel{2}} \epsilon^{\cancel{2}}} \cancel{S} d$$

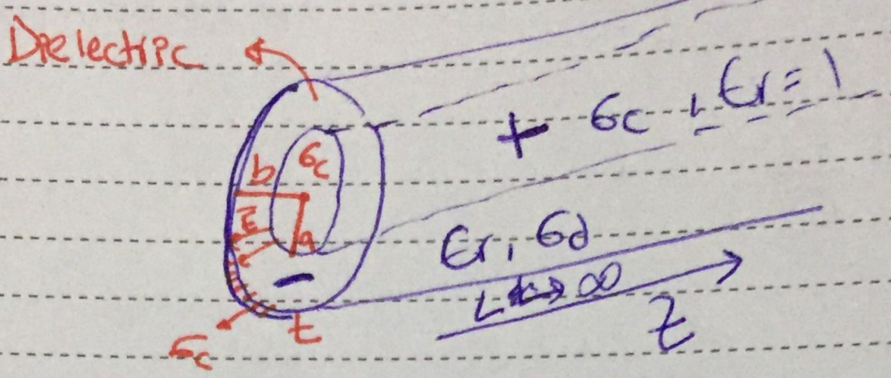
$$= \frac{Q^2 d}{2 \epsilon S} \quad \frac{1}{C}$$

$$= \frac{Q^2}{2C} \quad , \quad C = \frac{Q}{V} \rightarrow Q = CV$$

$$= \frac{C^{\cancel{2}} V^2}{2 \cancel{C}} = \frac{1}{2} C V^2 \; \#$$

if uniform.

2) Cylindrical capacitor (coaxial capacitor)
(coaxial cable)



Cylindrial coordinates.

$$\bar{E} = E_\rho \hat{a}_\rho$$

$$ds = \rho \, d\phi \, dz \, \hat{a}_\rho$$

$$\int_S \epsilon \bar{E} \cdot \bar{ds} = Q$$

$$\int_{z=0}^{L} \int_{\phi=0}^{2\pi} \epsilon \, E_\rho \cancel{\hat{a}_\rho} \cdot \rho \, dz \, d\phi \, \cancel{\hat{a}_\rho}$$

$$= \epsilon E_\rho \underbrace{2\pi \rho L}_{Area} = Q$$

$$\boxed{\bar{E} = \frac{Q}{2\pi \epsilon \rho L} \hat{a}_\rho} \quad V/m$$

$$V = -\int_{L} \bar{E}\cdot\bar{dL}$$

$$= -\int_{b}^{a} \frac{Q}{2\pi\epsilon L \rho}\, \cancel{\hat{a}_\rho} \cdot d\rho\, \cancel{\hat{a}_\rho}$$

$$V = \frac{Q}{2\pi\epsilon L} \ln\left(\frac{b}{a}\right) \quad (V)$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q\ln\left(\frac{b}{a}\right)}{2\pi\epsilon L}} = \frac{2\pi\epsilon L}{\ln\left(\frac{b}{a}\right)} \quad (F)$$

$$\boxed{\frac{C}{L} = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)} \quad F/m}$$

if ϵ_r is homo.

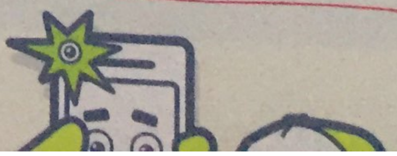
$$R_d = \frac{V}{I} \checkmark$$

$$I = \int_{S} \sigma_d \bar{E}\cdot\bar{ds} = \int_0^L\int_0^{2\pi} \frac{\sigma_d Q}{2\pi\epsilon \cancel{\rho}}\, \cancel{\hat{a}_\rho} \cdot \cancel{\rho}\, d\phi\, dz\, \cancel{\hat{a}_\rho}$$

$$\boxed{I = \frac{\sigma_d Q}{\epsilon}} \quad (A)$$

$$R_d = \frac{\frac{Q}{2\pi\epsilon L}\ln\left(\frac{b}{a}\right)}{\frac{\sigma_d Q}{\epsilon}} = \boxed{\frac{\ln(b/a)}{2\pi\sigma_d L}} \quad \Omega$$

$$R_d C = T_r = \frac{\epsilon}{\sigma_d}$$



$$* \; G = \frac{1}{R_d} = \frac{2\pi\sigma_d \ell}{\ln(b/a)} \; (S)$$

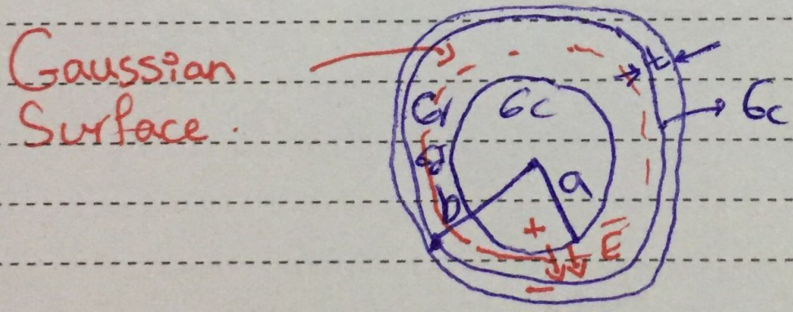
$$G/\ell = \frac{2\pi\sigma_d}{\ln(b/a)} \; (S/m) \quad (\text{Like } \sigma)$$

$$R_d \cdot \ell = \frac{\ln(b/a)}{2\pi\sigma_d} \; (\Omega \cdot m) \quad (\text{Like } \rho)$$

$$W_E = \frac{1}{2}\int_V \epsilon E^2 \, dv = \frac{1}{2}\boxed{C}V^2$$

$$C = \frac{2\pi\epsilon\ell}{\ln(b/a)}$$

3) Spherical Capacitor



- Spherical coordinates.

$$\oint_S \epsilon \bar{E} \cdot \bar{ds} = Q$$

$$\bar{E} = E_r \hat{a_r}$$

$$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$$

$\rightarrow 4\pi r^2 \epsilon E_r = Q$

$$\boxed{\bar{E} = \frac{Q}{4\pi\epsilon r^2}\,\hat{a_r}}$$

طلع معي لقانون زي قانون point charge
لأنه كل لشحنات بتجمع على لسطح
يعني point charge

$$V = -\int_{L} \bar{E}\cdot\bar{dL}$$

$$V = \frac{-Q}{4\pi\epsilon}\int_{b}^{a} \frac{1}{r^2}\,\cancel{\bar{a_r}}\cdot dr\,\cancel{\bar{a_r}}$$

$$\boxed{V = \frac{Q}{4\pi\epsilon}\left(\frac{1}{a}-\frac{1}{b}\right) \quad (V)}$$

$$C = \frac{Q}{V} = \boxed{\frac{4\pi\epsilon}{\left(\frac{1}{a}-\frac{1}{b}\right)}} \quad (F)$$

$$R_d = \frac{V}{I}$$

$$I = \int_{S} \sigma_d\,\bar{E}\cdot\bar{ds} = \int_{0}^{2\pi}\int_{0}^{\pi} \frac{\sigma_d Q}{4\pi\epsilon r^2}\,\cancel{\bar{a_r}}\cdot r^2 \sin\theta\, d\theta\, d\phi\, \cancel{\bar{a_r}}$$

$$\boxed{I = \frac{\sigma_d Q}{\epsilon}} \quad A$$

$$R_d = \frac{V}{I} = \boxed{\frac{\frac{1}{a}-\frac{1}{b}}{4\pi\sigma_d}} \quad \Omega$$

$$\boxed{G = \frac{1}{R_d} = \frac{4\pi\sigma_d}{\frac{1}{a}-\frac{1}{b}}}$$

$$R_d C = \frac{\epsilon}{\sigma_d} = T_r$$

$$W_E = \frac{1}{2}\, \underset{\frac{4\pi\epsilon}{\frac{1}{a}-\frac{1}{b}}}{\underset{\downarrow}{\boxed{C}}}\, V^2 \quad (J)$$

4) Isolated capacitor (Isolated sphere)

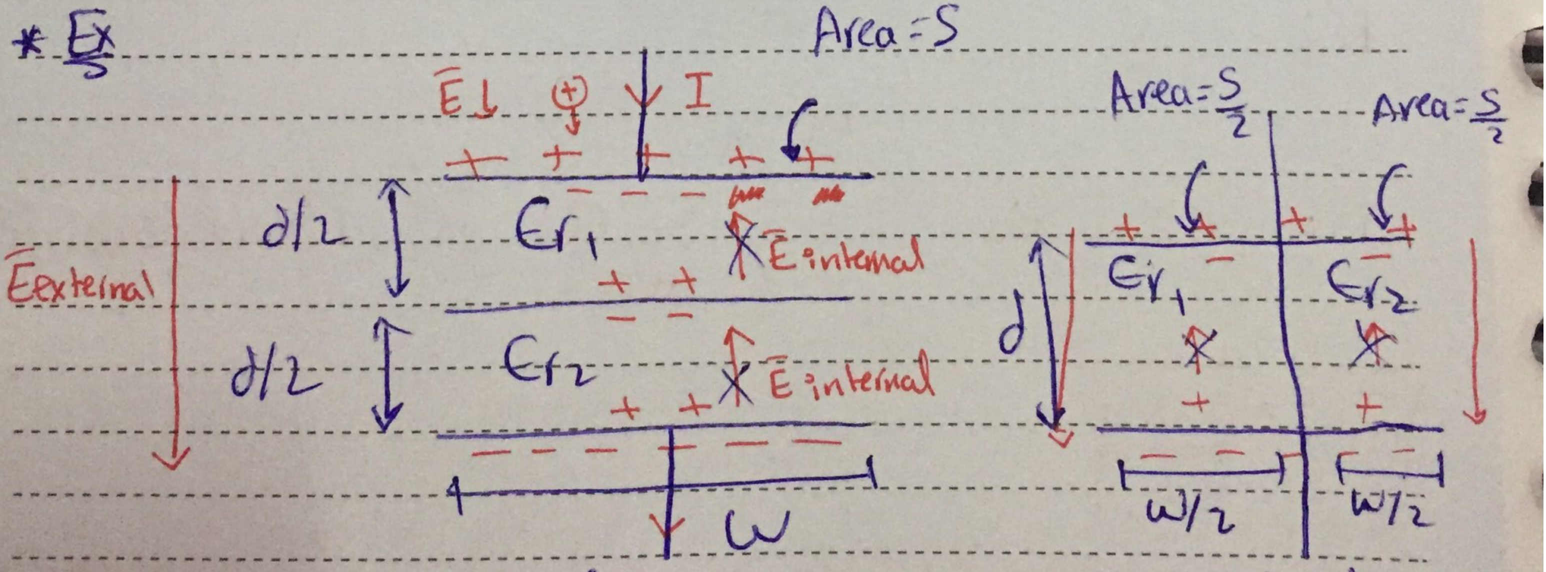
$b \rightarrow \infty$

$C = 4\pi\epsilon a \quad (F)$

$G = 4\pi\sigma_d\, a \quad (S)$

$R_d = \frac{1}{4\pi\sigma_d a} \quad (\Omega)$

* Ex 5

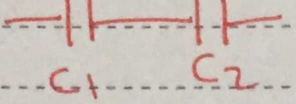


(a) Series

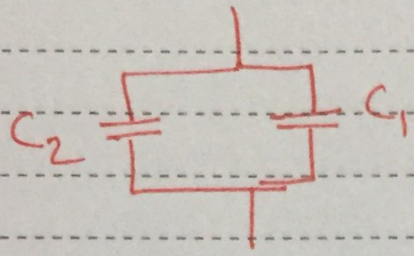
(b) Parallel

Sol

- Recall

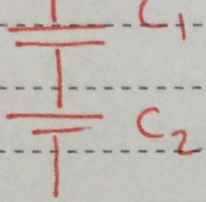


$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$



$C_{eq} = C_1 + C_2$

a)

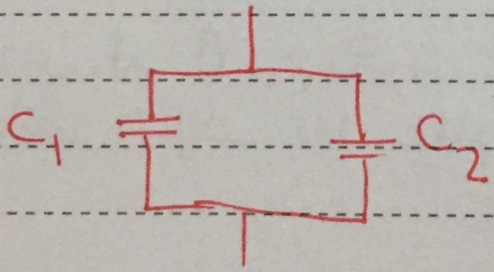


$C_1 = \frac{\epsilon_0 \epsilon_{r_1} S}{d/2}$

$C_2 = \frac{\epsilon_0 \epsilon_{r_2} S}{d/2}$

$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$

b)

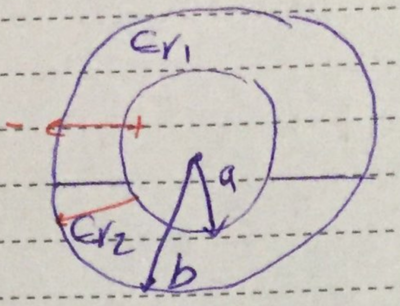


$C_1 = \frac{\epsilon_0 \epsilon_{r_1} S/2}{d}$

$C_2 = \frac{\epsilon_0 \epsilon_{r_2} S/2}{d}$

$C_{eq} = C_1 + C_2$

- Ex



Sol Parallel

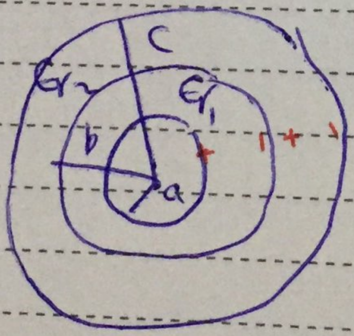
$C_{eq} = C_1 + C_2$

$$C_1 = \frac{4\pi \epsilon_0 \epsilon_{r_1}}{\frac{1}{a} - \frac{1}{b}} \cdot \boxed{\frac{1}{2}}$$ لأنه نصف كرة

$$C_2 = \frac{4\pi \epsilon_0 \epsilon_{r_2}}{\frac{1}{a} - \frac{1}{b}} \cdot \frac{1}{2}$$

التكامل لل θ رح يكون من 0 ـــ $\frac{\pi}{2}$ ، ومن $\frac{\pi}{2}$ لـ π
مشان هيك رح يطلع عندي ضرب $\frac{1}{2}$

- Ex



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_1 = \frac{4\pi \epsilon_0 \epsilon_{r_1}}{\frac{1}{a} - \frac{1}{b}} \ (F) \ , \ C_2 = \frac{4\pi \epsilon_0 \epsilon_{r_2}}{\frac{1}{b} - \frac{1}{c}} \ (F)$$

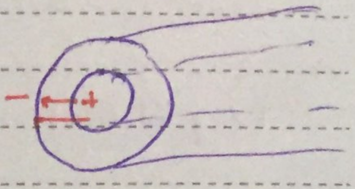
- Ex A Coaxial cable has $\epsilon_r = \frac{10+\rho}{\rho}$

where (ρ) is in cm

if $a = 1\,cm$

$b = 2.5\,cm$

find C/ℓ ??



Sol

$$\oint_S \epsilon \bar{E}\cdot \bar{ds} = Q$$

$$\int_0^{\ell}\int_0^{2\pi} \epsilon E_\rho \cancel{a_\rho} \cdot \rho\, d\phi\, dz\, \cancel{a_\rho} = Q$$

$$\bar{E} = \frac{Q}{2\pi\epsilon\rho\ell}\,\hat{a}_\rho$$

$$V = -\int_{\ell} \bar{E}\cdot \bar{dL}$$

$$= -\frac{Q}{2\pi\epsilon_0\ell}\int \frac{1}{\frac{10+\rho}{\cancel{\rho}}\cancel{\rho}}\,\cancel{a_\rho}\cdot d\rho\, \cancel{a_\rho}$$

$$= \frac{-Q}{2\pi\epsilon_0\ell}\left(\ln(10+\rho)\right)\Big|_b^a$$

$$= \frac{Q}{2\pi\epsilon_0\ell}\ln\left(\frac{10+b}{10+a}\right) \quad (V)$$

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_0\ell}{\ln\left(\frac{10+b}{10+a}\right)}$$

$$C/\ell = \frac{2\pi\epsilon_0}{\ln\left(\frac{12.5}{11}\right)} = \boxed{434.6\ PF/m}$$

CH 7 Magnetostatic Fields

- Sources :-

1) permenant Magnet

2) charge moving with uniform velocity (zero accelaration)

3) DC Current flowing in the wire

* Major laws :-

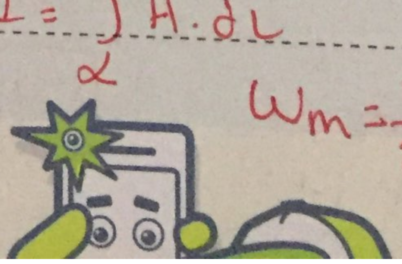
1) Biot - savart's Law (General)

2) Ampere's Law (Special case)

* Duality between Electrostatic and Magnetostatic

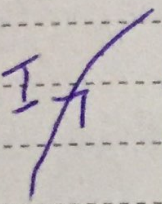
Elec.	Mag.
$\overline{E}$ (V/m)	$\overline{H}$ (A/m)
ectric flux ensity $\overline{D}$ (C/m^2)	$\overline{B}$ (wb/m^2) magnetic flux densit
$\psi_e = \int_s \overline{D} \cdot \overline{ds}$	$\psi_m = \int_s \overline{B} \cdot \overline{ds}$
ϵ	μ
$V = -\int_L \overline{E} \cdot \overline{dL}$	$I = \int_L \overline{H} \cdot \overline{dL}$

$W_E = \frac{1}{2} \int_v \overline{E} \cdot \overline{D} \, dv$ $\qquad$ $W_m = \frac{1}{2} \int_v \overline{H} \cdot \overline{B} \, dv$

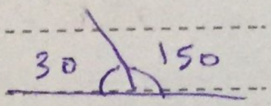


*Biot _ Savart's Law

- Orstead exp.

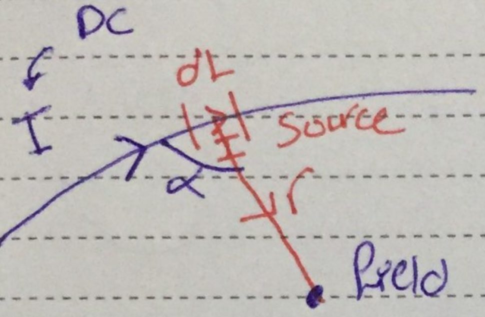


Magnetic field around the wire



$\sin(30) = \sin(150)$

*



$\bar{H}$: Magnetic field intensity

- by exp:-

$dH \propto \frac{I\,dL}{r^2} \sin(\alpha)$

α: is the angle between the conductor (dL) and the distance (r).

$$\boxed{dH = \frac{K\, I\, dL \sin(\alpha)}{r^2}}$$

K = proportionality constant $= \frac{1}{4\pi}$

لـ ما بتأثر بالوسط الي حوله

$$\boxed{H = \int_{L} \frac{I \sin\alpha}{4\pi r^2}\, dL}$$ Magnitude only.

