

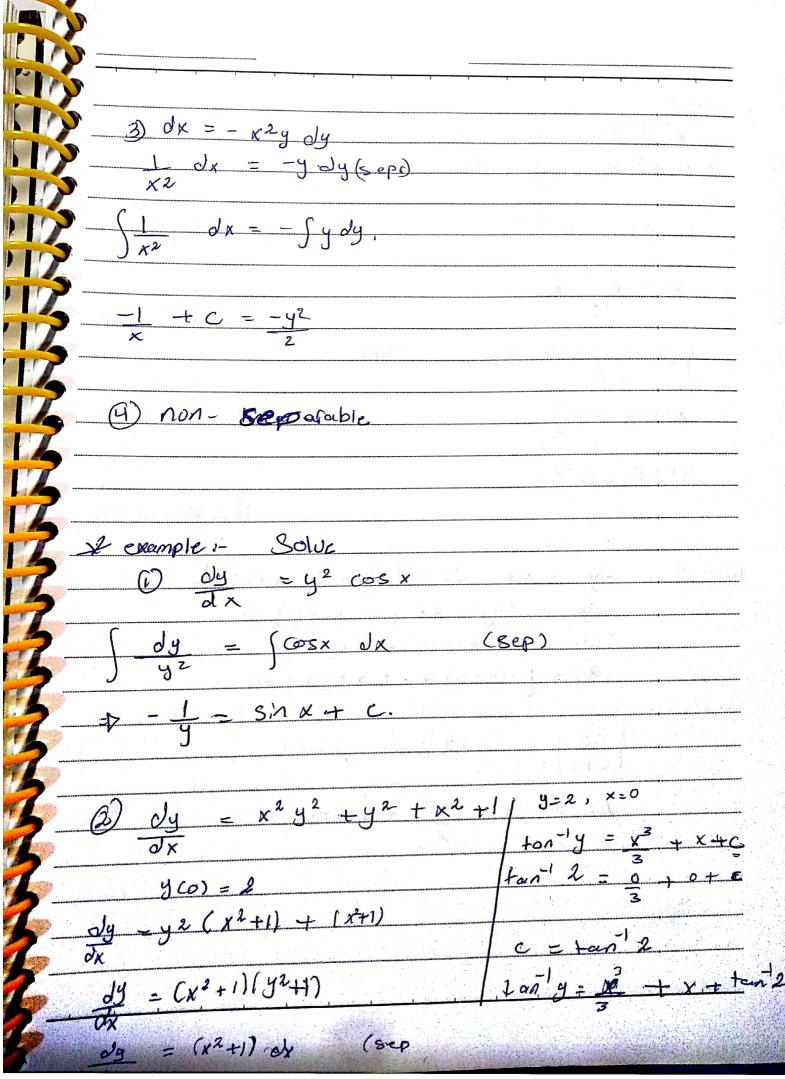
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Diff
* ordinary diff eq:-
1.1 Basic concepts
1) O.D.E eq. Which contains derivatives
of the unknown
ex: -2y' + y = 0
$y' = (x+1) e^{x} y^{2}$
$y''' = 2x (y'')^3 = 0$
The second secon
2) classification
II order => the highest order of derivative.
[2] linear and non linear
ex on non linear: - y2, Ty, ey, lny, siny, gy, g'g'
3) home and rowhome
an (x) yan + an, (x) yn-1) + + any = f(x)
$If f(a) = 0 \Rightarrow none$
& ten to non-homo (pure x function)

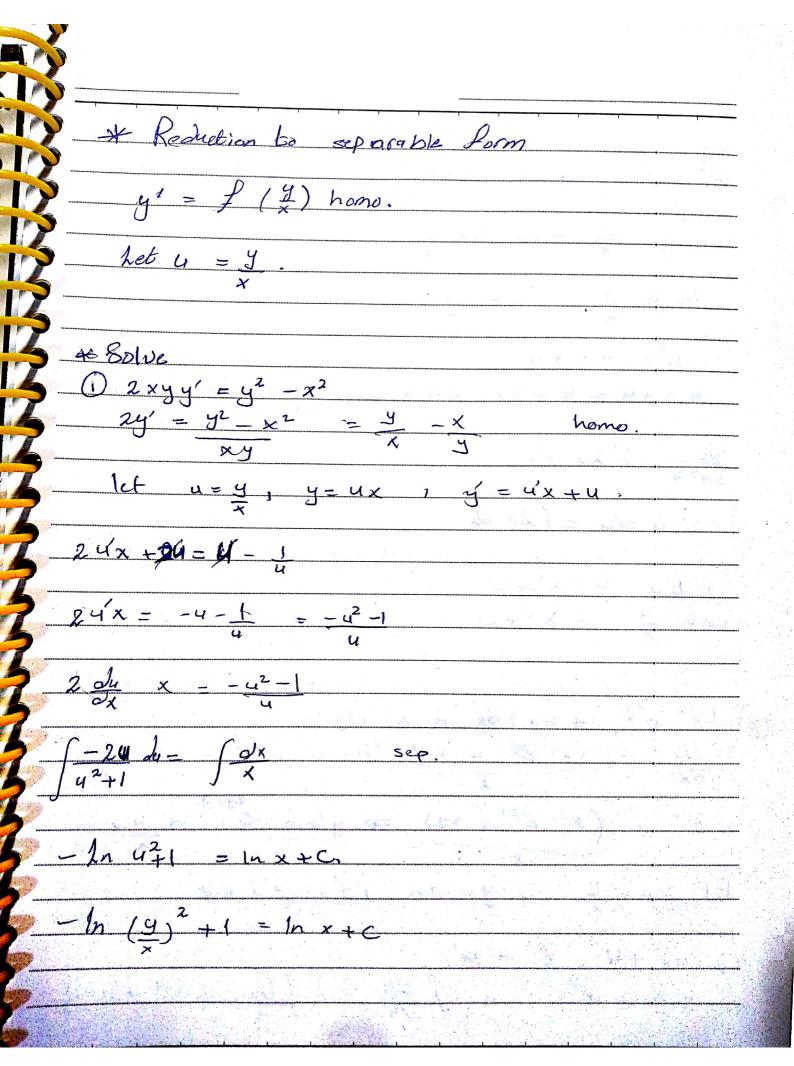
examples: - (D y" - Sin x y'	$-4 = \sin x$	
	٥	
(3) (x-1) y' + y = 0		•
Ø 4"+34 =2		***************************************
(6) (x + xy) dx + 2y dy	= 0	
J		7. 4
Sol. 1) 2nd Goder, linear, 2) 3rd order, pon-line	non-homo	
2) 3rd order, bon-line	or homo	
3) ist order linear	homo	
4) 2nd order, Linear, n	101 - homo	
5) 1st order, hinear, no	on-homo	
$\begin{array}{c} 6) \Rightarrow (x + xy) + 2y = 0 \\ \Rightarrow (x + xy) + 2yy' = 0 \end{array}$	× Charles	
4 1st order, non-lines		•
	Salat Maria	Taraki (Sa.
* In O.D.E we all	Looking for 801	y= Foo
80 the 801 of U.D.E	is y = f (x) sa	tisfying
o. D. F.	and the second	
	naun appropriation and a propriation of the state of the	

example: Show that g(x) = 10 - c ex with constants C, is ased to y'ty=10. y + y' = 10y = 10-60* 10 - cot + cox = 10 y cair's sol for 0:0.E * Initial Nolve proplem: J.V.P = O.D. E. Vinitial condition ex: Show that $y = ce^{2x}$ is a sol of the IVP y' = 2y, y(0) = 1 0.0.E 0.0.E y' = 2y y' = 2y $y(\alpha) = 1 = CE$ 1.C $2e^{2x} = 3 = 2e^{2x}$

1.3 separable O.D.E [a) dx = g(y) dy example: $(3) dx + x^2 y dy = 0$ (4) (x + y) dx + x2 siny dy =0 801, 1) xsiny dx = - x2dy 1 dx = 5-csc y dy $\ln x + c = -\ln |\cos + \cot |$ $x dx = -x^2y dy.$ $\int \frac{1}{x} dx = - \int \frac{1}{y} dy$ $hx + C = -y^2$



 $e^{x+y} dx = e^{x-2y} dy$ (Sup) 3x+ ====39. dy - xy dx = (4y + 3x +12) dx H. W 30 dy = (4y + 3x + 12 + xy) dx dy = (4y +12 + 3x +xy) dx $\frac{1}{2}$ $\frac{1}$

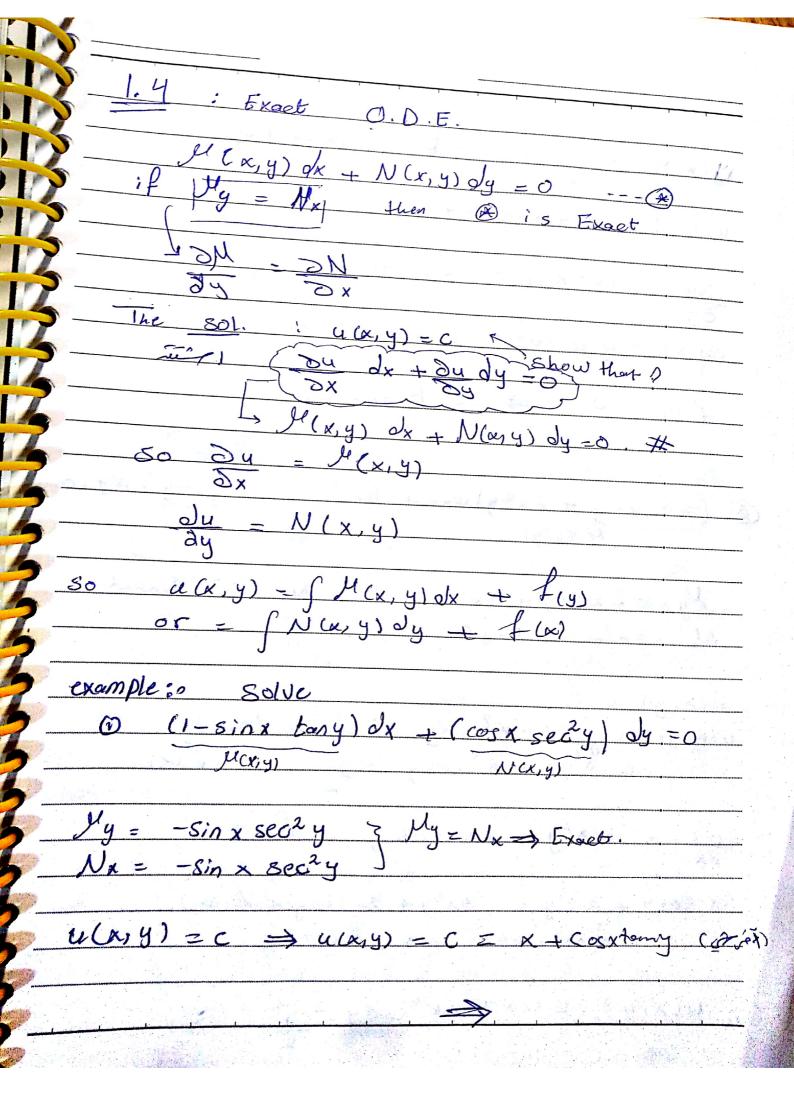


(2) $xy' = y + 2x^3 + 8 \ln^2 \frac{y}{x}$ $y' = \frac{y}{x} + 2x^2 \sin^2 \frac{y}{x}$ let u= y , ux = y 4x+4=4+2x2 sin24 - du = 2 x sin24 csez u du = 52x dx $y' = (x^2 e^{2y/x} + xy) \Rightarrow y' =$ let u = y , y = ux , y' = u'x + u $\Rightarrow ux + y = e^{2y} + y$ $u'x = e^{2y} = \int dy = \int dx$

