EM II

D.r yanal

By RAFAT TAYYEM

POWER UNIT

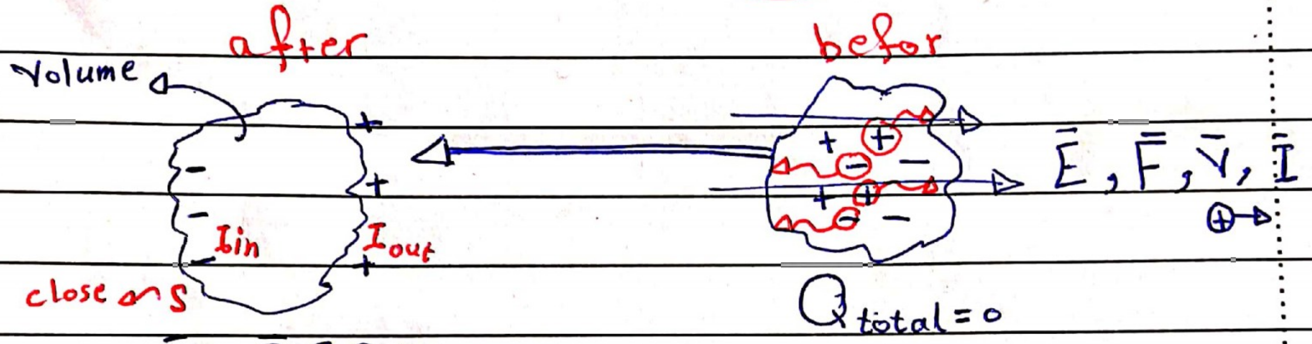
Scanned by CamScanner

22/9/2019

EM 2 :-

⇒ Continuity equation :-

$$\nabla\cdot\bar{J} = -\frac{\partial \rho_v}{\partial t}$$



$\bar{F} = \bar{Q}E$

KCL: $I_{out} = I_{in}$

$$\oint_s \bar{J}\cdot d\bar{s} = -\frac{\partial q_{in}}{\partial t}, \quad Q = \int_v \rho_v \, dv$$

$$\oint_s \bar{J}\cdot d\bar{s} = -\frac{\partial}{\partial t}\int \rho_v \, dv$$

$\nabla\cdot\bar{D} = \rho_v$ ⇒ Maxwell eq (Gauss's low)

$$\oint_s \bar{D}\cdot d\bar{s} = Q_{en.} = \int_v \rho_v \, dv$$

→ Divergence Thearem

$$\oint_s \Rightarrow \int_v$$

$$\cancel{\int_v} \nabla\cdot\bar{D}\,\cancel{dv} = \cancel{\int_v} \rho_v \,\cancel{dv}$$

⟹

Scanned by CamScanner

$$\oint_s \bar{J} \cdot d_s = -\frac{\partial}{\partial t} \int_v \nabla \cdot D \, dv$$

$$\oint_v \cancel{\nabla} \cdot \bar{J} \, \cancel{dv} = -\oint_v \cancel{\nabla} \cdot \frac{\partial \bar{D}}{dt} \, \cancel{dv}$$

$$\boxed{\bar{J}_d = -\frac{d\bar{D}}{\partial t}} \Longrightarrow \text{Displacement Current dencity } (A/m^2)$$

$$\boxed{\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{dt}}$$

Maxwell's eq. in time varying field,

Sourre :- charge moving with acceleration

& AC current flowing in the wire

Space time

$$\left.\begin{array}{l} \rho_v(x, y, z, t) \\ J(x, y, z, t) \end{array}\right\} \Longrightarrow \begin{array}{c} \bar{E} \\ \bar{D} \\ \bar{H} \\ \bar{B} \end{array} (x, y, z, t)$$

$\Longrightarrow$

Scanned by CamScanner

Subject Date No.

Diff	Integral
1 $\nabla \cdot \bar{D} = \rho_v$	$\oint_s \bar{D} \cdot d\bar{s} = \int_v \rho_v dv$
2 $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$	$\oint_l \bar{E} \cdot d\bar{l} = -\int_s \frac{\partial \bar{B}}{\partial t} \cdot ds$
3 $\nabla \times \bar{H} = \bar{J} + \bar{J}_d$	$\oint_l \bar{H} \cdot d\bar{l} = \oint_s \bar{J} \cdot ds + \int_s \frac{\partial \bar{D}}{\partial t} \cdot ds = \oint (\bar{J} + \frac{\partial \bar{D}}{\partial t}) \cdot d\bar{s}$
4 $\nabla \cdot \bar{B} = 0$	$\oint_s \bar{B} \cdot d\bar{s} = 0$

AC/DC: Conduction $\bar{J} = \sigma \bar{E}$ | DC: Convection $\bar{J} = \rho_v \bar{u}$ | AC: Displacement $\bar{J}_d = \frac{\partial \bar{D}}{\partial t}$

$$\bar{E}(x, y, z, t) = E_s(x, y, z) e^{j\omega t} \Longrightarrow \text{sinusoidal}$$

Euler's Identity

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2}$$

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

sec: 9.7

Maxwell's eq in Phasor Domain (No time)

1 $\nabla \cdot \bar{D}_s = \rho_v$	$\oint D_s \cdot d\bar{s} = \int \rho_v dv$
2 $\nabla \times \bar{E}_s = -j\omega \bar{B}_s$	$\oint E_s \cdot dl = -j\omega \int \bar{B}_s \cdot d\bar{s}$
3 $\nabla \times \bar{H}_s = (\sigma + j\omega\varepsilon) E_s$	$\oint_l \bar{H}_s \cdot d\bar{l} = \oint_s (\sigma + j\omega\varepsilon) E_s \cdot ds$
4 $\nabla \cdot B_s = 0$	$\oint_s B_s \cdot ds = 0$

NOTEBOOK

Scanned by CamScanner

9.6

Subject Date 24/9/2019 No.

Time Varying Potential :

in DC : $V = \dfrac{Q}{4\pi \epsilon_o r}$ → Point charge

$V = \int_v \dfrac{\rho_v \, dv}{4\pi \epsilon_o r}$, $\bar{A} = \int_v \dfrac{\mu_o \bar{J} \, dv}{4\pi r}$

electric Potential (scalar)

Magnatic potential (vector)

→ Maxwell eq:

$\nabla \cdot \bar{D} = \rho_v$

$\nabla \times \bar{E} = -\dfrac{\partial \bar{B}}{\partial t}$

$\nabla \times H = \bar{J} + \dfrac{d\bar{D}}{\partial t}$

$\nabla \bar{B} = 0$

$\bar{D} = \epsilon_o \bar{E}$

$\bar{B} = \mu_o \bar{H}$

$\bar{J} = r\bar{E}$

$\bar{B} = \nabla \times \bar{A}$

$\bar{E} = \nabla V$

$\nabla \times (-\nabla V) = 0$

$\nabla \times \nabla \times \bar{A} = \nabla^2 \bar{A} + \nabla(\nabla \cdot \bar{A})$

$\nabla \times \bar{E} = -\dfrac{\partial \bar{B}}{\partial t} = -\dfrac{\partial}{\partial t}(\nabla \times \bar{A})$

$\nabla \times \left(\bar{E} + \dfrac{\partial \bar{A}}{\partial t}\right) = 0$

$E + \dfrac{\partial \bar{A}}{\partial t} = -\nabla V$

$\boxed{\bar{E} = -\nabla V - \dfrac{\partial \bar{A}}{\partial t}}$ ← (1)

always

Scanned by CamScanner

Subject Date No.

$\nabla \bar{D} = \rho_v \rightarrow \nabla . \bar{E} = \frac{\rho_v}{\epsilon}$ ϵ is homogeneous

$\epsilon = \epsilon_o \epsilon_r$

Take div of eq(1)

$$\nabla \bar{E} = \frac{\rho_v}{\epsilon} = -\nabla^2 V - \frac{d}{\partial t} (\nabla . \bar{A})$$

$$\boxed{\nabla^2 V + \frac{\partial}{\partial t} (\nabla . \bar{A}) = \frac{-\rho_v}{\epsilon}} \text{ --- } (2)$$

$$\nabla^2 V = \frac{-\rho_v}{\epsilon} \rightarrow \text{Poisson's (DC)}$$

$$\nabla \times \bar{H} = \bar{J} + \epsilon \frac{d \bar{E}}{\partial t}$$

eq(i) $\mu = \mu_o \mu_r$

$$\nabla \times \bar{B} = \mu \bar{J} + \mu \epsilon \frac{\partial \bar{E}}{\partial t}$$

$$\nabla \times \bar{B} = \mu \bar{J} + \mu \epsilon \frac{\partial}{\partial t} \left(-\nabla V - \frac{\partial \bar{A}}{\partial t} \right)$$

$$\nabla \times \nabla \times \bar{A} = \mu \bar{J} + \mu \epsilon \frac{\partial}{\partial t} (-\nabla V) - \mu \epsilon \frac{\partial^2 \bar{A}}{\partial t^2}$$

$$\nabla (\nabla . \bar{A}) - \nabla^2 \bar{A} = \mu \bar{J} - \nabla \left(\mu \epsilon \frac{\partial V}{\partial t} \right) - \mu \epsilon \frac{\partial^2 \bar{A}}{\partial t^2}$$

NOTEBOOK

Scanned by CamScanner

Subject Date No.

assume:-

$$\nabla(\nabla\cdot\bar{A}) = \nabla\left(-\mu\epsilon\frac{\partial V}{\partial t}\right) \rightarrow \text{sub. in } 2$$

$$\boxed{\nabla^2\bar{A} = \mu\epsilon\frac{\partial^2 A}{\partial t^2} = -\mu\bar{J}} \quad ---4.$$

$$\boxed{\nabla^2 V - \mu\epsilon\frac{\partial^2 V}{\partial t^2} = -\frac{\rho_v}{\epsilon}} \quad --(2)'$$

→ wave eq

Helmholtz's eq

$$V = \int_v \frac{\rho_v \, dv}{4\pi\epsilon_0 r}, \quad \bar{A} = \int_v \frac{\mu_0 \bar{J}\,dv}{4\pi r}$$

sol to (4) and (2)

$$V = \int_v \frac{[\rho_v]\,dv}{4\pi\epsilon_0 r}, \quad \bar{A} = \int_v \frac{\mu_0 [\bar{J}]\,dv}{4\pi r}$$

$[\rho_v] \equiv$ retarted volume charge density (C/m^3)

$[J] \equiv$ " " current ~~d~~ (A/m^3)

(Delay)

Scanned by CamScanner

Subject Date No.

$\rho_v(x, y, z, t)$ in time domain

$[\rho_v](x, y, z, t')$ $\quad t' = t - \frac{r}{u}$

u: depends on media characteristic (ϵ, μ, σ)

$$u = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c = 3\times10^{6} \quad (\sigma = 0)$$

Ch.10:- EM wave propagation :-

Waves properties :-

- All travel without a mean.
- " " at high speeds.
- waves → Periodic → sinusoidal

* $\sin(\varphi \pm 180°) = -\sin\varphi$
* $\cos(\varphi \pm 180°) = -\cos\varphi$
* $\sin(\varphi \pm 90°) = \pm\cos\varphi$
* $\cos(\varphi \pm 90°) = \mp\sin\varphi$

* type of media :-

1. lossy media : $(\epsilon = \epsilon_0\epsilon_r, \mu = \mu_0\mu_r, \sigma \neq 0)$
2. Lossless media : $(\epsilon = \epsilon_0\epsilon_r, \mu = \mu_0\mu_r, \sigma = 0)$
3. Free space : $(\epsilon = \epsilon_0, \mu = \mu_0, \sigma = 0)$
4. Good conductors : $(\epsilon = \epsilon_0, \mu = \mu_0\mu_r, \sigma \simeq \infty)$

Not Magnets $\epsilon_r \cong .9999$

$\mu_r = 1$

NOTEBOOK

Scanned by CamScanner

Subject Date No.

wave eq:

take:

$\nabla^2 V - \mu \epsilon \frac{\partial^2 V}{\partial t^2} = \frac{-\rho_v}{\epsilon}$ $V = \int \bar{E}\, d\ell$, $\bar{E} = -\nabla V$

, $E = \frac{V}{d}$ (V/m

$\nabla^2 E - \mu \epsilon \frac{\partial^2 E}{\partial t^2} = V$ $\rho_v = 0 \rightarrow$ ~~surface~~ source free

convert to freq domain :-

$E(x,y,z,t) \rightarrow E_s(x,y,z)\, e^{j\omega t}$

E phaser

$\nabla^2 E_s + \mu \epsilon \omega^2 E_s = 0$

$\nabla^2 E_s + \frac{\omega^2}{u^2} E_s = 0$, $u = \frac{1}{\sqrt{\mu \epsilon}}$

wave eq for lossless or free space

$u = \frac{1}{\sqrt{\mu \epsilon}}$ $u = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$$\boxed{\nabla^2 E_s + \beta^2 E_s = 0}$$

NOTEBOOK

Scanned by CamScanner

Subject Date No.

wave eq

$$\nabla^2 E_s + \beta^2 E_s = 0$$

- source free ($\rho_v = 0, J = 0$)
- lossless ($\sigma = 0$)

β: phase constant (rad/m)

$\beta = \frac{\omega}{u}$, $u = \frac{1}{\sqrt{\mu\epsilon}}$ only if $\sigma = 0$

$$\nabla^2 H_s + \beta^2 H_s = 0$$

solution of wave eq :-

∇^2 = laplacian

$\nabla \cdot \nabla$

$$\nabla^2 E_s = \cancel{\frac{\partial^2 E_s}{\partial x^2}} + \cancel{\frac{\partial^2 E_s}{\partial y^2}} + \frac{\partial^2 E_s}{\partial z^2}$$

Assume the wave is travelling in one direction :-

i.e take the Z-direction

$$\frac{\partial^2 E_s}{\partial z^2} + \beta^2 E_s = 0$$

let $\frac{\partial}{\partial z} = m$

$$m^2 E_s + \beta^2 E_s = 0$$

$* \; m = \pm j\beta \rightarrow$ imagenary

sol $\rightarrow$ sin or e^{j}
cos e^{-j}

$$E_s = \underbrace{E_0^+}_{\text{forward wave}} e^{-j\beta z} + \underbrace{E_0^-}_{\text{Backward wave}} e^{j\beta z}$$

forward wave: +ve - Z axis

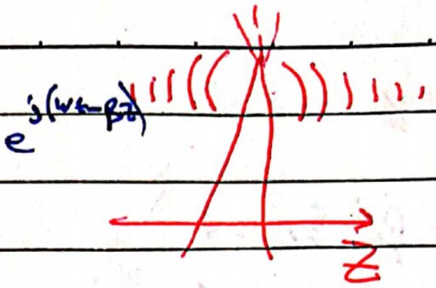
Backward wave: -ve - Z axis

NOTEBOOK

Scanned by CamScanner

tak the forward wave

$E_s = E_o^+ e^{-j\beta z}$ V/m



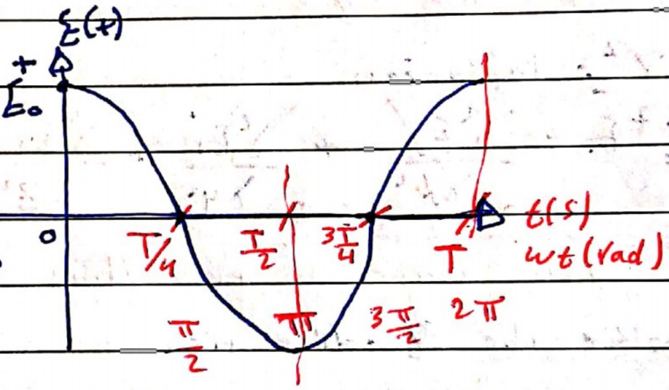
in time domain

$E(z,t) = Re\{E_s e^{j\omega t}\}$

$$\boxed{E(z,t)\ E_o^+ \cos(\underbrace{\omega}_{rad/s} \underbrace{t}_{s} - \underbrace{\beta}_{rad/m} \underbrace{z}_{m}) \text{ V/m}}$$

as a function of time

let z=0



Period ≡ T, freq = $\frac{1}{T}$, $\omega = 2\pi f$

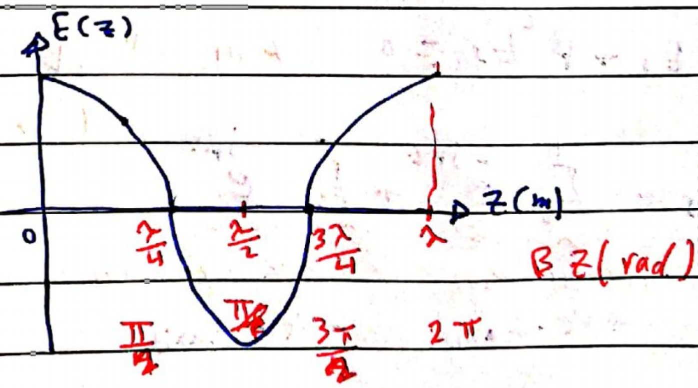
$\omega = \frac{2\pi}{T}$

as a function of space

let t=0

$\cos(-\theta) = \cos(\theta)$

$E(z) = E_o^+ \cos(\beta z)$



wave length ≡ λ $\quad u = \frac{\lambda}{T} = \frac{\omega}{\beta} = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}}$, $\lambda = \frac{2\pi}{\beta}$ (m)

NOTEBOOK

$u = \frac{\lambda}{T}$ ↓λ f↑

Scanned by CamScanner

Ex :- if $\bar{E} = 50 \cos(10^8 t + \beta x)\, \hat{a}_y$ V/m in free space

اشارة موجبة $\hat{a}_E$

$\hat{a}_k = -\hat{a}_x$ Backward اذا كانت β موجبة فهي

$\hat{a}_k \equiv$ unit vector in the wave direction.

Find :- [a] $\hat{a}_k$ → done

[b] calculate β and the time required to travel $\frac{\lambda}{2}$ m.

[c] sketch the wave at $t = 0, \frac{T}{4}, \frac{T}{2}$.

[b] $\beta = \frac{\omega}{u} = \frac{\omega}{c} = \frac{10^8}{3\times10^8} = \frac{1}{3}$ rad/m

$\lambda = \frac{2\pi}{\beta} = 6\pi$ m but in Example $\frac{\lambda}{2} = 3\pi$ m

$\frac{\lambda}{2} \longrightarrow \frac{T}{2}$ $\quad t = \frac{T}{2} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega} = \frac{3.14}{10^8} = 31.4$ ns

$c = \frac{\lambda/2}{T/2} = 3\times10^8$

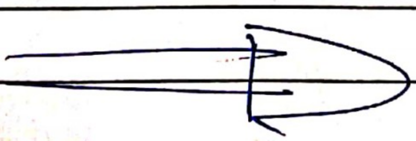
[C] at t=0 $\quad E(x) = 50 \cos(\beta x)\, \hat{a}_y$

at $t = \frac{T}{4}$ $\quad E(x) = 50 \cos(\beta x + 90^\circ)$ → $\omega = \frac{2\pi}{T} \cdot \frac{T}{4} = \frac{\pi}{2} = 90$

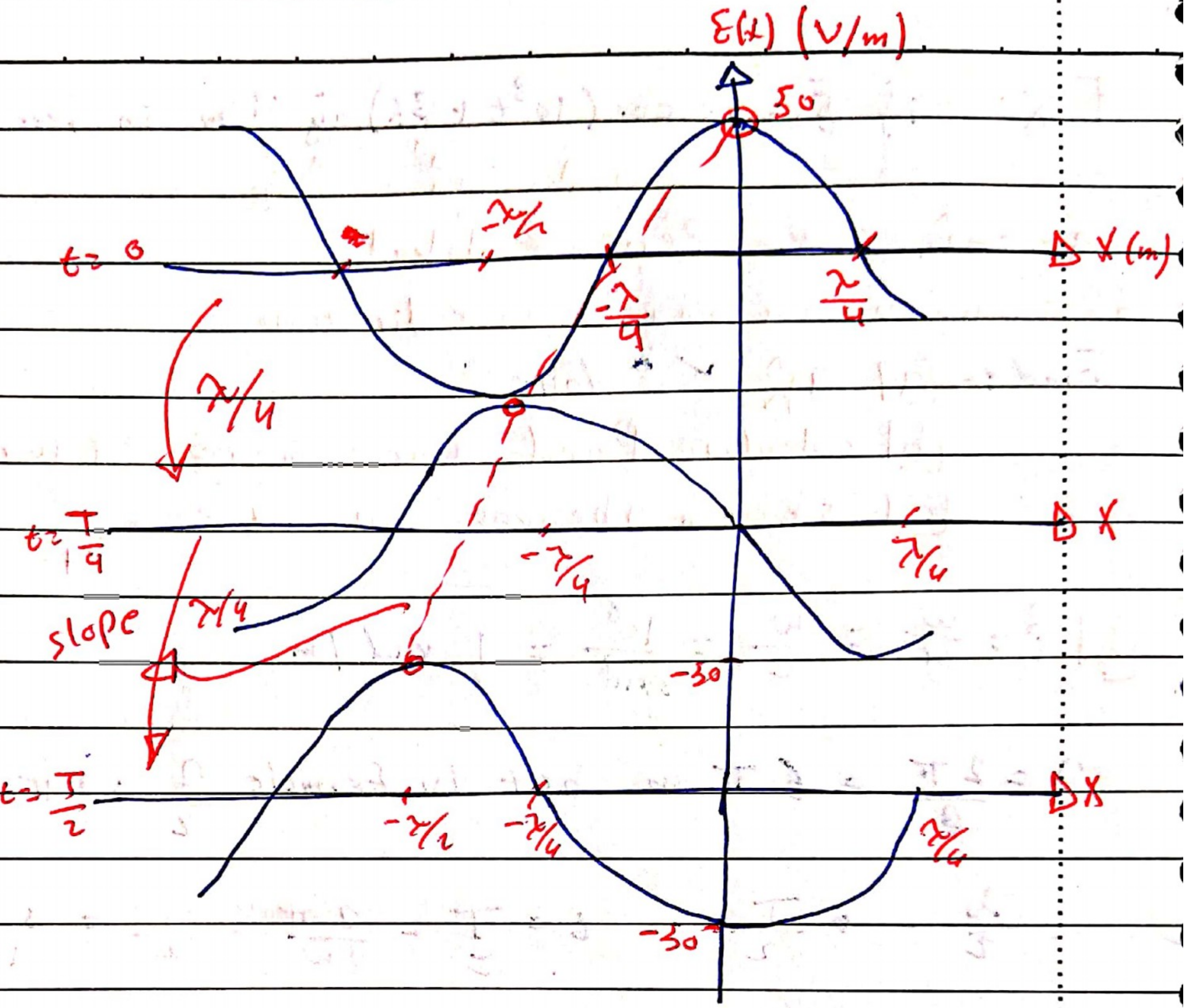
$= -50 \sin(\beta x)\, \hat{a}_y$

at $t = \frac{T}{2}$ $\quad E(x) = 50 \cos(\beta x + 180^\circ)\, \hat{a}_y$

$= -50 \cos(\beta x)\, \hat{a}_y$



Scanned by CamScanner



$\bar{H} \perp \bar{E}$

$\perp \hat{a_k}$

$\hat{a_E} = \hat{a_y}$

$\hat{a_k}$ $\hat{a_H}$

$= -\hat{a_x}$

$\hat{a_H} = \hat{a_k} \times \hat{a_E}$

$= -\hat{a_x} \times \hat{a_y}$

$= -\hat{a_z}$

wave $\perp \bar{E}, \bar{H}$

$\Downarrow$

TEM

Transverse

Electric

magnetic

Scanned by CamScanner

2019/9/29

* wave Propagation in lossy ($\sigma \neq 0$)
① ielectrics :-
(general Case)

→ Start from Maxwell's eq.

X $\nabla \cdot \epsilon E_s = 0$ ($\rho_v = 0$) ∴ source free

$$\begin{cases} \nabla \times \bar{E}_s = -j\omega\mu\bar{H}_s \quad (\bar{B}_s = \mu\bar{H}_s) \\ \nabla \times \bar{H}_s = (\sigma + j\omega\epsilon) E_s, \quad \bar{J} = \sigma\bar{E} \quad (\sigma \neq 0) \end{cases}$$

X $\nabla \cdot \bar{B}_s = 0$

$$\boxed{\nabla^2 E_s + \beta^2 E_s = 0}$$ → lossless
→ free space

Take the Curl for eq $\nabla \times \bar{E}_s$

$$\nabla \times \nabla \times \bar{E}_s = \nabla \times (-j\omega\mu\bar{H}_s)$$

$$\nabla(\cancel{\nabla \cdot \bar{E}_s}) + \nabla^2 \bar{E}_s = + j\omega\mu \nabla \times \bar{H}_s$$

$$\nabla^2 \bar{E}_s = \underbrace{j\omega\mu(\sigma + j\omega\epsilon)}_{\gamma^2} \bar{E}_s$$

$$\boxed{\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0}$$

γ = Propagation Constant

$\gamma = \alpha + j\beta$ → Phase Constant (rad/m) $\beta = \frac{\omega}{u}$

→

Scanned by CamScanner

Subject Date No.

$\alpha \equiv$ Attenuation constant

(NP/m) or dB/m

Neper

$1 NP = 8.686 dB$

$= 20 \log_{10} e$

given in exam

$$\boxed{\begin{aligned}\gamma &= \sqrt{-w^2\mu\epsilon + jw\mu\sigma} \\ &= \alpha + j\beta\end{aligned}}$$

To find α and β: solve the following eq^{n}s

$$Re\{\gamma^2\} \equiv Re\{\gamma^2\}$$

$$|\gamma^2| \equiv |\gamma^2|$$

$$\gamma^2 = -w^2\mu\epsilon + jw\mu\sigma$$

$$\gamma^2 = \alpha^2 - \beta^2 + 2j\alpha\beta$$

$$\beta^2 - \alpha^2 = w^2\mu\epsilon \quad ---- \boxed{1}$$

$$(w^2\mu\epsilon)^2 + (w\mu\sigma)^2 = \alpha^2 + \beta^2 \quad --- \boxed{2}$$

$$\alpha^2 = (w^2\mu\epsilon)^2 + (w\mu\sigma)^2 - \beta^2$$ ...... sub in eq $\boxed{1}$

$$\alpha = w\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{w\epsilon}\right)^2} - 1\right)}$$

$$\beta = w\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{w\epsilon}\right)^2} + 1\right)}$$

in Free space

$\sigma = 0$

$\alpha = 0$

$\beta = w\sqrt{\mu\epsilon}$

$= \frac{w}{u}$

NOTEBOOK

Scanned by CamScanner

Subject Date No.

$$\underset{\text{speed}}{u} = \frac{w}{\beta} \neq \frac{1}{\sqrt{\mu \epsilon}} \quad , \quad \lambda = \frac{2\pi}{\beta}$$

E - Expression: $\rightarrow$ wave travel in +ve z-dir

$$\bar{E}(z,t) = \mathcal{E}_o^+ e^{-\alpha z} \cos(\omega t - \beta z) \, \hat{a_x}$$

$$\mathcal{E}_s = \mathcal{E}_o^+ e^{-\gamma z} = \mathcal{E}_o^+ e^{-(\alpha + j\beta) z}$$

$$= \mathcal{E}_o^+ e^{-\alpha z} e^{-j\beta z}$$

H??

$$\nabla^2 \bar{H}_s - \gamma^2 \bar{H}_s = 0$$

or $\bar{H}(z,t) = H_o^+ e^{-\alpha z} \cos(\omega t - \beta z) \, \cancel{\hat{a_H}}$ $\hat{a_y}$

$$\hat{a_H} = \hat{a_k} \times \hat{a_\mathcal{E}}$$

$$= \hat{a_z} \times \hat{a_x}$$

$$= +\hat{a_y}$$

in ohm Low : $I = \frac{V}{Z}$

$$H = \frac{\mathcal{E}}{\eta}$$

$\eta \rightarrow$ Intrensic Impedance

eetta

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$ from maxwell's eq.

η is complex

$$\eta = |\eta| \angle \theta_\eta$$

$$\eta^2 = \frac{j\omega\mu}{\sigma + j\omega\epsilon} \cdot \frac{\sigma - j\omega\epsilon}{\sigma - j\omega\epsilon}$$

$$\eta = \underbrace{\frac{\sqrt{\mu/\epsilon}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}}_{|\eta|} \angle \underbrace{\frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)}_{\theta_\eta}$$

N O T E B O O K

Scanned by CamScanner

Subject θ between $\bar{E}$ & $\bar{H} = 90^\circ$ Date No.

$$\bar{H} = \frac{E_o^+}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \hat{a}_y \quad A/m$$

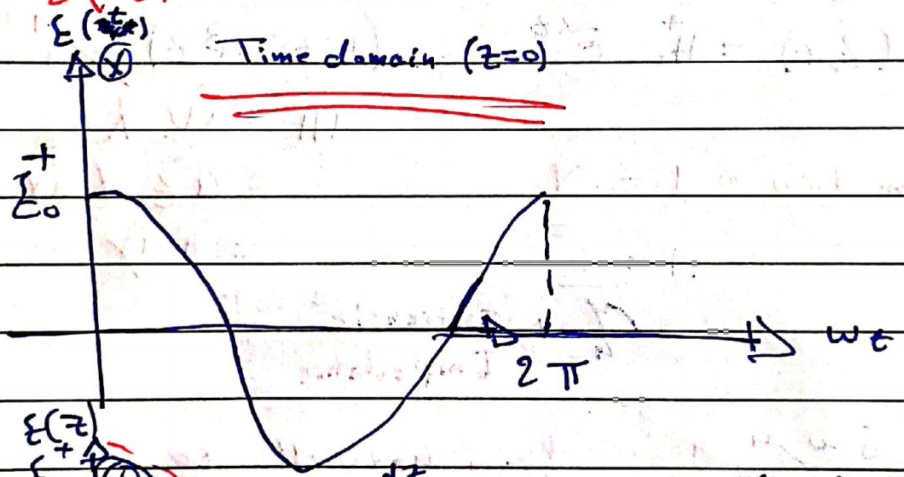
E leads $\bar{H}$ by $\theta_\eta \xrightarrow{\text{in CKT}}$ V leads I → inductive

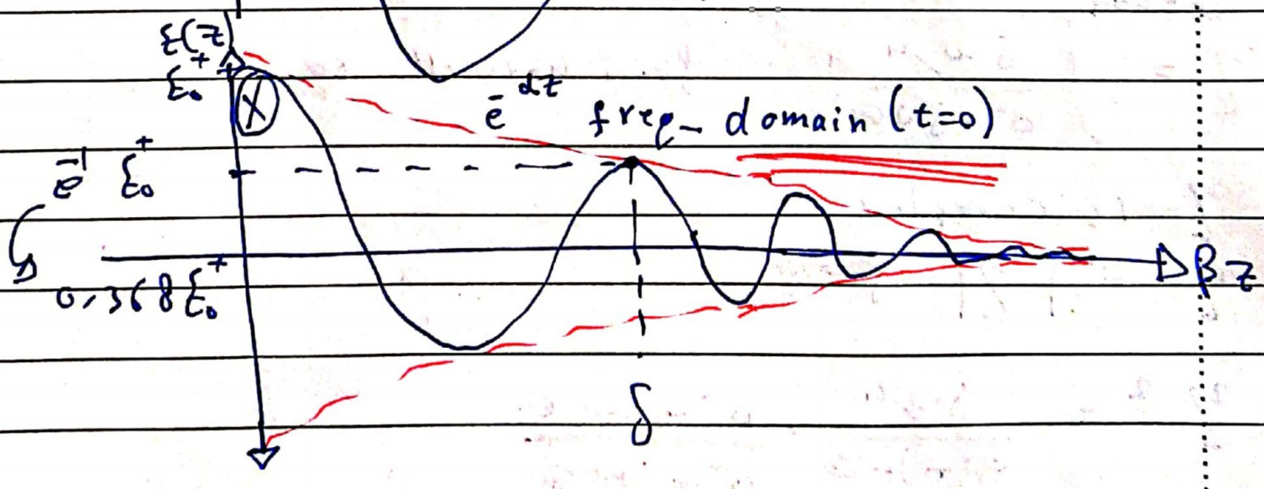
$$E = \eta H$$

if $\sigma = 0 \rightarrow \eta = \sqrt{\frac{\mu}{\epsilon}} \angle 0^\circ$

loss less or free space

* sketch $E(z,t) = E_o^+ e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x$





$\delta \equiv$ skin depth in (m) $\quad \delta = \frac{1}{\alpha}$, $z = \delta$

$E = E_o^+ e^{-\alpha\delta}$

$z = 2\delta \rightarrow e^{-2} = 0.14 E_o^+$

$z = 3\delta \rightarrow e^{-3} = 0.05 E_o^+$

$z = 5\delta \rightarrow e^{-5} =$ less than 1% E_o^+

Scanned by CamScanner

Subject Date No.

1/10/2019

[1] wave propagation in lossy media :- $[\sigma \neq 0,\ \epsilon = \epsilon_0 \epsilon_r,\ \mu = \mu_0 \mu_r]$

special cases :-

$$\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0$$

$$\bar{E} = E_0^+ e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x \ \text{V/m}$$

$$\bar{H} = \frac{E_0^+}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \hat{a}_y \ \text{A/m}$$

$$\gamma = \alpha + j\beta$$

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} - 1\right)}$$

$$\frac{E}{H} = \eta = \frac{\sqrt{\mu/\epsilon}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}} \angle \left[\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)\right] \frac{1}{2} \ \Omega$$

$|\eta|$ $\quad\theta_\eta$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} + 1\right)}$$

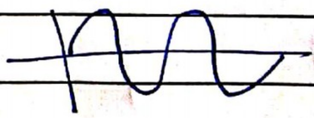
$$\boxed{\lambda = \frac{2\pi}{\beta},\quad u = \frac{\omega}{\beta},\quad \omega = 2\pi f,\quad \delta = \frac{1}{\alpha}}$$

[2] wave Propagation in lossless media : $[\epsilon = \epsilon_0 \epsilon_r,\ \mu = \mu_0 \mu_r,\ \sigma = 0]$

$$\bar{E} = E_0^+ \cos(\omega t - \beta z)\, \hat{a}_x \ \text{V/m}$$

$$\bar{H} = \frac{E_0^+}{\eta} \cos(\omega t - \beta z)\, \hat{a}_y \ \text{A/m}$$

$\alpha = 0$ $\quad \beta = \omega\sqrt{\mu\epsilon}\ ,\ \gamma = j\beta$



$\eta = \sqrt{\frac{\mu}{\epsilon}} \ \angle 0 \ \Omega \rightarrow$ Pure real

$$\lambda = \frac{2\pi}{\beta},\quad u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}},\quad \delta = \infty$$

$\bar{E}$ and $\bar{H}$ are in phase

N O T E B O O K

Scanned by CamScanner

Subject Date No.

[3] wave propagation in free space: $[\epsilon = \epsilon_o, \mu = \mu_o, \sigma = 0]$

$$\bar{E} = E_o^+ \cos(\omega t - \beta z)\, \hat{a}_x \; V/m$$

$$\bar{H} = \frac{E_o^+}{\eta_o} \cos(\omega t - \beta z)\, \hat{a}_y \; A/m$$

$\alpha = 0$

$\beta = \omega\sqrt{\mu_o \epsilon_o}$, $\gamma = j\beta$

$$\eta_o = \sqrt{\frac{\mu_o}{\epsilon_o}} = \sqrt{\frac{4\pi \times 10^{-7}}{10^{-9}/36\pi}} = 120\pi \; \Omega = 376.6 \; \Omega \simeq 377 \; \Omega$$

$$\lambda = \frac{2\pi}{\beta}, \quad u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_o \epsilon_o}} = c \simeq 3 \times 10^8$$

[4] wave propagation in good conductors: $[\epsilon = \epsilon_o, \mu = \mu_r \mu_o, \sigma \simeq \infty]$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}, \quad \beta = \sqrt{\frac{\omega\mu\sigma}{2}}, \quad \gamma = (1+j)\beta, \quad \eta = \sqrt{\frac{\omega\mu}{\sigma}} \measuredangle 45^\circ \; \Omega$$

$$\eta = \sqrt{\frac{2\pi f\mu}{\sigma}} \measuredangle 45^\circ = \sqrt{\frac{2\pi f\mu}{\sigma}}\, e^{j\pi/4} = \sqrt{\frac{\cancel{2}\pi f\mu}{\sigma}} \left(\frac{1}{\cancel{\sqrt{2}}} + j\frac{1}{\cancel{\sqrt{2}}}\right)$$

$$= \sqrt{\frac{\pi f\mu}{\sigma}}\,(1+j)$$

N O T E B O O K

Scanned by CamScanner

loss tangent :-

$$\eta = \frac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}} \angle \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)$$

loss tangent angle

$$\theta = 2\theta_\eta = \tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right), \quad \tan\theta = \frac{\sigma}{\omega\epsilon}$$

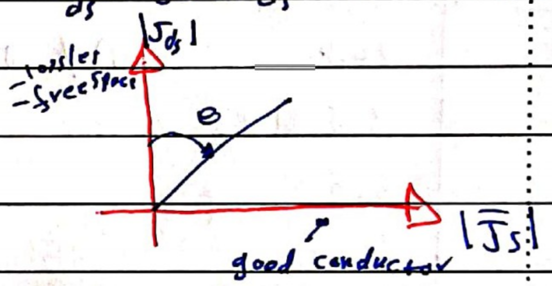
loss tangent

$\sigma \gg \omega\epsilon \to$ good conductor

Current Ratio:

- $\to$ conduction $J_s = \sigma \bar{E}_s$
- $\to$ Displacement $J_{ds} = j\omega\epsilon \bar{E}_s$

$$\frac{|\bar{J}_s|}{|J_{ds}|} = \frac{\sigma \bar{E}_s}{j\omega\epsilon \bar{E}_s} = \frac{\sigma}{j\omega}$$



$\theta = 90^\circ \to \theta_\eta = \frac{\theta}{2} = 45^\circ$

3rd Maxwell's eq: [include (σ)]

$$\nabla \times \bar{H}_s = (\sigma + j\omega\epsilon)\bar{E}_s$$

$$\nabla \times \bar{H}_s = j\omega\epsilon\left(1 - \frac{j\sigma}{\omega\epsilon}\right)\bar{E}_s$$

$$\nabla \times \bar{H}_s = j\omega\,\epsilon_c \bar{E}_s$$

ϵ_c = complex permittivity, $\boxed{\epsilon_c = \epsilon' - j\epsilon''}$

$\epsilon_c = \epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)$, $\epsilon_c = \epsilon - \frac{\sigma}{\omega}$

$\epsilon' = \epsilon$, $\epsilon'' = \frac{\sigma}{\omega}$

real part   Imagenary Part

$$\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon} = \text{lossles tangent}$$

NOTEBOOK

$$\tan\theta = \frac{|J_s|}{|J_{ds}|}\ I$$

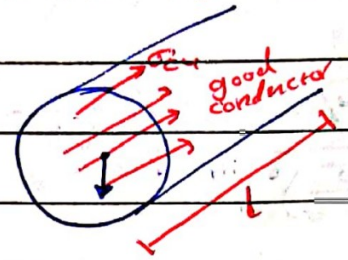
Scanned by CamScanner

Subject　　　　Date　　　　No.

3/10/2019

DC and AC Resistances :-

$R_{DC} = \frac{l}{\sigma A}$



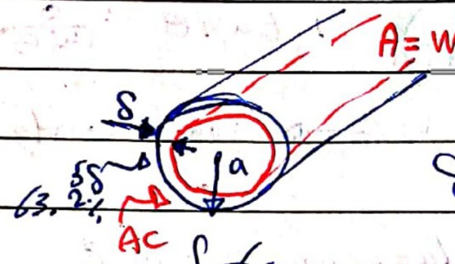
$A = \pi a^2$

$f = 0, \ \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

$= \frac{1}{0} = \infty = 2a$

$A = W\delta = 2\pi a \delta$

$R_{AC} = \frac{l}{\sigma A}$



$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$ $\sigma \simeq 5.8 \times 10^7$

$f \uparrow$ $\delta \downarrow$ $4\pi \times 10^{-7} \mu_r$ $\mu_r \approx 1$

$f \neq 0$

$a = 1\,mm$

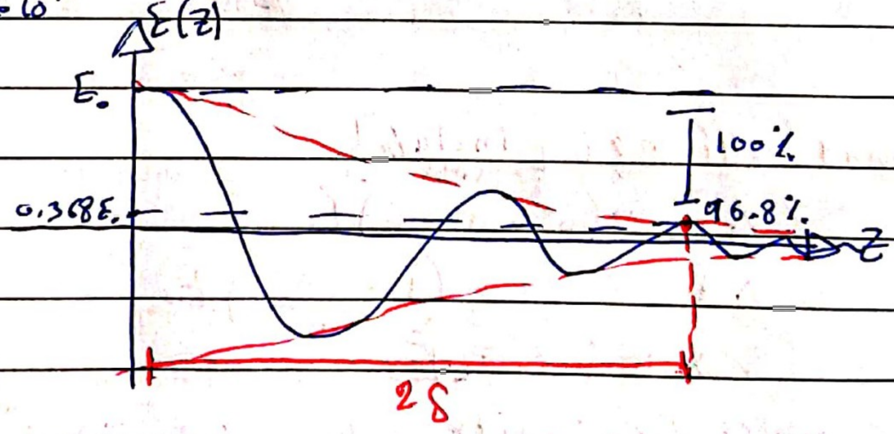
$2a = 2\,mm$

$A \neq \pi a^2$

f (Hz)	δ (mm)
10	20.8
60	8.6
100	6.6
500	2.99
100 GHz = 10^{10}	6.6×10^{-4}

$R_{ac} = R_{dc}$ if $\delta \geq a$

$R_{ac} > R_{dc}$ @ higher freq...



Scanned by CamScanner

Subject　　　　　　　　　　　　Date　　　　　No.

