* CH #1

* How to write a vector :-

↳ Cartesian : $\bar{A} = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$ OR (A_x, A_y, A_z)

Unit vector هدول اسمهم

قيمتهم = 1 بس بدلو على الاتجاه .

ومنهم بعرف إنو هاد ال vector

موجود بال Cartesian coordinate.

↳ Cylindrical : $\bar{A} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}$ OR (A_ρ, A_ϕ, A_z)

↳ Spherical : $\bar{A} = A_r \hat{a_r} + A_\theta \hat{a_\theta} + A_\phi \hat{a_\phi}$ OR (A_r, A_θ, A_ϕ).

Note

don't mix between points & vectors!

$p(x, y, z) \rightarrow$ point

$\bar{A} = (A_x, A_y, A_z) \rightarrow$ vector

* Vector Magnitude :- (Scalar)

$$|\bar{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Scalar Quantities :-

time / mass / distance / temperature / entropy / electric potential / population

هدول ما الهم اتجاه بس قيمة

* Unit vector along a vector :-

$$\hat{a_A} = \frac{\bar{A}}{A} = \frac{A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

لما يطلب مني هاد الإشي

1) بطلع ال magnitude لل vector

2) بقسم ال vector على ال magnitude

Vector Quantities :-

Velocity / force / displacement

Electric field

هدول الهم قيمة + اتجاه .

* Operations on vectors :-

[1] Addition :

$\bar{C} = \bar{A} + \bar{B}$

$= (A_x + B_x)\,\hat{a}_x + (A_y + B_y)\,\hat{a}_y + (A_z + B_z)\,\hat{a}_z$

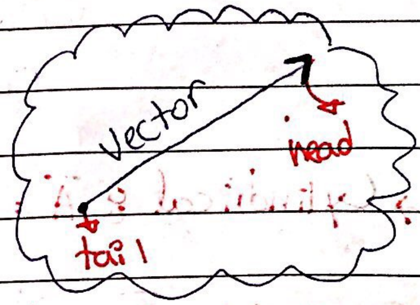
هاي بتدل على العملية الي عندي

from vector definition.

- Graphically :-

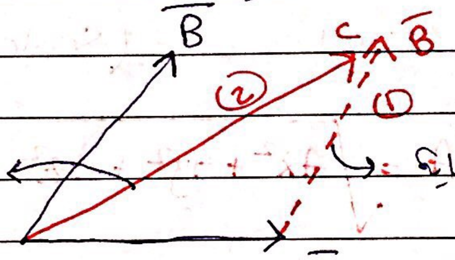
كيف ارسم جمع 2 vectors ؟

↳ Arrow method.



- عند نهاية ال vector الأول (يعني ال head)
بجيب ال vector الثاني و برسمه ، بعدين
حاصل الجمع بكون توصيل بين ال tail الأول
لـ head الثاني.

مثال



رسمت $\bar{B}$ عند نهاية $\bar{A}$ (عند head $\bar{A}$)

و وصلت بين tail $\bar{A}$ و head $\bar{B}$

[2] Subtraction

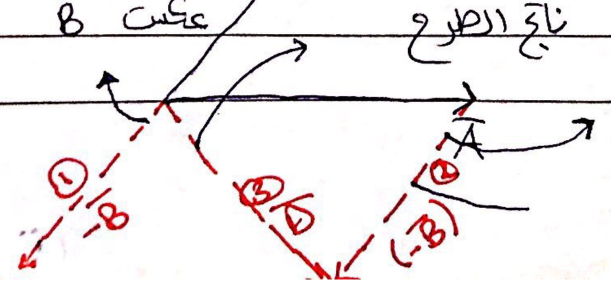
$\bar{C} = \bar{A} - \bar{B}$

$= (A_x - B_x)\,\hat{a}_x + (A_y - B_y)\,\hat{a}_y + (A_z - B_z)\,\hat{a}_z$

- Graphically :-

نفس طريقة الرسم، لكن لازم أعكس ال vector الي بعد إشارة الطرح.