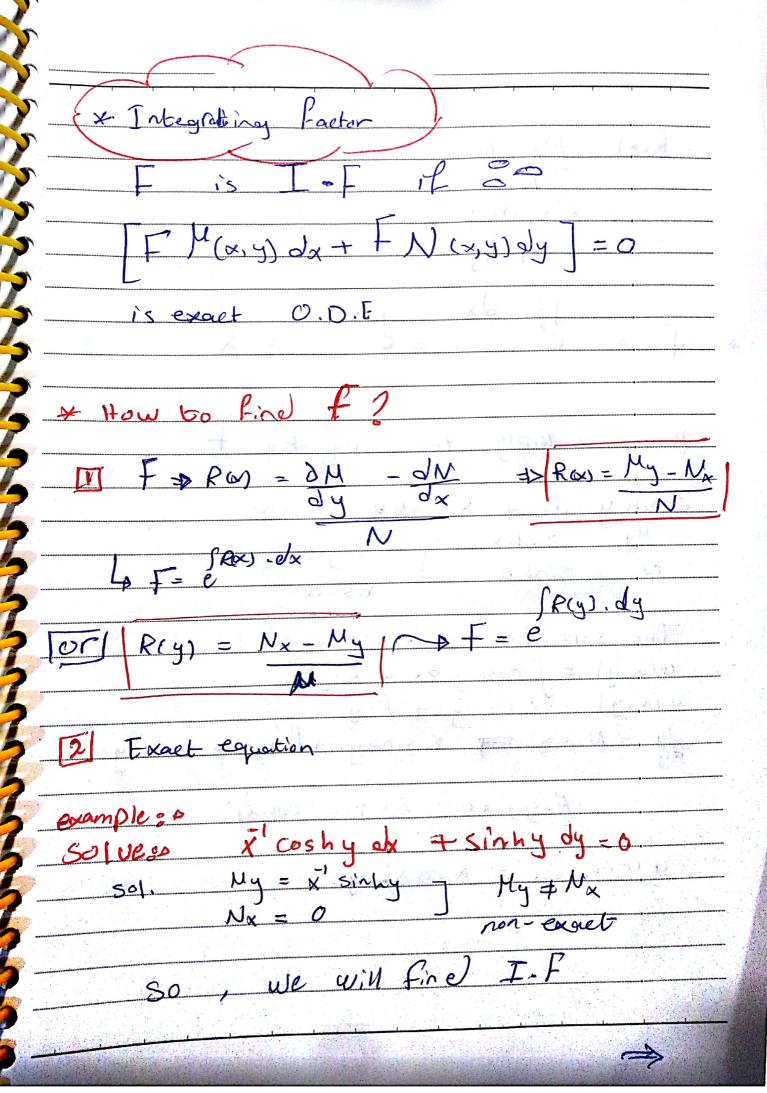
-24 4 X (1+4) جدعفا مات In- (1+24-42) = Inx+c

HEW OF Dy = (y + 4x)2	veriens opens
	عادر عن أنواء ك
$let \ \ U = Y + 4X$. When ~ 15 1/1 "216
y = u - 4x	
y' = u' - 4	
	* Control of the Cont
	#####################################
4-4 = 42	
$u' = 4^2 + 4$	•
$\int \frac{du}{u^2+4} = \int dx$	
	•
	<u> </u>
$(2) 11 (2)^2$	***************************************
(2) $y' = (x+y-2)^2$	•
u=x+y-2.	

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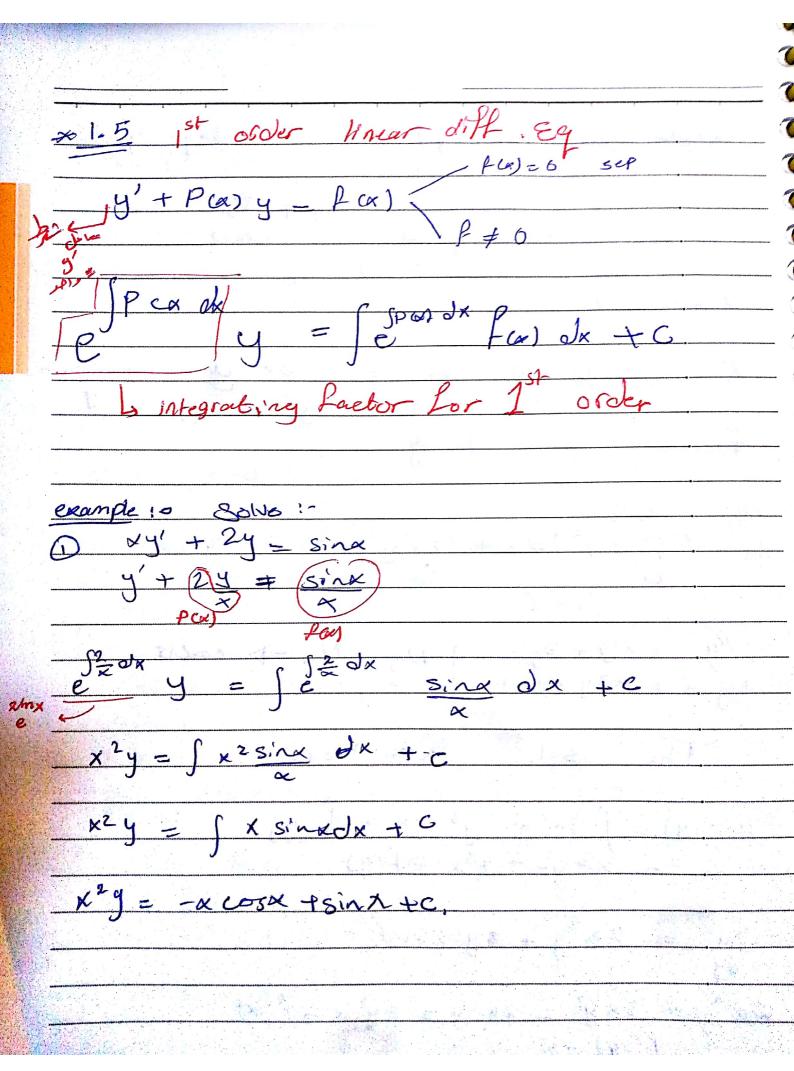


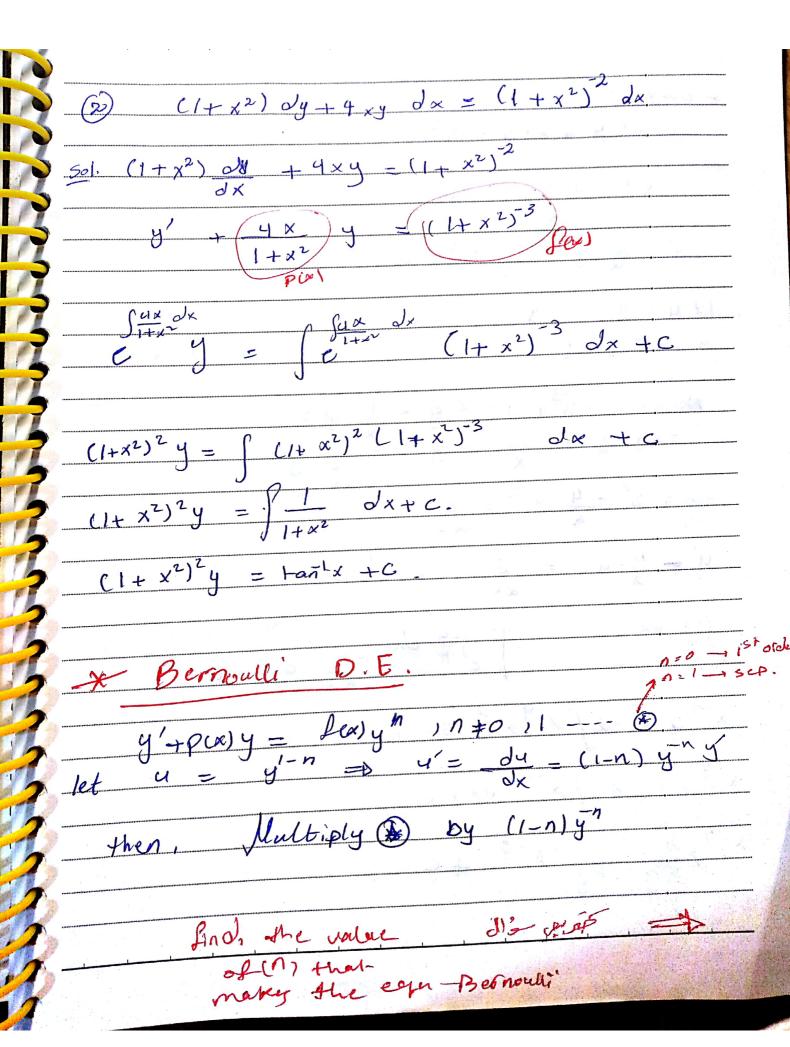
y(x,y) = \((1-sinx borny) dx - 1 Fly) u(x,y)= x+ cosx x tany + fly) Dy = cos x sec 2 y + p'(y) cos x sec2 y = cos x sec2 y + f'(y) $f'(y) = 0 \implies so f(y) = C.$ (2) $(2x \cos y + 3x^2y) d_x + (x^3 - x^2 \sin y - y) d_{y=0}$ $\frac{M_{y} = -2 \times \sin y + 3x^{2}}{N_{x} = 3x^{2} - 2x \sin y} + 3x^{2} - \frac{M_{y} - N_{x}}{1 + 3x^{2}} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2} \times \frac{1}{2}$ u(x,y) = C. $u(x,y) = \int (x^3 - x^2 \sin y - y) \, dy + \int (x)$ $= x^3 y + x^2 \cos y - y^2 + \int (x)$ $\frac{du}{dx} = 3a^2y + 2a \cos y + b \cos y$ 2x cosy + 3x2y = 3x2y + 2x cosy + 8'(a) fas = 0 -> fas=c $u(x,y) = c = x^3y + x^2 \cos y - y^2$



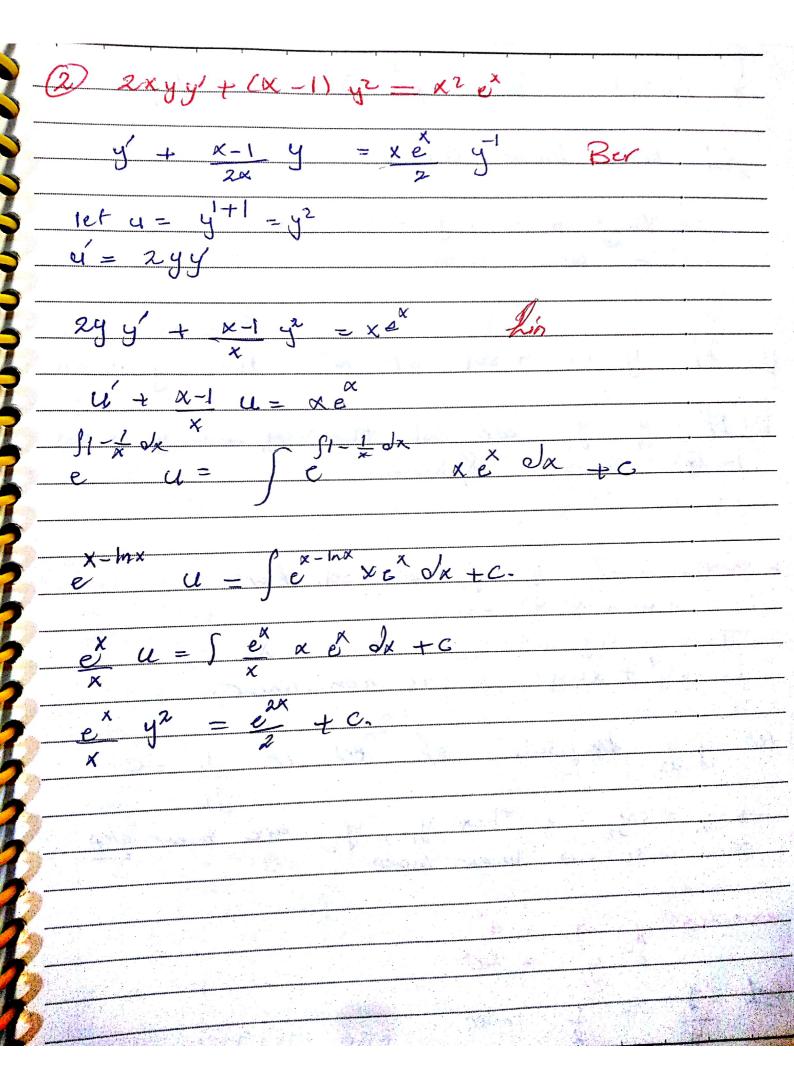
now, multiply the eq by coshy dx + xsinky dy = 0 The sol. is u(x,y) = c u(x,y) = s coshy dx + f(y) x sinky + f'(y) = x sinky f(y) = 0 , f(y) = const x co shy:

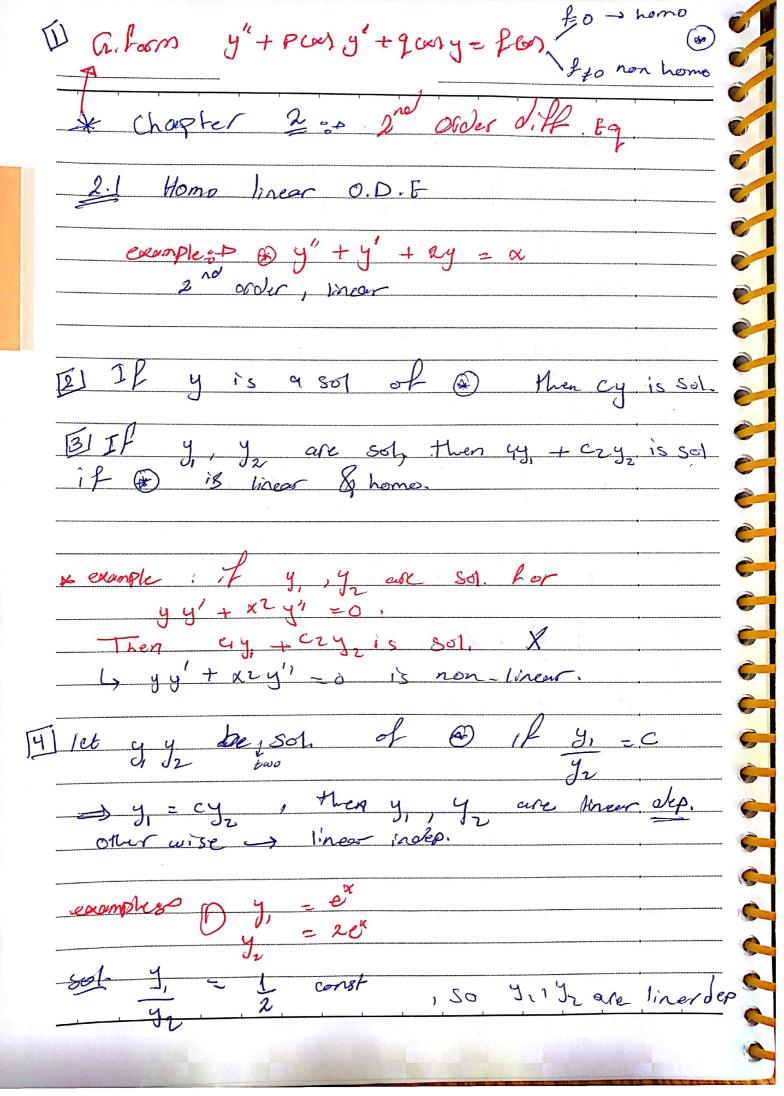
example $z = (2x^2y+y^2) dx + (2x^3 + 3xy) dy = 0$ * find I.F $R = 3x^2 + 19 - 6x^2 - 39$ $\int_{0}^{2} (3x^{2}y^{2} + y^{3}) dx + (2x^{3}y + 3xy^{2}) dy = 0$ The sol. 4004) = C. u(x,y) = \(3x^2y^2 + y^3 dx + \((y) \) Du = 2x3y+3y2x+P 2x3y+3xy2 = 2x2y = 3y2x+fas fcx1 = const , C= x342 +y3x

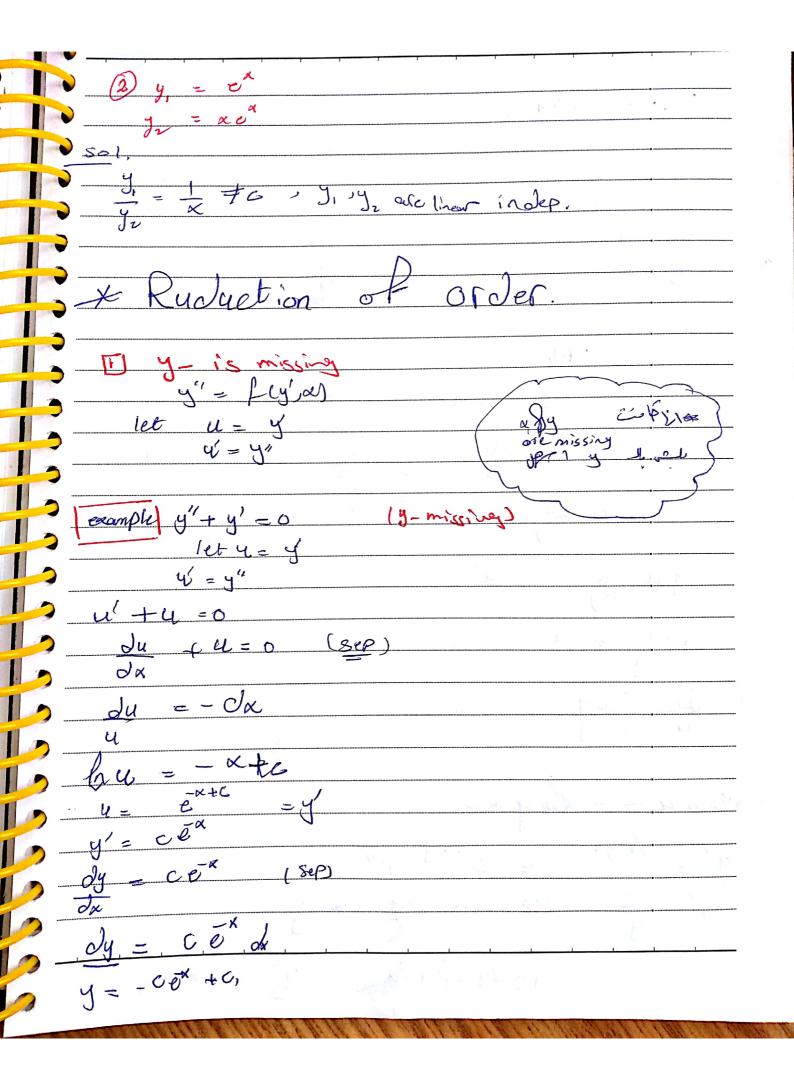




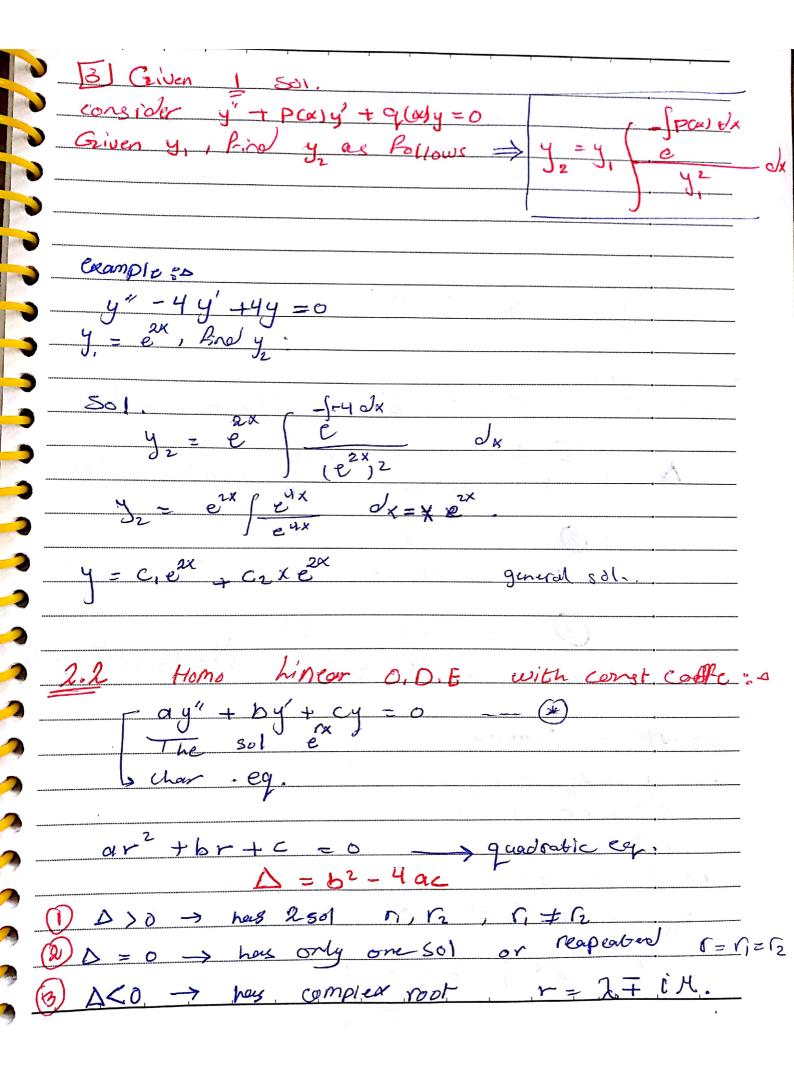
x2y' + 2ey = y3 y' + 2 y = 1 y^3 Ber $|e| y = y^{1-3} = y^2$ Now, $-2y^{3}(y'+2y=1)$ $-25^{3}9' - \frac{4}{4}9^{-2} = -2$ $\int_{-\infty}^{-4} dx = \int_{-\infty}^{-4} dx + G$ x-4 y = -2 | x = 2 dx + c x4 u=-2 fx6 date w-4,-2 = -2 x-3 + c







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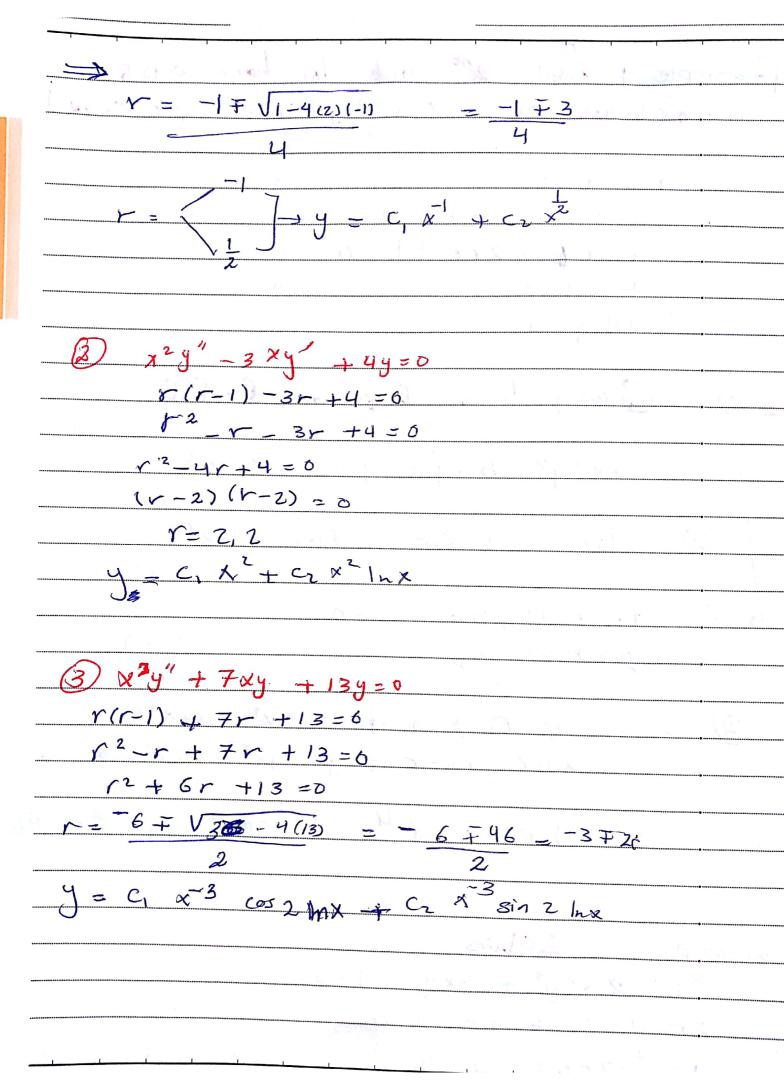


* Quick Reision &P $-\ddot{c} = \frac{1 \times c}{1 \times c} \qquad -\dot{c} = \frac{c}{c} \qquad V$ A Oular Rules DX ILLX λx ech+ily)x e (cosilatisin Mx) the sol wif & Jh = Cierx + czerx the sol of a ; y = c, ex + c, xe the solol @, y = cre costa + cre sintix example 1- Solve: (r+1) (r-3) 20 , e3x J basis sol.