Surface Resistance (R_s) :-

Ω

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{\pi f \mu}{\sigma}} = Re\{\eta\}$$

good conductor

$$\eta = \boxed{\sqrt{\frac{\omega\mu}{\sigma}}} \angle 45^\circ = \sqrt{\frac{2\pi f\mu}{\sigma}} = \frac{(1+j)}{\sqrt{2}}$$

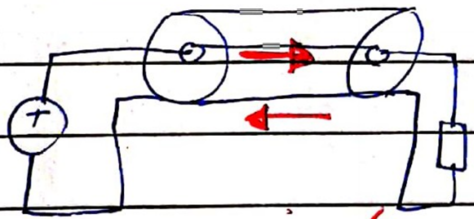
$$\sqrt{\frac{\omega\mu}{\sigma}}\frac{(1+j)}{\sqrt{2}} = Re = \sqrt{\frac{\pi f\mu}{\sigma}} = R_s$$

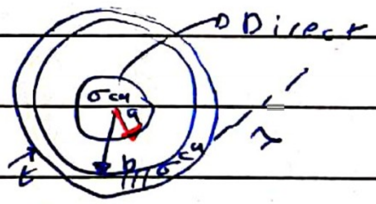
$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}$$

Ex :- for a copper coaxial cable $\sigma = 5.8\times10^7$ S/m has $a = 2$ mm, $b = 6$ mm, $t = 1$ mm and $L = 2$ m

find?? the Resistance at DC and 100MHz





$$R_{Total} = R_i + R_o$$

@ DC :-

$$R_i = \frac{L}{\sigma A} = \frac{L}{\sigma\pi a^2} =$$

$$R_o = \frac{L}{\sigma A} = \frac{1}{\sigma\pi[(b+t)^2-b^2]} =$$

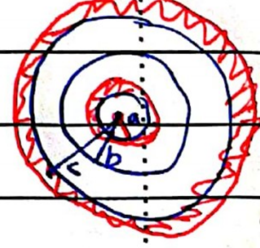
$$R_{total} = 3.587\ m\Omega$$

@ AC :-

$$\delta = \frac{1}{\sqrt{\pi f\mu\sigma}} = 6.6\mu m \ll t$$

$$R_i = \frac{L}{\sigma 2\pi a\delta} =$$

$$R_o = \frac{L}{\sigma 2\pi(b+t)\delta} =$$



$$R_{TOTAL} = 548.4\ m\Omega$$

NOTEBOOK

Scanned by CamScanner

Scanned by CamScanner

Subject Date No.

$$\therefore \frac{R_{ac}}{R_{dc}} = \frac{\ell}{\sigma 2\pi a \delta} \Big/ \frac{\ell}{\sigma \pi a^2} = \frac{a}{2\delta}$$

$$\frac{R_{ac}}{R_{dc}} \approx 150 \text{ times}.$$

H.w :-

$f = 300MHz$

find :-

$\frac{R_{ac}}{R_{dc}}$??

N O T E B O O K

Scanned by CamScanner

Subject　　　　　　　　　　Date　　　　No.

Ex:- In a Lossless medium $\eta = 60\pi$, $\mu_r = 1$

$\bar{H} = -0.1\cos(\omega t - z)\hat{a}_x + 0.5\sin(\omega t - z)\hat{a}_y$ A/m

Calculate:- ϵ_r, ω, $\bar{E}$??
(1) (2) (3)

$\bar{H} = \bar{H}_1 + \bar{H}_2$

Sol:-

$$\eta = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \frac{\eta_0}{\sqrt{\epsilon_r}} = \cancel{60\pi} \qquad \therefore \boxed{\epsilon_r = 4} \quad \boxed{1}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \boxed{120\pi}$$

$$\omega = \beta u, \quad \beta = 1, \quad \omega = u = \frac{c}{\sqrt{\epsilon_r}} \quad \left(\frac{1}{\sqrt{\mu_0 \epsilon_0 \mu_r \epsilon_r}}\right)$$

$$= \boxed{1.5\times 10^8} \text{ rad/s} \quad \boxed{2}$$

$$\bar{H} = -\underbrace{0.1}_{H_{10}}\cos(\omega t - z)\underbrace{\hat{a}_x}_{\hat{a}_{H_1}} + \underbrace{0.5}_{H_{20}}\sin(\omega t - z)\underbrace{\hat{a}_y}_{\hat{a}_{H_2}}$$

$\Rightarrow \bar{E} = \eta\bar{H} \Rightarrow |\bar{E}| = \eta|\bar{H}|$

$\Rightarrow E_{10} = 60\pi * 0.1 = 6\pi$ V/m, $E_{20} = 60\pi * 0.5 = 30\pi$ V/m

$\Rightarrow \hat{a}_{E_1} = \hat{a}_H \times \hat{a}_k = -\hat{a}_x \times \hat{a}_z = \hat{a}_y$

$\Rightarrow \hat{a}_{E_2} = \hat{a}_y \times \hat{a}_z = \hat{a}_x$

$\hat{a}_E$, $\hat{a}_k$, $\hat{a}_H$, R　　$\hat{a}_x$, $\hat{a}_z$, R, $\hat{a}_y$

$$\bar{E} = 6\pi\cos(\omega t - z)\hat{a}_y + 30\pi\sin(\omega t - z)\hat{a}_x \text{ V/m}$$

N O T E B O O K

Scanned by CamScanner

Subject Date No.

① Poynting Vector and Power :-

Poynting vector (time dependent) $P = v \cdot i$

$$\bar{P} = \bar{E} \times \bar{H} \quad \frac{w}{m^2}$$

in a lossy media :-

$$\bar{E} = E_o e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a_x} \quad V/m$$

$$\bar{H} = \frac{E_o}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \hat{a_y} \quad A/m$$

$$\bar{P} = \bar{E} \times \bar{H} = \frac{E_o^2}{|\eta|} e^{-2\alpha z} \left[\frac{1}{2} \cos(2\omega t - 2\beta z - \theta_\eta) + \frac{1}{2} \cos \theta_\eta \right] \hat{a_z} \quad W/m^2$$

$\bar{P}$ → instantaneous form

② Time average Poynting Vector :-

$$\bar{P}_{ave} = \frac{1}{T} \int_0^T \bar{E} \times \bar{H} \, dt \quad \text{------} \; (1)$$

⟹ Time invarient

$$= \frac{E_o^2}{2\eta} e^{-2\alpha z} \cos \theta_\eta \, \hat{a_z} \quad W/m^2$$

⟹ If lossless :- $\sigma = 0$, $\alpha = 0$, $\theta_\eta = 0$

$$\bar{P}_{ave} = \frac{E_o^2}{2\eta} \hat{a_z} \quad W/m^2$$

$\frac{Peak}{\sqrt{2}} = V_{rms}$

N O T E B O O K

Scanned by CamScanner

Subject Date No.

3 Total Power :- (scalar)

$$P_{ave} = \int_s \bar{P}_{ave} \cdot d_s \quad (W)$$

↳ normal surface area in ch2

$$\bar{ds} = ds\, \hat{a}_n$$

$$P_{ave} = \int_s \bar{P}_{ave} \cdot \hat{a}_n\, ds$$

$$\bar{P}_{ave} = \frac{1}{T}\int_0^T \bar{E}\times\bar{H}\, dt \Longrightarrow \text{Time domain}$$

$$\bar{E} \longrightarrow \bar{E}_s = E_o\, e^{-\alpha z} \cdot e^{-j\pi z}\, \hat{a}_x$$

$$\bar{H} \longrightarrow \bar{H}_s = \frac{E_o}{|\eta|}\, e^{-\alpha z}\, e^{-j\beta z}\, e^{-j\theta_\eta}\, \hat{a}_y$$

⟹ CH9: in 9.7

conjeger

$$Re\{\bar{E}\times\bar{H}^*\} = \frac{E_o^2}{|\eta|}\, e^{-2\alpha z} \cos(\theta_\eta)\, \hat{a}_z$$

②

Scanned by CamScanner

Subject Date No.

$\bar{P} = \bar{E} \times \bar{H}$

$P_{ave} = \frac{1}{T}\int_0^T \bar{E} \times H \, dt = \frac{1}{2} Re\{\bar{E}_s \times \bar{H}_s^*\}$

$P_{ave} = \int_s \bar{P}_{ave} \, \bar{d}_s$

Ex:- In anonmagnetic medium:-

$\bar{E} = 4 \sin(2\pi * 10^7 t - 0.8x)\hat{a}_z$ v/m

find:-

a) ϵ_r , η b) $\bar{P}_{ave}$ c) The total Power crossing $100 cm^2$ of plane

$2x + y = 5$

$f = 2x + y - 5$

a) $u = \frac{\omega}{\beta} = \frac{2\pi * 10^7}{0.8}$ $\beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c}\sqrt{\epsilon_r}$

$\neq 3*10^8$

$\boxed{\epsilon_r = 14.59}$ Lossless

Dielective constant (DK)

$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{\mu_0}{\sqrt{\epsilon_r}} = 10\pi^2 \Omega = 98.7 \Omega$

b) $\bar{P}_{ave} = \frac{E^2}{2\eta} e^{-2\alpha z} \cos\theta_\eta \, \hat{a}_x$

$= \frac{16^{-8}}{2(98.7)} \hat{a}_x$ w/m²

NOTEBOOK

Scanned by CamScanner

Subject Date No.

c) $P_{ave} = \int \bar{P}_{ave} \, \underset{\smile}{d_s} \, \hat{a_n}$

$= \bar{P}_{ave} \, \hat{a_n} \, (100 \times 10^{-4})$

$= \frac{8}{98.7} \, (100 \times 10^{-4}) \, \frac{2}{\sqrt{2}} \, (W)$

$\hat{a_n} = \frac{\nabla f}{|\nabla f|}$

$\hat{a_n} = \frac{(2,1,0)}{\sqrt{5}}$

* Polarization :- (in time domain) :-

⟹ Linearly Polarized wave (LPW)

1. Horizantal (HLPW)
2. vertical (VLPW)
3. other wise (LPW)

⟹ circularly Polarized wave (CPW)

1. Right Hand (RHCPW)
2. Left Hand (LHCPW)

⟹ Ellipticall Polarized wave (EPW)

1. right Hand
2. Left Hand

Scanned by CamScanner

Subject Date No.

Ex3:- $\bar{E} = \mathcal{E}_1 e^{j(\omega t - ky)} \hat{a}_x$ V/m

5 steps solutions :-

① Take either real or imaginary part:-

$$\bar{E} = \mathcal{E}_1 \sin(\omega t - ky)\, \hat{a}_x \text{ V/m}$$

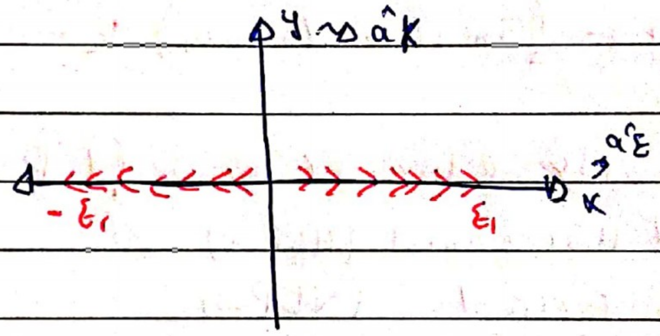
② fix the space

Let $y = 0 \longrightarrow \bar{E} = \mathcal{E}_1 \sin(\omega t)\, \hat{a}_x$ V/m

③ choose $\omega t = 0 \longrightarrow \bar{E} = 0$

$\omega t = 90° \longrightarrow \bar{E} = \mathcal{E}_1 \hat{a}_x$ V/m

④ Draw the 2D plane:-



⑤ HLPW

Scanned by CamScanner

Ex: $\bar{E} = \mathcal{E}_2 \, e^{j(\omega t - Ky)} \, \hat{a_z}$ V/m

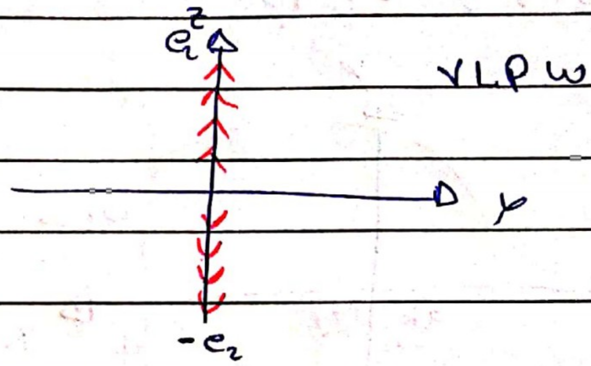
$\bar{E} = \mathcal{E}_2 \cos(\omega t - Ky) \, \hat{a_z}$

Let $y=0$

$\bar{E} = \mathcal{E}_2 \cos(\omega t) \, \hat{a_z}$

$\omega t = 0 \rightarrow \bar{E} = \mathcal{E}_2 \, \hat{a_z}$

$\omega t = 90 \rightarrow \bar{E} = 0$

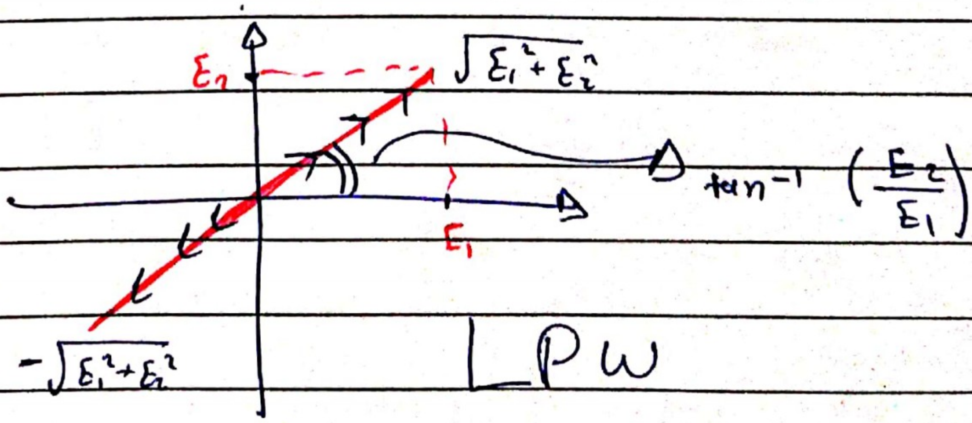


Ex: $\bar{E} = (\mathcal{E}_1 \cdot \hat{a_x} + \mathcal{E}_2 \, \hat{a_z}) \, e^{j(\omega t - Ky)}$ V/m

$\bar{E} = (\mathcal{E}_1 \, \hat{a_x} + \mathcal{E}_2 \, \hat{a_z}) \sin(\omega t) \Big|_{y=0}$

$\omega t = 0 \rightarrow \bar{E} = 0$

$\omega t = 90 \rightarrow \bar{E} = \mathcal{E}_1 \, \hat{a_x} + \mathcal{E}_2 \, \hat{a_z}$



Scanned by CamScanner

Subject Date No.

uniform plane wave ch.13

Ex:- A UPW has $\bar{E} = \mathcal{E}_1 e^{j(\omega t - kz)} \hat{a_x} + \mathcal{E}_1 e^{j(\omega t - kz + \frac{\pi}{2})} \hat{a_y}$ V/m

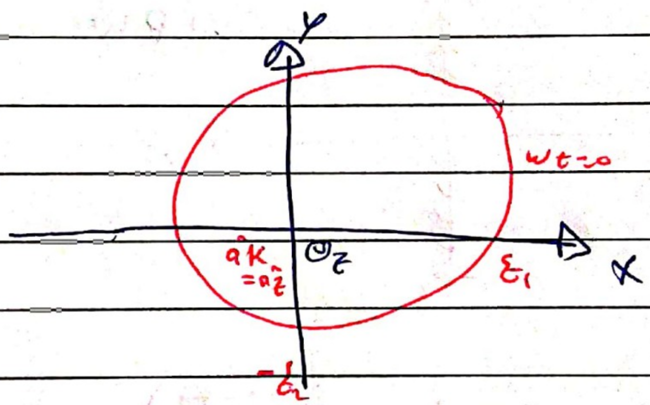
$\bar{E} = \mathcal{E}_1 \cos \omega t \, \hat{a_x} + \mathcal{E}_1 \cos(\omega t + \frac{\pi}{2}) \hat{a_y}$ (real + $z = 0$)

$90^\circ + \cos = -\sin$

$= \mathcal{E}_1 \cos(\omega t) \hat{a_x} - \mathcal{E}_1 \sin(\omega t \, \hat{a_y})$

$\omega t = 0 \rightarrow \mathcal{E} = \mathcal{E}_1 \hat{a_x}$

$\omega t = 90 \rightarrow \mathcal{E} = -\mathcal{E}_1 \hat{a_y}$



Scanned by CamScanner

2019/ 10/10

ex:-

$$\bar{E} = E_1 e^{j(\omega t - kz)} \hat{a_x} + E_2 e^{j(\omega t - kz \pm \psi)} \hat{a_y} \ \text{V/m}$$

$K = \beta$, if lossless

→ Linear polarization: E_1, E_2, ψ

$$\left.\begin{array}{l} \Rightarrow E_1 = 0 \ , \ E_2 \neq 0 \\ \Rightarrow E_1 \neq 0 \ , \ E_2 = 0 \end{array}\right\} \rightarrow \psi \equiv \text{any value}$$

$E_1 \neq 0$, $E_2 \neq 0$, $\psi = n\pi$, $n = 0, 1, 2, \ldots$

→ Circular Polarization:

$\Rightarrow E_1 = E_2$, $\psi = (2n+1)\frac{\pi}{2}$, $n = 0, 1, 2, \ldots$

Otherwise → EPW

→ electrical polarization:

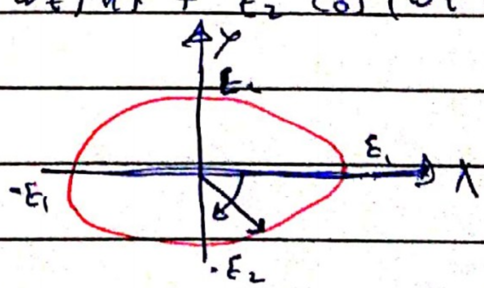
$\Rightarrow E_1 \neq 0$, $E_2 \neq 0$, $\psi = 90°$ $\Rightarrow E_1 > E_2$

for ex:

$$E(z,t)\big|_{z=0} = E_1 \cos(\omega t)\hat{a_x} + E_2 \cos(\omega t + 90°)\hat{a_y}$$

$\omega t = 0 \rightarrow \hat{E} = E_1 \hat{a_x}$

$\omega t = 90° \rightarrow \bar{E} = -E_2 \hat{a_y}$



LHEPW

Scanned by CamScanner

Subject Date No.

$$\text{Axial Ratio} = \frac{\text{Major axis}}{\text{minor axis}} = \frac{OA}{OB} = \frac{E_1}{E_2}$$

$$1 \leq AR \leq \infty$$

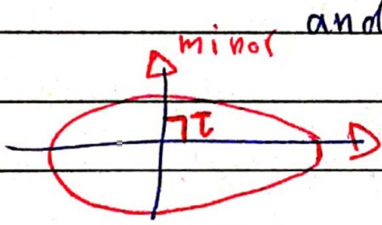
Circular Polarization — Linear Polarization

$$0 \leq AR \leq \infty$$

(0-3) — dB (10-∞

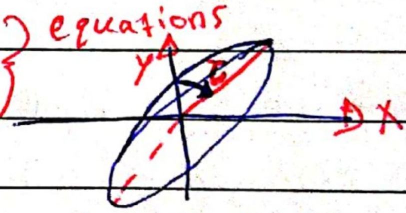
τ = Tilt angle :-

is the angle between the major axis and vertical axis.



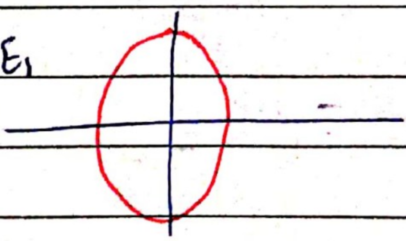
$\tau = 90°$

OA =
OB =
τ =
} equations

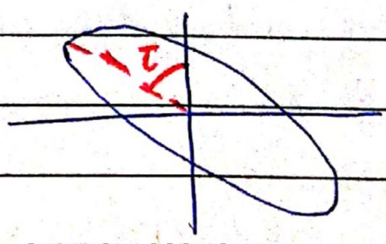


when $\varphi \neq 90°$

$E_2 > E_1$



$\tau = 0°$



NOTEBOOK

Scanned by CamScanner

ex:- A UPW has $\rightarrow$ in free space

$$\bar{E} = (3\hat{a}_x + j4\hat{a}_y)\, e^{-j0.5\pi z} \quad V/m$$

Find:-

a) f b) $\bar{H}$ c) Polarization, AR and τ

d) $\bar{P}_{ave}$ e) Total Power crossing a $(2\times 2)\,m^2$ plane along XY-Plane.

Sol:-

a) $\omega = \beta u = 1.5\pi\times 10^8$ m/s $= 2\pi f$, $f = \frac{\omega}{2\pi} = 75$ MHz

b) $\hat{a}_{H_1} = \hat{a}_k \times \hat{a}_z$

$\bar{H} = \frac{\bar{E}}{\eta_0}$ $\quad = \hat{a}_z \times \hat{a}_x = \hat{a}_y$

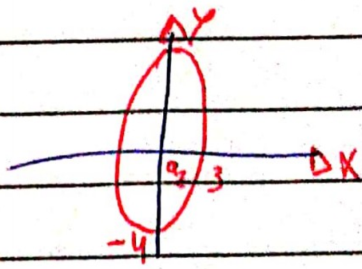
$\hat{a}_{H_2} = \hat{a}_z \times \hat{a}_y = -\hat{a}_x$

$$\bar{H} = \left(\frac{-j4}{120\pi}\hat{a}_x + \frac{3}{120\pi}\hat{a}_y\right) e^{-j0.5\pi z} \quad A/m$$

c) $\bar{E}|_{z=0} = 3\cos \omega z\, \hat{a}_x - 4\sin \omega t\, \hat{a}_y$

$\omega t = 0 \rightarrow 3\hat{a}_x$

$\omega t = 90 \rightarrow -4\hat{a}_y$



LHEPW

$AR = \frac{4}{3}$ $\qquad \tau = 0^\circ$

Scanned by CamScanner

Subject Date No.

d) $\bar{P}_{ave} = \frac{E^2}{2\eta} \cancel{e^{-2\alpha z}} \cos \theta_y \hat{a_z}$

$= \frac{25}{240\pi} \hat{a_z}$ (w/m²)

e) $P_{ave} = \int_s \bar{P}_{ave} \cdot \bar{d_s} = \int_s \bar{P}_{ave} \cdot \hat{a_n}\, d_s$

$\hat{a_n} = \hat{a_z}$

$= \frac{25}{240\pi} * 4$ (w)

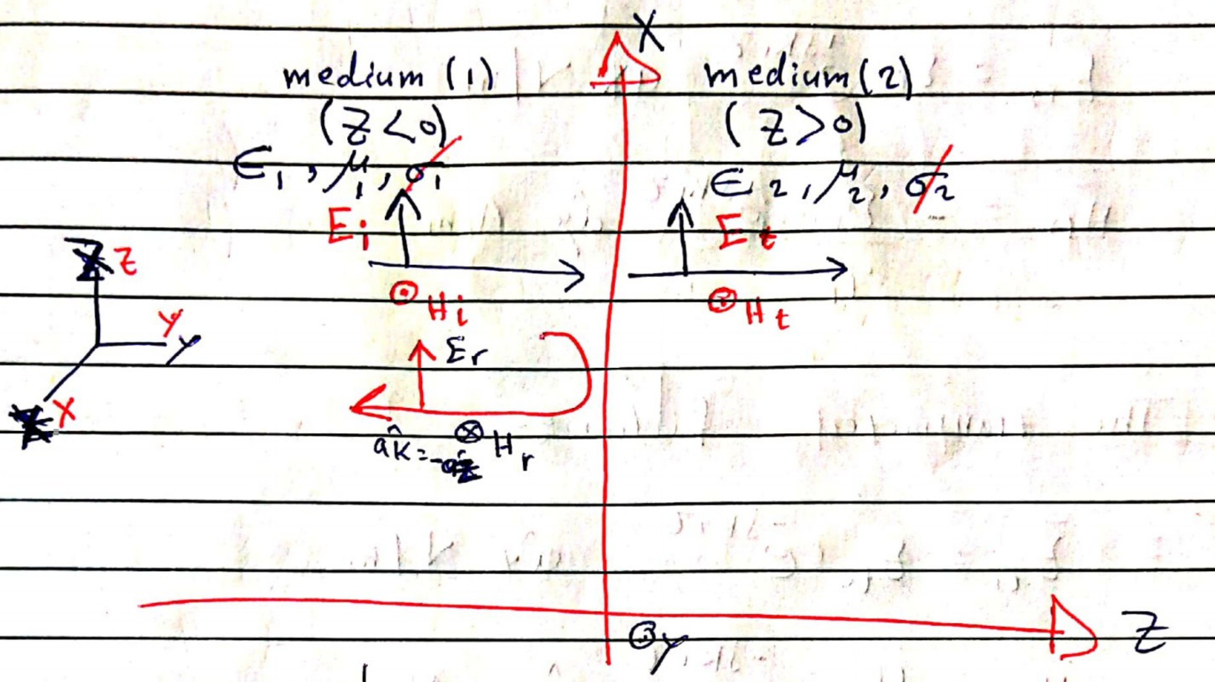
$= 0.417$ w

Scanned by CamScanner

Subject Date No.

13/10/2019

* Reflection of Em waves at Normal Incidence :-



$\eta_1 = \sqrt{\dfrac{\mu_1}{\epsilon_1}}$ $\sigma_1 = 0$ $z = 0$ xy-plane $\eta_2 = \sqrt{\dfrac{\mu_2}{\epsilon_2}}$ $\sigma_2 = 0$

if lossless media $\sigma_1 = \sigma_2 = 0$ write the fields :-

⇒ The incident fields :-

$$\bar{E}_{is} = E_{i0}\, e^{-jk_1 z}\, \hat{a}_x \ \text{V/m}$$

$$\bar{H}_{is} = \frac{H_{i0}}{\eta_1}\, e^{-jk_1 z}\, \hat{a}_y \ \text{A/m}$$

NOTEBOOK

Scanned by CamScanner

Scanned by CamScanner

⇒ The reflected fields :-

$$\bar{E}_{rs} = E_{r0}\, e^{+jk_1 z}\ \hat{a}_x \ \text{V/m}$$

$$\bar{H}_{rs} = -\frac{E_{r0}}{\eta_1}\, e^{+jk_1 z}\ \hat{a}_y \ \text{A/m}$$

⇒ The transmeted field :-

$$\bar{E}_{ts} = E_{t0}\, e^{-jk_2 z}\ \hat{a}_x \ \text{V/m}$$

$$\bar{H}_{ts} = \frac{H_{t0}}{\eta_2}\, e^{-jk_2 z}\ \hat{a}_y \ \text{A/m}$$

$$k_1 = \omega\sqrt{\mu_1 \epsilon_1} = \frac{\omega}{c}\sqrt{\mu_{r1}\epsilon_{r1}} \quad , \quad \eta_1 = \eta_0\sqrt{\frac{\mu_{r1}}{\epsilon_{r1}}}$$

$$k_2 = \omega\sqrt{\mu_2 \epsilon_2} = \frac{\omega}{c}\sqrt{\mu_{r2}\epsilon_{r2}} \qquad \eta_2 = \eta_0\sqrt{\frac{\mu_{r2}}{\epsilon_{r2}}}$$

To find E_{r0} and E_{t0} :-

⇒ Apply the B.C.S @ the interface

$$\bar{E}_{1t} = \bar{E}_{2t}$$
$$\bar{H}_{1t} = \bar{H}_{2t}$$

if K=0 curent

Scanned by CamScanner

Subject Date No.

$\bar{E}_1 \equiv$ The Electric field in region (1)

$\bar{E}_1 = \bar{E}_i + \bar{E}_r$

$\bar{H}_1 = \bar{H}_i + \bar{H}_r = \dfrac{\bar{E}_i}{\eta_1} - \dfrac{\bar{E}_r}{\eta_1}$

$$\bar{E}_2 = \bar{E}_t$$

$$\bar{H}_2 = \bar{H}_t$$

$\bar{E}_{1t} = \underset{\substack{\uparrow \\ \text{tangent}}}{\bar{E}_{2t}} \longrightarrow \oint_l \bar{E}\, d\bar{l} = 0$

$$\bar{E}_{is}(0) + \bar{E}_{rs}(0) = \bar{E}_{ts}(0)$$

$$E_{io}\cancel{a_x} + E_{ro}\cancel{a_x} = E_{to}\cancel{a_x}$$

$$\boxed{E_{io} + E_{ro} = E_{to}} \quad \text{\#} \ldots\ldots (1)$$

$\bar{H}_{1t}(0) = \bar{H}_{2t}(0)$

$\bar{H}_{is}(0) + \bar{H}_{rs}(0) = H_{ts}(0)$

$\dfrac{\bar{E}_{is}(0)}{\eta_1} - \dfrac{\bar{E}_{rs}(0)}{\eta_1} = \dfrac{\bar{E}_{ts}(0)}{\eta_1}$

$\dfrac{E_{io}}{\eta_1}\cancel{a_y} - \dfrac{E_{ro}}{\eta_1}\cancel{a_y} = \dfrac{E_{to}}{\eta_2}\cancel{a_y}$

$$\boxed{\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{1}{\eta_2} E_{to}} \quad \text{\#} \ldots\ldots (2)$$

N O T E B O O K

Scanned by CamScanner

Subject Date No.

$$\boxed{E_{ro} = E_{io}\left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right)}$$

$$\boxed{E_{to} = E_{io}\left(\frac{2\eta_2}{\eta_2 + \eta_1}\right)}$$

$$\frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma$$

$\Gamma \equiv$ reflection coefficient

$$\frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1} = \tau$$

$\tau \equiv$ Transmission coefficient

$$\boxed{|\tau| = 1 + |\Gamma|}$$

NOTEBOOK

Scanned by CamScanner

Subject Date No.

15/10/2019

* lossless :

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ from μ, ϵ , $\tau = 1 + \Gamma$

$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$, $E_r = \Gamma E_i$

$$E_t = \tau E_i$$

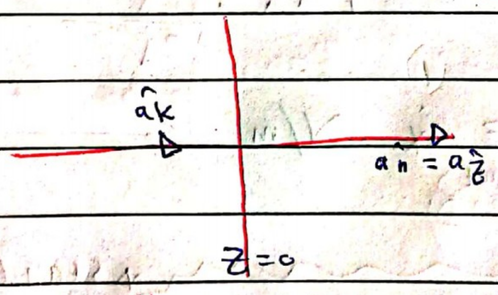
$$0 \le |\Gamma| < 1$$

No reflection $\eta_2 = \eta_1$

Total reflection η_1 or $\eta_2 = 0$

$\Gamma = 1 = 1\angle 0^\circ$

$\Gamma = -1 = 1\angle 180^\circ$



$$0 < \tau < 2$$

Scanned by CamScanner

Subject Date No.

* Special Case :-

medium(1) is lossless

medium(2) is pefect Conductor

medium(1)	medium(2)
$\epsilon_1, \mu_1, \sigma = 0$	$\epsilon_0, \mu_2, \sigma \approx \infty \rightarrow \delta = 0$
$\eta_1 = \eta_0 \sqrt{\frac{\epsilon_{r1}}{\mu_{r1}}}$	$\eta_2 = 0$

$\rightarrow \eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$

E_i, a_x

a_x

$E_t = E_2 = 0$

$E_r = \Gamma E_i = -E_i \longrightarrow z$

$\Gamma = 1$

$\tau = 0$

$E_{1s} = E_i + E_r$

$= E_{10} e^{-j\beta_1 z} \hat{a}_x$

$+ -E_{10} e^{+j\beta_1 z}$

$\therefore E_{1s} = E_{10}\left(e^{-j\beta_1 z} - e^{-j\beta_1 z}\right) \hat{a}_x$

$$E_{1s} = -E_{10}\left(e^{j\beta_1 z} - e^{-j\beta_1 z}\right) \hat{a}_x \cdot \frac{2j}{2j}$$

$$E_{1s} = -2j\, E_{10} \sin\beta_1 z\, \hat{a}_x \quad V/m$$

$$E_1 = \mathrm{Re}\{E_{1s} e^{j\omega t}\}$$

$$\boxed{\bar{E} = 2\, E_{10} \sin\beta_1 z \sin\omega t\, \hat{a}_x} \rightarrow \text{standing wave}$$

expresion :.

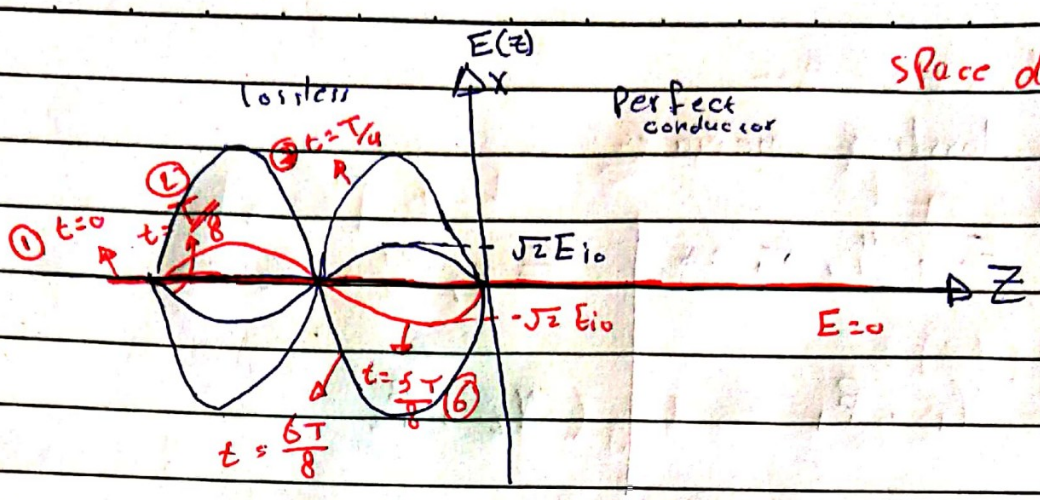
$$\boxed{\bar{E} = 2\, E_{10} \cos(\omega t - \beta z)\, \hat{a}_x} \rightarrow \text{Propagating wave}$$

Standing wave $\equiv$ forward wave (E_1) + Backward wave (E_r)

N O T E B O O K

Subject Date No.

Space domain



$\bar{E}_1 = 2E_{i0}\sin\beta_1 z \sin\omega t \; \hat{a}_x$ $\omega = \frac{2\pi}{T}\cdot\frac{T}{8} = \pi/4$

t=0 ①, t=T/8 ②, t=T/4 ③, t=3T/8 ④ same②, T/4 ⑤ same①, 5T/8 ⑥, 6T/8

$\bar{H}_1 = \frac{2E_{i0}}{\eta_1}\cos\beta_1 z \cos\omega t \; \hat{a}_y$ A/m

Max value:

$\sin\beta_1 z = 1$

$\beta_1 z = (2n+1)\frac{\pi}{2}$, n = 0, 1, 2 ...

$z = -\frac{(2n+1)\pi}{2\beta}$ since region(1) is located for z<0

first max (n=0)

$z_{max} - (2n+1)\frac{\lambda}{4}$, $n=0 \rightarrow -\frac{\lambda}{4}$, $n=1 \rightarrow -\frac{3\lambda}{4}$

$\eta_2 = 0$, $\eta_1 > 0$

$\beta_1 z_{min} = n\pi$, n = 0, 1, 2 ~ $\eta_1 > \eta_2$

$z_{min} = \frac{-n\pi}{\beta_1}$, $n=0 \rightarrow z_{min_1} = 0$, $n=1 \rightarrow z_{min} = \frac{-\lambda}{2}$

$z_{min} = \frac{-n\lambda}{2}$

NOTEBOOK

Scanned by CamScanner

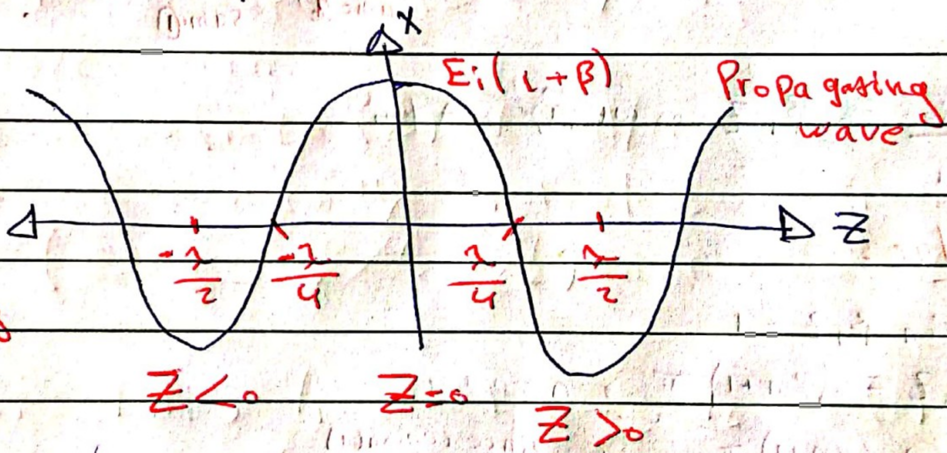
*** spical Case :-**

two lossless media :-

$\epsilon_1, \mu_1, \sigma_1 = 0$	$\epsilon_2, \mu_2, \sigma_2 = 0$
η_1 , β_1	η_2 , β_2

if $\eta_2 > \eta_1$, $1 > \Gamma > 0$, $Max\ \Gamma = 1$

$$E_1 = 2\, E_{10} \cos\beta_1 Z \cos\omega t\ \hat{a}_x$$



$Z_{min} = (2n+1)\frac{\lambda}{4}$, $n = 0, 1, 2 \ldots$

~~Zmax =~~ $\cos\beta_1 Z_{max} = n\pi$, $Z_{max} = \frac{n\pi}{\beta_1} = \frac{n\cancel{\pi}\lambda}{2\cancel{\pi}}$

$\beta_1 Z_{max} = n\pi$

$E_r = \Gamma E_i$, $E_2 = E_t$, $E_{t0} = \tau E_{10}$, $\tau = 1 + \Gamma$

Scanned by CamScanner

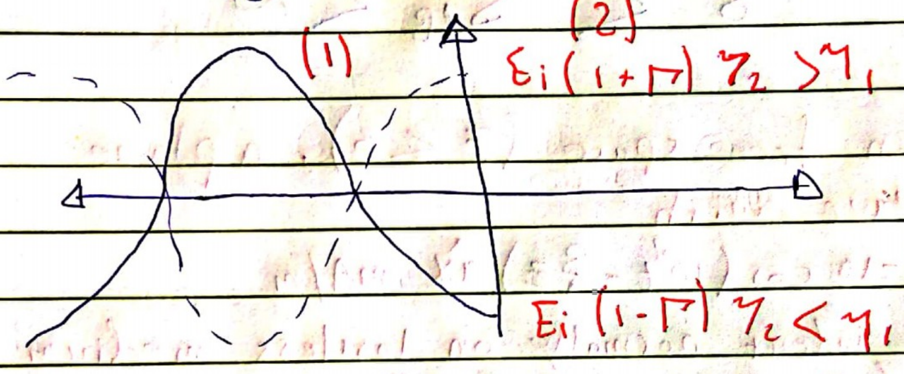
Subject Date No.

$\eta_2 < \eta_1 \Rightarrow \Gamma$ is $-ve$

$E_1 = 2\,E_{i0} \sin\beta_1 z \sin\omega t\ \hat{a}_x$ (when $\Gamma = -1$)

$Z_{max} = (2n+1)\frac{\lambda}{4}$

$Z_{min} = -\frac{n\lambda}{2}$ for $z < 0$



* إذا إلتقوا عند ال Zero يكون وسط ناقل (good conductor

$E_{max} \Rightarrow \bar{H}_{min}$ *

$E_{min} \rightarrow \bar{H}_{max}$ *

$E_{1t} = E_{2t}$, $E_i + E_r = E_2$, $E_i + \Gamma E_i = 0$, $E_i(1 + \Gamma)$ $-ve$

Voltage

$\boxed{*}$ Standing wave ratio (SWR) OR (S) :-

$$S = \frac{E_{max}}{E_{min}} = \frac{H_{max}}{H_{min}}, \quad S = \frac{\cancel{E_i}(1+|\Gamma|)}{\cancel{E_i}(1-|\Gamma|)} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

if $\Gamma = \pm 1$ good conductor

$$S = \frac{1+1}{1-1} = \infty$$

N O T E B O O K

Scanned by CamScanner

Scanned by CamScanner

Subject Date No.

$$\Gamma = 0 \leftarrow 1 \leq S \leq \infty \rightarrow \Gamma = \mp 1$$

Tn (dB):

$$S_{dB} = 20 \log (S)$$

$$0 \leq S_{dB} \leq \infty$$

Ex:- In free space $(Z \leq 0)$, a plane wave with
$\bar{H} = 10 \cos(10^8 t - \beta Z)\, \hat{a}_x$ mA/m
is indent normally on lossless medium
$(\epsilon = 2\epsilon_0, \mu = 8\mu_0)$ in region $Z \geq 0$

sol:
$E_i = \eta \bar{H}_i$, $\eta_1 = 120\pi\ \Omega$, $\eta_2 = 120(2) = 240\pi\ \Omega$

$\eta_2 > \eta_1$

$$\Gamma = \frac{2\eta_0 - \eta_0}{2\eta_0 + \eta_0} = 1/3$$

*Note: Propagting wave
$E_i = \sin \rightarrow H = \sin$

$\bar{E}_i = 10\,\eta_0 \cos(10^8 t - \beta Z)\, \hat{a}_y$

$\bar{E}_r = \frac{-10}{3}\,\eta_0 \cos(10^8 t - \beta Z)\, \hat{a}_y$

$E_t = \frac{-40}{3}\,\eta_0 \cos(10^8 - \beta Z)\, \hat{a}_y$

*Note:- Standing wave $E_i = \sin \rightarrow H = \cos$

$\Longrightarrow$

NOTEBOOK

Scanned by CamScanner

$E_i(0) + E_r(0) = E_t(0)$ to cheak

$-10\,\eta_0 - \frac{10\,\eta_0}{3} = -\frac{40\,\eta_0}{3}$ to cheak

$\bar{H} = \frac{\bar{E}_r}{\eta_1}$, $H_r = \Gamma_k \bar{H}_i$, $\Gamma_k = -\frac{1}{3}$

$H_r = \frac{\frac{-10\,\eta_0}{3}}{\eta_0} = -10/3 \cos(10^8 t - \beta z)\, a\hat{x}$

$\bar{H}_t = \frac{E_t}{\eta_2} = \frac{-40\,\eta_0}{3(2\eta_0)} \cos(10^8 t - \beta z)\, a\hat{y}$

$= \frac{-20}{3} \cos(10^8 t - \beta z)\, a\hat{x}$

$H_i(0) - H_r(0) = H_t(0)$

$H_i \cancel{a\hat{x}} + H_r(\cancel{-a\hat{x}}) = H_r \cancel{a\hat{x}}$

17/10/2019

NOTEBOOK

Scanned by CamScanner

20/10/2019

Ex:- Given a U.P.W in Air has

$\bar{E}_i = 40\cos(\omega t-\beta z)\,\hat{a}_x + 30\sin(\omega t-\beta z)\,\hat{a}_y$ V/m

find: $\boxed{a}$ $\bar{H}_i$

$\boxed{b}$ If the wave encounters a Perfectly conducting Plane normal to the z-axis at $z=0$, find $\bar{E}_r, \bar{H}_r$

$\boxed{c}$ The time average Poynting vector for $z \leq 0$ and $z \geq 0$

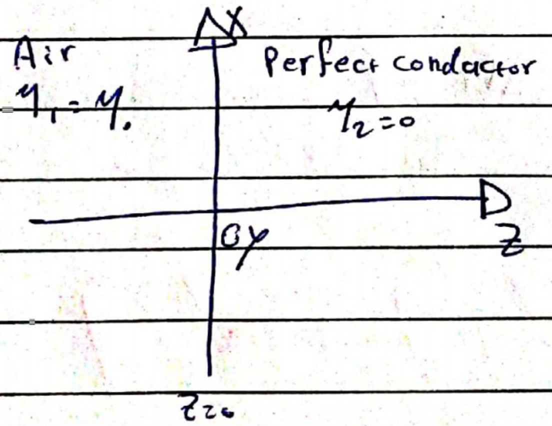
Solo.

