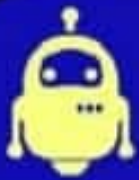


Doctor's Name :  
Hasan farahneh

# Circuits 2

Note Book



**YOUR  
GUIDE**

Rada Alasaad

①

sinusoidal  $\theta$   $\begin{cases} \text{sin} \\ \text{cos} \end{cases}$

$$V(t) = V_m \sin(\omega t + \theta)$$

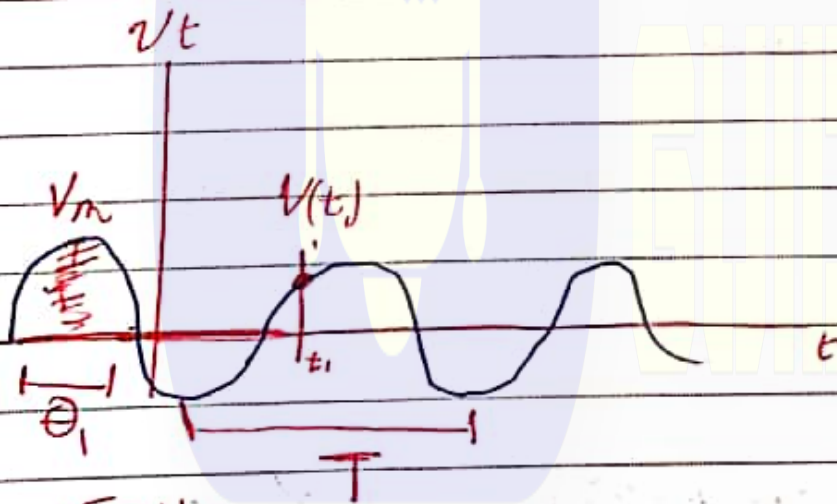
$\downarrow$  instantaneous       $\leftarrow$  max       $\leftarrow$  Angular freq       $\leftarrow$  phase shift

$$\omega = \frac{2\pi}{T}$$

periodic time

$$T = \frac{1}{f}$$

$$\omega = 2\pi f$$



Variable  
 $\begin{cases} t \\ \omega \end{cases}$

$t_1$   
 $\omega_0 \rightarrow$  freq

$V(t_1)$  - instantaneous value

2

No. \_\_\_\_\_

phasor/polar Form

v(t), Vm cos(ωt + θ) Time domain,

vector ← V = Vm ∠ θ

sin x = cos(x - 90°)
-sin x = cos(x + 90°)
-cos x = cos(x + 180°)

v(t), Vm cos(ωt + θ1)

i(t), Im cos(ωt + θ2)
amplitude

Two signals

sin, sin or cos, cos

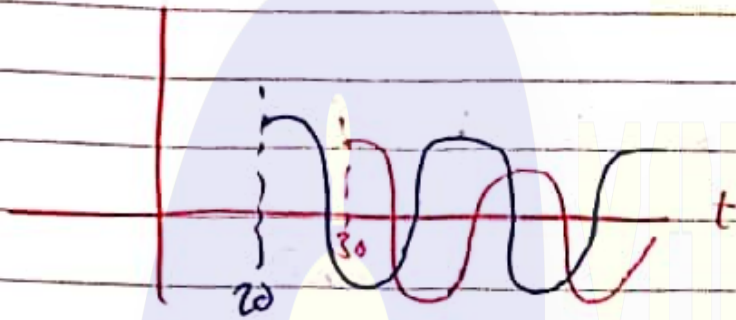
ω same

ex - v(t) = 4cos(100t - 30°) → 4 ∠ -30
i(t) = 6sin(100t + 70°)

i(t) = 6cos(100t + 70° - 90°)
i(t) = 6cos(100t - 20°)

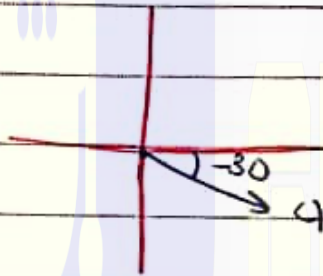
No. \_\_\_\_\_

3



$i$  leads  $V$  by  $10^\circ$   
 $V$  lags  $i$  by  $10^\circ$

$4 \angle -30$

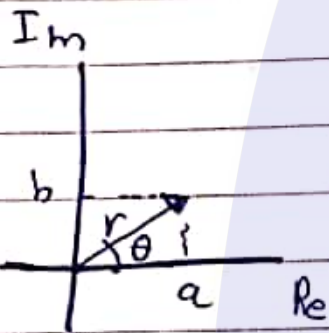


4

No. \_\_\_\_\_

## Complex / Cartesian

$$z_s = a + jb \rightarrow z_s = |r| \angle \theta$$
$$a = r \cos \theta$$
$$b = r \sin \theta$$



$$r = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1} \frac{b}{a}$$

ex:  $v_1 = 4 \cos(100t - 30^\circ)$        $v_2 = 6 \sin(100t)$   
 $6 \cos(100t - 90^\circ)$

$$v_1(t) + v_2(t)$$

Sol:  $v_1 = 4 \angle -30^\circ$   
 $v_2 = 6 \angle -90^\circ$

$$v_1 + v_2 = 8.7 \angle -66.5^\circ \quad \text{time domain } 8.7 \cos(100t - 66.5^\circ)$$

5

No. \_\_\_\_\_

Load = 

① Resistor	$R(\Omega)$	} Impedance
② Inductor	$\omega L$ L (H)	
③ Capacitor	$\frac{1}{\omega C}$ C (F)	

• Resistor  $Z_R = R \angle 0^\circ$

• Inductor  $Z_L = j\omega L = \omega L \angle 90^\circ$

• Capacitor  $Z_C = \frac{1}{j\omega C} = \frac{1}{\omega C} \angle -90^\circ$

$j = i = \sqrt{-1}$

$\begin{matrix} a & b \\ 0 & +j4 \end{matrix} = 4 \angle 90^\circ$

$\begin{matrix} a & b \\ 3 & +j4 \end{matrix} = 5 \angle 53.13^\circ$

$\begin{matrix} a & b \\ 0 & + -j4 \end{matrix} = 4 \angle -90^\circ$

①  $A \angle \theta_1 \cdot B \angle \theta_2 = |A||B| \angle \theta_1 + \theta_2$

②  $\frac{A \angle \theta_1}{B \angle \theta_2} = \frac{|A|}{|B|} \angle \theta_1 - \theta_2$

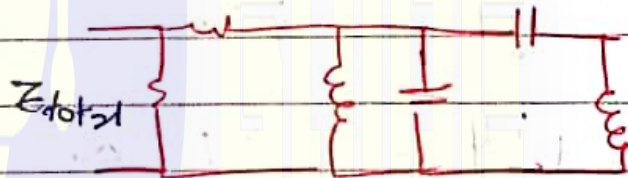
6

\* in  $L$  the  $V_L$  lead  $i_L$  by  $90^\circ$

\* in  $C$  the  $I_C$  lead  $V_C$  by  $90^\circ$

$$\left| Z_s \frac{V}{I} \right| \quad \left| Z_L \frac{V_L \angle \theta_L}{I_L \angle \theta_L} \right|$$

$$\left| Z_s \omega L \angle \theta_1 - \theta_2 = \omega L \angle 90^\circ \right|$$



\*  $\angle + \theta$   
inductive load

\*  $\angle - \theta$   
capacitive load

\*  $\angle 0$   
Resistive load      in phase the  $V, I$  are the same

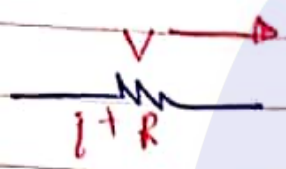
7

No. \_\_\_\_\_

Tuesday  
11/16/2019

# Chapter 11

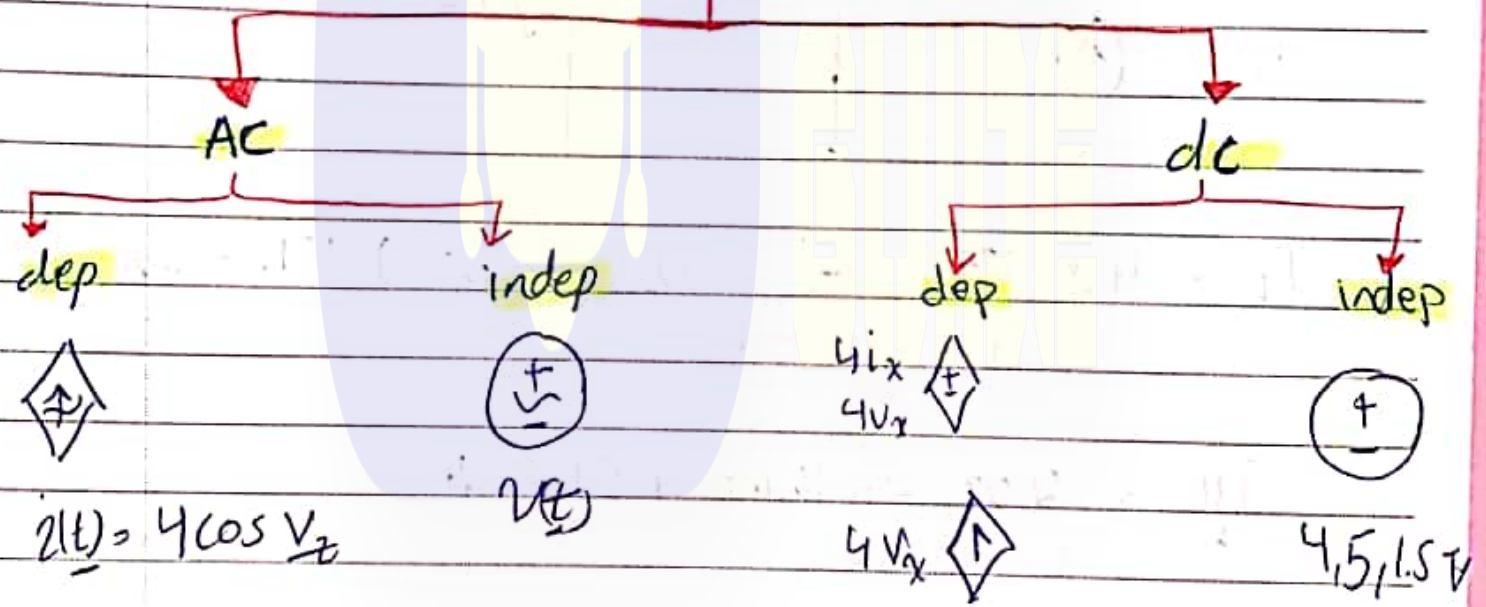
## AC - power



مقدار الشغل المبذول لكل  
سنة - مقدار الطاقة  
التي تفرغ من السعة

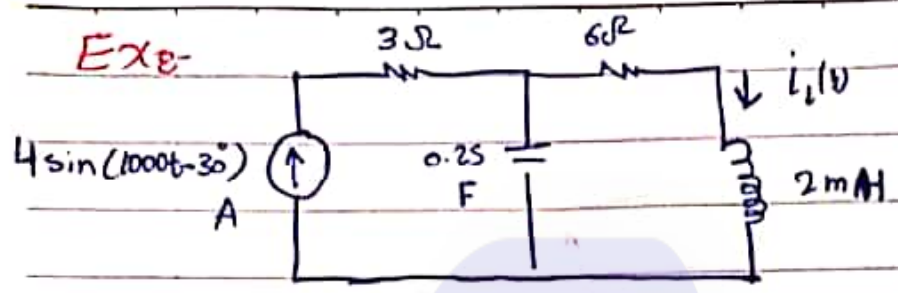
$$\begin{aligned}
 P_s &= I^2 R \\
 P_s &= V^2 / R \\
 P_s &= I C
 \end{aligned}
 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \rightarrow \text{DC}$$

## power supplied





Exe-



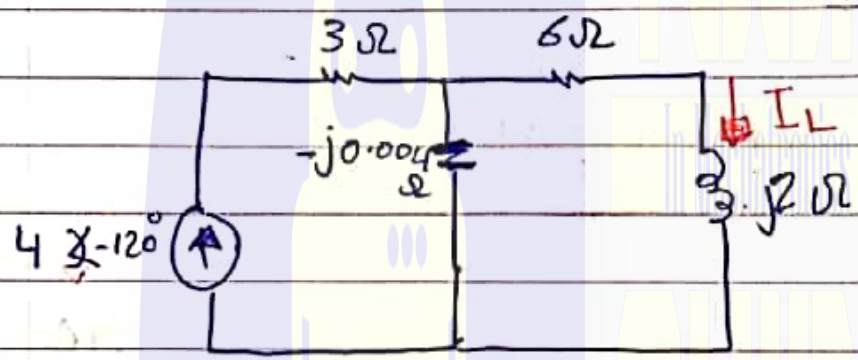
تایم سیر انتقال ای داره و ۵۰٪  
time domain  
phase form ←

sol:

①  $4 \cos(1000t - 30 - 90)$   
 $4 \cos(1000t - 120)$   
 $4 \angle -120$

②  $C \rightarrow \frac{-j}{\omega C} = -j4 \times 10^{-3} \Omega$

③  $L \rightarrow j\omega L = j2 \Omega$



$$I_L = \frac{4 \angle -120 (-j0.004)}{6 + j2 - j0.004} = 2.53 \angle 131.6 \text{ mA}$$

$$i_L(t) = 2.53 \cos(1000t + 131.6) \text{ mA}$$

$$V_L(t) = L \frac{di_L}{dt}$$

$$V_L(t) = j2 (2.53 \angle 131.6)$$

$$V_L(t) = 5.06 \angle 22.16 \text{ mV}$$

$$V_L = j2 + I_L$$

$$\downarrow$$

$$5.06 \cos(1000t + 22.16) \text{ mV}$$

**Instantaneous power**

$$p(t) = v(t) i(t)$$

$$p_L(t) = V_L(t) i_L(t)$$

للجهد  $V$  وللتيار  $i$

تساوي القوى  
السالبة

cosut...

$$\cos A \cdot \cos B =$$

$$0.5 \cos(A+B) + 0.5 \cos(A-B)$$

$$p(t) = v(t) i(t) \quad \theta_v \quad \theta_i$$

$$5.06 \cos(1000t + 221.6) (2.53 \cos(1000t + 131.6))$$

$$\frac{12.8}{2} \cos(1000t + 353.2) + \frac{12.8}{2} \cos(221.6 - 131.6)$$

Ac part + dc part

in general form:

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$p(t) = v(t) \cdot i(t)$$

$$V_m \cos(\omega t + \theta_v) I_m \cos(\omega t + \theta_i)$$

$$\frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

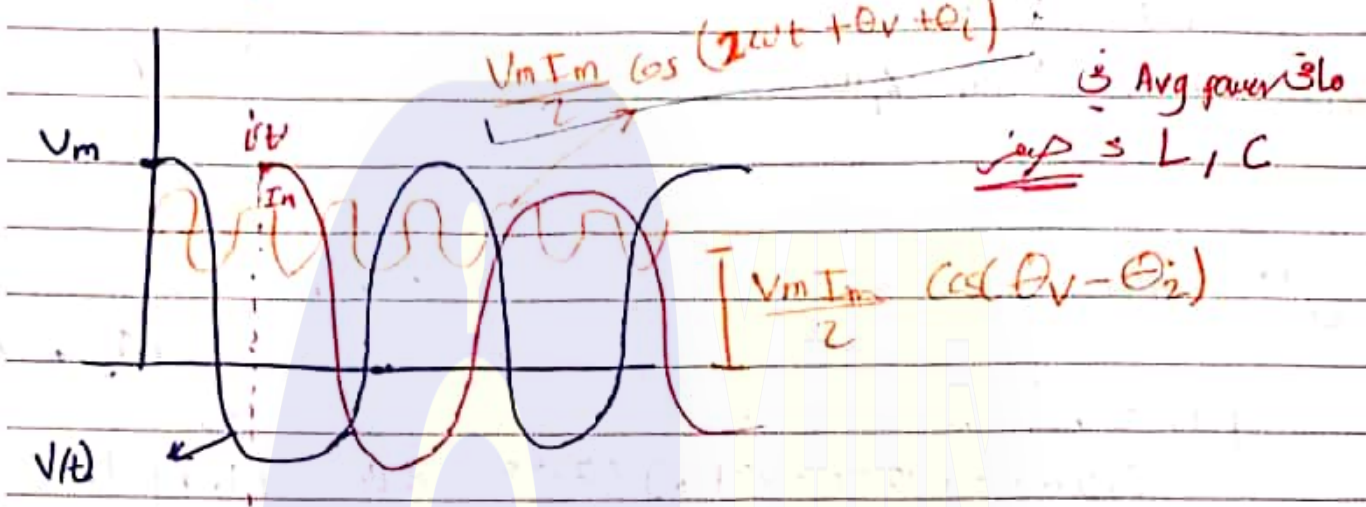
Ac part

Dc-part

Average power

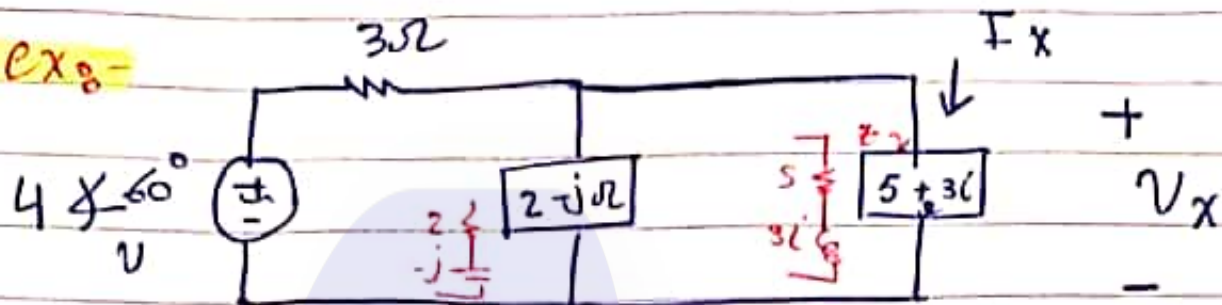
سوال السنه

The frequency of  $p(t)$  is twice of the frequency of  $v(t)$  or  $i(t)$

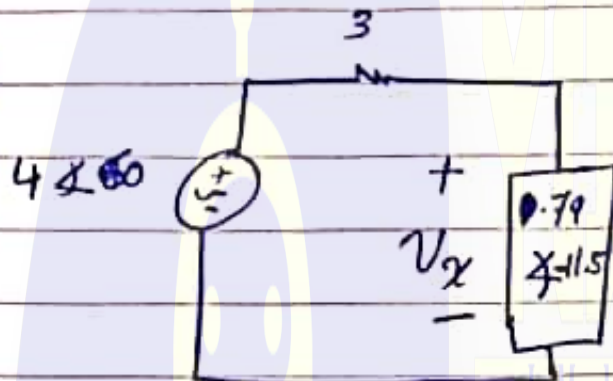


Arrange power في  
 $L, C$   
 $\cos \phi =$

ex:-



sol:-



$$V_x = \frac{4 \angle 60^\circ (1.79 \angle -11.5^\circ)}{3 + (1.75 - 0.35i)} \quad 1.75 - 0.35i$$

$$V_x = 1.5 \angle 53^\circ \text{ V}$$

$$I_x = \frac{1.5 \angle 53^\circ}{5 + 3i} = 0.25 \angle 22^\circ$$

power Average

$$P_x = 0.5 V_m I_m \cos(\theta_v - \theta_i)$$

$$= 0.5 \times 1.5 \times 0.25 (\cos(53^\circ - 22^\circ))$$

$$P_x = 0.16 \text{ W}$$

5 Ω resistor  
 $\angle_{22^\circ, 1.25}$

الزوايا  
 $\varphi = V, I$  Resistor shift  $\phi$   
in phase

(B)

No.

Sunday  
16/5/2019

$$I = I_m \angle \theta \rightarrow a + jb$$

$$I^* : \text{conjugate } I_m \angle -\theta \rightarrow a - jb$$

$$P_{av} = \frac{1}{2} I_m V_m \cos(\theta_v - \theta_i)$$

$$P_{av} = \frac{1}{2} V I^* \quad \text{as a vector}$$

$$\frac{1}{2} V_m \angle \theta_v \quad I_m \angle -\theta_i$$

$$\downarrow$$
$$V \cos(\omega t + \theta)$$

$$I \angle -\theta$$

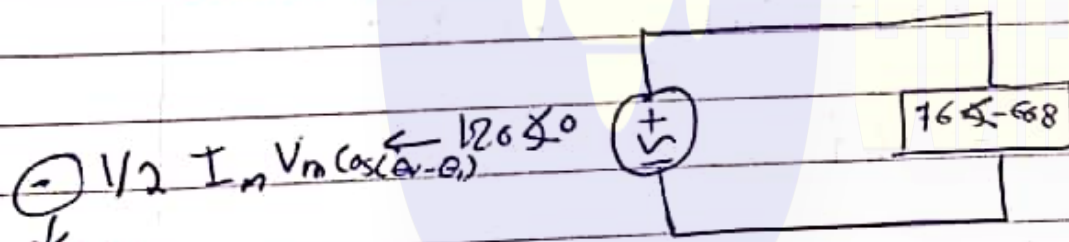
ex8-

$$Z = \frac{R}{30} - 70i \Omega$$
$$76.1 \angle -66.8$$

$$V_s = 120 \angle 0$$

Find  $I_s$  -

مجموع البورق في دائرة  
0 =



current  
جاري  
3  
(-)

$$I_s = \frac{V}{Z} = \frac{120 \angle 0}{76 \angle -66.8} = 1.576 \angle 66.8 \text{ A}$$

Final power average

$$P = \frac{1}{2} I_m V_m \cos(\theta_v - \theta_i)$$
$$\frac{1}{2} 1.576 120 (\cos(0 - 66.8))$$
$$37.25 \text{ W}$$

$$P = \frac{1}{2} I_m^2 R$$

يا جاري  
smile for me

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No. \_\_\_\_\_

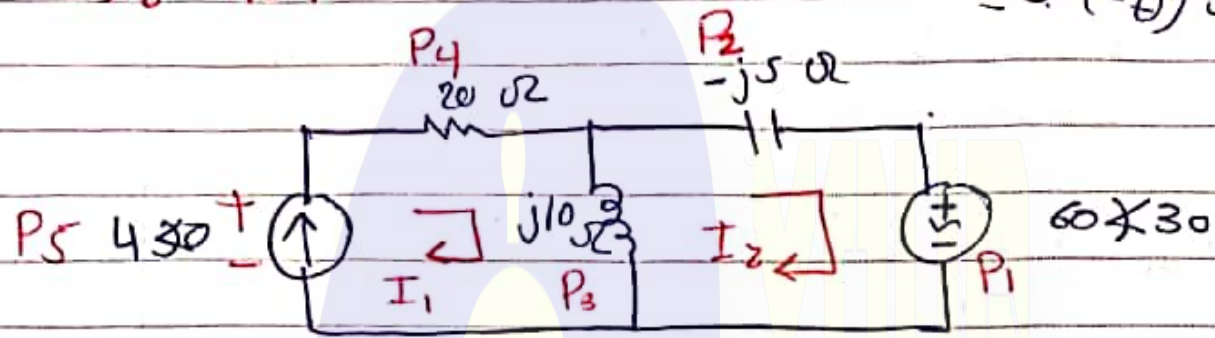
$4j$   
 $-4j$   
 $3$   
 $-3$

$4 \angle 90^\circ$   
 $4 \angle -90^\circ$   
 $3 \angle 0^\circ$   
 $3 \angle 180^\circ$

اذا كانت  $(-j)$  برتبة 180  
 اذا كانت  $(+j)$  برتبة 180

مثال في الزاوية اعلى من 180

ex 8-11.4



Find  $P_{average}$  in each element.

$I_1 = 4 \angle 0^\circ \text{ A}$

$-V_x + 20I_1 + j10(I_1 - I_2) = 0$

at mesh 2

$-j5I_2 + 60 \angle 30^\circ + j10(I_2 - I_1) = 0$

$5jI_2 - 10j(4 \angle 0^\circ) = 60 \angle -150^\circ$

$5jI_2 - 40j = 60 \angle -150^\circ$

$5jI_2 = 60 \angle -150^\circ + 40j$

$I_2 = 10.58 \angle 79^\circ \text{ A}$

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No. \_\_\_\_\_

$$P_1 = 0.5 I_m V_m \cos(\theta_v - \theta_i)$$

$$= 0.5 (60)(10.5) \cos(30^\circ - 79^\circ) = 206.65 \text{ W}$$

$$P_2 = 0 \text{ W}$$

$$P_3 = 0 \text{ W}$$

$$P_4 = 0.5 I_m^2 R = 0.5 (4)^2 \times 20 = 160 \text{ W}$$

$$P_5 = 0 \Rightarrow P_1 + P_2 + P_3 + P_4 + P_5 = 0$$

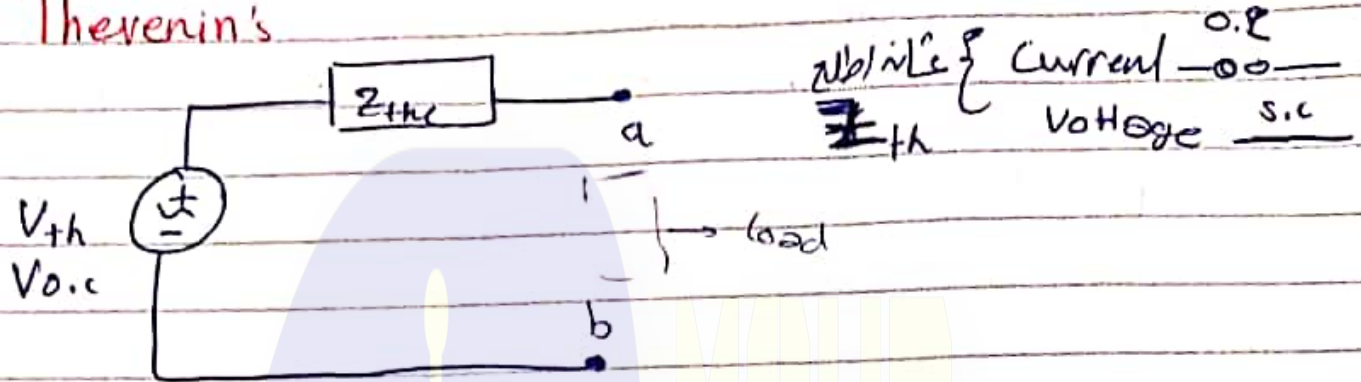
$$206.65 + 160 + 0 + 0 + P_5 = 0$$

$$P_5 = -366.65 \text{ W}$$

$$P_5 = 11.4 \text{ W}$$



Theremin's



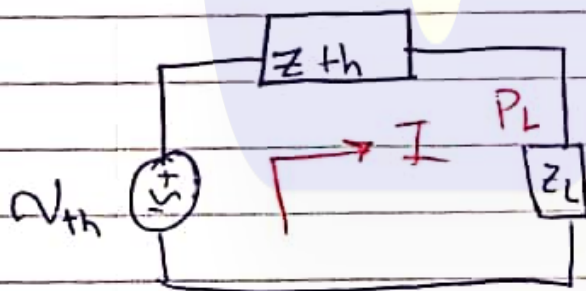
Max power transfer:

dc.

$$R_L = R_{th}$$

$$P_{max} = \frac{V_{th}^2}{4 R_{th}}$$

Theremin eq. Ac



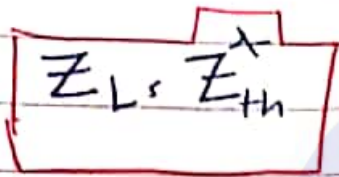
$$I = \frac{V_{th}}{Z_{th} + Z_L}$$

$$P_L = 1/2 V_L I = 0.5 \frac{V_{th} Z_L}{Z_L + Z_{th}} \frac{V_{th}}{Z_{th} + Z_L}$$

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No. \_\_\_\_\_

To have Max power



$$P_{max} = \frac{|V_{th}|^2}{8 R_{th}}$$

mag

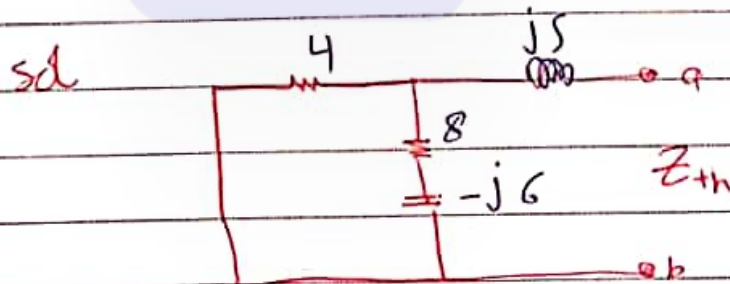
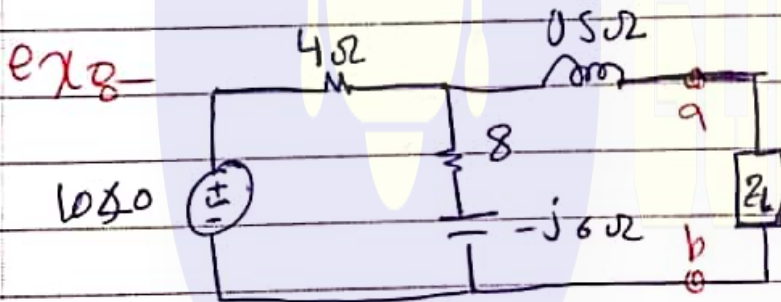
if

$$Z_{th} = 8 - j10 \Omega$$

$$Z_L = 8 - j10 \Omega$$

$$Z_{th} = 5 \angle -30^\circ \Omega$$

$$Z_L = 5 \angle 30^\circ \Omega$$



$$Z_{th} = 4 \parallel (8 - j6) + j5 = 2.933 + j4.46 \Omega$$

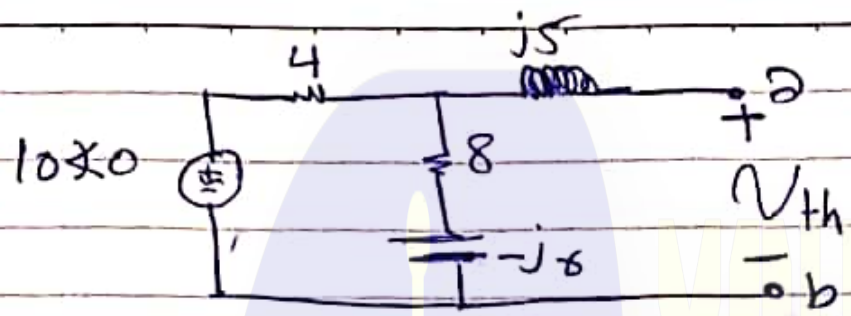
To have Max power

$$Z_L = Z_{th}^* = 2.933 - j4.46 \Omega \text{ smile for Me}$$

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No. \_\_\_\_\_

Current source 15V  
Capacitor



$$V_{th} = V_{8\Omega} + V_{j6\Omega} = \frac{10\angle 0^\circ (8 - j6)}{4 + 8 - j6}$$

$$V_{th} = (7.45) \angle -10.3^\circ$$

$$\text{Power max} = \frac{17.45^2}{8(2.933)}$$

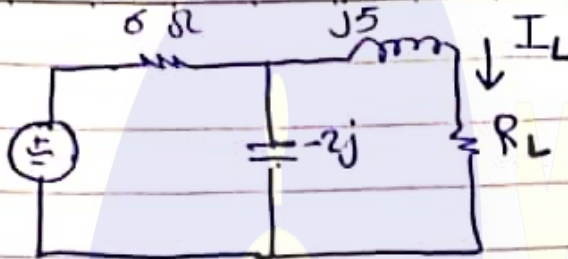
19

$Z_{th}$  = Real Part and Imag  
 $R_s$  = Real

No.