* Direction of H :-

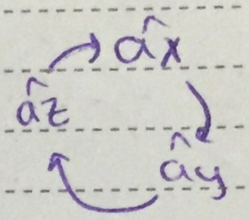
$$\boxed{\bar{H} = \int_L \frac{I \ \bar{dL} \times \hat{a}_r}{4\pi r^2}}$$ (A/m)

For Line current

$|\bar{dL} \times \hat{a}_r| =$
$dL \, (1) \, \sin\alpha$

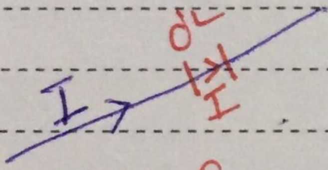
$\bar{dL} \times \hat{a}_r \perp \bar{dL}$

$\bar{dL} \times \hat{a}_r \perp \hat{a}_r$



* Types of current distributions:-

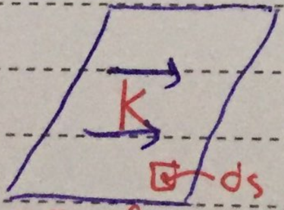
1) Line Current → 1D segment



$I\bar{dL}$ = Line current (A·m)

$$\bar{H} = \int_L \frac{I \bar{dL} \times \hat{a}_r}{4\pi r^2} = \boxed{\int_L \frac{I \bar{dL} \times \bar{r}}{4\pi r^3}}$$

2) Surface Current → 2D Surface.

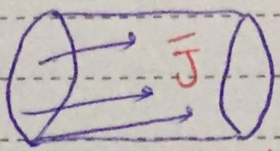


$\bar{K}ds$ = Surface current (A·m)

K: Surface current density (A/m)

$$\boxed{\bar{H} = \int_S \frac{\bar{K} \ ds \times \bar{r}}{4\pi r^3}} = \int_S \frac{\bar{K} \times \bar{r}}{4\pi r^3} \, ds$$

3) Volume Current distribution → 3D object



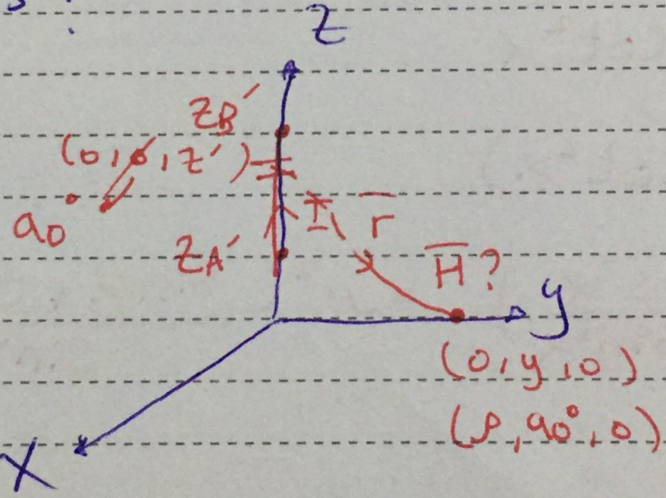
$\overline{J}$: Volume current density . (A/m²)

$\overline{J}\,dv$ = Volume current distribution . (A.m)

$$\boxed{\overline{H} = \int_V \frac{\overline{J}\times\overline{r}}{4\pi r^3}\,dv}$$

Ex Find $\overline{H}$ due to a Finite Straight wire along z-axis ?

Sol



$\overline{r} = (\rho, 90°, 0) - (0, 90°, z') = \rho\,\hat{a}_\rho - z'\,a_z$

$r = \sqrt{\rho^2 + z'^2}$ $\qquad$ $\overline{dL} = dz'\,\hat{a_z}$

$$\overline{H} = \int_L \frac{I\,\overline{dL}\times\overline{r}}{4\pi r^3} = \frac{I}{4\pi}\int_{z_A'}^{z_B'} \frac{\cancel{d}\,\hat{a}_z(\rho\,\hat{a}_\rho - z'\,\hat{a}_z)\,dz'}{(\rho^2+z'^2)^{3/2}}$$

$$\overline{H} = \frac{I\rho}{4\pi}\int_{z_A'}^{z_B'} \frac{\hat{a}_\phi}{[\rho^2+z^2]^{3/2}}\, dz'$$

$$\sin\alpha = \frac{\rho}{r}$$

$$\cos\alpha = \frac{z'}{r}$$

$$\tan\alpha = \frac{\rho}{z'} \rightarrow z' = \frac{\rho}{\tan\alpha} = \rho\cot\alpha$$

$$dz' = \rho\csc^2\alpha\, d\alpha$$

$$r^2 = \rho^2 + z'^2$$

$$= \rho^2 + \rho^2\cot^2\alpha$$

$$= \rho^2(1+\cot^2\alpha)$$

$$= \rho^2\left(1+\frac{\cos^2\alpha}{\sin^2\alpha}\right) = \rho^2\left(\frac{\overset{1}{\cancel{\sin^2\alpha+\cos^2\alpha}}}{\sin^2\alpha}\right)$$

$$r^2 = \rho^2\csc^2\alpha$$

$$\boxed{r^3 = \rho^3\csc^3\alpha}$$

sub. in integral

$$\overline{H} = \frac{I\rho}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{-\rho\csc^2\alpha}{\rho^3 cs^3\alpha}\, d\alpha\ \hat{a}_\phi$$

$$\overline{H} = \frac{I}{4\pi\rho}\int_{\alpha_1}^{\alpha_2} -\sin\alpha\, d\alpha\ \hat{a}_\phi$$

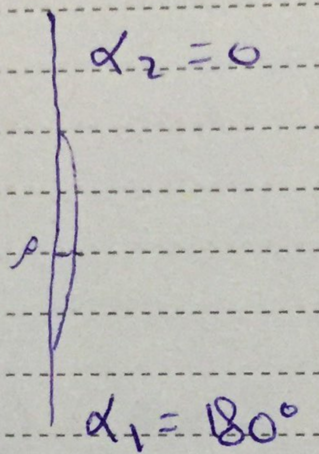
الزاوية الخارجية الزاوية الداخلية

$$\boxed{\bar{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\hat{a}_\phi}$$ A/m

$$\boxed{\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho}$$

for finite straight wire

- for infinite Line

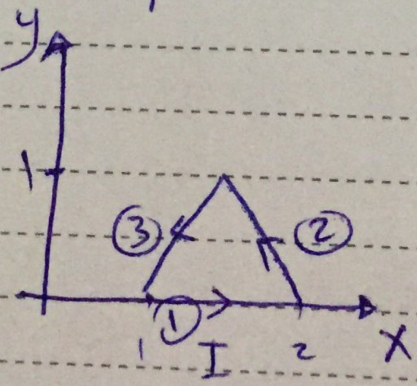


$$\boxed{\bar{H} = \frac{I}{2\pi\rho}\,\hat{a}_\phi}$$

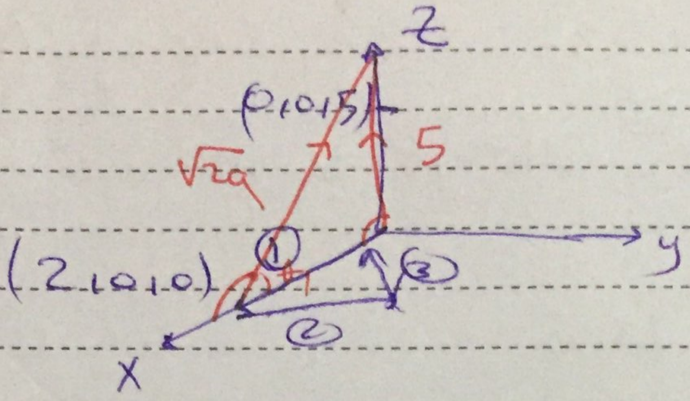
$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho$

for infinite line

Ex ★ The conducting Triangular loop carries a current of 10 A. Find $\bar{H}$ at (0,0,5) due to Side (1) of the loop.



Sol/



$$\bar{H} = \frac{I}{4\pi\rho}\left(\cos\alpha_2 - \cos\alpha_1\right)\hat{a}_\phi$$

$$\cos\alpha_1 = \frac{-2}{\sqrt{29}}$$

$\alpha_2 = 90°$ الزاوية بين الـ X-axis و Z-axis

$$\cos\theta_1 = \frac{2}{\sqrt{29}}$$

$\cos\alpha_1 = -\cos\theta_1$ (only if $\alpha_1 + \theta_1 = 180°$)

$\rho = 5$, $\bar{\rho} = 5\hat{a}_z$

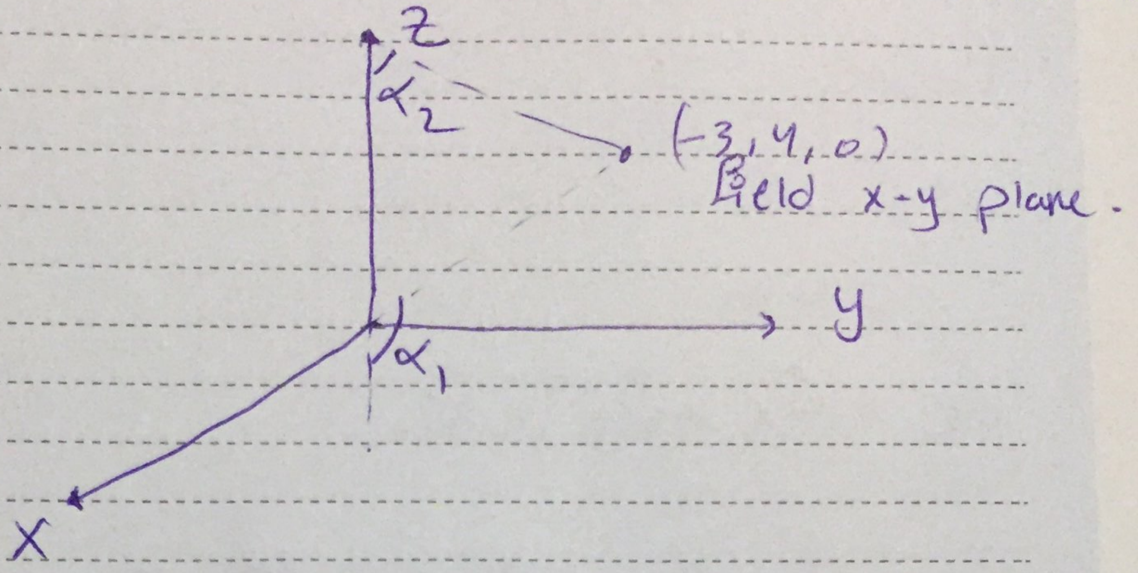
$$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho$$

$$= \hat{a}_x \times \hat{a}_z = -\hat{a}_y$$

$$\rightarrow \bar{H} = \frac{10}{4\pi(5)}\left(\cos 90 + \frac{2}{\sqrt{29}}\right)(-\hat{a}_y)$$

$$\boxed{\bar{H} = -59.1\,\hat{a}_y \text{ mA/m}}$$

Ex Find $\bar{H}$ at (-3,4,0) due to a current filament shown:



Sol $\bar{H} = \bar{H}_1 + \bar{H}_2$

$\alpha_2 = 0$ Since the line extends to ∞

$\alpha_1 = 90°$

$\bar{\rho} = (-3,4,0)$

$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho$

$= -\hat{a}_z \times \frac{\bar{\rho}}{\rho}$

$= -\hat{a}_z \times (\frac{-3}{5}\hat{a}_x + \frac{4}{5}\hat{a}_y)$

$\hat{a}_\phi = \frac{3}{5}\hat{a}_y + \frac{4}{5}\hat{a}_x$.

$\bar{H}_1 = \frac{I}{4\pi\rho}\hat{a}_\phi = 38.2\hat{a}_x + 28.65\hat{a}_y$ mA/m

For line ②

$\cos\alpha_1 = \frac{-3}{5}$

$\alpha_2 = 0$

$\rho = 4\,m$

$\hat{a}_\phi = \hat{a}_x \times \hat{a}_y = \hat{a}_z$

$\overline{H}_2 = \frac{I}{4\pi\rho}\left(1 + \frac{3}{5}\right)\hat{a}_\phi \quad \# \quad \frac{I}{4\pi\rho}\,\hat{a}_\phi$

$\overline{H}_2 = 23.88\,\hat{a}_z$ mA/m

$\overline{H} = \overline{H}_1 + \overline{H}_2$

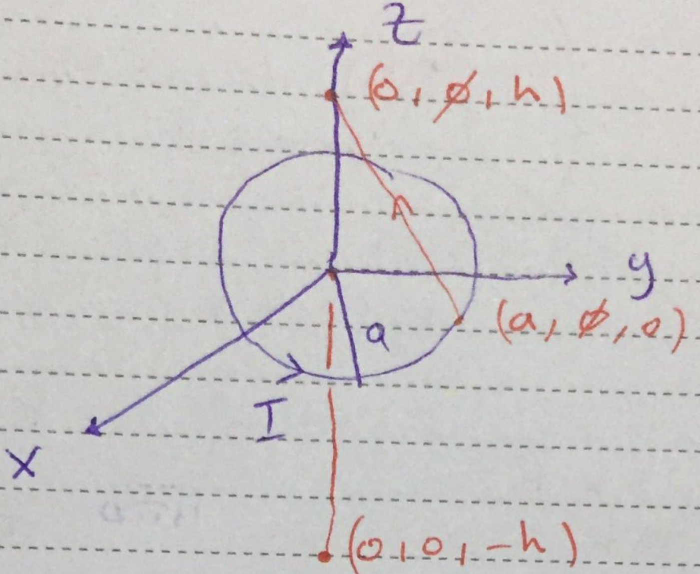
$= 38.2\,\hat{a}_x + 28.65\,\hat{a}_y + 23.88\,\hat{a}_z$ mA/m

(Carto)

in cyli

$\overline{H} = -47.75\,\hat{a}_\phi + 23.88\,\hat{a}_z$

Ex Find $\bar{H}$ at $(0,0,h)$ and $(0,0,-h)$ due to current ~~S~~ element shown if $I = 5A$.



Sol $\bar{H} = \int_L \frac{I\,\bar{dL} \times \bar{R}}{4\pi R3}$

$\bar{dL} = \rho\, d\phi\, \hat{a}_\phi \Big|_{\rho = a}$

$\bar{R} = -a\,\hat{a}_\rho + h\,\hat{a}_z$

$|R| = \sqrt{a^2+h^2}$

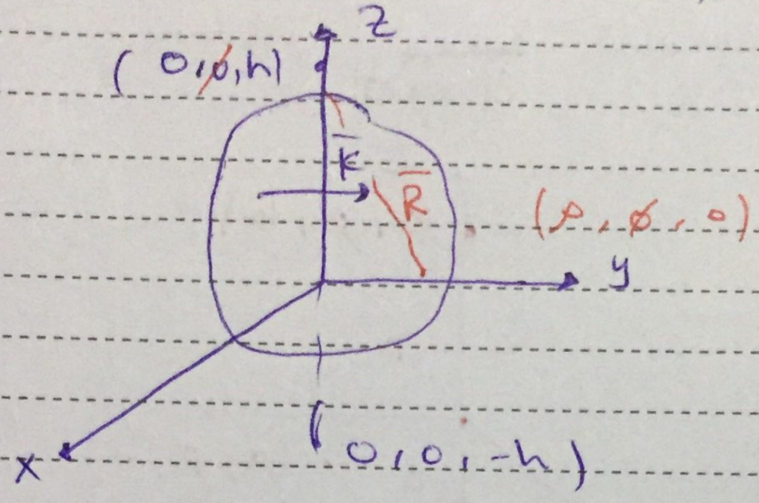
$$\bar{H} = \frac{Ia}{4\pi} \int_0^{2\pi} \frac{\hat{a}_\phi \times (-a\hat{a}_\rho + h\hat{a}_z)}{[a^2+h^2]^{3/2}}\ \hat{a}_\phi$$

due to Symmetry the ρ - Component will be cancelle

$$\bar{H} = \frac{Ia^2}{2[a^2+h^2]^{3/2}}\ \hat{a}_z \ \text{A/m}.$$

at $(0,0,h)$ Same for $(0,0,-h)$.

* **Ex** if a disk of radius (a)



$\bar{K} = k_y \hat{a}_y$

Sol $\bar{H} = \int_S \frac{\bar{K}\, ds \times \bar{R}}{4\pi R^3}$

$\bar{R} = -\rho \hat{a}_\rho + h\hat{a}_z$

$|R| = \sqrt{\rho^2 + h^2}$

$\bar{H} = \frac{k_y}{4\pi}\int_0^{2\pi}\int_0^{a} \frac{\hat{a}_y \times (-\rho\hat{a}_\rho + h\hat{a}_z)\,\rho\, d\rho\, d\phi}{[\rho^2 + h^2]^{3/2}}$

$\bar{ds} = \rho\, d\rho\, d\phi$

we need matrix to convert
$\hat{a}_y \rightarrow$ cyl:
or $\hat{a}_\rho \rightarrow$ cart.

*Ampere's Law:-
(special cases)

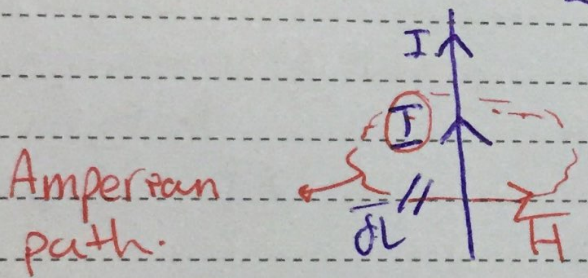
$$\oint_{L} \overline{H} \cdot \overline{dL} = I_{enc.}$$

↳ 3rd maxwell's eq. in integral form.

*Applications on Ampere's Law:-

① find $\overline{H}$ for an infinite line of current

$$\rightsquigarrow \overline{H} = \frac{I}{2\pi\rho}\,\hat{a_\phi}$$



Right hand Rule (R.H.R)
الابهام باتجاه ال I
وحركة الاصابع H

$$\oint_{L} \overline{H} \cdot \overline{dL} = I_{enc} = I$$

from R.H.R
$$\overline{H} = H_\phi\,\hat{a_\phi}$$

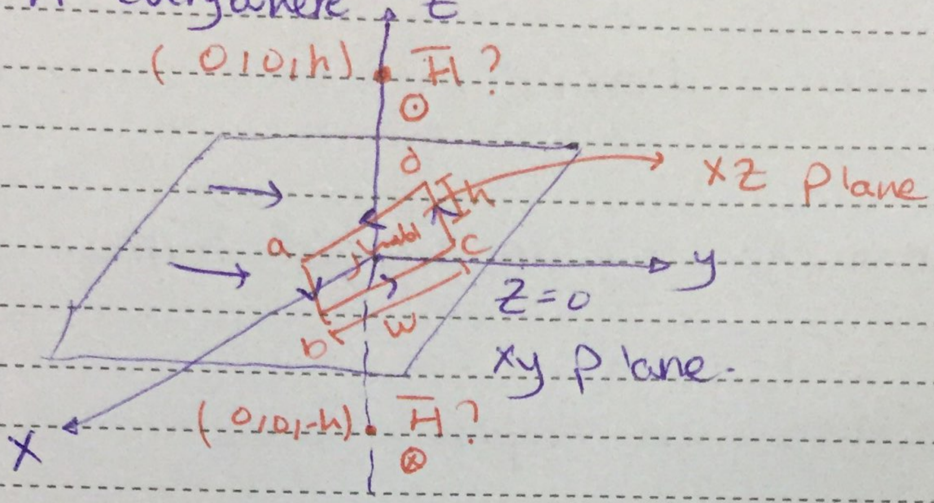
$$\overline{dL} = \rho\, d\phi\, \hat{a_\phi}$$

$$\rightarrow \int_0^{2\pi} H_\phi\, \hat{a_\phi} \cdot \rho\, d\phi\, \hat{a_\phi} = I$$

$$2\pi\rho H_\phi = I \longrightarrow \boxed{\overline{H} = \frac{I}{2\pi\rho}\,\hat{a_\phi}}$$ Same as Biot-Savar.

