

9

(9 points)

**Problem 1:** Solve the following short questions.

a)  $(42.3)_6$  is equal to  $(26.5)_{10}$

$$2 \times 6^0 + 4 \times 6^{-1} + 3 \times 6^{-2}$$

$$2 \times 1 + 24 + \frac{3}{18}$$

$$(26.5)$$

b)  $(368)_{10}$

is equal to  $(101110000)_2$

|     |     |   |
|-----|-----|---|
| 368 | 184 | 0 |
| 184 | 92  | 0 |
| 92  | 46  | 0 |
| 46  | 23  | 0 |

|      |    |   |
|------|----|---|
| 23/2 | 11 | 1 |
| 11/2 | 5  | 1 |
| 5/2  | 2  | 1 |
| 2/2  | 1  | 0 |
| 1/2  | 0  | 1 |

c)  $(375.47)_8$

is equal to  $(FD.9C)_{16}$

4 2 1  
4 4 2 1

$$(01111101.1001110)_2$$

POWERUNIT

- A 10
- B 11 = FD.9C
- C 12
- D 13

d)  $(00111001)_{\text{Excess3}}$

is equal to  $(012)_4$

39

~~37~~ (06)<sub>10</sub>

|     |   |   |
|-----|---|---|
| 6/4 | 1 | 2 |
| 1/4 | 0 | 1 |
| 0/4 | 0 | 0 |

3 2 1 0

(---)

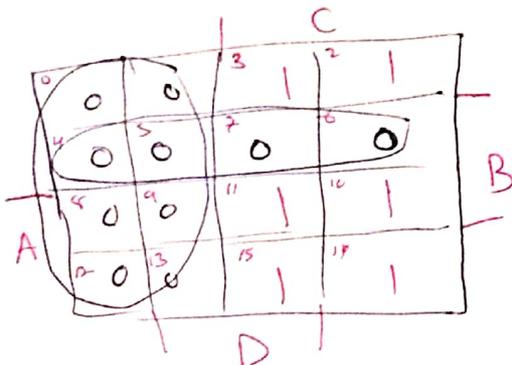
e) Assume  $N$  is a 4-digit number represented in numbering system of base 5 (i.e. radix=5). The minimum number of digits needed when representing  $N$  in decimal is 3

f) The Even parity bit for the following  $(010111011110111)_2$  code word is: 1

g) Given  $F(A, B, C, D) = (ABC + \bar{B}C)(C + A\bar{D})$ ,

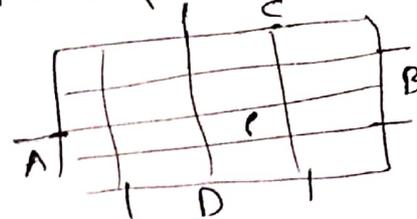
Determine  $F(A, B, C, D) = \prod_M(0, 1, 4, 5, 6, 7, 8, 9, 12, 13)$

$F = ABC + \bar{B}C + ABC\bar{D} + A\bar{B}C\bar{D}$



$\bar{F} = \bar{C} + \bar{A}\bar{B}$

$F = C \cdot (A + \bar{B}) = AC + \bar{B}C$



h) Given  $\bar{F}(A, B, C, D) = \sum_m(1, 2, 3, 5, 7, 10, 13, 15)$ ,

Determine  $F(A, B, C, D) = \sum_m(4, 6, 8, 9, 11, 12, 14)$

$\bar{C} + \bar{D}$

i) The Dual for the function  $F = (ABC + \bar{C}\bar{D}) \cdot B + C$  is:

$F = ((A+B+C) \cdot (\bar{C} \cdot \bar{D}) + B) \cdot C$

Problem 2. Using Boolean algebra, prove that:

$(A + \bar{B} + C)(A + C) + (A \oplus B) = A + B + C$

$A \oplus B = \bar{A}B + A\bar{B}$

(3 points)

$(A + \bar{B} + C)(A + C) + \bar{A}B + A\bar{B}$   
 $= AA + AC + A\bar{B} + \bar{B}C + AC + CC + \bar{A}B + A\bar{B}$   
 $A + AC + A\bar{B} + \bar{B}C + AC + C + \bar{A}B$   
 $A(1+C) + C(\bar{B}+1) + \bar{A}B + A\bar{B}$

$= A + C + \bar{A}B + \bar{A}B(A + \bar{A})$   
 $= A + C + \bar{A}B + \bar{A}B$   
 $A + C + \bar{A}B + \bar{A}B$   
 $= A + C + (\bar{A}B + \bar{A}) (\bar{A}B + B)$   
 $= A + C + (\bar{A} + \bar{B}) (\bar{B} + A) C$

