DIFF

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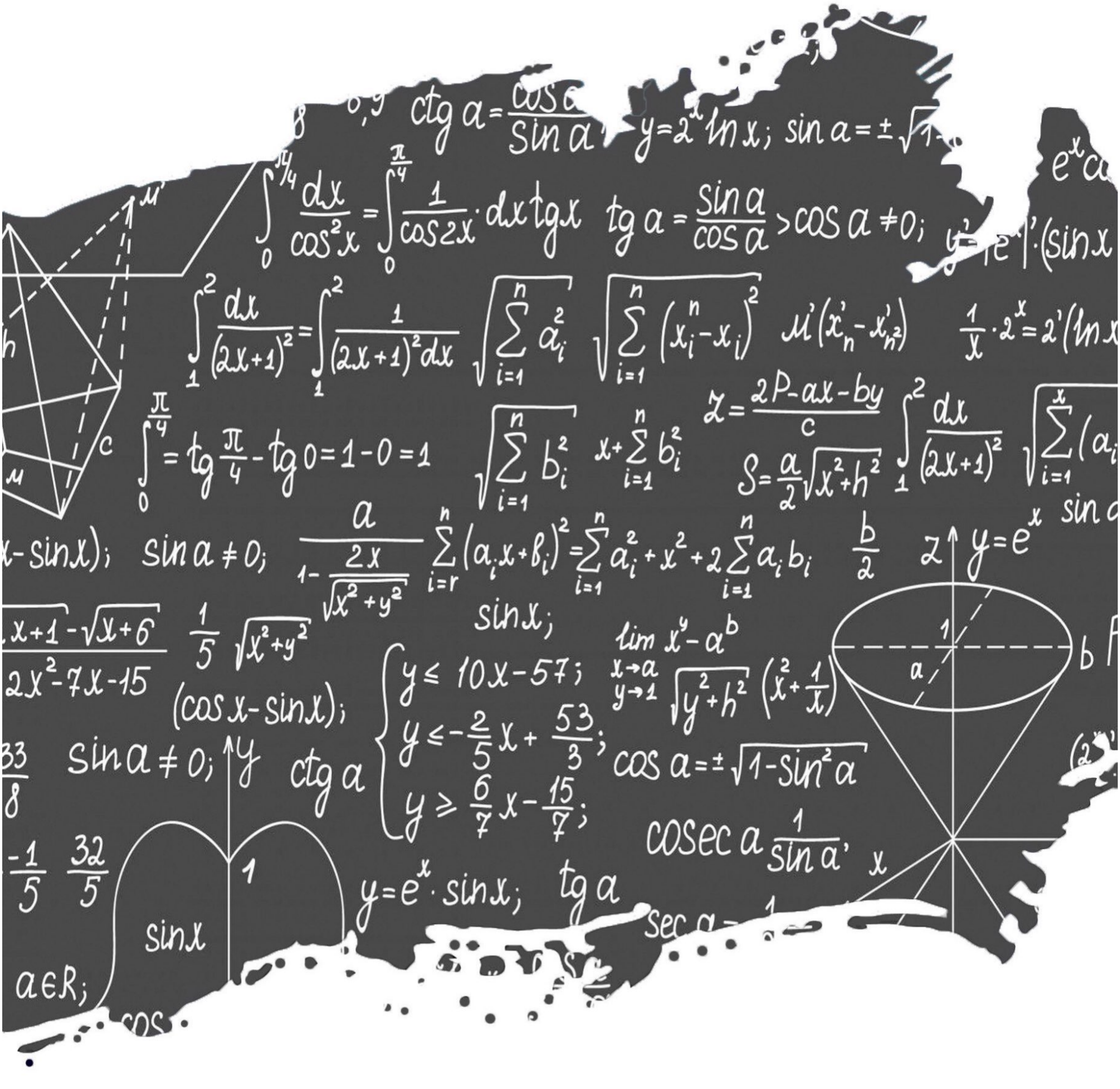

POWER UNIT

Ordinary diff equ:-

1.1 Basic concepts

① O.D.E: Eq wich contains the devirатives of thunknown y

ex: $2y' + y = 0$ O.D.E

$y' = (x+1)\, e^{-x}\, y^2$ O.D.E

$y'' - 2x(y'')^3 = 0$ O.D.E

- classifiction.

1) order the highest order of derivative

2) linear or non-linear

⊛ $a_n(x)\, y^{(n)} + a_{n-1}(x)\, y^{(n-1)} + \cdots + a(x)\, y' + a_0(x)\, y = f(x)$

ex: e^y, $\sqrt{y}$, $\sin y$, $y'y$, $(y'')^3$, y^2, ---- non-linear

3) homo or non homo

if $f(x) = 0 \Rightarrow$ ⊛ is homo

$f(x) \neq 0 \Rightarrow$ ⊛ is non homo

Ex:

1) $y'' - \sin x\, y' - y = \sin x$ 2^{nd} order, linear, non homo

2) $y''' - 2x(y'')^3 = 0$
3^{rd} order, N.Linear, homo

3) $(x-1)y' + y = 0$
1^{st} order, linear, homo

4) $y'' + 3y = 2$
2^{nd}, linear, non-homo

5) $y' + e^x y = x^2$
1^{st}, linear, non-homo

6) $(x + xy)\,dx + 2y\,dy = 0$ $\div dx$

$x + xy + 2y\,y' = 0 \rightarrow 1^{st}$, non linear, non homo

سؤال مشابه

$y'' - 4y' + 4y = 0$ 2^{nd}, linear, homo

$y'' - 4y' + 4 = 0$ 2^{nd}, linear, non-homo

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in O.D.E we are loking for sol $y(x)$ so the solu of O.D.E is $y(x)$ is satisfying the O.D.E.