$\boxed{2}$ Find $\bar{H}$ for an infinite sheet of current.

- Ex $Z=0$ carry current $\bar{K} = K_y \hat{a_y}$
Find $\bar{H}$ everywhere



Sol

$$\oint_L \bar{H}\cdot\bar{dL} = I_{enc} = K_y w$$

لازم نختار مسار عشان فيه K
عشان نعرف الجواب
المسار يكون عامودي على K
$\bar{K} = K_y \hat{a_y}$
المسار لازم يكون XZ

$$\bar{H} = \begin{cases} H_0 \hat{a_x} & , z > 0 \\ H_0 (-\hat{a_x}) & , z < 0 \end{cases}$$

$\bar{dL} = ---$

Apply Ampere's Law on path a-b-c-d

$$\oint_L \bar{H}\cdot\bar{dL} = \int_a^b \cancelto{0}{\bar{H}\cdot\bar{dL}_1} + \int_b^c \bar{H}\cdot\bar{dL}_2 + \int_c^d \cancelto{0}{\bar{H}\cdot\bar{dL}_3} + \int_d^a \bar{H}\cdot\bar{dL}_4 = K_y w$$

Since $\bar{dL} = dz\hat{a_z}$ and $\pm\hat{a_x} \cdot \hat{a_z} = 0$

$$\int_b^c \bar{H}\cdot dL_2 = \int_b^c H_0(-\hat{a_x})\cdot dx\,(-\hat{a_x}) = \boxed{H_0 w}$$

$$\int_d^a \bar{H}\cdot \bar{dL} = H_0 w$$

$$H_0 \cancel{w} + H_0 \cancel{w} = K_y \cancel{w}$$

$$\boxed{H_0 = \frac{K}{2}}$$

$$\boxed{\bar{H} = \begin{cases} \frac{K_y}{2}\,\hat{a_x}, & z>0 \\ \frac{K_y}{2}\,(-\hat{a_x}), & z<0 \end{cases}}$$

Always. for infinite sheet

$$\hookrightarrow \boxed{\bar{H} = \frac{1}{2}\bar{K}\times\hat{a_n}}$$

نفس لقانون بصورة اخرى

Ex: $\bar{K} = K_x\hat{a_x}$

$$\bar{H} = \frac{1}{2}\bar{K}\times\hat{a_n}$$

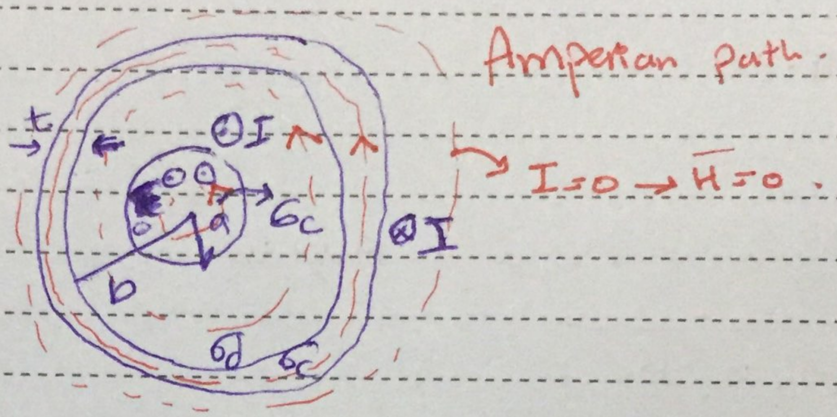
Ex: $\bar{K} = K_x\hat{a_x} + K_y\hat{a_y}$

$$\bar{H} = \frac{1}{2}\bar{K}\times\hat{a_n}$$

$\boxed{3}$ Find $\overline{H}$ for a volume current

- Ex Find $\overline{H}$ everywhere for a Coaxial cable.

(infinetly long)



Sol $\oint_L \overline{H}\cdot\overline{dL} = I_{enc}$.

- for $0<\rho<a$

$\overline{H} = H_\phi \hat{a}_\phi$

$\overline{dL} = \rho d\phi \hat{a}_\phi$

$$\oint \overline{H}\cdot\overline{dL} = \int_0^{2\pi} H_\phi \cancel{\hat{a}_\phi}\cdot \rho d\phi \cancel{\hat{a}_\phi}$$

$$\oint_L \overline{H}\cdot\overline{dL} = 2\pi\rho H_\phi$$

For $0 < \rho < a$

$$2\pi\rho H_\phi = I_{enc} = \int_S \overline{J} \cdot \overline{ds} \quad \cdots ①$$

at $\rho = a \longrightarrow I_{enc} = I$

$$I = \int_S \overline{J} \cdot \overline{ds} = \int_0^{2\pi}\int_0^{a} J_z \cancel{\hat{a_z}} \cdot \rho \, d\rho \, d\phi \, \cancel{\hat{a_z}}$$

$$I = J_z \frac{a^2}{\cancel{2}} (\cancel{2}\pi)$$

$$\overline{J} = \frac{I}{\pi a^2} \hat{a_z} \quad A/m^2 \longrightarrow \text{Sub in } ①$$

from equ ①

$$2\pi\rho H_\phi = \int_0^{2\pi}\int_0^{\rho} \frac{I}{\pi a^2} \cancel{\hat{a_z}} \cdot \rho \, d\rho \, d\phi \, \cancel{\hat{a_z}}$$

$$2\pi\rho H_\phi = \frac{I}{\pi a^2} \frac{\rho^2}{\cancel{2}} \cancel{2\pi}$$

$$2\pi\rho H_\phi = I \left(\frac{\cancel{\pi}\rho^2}{\cancel{\pi}a^2} \right)$$

$$\boxed{\overline{H} = \frac{I\rho}{2\pi a^2} \hat{a_\phi}}$$ for $0 < \rho \leq a$

*for $a \leq \rho < b$

$$2\pi\rho H_\phi = I_{enc} = I$$

$$\boxed{\bar{H} = \frac{I}{2\pi\rho}\hat{a_\phi}}$$ for $a \leq \rho \leq b$

infinite line.

-for $b < \rho < b+t$

$$2\pi\rho H_\phi = I_{enc.} = \underbrace{I}_{0\to a} - \underbrace{\int_S \bar{J}\cdot\bar{ds}}_{b\to b+t} \quad --- ②$$

to find $\bar{J}$

$\rho = b+t \rightarrow I_{enc} = I$

$$I = \int_S \bar{J}\cdot\bar{ds} = \int_0^{2\pi}\int_b^{b+t} J_z(\cancel{-\hat{a_z}}) . \rho\, d\rho\, d\phi\, (\cancel{-\hat{a_z}})$$

[illegible]

$$I = J_z \frac{(b+t)^2 - b^2}{\cancel{2}}(\cancel{2}\pi)$$

$$\bar{J} = \frac{I}{\pi((b+t)^2 - b^2)}\hat{a_z} \quad A/m^2$$ --- sub in ②

$$2\pi\rho H_\phi = I - \int_0^{2\pi}\int_b^{\rho} \frac{I}{\pi(2bt+t^2)} \cancel{\hat{a}_z} \cdot \rho\, d\rho\, d\phi\, \cancel{\hat{a}_z}$$

$$2\pi\rho H_\phi = I\left(1 - \frac{1}{\pi(2bt+t^2)} \frac{(\rho^2-b^2)}{\cancel{2}} \cancel{2}\pi\right)$$

$$\boxed{\bar{H} = \frac{I}{2\pi\rho}\left(1 - \frac{\rho^2-b^2}{2bt+t^2}\right)\hat{a}_\phi \quad A/m}$$

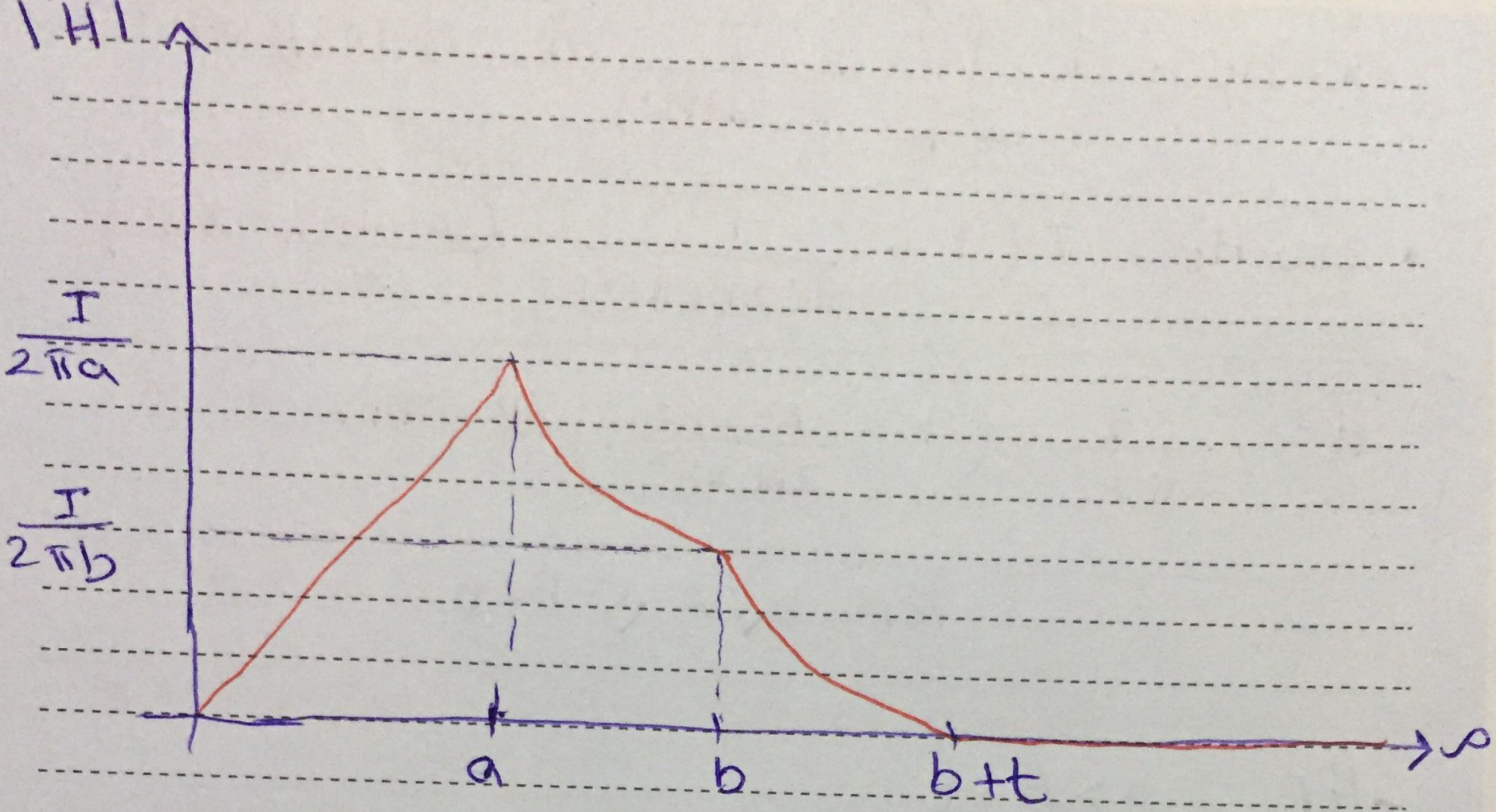
for $b \leq \rho \leq b+t$

- for $\rho > b+t$

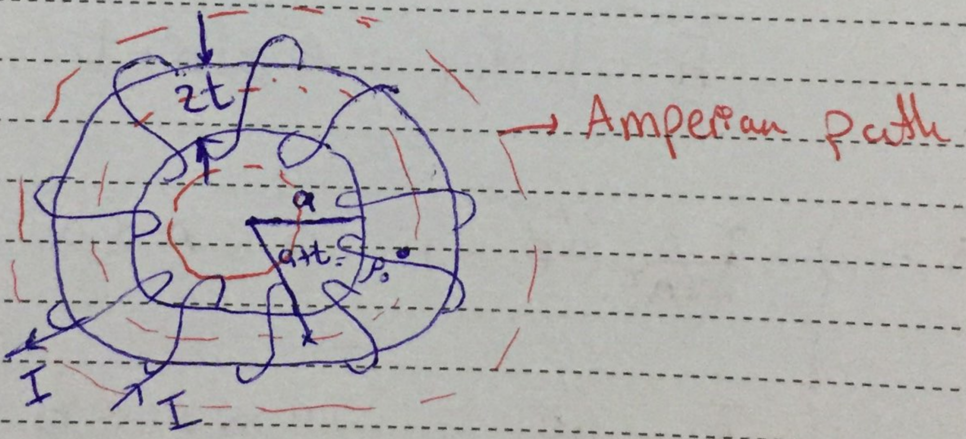
$$2\pi\rho H_\phi = I_{enc} = I - I = 0.$$

$\boxed{\bar{H} = 0}$ for $\rho \geq b+t$.

$$\bar{H} = \begin{cases} \frac{I\rho}{2\pi a^2}\hat{a}_\phi, & 0 < \rho \leq a \\ \frac{I}{2\pi\rho}\hat{a}_\phi, & a \leq \rho \leq b \\ \frac{I}{2\pi\rho}\left(1 - \frac{\rho^2-b^2}{2bt+t^2}\right)\hat{a}_\phi, & b \leq \rho \leq b+t \\ 0, & \rho \geq b+t \end{cases}$$



$\boxed{4}$ Toroid



Find $\bar{H}$ everywhere?

Sol $\oint_L \bar{H}\cdot d\bar{l} = I_{enc}$

$0 < \rho < a$

$I = 0 \rightarrow H = 0$

$\rho > \rho_0 + t \rightarrow I_{enc} = I - I = 0$

$\bar{H} = 0$

For $\overset{\rho_0 - t}{a} < \rho < \rho_0 + t$ ($a < \rho < a + 2t$)

$$\overline{H} = H_\phi \hat{a}_\phi$$

$$\overline{dL} = \rho \, d\phi \, \hat{a}_\phi$$

$$2\pi\rho H_\phi = I_{enc} = N I$$

↳ # of turns

$$\overline{H} = \frac{N I}{2\pi\rho} \hat{a}_\phi$$

$$\overline{H} = \frac{N I}{L} \quad , L = 2\pi\rho$$

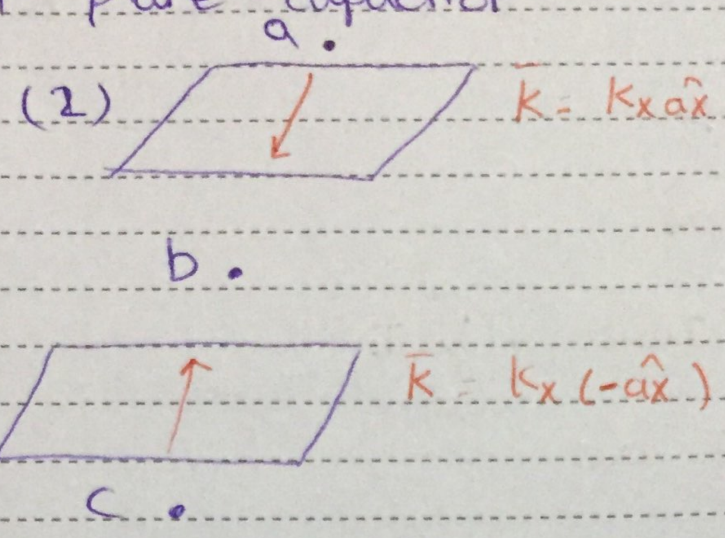
$$\overline{H} = n I \hat{a}_\phi \quad , \quad n = \frac{N}{L}$$

↓ turns density.

$$\boxed{\overline{H} \cdot \overline{L} = N I}$$ if uniform.

$$\boxed{\oint_L \overline{H} \cdot \overline{dL} = N I}$$ Ampere's Law for N-turns

Ex inf parallel plate capacitor



find $\bar{H}$ at a, b, c ?

Sol at point (b).

$$\bar{H} = \bar{H}_1 + \bar{H}_2$$

$$\bar{H}_1 = \frac{-1}{2} K \hat{a_x} \times \hat{a_z} + \frac{1}{2} K_x \hat{a_x} \times (-\hat{a_z})$$

$$= \frac{K_x}{2} \hat{a_y} + \frac{K_x}{2} \hat{a_y}$$

$$\boxed{\bar{H} = K_x \hat{a_y}}$$

at point (a) → zero

at point (c) → zero.

* Magnetic Flux Density ($\overline{B}$)

$\overline{B} = \mu_0 \overline{H}$

μ_0 : free space permeability = $4\pi * 10^{-7}$ H/m

i.e for finite straight line

$$\overline{B} = \frac{\mu_0 I}{4\pi \rho} (\cos\alpha_2 - \cos\alpha_1) \hat{a}_\phi$$

- for infinite sheet

$$\overline{B} = \frac{\mu_0}{2} \overline{k} \times \hat{a}_n$$

* Units of $\overline{B}$: (MKS)

1) $\frac{H}{m} \frac{A}{m} = \frac{H \cdot A}{m^2}$

2) $\frac{Wb}{m^2}$, Wb = Weber 1 wb = 1H.1A

3) T , Tesla

- Old unit (cgs)

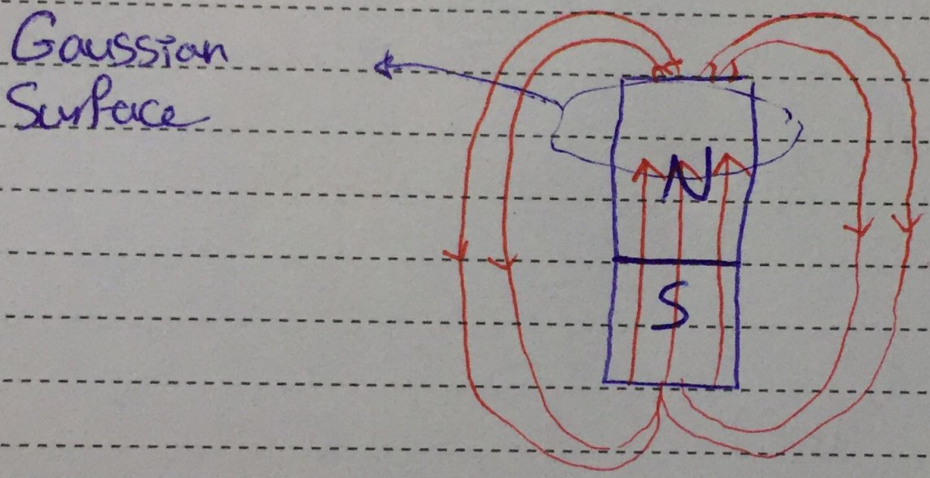
4) $G = 10^{-5} T$

* Magnetic Flux (Ψ_m)

$$\Psi_e = \int_S \overline{D}\cdot\overline{ds}$$

$$\boxed{\Psi_m = \int_S \overline{B}\cdot\overline{ds} \quad \text{in wb}}$$

* Flux Lines :-



خطوط المجال المغناطيسي

continuos [1]

from N → S [2]
outside and from
S → N inside.

$$* \boxed{\Psi_m = \oint_S \overline{B}\cdot\overline{ds} = 0}$$

because there is no single pole magnet.

