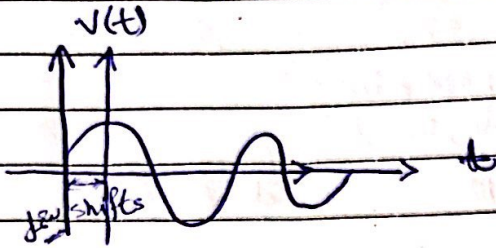


Circuits (2):-

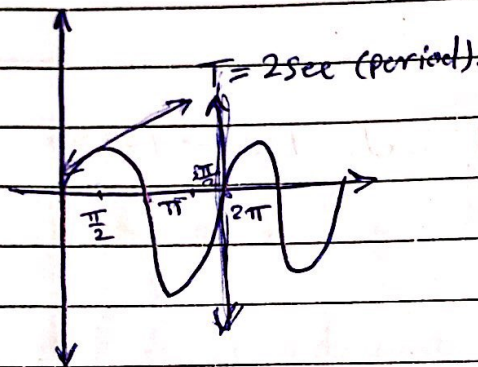
lecture (1):-

* AC - ckt:- sinusoidal, periodic.



① periodic:-

$$v(x) = \sin x.$$



* period = T (second). الزمن المستغرق (اللزيم) في تمام دورة كاملة واحدة.

* frequency = # of periods per 1 second. التردد :

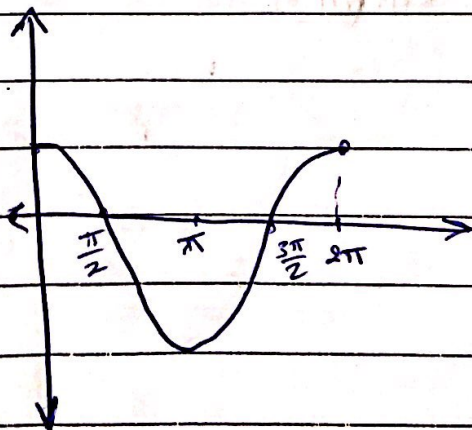
when * $T = 2 \text{ sec.}$

عدد الدورات في الثانية الواحدة.

$$* f = \frac{1}{2} \text{ Hz.}$$

$$\boxed{f = \frac{1}{T}} \Rightarrow \text{velocity. (سرعة الموجة).}$$

*

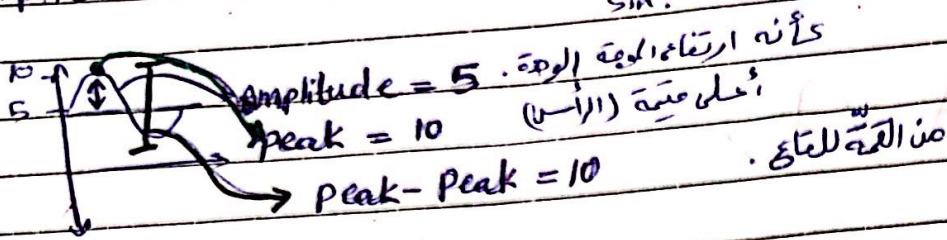


$$v(x) = \cos x.$$

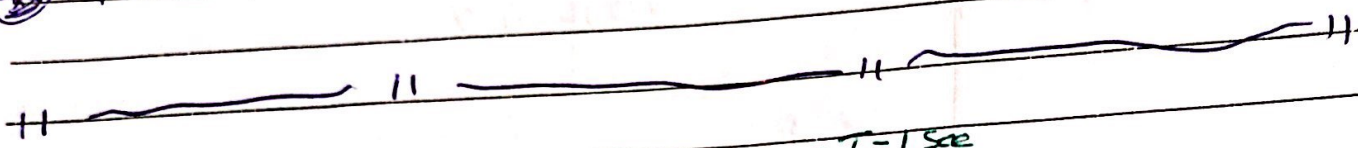
دورات (5) :-

(3) Peak: (Max value).
 أعلى قيمة.
 أعلى قمة (المتعة)
 أعلى قمة (المتعة)

(4) Amplitude . Avg \rightarrow Max .
 القيمة المتوسطة بال Sin.



(5) peak to peak . (peak - peak) : (Min \rightarrow Max).



$X = \text{angle}$. 1 period needs 1 sec $\rightarrow T = 1 \text{ sec}$

if $2\pi \rightarrow T \text{ sec}$

α (angle) \rightarrow ??

time ?? = $\frac{\alpha T}{2\pi}$ time .

α (angle) = $\frac{2\pi}{T}$ (rad/sec) = angular speed = ω : omega
 $\omega = \omega$ = angular frequency. $\omega = 2\pi f$
 while $\frac{1}{T} = f$.

which means that $\alpha = \omega T$

$v(x) = \sin x \rightarrow \sin \alpha \rightarrow \sin(\omega t)$.

* note ω always rad/sec

* Examples. motor rotates $300^\circ/\text{sec}$, @ how long does it take to complete 360° :- (b) period. (c) Frequency.

(a) $\omega = \frac{300 \times \pi}{180} = \omega = \frac{10\pi \text{ rad/sec.}}$

as well as:-

$\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{10\pi}{6}} = \frac{2\pi \times 6}{10\pi} = \frac{12}{10} = 1.2 \text{ sec.} = T$

(c) $\frac{1}{T} = \frac{1}{1.2} = \frac{10 \text{ Hz}}{12} \text{ or } \omega = 2\pi f$

* Examples- A sinusoidal current has { Amplitude = 10A.
(T) Period = 1.2 sec.

Determine the times at which $i = 5A$.

$i(t) = I \cdot \sin(\omega t) \Leftrightarrow 10 \sin(\omega t)$

$\omega = \frac{2\pi}{T} \rightarrow \omega = \frac{2\pi}{1.2} \text{ rad/sec} \rightarrow$

$i(t) = 10 \sin\left(\frac{2\pi}{1.2}\right) t$

$5 = 10 \sin\left(\frac{2\pi}{1.2}\right) t$

* $\sin x = \frac{1}{2}$
 $x = \frac{\pi}{6} = 30^\circ$

$\left(\frac{1}{2} = \sin\left(\frac{2\pi}{1.2}\right) t\right)$

للقيمة
للقيمة

$\frac{\pi}{6} = \frac{2\pi}{1.2} t \Rightarrow$

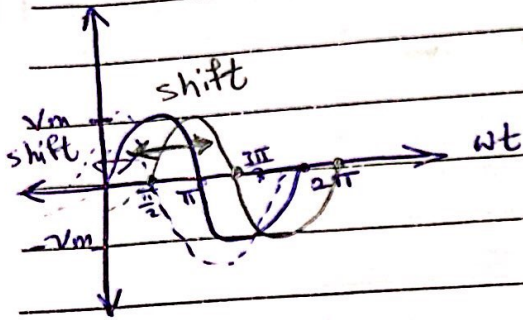
$t_1 = 0.1 \text{ sec} \# \rightarrow \text{الزمن} = \frac{\pi}{6} \text{ ١}$

$t_2 = \frac{5\pi}{6} \rightarrow 0.5 \text{ sec} \#$

* lecture (2) :-

* Phase shift.

$$V(t) = V_m \cdot \sin(\omega t)$$



$V(t)_* = V_m \cdot \sin(\omega t - \frac{\pi}{2})$: shift to the right.
phase shift $\rightarrow (\ominus)$

$V(t)_* = V_m \cdot \sin(\omega t + \frac{\pi}{2})$: shift to the left.

- \otimes lag $(-\ominus)$ $V_*(t) \rightarrow$ lags.
- lead $(+\ominus)$ $V_*(t) \rightarrow$ leads.

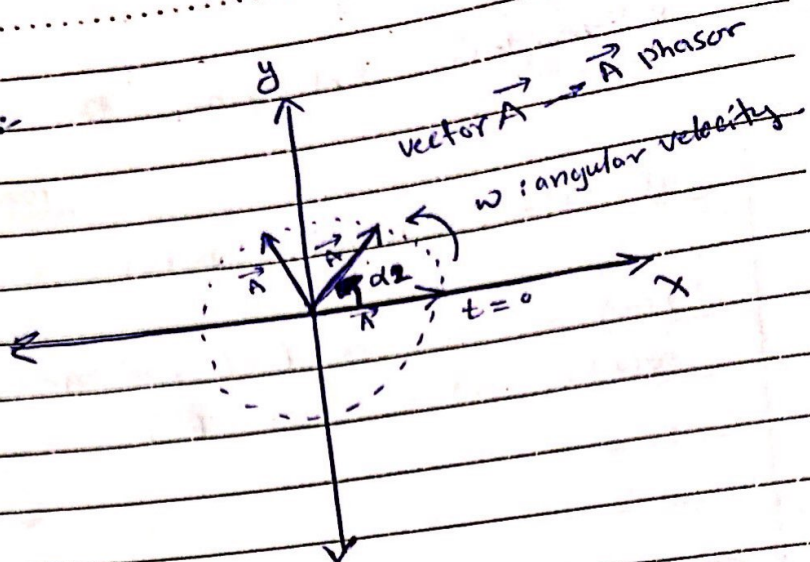
$V(t)$ leads $V_*(t)$ by $90^\circ (\frac{\pi}{2})$.

$V_*(t)$ lags $V(t)$ by $90^\circ (\frac{\pi}{2})$.