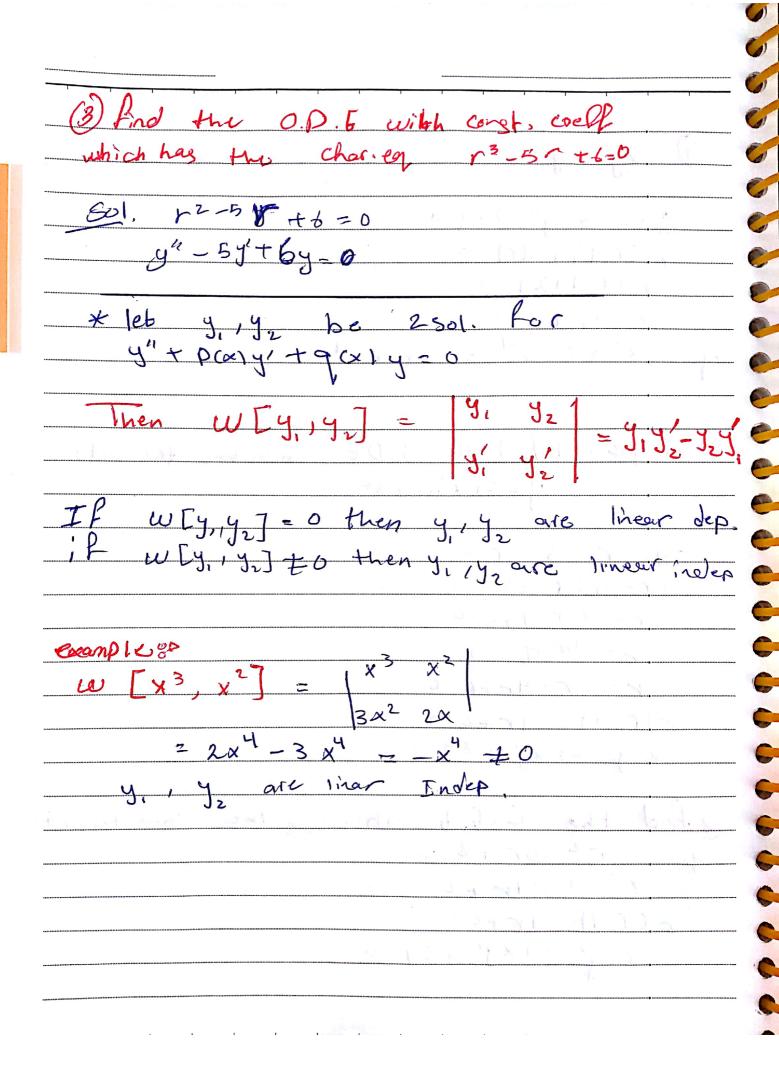
@ y" - 3y' = 6 r(r-3)=0 $\frac{\Gamma = 0, \Gamma = 3}{4} = \frac{3x}{c_1 e^x} + \frac{3x}{c_2 e^x} = \frac{c_1}{c_1} + \frac{3x}{c_2} = \frac{3x}{e^x}$ (3) 2y'' - 12y' + 18y = 0212-12 1+18=0 M2 - 6r +9 =0 (r-3)(r-3)=0example = Find the O.D. & Owhich has the 801. Y=C, 3x+cre2x sal 1=3, 1=2. char +2-1-6=0 y" - y' - 6y = 0 @) which has the sol, you = e2d (c+xcz) (r-2)(r-2)=0y'' - 4y' + 4y = 0

Solve: - 1 4 + 4 = 0 r2+1=0 = r2=1 (r=t [] cosx + creox sing 4 cos x + cr sinx @ 24" + 2y +44 =0 202 + 25+ 4=6 r2+r+2=0 D= b2-yae $= \mathbf{e}, \quad \mathbf{e}^{\frac{1}{2} \times} \cos \sqrt{7} \times + \mathbf{c}_{2} \quad \mathbf{e}^{\frac{1}{2} \times} \sin \sqrt{7}$ (3) y'' + 9y = c (3) (3) = 1 (3) = 3y'=0+3c2co53x >> 3-3c2 >> 52=17

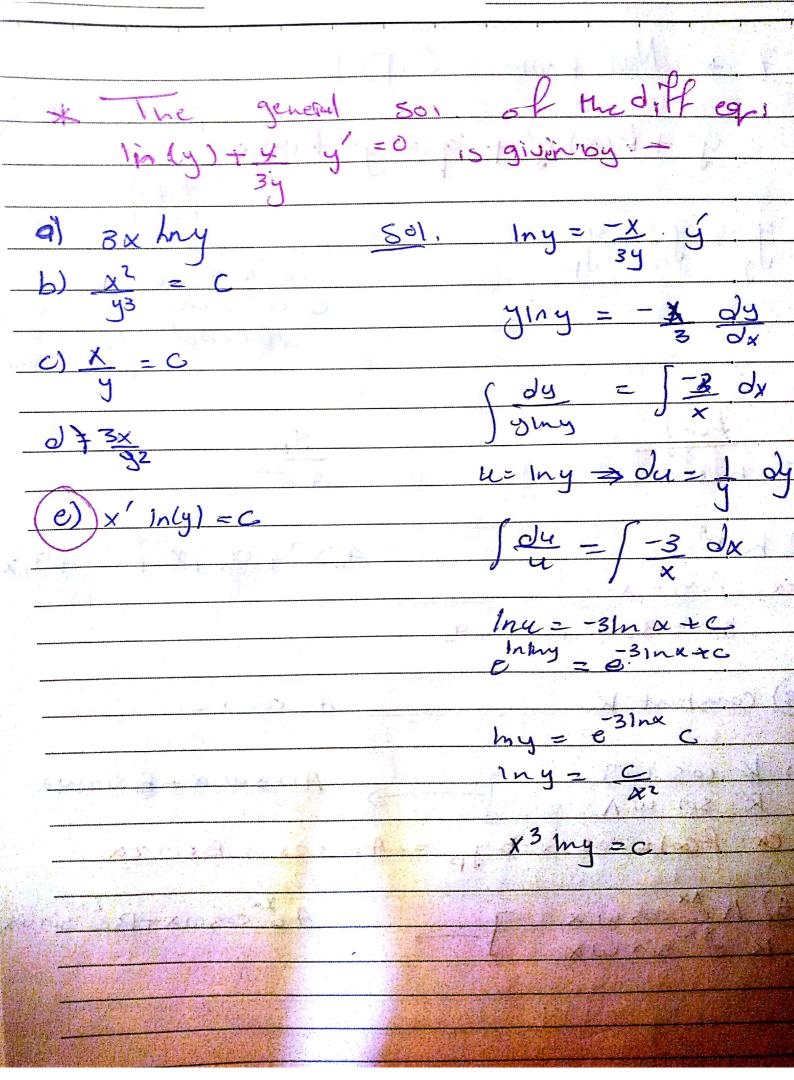
| example as find and order In. (C + 2i) = 0 $r^2 + 2r / - 2r + 4 = 0$ Cauchy Eulor Ecquation C.E.E. (1) ax2y"+ bxy+ey=0 C.E.E. The sol for C.E.E. Y=xT 2 => 4 = C1X1 + C2X12 => y= Cx + cz x lnx. 3) r= 27 ill = c, x cosl Inx+cxx sin Plax * examples solve 1) 2x2y" + 3xy - y=0 2r (r-1) + 3r 1-6 25-25+25-1=0



x2y"+2xy=0 example: - 1) find the O.D. & which has



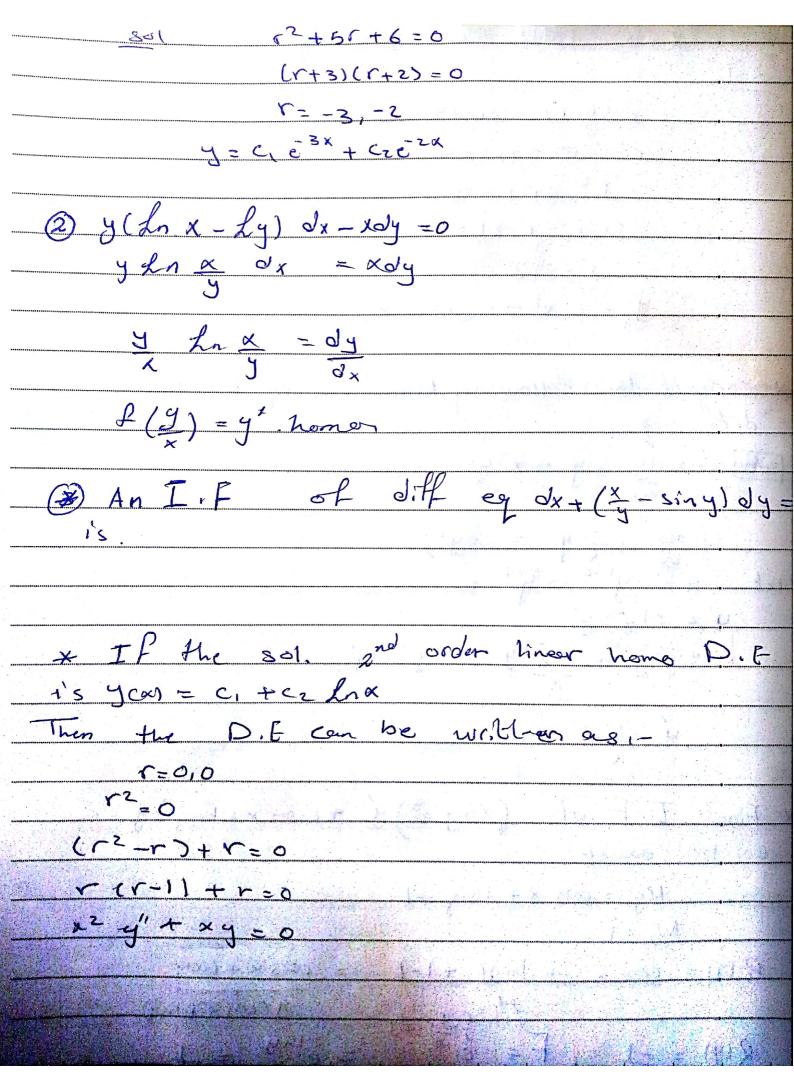
Hethod ex: flex) = cos 201.



* The general sol of diff eq x2"-34"+44=0, 2 > 0 C.E.E is givening 1-((1-1) - 3r +4 =0 7 - C102 + C2 x2 hx * convert the Pollowing Bernoull eq to linear

diff eq.

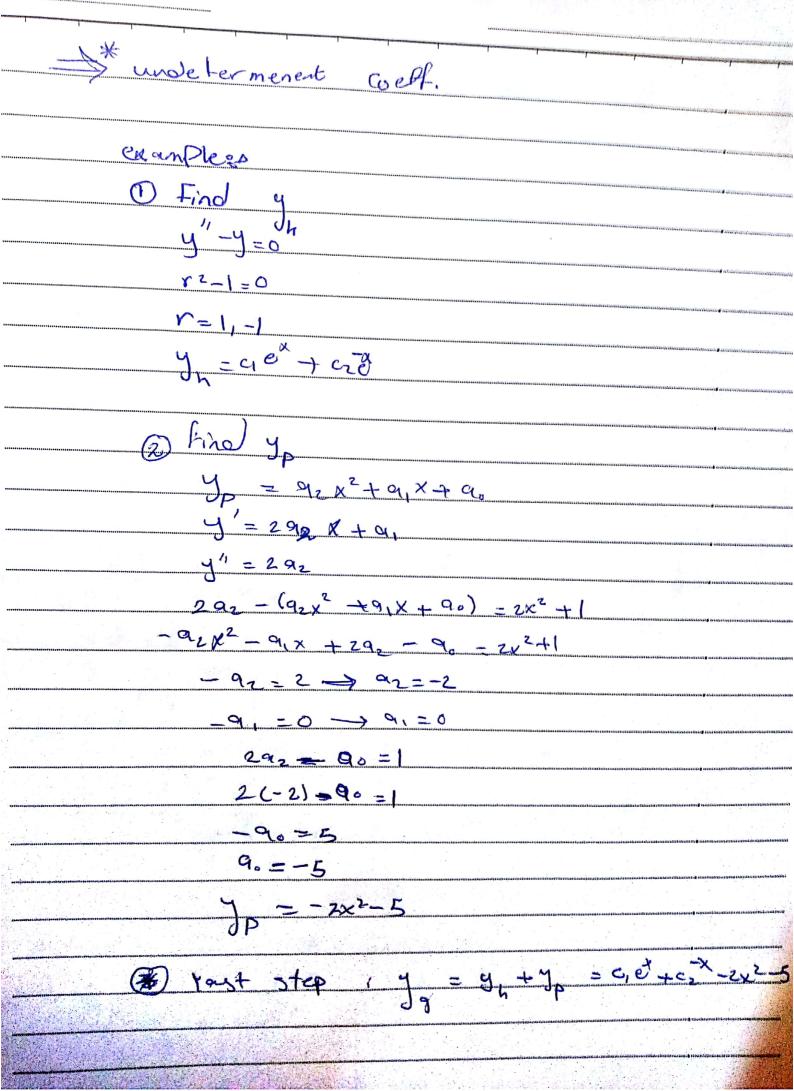
[SAI] X dy + y = x2y2 u - L u = -x* Find I.F of (x+y ey) y' + (cosx+ Iny) y =0 Gas, My = cos x + Imy +1 $R(y) = \cos x + \ln y + 1 - 1 = \cos x + \ln y$ $= -y \left(\cos x + \ln y \right)$ $R(y) = -\frac{y}{2} \left(\cos x + \ln y \right)$ $= -\frac{y}{2} \left(\cos x + \ln y \right)$ $= -\frac{y}{2} \left(\cos x + \ln y \right)$ $= -\frac{y}{2} \left(\cos x + \ln y \right)$

