* CH #1

* How to write a vector:-

↳ Cartesian: $\bar{A} = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$ OR (A_x, A_y, A_z)

Unit vector هدول اسمهم
قيمتهم = 1 بس بدلو على الاتجاه.
ومنهم بعرف إنو هاد ال Vector
موجود بال Cartesian coordinate.

↳ Cylindrical: $\bar{A} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}$ OR (A_ρ, A_ϕ, A_z)

↳ Spherical: $\bar{A} = A_r \hat{a_r} + A_\theta \hat{a_\theta} + A_\phi \hat{a_\phi}$ OR (A_r, A_θ, A_ϕ).

Note

don't mix between points & vectors!
$p(x, y, z) \rightarrow$ point
$\bar{A} = (A_x, A_y, A_z) \rightarrow$ vector

* Vector Magnitude:- (Scalar).

$$|\bar{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Scalar Quantities:-
time / mass / distance / temperature / entropy
electric potential / population
هدول ما الهم اتجاه بس قيمة

* Unit vector along a vector:-

$$\hat{a_A} = \frac{\bar{A}}{A} = \frac{A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

لما يطلب مني هاد الاشي
1) بطلع ال magnitude لل Vector
2) بقسم ال Vector على ال magnitude

Vector Quantities:-
Velocity / force / displacement
Electric field
هدول الهم قيمة + اتجاه.

* Operations on vectors :-

[1] Addition :

$\bar{C} = \bar{A} + \bar{B}$

$= (A_x + B_x)\,\hat{a_x} + (A_y + B_y)\,\hat{a_y} + (A_z + B_z)\,\hat{a_z}$

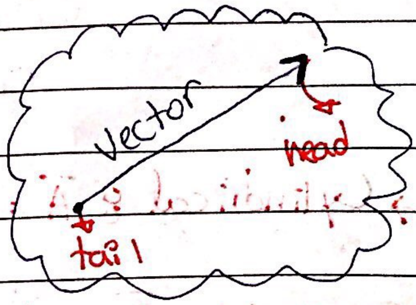
هاي بتدل على العملية الي عندي

from vector definition.

- Graphically :-

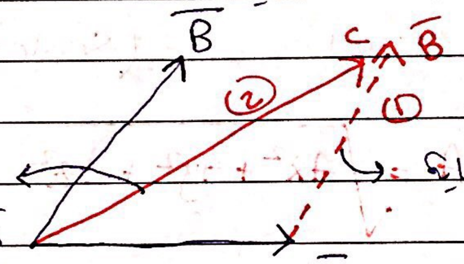
كيف ارسم جمع 2 vectors ؟

↳ Arrow method.



- عند نهاية ال vector الأول (يعني ال head)
بجيب ال vector الثاني، برسمه، بعد هيك
حاصل الجمع بكون توصيل بين ال tail الأول
ل head الثاني.

مثال



وصلت بينه tail $\bar{A}$ و head $\bar{B}$

رسمت $\bar{B}$ عند نهاية $\bar{A}$ (عند head $\bar{A}$)

[2] Subtraction

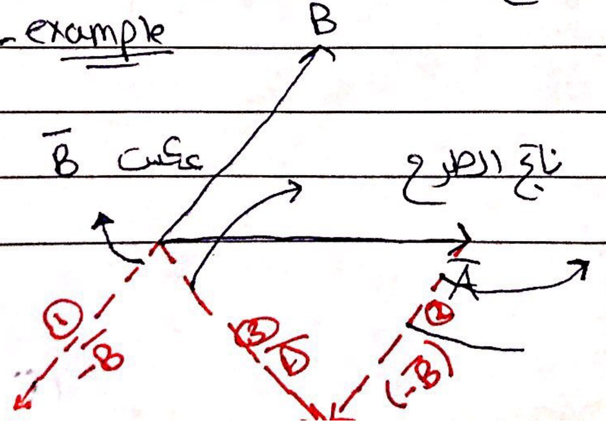
$\bar{C} = \bar{A} - \bar{B}$

$= (A_x - B_x)\,\hat{a_x} + (A_y - B_y)\,\hat{a_y} + (A_z - B_z)\,\hat{a_z}$

- Graphically :-

نفس طريقة الرسم، لكن لازم أعكس ال vector الي بعد إشارة الطرح.