$= A(1 + \bar{B}) + C + \bar{A}B$   
 $= A \cdot 1 + C + \bar{A}B$   
 $= A + \bar{A}B + C$   
 $= A + B + C \quad \#$

**Problem 3:** Given the following function:

(2 points)

$$F(A, B, C, D, E) = \underbrace{(A + \bar{B})}_{(1)} \cdot \underbrace{\bar{C}}_{(2)} + \underbrace{\bar{B}D}_{(3)} + \underbrace{\bar{A} \cdot (B + E)}_{(4) \text{ and } (5)}$$

2

Without any simplification, what is the literal cost (L), the gate-input cost (G) and the gate-input cost with inverters counted (GN), of F?

L = 8

G = 13

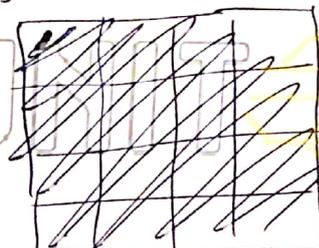
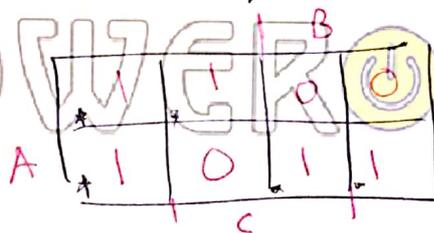
GN = 16

**Problem 4:** Fill the K-map of function F given by the following Boolean expression. You must label the K-map with the input variables. (2 points)

$$\begin{aligned} F(A, B, C) &= (A + \bar{B}) \cdot (\bar{A} + B + \bar{C}) \\ &= \cancel{A\bar{A}} + AB + A\bar{C} + \bar{A}\bar{B} + \cancel{B\bar{B}} + \bar{C}\bar{B} \\ &= AB + A\bar{C} + \bar{A}\bar{B} + \bar{C}\bar{B} \end{aligned}$$

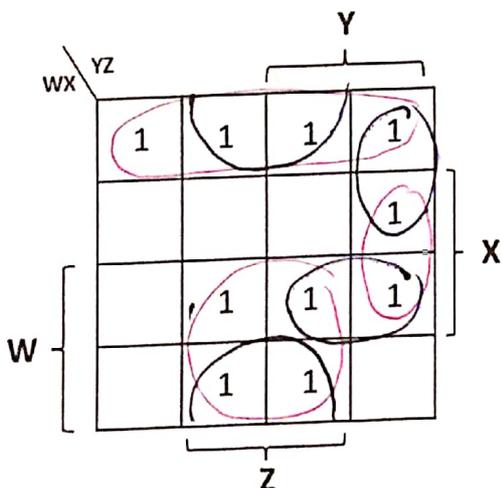
2

$F = \sum_m (0, 1, 4, 6, 7)$



**Problem 5:** Consider the following K-map for function F(W, X, Y, Z), identify the expressions of the six prime implicants and determine which are essential. (3 points)

3

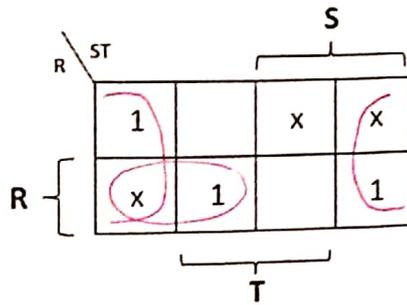


| Prime Implicant Expression | Is it Essential? |
|----------------------------|------------------|
| $\bar{W}\bar{X}$           | ✓                |
| $\bar{X}Z$                 | X                |
| $WZ$                       | ✓                |
| $\bar{W}Y\bar{Z}$          | X                |
| $XY\bar{Z}$                | X                |
| $WXY$                      | X                |

**Problem 6:** Given the K-map of function  $F(R,S,T)$ , write the optimized Boolean expression of  $F$  as Sum of Products (SoP). (2 points)

