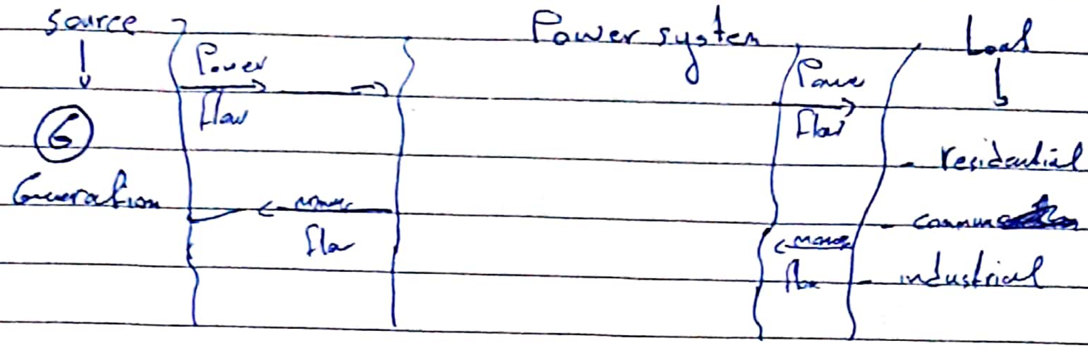
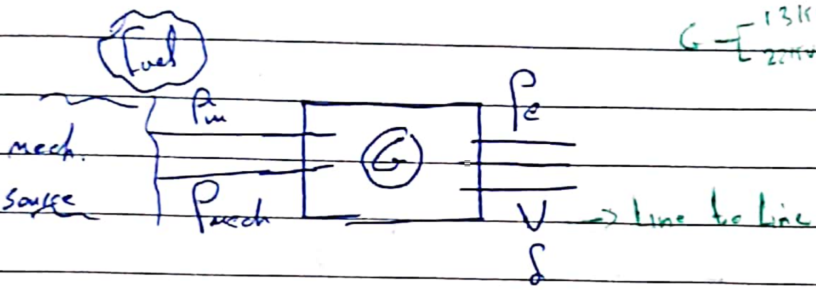
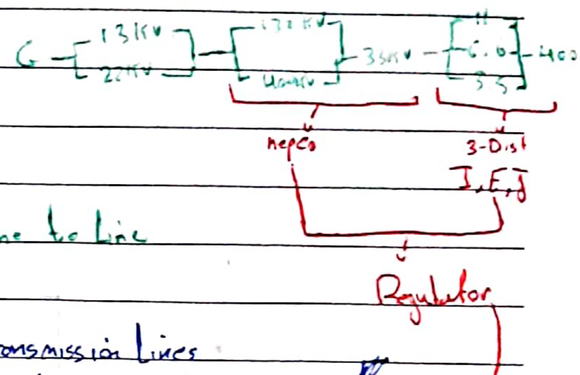
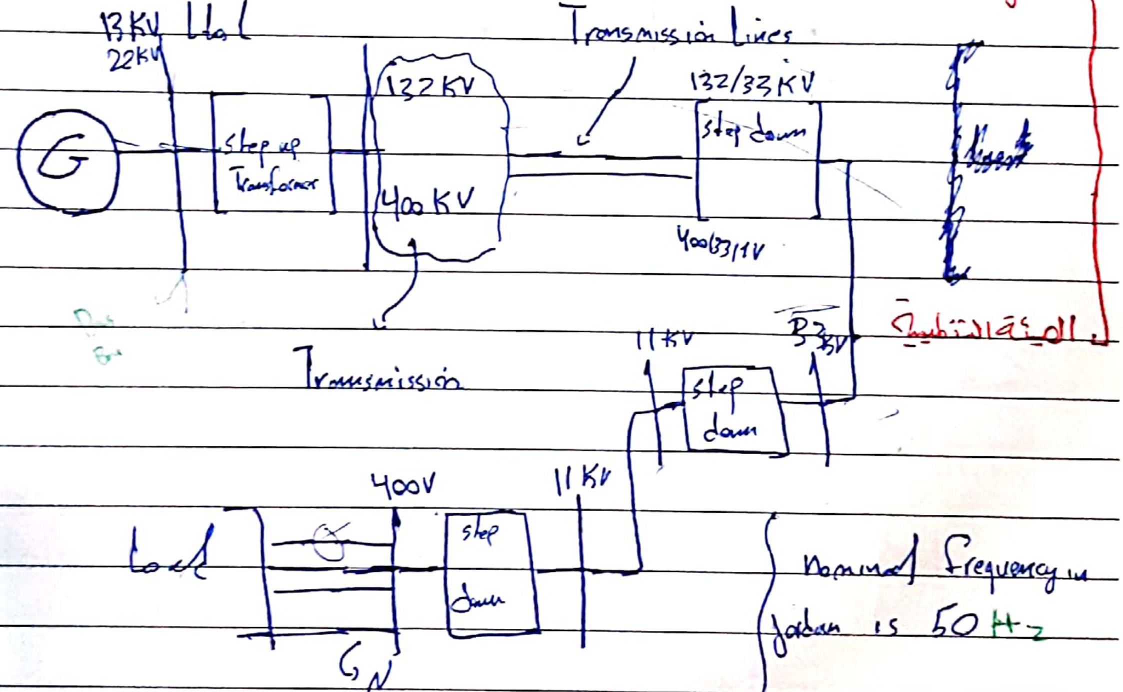
Power system analysis I

23/9/









nominal frequency in Jordan is 50 Hz

System op is controlled in Load – Generation Balance

in Jordan (nepco) is "Tr network operator" "The system operator"
control 132k, 400k

(1)

23-9-2019

Power 1

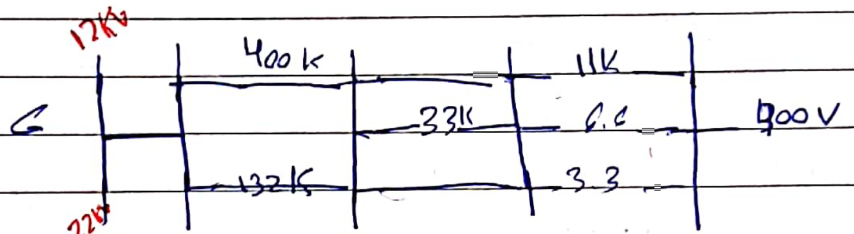
Distribution companies control (33k - (11k, 6.6, 3.3) - 400V)
(Retailers)

3 companies control it

Jepco → الوسط

IDeco → الشمال

EDco → الجنوب



PSA why? to keep light on.

Normal

emergency

Goals

energy

environment

energy Power

infrastructure

economy

investment

3E, 2I

(2)

Power 23 - 9 - 2019

Power system analysis I . why I ?

classification

1

static
- frequency const
- Phasor

Transient
- electromagnetic
- electromechanical

time scale of Power system analysis

Real time days weeks years

operation Planning → Planning

25/9 L2

structure of Power Systems

now → 1- Consumers
2- Prosumers

Transmission networks 132KV 400KV — transformer — 22KV up to → G

transformer

G — Distribution 33KV

new

→ tie two systems → saudia, Syria, egypt

→ Very large customer

→ Large customers

transformer

G — Distribution 11 KV

new

transformer

low voltage < 1K → comm, Resi, Small industries (3)

Five Apple

25/9

- Question → why $f = 50\,Hz$?

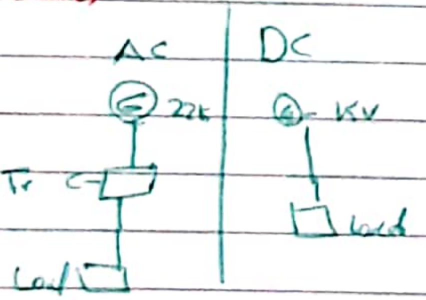
to lower ① Reactance for inductor → $2\pi f L$ → Generator and transformers Lines
↳ $f\uparrow$ → $X\uparrow$

↳ ② Reactance for capacitance → $\frac{1}{2\pi f C}$ → Lines
$f\downarrow$ $X\downarrow$
└ Leads to flicker in human eye

↳ ③ P_m losses = $P_h + P_e$
↳ frequency dependent

↳ using Power electronics we can control AC and change frequency to different levels

why such variety of Voltage Levels



why 3-phase dist?

→

why more than 1 Voltage Level?

why AC → We can control Voltage Level
Load → $P = \uparrow V I \downarrow$
↳ less power on lines
$P_{loss} = (I)^2 R$

25/9

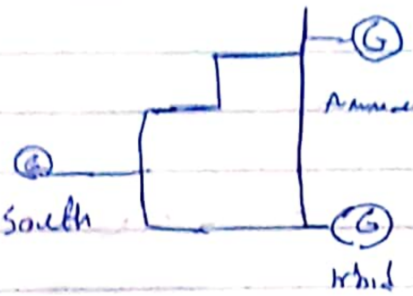
because of cost → higher voltage ↑ more cost
more physical constants

↳ more "capital cost"
more "operational cost"

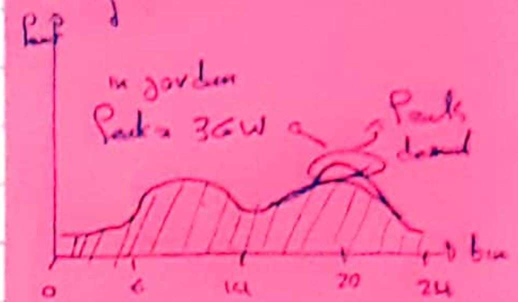
towers, lines, machine pay
↳ Generation, maintenance.

why connect with other types of systems as egypt and syria?
(why its when connected)

↳ to supply the demand in the network (that are unstable)



always design more than the load required → to ensure safety of system



↳ ISO independent system operator must ensure that (Load = Generation)

Single line diagram

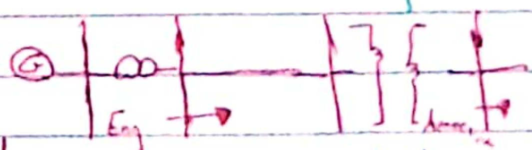
- Generators (G)
- transformers
- Lines "cable"
- Load
- motors (M)
- circuit breaker

P S

3 winding transformer

P S t

- Bus bar "node"



↳ this is called radial (5)

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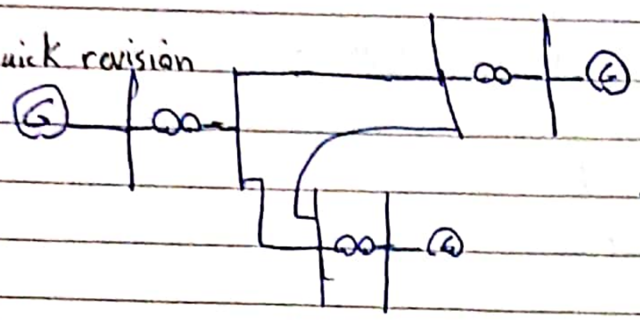
↳ quick revision



→ this is called mesh

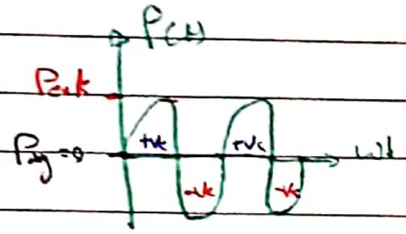
revision

$$Z = R + jX$$

$$Y = G + jB$$

$$Y = \frac{1}{Z}$$

Z = impedance
R = resistance
X = reactance
Y = addmitance
G = conductance
B = susceptance

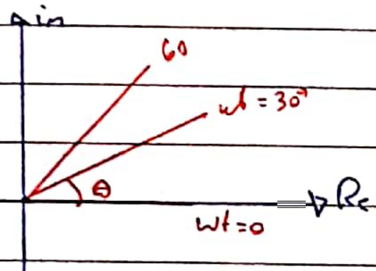


→ Pavg = 0 → load is not Ressistive

|Q| reactive = Peak Pw. ↳ could be
1- Capacitive
2- inductive

↳ Large electronic circuits

Motors, transformers & industrial usage



Phasor Diagram

L3

time —— phasor

$v(t) = V_m \cos(\omega t + \theta) \longrightarrow V_m \angle \theta$

because (cos) projection on x-axis

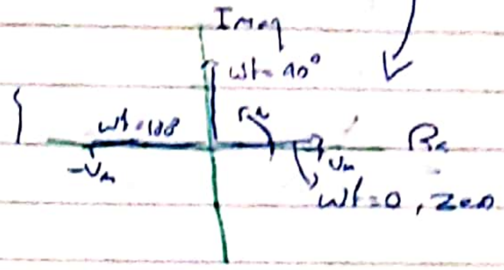
$v(t) = Re\{V_m e^{j\omega t} e^{j\theta}\}$

Phasor ال time

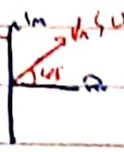
$\{V_m\}$ = x axis ال ظل

(= 0)

= 0



example

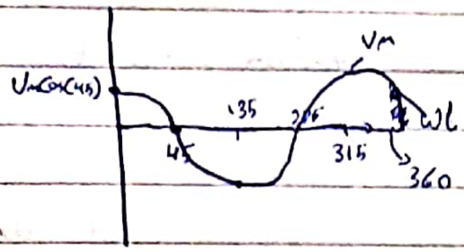


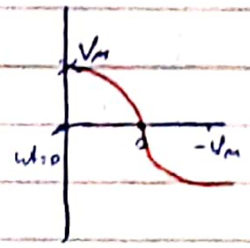
$V_m \angle 45$

$v(t) = V_m \cos(\omega t + 45°)$

$= Re\{V_m e^{j\omega t} e^{j45}\}$

Phasor

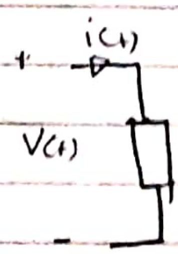




if we use sin

projection on y-axis

Power, reactive Power



$P(t) = i(t)\, v(t)$

$\because v(t) = V_m(\cos(\omega t))$

$i(t) = I_m(\cos(\omega t + \theta))$

$P(t) = V_m \cos(\omega t) \cdot I_m \cos(\omega t + \theta)$

$P(t) = \frac{1}{2} V_m I_m \cos\theta + \frac{1}{2} V_m I_m \cos(2\omega t + \theta)$