- example



عكس $\bar{B}$

ناتج الطرح

جبت $\bar{B}$ عند نهاية $\bar{A}$

- Application on Subtraction is Distance (vector)

$P(x_1, y_1, z_1)$ → Point

$\bar{r}_{PQ}$ → distance vector from P to Q

$Q(x_2, y_2, z_2)$ ← point

$\bar{r}_P = \bar{r}_{OP} = \bar{r}_P - \bar{r}_O$

المسافة لأي نقطة هي المسافة بين النقطة نفسها ونقطة الأصل

$$\bar{r}_{PQ} = \bar{r}_Q - \bar{r}_P$$
$$= (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$ → هيك طلعت الـ distance vector .

- إذا طلب الـ distance لحال يعني قيمة فقط بآخذ الـ magnitude للـ $\bar{r}_{PQ}$ حسب القانون .

$$\Rightarrow |\bar{r}_{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- إذا طلب الـ direction

$$\hat{a}_{r_{PQ}} = \frac{\bar{r}_{PQ}}{r_{PQ}} = -\hat{a}_{r_{QP}}$$

عكس الاتجاه

3) Multiplication.

(a) Dot product.

$$\bar{A} \odot \bar{B} = |\bar{A}|\,|\bar{B}| \cos\theta_{(AB)}$$ → هاد القانون بنستخدمه لما أكون بعرف الزاوية بين الـ Vectors.

إشارة الـ dot product

- إذا ما بعرف الزاوية بنفهم بستخدم هاد القانون ⟸

$$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z$$

رح يطلع معي الجواب رقم يعني scalar

$$\theta_{AB} = \cos^{-1}\left(\frac{\bar{A} \cdot \bar{B}}{|\bar{A}|\,|\bar{B}|}\right)$$ ⟸ لو طلب مني الزاوية بستخدم هاد القانون

* ليش الـ dot product بطلع قيمته Scalar ؟

لأنه القانون بعتمد على $\cos\theta$ ، ولما آجي أضرب vectors

بضرب الـ terms المتشابهة ، الزاوية بين الـ unit vector

الهم = صفر ، $\cos(0) = 1$

توضيح ↵ $A_x \hat{a}_x \cdot B_x \hat{a}_x$

$= A_x B_x (\hat{a}_x \cdot \hat{a}_x)$ أخذنا ثابتها عامل مشترك

$= A_x B_x \; (1)$

لأنه الزاوية بينهم صفر

$$\hat{a}_n \cdot \hat{a}_m = \begin{cases} 1, & n = m \\ 0, & n \neq m. \end{cases}$$

properties of dot product :-

1) $\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$

2) $\bar{A} \cdot \bar{A} = A^2 = A_x^2 + A_y^2 + A_z^2$

3) $\bar{A} \cdot (\bar{B} + \bar{C}) = \bar{A} \cdot \bar{B} + \bar{B} \cdot \bar{C}$

↳ b) Cross product (it has magnitude + direction).

$|\bar{A} \times \bar{B}| = AB \sin\theta_{AB}$ ← هاد القانون بستخدمه لما أعرف الزاوية بين $\bar{A}$ و $\bar{B}$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

⇐ إذا ما بعرف الزاوية بينهم بحل cross product بينهم باستخدام الـ matrix

أول صف بحط الـ unit vector

ثاني صف بحط أول vector

وثالث صف بحط ثاني vector

$\bar{A} \times \bar{B} = (-1)^{1+1} (A_y B_z - A_z B_y) \hat{a}_x$

$+ (-1)^{1+2} (A_x B_z - A_z B_x) \hat{a}_y$

$+ (-1)^{1+3} (A_x B_y - A_y B_x) \hat{a}_z$

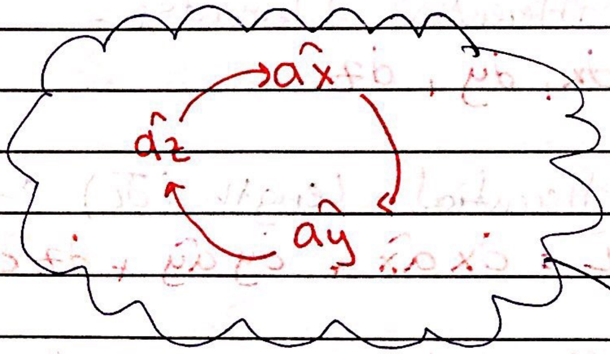
← أول مرة بخفي أول عامود ، بضرب الثاني والثالث ببعض ، ثاني مرة بخفي ثاني عامود ، بضرب الأول والثالث ، وآخر اشي بخفي ثالث عامود ، بضرب الأول والثاني .