→ Magnetic Gauss's Law

↳ 4th Maxwell equ. in integral form.

*Maxwell's equ. For $\boxed{\text{Static}}$ Fields :-

$\boxed{1}$ $\oint_S \overline{D} \cdot \overline{ds} = \int_V \rho v \, dv = 0$

$\boxed{2}$ $\oint_L \overline{E} \cdot \overline{dL} = 0$

$\boxed{3}$ $\oint_L \overline{H} \cdot \overline{dL} = I_{enc} = \int_S \overline{J} \cdot \overline{ds}$

$\boxed{4}$ $\oint_S \overline{B} \cdot \overline{ds} = 0$

+

$\overline{D} = \epsilon \overline{E}$

$\overline{J} = \sigma \overline{E}$

$\overline{B} = \mu \overline{H}$

* Magnetic Potentials:-

↳ Scalar potential (V_m) in (A)
↳ vector potential ($\bar{A}$) in (Wb/m)

- Duality:-

$$\bar{E} = -\nabla V \longrightarrow \boxed{\bar{H} = -\nabla V_m}$$

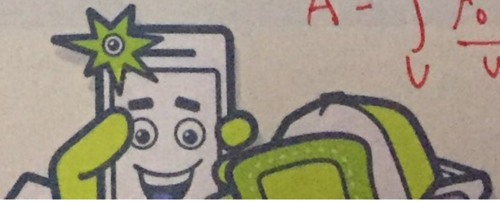
A/m 1/m A

$$V = -\int_L \bar{E} \cdot \bar{dL} \longrightarrow \boxed{I = \oint_L \bar{H} \cdot \bar{dL}}$$

$$V = \oint_L \bar{E} \cdot \bar{dL} = 0 .$$

* For $\bar{A}$ Duality:-

- for line charge	for line current
$V = \int_L \frac{\rho_L dL}{4\pi\epsilon_0 r}$	$\bar{A} = \int \frac{\mu_0 I \bar{dL}}{4\pi r}$
- for surface charge	
$V = \int_S \frac{\rho_s ds}{4\pi\epsilon_0 r}$	$\bar{A} = \int_S \frac{\mu_0 \bar{K} ds}{4\pi r}$
- for Volume charge	$\bar{A} = \int_V \frac{\mu_0 \bar{J} dv}{4\pi r}$
$V = \int_V \frac{\rho_V dv}{4\pi\epsilon_0 r}$	



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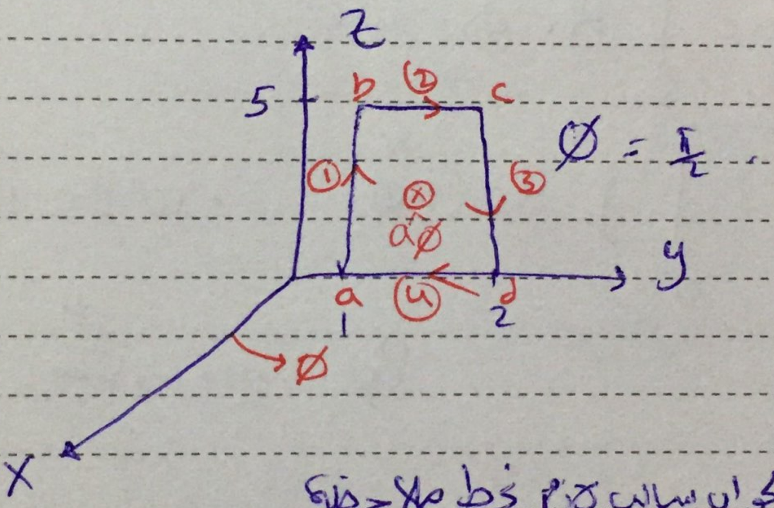
$$\boxed{* \ \Psi_m = \oint_L \bar{A}\cdot\overline{dL}}$$

$$\boxed{\Psi_m = \int_S \bar{B}\cdot\overline{ds}}$$

* <u>Ex</u> Given the magnetic potential $\bar{A} = -\frac{\rho^2}{4}\hat{a}_z$ wb/m Calculate the total flux crossing the surface $\phi = \frac{\pi}{2}$, $1 \le \rho \le 2$m , $0 \le z \le 5$m.

<u>Sol</u>



إذا طلع الجواب سالب كمان خط ملاحظة
انه التدفق ما طلع سالب معناه
تغير اتجاه مش

$\Psi_m = \oint_L \bar{A}\cdot\overline{dL}$ on path abcda

$$\Psi_m = \int_{L_1}\bar{A}\cdot\overline{dL_1} + \cancelto{0}{\int_{L_2}\bar{A}\cdot\overline{dL_2}} + \int_{L_3}\bar{A}\cdot\overline{dL_3} + \cancelto{0}{\int_{L_4}\bar{A}\cdot\overline{dL_4}}$$

$\overline{dL_1} = dz\,\hat{a}_z$

$\overline{dL_2} = d\rho\,\hat{a}_\rho$

صفر لأنه ال $\bar{A}$ باتجاه $\hat{a}_z$
و التكامل الثاني و الرابع باتجاه $\hat{a}_\rho$
← صفر :

$$\psi_m = \int_0^5 \frac{-\rho^2}{4} \cancel{\hat{a_z}} \cdot dz \, \cancel{\hat{a_z}} \Big|_{\rho=1} + \int_5^0 \frac{-\rho^2}{4} \cancel{\hat{a_z}} \cdot dz \, \cancel{\hat{a_z}} \Big|_{\rho}$$

$$= \frac{-1}{4}(5) - (1)(-5) = [illegible]$$

$$= \frac{-5}{4} + 5 = \boxed{\frac{15}{4} \; wb}$$

b) $\psi_m = \int_S \bar{B} \cdot \bar{ds}$

$$\frac{15}{4} = \int_0^5 \int_1^2 B_\phi \, \hat{a_\phi} \cdot d\rho \, dz \, \hat{a_\phi}$$

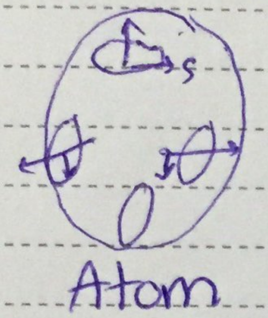
$\downarrow$

B_ϕ مشان نوجد

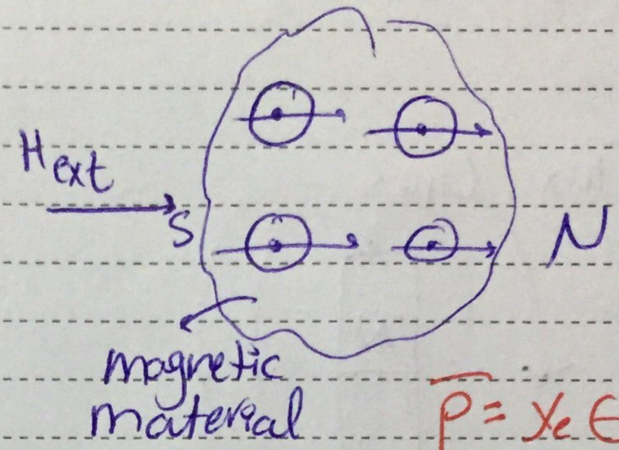
CH8 Magnetic fields in materials :-

Section 8.5 magnitization ($\bar{M}$) Unit (A/m)

* $\bar{M} = \lim_{\Delta V \to 0} \sum_{k=1}^{N} \frac{\bar{m}_k}{\nabla V}$



$\bar{M} = 0$



$\bar{P} = \chi_e \epsilon . \bar{E}$

- for some materials :-

$\bar{M} = \chi_m \bar{H}$

* $\bar{B} = \mu_0 \bar{H}$ in free space ($\mu_r = 1$)

- Currents

free	bound
$\bar{k} = \bar{H} . \hat{an}$	$\bar{k}_b = M \times \hat{an}$
$\bar{J} = \nabla \times \bar{H} \leftarrow X$	$\bar{J}_b = \nabla \times \bar{M} \leftarrow X$

$\oint \bar{H} . \bar{dL} = I_{enc}$

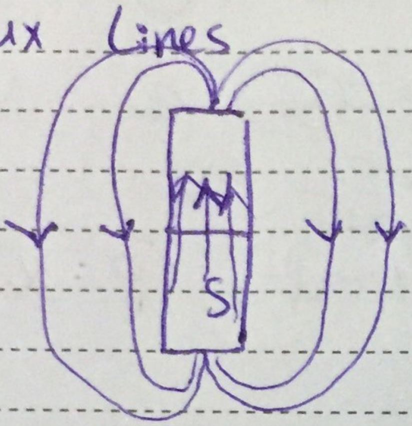
$\int \bar{J} . ds$

$* \bar{B} = \mu \bar{H}$ *

$\mu \rightarrow \mu_o \boxed{\mu_x}$ → ~~relation~~ relative permitivity (Unitless)

in general $X_m = \mu_r - 1$

* Flux Lines



- Classification of magnetic materials

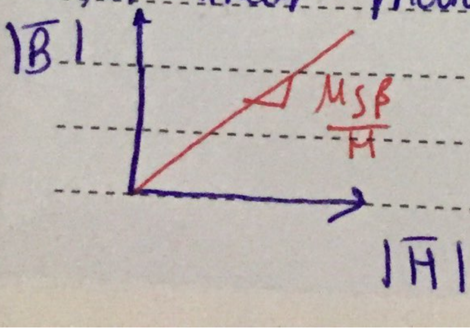
↳ Dia magnetic (Cu, AL, lead) ($\mu_r \leq 1$)

↳ Paramagnetic (Air) ($\mu_r \geq 1$)

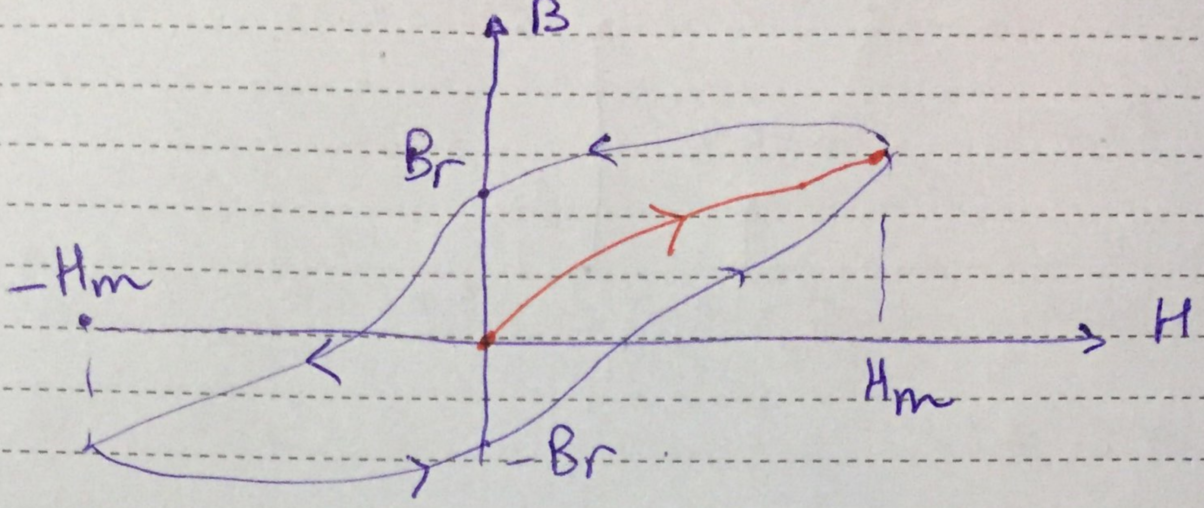
↳ Ferro magnetic ($\mu_r >>> 1$)
(Iron, Nickle, rebolt, ferrite,) → Non linear material

*B/H curve

For linear material



- for non-linear materials: -



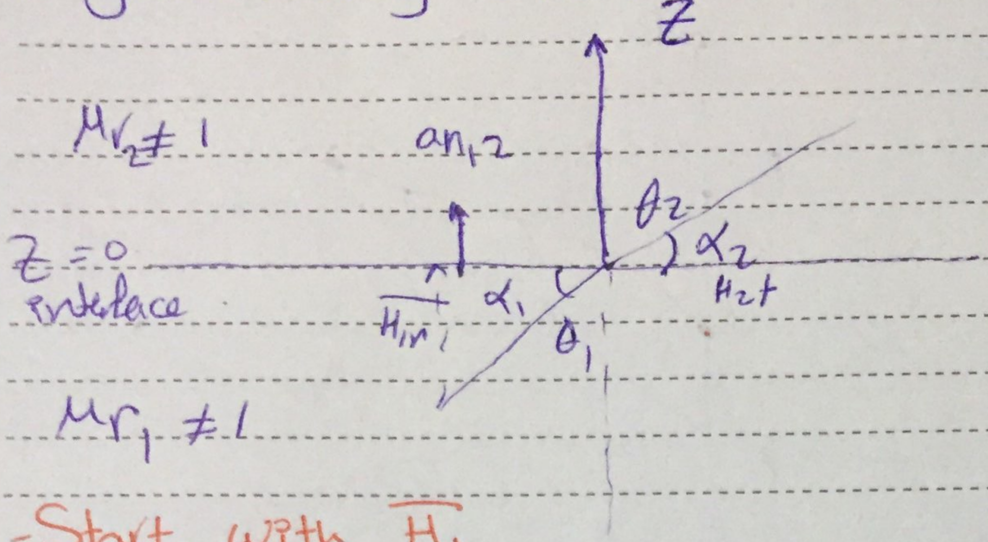
* T_c = Curie temp
  Iron = 790°c

* B_r = Resional magnetic flux density
  H_c = Demagnitization magnetic field Intensity

- Area ↑ → Had magnetic material ↑

$\bar{B} = \mu \bar{H}$
only at point (not in general).

*Magnetic boundry Conditions:-



- Start with $\overline{H_1}$

$\overline{H_1} = \overline{H_{1n}} + \overline{H_{1t}}$

$\overline{H_{1n}} = (H_1 . \hat{an})\, \hat{an}$

$\overline{H_{1t}} = \overline{H_1} - \overline{H_{1n}}$

$\sin\theta_1 = \frac{H_{1t}}{H_1}$

$\alpha_1 = 90 - \theta_1$

$\cos\theta_1 = \frac{H_{1n}}{H_1}$

$\tan\theta_1 = \frac{H_{1t}}{H_{1n}}$

- To find $\overline{H_2}$

$\overline{H_2} = \overline{H_{2n}} + \overline{H_{2t}}$

Apply the 3rd and 4th Maxwell's equation at the interface

$\oint_L H \cdot \overline{dL} = I_{enc}$

$\oint_s B \cdot \overline{ds} = 0.$

-Given that $\overline{H} = -2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z$ A/m
in region $y - x - 2 \le 0$ where $\mu_r = 5\mu_0$
calculate:-

1) $\overline{M}_1$ and $\overline{B}_1$

2) $\overline{H_2}$ and $\overline{B_2}$ in region $y - x - 2 \ge 0$ if $\mu_2 = 2\mu_0$

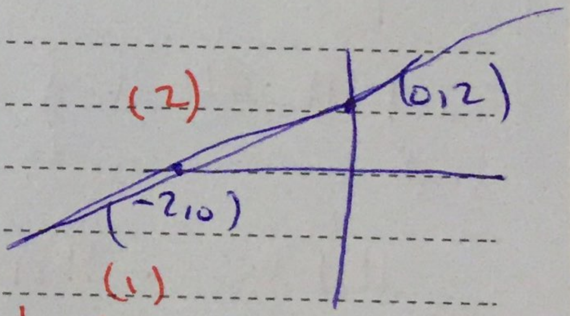
Sol:
1) $\overline{M_1} = (\mu_{r_1} - 1)\overline{H_1}$

$= 4\overline{H_1} = (-8, 24, 16)$ A/m

$\overline{B_1} = \mu_1 H_1$

$= 5\mu_0 H_1 = (-10, 30, 20)\mu_0$

2) $\overline{H_1} = \overline{H_{1t}} + \overline{H_{1n}}$



$\overline{H_{1n}} = (H_1 \cdot \hat{a}_n)\hat{a}_{n_{12}}$

$= \left((-2,6,4)\frac{(-1,1,0)}{\sqrt{2}}\right) \cdot \frac{(-1,1,0)}{\sqrt{2}}$

$\overline{H_{1n}} = (-4, 4, 0)$ A/m

$\hat{a}_n = \frac{\nabla f}{|\nabla f|}$

$= \frac{(-1,1,0)}{\sqrt{2}}$

$\overline{H_{1t}} = \overline{H_1} - \overline{H_{1n}}$

$= (2, 2, 4)$ A/m

$= \overline{H_{2t}}$ (since $\overline{K} = 0$)

$\overline{B_{1n}} = \overline{B_{2n}}$

$\overline{H_{2n}} = \frac{\mu_{r1}}{\mu_{r2}} \overline{H_{1n}}$

$\overline{H_{2n}} = (-10, 10, 0)$

$\overline{H_2} = -8\hat{a_x} + 12\hat{a_y} + 4\hat{a_z}$ A/m

$\overline{B_2} = 20\mu_0 H_2 = -16\hat{a_x} + 24\hat{a_y} + 8\hat{a_z}$ M-T

$\mu_2 = \overline{H_2}$

- $Z=0$ (infinite sheet) Apply $\oint_L \overline{H}\cdot\overline{dL} = I_{enc}$ on path a-b-c-d-a

$$\int_a^b \overline{H}\cdot\overline{dL} + \int_b^c \overline{H}\cdot\overline{dL} + \int_c^d \overline{H}\cdot\overline{dL} + \int_d^a \overline{H}\cdot dL = k_x W$$

