30

The University of Jordan
Department of Mathematics
Linear Algebra I, First Exam

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Q1) (4 points) Let $M = \{(a, b, c, d) : a = 3b - c,\ d = 2c + b\}$. Find a basis for M and find $dim(M)$.

$M = \{(3b - c, b, c, 2c + b)\}$

$dim(M) = 2$ number of free elements

basis for $M = \{(3,1,0,1), (-1,0,1,2)\}$

4

Q2) (3 points) Let $s = \{2x^2 - 4x, 3x + 1, 5 - 2x\}$ be a basis of P_2 and let $u \in P_2$, if $(u)_s == \begin{bmatrix} -3 \\ 2 \\ 4 \end{bmatrix}$, find u.

$u = -3(2x^2 - 4x) + 2(3x + 1) + 4(5 - 2x)$

$u = -6x^2 + 12x + 6x + 2 + 20 - 8x$

$u = -6x^2 + 10x + 22$

3

Q3) (3 points) Let $W = \{(x, 2, y) : x, y \in R\}$, determine whether W a subspace of R^3 or not.

Let $u = (x_1, 2, y_1)$ $u \in W$

$v = (x_2, 2, y_2)$ $v \in W$

1) $u + v \stackrel{?}{=} v + u$

$(x_1 + x_2, 4, y_1 + y_2) = (x_2 + x_1, 4, y_2 + y_1) \notin W$

3

2) $ku = (kx_1, 2k, ky_1)$ $k \in \mathbb{R}$

$ku \notin W$ since $2k \neq 2$

$\therefore$ W is not subspace of R^3

Q6) (5 points) If $\| u \| = 2$, $\| v \| = 3$ and $\| 3u + v \| = 5$. Find $< u, v >$.

$$\|3u+v\|^2 = \langle 3u+v, 3u+v \rangle$$

$$25 = \langle 3u, 3u \rangle + \langle 3u, v \rangle + \langle v, 3u \rangle + \langle v, v \rangle$$

$$25 = 9 * 4 + 3 * \langle u, v \rangle + 3 \langle v, u \rangle + 9$$

$$25 = 36 + 6 \langle u, v \rangle + 9$$

$$25 = 45 + 6 \langle u, v \rangle$$

$$\boxed{\langle u, v \rangle = \frac{-20}{6}}$$

5

Q7) (5 points) Let $s = \{x^2 - 1, 3x - 2, x^2 + 2x - 1\}$, show that s is a basis of P_2.

$$k_1 (x^2 - 1) + k_2 (3x - 2) + k_3 (x^2 + 2x - 1) = 0$$

$x^2 \Rightarrow k_1 + k_3 = 0$ ---- (1)

$x \Rightarrow 3k_2 + 2k_3 = 0$ --- (2)

$1 \Rightarrow -k_1 - 2k_2 - k_3 = 0$ --- (3)

(1) and (3)

$$\begin{aligned} \cancel{k_1} + \cancel{k_3} &= 0 \\ -\cancel{k_1} - 2k_2 - \cancel{k_3} &= 0 \quad + \\ \hline -2k_2 &= 0 \end{aligned}$$

$\boxed{k_2 = 0}$

$\boxed{k_3 = 0}$

$\boxed{k_1 = 0}$

Linearly Independent

$$ax^2 + bx + c = k_1(x^2 - 1) + k_2(3x - 2) + k_3(x^2 + 2x - 1)$$

$a = k_1 + k_3$

$b = 3k_2 + 2k_3$

$c = -k_1 - 2k_2 - k_3$

$-2k_2 = a + c$

$$\boxed{k_2 = \frac{a+c}{-2}}$$

$$b = 3\left(\frac{a+c}{-2}\right) + 2k_3$$

$k_3 = \dots$

$k_1 = \dots$

s span P_2

5

s is a basis of P_2

Q4) (4 points) If A is 6×5 matrix and $nullity(A^t) = 3$, find $rank(A)$ and $nullity(A)$.

rank (A^t) + nullity (A^t) = number of columns (A^t)

rank (A^t) + 3 = 6

rank (A^t) = 3 = rank (A)

$\boxed{\text{rank } (A) = 3}$

4

rank (A) + nullity (A) = no of columns (A)

3 + nullity (A) = 5

$\boxed{\text{nullity } (A) = 2}$

Q5) (6 points) Let $W = \{(1,-1,1,-2), (-1,2,2,3), (1,0,4,-1)\}$. Find $W^{\perp}$ and find $dim(W^{\perp})$.

$WX = 0$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -2 & 0 \\ -1 & 2 & 2 & 3 & 0 \\ 1 & 0 & 4 & -1 & 0 \end{array}\right]$$

$$\begin{array}{c} r_1 + r_2 \\ -r_1 + r_3 \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 1 & -2 & 0 \\ 0 & 1 & 3 & 1 & 0 \\ 0 & 1 & 3 & 1 & 0 \end{array}\right]$$

$$-r_2 + r_3 \left[\begin{array}{ccccc} 1 & -1 & 1 & -2 & 0 \\ 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

6

$X_2 + 3X_3 + X_4 = 0$ Let $\boxed{X_4 = t}$

$\boxed{X_3 = s}$

$X_2 + 3s + t = 0$

$\boxed{X_2 = -3s - t}$

$X_1 - X_2 + X_3 - 2X_4 = 0$

$X_1 - (-3s - t) + s - 2t = 0$

$X_1 + 3s + t + s - 2t = 0$

$\boxed{X_1 = -4s + t}$

$\boxed{W^{\perp} = \text{null space} = \{(-4s+t), (-3s-t), (s), (t)\}}$

$\boxed{dim(W^{\perp}) = 2}$ number of free elements