$$\theta = \sin^{-1}\left(\frac{|\bar{A}\times\bar{B}|}{|\bar{A}||\bar{B}|}\right)$$

* إذا طلب مني الزاوية ←
بطلع $\bar{A}\times\bar{B}$ باستخدام الـ matrix حسب القانون ، بعدين باخذ الـ magnitude للـ vector اللي طلع معي من الـ matrix ، وباخذ الـ magnitude لـ $\bar{A}$ و $\bar{B}$ ، وبقسم $\frac{|\bar{A}\times\bar{B}|}{|\bar{A}||\bar{B}|}$ ، وبعدين باخذ الـ $(\sin^{-1})$ للناتج القسمة.

Properties of cross product

1) $\bar{A}\times\bar{B} = -\bar{B}\times\bar{A}$ ⟶ السالب للاتجاه.
2) $\bar{A}\times(\bar{B}\times\bar{C}) \neq (\bar{A}\times\bar{B})\times\bar{C}$ ⟶ الأولوية للأقواس
3) $|\hat{a}_x \times \hat{a}_x| = 0$
لأنه الزاوية بينهم صفر
، $(\sin 0) = 0$



إذا مشيت مع عقارب الساعة بكون الجواب موجب ، وإذا عكس عقارب الساعة سالب .

* Vector projection along another vector:—

1) Scalar projection:-
$A_B = \bar{A}\cdot\hat{a}_B$

2) Vector projection:-
$\bar{A}_B = (\bar{A}\cdot\hat{a}_B)\,\hat{a}_B$, $\hat{a}_B = \frac{\bar{B}}{B}$

- لما يطلب مني Vector projection أول اشي بوجد $\hat{a}_B = \frac{\bar{B}}{B}$ اللي رح تكون Vector ، بعدين رح أعمل dot product بين $\bar{A}$ و $\hat{a}_B$ ، بطلع معي جواب scalar بضرب الجواب هاد بالـ $\hat{a}_B$.

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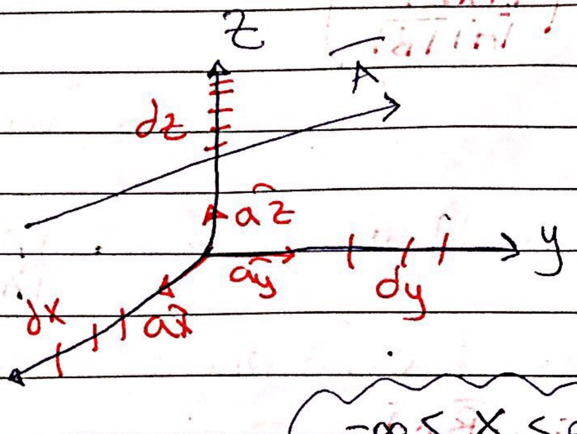
* CH #2

1) Cartesian Coordinates



$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$-\infty < x < \infty$
$-\infty < y < \infty$
$-\infty < z < \infty$

3D object
Infinite Box

- Differential Elements :-
dx, dy, dz

* differential Length ($\bar{dL}$) :-
$\bar{dL} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

- Differential Normal Surface Area ($\bar{ds}$).
$\bar{ds}_{front} = dy\,dz\,(\hat{a}_x)$ هاي اسمها العامودي

بالواجهة الأمامية بكون عندي الي طالع من ال object هو العامودي هو $\hat{a}_x$.

$\bar{ds}_{back} = -dy\,dz\,\hat{a}_x$ السالب للاتجاه لأنه العامودي
$\bar{ds}_{right} = dx\,dz\,\hat{a}_y$ للداخل مش للخارج .
$\bar{ds}_{left} = -dx\,dz\,\hat{a}_y$
$\bar{ds}_{top} = dx\,dy\,\hat{a}_z$
$\bar{ds}_{bottom} = -dx\,dy\,\hat{a}_z$

Differential Volume (dv)
$dv = dx\,dy\,dz$ دايمًا الحجم scalar مش vector.

* 2D Surface (we fix one variable only).

- if x = constant (not zero).
  ↳ infinite plane parallel to yz plane.

لو سألني شو العامودي على هاد ال plane ؟
لازم تكون ($\hat{a}_x$) أو ($-\hat{a}_x$) حسب إذا النقطة فوق ال plane أو تحته.

- if $x = 0$
  ↳ infinite plane along yz plane.