$\boxed{a}$ $\bar{H}_i = \frac{-30}{120\pi}\sin(\omega t-\beta z)\,\hat{a}_x + \frac{40}{120\pi}\cos(\omega t-\beta z)\,\hat{a}_y$ A/m

$\hat{a}_{H_1} = \hat{a}_k \times \hat{a}_E = \hat{a}_z \times \hat{a}_x = \hat{a}_y$

$\hat{a}_{H_2} = \hat{a}_z \times \hat{a}_y = -\hat{a}_x$

$\boxed{b}$



$\Gamma = -1 \quad \bar{E}_r = \Gamma \bar{E}_i$

$\Gamma = \frac{\cancel{\eta_2} - \eta_1}{\cancel{\eta_2} + \eta_1}$

$\bar{E}_r = -40\cos(\omega t-\beta z)\,\hat{a}_x - 30\sin(\omega t-\beta z)\,\hat{a}_y$ V/m

$\bar{H}_r = \Gamma_H H_i = \bar{H}_i$

$\bar{H}_r = 40\cos(\omega t-\beta z)\,\hat{a}_{H_1} - \frac{30}{120\pi}\sin(\omega t-\beta z)\,\hat{a}_x$

$\hat{a}_{H_1} = \hat{a}_z \times \hat{a}_x = \hat{a}_y \qquad \hat{a}_{H_2} = \hat{a}_z \times \hat{a}_y = -\hat{a}_x$

$\bar{H}_r = \frac{-E_r}{\eta}$

$\hat{a}_z \times \hat{a}_y = -\hat{a}_x$

$\hat{a}_y \times \hat{a}_z = -\hat{a}_x$

Scanned by CamScanner

(c) $\bar{P}_{av} = \frac{E^2}{2\eta} e^{-2\alpha z} \cos\theta_\eta \, \hat{a}_k$ W/m^2

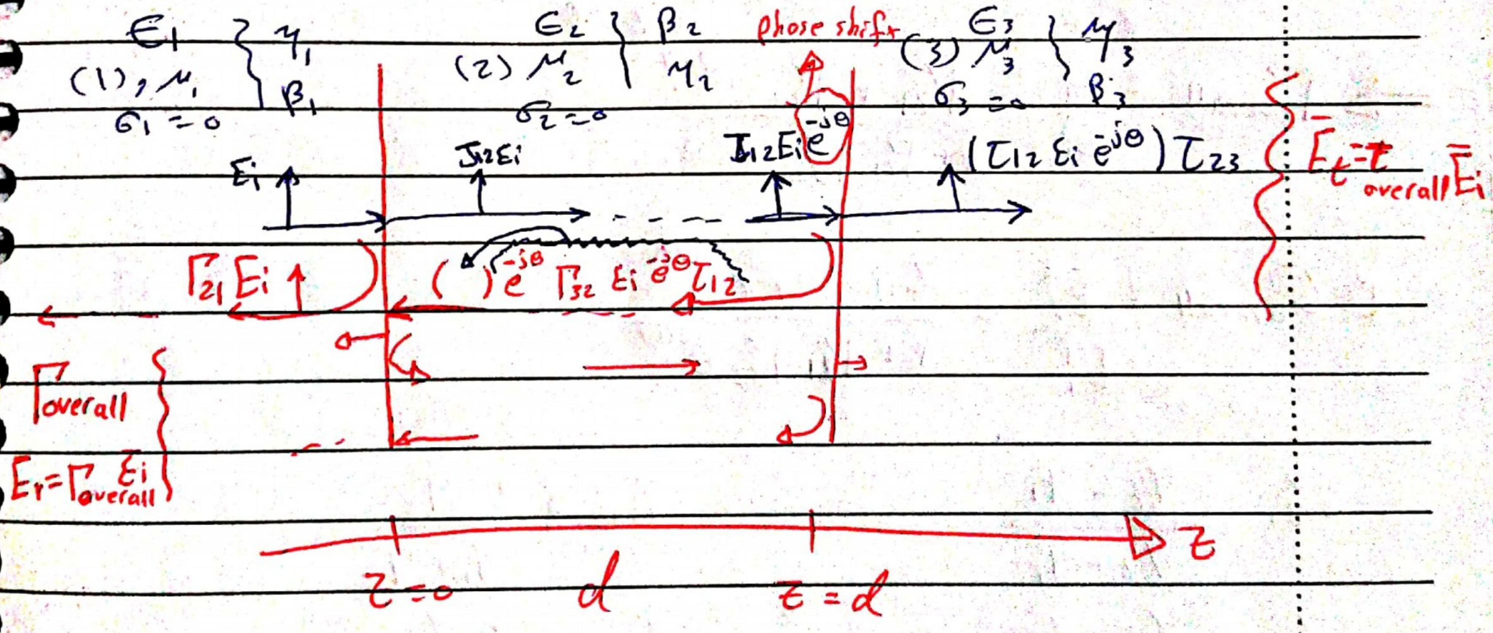
for $z < 0 \Longrightarrow$ Air, $\bar{E}_1 = \bar{E}_i + \bar{E}_r$

$$\bar{P}_{av} = \frac{E_i^2}{2\eta}\hat{a}_z + \frac{E_r}{2\eta_0}(-\hat{a}_z)$$

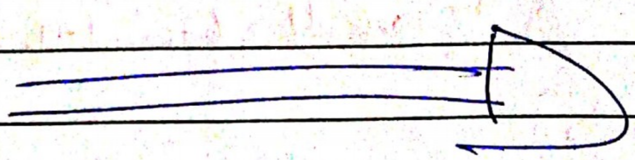
$$= \frac{30^2+40^2}{2(120\pi)}\hat{a}_z + \frac{30^2+40^2}{2(120\pi)}(-\hat{a}_z) = 0$$

$P_{ave}\big|_{z \geq 0} = 0$, $E_t = 0$

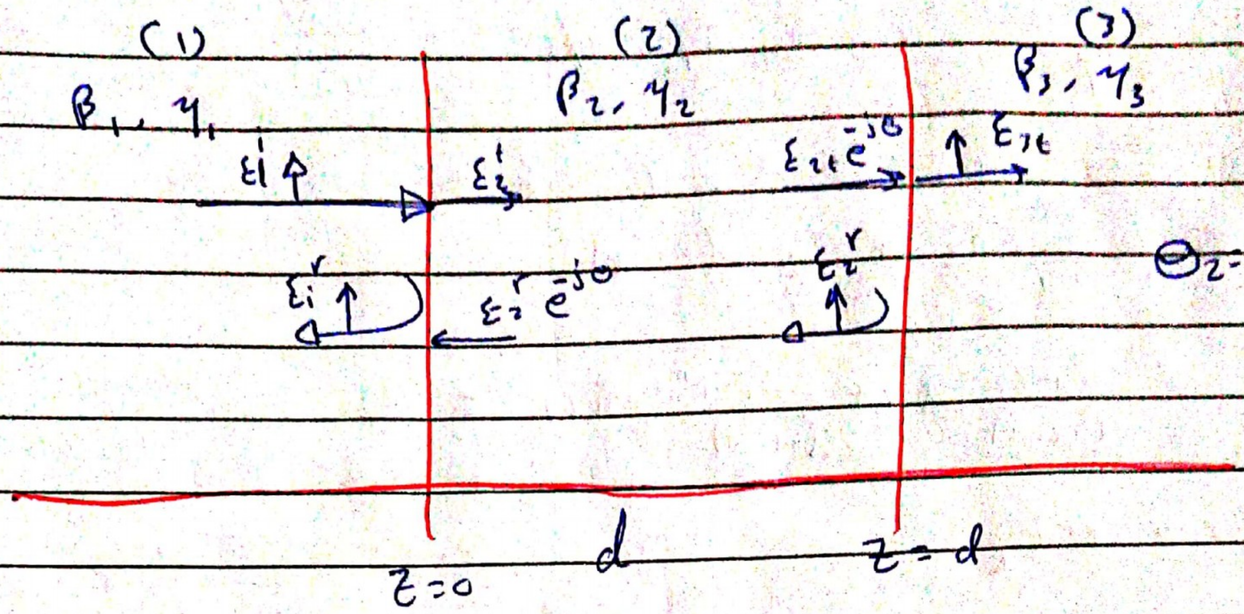
* Normal Incidence at multiple reflections :-



$\theta = \beta_2 d \equiv$ electrical length (Degrees or radins)



Scanned by CamScanner



$\theta_2 = \beta_2 d$

$$\left.\begin{aligned} \mathcal{E}_{1t} &= \mathcal{E}_{2t} \\ H_{1t} &= H_{2t} \end{aligned}\right|_{z=0} \qquad \left.\begin{aligned} \mathcal{E}_{2t} &= \mathcal{E}_{3t} \\ H_{2t} &= H_{3t} \end{aligned}\right|_{z=d}$$

$$\Rightarrow \Gamma_{overall} = \frac{\mathcal{E}_i^r}{\mathcal{E}_i^i} = \frac{\Gamma_{21} + \Gamma_{32} e^{-j2\theta}}{1 + \Gamma_{21}\Gamma_{32} e^{-j2\theta}}$$

$$\Rightarrow \tau_{overall} = \frac{\mathcal{E}_3^t}{\mathcal{E}_1^i} = \frac{\tau_{12}\tau_{23} e^{-j2\theta}}{1 + \Gamma_{21}\Gamma_{32} e^{-j2\theta}}$$

$$\Gamma_{21} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau_{12} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$\Gamma_{32} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2}, \qquad \tau_{23} = \frac{2\eta_3}{\eta_2 + \eta_3}$$

$\theta = \beta_2 d$

X $1 + \Gamma_{overall} = \tau_{overall} \Rightarrow$ depends on θ

$\Rightarrow$

Scanned by CamScanner

Subject Date No.

* To have $\Gamma_{overall} = 0$

$\Rightarrow$ No reflection

when $\Gamma_{21} + \Gamma_{32} e^{-j2\theta} = 0$

$$\Gamma_{21} = -\Gamma_{32} e^{-2j\theta}$$

$$\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} \left(-e^{-j2\theta} \right)$$

$\theta = \beta_2 \theta$

ϵ, μ

either control if ϵ_s', μ_s^* are fixed

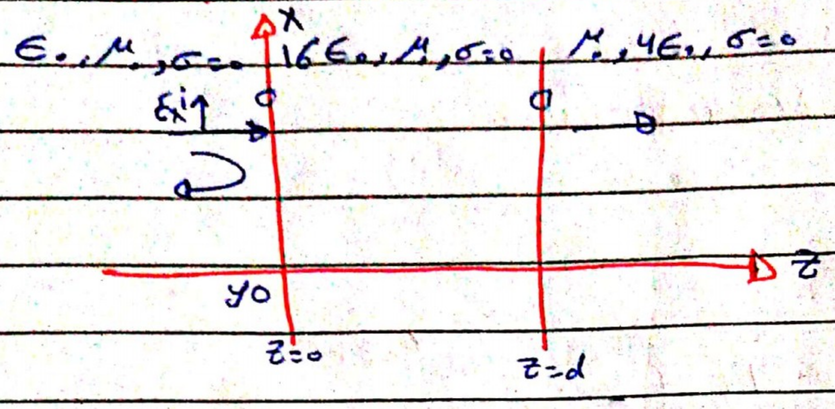
or control ϵ_2, μ_2 if (d) is fixed

$\Rightarrow$ sheilding Effectiveness :-

$$S.E = 20 \log_{10} (T_{overall})$$

Scanned by CamScanner

22/10/2019



Find $\Gamma_{overall}$
$\tau_{overall}$
S.E
for $d = \frac{\lambda}{4}$ & $d = \frac{\lambda}{2}$??

$$\eta_1 = \eta_0 = 120\pi$$

$$\eta_2 = \frac{\eta_0}{4} = 30\pi \quad , \quad \beta_2 = \frac{2\pi}{\lambda}$$

$$\eta_3 = \frac{\eta_0}{2} = 60\pi$$

$$\Gamma_{overall} = \frac{\Gamma_{21} + \Gamma_{32}\, e^{-2\theta j}}{1 + \Gamma_{21}\Gamma_{32}\, e^{-j2\theta}} \quad , \quad \tau_{overall} = \frac{\tau_{12}\tau_{23}\, e^{-j2\theta}}{1 + \Gamma_{21}\Gamma_{32}\, e^{-j2\theta}}$$

$$\Gamma_{21} = \frac{30-120}{30+120} = -\frac{9}{15} = -\frac{3}{5} \quad , \quad \tau_{12} = \frac{2(30\pi)}{120\pi + 30\pi} = \frac{6}{15} = \frac{2}{15}$$

$$\Gamma_{32} = \frac{60-30}{60+30} = \frac{1}{3} \qquad \tau_{23} = \frac{2(60)}{60+30} = \frac{12}{9} = \frac{4}{3}$$

$$1 + \Gamma_{21} = \tau_{12}$$

$$1 + \Gamma_{32} = \tau_{23}$$

at $d = \frac{\lambda}{4}$

$$e^{-j2\theta} = e^{-j2\beta d} = e^{-j(2)\frac{2\pi}{\lambda}\frac{\lambda}{4}} = e^{-j\pi} = \cos\pi - j\sin\pi = -1$$

$\tau_{overall} = -0.44$??

$\Gamma_{overall} = -0.78$ S.E $= 20\log(0.44) = (\ )$ dB

Scanned by CamScanner

ubject Date No.

H.w → Painting لنهاية
Quis → normal لنهاية

at $d = \frac{\lambda}{2}$

$e^{-j2\theta} = e^{-j\pi \frac{2\pi}{\lambda} \frac{\lambda}{2}} = 1$

$\Gamma_{overall} = -0.33 = 33\%$

$\tau_{overall} = 0.67 = 67\%$

$$\Gamma_{overall} = \frac{\Gamma_{21} + \Gamma_{32} e^{-2\theta j}}{1 + \Gamma_{21}\Gamma_{32} e^{-2\theta j}}$$

$\Gamma_{overall} = 0$

$\Gamma_{21} + \Gamma_{32} e^{-j2\theta} = 0$

$\Gamma_{21} = \Gamma_{32}$

$$\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2}$$

$$\eta_2 = \sqrt{\eta_1 \eta_3}$$

$$\eta_2 = \sqrt{(120\pi)(60\pi)} = 60\pi\sqrt{2} = \eta_0 / \sqrt{\epsilon_{r2}}$$

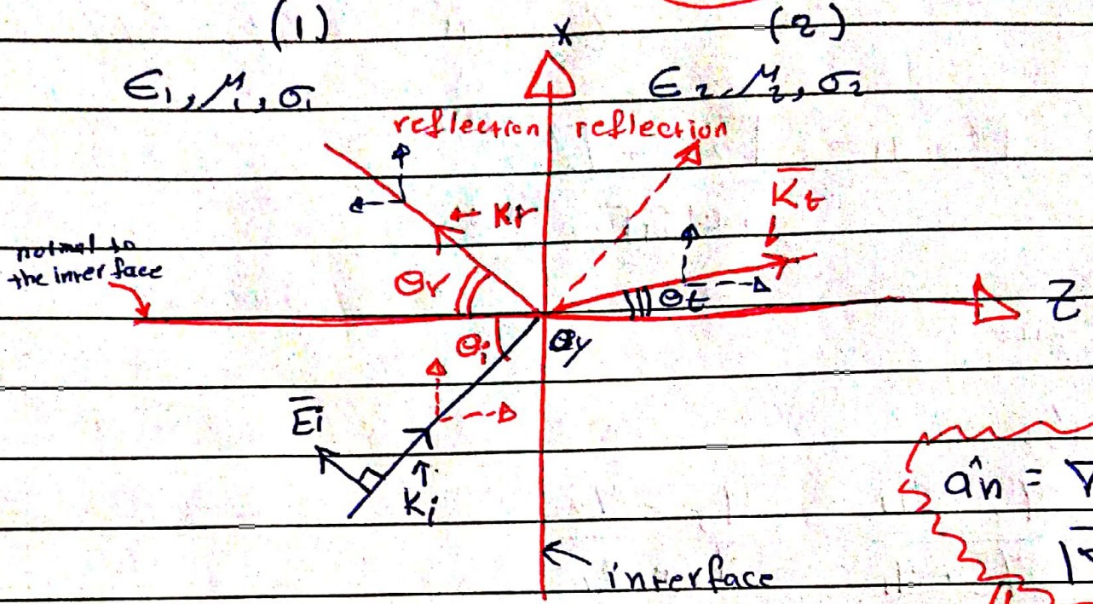
$$\epsilon_r = 2$$

$$\tau_{overall} = \frac{\tau_{12}\tau_{23}(-1)}{1 + \Gamma_{21}\Gamma_{32}(-1)}$$

Scanned by CamScanner

Scanned by CamScanner

EM wave reflections at oblique incidence:



$z = 0$

XY-plane

$\hat{a}_n = \hat{a}_z$

$$\hat{a}_n = \frac{\nabla f}{|\nabla f|}$$

لو كان السطح
مائل

*** in general:-**

$$\bar{E}(r,t) = \bar{E}_0 \cos(\bar{K} \cdot \bar{r} - \omega t) \ V/m$$

$$\bar{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$$

position vector

$$\bar{K} = K_x\,\hat{a}_x + K_y\,\hat{a}_y + K_z\,\hat{a}_z$$

phase constant

$$\bar{K} \cdot \bar{r} = K_x x + K_y y + K_z z$$

if lossless

$$K = \sqrt{K_x^2 + K_y^2 + K_z^2} = \beta = \omega\sqrt{\mu\epsilon}$$

Scanned by CamScanner

fields :-

$$\bar{E}_i = \bar{E}_{io} \cos(K_{ix}X + K_{iy}y + K_{iz}Z - \omega_i t)$$

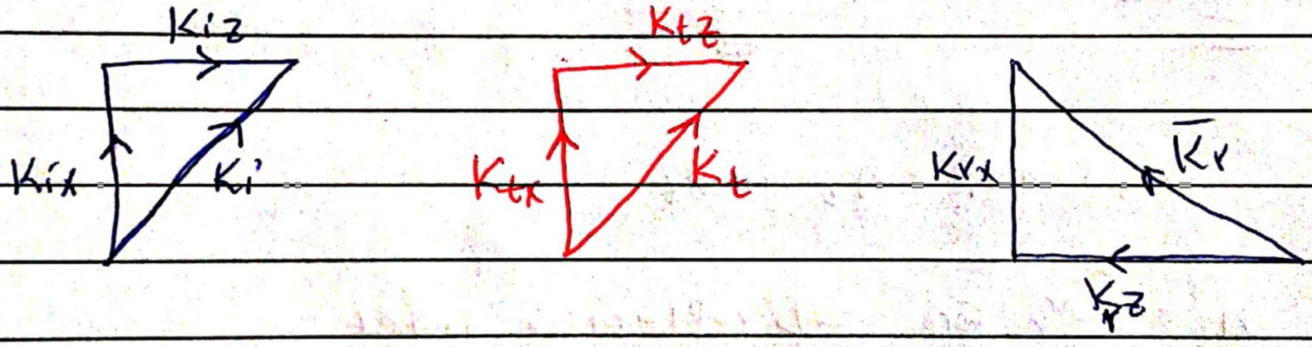
$$\bar{E}_r = \bar{E}_{ro} \cos(K_{rx}X + K_{ry}y + K_{rz}Z - \omega_r t)$$

$$\bar{E}_t = E_{to} \cos(K_{tx}X + K_{ty}y + K_{tz}Z - \omega_t t)$$

Conditions :-

$\Rightarrow$ $\omega_i = \omega_r = \omega_t \Longrightarrow$ Same Sourc

The tangent components of $\bar{E}$ & $\bar{H}$ must be continuous.



$\Rightarrow$ $K_{ix} = K_{rx} = K_{tx}$
$K_{iy} = K_{ry} = K_{ty}$ } both are tangent + $Z=0$

$K_{ix} = K_i \sin\theta_i$
$K_{rx} = K_r \sin\theta_r$,
$K_{tx} = K_t \sin\theta_t$

$K_{ix} = K_{rx}$
$K_i = K_r$ = same media
$\omega\sqrt{\epsilon_1\mu_1}$ $\omega\sqrt{\mu_1\epsilon_1}$

$\boxed{\theta_i = \theta_r}$ Snell's law for reflection

Scanned by CamScanner

$$K_{ix} = K_{tx}$$

$$\boxed{K_i \sin\theta_i = K_t \sin\theta_t}$$ # snell's law for reflection

$$\theta_t = \sin^{-1}\left(\frac{K_i}{K_t}\sin\theta_i\right)$$

$$\omega\sqrt{\mu_1\epsilon_1}\,\sin\theta_i = \omega\sqrt{\mu_2\epsilon_2}\,\sin\theta_t$$

$$\frac{\sqrt{\mu_{r1}\epsilon_{r1}}}{\epsilon}\sin\theta_i = \frac{\sqrt{\mu_{r2}\epsilon_{r2}}}{\epsilon}$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

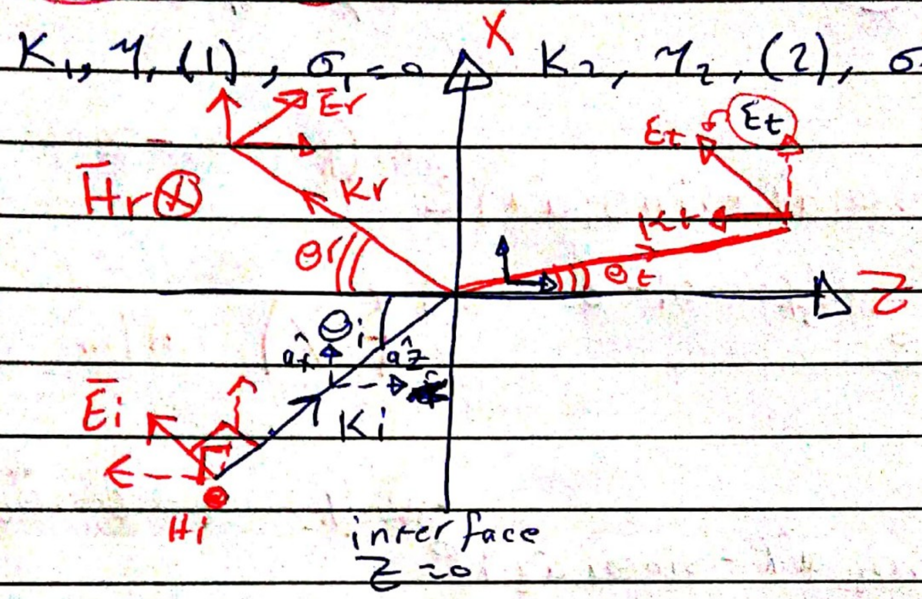
$n_1 = \sqrt{\mu_{r1}\epsilon_{r1}}$ $\Rightarrow$ reflection index

Scanned by CamScanner

24/10/2019

oblique incidence :-

$$\theta_i = \theta_r$$
$$K_i \sin\theta_i = K_t \sin\theta_t$$



in general :-

$$\bar{E} = \bar{E}_o \cos(\bar{K}\cdot\bar{r} - \omega t) \ V/m$$

Based on the $\bar{E}$ - field component with in the wave

either :-

a) Parallel Polarization

b) Perpendicular.

for a) $\hat{a}_H = \hat{a}_K \times \hat{a}_E$ * Parallel component will not charge

or

* the perpendicular component will be reflected.

Scanned by CamScanner

$\bar{E}_i$ & $\bar{H}_i$ fields:-

$\Rightarrow \bar{E}_i = E_{io}(\cos\theta_i \, \hat{a_x} - \sin\theta_i \, \hat{a_z}) \, e^{-jk_1(\sin\theta_i x + \cos\theta_i z)}$ V/m

$\bar{H}_i = \dfrac{E_{io}}{\eta_1} \, e^{-jk_1(y\sin\theta_i + z\cos\theta_i)} \, \hat{a_y}$ A/m

$\Rightarrow \bar{E}_r = E_{ro}(\cos\theta_i \, \hat{a_x} + \sin\theta_i \, \hat{a_z}) \, e^{-jk_1(x\sin\theta_i - z\cos\theta_i)}$ V/m

$\bar{H}_r = \dfrac{E_{ro}}{\eta_1} \, e^{-jk_1(x\sin\theta_i - z\cos\theta_i)} \, (-\hat{a_y})$ A/m

$\Rightarrow \bar{E}_t = E_{to}(\cos\theta_t \, \hat{a_x} - \sin\theta_t \, \hat{a_z}) \, e^{-jk_2(x\sin\theta_t + z\cos\theta_t)}$ V/m

$\bar{H}_t = \dfrac{E_{to}}{\eta_2} \, e^{-jk_2(x\sin\theta_t + z\cos\theta_t)} \, (\hat{a_y})$ A/m

E_{ro} & E_{to} :-

Apply $E_{1\,tangent}^{(t=0)} = E_{2\,tangent}^{(t=0)}$

$\bar{H}_{1\,tangent}^{(z=0)} = \bar{H}_{2\,tangent}^{(z=0)}$

$E_{1\,tang}(z=0) = \bar{E}_{i\,tang}^{(z=0)} + \bar{E}_{r\,tang}^{(z=0)}$

$E_{io}\cos\theta_i \, \hat{a_x} \, \cancel{e^{-jk_1 x\sin\theta_i}} + E_{ro}\cos\theta_i \, \hat{a_x} \, \cancel{e^{-jk_1 x\sin\theta}} = E_{to}\cos\theta_t \, \cancel{e^{-jk_2 x\sin\theta_t}} \, \hat{a_x}$

$\boxed{\cos\theta_i(E_{io} + E_{ro}) = E_{to}\cos\theta_t}$ 1

Scanned by CamScanner

$$H_{i_{tang}}(z=0) + H_{r_{tang}}(z=0) = H_{t_{tang}}(z=0)$$

$$\frac{E_{io}}{\eta_1} e^{-jk_1 x \sin\theta_i} \hat{a}_y - \frac{E_{ro}}{\eta_1} e^{-jk_1 x \sin\theta_i} \hat{a}_y$$

$$= \frac{E_{to}}{\eta_2} e^{-jk_2 x \sin\theta_t} \hat{a}_y$$

$$\boxed{\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{E_{to}}{\eta_2}} \quad \ldots\ldots (2)$$

Scanned by CamScanner

Subject Date No.

27/10/2019

$$(E_{io} - E_{ro}) \cos\theta_i = E_{to} \cos\theta_t \quad (1)$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{E_{to}}{\eta_2} \quad (2)$$

$$\theta_t = \sin^{-1}\left(\frac{K_i}{K_t} \sin\theta_i\right)$$

$$K_i = \frac{\omega}{c}\sqrt{\mu_{r1}\epsilon_{r1}} \quad , \quad K_t = \frac{\omega}{c}\sqrt{\mu_{r2}\epsilon_{r2}}$$

$$\eta_1 = \eta_o\sqrt{\frac{\mu_{r1}}{\epsilon_{r1}}} \quad , \quad \eta_2 = \eta_o\sqrt{\frac{\mu_{r2}}{\epsilon_{r2}}}$$

$$\frac{E_{ro}}{E_{io}} = \Gamma_{11} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\frac{E_{to}}{E_{io}} = \tau_{11} = \frac{2\eta_2 \cos\theta}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$1 + \Gamma_{11} = \tau_{11}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$$

$E_{ro} = \Gamma_{11} E_{io}$, $E_{to} = \tau_{11} E_{io}$

No reflection

$\Gamma_{11} = 0$, this is occurs when $\theta_i = \theta_{B11}$

$$\eta_2 \cos\theta_t = \eta_1 \cos\theta_{B11}$$

Scanned by CamScanner

$$\eta_2^2 \left(1 - \sin^2\theta_t\right) = \eta_1^2 \left(1 - \sin^2\theta_{\beta_\parallel}\right)$$

$$\cancel{\frac{K_1}{K_2} \sin\theta_{\beta_\parallel} = K_2 \sin\theta_t}$$

$\theta_{\beta_\parallel}$ = Brewster angle (Polarizing angle)

@ which $\Gamma_\parallel = 0$

$$\boxed{\sin^2\theta_{\beta_\parallel} = \frac{1 - \dfrac{\mu_2 \epsilon_1}{\mu_1 \epsilon_2}}{1 - \left(\dfrac{\epsilon_1}{\epsilon_2}\right)^2}}$$

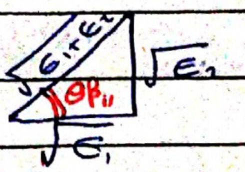
$\triangle$ general

& special case when both media are non-magnetic

$$\mu_{r_1} = 1 = \mu_{r_2} \qquad \mu_1 = \mu_2$$

$$\sin^2\theta_{\beta_\parallel} = \left(\frac{\epsilon_2 - \epsilon_1}{\epsilon_2}\right) \Big/ \left(\frac{\epsilon_2^2 - \epsilon_1^2}{\epsilon_2^2}\right) = \frac{\dfrac{\epsilon_2 - \epsilon_1}{\epsilon_2}}{\left(\dfrac{\epsilon_2 - \epsilon_1}{\epsilon_2}\right)\left(\dfrac{\epsilon_2 + \epsilon_1}{\epsilon_2}\right)}$$

$$\sin\theta_\beta = \sqrt{\epsilon_2 / (\epsilon_1 + \epsilon_2)}$$



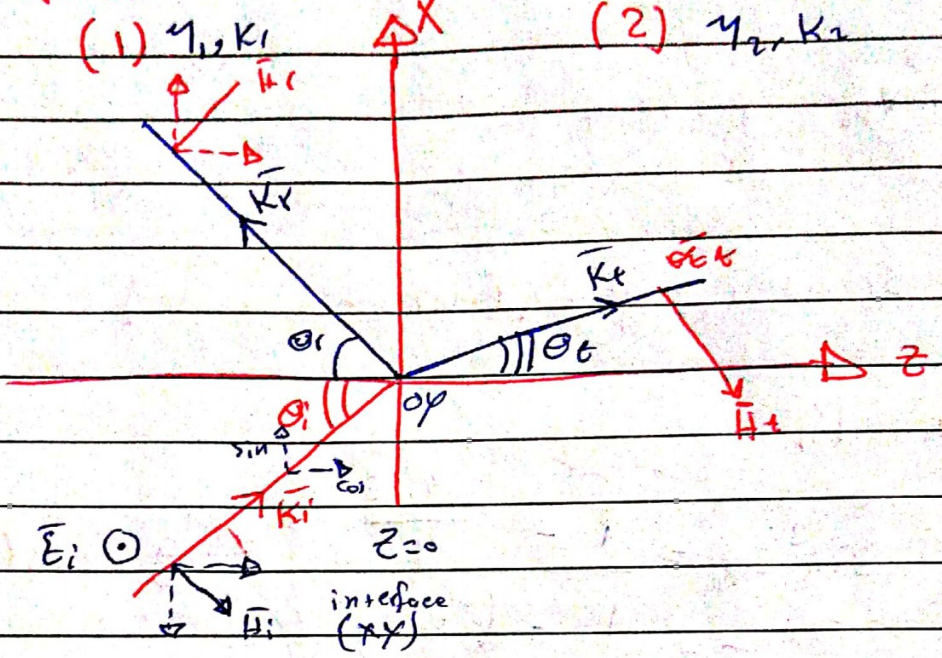
$$\tan\theta_{\beta_\parallel} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\boxed{\theta_{\beta_\parallel} = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)}$$

if $\mu_1 = \mu_2$

Scanned by CamScanner

for Perpendicular Polarization :-



phaser

$$\bar{E}_{is} = E_{io}\, e^{-jK_1(x\sin\theta_i + z\cos\theta_i)}\, \hat{a}_y \ \ V/m$$

$$\bar{H}_{is} = \frac{E_{io}}{\eta_1}\left(\cos\theta_i(-\hat{a}_x) + \sin\theta_i\, \hat{a}_z\right) e^{-jK_1(x\sin\theta_i + z\cos\theta_i)} \ \ A/m$$

$$\bar{E}_r = E_{ro}\, e^{-jK_1(x\sin\theta_i - z\cos\theta_i)}\, \hat{a}_y \ \ V/m$$

$$\bar{H}_r = \frac{E_{ro}}{\eta_1}\left(\cos\theta_i\, \hat{a}_x + \sin\theta_i\, \hat{a}_z\right) e^{-jK_1(x\sin\theta_i - z\cos\theta_i)} \ \ A/m$$

$$\bar{E}_t = E_{to}\left(e^{-jK_2(x\sin\theta_t + z\cos\theta_t)}\right) \hat{a}_y \ \ V/m$$

$$\bar{H}_t = \frac{E_{to}}{\eta_2}\left(\cos\theta_t(-\hat{a}_x) + \sin\theta_t\, \hat{a}_z\right) e^{-jK_2(x\sin\theta_t + z\cos\theta_t)} \ \ A/m$$

Scanned by CamScanner

Subject Date No.

$\Rightarrow$ To find E_{ro} & E_{to} $\rightarrow$ B.C'S

$$E_1(z=0) = E_2(z=0)$$ $$k_1 \sin\theta_i = k_2 \sin\theta_t$$

$$E_i(z=0) + E_r(z=0) = E_t(z=0)$$

$$E_{io} + E_{ro} = E_{to} \text{ --- } (1)$$

$$\bar{H}_1(z=0) = \bar{H}_2(z=0)$$

$$\bar{H}_i(z=0) + \bar{H}_r(z=0) = \bar{H}_t(z=0)$$

$$\frac{E_{io}}{\eta_1}\cos\theta_i(-\hat{a}_x) - \frac{E_{ro}}{\eta_1}\cos\theta_i(-\hat{a}_x) = \frac{E_{to}}{\eta_2}\cos\theta_t(-\hat{a}_x)$$

$$\boxed{\frac{\cos\theta_i}{\eta_1}(E_{io} - E_{ro}) = \frac{E_{to}}{\eta_2}\cos\theta_t} \text{ --- } (2)$$

solve (1) & (2)

To find E_{ro} & E_{to} ? ?

$$\frac{E_{ro}}{E_{io}} = \Gamma_{\perp} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\frac{E_{to}}{E_{io}} = \tau_{\perp} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$[1 + \Gamma_{\perp} = \tau_{\perp}]$$

$$E_{ro} = \Gamma_{\perp} E_{io}$$

$$\Gamma_{\parallel} \text{ and } \Gamma_{\perp}\Big|_{\theta_i = n} = \Gamma_{normal}$$

$$E_{to} = \tau_{\perp} E_{io}$$

NOTEBOOK

Scanned by CamScanner

29/10/2019

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$\Gamma_{\parallel}$, $\Gamma_{\perp}$ $\rightarrow$ Γ_{normal} | $\theta_i = 0$, $\theta_t = 0$

$$1 + \Gamma_{\perp} = \tau_{\perp}$$

Brewster angle :-

$$\Gamma_{\perp} = 0 , \quad \theta_i = \theta_{B\perp}$$

$$\eta_2 \cos\theta_{B\perp} = \eta_1 \cos\theta_t$$

$$\eta_2^2 (1 - \sin^2\theta_{B\perp}) = \eta_1^2 (1 - \sin^2\theta_t)$$

$k_1 \sin\theta_{B\perp} = k_2 \sin\theta_t \Rightarrow$ Snell's

$$\boxed{\sin^2\theta_{B\perp} = \frac{1 - \frac{\mu_1 \epsilon_2}{\mu_2 \epsilon_1}}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2}}$$

if $\epsilon_1 = \epsilon_2 = \epsilon_0$

$$\tan\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_1}}$$

if non magnatic media :-

$\mu_1 = \mu_2 = \mu_0$

$\theta_{B\perp}$ does not exist

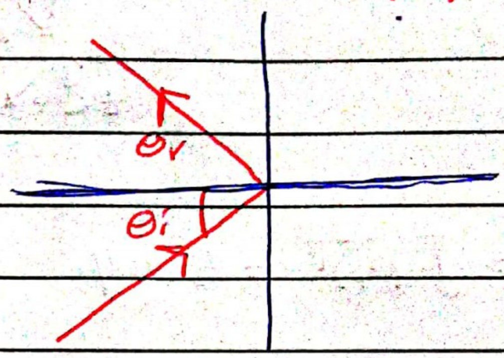
Scanned by CamScanner

* Critical Angle (θ_c)

Total reflection angle

$\tau_\perp = 0$ @ $\theta_i = \theta_c$ $\theta_t = 90^\circ$

(1) (2)



$K_1 \sin\theta_c = K_2 \sin 90^\circ$

$$\theta_c = \sin^{-1}\left(\frac{K_2}{K_1}\right)$$

$$\boxed{\theta_c = \sin^{-1}\left(\sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}}\right)}$$

for parallel
& Perpendicular

metomaterial
μ, ϵ_r
are $-ve$

Ex:- A U.P.W in air with

$$\bar{E} = 8\cos(\omega t - 4x - 3z)\,\hat{a}_y \ V/m$$

is incident on dielectric slab ($z \geq 0$)
with $\mu_r = 1$, $\epsilon_r = 2.5$, $\sigma = 0$

find:-

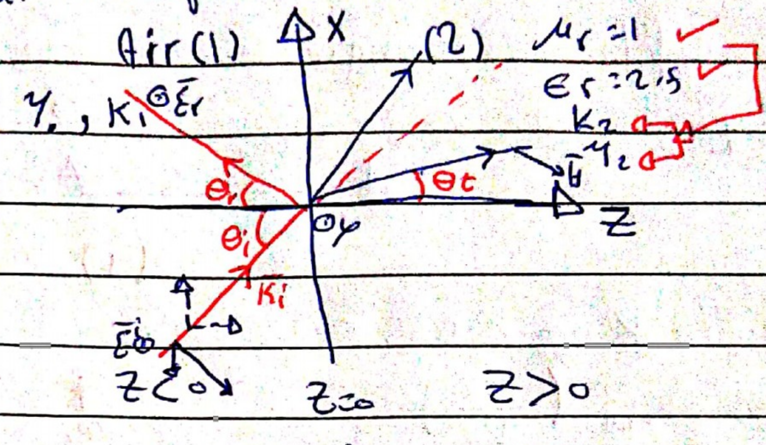
a) The Polarization of wave
b) θ_i
c) $\bar{E}_r$
d) $\bar{H}_t$

Scanned by CamScanner

Subject Date No.

Sol :-

(a) Perpendicular Polarization



(b) $K_1 = \sqrt{4^2 + 3^2} = 5$ rad/m , $K_1 = \frac{\omega}{c}$, $\omega = 15 \times 10^8$ rad/s

$K_1 \cos\theta_i = 3$

$\theta_i = \cos^{-1}\frac{3}{5}$ or $\theta_i = \sin^{-1}\frac{4}{5} = 53.13°$

(c) $E_r = E_{ro}\, e^{-j(K_1)(\omega t + 4x - 3z)}\, \hat{a}_y$ V/m

5

(d) θ_t , $K_2 = \frac{\omega}{c}\sqrt{2.5} = 5\sqrt{2.5} = 7.93$ rad/m

$K_{2x} = K_2 \sin\theta_t = 4$

$\theta_t = \sin^{-1}\left(\frac{4}{7.9}\right) = 30.39 < \theta_1$

$\epsilon_{r2} > \epsilon_{r1}$

$K_{2z} = K_2 \cos\theta_t = 6.819$ rad/m

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 0.389$$

$E_{ro} = \Gamma_\perp E_{io} = -3.112$ V/m

E_r وعوضها بال

NOTEBOOK

Scanned by CamScanner

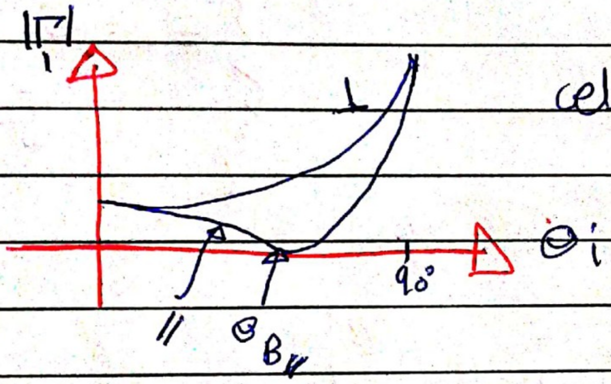
Scanned by CamScanner

Subject Date No.