- example



عكس $\bar{B}$

ناتج الطرح

حطيت $\bar{B}$ عند نهاية $\bar{A}$

- Application on Subtraction is Distance (vector)

$P(x_1, y_1, z_1)$ ← Point

$\bar{r}_{PQ}$ → distance vector from P to Q

$Q(x_2, y_2, z_2)$ ← point

$\bar{r}_P = \bar{r}_{OP} = \bar{r}_P - \bar{r}_O$

المسافة لأي نقطة هي المسافة بين النقطة نفسها ونقطة الأصل

$$\bar{r}_{PQ} = \bar{r}_Q - \bar{r}_P$$

$$= (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$ → هيك طلعت ال distance vector.

- إذا طلب ال distance لحال يعني قيمة فقط باخذ ال magnitude لل $\bar{r}_{PQ}$ حسب القانون. ⇐ $|\bar{r}_{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

- إذا طلب ال direction

$$\hat{a}_{r_{PQ}} = \frac{\bar{r}_{PQ}}{|\bar{r}_{PQ}|} = -\hat{a}_{r_{QP}}$$

$-$ ← عكس الاتجاه

3 Multiplication.

↳ a) Dot product.

$$\bar{A} \odot \bar{B} = |\bar{A}|\,|\bar{B}| \cos\theta_{(AB)}$$ → هاد القانون بستخدمه لما أكون بعرف الزاوية بين ال Vectors.

$\odot$ ← إشارة ال dot Product

- إذا ما بعرف الزاوية بنهم بستخدم هاد القانون ⇐

$$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z$$

رح يطلع يعني الجواب رقم يعني scalar

- لو طلب مني الزاوية بستخدم هاد القانون ⇐

$$\theta_{AB} = \cos^{-1}\left(\frac{\bar{A} \cdot \bar{B}}{|\bar{A}|\,|\bar{B}|}\right)$$

* ليش ال dot product بطلع قيمته Scalar ؟
لأنه لقانون بعتمد على $\cos\theta$, ولما آجي أضرب vectors
بضرب ال terms المتشابهة , الزاوية بين Unit vector
الهم = صفر , $\cos(0) = 1$

توضيح ↲ $A_x \hat{a}_x \cdot B_x \hat{a}_x$

$= A_x B_x (\hat{a}_x \cdot \hat{a}_x)$ أخذنا طلعنا بعامل مشترك

$= A_x B_x \ (1)$

لأنه الزاوية بينهم صفر

$$\hat{a}_n \cdot \hat{a}_m = \begin{cases} 1, & n=m \\ 0, & n \neq m. \end{cases}$$

properties of dot product :-

1) $\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$

2) $\bar{A} \cdot \bar{A} = A^2 = A_x^2 + A_y^2 + A_z^2$

3) $\bar{A} \cdot (\bar{B} + \bar{C}) = \bar{A} \cdot \bar{B} + \bar{B} \cdot \bar{C}$

↳ b) Cross product (it has magnitude + direction).

$|\bar{A} \times \bar{B}| = AB \sin\theta_{AB}$ ← هاد لقانون بستخدمه لما أعرف الزاوية بين $\bar{A}$ و $\bar{B}$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

إذا ما بعرف الزاوية بينهم بعمل cross product بينهم باستخدام ال matrix ⇐

أول صف بحط ال Unit vector
ثاني صف بحط أول vector
ثالث صف بحط ثاني vector

$\bar{A} \times \bar{B} = (-1)^{1+1} (A_y B_z - A_z B_y)\, \hat{a}_x$
$+ (-1)^{1+2} (A_x \cancel{B_z}^{B_z} - \cancel{A_z}^{A_z} B_x)\, \hat{a}_y$
$+ (-1)^{1+3} (A_x B_y - A_y B_x)\, \hat{a}_z$

← أول مرة بخفي أول عامود و بضرب الثاني والثالث ببعض , ثاني مرة بخفي ثاني عامود و بضرب الأول والثالث , وآخر اشي بخفي ثالث عامود و بضرب الأول والثاني .

$$\theta = \sin^{-1}\left(\frac{|\bar{A}\times\bar{B}|}{|\bar{A}||\bar{B}|}\right)$$

* إذا طلب مني الزاوية ←
بطلع $\bar{A}\times\bar{B}$ باستخدام الـ matrix حسب
القانون، بعدين باخد الـ magnitude للـ vector
الي طلع معي من الـ matrix ، و باخد
الـ magnitude لـ $\bar{A}$ و $\bar{B}$ ، بقسم $\frac{|\bar{A}\times\bar{B}|}{|\bar{A}||\bar{B}|}$ ، وبعدين باخذ الـ ($\sin^{-1}$) للناتج القسمة.