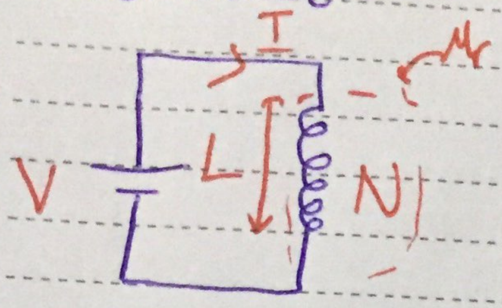
$$H_{2t} W - H_{2n}\frac{h}{2} - H_{1n}\frac{h}{2} - H_{1t} W + H_{1n}\frac{h}{2}$$
$$+ H_{2n}\frac{h}{2} = k_x W$$

$$\boxed{H_{2t} - H_{1t} = k_x}$$

$$\boxed{\text{if } k=0 \rightarrow \overline{H_{2t}} = \overline{H_{1t}}}$$

$$\boxed{\overline{H_{2n}} = \frac{\mu_{r1}}{\mu_{r2}} \overline{H_{1n}}}$$

* Inductors:-



$\lambda \propto I$

$\lambda = L I$

L: proportionality constant

L = inductance

$I \propto B \propto \Psi \propto \lambda$

$$\boxed{\Psi = \int_S \overline{B} \cdot \overline{ds}}$$

$$\boxed{\lambda = N \Psi}$$

↳ Flux linkage (Wb)

$$\boxed{L = \frac{\lambda}{I} \quad (H)}$$

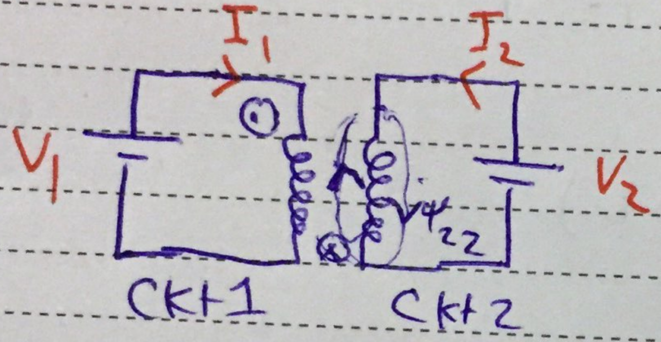
* Magnetic energy (Wm) in (J).

$$\left.\begin{aligned} W_m &= \frac{1}{2} \int_V \overline{H} \cdot \overline{B} \, dv \\ &= \frac{1}{2} \int_V \mu H^2 \, dv \\ &= \frac{1}{2} \int_V \frac{B^2}{\mu} \, dv \end{aligned}\right\} \text{in general}$$

- if uniform: -

$$\boxed{\begin{aligned} W_m &= \frac{1}{2} L I^2 \\ I &= \frac{2 W_m}{I^2} \end{aligned}}$$

- Self and mutual Inductance: -



Ψ_{11} Ckt 1 ↲ ↳ current 1

Ψ_{22} : Self fluxes

Ψ_{12}, Ψ_{21} : Mutual flux

- Self Inductance: -

$$\boxed{L_{11} = \frac{\lambda_{11}}{I_1} = \frac{N_1 \Psi_{11}}{I_1}}$$

$$\boxed{L_{22} = \frac{\lambda_{22}}{I_2} = \frac{N_2 \Psi_{22}}{I_2}}$$

→ Mutual Inductance

$$\boxed{M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \Psi_{12}}{I_2}}$$

$$\boxed{M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2 \Psi_{21}}{I_1}}$$

- Total flux :-

Ψ_1 : total flux in ckt 1

$$\Psi_1 = \Psi_{11} \;\textcircled{+}\; \Psi_{12}$$

⊕ → if in the same direction

⊖ → if in the opposite direction

- L_1 : total Inductance of ckt 1

$$L_1 = L_{11} \;\textcircled{+}\; M_{12}$$

⊕ → if flux +

⊖ → if flux −

L_1 ↓ must be positive.

$$W_m = \frac{1}{2} L_{11} I_1^2 + \frac{1}{2} L_{22} I_2^2 \pm \left(\frac{1}{2} M_{12} I_2^2 + \frac{1}{2} M_{21} I_1^2 \right)$$

$$W_m = W_{m_1} + W_{m_2}$$

$$= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \longrightarrow \boxed{\frac{I_2}{I_1} = \frac{N_1}{N_2}}$$

*procedure to find (L) or (M):-

1) Choose suitable coordinates.

2) Assume current

3) find B (As you did in CH7)

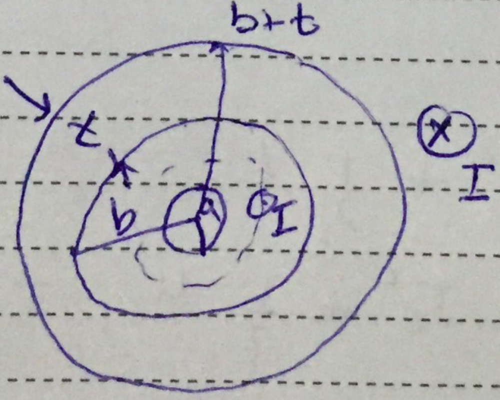
4) $\Psi = \int \vec{B} \cdot ds$

5) $L = \frac{N\Psi}{I} = \frac{\lambda}{I}$

4) $W_m = \frac{1}{2}\int_V \frac{B^2}{\mu}\, d$

5) $L = \frac{2W_m}{I^2}$

- Ex For a coaxial cable, find all self inductance



Sol 1) Cylindrical

2) on the figure .

from CH7 $\rightarrow \bar{B} = \begin{cases} \dfrac{\mu_0 I \rho}{2\pi a^2}\,\hat{a}_\phi , & 0 \le \rho \le a \\ \dfrac{\mu_0 \mu_r I}{2\pi \rho}\,\hat{a}_\phi , & a \le \rho \le b \end{cases}$

$$L = L_{int} \overset{\text{series}\,\uparrow}{\oplus} L_{ext}$$

for L_{int}

$$W_m = \frac{1}{2} \int_0^L \int_0^{2\pi} \int_0^a \frac{\mu_0^{\cancel{2}} I^2 \rho^2}{\cancel{\mu_0}\, 4\pi^2 a^4}\, \rho \, d\rho \, d\phi \, dz$$

$$\frac{\mu_0 I^2}{2 \cdot 4\pi^2 \cancel{a^4}} \left(\frac{\cancel{a^4}}{4} \right) (\cancel{2}\pi)(L)$$

$$W_m = \frac{\mu_0 I^2 L}{16\pi} \ (J)$$

5) $L_{ext} = \dfrac{\mu_0 L}{8\pi} L \rightarrow \dfrac{L_{int}}{L} = \dfrac{\mu_0}{8\pi}$ H/m

For Lext

$$\psi = \int_S \bar{B} \cdot \bar{ds}$$

$$= \int_0^L \int_a^b \frac{\mu_0 \mu_r I}{2\pi \rho} \hat{a}_\phi \, d\rho \, dz \, \hat{a}_\phi$$

$$= \frac{\mu_0 \mu_r I L}{2\pi} \ln\left(\frac{b}{a}\right) \quad (wb)$$

$$L_{ext} = \frac{\psi}{I} = \frac{\mu_0 \mu_r L}{2\pi} \ln\left(\frac{b}{a}\right) \quad (H)$$

$$\frac{L_{ext}}{L} = \frac{\mu_0 \mu_r}{2\pi} \ln\left(\frac{b}{a}\right) \quad (H/m)$$

$$\frac{L}{l} = \frac{\mu_0}{2\pi} \left[\frac{1}{4} + \mu_r \ln\left(\frac{b}{a}\right) \right] \quad (H/m)$$

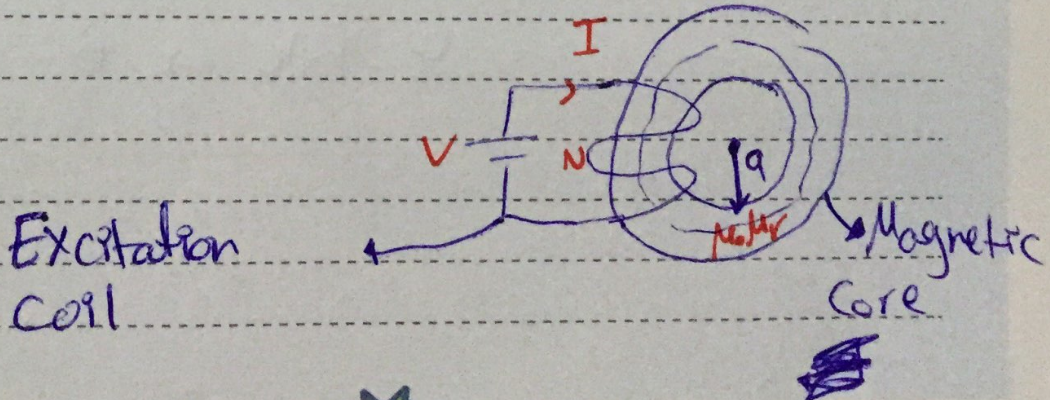
- Magnetic Circuits

Electrical Circuit	Magnetic Circuit
Vem	$V_{emf} = F$
I	Ψ_m or Ψ
$R = \frac{V}{I} = \frac{l}{\sigma s}$	$R_m = \frac{F}{\Psi} = \frac{l_c}{\mu A_c}$
$G = \frac{1}{R}$	$P = \frac{1}{R_m}$
$I = \int_S \overline{J} \cdot \overline{ds}$	$\Psi = \int_S \overline{R_B} \cdot \overline{ds}$
$W_E = \frac{1}{2} \int_v \epsilon E^2 dv$	$W_m = \frac{1}{2} \int_v \mu H^2 dv$
ϵ	μ

* Magnetic structures:-

↳ magnetic core

↳ Excitation coil



- R_m: magnetic resistance (Reluctance)

$$\boxed{R_m = \frac{\ell_c}{\mu_0 \mu_r A_c}}$$

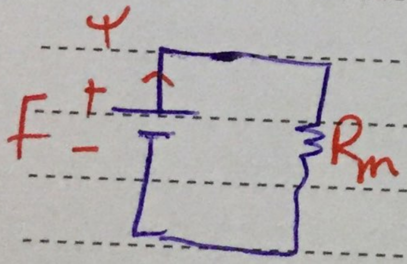
$$V_{mmf} = F = NI$$
$$= \Psi R_m$$
$$= H \ell_c$$

$$\boxed{\ell_c = 2\pi \rho_0 \quad (\text{Toroid})}$$

A_c: Cross sectional Area of the core

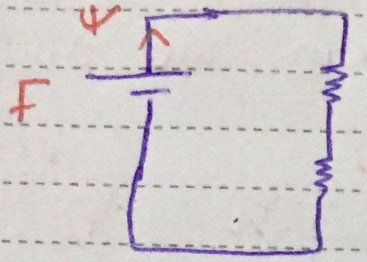
$$\boxed{A_c = \pi t^2 \quad (\text{Toroid})}$$

- Convert magnetic structure to a mgnetic ckt :-



$$\boxed{\begin{aligned} \Psi &= \frac{F}{R_m} = \frac{NI}{\frac{\ell_c}{\mu_0 \mu_r A_c}} \\ \Psi &= B A_c \longrightarrow B = \frac{\Psi}{A_c} \end{aligned}}$$

- Magnetic structure with air gap



$R_m = \frac{L_c}{\mu_0 \mu_r A_c}$ (H^{-1})

$R_{AG} = \frac{\ell_{CA}}{\mu_0 \underset{\downarrow \mu_r}{(1)} A_c}$

$$R_{eq} = R_m + R_{AG} \quad , \quad \Psi = \frac{F}{R_{eq}}.$$

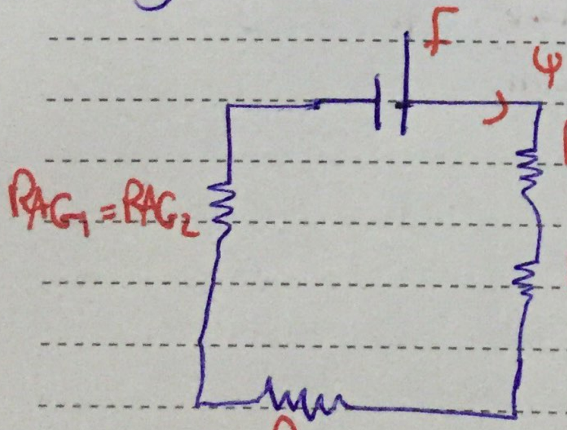
- Force in the air gap -

Magnetic Tractive Force ($F_{\mu T}$) in (N)

$$F_{\mu T} = \frac{B^2 A_c}{2\mu_0} \qquad B = \frac{\Psi}{A_c}$$

$$\text{Pressure} = \frac{F_{mt}}{A_c} \qquad (\text{pascal})$$

* Magnetic Limitations



$R_{m_1} = \frac{\ell_{c1}}{\mu_0 \mu_{r_1} A_c}$

$R_{AG_1} = \frac{\ell_{AG}}{\mu_0 A_c}$

$R_{m_2} = \frac{\ell_{c2}}{\mu_0 \mu_{r_2} A_c}$

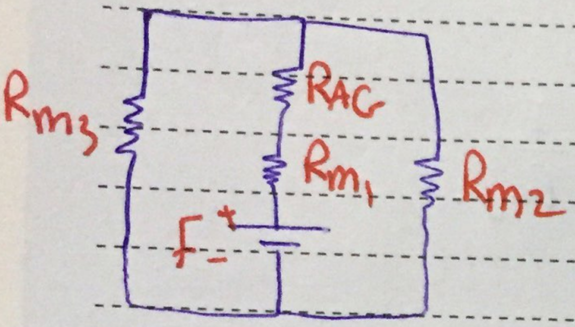
$$R_{eq} = 2R_{AG} + R_{m_1} + R_{m_2}$$

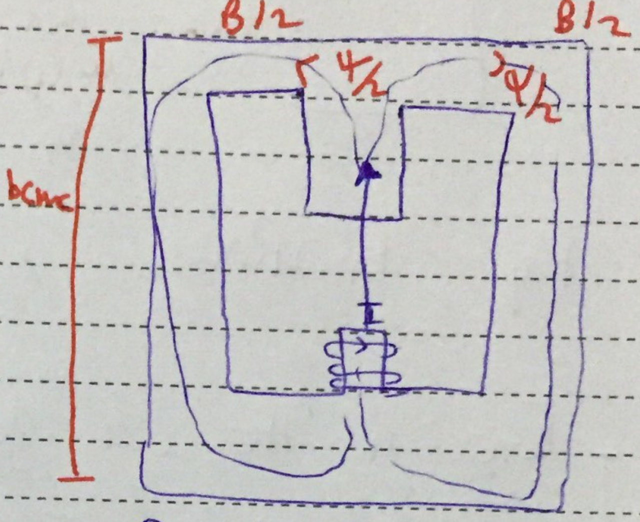
$$F_{mT} = mg = \left(\frac{B^2 A_c}{2\mu_0}\right) \times 2$$

$$\Psi = BA_c = \frac{NI}{R_{eq}}$$

- **Ex** Calculate the current in the coil to produce 1.5 Wb/m² in the air gap, if $\mu = 50 \mu_o$, $N = 400$ turs, $\ell AC =$ 10cm and all the branches have $A_c = 10$ cm².

Sol





ن تفرع ال Ψ ← Parallel

$$R_{m_1} = \frac{9 * 10^{-2}}{(4\pi * 10^{-7})(50)(10 * 10^{-4})}$$

$$R_{m_2} = \frac{30 * 10^{-2}}{(4\pi * 10^{-7})(50)(10 * 10^{-4})}$$

$$R_{eq} = R_{m_1} + R_{AG} + (R_{m_2} // R_{m_3}) = \frac{7.4 * 10^{8}}{20\pi} \ H^{-1}$$

$$F = N I = \Psi R_{eq}$$

$$\Psi = B A$$

$$I = \frac{\Psi R_{eq}}{N} = 44.16 \ A$$

CH9 Maxwell's equations (Time Varying fields) (Electromagnetic

- Source for time varying fields:-

1) Charge moving with accelaration
2) Ac current flowing in the wire

- Maxwell's eq. → for static field

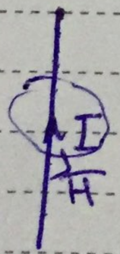
$$\oint_S \bar{D} \cdot \bar{ds} = \int_V \rho v \, dv$$

$$\oint_L \bar{E} \cdot \bar{dL} = 0$$

$$\oint_L \bar{H} \cdot \bar{dL} = I_{enc} = \oint_S \bar{J} \cdot \bar{ds}$$

$$\oint_S \bar{B} \cdot \bar{ds} = 0$$

- Faraday's Law:-



to find Vemf from $\bar{H}$
↳ generator principle.

$$V_{emf} = -\frac{d\lambda}{dt} \qquad V_{emf} = \frac{-d\Psi}{dt}$$

$$\Psi(\underbrace{x, y, z}_{space}, \underset{\hookrightarrow time}{t})$$

$$V_{emf} = -\frac{d}{dt} \int_S \bar{B} \cdot \bar{ds}$$

from CH4

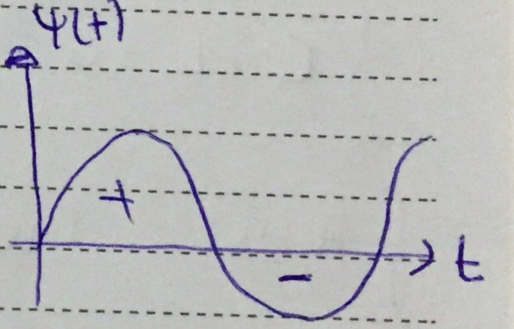
$$\hookrightarrow V = -\int_L \bar{E} \cdot \bar{dL}$$

$$\hookrightarrow V = \oint_L \bar{E} \cdot \bar{dL} = 0$$

$$\boxed{V_{emf} = \oint_L \bar{E} \cdot \bar{dL} = -\frac{d}{dt} \int_S \bar{B} \cdot \bar{ds}}$$

Faraday's Law for N=1 turn

- Ac Source (Time Varying $\bar{B}$)



$$v(t) \rightarrow i(t) \rightarrow B(t) \rightarrow \psi(t) \rightarrow \lambda(t)$$

$$V_{emf_1} = -\frac{N\,d\psi}{dt}$$

$$V_{emf} = \oint_L E \cdot dL = \int_S \frac{-dB}{dt} \cdot dS$$

2) Static fields on a moving loop

$$V_{emf} = \oint_L \bar{E} \cdot \bar{dL} = \oint_L \underbrace{\bar{u}}_{\text{velocity}} \times \bar{B} \cdot dL$$

$$\bar{E} = \bar{u} \times \bar{B}$$

3) Time varying field on a moving loop

$$V_{emf} = \oint_L \bar{E} \cdot \bar{dL} = \oint_S \frac{-d\bar{B}}{dt} \cdot \bar{ds} + \oint_L \bar{u} \times \bar{B} \cdot dl$$

$$\boxed{V_{emf} = \oint_L \bar{E} \cdot \bar{dL} = \frac{-d}{dt} \int_S \bar{B} \cdot \bar{ds}}$$

* Displacement current 8 (I_d)
found by Maxwell.