[d] $\bar{H}_t = \frac{E_{t0}}{\eta_2}\left(\cos\theta_t(-\hat{a}_x) + \sin\theta_t \hat{a}_z\right) e^{-jk_2(\omega t - 4x - 6.819z)}$ A/m

$\eta_2 = \frac{\eta_0}{\sqrt{2.5}} = 238.4\,\Omega$, $E_{t0} = \tau_\perp E_{i0} = 4.888$ V/m , $\tau_\perp = 1 + \Gamma_\perp = 0.611$

$\bar{H}_t = (-17.69\hat{a}_x + 10.37\hat{a}_z)\cos(15\times10^8 t - 4x - 6.819z)$ mA/m



* نقطتين مهمات عشان تحلوا

CH: 10

(1)

(2)

Maxwells' eq:-

$\bar{K}\cdot\bar{E} = 0$, $\bar{K}\cdot\bar{H} = 0$

$\bar{H} = \frac{\hat{a}_k \times \bar{E}}{\eta}$, $\bar{E} = \frac{\bar{H} \times \hat{a}_k}{j\omega\epsilon}$

$\bar{K}\times\bar{E} = \omega\mu\bar{H}$, $\bar{K}\times\bar{H} = -\omega\epsilon\bar{E}$

N O T E B O O K

Scanned by CamScanner

Scanned by CamScanner

$$\bar H = \frac{\bar k \times \bar E}{\omega \mu} = \frac{\overset{\omega\sqrt{\mu\epsilon}}{k}\,\hat a_k \times \bar E}{\omega \mu} = \sqrt{\frac{\epsilon}{\mu}}\,\hat a_k \times \bar E$$

$$\bar H = \frac{\hat a_k \times \bar E}{\eta} \rightarrow E \hat a_E$$

$$H = \frac{E}{\eta}, \quad \hat a_H = \hat a_k \times \hat a_E$$

$$\bar E = \frac{\bar H \times \bar k}{\omega \epsilon} = \frac{\bar H \times k \hat a_k}{\omega \epsilon} = \hat H \times a_k \sqrt{\frac{\mu}{\epsilon}} = \eta \bar H \times \hat a_k$$

CH:9+10 [illegible] H_0

$$\epsilon_c = \epsilon' - j\epsilon''$$

$$\epsilon'' \rightarrow \frac{\sigma}{\omega}, \quad \sigma \neq 0$$

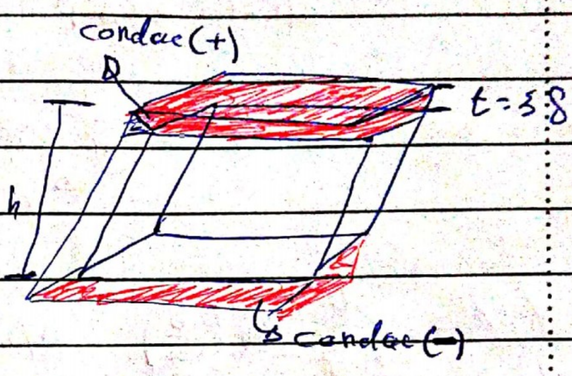
END of CH-10

Scanned by CamScanner

31/10/2019

CH-11 :- Transmission lines :- (T.L)

→ Tow or mor conductor
- → coaxil cable ($f = 300 MHz$)
- → Twin-wire cable ($f \simeq 30 MHz$)
- → planer lines
  - → strip line
  - → sor line
  - → slot line
  - → micro strip
  - → co planner
  - wave guide line



* Transmission line Parameters :-

i.e ⇒ Twin wire cable

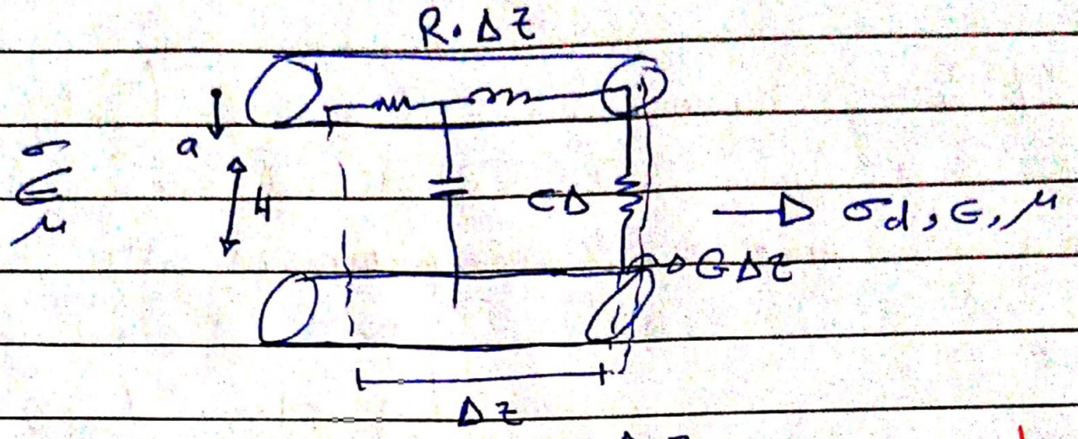
* for a coaxal cable :-

$$\Rightarrow \frac{R}{L} = \frac{1}{2\pi \delta_c}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$\Rightarrow \frac{C}{L} = \frac{2\pi \epsilon_0 \epsilon_r}{\ln\left(\frac{b}{a}\right)} \quad f/m$$

$$\Rightarrow \frac{G}{L} = \frac{2\pi \sigma_d}{\ln\left(\frac{b}{a}\right)}$$

Scanned by CamScanner



CH - 10 → (AC)

$$\frac{1}{\alpha} = \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$\frac{1}{\sigma} = Rd \rightarrow$ CH - 6 $\rightarrow T_r = Rd\,c = \frac{\epsilon}{\sigma_d}$

↳ relation time

$$\Rightarrow L/l = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \quad H/m$$

↳ $L_{internal}$ ✗

↳ $L_{external}$ ✓

Note:- $Rd \cdot C = \frac{C}{G} = \frac{\epsilon_0 \epsilon_r}{\sigma}$

$$\frac{L_{ext}}{l} \cdot \frac{C}{l} = \mu \epsilon$$

$X_L = \omega L$

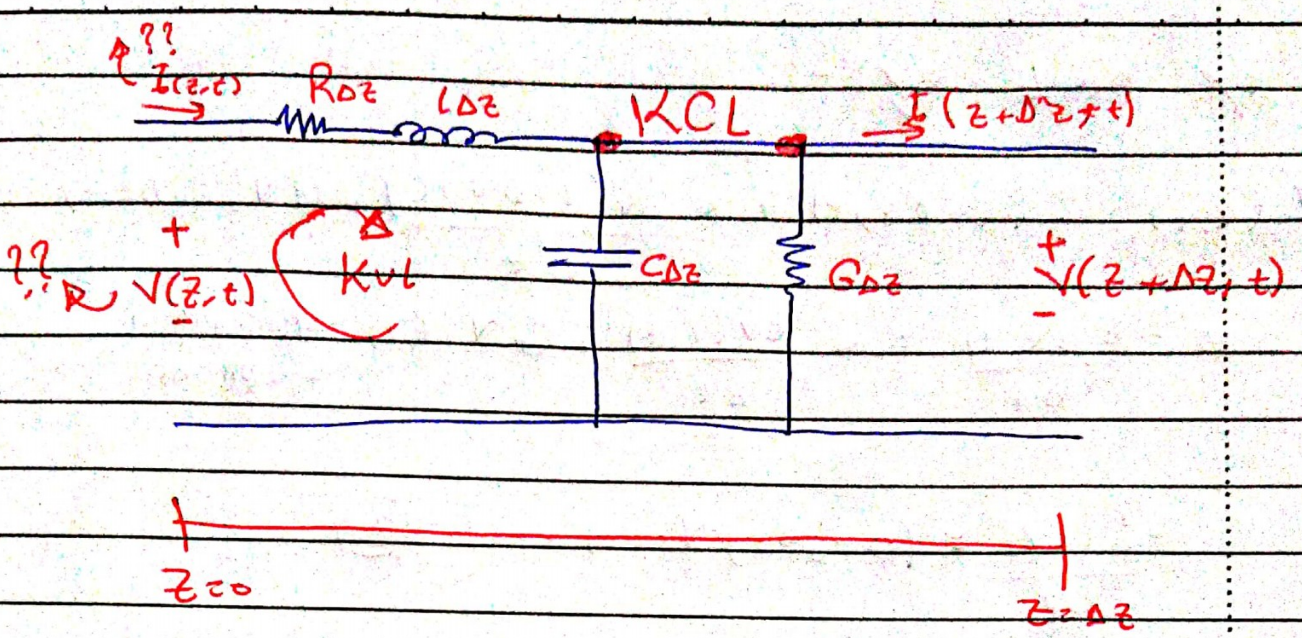
$L = \frac{X_L}{\omega}$

⇒ equivelant CKT for tranmision line:-

→ L-type → π-type → T-type

Scanned by CamScanner

Subject Date No.



by KVL eq:-

$$-V(z,t) + R\Delta z\, I(z,t) + L \frac{dI(z,t)}{dt} + V(z+\Delta z,t) = 0$$

$$-\left[V(z+\Delta z,t) - V(z,t)\right] = R\Delta z\, I(z,t) + L\Delta z \frac{dI(z,t)}{\partial t}$$

⇒ Divide by Δz:-

⇒ take lim $\qquad \frac{-\partial v(z,t)}{\partial t} = R\, I(z,t) + L \frac{\partial I(z,t)}{\partial t}$

⇒ convert to freq.-

$$\frac{\partial}{\partial t}\left(e^{j\omega t}\right) = j\omega\, e^{j\omega t}$$

$$\therefore -\frac{\partial V_s(z)}{\partial z} = R\, I_s(z) + j\omega L\, I_s(z)$$

$$\therefore -\frac{\partial V_s(z)}{\partial z} = (R + j\omega L)\, I_s(z) \quad \text{-----} \; (1)$$

⇒

Scanned by CamScanner

Subject Date No.

by KCL eq:-

$$I(z,t) = I(z+\Delta z, t) + C\frac{\partial v(z+\Delta z, t)}{\partial t} + G\,V(z+\Delta z, t)$$

$$\Rightarrow -\frac{\partial I(z,t)}{\partial t} = C\frac{\partial v(z,t)}{\partial t} + G\,V(z,t)$$ $\Rightarrow$ convert to phasor

$$-\frac{I_s(z)}{\partial z} = (G + j\omega C)\,V_s(z) \quad \cdots \quad (2)$$

$\Rightarrow$ To solve for $V_s(z)$ & $I_s(z)$

$$\Rightarrow -\frac{\partial^2 V_s(z)}{\partial z^2} = (R + j\omega L)(G + j\omega C)\,V_s(z)$$

$\gamma = \alpha + j\beta$

Voltage wave eq:- $\Rightarrow$ $$\frac{\partial^2 V_s(z)}{\partial z^2} - \gamma^2 V_s(z) = 0$$

$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

Current wave eq:- $\Rightarrow$ $$\frac{\partial^2 I_s(z)}{\partial z^2} - \gamma^2 I_s(z) = 0$$

N O T E B O O K

Scanned by CamScanner

Subject Date No.

* Characteristic Impedance (Z_o) in Ω :-

$$\sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\frac{\gamma}{G+j\omega C} = \frac{R+j\omega L}{\gamma}$$

$$Z_o = \frac{\text{Voltage}}{\text{Current}} = \frac{V_o^+}{I_o^+} = \frac{-V_o^-}{I_o^-}$$

reflected (Backward)

* Solution to wave eq :-

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$$

$$I_s(z) = \underbrace{I_o^+ e^{-\gamma z}}_{\text{forward wave } +ve\ z} + \underbrace{I_o^- e^{\gamma z}}_{\text{Backword wave } -ve\ z}$$

N O T E B O O K

Scanned by CamScanner

3/11/2019

Transmeition line equations :-

$$\frac{\partial^2 V_s(z)}{\partial z^2} - \gamma^2 V_s(z) = 0$$

$$\frac{\partial^2 I_s(z)}{\partial z^2} - \gamma^2 I_s(z) = 0$$

$$V = -\int_l \bar{E} \cdot \bar{dl}$$

$$I = \int_l \bar{H} \cdot \bar{dl}$$

Sol:-

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad (V)$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad (A)$$

$$\gamma = \alpha + j\beta \quad , \quad e^{-\gamma z} = \underbrace{e^{-\alpha z}}_{\text{Amp}} \ \& \ \underbrace{e^{-j\beta z}}_{\text{phare}}$$

in time domain:-

$$V(z,t) = Re\{V_s(z) e^{j\omega t}\}$$

$$V(z,t) = V_0^+ e^{-\alpha t} \cos(\omega t - \beta z) + V_0^- e^{\alpha z} \cos(\omega t + \beta z) \ (V)$$

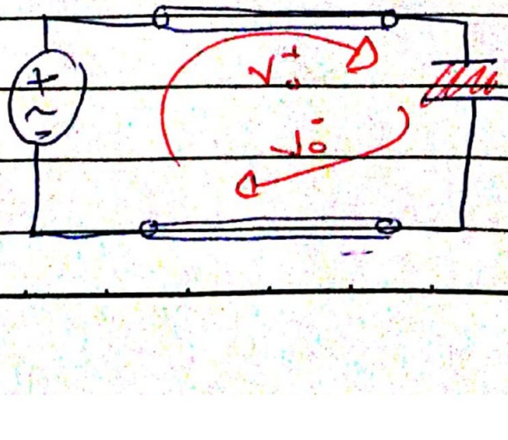
$$I(z,t) = I_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + I_0^- e^{\alpha z} \cos(\omega t + \beta z) \ (A)$$

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0$$

characteristics Addmittans (Y)

$$Y_0 = \frac{1}{Z_0} = G_0 + jB_0$$

$$I(z,t) = \frac{V_0^+}{Z_0} e^{-\alpha z} \cos(\omega t - \beta z) - \frac{V_0^-}{Z_0} e^{\alpha z} \cos(\omega t + \beta z) \ (A)$$



Scanned by CamScanner

Subject Date No.

In general :- lossy T.L

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$\lambda = 2\pi/\beta$, $u = \frac{\omega}{\beta}$, $T = \frac{1}{f}$

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \ (V)$$

$$I_s(z) = \frac{1}{Z_0}\left[V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right] \ (A)$$

Special Case :-

A] lossless T.L.

$R = 0 = G \left(= \frac{1}{R_d}\right)$

$\sigma_c = \infty$ $\sigma_d = 0$

$\gamma = j\omega\sqrt{LC}$ ch:10

$\alpha = 0$, $\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\epsilon}$

$Z_0 = \sqrt{\frac{L}{C}} = R_0$ (Pure real)

$X_0 = 0$

$\eta = \sqrt{\frac{\mu}{\epsilon}} \rightarrow$ ch:10

$$R = \frac{1}{2\pi\sigma_c\delta}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$\sigma_c = \infty$

$$G = \frac{2\pi \cancel{\sigma_d}^{\,0}}{\ln \frac{b}{a}}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

N O T E B O O K

Scanned by CamScanner

B Distortionless T.L :-

$$\boxed{\frac{R}{L} = \frac{G}{C}} \quad , \quad R = \frac{LG}{C}$$

$$\gamma = \sqrt{R\left(1+\frac{j\omega L}{R}\right) G\left(1+\frac{j\omega C}{G}\right)} = \sqrt{RG}\left(1+j\omega\frac{L}{R}\right)$$

$$= \sqrt{RG} + j\sqrt{RG}\,\omega\frac{L}{R}$$

$\alpha \neq 0$

$$\beta = \omega\sqrt{\frac{RGL^2}{R}} \quad , \quad \frac{G}{R} = \frac{C}{L}$$

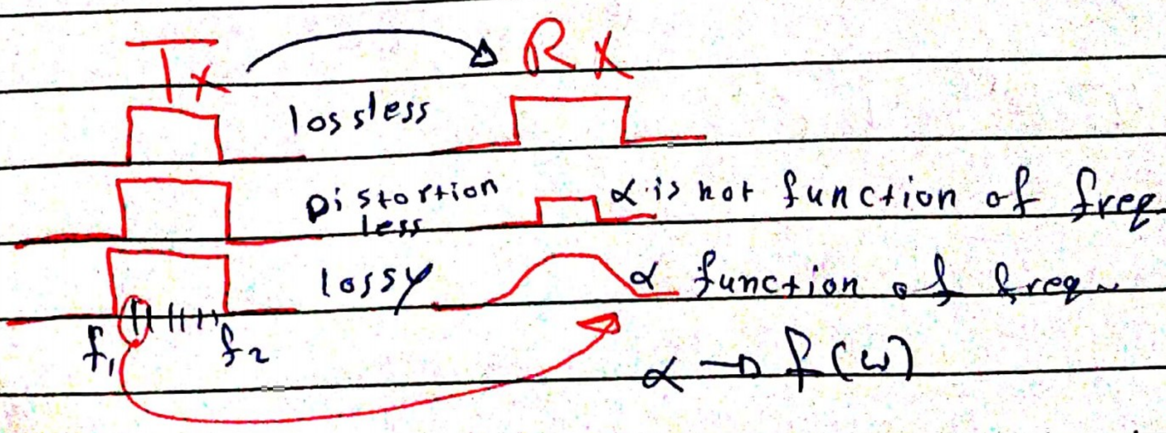
$$\beta = \omega\sqrt{CL}$$

$$Z_0 = \sqrt{\frac{R(1+j\omega L/R)}{G(1+j\omega C/G)}} = \sqrt{\frac{L}{C}}$$

$$Z_0 = R_0 = \sqrt{\frac{L}{C}} \text{ real} \qquad X_0 = 0$$

$$V_s(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z} \ (V)$$

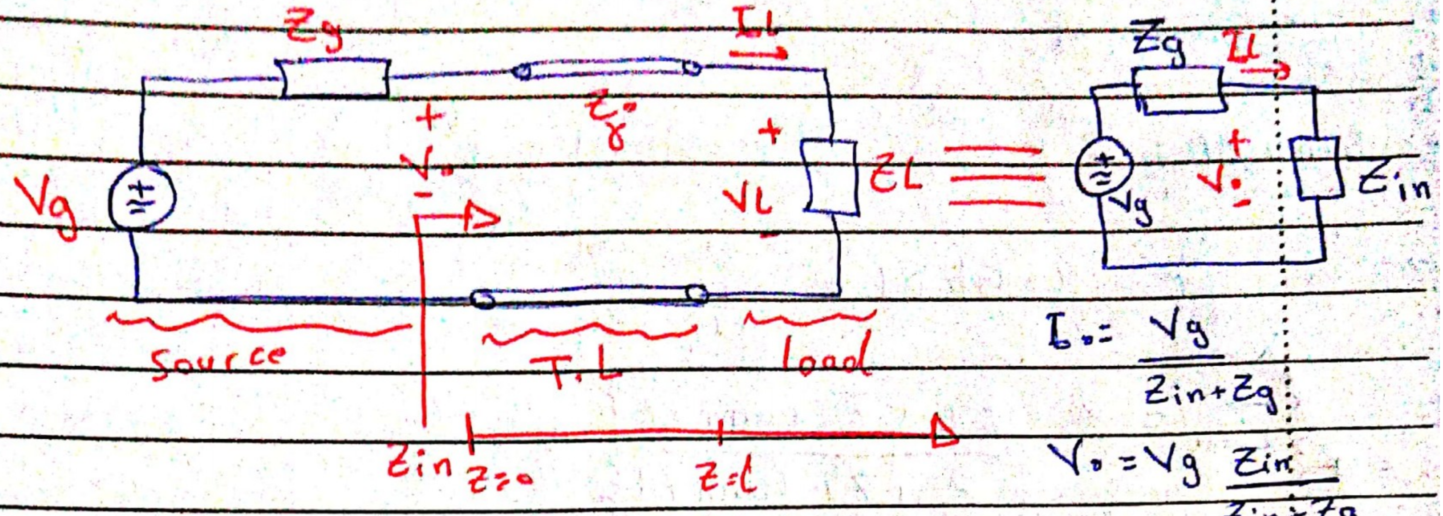
$$u = \frac{1}{\sqrt{LC}} \quad , \quad \lambda = \frac{2\pi}{\beta}$$



Scanned by CamScanner

Subject　　　　　　　　　　　　　　　　Date　　　　　No.

* Input Impedance, SWR and Power



$$I_o = \frac{V_g}{Z_{in}+Z_g}$$

$$V_o = V_g \frac{Z_{in}}{Z_{in}+Z_g}$$

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$

$$I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z}$$

@ $z=0$

$$V_s(0) = V_o = V_o^+ + V_o^- \quad \ldots (1) \quad \text{Solve for } V_o^+, V_o^-$$

$$I_s(z) = I_o = \frac{V_o^+ - V_o^-}{Z_o} \quad \ldots (2) \quad V_o^+ = \frac{1}{2}(V_o + I_o Z_o)$$

$$V_o^- = \frac{1}{2}(V_o - I_o Z_o)$$

@ $z=l$

$$V_s(l) = V_L = V_o^+ e^{-\gamma l} + V_o^- e^{\gamma l} \quad \ldots (1) \quad \text{Solve for } V_o^+, V_o^-$$

$$I_s(l) = I_L = \frac{V_o^+ e^{-\gamma l} - V_o^- e^{\gamma l}}{Z_o} \quad \ldots (2) \quad V_o^+ = \frac{1}{2}(V_L + I_L Z_o) e^{\gamma l}$$

$$V_o^- = \frac{1}{2}(V_L - I_L Z_o) e^{-\gamma l}$$

$$\frac{V_o}{I_o} = Z_{in}\Big|_{z=0}, \quad \frac{V_L}{I_L} = Z_L, \quad Z_{in} = \frac{V(z)}{I(z)}$$

Scanned by CamScanner

$$Z_{in}\Big| = \frac{V_s}{I_s} = Z_o \frac{V_o^+ e^{-\gamma l} + V_o^- e^{\gamma l}}{V_o^+ e^{-\gamma l} - V_o^- e^{\gamma l}}$$

as function of (l) →

$$\sinh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{2}, \quad \cosh \gamma l = \frac{e^{\gamma l} + e^{-\gamma l}}{2}$$

$$\tanh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}}$$

lossy →
$$\boxed{Z_{in} = Z_o \frac{Z_L + Z_o \tanh \gamma l}{Z_o + Z_L \tanh \gamma l}} \; \Omega \; !$$

if lossless → $\gamma = j\beta$

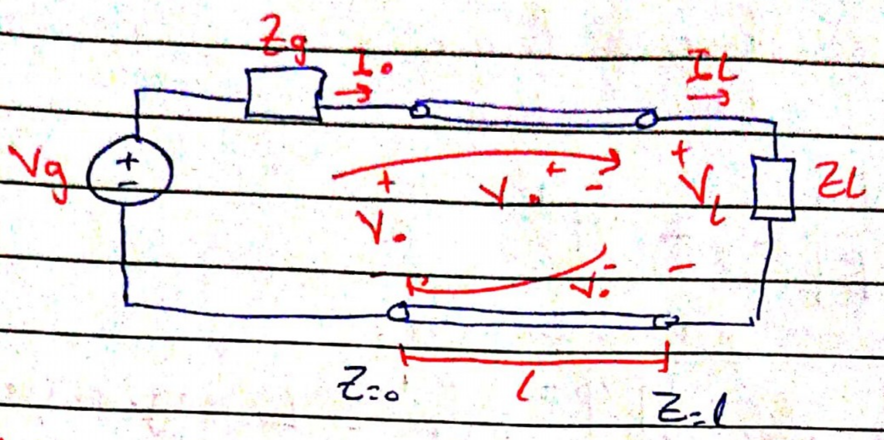
$$\tanh(j\beta l) = j\tan \beta l$$

lossless →
$$\boxed{Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l}} \; \Omega$$

Scanned by CamScanner

Subject　　Date　　No.

5/11/2019



lossless:-

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}$$

$$V_s(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

$$I_s(z) = \frac{V_o^+}{Z_o} e^{-j\beta z} + \frac{-V_o^-}{Z_o} e^{j\beta z}$$

Reflection Coefficient:-

$$\Gamma = \frac{\text{reflected signal}}{\text{Incident signal}}$$

$$= \frac{V_o^-}{V_o^+} \frac{e^{+\gamma L}}{e^{-\gamma l}} \quad \text{if lossy}$$

$$= \frac{V_o^-}{V_o^+} e^{2\gamma L}$$

$$= \frac{V_o^-}{V_o^+} e^{2j\beta l} \quad \text{if lossless}$$

$$|\Gamma| = \frac{V_o^-}{V_o^+} \Longrightarrow |V_o^-| = |\Gamma| |V_o^+| \quad \text{---} (1)$$

$$\Gamma_i = -\Gamma_{voltage} \qquad \Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} \quad \text{----} (2)$$

$$= |\Gamma| \angle \theta_\Gamma$$

N O T E B O O K

Scanned by CamScanner

Subject Date No.

$$SWR = S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}}$$

$$= \frac{|V_0^+| + |V_0^-|}{|V_0^+| - |V_0^-|} \div V_0^+$$

$$\boxed{S = \frac{1 + |\Gamma|}{1 - |\Gamma|}} \qquad \boxed{|\Gamma| = \frac{S-1}{S+1}} \rightarrow \text{mag. only}$$

No reflection $\leftarrow$ $0 \leq |\Gamma| \leq 1 \rightarrow$ total reflection

$$1 < S < \infty$$

$$0 < S(dB) < \infty$$

Power (real) $\Rightarrow$ $P = \frac{Re}{2}\{ V_s(z)\, I_s^*(z) \}$ Peak

$$V_s(z) = V_0^+ e^{-j\beta z} + |\Gamma V_0^+| e^{+j\beta z}$$

$$I_s^*(z) = \frac{V_0^+}{Z_0} e^{+j\beta z} - \frac{|\Gamma V_0^+|}{Z_0} e^{-j\beta z}$$

$$P = \frac{1}{2}\left(\frac{V_0^{+2}}{Z_0} - \frac{|\Gamma V_0^+|^2}{Z_0} \right)$$

$$\boxed{P = \frac{V_0^{+2}}{2Z_0}\left(1 - |\Gamma|^2\right)} \; W$$

$$P = \frac{V_0^{+2}}{2Z_0} - |\Gamma|^2 \frac{V_0^{+2}}{2Z_0}, \quad P_t = P_i - P_r$$

$P_r = 0$ if $\Gamma = 0 \rightarrow P_t = P_i \Rightarrow$ M.P.T

NOTEBOOK

Scanned by CamScanner

Subject Date No.

$$Z_{in_{max}} = \frac{V_{max}}{I_{min}} \cdot \frac{V_{min}}{V_{min}}$$

$$\boxed{Z_{in_{max}} = S Z_0}$$

$$Z_{in_{min}} = \frac{V_{min}}{I_{max}} \cdot \frac{I_{min}}{I_{min}}$$

$$\boxed{Z_{in_{min}} = \frac{Z_0}{S}}$$

كل ماسبق كان

for general load

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}, \quad S = \frac{1-|\Gamma|}{1+|\Gamma|}$$

* Special Case :-

$\boxed{1}$ short-circuit load :- $Z_L = 0$

$$Z_{in_{sc}} = j Z_0 \tan\beta l \ \Omega$$

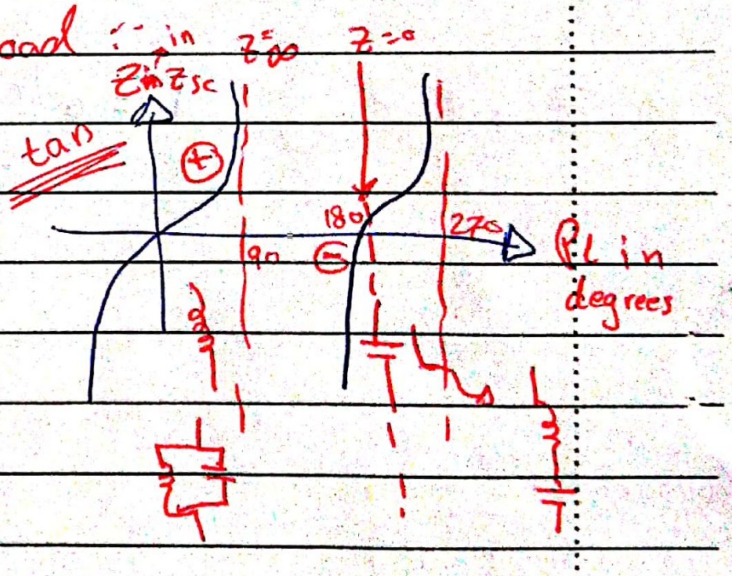
→ pure imagenary

$$\Gamma = -1 = 1 \angle 180^\circ$$

$$S = \infty$$

$\frac{2\pi}{\lambda}$

terms of λ



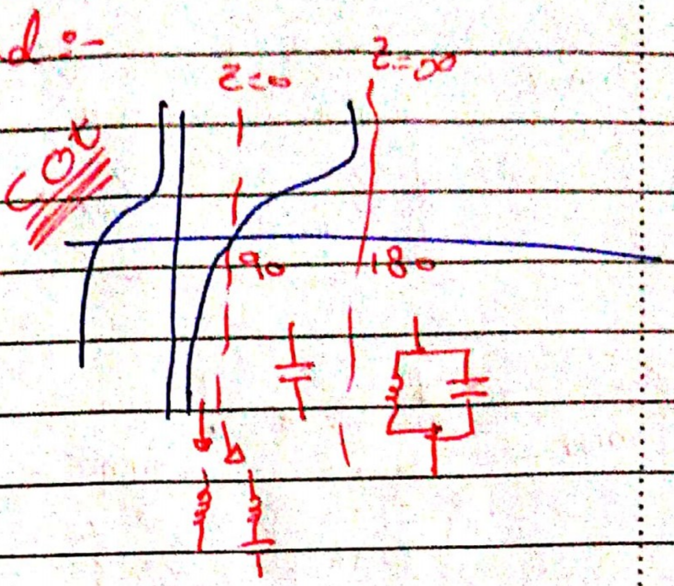
Scanned by CamScanner

Subject Date No.

[2] open-circuit load :-

$Z_L = \infty$

$Z_{in_{oc}} = -jZ_0 \cot \beta l$ → pure imagenary



$$\boxed{Z_0 = \sqrt{Z_{in_{sc}} * Z_{in_{oc}}}}$$

[3] matched load :-

$Z_L = Z_0$

M.P.T

$Z_{in} = Z_0$ $\Gamma = 0$ $S = 1$ $P = \frac{{V_0^+}^2}{2Z_0}$ W

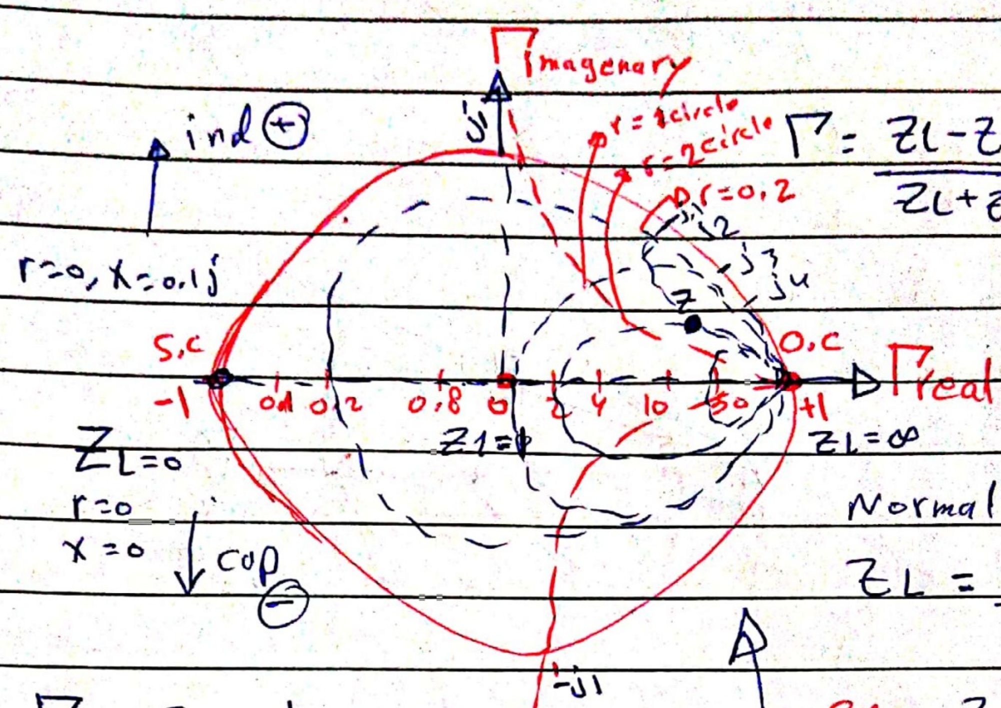
lossless → smith chart

lossy → equation

Scanned by CamScanner

Subject Date No.

7/11/2019



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$, Z_0 is real lossless

Normalization

$$z_L = \frac{Z_L}{Z_0}$$

$$\Gamma = \frac{z_L - 1}{z_L + 1} = \Gamma_r + j\Gamma_i = |\Gamma| \angle \theta_\Gamma$$

ex:- $Z_L = 50 + j100$, $Z_0 = 50$

$z_L = 1 + 2j$

$\Gamma = 1$ ($r = 0$)
circle

circle

$S = \infty$
circle

$z_L < 1$ | $z_L > 1$

← | →

$Z_L < Z_0$ | $Z_L > Z_0$

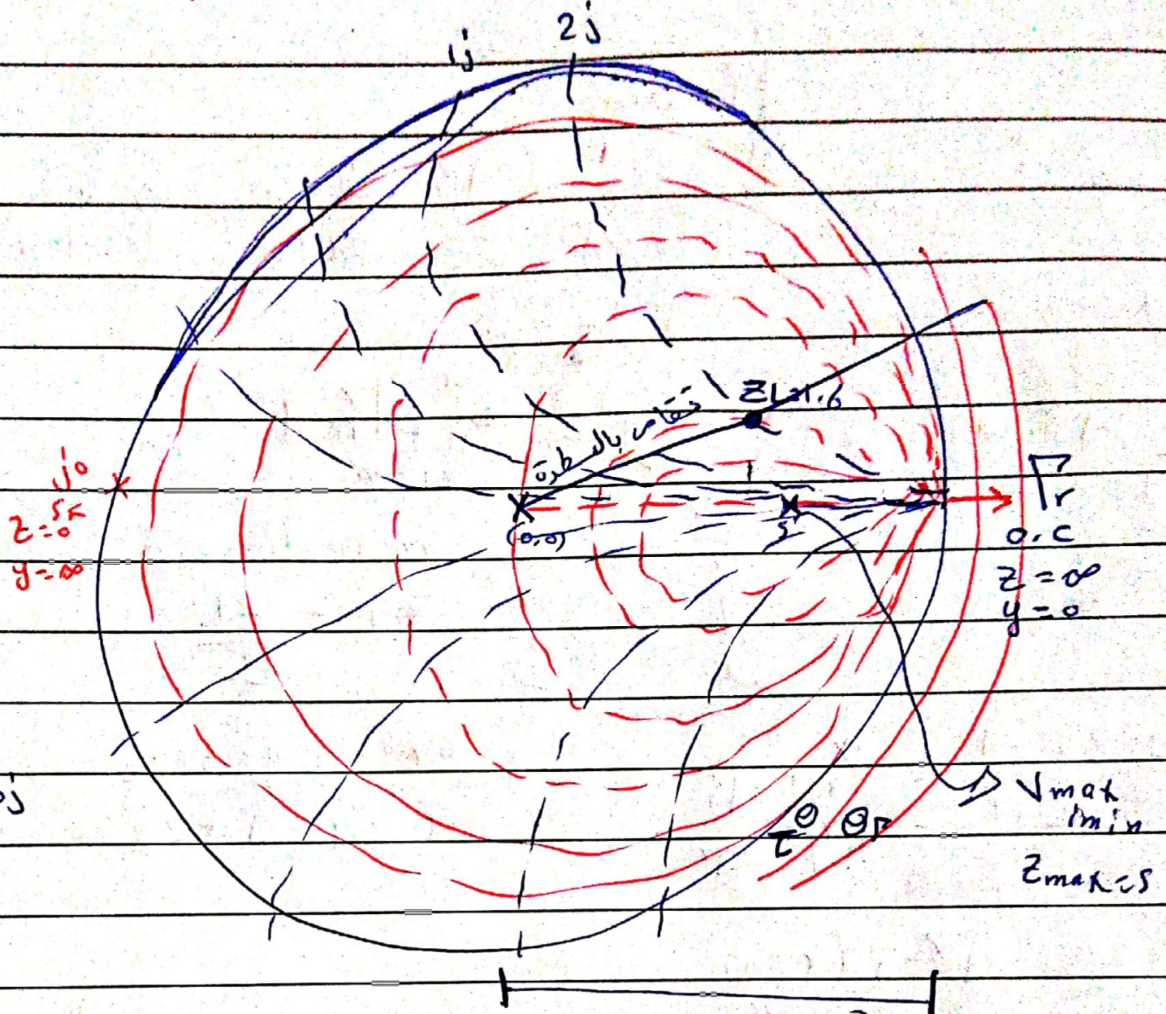
N O T E B O O K

Scanned by CamScanner

Scanned by CamScanner

Subject Date No.

7/11/2019



ind
+ve

-ve
cap

$\Gamma = 0 + 0j$

$\Gamma = \frac{Z_L - 1}{Z_L + 1}$, $Z_L = \frac{Z_L}{Z_0}$

ex:- $Z_L = 80 + j100\,\Omega$, $Z_0 = 50\,\Omega$

$Z_L = 1.6 + 2j$

$\Gamma = |\Gamma| \angle \theta_\Gamma$

مثلاً قياس المسطرة

3cm

$\Gamma = 1 \rightarrow 7.7\text{cm}$

$\Gamma = ?? \rightarrow 3\text{cm}$

$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$

$Z_{inmax} = S Z_0 \rightarrow Z_{inmax} = S$

$Z_{inmin} = \frac{Z_0}{S} \rightarrow Z_{inmin} = \frac{1}{S}$

NOTEBOOK

Scanned by CamScanner

$$Y_0 = \frac{1}{Z_0} \qquad y_L = \frac{Y_L}{Y_0} = \frac{Y_L}{\frac{1}{Z_0}}$$

$$\therefore Y_L = Z_0 \cdot y_L$$

$$y_L = \frac{Y_L}{Z_0} \ (S)$$

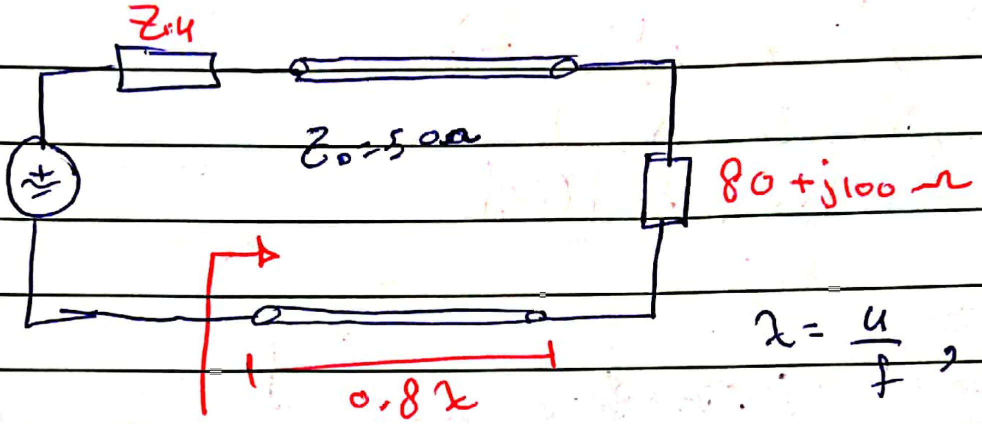
END of MID MATERIAL

Scanned by CamScanner

$$Y_o = \frac{1}{Z_o} \qquad y_L = \frac{Y_L}{Y_o} = \frac{Y_L}{\frac{1}{Z_o}}$$

$$y_L = Z_o Y_L$$

$$Y_L = \frac{y_L}{Z_o} \ (S)$$



$$\lambda = \frac{u}{f}, \quad u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{\omega}{\beta}$$

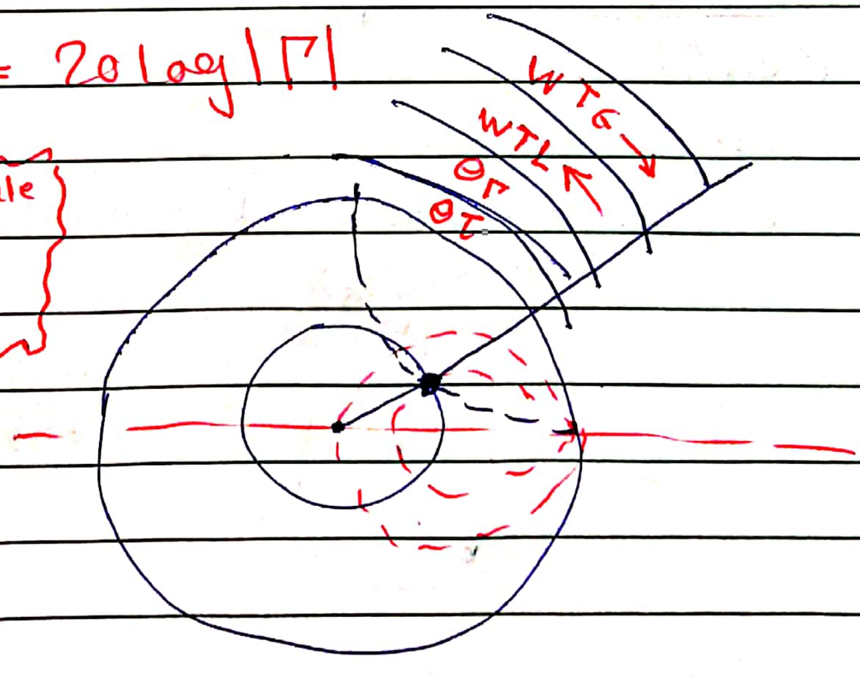
$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta L}$$

$$l \to \lambda$$

$$\beta = 2\pi/\lambda$$

Return loss = $20\log|\Gamma|$

$$\boxed{\begin{array}{c} 360^\circ = \frac{\lambda}{2} \text{ scale} \\ \Rightarrow \lambda = 720^\circ \end{array}}$$



Subject Date No.