SoP

2

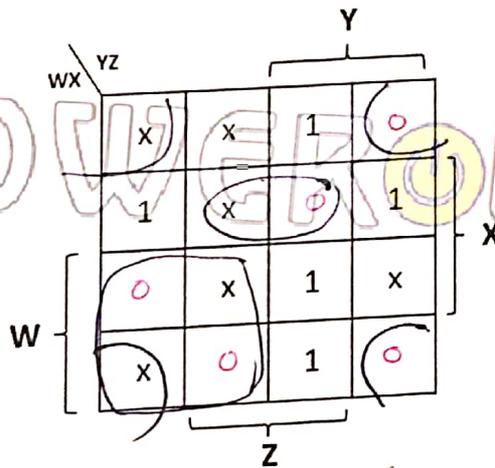


$$F(R,S,T) = \overline{T} + R\overline{S}$$

**Problem 7:** Given the K-map of function  $F(W,X,Y,Z)$ , write the optimized Boolean expression of  $F$  as Product of Sums (PoS). (3 points)

POS

3

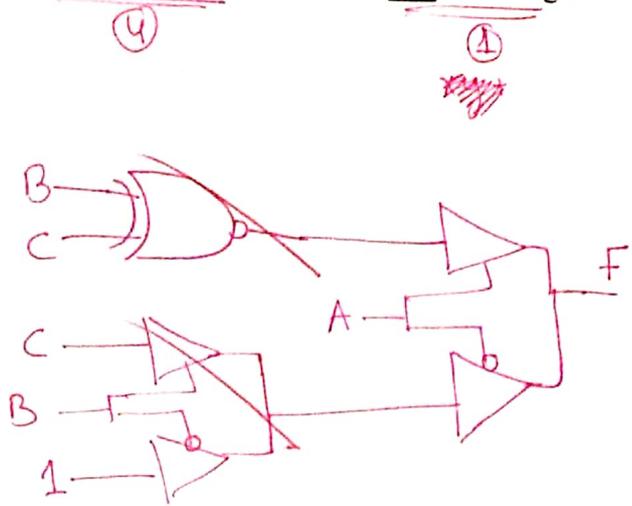


$$\overline{F} = \overline{X}\overline{Z} + \overline{W}XZ + W\overline{Y}$$

$$F = (X+\overline{Z})(W+\overline{X}+\overline{Z})(\overline{W}+Y)$$

$$F(W,X,Y,Z) = (X+\overline{Z})(W+\overline{X}+\overline{Z})(\overline{W}+Y)$$

**Problem 8.** The following table is the truth table for  $F(A,B,C)$ . Draw the implementation of the function using only four Tri-state buffers and one XNOR gate.

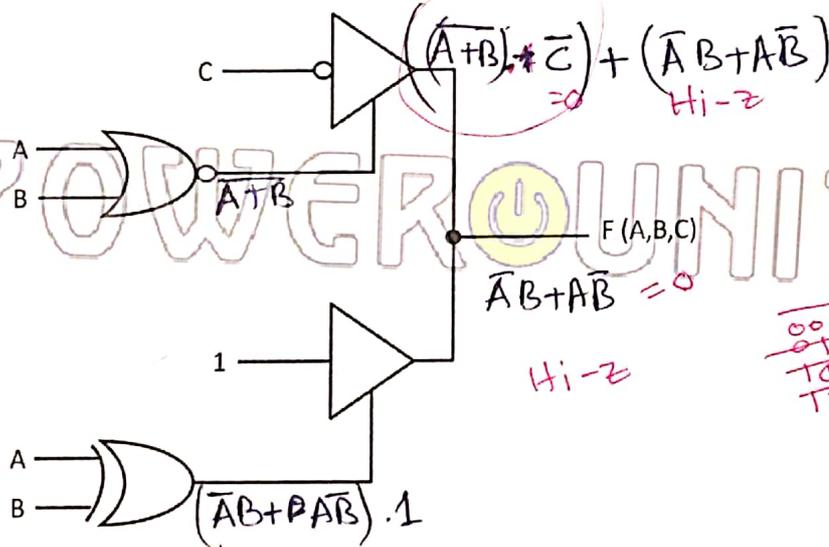


(3 points)

| A | B | C | F |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

**Problem 9.** Considering the following combinational logic circuit, answer the questions below:

(3 points)



a) Write the equation of output  $F$  without simplification:

$$F(A, B, C) = ((\overline{A+B}) \cdot \overline{C}) + (\overline{A}B + A\overline{B}) = ((\overline{A+B}) \cdot \overline{C}) + A \oplus B$$

b) If output  $F$  is at high-impedance state, what are the values of  $A$  and  $B$ :

A = ~~1~~  
 B = ~~1~~

# Digital Logic , First Exam, Summer 2019.

problem ① Solve the following short questions. [9 points]