$P(t) = \frac{1}{2} V_m I_m \cos\theta + \frac{1}{2} V_m I_m \cos\theta \cos 2\omega t - \frac{1}{2} V_m I_m \sin\theta \sin 2\omega t$

30/9/2019

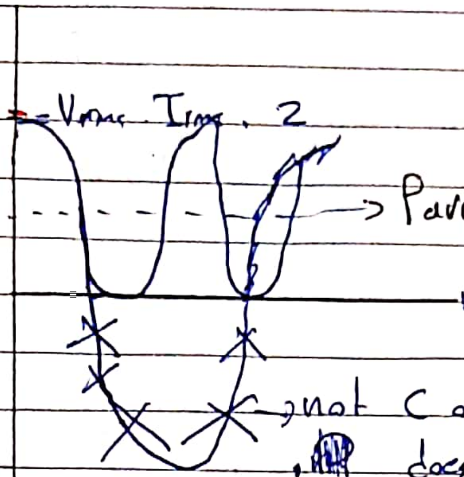
$$P(t) = \frac{1}{2} V_m I_m \cos\theta \,(1 + \cos(2wt)) - \frac{1}{2} V_m I_m \sin\theta \,\sin(2wt)$$

if θ = zero → Resistive $P(t) = \frac{1}{2} V_m I_m (1 + \cos 2wt$

consume

$V_m I_m = V_{rms} I_{rms} \cdot 2$

$\frac{1}{2} V_m I_m = V_{rms} I_{rms}$



→ Pavg

→ wt

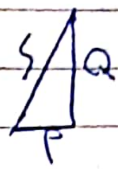
→ not C or L, doesn't consume

$S' = V_{rms} I_{rms}$

$V_{rms} = \frac{V_m}{\sqrt{2}}$

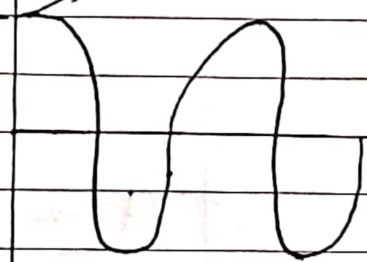
$I_{rms} = \frac{V_m}{\sqrt{2}}$

S' = apparent power



Q → reactive Power

$\frac{1}{2} V_m I_m$



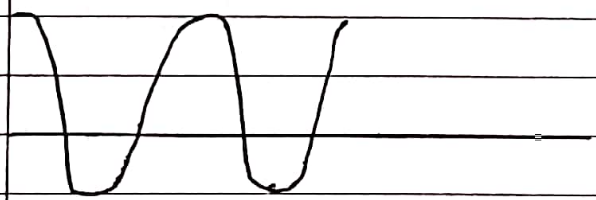
→ $P_{avg} = 0$

$\theta = -90$ ↳ inductor

$P(t) = \frac{1}{2} V_m I_m \sin(2wt)$

$v(t) = V_m \cos wt$

$i(t) = I_m \cos(wt + \theta_x)$



Peak$_{max}$ to Peak$_{min}$ ↳ $2S'$

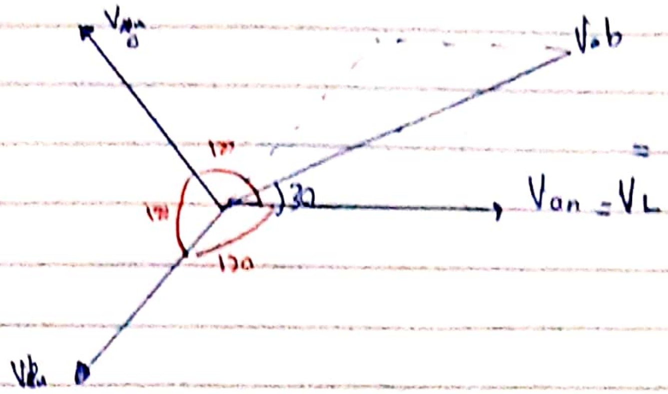
$$P(t) = \frac{1}{2} V_m I_m \cos\theta\,(1 + \cos(2wt)) - \frac{1}{2} V_m I_m \sin\theta \sin 2wt$$

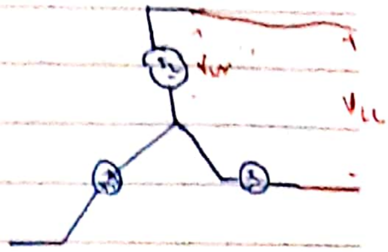
(8)

Power 1

30/9/2010

three phase system





$V_{LL} = V_{LN}\sqrt{3}\ \angle 30$

↳ only if Balance

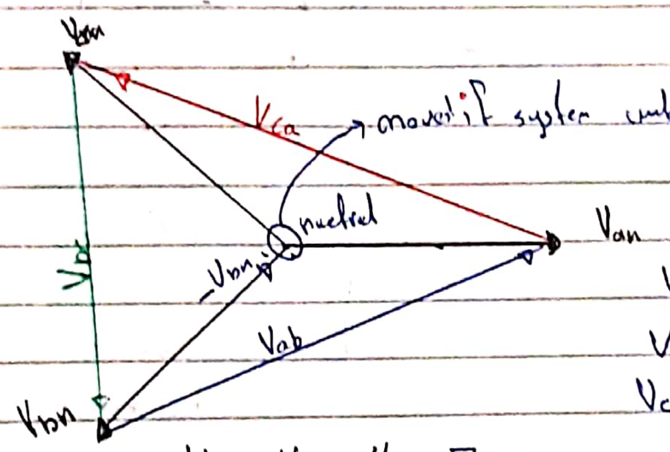
↳ if not

$V_{LL} = V_{L1N} - V_{L2N}$

$V_{ab} = V_{an} - V_{bn}$

↳ General case

$S_{3\phi} = 3S_{1\phi} = \sqrt{3}\,V_L I_L$



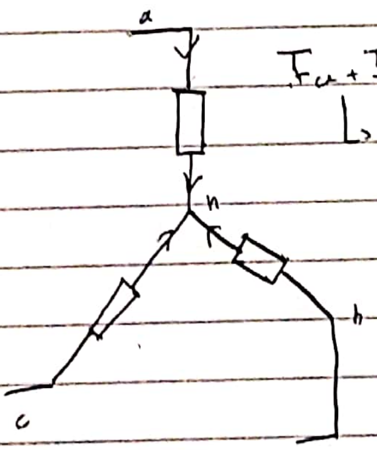
→ moves if system unbalanced → move away from Geometric Reference Point.

$V_{ab} = V_{an} - V_{bn}$

$V_{bc} = V_{bn} - V_{cn}$

$V_{ca} = V_{cn} - V_{an}$

$V_{ab} + V_{bc} + V_{ca} = \text{Zero}$

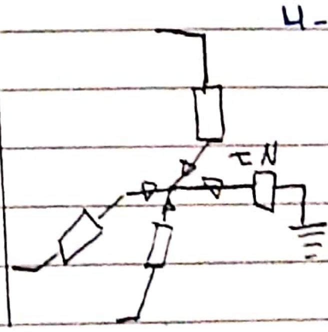


$I_a + I_b + I_c = 0$

↳ ungrounded neutral

3-wire system

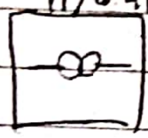
4-wire sys

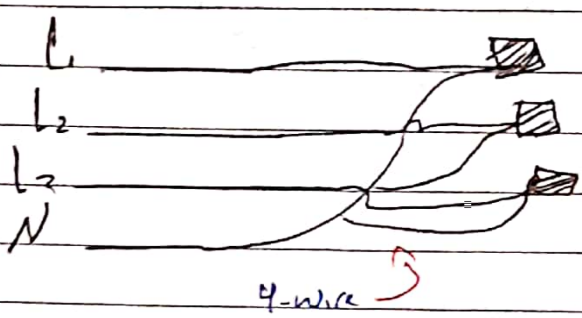


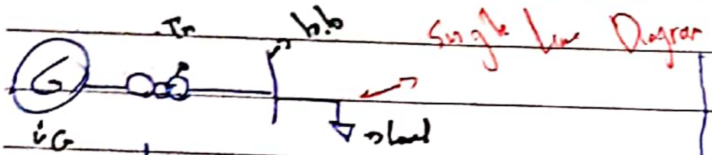
30/9/2019

"Low Voltage" LV

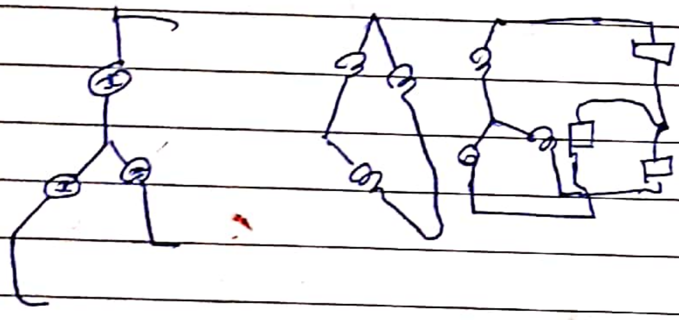
11/0.415 → compensate تعويض لطول الاسلاك

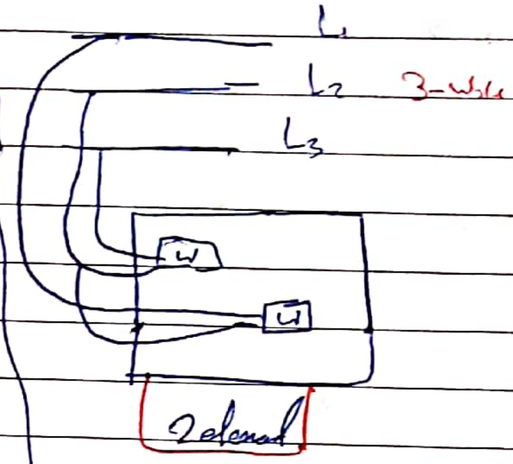






remember





why 0.415?

→ to compensate Power loss in wire to homes.

end of L3

2/10/2019

L4 → Dr didn't come.

L5

7-10-2019: Power 1

Per unit system

V, I, P — Power Flow

① Per Unit = $\frac{\text{Actual Value}}{\text{Base Value}}$

Transient — Fault analysis

Let V = 240, V_{Base} = 230

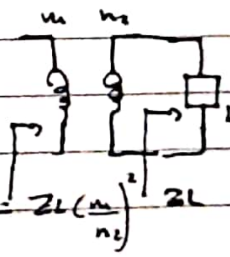
$V_{p.u} = \frac{240}{230}$ P.U = { Voltage rise

if V = 240∠30 $V_{p.u} = \frac{240∠30}{230}$ P.U. → always θ = 0

* models — Generator
→ Transformer
→ T.L
→ Loads

why Per unit ?

① Power system ⟹ multi-Voltage Levels
→ impedance of the transformer depends on the side which it is observed.
→ Equipment with same type have the same P.U impedance regardless with size



- 250 KVA, 11/0.415 KV
- 630 KVA 11/0.415 KV
- 1100 KVA 11/0.415 KV
- 100 KVA 11/0.415 KV

→ different size (rating) but same P.U impedance

$Z_t(PU) \approx 4\% - 6\%$

Power I

7-10-2019

L.L

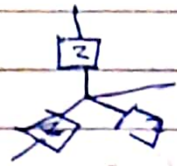
S V I, Z → chose 2.

3φ

$$I_{base} = \frac{S_{base}}{\sqrt{3}\, V_{L\text{-}L\,base}} \qquad S = \sqrt{3}\, V_L I_L$$

$$Z_{base} = \frac{V_{LN\,base}}{I_{base}} = \frac{(V_{L\text{-}L})/\sqrt{3}}{S_{base}/(\sqrt{3}\, V_{L\text{-}L\,base})} = \frac{(V_{L\text{-}L\,base})^2}{S_{base}}$$

assuming



$$S_{actual} = \sqrt{3}\, V_L I_L$$

$$S_{p.u} = \frac{S_{actual}}{S_{base}} = \frac{\cancel{\sqrt{3}}\, V_L I_L}{\cancel{\sqrt{3}}\, V_{L\,base}\, I_{L\,base}}$$

$$|S_{pu}| = |V_{PU}| \cdot |I_{PU}|$$

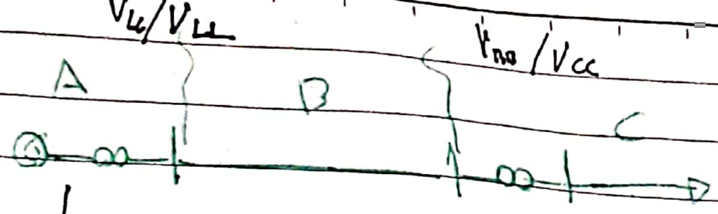
$$S_{p.u} = V_{pu} I_{pu}^*$$

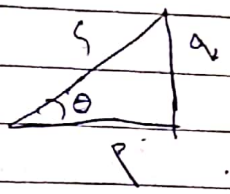
why PU? ⇒ $\cancel{\sqrt{3}}$; Does $V_{LL}(PU)$ differs from $V_{LN}(PU)$

$$V_{LL}(pu) = \frac{V_{LL\ actual}}{V_{LL,\ base}} = \frac{\cancel{\sqrt{3}}\, V_{LN}(actual)}{\cancel{\sqrt{3}}\, V_{LN}(base)} = V_{LN}(PU)$$

Five App

7 - 10 - 2019 Power ...