10/11/2019

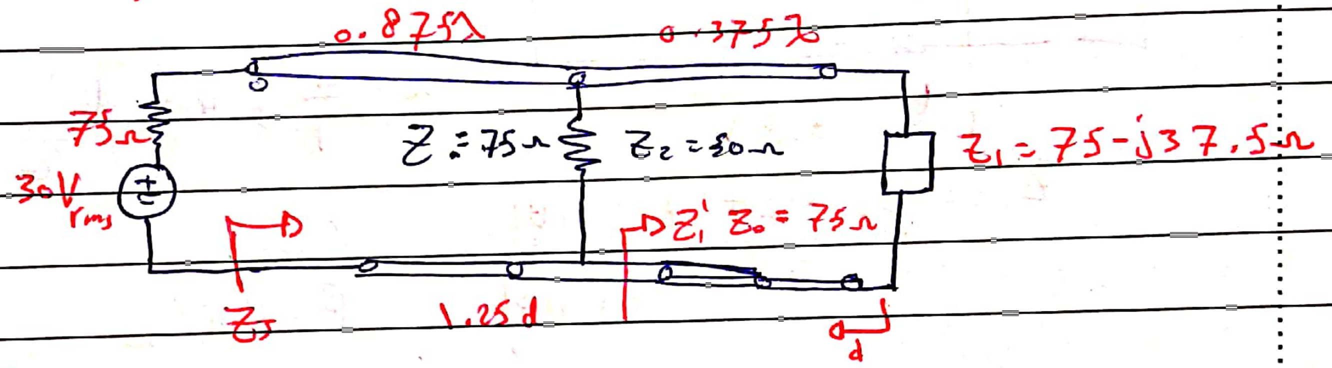
Ex:- A lossless T.L is connected to 30 V_{rms} with a 75 Ω internal impedance & to two load Z_1 & Z_L if $Z_0 = 75$ Ω

Find:-

a) Z_i', Y', Z_2', Y_2', Z_s, Y_s

b) I_1, V_2, I_2, V_s, I_s

c) sketch $|V(d)|$ & $|I(d)|$



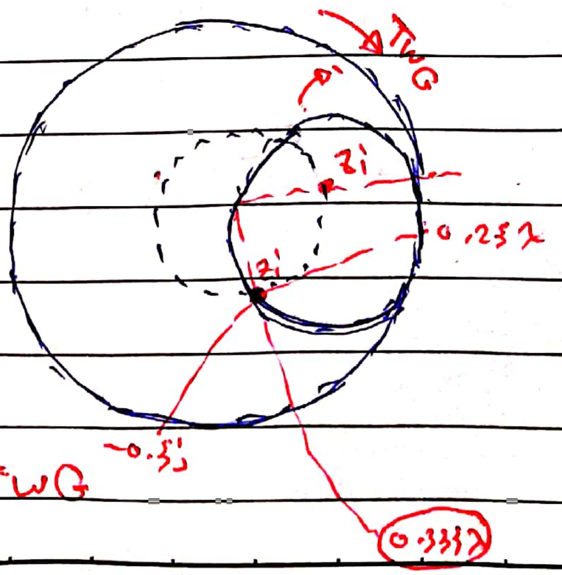
1) Normalization

$$z_1 = \frac{75 - j37.5}{75} = 1 - j0.5$$

2) Draw Γ circle

3) Move 0.357λ on TWG

till 0.357λ + 0.355 − 0.5λ = 0.23λ



Smith Chart

4) $Z_1' = 1.6 + 0.2j$ $\quad Z_1 = Z_1' Z_0$

$= 120 + 15j\,\Omega$

$y_1' = \frac{1}{Z_1'} = 0.61 - 0.075j$

$y_1' = \frac{1}{Z_1'}$ oR $\frac{y_1}{Z_0} = 0.008 - 0.00053j\,\Omega^{-1}$

5) $Z_2' = Z_1 // Z_2 = \frac{Z_1 * Z_2}{Z_1 + Z_2} = \frac{120+15j * 50}{120+15j+50} = 37.5 + j2.85\,\Omega$

$y_i' = y_1' + y_2$

$= 0.01 + 0.075 + \frac{75}{50} =$

$y_2' = \frac{y_2}{75} = \frac{1}{Z_0} = 0.028 - 0.001j\,\Omega^{-1}$

6) $Z_2' = \frac{Z_i}{75} = 0.5 + j0.03$

7) Move TWG a distance

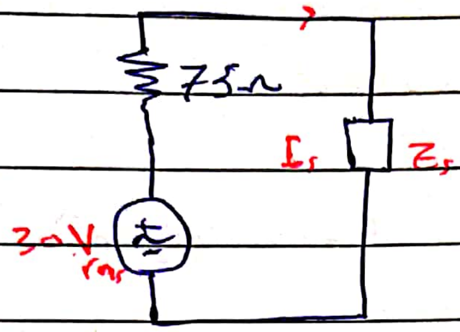
$0.88\lambda - 0.5\lambda = 0.38\lambda$

8) $Z_s = 0.765 - j0.575$

$Z_s = 57.4 - j43.1\,\Omega$

$Y_s = \frac{1}{Z_s} = 0.11 + j0.008\,\Omega^{-1}$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \qquad \beta = \frac{2\pi}{\lambda}$$



$$V_s = \frac{30(57.4 - j43.1)}{57.4 - j43.1 + 73}$$

$$= |V_s| \angle \theta_{V_s}$$

$$= 15.5 \angle -18.9 \ V_{rms}$$

$$I_s = \frac{V_s}{Z_s + 75} = 0.216 \angle 18 \ A$$

V_s or smith chart is 8cm Drawing scale

$$\frac{15.5 \ V}{8 \ cm} = 1.9375 \ V/cm$$

$$V_2' = 1.935 \ V/\cancel{cm} \times 5.12 \ \cancel{cm}$$

$$= 9.9 \ V$$

$$V_2 = V_1'$$

I Drawing scale

$I_s = 8.2$ cm

Subject Date No.

$$\frac{0,216 \text{ A}}{0,2 \text{ cm}} = 0,026 \text{ A/cm}$$

$$E_2' = 0,026 \text{ A/cm} \times 10,25$$

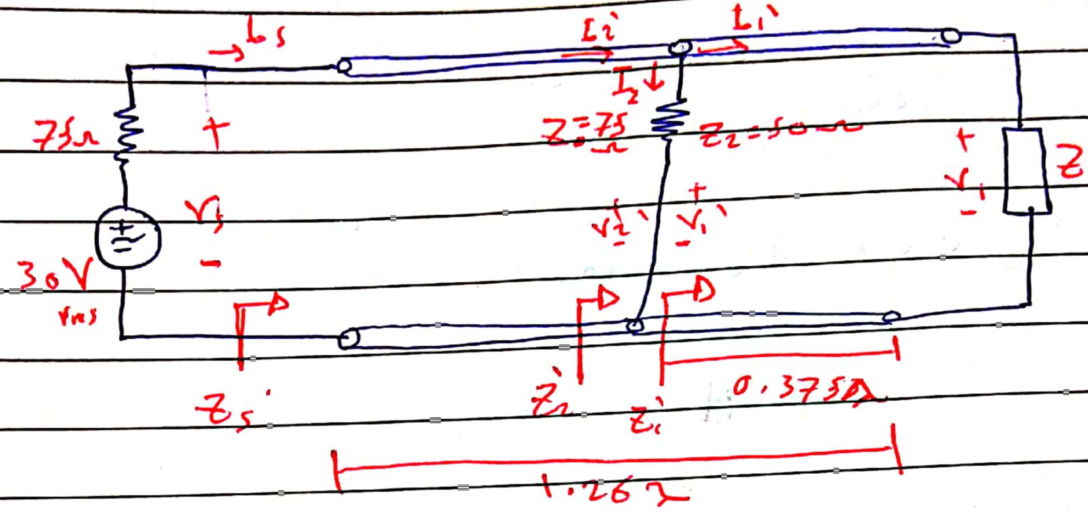
$$= 0,27 \text{ A}$$

N O T E B O O K

Subject Date No.

14/11/2019

Ex:-



$|V_s| = 15.5\ V \longrightarrow 1.9375\ V/cm$

$|I_s| = 0.216\ A \longrightarrow 0.026\ A/cm$

$V_2' = 1.9375\ V/cm \times 5.1\ cm = 9.9\ V = V_2 = V_1'$

$|I_2| = 0.026\ A/cm \times 10.25\ cm = 0.27\ A = I_2 + I_1'$

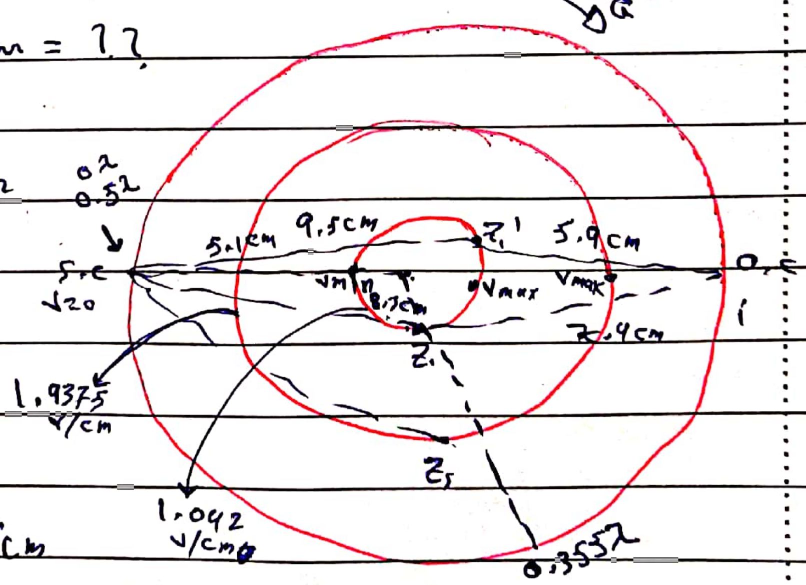
$|V_1'| = 9.9\ V$ @ $9.5\ cm$

$\frac{9.9}{9.5} = 1.042\ V/cm$

$|V_1| = 8.3\ cm \times 1.042\ V/cm = 8.65\ V$ → To G

$I_2' = \frac{V_2}{Z_2} = 9.9/50\ \Omega = ??$

$|I_1| = \frac{|V_1'|}{|Z_1'|} = I_2' - I_2$



$|I_1'| = \frac{9.9}{120.9} = 0.082\ A$

$Z_1' = 120 + j15\ \Omega$

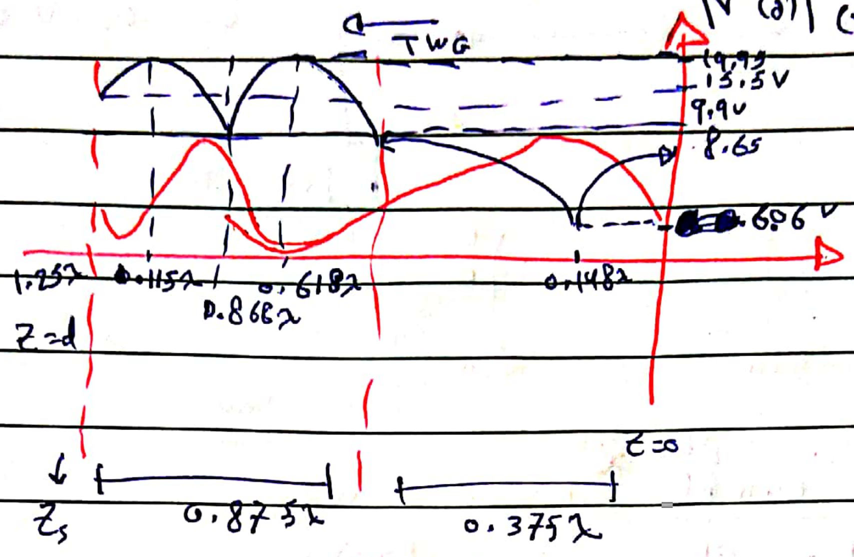
$\frac{0.082\ A}{5.9\ cm} = 0.0139\ A/cm$

$I_1 = 0.0139\ A/cm \times 7.4 = 0.1\ A$

→ D NOTEBOOK

Subject Date No.

$V_g = 30\,V$ $V_s = 15.5\,V$ $V_2' = 9.9\,V = V_1'$ $V_1 = 8.65\,V$

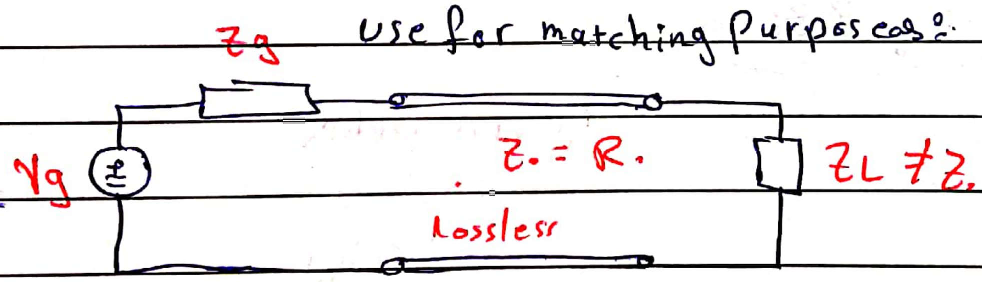


APPlications on T.L :-

[1] Quarter wavelength Tl :-

$\frac{\lambda}{4}$ Tr

use for matching Purposes :-



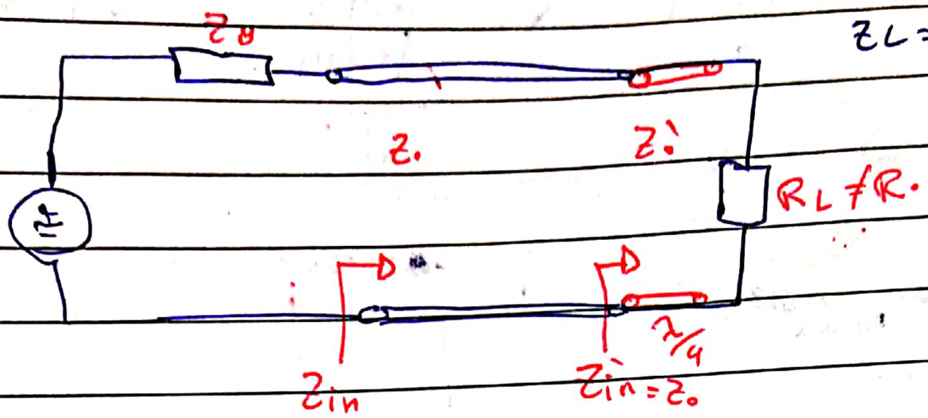
if the load is resistance $Z_L = R_L \neq R_o$

$$\Gamma = \frac{R_L - R_o}{R_L + R_o} \neq 0$$

NOTEBOOK

Subject Date No.

$Z_L = Z_0 \Rightarrow Z_{in} = Z_0$



$$Z_{in}' = Z_0' \, \frac{\cancel{Z_L} + j Z_0' \tan \beta l}{\cancel{Z_0'} + j Z_L \tan \beta l}$$

$\rightarrow \beta = \frac{2\pi}{\lambda}$

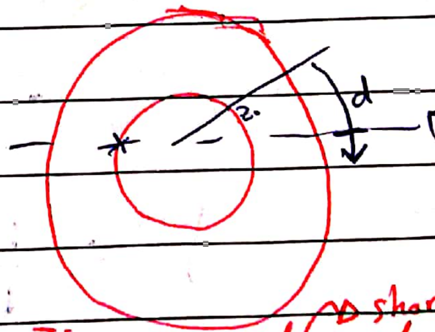
divide all termes by $\tan(90°)$

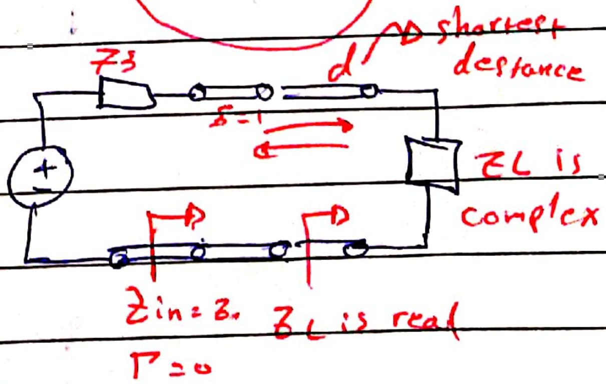
$$\tan(\beta l) = \tan\left(\frac{2\pi}{\cancel{\lambda}} \frac{\cancel{\lambda}}{4}\right)$$

$$Z_0 = Z_{in}' = \frac{Z_0'^2}{Z_L} \Rightarrow Z_0' = \sqrt{Z_{in}' Z_L}$$

$$\boxed{Z_0' = \sqrt{Z_0 Z_L}}$$

if Z_L is complex



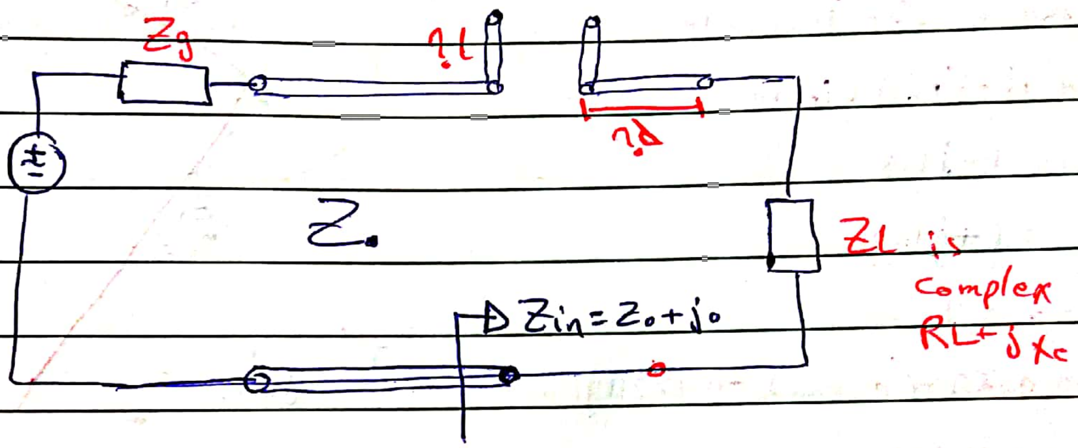


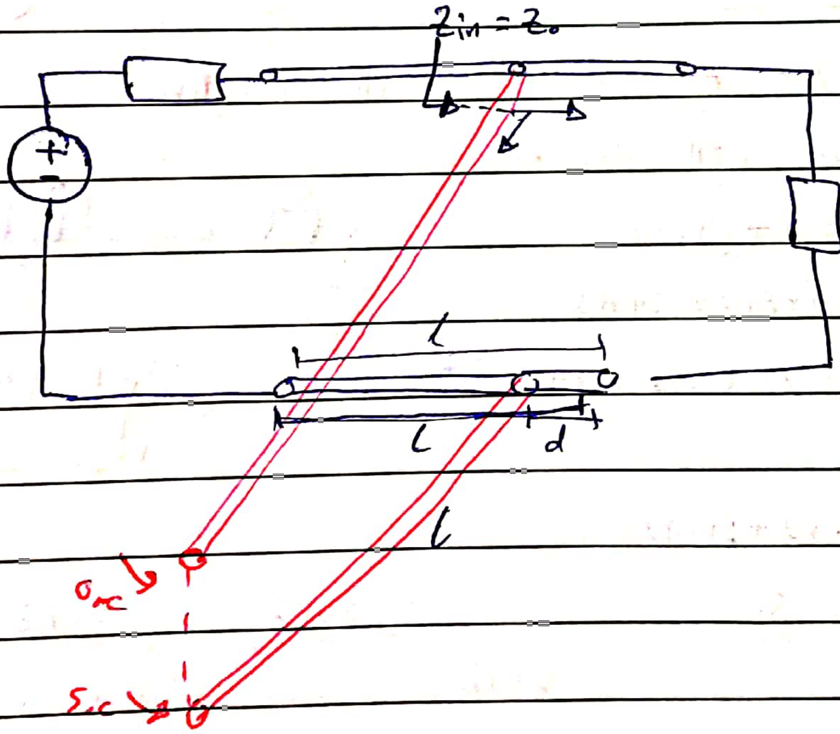
Subject Date No.

17/10/2019

* Single stub matching :-

- series stub
  - S.C
  - O.C
- shunt (Parallel) stub
  - S.C
  - O.C



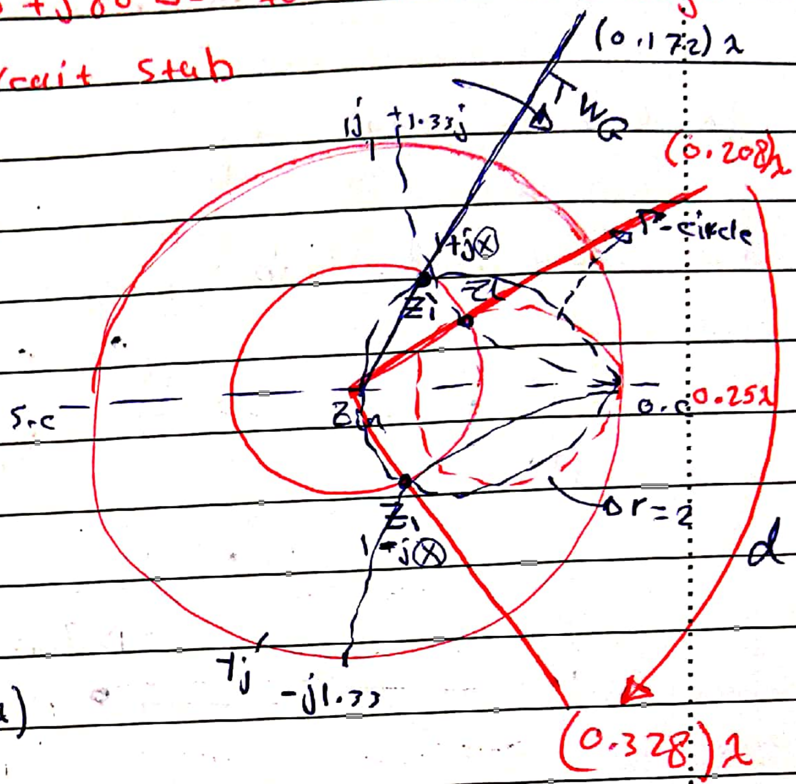


$$Z_{in\,s.c} = jZ_0 \tan\beta d$$

$$Z_{in\,o.c} = -jZ_0 \cot\beta l$$

Ex:- Match a load $Z_L = 100 + j80\ \Omega$ to a $50\ \Omega$ Line using:-

[A] single series open-circuit stub



Steps:- Solution.

[1] $Z_L = 2 + j1.6$

[2] Draw the Γ-circle

[3] Find the intersections between the Γ-circle and the (r=1) circle

∴ $Z_1 = 1 - j1.33$

$Z_1' = 1 + j1.33$

[4] $d = 0.12\lambda$

$d' = 0.5\lambda - (0.208\lambda - 0.172\lambda)$

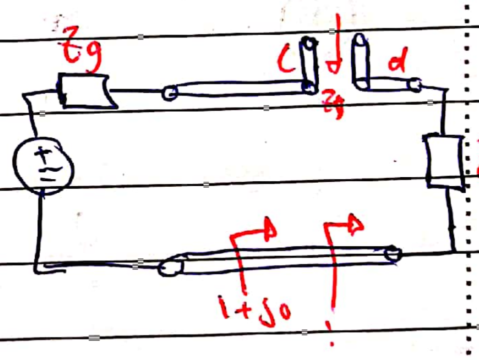
$d' = 0.463\lambda$

[5] for Z_1 find (l) by calculating the length from o.c point till you reach

$Z_s = 0 + j1.33$

$l_1 = 0.25\lambda + 0.148\lambda = 0.398\lambda$

$l_1' = 0.352\lambda - 0.25\lambda = 0.102\lambda$



[6] $Z_{in} = 1 - j1.33 + j1.33$

$= 1 + j0$ #

	Sol 1	Sol 2
Stub:-	$d = 0.12\lambda$	$d' = 0.463\lambda$
	$l_1 = 0.398\lambda$	$l_1' = 0.102\lambda$

The shortest distance and shortest length

[B]

Subject Date No.

[B] single series short-circuit stub:-

{from step 1 to 4 the same}

$Z_1 =$

$Z_1' =$

$d = 0.12\lambda$

$d' = 0.463\lambda$

$l = 0.148\lambda$

$l' = 0.352\lambda$

$l + l' = 0.5\lambda$

differance between $l_{s.c}$ & $l_{o.c}$ is alwys 0.25λ

19/11/2019

Match $Z_L = 100 + j80\ \Omega$ to a 50Ω T.L using shunt:- stub.

$Z_L = 2 + 1.6j$

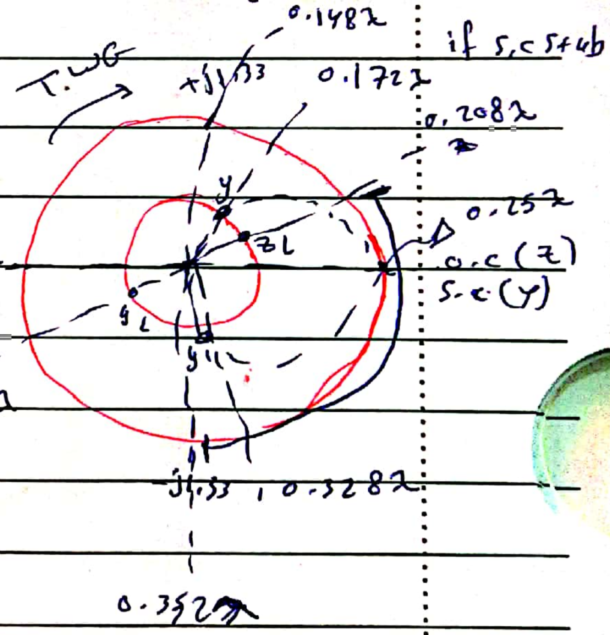
$y_L = \frac{1}{Z_L} = 0.3 - 0.24j$

$y = 1 + 1.33j$

$y' = 1 - 1.33j$

$d = [0.172 + (0.5 - 0.458)]\lambda = 0.214\lambda$

$d' = 0.37\lambda$



if s.c stub

if $y = 1 + j1.33$ $\quad l = 0.352\lambda - 0.25\lambda = 0.102\lambda$

$y_s = -j1.33$ $\quad$ if $y' = 1 - 1.33j$

$Z_{in} = 1 + j0$ $\quad y'_s = 1 + 1.33j$

$l = 0.25\lambda + 0.148\lambda = 0.398\lambda$

s.c stub

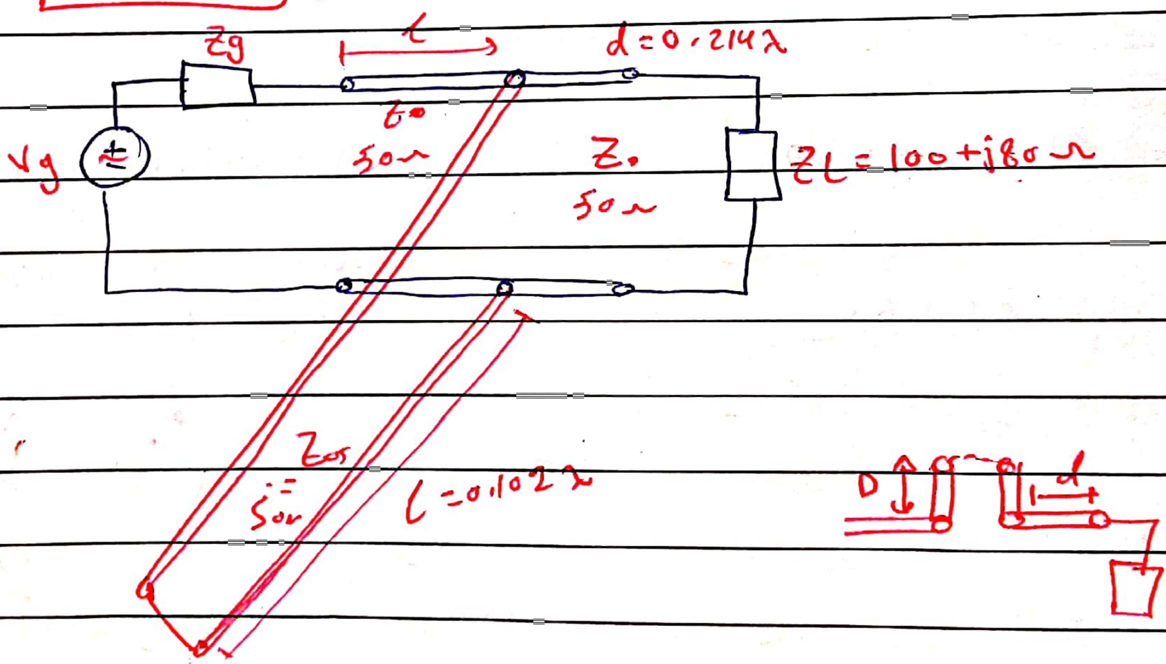
sol 1 → $\boxed{d = 0.214\lambda}$, $\boxed{l = 0.102\lambda}$

sol 2 → $\boxed{d' = 0.37\lambda}$, $\boxed{l = 0.398\lambda}$

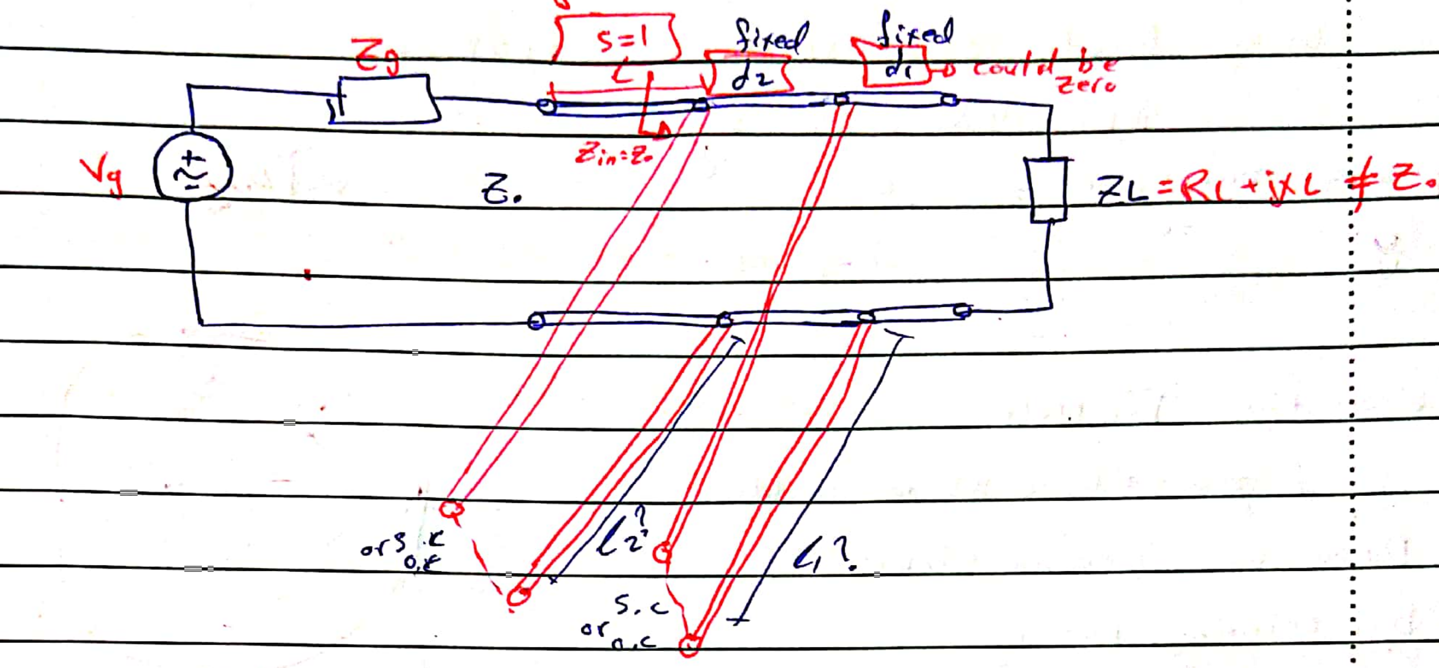
if o.c stub

sol 1 → $\boxed{d = 0.214\lambda}$, $\boxed{l = 0.352\lambda}$

sol 2 → $\boxed{d = 0.37\lambda}$, $\boxed{l = 0.148\lambda}$



Double Stub Matching :- → غير موجود بالكتاب



Ex:- $Z_0 = 50\Omega$, $Z_L = 60 - j80\,\Omega$, shunt, O.C , $d_2 = \lambda/8$, $d_1 = 0$

Steps sol :-

~~1) $z_L = \frac{60-j80}{50} = 1.2 - j1.6$ shunt → $y_L = 0.3 + j0.4$~~

H.W This Example.

answer :- Sol 1 $l_1 = 0.146\lambda$, $l_2 = 0.204\lambda$

Sol 2 $l_1' = 0.482\lambda$, $l_2' = 0.35\lambda$

Subject Date No.

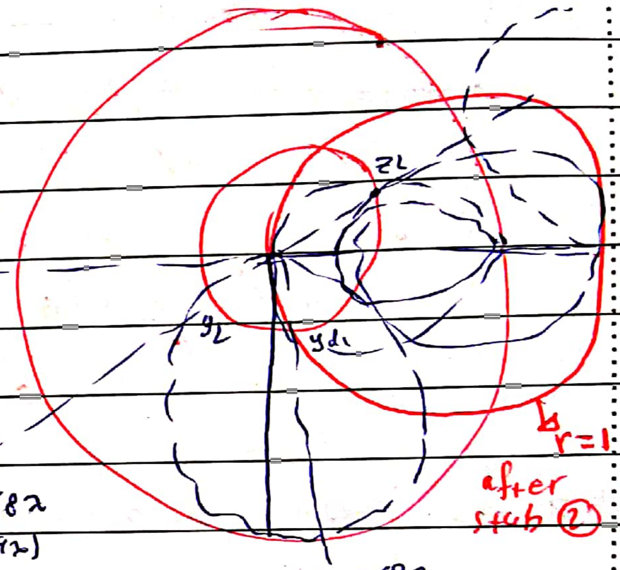
Ex1. Match the load $100+j100\,\Omega$ to a $50\,\Omega$ T.L using double $50\,\Omega$ short-CKT stub if

$d_1 = 0.4\lambda$, $d_2 = \frac{3\lambda}{8}$

TW

[1] $Z_L = \frac{100+j100}{50} = 2+j2$

$y_L = \frac{1}{Z_L} = \frac{1}{2+j2} = 0.25-j0.25$



[2] Draw the Γ circle

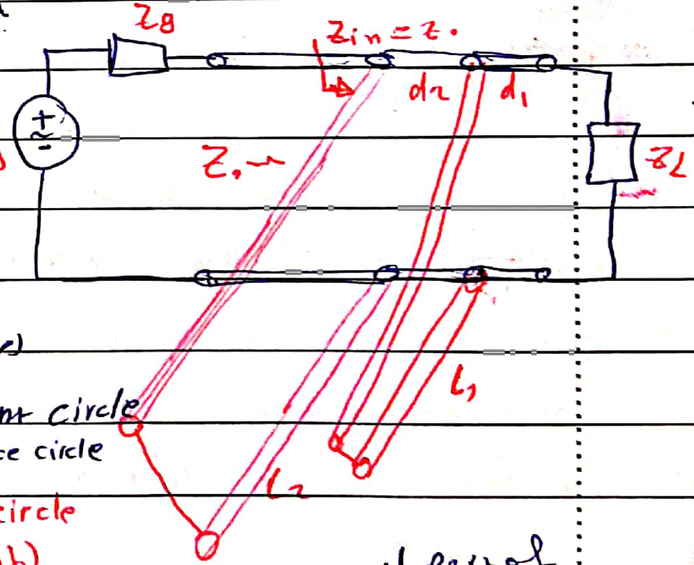
[3] if $d_1 \neq 0 \rightarrow$ move y_L a distance (d_1)

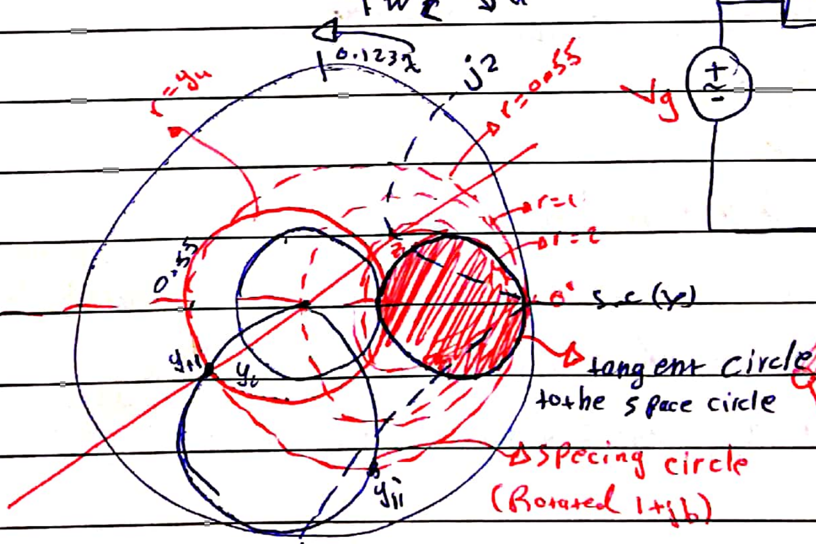
[4] Draw the spacing circle by rotating the r=1 circle a distance of (d_2) SWL

$d_2 = \frac{3\lambda}{8} = \frac{3}{8}(720°) = 270°$

0.458λ
+d1 (0.4λ)
−0.5λ
0.358λ

TWL TWG





[5] Find the intersections betwen the spacing circle & (r=0.55 circle) ← real part of the load (moved load)

$y_{11} = 0.55 - j0.11$, $y'_{11} = 0.55 - j1.88$

Subject Date No.

[6] find stub impedance

$y_{11} = y_{s1} + y_{d1}$

$y_{s1} = y_{11} - y_{d1} = j0.97$

$y'_{s1} = y'_{11} - y_{d1} = -j0.8$

[7] find the lengths of the first stub

$l_{s1} = 0.25 + 0.123 = 0.373\lambda$

$l'_{s1} = 0.225\lambda$? $(-j0.08)$

[8] Draw the Γ-circle for y_{11} and y'_{11}

& find the intersection with the unity circle that has d distance of (d_2)

δ → continue

$y_{d2} = 1 - j0.61$

$y'_{d2} = 1 + j2.6$ check

[9] find $y_{s2} = y_{22} - y_{d2}$

$y_{22} = 1 + j0 - (1 - j0.61)$

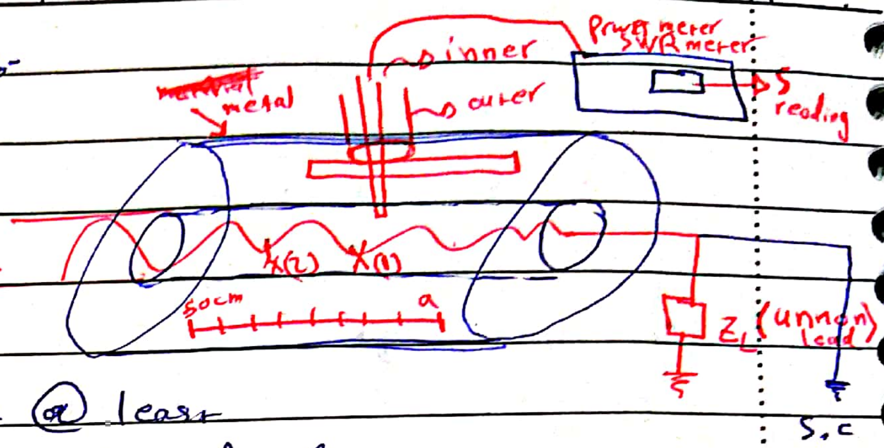
$y_{s2} = j0.61$

$l_2 = 0.25\lambda + 0.08\lambda$

$l_2 = 0.337\lambda$

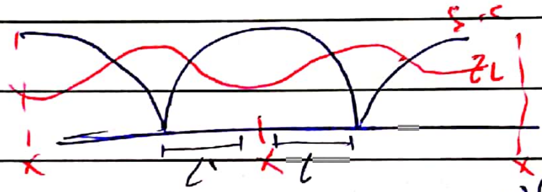
$l'_2 = \text{- - - -}$

* Impedance measurements:-
(slotted line)

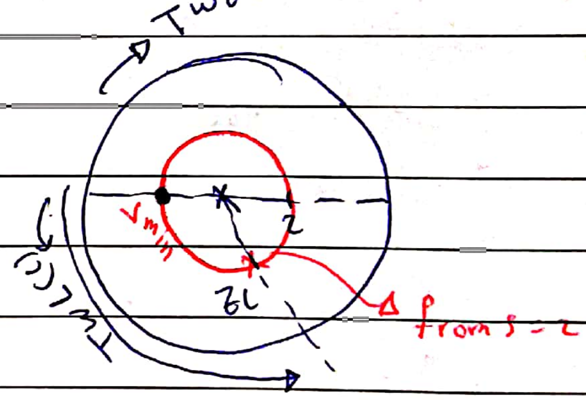


⇒ Ploced use:-

1. connect the s.c load
2. Move the slider & record @ least two minimum → $\lambda/2$ & S will be found
   ↳ move sharper
3. connect the unknown load and record the minimum value



4. find (λ)



Subject Date No.