- if y = constant (not zero)
  ↳ infinite plane parallel to xz plane

- if $y = 0$:
  ↳ infinite plane along xz plane

- if z = constant (not zero)
  ↳ infinite plane parallel to xy plane

- if $z = 0$
  ↳ infinite plane along xy plane.

** 1D Segment (we fix two variables).

- if x, z are constants (not zero) ($x \neq 0$, $z \neq 0$).
  ↳ infinite line parallel to y-axis.

- if $x = 0$, $z = 0$
  ↳ infinite line along y-axis.

- if y, z constants ($y \neq 0$, $z \neq 0$).
  ↳ infinite line parallel to x-axis.

- if $y=0,\ z=0$.
  ↳ infinite Line along x-axis.

- if x, y are constants ($x \neq 0,\ y \neq 0$)
  ↳ infinite Line parallel to z-axis.

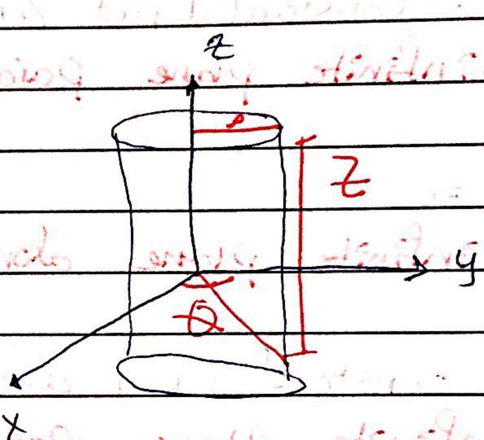
- if $x=0,\ y=0$.
  ↳ infinite Line along z-axis.

* Cylindrical coordinates.

$0 \leq \rho < \infty$

$0 \leq \phi \leq 2\pi$

$-\infty < z < \infty$



Infinite solid Cylinder.

$\bar{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$

* Differential Elements:-

$d\rho,\ \rho\, d\phi,\ dz$.

$\bar{dL} = d\rho\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$

$\bar{dS}_{top} = \rho\, d\rho\, d\phi\, \hat{a}_z$

$\bar{dS}_{bottom} = -\rho\, d\rho\, d\phi\, \hat{a}_z$

$\bar{dS}_{side} = \rho\, d\phi\, dz\, \hat{a}_\rho$

$\bar{dS}_{cut}$ (ϕ = constant) $= d\rho\, dz\, \hat{a}_\phi$

$dV = \rho\, d\rho\, d\phi\, dz$.

كيف عرفته؟ بحدد العامودي على السطح و بضرب بالباقي بيطلع

لما يحكيلي اشي هو const بكون عامودي

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* 2D Surface (we fix one variable).
  - ρ = constant → infente hollow cylinder
    $\rho = 0$ → inf line along z-axis

  - ϕ = constant → semi-infinite plane
    $\phi = 90°$ → semi-inf plane along yz plane.

  - z = constant → inf. Disk // xy plane
    z = 0 → inf. Disk along xy plane.

* 1 D Surface (we fix 2 variables).
  - ρ, ϕ constants & $\rho \neq 0$
    ↳ inf. Line parallel to z-axis

  - ρ, ϕ constants & $\rho = 0$
    ↳ inf. Line along ~~y~~ z-axis

  - ρ, z constants ($z \neq 0$, $\rho > 0$).
    ↳ Circle // xy plane

  - ρ, z constants ($z = 0$, $\rho > 0$)
    ↳ Circle along xy plane

  - ρ, z constants (& $\rho = 0$)
    ↳ point

  - ϕ, z constants
    ↳ semi-inf Line (Ray)

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* Spherical coordinates:-

$0 \leq r < \infty$
$0 \leq \theta \leq \pi$
$0 \leq \phi \leq 2\pi$

3D object
Infinite Solid Sphere.

$\bar{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$

* Differential Elements:-
dr, $r d\theta$, $r\sin\theta \, d\phi$

- $\bar{dL} = dr \, \hat{a}_r + r d\theta \, \hat{a}_\theta + r\sin\theta \, d\phi \, \hat{a}_\phi$

$\bar{ds}$ surface $= r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$

$\bar{ds}_\theta = r\sin\theta \, dr \, d\phi \, \hat{a}_\theta$

$ds_\phi = r dr \, d\theta \, \hat{a}_\phi$

$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$

* 2D Surface (we fix one variable).