Properties of cross product

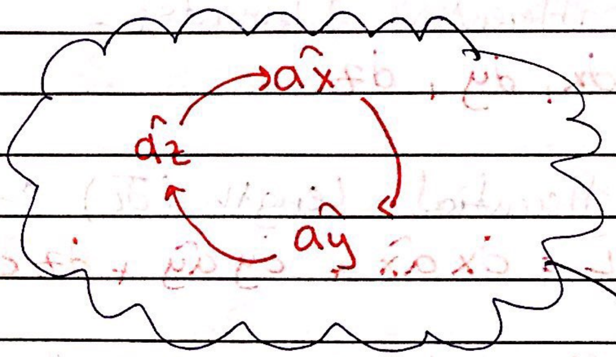
1) $\bar{A}\times\bar{B} = -\bar{B}\times\bar{A}$ → السالب للاتجاه.

2) $\bar{A}\times(\bar{B}\times\bar{C}) \neq (\bar{A}\times\bar{B})\times\bar{C}$ → الأولوية للأقواس

3) $|\hat{a}_x \times \hat{a}_x| = 0$

لأنه الزاوية بينهم صفر

$0 = (\sin 0)$



إذا مشيت مع عقارب الساعة بكون الجواب موجب، وإذا عكس عقارب الساعة سالب

* Vector projection along another vector: —

↳ 1) Scalar projection: —

$$A_B = \bar{A}\cdot\hat{a}_B$$

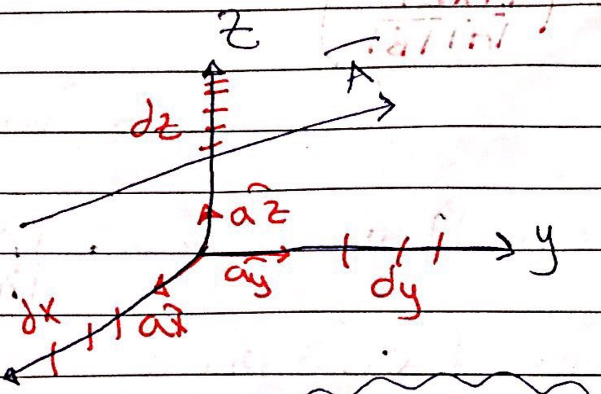
2) Vector projection: —

$$\bar{A}_B = (\bar{A}\cdot\hat{a}_B)\,\hat{a}_B \quad , \quad \hat{a}_B = \frac{\bar{B}}{B}$$

- لما يطلب مني Vector projection أول اشي بوجد $\hat{a}_B = \frac{\bar{B}}{B}$ الي رح تكون Vector ، بعدين رح أعمل dot product بين $\bar{A}$ و $\hat{a}_B$ ، بطلع معي جواب scalar بضرب الجواب هاد بالـ $\hat{a}_B$.

* CH #2

1) Cartesian Coordinates



$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$-\infty < x < \infty$

$-\infty < y < \infty$

$-\infty < z < \infty$

3D object
Infinite Box

-Differential Elements:-

dx, dy, dz

* differential Length ($\bar{dL}$) :-

$\bar{dL} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

-Differential Normal Surface Area ($\bar{ds}$).

$\bar{ds}_{front} = dy\,dz\,(\hat{a}_x)$ هاي اسمها العامودي

بالواجهة الأمامية بكون عندي اللي طالع من ال object هو العامودي هو $\hat{a}_x$.

$\bar{ds}_{back} = -dy\,dz\,\hat{a}_x$ السالب للاتجاه لأنه العامودي للداخل مش للخارج.