24/11/2019

* Impedance measurements :-

⇒ Procedure :-

[1] with the load is connected to the slotted line

⟹ read value of (S) from the meter

⟹ record the value of voltage minimum (more shorter than maximum)

[2] Replace the load with S.C & record the location of voltage minimum.

[3] Determine (ℓ) which is the distance between the load minimum & S.C minimum.

[4] on the smith chart move a distance (ℓ) from the minimum point on TWL or ℓ or TWG

Ex:- unknown load is connected to a slotted air line has S=2 & minimum are found at 11cm, 19cm, ... on the scale.

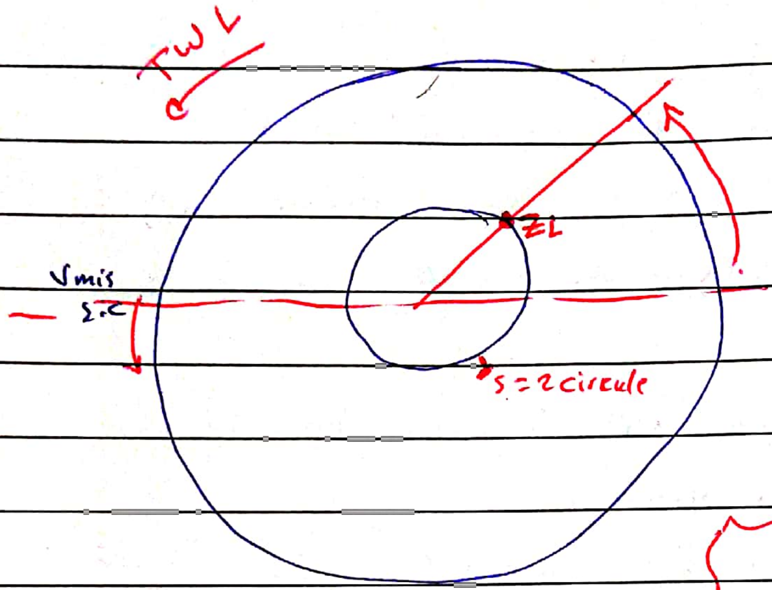
when the load is replaced by a S.C the minimum are located at 16 cm, 24 cm, ...

If $Z_o = 50\Omega$ find

λ, f & Z_L

Sol -

$\frac{\lambda}{2} = 19 - 11 = 8\,cm \rightarrow \lambda = 16\,cm$



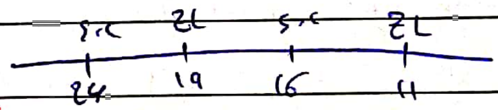
$f = \frac{c}{\lambda} = \frac{3\times10^8}{16\times10^{-2}}$

$= 18755\,Hz$

$16\,cm \rightarrow 1\lambda$

$5\,cm \rightarrow ??$

$l = \frac{5}{16} = 0.3125\lambda$



TWL: $l = min|_{s.c} - min|_{ZL}$

TWG: $l' = min|_{ZL} - min|_{s.c}$

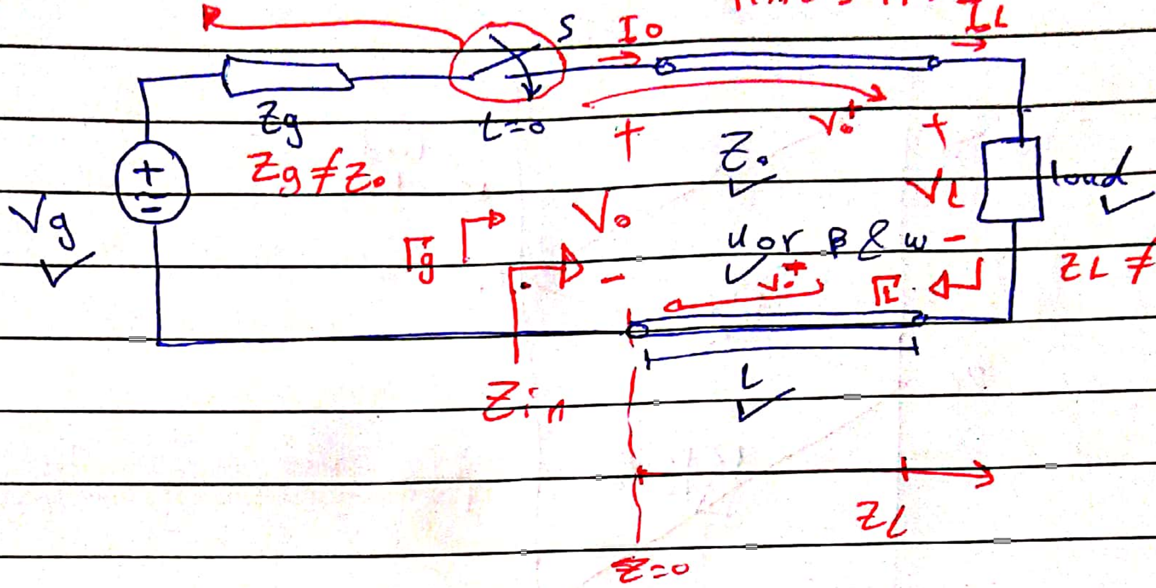
1 $\lambda = 16\,cm$ ✓

2 $f = 18755\,Hz$ ✓

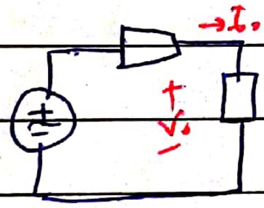
3 ZL in smith chart ✓

* Transients on T.L :-

Time ≡ T_1 ≡ Transient



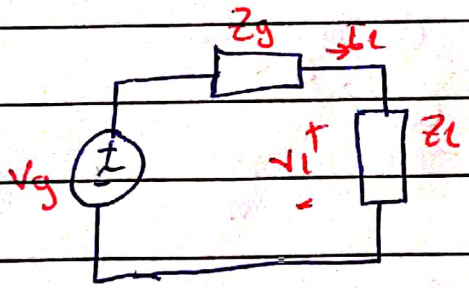
at the generator (source) end $\begin{pmatrix} z=0 \\ t=0^+ \end{pmatrix}$



$$V_0 = \frac{V_g Z_0}{Z_0+Z_g} , \quad I_0 = \frac{V_g}{Z_0+Z_g}$$

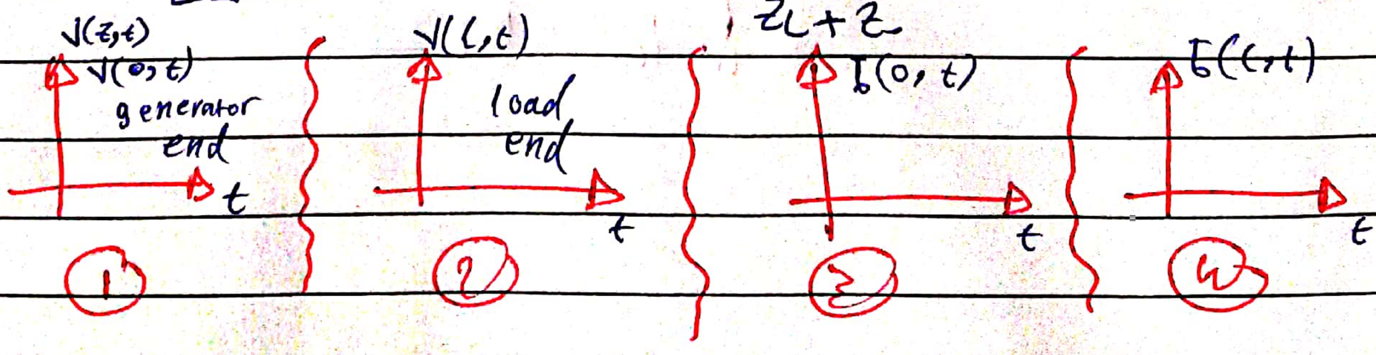
$$\Gamma_g = \frac{Z_g - Z_0}{Z_g+Z_0}$$

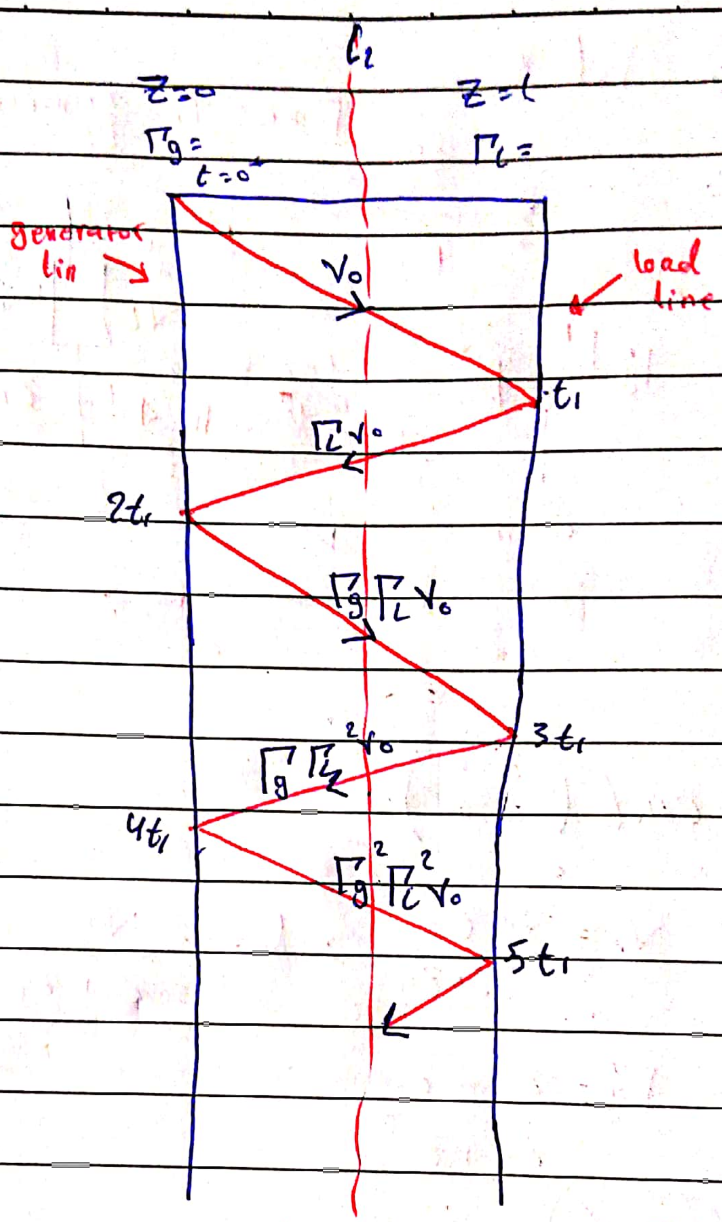
at the load end $\begin{pmatrix} z=l \\ t=t \end{pmatrix}$ $\qquad t_1 = \frac{l}{u}$



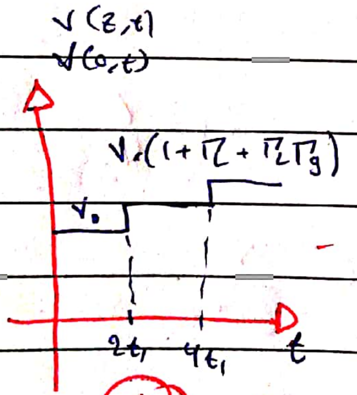
$$V_L = \frac{V_g Z_L}{Z_L+Z_g} , \quad I_L = \frac{V_g}{Z_L+Z_g}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L+Z_0} , \quad \Gamma_{Li} = -\Gamma_L$$





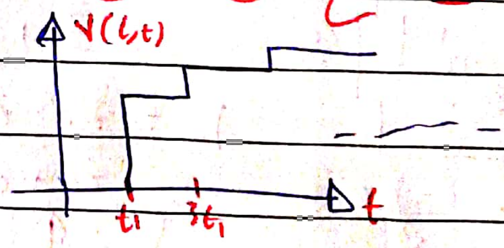
ℓ_2
$z=0$
$z=\ell$
$\Gamma_g =$
$\Gamma_L =$
$t=0^+$
generator lin
load line
V_0
t_1
$\Gamma_L V_0$
$2t_1$
$\Gamma_g \Gamma_L V_0$
$3t_1$
$\Gamma_g \Gamma_L^2 V_0$
$4t_1$
$\Gamma_g^2 \Gamma_L^2 V_0$
$5t_1$



$V(z,t)$
$V(0,t)$
$V_0(1+\Gamma_L+\Gamma_L\Gamma_g)$
V_0
$2t_1$
$4t_1$
t

1

- - - - - - - - 2 , 3 & 4

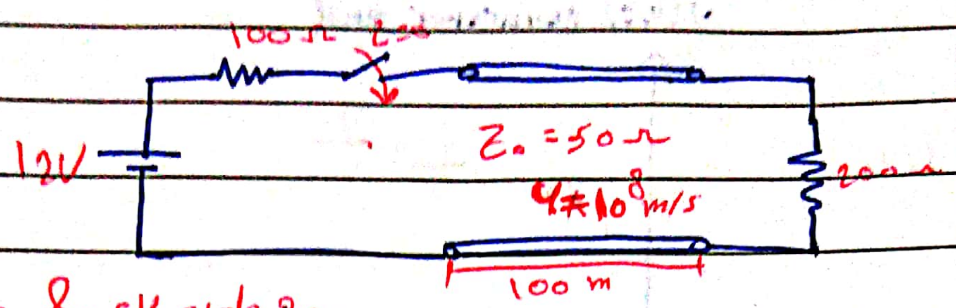


$V(\ell,t)$
t_1
$3t_1$
t

Subject Date No.

26/11/2019

Ex:-



calculate & sketch:-

[a] The voltage at the load and generator ends for $0 < t < 6\mu s$

[b] The current at the load and generator terminals for $0 < t < 6\mu s$

Sol:-

$$V_0 = \frac{12(50)}{150} = 4V \quad , \quad V_\infty = V_{ss} = V_L = \frac{12(200)}{300} = 8V$$

$$I_0 = \frac{12}{150} = 80mA \quad , \quad I_\infty = I_{ss} = I_L = \frac{12}{300} = 40mA$$

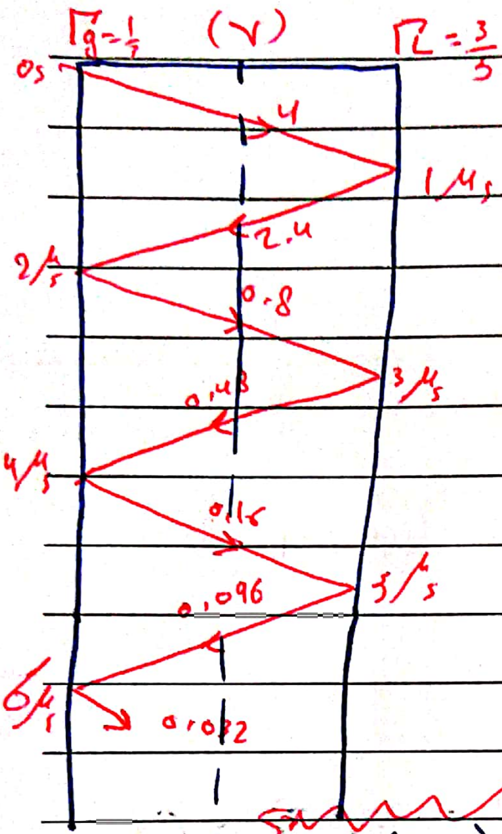
$$\Gamma_L = \frac{200-50}{200+50} = \frac{3}{5} \quad , \quad \Gamma_g = \frac{100-50}{100+50} = \frac{1}{3}$$

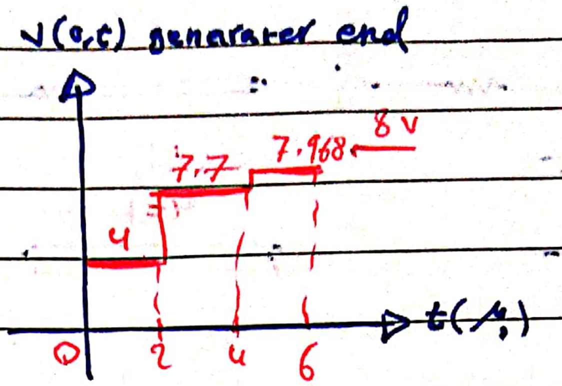
$$t = \frac{100}{10^8} = 1\mu s$$

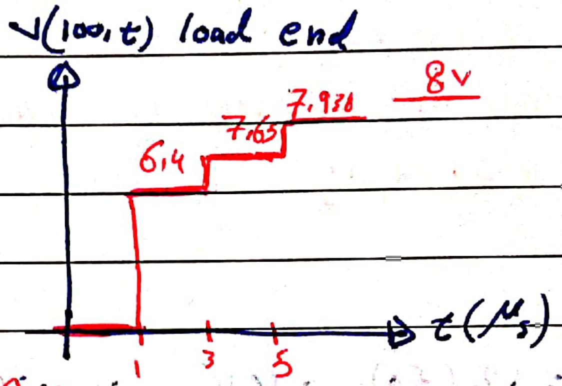
$\Longrightarrow$

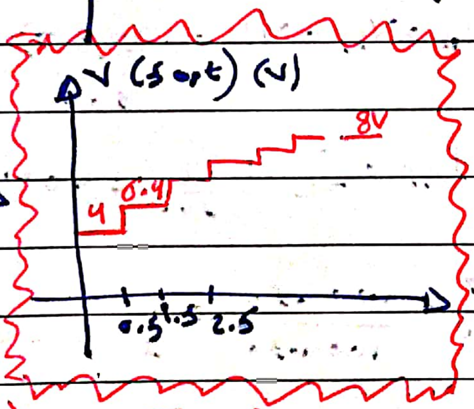
N O T E B O O K

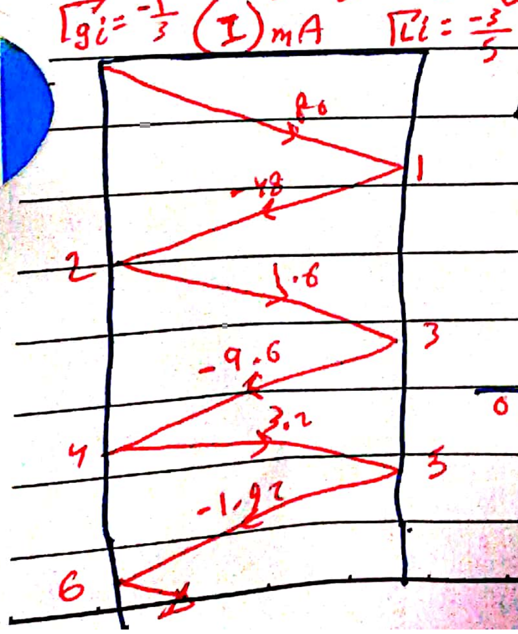
Scanned by CamScanner

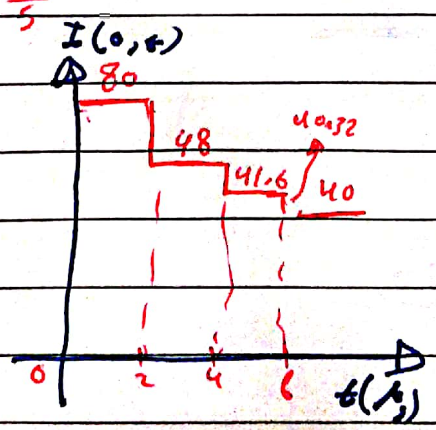

$\Gamma_g = \frac{1}{5}$
(V)
$\Gamma_L = \frac{3}{5}$
0s
4
1μs
2.4
2μs
0.8
3μs
0.48
4μs
0.16
0.096
5μs
6μs
0.032

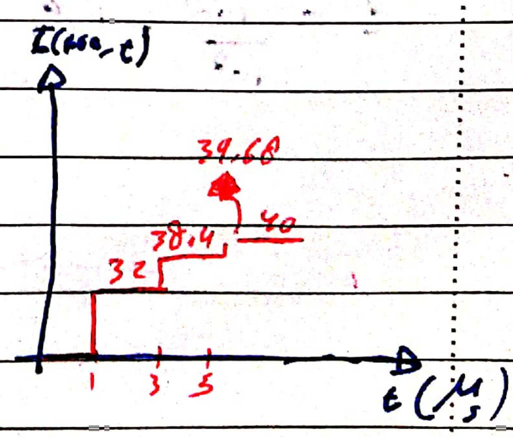

V(0,t) generater end
8V
7.968
7.7
4
t(μs)
0
2
4
6


V(100,t) load end
8V
7.938
7.65
6.4
t(μs)
1
3
5

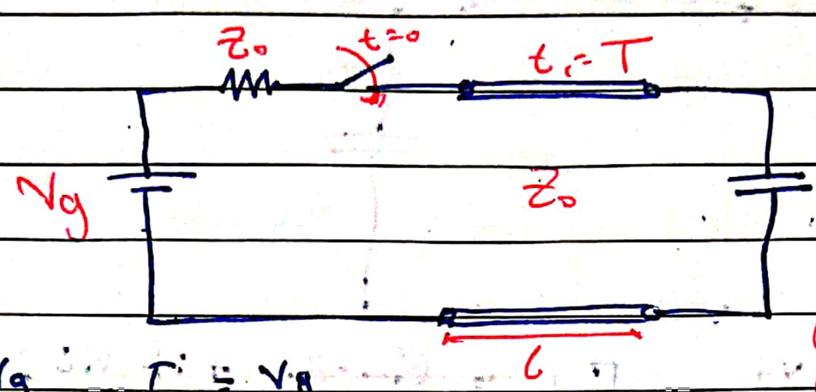

V(50,t) (V)
8V
6.4
4
0.5
1.5
2.5


$\Gamma_{gi} = \frac{-1}{5}$
(I) mA
$\Gamma_{Li} = \frac{-3}{5}$
80
1
-48
2
1.6
3
-9.6
4
3.2
5
-1.92
6


I(0,t)
80
48
41.6
40.32
40
0
2
4
6
t(μs)


I(100,t)
39.68
40
38.4
32
1
3
5
t(μs)

Ex:-



if inductor بنعكس الرسمات

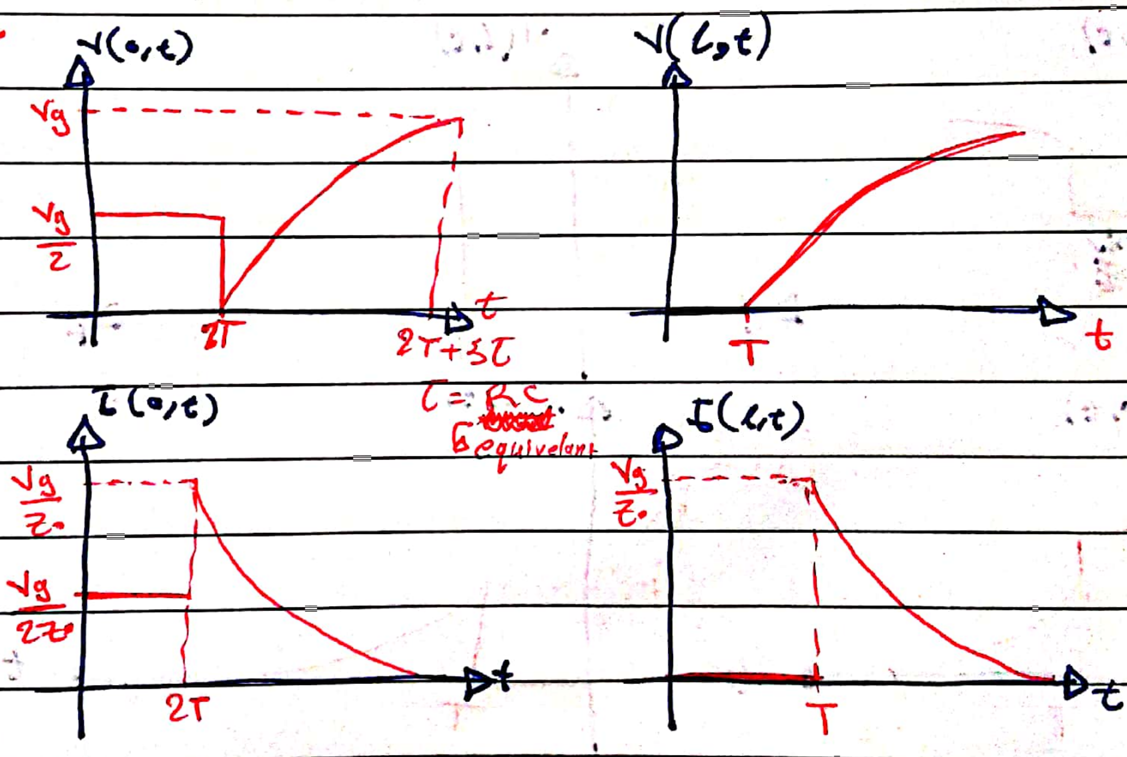
$V \to I$

$I \to V$

$V_0 = \frac{V_g}{2}$, $I_0 = \frac{V_g}{2Z_0}$

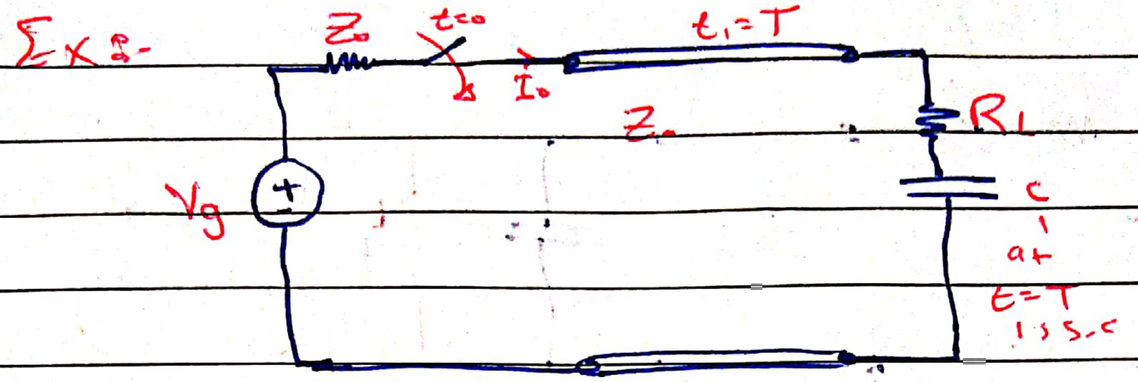
$\Gamma_g = 0$, $\Gamma_L = -1$ S.C , $\Gamma_L = 1$ O.C

$\frac{Z_0 - Z_0}{Z_0 + Z_0}$ $\quad t<0$



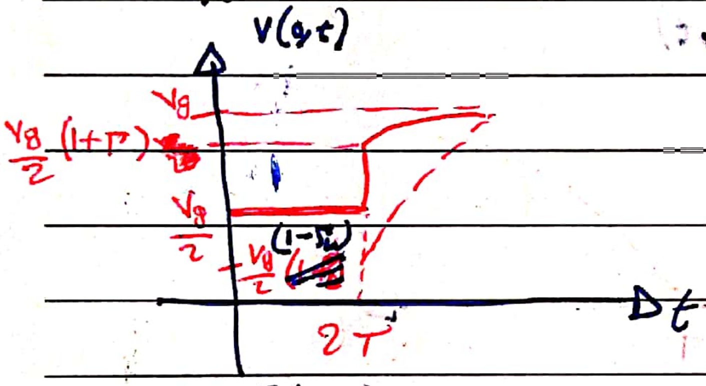
28/11/2019

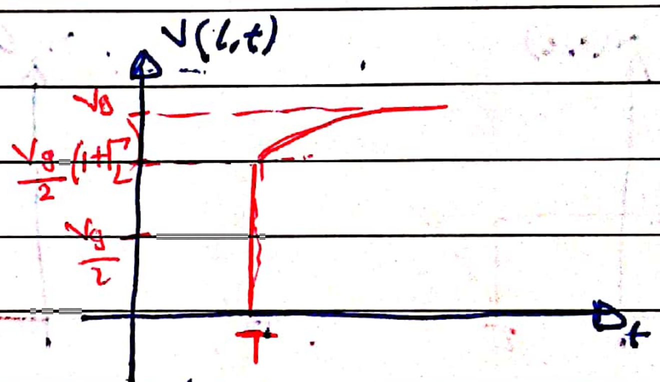
Ex:-

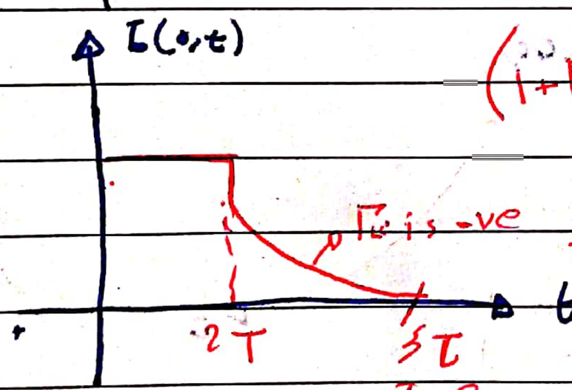


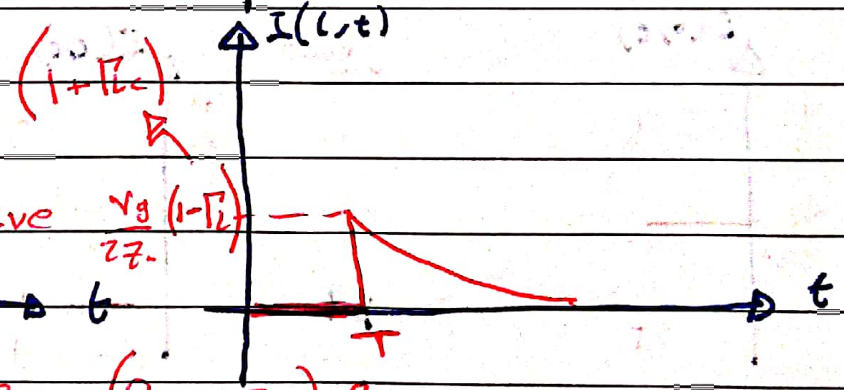
$V_o = \frac{V_g}{2}$, $I_o = \frac{V_g}{2Z_o}$, $\Gamma_L \Big|_{t=T} = \frac{R_L - Z_o}{R_L + Z_o}$ + if $R_L > Z_o$, − if $R_L < Z_o$

$\Gamma_L \Big|_{s.c} = 1$ o.c



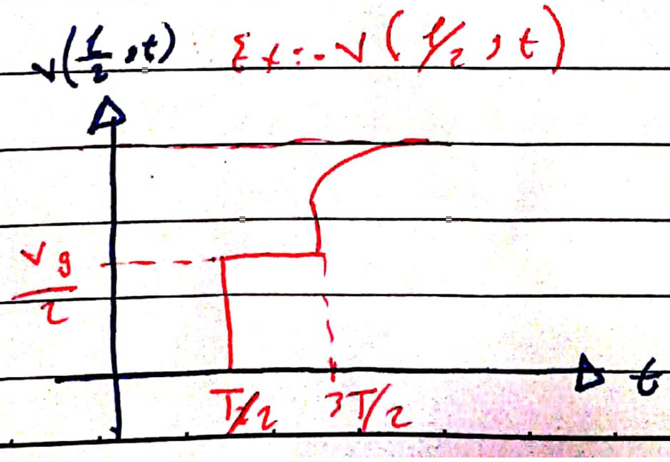




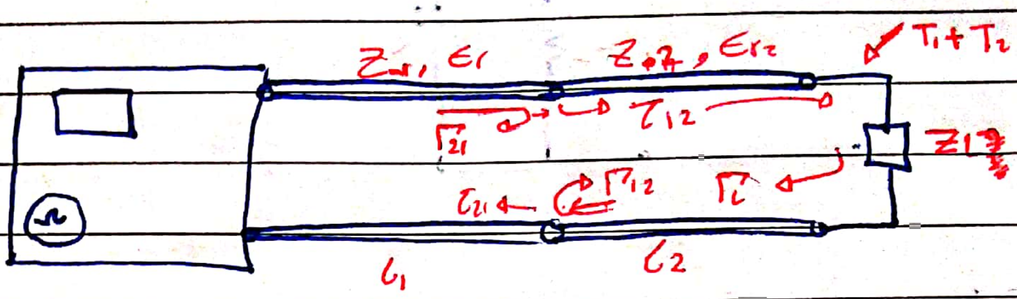


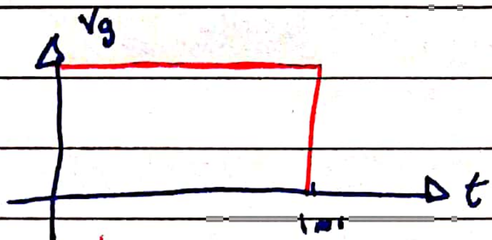
$\tau = R_{eq} C = (R_L + Z_g) C$

Ex:- $V(l/2, t)$

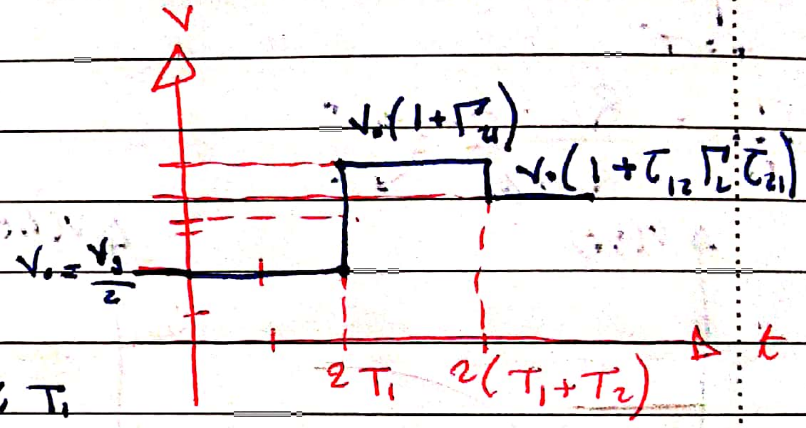


* Time domain Reflectometer (TDR)





↳ considered as DC



$$u_1 = \frac{c}{\sqrt{\epsilon_{r1}}}, \quad l_1 = u_1 T_1$$

$$u_2 = \frac{c}{\sqrt{\epsilon_{r2}}}, \quad l_2 = u_2 T_2$$

$$\Gamma_{21} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}, \quad \tau_{12} = \frac{2 Z_{02}}{Z_{01} + Z_{02}}, \quad \tau_{21} = \frac{2 Z_{01}}{Z_{01} + Z_{02}}$$

$$\Gamma_L = \frac{Z_L - Z_{02}}{Z_L + Z_{02}}$$ find Z_L

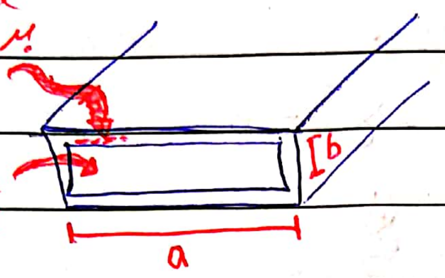
END of CH: 11

CH 12 :- Waveguides (W.G)

one conductor T.L

cond $\sigma_c, \epsilon_c, \mu$

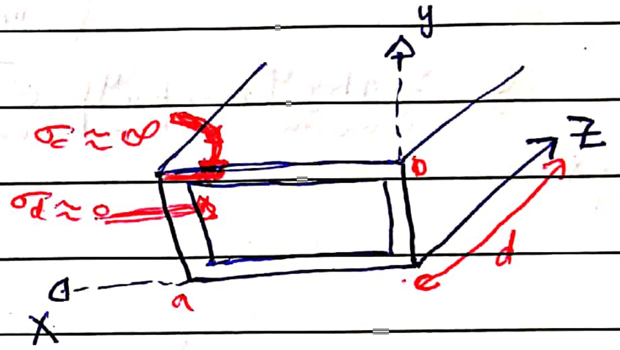
Dielectric $\sigma_d, \epsilon_r, \mu$



R.W.G

rectangular wave guide

⟹ lossless R.W.G



* Based on Electromagnetic theory

$$\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0$$

or $\nabla^2 \bar{E}_s + K^2 \bar{E}_s = 0$ if lossless

$$\frac{\partial^2 E_s}{\partial x^2} + \frac{d^2 E_s}{\partial y^2} + \frac{d^2 E_s}{\partial Z} + K^2 E = 0$$

Assume the wave propagates in the +ve Z-direction

⟶ solve using separation of variables [Ex: 6.5 CH 6]

let: $E_s = X_{(x)} Y_{(y)} Z_{(z)} = XYZ$

choose a suitable value for (K)

$$K^2 = K_x^2 + K_y^2 - \gamma^2$$

Propagation الـ مع γ [illegible]

$$X''YZ + XY''Z + XYZ'' + K^2 XYZ = 0$$

⟹

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} + K^2 = 0$$

① ② ③

$$\frac{X''}{X} + K_x^2 = 0, \quad \frac{Y''}{Y} + K_y^2 = 0, \quad \frac{Z''}{Z} - \gamma^2 = 0$$

$\Downarrow$

$X'' + K_x^2 X = 0$

$m^2 + K_x^2 = 0$

sol: $m = \pm j K_x$

imagenary roots

$\sin K_x X, \cos K_x X$

$\Downarrow$

$Y'' + K_y^2 = 0$

imagenery roots

$\sin K_y y, \cos K_y y$

$\Downarrow$

$Z'' - \gamma^2 Z = 0$

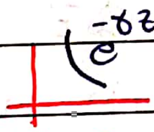
$m^2 - \gamma^2 = 0$

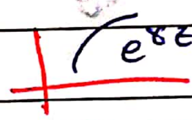
$m = \pm \gamma$ real

$e^{-\gamma z}, e^{\gamma z}$

$$E_s = (c_1 \cos K_x X + c_2 \sin K_x X) \cdot (c_3 \cos K_y y + c_4 \sin K_y y) \cdot (c_5 e^{\gamma z} + c_6 e^{-\gamma z})$$

based on the Boundary condition





$C_5 = 0$ since $E \to 0$ as $Z \to \infty$

$$E_{zs} = (A_1 \cos K_x X + A_2 \sin K_x X) \cdot (A_3 \cos K_y Y + A_4 \sin K_y Y) e^{-\gamma z}$$

$$H_{zs} = (B_1 \cos K_x X + B_2 \sin K_x X) \cdot (B_3 \cos K_y y + B_4 \sin K_y y) e^{-\gamma z}$$

* need to find:

$E_{xs}, E_{ys}, H_{xs}, H_{ys}$

Scanned by CamScanner

* Use Maxwell's eqs:-

$$\nabla \times \bar{E}_s = -j\omega\mu \bar{H}_s$$

$$\nabla \times \bar{H}_s = (\cancel{\sigma_d} + j\omega\epsilon)\,\bar{E}_s \;\rightarrow\; \bar{E}_s = \frac{1}{j\omega\epsilon}\nabla \times \bar{H}_s$$

$$\begin{bmatrix} H_{xs} \\ H_{ys} \\ H_{zs} \end{bmatrix} = -\frac{1}{j\omega\mu}\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_{xs} & E_{ys} & E_{zs} \end{vmatrix}$$

$H_{xs} = +$ ①ⓓ

$H_{ys} = \frac{1}{j\omega\mu}\left(\frac{\partial E_{zs}}{\partial x} - \frac{\partial E_{xs}}{\partial z}\right)$ ①ⓔ

$H_{zs} = +$ ①ⓕ

$$\begin{bmatrix} E_{xs} \\ E_{ys} \\ E_{zs} \end{bmatrix} = \frac{1}{j\omega\epsilon}\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_{xs} & H_{ys} & H_{zs} \end{vmatrix}$$

$E_{xs} = \frac{1}{j\omega\epsilon}\left(\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z}\right)$ ①ⓐ

$E_{ys} =$

$E_{zs} =$

from ①ⓐ & ①ⓔ :-

$$E_{xs} = \frac{1}{j\omega\epsilon}\left[\frac{\partial H_{zs}}{\partial y} - \frac{1}{j\omega\mu}\left(\frac{\partial^2 E_{zs}}{\partial x \partial z} + \frac{\partial^2 E_{xs}}{\partial z^2}\right)\right]$$

$$\frac{\partial E_{zs}}{\partial z} \propto -\gamma E_{zs}, \quad E_{xs} \rightarrow e^{-\gamma z}$$

$$\frac{\partial^2 E_x}{\partial z} \propto \gamma^2 E_{xs}$$

$$E_{xs} = \frac{1}{j\omega\epsilon}\frac{\partial H_{zs}}{\partial y} - \frac{\gamma}{\omega^2\mu\epsilon}\frac{\partial E_{zs}}{\partial x} - \frac{\gamma^2}{j\omega\mu}E_{xs}$$

$$E_{xs}\left(1+\frac{\gamma^2}{j\omega\mu}\right) = \frac{1}{j\omega\epsilon}\frac{\partial H_{zs}}{\partial y} - \frac{\gamma}{\omega^2\mu\epsilon}\frac{\partial E_{zs}}{\partial x}$$

$$= \frac{j\omega\mu+\gamma^2}{j\omega\mu} \qquad \div \qquad \div$$

$$\boxed{E_{xs} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}}$$

$$h^2 = \gamma^2 + K^2 = K_x^2 + K_y^2$$

$$E_{xs} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}$$

$$E_{ys} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial y} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x}$$

$$H_{xs} = \frac{j\omega\epsilon}{h^2}\frac{\partial E_{zs}}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial x}$$

$$H_{ys} = -\frac{j\omega\epsilon}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial y}$$

* $\Rightarrow$ الحل مهمة *

مهمة ^.^

$E_{xs} \times H_{ys} \rightarrow +\hat{a}_z$, $E_{ys} \times H_{xs} \rightarrow -\hat{a}_z$

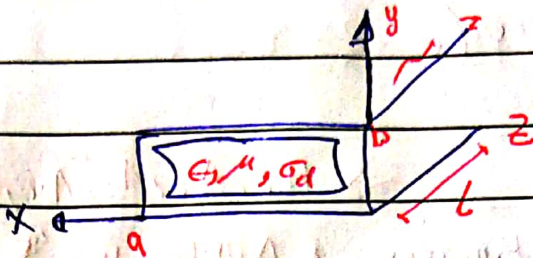
Scanned by CamScanner

3/11/2019

R.W.G

$\sigma_c \simeq \infty$

$\sigma_d \simeq 0$



$\nabla^2 \bar{E}_s + K^2 \bar{E}_s = 0, \quad \nabla^2 \bar{H}_s + K^2 \bar{H}_s = 0$

$E_{zs} = (A_1 \cos K_x x + A_2 \sin K_x x)(A_3 \cos K_y y + A_4 \sin K_y y)\, e^{-\gamma z}$

$H_{zs} = (B_1 \cos K_x x + B_2 \sin K_x x)(B_3 \cos K_y y + B_4 \sin K_y y)\, e^{-\gamma z}$

$E_{xs} = \frac{-\gamma}{h^2} \frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_{zs}}{\partial y}$

$E_{ys} = \frac{-\gamma}{h^2} \frac{\partial E_{zs}}{\partial y} - \frac{j\omega\mu}{h^2} \frac{\partial H_{zs}}{\partial x}$

$\boxed{\eta = \frac{E_x}{H_y} = \frac{-E_y}{H_x}, \quad \hat{a}_E \times \hat{a}_H = \hat{a}_k = \hat{a}_z}$

$H_{xs} = \frac{j\omega\epsilon}{h^2} \frac{\partial E_{zs}}{\partial y} - \frac{\gamma}{h^2} \frac{\partial H_{zs}}{\partial x}$

$H_{ys} = \frac{-j\omega\epsilon}{h^2} \frac{\partial E_{zs}}{\partial x} - \frac{\gamma}{h^2} \frac{\partial H_{zs}}{\partial y}$

$h^2 = K^2 + \gamma^2 = K_x^2 + K_y^2$

There are four cases:-

$\boxed{1}$ $E_{zs} = 0, \; H_{zs} = 0$

TEM mode ($h = 0$)

$E_{xs}, E_{ys}, H_{xs}, H_{ys}$ all are zeros

so, W.G can't support TEM mode

$\boxed{3}$ $E_{zs} \neq 0, \; H_{zs} = 0$

TM Mode

$E_{xs}, E_{ys}, H_{xs}, H_{ys}$ exists

$\boxed{2}$ $E_{zs} = 0, \; H_{zs} \neq 0$

TE mode — no E-field with the wave

$E \perp \hat{a}_k$

$E_{xs}, E_{ys}, H_{xs}, H_{ys}$ exists

$\boxed{4}$ $E_{zs} \neq 0, \; H_{zs} \neq 0$

all fields exist

HE Mode or EH Mode

optical fiber

NOTEBOOK

Scanned by CamScanner

* TM mode :

$H_{zs} = 0$

$E_{zs} \neq 0 \rightarrow A_1, A_2, A_3, A_4, K_x, K_y$

To find the unknowns $\rightarrow$ Apply B.C's

$E_{zs} = 0$ at $x = 0$

$x = a$

$y = 0$

$y = b$

$E_{zs} = 0$ at $\boxed{x = 0}$ نعوض بالمعادلة

$A_1 = 0$

$E_{zs} = 0$ at $\boxed{x = a}$

$A_2 \cdot \sin K_x a = 0$

$A_2 \neq 0$

$\sin K_x a = 0$

$K_x a = m\pi$

$$\boxed{K_x = \frac{m\pi}{a}}$$

$m = 1, 2, \dots$

not zero

$E_{zs} = 0$ at $y = 0$

$A_3 = 0$

$E_{zs} = 0$ at $y = b$

$A_4 \sin K_y b = 0$

$A_4 \neq 0$

$\sin K_y b = 0$

$K_y b = n\pi$

$$\boxed{K_y = \frac{n\pi}{b}}$$

$n = 1, 2, 3 \dots$

not zero

$$E_{zs} = \overset{A_2 A_4}{E_o} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$H_{zs} = 0$

$$E_{xs} = \frac{-\gamma}{h^2} E_o \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$E_{ys} =$

$H_{xs} =$

$H_{ys} =$

NOTEBOO

set of solutions .