17/6/2019  
Monday

ex 2-



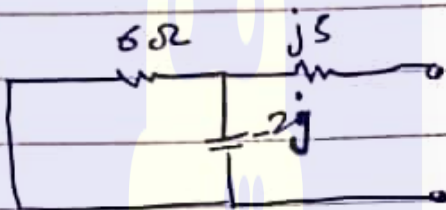
$Z_{th} = R_{th} + jX_{th}$   
 For pure resistive load  
 $R_L = |Z_{th}| = \sqrt{R_{th}^2 + jX_{th}^2}$

\* Assume  $Z_{th} = 3 + j4 \Omega$

$R_L = \sqrt{3^2 + 4^2} = 5 \Omega$

Find  $R_L$  to transfer Max power

soln



$Z_{th} = (6 || -2j) + j5 = 3.2 \angle 79.38^\circ$

$Z_{th} = 0.6 + j3.2 \Omega$

$R_L = (\text{for max power}) = |Z_{th}^*| = \sqrt{0.6^2 + 3.2^2} = 3.255 \Omega$

جواب السؤال

Power max =  $\frac{|V_{th}|^2}{8 R_{th}} = 0.5 I_{0L}^2 R_L$

### \* Root Mean squared value (rms)

$$V(t) = \underline{V_m} \cos(\omega t + \theta)$$

max value

$$dc \cos/\sin = 0$$

$$x(t) = X_m \cos(\omega t + \theta)$$

effective value  $\leftarrow X_{eff} = X_{rms} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt}$

For sinusoidal signal

$$v(t) = V_m \cos(\omega t)$$

Find  $V_{rms}$  &  $V_{eff}$

alok dya (relax)

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T V_m^2 \cos^2 \omega t dt} = \sqrt{\frac{V_m^2}{2T} \int_0^T (1 - \cos 2\omega t) dt}$$

$$= \sqrt{\frac{V_m^2}{2T} t} \Big|_0^T$$

$$= \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

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No. \_\_\_\_\_

$$e_x = v(t) = 10 \cos(\omega t + 30^\circ)$$

$$i(t) = 8 \cos(\omega t + 60^\circ)$$

$$V_{\max} = 10$$

$$V_{\text{rms}} = \frac{10}{\sqrt{2}}$$

$$* P_{\text{av}} = \frac{1}{2} V I^*$$

جواباً  
 $V_{\text{rms}}$

$$* P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$\frac{V_{\text{rms}} V_m}{V_{\text{rms}} \sqrt{2} V_{\text{rms}}}$$

$$= \frac{1}{2} (\sqrt{2} V_{\text{rms}} \sqrt{2} I_{\text{rms}} \cos(\theta_v - \theta_i))$$

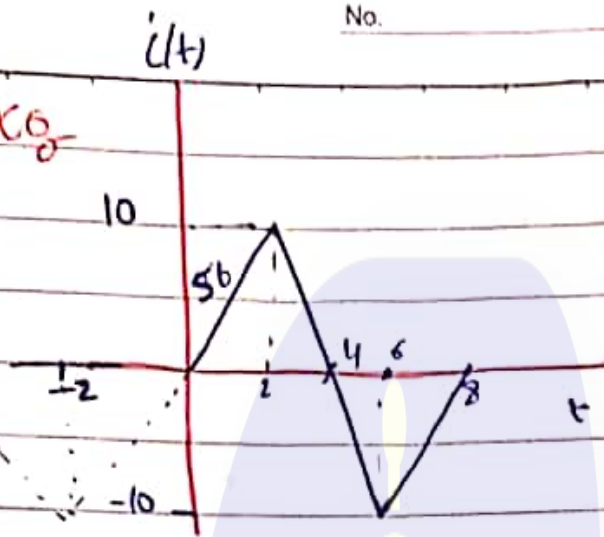
$$\textcircled{1} \left[ P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) \right]^*$$

$$\textcircled{2} \left[ P_{\text{av}} = V_{\text{eff}} * I_{\text{eff}} \right]^*$$

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No. \_\_\_\_\_

Exo



$$i_{rms}, i_{eff} = \sqrt{\frac{1}{T} \int_0^T i^2 dt} = \sqrt{\frac{1}{8} \int_{-2}^2 5t^2 dt + \int_2^6 (5t+20)^2 dt}$$

$$= \sqrt{\frac{1}{8} \int_{-2}^2 25t^2 dt + \int_2^6 25t^2 - 200t + 400 dt}$$

$$= \sqrt{\frac{1}{8} \left( \left[ \frac{25}{3} t^3 \right]_{-2}^2 + \left[ \frac{25}{3} t^3 - \frac{200}{2} t^2 + 400t \right]_2^6 \right)}$$

5.77 A

dc = 0

Max values 10

$$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \operatorname{Re}[V I^*]$$

### \* Apparent power

$$S = V_{eff} I_{eff} \quad (\text{VA})$$

$$S = \frac{1}{2} V_m I_m \quad (\text{VA})$$

### \* reactive power

$$Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i) \quad (\text{VAR})$$

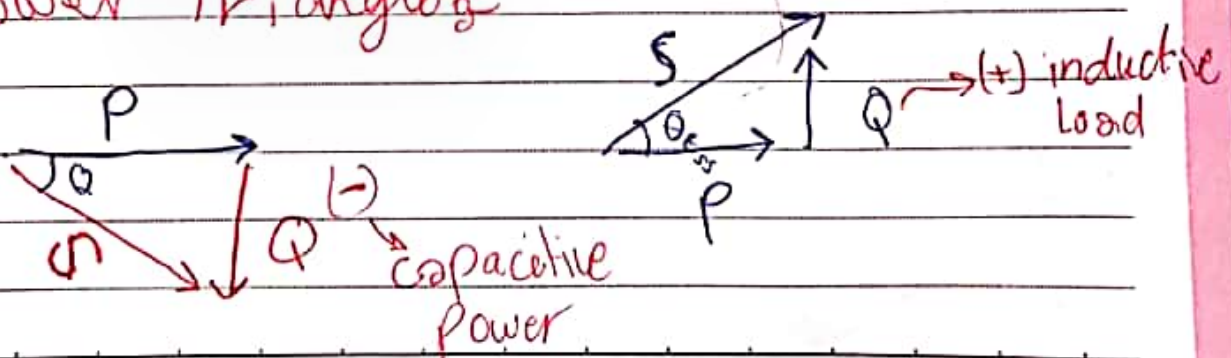
$$Q = I_{eff} V_{eff} \sin(\theta_v - \theta_i) \quad (\text{VAR})$$

### \* Complex power:-

$$S = \overset{\substack{\text{average power} \\ \uparrow}}{P} + jQ \rightarrow \text{reactive power} \quad (\text{VA})$$

Apparent power  $|S|$

### \* power Triangles





\* power factor (PF)

$\cos(\theta_v - \theta_i)$  s  $\cos(\theta_z)$   
↳ total

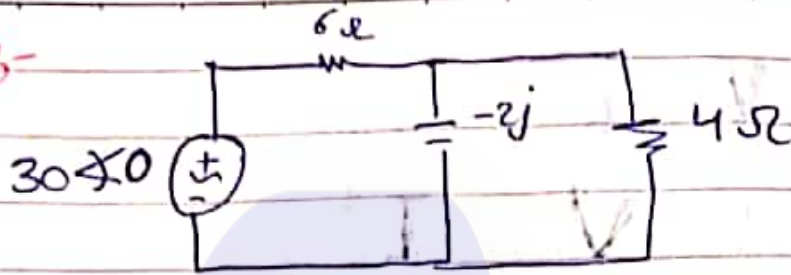
PF  $\theta_z < 0$  leading power factor  
\* (capacitive load)

$\cos \theta_z \frac{P}{S}$

$\theta_z > 0$  lagging power factor  
(inductive load).

$\theta_z = 0$  pure resistive ~~load~~  
load.

\* ex 8 -



P.F  
( $0 < pf < 1$ )

find p.f.

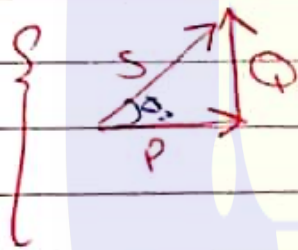
کل بازار P.F سٹوڈینٹ

$$Z_{total} = (4 \parallel -2j) + 6$$

$$Z_{Ts} = 7 \angle -13.24$$

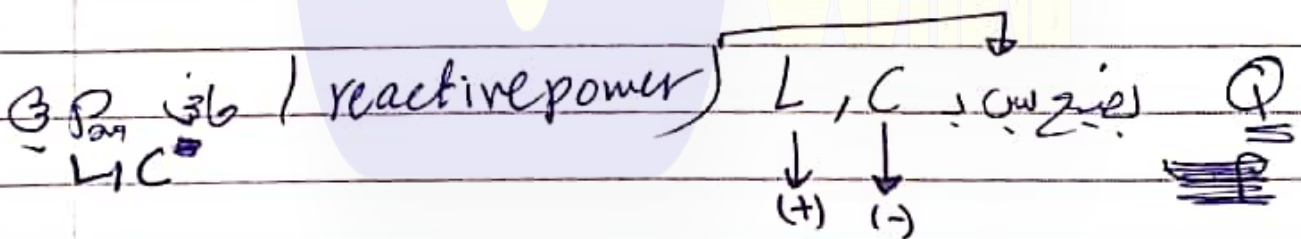
power factors  $\cos(-13.24) = 0.9734$

leading



$$P_{avg} = |S| \cos \theta$$

$$Q = |S| \sin \theta$$



reactive power (Q) is stored in inductor (+) and capacitor (-) and resistor power is real power (P)

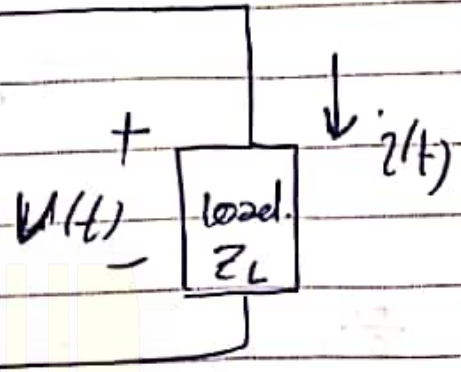
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No. \_\_\_\_\_

$$P = V_{rms} I_{rms}^*$$

$$= \frac{1}{2} V_{max} I_{max}^*$$

exg-



$$v(t) = 60 \cos(\omega t - 10^\circ) \quad 60 \angle -10^\circ$$

$$i(t) = 1.5 \cos(\omega t + 50^\circ) \quad 1.5 \angle 50^\circ \quad I^* = 1.5 \angle -50^\circ$$

Find  $S$ , apparent  $P$ ,  $Q$ , P.F.,  $Z$   
draw power triangle

Sol<sup>n</sup> -  $V_{rms} = \frac{60}{\sqrt{2}} = 42.42 \text{ V}$

$$I_{rms} = \frac{1.5}{\sqrt{2}} = 1.06 \text{ A}$$

$$S = V_{rms} I_{rms}^* = 42.42 \angle -10^\circ \cdot 1.06 \angle -50^\circ$$

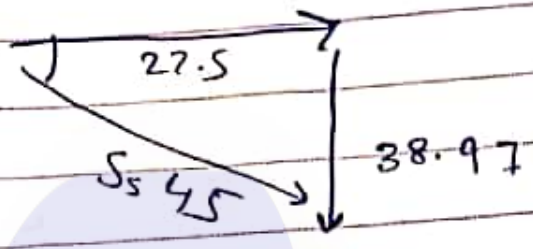
$$45 \angle -60^\circ$$

$$P.F. = \cos(-60^\circ) = 1/2 \text{ leading}$$

$$\text{apparent power } |S| = 45 \text{ VA}$$

$$P = |S| \cos(-60^\circ) = 22.5 \text{ W}$$

$$Q = |S| \sin(-60^\circ) = -38.97 \text{ VAR}$$



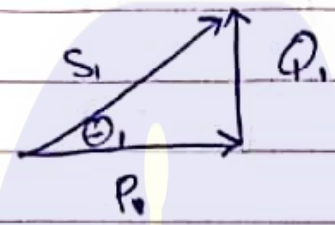
$$Z_s = \frac{V}{I} = \frac{60 \angle -10^\circ}{1.5 \angle -60^\circ} + \frac{40 \angle -60^\circ}{20 - j31.64 \Omega}$$

$Z_{total}$  زاویه  $\angle$  ~~...~~  $\angle$

$$P_{ave} = \frac{1}{2} |I|^2 R$$

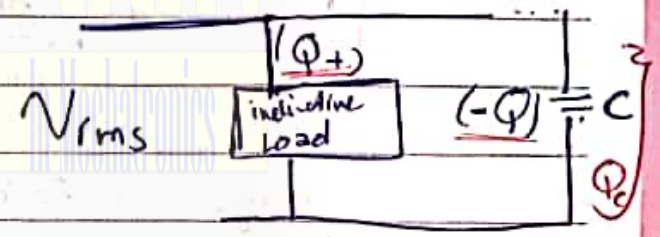
$$Q_s = \frac{1}{2} (I^2) |I_m|^2$$

\* Power Factor correction

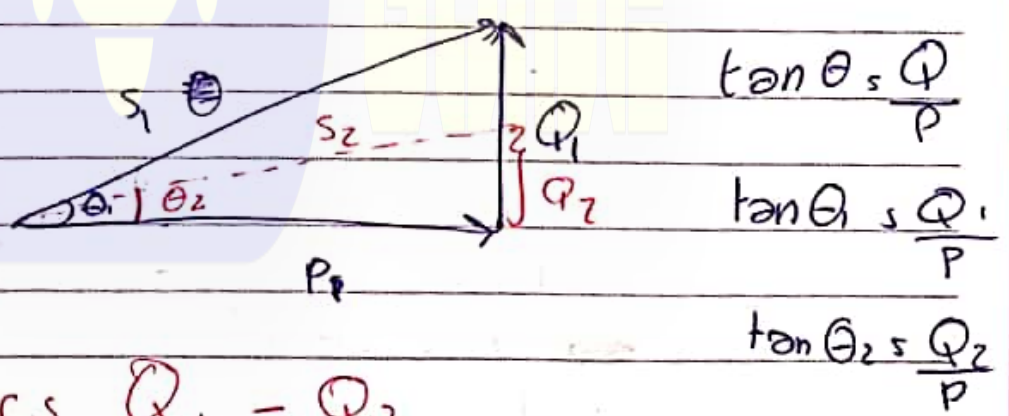


$$P_s = S_1 \cos \theta_1$$

$$Q_1 = S_1 \sin \theta_1$$



\* if we have an inductive load we have to connect a C in parallel



$Q_c = Q_1 - Q_2$

$P_s = S_2 \cos \theta_2$

~~$Q_1 = Q_1 = Q_2$~~

$Z_c = \frac{1}{j\omega C}$  impedance  
 $|Z_c| = X_c = \frac{1}{\omega C}$  No. reactance

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$Q_c = Q_1 - Q_2$

$P = \frac{V^2}{R}$

$Q_c = p(\tan \theta_1 - \tan \theta_2)$

$Q = \frac{V_{rms}^2}{X_c}$   
 $Q = \frac{V_{rms}^2}{X_c}$

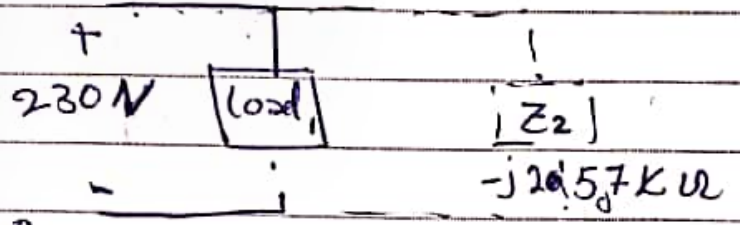
$\frac{V_{rms}^2}{1/\omega C} = p(\tan \theta_1 - \tan \theta_2)$

$\omega C V_{rms}^2 = p(\tan \theta_1 - \tan \theta_2)$

$C = \frac{p(\tan \theta_1 - \tan \theta_2)}{V_{rms}^2 \omega}$

exo-  $P = 50 \text{ kW}$ ,  $p.f. = (0.8) \text{ Lag (inductive load)}$   
 $V_{rms} = 230 \text{ V}$   
 $P.f._{new} = (0.95 \text{ Lag})$

Sol



$\theta_1 = \cos^{-1} 0.8 = 36.9^\circ$

$\theta_2 = \cos^{-1} 0.95 = 18.19^\circ$

$$P_s = S \cos \theta_1$$

$$1 \text{ S} \cdot \frac{P}{\cos \theta} = 62.5$$

$$S = 62.5 \times 36.9 \text{ KVA}$$

$$Q_{s1} = S \sin \theta_1 = 62.5 \sin \theta (36.9), 37.5 \text{ KVA}$$

$$S_{2s} = \frac{P}{\cos \theta_2} = \frac{50}{\cos 18.9} = 52.6 \text{ KVA}$$

$$Q_{2s} = S_2 \sin \theta_{2s} = 16.4 \text{ KVA}$$

$$Q_{cs} = 37.5 - 16.4 = 21.1$$

capit

$$Q_c = \frac{V_{rm}^2}{|Z_c|} = Z_c = \frac{230^2}{21.1} = 2507.1 \text{ VAR}$$

2.507 KVAR



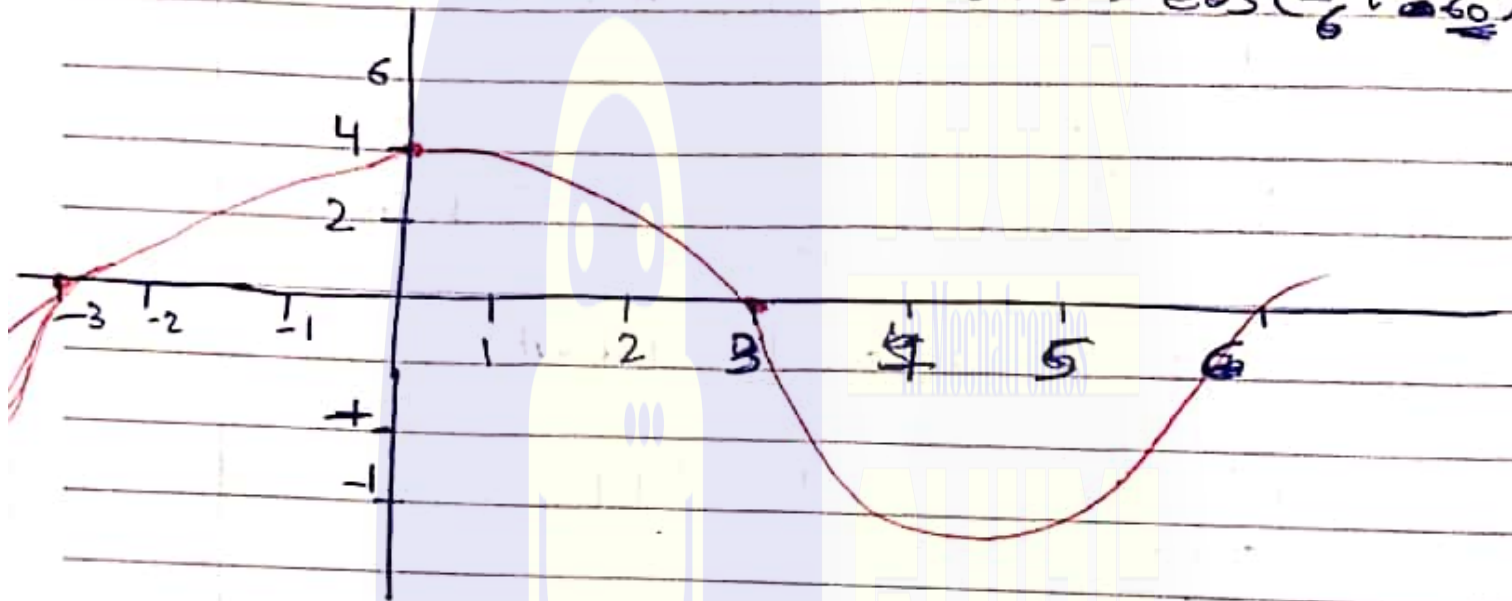
سوال افغانه  $ex_8$

$$v = 4 \cos\left(\frac{\pi}{6} t\right)$$

$$I = \frac{4 \angle 0^\circ}{2 \angle -60^\circ}$$

$$Z = 2 \angle 60^\circ \Omega$$

$$i(t) = 2 \cos\left(\frac{\pi}{6} t + 60^\circ\right)$$

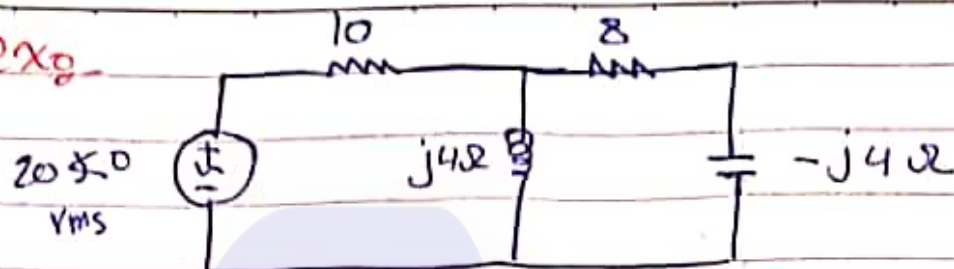


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No.

Sunday  
23/6/2019

exg.



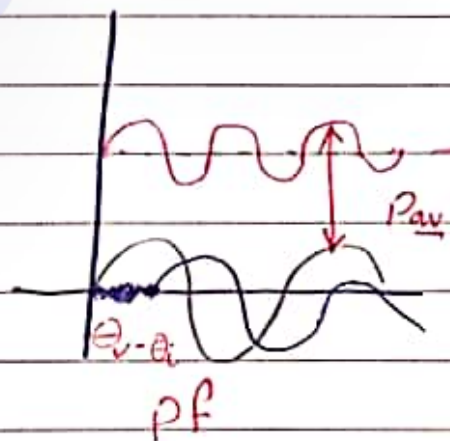
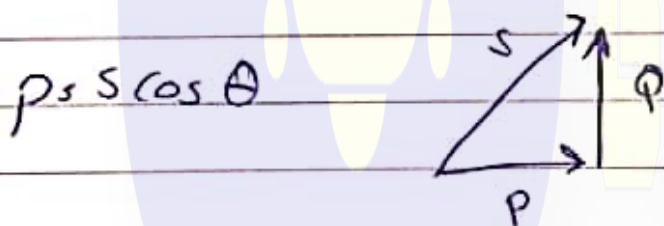
Find P.F.?

Sol:-  $Z_{total} = [-j4 + 8 || j4] + 10 = 12 + 4j$   
 $12.64 \angle 18.4^\circ$

① p.f.,  $\cos(18.43) = 0.95$  Lag

②  $I = \frac{V_s}{Z} = \frac{20 \angle 0}{12.64 \angle 18.43} = 1.58 \angle -18.43^\circ$

$S = V I^* = 20 \angle 0 \times 1.58 \angle 18.43^\circ = 31.6 \angle 18.43^\circ$  VA



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11.6 ex 2

Find  $V_o$  and

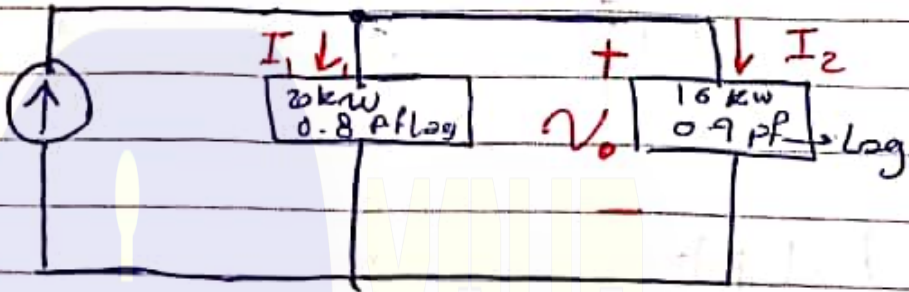
input p.f

12.50 A

Power

rms

Source



SOL

pf

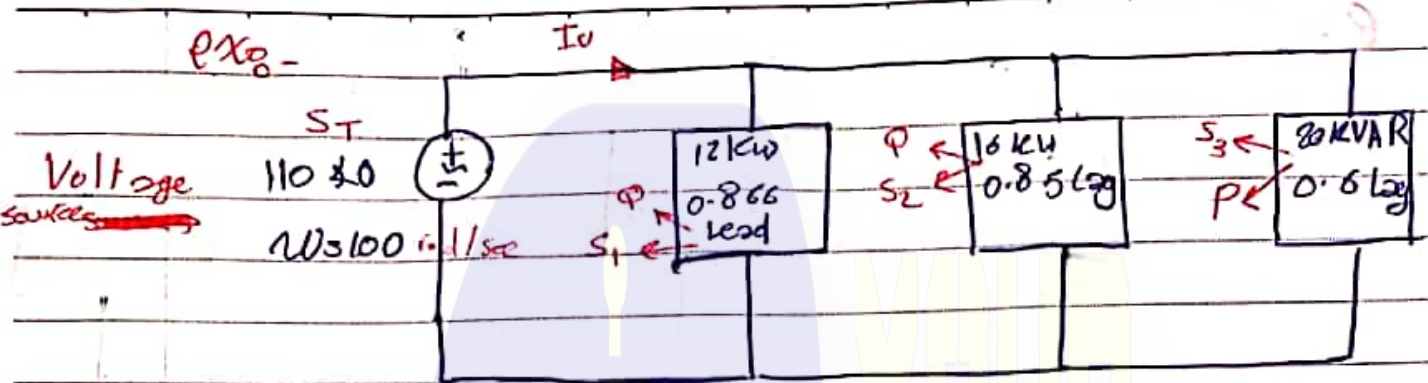
$$P_s = I_s V_o (\cos(\theta_v - \theta_i))$$

$$P_1 = I_1 V_o 0.8 = 20 \text{ kW}$$

$$P_2 = I_2 V_o 0.9 = 16 \text{ kW}$$

$$I_1 + I_2 = 12.50 \text{ A}$$

بس آطرح  $V_o$  بقدر آطرح  $P$  كى  $I$



Sol<sup>n</sup>  $Z = R \pm jX$

$P = \frac{V^2}{R}$ ,  $Q = \frac{V^2}{X}$

$S_1 = \frac{P_1}{\cos \theta} = \frac{12}{0.866} = 13.8 \angle -\cos^{-1} 0.8$   $S_1 = 13.8 \angle -30^\circ$  KVA

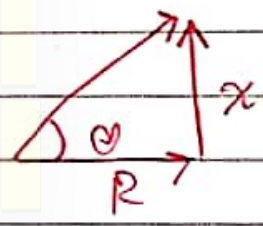
$S_2 = \frac{16}{0.85} = 18.8 \angle +\cos^{-1} 0.8$   $S_2 = 18.8 \angle 31.7^\circ$

$S_3 = \frac{20}{0.6} = 33.3 \angle +\cos^{-1} 0.6$   $S_3 = 33.3 \angle 53.1^\circ$

$\sin \theta$

$Z = R + jX$   
 $|Z| = \sqrt{R^2 + X^2}$

$\theta = \cos^{-1}(0.6) = 53.1$



$S_T = S_1 + S_2 + S_3 = 48.7 \angle 28.09$  KVA

$= P + jQ$

P.F.  $\cos(28.09) = 0.88$

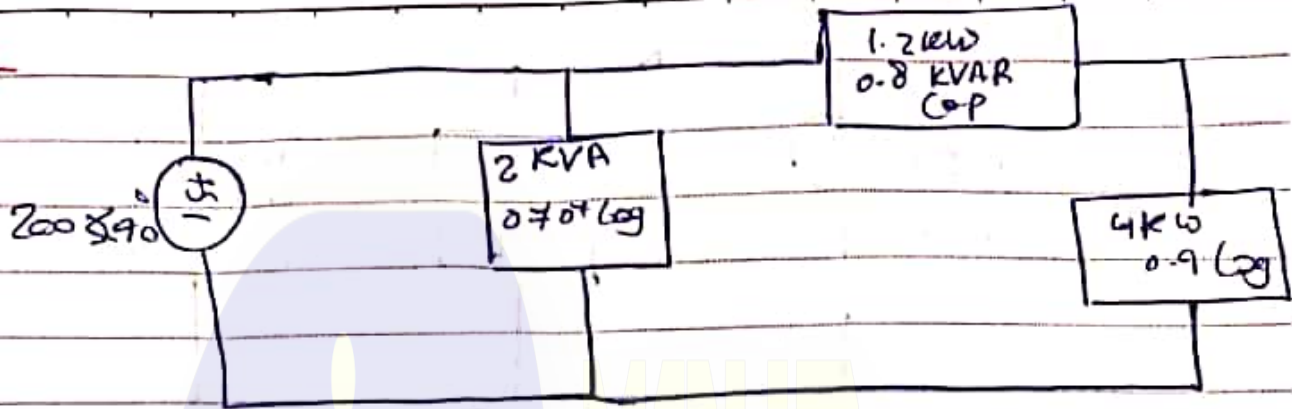
Let correct the P.F.  $\rightarrow 0.95$

$C_s P (\tan \theta_{old} - \tan \theta_{new})$   
 $V_{rms}^2 W$

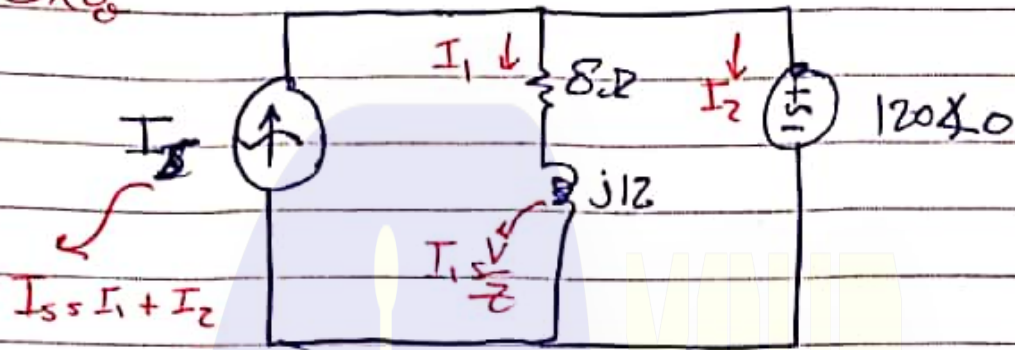
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Ans-



ex 98

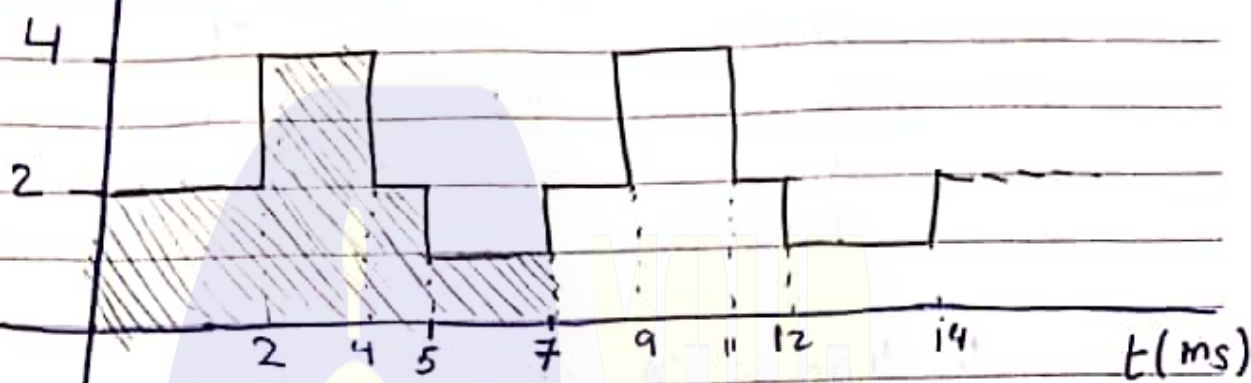


\* 2.5 kW, 0.4 KVAR Loading

Find  $I_s$  ?

$$S_s = 2.5 + 0.4j$$

$$S_s = VI_2^*$$

ex.  $i(t)$  (mA)Find  $T, f, \omega, I_{av}, I_{rms}$ 

soln

$$T = 7 \text{ ms}$$

$$f = \frac{1}{T} = \frac{1}{7} \times 10^3 \text{ Hz}$$

$$\omega = 2\pi f = \frac{2\pi}{7} \times 10^3 \text{ rad/sec}$$

$$I_{av} = \frac{1}{T_0} \int_0^T i(t) dt = \text{Area under the curve.}$$

$$\frac{(2 \times 2) + (2 \times 4) + (1 \times 2) + (2 \times 1)}{7}$$

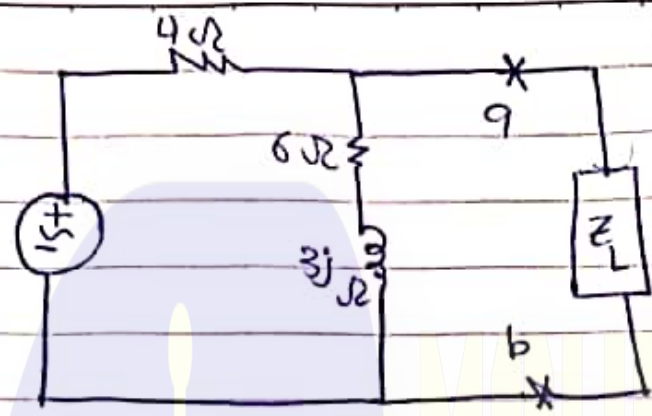
$$\frac{16}{7} = 2.28 \text{ A}$$

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt} = \sqrt{\frac{1}{7} \left( \int_0^2 2^2 dt + \int_2^4 4^2 dt + \int_4^5 2^2 dt + \int_5^7 1^2 dt + \int_{11}^{12} 2^2 dt \right)}$$

39

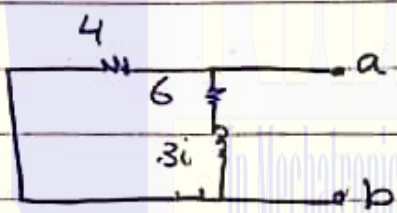
No. \_\_\_\_\_

ex 8



Find the thevenin equivalent?