$\bar{ds}_{right} = dx\,dz\,\hat{a}_y$

$\bar{ds}_{left} = -dx\,dz\,\hat{a}_y$

$\bar{ds}_{top} = dx\,dy\,\hat{a}_z$

$\bar{ds}_{bottom} = -dx\,dy\,\hat{a}_z$

Differential Volume (dv)

$dv = dx\,dy\,dz$ دايماً الحجم scalar مش vector

Scanned with CamScanner

* 2D Surface (we fix one variable only).

- if x = constant (not zero).
  ↳ infinite plane parallel to yz plane.

لو سألني شو العامودي على هاد ال plane ؟

لازم تكون $(\hat{a}_x)$ أو $(-\hat{a}_x)$ حسب إذا النقطة فوق ال plane أو تحته.

- if $x = 0$
  ↳ infinite plane along yz plane.

- if y = constant (not zero)
  ↳ infinite plane parallel to xz plane

- if $y = 0$:
  ↳ infinite plane along xz plane

- if z = constant (not zero)
  ↳ infinite plane parallel to xy plane

- if $z = 0$
  ↳ infinite plane along xy plane.

** 1D Segment (we fix two variables).

- if x, z are constants (not zero) $(x \neq 0,\ z \neq 0)$.
  ↳ infinite line parallel to y-axis.

- if $x = 0,\ z = 0$
  ↳ infinite line along y-axis.

- if y, z constants $(y \neq 0,\ z \neq 0)$.
  ↳ infinite line paralle to x-axis.

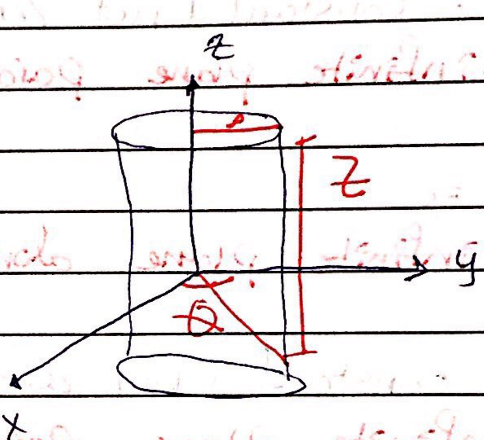
- if $y=0$, $z=0$.
  ↳ infinite Line along x-axis.

- if x, y are constants ($x \neq 0$, $y \neq 0$)
  ↳ infinite Line parallel to z-axis.

- if $x=0$, $y=0$.
  ↳ infinite Line along z-axis.

* Cylindrical coordinates.

$0 \leq \rho < \infty$
$0 \leq \phi \leq 2\pi$
$-\infty < z < \infty$



Infinite solid Cylinder.

$\bar{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$

* Differential Elements:-
* $d\rho$, $\rho\, d\phi$, dz.

$\bar{dL} = d\rho\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$.

$\bar{dS}_{top} = \rho\, d\rho\, d\phi\, \hat{a}_z$

$\bar{dS}_{bottom} = -\rho\, d\rho\, d\phi\, \hat{a}_z$

$\bar{dS}_{side} = \rho\, d\phi\, dz\, \hat{a}_\rho$

$\bar{dS}_{cut}$ (ϕ = constant) $= d\rho\, dz\, \hat{a}_\phi$

$dV = \rho\, d\rho\, d\phi\, dz$.

كيف عرفته ؟ بحدد العاموري على السطح و بضرب الباقيين ببعض

لما يحكيلي اشي هو const بكون عاموري

* 2D Surface (we fix one variable).
  - ρ = constant → infinte hollow cylinder
    $\rho = 0$ → inf line along z-axis

  - ϕ = constant → semi-infinite plane
    $\phi = 90°$ → semi-inf plane along yz plane.

  - z = constant → inf. Disk // xy plane
    $z = 0$ → inf. Disk along xy plane.

* 1 D Surface (we fix 2 variables).
  - ρ, ϕ constants & $\rho \neq 0$
    ↳ inf. Line parallel to z-axis

  - ρ, ϕ constants & $\rho = 0$
    ↳ inf. Line along z-axis

  - ρ, z constants ($z \neq 0$, $\rho > 0$).
    ↳ Circle // xy plane

  - ρ, z constants ($z = 0$, $\rho > 0$)
    ↳ Circle along xy plane

  - ρ, z constants (& $\rho = 0$)
    ↳ point

  - ϕ, z constants
    ↳ semi-inf Line (Ray)

* Spherical coordinates:-

$0 \leq r < \infty$

$0 \leq \theta \leq \pi$

$0 \leq \phi \leq 2\pi$

3D object
Infinite Solid Sphere.