- if r = constant $(r \neq 0)$ → hollow sphere.
- if $r = 0$ → point
- if θ = constant
  - ↳ $\theta \in (0, 90)$ and $(90, 180)$ → inf. hollow ~~sphere~~ cone.
  - ↳ $\theta = 90°$ → inf. disk along xy plane
  - ↳ $\theta = 0°$ → Semi-inf. Line in the +ve z-axis
  - ↳ $\theta = 180°$ → semi-inf Line in the -ve z-axis.

- if ϕ = constant → semi-inf. Disk
  - ↳ $\phi = 90°$ → semi-inf Disk along yz plane.

* 1D Segment (we fix 2 variables).

- if r, θ constants
  - ↳ $(r > 0), (\theta \neq 90°) \rightarrow$ Circle // xy plane
  - ↳ $(r > 0), (\theta = 90°) \rightarrow$ Circle along xy plane
  - ↳ $r = 0 \rightarrow$ point

- if r, ϕ constants.
  - ↳ $(r > 0) \rightarrow$ half circle
  - ↳ $(r = 0) \rightarrow$ point

- if θ, ϕ are constants
  - ↳ semi-inf. Line (Ray).

* Transformation between coordinates :-

المعطى   المطلوب

- $(x, y, z) \longrightarrow (\rho, \phi, z)$

$$\rho = \sqrt{x^2 + y^2} \quad , \quad \phi = \tan^{-1}\left(\frac{y}{x}\right) \quad , \quad z = z.$$

- $(\rho, \phi, z) \longrightarrow (x, y, z)$

$$x = \rho \cos\phi \quad , \quad y = \rho \sin\phi \quad , \quad z = z$$

- $(x, y, z) \longrightarrow (r, \theta, \phi).$

$$r = \sqrt{x^2 + y^2 + z^2} \quad , \quad \theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) \quad , \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

* $(r, \theta, \phi) \rightarrow (x, y, z)$

$x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

~~*~~

* $(\rho, \phi, z) \rightarrow (r, \theta, \phi)$

$r = \sqrt{\rho^2 + z^2}$, $\theta = \tan^{-1}\left(\frac{\rho}{z}\right)$, $\phi = \phi$

* $(r, \theta, \phi) \rightarrow (\rho, \phi, z)$

$\rho = r\sin\theta$, $\phi = \phi$, $z = r\cos\theta$

القوانين الي قبل كلها لتحويل نقاط فقط مش vectors

* Vectors Transformation

$$\begin{bmatrix} A\rho \\ A\phi \\ Az \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Ax \\ Ay \\ Az \end{bmatrix}$$

المطلوب — المعطى

هاي ال matrix بتكون معطاه بالامتحان.

$A\rho = \cos\phi\, Ax + \sin\phi\, Ay$.

:

لو طلب من ال cartesian وكان معطيني ال cylindrical كيف أحلها؟

باخذ ال transpose للماتركس بعني كل عامود بصير صف وزكل بنفس الطريقة.

* CH # 3

* Line Integral :-

$$\int_{L} \bar{A} \cdot \bar{dL}$$

ممكن تكون أي وحدة من القوانين الي أخذناهم .

* Surface Integral :-

$$\int_{S} \bar{A} \cdot \bar{ds}$$

* Volume Integral :-

$$\int |\bar{A}| \, dv$$

* Del operator (∇)

- in Cartesian

هاي اسمها Partial

$$\nabla v = \frac{\partial v}{\partial x}\hat{a}_x + \frac{\partial v}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

تطبيق على ∇

Gradient

$\nabla * V = \text{vector}$

vector ↓ scalar

- in cylindrical

$$\nabla u = \frac{\partial u}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial u}{\partial \phi}\hat{a}_\phi + \frac{\partial u}{\partial z}\hat{a}_z$$

- in spherical

$$\nabla T = \frac{\partial T}{\partial r}\hat{a}_r + \frac{1T}{r}\frac{\partial}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{a}_\phi$$

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* CH 4

- coulomb's Law

↳ the Force between two point charges is along the line joining them, directly proportional to Q_1, Q_2 and inversely proportional to the square of the distance between them.

$$F = k\,\frac{Q_1 Q_2}{R^2}\ (N), \quad k = \frac{1}{4\pi\epsilon_0}, \quad \epsilon_0 = \frac{10^{-9}}{36\pi}\ (F/m).$$