$I_c = \int_S \bar{J} \cdot \bar{ds}$, $\underset{\text{conduction}}{\bar{J} = \sigma \bar{E}}$, $\underset{\text{convention}}{\bar{J} = \rho_v \bar{u}}$

$$I_d = \int_S \underbrace{\bar{J_d}}_{\text{displacement current density.}} \cdot \bar{ds}$$

- Continuity equations :-

$$\oint_S \overline{J} \cdot \overline{ds} = - \int_S \frac{d\rho v}{dt} \, dv$$

$$I_c + I_d = 0 \rightarrow \overline{H} = 0 .$$

$$I_c = - I_d$$

$$I_d = \oint_S \overline{J} \cdot \overline{ds} = \oint_v \frac{d\rho v}{dt} \, dv .$$

$\overline{J_d} = \frac{d\overline{D}}{dt}$ → electric flux density ($\overline{D} = \epsilon_0 \overline{E}$)

$$\oint_L H \cdot \overline{dL} = (\overline{J_m} + \overline{J_d}) \cdot \overline{ds}$$

3rd maxwell eq. in Time varying fields

[illegible]

~~9.5 Maxwell's eq~~

9.5 Maxwell's equ in Time Varying fields :-

$$\oint_S \bar{D} \cdot \overline{ds} = \int_V \rho v \, dv$$

$$\oint_L \bar{E} \cdot \overline{dl} = \frac{-\partial}{\partial t} \int_S \bar{B} \cdot ds$$

$$\oint_L \bar{H} \cdot dl = \int_S (\bar{J} + \overline{J_d}) \cdot \overline{ds}$$

$$\oint_S \bar{B} \cdot \overline{ds} = 0 \rightarrow \int_S \bar{H} \cdot \overline{ds} = 0$$

9.7 Time Harmonic Fields
periodic (Sinusoidals)

- Start with the revision of complex No

* How to convert between time domain and frequency domain and vice - versa :-

– Maxwell's equ.

$$\oint_S \overline{D}\cdot\overline{ds} = \int_v \rho_v\, dv$$

$$\oint_L \overline{E}\cdot\overline{dL} = -\int_S \boxed{\frac{d\overline{B}}{dt}}\cdot\overline{ds}$$

$$\oint_L \overline{H}\cdot\overline{dL} = \int_S \left(\sigma\overline{E} + \boxed{\frac{d\overline{D}}{dt}}\right)\cdot\overline{ds}$$

$$\oint_S \overline{B}\cdot\overline{ds} = 0$$

Note:

* $\boxed{\frac{d}{dt}}\left(\underline{\underline{e^{j\omega t}}}\right) = \boxed{j\omega}\,\underline{\underline{e^{j\omega t}}}$

* $\boxed{\int_t} \underline{\underline{e^{j\omega t}}}\,dt = \boxed{\frac{1}{j\omega}}\,\underline{\underline{e^{j\omega t}}}$

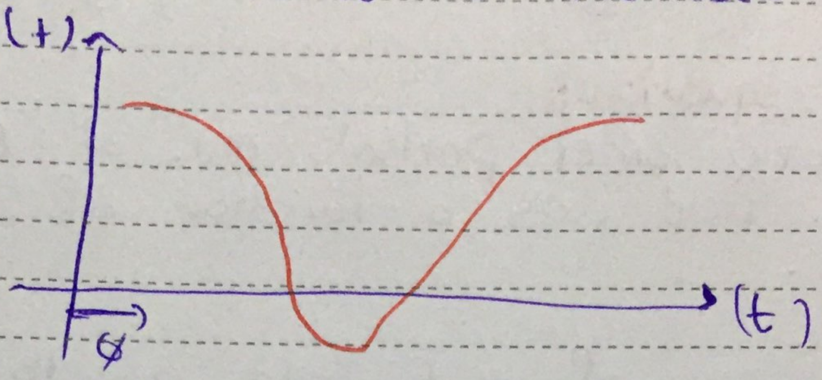
- i.e. $I(t) = \boxed{I_0}\cos(\boxed{\omega}\boxed{t} + \boxed{\phi})$

Phase Shift (Argument)

Amplitude
Magnitude
Absolute

radian freq (rad/s)
$\omega = 2\pi f$
$f = \frac{1}{T}$
T → period

- in time domain called instantaneous form



- Euler's identity :-

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

$$\boxed{e^{\textcircled{-}j\theta} = \cos\theta \textcircled{-} j\sin\theta} = (e^{j\theta})^{*} \leftarrow \text{conjugate}$$

$$I(t) = I_o \cos(\omega t + \phi) \Rightarrow I_s = I_o e^{j\phi}$$
$$= I_o \angle\phi$$

$$\boxed{\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}}$$

$$\boxed{\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2}}$$

$$I(t) = Re\left\{I_o e^{j(\omega t + \phi)}\right\}$$

$$I(t) = Re\left\{I_o e^{j\phi} \underline{e^{j\omega t}}\right\}$$

Isolate the time ~~[illegible]~~ term.

$$I_{\textcircled{s}} = I_o e^{j\phi}$$

Phasor

CH 10 Electromagnetic wave Propagation :-

- Wave equation :-
  Second order partial equ. of $\overline{E}$-field or mag. field as a function of space and time.

- wave equ. of an $\overline{E}$-field in the Z-direction

$\downarrow$ wave direction

- partial $\frac{d^2E}{dt^2} - u^2\frac{d^2E}{dz^2} = 0$

← time ← space

- diff $\frac{d^2E}{dt^2} - u^2\frac{d^2E}{dz^2} = 0$

$$E(x,y,z;t) = R_e\{E_s e^{j\omega t}\}$$

$$(j\omega)^2 \cancel{e^{j\omega t}} E_s - u^2 \frac{d^2E_s}{dz^2} \cancel{e^{j\omega t}} = 0$$

source

$$\frac{d^2E_s}{dz^2} + \frac{\omega^2}{u^2} E_s = 0$$

بعتمد على الوسط

Let $\beta = \frac{\omega}{u}$ (phase constant) (rad/m).

$$\boxed{\frac{d^2E_s}{dz^2} + \beta^2 E_s = 0}$$ wave in z-direction.

-let $\frac{d}{dz} = m$

$$m^2 \cancel{E_s} + \beta^2 \cancel{E_s} = 0$$

$$\sqrt{m^2} = \sqrt{-\beta^2}$$

$$m = \pm\sqrt{-1}\,\sqrt{\beta^2}$$

$$\boxed{m = \pm j B}$$

if the roots are imagenary then the Solution is either Sin(B?) or cos(B?) or $e^{+jB?}$ or e^{-jB-}

- Solution to the wave equ. is:-

$$E_s^{(2)} = \underbrace{E_0^{+} e^{-jBz}}_{\text{wave travels in the +ve z-dir.}} + \underbrace{E_0^{-} e^{+jBz}}_{\text{wave travels in the -ve z-dir.}} \quad V/m$$

(Forward-wave)
(Incident wave)

(Backward wave)
(Reflected wave)

if the wave encounters different media.

-Convert to time domain :-

$$E(z;t) = Re\left\{ E_o^+ e^{-j(wt-\beta z)} + E_o^- e^{+j(wt+\beta z)} \right\}$$

$$E(z;t) = \underbrace{E_o^+ \cos(wt-\beta z)}_{\substack{\text{Forward} \\ \hat{a}k = +\hat{a}z}} + \underbrace{E_o^- \cos(wt+\beta z)}_{\substack{\text{Backward} \\ \hat{a}k = -\hat{a}z}} \quad V/m$$

$\hat{a}k$: unit vector in the wave direction

$$* \; u = \frac{\text{distance}}{\text{time}} = \frac{w}{\beta}$$

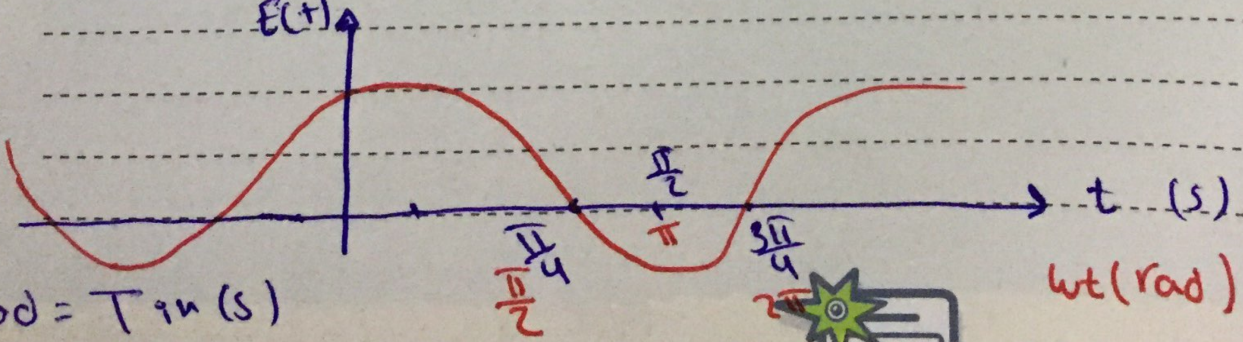
$$wt - \beta z = 0$$
$$wt = \beta z$$
$$w\,dt = \beta\,dz \rightarrow \overset{u}{\boxed{\frac{dz}{dt}}} = \overset{u}{\boxed{\frac{w}{\beta}}}$$

-continue for exp. with the forward wave:

$$E(z;t) = E_o^+ \cos(wt-\beta z) \quad V/m$$

Sketch:-
in Time domain
z is constant
i.e. $z=0$

$$E(t) = E_o^+ \cos(wt) \quad V/m$$



Period = T in (s)

-in Freq. domain:-

Let $t = 0$

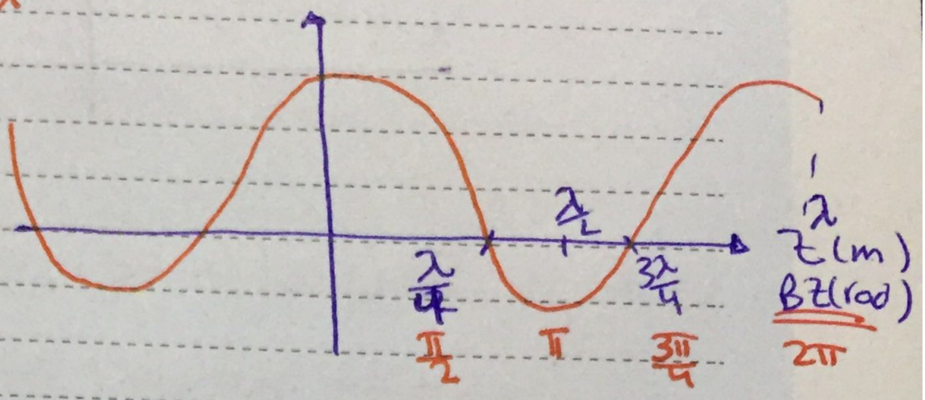
$E(z) = E_o^+ \cos(\beta z)$

$\frac{2\pi}{\lambda}$ ↙ ↘ λ

$u = \frac{\omega}{\beta} = \frac{\lambda}{T}$

$B = \frac{\omega T}{\lambda}$

λ: wave length (m)



-Ex: Given $\bar{E} = 50\cos(10^8 t + \beta x)\,\hat{a}_y$ V/m in free space $u = 3*10^8$

a) find the direction of wave propagation

b) calculate β and time it takes to travel a distance of $\frac{\lambda}{2}$

c) sketch the wave at $t = 0, \frac{T}{4}, \frac{T}{2}$ in space domain.

Sol: a) $\hat{a}_k = -\hat{a}_x$

(اتجاه عكس معامل β)

b) $\beta = \frac{\omega}{u} = \frac{2\pi}{\lambda} = \frac{10^8}{3*10^8} = \frac{1}{3}$ rad/m.

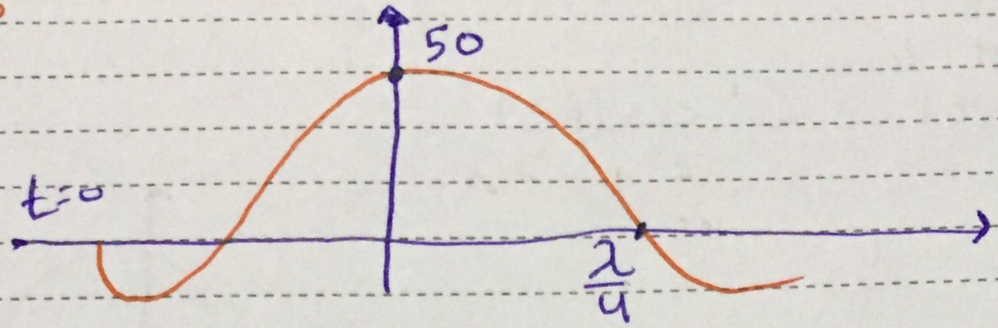
to travel a $\frac{\lambda}{2}$ distance it requires $\frac{T}{2}$ (s)

$T? \rightarrow \omega = \frac{2\pi}{T} \rightarrow \frac{T}{2} = \frac{2\pi}{2 10^8}$

Time required = 31.4 ns

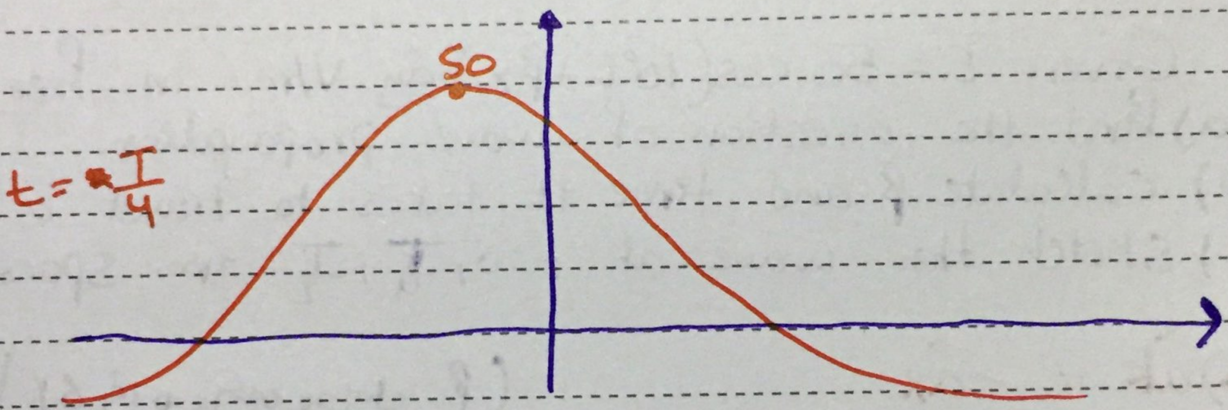
Period = 62.8 ns

c) ① $\bar{E}\Big|_{t=0} = 50\cos(\beta x)\,\hat{a_y}$



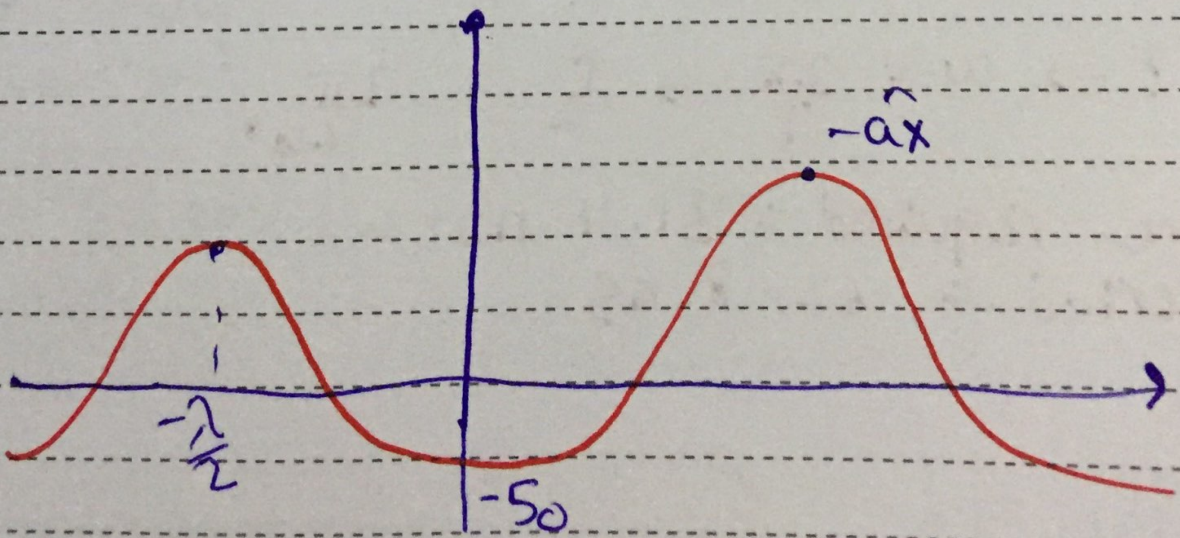
② $\bar{E}\Big|_{\frac{T}{4}} = 50(\cos(\beta x + 90°))\,\hat{a_y}$

$= -50\sin(\beta x)\,\hat{a_y}$



③ $\bar{E}\Big|_{\frac{T}{2}} = 50\cos(\beta x + 180°)\,\hat{a_y}$

$= -50\cos(\beta x)\,\hat{a_y}$



-Types of media :-

1) Lossy media ($\epsilon = \epsilon_0 \epsilon_r$, $\mu = \mu_0 \mu_r$, $\sigma \neq 0$)

2) Lossless media ($\epsilon = \epsilon_0 \epsilon_r$, $\mu = \mu_0 \mu_r$, $\sigma = 0$)

3) Free space ($\epsilon = \epsilon_0$, $\mu = \mu_0$, $\sigma = 0$)

4) Good (Super)(perfect) conductor ($\epsilon = \epsilon_0$, $\mu = \mu_0 \mu_r$, $\sigma \simeq \infty$)

* Wave propagation in lossless media:-

$$\gamma = \alpha + j\beta$$

γ : propagation constant
α : attenuation constant
β : phase constant

$$\boxed{\alpha = 0 \quad \text{Since } \sigma = 0}$$

$$\boxed{\beta = \omega \sqrt{\mu \epsilon}} \quad , \quad \boxed{u = \frac{\omega}{\beta}}$$

$$u = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\mu_r \epsilon_r}}$$, $c = 3 \times 10^8$ Speed of light

$$\lambda = \frac{2\pi}{\beta}$$

η : indrensie impedance in (Ω)

$$\boxed{\eta = \frac{\epsilon}{\mu}}$$ as mag.

$$\eta = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} \qquad \eta = |\eta| \angle \theta_\eta$$

$$= \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{\frac{\mu_r}{\epsilon_r}} \quad \angle 0°$$

$$\eta = 120\pi\ \Omega$$

- $\bar{E}$ and $\bar{H}$ are in phase → $\theta_\eta = 0$

$$\boxed{\bar{E} = E_0 \cos(\omega t - \beta z)\, \hat{a}_x}$$

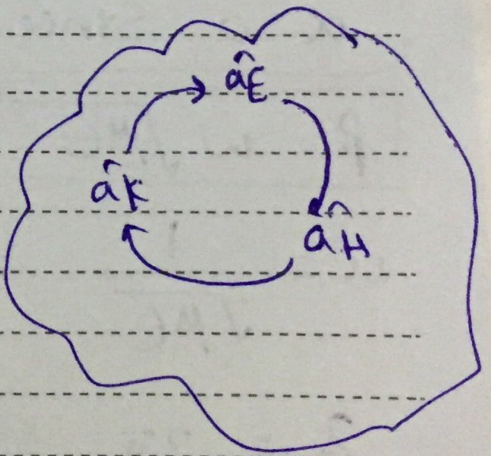
$$\boxed{\bar{H} = \underbrace{\frac{E_0}{\eta}}_{H_0} \cos(\omega t - \beta z)\, \hat{a}_y}$$

2) Free space :-

$\alpha = 0$, $\boxed{\beta = \omega \sqrt{\mu_0 \epsilon_0} = \frac{\omega}{c}}$

$\boxed{u = \frac{1}{\sqrt{w_0 \epsilon_0}}} = c$

$\boxed{\eta = \sqrt{\frac{\mu_0}{\epsilon_0}}} = 120\pi\ \Omega$



$\boxed{\lambda = 2\pi\beta}$, $\bar{E}$ & $\bar{H}$ are in phase

$\boxed{\text{Skindepth } (\delta) = \frac{1}{\alpha} = \infty}$

TEM: Transver Electro magnetic.