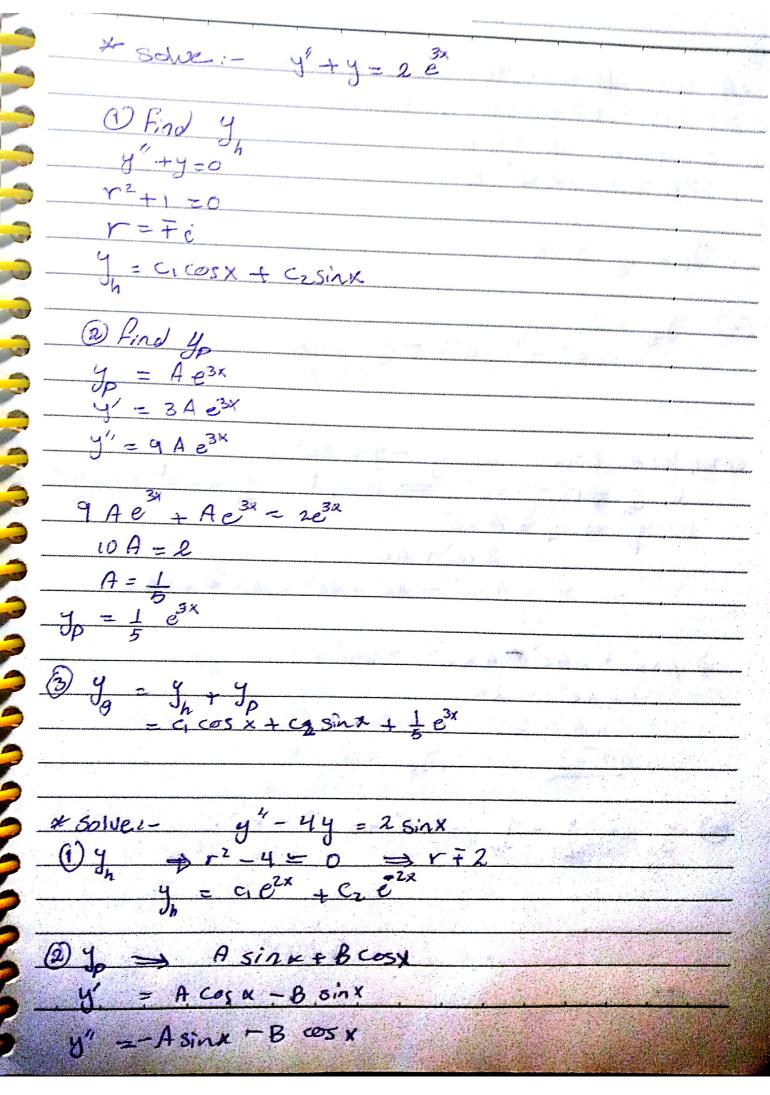


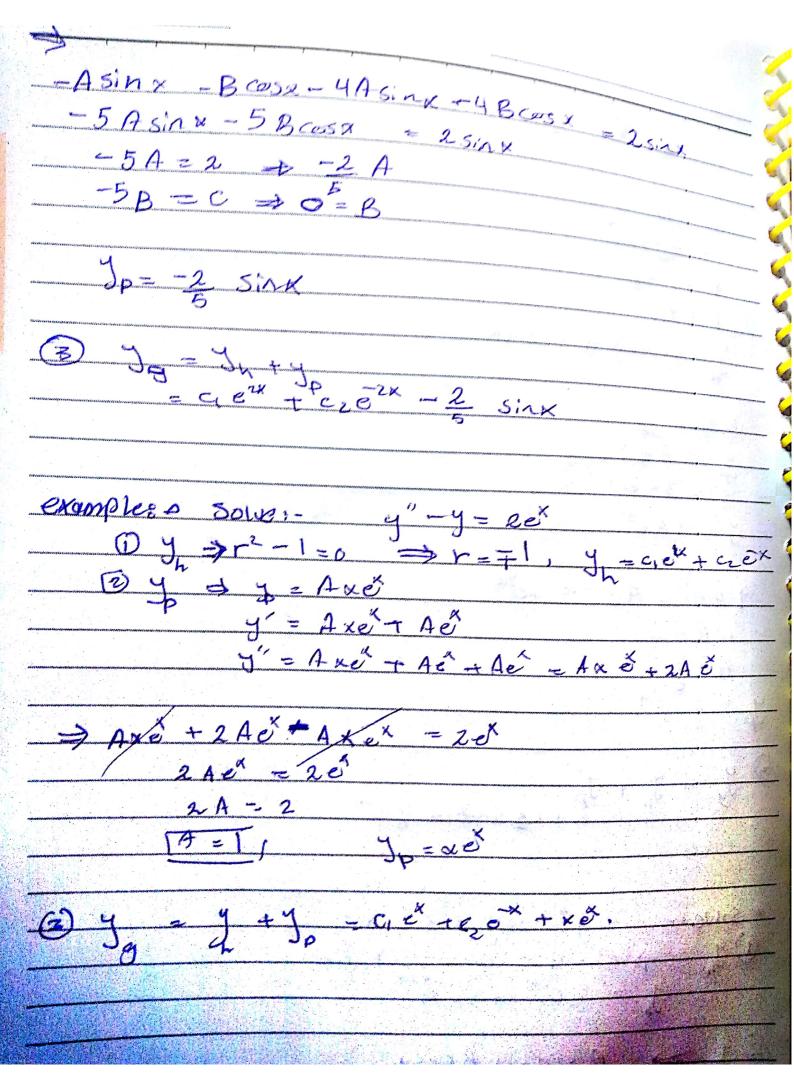
(2xy" +2y) dx + (2x2y+2x) dy =0 2 m x ym-1 + 2 = 4x4 +2 w (1,9(a))= ux3, Theng(a)= 2 219'=4x3 B197 = 4x3 [4] dg = [4x3dx 9 cm = X"+c

* The 301. of the Problem. 2x dy = 1+y2 163 $e^{\int \frac{2y}{1+y^2}} dy = \int \frac{dx}{x}$ $e^{\int \frac{1+y^2}{x}} = e^{\int \frac{x}{x}}$ 9(d) 3-1, what 7(2) ? -1 ln 1-24 = lnx+0 -1 ls(1-24) = lsx+6 -1 ln (1-24-1) = for to

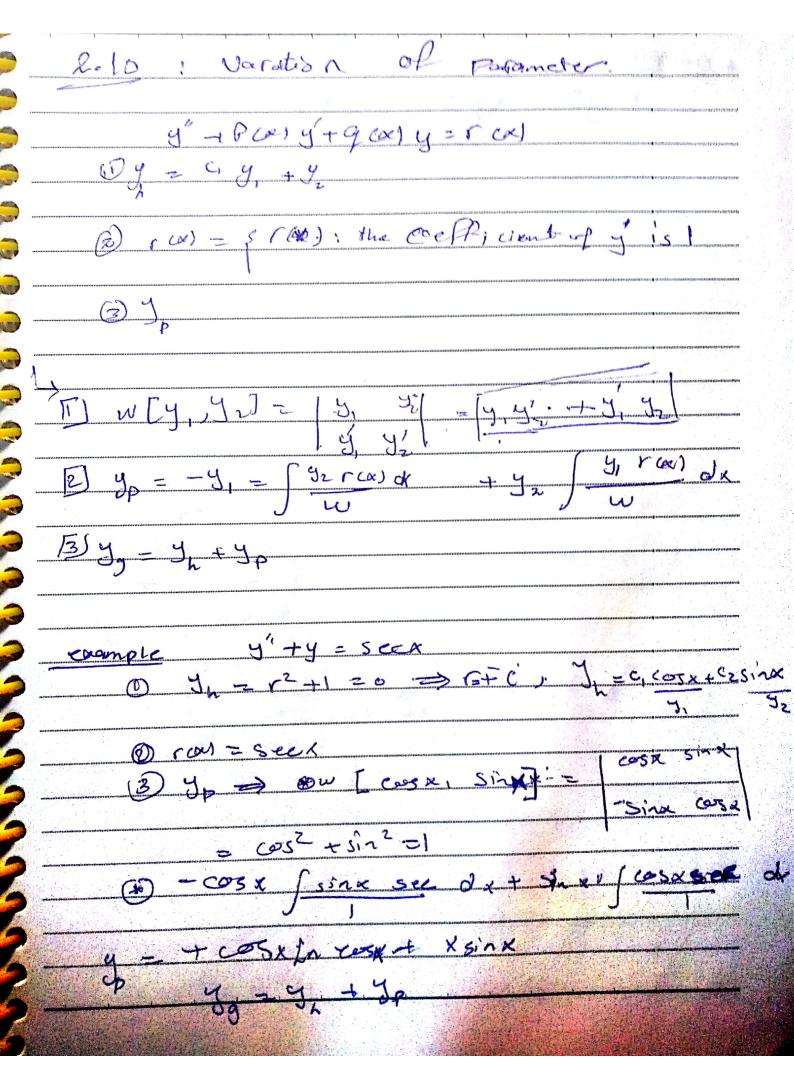
-1 ln (1-24) = Ln x - 1 ln 3 -1 fr (1-24) = Ln(2) -1 Ln 3 -1 ln (1-24) = Ln(8)-1km3 # ln (1-y) = 2 Ln (2) . + 2 n 3 In (1-4) = Ln 3 - hrg e (1-8) = ln (3) 4/2 4 1-y= 3 -> 1-3 - y => J= 4 Sol. Set for I The fundamental y" + y = 0 4 = C1 CO3X4 Cremx