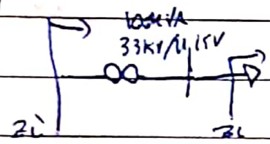


$V_{base\,A}$, $V_{base}\Big|_B * \left(\dfrac{V_{LL}}{V_{LL}}\right)$ Tr. ratio

assume

$$V_{base}\Big|_C = V_{base}\Big|_B * \left(\frac{V_{CC}}{V_{BB}}\right) \text{ Tr. ratio}$$

example ①



Z_L (phase) = 1 Ω

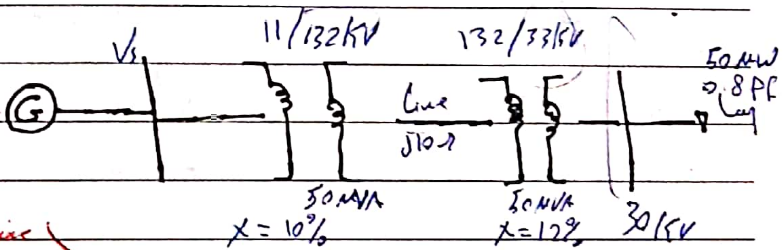
① $Z_L(pu) = \dfrac{1}{(11)^2/10} = \dfrac{10}{(11)^2}$ P.U

Same

② $Z_{L\,pu}' = \dfrac{Z_L'}{Z_{base}}$ ∴ $Z_{pu}' = 9 * \dfrac{1}{(33)^2/10} = \dfrac{90}{(33)^2}$

$Z_L' = 1 \times \left(\dfrac{33}{11}\right)^2 = 9\ \Omega$

example ②



① find Vs required to maintain Voltage level @ 30KV?
// use PU

$S_{base} = 100MVA$

$$Z_{Line}(P.U) = \frac{j10}{(132)^2/100} = j0.575$$

7 - 10 - 2019

Tr 1 $\Rightarrow X = 10\%$ [50 MVA , 11/132 KV]

$X = ??$ [100 MVA , 11/132 KV]

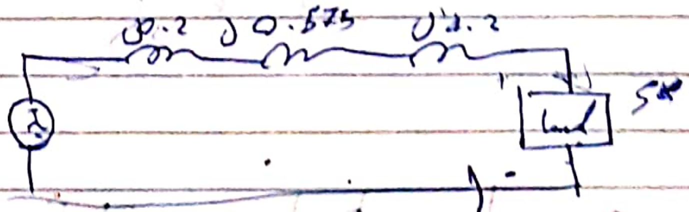
$$X_{P.U(new)} = X_{PU(old)} \times \left[\frac{S_{base\ new}}{S_{base\ old}}\right] \times \left(\frac{V_{base\ old}}{V_{base\ new}}\right)^2$$

$\therefore X_{pu(new)}\Big|_{T1} = 0.1 \left(\frac{100}{50}\right) \times \left(\frac{11}{11}\right)^2$ → if change you change it

$= 0.2$

$X(T_2) = 0.12 * (100/50) = 0.24$

S/L = $Z_{Load}\ (P.U) = \dfrac{Z_{actual}}{Z_{base}} = \dfrac{30^2/(50/0.8)}{(33)^2/100} =$



Load,

$S_{PU} = V_{PU} \cdot I_{PU}$

$\dfrac{50/0.8}{100} = \dfrac{30}{33} \cdot I_{PU}$

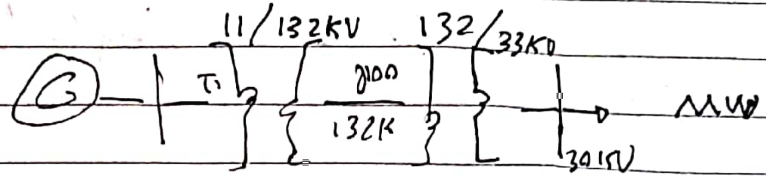
$\therefore I_{PU} = 0.687 \angle -36.8$

$V_s = 30/33 \angle 0 + I_{PU}(0.2 + 0.575 + 0.2)$

$= 15.84\ KV$

9-10-2019

L-6



$x = 0.1$ $x = 0.12$

50MVA   50MVA

set 11.5kV base • find V_s , base = 100 MVA

$T_1 \Rightarrow X_{pu(new)} = X_{pu(old)} \cdot \frac{S_{new}}{S_{old}} \cdot \left(\frac{V_{old}}{V_{new}}\right)^2 = 0.1 \cdot \left(\frac{100}{50}\right) \cdot \left(\frac{11}{11.5}\right)^2$

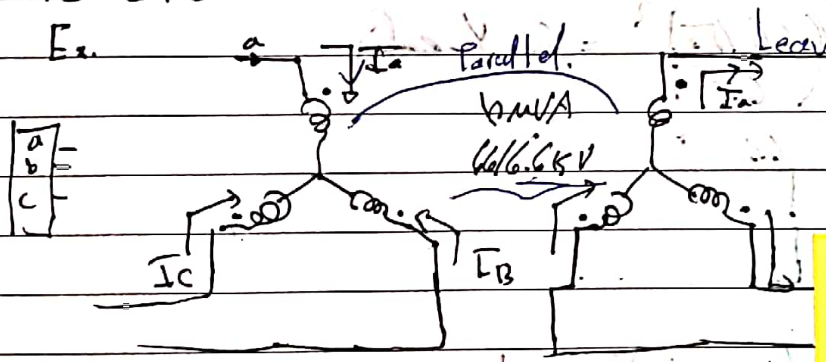
or $= 0.1 \cdot (100/50) \cdot \left(\frac{132}{11}\right)$

$\therefore X_{pu(new)} = 0.183$

$V_{new} = \left(\frac{132}{11}\right) \cdot 11.5$

enters the dot on one side
Leaves the other dot on the other side

Ex.



V_{LL}/V_{LL} = transformation ratio

V_ϕ/V_ϕ = turns ratio

$R_{PU} = \frac{\overset{actual}{0.6}}{(6.6)^2/10} = 0.137$

Conc. R Per unit has the same Value and doesn't depend on side, to solve complex system turn every thing to P.U

another way is to reflected R on P

$R_p = 0.6 \cdot \left(\frac{66/\sqrt{3}}{6.6/\sqrt{3}}\right)^2 = 60\,\Omega$ $= 0.6 \cdot \left(\frac{66}{6.6}\right)^2 = 60\,\Omega$ using turns ratio

$R_p\,PU = \frac{60}{66^2/10} = 0.137$

9-10-2019

$66/\sqrt{3} - 6.6/\sqrt{3}$

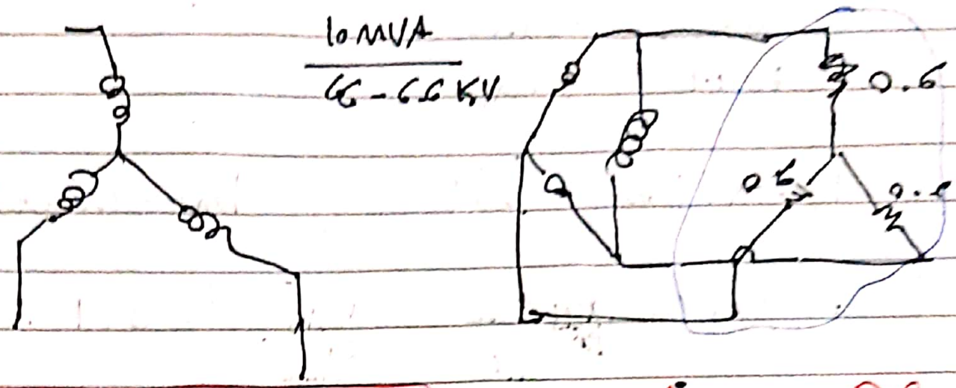
$$R' = 0.6\left(\frac{66/\sqrt{3}}{6.6/\sqrt{3}}\right)^2 = 60\,\Omega \rightarrow \text{single line}$$

ex.

Y–Δ 3φ Tr

10 MVA

66 – 6.6 kV



$$R_p = 0.6 * \left(\frac{66}{6.6}\right)^2 = 60 \rightarrow \text{only gives Ref}$$

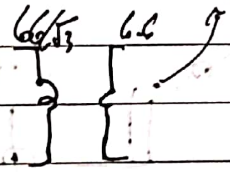
if load in Y config

another way. make load Δ

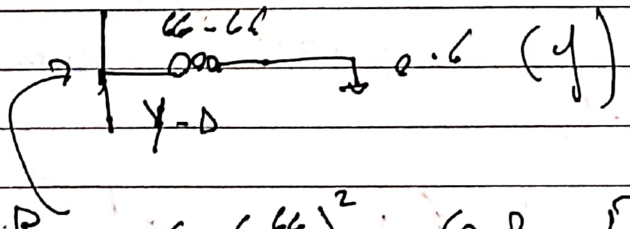
$$Z_\Delta = Z_Y \cdot 3 = 0.6 * 3 = 1.8\,\Omega$$

$$R_p = 1.8 \times \left(\frac{66/\sqrt{3}}{6.6}\right)^2 = 60 \text{ Y-config}$$

single phase $66/\sqrt{3}$ 6.6



$$Y_{eq\,\text{primary}} = (\text{Tr ratio})^2 \rightarrow Y_{eq\,\text{secondary}}$$



$$R_p = 0.6 \cdot \left(\frac{66}{6.6}\right)^2 = 60\,\Omega \rightarrow \text{Refered to Primary}$$

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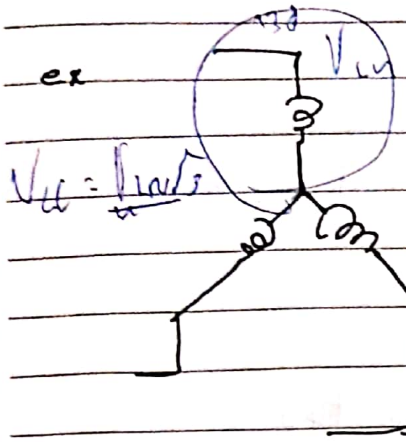
ex

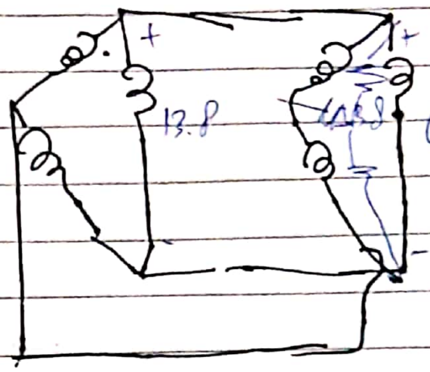


Y - Δ Tr

10 MVA

138/13.8 KV

Δ load



800 KW @ rated Voltage "balanced"

φeq res seen from (HV side) Primary?

2 ways: [Δ load → Y load → balanced load @ rated Voltage which indicates resistive load

$$P = 3 \cdot P_1 = 3 \cdot \frac{V^2}{R}$$

$\frac{P}{3}$

$$\therefore R = \frac{3V_1^2}{P_1} = \frac{3 \cdot (13.8)^2}{800\,\text{KW}}$$

$$R = 0.714\ \Omega$$

bec balance $R_Y = \frac{R_\Delta}{3}$

$$\therefore R_Y = \frac{0.714}{3} = 0.23805\ \Omega$$

$$R_p' = 0.23805\ (\text{Transformation ratio})^2$$

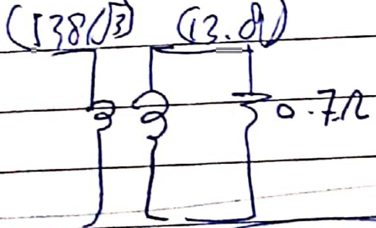
$$: \left(\frac{138}{13.8}\right)^2$$

$$= 23.8\ \Omega$$

faster

using ...

Or



$$R_p' = 0.714 \times \left(\frac{138/\sqrt{3}}{13.8}\right)^2 = 23.8\ \Omega$$

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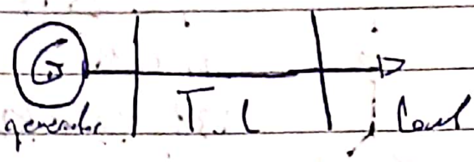
Synchronous Generator

Generator
Power Plant



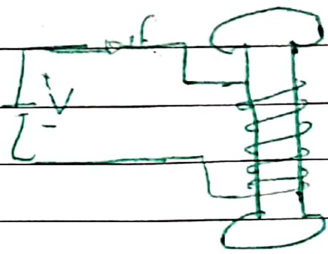
Mechanical P_m → G → electrical Power P_e

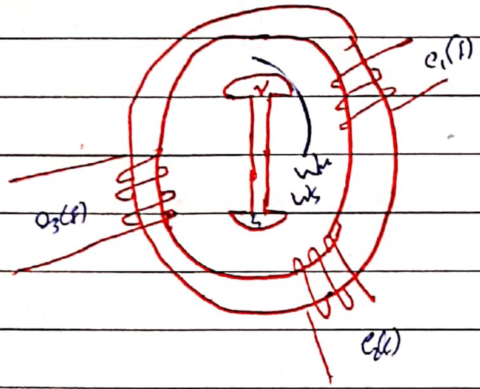
Fuel

solar, wind, gas, nuclear

Synchronous Generator

- stator — winding / armature
- rotor — field





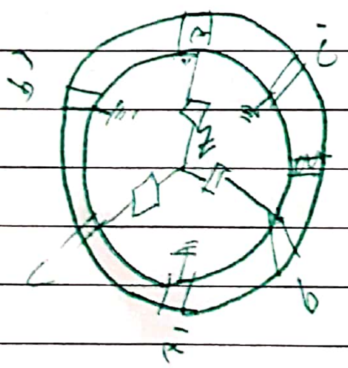
$$emf = N\frac{d\phi}{dt} \Rightarrow \text{Faraday}$$

$$e_1(t) = E_{m_1}\sin(wt)$$

$$e_2(t) = E_{m_2}\sin(wt - 120)$$

$$= E_{m_3}\sin(wt - 240)$$

for Y - connection



$\longrightarrow P_m \longrightarrow (G) \longrightarrow P_e$

stable

not stable

steady state

$\hookrightarrow P_e = P_m$

$\hookrightarrow$ Porpous of Power

$\hookrightarrow$ we need this

* Linear

$\Sigma F = ma$

x ~~Rotational~~ Rotational

$$J \frac{dw}{dt} = T_{mechanical} - T_{electrical}$$

inertia

$$\hookrightarrow K \frac{d^2 \theta_m}{dt^2} = P_m - P_e$$

θ_m $\rightarrow$ rotor angle.

$$\vec{\phi}_{stator} = \vec{\phi}_a + \vec{\phi}_B + \vec{\phi}_c$$

$$= \phi_{rms} \sin(\omega t) + \phi_{rms} \sin(\omega t - 120) + \phi_{rms}(\omega t + 120)$$

convention this equals 0

but its not

why?