$\bar{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$

* Differential Elements:-

dr, $r d\theta$, $r\sin\theta \, d\phi$

- $\bar{dL} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$

$\bar{ds}$ surface $= r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r$

$\bar{ds}_\theta = r\sin\theta\,dr\,d\phi\,\hat{a}_\theta$

$ds_\phi = r\,dr\,d\theta\,\hat{a}_\phi$

$dv = r^2\sin\theta\,dr\,d\theta\,d\phi$

* 2D Surface (we fix one variable).

- if r = constant ($r \neq 0$) → hollow sphere.
- if $r = 0$ → point
- if θ = constant
  - ↳ $\theta \in (0, 90)$ and $(90, 180)$ → inf. hollow ~~sphere~~ cone.
  - ↳ $\theta = 90°$ → inf. disk along xy plane
  - ↳ $\theta = 0°$ → Semi-inf. Line in the +ve z-axis
  - ↳ $\theta = 180°$ → semi-inf Line in the -ve z-axis.

- if ϕ = constant → semi-inf. Disk
  - ↳ $\phi = 90°$ → semi-inf Disk along yz plane.

* 1D Segment (we fix 2 variables).

- if r, θ constants
  - $(r > 0), (\theta \neq 90^\circ) \rightarrow$ Circle // xy plane
  - $(r > 0), (\theta = 90^\circ) \rightarrow$ Circle along xy plane
  - $r = 0 \rightarrow$ point

- if r, ϕ constants.
  - $(r > 0) \rightarrow$ half circle
  - $(r = 0) \rightarrow$ point

- if θ, ϕ are constants
  - Semi-inf. Line (Ray).

* Transformation between coordinates :-

المعطى ... المطلوب

- $(x, y, z) \longrightarrow (\rho, \phi, z)$

$$\rho = \sqrt{x^2 + y^2} \quad , \quad \phi = \tan^{-1}\left(\frac{y}{x}\right) \quad , \quad z = z.$$

- $(\rho, \phi, z) \longrightarrow (x, y, z)$

$$x = \rho \cos\phi \quad , \quad y = \rho \sin\phi \quad , \quad z = z$$

- $(x, y, z) \longrightarrow (r, \theta, \phi).$

$$r = \sqrt{x^2 + y^2 + z^2} \quad , \quad \theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) \quad , \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

* $(r, \theta, \phi) \rightarrow (x, y, z)$

$x = r\sin\theta\cos\phi \quad , \quad y = r\sin\theta\sin\phi \quad , \quad z = r\cos\theta$

* $(\rho, \phi, z) \rightarrow (r, \theta, \phi)$

$r = \sqrt{\rho^2 + z^2} \quad , \quad \theta = \tan^{-1}\left(\frac{\rho}{z}\right) \quad , \quad \phi = \phi$

* $(r, \theta, \phi) \rightarrow (\rho, \phi, z)$

$\rho = r\sin\theta \quad , \quad \phi = \phi \quad , \quad z = r\cos\theta$

القوانين الي قبل كلها لتحويل نقاط فقط مش Vectors

* Vectors Transformation

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

المطلوب

المعطى

هاي لـ matrix بتكون معطاة بالامتحان.

$A_\rho = \cos\phi \, A_x + \sin\phi \, A_y$.

لو طلب مني ال cartesian وكان معطيني ال cylindrical كيف أحلها؟

باخد ال transpose للماتركس يعني كل عامود بصير صف وبكمل بنفس الطريقة.

* CH # 3

* Line Integral :-

$$\int_{l} \bar{A} \cdot \bar{dL}$$

ممكن تكون أي وحدة من القوانين اللي أخذناهم .