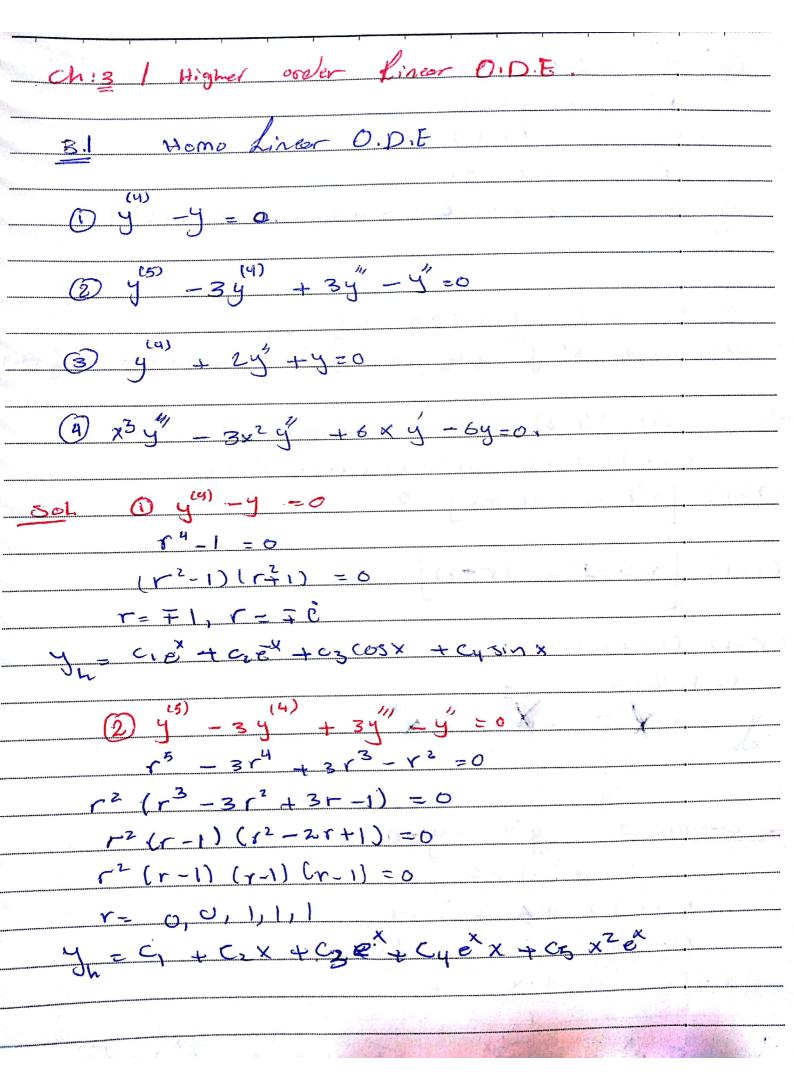






* Determine the form of particular soly 回y - Adsina+Bécosx. D $y'' + 2y' = x'' - 2x^3 + 1 + x^2 e^{-2x} + \cos 2x$ $[2] y = (a_4 x^4 + a_2 x^3 + a_2 x^2 + a_1 x + a_0) x$ $+ (b_1 x^2 + b_1 x + b_0) = 2x x$ " + 24 + 24 = E + BE sind Hall r=-176 G EXCOSK + O2 EX Sins





```
6
                                         (r2+1)(r+1)=0
                                          ア= ti ノド=ti
                  y = c, cosx + c2x60s x + c3sinx + C4xsinx
4) x3 y" - 3 x2 y" + 8 x y - 6 y = 0

x elw couchy ) an x. or y (n) + 9 x x y - 6 y = 0

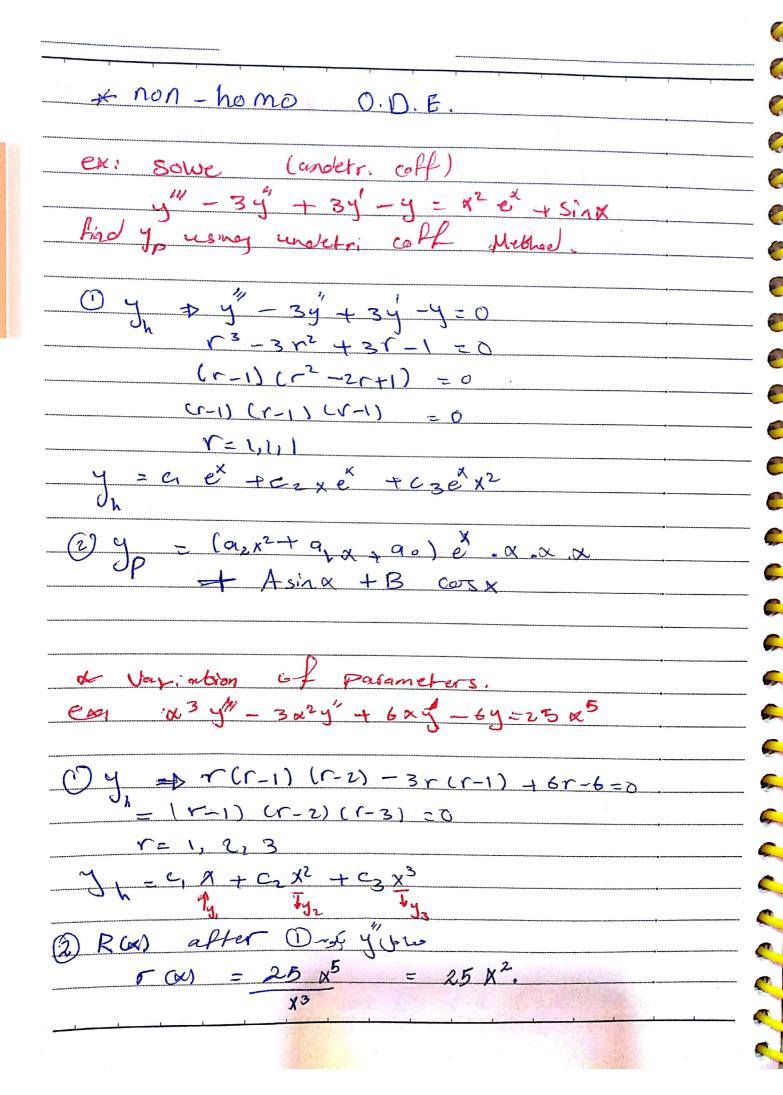
+ elw couchy ) an x. or y (n) + 9 x x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x y + 9 x
