Partner

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The University of Jordan
Department of Mathematics
Linear Algebra I, First Exam

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Q1) (1+2+2 points) Let = A $\begin{bmatrix} 3 & 6 \\ 2 & 3 \end{bmatrix}$

a) Find A^{-1}.
b) If $(3B - 2I)^{-1} = A$, find B.
c) If $AX = \begin{bmatrix} 2 & 1 & 4 \\ -1 & -2 & 3 \end{bmatrix}$. Find X.

a) $A^{-1} = \frac{1}{9-12}\begin{bmatrix} 3 & -6 \\ -2 & 3 \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} 3 & -6 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2/3 & -1 \end{bmatrix}$

b) $(3B - 2I)^{-1} = A$

$3B - 2I = A^{-1}$

$3B = A^{-1} + 2I \Rightarrow 3B = \begin{bmatrix} -1 & 2 \\ 2/3 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2/3 & 1 \end{bmatrix}$

$B = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ 2/3 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 2/3 \\ 2/9 & 1/3 \end{bmatrix}$

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c) $AX = \begin{bmatrix} 2 & 1 & 4 \\ -1 & -2 & 3 \end{bmatrix}$

$X = \begin{bmatrix} 2 & 1 & 4 \\ -1 & -2 & 3 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} -1 & 2 \\ 2/3 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ -1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} -4 & -5 & 2 \\ 4/3+1 & 2/3+2 & 8/3-3 \end{bmatrix}$

Q2) (2 points) Let A be a square matrix such that $A^2 + 5A - I = 0$. Show that A is invertible and $A^{-1} = A + 5I$.

$A^2 + 5A - I = 0$

$A^2 + 5A = I$

$A(A + 5I) = I$

$\therefore A^{-1} = A + 5I$ #

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$B = A + 5I$

$AB = I$

$\therefore B = A^{-1} = A + 5I$

Q5) (3 points) Let $A = \begin{bmatrix} 3 & -1 & 5 \\ 1 & k & 2 \\ -2 & 1 & 3 \end{bmatrix}$.

Find all values of k such that the system $AX = 0$ has only the trivial solution.

A shouldn't be invertible

$|A| \neq 0$

$|A| = 3(3k-2) + 1(3+4) + 5(1+2k)$

$= 9k - 6 + 7 + 5 + 10k$

$= 19k + 6 = 0$

$k \neq \frac{-6}{19}$

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Q6) (3 points) If A and B are square matrices of same order. Prove that $adj(AB) = adj(A)\, adj(B)$.

$(AB)^{-1} = \frac{adj(AB)}{det(AB)}$

$adj(AB) = (AB)^{-1} \, det(AB)$

$adj(A)\, adj(B) =$

$A^{-1}(det(A)) \cdot B^{-1}(det(B))$

$adj(A)\, adj(B) = (AB)^{-1} \, det(AB)$

$(BA)^{-1}$

$\therefore adj(AB) = adj(B)\, adj(B)$ #

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Q3) (4 points) Let $C = \begin{bmatrix} 2 & 1 & 4 \\ -1 & -2 & 3 \\ 2 & 1 & 6 \end{bmatrix}$. Use the adjoint method to find C^{-1}

$C_{11} = (-1)^2 \begin{vmatrix} -2 & 3 \\ 1 & 6 \end{vmatrix} = -12 - 3 = -15$

$C_{12} = (-1)^3 \begin{vmatrix} -1 & 3 \\ 2 & 6 \end{vmatrix} = -1(-6-6) = 12$

$C_{13} = (-1)^4 \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = -1 + 4 = 3$

$C_{21} = (-1)^3 \begin{vmatrix} 1 & 4 \\ 1 & 6 \end{vmatrix} = -1(6-4) = -2$

$C(c) = \begin{bmatrix} -15 & 12 & 3 \\ -2 & 4 & 0 \\ 11 & -10 & -3 \end{bmatrix}$ 4 $\quad adj(c) = \begin{bmatrix} -15 & -2 & 11 \\ 12 & 4 & -10 \\ 3 & 0 & -3 \end{bmatrix}$

$det(c) = 2(-15) - (-12) + 4(3)$

$= -30 + 12 + 12$

$= -6$

$C^{-1} = \frac{adj(c)}{det(c)} = \frac{1}{-6} \begin{bmatrix} -15 & -2 & 11 \\ 12 & 4 & -10 \\ 3 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 15/6 & 1/3 & -11/6 \\ -2 & -2/3 & 10/6 \\ -\frac{1}{2} & 0 & 1/2 \end{bmatrix}$

Q4) (3 points) Use Gaussian elimination to solve the following system

$-x - 10y + 10z = -31$
$2x - y + z = -1$
$3x + 2y - 2z = 9$

$\left[\begin{array}{ccc|c} -1 & -10 & 10 & -31 \\ 2 & -1 & 1 & -1 \\ 3 & 2 & -2 & 9 \end{array}\right] -r_1$

$\left[\begin{array}{ccc|c} 1 & 10 & -10 & 31 \\ 2 & -1 & 1 & -1 \\ 3 & 2 & -2 & 9 \end{array}\right] \begin{array}{l} -2r_1 + r_2 \\ -3r_1 + r_3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 10 & -10 & 31 \\ 0 & -21 & 21 & -63 \\ 0 & -28 & 28 & -84 \end{array}\right] \frac{-1}{21} r_2$

$\left[\begin{array}{ccc|c} 1 & 10 & -10 & 31 \\ 0 & 1 & -1 & 3 \\ 0 & -28 & 28 & -84 \end{array}\right] 28r_2 + r_3$

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$\rightarrow \left[\begin{array}{ccc|c} 1 & 10 & -10 & 31 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$y - z = 3 \quad \boxed{z = t} \quad t \in \mathbb{R}$

$\boxed{y = 3 + t}$

$x + 10y - 10z = 31$

$x + 10(3+t) - 10t = 31$

$x + 30 + 10t - 10t = 31$

$\boxed{x = 1}$

$\{(1, 3+t, t) : t \in \mathbb{R}\}$

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