* Surface Integral :-

$$\int_{s} \bar{A} \cdot \bar{ds}$$

* Volume Integral :-

$$\int |\bar{A}| \, dv$$

* Del operator (∇)

- in Cartesian

هاي اسمها Partial

$$\nabla v = \frac{\partial v}{\partial x} \hat{a_x} + \frac{\partial v}{\partial y} \hat{a_y} + \frac{\partial}{\partial z} \hat{a_z}$$

تطبيق على ∇

Gradient

$\nabla * V$ = vector

vector ↓ scalar

- in cylindrical

$$\nabla u = \frac{\partial u}{\partial \rho} \hat{a_\rho} + \frac{1}{\rho} \frac{\partial u}{\partial \phi} \hat{a_\phi} + \frac{\partial u}{\partial z} \hat{a_z}$$

- in spherical

$$\nabla T = \frac{\partial T}{\partial r} \hat{a_r} + \frac{1T}{r} \frac{\partial}{\partial \theta} \hat{a_\theta} + \frac{1}{r \sin\theta} \frac{\partial T}{\partial \phi} \hat{a_\phi}$$

*CH 4

- coulomb's Law

↳ the Force between two point charges is along the line joining them, directly propotional to Q_1, Q_2 and inversely propotional to the square of the distance between them.

$$F = k \frac{Q_1 Q_2}{R^2} \ (N), \quad k = \frac{1}{4\pi\epsilon_0}, \quad \epsilon_0 = \frac{10^{-9}}{36\pi} \ (F/m).$$

↳ هاد القانون magnitude only

*F as a vector quantity:-

↳ ① $\overline{F}_{12} = \dfrac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \hat{a}_{R_{12}}$ $\qquad \hat{a}_{R_{12}} = \dfrac{\overline{R}_{12}}{|R_{12}|}$

Force on Q_2 due to Q_1

↳ ② $\overline{F}_{12} = \dfrac{Q_1 Q_2 . \overline{R}_{12}}{4\pi\epsilon_0 R^3}$

* ملاحظة :-
قانون ① و ② و ③ نفس الاشي
ولكن بأشكال مختلفة

↳ ③ $\overline{F}_{12} = \dfrac{Q_1 Q_2 (\overline{r}_2 - \overline{r}_1)}{4\pi\epsilon_0 |r_2 - r_1|^3}$

$\overline{F}_{12} = - \overline{F}_{21}$
نفس المقدار، عكس الاتجاه.

*Force due to N-Point charges:-

$$\overline{F} = \frac{Q}{4\pi\epsilon_0} \sum_{k=1}^{N} \frac{Q_k (\overline{r} - \overline{r}_k)}{|r - r_k|^3}$$

↳ Using superposition principle.

Field → الي بدي أحسب عندها (الي بأثروا عليها)

ote

1. the distance between Q_1 & Q_2 must be large compared to their bodies (point charges)

2. Q_1 & Q_2 must be static (at rest)

Source → الي بأثروا على النقطة الي بدي أحسب عندها.

* Electric field intensity : the force that a +ve charge experiences when placed in an Electric field.

$$\bar{E} = \frac{\bar{F}}{Q} \quad (N/C) \text{ or } (V/m)$$

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r = \frac{Q(\bar{r}-\bar{r}')}{4\pi\epsilon_0|r-r'|^3}$$

$r \rightarrow$ field
$r' \rightarrow$ source

* For N- point charges:-

$$\bar{E} = \frac{1}{4\pi\epsilon_0}\sum_{k=1}^{N}\frac{Q_k(\bar{r}-\bar{r}_k)}{|r-r_k|^3}$$

Electric fields due to continuous charge distributions:-

$\rho_L \rightarrow$ Line charge density (C/m)

$\rho_s \rightarrow$ Surface charge density (C/m²)

$\rho_V \rightarrow$ Volume charge density (C/m³)

don't mix between ρ for cylindrical radial and ρ_L, ρ_s, ρ_V.