Ex: show that $y(x) = 10 - ce^{-x}$, with c is constant is a solution of $y' + y = 10$

$y = 10 - ce^{-x}$

$y' = ce^{-x}$

$\cancel{ce^{-x}} + 10 - \cancel{ce^{-x}} = 10$

$10 = 10$ ✓

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* initial value problem:- (ivp)

$$O.D.E + IC = IVP$$

IC → $y(x_0) = y_0$ → to find c

ex: show that $y = Ce^{2x}$ is solu of the IVP $y' = 2y$, $y(0) = 1$

$y = ce^{2x}$

$1 = Ce^{2(0)}$

$\boxed{C = 1}$

$y = e^{2x}$

$y' = 2e^{2x}$

$y' = 2y$

$2e^{2x} = 2e^{2x}$ ✓

1.3 separable D.E

$f(x)\,dx = g(y)\,dy$ sep

the solu $\int f(x)\,dx = \int g(y)\,dy$

ex wich of the following D.E: sep

① $x \sin y\,dx + x^2\,dy = 0$

$$\frac{x \sin y\,dx}{-x^2 \sin y} = \frac{-x^2\,dy}{-x^2 \sin y} \Rightarrow -\frac{1}{x}\,dx = \frac{1}{\sin y}\,dy \text{ sep}$$

2) $x\,dx + x^2 y\,dy = 0$

$$\frac{x\,dx}{-x^2} = \frac{-x^2}{-x^2}\,y\,dy$$

$$\frac{-1}{x}\,dx = y\,dy \quad \text{sep}$$

3) $dx + x^2 y\,dy = 0$

$$\frac{dx}{-x^2} = \frac{-x^2}{x^2}\,y\,dy$$

$$\frac{dx}{-x^2} = y\,dy \quad \text{sep}$$

4) $(x+y)\,dx + x^2 \sin y\,dy = 0$ $\quad X$

ex solve

1) $\frac{dy}{dx} = y^2 \cos x \longrightarrow \int \frac{dy}{y^2} = \int \cos x \, dx$

$-\frac{1}{y} = \sin x + c \longrightarrow y = \frac{1}{-\sin x + c}$

إذا طلب y موضوع القانون

2) $\frac{dy}{dx} = x^2 y^2 + y^2 + x^2 + 1 \quad , \quad y(0) = 2$

$dy = \left(y^2 (x^2+1) + (x^2+1) \right) dx$

$dy = (y^2+1)(x^2+1) \, dx$

$\int \frac{dy}{y^2+1} = \int x^2+1 \, dx \rightarrow \tan^{-1} y = \frac{x^3}{3} + x + C$

$\tan^{-1} 2 = \frac{0^3}{3} + 0 + c \rightarrow \boxed{c = \tan^{-1} 2}$

3) $e^{x+y} dx = e^{x-2y} dy \rightarrow \frac{e^x e^y \, dx}{e^x e^y} = \frac{e^x e^{-2y} \, dy}{e^x e^y}$

$\int dx = \int e^{-3y} dy$ sep $\rightarrow x + c = e^{-3y} \times \frac{-1}{3}$

4) $dy - xy \, dx = (4y + 3x + 12) \, dx$

$dy = (xy + 4y + 3x + 12) dx \rightarrow dy = \left(y(x+4) + 3(x+4) \right) dx$

$\int dy = \int (x+4)(y+3) \, dx \rightarrow \ln|y+3| = \frac{x^2}{2} + 4x + c$

sep + integration

* Reduction to separable : (Homo D.E)

$y' = f\left(\frac{y}{x}\right)$ Homo D.E

let $u = \frac{y}{x}$, $y = ux$

$y' = u'x + u$

$u'x + u = f(u) \rightarrow u'x = f(u) - u$

$\frac{du}{dx} x = f(u) - u$

$\frac{du}{f(u) - u} = \frac{dx}{x}$ sep

ex: solu $2xy\,y' = y^2 - x^2$ مانيزط sep

$y' = \frac{y^2 - x^2}{2xy} = \frac{1}{2}\left(\frac{y}{x} - \frac{x}{y}\right)$

$y' = \frac{1}{2}\left(\frac{y}{x} - \frac{1}{\frac{y}{x}}\right)$ Homo D.E

من صورة بقدر أحول لـ sep

let $u = \frac{y}{x}$, $y = ux$, $y' = u'x + u$

$u'x + u = \frac{1}{2}\left(u - \frac{1}{u}\right) \rightarrow u'x = \frac{1}{2}u - u - \frac{1}{2u}$

$\frac{du}{dx} x = -\frac{1}{2}u - \frac{1}{2u} = \frac{-u^2 - 1}{2u} \longrightarrow \frac{du}{dx} x = -\left(\frac{u^2+1}{2u}\right)$

$\int \frac{2u}{u^2+1} du = \int -\frac{dx}{x} = \ln u^2 + 1 = -\ln |x| + c$

$\left(\frac{y}{x}\right)^2$

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