⊗ phasor = rotation:

$$V_L = L \frac{di}{dt}$$

$$i_C = C \frac{dv}{dt}$$



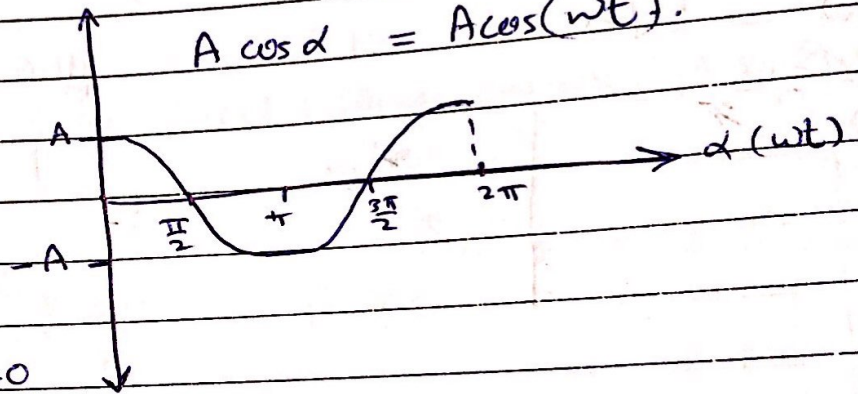
$$t=0 \Rightarrow A \cos(\alpha_1 = 0)$$

$$t=1 \Rightarrow A \cos \alpha_2$$

$$t=t_0 \Rightarrow A \cos \alpha t_0$$

$$(\alpha = \omega t)$$

$$A \cos \alpha = A \cos(\omega t)$$



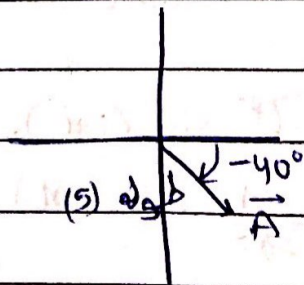
phasor. $|A| \angle \theta$ $t=0$



Time domain. $A \cos(\omega t + \theta)$

→ التردد
فاز
فاز
(cos)

$$\vec{A} = 5 \cos(\omega t - 40^\circ)$$



* Example:- Determine the phasor for the following sources:-

and plot it in phasor diagram.

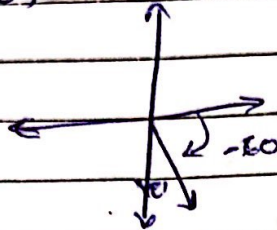
⊕ Known Identity:-

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$

① $v(t) = 10 \sin(\omega t + 30)$

$v(t) = 10 \cos(\omega t + 30 - 90)$

$v(t) = 10 \cos(\omega t - 60)$

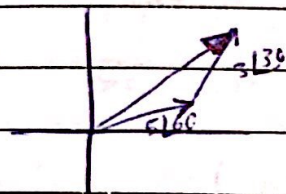


② $v(t) = 5 \cos(\omega t + 60) + 3 \cos(\omega t + 30)$

→ \int_{SS}
polar to form.

$5/60 + 3/30 = 11/0 \rightarrow 7.7/49^\circ$

How to do



or polar → rect → polar

* Trigonometric Identities:-

لإيجاد الأضداد

1. $\sin(\omega t \mp 180) \Rightarrow -\sin(\omega t)$

2. $\cos(\omega t \mp 180) = -\cos(\omega t)$

3. $\cos(\omega t \mp 90) = \mp \sin(\omega t)$

4. $\sin(\omega t \pm 90) = \pm \cos(\omega t)$

Same lines

* Lecture (3) :- phasor

vector \Rightarrow polar form.

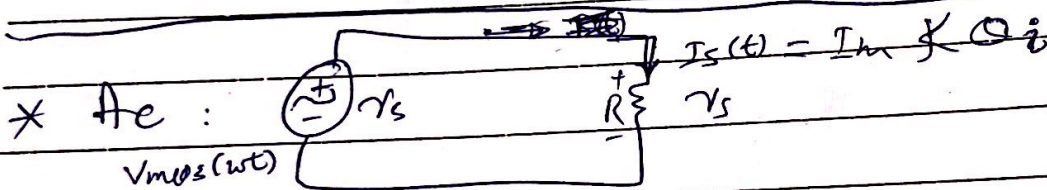
$|A| \angle \theta \rightarrow t=0$

angular speed = ω rad/sec. = angular frequency.

$$\omega = \frac{2\pi}{T} = 2\pi f.$$

* Euler rule:- $A e^{j(\omega t + \theta)} = A e^{j\omega t} e^{j\theta}$

$$\operatorname{Re} \{ A e^{j(\omega t + \theta)} \} = \operatorname{Re} \{ \cos(\omega t + \theta) + j \sin(\omega t + \theta) \}$$



Ohm's law:-

$$V_s = I_s \times R$$

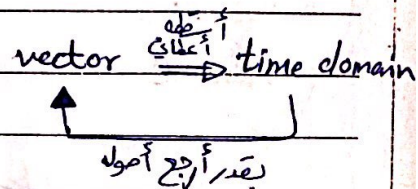
lecture (3) :- phasor \rightarrow
 vector \rightarrow polar form.
 $|A| \angle \theta$

* angular speed = $\omega \rightarrow$ (rad/sec) = $\frac{2\pi}{T} = 2\pi f$

* Euler rule :- جمع الاجزاء
 $A e^{j(\omega t + \theta)} = A e^{j\omega t} e^{j\theta}$

Real $\left\{ \begin{aligned} &= \text{Real} \left\{ \underbrace{A \cos(\omega t + \theta)}_{\text{Real part}} + j \underbrace{A \sin(\omega t + \theta)}_{\text{imaginary part}} \right. \end{aligned} \right.$

Real $\left\{ A e^{j(\omega t + \theta)} = A \cos(\omega t + \theta) \right.$
 time domain



// ~~~~~ //

* 2 vectors بقية
 أحدهم (أو لهما نفس)
 ω و قدره أو سعة
 على نفس ال (Phasor)
 (Diagram)

$\left. \begin{aligned} &* A \angle \theta \text{ (polar.)} \# (1) \\ &* \text{Real} + j\text{imag} \text{ (Rectangular)} \# (2) \end{aligned} \right\}$

$\left. \begin{aligned} &* \text{Real} = A \cos \theta \\ &* \text{Imaginary} = A \sin \theta \end{aligned} \right\} \begin{aligned} &A = \sqrt{R^2 + I^2} \\ &\theta = \tan^{-1} \left(\frac{I}{R} \right) \end{aligned}$

// ~~~~~ //

X الحالات الكسرية - قبل ضربها بقية
 بقية polar \rightarrow بقية الكسرية $|A \times B| \angle \theta_1 + \theta_2$
 وجمع الزاويتين

بقية polar \rightarrow بقية الكسرية $|A| \angle \theta_1 - \theta_2$
 ونظره الزاوية $|B|$

جمع الكسرية Rectangular \rightarrow بقية الكسرية
 كالتالي Real \angle كالتالي

* Rect:-

$$(A+jB) \cdot (C+jD) = AC + jAD + jBC - BC$$

(Rationalizing)

$$\frac{A+jB}{C+jD} \times \frac{C-jD}{C-jD} = \frac{(A+jB) \cdot (C-jD)}{C^2+D^2}$$

$$* Ae^{j\theta} = A \cos \theta + jA \sin \theta$$

$$* \cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$* \sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

V. imp Identities

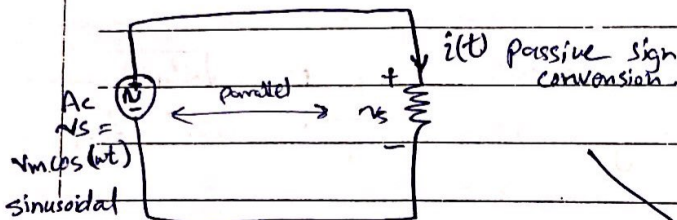
* R.L.C - CKT:-

① pure {R}:-

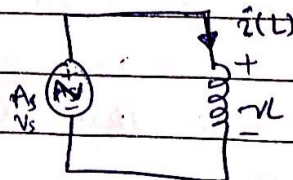
Ohm's law $\Rightarrow V_s = I_s \cdot R$

$$V_m \angle 0^\circ = I_m \angle 0^\circ$$

V_R and I_R in phase.



② inductor {L}:-



$$v_L = L \cdot \frac{di}{dt}$$

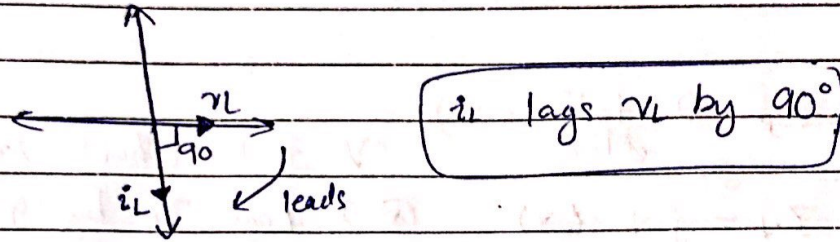
$$i = \frac{1}{L} \int v_L(t) \cdot dt$$

$$i = \frac{1}{L} \int v_m \cos(\omega t) dt$$

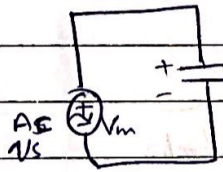
$$i_L = \frac{V_m}{L} \cdot \frac{\sin(\omega t)}{\omega} \Rightarrow \frac{V_m}{L} \cos(\omega t - 90^\circ)$$

$$\frac{V_L}{i_L} = \frac{V_m \angle 0^\circ}{\frac{V_m}{\omega L} \angle 90^\circ} \rightarrow \frac{V_L}{i_L} = \omega L \angle 90^\circ \rightarrow j\omega L = \text{impedance} = Z = \boxed{j\omega L}$$

Phasor diagram of the conductor:-



Capacitor :-

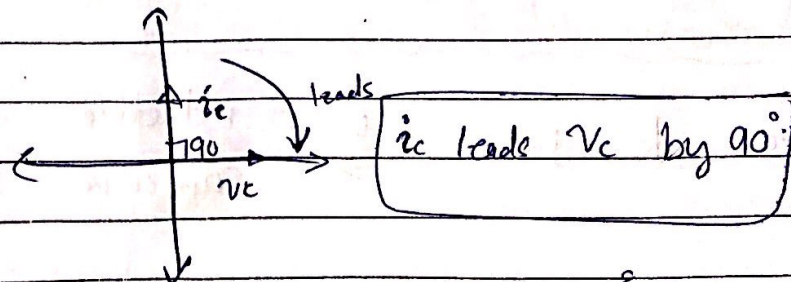


$$V_C = V_m \angle 0^\circ$$

$$i_C = C \frac{dV}{dt} \rightarrow i_C = C \cdot \omega V_m \sin(\omega t) = -V_m \omega C \cdot \cos(\omega t - 90^\circ)$$

$$Z = \frac{V_C}{i_C} = \frac{V_m \angle 0^\circ}{\frac{V_m \omega C \angle -90^\circ}{-1}} \rightarrow \frac{1}{\omega C \angle 90^\circ} = \frac{\angle 90^\circ}{\omega C} = \boxed{\frac{-j}{\omega C}}$$