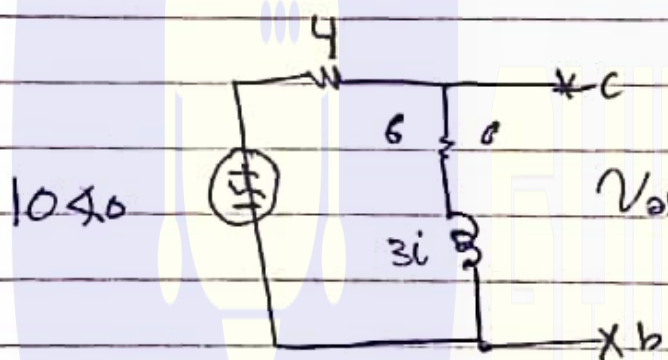
Sol:  $Z_{th}$  →



$$Z_{th} = 4 \parallel (6 + 3j)$$

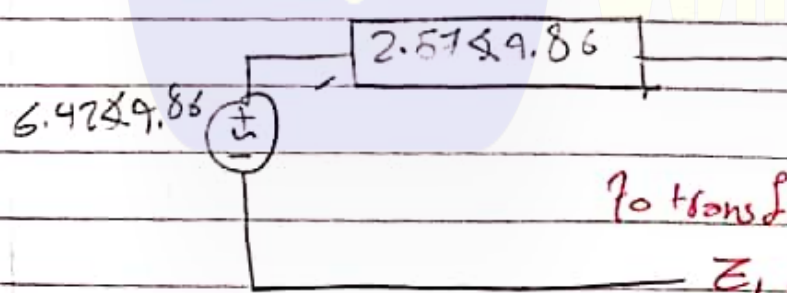
$$= \frac{24 + 12j}{10 + j3}$$

$$= 2.57 \angle 9.86^\circ \Omega$$



$$V_{ob} = V_{o.c} + V_{th} \cdot \frac{10 \parallel (6 + 3j)}{4 + 6 + 3j}$$

$$= 6.42 \angle 9.86^\circ$$



To transfer max power:

$$Z_L = Z_{th}^* = 2.57 \angle -9.86^\circ$$

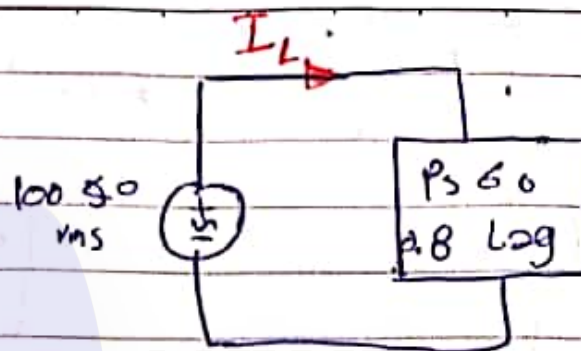
$$P_{max} = \frac{|V_{th}|^2}{8R_L}$$



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No. \_\_\_\_\_

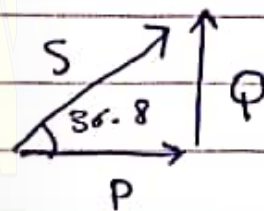
exg



how to find  $I_L$

soln-

$$|S| = \frac{60}{0.8} = 75$$



$$S = 75 \angle 36.8^\circ = I_{rms}^* V_{rms}^*$$

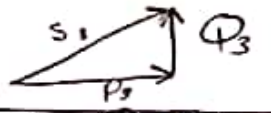
$$75 \angle 36.8^\circ = 100 \angle 0^\circ I_{rms}^*$$

$$I_{rms}^* = 0.75 \angle -36.8^\circ$$

$$I_L = 0.75 \angle -36.8^\circ$$

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$S_1 = 100 + j200$   
 $223 \angle 63.4 \text{ VA}$



$\sin \theta_3 = \frac{Q}{S} = \frac{1}{2}$

$\theta_3 = 30^\circ$   
 $S_3 = 600 \angle 30^\circ$

exg

$I_s$

$P_1 = 100 \text{ W}$   
 $Q_1 = 200 \text{ VAR (cap)}$

$100 \angle 0$   
 rms

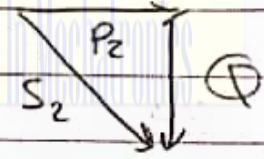


$P_2 = 400 \text{ W}$   
 $\text{P.F.} = 0.8 \text{ leads}$

$S_3 = 600 \text{ VA}$   
 $Q_3 = 300 \text{ VAR ind}$

Sol:

$\cos^{-1}(0.8) = 36.8^\circ = \theta = \frac{P}{S} = \frac{500}{S} \angle -36.8$   
 VA



$S_T = S_1 + S_2 + S_3 = 1039.6 \angle 11.11 \text{ VA}$

$S = V \angle I_s^*$

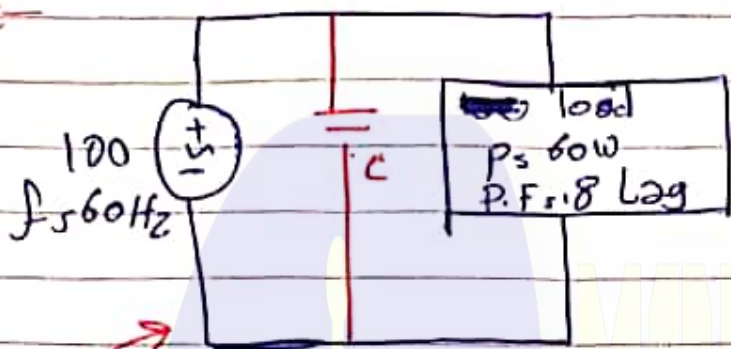
$1039.6 \angle 11.11 = 100 \angle 0 \angle I_s^*$

$I_s = 10.39 \angle -11.11 \text{ A}$

$I_s = 10.39 \angle -11.11 \text{ A}$

$\text{P.F. source} = \cos(11.11) = 0.981 \text{ Lag}$

Q7e



To correct the power factor to be 0.9:  
 We add a capacitive load in parallel

Sol<sup>n</sup>  $P = 0.5 I V \cos(\theta_v - \theta_i)$  <sup>P.F.</sup>

$60 = 0.5 I (100) (0.8)$

$I = \frac{120}{80} = 1.5 A$

$C = \frac{P (\tan(\theta_{old}) - \tan(\theta_{new}))}{V_{rms}^2 \omega}$

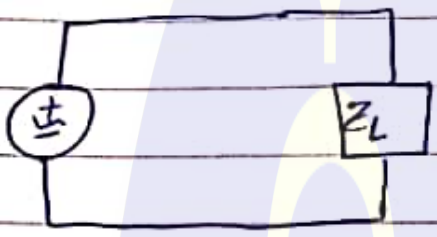
$X_c = \frac{1}{\omega C}$

$\theta_{old} = \cos^{-1}(0.8)$   
 $\theta_{new} = \cos^{-1}(0.9)$

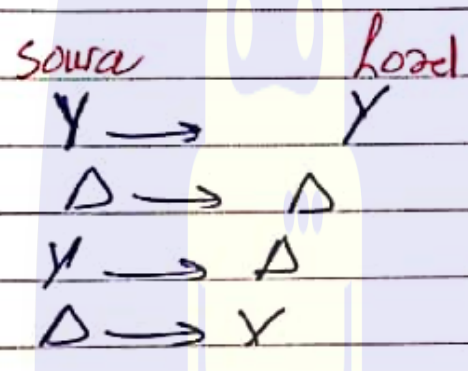
$I_c = \frac{V_c}{\frac{1}{j\omega C}} = \frac{100}{\frac{1}{j\omega C}}$

$P_s = 60$   
 $\omega = 60 (2\pi)$   
 $V_{rms} = \frac{100}{\sqrt{2}}$

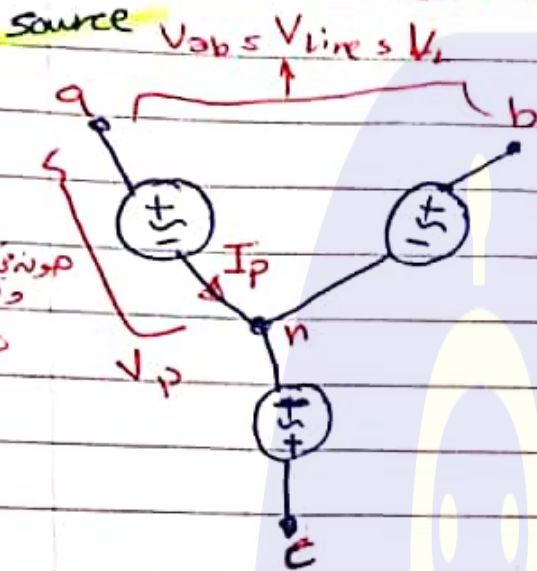
CH # 12 & Three-Phase - CKTs



one



① Y → Y Connection



$V_{an} = V_{phase} = V_p$   
 $V_{an}, V_{bn}, V_{cn}$  sequence phase

Between A, B,  $V_{ab}, V_{line} = V_L$   
 Between A, n:  $V_p, V_{phase}$   
 neutral

Current passing =  $I_p$   
 $V_L = \sqrt{3} V_p$

Sequence

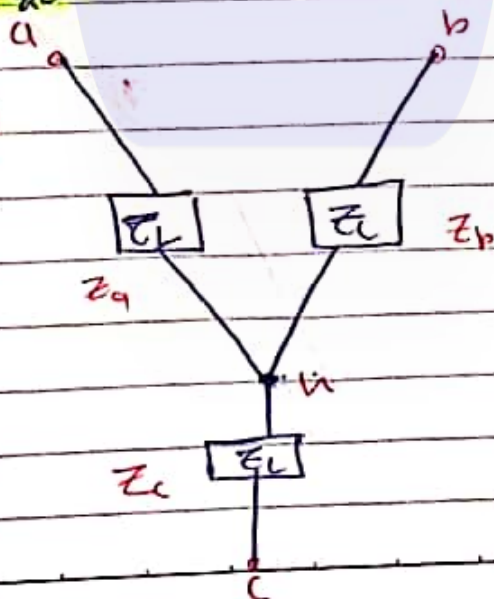
Positive sequence

$V_{an} \angle 0$   
 $V_{bn} \angle -120^\circ$   
 $V_{cn} \angle -240^\circ$

Negative sequence

$V_{an} \angle 0$   
 $V_{bn} \angle +120^\circ$   
 $V_{cn} \angle +240^\circ$

Load



Balanced Load

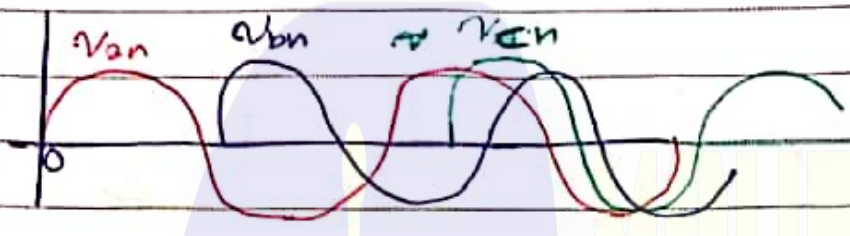
$Z_a = Z_b = Z_c$

Balanced system:

$(V_{an} + V_{bn} + V_{cn}) = 0$

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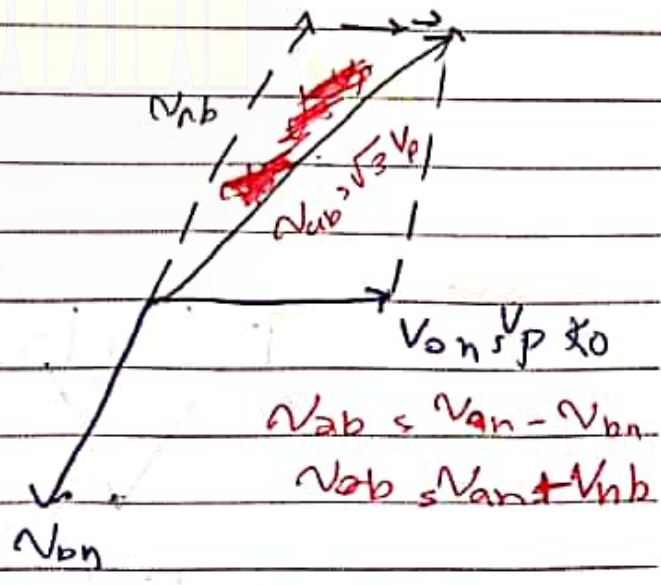


He

- $V_{an} = 10 \angle 0^\circ$
- $V_{bn} = 10 \angle -120^\circ$
- $V_{cn} = 10 \angle -240^\circ$

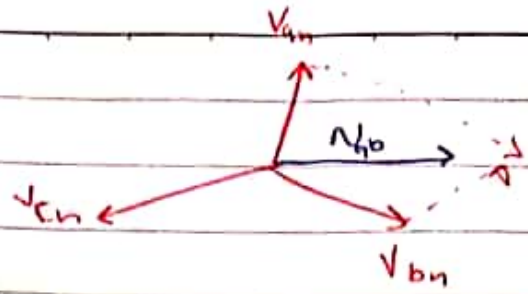
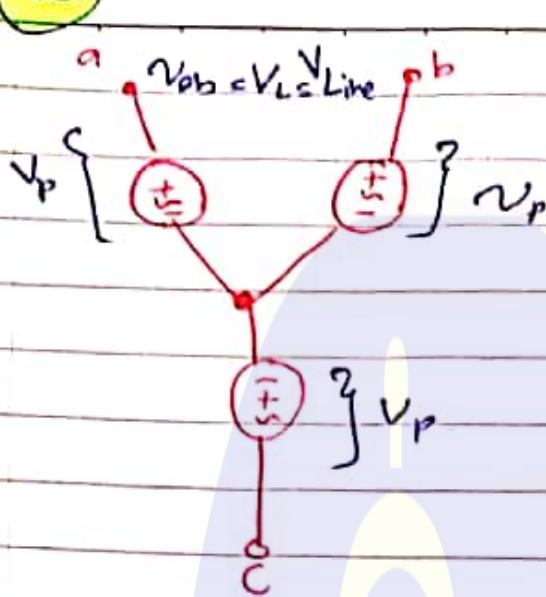
$\phi_a = \text{Let } +ve$

- $V_{an} = 10 \angle 135^\circ$
- $V_{bn} = 10 \angle 15^\circ$
- $V_{cn} = 10 \angle -105^\circ$



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No. \_\_\_\_\_



$$V_{ab} = \sqrt{3} V_p$$

$$V_{ca} = \sqrt{3} V_p$$

$$V_{bc} = \sqrt{3} V_p$$

$$V_{ab} = \sqrt{3} V_p \angle 30^\circ + \theta$$

ex 5  $V_{an} = 5 \angle 60^\circ$

sol:  $V_{ab} = \sqrt{3} 5 \angle 90^\circ$

+ve = -12

ex 100  $V_{an} = 10 \angle 0^\circ$

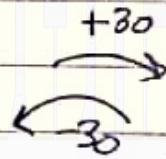
$V_{ab} = \sqrt{3} 10 \angle 30^\circ$

$V_{bn} = 10 \angle -120^\circ$

$V_{bc} = \sqrt{3} 10 \angle -90^\circ$

$V_{cn} = 10 \angle -240^\circ$

$V_{ca} = \sqrt{3} 10 \angle -210^\circ$



ex 100  $V_{an} = 5/\sqrt{3} \angle 150^\circ$

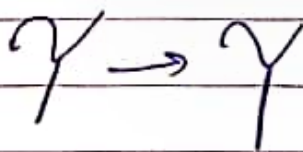
$V_{ab} = 5 \angle 180^\circ$

$V_{bn} = 5/\sqrt{3} \angle 30^\circ$

$V_{bc} = 5 \angle 60^\circ$

$V_{cn} = 5/\sqrt{3} \angle -90^\circ$

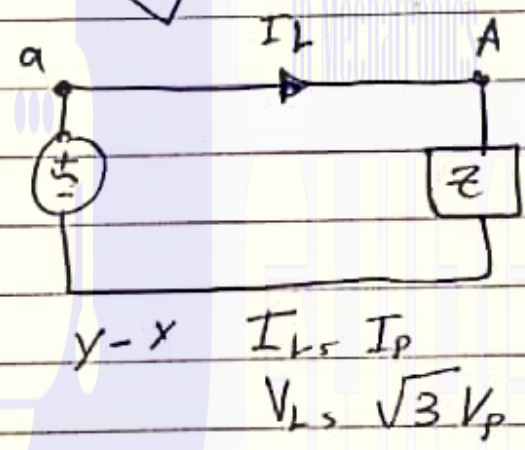
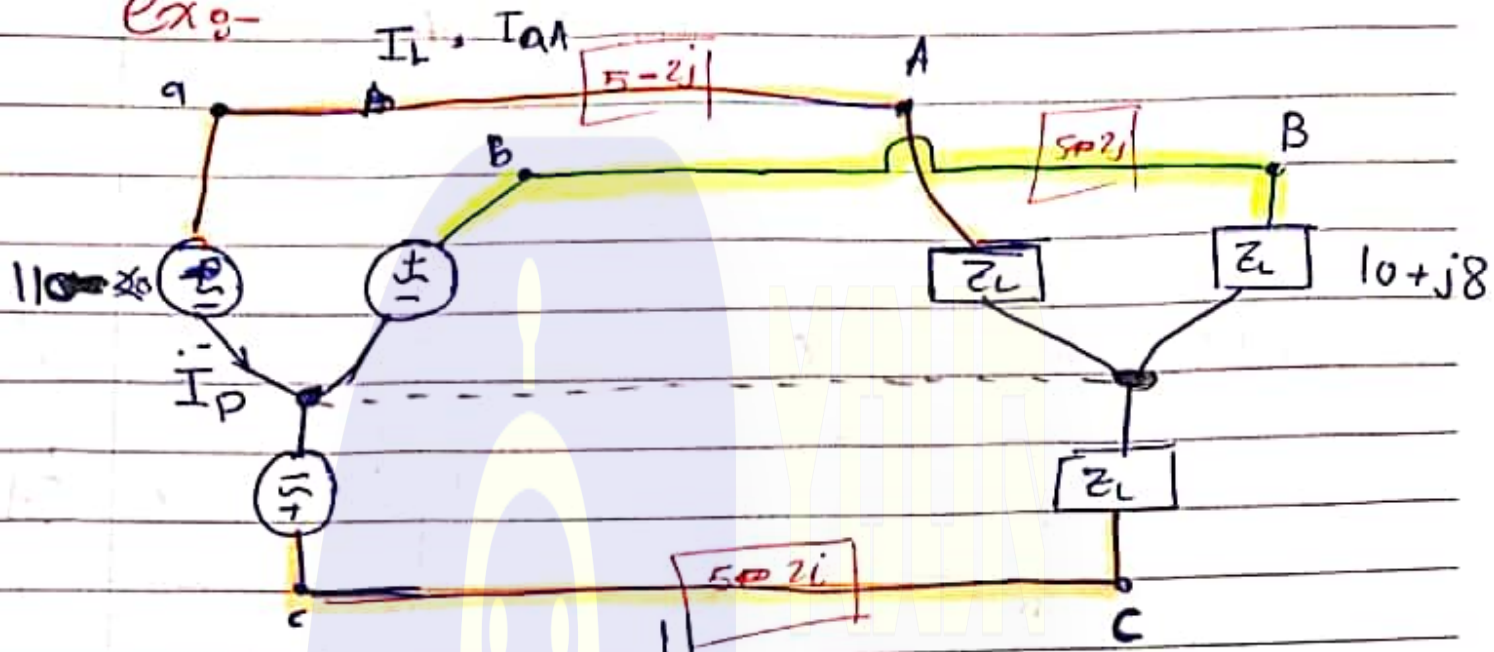
$V_{ca} = 5 \angle -60^\circ$



$$I_L = I_p$$

$$V_L = \sqrt{3} V_p$$

Exo-



$I_{aA} \angle 0^\circ$   
 $I_{bB} \angle -120^\circ$   
 $I_{cC} \angle -240^\circ$

$\downarrow$   $\omega L$  in  $\angle 90^\circ$   $\rightarrow$   $5 - 2j$   $\angle 10^\circ$   $\rightarrow$   $aA, bB, cC$

$I_{aA} = \frac{100 \angle 0^\circ}{5 - 2j + 10 + j8} = 8.6 \angle -21.8^\circ$   
 $I_{bB} = 8.6 \angle -158.6^\circ$   
 $I_{cC} = 8.6 \angle -278.6^\circ$

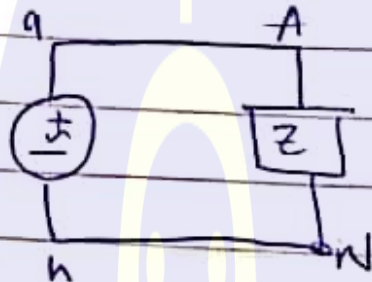
$I_{aA} + I_{bB} + I_{cC} = 0 = I_{nN}$



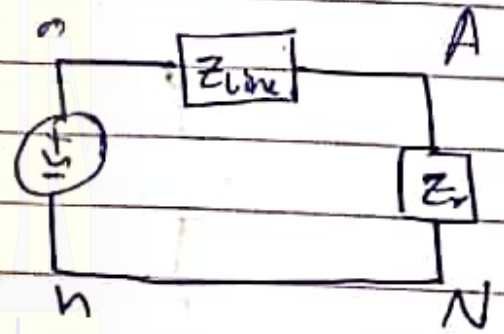
① power per Phase  $\leq S_{\phi} V I^*$   
 $\leq V_{AN} I_{\phi}^*$

$1100 \times 0 + (8.6 \times 38.6) \text{ VA}$

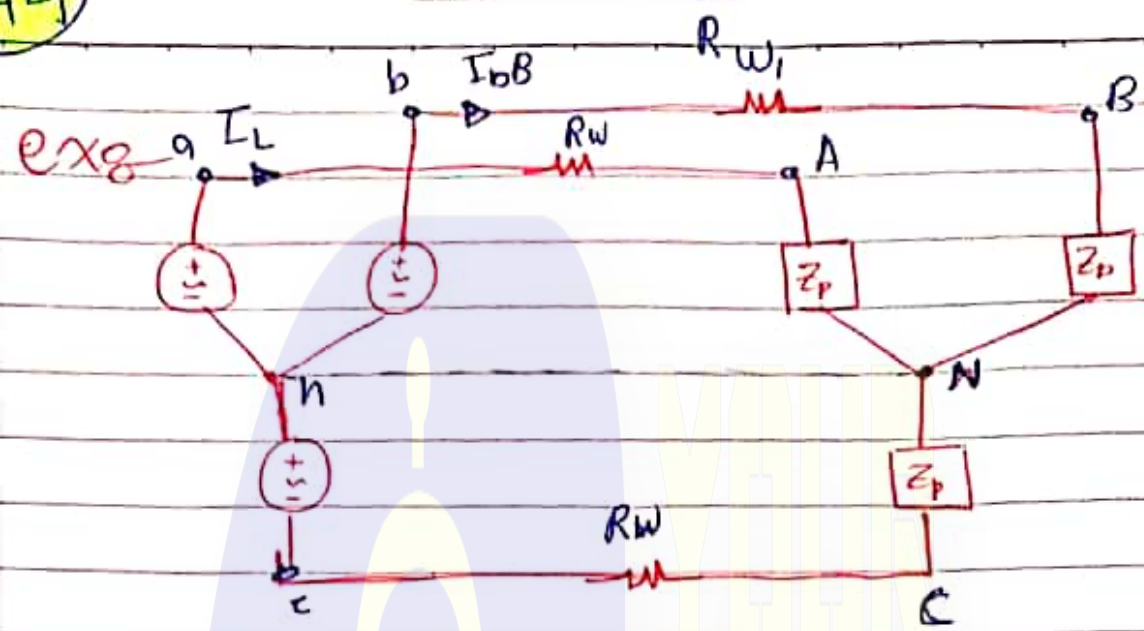
Note



$V_{AN} = V_{an}$



$V_{AN} \neq V_{an}$

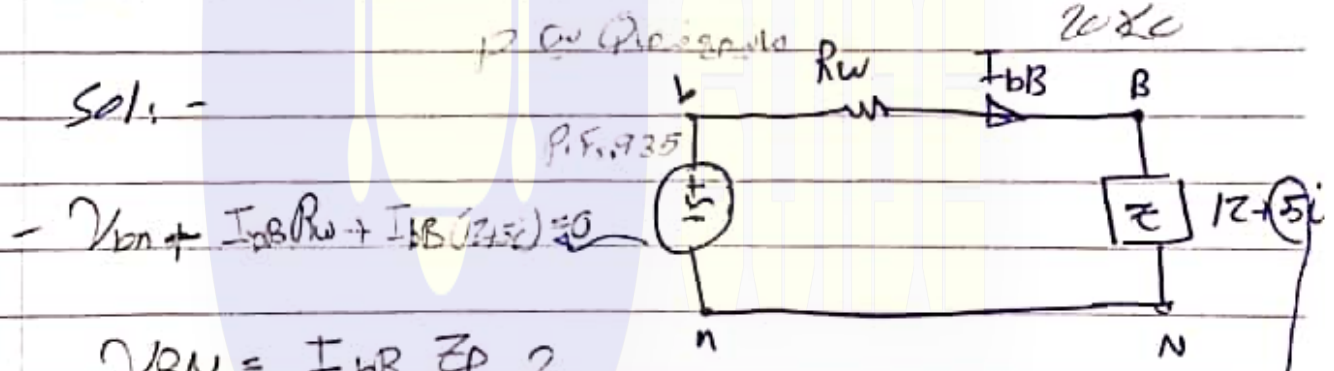


$Z_p = 12 + j5 \Omega$   
 $I_{bB} = 20 \text{ A}$   
 P.F.s 0.935 from source.