↳ هنا، القانون magnitude only

* F as a vector quantity:-

↳ ① $\bar{F}_{12} = \dfrac{Q_1 Q_2}{4\pi\epsilon_0 R^2}\,\hat{a}_{R_{12}}$ $\qquad \hat{a}_{R_{12}} = \dfrac{\bar{R}_{12}}{|R_{12}|}$

Force on Q_2 due to Q_1

↳ ② $\bar{F}_{12} = \dfrac{Q_1 Q_2 \cdot \bar{R}_{12}}{4\pi\epsilon_0 R^3}$

* ملاحظة:- قانون ① و ② و ③ نفس الشيء ولكن بأشكال مختلفة

↳ ③ $\bar{F}_{12} = \dfrac{Q_1 Q_2\,(\bar{r}_2 - \bar{r}_1)}{4\pi\epsilon_0\,|r_2 - r_1|^3}$

$\bar{F}_{12} = -\bar{F}_{21}$

نفس المقدار، عكس الاتجاه.

* Force due to N-Point charges:-

$$\bar{F} = \frac{Q}{4\pi\epsilon_0}\sum_{k=1}^{N}\frac{Q_k(\bar{r} - \bar{r}_k)}{|r - r_k|^3}$$

↳ Using superposition principle.

Field → اللي بدي أحسب عندها (اللي بأثروا عليها)

Source → اللي بأثروا على النقطة اللي بدي أحسب عندها.

Note:

1. the distance between Q_1 & Q_2 must be large compared to their bodies (point charges)

2. Q_1 & Q_2 must be static (at rest)

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* Electric field intensity: the force that a +ve charge experiences when placed in an Electric field.

$$\bar{E} = \frac{\bar{F}}{Q} \quad (N/C) \text{ or } (V/m)$$

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r = \frac{Q(\bar{r}-\bar{r}')}{4\pi\epsilon_0|r-r'|^3}$$

$r \rightarrow$ field
$r' \rightarrow$ source

* For N-point charges:-

$$E = \frac{1}{4\pi\epsilon_0}\sum_{k=1}^{N}\frac{Q_k(\bar{r}-\bar{r}_k)}{|r-r_k|^3}$$

Electric fields due to continuous charge distributions:-

$\rho_L \rightarrow$ Line charge density (C/m)
$\rho_s \rightarrow$ Surface charge density (C/m²)
$\rho_v \rightarrow$ Volume charge density (C/m³)

don't mix between ρ for cylindrical radial and ρ_L, ρ_s, ρ_v.

* Total charge:-

1) Line charge.

$$dQ = \rho_L\, dL \rightarrow Q = \int_L \rho_L\, dL$$

2) Surface charge

$$dQ = \rho_s\, ds \rightarrow Q = \int_S \rho_s\, ds.$$

3) Volume charge

$$dQ = \rho_v\, dv \rightarrow Q = \int_V \rho_v\, dv.$$

- Electric field intensity

1) Line charge.

$$\bar{E} = \int_L \frac{\rho_L\, dL}{4\pi\epsilon_0 R^2}\,\hat{a}_r$$

2) Surface charge

$$\bar{E} = \int_S \frac{\rho_s\, ds}{4\pi\epsilon_0 R^2}\,\hat{a}_r$$

3) Volume charge.

$$\bar{E} = \int_V \frac{\rho_v\, dv}{4\pi\epsilon_0 R^2}\,\hat{a}_r$$

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* Line Charge
- Finite line charge

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0\rho}\left[-(\sin\alpha_2 - \sin\alpha_1)\hat{a_\rho} + (\cos\alpha_2 - \cos\alpha_1)\hat{a_z}\right] \cdot (V/m)$$

- infinite line charge.

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a_\rho} \quad (V/m)$$

* Surface Charge.
- infinite sheet of charge

$$\bar{E} = \frac{\rho_s}{2\epsilon_0}\hat{a_n} \quad (V/m)$$, $\hat{a_n}$: Unit vector normal to the sheet.

- For parallel plate capacitor.

$$\bar{E} = \frac{\rho_s}{\epsilon_0}\hat{a_n} \quad (V/m)$$

Volume Charge

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a_r} \quad (V/m)$$

Electric Flux Density :-

$$\bar{D} = \epsilon_0 \bar{E} \quad (c/m^2)$$

infinite sheet of ~~charge~~ charge

$$\bar{D} = \frac{\rho_s}{2}\hat{a_n} \quad (c/m^2)$$

نفس قانون E بس بنشيل ϵ_0

infinite line charge

$$\bar{D} = \frac{\rho_L}{2\pi\rho}\hat{a_\rho} \quad (c/m^2)$$

Volume charge distribution

$$D = \int_v \frac{\rho_v\, dv}{4\pi R^2}\hat{a_r}$$

The flux ($\bar{D}$) is independent of the meduim
بيعتمد فقط على الشحنة وموقعها :

* Gauss's Law:-
↳ total electric flux through closed surface is equal to the total charge enclosed by that surface.

$$Q = \oint_S \overline{D} \cdot ds = \int_V \rho_v \, dv$$

سطح
مغلق
enclosed surface.