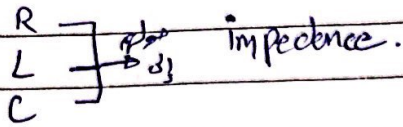
Phasor diagram of the capacitor:-



leads

* lecture (5) :-

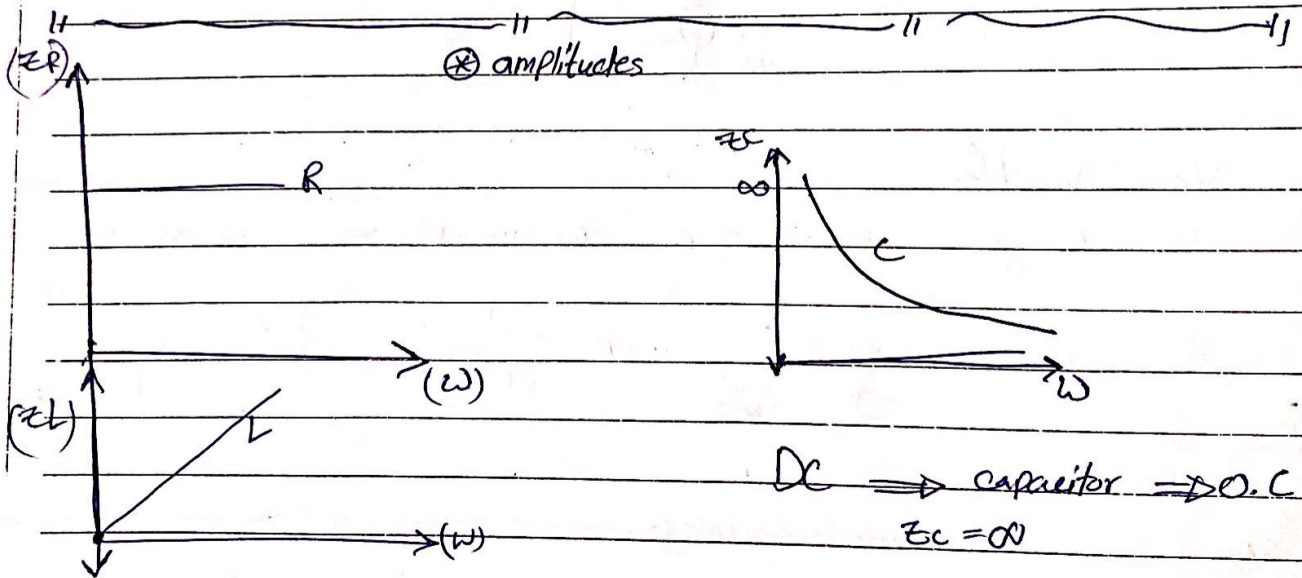
Phasor



$R \Rightarrow Z_R = 0$ (resistor) $V = Z I$ (Ohm's law)

$L \Rightarrow Z_L = j\omega L$ (inductor) $\otimes I_L$ lags V_L by (90°)

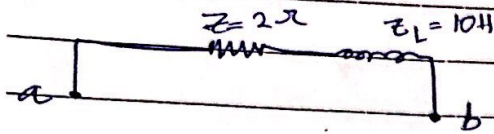
$C \Rightarrow Z_C = j\frac{1}{\omega C}$ (capacitor) $\otimes I_C$ leads V_C by (90°)



$\omega = 0 \Rightarrow f = 0 \Rightarrow$ DC source \Rightarrow inductor \Rightarrow S.C.
 $Z_L = 0$

* $Z = R \pm jX$ \rightarrow \oplus inductive load.
 \rightarrow \ominus capacitive load.

* Series:-



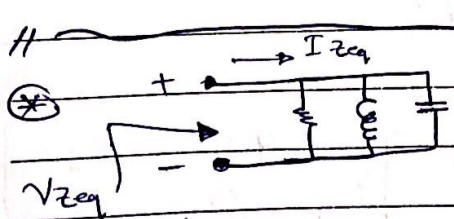
* parallel :-

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots$$

$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$Z = \text{Real } \mp \underbrace{j \text{ imagine}}_{\text{Reactance}}$$

$$Z = R \mp jX$$



$$Z_{eq} = 5 \oplus j3 \quad \text{inductive}$$

I_L lags V_L by $\phi_{Z_{eq}}$

$$Z_{eq} = 5 - j3 \quad \text{capacitive}$$

I_C leads V_C by $\phi_{Z_{eq}}$

* $\frac{1}{Z} = Y$ (admittance).

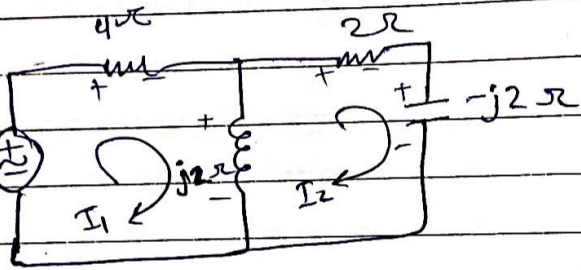
$Z = R + jX$

$Y = G + jB$
conductance = substance

$Y = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2}$

Example:-

Find the Mesh currents



Mesh 1:

$5\angle 0^\circ = + (4 + j2)I_1 - (j2) I_2$

$0 = -(j2)I_1 + 2I_2$

$5\angle 0^\circ = (4 + j2 + 2)I_1$

$I_1 = \frac{5}{6 + j2} \text{ A}$

$I_2 = \left(\frac{j2}{2}\right) \left(\frac{5}{6 + j2}\right) = \frac{j5}{6 + j2}$

$$\begin{bmatrix} 5\angle 0^\circ \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j2 & -j2 \\ -j2 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = A^{-1} \begin{bmatrix} 5\angle 0^\circ \\ 0 \end{bmatrix} =$$

$$A^{-1} = \frac{1}{\Delta} \begin{bmatrix} 2 & +j2 \\ +j2 & -4 - j2 \end{bmatrix}$$

$$\Delta = (4 + j2) \times 2 - (-j2) \times (+j2)$$

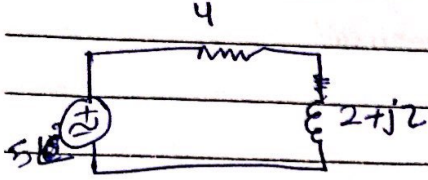
* محلولة

بدون اسطر

Mesh analysis

$$(2-j2) \parallel j2$$

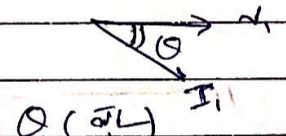
$$\frac{(2-j2)(j2)}{2} = \frac{4j+4}{2} = \boxed{2+j2}$$



$$I_1 = \frac{5\angle 0^\circ}{6+j2}, \quad I_2 = I_1 \cdot \frac{Z_{eq}}{2-j2}$$

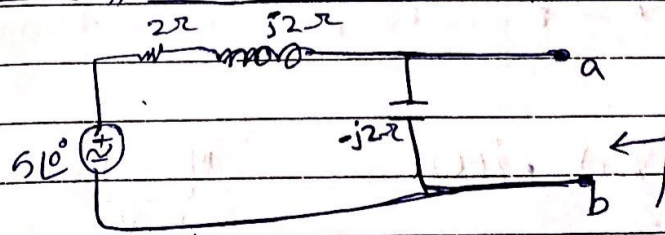
$$I_2 = \frac{5}{6+j2} \left(\frac{2+j2}{2-j2} \right), \text{ current division.}$$

$$I_1 = \frac{5\angle 0^\circ}{6+j2} \xrightarrow{H\angle 0^\circ} I_1(t) = I_m \cos(\omega t + \theta)$$



$\theta < 0^\circ$
 Inductive load.
 I_1 lags V_1 .

* Example 8-1



find Z_{Th} in

find Z_{Th} from a, b:-

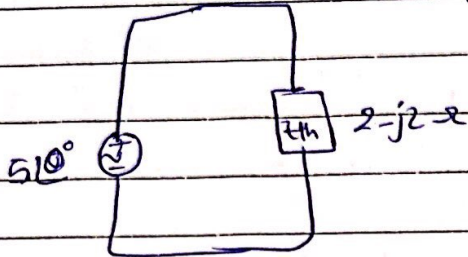
V_{Th} , I_{sc}

$$\frac{(2+j2) \parallel -j2}{2} = \frac{(2+j2)(-j2)}{2} = \frac{-4+4}{2} = \boxed{2-j2 \Omega}$$

$$V_{o.c} = V_{ab} = V_c =$$

$$V_c = \frac{5 \angle 0^\circ}{2} (-j2) = V_{th} = V_c = -j5V = 5 \angle 90^\circ$$

Thevenin equivalent circuit.



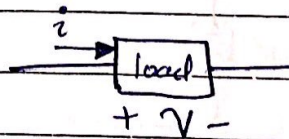
$$I_{sc} = \frac{V_{th}}{Z_{th}} = \frac{-j5}{2-j2} \quad \#$$

* lecture (6) :-

* CH. 9 :- power of AC.

$P = VI$
 + : absorb power (load).
 - : generates power, supply, delivers. (source).

* $p(t) = v(t) \cdot i(t)$.
 instantaneous power.