Find:  $R_w, V_{bn}, V_{AB} - V_T$

Sol: -

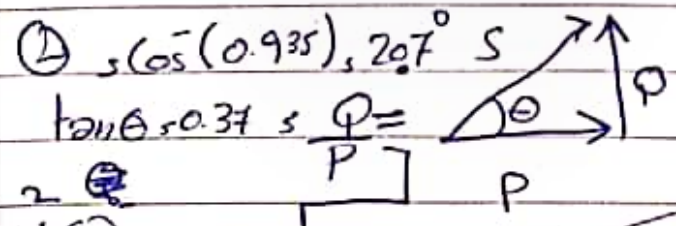
12 ohm resistor



$-V_{bn} + I_{bB}R_w + I_{bB}(12 + j5) = 0$

$V_{bN} = I_{bB} Z_p$

$V_{AB} = V_{An} - V_{bN}$



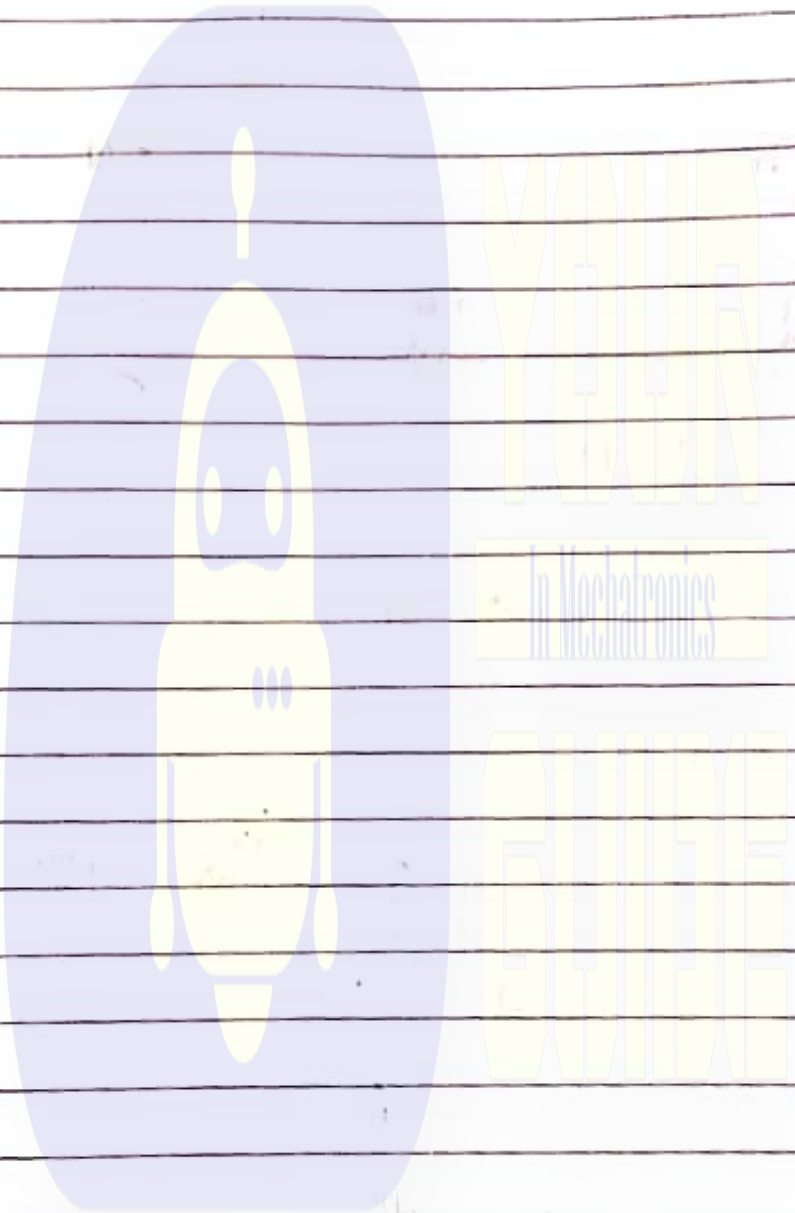
$P = \frac{20^2 \cdot 5}{0.937} = 5405 \text{ W}$

$P = I^2 (R_w + 12)$   
 $R_w = 1.5 \Omega$

50

No. \_\_\_\_\_

لازم آفرینا - 3 - 1

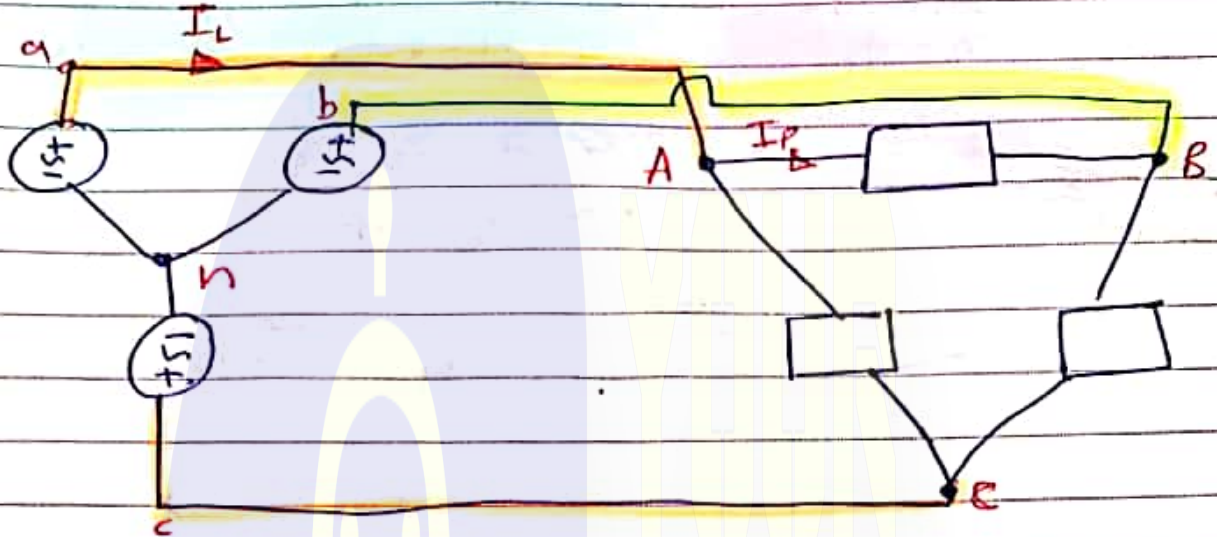


(51)

No. \_\_\_\_\_

wednesday  
26/6/2019

$Y \rightarrow \Delta$  connection



$I_L \neq I_P$   
 $V_L = V_P$

- ①  $I_P = |I_{AB}|, |I_{BC}|, |I_{CA}|$
- ②  $V_{AB}, V_{BC}, V_{CA}$  (ident trans line)  
 و الجهد على الجهد و الجهد على الجهد

$I_{\phi A} = I_{AB} - I_{CA}$   
 $I_{AB} = \frac{V_{AB}}{Z_P}$

Ex: - Van  $200 \angle 60^\circ$ , Sp = 2-j KVA,  
 Find  $V_{BC}$ ,  $Z_P$ ,  $I_{\phi A}$  &  $I_L$ ?

sol

$V_{an} = 200 \angle 60^\circ$        $V_{ab} = \sqrt{3} 200 \angle 90^\circ$   
 $V_{bn} = 200 \angle -60^\circ$        $V_{bc} = \sqrt{3} 200 \angle -30^\circ = V_{BC}$   
 $V_{cn} = 200 \angle -180^\circ$        $V_{ca} = \sqrt{3} 200 \angle -150^\circ$  (ident)

$V_{AB} = V_{ab} \rightarrow V_{BC} = V_{bc} \rightarrow V_{CA} = V_{ca}$

$SP = V_P \cdot I_{AB}$

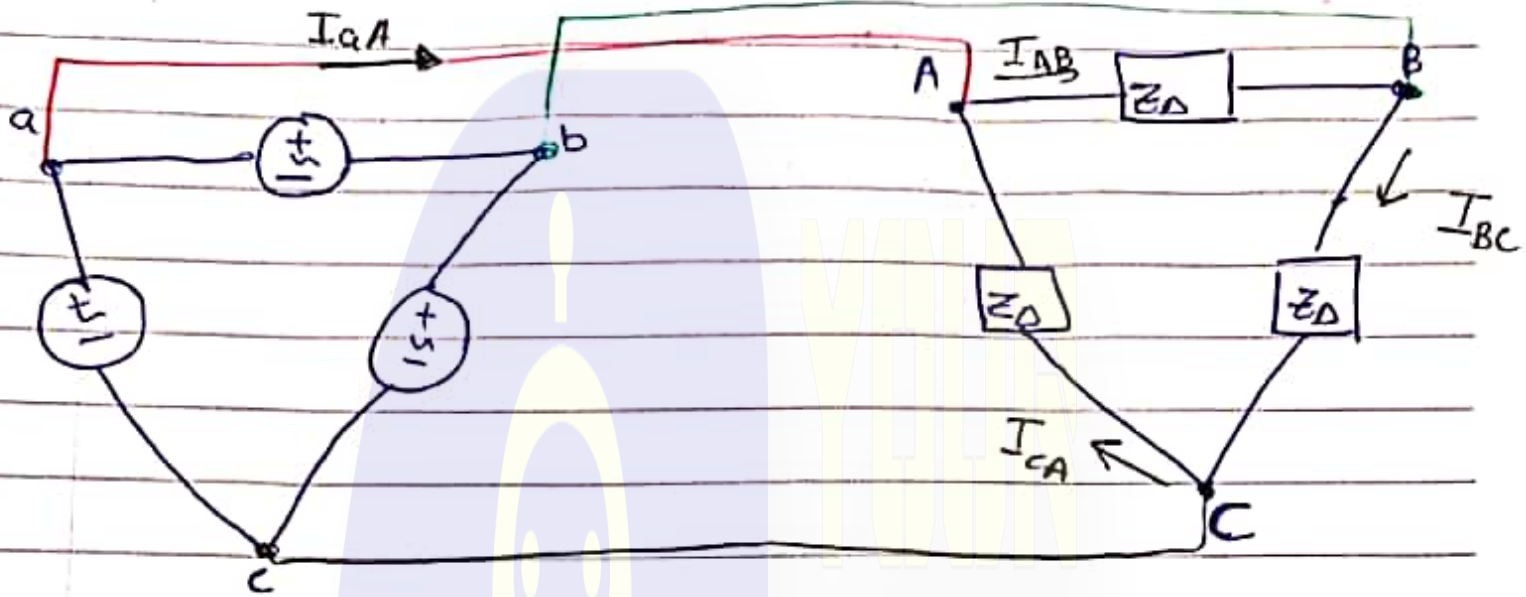
$$Y \rightarrow \Delta$$
$$I_L \neq I_p$$

$$V_L \approx V_p$$

$$Y \rightarrow Y$$
$$I_L \approx I_p$$

$$V_L \approx \sqrt{3} V_p$$

$\Delta \rightarrow \Delta$  Connection



$$* I_{AB} = \frac{V_{AB}}{Z_D} = \frac{V_{ab}}{Z_D}$$

$$* I_{aA} = I_{AB} - I_{CA} = \sqrt{3} I_P$$

54

No. \_\_\_\_\_

*solusi*

*Cr*  $Z_D = Z_0 - j15$ ,  $V_{ab} = 330 \angle 0^\circ V$

calakafethe phase current & the line current

Sol,  $I_{AB} = \frac{V_{ab}}{Z_D} = \frac{330 \angle 0^\circ}{20 - j15} = 13.2 \angle 36.8^\circ A$

$I_{BC} = 13.2 \angle -83.2^\circ A$

$I_{CA} = 13.2 \angle -203.2^\circ A$

$I_L = \sqrt{3} I_P$

$I_{aA} = I_{AB} - I_{CA} = 22.6 \angle 6.87^\circ A$

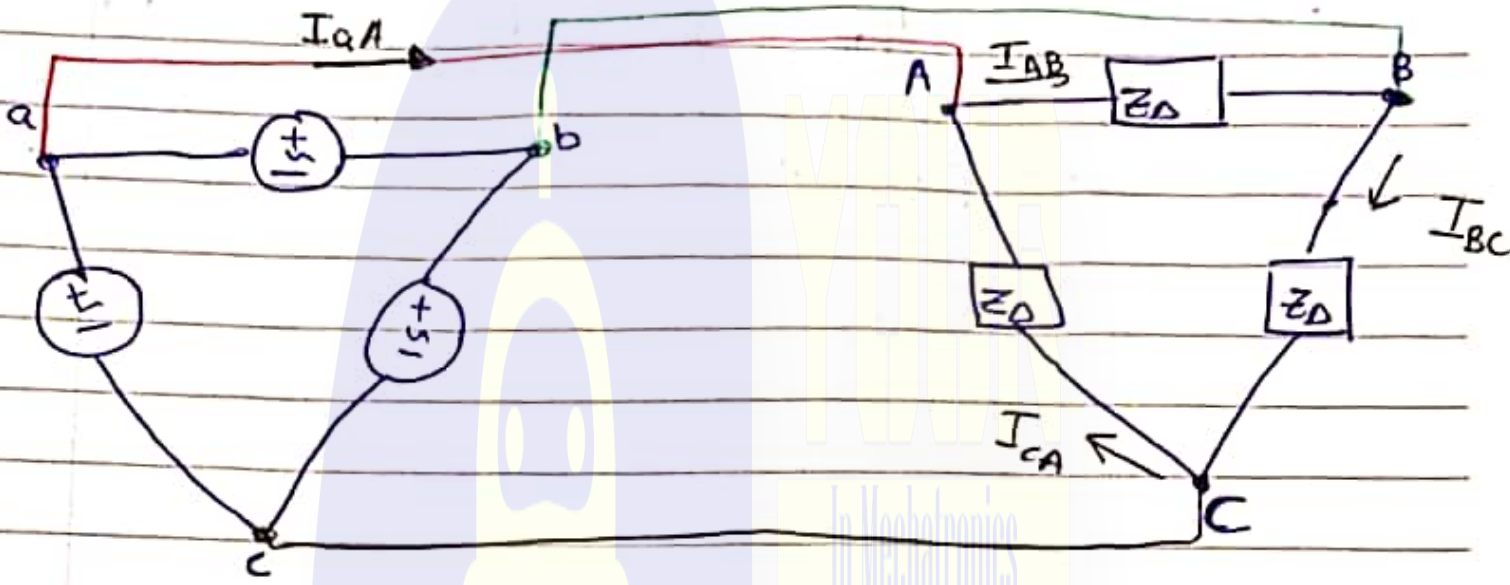
OR  $\sqrt{3} (I_{AB}) \angle \ominus -30^\circ$

*you can't use them if it wasn't abalanced load.*

$I_{bB} = \sqrt{3} I_{BC} \angle \ominus -3$

$S_P = 0.5 V_{AB} I_{AB}$

$\Delta \rightarrow \Delta$  Connection

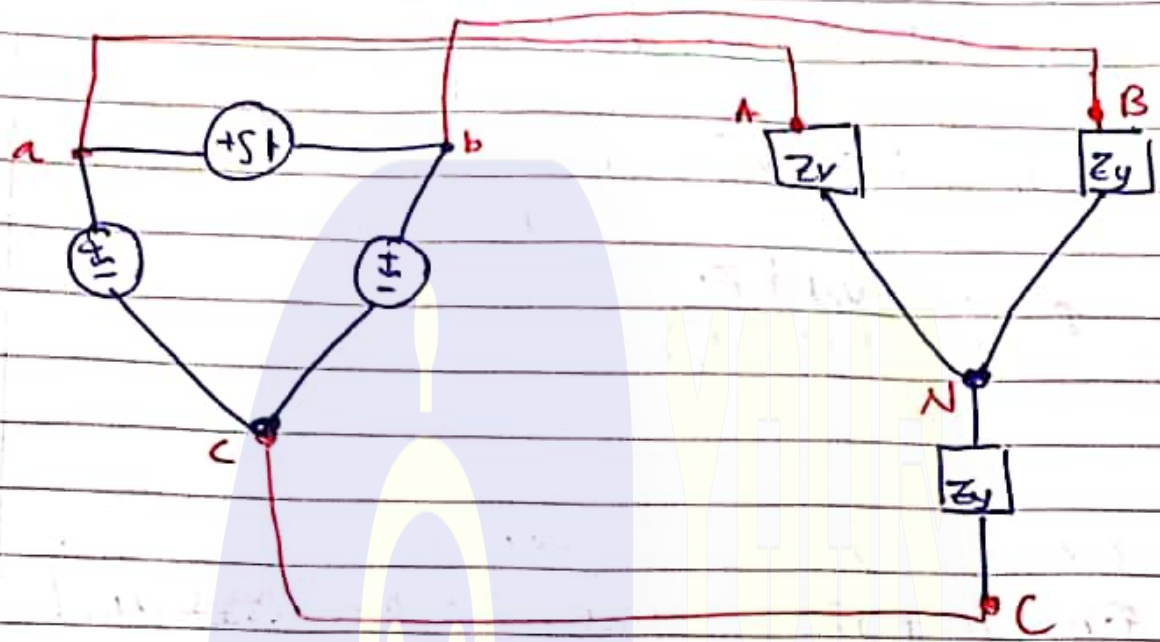


\*  $I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{V_{ab}}{Z_{\Delta}}$

\*  $I_{aA} = I_{AB} - I_{CA} = \sqrt{3} I_p$

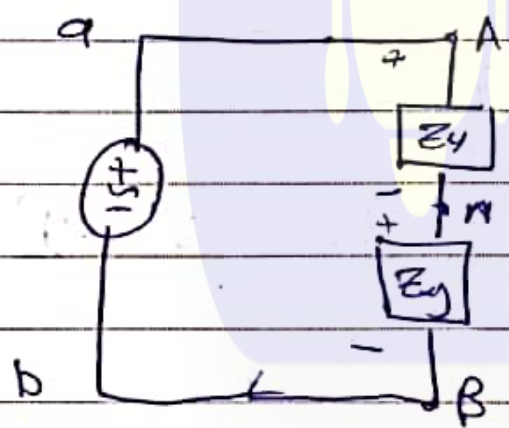


$\Delta \rightarrow Y$  connection



$$I_{AA} = \frac{V_{an}}{Z_Y}$$

$$V_{an} = \frac{V_{ab}}{\sqrt{3}} \angle -30^\circ$$



$$-V_{ab} + I_{AN} Z_Y + I_{BN} Z_Y = 0$$

$$Z_Y (I_{AN} - I_{BN}) = V_{ab}$$

$$\rightarrow I_{AN} - I_{BN} = \frac{V_{ab}}{Z_Y}$$

$$I_{AN} = K \angle 0^\circ$$

$$I_{BN} = K \angle -120^\circ$$

$$K \cos(-120^\circ) + jK \sin(-120^\circ)$$

$$0.5K - j \frac{\sqrt{3}}{2} K$$

$$= \frac{-K}{2} (1 + j\sqrt{3})$$

$$= I_{AN} (1 + j\sqrt{3})$$

smile...

$$I_a = \frac{-I_a (1+j\sqrt{3})}{2} = \frac{V_{ab}}{Z_y}$$

$$I_{aA} (1.5 + j\frac{\sqrt{3}}{2}) = \frac{V_{ab}}{Z_y}$$

$$I_{aA} = \frac{V_{ab} | Z_y}{(1.5 + j\frac{\sqrt{3}}{2})}$$

ex:  $\Delta \rightarrow Y$  بداية التحويل من دلتا إلى ي  
 $Z_y = 40 + j25$      $V_{ab} = 210 \angle 0^\circ$     ~~Find~~ Find  $I_{aA}$ ?

Sol

~~$I_{aA} = \frac{210 \angle 0^\circ}{\sqrt{3}}$~~      $V_{ab} = \frac{210}{\sqrt{3}} \angle 0-30^\circ$

$$V_{ab} = 121.2 \angle -30^\circ$$

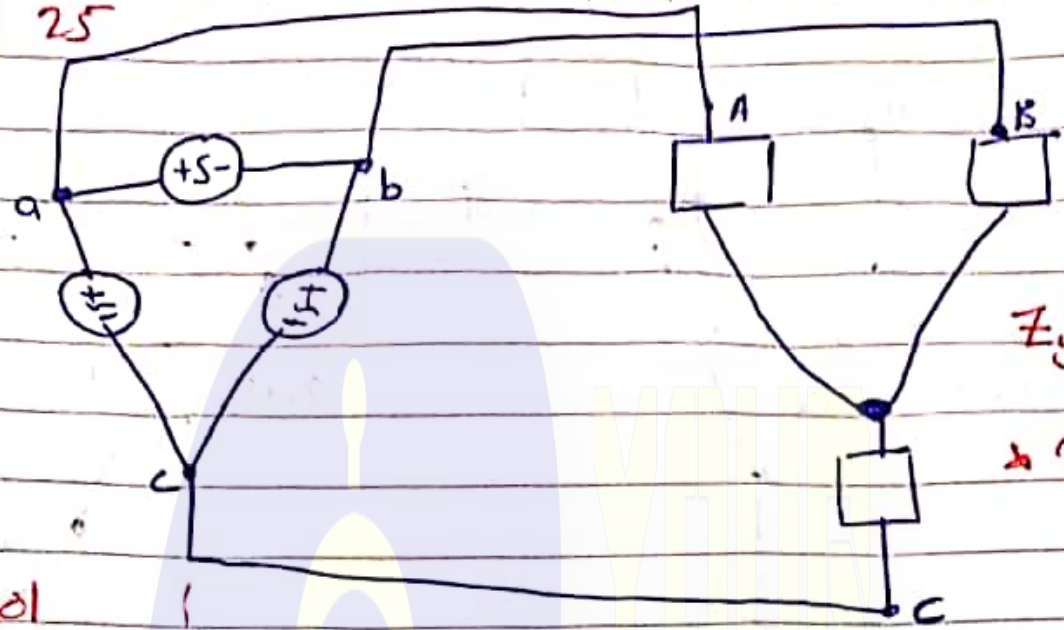
$$I_{aA} = \frac{121.2 \angle -30^\circ}{40 + j25} = 2.57 \angle -62^\circ \text{ A}$$

58

No.

Monday  
1/7/2019

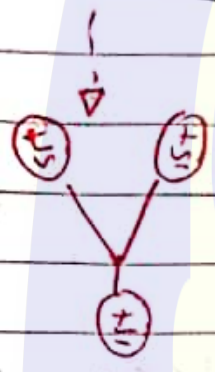
Q 25



$$Z_T = 10 - j8$$

$$V_{ab} = 440 \text{ V}$$

sol



$$V_{an} = \frac{440 \angle -20}{\sqrt{3}}$$

$$I_{aA} = \frac{V_{an}}{Z_{AN}} = \frac{440/\sqrt{3} \angle -20}{10 - j8} = 19.8 \angle 18.4^\circ$$

$$I_{bB} = 19.8 \angle 10.35^\circ$$

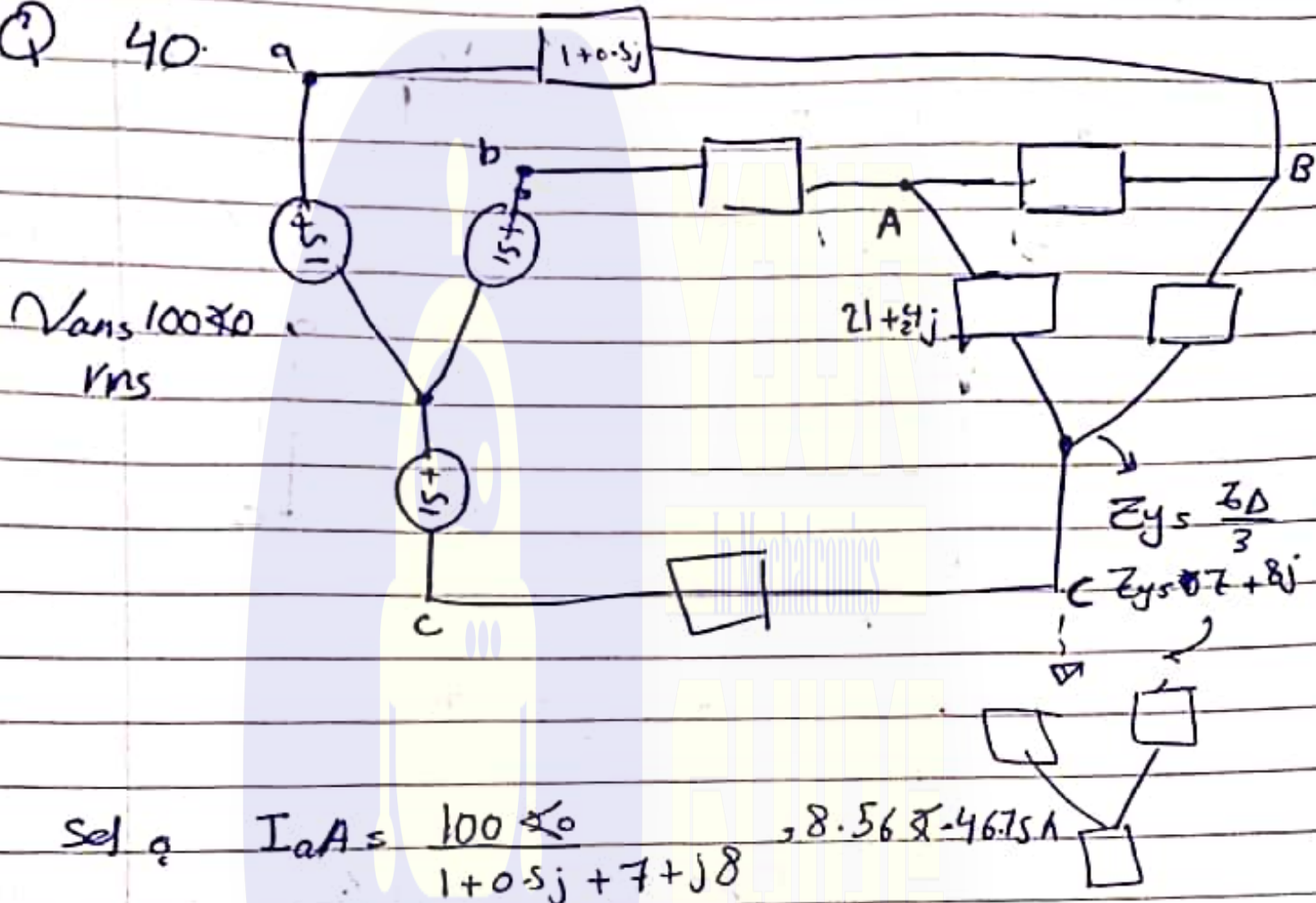
if there was a line imp that is equal to  $3 + j2$  in each line.

$$\text{sol. } I_{aA} = \frac{440 \angle -20}{3 + j2} = 177 \angle 4.68^\circ$$

(59)

No. \_\_\_\_\_

Q 40



sel a  $I_{aA} = \frac{100 \angle 0}{1+0.5j + 7+8j}$

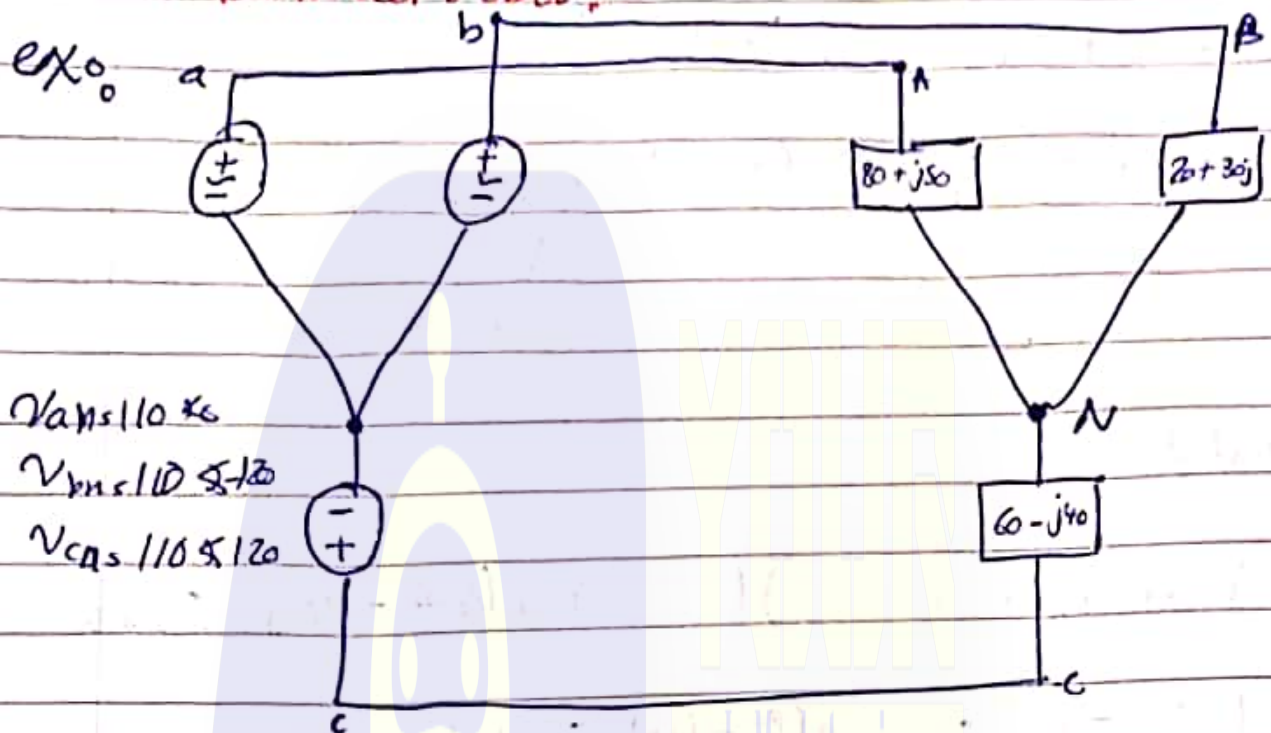
$= 8.56 \angle -46.75^\circ$

Find the average power absorbed by the load?  $S_p = V I_{aA}^*$   
 $S_p = 3V I_{aA}^*$   
 $P_{avg} = |I_{aA}|^2 R_L \times 3$  for load

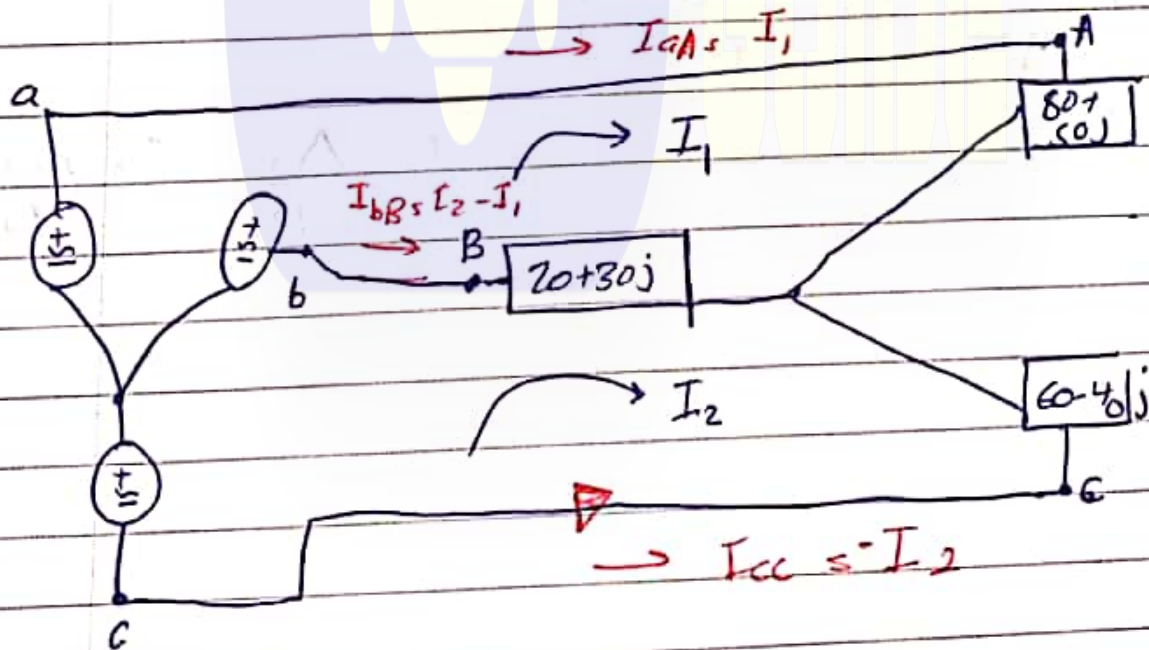
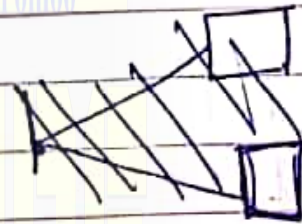
(60)