$\hookrightarrow$ ϕ_{rms} depends on rotor so it has a resultant armature or stator magnetic field

$$N_s = \frac{120 f}{P}$$

P = # of poles

f = Frequency

N_s = speed in RPM

Page 1

15-10

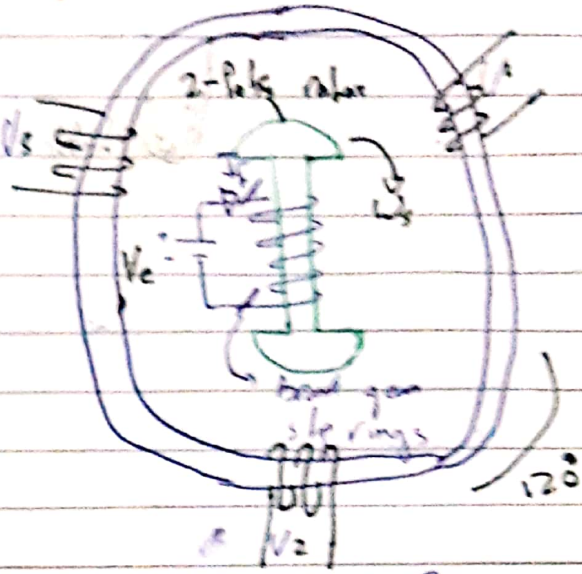
Synchronous Generator (alternator):

a type of G that uses moving magnetic field to induce voltage

V_e = excitation DC voltage
controls I which in return
controls ϕ_{rotor}

↳ uses AVR
automatic Voltage Regulator
to E_a suit the load on the generator

↳ most cases, uses a DC generator applied on rotor to supply this voltage
(self exciting synchronous generator)



ω_s = synchronous frequency
$V_1(t) = E_{max} \sin(\omega t)$
$V_2(t) = E_{max} \sin(\omega t - 120)$
$V_3(t) = E_{max} \sin(\omega t + 120)$

why its called synchronous?

because the wave form of the generated Voltage is synchronized with rotation of the generator, each peak of the sinusoidal wave form corresponds to a physical position of the rotor.

the f of said induced Voltage E(t) is calculated from

$$f = \frac{N_s . P}{120}$$

- so for a 4 Pole - 60 Hz Power
- N_s should be 1800 RPM
- which is Very high and leads to machine destruction.

why used alot in Power stations?

1- the output frequency can be easily controlled in synchronous motors. → remember $E_{rf} = 4.44 \phi N f$ and ϕ in synchronous depends on Dc excitation circuit.

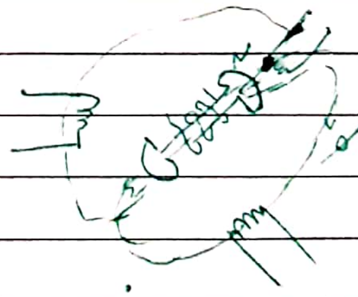
2- can accommodate different load Power factors

Usually synchronous Generators have alot of Poles

$N > 20$

↳ to limit centrifugal force on structure

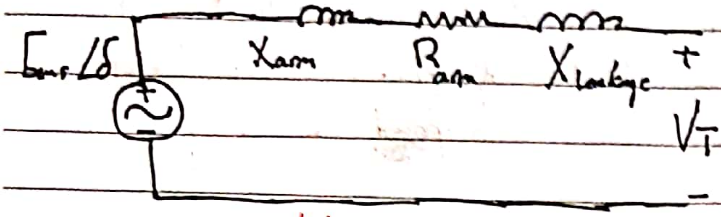
Lowering N_s

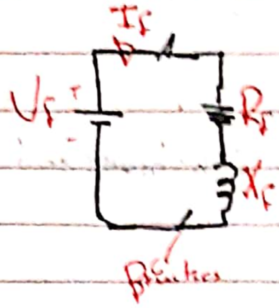


15-10

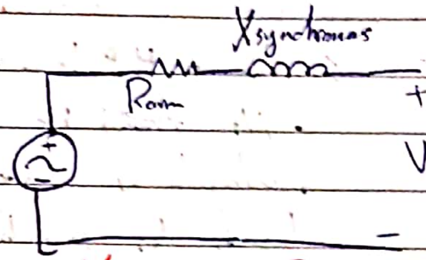
Modeling "eq. circuit"

Per phase calculations

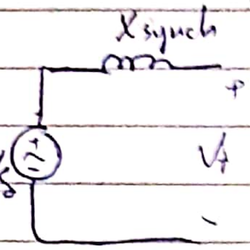




→ stator eq circuit



$V_T = V_{rms}\angle\delta$



$X_{sync} >>> R_{arm}$
↳ neglected.

simplest model

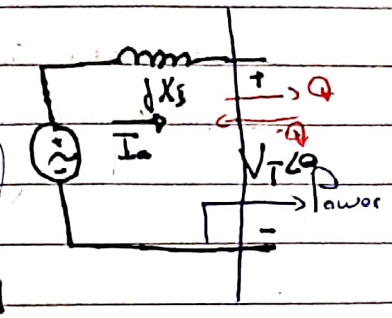
What we need from this model?

Power, Reactive Power

Generator equivalent ckt.

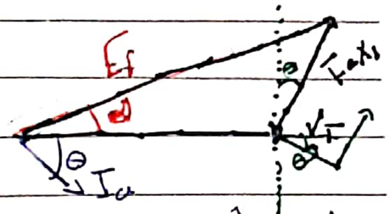
$E_f \angle \delta$

Power angle δ Depends on mech. Position of rotor



Gen always gives P

Mot always takes P



δ is always + in Gen

3ϕ

$$P = 3 \times V_T I_a \cos\theta$$

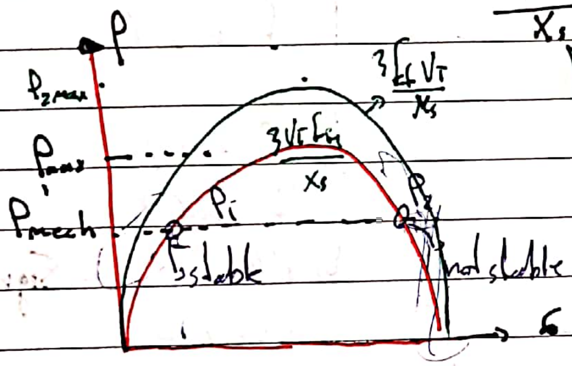
$$I_a X_s \cos\theta = E_f \sin\delta$$

$$I_a \cos\theta = \frac{E_f \sin\delta}{X_s}$$

$$P = \frac{3 \times V_T \times E_f \sin\delta}{X_s}$$

→ only Viable for HV systems

↳ if LV R will have an effect



$$K \frac{d^2\delta}{dt^2} = P_{mech} - P_e$$

at P_1 if $P_{mech} \uparrow$ by conv δ should increase, if it increase P_e will increase; but at P_2 P_{ele} will go down.

$$Q = 3 V_T I_a \sin\theta$$

from (phasor) $I_a X_s \sin\theta = E_f \cos\delta - V_T$

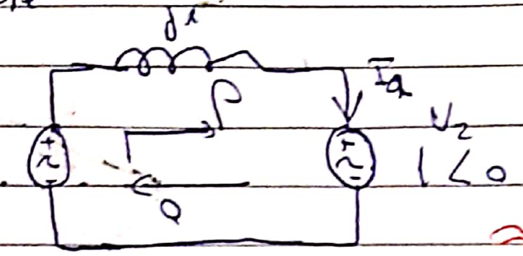
$$\therefore Q = \frac{3 V_T (E_f (\cos\delta) - V_T)}{X_s}$$

16 - 10

$E_f \cos\delta - V_T > 0 \Rightarrow Q > 0$ → Generator supplies Q → Load consumes Q

$E_f \cos\delta - V_T < 0 \Rightarrow Q < 0$ → Generator consumes Q → Load supplies Q

example



⟹ which is Gen and which is motor

Find V_1, V_2

$$I_a = \frac{V_1 - V_2}{jx} \Rightarrow S_2 = V_2 I_a^* = P + jQ$$

→ if + values, it's consume
→ if − values, it injects (supplies)
→ follow passive sign convention.

P_{+ve}, Q_{+ve} → motor

P_{-ve}, Q_{-ve} → Generator.

→ following passive sign convention

as a summery

$$P = \frac{3 V_T E_f}{X_s} \sin(\delta)$$

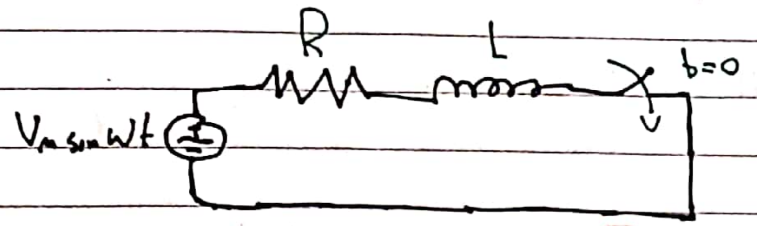
L,N values

$$\hookrightarrow P = \frac{3 \frac{V_{T\,LL}}{\sqrt{3}} \cdot \frac{E_{f\,LL}}{\sqrt{3}} \sin\delta}{X_s} = \frac{V_{T\,LL} \cdot E_{f\,LL} \sin\delta}{X_s}$$

$$Q = \frac{3 V_T}{X_s} \cdot (E_f \cos\delta - V_T)$$

$$= \frac{3}{\sqrt{3}} \frac{V_{T\,LL}}{X_s} \cdot \frac{(E_f \cos(\delta) - V_T)}{\sqrt{3}} = \frac{V_{T\,LL}}{X_s} \cdot (E_{f\,LL} \cos\delta - V_{T\,LL})$$

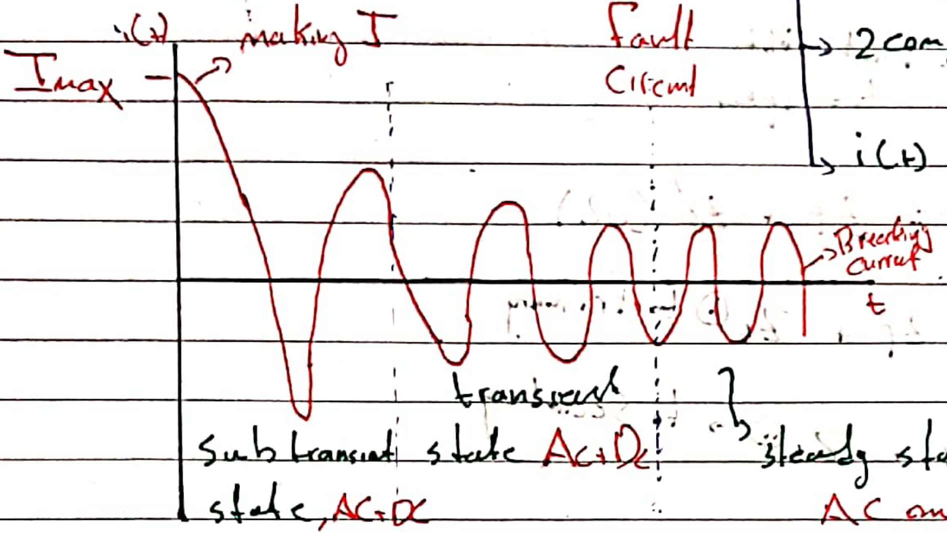
16 - 10



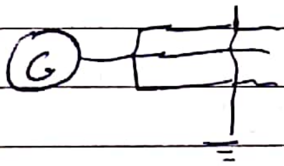
$i(t) = ??$

→ 2 components

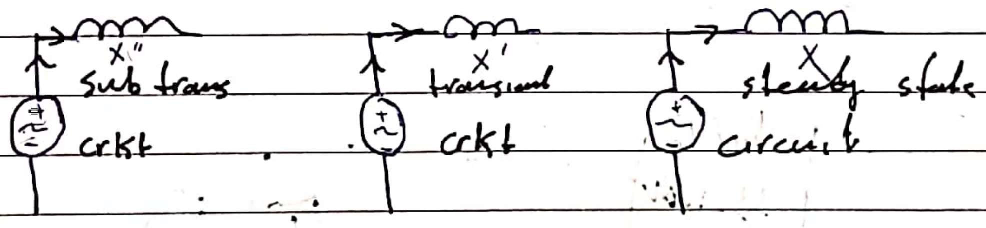
→ $i(t) = i_{steady\ state} + i_{transient}.$



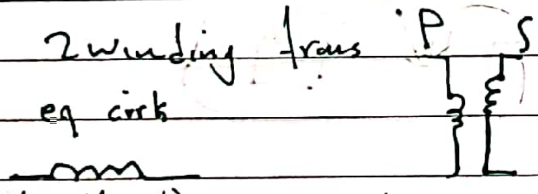
Breaker circuit the model.