                                                  r(r-1)(r-2) - 3 r(r-1) + 6 r - 8 = 0
                                               r (1-1) (r-2) -3r(1-1) +6(1-1)=0
                 (r-1) [r(r-2)-3r+6]=0
                  (r-1) [r(r-2)-3(r+2)] = 0
                  (r-1) (r-2) (r-3) = 0
                     (=1,2,3

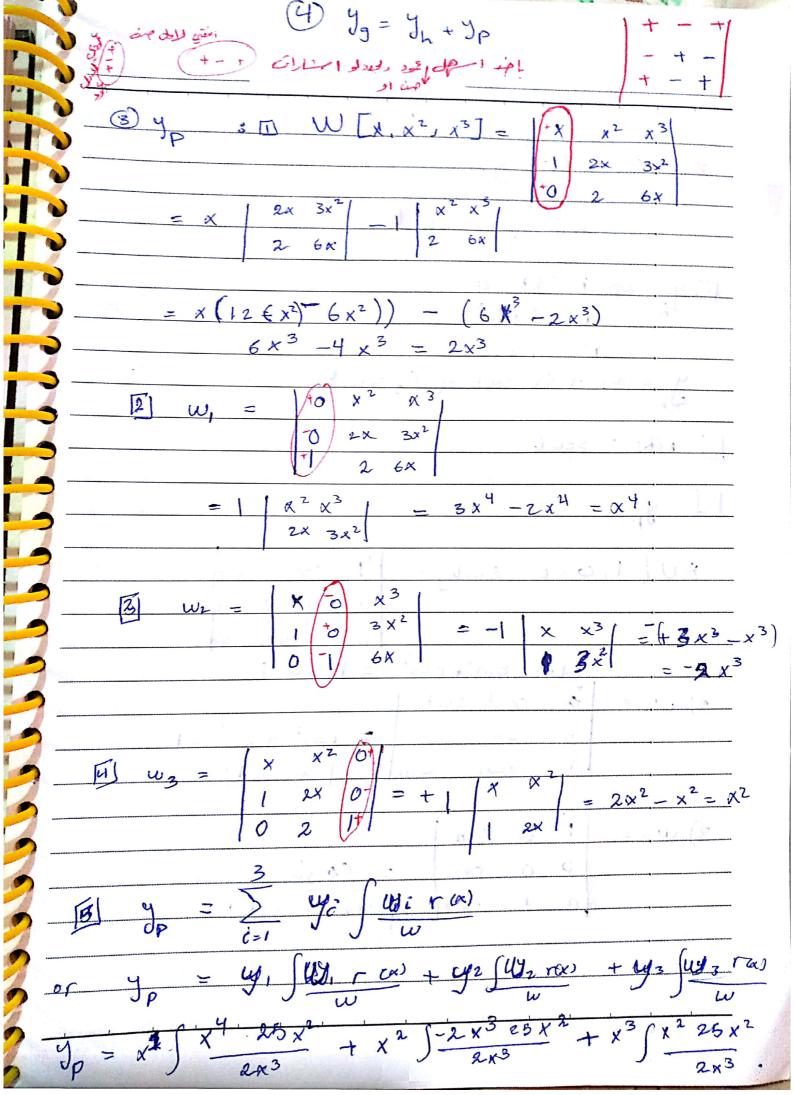
- cyl + cyl + C3 3 4
                        = example: y''l -y = 0
                                           [r-1) (r2+4541) =0
                                                r=1 ) r= -1 = VI-4
                                 r=1 , r=-1731 2
                      Jh 7 C1 ex + C2 ex cos V3 x 4 C3 ex sin [3]
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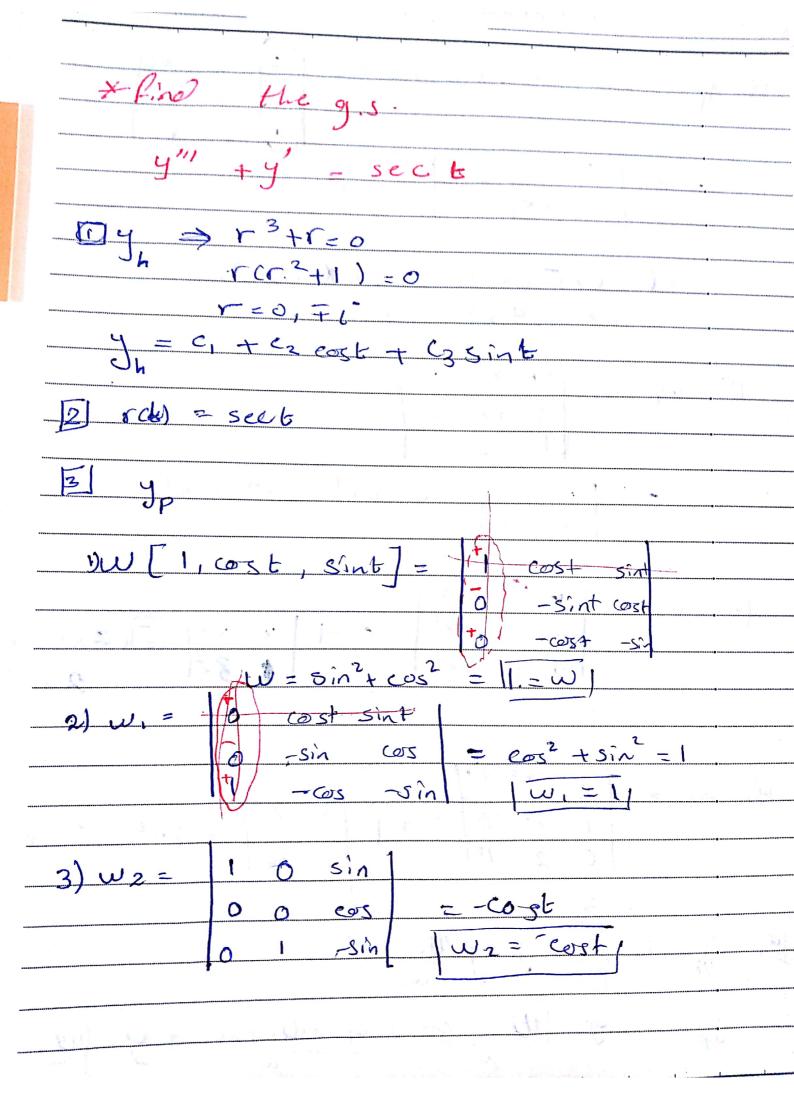
H.w=> y = C1 x + c1x ln x + c3x (hns)²