No. \_\_\_\_\_

### Unbalanced Load :-



AC circuit  
abdomen



61

No.

mesh 1

$$V_{bn} - V_{an} + I_1(80 + 50j)(20 + j30)(I_1 - I_2) = 0$$

$$(100 + j80) I_1 - I_2(20 + 30j) = V_{an} - V_{bn}$$

$= V_b$

$$165 + j95.263$$

mesh 2

$$V_{cn} + V_{bn} + (20 + 30j)(I_2 - I_1) + (60 \angle -40^\circ) I_2 = 0$$

$$(-20 - j30)I_1 + (80 - j10)I_2 = V_{bc} = 110 \angle -120^\circ - 110 \angle 120^\circ = 190.5 \angle -90^\circ$$

$$\Delta_s = \begin{bmatrix} 100 + j80 & -20 - j30 \\ -20 - j30 & 80 - j10 \end{bmatrix}$$

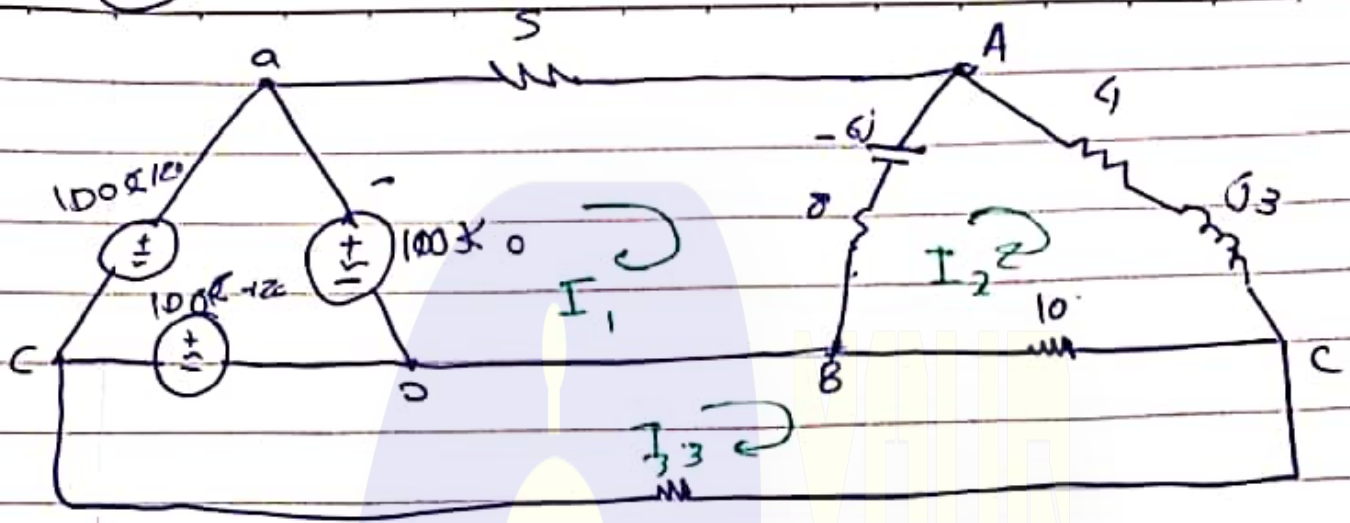
$$\Delta_1 = \begin{bmatrix} 165 + j95.263 & -20 - j30 \\ -j190.5 & 80 - j10 \end{bmatrix}$$

$\Delta_2$  :

$$I_1 = \frac{\Delta_1}{\Delta} \quad I_2 = \frac{\Delta_2}{\Delta}$$

62

No. \_\_\_\_\_



Mesh 1, 2, 3,  $I_1, I_2, I_3$

$$I_2 = -I_{CA}$$

$$I_1 - I_2 = I_{AB}$$

$$I_3 - I_2 = I_{BCA}$$

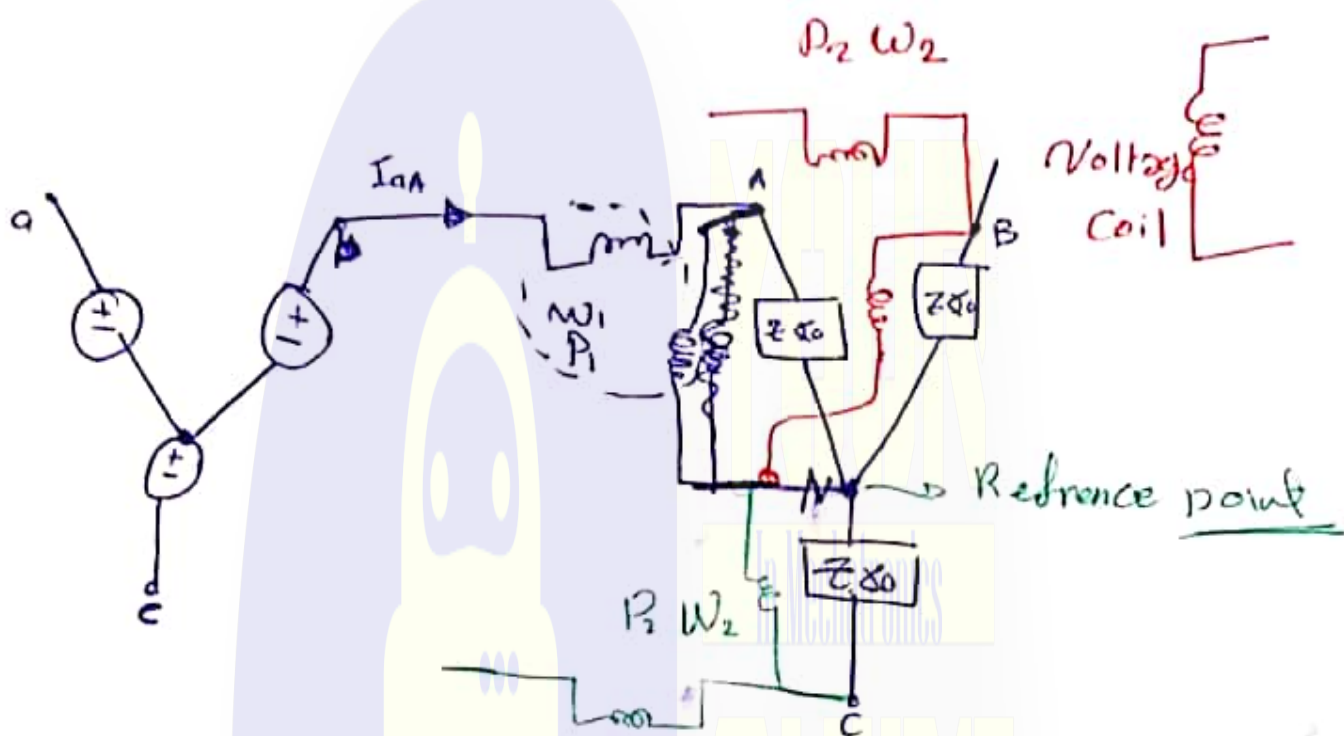
CH11 + Y → Y & Y → Δ | exam 7/17

(63)

Tuesday  
2/7/2014

Watt-meter (Average Power)

current coil



$$P_T = P_1 + P_2 + P_3$$

$$P_1 = P_2 = P_3 = V_p I_p \cos(\theta_v - \theta_i)$$

$$= \frac{V_L}{\sqrt{3}} I_p \cos(\theta_v - \theta_i)$$

$$= 3 I_p \frac{V_L}{\sqrt{3}} \cos(\theta_v - \theta_i)$$

$$P_T = 3 I_L \frac{V_L}{\sqrt{3}} \cos \theta$$

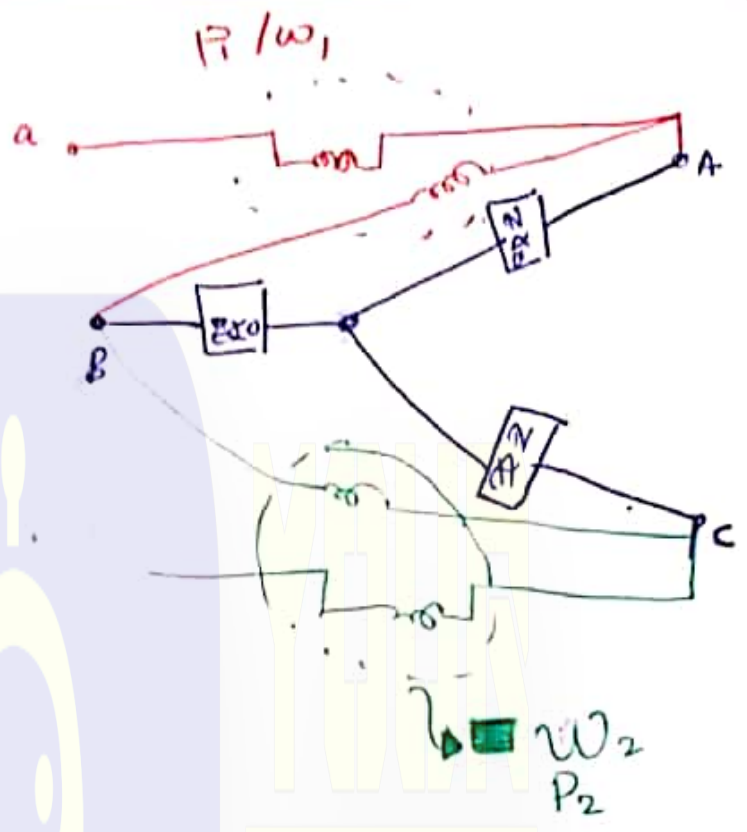
$$P_T = \sqrt{3} I_L V_L \cos \theta$$



If we use 2 wattmeters (64)

180  
180

BC / CB  
180



$$P_T = P_1 + P_2$$

$$P_1 = V_L I_L \cos(\theta + 30^\circ)$$

$$P_2 = V_L I_L \cos(\theta - 30^\circ)$$

$$P_T = V_L I_L (\cos(\theta + 30^\circ) + \cos(\theta - 30^\circ))$$

$$P_T = V_L I_L (2 \cos 30^\circ \cos \theta)$$

$P_T = V_L I_L \cos \theta$   
 Wattmeter 3  
 Ref 180

$$V_{ab} = V_L \angle 30^\circ$$

$$V_{bc} = V_L \angle -90^\circ$$

$$V_{ca} = V_L \angle -210^\circ$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

# For Two Wattmeter

(65)

- ①  $P_T = P_1 + P_2$
- ②  $Q = \sqrt{3}(P_2 - P_1)$
- ③  $S_T = \sqrt{P_T^2 + Q^2}$
- ④  $\tan \theta = \frac{Q_T}{P_T} = \sqrt{3} \frac{P_2 - P_1}{P_2 + P_1}$

$$Z_{\theta} = \frac{V_{ab}}{I_{an}}$$

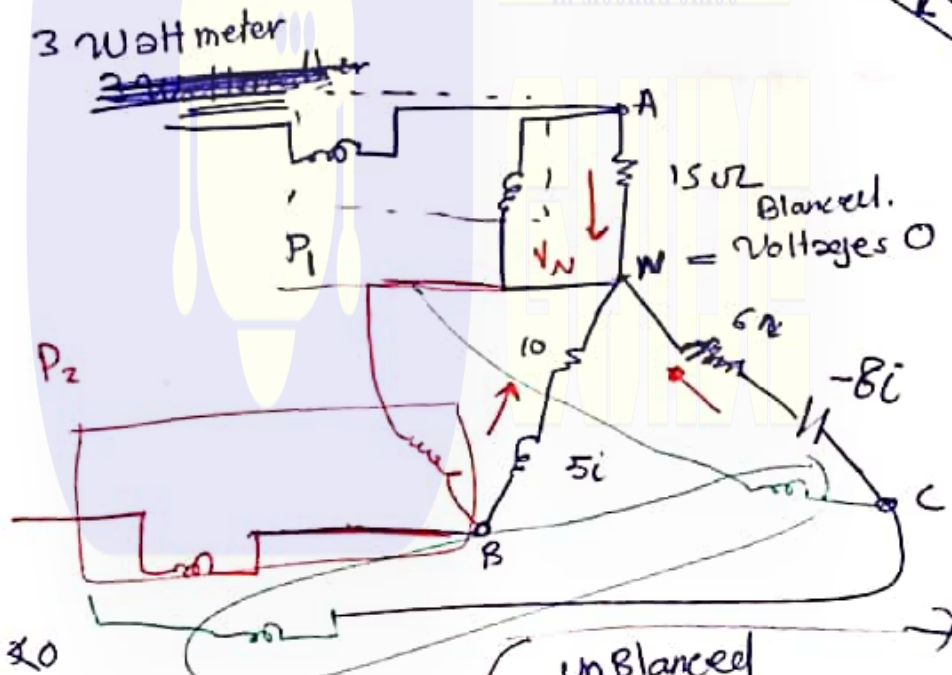
$$I = \frac{V_{ab}}{Z_{\theta}}$$

Use  $\cos \phi$  (30-60-90)

if  $P_1 > P_2$  Resistive load  
 $P_2 > P_1$  Inductive load  
 $P_2 < P_1$  Capacitive load



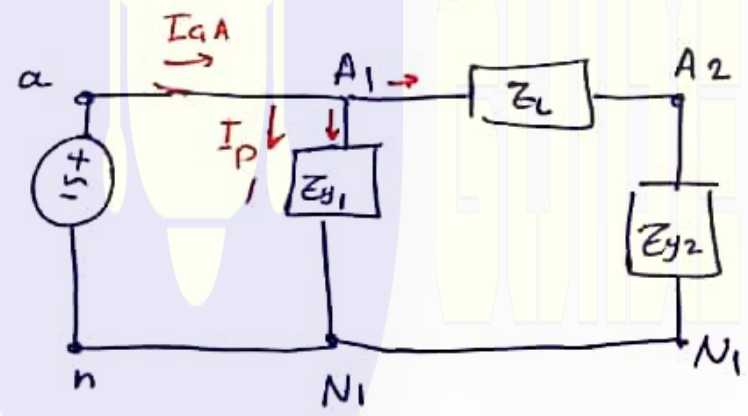
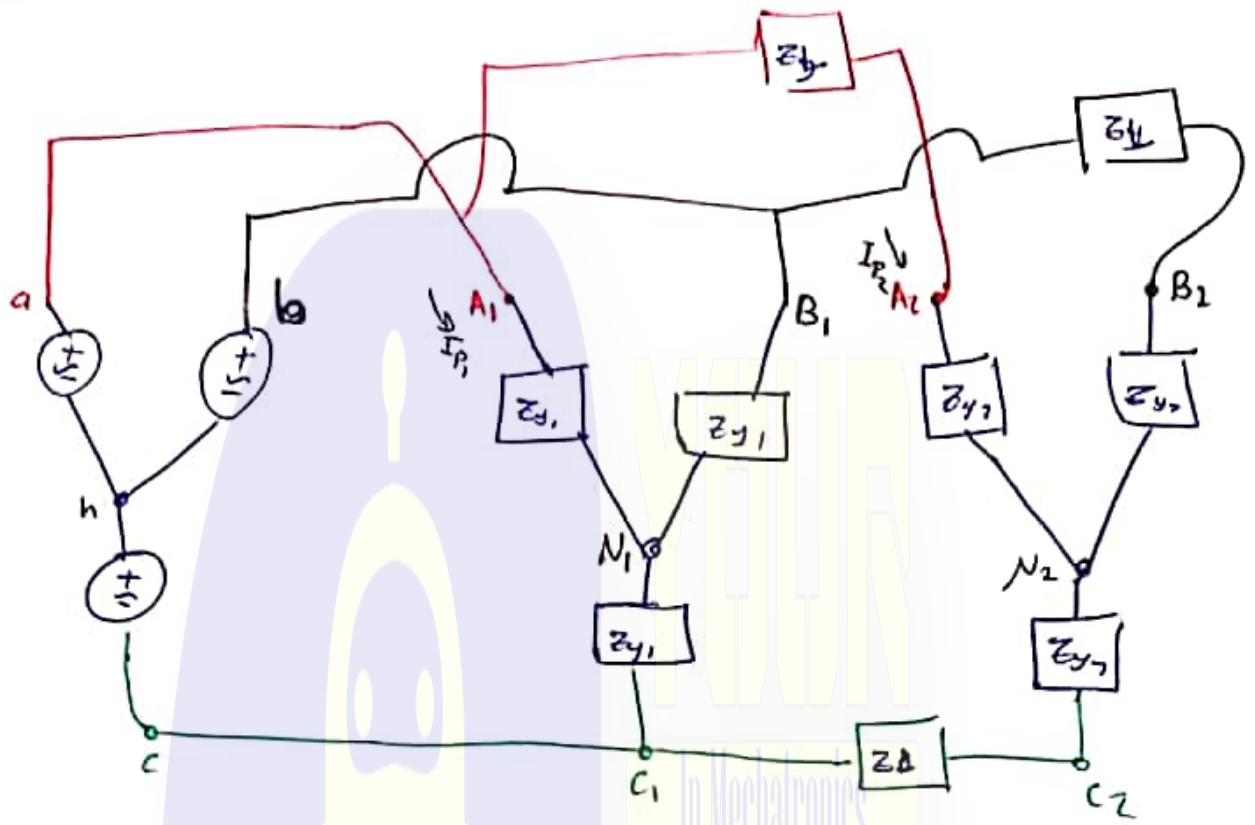
ex: 12.13



- $V_{AN} = 100 \angle 0$
- $V_{BN} = 100 \angle 120$
- $V_{CN} = 100 \angle -120$
- $I_{aA} = 6.67 \angle 0$
- $I_{bB} = 8.14 \angle 43.44$
- $I_{cC} = 10 \angle -66.84$

$$\frac{V_{AN} - V_N}{Z_a} + \frac{V_B - V_N}{Z_b} + \frac{V_C - V_N}{Z_c} = 0$$

③



$$I_{A1} = \frac{V_{an}}{Z_{Y1}}$$

$$I_{P2} = \frac{V_{an}}{Z_L + Z_{Y2}}$$

$$I_{QA} = I_{P1} + I_{P2}$$

$$P_T = P_1 + P_2$$

$$Q_T = \sqrt{3} (P_2 - P_1)$$

$$S_T = P_T \pm j Q_T = \sqrt{P_T^2 + Q_T^2}$$

load & power conservation

$$\tan \theta = \sqrt{3} \frac{P_2 - P_1}{P_1 + P_2}$$

Load  $\downarrow$   $\uparrow$   $\downarrow$   $\uparrow$

12.70

$$P_1 = 1200 \text{ W}$$
$$P_2 = -400 \text{ W}$$

$$V_L = 240 \text{ V}$$
$$I_L = 6 \text{ A}$$

Find P, F &  $\theta$  &  $Z_L$

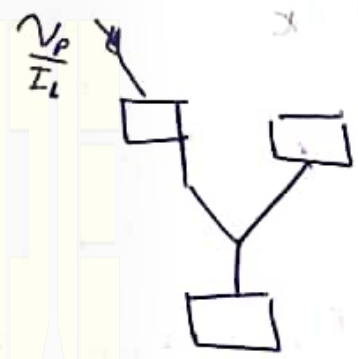
Sol  $P_T = 1200 - 400 = 800 \text{ W}$

$$Q = \sqrt{3} (-400 - 1200) = -2771.28 \text{ VAR}$$

$$\tan \theta = \frac{Q_T}{P_T} = \frac{-2771.28}{800}$$

$$\theta = -73.89^\circ$$

$$Z = \frac{(240/\sqrt{3})}{6} \angle -73.89^\circ$$



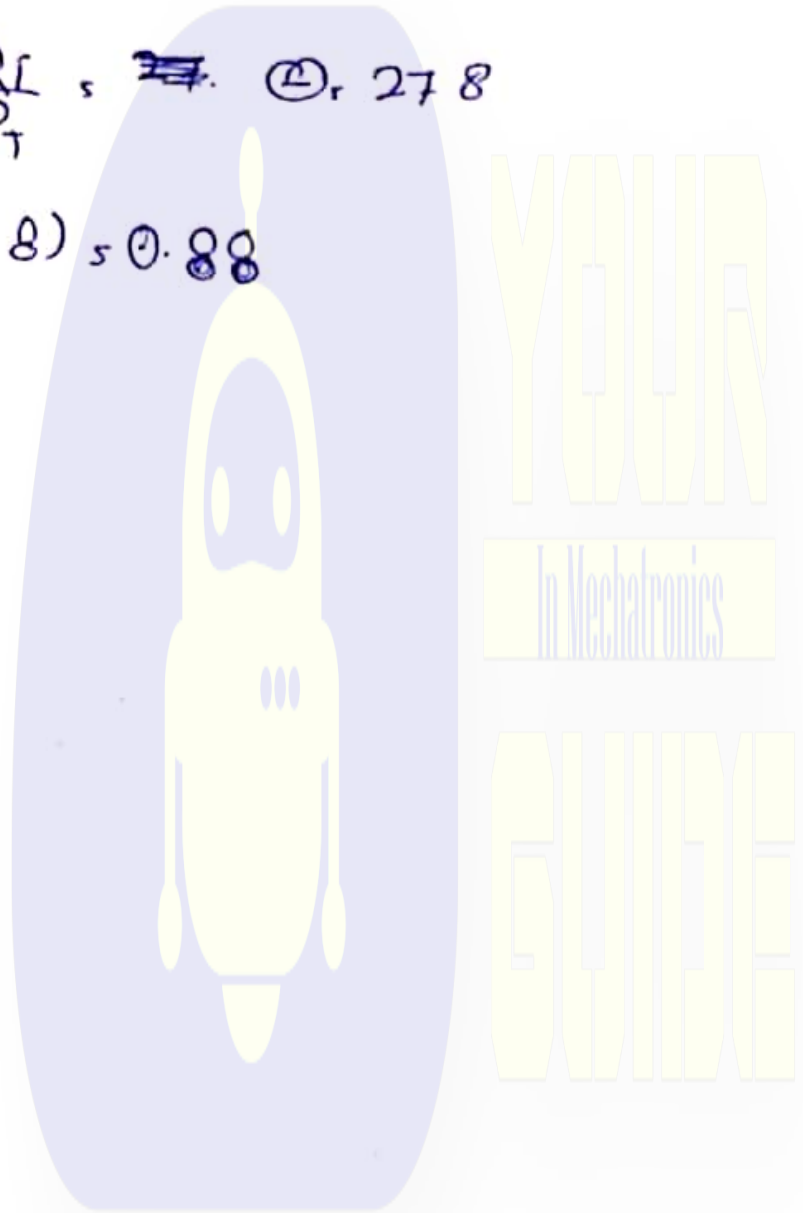
$$P_T = P_1 + P_2 = 7394.5 \text{ W}$$

$$Q = \sqrt{3} (4803.5 - 2591) = 3832.5 \text{ VAR}$$

$$|S| = \sqrt{P_T^2 + Q^2} = 8328.5$$

$$\tan \theta = \frac{Q}{P_T} = 0.519 \Rightarrow \theta = 27.8$$

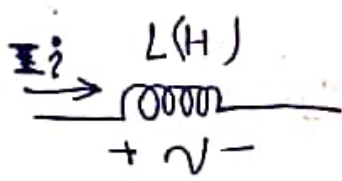
$$\cos(\theta) = 0.88$$



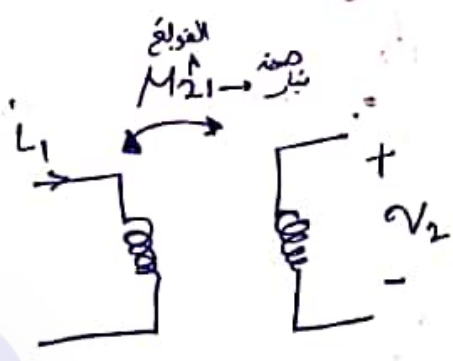
CH 13

(71)

Magnetic Coupling CKTs

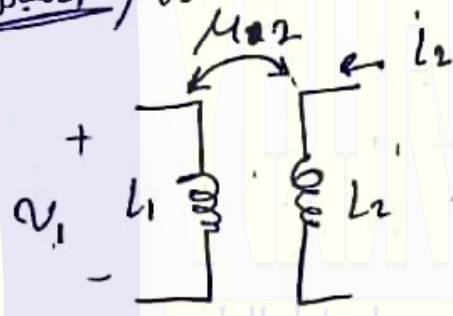


$$v(t) = L \frac{di(t)}{dt}$$



$$v_2 = M_{21} \frac{di_1}{dt}$$

دباؤ  $\mu_s$  Mutual inductance



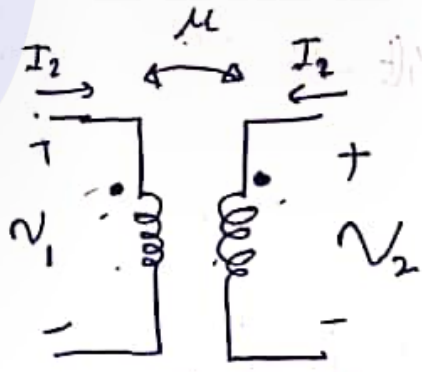
$$v_1 = M_{12} \frac{di_2}{dt}$$

$$M_{12} = M_{21} = \mu$$

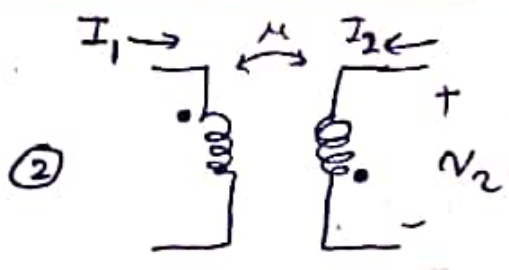
$\mu_s$  Mutual inductance: - ~~The coil~~  $\mu$

↳ The ability of a coil to induce a voltage in other coils

\* Dot convention 8- ①



$$v_2 = \mu \frac{di_1}{dt}$$



$$V_2 = -M \frac{di_1}{dt}$$

تیار داخل بی • پوزیٹو علامت

(3)



$$V_2 = -M \frac{di_1}{dt}$$

(4)



$$V_2 = M \frac{di_1}{dt}$$

اذا اعمارنا  
تغییرت بطل



$$i_2(t) = 5 \sin 45t$$

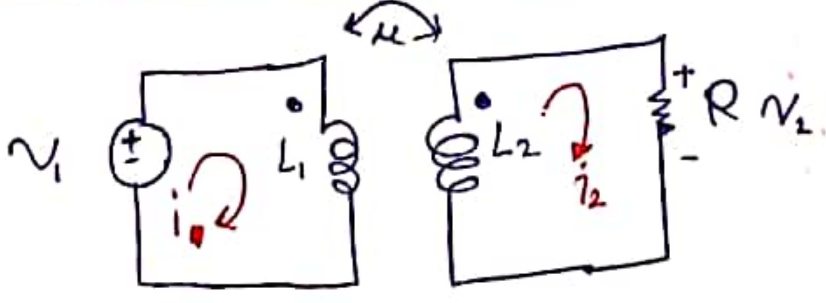
$$V_1(t) = M \frac{di_2}{dt} = (-2(5(45) \cos 45t)) = -450 \cos 45t \text{ V}$$

Current source 1

Current source 1

let  $i_1 = 4e^{-6t}$  find  $V_2$

$$V_2 = -M \frac{di_1}{dt} = -2(4(-6)e^{-6t}) = 48e^{-6t} \text{ V}$$



Mesh 1

$$-v_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = 0$$

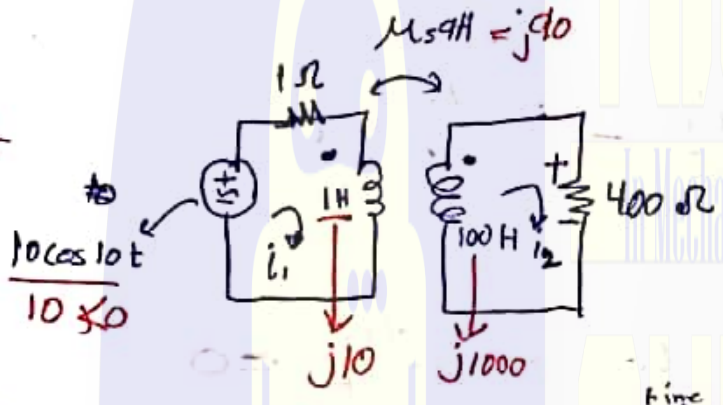
$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

Mesh 2

$$-v_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = 0$$

$$v_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

ex



Sol

Mesh 1

$$v_1 = i_1(1) + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_1 = I_1(1) + j10 I_1 - j90 I_2$$

$$10 \angle 0 = I_1 + j10 I_1 - j90 I_2$$

$$(1 + 10j) I_1 - j90 I_2 = 10 \angle 0 \quad \text{--- (1)}$$

Mesh 2

$$400 I_2 + 100j I_2 - 90j I_1 = 0$$

$$-90j I_1 + (400 + 100j) I_2 = 0 \quad \text{--- (2)}$$

Solving by Cramer rule

Time domain  $i \rightarrow d/dt \rightarrow \frac{di}{dt}$   
 Freq  $i \rightarrow j\omega \rightarrow I, j\omega$



$$\Delta = \begin{vmatrix} & \\ & \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} & \\ & \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} & \\ & \end{vmatrix}$$

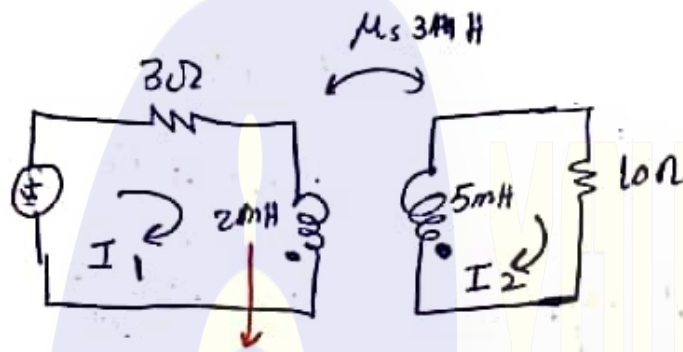
$$I_1 = \frac{\Delta_1}{\Delta}$$

$$I_2 = \frac{\Delta_2}{\Delta}$$

24

ex:

$v_1$   
 $-1000t$   
 $20e$   
 ما لا نألف  
 Solg  
 Mesh

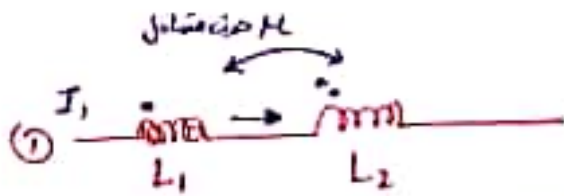


3  
 حون  
 تعرات

$$-v_0 + 3i_1 + 2 \frac{di_1}{dt} - 3 \frac{di_2}{dt} = 0$$

Mesh 2

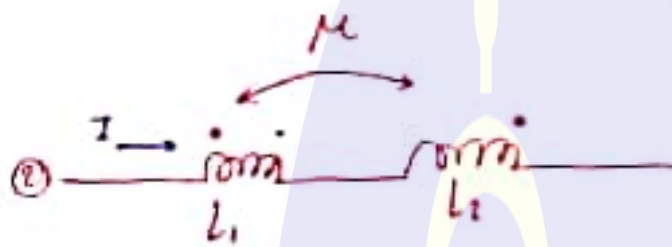
$$10i_2 + 5 \frac{di_2}{dt} - 3 \frac{di_1}{dt} = 0$$



$$L_{T1} = L_1 + M$$

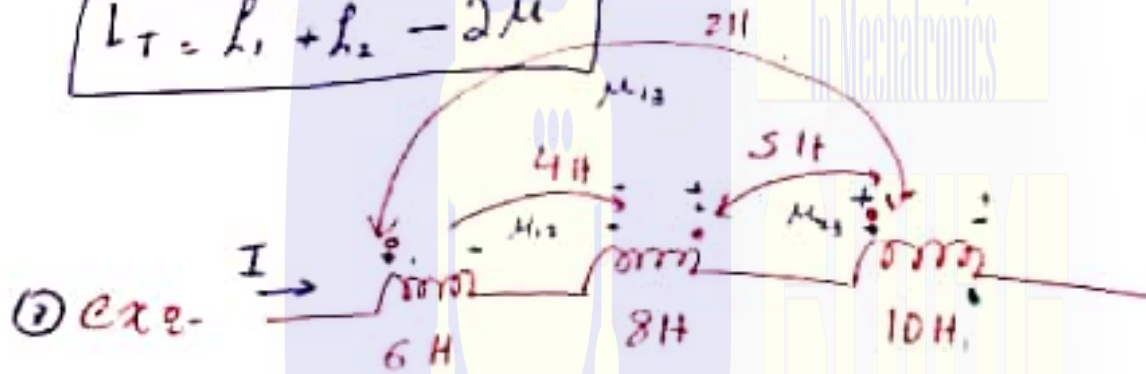
$$L_{T2} = L_2 + M$$

$$L_T = L_1 + L_2 + 2M$$



$$L_T = L_1 - M + L_2 - M$$

$$L_T = L_1 + L_2 - 2M$$



(+) = positive

لشوف فيه  
دوب داخل  
+ 6H

مكلا  
دوسر من ال 10H

(+) = positive

دوسر

Find  $L_T$  ?