@ sinusoidal source :-

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$P(t) = V_m \cdot I_m \cdot \cos(\omega t + \theta_v) \cdot \cos(\omega t + \theta_i) \rightarrow$$

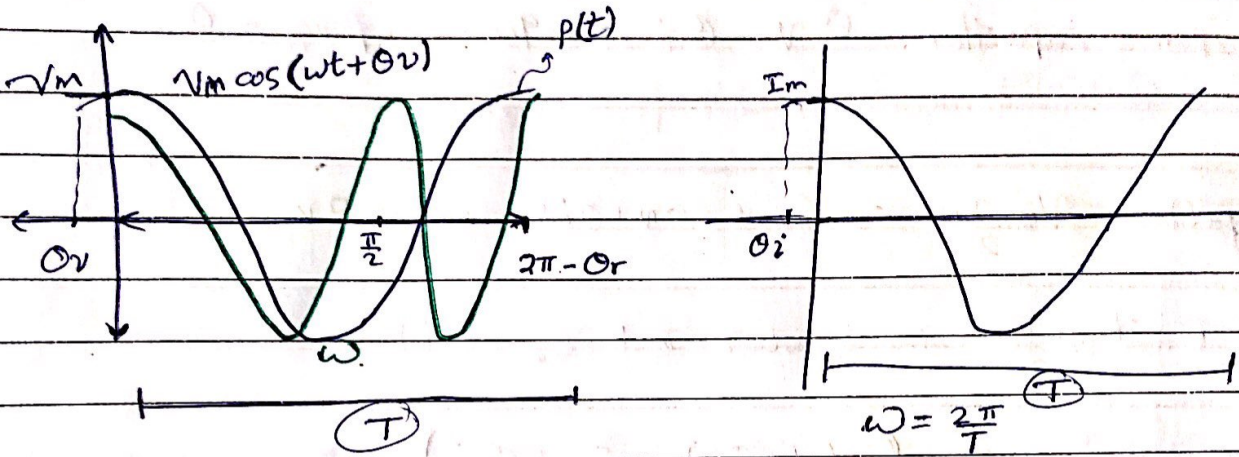
$$P(t) = \frac{1}{2} V_m \cdot I_m \left[\cos(2\omega t + \theta_v + \theta_i) + \cos(\theta_v - \theta_i) \right]$$

* defn: $\cos x \cdot \cos y = \frac{1}{2} [\cos(x+y) + \cos(x-y)]$.

$$p(t) = \frac{V_m \cdot I_m}{2} \left[\cos(2\omega t + \theta_v + \theta_i) + \cos(\theta_v - \theta_i) \right] \Rightarrow$$

$$p(t) = \frac{V_m \cdot I_m}{2} \cos(2\omega t + \theta_v + \theta_i) + \frac{V_m \cdot I_m}{2} \cos(\theta_v - \theta_i)$$

constant
(Average power),



$$P_{\text{Peak}} = \frac{V_m \cdot I_m}{2} + \frac{V_m \cdot I_m}{2} \cos(\theta_v - \theta_i) \quad \boxed{\omega_p = 2\omega}$$

periodic function T

$$P_{\text{avg}} = \frac{1}{T} \int p(t) \cdot dt \Rightarrow$$

$$\frac{V_m \cdot I_m}{2T} \left[\int_0^T \cos(2\omega t + \theta_v + \theta_i) \cdot dt + \int_0^T \cos(\theta_v - \theta_i) \cdot dt \right]$$

$$\Rightarrow \frac{V_m \cdot I_m}{2T} \cdot \cos(\theta_v - \theta_i) T \Rightarrow \frac{V_m \cdot I_m}{2} \cos(\theta_v - \theta_i)$$

$$\square \text{ Avg power} := \frac{V_m \cdot I_m \cos(\theta_v - \theta_i)}{2}$$

$$* \text{ Pure Resistance: } \theta_v - \theta_i = 0 \quad , \quad P_{avg} = \frac{V_m \cdot I_m}{2}$$

$$* \text{ Pure inductance: } \theta_v - \theta_i = +90 \quad , \quad P_{avg} = 0$$

$$* \text{ Pure capacitor: } \theta_v - \theta_i = -90 \quad , \quad P_{avg} = 0$$

$$p(t) = \frac{V_m \cdot I_m}{2} \cos(2\omega t + \theta_v + \theta_i) \cdot (-\theta_i + \theta_v)$$

$$\Rightarrow \frac{V_m \cdot I_m}{2} \cos(2\omega t + \theta_v + \theta_i)$$

$$\Rightarrow \frac{V_m \cdot I_m}{2} \cos(\underbrace{2(\omega t + \theta_v)}_x + \underbrace{\theta_i}_y) \rightarrow$$

$$* \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\frac{V_m I_m}{2} \cos(2(\omega t + \theta_i)) \cdot \cos(\theta_v - \theta_i) - \frac{V_m I_m}{2} \sin(2\omega t + \theta_i) \cdot \sin(\theta_v - \theta_i)$$

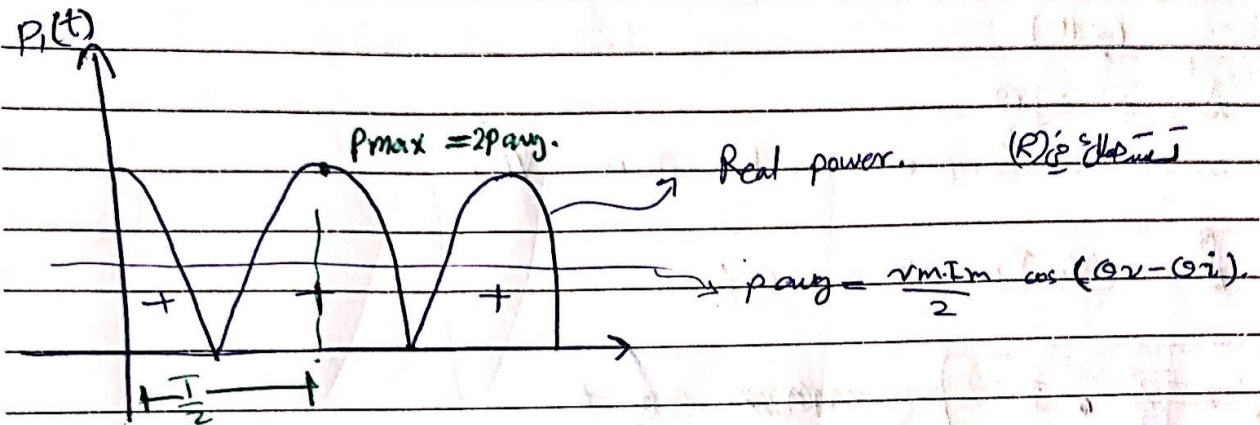
$$p(t) = \frac{I_m \cdot V_m}{2} \cos(\theta_v - \theta_i) [1 + \cos 2(\omega t + \theta_v)] \rightarrow P_1(t)$$

$$+ \left[\frac{V_m \cdot I_m}{2} \sin(\theta_v - \theta_i) \cdot \sin 2(\omega t + \theta_v) \right] \rightarrow P_2(t)$$

$$* P_1(t) = \frac{V_m \cdot I_m}{2} \cos(\theta_v - \theta_i) [1 + \cos 2(\omega t + \theta_v)] > 0$$

+ positive avg power. > 0

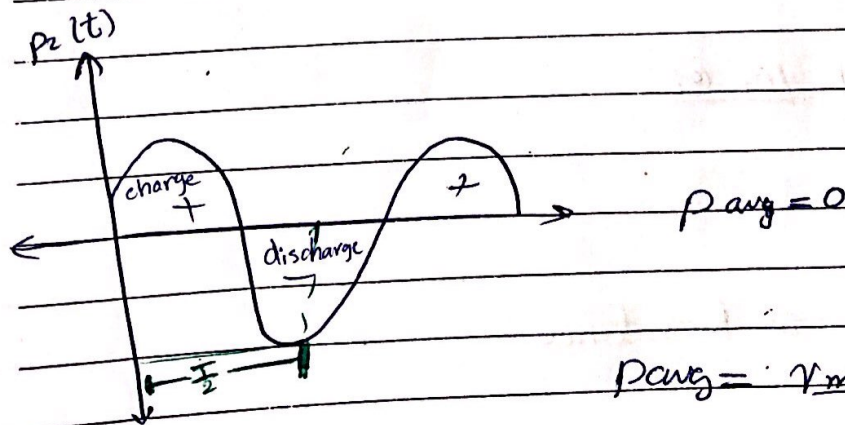
$$-90 < \theta_v - \theta_i < 90$$



$$P_{max} = \frac{V_m \cdot I_m}{2} \cos(\theta_v - \theta_i) [1+1] = \underline{\underline{2 p_{avg}}}$$

+ (positive) reactive power.

$$* P_2(t) = \frac{V_m \cdot I_m}{2} \sin(\theta_v - \theta_i) \cdot \sin 2(\omega t + \theta_v)$$



$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \rightarrow \text{real}$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \rightarrow \text{imag}$$

amplitude of reactive power.

$$P + j \text{img}$$

$$\frac{V_m I_m}{2} \left[\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i) \right]$$

\downarrow
 complex power (VA)

$$S = P + jQ$$

\downarrow
 amp of reactive power (VAR)

$$S = \frac{V_m I_m}{2} \angle (\theta_v - \theta_i)$$

$$S = \frac{\vec{V}}{2} \cdot \vec{I}^*$$

→ complex power

* lecture (7): - $S = \frac{V_m I_m}{2} \angle (\theta_v - \theta_i)$ (VAR) voltage Ampere reactive.

$$S = \frac{1}{2} \vec{V} \cdot \vec{I}^*$$

$$Z = \frac{V \angle \theta_v}{I \angle \theta_i} = \frac{|V|}{|I|} \angle (\theta_v - \theta_i)$$

① $\theta_z = 0$.

$\theta_z = 0, \theta_s = 0 \rightarrow$ Resistance.

$Q = 0$

② $\theta_z > 0$.

$\theta_s > 0, Q > 0 \rightarrow$ Inductive load.

~~$\theta_z > 0, \theta_s > 0$~~

③ $\theta_z < 0 \rightarrow$ capacitive load.

$Q < 0$

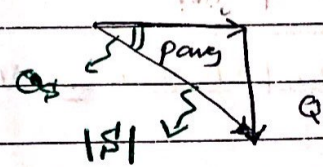
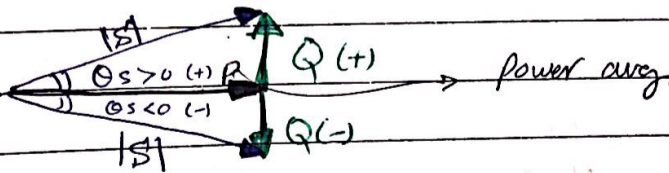
$P_{avg} = 0$ $\begin{cases} \text{bc} \rightarrow \text{Pure cap } Q < 0. \\ \text{or} \\ \rightarrow \text{Pure indu. } Q > 0. \end{cases}$

* $P_{avg} = \text{Watt}$

* $Q = \text{VAR}$

* $S = \text{VA}$

* Complex power [Power Triangle] :-

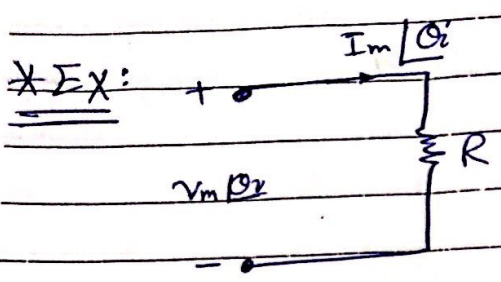


$\theta_v - \theta_i = \theta_s$

$|S| = \sqrt{P^2 + Q^2}$

$\theta_s = \tan^{-1} \left(\frac{|Q|}{|P|} \right)$

وہاں θ_s C_{si}
 اور C_{li}

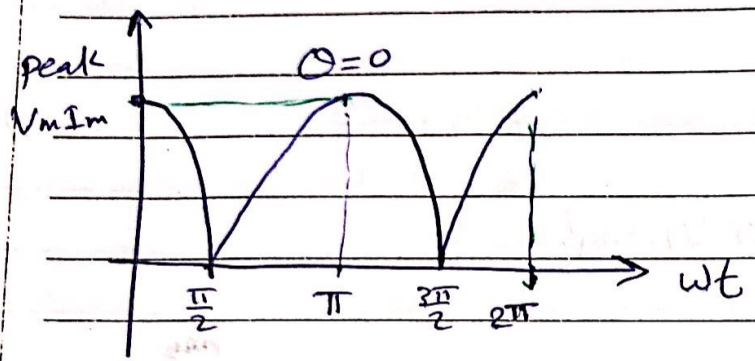


plot $p(t)$ by R.