-Wave propagation in a good conductor :-

$\epsilon = \epsilon_0$, $\mu = \mu_0 \mu_r$, $\sigma \simeq \infty$

$\alpha \neq 0$ since $\sigma \neq 0$

$\alpha = \sqrt{\frac{\omega \mu \sigma}{2}}$

$\alpha = \sqrt{\pi f \mu \sigma}$

$\beta = \sqrt{\pi f \mu \sigma}$

$u = \frac{\omega}{\beta} \neq \frac{1}{\sqrt{\mu \epsilon}}$

$\lambda = \frac{2\pi}{\beta}$

$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$

$= \eta_1 \angle \theta_\eta$

η is complex.

$\alpha = 0$ in good Conductor

$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$

$-\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45°$

$z = r \angle \theta$

$\sqrt{z} = \sqrt{r} \angle \theta/2$

$j = 1 \angle \frac{\pi}{2} = 1 \angle 90°$

$\theta_\eta = 0$ if lossless or free space

$0 < \theta_\eta < 45°$ if lossy

$\theta_\eta = 45°$ if good rond

$\bar{E}_s = E_0 \, e^{-\gamma z} \, \hat{a}_x$ forward wave

$\bar{E}_s = \underbrace{E_0 \, e^{-\gamma z}}_{\text{Amplitud}} \, \underbrace{e^{-j\beta z}}_{\text{phase}} \, \hat{a}_x$

- convert to time domain

$\bar{E}(z,t) = Re\{\bar{E}_s \, e^{j\omega t}\}$

$\bar{E}(z,t) = E_0 \, e^{-\alpha z} \cos(\omega t - \beta z) \, \hat{a}_x$ V/m

$\bar{H} = \frac{\bar{E}}{\eta} = \frac{\bar{E}}{|\eta|\angle 45°}$

$\bar{H} = \frac{E_0}{|\eta|} \, e^{-\alpha z} \cos(\omega t - \beta z - 45°) \boxed{\hat{a}_y}$ A/m

$\hat{a}_H = \hat{a}_k \times \hat{a}_E$

$= \hat{a}_z \times \hat{a}_x$

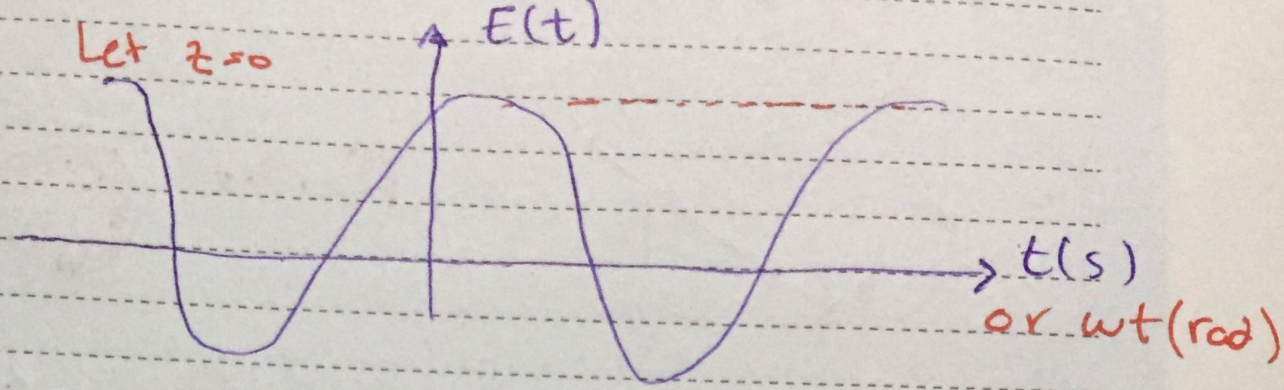
$= \hat{a}_y$

- $\bar{E}$ lead $\bar{H}$ by 45°

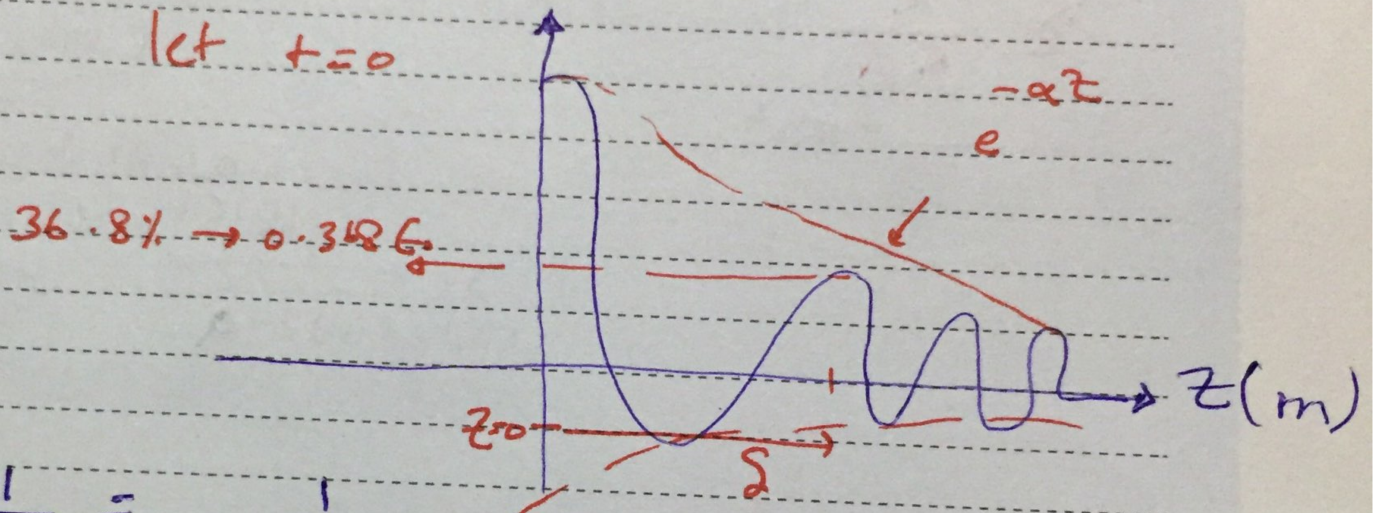
↓

Inductive media.

- in time domain



- in space domain



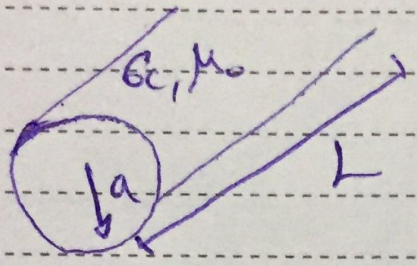
$$\delta = \frac{1}{\boxed{\alpha}} = \frac{1}{\sqrt{\pi f \mu \sigma}} \neq \infty$$

α → Np/m

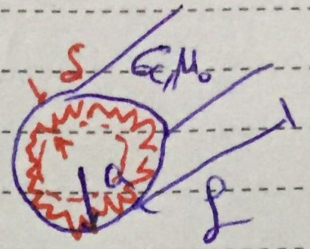
$$\boxed{\begin{aligned} 1\,Np &= 20 \log_{10} e \\ 1\,Np &= 8.686\,dB \end{aligned}}$$

- Application of Skin effect :-

DC
$f=0$



$f\uparrow \rightarrow \delta\downarrow$



AC
$f>0$

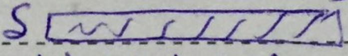
Diameter = 2a

$$R_{dc} = \frac{L}{\sigma_0 \underset{\downarrow \pi a^2}{\boxed{A}}}$$

$$R_{ac} = \frac{L}{\sigma_c \boxed{A}}$$

$$A \rightarrow \underbrace{(2\pi a)\,(\delta)}$$

ده اجا لما أعمل منطقة
المظللة بالأحمر ا كأنها
مستطيل



$2\pi a$ (من محيط الدائرة)

$$\frac{R_{ac}}{R_{dc}} = \frac{\dfrac{\cancel{L}}{\cancel{\sigma}\, 2\cancel{\pi} a \delta}}{\dfrac{\cancel{L}}{\cancel{\sigma}\, \cancel{\pi} a^2}} = \frac{a}{2\delta}$$

$R_{ac} = R_{dc}$ if $\delta \geq 2a$

$R_{ac} \gg R_{dc}$ at higher Freq.

- Loss tangent :-

$0 \leq \theta_\eta \leq 45^\circ$

$\tan(\theta) = \tan(2\theta_\eta) = \left(\frac{\sigma}{\omega\varepsilon}\right)$

$0 \leq \theta \leq 90^\circ$

$\theta = 2\theta_\eta$ = loss tangent angle

- Note

$\sigma = 0 \longrightarrow \theta = 0 \longrightarrow \theta_\eta = 0$

$* \tan\theta = \frac{\sigma}{\omega\varepsilon} = \frac{|\overline{J_s}|}{|\overline{J}_{ds}|} = \frac{\sigma|\cancel{E_s}|}{|j\omega\varepsilon \cancel{E_s}|} = \frac{\sigma}{\omega\varepsilon}$ $\left\{ \overline{J_d} = \frac{\partial D}{\partial t} \right.$

[redacted]

$\boxed{\overline{P} = \overline{E} \times \overline{H} \quad (\text{power})}$ (w/m2)

↳ poynting vector ↳ in time domain.

$* \boxed{\overline{P}_{avg} = \frac{1}{T}\int_{t} \overline{E} \times \overline{H}\, dt}$

$\boxed{\overline{P}_{avg} = \frac{1}{2}\, \mathrm{Re}\left\{ \overline{E_s} \times \overline{H_s}^* \right\}}$

time avg poynting vector. (w/m2)

$$\boxed{P_{avg} = \int_s \overline{P_{avg}} \cdot \overline{ds} \qquad (w)}$$

total power

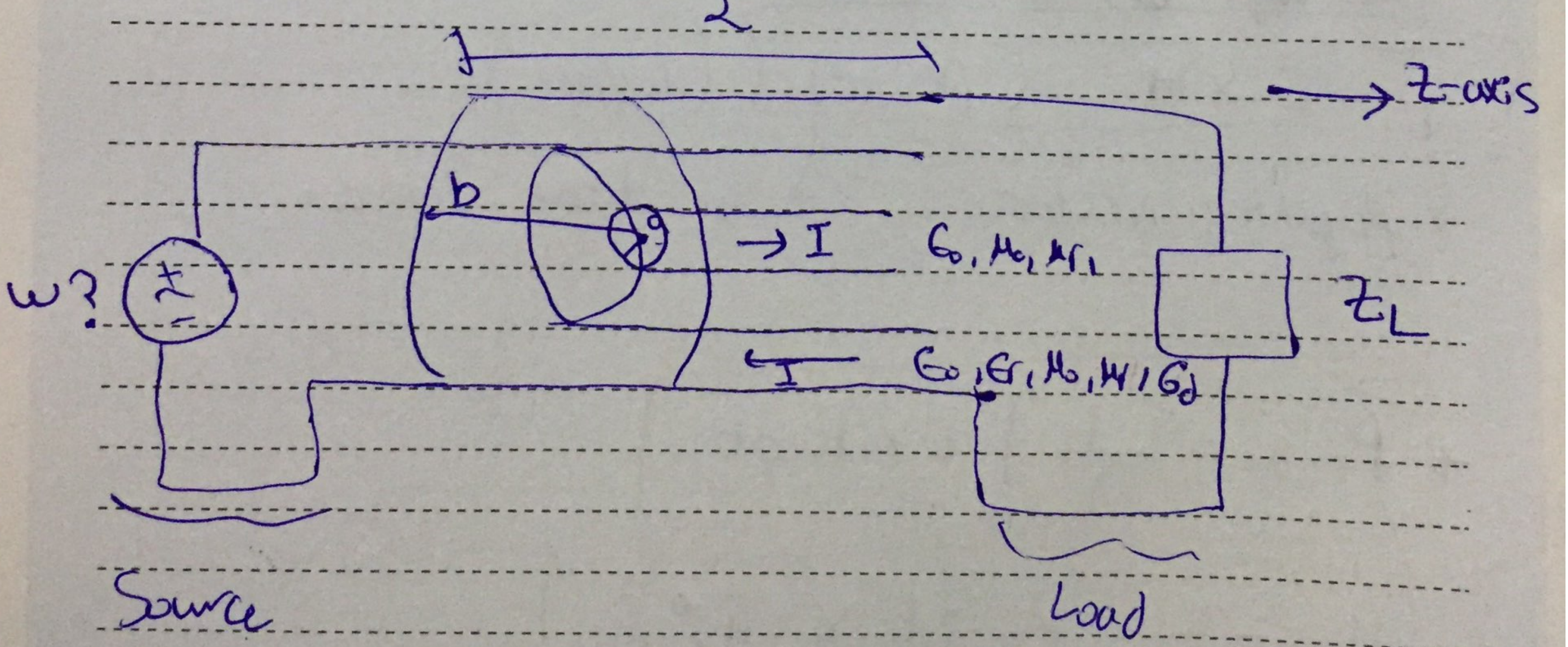
- CH 11 Transmission Lines :-

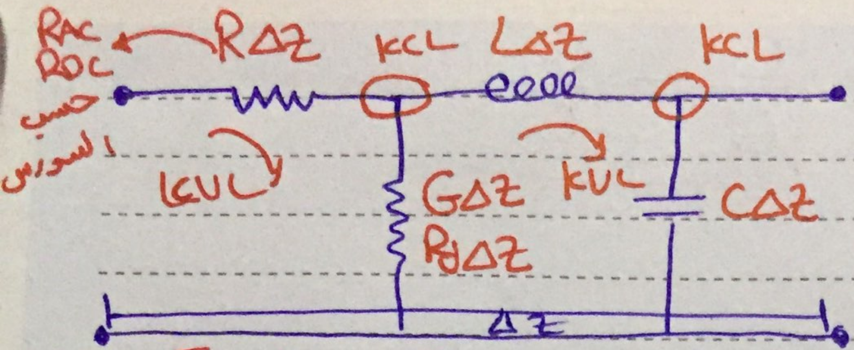
(2 or more conductor (T.L.)) → Transmission Line

Based on Maxwell's equ.

- Coaxial cable
- Twin wire cable
- Planar Lines
  - Strip line
  - micro Strip line
  - Slot line

- T.L. parametrs





$R_d = \frac{1}{G}$

Exact circuit.

- for a coaxial cables -

$$\frac{R}{l} = \frac{1}{2\pi \underset{\text{Conductor}}{\boxed{\sigma_c}}\, \delta}\left(\frac{1}{a} \underset{\text{Series}}{\boxed{+}} \frac{1}{b}\right) \quad (\Omega/m)$$

$$\frac{G}{l} = \frac{2\pi \boxed{\sigma_d}}{\ln\left(\frac{b}{a}\right)} \quad \text{Dielectric} \quad (S/m)$$

$$\frac{C}{l} = \frac{2\pi \boxed{\epsilon_0 \epsilon_r}}{\ln\left(\frac{b}{a}\right)} \quad \text{Dielectric}$$

ext

$$\frac{\boxed{L}}{l} = \frac{\boxed{\mu_0 \mu_r}}{2\pi} \ln\left(\frac{b}{a}\right) \quad (H/m)$$

dielectric

* $L_{int} \to 0$ as $\omega \to \infty$

Scanned with CamScanner

$$\tau_r = \frac{\epsilon_0 \epsilon_r}{\sigma_d} = R_d C = \boxed{\frac{C}{G} = \frac{\epsilon}{\sigma_d}}$$

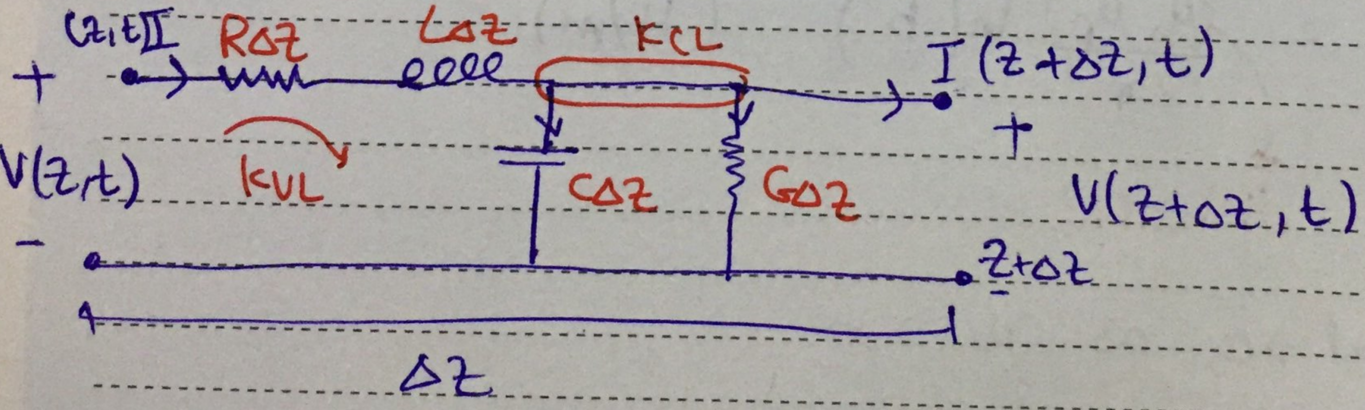
Relaxation Time

$$\boxed{* \frac{L_{ext}}{l} \quad * \frac{C}{l} = \mu\epsilon}$$

- Note:
$R \rightarrow$ conductor
$R_d \rightarrow$ dielectric

- T.L. equivalent ckt (Approximation):-
  - ↳ L- Type (95% accurate)
  - ↳ π- Type (99% accurate)
  - ↳ T- Type (99% accurate)

* L- Type equ. ckt:-



عنا V, I تجريبي

T.L. equation through Solving KVL & KCL equ.