Soln-

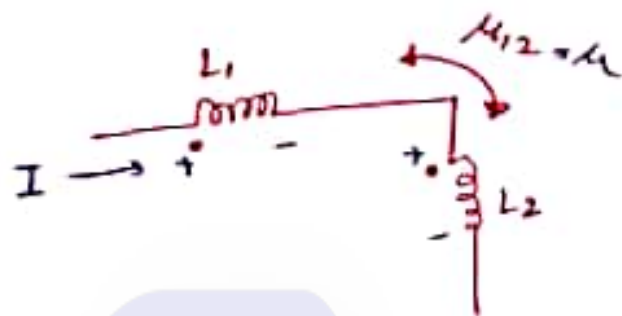
$$L_{T1} = 6 - 4 + 2 = 4$$

$$L_{T2} = 8 - 4 - 5 = -1$$

$$L_{T3} = 10 + 2 - 5 = 7$$

$$L_T = 10 H$$

ex-9



$$L_T = L_1 + M + L_2 + M$$

$$L_T = L_1 + L_2 + 2M$$

ex-10 find  $Z_{eq}$



$$\underline{Z}_{S, total} = \frac{V_s}{I_s} = j\omega L_T$$

$$I_s = I_1 + I_2$$

$$\rightarrow V_s = j\omega L_1 I_1 + j\omega M I_2$$

$$\rightarrow V_s = j\omega L_2 I_2 + j\omega M I_1$$

$$\begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_s \\ V_s \end{bmatrix}$$

$$\Delta = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} V_s & j\omega M \\ V_s & j\omega L_2 \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} j\omega L_2 & V_s \\ j\omega M & V_s \end{bmatrix}$$

Solve for  $I_1, I_2$

$$\Delta = \begin{bmatrix} j\omega L_1 & jM\omega \\ jM\omega & j\omega L_2 \end{bmatrix} = -\omega^2 L_1 L_2 + \omega^2 M^2$$

$$\Delta_1 = \begin{bmatrix} V_s & jM\omega \\ V_s & j\omega L_1 \end{bmatrix} = jV_s \omega L_2 - jV_s \omega M$$

$$\Delta_2 = \begin{bmatrix} j\omega L_1 & V_s \\ jM\omega & V_s \end{bmatrix} = jV_s \omega L_1 - jV_s \omega M$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{jV_s \omega [L_2 - M]}{\omega^2 [M^2 - L_1 L_2]}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{jV_s \omega [L_1 - M]}{\omega^2 [M^2 - L_1 L_2]}$$

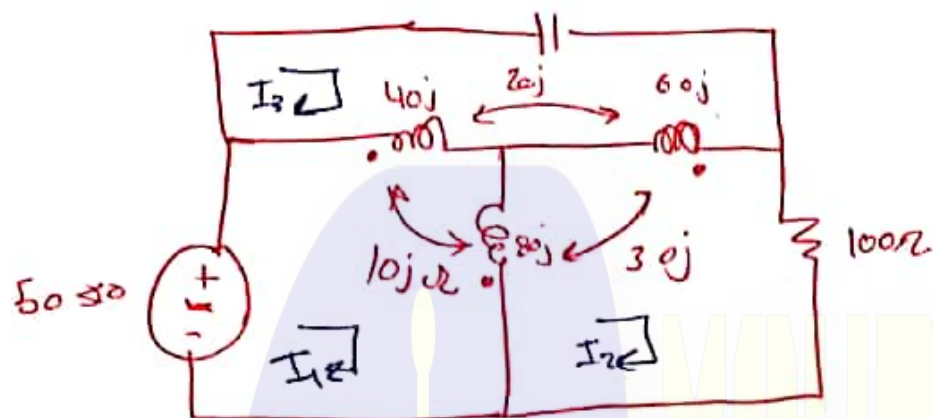
$$I_s = I_1 + I_2 \rightarrow \frac{jV_s \omega [L_2 - M] + jV_s \omega [L_1 - M]}{\omega^2 [M^2 - L_1 L_2]} = \frac{jV_s \omega [(L_1 - M) + (L_2 - M)]}{\omega^2 [M^2 - L_1 L_2]}$$

$$Z_{\text{total}} = \frac{V_s}{I_s} = \frac{V_s}{\frac{jV_s \omega [(L_1 - M) + (L_2 - M)]}{\omega^2 [M^2 - L_1 L_2]}} = \frac{\omega^2 [M^2 - L_1 L_2]}{j\omega [(L_1 - M) + (L_2 - M)]}$$

$$Z_s = \frac{-j\omega [M^2 - L_1 L_2]}{[(L_2 - M) + (L_1 - M)]}$$

$$L_T = \frac{Z_s}{j\omega} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

ex- write the mesh eqs



Mesh 1

$$-50\angle 0 + j40(I_1 - I_3) + j80(I_1 - I_2) + j20(I_3 - I_2) + j10(I_2 - I_1) + 30j(I_2 - I_3) + 10j(I_3 - I_1) = 0$$

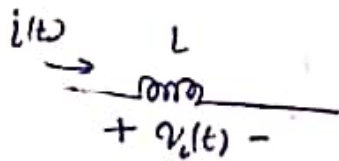
Mesh 2

$$100I_2 + j80(I_2 - I_1) + j40(I_2 - I_3) + 10j(I_1 - I_3) + 30j(I_3 - I_2) + j20(I_3 - I_1) + 30j(I_1 - I_2) = 0$$

Mesh 3

$$-j50I_3 + j60(I_3 - I_2) + j40(I_3 - I_1) + 30j(I_2 - I_1) + 20j(I_1 - I_3) + j20(I_2 - I_3) + j10(I_1 - I_2) = 0$$

# # Energy in a coupled ckt's



$$p(t) = i(t) v(t)$$

$$= i(t) L \frac{di}{dt}$$

$$W = E = \int p(t) dt = \int i(t) L \frac{di}{dt} dt$$

$$= \frac{1}{2} L i^2 \Rightarrow J$$

## ⇒ For Mutual coupling ckt's

$$E, W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$

~~energy should be~~

energy should  $\geq 0$

$$M \leq \sqrt{L_1 L_2}$$

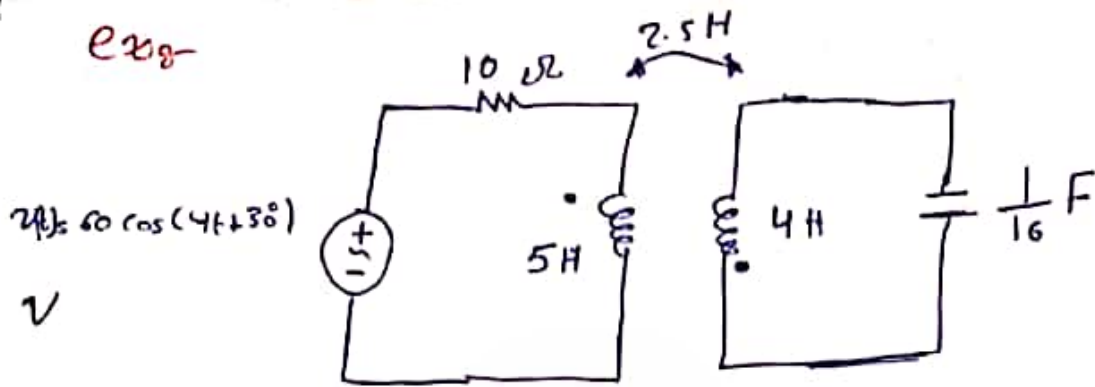
## \* coupling coefficient

← دالة ما بين  
2019

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$0 \leq k \leq 1$$

Ex 9-

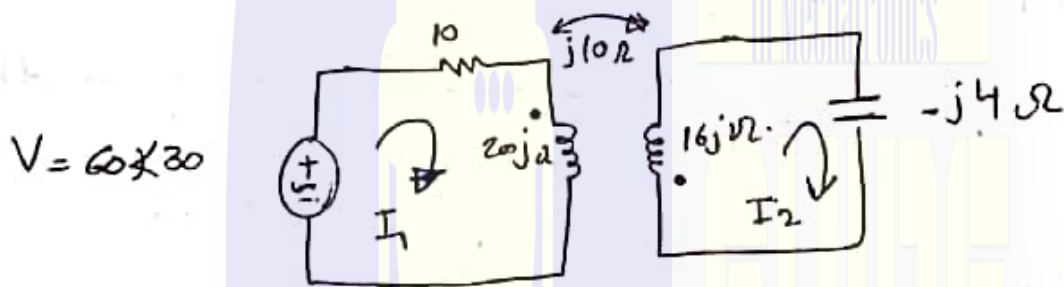


Find  $K$  &  $w$  at  $t = 1 \text{ sec}$  ?

Sol 9-

$$K = \frac{\mu}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{(4)(5)}} = \frac{2.5}{\sqrt{20}} = 0.56$$

$$E = \frac{1}{2} L_1 \dot{i}_1^2 + \frac{1}{2} L_2 \dot{i}_2^2 \pm \mu \dot{i}_1 \dot{i}_2$$



پارغا تعبيرين  
=  $\frac{1}{2} \mu \dot{i}_1 \dot{i}_2$

$$\textcircled{1} (10 + j20) I_1 + j10 I_2 = 60 \angle 30$$

$$\textcircled{2} (j16 - j4) I_2 + j10 I_2 = 0$$

Solve for  $I_1$  &  $I_2$

$$I_1 = 3.9 \angle -19.4^\circ \text{ A}$$

$$I_2 = 3.254 \angle 160.6^\circ \text{ A}$$

$$i_1(t) = 3.9 \cos(4t - 19.4)$$

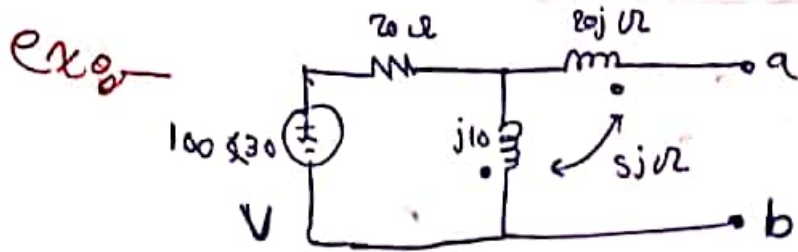
$$i_2(t) = 3.254 \cos(4t + 160.6)$$

$$i_1(t=1) = -3.389 \text{ A}$$

$$i_2(t=1) = 2.824 \text{ A}$$

$$E = 0.5(5) (-3.389)^2 + 0.5(4) (2.824)^2 + (2.5)(-3.389)(2.824)$$

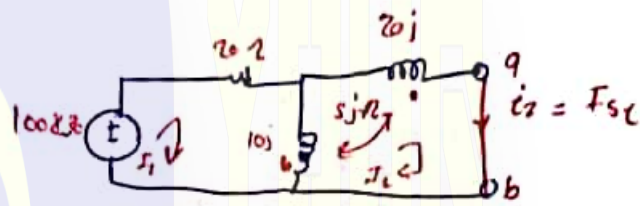
(2.824)



find  $Z_{eq}$

Solve

$$Z_{eq} = \frac{V_{oc}}{I_{s.c}}$$



mesh 1

$$(z_0 + j10) I_1 - j10 I_2 + js I_2 = 100 \angle 30^\circ$$

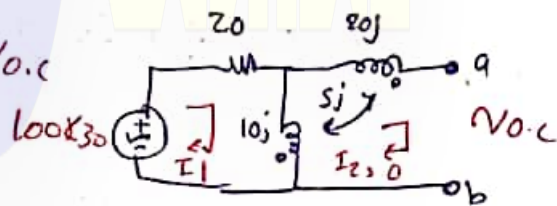
mesh 2

$$(jz_0 + j10) I_2 - j10 I_1 + js I_1 = 0$$

Solve for  $I_2$

$$I_2 = 1.145 \angle 6.37^\circ \text{ A}$$

Now we have to find  $V_{oc}$



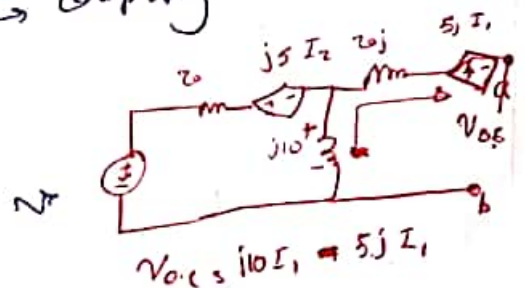
$$(z_0 + j10) I_1 = 100 \angle 30^\circ$$

$$I_1 = 4.47 \angle 3.43^\circ \text{ A}$$

$$V_{oc} = 10j I_1 + sj I_1$$

coupling

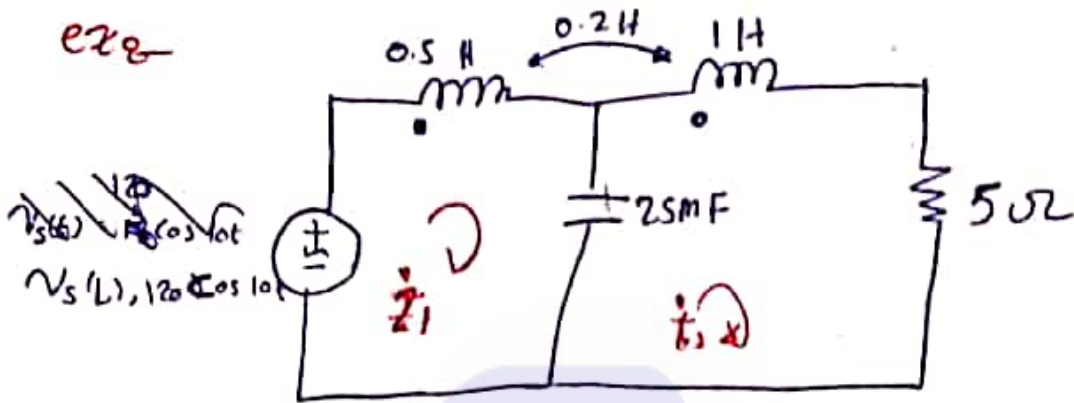
$$Z_{eq} = \frac{V_{oc}}{I_{s.c}}$$



$$V_{oc} = 10j I_1 + sj I_1$$



ex 2

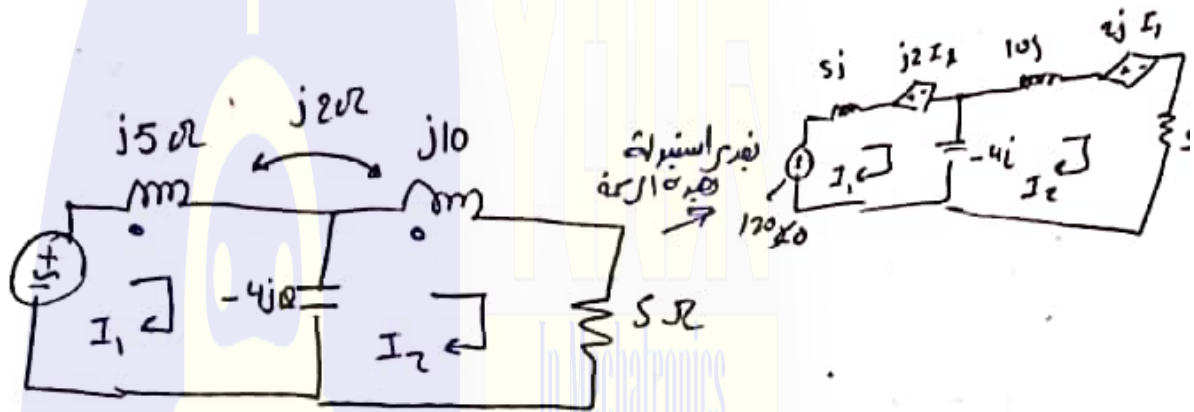


$i_1(t) ?$   
 $i_2(t) ?$

$W |_{t=15ms} = ?!$

Sol

$V_s = 120 \angle 0^\circ$   
 $\omega = 10$



mesh 1

$$(j5 - 4j) I_1 + 4j I_2 + 2j I_2 = 120 \angle 0^\circ$$

mesh 2

$$(5 + 10j - 4j) I_2 + j4 I_2 + j2 I_1 = 0$$

so solve for  $I_1, I_2$

$$I_1 = 3.081 \angle 40.74^\circ \text{ A} \rightarrow i_1(t) = 3.081 \cos(10t + 40.74^\circ)$$

$$I_2 = 2.367 \angle -99.46^\circ \text{ A} \rightarrow i_2(t) = 2.367 \cos(10t - 99.46^\circ)$$

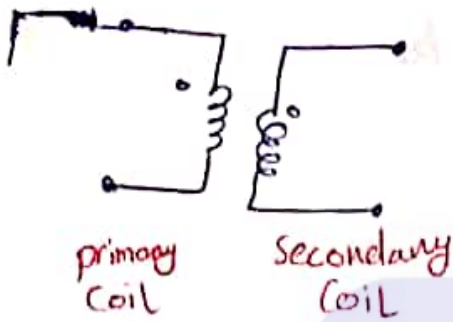
$i_1(t) =$  \_\_\_\_\_

$i_2(t) =$  \_\_\_\_\_

$W_s =$  \_\_\_\_\_

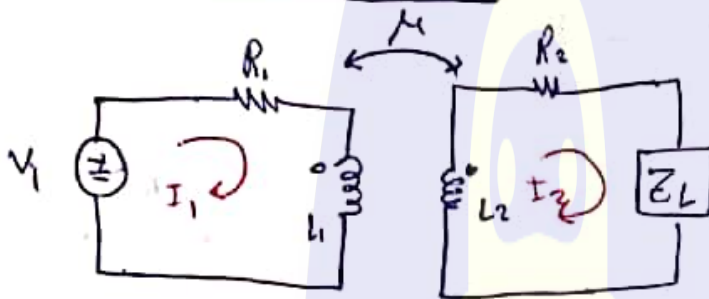
# Linear Transformer

Monday  
15/7/2019



used with high frequencies ckt's. (RF) Radio Frequency

~~has~~  
has 4 terminals.



mesh 1

$$V_i - I_1(R_1 + j\omega L_1) - j\omega M I_2 = 0 \quad (1)$$

mesh 2

$$0 = -j\omega M I_1 + (R_2 + Z_L + j\omega L_2) I_2 \quad (2) \quad I_2 = \frac{j\omega M I_1}{R_2 + Z_L + j\omega L_2}$$

$$Z_{in} = \frac{V_i}{I_1}$$

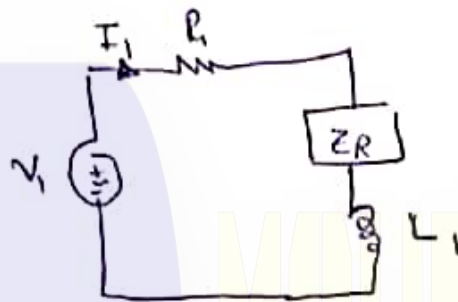
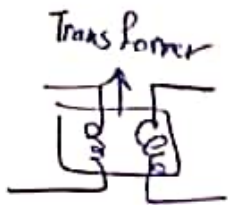
$$Z_{in} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + Z_L + j\omega L_2}$$

# Reflected Impedance

$$Z_R = \frac{\omega^2 \mu^2}{R_2 + Z_L + j\omega L_2}$$



كيف شكل الدارة

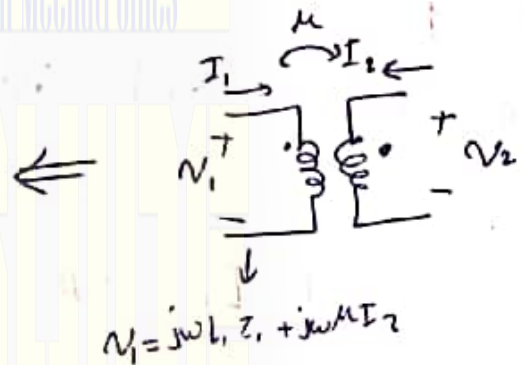
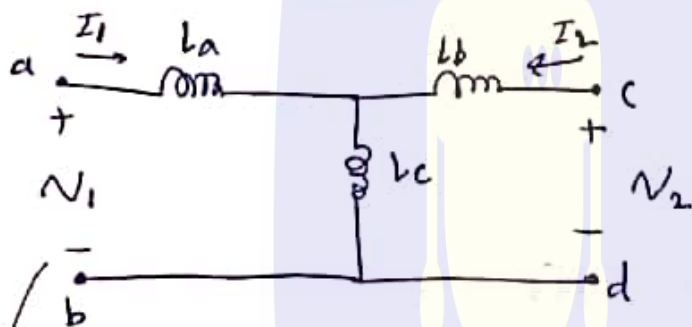


$$Z_{in} = \frac{V_1}{I_1} = R_1 + j\omega L_1 + Z_R$$

كيف شكل الدارة

$$\frac{\omega^2 \mu^2}{R_2 + Z_L + j\omega L_2}$$

## T: equivalent CKT.



$$V_1 = j\omega L_1 I_1 + j\omega \mu I_2$$

$$L_a = L_1 - \mu$$

$$L_b = L_2 - \mu$$

$$L_c = \mu$$

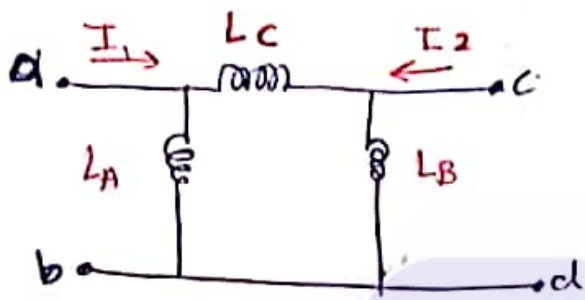
الدائرة  
تكون  
مكافئة  
للمحول  
والتي  
تحتوي  
على  
المحثات  
والتي  
تحتوي  
على  
المحثات  
والتي  
تحتوي  
على  
المحثات

$$V_1 = j\omega L_a I_1 + j\omega L_c (I_1 + I_2)$$

$$V_1 = j\omega I_1 (L_1 - \mu) + j\omega \mu I_1 + j\omega \mu I_2$$

$$V_1 = j\omega I_1 L_1 + j\omega \mu I_2$$

π equivalent :-



Δ ← Y من

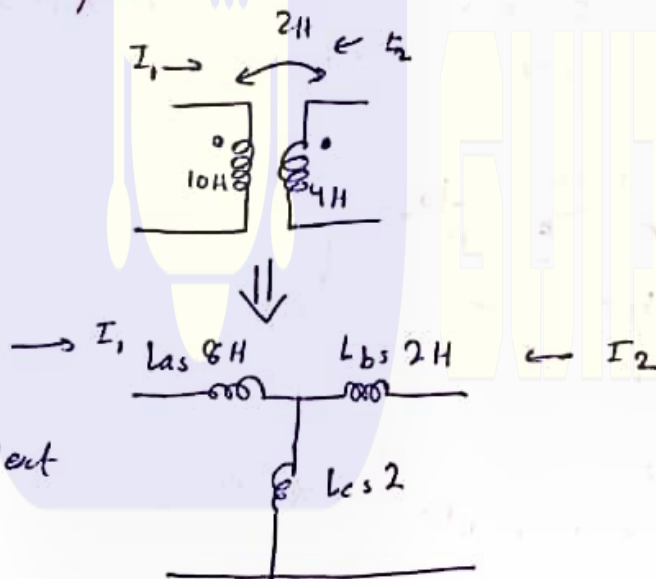
Y → Δ conversion

$$L_A = \frac{L_1 L_2 - \mu^2}{L_2 - \mu}$$

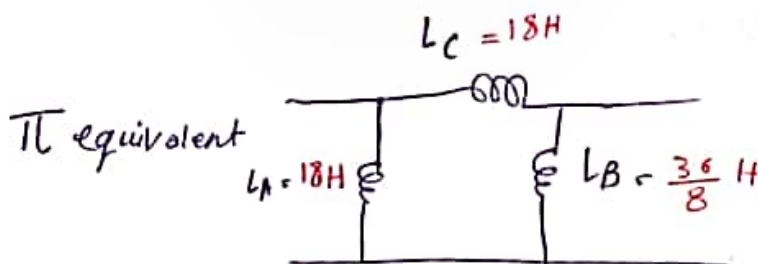
$$L_B = \frac{L_1 L_2 - \mu^2}{L_1 - \mu}$$

$$L_C = \frac{L_1 L_2 - \mu^2}{\mu}$$

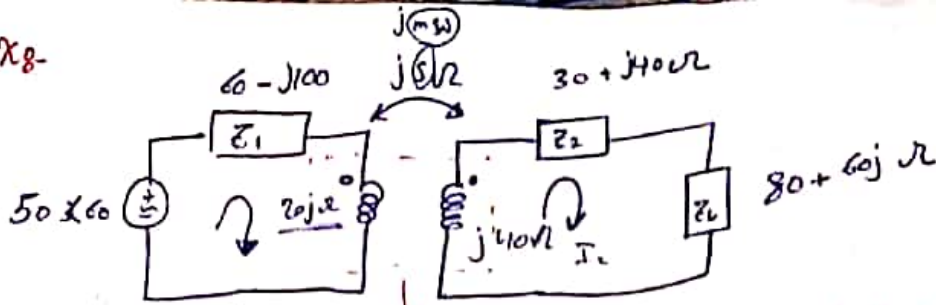
ex 8-



Tequivalent



ex 8-



find  $Z_{in}$  &  $I_1$  ?

همین بطن انرژی را  
ببینیم چون اینها هم به هم  
متصلند و باید جمع دارن بریم ما می

Sol 8-

$$Z_{in} = Z_1 + j\omega L_1 + Z_R \rightarrow \frac{(VM)^2}{Z_2 + Z_L + j\omega L_2}$$

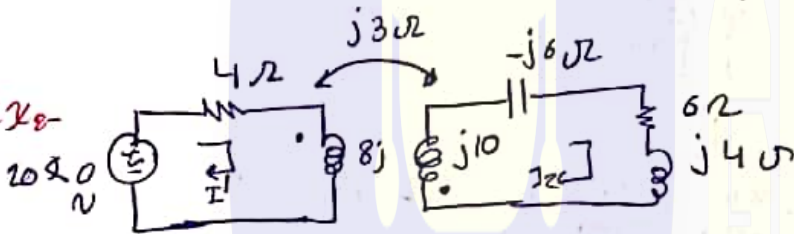
$$= 60 - j100 + 20j + \frac{5^2}{30 + 40j} + 80 + 60j + 40j$$

~~$Z_{in} = 100 \angle 53.1^\circ$~~

$$Z_{in} = 100.14 \angle -53.1^\circ \Omega$$

$$I_1 = \frac{V}{Z_{in}} = \frac{50 \angle 60}{100.14 \angle -53.1} = 0.5 \angle 113.1^\circ \text{ A}$$

ex 8-

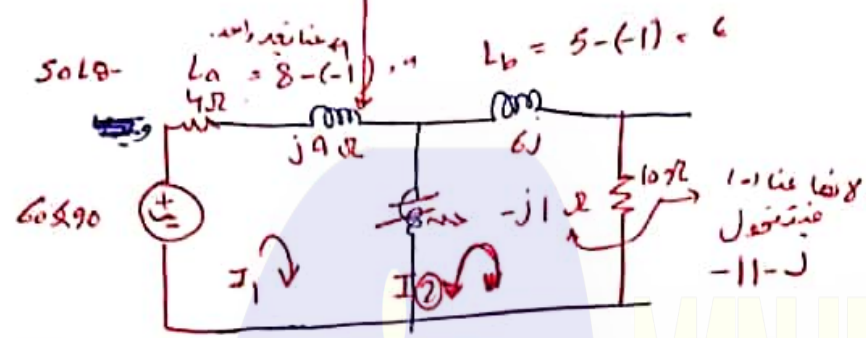


sol

$$Z_{in} = 4 + j8 + \frac{3^2}{-j6 + 6 + j4 + j10} \Rightarrow Z_{in} = 8.57 \angle 58.05^\circ \Omega$$

$$I_1 = \frac{V}{Z_{in}} = \frac{20 \angle 0}{8.57 \angle 58.05} = 2.3 \angle -58.05^\circ \text{ A}$$

ایکٹہ ایہ اصلیت  
ایک آسٹہ نامہ  
اول ۱۱ خاصیت



پر بروج کرستی  
زی عاکز

mesh 1

$$60\angle 90 = (4 + j9 - j)I_2 + -j1 I_2$$

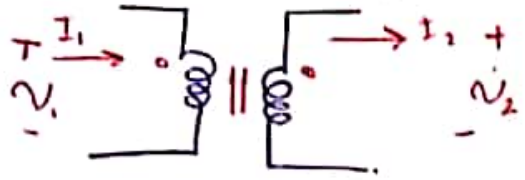
mesh 2

$$0 = (10 + j6) I_2 + -j1 I_1$$

تبدل راحل  
عادة زی کلنفا  
دارت عادی  
بضلع  $Z_{eq}$   
بعین بطلج  $I_1$   
بعد  
current division

# Ideal Transformer

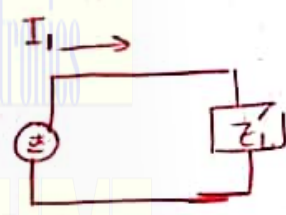
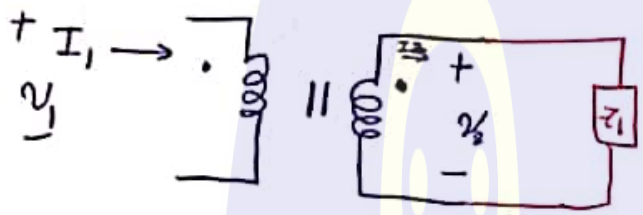
Tuesday  
16/7/2019



المطابق عدد لفات  
لبراهين في عدد لفات سكندري

$N_1, N_1$  - number of turns of primary coil  
 $N_2, N_2$  - number of turns of secondary coil

$$\frac{N_2}{N_1} = n \text{ turns ratio}$$



we have

- ① step up Transformer
- ② step down transformer.