$1 \cdot \cos(\theta_v - \theta_i) = 1$. pure Resistance.

$p(t) = \frac{1}{2} V_m I_m + \frac{1}{2} V_m I_m \cos(2\omega t + 2\theta)$ $(\theta_v = \theta_i = \theta)$

$= \frac{1}{2} V_m I_m \left[\underbrace{1 + \cos 2(\omega t + \theta)}_{>0} \right]$



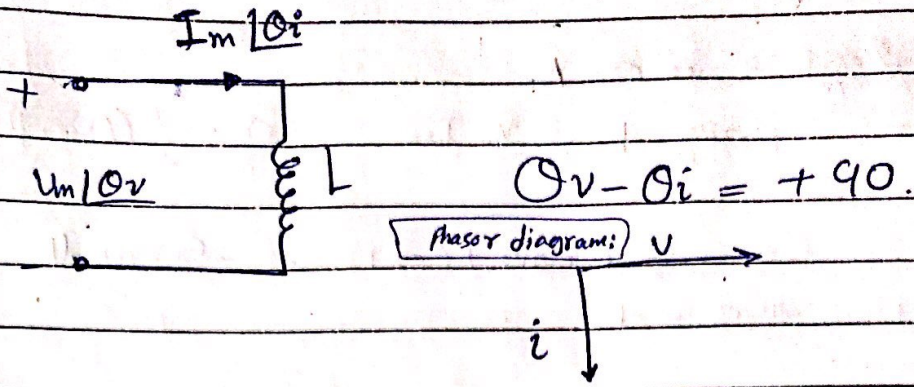
peak (max) = $V_m I_m$.

Peak to peak = $2|S| = V_m I_m$

$S = \frac{V_m I_m}{2}$
 Peak
 Peak-peak

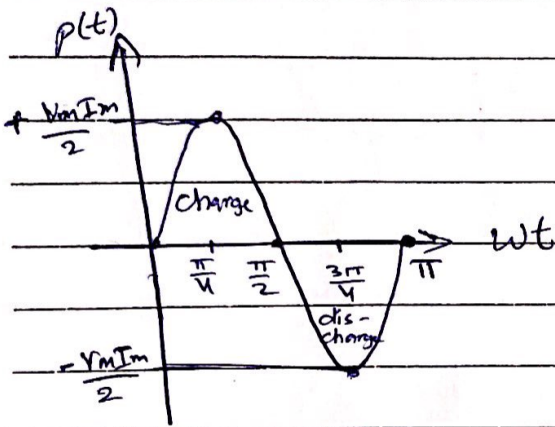
$p_{avg} = \frac{V_m I_m}{2}$

* Ex:-



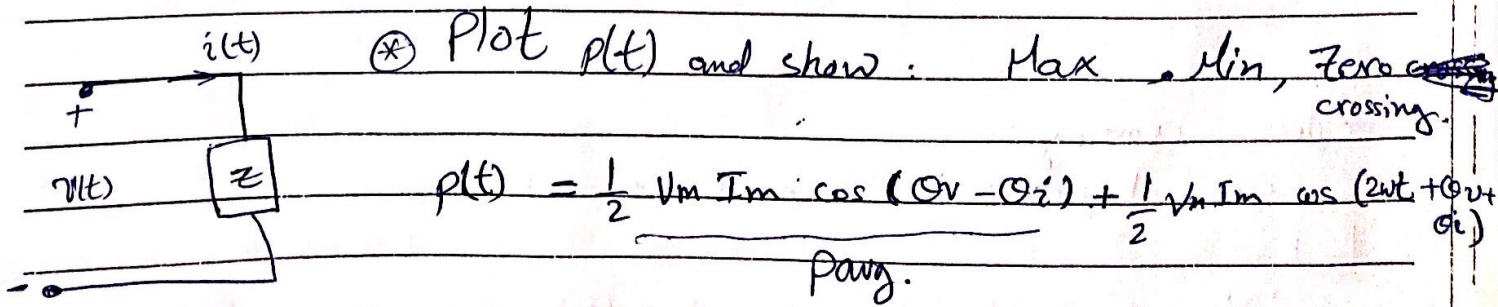
$$p(t) = 0 + \frac{1}{2} V_m I_m \cos(2\omega t - 90^\circ)$$

Assume $\theta_v = 0 \rightarrow \theta_i = -90$ avg power = 0
 peak - Peak = $V_m I_m$



* lecture (7):-

* Ex:- $v(t) = 100\sqrt{2} \cos(\omega t)$
 $i(t) = \sqrt{2} \cos(\omega t - 60)$



$$P_{avg} = \frac{1}{2} (100\sqrt{2}) (\sqrt{2}) \cos(0 - 60) \Rightarrow 100 \cos 60 = \boxed{50 \text{ watt}}$$

~~bit~~ $\cos \theta = 1$

$$P_{\text{Max}} := p_{\text{avg}} + \frac{1}{2} V_m I_m = 50 + \frac{1}{2} (100\sqrt{2})(\sqrt{2}) = 150 \text{ watt}$$

$$P_{\text{Min}} := 50 - \frac{1}{2} (100\sqrt{2})(\sqrt{2}) = -50 \text{ watt}$$

\hookrightarrow bit $\cos \theta = -1$

① $p(t) = p_{\text{avg}}$ $\omega t = ??$

$$\cos(2\omega t - 60) = 0$$

$$2\omega t - 60 = 90^\circ$$

$$\omega t = \frac{90 + 60}{2} = 75^\circ$$

② $p(t) = 50$ $\omega t = ??$

$$\cos(2\omega t - 60) = 1$$

$$2\omega t - 60 = 0$$

$$\omega t = \frac{60}{2} = 30^\circ$$

③ $p(\omega t = ??) = -50$

$$\cos(2\omega t - 60) = -1$$

$$2\omega t - 60 = 180$$

$$\omega t = \frac{180 + 60}{2} = \frac{240}{2} = 120^\circ$$

Zero-crossing :-

$$-50 = \frac{1}{2} (100\sqrt{2})(\sqrt{2}) \cos(2\omega t - 60)$$

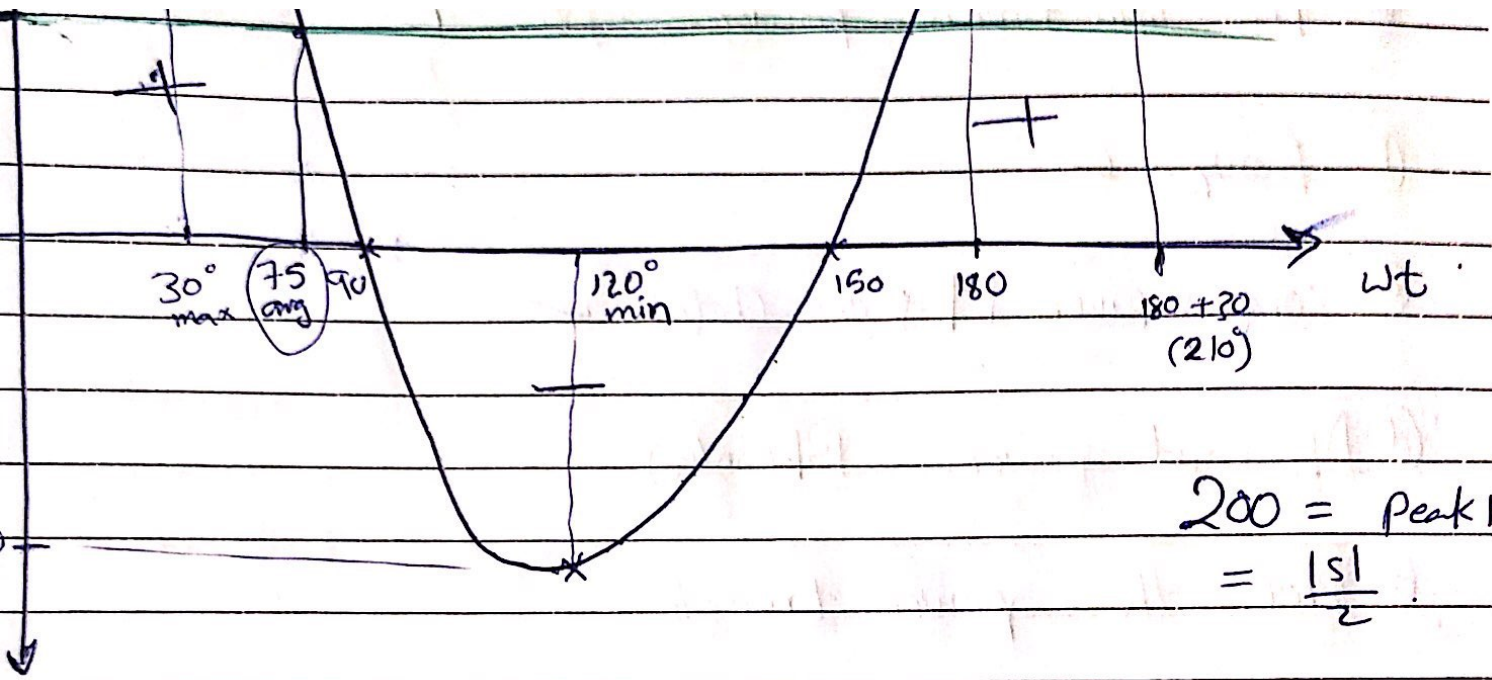
$$\frac{-50}{100} = -0.5 = \cos(2\omega t - 60)$$

$$120^\circ = 2\omega t - 60$$

$$\omega t = \frac{120 + 60}{2} = 90^\circ$$

$$240 = 2\omega t - 60$$

$$\omega t = \frac{300}{2} = 150^\circ$$



$$200 = \text{Peak to Peak} \\ = \frac{|s|}{2}$$

$$v(t) = 100\sqrt{2} \cos(\omega t)$$

$$i(t) = \sqrt{2} \cos(\omega t - 60^\circ)$$

→ draw it.