$m = 1, 2, 3, \dots$

$n = 1, 2, 3, \dots$

Subject Date No.

m = Number of half cycle Variation along x-axis

n = " " " " " " y-axis

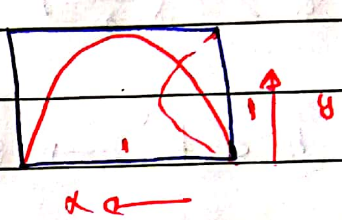
The lowest mode TM_{mn}

$m=1, n=1 \rightarrow TM_{11}$

TM_{12} TM_{21} TM_{22}

TM_{13} TM_{31} TM_{23}

TM_{14} TM_{41} TM ...



$$h^2 = K^2 + \gamma^2 ,\quad h^2 = K_x^2 + K_y^2 ,\quad K = \beta' \text{ if } \beta = \omega\sqrt{\mu\epsilon}$$

$\boxed{\beta \text{ in CH 10}}$

β'

$$\gamma = \sqrt{h^2 - K^2}$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

There are 3 cases :-

1) when $h^2 = K^2 \rightarrow \gamma = 0$

Cut off mode → no propagation

$$h^2 = K^2 \rightarrow \sqrt{\omega_c^2\mu\epsilon} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\omega_c = 2\pi f_c ,\quad f_c = \frac{\omega_c}{2\pi}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_{c_{mn}} = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$u' = \frac{1}{\sqrt{\mu\epsilon}} \rightarrow$ velocity of TEM mode

The lowest freq. is $f_{c_{11}}$ in TM mode

N O T E B O O K

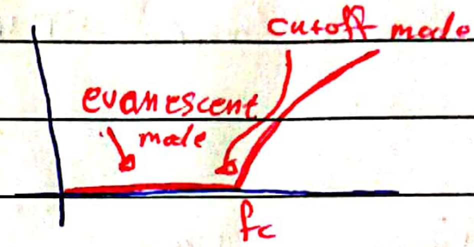
(2) when $h^2 > K^2 \rightarrow \gamma$ is real $\rightarrow \gamma = \alpha + j\beta$

$\beta = 0 \rightarrow$ No propagation

$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 > \omega^2\mu\epsilon$ [Evanescent mode]

occurs when $\omega < \omega_c$

$$\alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$



(3) when $h^2 < K^2 \rightarrow$ Propagation Mode occurs when $h^2 < K^2$

$\omega > \omega_c$

γ is -ve (Pure imagenary) $\rightarrow \gamma = j\beta$, $\alpha = 0$

$$\cancel{j}\beta_{min} = \cancel{j}\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}$$

Subject Date No.

5/12/2019

* TM mode : $h^2 = \gamma^2 + K$, $\gamma = \sqrt{h^2 - k^2}$

$$\beta_{mn} = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

→ Propagation mode

at

$\omega = \omega_c$, $f = f_c$, $h^2 = k^2$, $f_{c_{min}} = \frac{u'}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$

$u' \rightarrow \frac{1}{\sqrt{\mu\epsilon}}$ lossless

$$\beta_{mn} = \beta' \sqrt{1 - \left(\frac{f_{cmn}}{f}\right)^2} , \quad \lambda = \frac{2\pi}{\beta} = \frac{u}{f}$$

$f \rightarrow$ operating freq.

$\beta'\big|_{TEM} = \omega\sqrt{\mu\epsilon}$

$\lambda_c = \frac{u'}{f_c}$ → cut off wavelength

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\gamma}{j\omega\epsilon} = \frac{j\beta}{j\omega\epsilon} = \frac{\beta}{\omega\epsilon}$$

$\therefore \eta_{TM} = \eta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$, $\eta' = \sqrt{\mu/\epsilon}$

* TE mode :- $E_{zs} = 0$, $H_{zs} \neq 0$

$~~E_{zs}~~$ =

$$H_{zs} = (B_1 \cos K_x X + B_2 \sin K_x X) \cdot (B_3 \cos K_y y + B_4 \sin K_y y) e^{-\gamma z}$$

$$E_{xs} = \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}$$

$$E_{ys} = \frac{-j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x}$$

$$H_{xs} = \frac{-\gamma}{h^2}\frac{\partial H_{zs}}{\partial x} , \quad H_{ys} = \frac{-\gamma}{h^2}\frac{\partial H_{zs}}{\partial y}$$

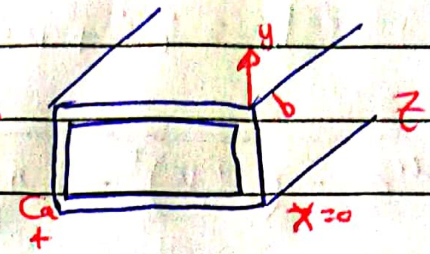
NOTEBOOK

Scanned by CamScanner

Subject Date No.

B.C.S ??

at $X=0$ and $X=a$ $\rightarrow$ $E_{ys}=0 \rightarrow \frac{\partial H_{zs}}{\partial x}=0$



at $y=0$ and $y=b$ $\rightarrow$ $E_{xs}=0 \rightarrow \frac{\partial H_{zs}}{\partial y}=0$

$\frac{\partial H_{zs}}{\partial x}=0$ at $X=0$

$-B_1 K_x \sin K_x(0) + B_2 K_x \cos K_x(0) = 0$

$\boxed{B_2=0}$

at $X=a$

$+B_1 K_x \sin K_x a = 0$

$B_1 \neq 0 \rightarrow K_x a = m\pi \rightarrow \boxed{K_x = \frac{m\pi}{a}}$ & $m=0,1,2 \ldots$

$\frac{\partial H_{zs}}{\partial y}=0$ at $y=0$

$-B_3 K_y \sin K_y y + B_4 K_y \cos K_y y = 0$, $y=b$

$\boxed{B_4=0}$ $\rightarrow \sin K_y b = 0 \rightarrow \boxed{K_y = \frac{n\pi}{b}}$ & $n=0,1,2,$

$\boxed{B_3 \neq 0}$

TE

$H_{zs} = H_0 \cos K_x X \cos K_y y \, e^{-\gamma z}$

$E_{zs} = E_0 \sin K_x X \sin K_y y \, e^{-\gamma z}$

TM

Lowest mode in TE

$f_{c_{mn}} = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

TE_{00} ✗ $\rightarrow$ TEM

$a > b$

TE_{10} ~> Dominant mode

on if $a>b$

$\frac{1}{a} < \frac{1}{b}$

N O T E B O O K

Scanned by CamScanner

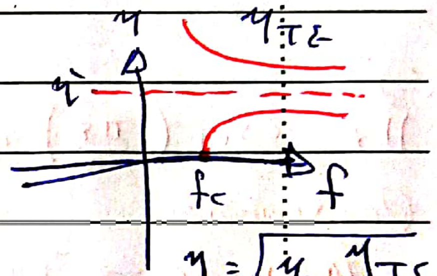
Subject Date No.

$f_{c_{mn}}$, β_{mn}, λ_{mn} $\Big\} \Rightarrow$ Same as TM

but $\eta_{TE} = \dfrac{E_x}{H_y} = -\dfrac{E_y}{H_x} = \dfrac{j\omega\mu}{\gamma} = \dfrac{j\omega\mu}{j\beta}$, $\beta_{mn} = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}$

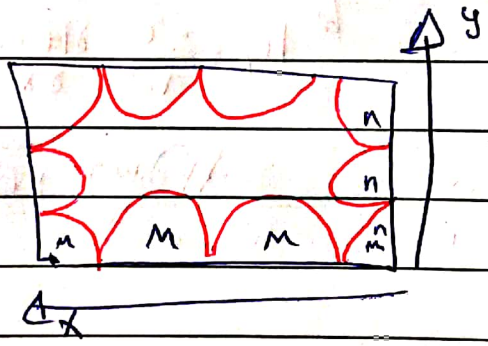
$E_{xs} = \cancel{+}\dfrac{j\omega\mu}{\cancel{h^2}} \dfrac{\partial H_{zs}}{\cancel{\partial y}}$, $H_{ys} = \cancel{+}\dfrac{\gamma}{\cancel{h^2}} \dfrac{\partial H_{zs}}{\cancel{\partial y}}$

$\boxed{\eta_{TE} = \dfrac{\eta'}{\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}}}$, $\boxed{\eta_{TM} = \eta'\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}}$

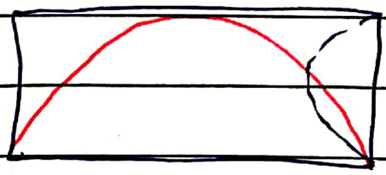


$\eta = \sqrt{\eta_{TM}\,\eta_{TE}}$

i.e TE_{32}



TM_{11}



ex : in Book

$f < 15.1$ GHz

$\sigma = 0$
$\epsilon = 4\epsilon_0$
$\mu_r = 1$

$b = 1$ cm

$a = 2.5$ cm

① $T_{10} =$ ✓

③ $T_{01} =$ ✓

② $TE_{20} =$ ✓

$TE_{70} =$ ✓

$TE_{02} =$ ✓

$TM_{11}, TE_{11} =$ ✓

$f_{c40} =$ ✓

$f_{c50} =$ ✓

NOTEBOOK

Subject Date No.

Degenerecte modes / non-degenerecte mode

if $\square$ > TM_{11}, TE_{11} Degenerecte

if no → non

Ex:- In a R.W.G for which $a = 1.5 cm$

$b = 0.8 cm$

$\sigma = 0$, $\mu = \mu_o$, $\epsilon = 4\epsilon_o$

$H_x = 2 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) \sin\left(\underset{\omega}{\pi \times 10^{11}} t - \beta z\right) A/m$

Determine:-

$f = 50 GHz$

[a] mode → TE_{13}, TM_{13}

[b] f_c ✓ if $f_c < f$

28 GHz

[c] β ✓

[d] γ ✓

[e] η → ... out fields

$H_x = \frac{-\gamma}{h^2} \frac{\partial H_{zs}}{\partial x}$, $H_{zs} = H_o \cos \cos$

$\frac{\gamma}{h^2} H_o \frac{m\pi}{a} \sin\left(\frac{\pi x}{a}\right) = 2$

?

Subject   Date   No.

8/12/2019

* wave Propagation in the guide :-

i.e For TE_{10} mode :-

$$E_{ys} = \frac{-j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right)H_o \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$\alpha = 0$

$h^2 \rightarrow \left(\frac{\pi}{a}\right)^2 + \cancel{\left(\frac{n\pi}{b}\right)^2}$

for $H_o = 1$

$$E_{ys} = \frac{-j\omega a}{\pi}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

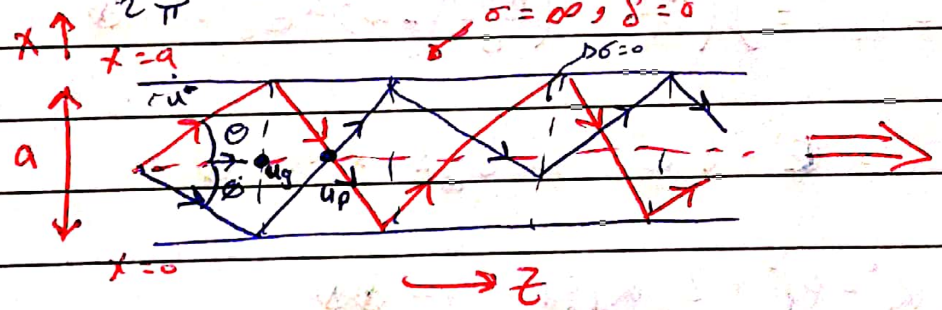
Note :- $E_{zs} = E_{xs} = H_{ys} = 0$, $n = 0$

Euler's Edentity :-

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2} , \quad \sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2}$$

$$E_{ys} = \frac{-\omega\mu a}{2\pi}\left(e^{\frac{j\pi x}{a}} - e^{-\frac{j\pi x}{a}}\right)e^{-j\beta z}$$

$$E_{ys} = \frac{\omega\mu a}{2\pi}\left(e^{-j\beta\left(z+\frac{\pi x}{\beta a}\right)} - e^{-j\beta\left(z-\frac{\pi x}{\beta a}\right)}\right)$$

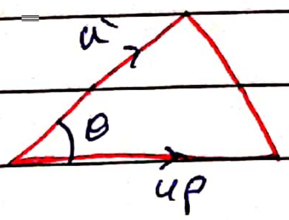


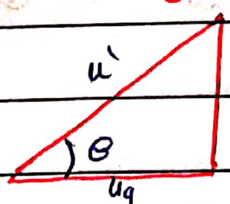
$u_p \equiv$ Phase Velocity

$u' \equiv$ TEM Velocity

$u_g \equiv$ Group Velocity

alawes $< u_p$





$$u_p = \frac{u'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\omega}{\beta} , \quad \beta = \beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$\beta' = \omega\sqrt{\mu\epsilon}$

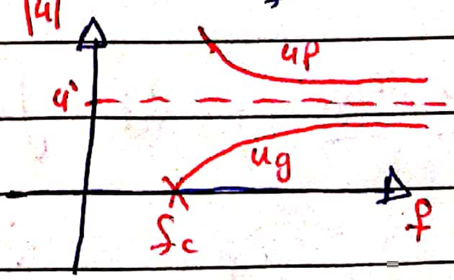
$$\cos\theta = \frac{u_g}{u'} \qquad \cos\theta = \sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\cos\theta = \frac{u'}{u_p}$$

$$u_p = \frac{u'}{\cos\theta} \qquad u_g = u'\cos\theta$$

$u_p > u'$

$$\sqrt{u_p u_g} = u'$$



N O T E B O O K

SEC 12.6 :-

* Power Transmission & Attenuation :-

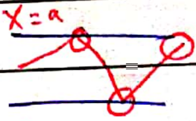
$$\bar{P}_{ave} = \frac{1}{2} Re\{ E_s \times H_s^* \} \Rightarrow \bar{P}_{ave} = \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\eta} \hat{a}_k$$

$\hat{a}_k \downarrow \hat{a}_z$

$$E_{xs} , \; E_{ys} , \; H_{xs} = \frac{-E_{ys}}{\eta} , \; H_{ys} = \frac{E_{xs}}{\eta} , \; \eta = \frac{E_{xs}}{H_{ys}} = -\frac{E_{ys}}{H_{xs}}$$

* Attenuation (lossy w.G)

$\sigma_c \neq \infty \rightarrow \alpha_c \rightarrow \delta = \frac{1}{\alpha_c}$, $\sigma_d \neq 0 \rightarrow \alpha_d$



$P_{Loss} \equiv P_L$ in w , $\alpha = \alpha_c + \alpha_d \equiv$ Total attenuation

$$\gamma = \alpha_d + j\beta_d = \sqrt{h^2 - K^2} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon_c} \text{ -----} *$$

replace ϵ by $\epsilon_c = \epsilon' - j\epsilon'' = \epsilon' - j\frac{\sigma_d}{\omega} \rightarrow$ sub in $*$.

$$\gamma^2 = (\alpha_d + j\beta_d)^2 = \alpha^2 - \beta_d^2 + 2j\alpha_d\beta_d \text{ -------} **$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon' + j\mu\sigma_d$$

$2\alpha_d\beta_d = \omega\mu\sigma_d$ equating the imagenary Part

$$\alpha_d = \frac{\omega\mu\sigma_d}{2\beta_d} , \quad \beta_d = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right) - \left(\frac{n\pi}{b}\right)^2}$$

$$\alpha_d = \frac{\sigma_d \, \eta'}{2\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \quad \eta' = \sqrt{\frac{\mu}{\epsilon}}$$

Subject Date No.

For $\alpha_c \longrightarrow$ $P_{Loss} = P_L\Big|_{y=0} + P_L\Big|_{y=b} + P_L\Big|_{x=0} + P_L\Big|_{x=a}$

usually given

$y=0 \downarrow E_{xs}, E_{zs}$ $\quad x=0 \downarrow E_{ys}, E_{zs}$

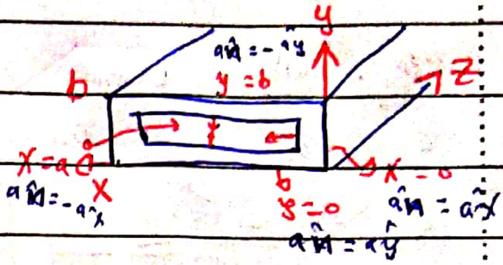
$$\alpha_c\Big|_{TE} = \frac{2R_s}{b\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[\left(1+\frac{b}{a}\right)\left(\frac{f_c}{f}\right)^2 + \frac{\frac{b}{a}\left(\frac{b}{a}m^2+n^2\right)}{\frac{b^2}{a^2}m^2+n^2}\left(1-\left(\frac{f_c}{f}\right)^2\right)\right]$$

Subject Date No.

10/12/2019

* wave guide Current & mode excitation :-

Lossless R.W.G



$\sigma_c = \infty$, $\sigma_d = 0$

For TE_{10} mode

ρ_s on E_{ys} $\Rightarrow$ at $x = 0$, $x = a$, $\rho_s = 0$ since $\hat{a_y} \cdot \hat{a_x} = 0$

at $y = 0 \rightarrow \rho_s$ is $+ve$, $\rho_s = \epsilon |E_{ys}|$ C/m^2

at $y = b \rightarrow \rho_s$ is $-ve$, $\rho_s = -\epsilon |E_{ys}|$ C/m^2

B.C

$(\bar{H}_1 - \bar{H}_2) \times a_{n12} = \bar{K}$

$D_{1n} - D_{2n} = \rho_s$

$D \cdot \hat{a_n} = \rho_s$, $D = \epsilon \cdot \bar{E}$

$\bar{H} \times \hat{a_n} = \bar{K}$

→ at $x = 0 \rightarrow \hat{a_n} = \hat{a_x}$

$K = \hat{a_z} \times \hat{a_x}$, $H_x \times \hat{a_x} = 0$

$= \hat{a_y}$

→ at $x = a \rightarrow \hat{a_n} = -\hat{a_x}$, $K = \hat{a_z} \times -\hat{a_x} = -\hat{a_y}$

→ at $y = 0 \rightarrow \hat{a_n} = \hat{a_y}$

$H_{xs} \rightarrow \bar{K} = \hat{a_x} \times \hat{a_y} = \hat{a_z}$

$H_{zs} \rightarrow \bar{K} = \hat{a_z} \times \hat{a_y} = -\hat{a_x}$

→ at $y = b$

$H_{xs} \rightarrow \bar{K} = \hat{a_x} \times -\hat{a_y} = -\hat{a_z}$

$H_{zs} \rightarrow \bar{K} = \hat{a_z} \times -\hat{a_y} = +\hat{a_x}$

Subject Date No.

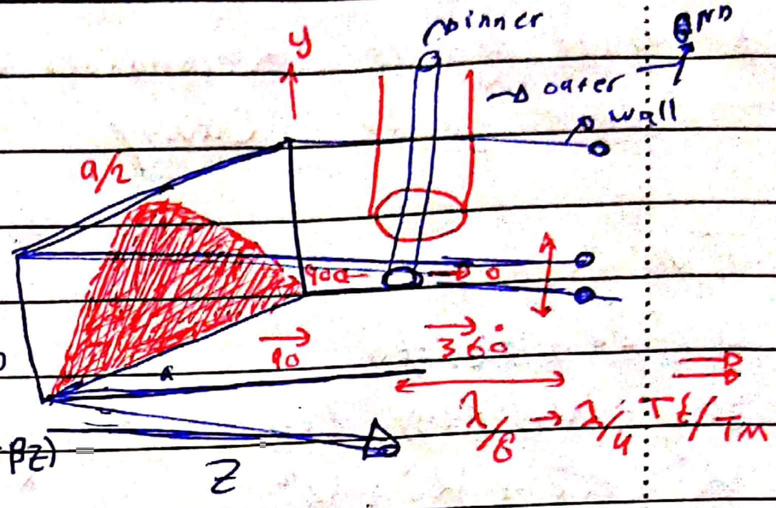
Adopter :-

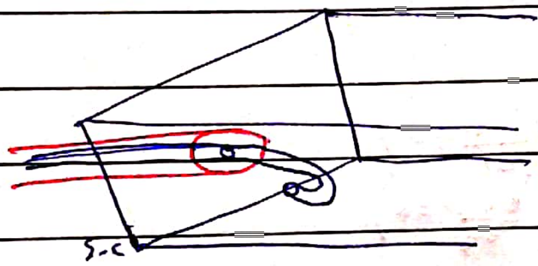
$$\lambda = \frac{u}{f}$$

Coaxial cable waveguide

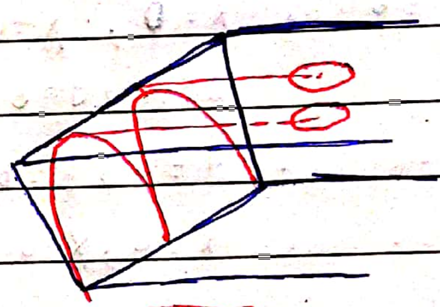
To excite TE_{10} mode

$$E_{y_r} = \frac{w\mu_a}{\Gamma} \sin\left(\frac{\pi x}{a}\right) \sin(wt - \beta z)$$





TM_{11}



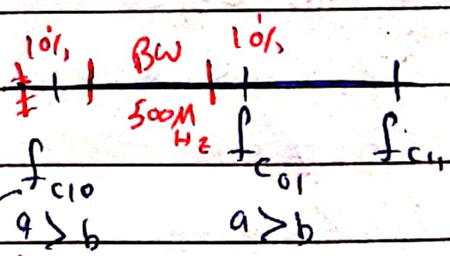
TE_{20}

* Design Procedure :-

a? , b?

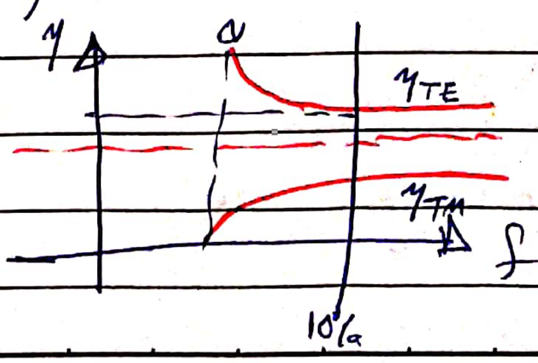
BW = 500MHz

Safe margin = 10%



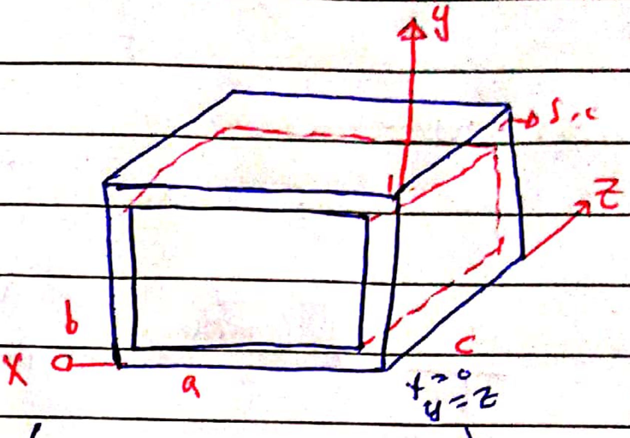
TE_{10}

$$BW = \left(\overset{f_H}{f_{c01} - 10\% f_{c10}}\right) - \left(\overset{f_L}{f_{c10} + 10\% f_{c10}}\right)$$



Subject   Date   No.

* wave guide Resonators :
wave guide Cavity



TM mode :-

$H_{zs} = 0 , E_{zs} \neq 0$

$E_z = X_{(x)} Y_{(y)} , Z_{(z)}$

$K^2 = K_x^2 + K_y^2 + K_z^2$

$$E_{zs} = (A_1 \cos K_x + A_2 \sin K_x X) \cdot (A_3 \cos K_y y + A_4 \sin K_y y) \cdot (A_5 \cos K_z z + A_6 \sin K_z z)$$

→ $E_{zs} = 0$ at $x = 0 , x = a$
$y = 0 , y = b$

→ $E_{xs} = 0$ at $y = 0 , y = b$ → $\left(\frac{\partial E_{zs}}{\partial x} = 0\right)$

→ $E_{ys} = 0$ at $x = 0 , x = a$ → $\left(\frac{\partial E_{zs}}{\partial y} = 0\right)$

$$E_{xs} = \frac{-\gamma}{h^2} \frac{\partial E_{zs}}{\partial x} , \quad E_{ys} = \frac{-\gamma}{h^2} \frac{\partial E_{zs}}{\partial y}$$

at $x = a$ → $E_{zs} = 0$

$A_2 \sin K_x a = 0$

$A_2 \neq 0$

$\sin K_x a = 0$

$K_x = \frac{m\pi}{a} , m = 1, 2, \ldots$

at $y = b$ → $E_{zs} = 0$

$A_4 \sin K_y b = 0 \rightarrow K_y = \frac{n\pi}{b} , n = 1, 2, \ldots$

$A_4 \neq 0$

rest

B.c $E_{xs} = 0$, $z = 0$, $z = c$

$E_{ys} = 0$, $z = 0$, $z = c$

$$E_{xs} \Rightarrow A \left. \frac{\partial E_{zs}}{\partial x} = 0 \right|_{z=0}$$

$$E_{zs} = (A_2 K_z \cos K_x)(\quad)(\overset{0}{\cancel{A_5}} \cos K_z(0) + A_6 \sin K_z(0))$$

Scanned by CamScanner

Subject Date No.

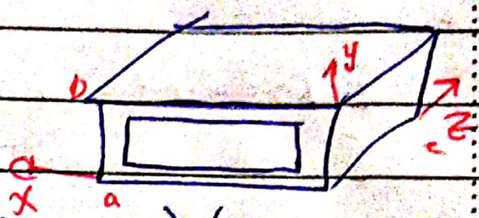
12/12/2019

* wave guide Resonators :-

~~TM mode~~

$E_{zs} \neq 0$, $H_{zs} = 0$

$E_{zs} = (A_1 \cancel{\cos} K_x x + A_2 \sin K_x x) \cdot (A_3 \cancel{\cos} K_y y + A_4 \sin K_y y) \cdot (A_5 \cos K_z z + A_6 \sin K_z z)$



B. C's are:-

$E_{zs} = 0$ at $x = 0, x = a, y = 0, y = b$

$E_{xs} = 0$ at $z = 0, z = c$

$E_{ys} = 0$ at $z = 0, z = c$

$E_{zs} = 0$ at $x = 0 \rightarrow A_1 = 0$

$E_{zs} = 0$ at $x = a \rightarrow K_x = \frac{m\pi}{a}$, $m = 1, 2, 3 \ldots$

$E_{zs} = 0$ at $y = 0 \rightarrow A_3 = 0$

$E_{zs} = 0$ at $y = b \rightarrow K_y = \frac{n\pi}{b}$, $n = 1, 2, 3 \ldots$

$E_{xs} = \frac{-\gamma}{h^2} \frac{\partial E_{zs}}{\partial x} \rightarrow$ T.L

$$j\omega\epsilon E_{xs} = \frac{1}{j\omega\mu}\left[\frac{\partial^2 E_{xs}}{\partial z^2} - \frac{\partial^2 E_{zs}}{\partial x \partial z}\right] + \cancel{\frac{\partial H_{zs}}{\partial y}}^{\,0}(\ldots\ldots)$$

$$j\omega\epsilon E_{ys} = \frac{-1}{j\omega\mu}\left[\frac{\partial^2 E_{zs}}{\partial y \partial z} - \frac{\partial^2 E_{ys}}{\partial z^2}\right] + \cancel{\frac{\partial H_{zs}}{\partial x}}^{\,0}(\ldots\ldots)$$

$$E_{xs} = 0\Big|_{\substack{z=0 \\ z=c}} \rightarrow \frac{\partial E_{zs}}{\partial z} = 0\Big|_{\substack{z=0 \\ z=c}}$$

$$E_{ys} = 0\Big|_{\substack{z=0 \\ z=c}} \rightarrow \frac{\partial E_{zs}}{\partial z}\Big|_{\substack{z=0 \\ z=c}} = 0$$

NOTEBOOK

derive the z-comp of E_{zs}

$$A_5 K_z \sin K_z z + A_6 K_z \cos K_z z = 0 \Big|_{z=0}$$

at $z = 0 \longrightarrow A_6 = 0$

at $z = c \longrightarrow A_5 K_z \sin K_z c = 0$

$A_5 \neq 0$

$\sin K_z c = 0$

$K_c = \dfrac{P\pi}{c}$, $P = 0, 1, 2 \ldots$

$$E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{P\pi z}{c}\right)$$

Lowest mode :-

TM_{110}

$$f_r = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{P}{c}\right)^2}$$

$$\lambda_r = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{P}{c}\right)^2}}$$

$$\beta^2 = K^2 = K_x^2 + K_y^2 + K_z^2$$

$$\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{P\pi}{c}\right)^2}$$

Subject Date No.

* TE mode :-

$E_{zs} = 0 \quad , H_z \neq 0$

$H_{zs} = (B_1 \cos K_x x + B_2 \sin K_x x) \cdot (B_3 \cos K_y y + B_4 \sin K_y y) \cdot (B_5 \cos K_z z + B_6 \sin K_z z)$

B.C's are :-

at $x = 0, x = a \rightarrow E_{ys} = 0 \Rightarrow \frac{\partial H_{zs}}{\partial x} = 0$

$y = 0, y = b \rightarrow E_{xs} = 0 \Rightarrow \frac{\partial H_{zs}}{\partial y} = 0$

$z = 0, z = c \rightarrow E_{xs} = 0, E_{ys} = 0 \Rightarrow H_{zs} = 0$

$H_{zs} = 0 \Big|_{z=0} \rightarrow B_5 = 0$

$H_{zs} = 0 \Big|_{z=c} \rightarrow K_z = \frac{P\pi}{c}, \; P = 1, 2, 3 \ldots$

$\frac{\partial H_{zs}}{\partial x} = 0 \Big|_{x=0} \Rightarrow B_2 = 0 \quad , \quad \frac{\partial H_z}{\partial x} = 0 \Big|_{x=a} \Rightarrow K_x = \frac{m\pi}{a}, \; m = 0, 1, 2, 3 \ldots$

$\boxed{m, n \neq 0, 0}$ same time

$\frac{\partial H_{zs}}{\partial y} = 0 \Big|_{y=0} \Rightarrow B_4 = 0 \quad , \quad \frac{\partial H_z}{\partial y} = 0 \Big|_{y=b} \Rightarrow K_y = \frac{n\pi}{b}, \; n = 0, 1, 2, 3 \ldots$

$H_{zs} = H_0 \cos\left(\frac{m\pi x}{b}\right) \sin\left(\frac{P\pi z}{c}\right)$

lowest mode

$TE_{101} \Big|_{\substack{\text{if } a > b \\ a > c}} \Rightarrow$ domainant mode

TE_{001} X

TE_{010} X

TE_{100} X

Subject Date No.

* Quality Factor (Q) :-

$$Q = 2\pi \frac{\text{Time average Energy Stored}}{\text{Energe loss Per Cycle of oscillator}}$$

$$Q = 2\pi \frac{W_E + W_m}{P_L T} = \frac{1}{BW} = \overset{2\pi f}{\omega} \frac{W}{P_L}$$

$$Q_{TE_{101}} = \frac{(a^2+c^2)\,abc}{\delta[2b(a^3+c^3)+ac(a^2+c^2)]}$$

$\alpha \neq 0$

$$\delta = \frac{1}{\sqrt{\pi f_{r_{101}} \mu_0 \sigma_c}}$$

Ex:- An Air filled resonant cavity with $a = 5cm$, $b = 4cm$, $c = 10cm$ is made of copper ($\sigma_c = 5.8\times10^7 \, S/m$)

Find:- [a]. The five lowest modes in order:-

$$f_r = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{c}\right)^2}$$

, $f_{r\,101} = 3.335 \, GHz \rightarrow TE$ (1)

$f_{r\,011} = 4.04 \, GHz \rightarrow TE$ (2)

$f_{r\,102} = 4.2435 \, GHz \rightarrow TE$ (2)

$f_{r\,110} = 4.8 \, GHz \rightarrow TE$ (4)

$f_{r\,111} = 5.031 \, GHz$ TM/TE (5)

[b] The Q for TE_{101}

$$Q_{TE_{101}} = 14358$$

$$\delta = 1.14176 \, \mu m$$

END of CH-12

NOTEBOOK

Subject　　　　　　　　　　　　　　　Date　　　　　　　No.