Complex power

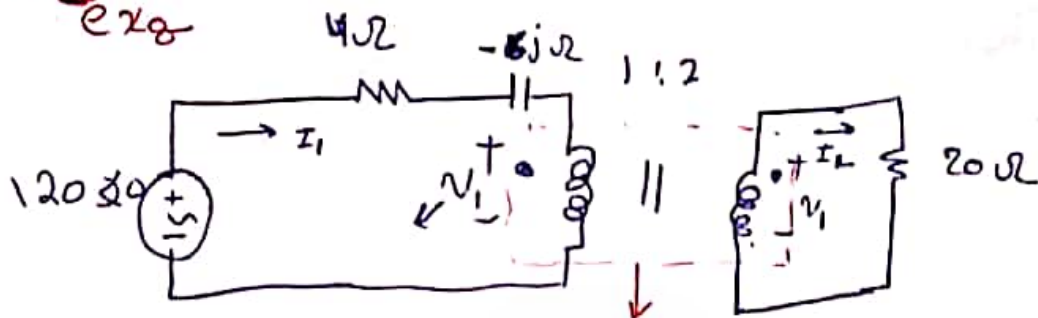
$$\begin{cases} S_1 = S_2 \\ I_1^* V_1 = I_2^* V_2 \end{cases}$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} *$$

if we reflect the [secondary impedance] to the primary

$$Z'_L = \frac{Z_L}{n^2}$$

120∠0°  
E 2g



Transformer

$V_1, V_2$  کے  
لیا

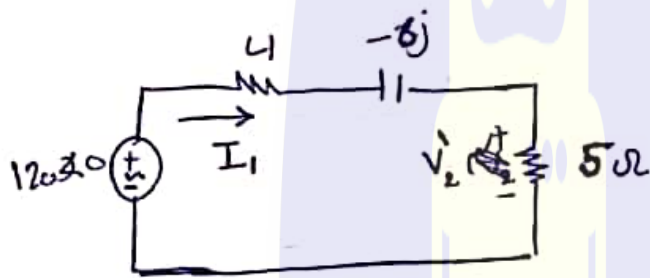
$\frac{2}{2} = \frac{4}{2}$   
1 (2)  
n

~~...~~ = ~~...~~

Sol:

$$Z'_L = \frac{Z_L}{n^2} = \frac{20}{4} = 5\Omega$$

Coil پہلے اپنی جگہ پر



تکون

$$I_2 = -\frac{I_1}{n}$$

$$I_1 = \frac{120\angle 0^\circ}{4 + 5j} = 11.09 \angle 33.69^\circ \text{ A}$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = n \rightarrow I_2 = \frac{I_1}{n} = \frac{I_1}{2} = 5.5 \angle 33.69^\circ \text{ A}$$

$$V_2 = I_2 (20) = 110.9 \angle 33.69^\circ \text{ V}$$

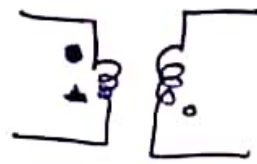
$$V'_2 = 5 I_1 = 55.45 \angle 33.69^\circ \text{ A}$$

$$\frac{V_2}{V'_2} = n \rightarrow \boxed{V'_2 = \frac{V_2}{n}} \rightarrow \boxed{I'_2 = I_2 n}$$

$$P_{20} = |I_2|^2 R = 5.5^2 \times 20 = 613.8 \text{ W}$$



إذا غيرنا =



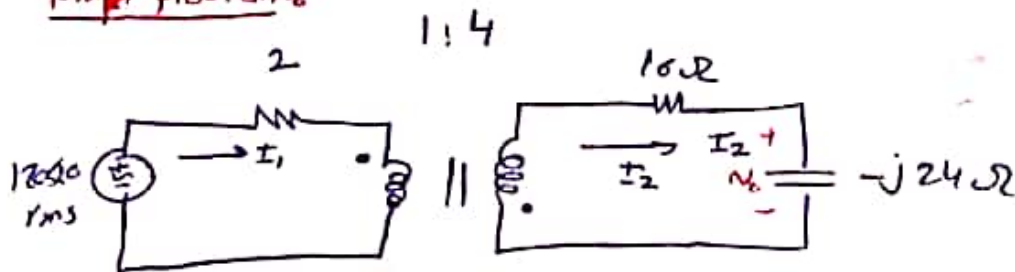
$$I_2 = \frac{-I_2}{2} = 5.54 \angle -146.31^\circ = I_2 \text{ بغيرنا}$$

$$V_2 = I_2 (r_2) = 110.9 \angle -146.31^\circ$$

$$\frac{V_2}{V_1} = n \rightarrow V_1 = \frac{V_2}{n} = 55.45 \angle -146.31^\circ$$

$$S_1 = V_1 I_1^* = 615.9 \angle -179.9^\circ$$

Practical problems



Sol.

$$Z'_L = \frac{16 - j24}{4} = 4 - j6 \Omega$$

$$I_1 = \frac{120 \angle 0}{2 + 4 - j6} = 35.77 \angle 26.5^\circ \text{ A}$$

$$I_2 = \frac{-I_1}{n} = 8.9 \angle -153.5^\circ \text{ A}$$

$$V_o = I_2 (-j24) = 214.66 \angle 116.56^\circ \text{ V}$$

$$S_{\text{source}} = P_{\text{source}} I_1^* = 120 \angle 0 (35.77 \angle -26.5) = 4292.4 \angle -26.5 \text{ VR}$$

16 - j24j

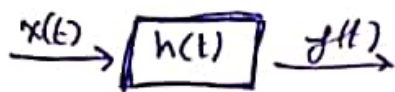
~~Reflect~~ Reflect <sup>Jaw</sup>

$\frac{N_2 < N_1}{\text{step up}}$

# CH 148 Freq. Responses

21/7/2019  
sunday

## Linear system



$x(t)$  = input  
 $y(t)$  = output  
 $h(t)$  = Impulse response

(convolution) output = input convolution



## Linear time invariant

$$y(t), x(t) * h(t)$$



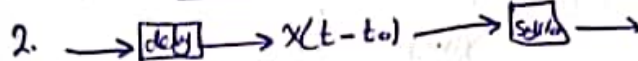
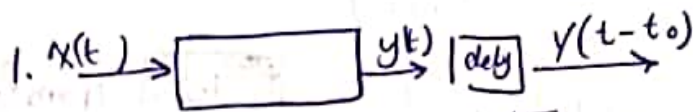
$$Y(\omega) = X(\omega) \times H(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

Transfer function

output / input  $\rightarrow$  in freq domain

## Time invariant system



linear time invariant



$$Y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt$$

Power

$$H(\omega) = \frac{I_o(\omega)}{I_{in}(\omega)} \rightarrow \text{current gain}$$

$$H(\omega) = \frac{V_o(\omega)}{V_{in}(\omega)} \rightarrow \text{Voltage gain}$$

$$H(\omega) = \frac{y(\omega)}{x(\omega)} \rightarrow \text{transfer function}$$

$$H(\omega) = \frac{\omega - 4}{\omega - 6}$$

$$\omega - 4 = 0$$
$$\omega = 4 \text{ (zero)} (\times)$$

$$\omega - 6 = 0$$
$$\omega = 6 \text{ (poles)} (o)$$

ex 00

$$H(\omega) = \frac{4\omega - 8}{\omega^2 + 7\omega + 12}$$

find zero, poles:-

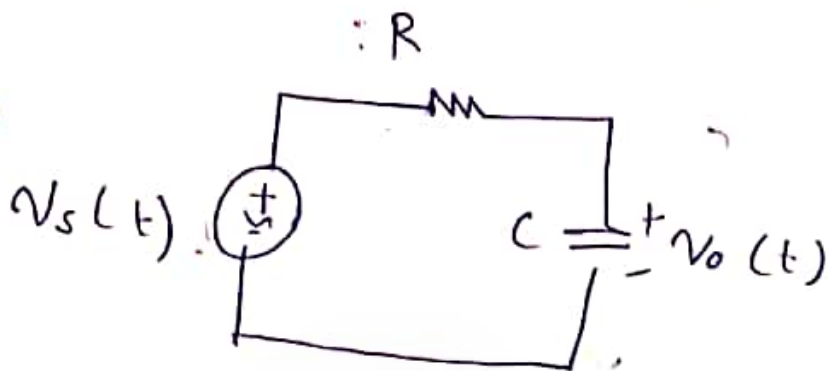
zero:  $4\omega - 8$

$$\omega = \frac{8}{4} = 2 \quad \omega = 2 \text{ rad/sec}$$

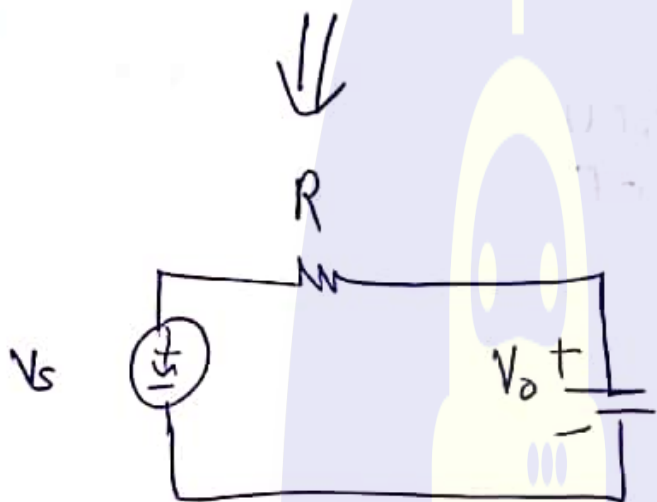
poles:  $\omega^2 + 7\omega + 12$

$$(\omega + 3)(\omega + 4)$$
$$\omega = -4$$
$$\omega = -3$$

ex 20



linear  $\frac{V_o(\omega)}{V_{in}(\omega)} = H(\omega)$



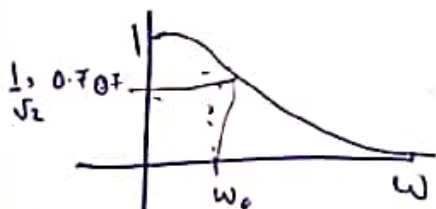
$$V_o = \frac{V_s \left( \frac{-j}{j\omega C} \right)}{R + \frac{-j}{j\omega C}}$$

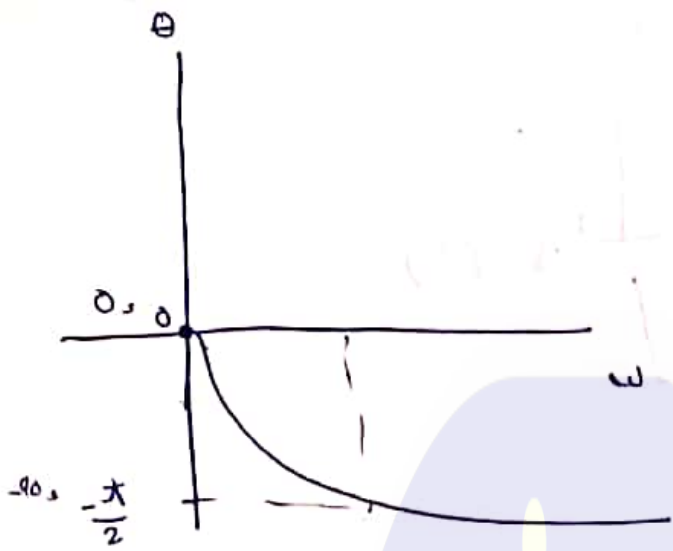
$$\frac{V_o}{V_s} = \frac{j\omega C}{R + \frac{1}{j\omega C}} \rightarrow \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

$$\omega_0 = \frac{1}{RC}$$

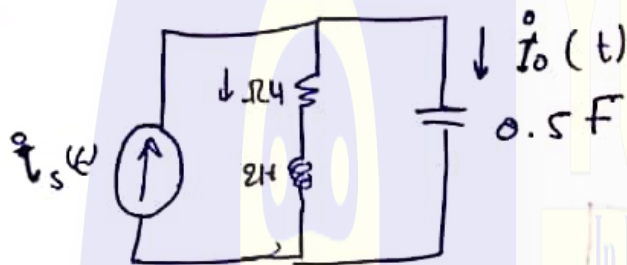
$$\Rightarrow |H| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$\angle = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

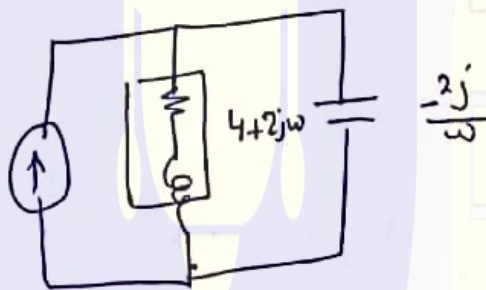




Exo



find  $\frac{I_0(\omega)}{I_s(\omega)} \equiv H(\omega)$



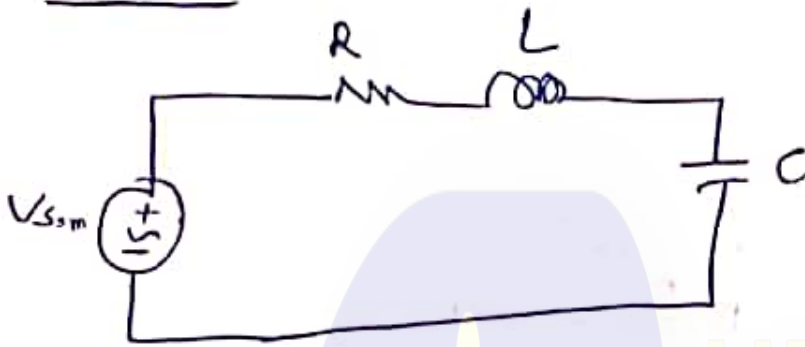
$$I_0 = I_s \frac{(4 + j2\omega)}{4 + 2j\omega - \frac{2j}{\omega}}$$

$$H(\omega) = \frac{I_0}{I_s} = \frac{(4 + j2\omega)}{4 + 2j\omega - \frac{2j}{\omega}}$$

# \* Series Resonance

Monday  
22/7/2019

## RLC

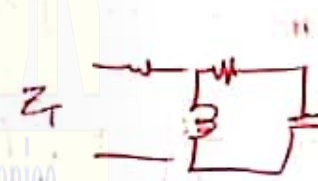


~~Resonance~~ Resonance occurs in RLC CKTs

$$\Rightarrow \boxed{X_L = X_C}$$

$$\Rightarrow \boxed{Z_T = R}$$

$I_{mag} \neq 0$



$$R + j I_{mag} \neq 0$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

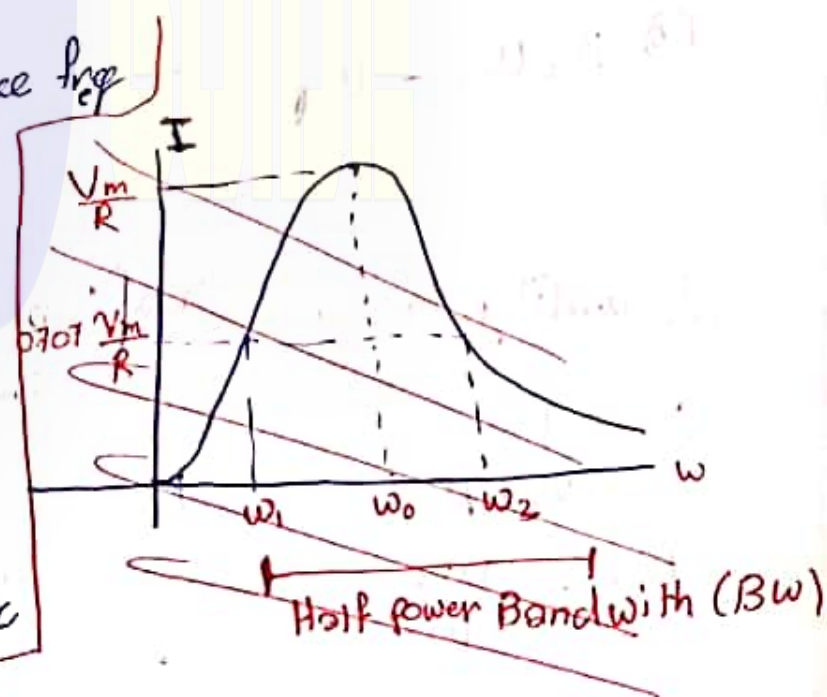
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Resonance freq

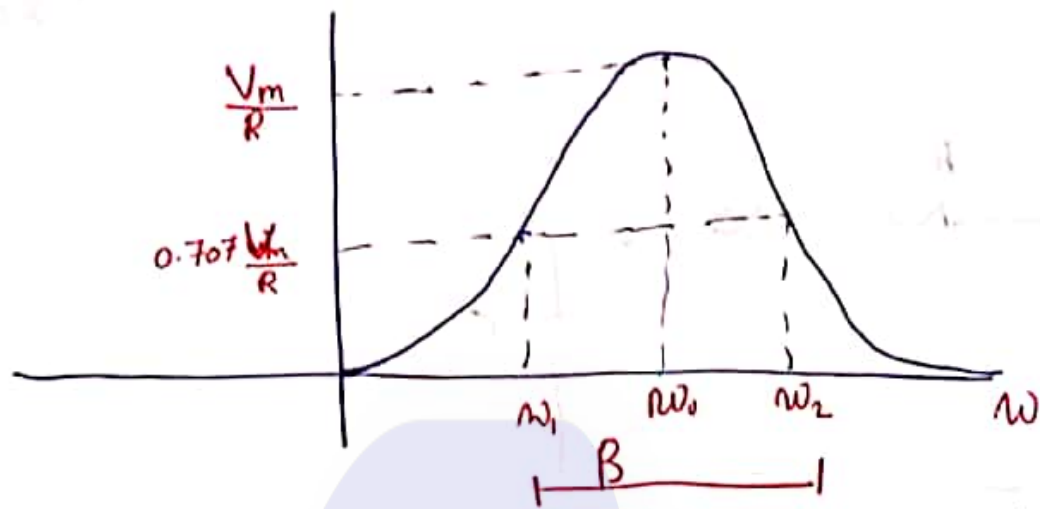
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

At Resonance

- ①  $Z_T = R$  ( $I_{mag} \neq 0$ )
- ②  $V, I$  are in phase
- ③  $\begin{cases} V_L, V_C > V_s \end{cases}$



~~Q factor~~



Half power Bandwidth (BW)

$\omega_1, \omega_2$  : Half power freq

①  $P_{max} = 0.5 I^2 R$

$\omega_0$  is Max @  $f_{res}$

②  $\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$

③  $\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$

④  $\omega_0 = \sqrt{\omega_1 \omega_2}$

⑤  $B = \omega_2 - \omega_1$

Quality factor  $Q = \frac{\text{Total energy stored (EKT)}}{\text{The energy loss per cycle}}$

$Q = \frac{1}{\omega_0 CR}$

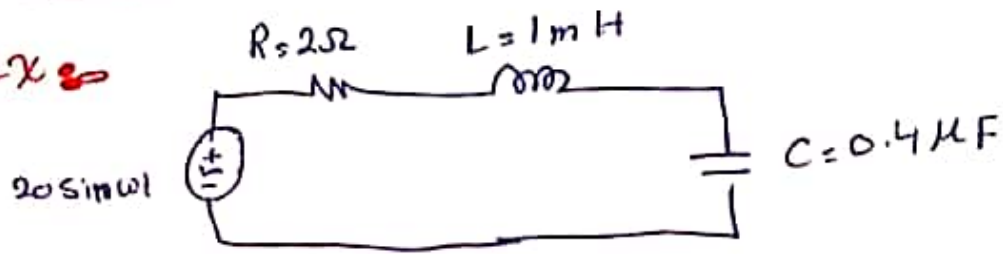
$Q = \frac{\omega_0}{B}$

$\omega_1 = \omega_0 - \frac{B}{2}$

$\omega_2 = \omega_0 + \frac{B}{2}$



ex 2



Sol -

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1 \times 10^{-3}) \times (0.4 \times 10^{-6})}} \approx 50 \text{ krad/sec}$$

$$\omega_1 = 49 \text{ krad/sec}$$

$$\omega_2 = 51 \text{ krad/sec}$$

$$B = \omega_2 - \omega_1$$

$$51 - 49 = 2 \text{ krad/sec}$$

$$P_{\max} = \frac{1}{2} I^2 R$$

$$I_{\max} = \frac{V_m}{R} = \frac{20}{2} = 10 \text{ A}$$

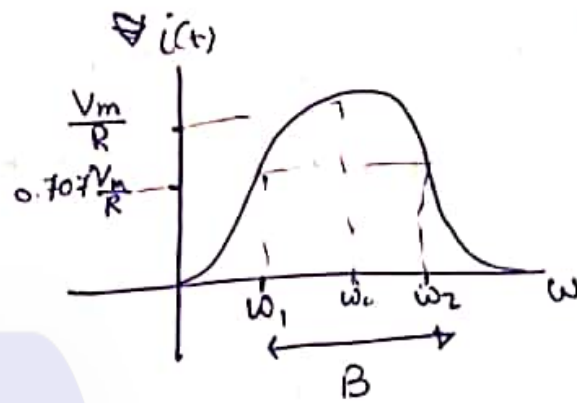
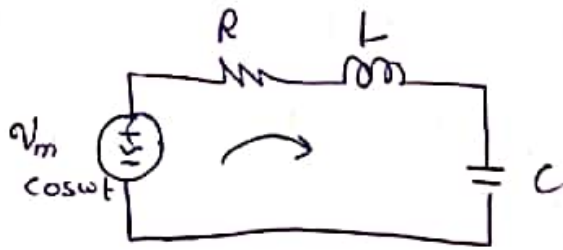
$$Q = 25 \quad \text{wb Unit}$$

ex 2

# Series Resonant

Tuesday

23/7/2019



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_1 = \omega_0 - \beta/2$$

$$\omega_2 = \omega_0 + \beta/2$$

$$Q \geq 10$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

$$B = \frac{R}{L} = \frac{\omega_0}{Q}$$

$$Z = R + j\omega L - \frac{j}{\omega C} = R + j\left[\omega L - \frac{1}{\omega C}\right]$$

$\downarrow = 0$   
~~Resonant~~  
 Resonant

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q \leq 10$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \dots$$

$$\omega_1, \omega_2 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$$

ex:  $R = 4 \Omega$   
 $L = 25 \text{ mH}$   
 $C = ?$

$$Q = 50$$

sol:

$$Q = \frac{\omega_0 L}{R} = 50 = \omega_0 \frac{25 \times 10^{-3}}{4} = \omega_0 = \frac{200}{25 \times 10^{-3}}$$

$$\omega_0 = 8 \text{ krad/s}$$

$$\omega_0^2 = \frac{1}{LC} \rightarrow C = \frac{1}{L \omega_0^2} \rightarrow C = \frac{1}{25 \times 10^{-3} \times (8 \times 10^3)^2}$$

$$V_m = 100$$

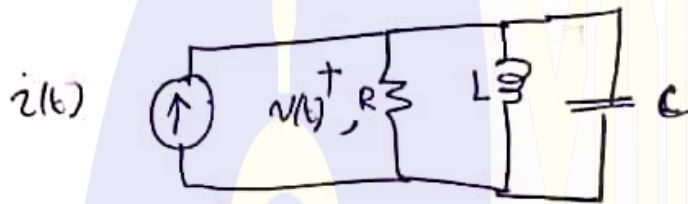
$$I_{max} = \frac{100}{4} = 25$$

$$\rightarrow P_{wc} = \frac{1}{2} \times 25^2 \times 4$$

$$I_{w_1, w_2} = 25 \times 0.707$$

$$P_{w_1, w_2} = \frac{1}{2} (17.6)^2 \times 4$$

### \* Parallel Resonance.

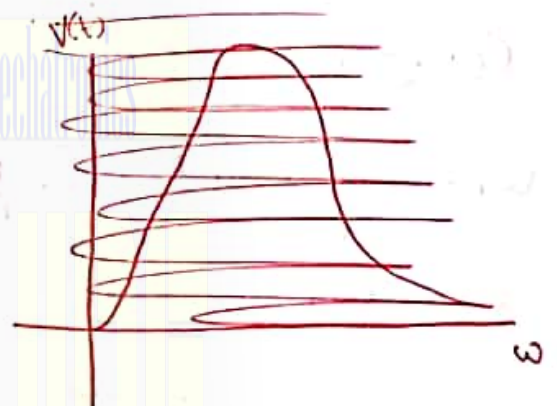


$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$\frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$

$$\omega C - \frac{1}{\omega L} = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



$$\textcircled{1} \omega_1 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\textcircled{2} \omega_2 = \frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_1 \omega_2 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$$

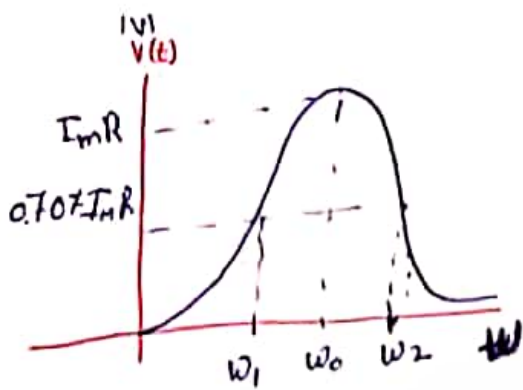
$$\textcircled{3} B = \omega_2 - \omega_1 = \frac{1}{RC}$$

$$\textcircled{4} Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

if  $Q \gg 10$

$$\textcircled{1} \omega_1 \approx \omega_0 - B/2$$

$$\textcircled{2} \omega_2 \approx \omega_0 + B/2$$

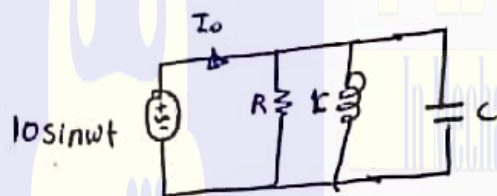


$$I_L = -I_C$$

$$I_L = \frac{I_m R}{\omega_0 L} = Q I_m$$

$$I_C = \omega_0 C I_m R = Q I_m$$

ex



$$R = 8 \text{ k}\Omega \quad L = 0.2 \text{ mH} \quad C = 8 \mu\text{F}$$

find  $\omega_0$ ,  $Q$  &  $\beta$

$$I_0 = \frac{10 \angle -90}{8 \times 10^3} = 1.25 \angle -90 \text{ A}$$

$$P_{\max} = 0.5 \times (1.25 \times 10^{-3})^2 \times (8 \times 10^3)$$

soln

$$\omega_0 = \frac{1}{\sqrt{LC}} = 25 \text{ k rad/s}$$

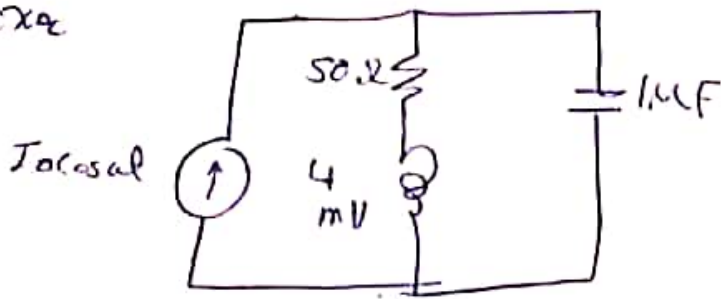
$$Q = \frac{R}{\omega_0 L} = \frac{8 \times 10^3}{25 \times 10^3 (0.2 \times 10^{-3})} = 16000$$

$$\beta = \frac{\omega_0}{Q} = \frac{25 \times 10^3}{16000} = 15.625 \text{ rad/s}$$

$$\omega_1 = \omega_0 - \beta/2 = 25000 - \frac{15.625}{2} = 24992 \text{ rad/s}$$

$$\omega_2 = \omega_0 + \beta/2 = 25000 + \frac{15.625}{2} = 25008 \text{ rad/s}$$

18 = exa



$$Z_{total} = \frac{(R + j\omega L) \cdot \frac{1}{j\omega C}}{(R + j\omega L) + \frac{1}{j\omega C}} = \frac{\frac{-jR}{\omega C} + \frac{\omega L}{\omega C}}{R + j(\omega L - \frac{1}{\omega C})}$$

$$Z_{total} = \frac{(50 + j40) \cdot \frac{1}{j\omega C}}{(50 + j40) + \frac{1}{j\omega C}} = \frac{\frac{-jR}{\omega L}}{R + j(\omega L - \frac{1}{\omega C})}$$

$$= \frac{-jR^2}{\omega L} - \frac{R}{\omega C} (\omega L - \frac{1}{\omega C}) + \frac{RL}{C} - j \left[ \frac{\omega L^2}{C} - \frac{L}{\omega C^2} \right]$$

---


$$R^2 + (\omega L - \frac{1}{\omega C})^2$$

$$= \frac{-j \left[ \frac{R^2}{\omega L} + \frac{\omega L^2}{C} - \frac{L}{\omega C^2} \right] + \frac{RL}{C} - \frac{R}{\omega C} (\omega L - \frac{1}{\omega C})}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

---

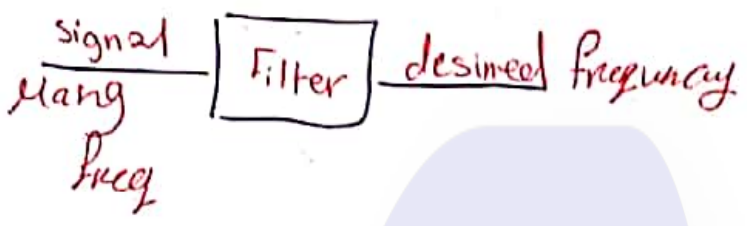

$$R^2 + (\omega L - \frac{1}{\omega C})^2$$

$$\omega_0 = \sqrt{\frac{1}{LC} + \frac{R^2 C}{L^3}}$$

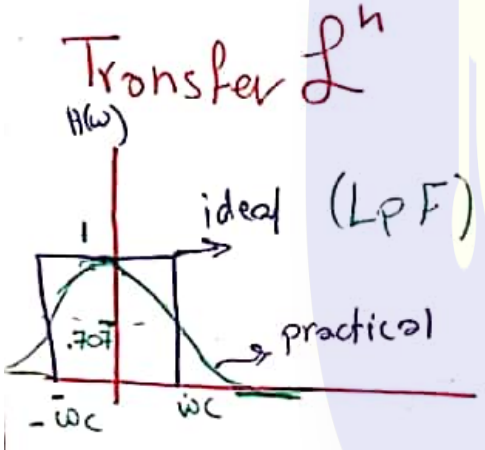
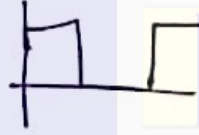
# passive filters