A)  $(42.3)_6$  is equal to  $(26.5)_{10}$   
 $42.3 \Rightarrow 4 * 6^1 + 2 * 6^0 + 3 * 6^{-1} = 24 + 2 + 0.5 = 26.5$

B)  $(368)_{10}$  is equal to  $(101110000)_2$

|       |     |   |     |
|-------|-----|---|-----|
| 368/2 | 184 | 0 | LSD |
| 184/2 | 92  | 0 |     |
| 92/2  | 46  | 0 |     |
| 46/2  | 23  | 0 |     |
| 23/2  | 11  | 1 |     |
| 11/2  | 5   | 1 |     |
| 5/2   | 2   | 1 |     |
| 2/2   | 1   | 0 |     |
| 1/2   | 0   | 1 | MSD |

$= (101110000)_2$

C)  $(375.47)_8$  is equal to  $(FD.9C)_{16}$

$(375.47)_8 = (01111101.10011100)_2 = (FD.9C)_{16}$

D)  $(00111001)_{\text{Excess}_3}$  is equal to  $(12)_4$

\* أول (تس) بنحو من BCD و Excess 3

\* و بعد من BCD و decimal و من عشری ل [4]

$= (0000110)_2$   
 $= (06)_{10}$

$\frac{6}{4}$  1 2 LSD

$\frac{1}{4}$  0 1 MSD

$\Rightarrow (012)_4$



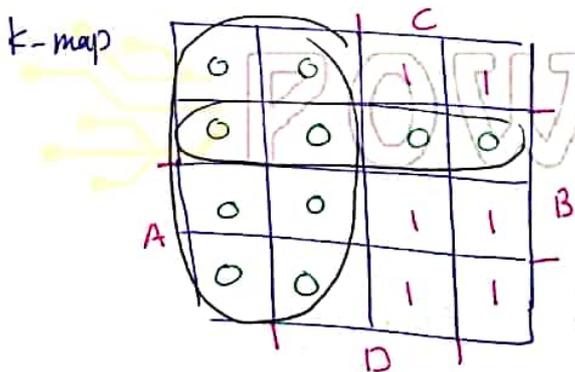
E) Assume  $N$  is a 4-digit number represented in numbering system of base 5 (i.e radix = 5). The minimum number of digits needed when representing  $N$  in decimal is  $\boxed{3}$

بقانونه  $n = \lceil \log_2 M \rceil$  ويمكن إذا ما عرفنا كم الجواب  
 نخطو اللوغاريتم بـ 2 ما كانه  $\boxed{3}$

F) The Even parity bit for the following  $(010111011110111)_2$  is  $\boxed{1}$

G) Given  $F(A, B, C, D) = (ABC + \bar{B}C)(C + A\bar{D})$   
 determine  $F(A, B, C, D) = \sum_m(0, 1, 4, 5, 6, 7, 8, 9, 12, 13)$

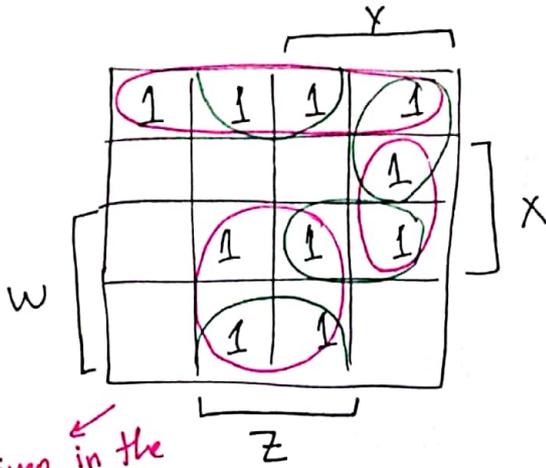
$$F = ABC + \bar{B}C + ABC\bar{D} + A\bar{B}C\bar{D}$$



H) Given  $\bar{F}(A, B, C, D) = \sum_m(1, 2, 3, 5, 7, 10, 13, 15)$ .  
 determine  $F(A, B, C, D) = \sum_m(0, 4, 6, 8, 9, 11, 12, 14)$

I) The Dual for the function  $F = (ABC + \bar{C}\bar{D}). B + C$  is  
 $F = ((ABC) + (\bar{C} + \bar{D})) B + C$   
 dual  $F = (((A + B + C) \cdot (\bar{C} \cdot \bar{D})) + B) \cdot C$

problem [5]: Consider the following K-map for function  $F(w, x, y, z)$ , identify the expressions of the Six prime implicants and determine which are essential. [3 points]