- Total Charge:-

1) Line charge.

$$dQ = \rho_L dL \rightarrow Q = \int_L \rho_L dL$$

2) Surface charge

$$dQ = \rho_s ds \rightarrow Q = \int_S \rho_s ds.$$

3) Volume charge

$$dQ = \rho_v dv \rightarrow Q = \int_V \rho_v dv.$$

- Electric field intensity

1) Line charge.

$$\bar{E} = \int_L \frac{\rho_L dL}{4\pi\epsilon_0 R^2}\hat{a}_r$$

2) Surface charge

$$\bar{E} = \int_S \frac{\rho_s ds}{4\pi\epsilon_0 R^2}\hat{a}_r$$

3) Volume charge.

$$\bar{E} = \int_V \frac{\rho_v dv}{4\pi\epsilon_0 R^2}\hat{a}_r$$

Scanned with CamScanner

* Line Charge

- finite line charge

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0\rho}\left[-(\sin\alpha_2 - \sin\alpha_1)\,\hat{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\,\hat{a}_z\right] \; (V/m)$$

- infinite line charge.

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\,\hat{a}_\rho \quad (V/m)$$

* Surface Charge.

- infinite sheet of charge

$$\bar{E} = \frac{\rho_s}{2\epsilon_0}\,\hat{a}_n \quad (V/m)$$, $\hat{a}_n$: Unit vector normal to the sheet.

- for parallel plate capacitor.

$$\bar{E} = \frac{\rho_s}{\epsilon_0}\,\hat{a}_n \quad (V/m)$$

Volume Charge

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r \quad (V/m)$$

Electric Flux Density :-

$$\bar{D} = \epsilon_0\,\bar{E} \quad (C/m^2)$$

infinite sheet of charge

$$\bar{D} = \frac{\rho_s}{2}\,\hat{a}_n \quad (C/m^2)$$

نفس قانون E بس بنشيل ϵ_0

infinite line charge

$$\bar{D} = \frac{\rho_L}{2\pi\rho}\,\hat{a}_\rho \quad (C/m^2)$$

Volume charge distribution

$$\bar{D} = \int_v \frac{\rho_v\,dv}{4\pi R^2}\,\hat{a}_r$$

The flux $(\bar{D})$ is independent of the meduim يعتمد فقط على الشحنة وموقعها :

* Gauss's Law:-

↳ total electric flux through closed surface is equal to the total charge enclosed by that surface.

$$Q = \oint_S \overline{D} \cdot ds = \int_V \rho v \, dv$$

دائرة

معناها

enclosed surface.

- First Maxwell's equation in integral form

$$\oint \overline{D} \cdot ds = \int_V \rho v \, dv$$

- First Maxwell's equation in point form

$$\rho v = \nabla \cdot \overline{D}$$

* Electric Potential:-

$$W = -Q \int_A^B \overline{E} \cdot \overline{dL} \quad (J)$$

: W: work (potential energy) required to move Q from point A to point B.

negative sign means the work is done by an external agent.

$$V_{AB} = \frac{W}{Q} = -\int_A^B \overline{E} \cdot \overline{dL} \quad (J/C) \; (V)$$

V_{AB}: potential difference between A & B

A is the initial point

B is the final point

if V_{AB} is positive

↳ gain in potential

↳ external agent performs the work

if V_{AB} is negative

↳ loss in potential

↳ work is being done by the field.

$$V_{AB} = V_B - V_A$$

* For a point charge:-

$$V_{AB} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

إذا ما كان معطيني ref point بالسؤال بعتبرها ∞ ،
$V_\infty = 0$

* Line charge

$$V(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho_L \, dl'}{|r-r'|}$$

- Surface charge

$$V(r) = \frac{1}{4\pi\epsilon_0}\int_s \frac{\rho_s \, ds'}{|r-r'|}$$

- Volume charge

$$V(r) = \frac{1}{4\pi\epsilon_0}\int_v \frac{\rho_v \, dv'}{|r-r'|}$$

* $\boxed{E = -\nabla V}$

→ $\bar{E}$ is opposite to the direction in which V increases

* Electric dipole:-

$$V = \frac{Q\,(r_2 - r_1)}{4\pi\epsilon_0 r_1 r_2}$$

or

$$V = \frac{Q\,d\cos\theta}{4\pi\epsilon_0 r^2}$$

- define dipole moment $\bar{P} = Q\bar{d}$

$$V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2}$$

or

$$V = \frac{\bar{P}\cdot\hat{a}_r}{4\pi\epsilon_0 r^2}$$

$$\bar{E} = \frac{P}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right)$$