3ϕ fault



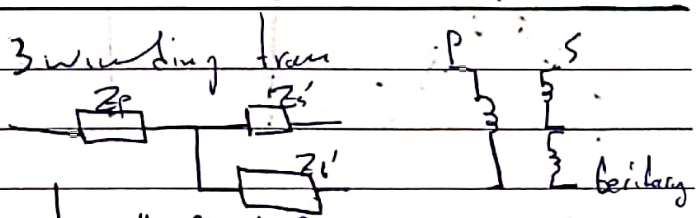
2 winding trans
eq circk



$X_{eq} : X_1 + X_2'$

2 tests oc, sc

3 winding tran



→ all ref to P

→ 3 test for 3 eqns

$Z_{ps} = Z_p + Z_s'$ → ref to S

$Z_{pt} = Z_p + Z_t'$

$Z_{st} = Z_s + Z_t''$

16-10

3 winding T.R. [illegible]

loads on 3W T

1- auxiliary voltage [illegible]
2- Reactive Power Reduce
3- harmonics reduce

① Z_{ps} → see short-circuit, t = open
primary

$= Z_p + Z_s'$ (Ω)

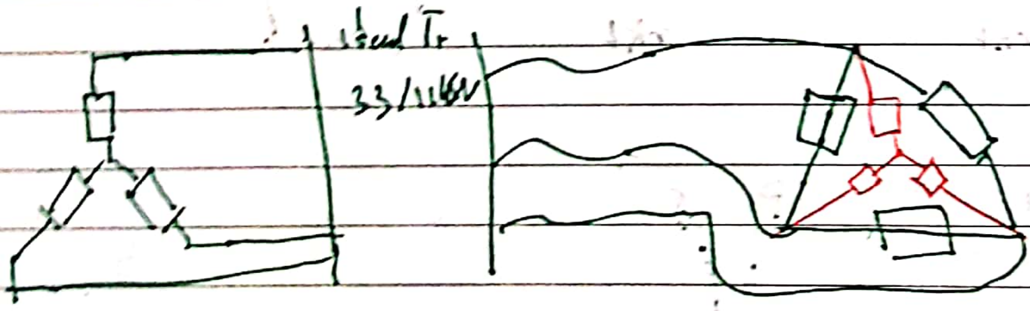
② $Z_{pt} = Z_p + Z_t$ ① to Primary

③ $Z_{st} = Z_s + Z_t$ ① → to Secondary

objective? : Z_p, Z_s', Z_t' → [illegible]

$Z_p = \frac{1}{2}(Z_{ps} + Z_{pt} - Z_{st}')$, $Z_s' = \frac{1}{2}(Z_{ps} + Z_{st}' - Z_{pt})$

$Z_t' = \frac{1}{2}(Z_{pt} + Z_{st}' - Z_{ps})$



turn every thing to Y

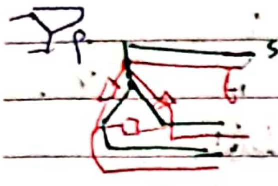
$Z_{eq_Y} = 3 \times \left(\frac{33}{11}\right)^2 = 27\Omega$

$Z_{eq_\Delta} = 3 Z_Y = 81 \Omega$

16-10

ex

Primary: Y-connected, 66 KV, 15 MVA
Secondary Y-connected, 13.2 KV, 10 MVA
Tertiary Δ-connected, 2.3 KV, 5 MVA
and 3-S.C. test applied:



$Z_{ps} = 7\%$ (15 MVA, 66 KV)
$Z_{pt} = 9\%$ (15 MVA, 66 KV)
$Z_{st} = 8\%$ (10 MVA, 13.2 KV)

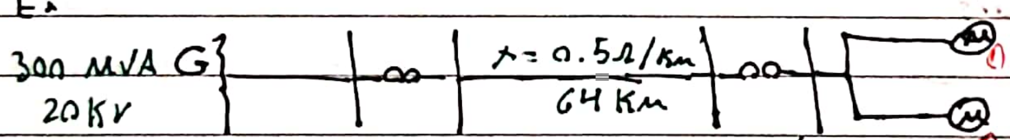
2 methods

1- $Z_{st\,new(pu)} = Z_{st\,old} \cdot \left(\frac{15}{10}\right) \cdot \left(\frac{13.2}{13.2}\right)^2 = 0.12$ pu — should be close to each other

2- First at on base then Reflect.
$Z_{st} = 0.08 \times \left(\frac{13.2^2}{10}\right) = 1.39\,\Omega$
$Z_{st}' = 1.39 \times \left(\frac{66}{13.2}\right)^2 = 34.848\,\Omega$
$Z_{st}'(pu) = \frac{34.848}{(66)^2/15} = 0.12$ pu

find the rest at home

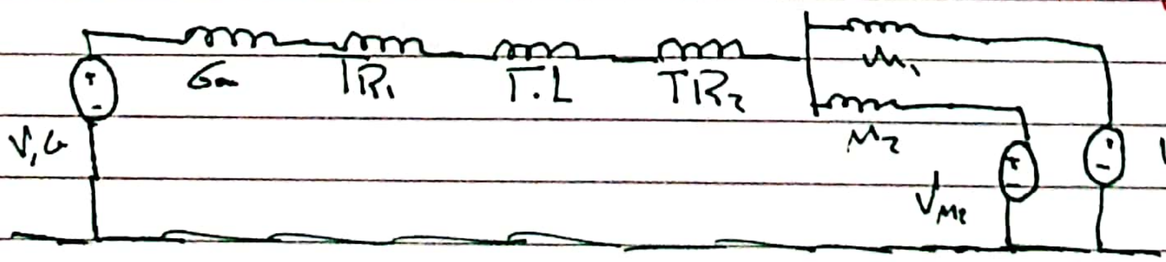
Ex



300 MVA G, 20 KV, $x'' = 20\%$
3φ 350 MVA, 20/230 KV, $x = 0.1$ (V_L)
$x = 0.5\,\Omega$/Km, 64 Km
3-1φ, 127/13.2 KV (V_p), 100 MVA, $x = 10\%$
M1: 13.2 KV, 200 MVA, $x'' = 20\%$
M2: 13.2 KV, $x'' = 20\%$, 100 MVA

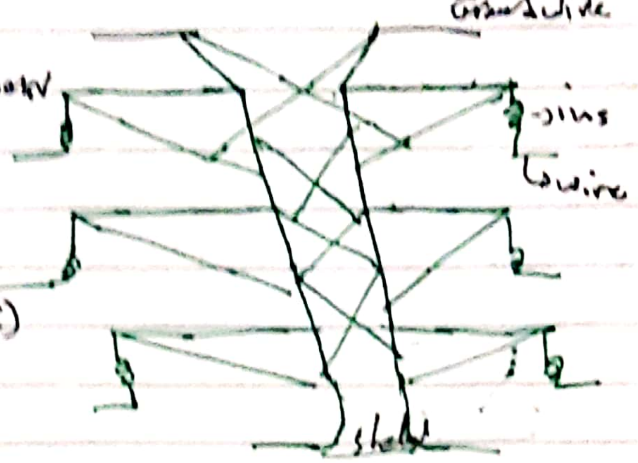
Base 300 MVA, 20 KV

1- Draw the impedance diagram.
2- input Power to Load. (13.2 KV, unity PF) $M_1 \to P = 120$ MW
$M_2 = P = 60$ MW



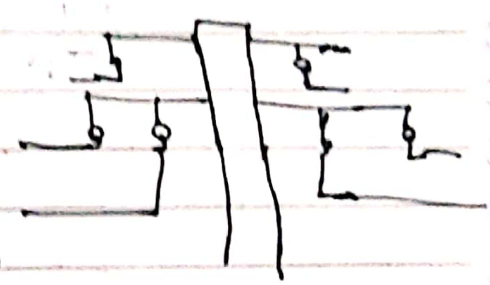
21 - 10

Transmission Lines 132-400kV



2 Types
- Over head Line (OH)
- Under ground cable (UGC)

Tower should withstand environmental condition



heat, ice and water may affect them

Conductor Material "Al"
 is cheaper
 Lighter

23kV → 3x 240mm² Cu cable
3x 500mm² Al cable
cross Sectional Area

Conductors

steel reinforcement inside conductor

① AAC : All aluminium conductor (no reinforcement steel)
② ACSR : All conductor steel reinforced
③ AAAC : All Al Alloy Conductor

e. ACSR: 24/7 : 24 Al strand
 7 steel strand

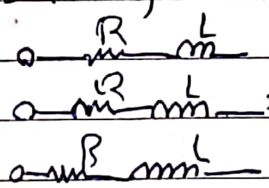
21 - 10

* Bundeled conductors
400 KV multiple Conductors/phase
Z ↓

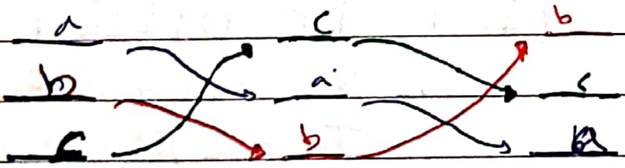
* Corona
in some certain conditions
: Localized electric field near components on a TL network
Result in a tiny electric discharge or "Corona"
Causing the air to Ionize (becomes a conductor
rather an insulator
- solved primarly by Bundeling

* Conductors
↳ Bundelled conductors
4 cond /phase
: cross section ↑ → less Corona ↓

* Transposition (TL model)

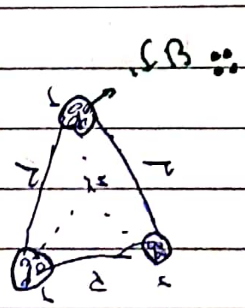


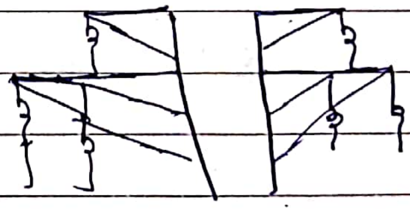
Balanced system



change in position every tower

* model





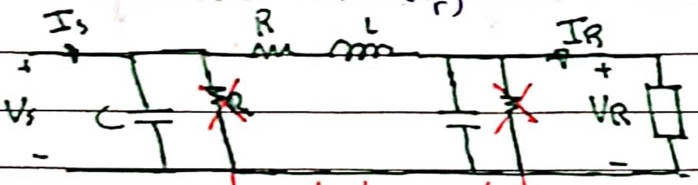
21-10

$$L = \frac{\mu_0}{8\pi}\left[1 + 4\ln\left(\frac{d}{r}\right)\right] \; H/m \Big\} \text{ from EM}$$

$d = \sqrt[3]{d_{12} \cdot d_{23} \cdot d_{13}}$ r: radius

↳ Geometric mean distance

$$C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{d}{r}\right)} \Big\} \text{ from EM}$$



Leakage current from Corona
↳ we neglect it

V_R = Received Voltage
I_R = " current

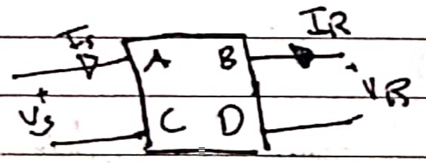
V_s = Send Voltage
I_s = " current

Length T.L

L ≤ 80 KM	short	Freq * type
240 km ≥ L ≥ 80 KM	medium	60 or 50 * OHL or UG
L ≥ 240 km	long	

23-10 Power 1

* 2 port networks



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$\therefore V_s = AV_R + BI_R$

$I_s = CV_R + DI_R$

$$A = \frac{V_s}{V_R}\Big|_{I_R = \text{zero}}$$ (open circuit) (no load)

$$C = \frac{I_s}{V_R}\Big|_{I_R = \text{zero}}$$

23 - 10

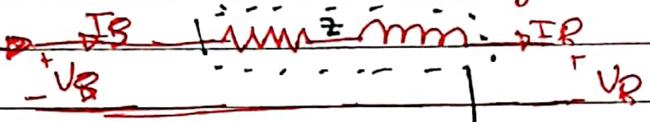
$B = \left.\frac{V_s}{I_R}\right|_{V_R = \text{zero}}$ (short circuit)

$D = \left.\frac{I_s}{I_R}\right|_{V_R = \text{zero}}$

$\underbrace{AD - BC}_{\text{determine of matrix}} = 1$

Representations (models)

1- short T.L (Length $\leq$ 80Km)



↳ Lumped Parameters

$Z = z \cdot \text{Length} = (r + j\omega l)^{\Omega} \cdot \text{Length}$

↳ (Ω / Km)

ex $z = 0.05\ \Omega/\text{Km}$

$l = 10\ \text{Km}$

$\therefore Z = 0.05 \times 10 = 0.5\ \Omega$

- ABCD -

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$V_s = \cancel{A} V_R + I_R Z = V_R + I_R Z$

$I_s = I_R$

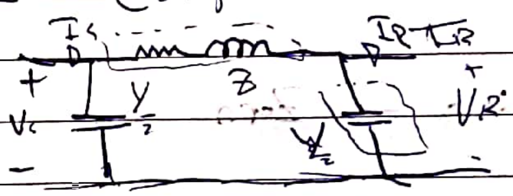
↳ From circuit above $\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

$AD - BC = 1$ ✓

2- medium T.L (Length $240 \geq L \geq 80$ Km)

$Y = y \cdot \text{Length}$

↳ $j\omega c$



$Z = z \cdot \text{Length}$

↳ $(r + j\omega l)^{\Omega}$

23 - 10

- ABCD - $\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$

1. $V_s = (V_R \frac{Y}{2} + I_R) Z + V_R$

or $V_s = \underbrace{V_R (1 + \frac{YZ}{2})}_{} + \underbrace{Z I_R}_{}$

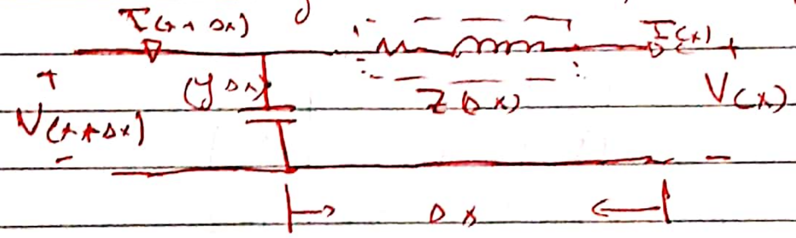
2. $I_s = I_R + V_R \frac{Y}{2} + V_s \frac{Y}{2}$