need:- we get from $p(t)$
 $p(t) \Rightarrow \text{Peak - Peak} = 2|s|$

$$|s| = \frac{P-P}{2}$$

$$\text{avg} \Rightarrow P_{\text{max}} = P_{\text{avg}} + |s|$$

$$i \Rightarrow P_{\text{max}} \Rightarrow p(t) = P_{\text{avg}} + |s| \cos(2\omega t + \theta_v + \theta_i)$$

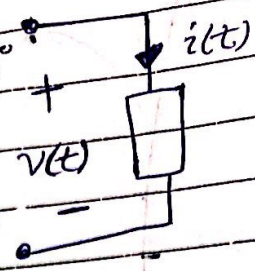
$$v(t) \Rightarrow \theta_v$$

$$\textcircled{7} Z = \frac{v(\theta_v)}{I(\theta_i)}$$

$$Z = |s| \angle \theta_v - \theta_i = P_{\text{avg}} + j0$$

$$\textcircled{8} I = \frac{2 \cdot P_{\text{avg}}}{v_{\text{rms}} \cos(\theta_v - \theta_i)}$$

* Ex: $v(t) = 80 \cos(10t + 20^\circ)$ $80 \angle 20^\circ$
 $i(t) = 15 \cos(10t - 30^\circ)$ $15 \angle -30^\circ$



Find:

- (a) The instantaneous power: $\rightarrow p(t) :- \checkmark$
- (b) $P_{avg} \checkmark$
- (c) complex power. $|S| \& \theta_s$ (both) \checkmark
- (d) Apperant power. $|S|$ (only)
- (e) plot the power triangle.
- (f) plot the phasor diagram.

Soln:

a) $p(t) = \frac{1}{2} \cdot V_m \cdot I_m \cos(\theta_v - \theta_i) + \frac{1}{2} \cdot V_m \cdot I_m \cos(2\omega t + \theta_v + \theta_i) =$

$p(t) = \frac{1}{2} \times (80)(15) \cos(20 - (-30)) + \frac{1}{2} (80)(15) \cos(2 \times 10t + 20 - 30)$

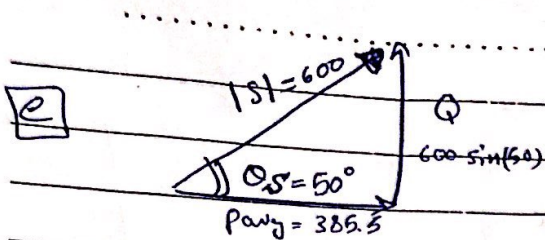
$p(t) = 385.5 + 600 \cos(20t - 10^\circ)$ watts.

b) $P_{avg} = 385.5$ watt.

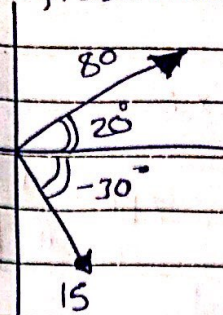
c) $S = \frac{1}{2} \cdot \vec{V} \cdot \vec{I}^*$
 because $\theta_s = + \rightarrow$ so lags $\rightarrow i$ lags v so inductive load in the ckt
 $= \frac{1}{2} \times (80 \angle 20^\circ) \times (15 \angle 30^\circ) = S = 600 \angle 50^\circ \text{ VA}$

$S = 600 \cos(50) + j 600 \sin(50)$
 (VA) $P_{avg} = 385.5$ watt Q (VAR)

d) 600 VA

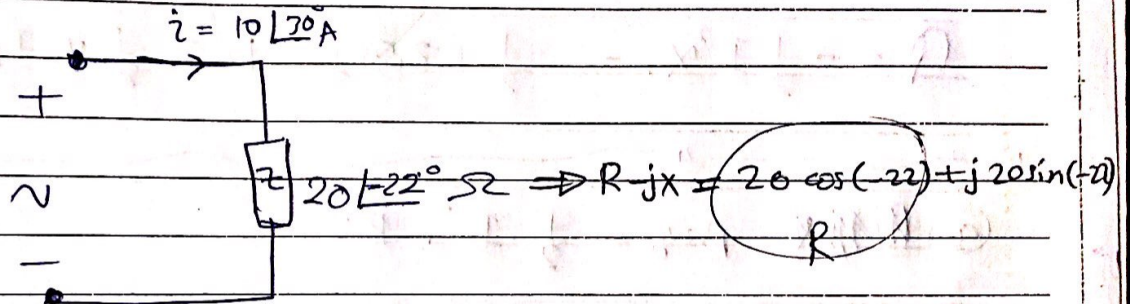


phasor diagram:-



i lags V by (50°)

* Ex:-



$$P_{avg} = ?? = \frac{1}{2} V_m I_m \cos(\theta_V - \theta_i)$$

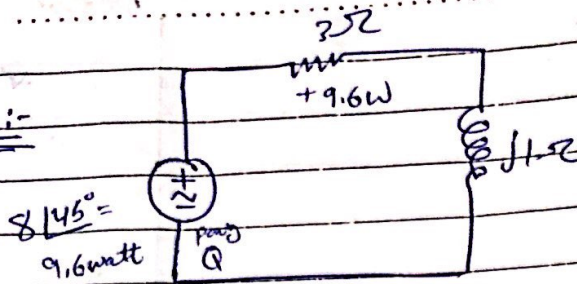
$$V = I Z \rightarrow (10 / 30^\circ)(20 / -22^\circ) = \boxed{V = 200 / 8^\circ}$$

$$P_{avg} = \frac{1}{2} (200)(10) \cos(-22) = 927 \text{ watt}$$

$$\textcircled{*} P_{avg} = \frac{1}{2} \cdot I^2 \cdot R \rightarrow P_{avg} = \frac{1}{2} (10)^2 \cdot 20 \cos(-22) \text{ watt}$$

$$Q = \frac{1}{2} I^2 X = \frac{1}{2} (10)^2 \cdot 20 \cos(-22) \text{ VAR}$$

Ex:-



Find:-

(a) avg power absorbed by $3\Omega, j1\Omega$;

(b) " " supplied by the source;

(c) Find reactive power (Q) supplied by the source:-

$$Q = -\frac{1}{2} I^2 X = -\frac{1}{2} (2.53)^2 \times 1 = \frac{1}{2} V_1 \cdot I_1 \cdot \sin(\theta_v - \theta_i)$$

" " " " " "

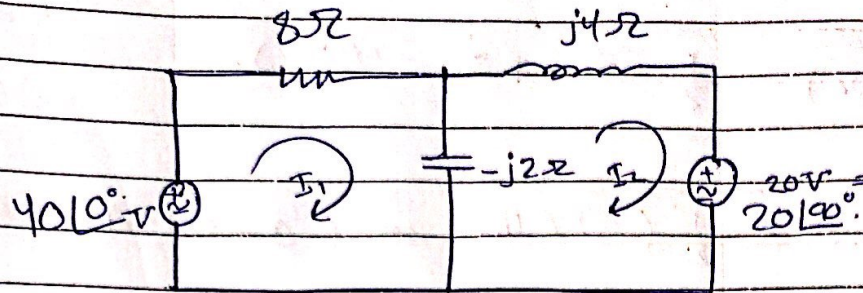
$$(a) \text{ ~~avg~~ } P_{avg} = \frac{1}{2} \cdot I^2 \cdot R(3)$$

$$I = \frac{8/45^\circ}{3+j} = 2.53 \angle 26.56^\circ$$

$$P = \frac{1}{2} (2.53)^2 \times 3 = 9.6W. \rightarrow \text{same source supplied}$$

$$P(j1\Omega) = 0.$$

* Ex:



* Calculate the avg power absorbed / generated by each element:-

$$P_{avg}(8\Omega) = \frac{1}{2} I_1^2 \cdot 8 = \text{---}$$

$$P_{avg}(j4\Omega) = \frac{-1}{2} V_1 I_1 \cos(0^\circ - 0^\circ)$$

$$P_{avg}(20V) = \frac{+1}{2} V \cdot I_2 \cos(90^\circ - 0^\circ)$$

||-----||

$$40\angle 0^\circ = (8 - j2) I_1 - (-j2) I_2$$

$$-j20 = -(-j2) I_1 + (-j2 + j4) I_2$$

$$40 = (8 - j2) I_1 + j2 I_2$$

$$(-j20 = +j2 I_1 + j2 I_2) \times -1$$

$$I_1 = \frac{40 + j20}{8 - j4} = 5\angle 53.14^\circ$$

$$I_2 = 13.6\angle 17.11^\circ \rightarrow$$

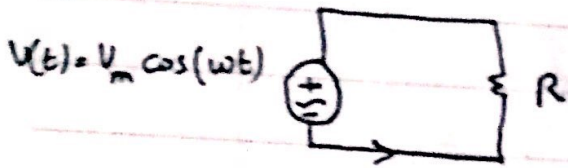
~~Ans~~

$$P(8\Omega) = \frac{1}{2} (5)^2 \times 8 = +100 \text{ W}$$

$$P(40\Omega) = -\frac{1}{2} (5)(40) \cos(0 - 53.14) = -60 \text{ W}$$

$$P(j20) = +\frac{1}{2} (13.6)(20) \cos(90 + 17.11) = -40 \text{ W}$$

effective Value

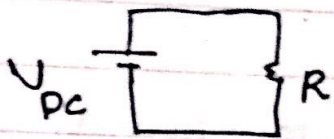


$$P_{avg} = \frac{1}{2} I_m^2 R$$

$i(t) = I_m \cos(\omega t)$

$$P_{avg} = \frac{1}{2} \frac{V_m^2}{R}$$

$\iff P_{avg}$



$$P = \frac{V_{DC}^2}{R}$$

$$\frac{1}{2} \frac{V_m^2}{R} = \frac{V_{DC}^2}{R}$$

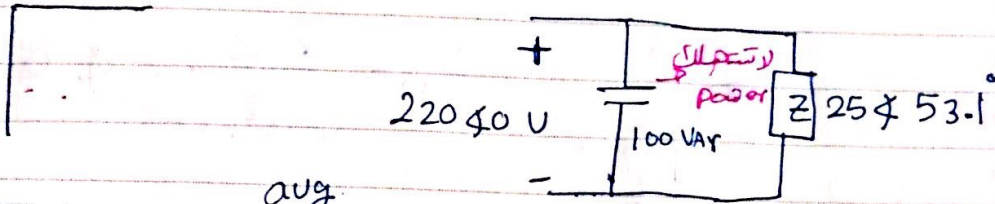
$$V_{DC} = \frac{V_m}{\sqrt{2}}$$

$$I_{effec}^2 R = P_{avg} = \frac{1}{T} \int_0^T I^2(t) R dt$$

$$I_{effec} = \sqrt{\frac{1}{T} \int_0^T I^2(t) dt}$$

Root mean Square

$$P = \frac{1}{2} \frac{V_m^2}{R} = \frac{1}{2} \frac{(V_{rms} \sqrt{2})^2}{R}$$



Find the net ^{avg.} power supplied by the source.