- KVL : $\sum_{loop} V = 0$

$$-V(z,t) + R\Delta z\, I(z,t) + L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z,t) = 0$$

Simplify (خطوات التبسيط بالكتاب)

phasor

$$-\frac{\partial V_s(z)}{\partial z} = (R + j\omega L)\, I_s(z) \quad \text{--- ① Voltage wave equ.}$$

- KCL : $I_{in} = I_{out}$ at node.

$$I(z,t) = I(z+\Delta z,t) + G\Delta z V(z+\Delta z,t) + C\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t}$$

Simplify

hasor

$$-\frac{\partial I_s(z)}{\partial z} = (G + j\omega C)\, V_s(z) \quad \text{--- ② Current wave equ.}$$

- To find V, I ?

derive equ ① and substitute ② in ①

or derive equ ② and substitute ① in ②.

$$\Rightarrow -\frac{\partial^2 V_s(z)}{\partial z^2} = (R + j\omega L)\underbrace{\left(\frac{\partial I_s(z)}{\partial z}\right)}_{\text{equ ②}}$$

$$+\frac{d^2V_s(z)}{dz^2} = \underbrace{(R+jwL)(G+jwC)}\ V_s(z)$$

Constant called γ^2

$$\boxed{\frac{d^2V_s(z)}{dz^2} - \gamma^2 V_s(z) = 0} \rightarrow \text{T.L equ.}$$

$\gamma = \sqrt{(R+jwL)(G+jwC)}$

↳ Propogation constant

$\gamma = \alpha + j\beta$

α ↓ attenuation constant (Np/m)

β → phase constant (rad/m)

1 Np = 8.686 dB

$$\boxed{\frac{d^2I_s(z)}{dz^2} - \gamma^2 I_s(z) = 0} \rightarrow \text{T.L equ}$$

* Solution to the T.L. equation:-

$$\frac{d^2V_s(z)}{dz^2} - \gamma^2 V_s(z) = 0$$

let $m = \frac{d}{dz}$

$m^2 \cancel{V_s(z)} - \gamma^2 \cancel{V_s(z)} = 0$

$m^2 = \gamma^2$ (real)

Sol → $e^{\gamma z}$

$e^{-\gamma z}$

$$V_s(z) = V_0^{+} e^{-\gamma z} + V_0^{-} e^{+\gamma z}$$

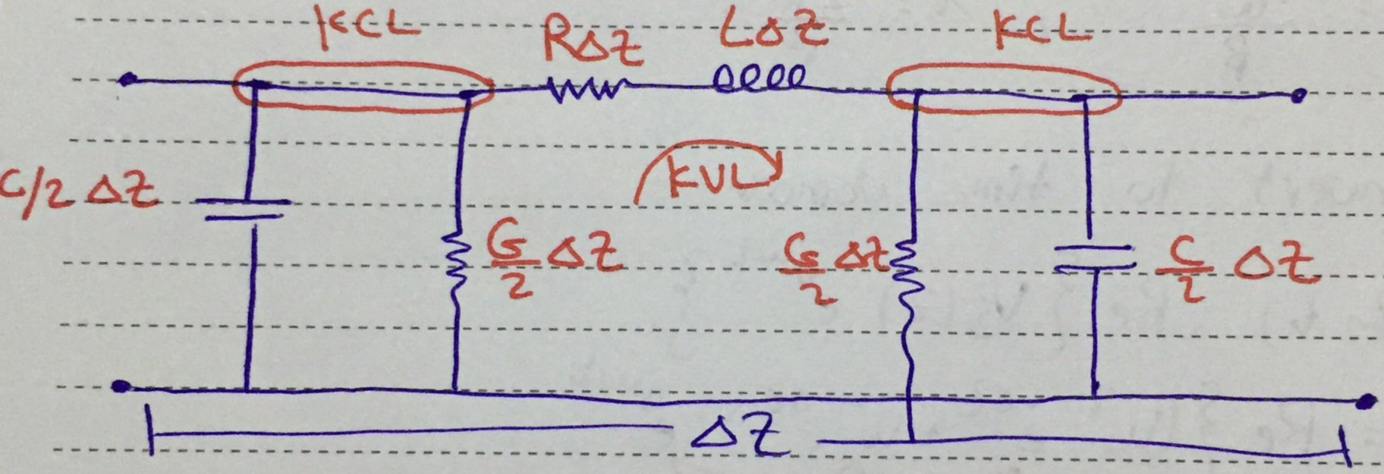
$$I_s(z) = I_0^{+} e^{-\gamma z} + I_0^{-} e^{+\gamma z}$$

Forward
عكس إشارة γ
(+ve z-dir)

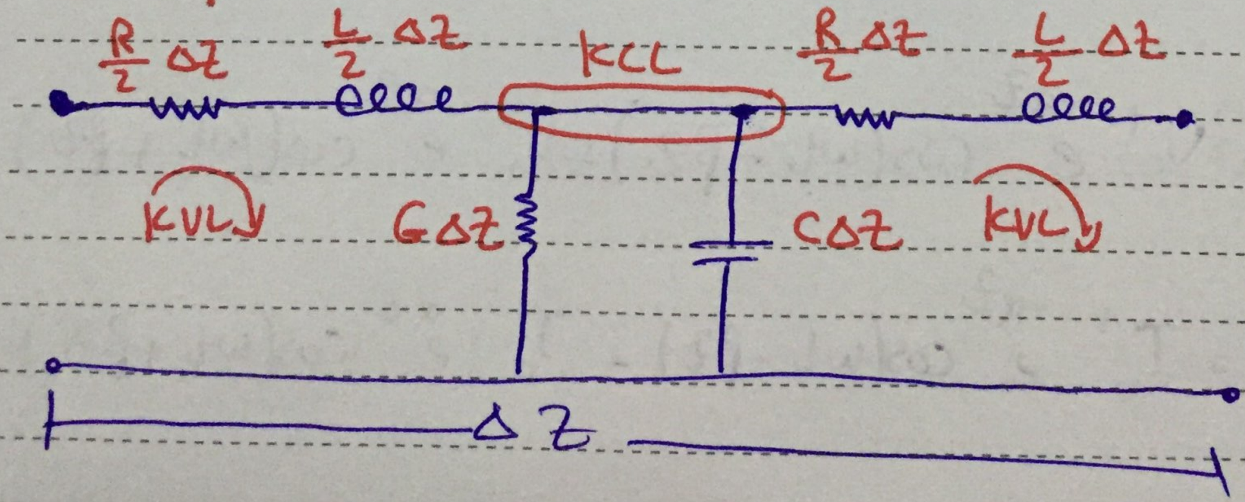
Backward
(-ve z-dir)

Backward and Forward لازم ناخد كلهم

* Π - equ. ckt.



* T- equ. ckt:-



- Voltage equ.

$$\frac{d^2 V_s(z)}{dz^2} - \gamma^2 V_s(z) = 0$$

Solution is:

$$V_s(z) = \underbrace{V_0^+ e^{-\gamma z}}_{\text{Incedent}} + \underbrace{V_0^- e^{\gamma z}}_{\text{Reflected}}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta$$

$$u = \frac{\omega}{\beta} \quad , \quad \lambda = \frac{2\pi}{\beta}$$

Convert to time domain -

$$V(z,t) = \mathrm{Re}\left\{ V_s(z)\, e^{j\omega t} \right\}$$

$$= \mathrm{Re}\left\{ \left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) e^{j\omega t} \right\}$$

$$= \mathrm{Re}\left\{ V_0^+ e^{-\alpha z} e^{-j\beta z} e^{j\omega t} + V_0^- e^{\alpha z} e^{j\beta z} e^{j\omega t} \right\}$$

$$V(z,t) = V_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_0^- e^{\alpha z} \cos(\omega t + \beta z)$$

$$I(z,t) = I_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + I_0^- e^{\alpha z} \cos(\omega t + \beta z)$$

-define Z_o: Characteristic impedance (Ω)

$$Z_o = \frac{V_o^+}{\underbrace{I_o^+}_{\text{+ve } z \text{ direction}}} = \ominus \frac{V_o^-}{\underbrace{I_o^-}_{\text{-ve } z \text{- direction}}}$$

$$Z_o = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

$$\hookrightarrow \boxed{I_o^+ = \frac{V_o^+}{Z_o}} \quad , \quad \boxed{I_o^- = \frac{V_o^-}{Z_o}}$$

-define Γ: reflection coefficient

$$\Gamma = \frac{\text{reflected Signal}}{\text{Incident Signal}} = \frac{V_o^-}{V_o^+}$$

$$\boxed{V_o^- = \Gamma V_o^+}$$

Note All the equations before are for a Lossy T.L. Since $(\alpha \neq 0)$

$(\sigma_c \neq \infty)$

$(\sigma_d \neq 0)$

- Special Types of T.L.:-

- Lossless T.L. $\rightarrow \sigma_c = \infty \rightarrow R = 0$
  $\hookrightarrow \sigma_d = 0 \rightarrow G = 0$

$R = 0 = G$

$$\gamma = \sqrt{(\underset{0}{\cancel{R}} + jwL)(\underset{0}{\cancel{G}} + jwC)} = \sqrt{-w^2 LC}$$

$$\boxed{\gamma = jw\sqrt{LC}} = \alpha_j \beta$$

$$\boxed{\alpha = 0} , \boxed{\beta = w\sqrt{LC}}$$

$$Z_0 = \sqrt{\frac{L}{C}} = R_0 + j X_0$$

$$R_0 = \sqrt{\frac{L}{C}} , X_0 = 0$$

$$u = \frac{w}{\beta} , \lambda = \frac{2\pi}{\beta}$$

$$V(z,t) = V_0^+ \cos(wt - \beta z) + V_0^- \cos(wt + \beta z)$$

$$I(z,t) = \underbrace{\frac{V_0^+}{Z_0}}_{I_0^+} \cos(wt + \beta z) - \underbrace{\frac{V_0^-}{Z_0}}_{I_0^-} \cos(wt + \beta z)$$

- Distortionless T. L. 8 -

$\boxed{\frac{R}{L} = \frac{G}{C}}$ شرط $\quad R \neq 0 , G \neq 0$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{R\left(1+j\omega\frac{L}{R}\right)G\left(1+j\omega\frac{C}{G}\right)}$$

$$= \left(1+j\omega\frac{L}{R}\right)\sqrt{RG}$$

$$= \underbrace{\sqrt{RG}}_{\alpha \neq 0} + \underbrace{j\omega\frac{L}{R}\sqrt{RG}}_{\beta}$$

$$\beta = \omega\sqrt{\frac{L^2 \cancel{R}G}{R^{\cancel{2}}}}$$

$$= \omega\sqrt{\frac{LLG}{R}}$$

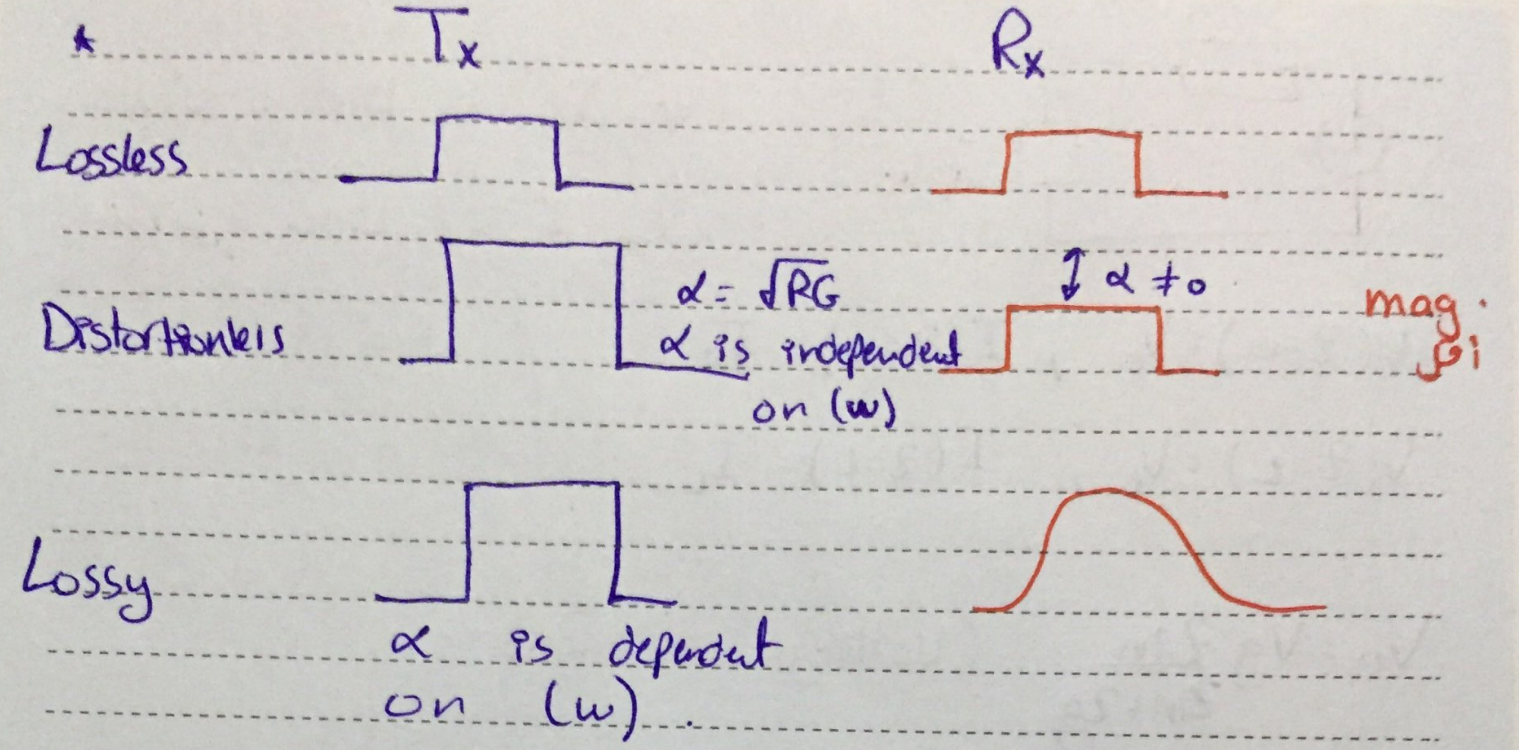
$$= \omega\sqrt{\frac{CL\cancel{G}}{\cancel{G}}}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

$$Z_0 = \sqrt{\frac{R\left(1+j\omega\frac{L}{R}\right)}{G\left(1+j\omega\frac{C}{G}\right)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

$u = \frac{\omega}{\beta}$, $\lambda = \frac{2\pi}{\beta}$

$$V_s(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z} \quad (V)$$

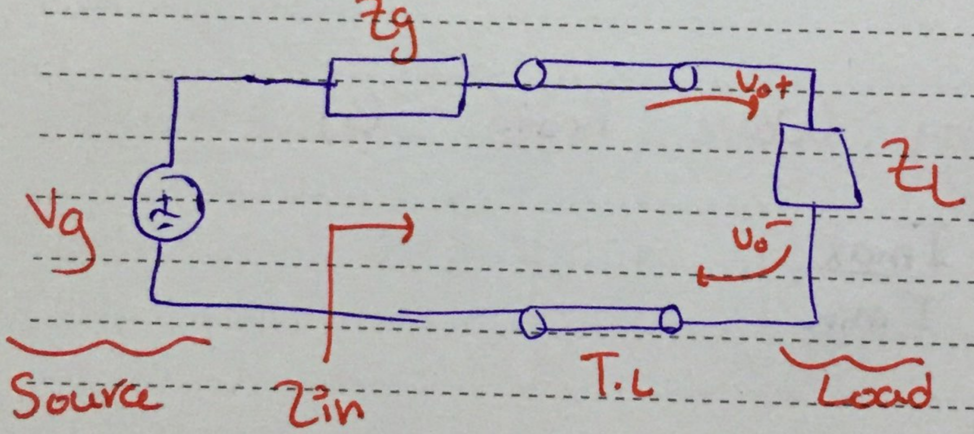


$\alpha = \sqrt{RG}$

α is independent on (ω)

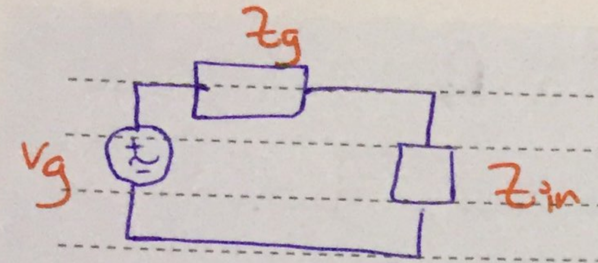
α is dependent on (ω).

* Input impedance, SWR and power



Zg: internal impedance of the src.

$$Z_{in} = \frac{Z_0 Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$ → Lossy and distortionless T.L.



$V_0, I_0 \rightarrow$ عند اول ال line

$V_L, I_L \rightarrow$ عند آخر ال line

$V(z=0)=V_0$, $I(z=0)=I_0$

$V(z=L)=V_L$, $I(z=L)=I_L$

$$V_0 = V_g \frac{Z_{in}}{Z_{in}+Z_g}$$ (voltage division).

$$I_0 = \frac{V_g}{Z_g+Z_{in}}$$ (ohm's Law)

- SWR 8 Standing Wave Ratio (s)

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}}$$

$$S = \frac{|V_0^+| + |V_0^-|}{|V_0^+| - |V_0^-|}$$

divide by $|V_0^+|$

$$S = \frac{1 + \left|\frac{V_0^-}{V_0^+}\right|}{1 - \left|\frac{V_0^-}{V_0^+}\right|}$$

$$|P| = \frac{|V_0^-|}{|V_0^+|}$$

$$S = \frac{1 + |\Gamma|}{1 + |\Gamma|} \rightarrow \boxed{|\Gamma| = \frac{S-1}{S+1}} \rightarrow \text{mag. ony}$$

$$* \; \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} \quad \text{if } Z_L \neq Z_o$$

$\downarrow$ reflection coefficient at the load.

$$\text{if } Z_g \neq Z_o \rightarrow \Gamma_g = \frac{Z_g - Z_o}{Z_g + Z_o}$$

- to cancel (Γ) $\rightarrow$ $Z_L = Z_o$, $Z_g = Z_o$

(matched T.L.)

(max power transfer)

* <u>Powers</u>

$$\text{Power} = \frac{1}{2} \, \text{Re}\left\{ \underset{\downarrow \atop \text{Peak}}{V_s} \; * \; \underset{\downarrow \atop \text{Peak}}{I_s}^{\overset{\text{conjugate}}{*}} \right\} \quad (\omega).$$

$$\boxed{V_{rms} = \frac{V_{peak}}{\sqrt{2}}}$$

- for lossless T.L.

$$V_s(z) = V_o^+ e^{-j\beta z} + \Gamma V_o^+ e^{+j\beta z}$$

$$I_s^*(z) = \frac{V_o^+}{Z_o} e^{+j\beta z} - \Gamma \frac{V_o^-}{Z_o} e^{-j\beta z}$$

$$P = \frac{|V_0^+|^2}{2Z_0}\left(\overset{P_i}{1} - \underset{P_r}{|\Gamma_L|^2}\right)$$

$P_t = P_i - P_r$

→ if $P_r = 0$ → $P_t = P_i$
(max power transfer)

$$\boxed{P_t = \frac{|V_0^+|^2}{2Z_0}}$$ (Max power transfer occurs when $Z_L = Z_0$)

* $Z_{in\ \underline{\underline{S.C}}} = jZ_0 \tan(\beta \ell)$ (if lossless)
Short circuit

$Z_{in\ \underline{\underline{O.C}}} = -jZ_0 \cot(\beta \ell)$ (if lossless)
open circuit

→ $S = \infty$, $\Gamma_L\ S.C = -1 = 1\angle 180°$

$\Gamma_L\ O.C = 1 = 1\angle 0°$

$$Z_0 = \sqrt{Z_{in\,S.C} * Z_{in\,O.C}}$$

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}}$$

الامتحان لنهاية 11.4
بالتوفيق ☺