- First Maxwell's equation in integral form

$$\oint \overline{D} \cdot ds = \int_V \rho_v \, dv$$

- First Maxwell's equation in point form

$$\rho_v = \nabla \cdot \overline{D}$$

* Electric Potential:-

$$W = -Q \int_A^B \overline{E} \cdot \overline{dL} \quad (J)$$

W: work (potential energy) required to move Q from point A to point B.

negative sign means the work is done by an external agent.

$$V_{AB} = \frac{W}{Q} = -\int_A^B \overline{E} \cdot \overline{dL} \quad (J/C) \; (V)$$

V_{AB}: potential difference between A & B
A is the initial point
B is the final point

if V_{AB} is positive
↳ gain in potential
↳ external agent performs the work

if V_{AB} is negative
↳ loss in potential
↳ work is being done by the field.

$$V_{AB} = V_B - V_A$$

* For a point charge:-

$$V_{AB} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

إذا ما كان معطيني ref. point
بالسؤال بعتبرها ∞ و
$V_\infty = 0$

$$V(r) = \frac{Q}{4\pi\epsilon_0 |r - r'|}$$

* Line charge

$$V(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho_L \, dL'}{|r - r'|}$$

- Surface charge

$$V(r) = \frac{1}{4\pi\epsilon_0}\int_S \frac{\rho_s \, ds'}{|r - r'|}$$

- Volume charge

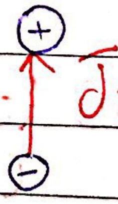
$$V(r) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho_v \, dV'}{|r - r'|}$$

- $\boxed{E = -\nabla V}$

$\bar{E}$ is opposite to the direction in which V increases

Electric dipole:-

$V = \frac{Q\,(r_2 - r_1)}{4\pi\epsilon_0 r_1 r_2}$ when locations are given.



or

$$V = \frac{Q\,d\cos\theta}{4\pi\epsilon_0 r^2}$$

- define dipole moment $\bar{P} = Q\bar{d}$

$$V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2} = \frac{P\cdot(\bar{r} - \bar{r}')}{4\pi\epsilon_0 |r - r'|^3}$$

or

$$V = \frac{\bar{P}\cdot\hat{a}_r}{4\pi\epsilon_0 r^2} \qquad \bar{E} = \frac{P}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right)$$

* Energy Density of Electrostatic fields:-

$$W_E = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k \quad (J)$$ for point charges.

- for line charge

$$W_E = \frac{1}{2}\int_L \rho_L V dL \quad (J)$$

- for surface charge

$$W_E = \frac{1}{2}\int_S \rho_s V ds \quad (J)$$

- for volume charge

$$W_E = \frac{1}{2}\int_V \rho_v V dv \quad (J)$$

* To find energy from $\bar{E}$

$$W_E = \frac{1}{2}\int_V \bar{D}.\bar{E}\, dv \quad (J)$$

or

$$W_E = \frac{1}{2}\int_V \epsilon_0 E^2 dv \quad (J)$$ $\bar{D} = \bar{E}\epsilon_0$

or

$$W_E = \frac{1}{2}\int_V \frac{D^2}{\epsilon_0} dv \quad (J)$$ $\bar{E} = \frac{\bar{D}}{\epsilon_0}$

* Energy density (w_E)

$$w_E = \frac{dW_E}{dv} = \frac{1}{2}\bar{E}.\bar{D} = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\frac{D^2}{\epsilon_0} \quad (J/m^3)$$

$$W_E = \int_V w_E\, dv \quad (J/m^3)$$

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CH5 - Materials based on it's electrical properties

↳ Conductors ($\sigma >>> 1$)

i.e. (Cu, Al, Ag, Lead)

↳ Semi-conductors ($\sigma > 1$)

i.e. (Si, GaAs, Ge)

↳ Dielectrics (Insulators) ($\sigma <<< 1$)

i.e (Li, Mica, polythelene, polyst..)