$$I = \frac{220 \angle 0}{25 \angle 53.1} = 8.8 \angle -53.1^\circ$$

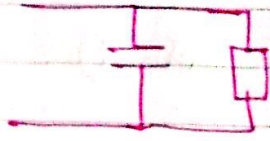
$$Z = \underbrace{25 \cos 53.1}_R + \underbrace{25j \sin 53.1}_{XL}$$

$$P = (8.8)^2 * 25 \cos 53.1 \text{ watt.}$$

$$S = P + jQ$$

$$Q_L = I^2 \left(\frac{WL}{XL} \right)$$

$$= (8.8)^2 (25 \sin 53.1)$$



* parallel
 $P = \frac{V^2}{R} \rightarrow P = I^2 R$

$$P = I^2 R$$

$$\text{VAR} \rightarrow F$$

$$P_{old} = P_{new} = 10 \text{ MW}$$

$$Q_{old} = P_{old} \tan \theta_{old} = 10 \text{ M} \tan (53.1) = +13.3 \text{ VAR}$$

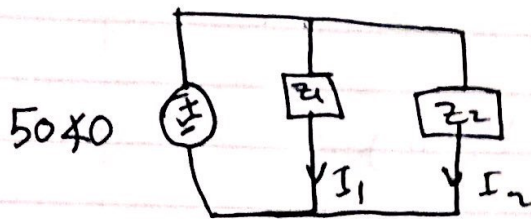
$$Q_{new} = P \tan \theta_{new} = 10 \text{ MW} \tan (18.12) \\ = 3.29 \text{ MVAR}$$

$$Q_C = Q_{old} - Q_{new} \\ = 13.3 - 3.3 \cong 10 \text{ MVAR}$$

$$Q = \frac{V^2}{X} = \frac{V^2}{\frac{1}{\omega C}} \rightarrow Q = \omega C V^2$$

$$10 \text{ M} = (2\pi \times 50) C (\cancel{33} \text{ k})^2$$

$$\theta_s = \theta_z$$



$$Z_1 = 50 \angle 45^\circ$$

$$Z_2 = 25 \angle 30^\circ$$

PF :- $\cos \theta_{z_1}$
or $\cos \theta_{z_2}$ ind lag or lead

① Find I_1, I_2, I

$$I_1 = \frac{50 \angle 0}{50 \angle 45} = 1 \angle -45^\circ$$

$$\begin{aligned} \text{PF}_{z_1} &= \cos \theta_{z_1} \\ &= \cos(45) \\ &= 0.707 \text{ lag.} \end{aligned}$$

$$I_2 = \frac{50 \angle 0}{25 \angle 30} = 2 \angle -30^\circ$$

$$I = I_1 + I_2 =$$

② S_{z_1}, S_{z_2}

$$S_{z_1} = V_{\text{rms}} I_{\text{rms}}^* = 50 \angle 0 (1 \angle +45) = 50 \angle 45 \text{ V.A}$$

$$S_{z_2} = V_{\text{rms}} I_{\text{rms}}^* = 50 \angle 0 (2 \angle 30) = 100 \angle 30 \text{ V.A}$$

③ S_{source}
 $S_1 + S_2$

④ PF source

$$\cos(0) = 1$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

Sinusoidal $V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$

$$S = \frac{1}{\sqrt{2}} \frac{V_m I_m^*}{\sqrt{2}} = V_{\text{rms}} I_{\text{rms}}^*$$

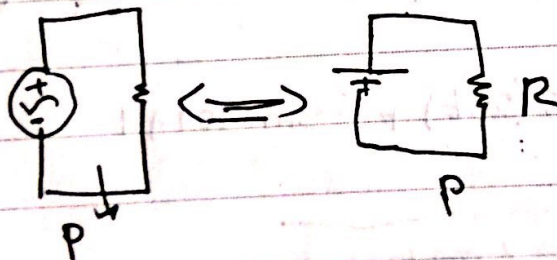
$$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

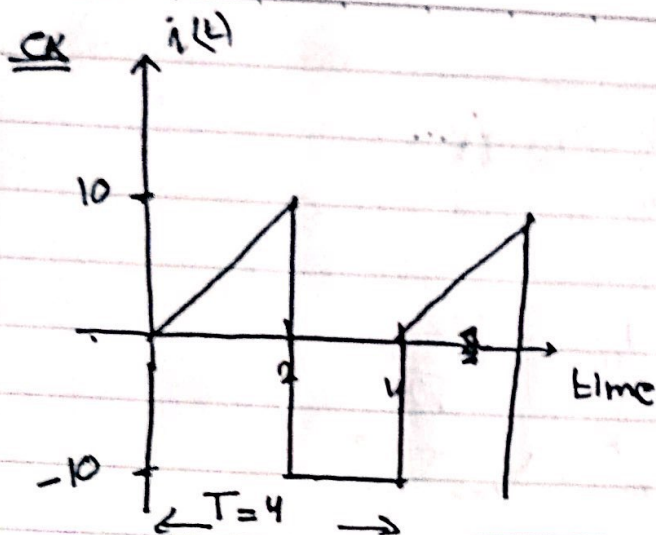
* With different frequency:

$$v(t) = V_1 \cos \omega_1 t + V_2 \cos \omega_2 t + \dots$$

$$V_{\text{rms}} = \sqrt{\left(\frac{V_1}{\sqrt{2}}\right)^2 + \left(\frac{V_2}{\sqrt{2}}\right)^2 + \dots}$$

$$V_{\text{rms}} = V_{\text{DC}}$$





$$\textcircled{1} I_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt} \quad i(t) = \begin{cases} \frac{10}{2}t, & 0 \leq t < 2 \\ -10, & 2 \leq t < 4 \end{cases}$$

$$= \sqrt{\frac{1}{4} \left[\int_0^2 \left(\frac{10t}{2}\right)^2 dt + \int_2^4 (-10)^2 dt \right]}$$

$$= \sqrt{\frac{1}{4} \left[\frac{25}{3}(2)^3 + 200 \right]} = 8.65 \text{ Amp.}$$

Ex $V(t) = 6 \cos(25t) + 5 \sin(30t) + 4$

$V_{\text{rms}}?$

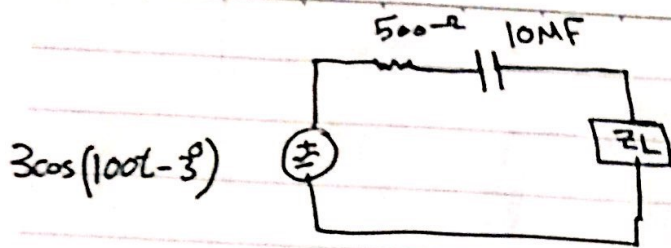
$$V_{\text{rms}} = \sqrt{\left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2 + (4)^2} = 6.82 V_{\text{rms}}$$

Ex $V(t) = 6 \cos 25t + 4 \sin(25t + 30^\circ)$, $V_{\text{rms}}?$

$$6 \angle 0^\circ + 4 \angle -60^\circ$$

$$= 8.717 \angle -23^\circ$$

$$V_{\text{rms}} = \frac{8.717}{\sqrt{2}} = 6.16 V_{\text{rms}}$$



What is the value of the inductive components of Z_L to absorb max power

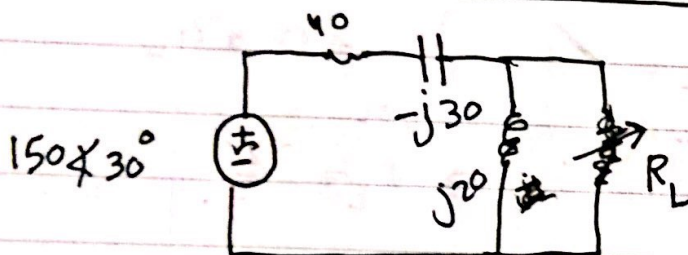
$$Z_L = Z_{Th}^* \rightarrow Z_{Th} = 500 - j \frac{1}{100 (10 \times 10^{-6})} = 500 - j1000 \Omega$$

$$Z_L = 500 + j1000 \Omega$$

X_L

$$X_L = jL \rightarrow L = \frac{1000}{100} = 10 \text{ mH}$$

$$P_{max} = \frac{1}{8} \frac{(3)^2}{500} = \frac{9}{4000} \text{ watt}$$



① Find R_L that will absorb the maximum power from the source

② Find P_{max}

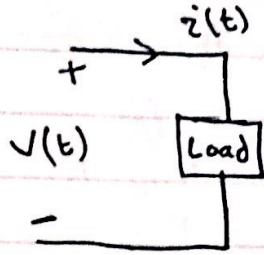
$$R_L = |Z_{Th}|$$

$$(40 - 30j) \parallel 20j$$

$$\frac{Z_{Th}}{Th} = \frac{(40 - 30j)(20j)}{40 - j30 + 20j} = 9.412 + 22.35j \Omega$$

$$R_L = |Z_{Th}| = \sqrt{(9.412)^2 + (22.35)^2} = 24.25 \Omega$$

Ex



$$v(t) = 110 \cos(\omega t + 65^\circ)$$

$$i(t) = 15 \sin(\omega t - 20^\circ)$$

- Ⓐ $P(t)$ Ⓑ P_{avg} Ⓒ reactive power
Ⓓ PF.