15/12/2019

CH-13 :- Antennas :-

Transducer

$$V = \int_V \frac{[\rho_v]\,\partial v}{4\pi\epsilon r} \quad , \quad \bar{A} = \int_V \frac{\mu [J]\,\partial v}{4\pi r}$$

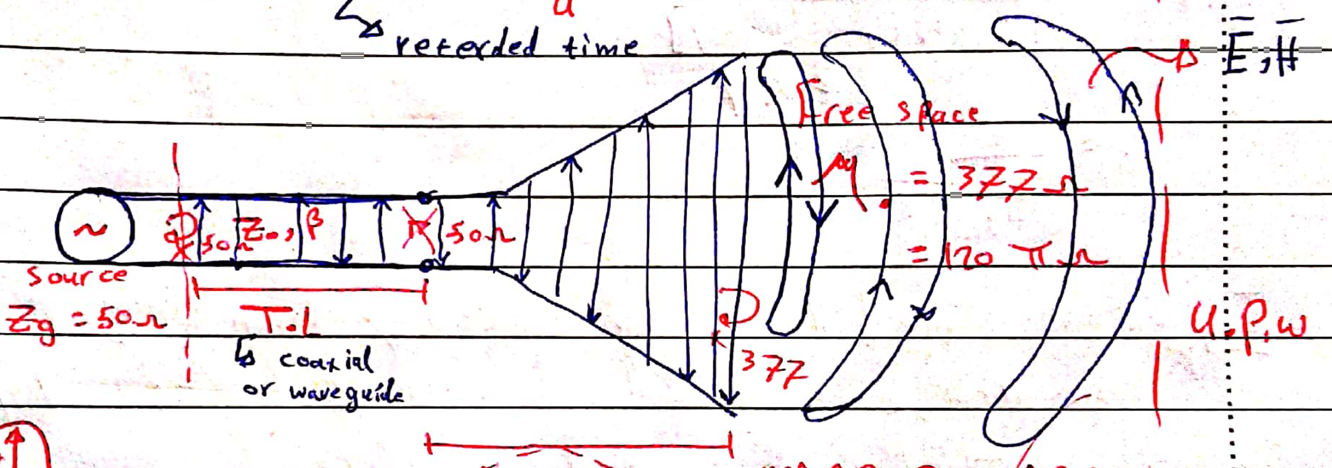
$[\rho_v]$ = returded volume charse dist. (c/m^3)

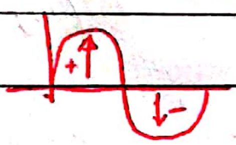
$[J]$ = " " current ". (A/m^2)

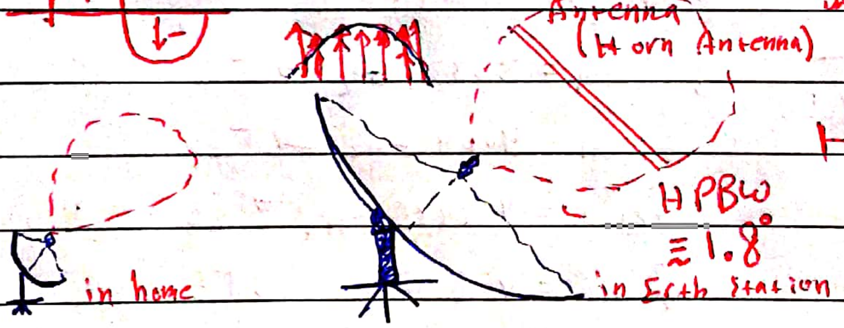
lag

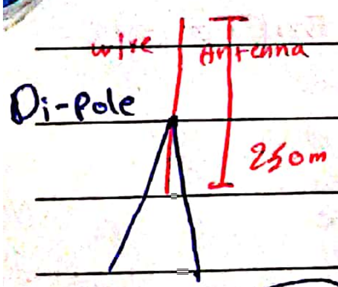
$t' = t - \frac{r}{u}$

retorded time

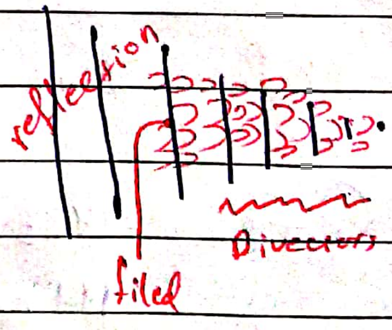


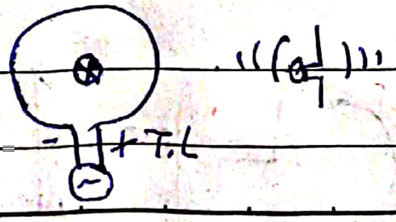




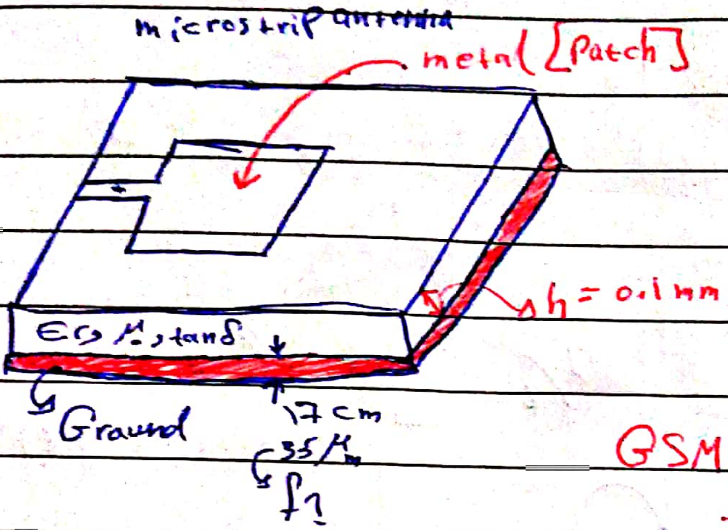


$$\lambda = \frac{c}{f_{lowest}}$$





NOTEBOOK



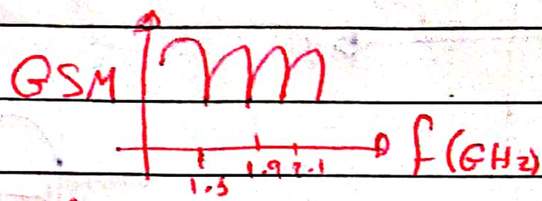
Rogers $\rightsquigarrow$ Daroid $\rightarrow \epsilon_r = 2.2$

$\tan\delta = 0.009$

FR-4 $\rightarrow \epsilon_r = 4.4$

$\mu_r = 1$

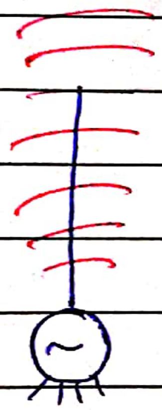
$\tan\delta = 0.02$



wifi 2.4

Bluetooth 2.45

GPS $\begin{cases} \rightarrow 1.712 \text{ band} \\ \rightarrow 1.761 \text{ band} \end{cases}$



mono-Pole

in this Course :-

[1] Di-Pole

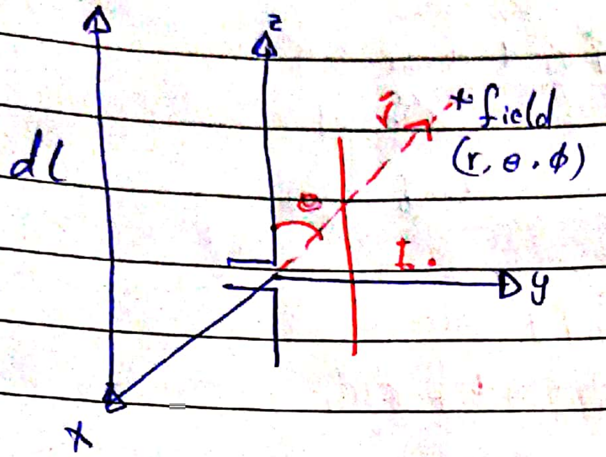
[2] mono-Pole

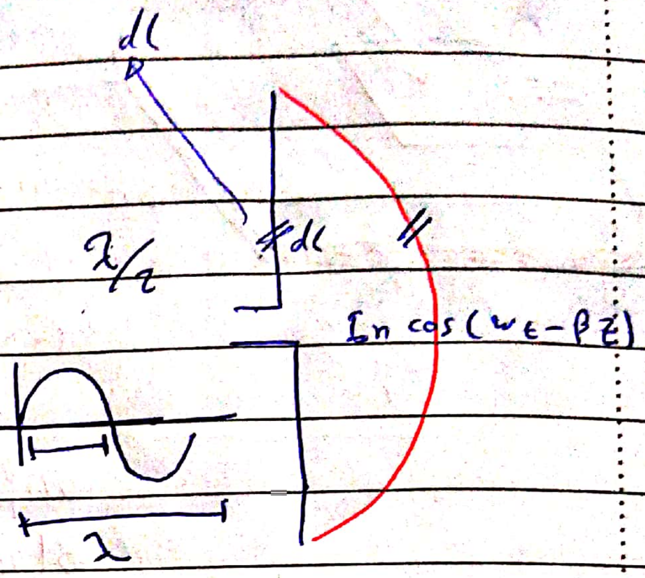
[3] Loop

17/12/2019

* Infinit small Dipol :-

Hertzian Dipole





$$dL < \lambda_o \ , \ \lambda = \frac{c}{f\sqrt{\epsilon_r}}$$

Assume the current is uniform & (I_o) :-

$$A = \frac{\mu [I] \overline{dL}}{4\pi R} \ , \ [I] = I(x, y, z, t')$$

$$t' = t - \frac{r}{u}$$

$$[I] = I_o \cos \omega\left(t - \frac{r}{u}\right)$$

$[I] = I_o \cos(\omega t - \beta r)$ +ve r-dir...

$$u = \frac{1}{\sqrt{\mu\epsilon}} \ , \ \beta = \frac{\omega}{u} = \frac{2\pi}{\lambda}$$

$$I_s = I_o e^{-j\beta r}$$

Since the dipole is along Z-axis

$$A_{zs} = \frac{\mu I_o dL e^{-j\beta r}}{4\pi r}$$ convert to sph. coord. —

$A_{rs} = A_{zs} \cos\theta$

$A_{\theta s} = -A_{zs} \sin\theta$ & $A_{\phi s} = 0$

$$\bar{B} = \nabla \times \bar{A}_s = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a_r} & r\hat{a_\theta} & r\sin\theta\, \hat{a_\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_{z_1}\cos\theta & -A_{z_1}\sin\theta & 0 \end{vmatrix}$$

$$H_s = \frac{\bar{B}_s}{\mu}, \quad H_{rs} = 0, \quad H_{\theta s} = 0$$

$$H_{\phi s} = \frac{I_0 dl}{4\pi} \sin\theta \left(\frac{j\beta}{r} + \frac{1}{r^2} \right) e^{-j\beta r}, \quad \bar{H}_s = H_{\phi s} \hat{a_\phi}$$

Using Maxwell's eq:

$$\nabla \times \bar{H}_s = (\sigma + j\omega\epsilon) \bar{E}_s, \quad \nabla \times \bar{E}_s = -j\omega\mu \bar{H}_s$$

$$\bar{E}_s = \frac{1}{j\omega\epsilon} \nabla \times \bar{H}_s$$

$$E_{rs} = \frac{\eta I_0 dl}{2\pi} \cos\theta \left(\frac{1}{r^2} - \frac{j}{\beta r^3} \right) e^{-j\beta r}$$

$$E_{\theta s} = \frac{\eta I_0 dl}{4\pi} \sin\theta \left(\frac{j\beta}{r} + \frac{1}{r^2} - \frac{j}{\beta r^3} \right) e^{-j\beta r}$$

$$E_{\phi s} = 0, \quad \eta = \sqrt{\mu/\epsilon}, \quad \beta = \frac{2\pi}{\lambda}$$

$\bar{E} \perp \bar{H} \rightarrow$ TEM for sure after the far-fieled distans

$$a_\theta \times \hat{a_\phi} = \hat{a_r}$$

$$\eta = \frac{E_\theta}{H_\phi}$$

$E_\theta = \eta H_\phi \rightarrow$ in far field

Subject Date No.

17/12/2019

* regions surrounding any antenna :-

[1] reactive Near field

[2] radiating Near field → NFC $\sqrt{\frac{D^2}{\lambda}} \leq r \leq \frac{2D^2}{\lambda}$

[3] radiating far field → U.P.W $r \geq \frac{2D^2}{\lambda}$

where D: biggest dimension of the antenna

* Power Calculation (far-field)

$$\bar{P} = \bar{E} \times \bar{H} \quad \text{in time domain}$$

$$= E_\theta H_\phi \hat{a}_r$$

$$\bar{P}_{ave} = \frac{1}{2} Re\{\bar{E}_s \times \bar{H}_s^*\} \; W/m^2$$

$$P_{ave} = P_{rad} = \{\bar{P}_{ave} \cdot \bar{ds}\} \; W$$

$$\boxed{\begin{aligned} H_{\phi s} &= \frac{jI_o \beta dl}{4\pi r} \sin\theta \, e^{-j\beta r} \\ E_{\theta s} &= \eta H_{\phi s} \end{aligned}}$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{2} \frac{I_o^2 \beta^2 dl^2}{16\pi^2 r^2} \sin^2\theta \, r^2 \sin\theta \, d\theta \, d\phi \, \cancel{dr}$$

$$\int_0^{\pi} \sin^3\theta \, d\theta = 4/3$$

$$P_{rad} = \frac{I_o^2 \pi \eta}{3}\left(\frac{dl}{\lambda}\right)^2 W = \frac{1}{2} I_o^2 R_{rad}$$

$$P_{rad} = 40\pi^2 \left(\frac{dl}{\lambda}\right) I_o^2 (W)$$

in free space $\eta_o = 120\pi$

$$R_{rad} = 2\frac{P_{rad}}{I_o^2}$$

$$R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 \Omega$$

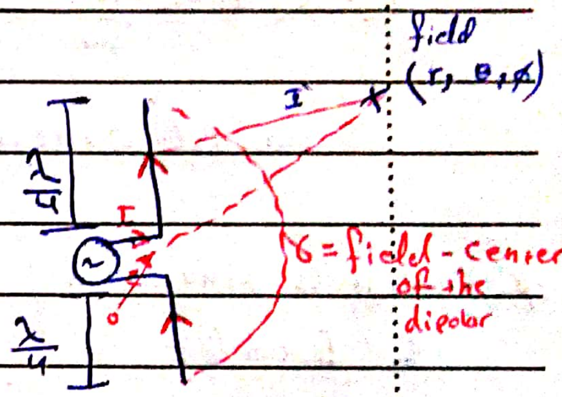
in free space

✳ Half wave-length dipole :-

$$l = \frac{\lambda}{2}$$

$$\bar{A} = \int_l \frac{\mu_o [I]\, dl}{4\pi r}$$

$$[I] = I_o \cos(\omega t - \beta r)$$



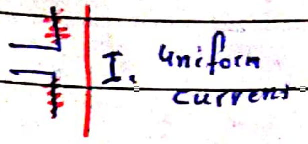
must I equal zero of the termend to satisfy the B.C.s

Subject Date No.

19/12/2019

* λ/2 Dipole Antenna :-

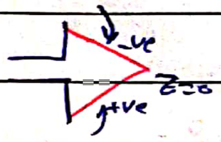
Hertzian Dipole



$dl << \frac{\lambda}{10}$ $\left(dl < \frac{\lambda}{50}\right)$

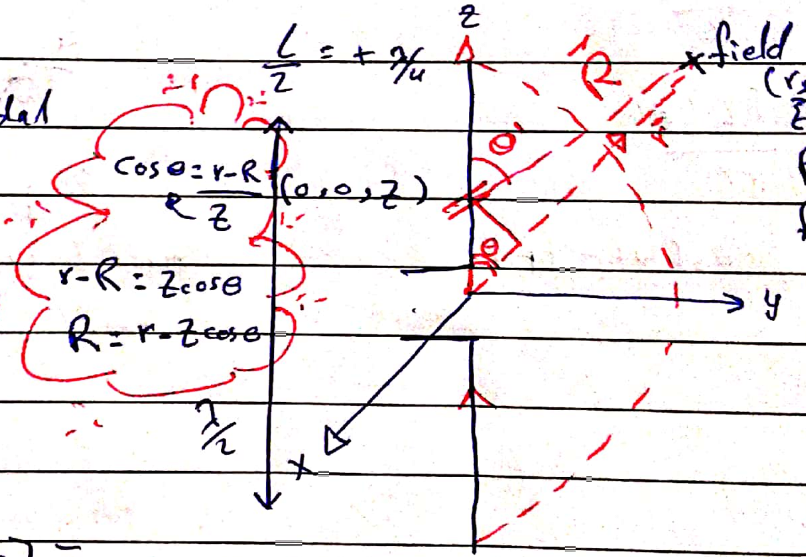
Short Dipole

$\frac{\lambda}{50} < d < \frac{\lambda}{10}$



if $l > \frac{\lambda}{10}$ finite Length Dipole

Sinusoidal current



$r \gg \frac{2D^2}{\lambda}$

* current is sinusoidal

$I = I_o \cos \beta z$

$[I] = I_o \cos \beta z \cos(\omega t - \beta r), t' = t - \frac{l}{4}$

$$A_z = \int_l \frac{\mu_o [I] \bar{dl}}{4\pi R}$$

$$A_{zs} = \int_l \frac{\mu_o I_o \cos \beta z \, e^{-j\beta R}}{4\pi R} dl, \quad dl = dz \, \hat{a}_z$$

Approximation

$|R| = |r|$ in amplitude term

$R = r - z\cos\theta$ in phase term

$$A_{zs} = \mu_0 I_0 e^{-j\beta r} \int_{-\lambda/4}^{\lambda/4} \cos\beta z \; e^{+j\beta z\cos\theta} \, dz$$

$$\boxed{\int e^{az} \cos bz = \frac{e^{az} \cdot (a\cos bz + b\sin bz)}{a^2 + b^2}} \; *$$

$a = j\beta\cos\theta$, $b = \beta$

$$A_{zs} = \frac{\mu I_0 e^{-j\beta r} \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \beta \sin^2\theta} \; \text{wb/m}$$

convert to sph.

$A_{rs} = A_{zs}\cos\theta$, $A_{\theta s} = -\sin\theta \, A_{zs}$, $A_{\phi s} = 0$

$\bar{B} = \nabla \times \bar{A}$, $\bar{H} = \bar{B}/\mu$

The fields in the far field

$\cancel{1/r^2}$, $\cancel{\frac{1}{r^3}}$

$$\boxed{\begin{aligned} H_{\phi s} &= \frac{j I_0 e^{-\beta r} \cos(\pi/2 \cos\theta)}{2\pi r \sin\theta} \quad \text{max} - 90^\circ \\ E_{\theta s} &= \eta H_{\phi s} \end{aligned}}$$

$H_{rs} = H_{\theta s} = E_{\phi s} = E_{rs} = 0$

$E_\theta \perp H_\phi \perp a_r \rightarrow$ TEM

$P_{rad} = \int_s Re\{ E_i \times H_i^* \} d_i$ W

$\bar{P}_{ave} = \frac{\eta I_0^2 \cos^2(\frac{\pi}{2}\cos\theta)}{8\pi^2 r^2 \sin^2\theta}$ W , $P_{rad} \cong 36.56\, I_0^2$

$R_{rad} = \frac{2 P_{rad}}{I_0^2} \cong 73\,\Omega \rightarrow$ real Power

$Z_A = R_{rad} + \cancel{R_L}^{\,\sigma=\infty} + jX_A$, $X_A = 42.5\,\Omega$ $= 73 + j42.5\,\Omega$

$l = \frac{\lambda}{2} = 0.5\lambda$

$l = 0.485\lambda \Rightarrow Z_A = 73\,\Omega + j0$

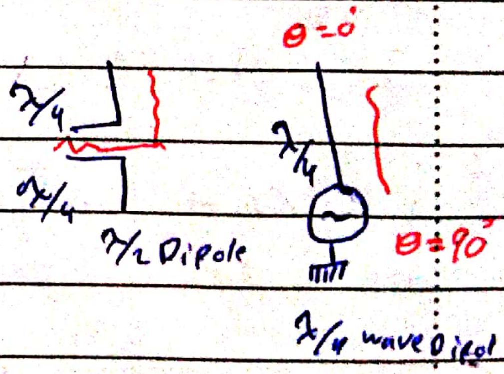
* Quarter wave Length monopole antenna, $\frac{\lambda}{4}$ monopole :-

$A_{zs} = \checkmark$

$H_{\phi s}$ = same

$E_{\theta s} = \eta H_{\phi s}$ } $0 \leq \theta \leq 90°$

all fields = 0 for $90° \leq \theta \leq 180°$



$P_{rad} = 18.28\, I_0^2$ W

$R_{rad} = 36.56\,\Omega$

$Z_m = 36.56 + j21.25\,\Omega$

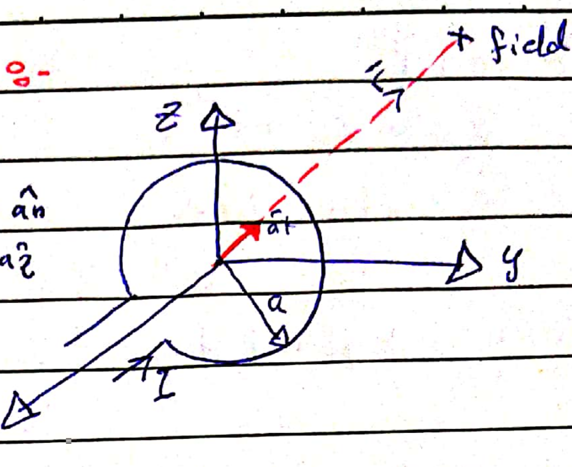
$l = 0.2425\lambda$ to cancel XA

$Z_A = 36.5\,\Omega$

22/12/2019

* Small loop antenna :-

$\bar{m} = \hat{a}_n$
$= \hat{a}_z$



in ch. 8 $\rightarrow f = 0\ Hz$

$$\bar{A} = \frac{\mu_0 I \pi a \sin\theta}{4\pi r^2} \hat{a}_\phi$$

magnetic moment $\quad \bar{m} = I S \hat{a}_r$

$$\bar{A} = \oint \frac{\mu_0 [I]\, d\bar{L}}{4\pi r}$$

for $f \neq 0$

$$A_{\phi s} = \frac{\mu_0 I_s (\sin\theta)}{4\pi r^2} (1 + j\beta r) e^{-j\beta r}$$

$A_{rs} = 0 \,,\; A_{\theta s} = 0$

$\bar{B}_s = \nabla \times \bar{A}_s \quad , \quad \bar{H}_s = B_s / \mu_0$

$$H_{rs} = \frac{j\omega\mu I_0 S}{2\pi\eta} \cos\theta \left[\cancelto{0}{\frac{1}{r^2}} - \cancelto{0}{\frac{j}{\beta r^3}} \right] e^{-j\beta r}$$

$$H_{\theta s} = \frac{j\omega\mu I_0 S}{4\pi} \sin\theta \left[\frac{j\beta}{r} + \cancelto{0}{\frac{1}{r^2}} - \cancelto{0}{\frac{1}{\beta r^3}} \right] e^{-j\beta r}$$

$H_{\phi s} = 0$

Subject Date No.

$\nabla \times \bar{H} = (\cancel{\sigma}^{0} + j\omega\epsilon)\,\bar{E}_r$

$J = \sigma \bar{E}_r = 0$ source free

$\bar{E}_r = \dfrac{1}{j\omega\epsilon} \nabla \times \bar{H}_r$ in far fields

$E_{rs} = 0$, $E_{\theta s} = 0$, $E_{\phi s} = \eta H_{\theta s}$ ($\eta = -E_{\phi}/H_{\theta}$), $\hat{a}_K = \hat{a}_E \times \hat{a}_H$

$\bar{P}_{ave} = \bar{E}_r \times \bar{H}_r = \eta H_{\theta s}^2\, \hat{a}_r$

$P_{rad} = \int_s \bar{P}_{ave} \cdot \bar{d}_s$ ($d_s \to r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$)

in free space $(\eta_o = 120\pi) = \dfrac{320\,\pi^4 S^2 I_o^2}{2\lambda^4}$ W

$= \dfrac{1}{2} I_o^2 R_{rad}$

$R_{rad} = \dfrac{320\,\pi^4 S^2}{\lambda^4}\ (\Omega)$ $\Longrightarrow S = N(\pi a^2)$

if $N > 1$ ($N \to$ number of terms)

Ex :- A magnetic field strength of $5\,\mu$ A/m is required at Point on $\theta = \pi/2$, 2 Km from an antenna in air (neglect ohmic losses).
Calculated ~~the~~ How much Power radiated if the antenna is:-

$\boxed{a}$ a Hertzion Dipole of $\lambda/25$

$\boxed{b}$ $\lambda/2$ dipole

$\boxed{c}$ $\lambda/4$ monople

$\boxed{d}$ loop of radius $\rho = \lambda/20$ with $N = 10$ terms.

Subject Date No.

Sol :- (a) $|H_\phi| = \dfrac{I_o \beta\, dl \sin\theta}{4\pi r}$ $I_o = 0.5A$

$R_{rad} =$

$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$

$= 40\pi^2 \left(\frac{dl}{\lambda}\right) I_o^2$

$= 158\ mW$

(b) $\lambda/2$ dipole:

$|H_{\phi s}| = \dfrac{I_o \cos(\frac{\pi}{2}\cos\theta)}{2\pi r \sin\theta}$

$I_o = 20\pi\ mA$

$P_{rad} = \frac{1}{2} I_o^2 \overbrace{R_{rad}}^{73\Omega}$

$= 144\ mw$

(c) $\lambda/4$ monopole

$P_{rad} = 72\ mw$

(d) loop

$|H_{\phi s}| = \dfrac{\pi I_o S}{r\lambda^2} \sin\theta$

$S = N\pi \rho_o^2$ $I_o = 40.53\ mA$

$R_{rad} = \dfrac{320\pi^4 S^2}{\lambda^4} = 192.3\Omega$

$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$

$= 158\ mw$

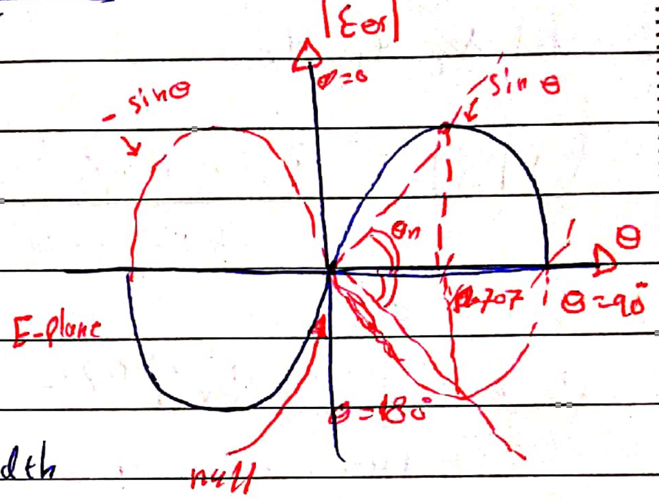
* Antenna Characteristics :-

[1] Radiation Pattern :- (in the far-field) → Power Pattern; → Field Pattern → E-plane; → H-plane

For Hertzian dipole

$$E_{\phi s} = \eta H_{\phi s} = \frac{j\eta I_o \beta\, dl \sin\theta}{4\pi r} e^{-j\beta r}$$

$$|E_{\theta s}| = |\sin\theta|$$
(normalized)



FNBW = 180°

First null Beam width

HPBW

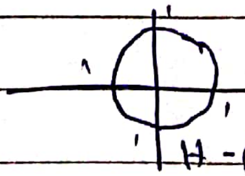
Half Power Beam width $\left[\left[\frac{1}{\sqrt{2}}\right]^2 = \frac{1}{2}\right]$

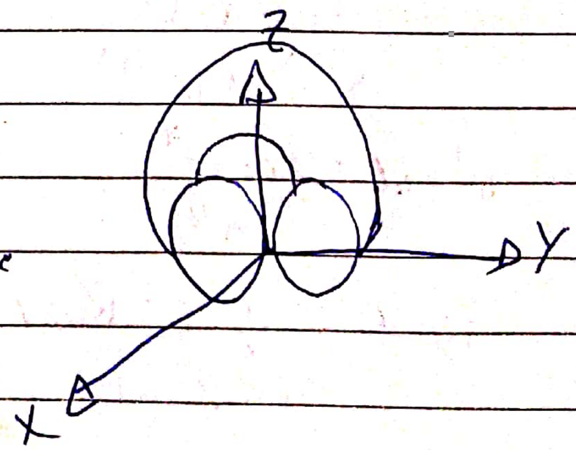
E-Plane → as a function of θ

H-Plane → as a function of ϕ

usually $\theta = \pi/2$

$$|E_{\theta}| = 1$$



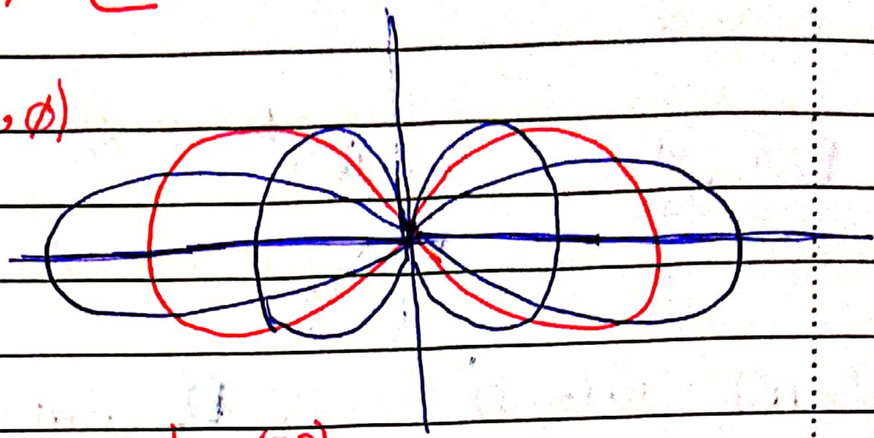


Subject Date No.

24/12/2019

* Radiation Intensity $\boxed{2}$ هامة

Far-field

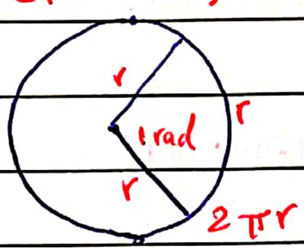


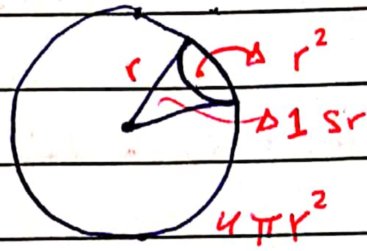
$U(\theta, \phi) \Longleftarrow f(\theta, \phi)$

$U = r^2 |\bar{P}_{ave}| = \frac{W}{Sr}$ $\hat{a}_r$

Steraidian

circle(1D) Sphere(3D)





whole sphere $= 4\pi Sr$

$\Omega = 4\pi Sr$

solid angle

$$P_{rad} = \oint_S \bar{P}_{ave} \cdot ds$$

$$= \int_0^{2\pi}\int_0^{\pi} P_{ave}\, \cancel{\hat{a}_r}\, r^2 \sin\theta\, d\theta\, d\phi\, \cancel{\hat{a}_r} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta\, d\theta\, d\phi$$

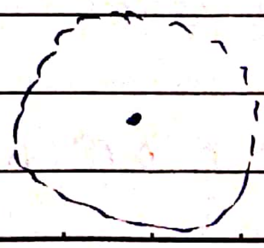
$$= \int_0^{2\pi}\int_0^{\pi} U\, d\Omega \quad , \quad \Omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\phi = 4\pi$$

$$P_{rad} = U_{ave}\, 4\pi \quad , \quad \boxed{U_{ave} = P_{rad}/4\pi \text{ or } U_o}$$

Ideal Source

Isotropic Antenna

(Point source) $D = 1$



[3] Directive Gain $G_d(\theta,\phi)$ or $D(\theta,\phi)$

far-field

$$D = \frac{U}{U_{ave}} = \boxed{\frac{4\pi U}{P_{rad}}} \rightarrow G_d(\theta,\phi)$$

Dimensionless

$D(dB) = 10\log_{10} D_0$, D_0 = Maximum Directivity

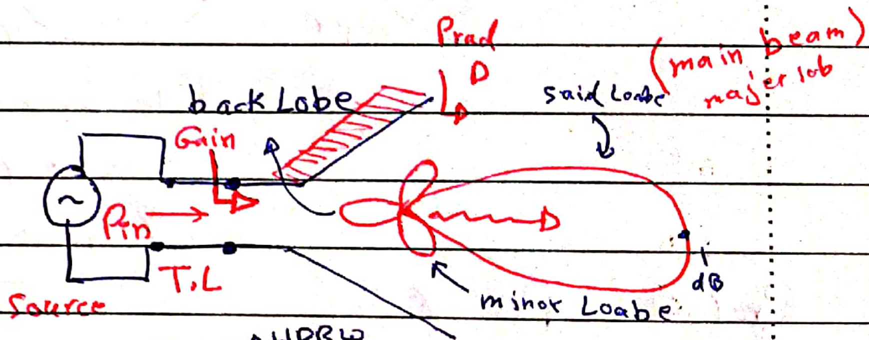
→ dimensionless or in (dB)

[4] Power Gain $G_p(\theta,\phi)$; Gain $G(\theta,\phi)$

$P_{in} = P_L + P_{rad}$

$\sigma_c = \sigma$

$\sigma_d = 0$ if air



$E \begin{cases} \rightarrow \text{HPBW} \\ \rightarrow 0.707 \\ \rightarrow \frac{1}{\sqrt{2}} \end{cases}$, $P \rightarrow \frac{1}{2}$, $dB \rightarrow -3dB$

$$G = \frac{4\pi U}{P_{in}} = \frac{4\pi U}{P_L + P_{rad}}$$

$G < D$, $G = D$ if $P_L = 0$

$G = \eta_r D$, η_r = radiation Efficiency

$$\eta_r = P_{rad}/P_{in} = \frac{P_{rad}}{P_{rad}+P_L} = \boxed{\frac{R_{rad}}{R_{rad}+R_L}}$$

→ conduction & Dielectric losses

→ Dimensionless → $G(dB) = 10\log_{10} G_0$, G_0 ≡ Maximum Gain

Subject Date No.

Ex:- Show that

[a] for a Hertzian dipole $\rightarrow D = 1.5 \sin^2\theta$

[b] for $\lambda/2$ dipole $\rightarrow D = 1.64 \dfrac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta}$

sol:

[a] $E \rightarrow \bar{P}_{ave} = 1/2 \, Re\{\bar{E}_s \times \bar{H}_s^*\}$ ($\eta \bar{H}_s$)

$$D = \frac{4\pi u}{P_{rad}}, \quad u = r^2 |\bar{P}_{ave}|, \quad |\bar{P}_{ave}| = \frac{\eta I_o^2 \beta^2 dl \sin^2\theta}{2 \times 16\pi^2 r^2}$$

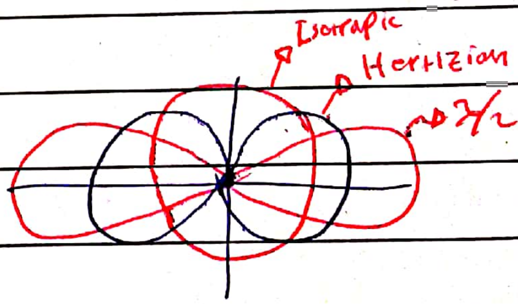
$u = B_o \sin^2\theta$

$$D = \frac{4\pi \cancel{B_o} \sin^2\theta}{\int_0^{2\pi}\int_0^{\pi} \sin^3\theta \, d\theta \, d\phi} = \frac{4\pi \sin^2\theta}{2\pi \cdot \frac{4}{3}} = \frac{3}{2} = 1.5 \sin^2\theta$$

$D_o = 1.5$

$D_o = 10 \log_{10}(1.5)$

[b] $\int_0^{\pi} \dfrac{\cos^2(\pi/2 \cos\theta)}{\sin\theta} \, d\theta = 1.2188$



Note:- $u = \sin^2\theta$

$\sqrt{0.5} = \sqrt{\sin^2\theta_h}$

$\sin\theta_h = 1/\sqrt{2}$

$\theta_h = 45^\circ$

HPBW $= 90^\circ$

Subject Date No.

26/12/2019

* Ex :- The radiation intensity of a certain antenna

$$U(\theta,\phi) = \begin{cases} 2\sin\theta\sin^3\phi, & 0\leq\theta\leq\pi \\ & 0\leq\phi\leq\pi \\ 0, & \text{eles where} \end{cases}$$

D ?

$$D = \frac{4\pi U}{P_{rad}} \quad , \quad P_{rad} = \int_0^{\pi}\int_0^{\pi} \sin^2\theta \sin^3\phi \, d\theta d\phi$$

$$P_{rad} = \int_0^{\pi}\int_0^{\pi} U \, d\theta d\phi \qquad = (2)\left(\frac{4}{3}\right)\left(\frac{\pi}{2}\right) = \frac{4\pi}{3}$$

$$= \iint \underbrace{|P_{ave}|r^2}_{U} \sin\theta \, d\theta d\phi \qquad U_{ave} = \frac{P_{rad}}{4\pi} = \frac{1}{3}$$

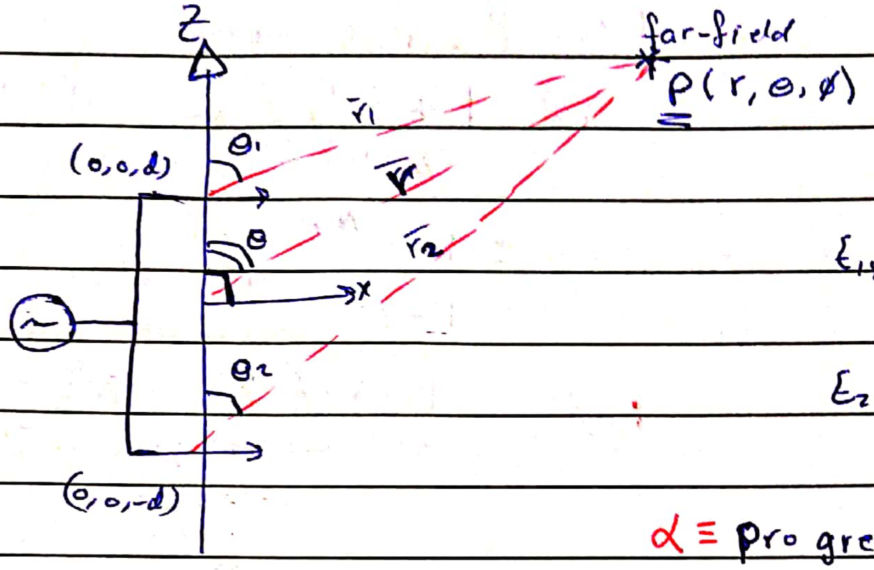
$$D_o = \frac{4\pi(2)}{\frac{4\pi}{3}} = 6$$

$$D_o\Big|_{dB} = 10\log_{10}(6) = ---(dB)$$

N O T E B O O K

* Antenna Arrays :-

more than (1) antenna



$$\bar{E}_s = \bar{E}_{1s} - \bar{E}_{2s}$$

$$E_{1s} = \frac{j\eta\beta I_o dl}{4\pi}\cos\theta_1 \frac{e^{-j\beta r_1}}{r_1} e^{j\alpha}$$

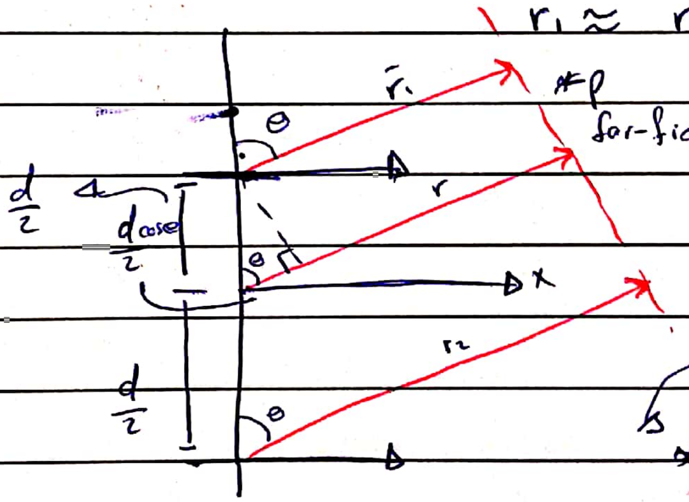
$$E_{2s} = \frac{j\eta\beta I_o dl}{4\pi}\cos\theta_2 \frac{e^{-j\beta r_2}}{r_2} \cancel{e^{j\alpha}}$$

$\alpha \equiv$ progressive phase

Approximation: if point (P) is in the far-field

$$\theta_1 \simeq \theta \simeq \theta_2$$

$$r_1 \simeq r \simeq r_2 \Longrightarrow \text{Amplitude terms}$$



$$\bar{E}_s = \frac{j\eta\beta I_o dl}{4\pi r}\cos\theta\, e^{-j\beta r} * e^{\frac{j\alpha}{2}}$$

$$* \left[e^{+j(\beta d\cos\theta)/2}\, e^{j\alpha/2} + e^{-j(\beta d\cos\theta)/2}\, e^{\frac{-j\alpha}{2}} \right]$$

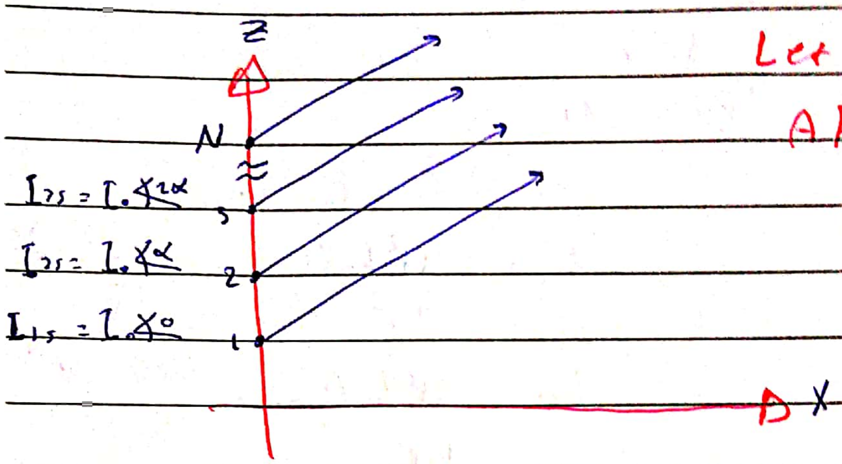
$$\bar{E}_s = \boxed{\frac{j\eta\beta I_o dl\cos\theta}{4\pi r}}\ \boxed{e^{-j\beta r}\, 2\cos\left(\frac{1}{2}(\beta d\cos\theta \pm \alpha)\right)}$$

Element factor   Array factor (AF)

Total pattern = Element pattern × Group pattern

pattern multiplication

for N-element Array



Let $\psi = \beta d \cos\theta + \alpha$

$$AF = 1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(N-1)\psi}$$

↳ $e^{j\alpha 1}$

$$= \frac{1 - e^{jN\psi}}{1 - e^{j\psi}}$$

$$\boxed{1 + x + x^2 + x^3 + \dots + x^N = \frac{1 - x^N}{1 - x}}$$

* type of Excitation:-
  1. uniform.
  2. Binomial (Butterworth) (maximally flat).
  3. chebyshev.

* type of array
  1. linear.
  2. planar.
  3. circular.

usually Controlling (α) and (d) to design an arry

$$AF = \frac{e^{jN\frac{\psi}{2}}}{e^{j\frac{\psi}{2}}} \cdot \frac{e^{jN\frac{\psi}{2}} - e^{-jN\frac{\psi}{2}}}{e^{j\psi/2} - e^{-j\psi/2}}$$

$$\boxed{e^{jN\psi} = e^{jN\frac{\psi}{2}} e^{jN\frac{\psi}{2}}}$$

$$= e^{j(N-1)\frac{\psi}{2}} \cdot \frac{\sin N\frac{\psi}{2}}{\sin\frac{\psi}{2}}, \quad |AF| = \left|\frac{\sin N\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right|$$

normalized

$$|AF|_n = \frac{1}{N}\left|\frac{\sin N\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right| = \left|\frac{\sin N\frac{\psi}{2}}{N\sin\frac{\psi}{2}}\right|$$

N
α
d

Nulls :-

$AF = 0$

$\sin \frac{N\psi}{2} = 0$

$\frac{N\psi}{2} = \pm K\pi \quad , K = 0, 1, 2, \ldots$

except $K \equiv$ multiple of (N) $\rightarrow \frac{0}{0}$

Principle Maxima :

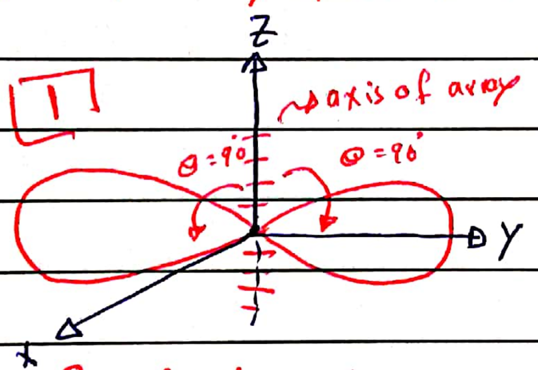
$\sin \frac{\psi}{2} = 0$

$\psi = \pm 2K\pi$ at $\psi = 0 \rightarrow$ Maxima $\rightarrow \cos\theta = \frac{-\alpha}{\beta d}$

$\beta = \frac{2\pi}{\lambda}$

* type of Array Patterns :-

[1]



[1] Broad side radiation

$\theta = 90° \rightarrow$ Principle Maxima

null : at $\theta = 0 \rightarrow \alpha = -\beta d$

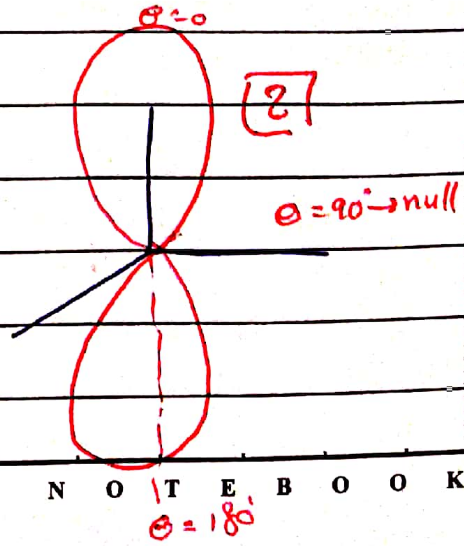
$\theta = 180° \rightarrow \alpha = \beta d$

[2] End-fire Pattern Maxima

at $\theta = 0$

or $\theta = 180$

[2]



[3] scanning Array

[4] Hansen - woodyard end-fire

END

of the

material