σ: conductivity (S/m)

↳ depends on:-

1) Temperature ($T\uparrow, \sigma\downarrow$)

2) Frequency.

$$\boxed{I = \frac{\Delta Q}{\Delta t} \quad (A)}$$

Current: the electric charge passing through an area per unit time.

- Current Density ($\bar{J}$)

$$\boxed{\bar{J} = \frac{\Delta I}{\Delta S}} \quad (A/m^2)$$

- Types of current:-

1] Convection Current

$I = \rho_v u_y \Delta S \quad (A)$

$\bar{J} = \rho_v \bar{u} \quad (A/m^2)$

2] Conduction Current

$\bar{J} = \sigma \bar{E}$

$\sigma = \frac{n^2 e \tau}{m}$

$\rho_v = ne$

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* Conductors:

↳ if the conductor is isolated.

↳ $\bar{E}$ inside $=0$.

$\bar{D}$ inside $=0$

ρ_v inside $=0$.

V_{ab} inside $=0$.

- The conductor is called equipotential body.

↳ if the conductor is not isolated

$$R = \frac{V}{I} = \frac{L}{\sigma S} = \frac{\rho_c L}{S} \quad \left(\rho = \frac{1}{\sigma}\right)$$

⇒ هاد القانون لما ال cross section لل conductor تكون uniform

$$R = \frac{V}{L} = \frac{\int_L \bar{E} \cdot d\bar{L}}{\int_S \sigma \bar{E} \cdot d\bar{S}}$$ → in general.

* power: -

$$P = \int_V \bar{J} \cdot \bar{E} \, dV$$

or

$$P = \sigma \int_V E^2 dV \ (W) \quad (\bar{J} = \sigma \bar{E})$$

or

$$P = \int_V \frac{J^2}{\sigma} dV \ (W) \quad \left(\bar{E} = \frac{\bar{J}}{\sigma}\right)$$

* Power Density.

$$w_p = \frac{dP}{dV} = \bar{J} \cdot \bar{E} = \sigma E^2 = \frac{J^2}{\sigma} \ (W/m^3)$$

$$P = \int_V w_p \, dV$$

- Non-polarized dielectrics:-

$$\bar{P} = \lim_{\Delta V \to 0} \frac{\sum_{k=1}^{N} Q_k \bar{d}_k}{\Delta V}$$

- Polarized dielectric:-

$$V = \int_S \frac{\rho_{ps}\, ds}{4\pi\epsilon_o r} \qquad V = \int_V \frac{\rho_{pv}\, dv}{4\pi\epsilon_o r}$$ هدول المعادلات للشحنات جوا ال dielectric

ρ_{ps}: polarized (bound) surface charge density
ρ_{pv}: polarized (bound) volume charge density

$$V = \int_S \frac{\rho_s\, ds}{4\pi\epsilon_o r} \qquad V = \int_V \frac{\rho_v\, dv}{4\pi\epsilon_o r}$$ free charge هدول المعادلات لل

$$* \boxed{\rho_{ps} = \bar{P} \cdot \hat{a}_n} \qquad \boxed{\rho_s = \bar{D} \cdot \hat{a}_n}$$

$$* \boxed{\bar{D} = \epsilon_0 \bar{E} + \bar{P}}$$

$$\boxed{\bar{P} = \chi_e \epsilon_0 \bar{E}}$$

$$\boxed{\bar{D} = \epsilon \bar{E}} \;,\; \boxed{\epsilon = \epsilon_r \epsilon_0}$$

$$\downarrow$$

$$\boxed{\epsilon_r = 1 + \chi_e}$$

$\hookrightarrow$ Relative permativity

$$* \boxed{\bar{P} = (\epsilon_r - 1)\epsilon_0 \bar{E}}$$ إذا مش معطيني χ_e بالسؤال

$$Q_{b^+} = \int_S \rho_{ps}\, ds$$

$$Q_{b^-} = \int_V \rho_{pv}\, dv$$

$Q_{btot} = 0 \longrightarrow$ material is electrically neutralized.

* Continuity Equation

$$\oint_S \overline{J} \cdot \overline{ds} = - \int_V \frac{d\rho v}{dt} \, dv$$

* Relaxation Time

$$T_r = \frac{\epsilon_o \epsilon_r}{\sigma} \quad (s)$$

$$\rho v = \rho v_o \, e^{-t/T_r} \quad (c/m^3)$$

ρv_o : initial charge density.