∴ $I_s = V_R \underbrace{(Y(1 + \frac{YZ}{4}))}_{} + I_R \underbrace{(1 + \frac{YZ}{2})}_{}$

∴ $A = 1 + \frac{YZ}{2} = D$

$B = Z$ $\qquad$ $C = Y(1 + \frac{YZ}{4})$

Long T.L Length ($240 \le L$) km



relations

$$V(x+\Delta x) = V(x) + I(x) \cdot Z \Delta x$$

$$I(x+\Delta x) = I(x) + V(x+\Delta x)\, Y \cdot \Delta x$$

$$\frac{V(x+\Delta x) - V(x)}{\Delta x} = I(x)\, Z \quad , \quad \frac{I(x+\Delta x) - I(x)}{\Delta x} = V(x+\Delta x) \cdot Y$$

$\Delta x \rightarrow$ Zero

$$\frac{\partial V(x)}{\partial x} = Z\, I(x) \quad , \quad \frac{\partial I(x)}{\partial x} = V(x)\, Y$$

23 - 10

Plug in $\frac{\partial^2 V(x)}{\partial x^2} = z \frac{\partial I(x)}{\partial x}$

$\frac{\partial^2 V(x)}{\partial x^2} = \underbrace{zy}_{\gamma} V(x)$ → Differential eqn

Sol:-

1- $V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$

2- $I(x) z = \frac{\partial V(x)}{\partial x}$

∴ $I(x) = \frac{A_1}{z} \gamma e^{\gamma x} - \frac{A_2}{z} \gamma e^{-\gamma x}$

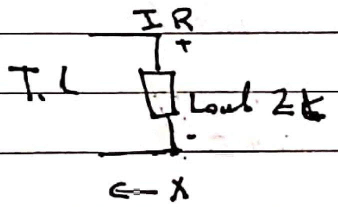
∴ $I(x) = \frac{A_1}{\frac{z}{\gamma}} e^{\gamma x} - \frac{A_2}{\frac{z}{\gamma}} e^{-\gamma x}$

$\gamma \triangleq$ Propagation constant $= \sqrt{zy} = \alpha + j\beta$

∴ α = attenuation constant (nep/m)

β = phase constant (rad/m)

Value of [A_1, A_2]?



$A_1 + A_2 = V_R$

$\frac{A_1 - A_2}{Z_c} = I_R$

$A_1 = \frac{V_R + Z_c I_R}{2}$

$A_2 = \frac{V_R - Z_c I_R}{2}$

$\frac{\gamma}{z} = \frac{\sqrt{zy}}{z} = \sqrt{\frac{y}{z}}$

and $\frac{z}{\gamma} = \sqrt{\frac{z}{y}}\ \Omega = Z_c$ C/s impedance

$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x} = \left(\frac{V_R + Z_c I_R}{2}\right) e^{\gamma x} + \left(\frac{V_R - Z_c I_R}{2}\right) e^{-\gamma x}$

$= \frac{e^{\gamma x} + e^{-\gamma x}}{2} V_R + Z_c \frac{e^{\gamma x} - e^{-\gamma x}}{2} I_R$, $A = \frac{e^{\gamma x} + e^{-\gamma x}}{2} = \cosh \gamma x$

$B = Z_c \frac{e^{\gamma x} - e^{-\gamma x}}{2} = Z_c \sinh \gamma x$

23 - 10

$C = \frac{\sinh \gamma x}{Z_c}$, $D = \cosh \gamma x = A$

$B = Z_c \sinh \gamma x$

in Summary

$\gamma = \sqrt{zy} = \alpha + jB$, $Z_c = \sqrt{\frac{z}{y}}$

$\cosh \gamma x = \frac{1}{2}\left[e^{\alpha x} \angle Bx + e^{-\alpha x} \angle -Bx \right]$

$\sinh \gamma x = \frac{1}{2}\left[e^{+\alpha x} \angle +Bx - e^{-\alpha x} \angle -Bx \right]$

Ex: $\underbrace{370\ \text{Km}}_{\text{Length}}$, $z = 0.5240 \angle 79.04\ \Omega/\text{Km}$

$y = 3.1728 \times 10^{-6}\ S/\text{Km}$

Load 125 MW , unity PF , 215 KV

Find: $V_s, V_R, P_s, V.R\%$

Sol: $\gamma = \sqrt{zy}$

$\gamma L = \sqrt{zy} \cdot L = \sqrt{(zL)\cdot(yL)} = \sqrt{ZY}$

$\gamma L = \underset{\alpha l}{0.0456} + j \underset{BL}{0.475}$

$Z_c = \sqrt{\frac{Z}{y}} = 406 \angle -5.48$

$Bl = 0.475$ rad

$BL = 0.475 \cdot \frac{180}{\pi}$

$= 27.22^\circ$

$V_s = A V_R + B I_R$

$A = \cosh \gamma x = \frac{1}{2} e^{0.0456} \angle 27.22 + \frac{e^{-0.0456}}{2} \angle -27.22$

$= \underbrace{0.89 \angle 1.34}_{\text{notice}}$

|mag| close to 1

and θ close to 0

$B = Z_c \sinh \gamma x = \left[0.5\, e^{0.0456} \angle 27.22 - 0.5\, e^{-0.0456} \angle -27.22\right]$

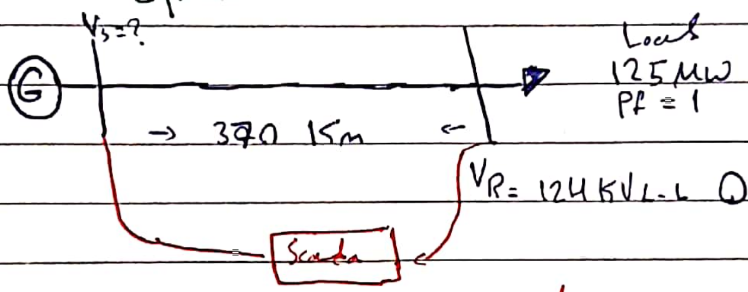
$= Z_c \left[0.0405 + j0.4578\right]$

$= (406 \angle -5.48)(0.0405 + j0.4578) = 34.14 + j183.47$

$= 186.66 \angle 79.457$

$C = \dfrac{\sinh \gamma x}{406 \angle -5.48} = 1.1322 \times 10^{-3} \angle 90.417$

① $\underbrace{V_s = AV_R + BI_R}_{\text{this is L-N equation}} = A\left[\dfrac{215}{\sqrt{3}}\right] + BI_R$



* scada is a monitoring system to change V_s depending on V_R to keep V_R constant

$I_R = ??$

$S = \sqrt{3}\, V_L I_L$

$\dfrac{125}{1} = \sqrt{3}\,(215)\, I_L$

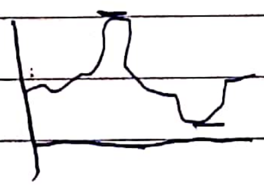
$I_L = 335.66 \angle 0$ A

now $V_s = AV_R + BI_R = 137 \angle 27.77°$

$V.R\% = \dfrac{V_{RNL} - V_{RFL}}{V_{RFL}} \times 100\%$ → Worst from Voltage drop

Worst from Voltage design

in jordan the VR is ∓10%



how to get V_{RNL}, V_{Rfl}

no load

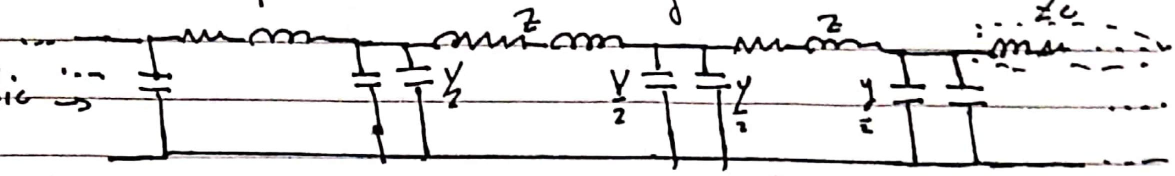
$$V_s = A V_R + \cancel{B I_R}$$

$$V_R = \frac{V_s}{A} = \frac{237}{0.89}$$

Five Apple

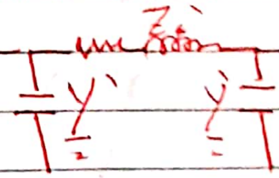
π equivalent of a long T.L

Realistic model



in our assumption the degree of error increases with Length.

We want to turn this to a π circuit



$$A = D = \cosh \gamma L$$
$$B = Z_c \sinh \gamma L$$
$$C = \sinh \gamma L / Z_c$$

$$Z' = Z_c \sinh \gamma L$$
$$= \sqrt{\frac{Z}{y}} \sinh \gamma L \cdot \frac{ZL}{ZL}$$
$$= \sqrt{\frac{Z}{Z^2 \cdot y}} \sinh \gamma L \cdot \frac{Z}{\gamma}$$
$$\frac{1}{\gamma} = \frac{\sinh \gamma L}{\gamma L} \cdot Z \longrightarrow \text{Medium T.L model}$$

↳ correction factor

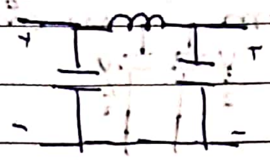
$$\therefore \frac{Y'}{2} = \frac{Y}{2} \times C.F \quad , \quad \cosh \gamma l = 1 + \frac{Y'Z}{2} \longrightarrow A \text{ of medium T.L}$$

↳ A of Long T.L

$$\frac{Y'}{2} = \frac{\cosh \gamma L - 1}{Z'_c} \cdot \frac{\frac{YL}{2}}{\frac{YL}{2}}$$

$$\therefore \frac{Y'}{2} = \frac{Y}{2} \left[\frac{\cosh \gamma L - 1}{\sinh \gamma L} \right] \frac{\gamma L}{2}$$

Lossless T.L



lossless $\alpha = 0$.

$$A = \cosh \gamma L = \frac{1}{2}\left(e^{\alpha L}\angle \beta l + e^{-\alpha L}\angle -\beta l\right)$$

$$= \left(\frac{1}{2} e^{j\beta L} + \frac{1}{2} e^{-j\beta L}\right) \rightarrow \cos \beta L$$

$\therefore$ $A = \cos \beta L$

$\beta = \frac{2\pi}{\lambda}$

$\gamma = \sqrt{ZY}$

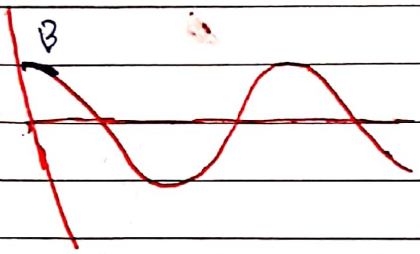
$= \sqrt{j\omega C \, j\omega L}$

$= j\omega \sqrt{LC}$

$\alpha = 0$

$\beta = j\omega\sqrt{LC}$

$\therefore$ $\gamma = j\beta$



the value of βL needs to be very large for the cosine to change value $\gg 500$ km

- So value of A is relatively not changed

in Power Sys

$\lambda f = c$

$\lambda = \frac{3 \times 10^8}{50}$

$\lambda = 6 \times 10^6$ m

wavelength

$B = Z_c \sinh \gamma L$, $Z_c = \sqrt{\frac{Z}{Y}}$

Lossless

$Z_c = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$ real value

$\therefore$ $\sinh \gamma = \frac{1}{2}\left(e^{\alpha L}\angle \beta L - e^{-\alpha L}\angle -\beta L\right)$

this is j sin

$B = \sinh \gamma L = \theta \sin \beta L \times \sqrt{\frac{L}{C}}$

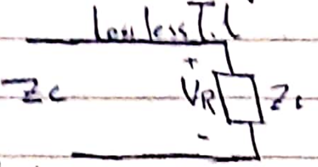
$\theta = 90° \Rightarrow$ so θ of B close to 90

$\therefore$ in Summary Lossless

$A = D = \cos \beta L$, $B = j Z_c \sin \beta L$, $C = j \sin \beta L / Z_c$

$\beta = \frac{2\pi}{\lambda}$, $\lambda f = c$, $Z_c = \sqrt{\frac{L}{C}}$, $j\omega\sqrt{LC} = \gamma$

SIL : Surge Impedance Loading



. Zc , real value

$$\text{Power}\,\Big|_R = \frac{3 \times V_{RL-N}^2}{Z_c} = \frac{V_{RL-L}^2}{Z_c}$$

when load Z_L = Z_c

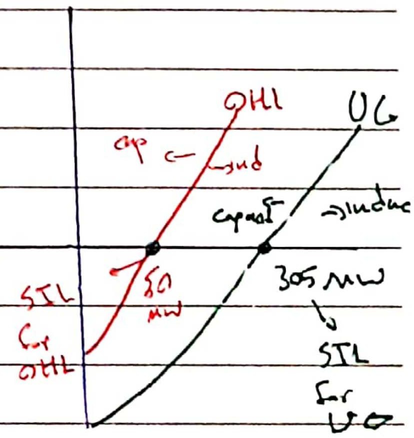
Z_L is called natural load

SIL is Primarly the Power used by the Load if it has the Same Value as the lossless Transmission Lines "Z_c" $= \sqrt{\frac{L}{C}}$