$$i(t) = 15 \cos(\omega t - 110^\circ)$$

Ⓑ + Ⓐ

$$P(t) = \frac{1}{2} (110) (15) \cos(65^\circ - 110^\circ)$$

$$+ \frac{1}{2} (110) (15) \cos(2\omega t + 65^\circ - 110^\circ)$$

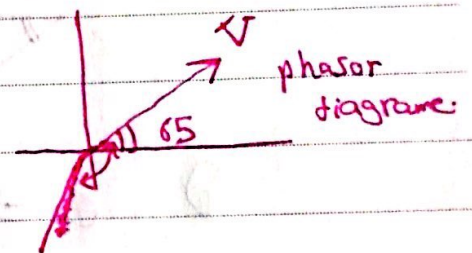
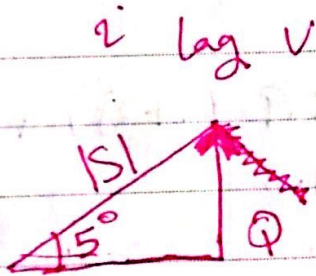
P_{avg}

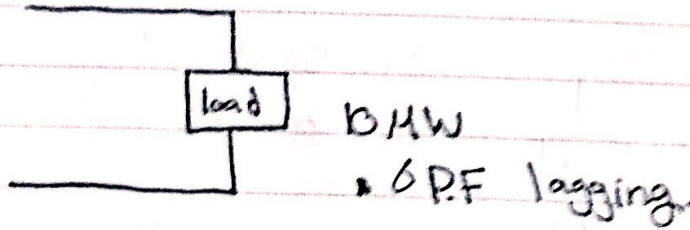
↳ reactive.

as amplitude.

Ⓒ $\frac{1}{2} (110) (15) \sin 175 = +71.9 \text{ VAR (absorb)}$
 generate power = -821.9 watt

Ⓓ





(A) power, Q absorbed by the load

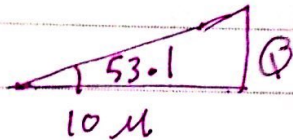
$$P = 10 \text{ MW}$$

$$\text{Lagging} \Rightarrow \cos \theta_s = 0.6$$

$$\Rightarrow \cos^{-1} 0.6 = \theta_s$$

$$\theta_s = 53.1$$

(B) Draw power triangle



$$\tan \theta_s = \frac{Q}{P} \Rightarrow Q = P \tan \theta_s$$

$$13.33 \text{ MVar} = 10 \text{ M} \tan(53.1)$$

Power factor

$P(t) + P_i(t)$ active power
~~or~~ reactive power $P_2(t)$.

$$\underline{S} = \underline{P}_{avg} + jQ$$

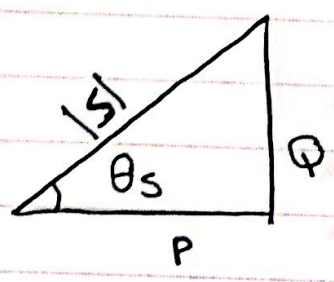
complex power active power Q

$|S| \rightarrow$ apparent power

$$P.F = \frac{P_{avg}}{|S|} = \frac{P_{avg}}{\sqrt{(P_{avg})^2 + Q^2}}$$

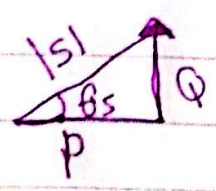
(The Best) $PF = 1 \rightarrow Q = 0 \rightarrow$ Unity PF
(pure resistive).
(مقاوم تكون فيه C و L من غير تأثير)

$$\frac{P_{avg}}{\sqrt{(P_{avg})^2 + Q^2}} = \frac{\cos(\theta_s)}{\cos(\theta_v - \theta_i)}$$

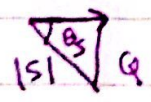


$$0 \leq PF \leq 1$$

i lag $V \Rightarrow$ ind



i lead $V \Rightarrow$ capa



unity \Rightarrow resistor (in phase) $\rightarrow P$

power factor correction

$$PF = \frac{P}{|S|}$$

PF = unity (optimal)

- Max power transfer → Min losses
Pure resistance

Inductive load

$$PF < 1$$

$$PF_{min} = .85$$

$$.88$$

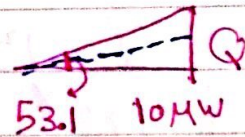
ex



6 pF lagging → PF_{new} = .95 lagging

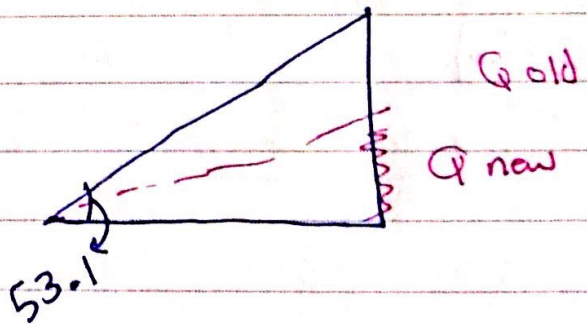
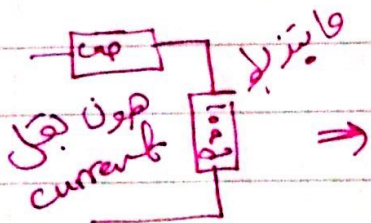
$$\theta_s = + \cos^{-1} (.6) = 53.1^\circ$$

$$+ \cos^{-1} (.95) = 18.7^\circ$$



* بتغير P نسبة adds capacitor

$$Q_{old} + \frac{Q_c}{neg} = Q_{new}$$



$$I_{new} = \frac{V_s}{|Z|} = \frac{V_s}{\sqrt{R^2 + (WL - \frac{1}{Wc})^2}}$$

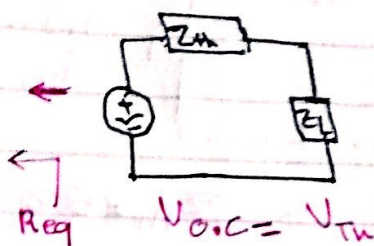
Z ↓ I ↑ P ↑

So, منه
فباله

Thevenin ckt

↓

Kill all sources:

V.S. \Rightarrow S.CI.S. \Rightarrow O.C

Max power transfer

$$P_{\text{max}} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

avg

Load max

$$Z_L = R_L + jX_L$$

$$Z_{th} = R_{th} + jX_{th}$$

$$P = \frac{1}{2} I_m^2 R_L$$

$$I = \frac{V_{th} \angle \theta_v}{Z_{th} + Z_L}$$

$$\frac{dP}{dZ_L} = 0 \Rightarrow Z_L$$

$$\Rightarrow Z_L = Z_{th}^*$$

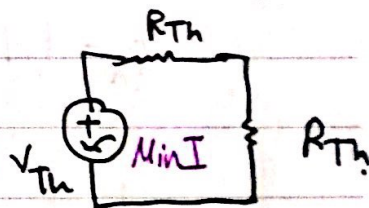
* If $Z_{th} = R_{th} + jX_{th}$

$$Z_L \neq Z_{th}^* \Rightarrow Z_L = Z_{th}^* = R_{th} - jX_{th}$$

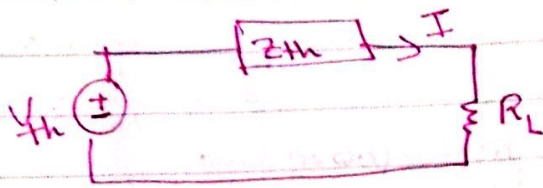
$$P_{\text{max}} = \frac{1}{2} \left(\frac{V_{th}}{2R_{th}} \right)^2 * R_{th}$$

$$= \frac{1}{2} \frac{V_{th}^2}{4R_{th}^2} * R_{th}$$

$$= \frac{1}{8} \frac{V_{th}^2}{R_{th}}$$

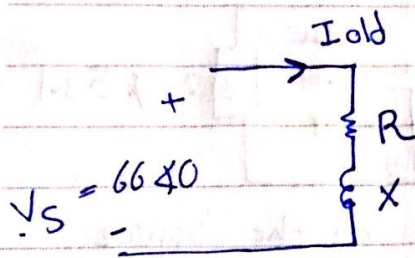


No.

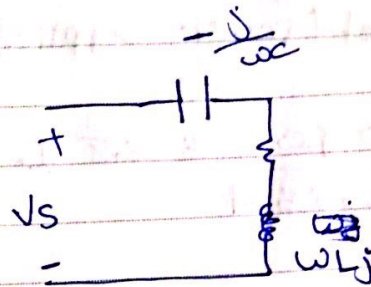


$$P = \frac{1}{2} \times I^2 \times R_L$$

$$= \frac{1}{2} \left(\frac{V_{th}}{Z_{th} + |Z_{th}|} \right)^2 \times |Z_{th}|$$



$$I_{old} = \frac{6640}{\sqrt{R^2 + X^2}} \angle 0 - 53.1^\circ$$



$$Z = R + j\omega L - \frac{j}{\omega C}$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$I = \frac{6640}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \rightarrow j52^\circ \quad I \uparrow \text{PT.}$$

Add capacitance in series to improve PF to unity
Find value of cap.

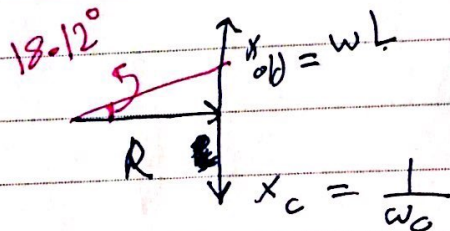
$$j\omega L = \frac{1}{j\omega C} \rightarrow \omega L = \frac{1}{\omega C}$$

$$C = \frac{1}{\omega^2 L}$$

$(2\pi \times 50)$

PF = .95 PF \rightarrow add series cap.

$$\theta_{S_{new}} = \theta_Z = 18.1^\circ$$



$R \rightarrow$ Constant

$$X_{new} = R \tan(18.12)$$

$$X_C = X_{old} - X_{new} =$$

$$X_C = \frac{1}{\omega C}$$