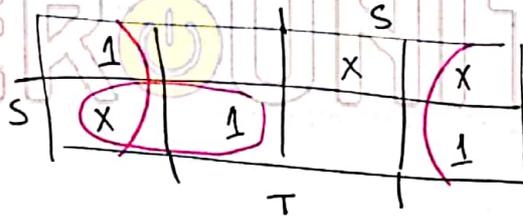


Given in the question.

| prime implicant Expression | Is it Essential |
|----------------------------|-----------------|
| $\bar{w} \bar{x}$          | ✓               |
| $\bar{x} z$                | X               |
| $w z$                      | ✓               |
| $\bar{w} y \bar{z}$        | X               |
| $x y \bar{z}$              | X               |
| $w x y$                    | X               |

problem [6]: Given the K-map of function  $F(R, S, T)$ , write the optimized boolean expression of  $F$  as sum of product: [3 points]

SOP



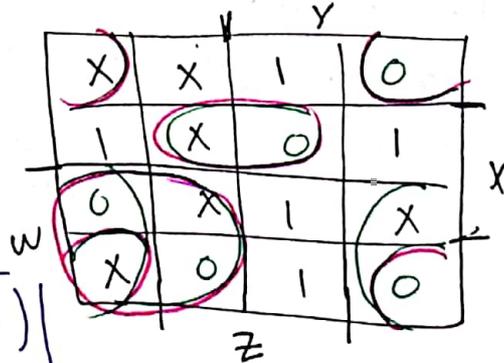
$$F(R, S, T) = \bar{T} + R\bar{S}$$

problem [7]: Given the K-map of function  $F(w, x, y, z)$ , write the optimized Boolean expression of  $F$  as product of sum: [3 points]

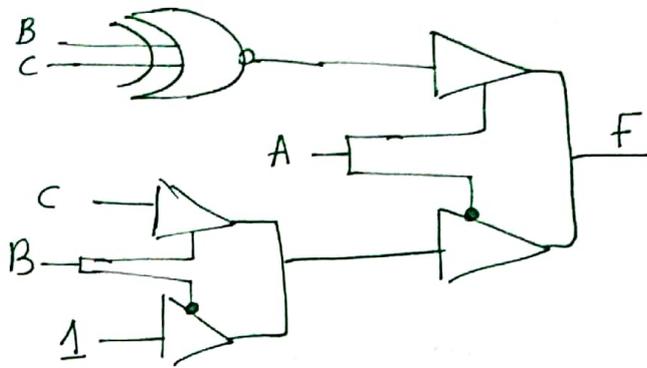
POS

$$\bar{F} = \bar{x}\bar{z} + \bar{w}xz + w\bar{y}$$

$$F = (x+z)(w+\bar{x}+\bar{z})(\bar{w}+y)$$



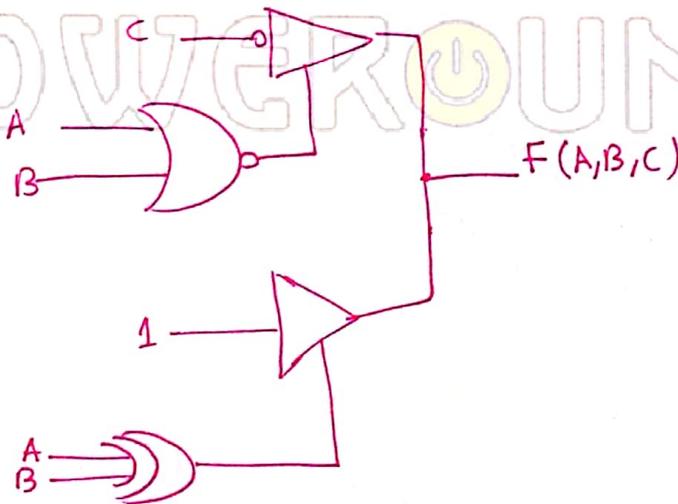
problem [8] : The following table is the truth table for  $F(A,B,C)$ .  
 Draw the implementation of the function using only four Tri-state buffers and one XNOR Gate. [3 points]



| A | B | C | F |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

B XOR C

problem [9] : Considering the following Combinational logic Circuit, answer the questions below: [3 points]



A) write the equation of the output F:

$$F(A,B,C) = ((\overline{A+B}) \cdot \overline{C}) + A \oplus B$$

B) IF the output F is at high-impedance state what is the values of A and B:

$$A = 1$$

$$B = 1$$