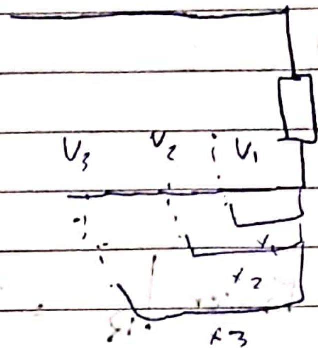
example Lines → UG
→ OHL

OHL	132 KV	275 KV
SIL	50 MW	240 MW
Zc	150 Ω	315 Ω
UG	132 KV	400 KV
SIL	325 MW	142.6 MW
Zc	36.1 Ω	37.2

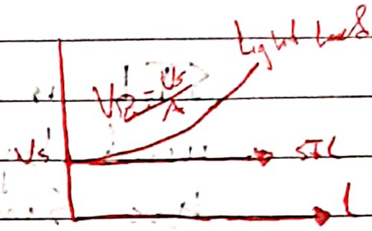
- as you can see SIL for UG is Very Large compared to OHL
- using UG Levels for a OHL Will make it view as a capacitive Load
- Voltage will rise



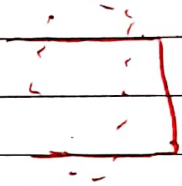
example for circuit →
prove $|V|$ across Lossless T.L is const with $Z_L = Z_c$:



$$V_s = V_R A + I_R B$$
$$V_s = \cos\beta l \, V_R + V_R \, j \frac{\sin\beta l}{Z_c}$$
$$V_s = V_R (\cos\beta l + j \sin\beta l)$$
$$V_s = V_R e^{j\beta l}$$
$$|V_s| = |V_R|$$



$V_s = A \cancel{V_R} + B I_R$ in SC
$I_s = C \cancel{V_R} + D I_R$



- So Don't forget even if SC happen V_s still has Voltage and $I_s \neq 0$

* Complex power flow through T.L:

$V_s \quad V_R$

$= V_s\angle\delta \quad = V_r\angle 0^\circ$

2 control subjects
- control voltage
- control Q

$S_R = V_R I_R^*$, $V_s = AV_R + B I_R$

$\therefore I_R = \frac{V_s - AV_R}{B} = \frac{|V_s|}{|B|}\angle\delta-\theta_B$

$- \frac{|V_R||A|}{|B|}\angle\theta_A-\theta_B$

$\therefore S_R = \frac{|V_s||V_R|}{|B|}\angle\theta_B-\delta$

$- \frac{|A||V_R|^2}{|B|}\angle\theta_B-\theta_A$

$$S_{R_{1\phi}} = \frac{|V_s||V_R|}{|B|}\angle\theta_B-\delta - \frac{|V_R|^2|A|}{|B|}\angle\theta_B-\theta_A$$

$\angle\theta_B-\delta \quad \angle\theta_B-\theta_A$

$S_{3\phi} = 3S_{1\phi} = \frac{|V_{s\,L-L}||V_{R\,L-L}|}{|B|} - \frac{|V_{RL}|^2|A|}{|B|}$ all V due V_{L-N}

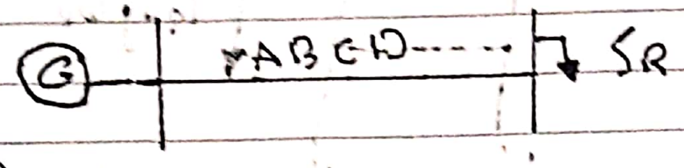
$\therefore S_{3\phi} = 3S_{1\phi}$ but we use L-L

so $3 \times \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = 1$ so. $S_{3\phi} = S_{1\phi}$

$S_{3\phi} = P_{3\phi} + Q_{3\phi}$, $P_{3\phi} = \frac{|V_{s\,L-L}||V_{R\,L-L}|}{|B|}\cos(\theta_B-\delta)$

$\frac{|A||V_{R\,L-L}|^2}{|B|}\sin(\theta_B-\theta_A)$

$$Q_{sd} = \frac{|V_{sL-L}||V_{RL-L}|}{|B|}\sin(\theta_B - \delta) - \frac{|V_{RL-L}|^2 |A|}{|B|}\sin(\theta_B - \theta_A)$$

1- 

2- P_{Rmax}?

to simplify → take Lossless → $\theta_A = 0$, $\theta_B = 90$

$$P_R = \frac{|V_{sL-L}||V_{RL-L}|}{Z_c \sin \beta l} \sin\delta \quad \ldots (1)$$

$\sin(90-\delta)$

Q depends on V difference

from equation (1)

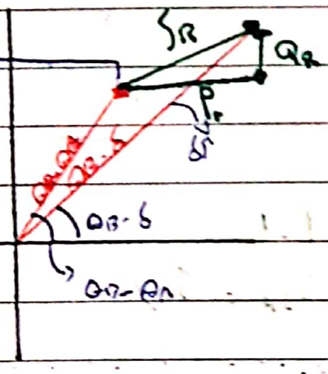
1- Power depends on δ
2- P_{max} happens on $\delta = 90$
3- as $L\uparrow \rightarrow \sin\beta l \uparrow \rightarrow P_R\downarrow$
4- P_R depends on V^2

Power Circle Diagram:

$$S_R = P_R + jQ_R = \frac{|V_s||V_R|}{|B|}\angle(\theta_B - \delta) - \frac{|A||V_R|^2}{|B|}\angle(\theta_B - \theta_A)$$

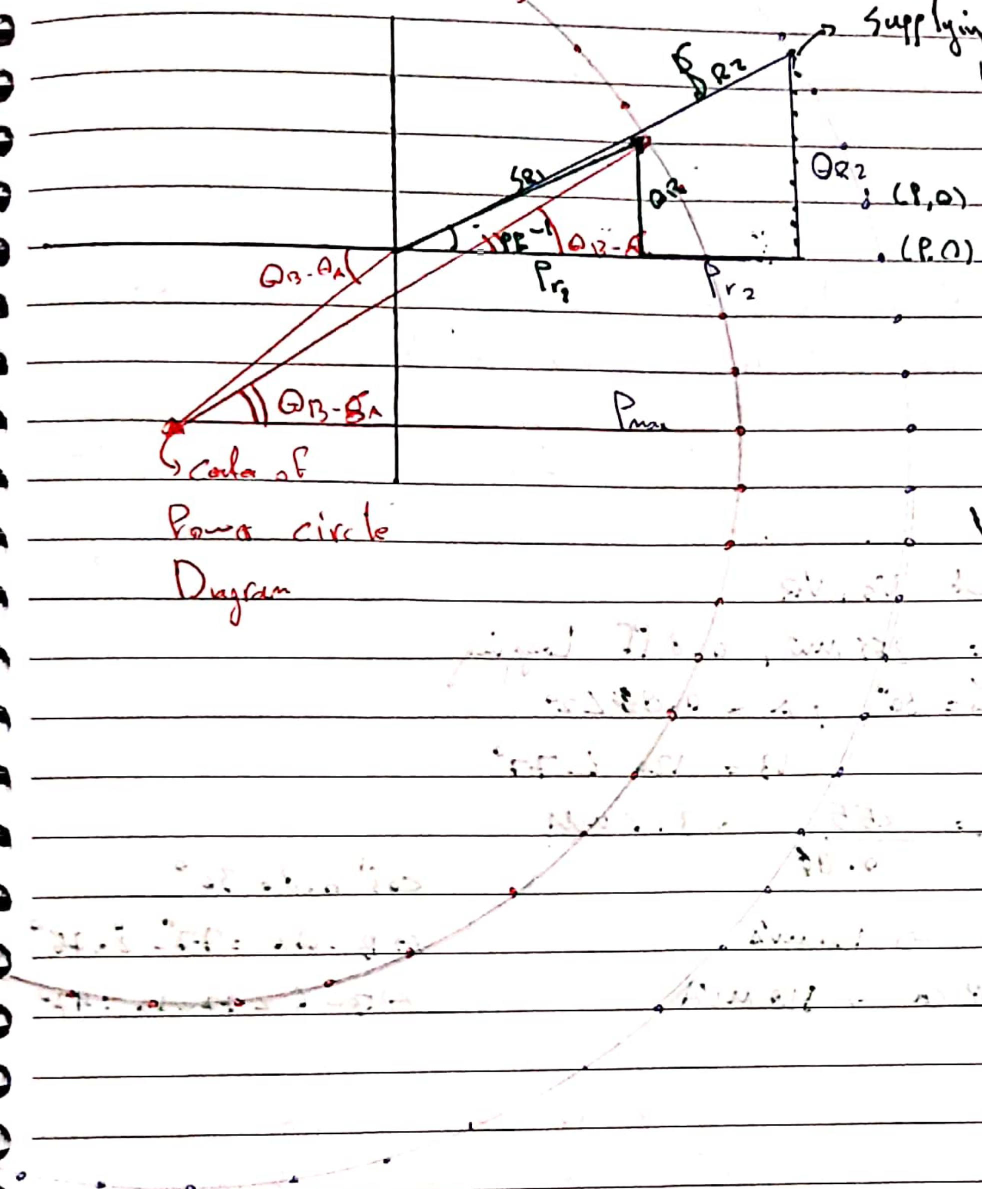
draw

- Point of Ref in circuits?
- dependent $|V_R|^2$



$\underbrace{(\theta_B - \theta_A)}_{\text{closer to } 90^\circ} > (\theta_B - \delta)$

- as you can see, its Vector addition Basically
- $\delta\uparrow \rightarrow P_R\uparrow$



→ supplying a Larger load
$|V_R| \to |V_{s1}| \to \delta$

P_1	Q_1
P_2	Q_2
P_3	Q_3

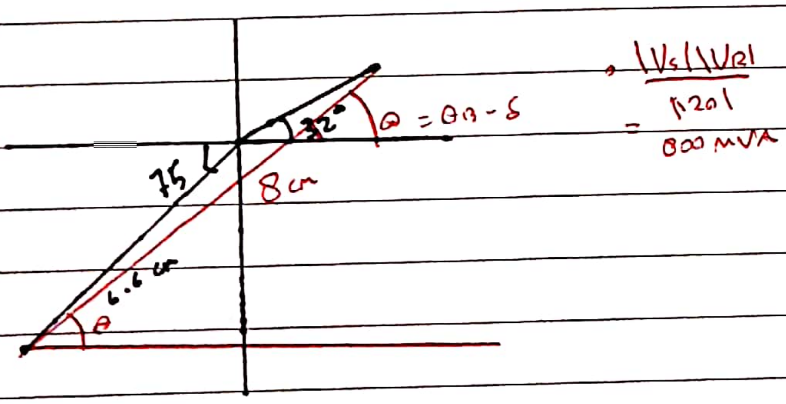
our objective is to keep V_R constant

1- if moving Vertically adding a Capacitor Bank will help.

2- historically a Capacitor was much cheaper than a Battery Bank

example: $A = 0.88 \angle 2$, $B = 120 \angle 77$, load: 170 MW, 0.85 PF @ 300 kV
Find V_s, δ

Load $\Rightarrow S_R \Rightarrow \frac{170\text{ MW}}{0.85} = 200\text{ MVA}$



$\frac{|V_s||V_R|}{|B|}$

$= \frac{|120|}{200\text{ MVA}}$

$\frac{|A||V_R|^2}{|B|} = \frac{0.88(300\text{ kV}^2)}{120}$

$= 660\text{ MVA}$

1 cm → 100 MVA

x = 6.6 cm → 6.6 MVA

② S_R load $= \frac{170}{0.88} = 200$ MVA , $\cos^{-1} 0.88 = 28$

P_{max} ?? $(\theta_B - \delta) = 0$

③ if the load is increased to 200 MVA at the same PF and $V_r = 300$ KV , find $V_s = ?$

Same way → $\frac{200}{0.88} = 235$ MW → 2.35 cm

④ find Q_c to keep V_R, V_s as in ① Graphically

example : find V_s, V_R

$P_R = 255$ MW , 0.8 PF lagging

$\delta = 30°$, $A = 0.8 \angle 2°$

$B = 120 \angle 77°$

Soln: $S_R = \frac{255}{0.8} = 316$ MVA

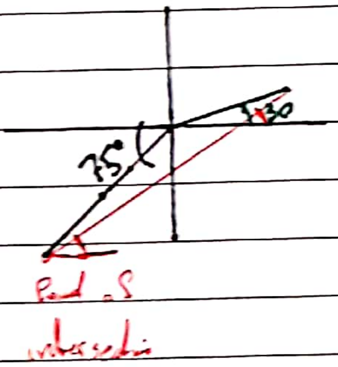
$\cos^{-1} 0.8 = 36°$

1 cm → 100 MVA

$\theta_B - \theta_A = 77° - 2° = 75°$

$X = 3.18$ cm → 318 MVA

$\theta_B - \delta = 77 - 30 = 47°$



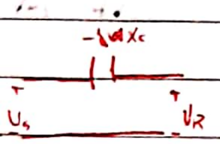
→ use graphical method to find value of vector length and then use $L\,(100\,\text{MVA}) = \frac{|V_S||V_R|}{|B|} \angle \theta_B - \delta$

Reactive Power compensation → controls Voltage by $\begin{cases} \text{Capacitor} \\ \text{inductor} \end{cases}$

Voltages → Peak demand "Voltage drop"
↳ Light load "Voltage rise"

∴ Series Capacitor is placed for Peak demand.
∴ Shunt ~~Capacitor~~ inductor → placed for Light load

Series Capacitor



$V_s = V_R + I_R(-jX_c)$

$I_s = I_R$

short T.L

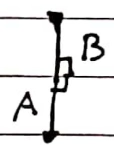
$$\begin{bmatrix} 1 & -jX_c \\ 0 & 1 \end{bmatrix}$$

$D = A = 1,\ B = -jX_c,\ C = 0$

T.L S.C

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} 1 & -jX_c \\ 0 & 1 \end{bmatrix} \quad V_R$$

$$\begin{bmatrix} A_{eq} & B_{eq} \\ C_{eq} & D_{eq} \end{bmatrix} = \begin{bmatrix} \ \end{bmatrix} \times \begin{bmatrix} \ \end{bmatrix}$$