Monday  
29/7/2019

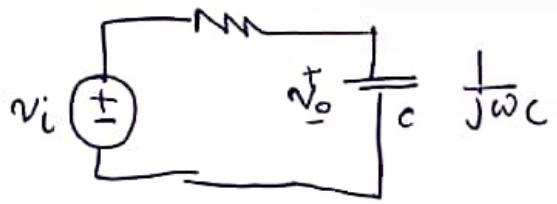
R, L, C



- ① Low pass filter [LPF] - Low frequency to pass.
- ② Band pass filter Modrate (BPF)
- ③ High pass filter [HPF] - high freq to pass
- ④ Band stop filter



$\omega_c$  = corner / cutoff freq



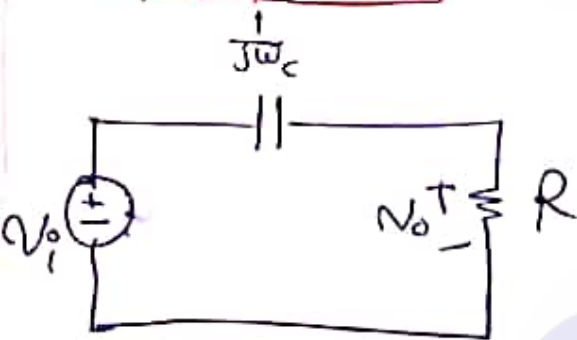
$$V_o = V_i \frac{1}{R + \frac{1}{j\omega C}}$$

$$\frac{V_o}{V_i} = H(\omega) = \frac{1}{R + \frac{1}{j\omega C}} \Rightarrow \frac{1}{1 + j\omega RC}$$

$\omega_c = \frac{1}{RC}$   
 $\therefore H(\omega_c) = \frac{1}{\sqrt{2}}$

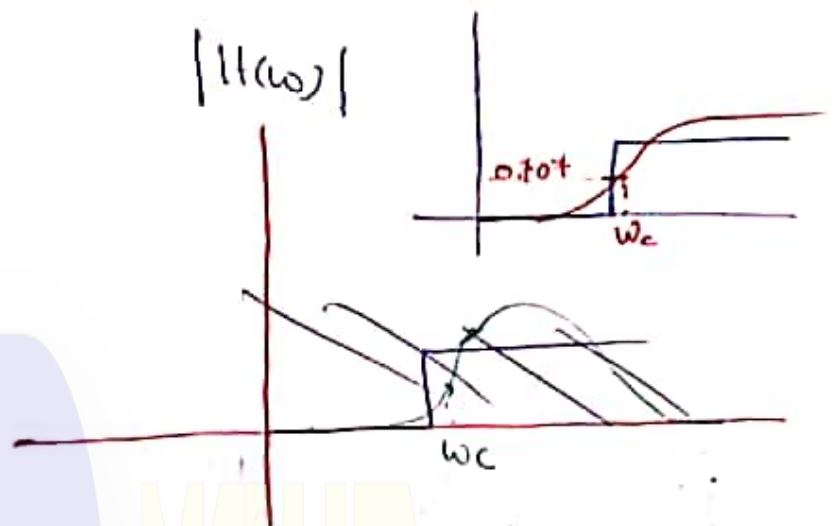
$H(0) = 1$   
 $H(\infty) = 0$

• high pass filter (HPF)



$$V_o = \frac{V_p R}{R + \frac{1}{j\omega c}}$$

$$\frac{V_o}{V_p} = H(\omega) = \frac{R}{R + \frac{1}{j\omega c}} = \frac{j\omega c R}{1 + j\omega c R}$$

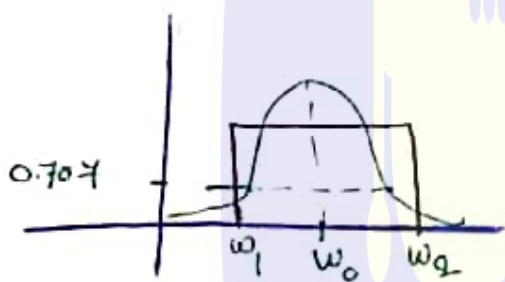


$$\omega_c = \frac{1}{RC}$$

$$H(0) = 0$$

$$H(\infty) = 1$$

⊗ Band Pass Filter (BPF)



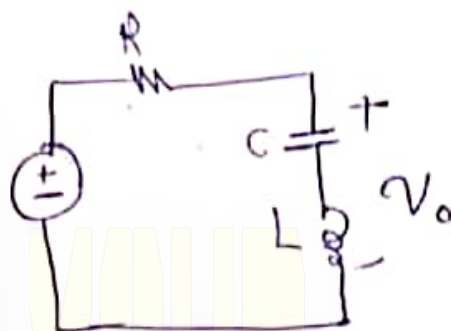
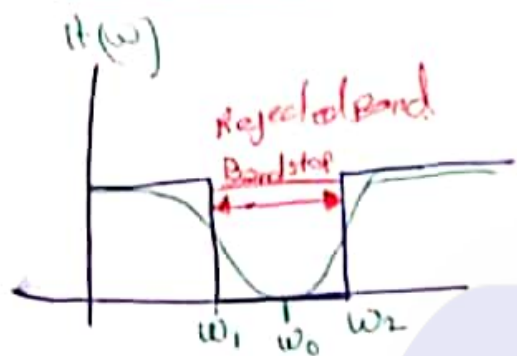
$$\frac{V_o}{V_p} = H(\omega) = \frac{R}{R + j\omega L + \frac{1}{j\omega c}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$H(0) = 0$$

$$H(\infty) = 0$$

# Band stop filter (~~BSF~~) (Notch Filter)



$$\frac{v_o}{v_i} = H(\omega) = \frac{1}{j\omega C + j\omega L} \cdot \frac{R + \frac{1}{j\omega C} + j\omega L}{R + \frac{1}{j\omega C} + j\omega L}$$

	$H(0)$	$H(\infty)$
LPF	1	0
HPF	0	1
BPF	0	0
BSF	1	1

Note  $S = j\omega$   
 $R = 2k\Omega$   
 $L = 2H$   
 $C = 2\mu F$



$$\frac{v_o}{v_i} = \frac{(R \parallel \frac{1}{sC}) v_i}{sL + (R \parallel \frac{1}{sC})} \rightarrow \frac{v_o}{v_i} = \frac{R}{s^2 RLC + sL + R}$$

$$H(\omega) = \frac{R}{-\omega^2 RLC + j\omega C + R}$$

$(\omega = \frac{1}{RC}) H(\omega) = 0$  LPF



$$|H|'s \sqrt{\frac{R^2}{(R - \omega^2 RL C)^2 + \omega^2 L^2}}$$

$$H = \frac{R}{\sqrt{(R - \omega^2 RL C)^2 + \omega^2 L^2}}$$

at  $\omega = \omega_c$   $|H(\omega) = \frac{1}{\sqrt{2}}$  *response*

$$\frac{1}{2} = \frac{2 \times 10^3}{(2 \times 10^3 - \omega_c^2 (2 \times 10^3 \times 2 \times 2 \times 10^{-6}))^2 + \omega_c^2 2^2}$$

Store for  $\omega_c$

$$\omega_c = 742 \text{ rad/s}$$

4/8/2019  
Sunday

• Decible scale

$$G = 10 \log_{10} \frac{P_2}{P_1}$$

$$G = 20 \log_{10} \frac{V_2}{V_1}$$

• Bode - plots

poles, zeroes  $\rightarrow (+)$   
 $H(\omega) = \frac{V_o}{V_i}$   
 slope (-)

ex:  $H(s) = \frac{5000s + 50000}{s^2 + 501s + 500}$

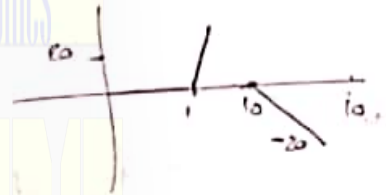
sol:  $\frac{5000s + (50000)}{(s+1)(s+500)}$

$$= \frac{50000 \left( \frac{s}{10} + 1 \right)}{500 \left( \frac{s}{1} + 1 \right) \left( \frac{s}{500} + 1 \right)}$$

$$= \frac{100 \left( \frac{s}{10} + 1 \right)}{\left( \frac{s}{1} + 1 \right) \left( \frac{s}{500} + 1 \right)}$$

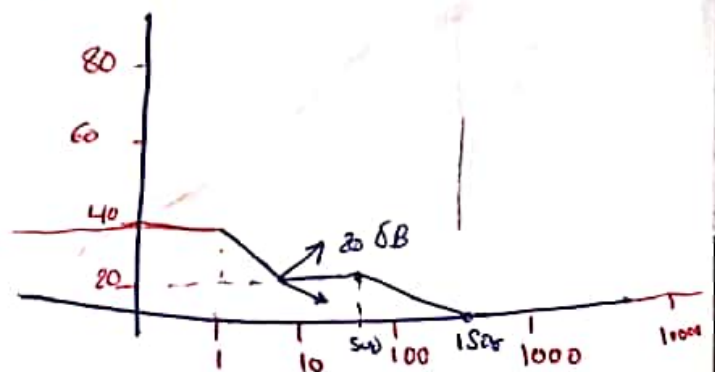
$$40 \text{ dB} = 20 \log 100 + 20 \log \left( \frac{s}{10} + 1 \right) - 20 \log \left( \frac{s}{1} + 1 \right) - 20 \log \left( \frac{s}{500} + 1 \right)$$

$$\frac{\left( \frac{s}{10} + 1 \right) \left( \frac{s}{500} + 1 \right)}{\left( \frac{s}{1} + 1 \right) \left( \frac{s}{10} + 1 \right)}$$



$$\log(AB) = \log A + \log B$$

$$\log\left(\frac{A}{B}\right) = \log A - \log B$$

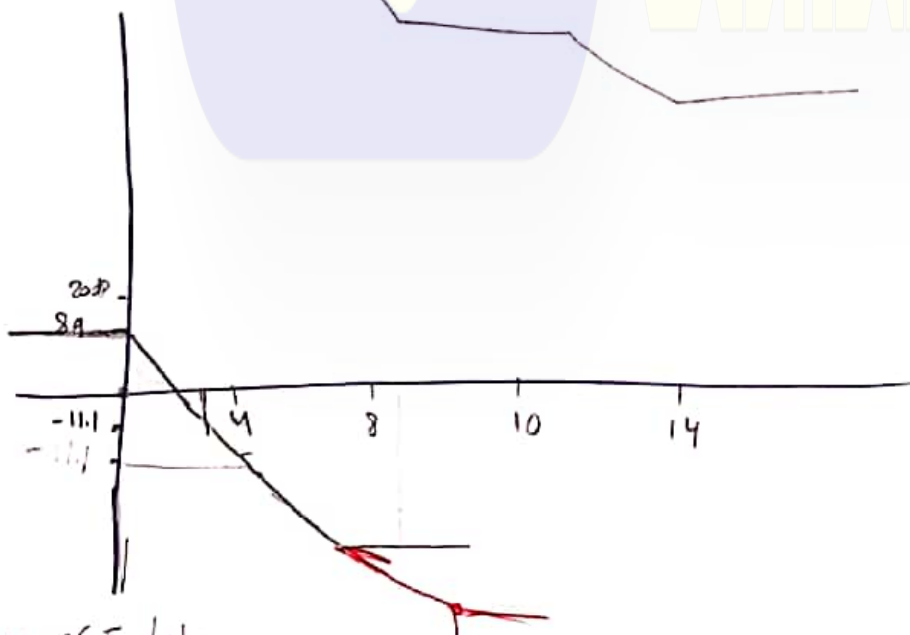
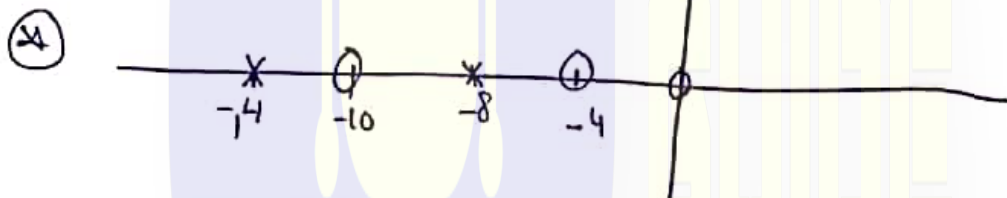


ex 2

$$H(s) = \frac{(s+8)(s+14)}{s(s+4)(s+10)}$$

$$\left| \frac{8 \left( \frac{s}{8} + 1 \right) \cdot 14 \left( \frac{s}{14} + 1 \right)}{4s \left( \frac{s}{4} + 1 \right) 10 \left( \frac{s}{10} + 1 \right)} \right|$$

$$= \frac{2.8 \left( \frac{s}{8} + 1 \right) \left( \frac{s}{14} + 1 \right)}{s \left( \frac{s}{4} + 1 \right) \left( \frac{s}{10} + 1 \right)}$$

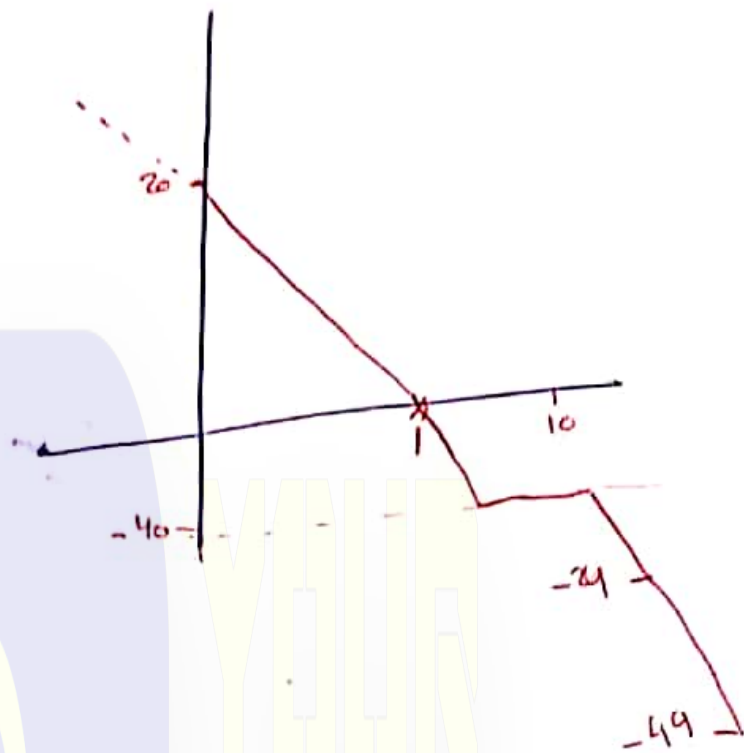


$$14.3 \left( \frac{s}{14.3} + 1 \right)$$

20dB

① exo

$$H(\omega) = \frac{5(j\omega + 2)}{j\omega(j\omega + 10)}$$
$$= \frac{5(2) \left(\frac{s}{2} + 1\right)}{10s \left(\frac{s}{10} + 1\right)}$$



# Two Ports Networks

Monday  
5/8/2019

## Impedance parameters (Z)



$$V_1 = I_1 Z_{11} + I_2 Z_{12} \rightarrow \textcircled{1}$$

$$V_2 = I_1 Z_{21} + I_2 Z_{22} \rightarrow \textcircled{2}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

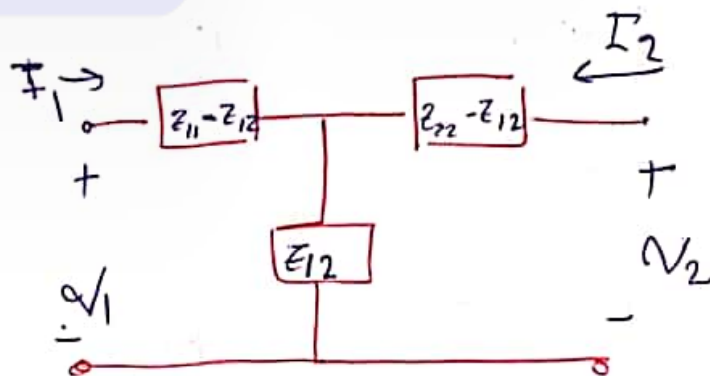
$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

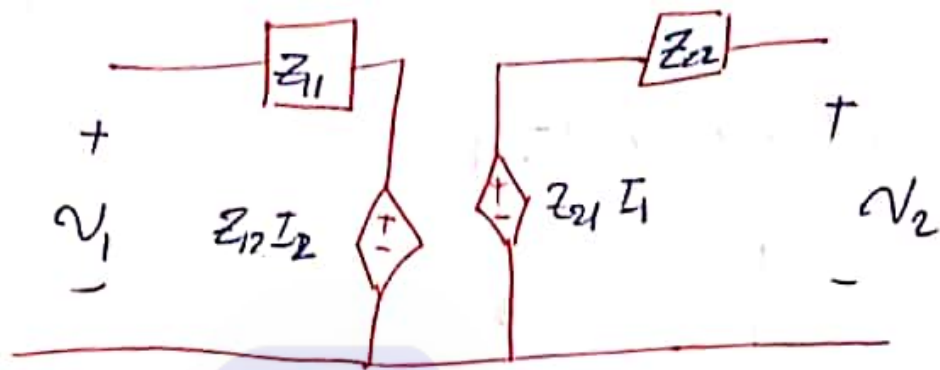
if  $Z_{11} = Z_{22}$  symmetrical network

if the network is linear [No dependent source]

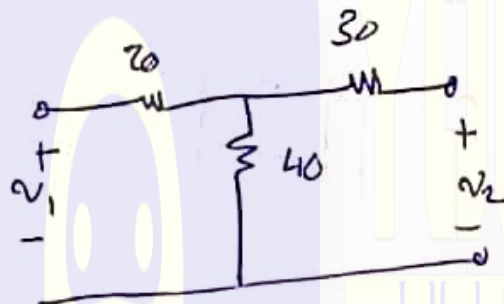
$Z_{21} = Z_{12}$  reciprocal network

①





exa-



sol:-

$$Z_{12} = 40 \Omega = Z_{21}$$

$$Z_{11} = 40 + 20 = 60 \Omega$$

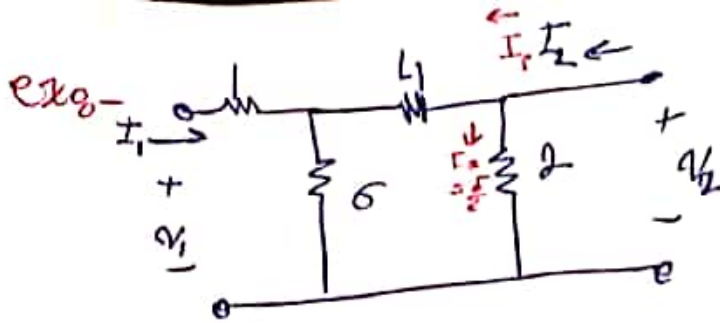
$$Z_{22} = 30 + 40 = 70 \Omega$$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = \frac{(20+40)I_1}{I_1} = 60 \Omega$$

$$Z_{21} = \frac{40 I_1}{I_2} \Big|_{I_2=0} = 40 \Omega$$

$$Z_{12} = \frac{40 I_1}{I_2} \Big|_{I_2=0} = 40 \Omega$$

$$Z_{22} = \frac{(30+40) I_2}{I_2} = 70 \Omega$$



Sol

to find  $Z_{11}$

$$v_2 = 2 \frac{I_1}{2} = I_1$$

$$Z_{11} = \frac{v_1}{I_1} \Big|_{I_2=0} = 1 \Omega$$

to find  $Z_{22}$

$$v_2 = \frac{5}{6} I_2 (2) = \frac{10}{6} I_2$$

$$Z_{22} = \frac{v_2}{I_2} \Big|_{I_1=0} = \frac{10}{6} \Omega = \frac{5}{3} \Omega$$



$$I_2 = \frac{I_2 \cdot 2}{10} = \frac{I_2}{5}$$

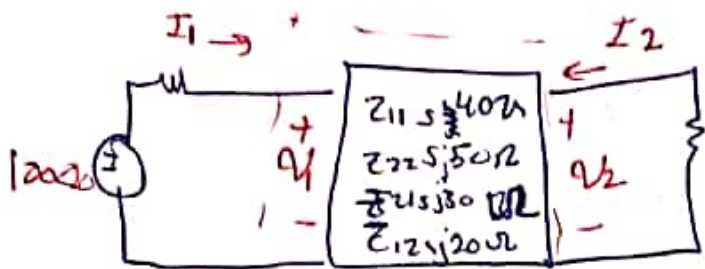
$$v_1 = \frac{I_2}{6} \times 6 = I_2$$

$$Z_{22} = \frac{v_2}{I_2} = 1 \Omega$$

$$v_1 = I_1 (1) + \frac{3}{2} (I_1)$$

$$v_1 = 4 I_1$$

$$Z_{11} = \frac{4 I_1}{I_1} \Big|_{I_2=0} = 4 \Omega$$



sol)  $v_1 = 100 + 5 I_1$

$$v_1 = I_1 Z_{11} + I_2 Z_{21}$$

$$v_2 = I_1 Z_{12} + I_2 Z_{22}$$

$$100 \angle 0^\circ = 40 I_1 + j20 I_2$$

$$-10 I_2 = 30j I_1 + j50 I_2$$

$$0 = 30j I_1 + (10 + 50j) I_2$$

$$v_2 = -10 I_2$$

$$A = \begin{bmatrix} 40 & 20j \\ -30j & 10 + j50 \end{bmatrix} \quad I_1 = \frac{\Delta_1}{\Delta}$$

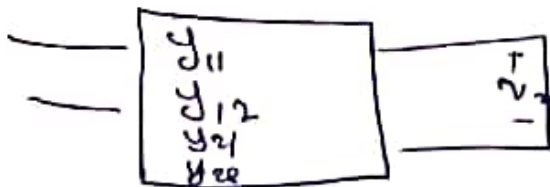
$$D_1 = \begin{bmatrix} 100 \angle 0^\circ & 20j \\ 0 & 10 + 50j \end{bmatrix} \quad I_2 = \frac{\Delta_2}{\Delta}$$

$$D_2 = \begin{bmatrix} 20 & 100 \angle 0^\circ \\ -j30 & 0 \end{bmatrix}$$



Tuesday  
6/8/2019

⊗ Admittance parameter



$$\begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_2 \end{aligned} \quad \otimes$$

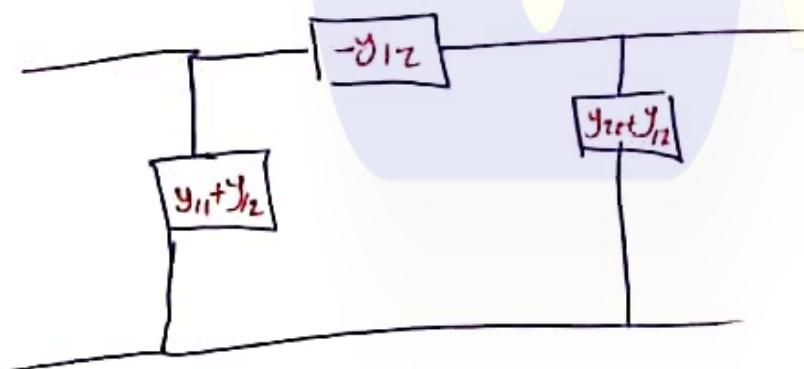
$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$  S.C Input impedance.

$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$   
 $Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$  } S.C transfer admittance.

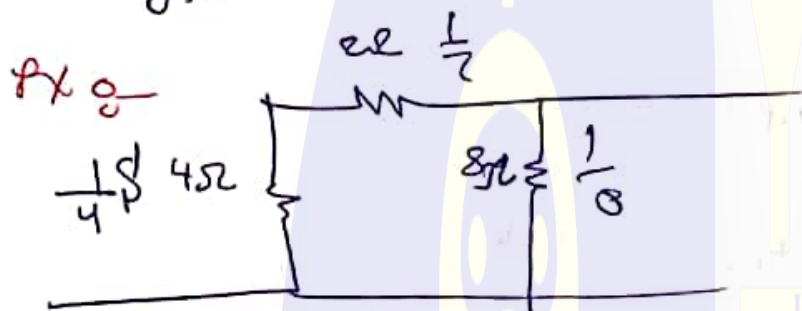
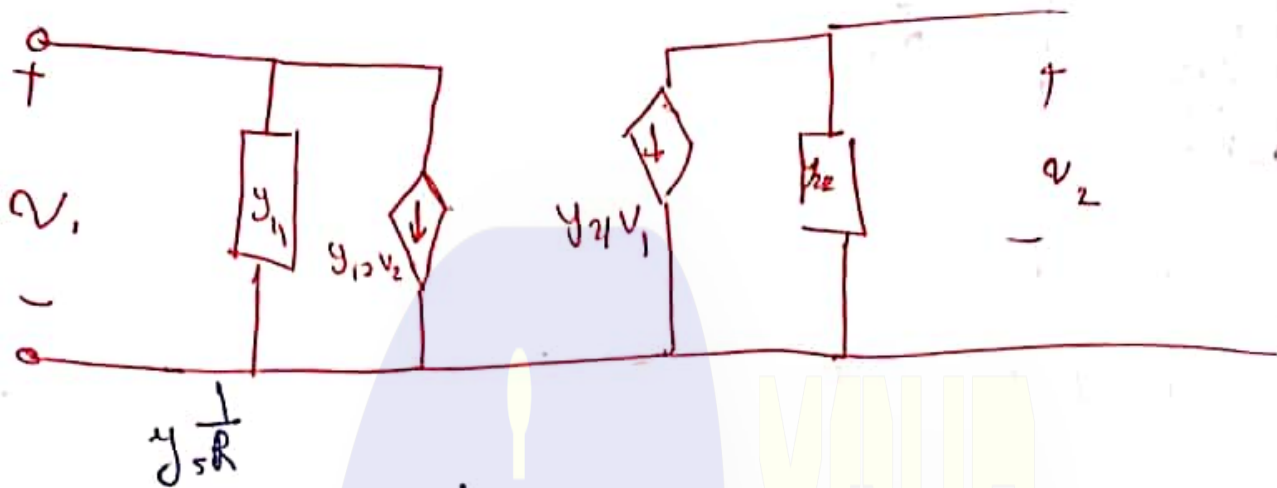
$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$  } S.C o/p admittance  
 output

~~Linear~~

Linear network (No dependent source)



If the network is not reciprocal



$$y_{21} y_{12} = -\frac{1}{2} \text{ S}$$

$$y_{11} + y_{12} = \frac{1}{4}$$

$$y_{11} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \text{ S}$$

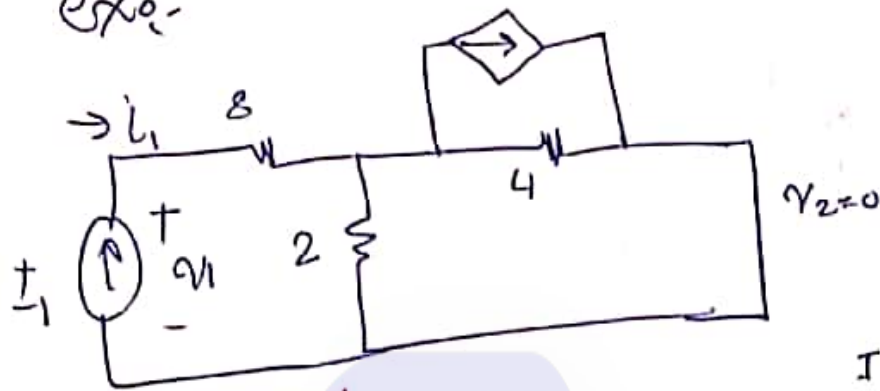
$$\rightarrow y_{22} + y_{12} = \frac{1}{8}$$

$$\rightarrow y_{22} = \frac{5}{8}$$

or  $y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$

(  
)  
)

exo:



$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$I_2 = \frac{I_1}{V_1} \Big|_{V_2=0}$$

$$y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

Meth 2

$$4I_2 - 8I_1 + 2(I_2 - I_1) = 0$$

$$6I_2 - 10I_1 = 0$$

$$I_2 = \frac{10}{6} I_1$$

$$I_1 = \frac{6}{10} I_2$$

$$V_1 = 8I_1 + 2\left(4 - \frac{10}{6} I_1\right)$$

$$V_1 = 8I_1 + 4I_2 - 8I_1$$

$$V_1 = 4I_2 = \frac{10}{3} I_1$$

$$V_1 = \left(\frac{30}{3} - \frac{10}{3}\right) I_1$$

$$V_1 = \frac{20}{3} I_1$$

$$y_{11} = \frac{I_1}{V_1} = \frac{3}{20} = 0.15 \text{ S}$$

$$V_1 = 8I_1 + 2(I_1 - I_2)$$

$$= 8\left(-\frac{3}{5} I_2\right) + 2\left(-\frac{3}{5} I_2 - 2\left(-\frac{I_2}{3}\right)\right)$$

$$V_1 = -\frac{4}{5} I_2 + \frac{6}{5} I_2 - \frac{4}{3} I_2$$

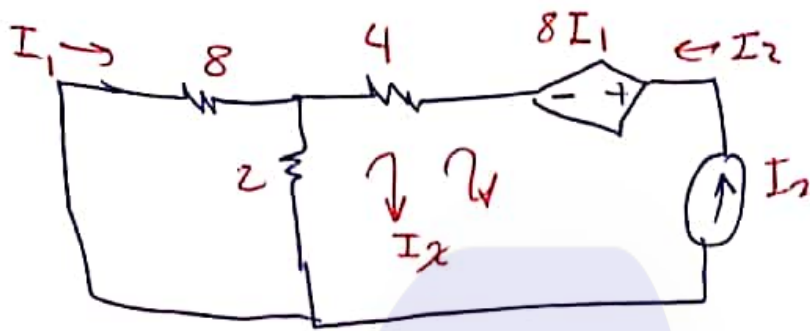
$$V_1 = I_2 \left( \frac{-4}{5} - \frac{4}{3} + \frac{6}{5} \right)$$

$$V_1 = I_2 \left( \frac{-20}{5} \right)$$

$$V_1 = -4 I_2$$

$$y_{21} = \frac{I_2}{V_1} = -\frac{1}{4} \text{ S}$$

$\int \mathbf{I}_2 \cdot d\mathbf{S}$



$$I_x = -I_2$$

$$Y_{12} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$

$$I_1 = -I_2 \frac{2}{10} = -\frac{1}{5} I_2$$

$$I_2 = -5 I_1$$

$$V_2 = 8I_1 + 4I_2 = 8I_1$$

$$s \cdot 8I_1 + 4I_2 = 8I_1$$

$$V_2 = -20 I_1$$

$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0} = -\frac{1}{20} \text{ S}$$

$$V_2 = -20 I_1$$

$$V_2 = -20 I_1$$

$$V_2 = -20 \left( -\frac{1}{5} I_2 \right)$$

$$V_2 = +4 I_2$$

$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{1}{4} \text{ S}$$