$\therefore A_{eq} = A,\quad B_{eq} = \underset{\angle -90}{A(-jX_c)} + \underset{\angle 90}{B}$

so $B_{eq} \downarrow$

$V_s = AV_R + BI_R$ → let V_r = const and I_R fixed

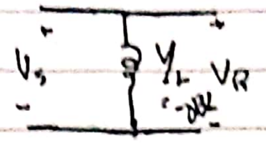
$B_{old} \Rightarrow V_{s1} = AV_R + BI_R$

$B_{eq\,new} \Rightarrow V_{s2} = AV_R + B_{eq}I_R$

$\therefore V_{s2} < V_{s1}$

$\therefore V_{s2} - V_R < V_{s1} - V_R \qquad V.D \downarrow$

shunt inductor



$V_s = V_R$
$I_s = I_R + V_R(jB_L)$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -jB_L & 1 \end{bmatrix}$$

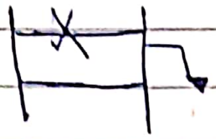
@ no load

$V_s = AV_R \Rightarrow V_R\Big|_{nl} = \frac{V_s}{A}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -jB_L & 1 \end{bmatrix}$$

if $A < 1 \rightsquigarrow V_R\uparrow$

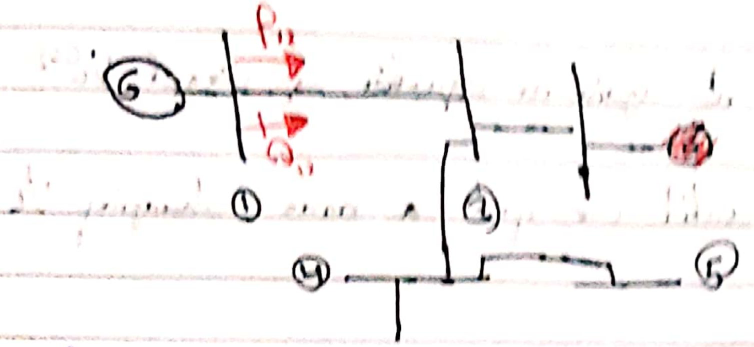
$\therefore$ obj $A\uparrow$



$A_{eq} = A + B(-jB_L)$
$\quad\quad\quad\quad\; 90^\circ \quad -90^\circ$

$\therefore A_{eq} > A$
$\hookrightarrow$ results in lower $V_R\downarrow = \frac{V_s}{A_{eq}\uparrow}$

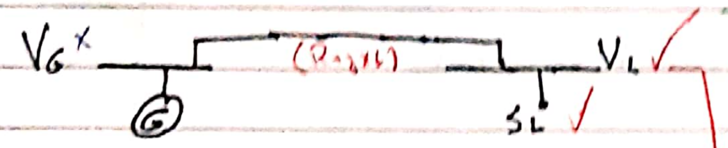
Power flow / Load flow (M) used in (1) planning (2) operation



we use this to find voltages
$V_1\angle\delta_1$, $V_3\angle\delta_3$
$V_2\angle\delta_2$, $V_4\angle\delta_4$
$V_5\angle\delta_5$ ... volts

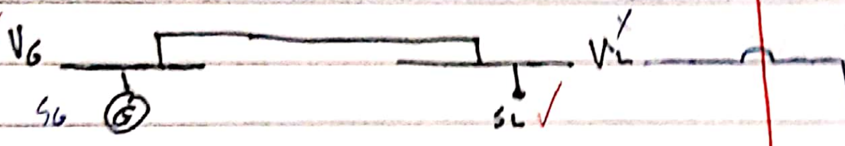
to find P, Q through transmission lines

(1)



Voltage and power known for same node

(2)



Voltage on 1 point, Power on 2nd point

$S_L = V_L I_L^*$

$$I_L = \frac{S_L^*}{V_L^*} = \frac{(P+jQ)^*}{V_L^*} = \frac{P-jQ}{(V_L^*)}$$

$V_G = V_L + I_L(R+jX) \rightarrow$ complete in here (we solved it the machines)

(4) $= (PR+QX)$, V_G??

V_L??

$$I_L = \frac{P_L - jQ_L}{V_L^*}$$

$$\therefore V_L = V_G - \frac{(P_L - jQ_L)(R+jX)}{V_L^*}$$

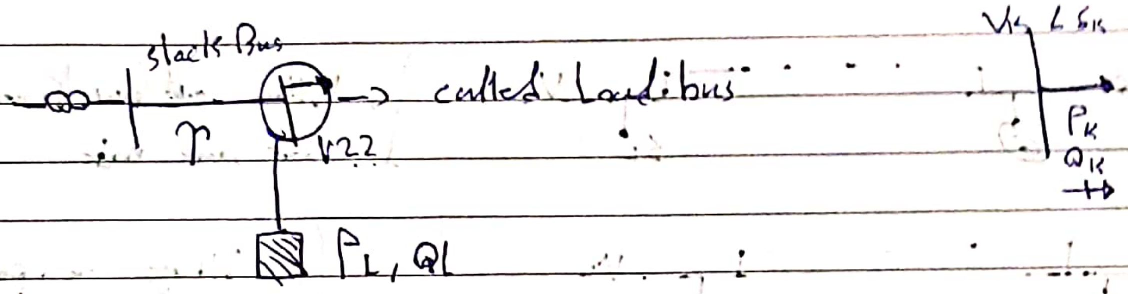
$x = f(x)$?? use numerical methods, to use iteration to solve this iteration set $V_L^{(0)} = 1\angle 0 \Rightarrow$ we get $V_L^{(1)}$

$V_L^{(1)}$ use it again in equation $\Rightarrow$ we get $V_L^{(2)}$

$V_L^{(2)}$ .... until we get a none changing V_L

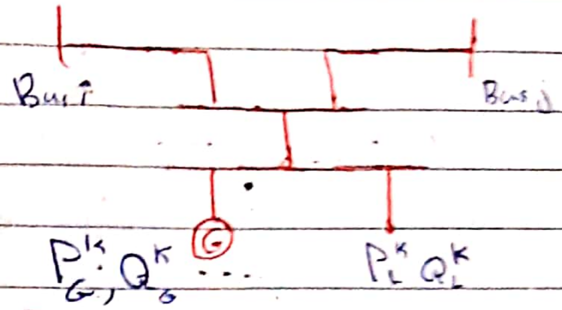
Power Flow Problem

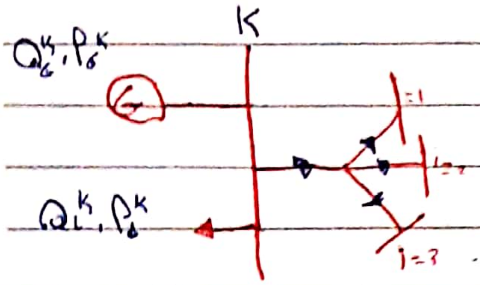
4 elements $|V_K|, \angle\delta_K, P_K, Q_K$



20/11

Power flow Problem





$$P_K^{injected} = P_G^k - P_L^k = \sum_{m=1}^{N} P_{km}$$
$$Q_K^{injected} = Q_G^k - Q_L^k = \sum_{m=1}^{N} Q_{km}$$

KCL

$$\text{so } P_2 = P_G^2 - P_L^2$$

Bus types.

1. PQ Bus → load Bus:
2. PV Bus
3. slack Bus, Ref Bus

	PQ Bus	PV Bus	slack
Known	P_K, Q_K	$P_K, \lvert V_K\rvert$	$\lvert V_K\rvert, \delta_K$
unknown	$\lvert V_K\rvert, \delta_K$	Q_K, δ_K	P_K, Q_K

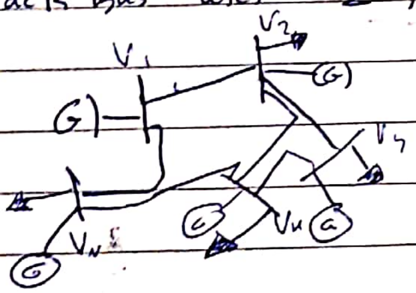
PQ Bus

- slack is the Ref Bus
- Only one slack Bus in a closed system

PV Bus: Q_K, δ_K → $P, \lvert V\rvert$ fixed → $\boxed{\text{Voltage control}}$ → controls $\lvert V\rvert$ by Q

→ P, Q

slack Bus uses $\sum P, Q$ Generated $= \sum P, Q$ Loads $+ \sum P, Q$ losses
↳ $f(V)$



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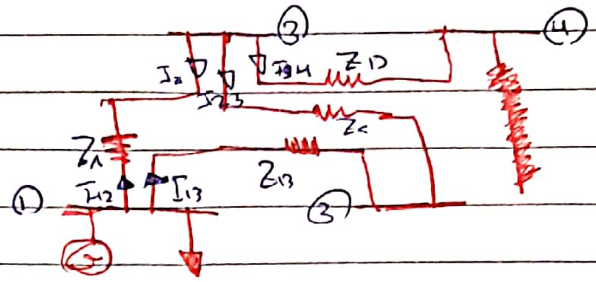
* Admittance Matrix



Bus ①

$$\sum I_{in} = \sum I_{out}$$

$$I_{G_1} - I_1 = I_{12} + I_{13}$$

$$I_{G_1} - I_L = \frac{(V_1 - V_2)}{Z_A} + \frac{V_1 - V_3}{Z_{13}}$$

$$\therefore \underbrace{I_{G_1} - I_{L_1}}_{\text{Injected}} = V_1(y_{13} + y_A) - V_2(y_A) - V_3(y_{13})$$

Bus ②

$$I_2 = I_{21} + I_{23} + I_{24} = \frac{V_2 - V_1}{Z_A} + \frac{V_2 - V_3}{Z_c} + \frac{V_2 - V_4}{Z_D}$$

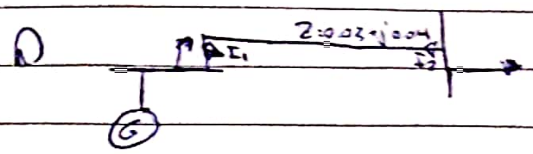
$$= V_1(-Y_A) + V_2(Y_A + Y_c + Y_D) - V_3(Y_c) - V_4(Y_D)$$

$$\therefore I = Y \cdot V$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} y_A + y_{13} & -y_A & -y_{13} & 0 \\ -y_A & Y_A + y_{13} + y_c & -y_c & -y_D \\ -y_B & -y_c & y_{13} + y_c & 0 \\ 0 & -y_D & 0 & y_D \end{bmatrix}$$

First col = first row

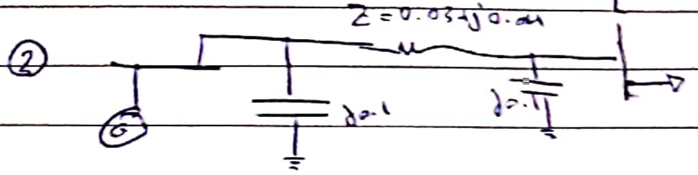
1



$$y = \frac{1}{Z} = 12 - j16$$

$$\begin{bmatrix} 12 - j16 & -12 + j16 \\ -12 + j16 & 12 - j16 \end{bmatrix}$$

2



$$\begin{bmatrix} 12 - j16 + 0.1 & -(12 - j16) + 0.1 \\ -(12 - j16) & 12 - j16 + j0.1 \end{bmatrix}$$

$S_1 = V_1 . I_1^*$ → S_1 injected , $S_2 = V_2 I_2^*$ → S_2 injected

Power Flow Solution

* Gauss Seidel → iteration

Solve $x - \sqrt{x} - 1 = \text{zero}$

0	1
1	2
2	2.4
:	
9	2.617398

$$S_i = V_i I_i^* = V_i \left[\sum_{j=1}^{n} y_{ij} V_j\right]^*$$

$$S_i^* = V_i^* \left[\sum_{j=1}^{n} y_{ij} V_j\right] = V_i^* \left[y_{ii} V_i + \sum_{\substack{j \neq i \\ j=1}}^{n} y_{ij} V_j\right]$$

$$V_i = \frac{1}{y_{ii}} \left[\frac{S_i^*}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}} y_{ij} V_j\right]$$

and $S_i = S_i^G - S_i^L$

$$S_i = V_i I_i^*$$

$$S_i = V_i \left[\sum_{j=1}^{N} y_{ij} V_j \right]^*$$

$$S_i^* = V_i^* \left[\sum_{j=1}^{N} y_{ij} V_j \right]$$

ex

$Z = 0.03 + j0.3$

① ——— ② $\longrightarrow$ short

$V_1 = 1\angle 0$ $\quad$ $0.1 + j0.4 = S$

Bus	Known	Type
①	$1\angle 0$	slack
②	S_2	PQ

$$P_2 = P_2^G - P_2^L = 0 - 1 = -1$$

$$Q_2 = Q_2^G - Q_2^L = 0 - 0.4 = -0.4$$

$$\therefore S_2 = -1 - j0.4 \quad , \quad y = \frac{1}{0.03 + j0.3} = 0.33 - j3.3$$

$$\therefore Y = \begin{bmatrix} 0.33 - j3.3 & -0.33 + j3.3 \\ -0.33 + j3.3 & 0.33 - j3.3 \end{bmatrix}$$

$$V_2 = \frac{1}{Y_{22}} \left[\frac{S_2^*}{V_2^*} - y_{21} V_1 \right]$$ 1st iteration $V_2^{(0)} = 1\angle 0$

$$V_2^{(1)} = \frac{1}{0.33 - j3.3} \left[\frac{-1 + j0.4}{1\angle 0} - (-0.33 + j3.3)\cdot(1\angle 0) \right]$$

$$V_2^{(1)} = 0.99\angle -16^\circ$$