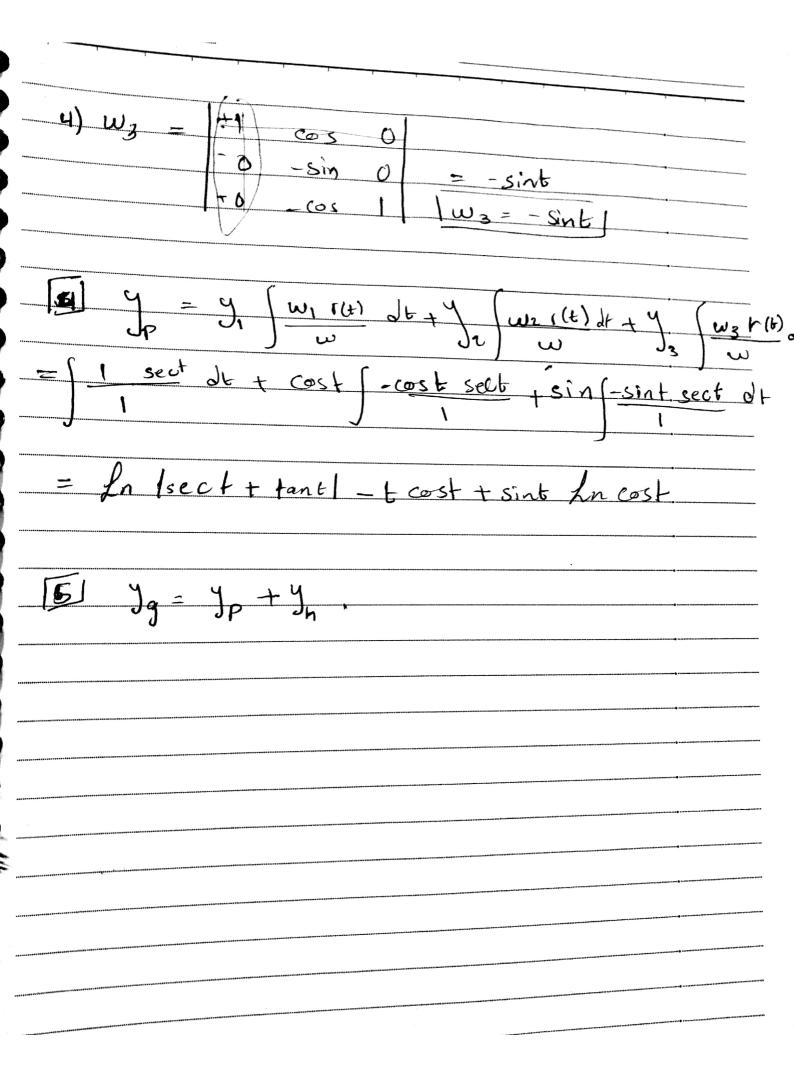
Find O.D. F

sol. x³y^m + xy' - y - 0

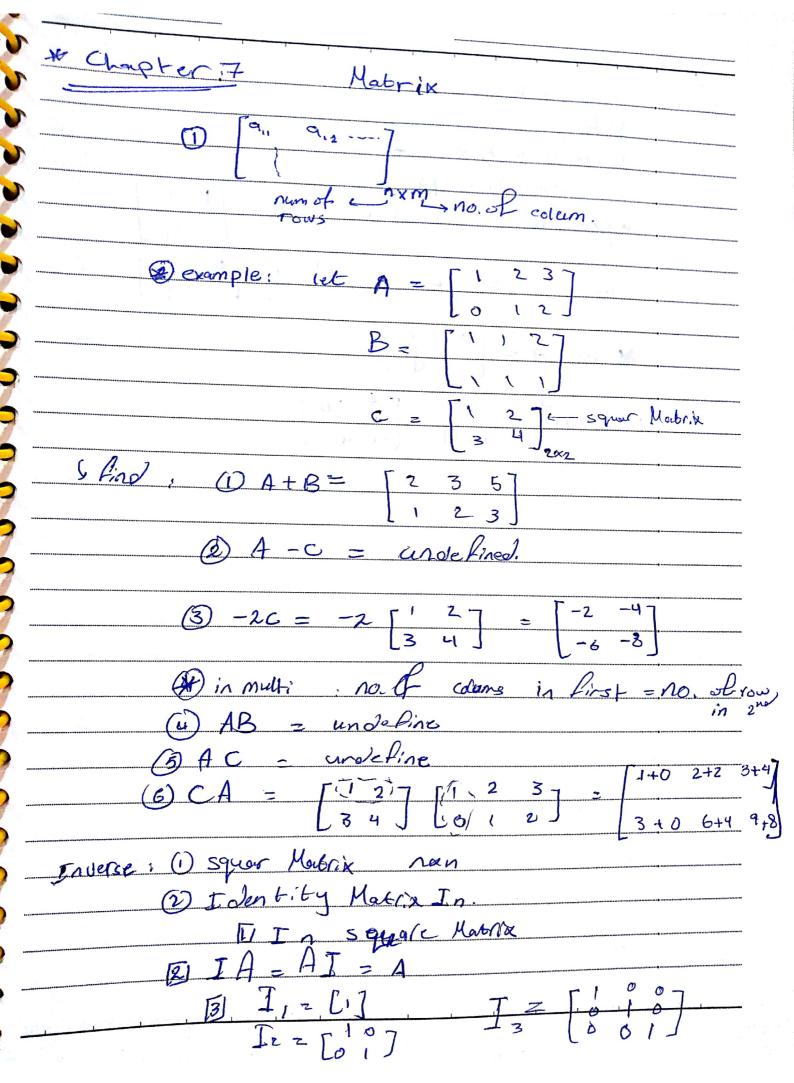


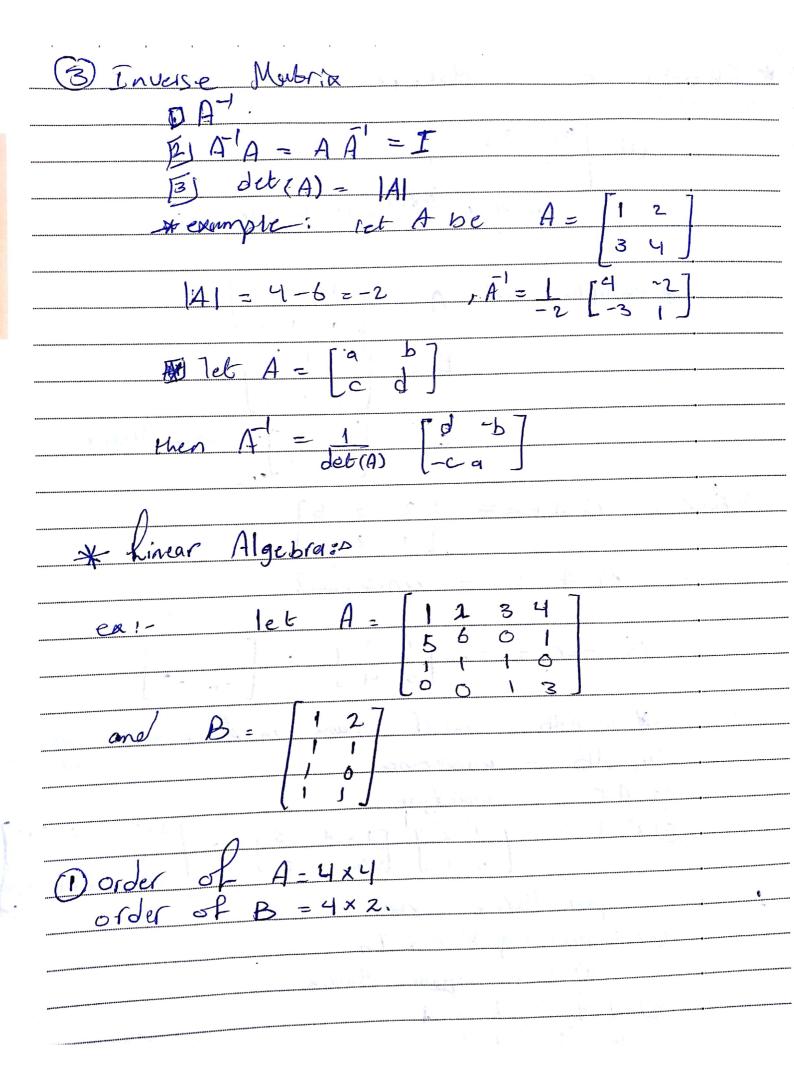


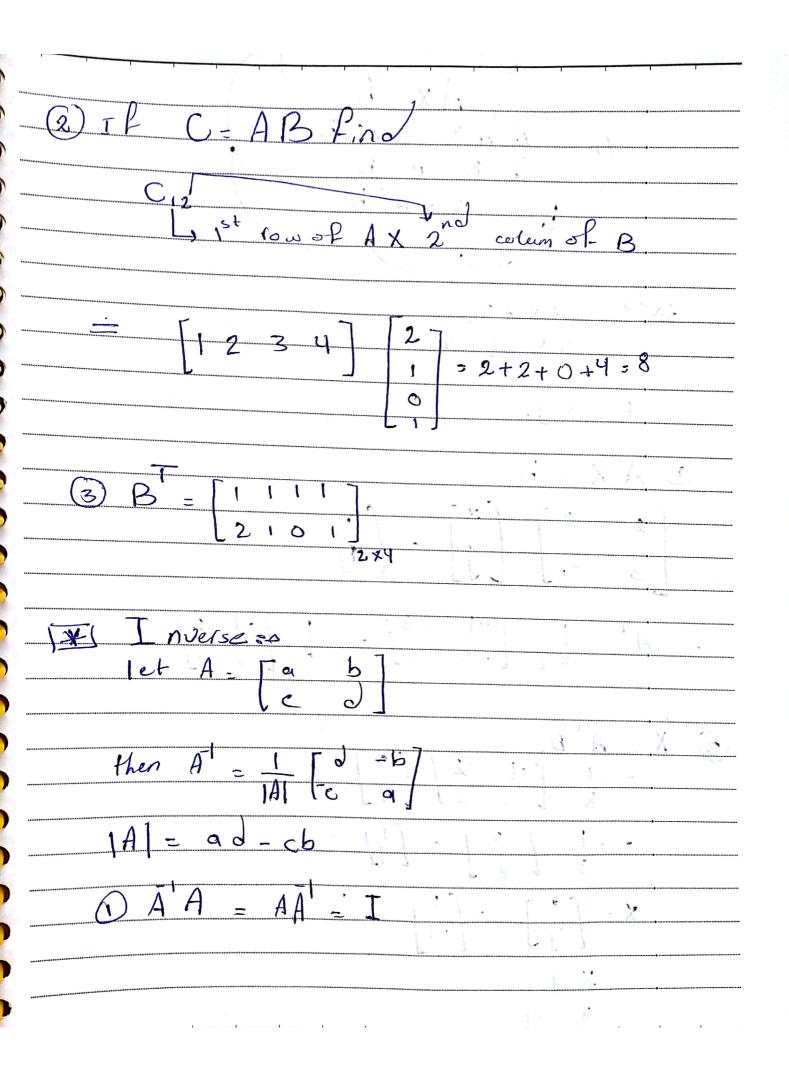


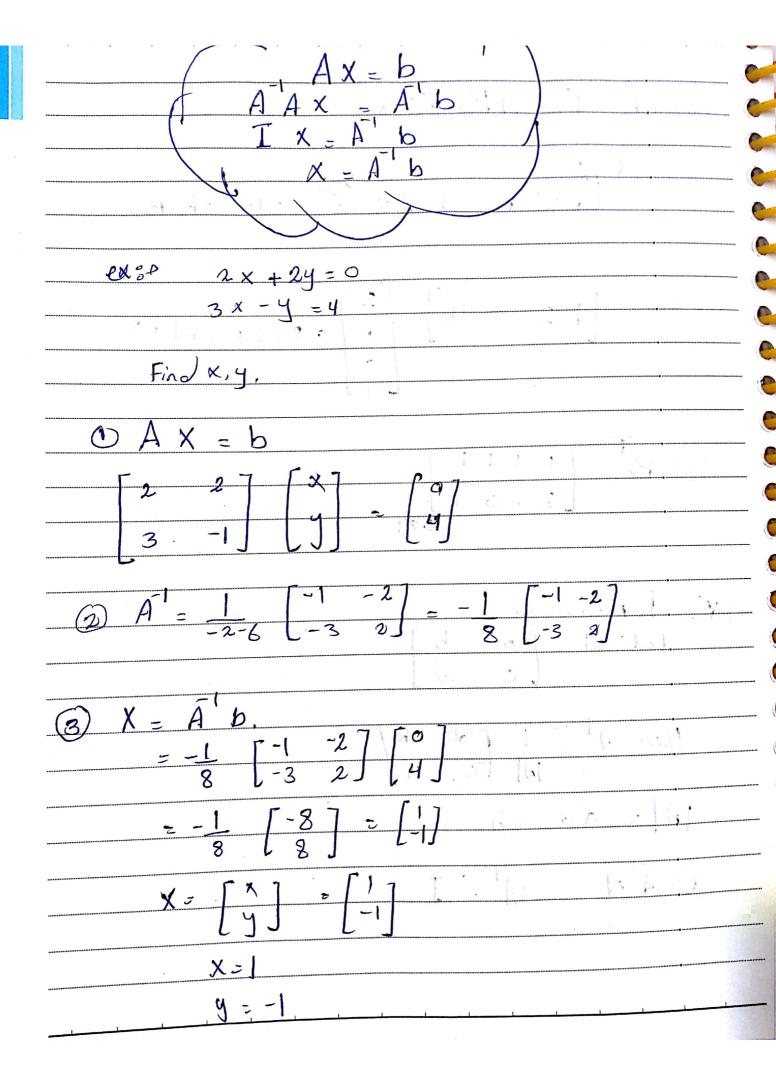


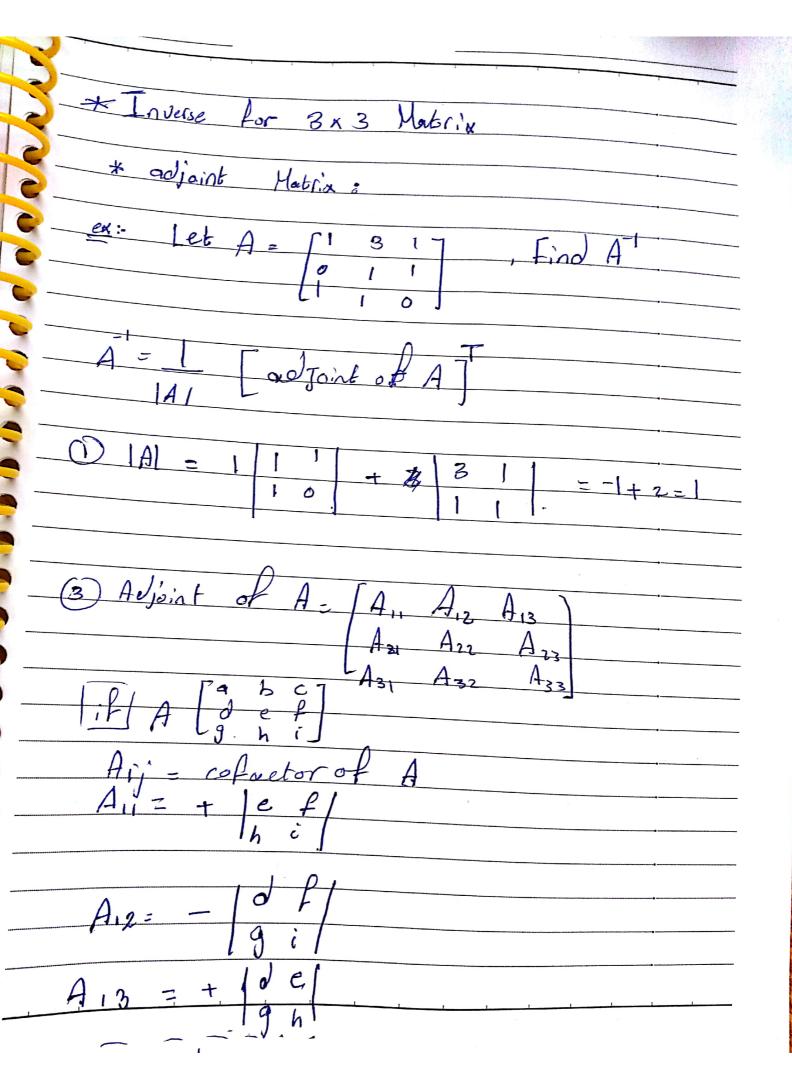
Ever $y''' + y' = 1 + 2x + \cos x \operatorname{Findy} ?$	•
	-
(i) $y \Rightarrow r^3 + 6 = 0$. $r = 0, + 0$.	-
$\Gamma = 0$, $\mp \dot{c}$	
$\frac{y}{h} = c_1 + c_2 \cos x + c_2 \sin x$	
2) y = (a, x + ax) x + (Acosx + Bsinx)x	
	1
	,

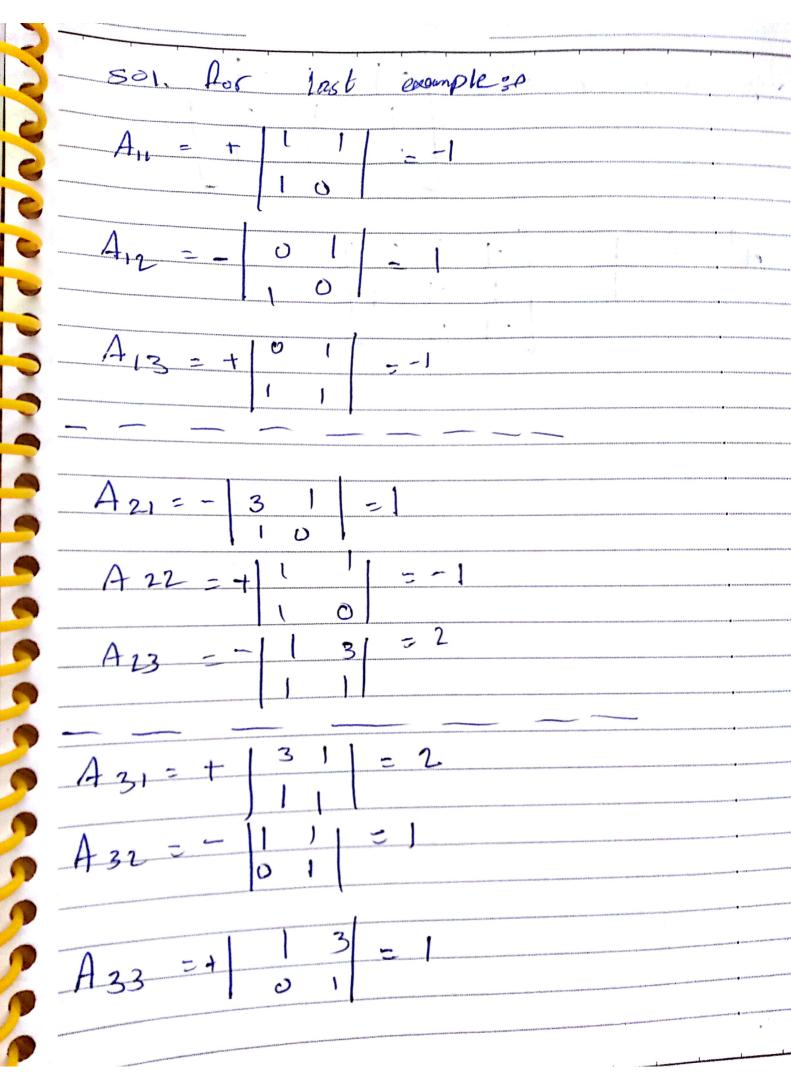


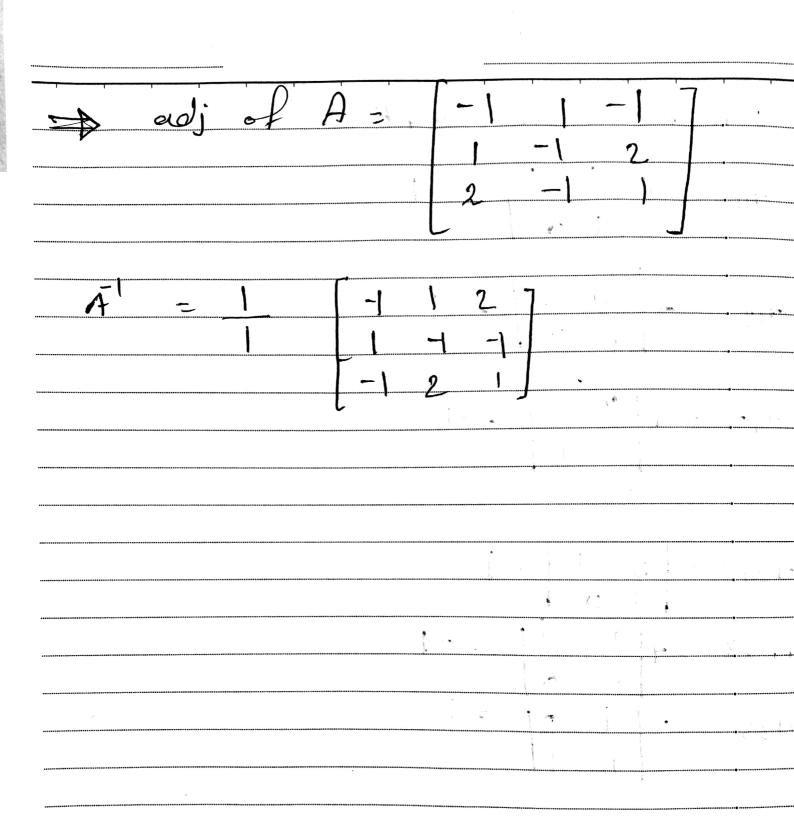


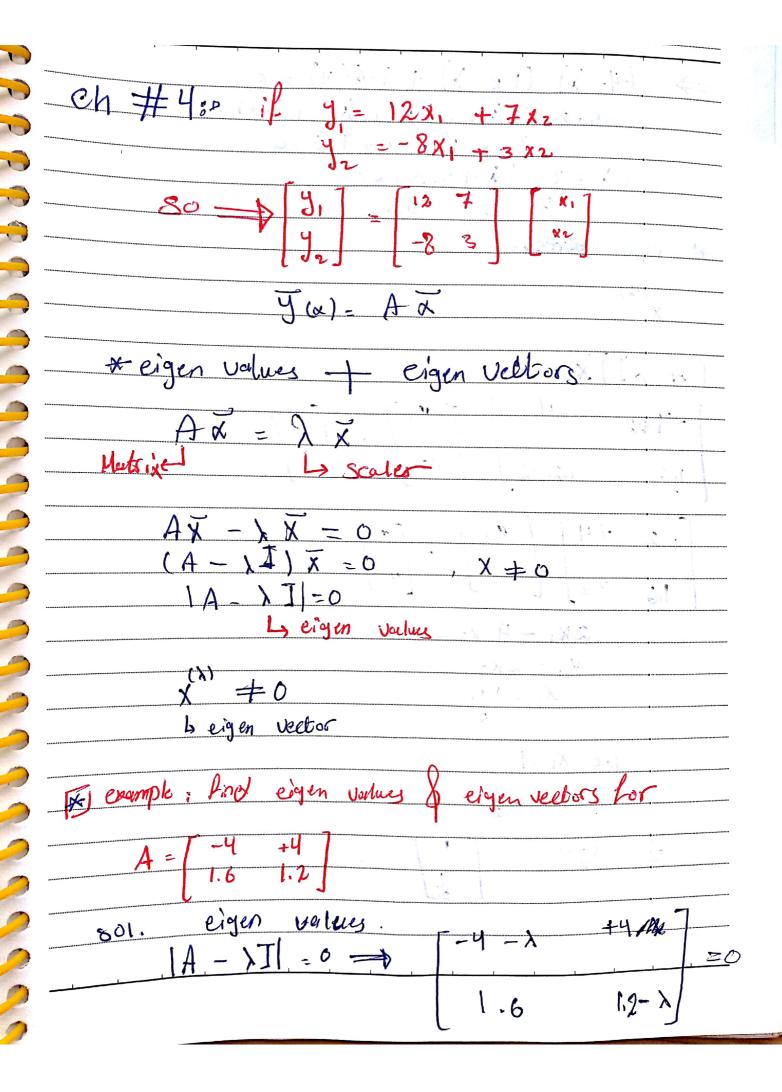




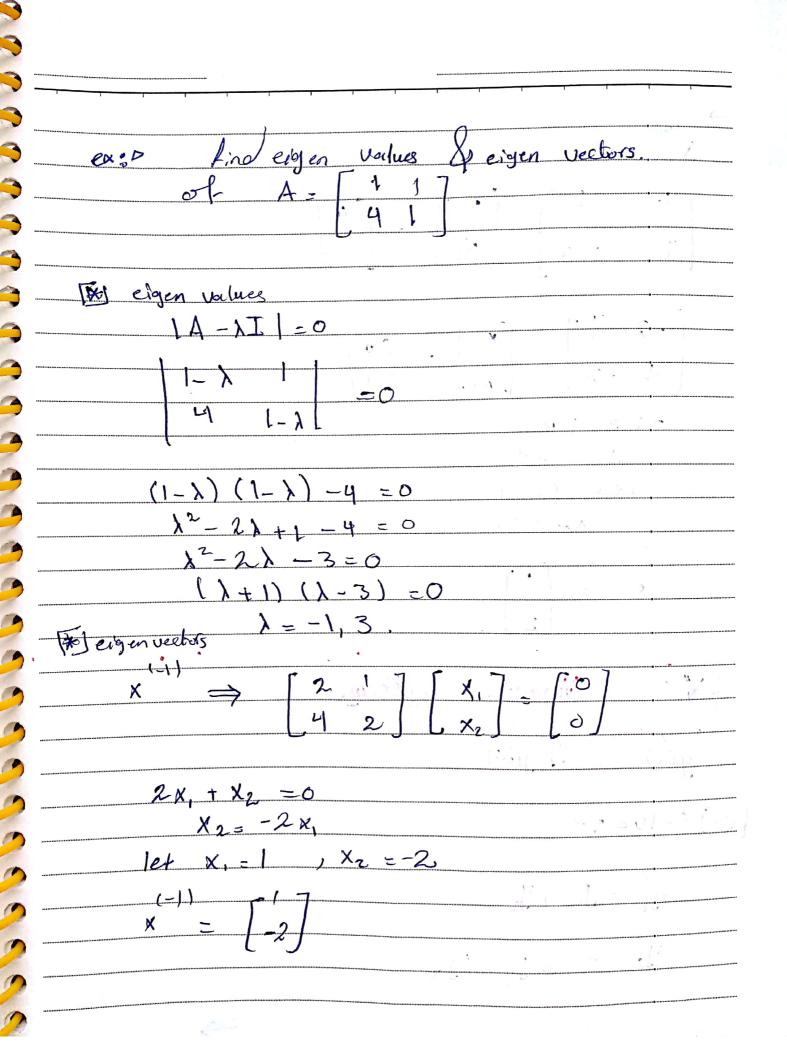


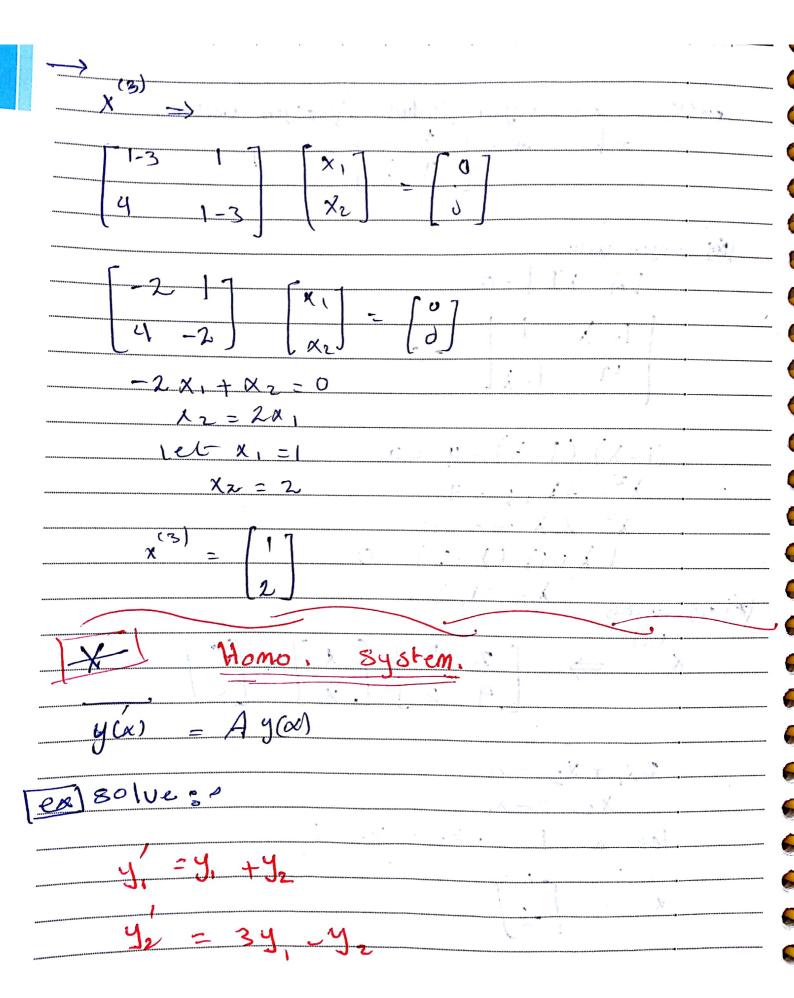


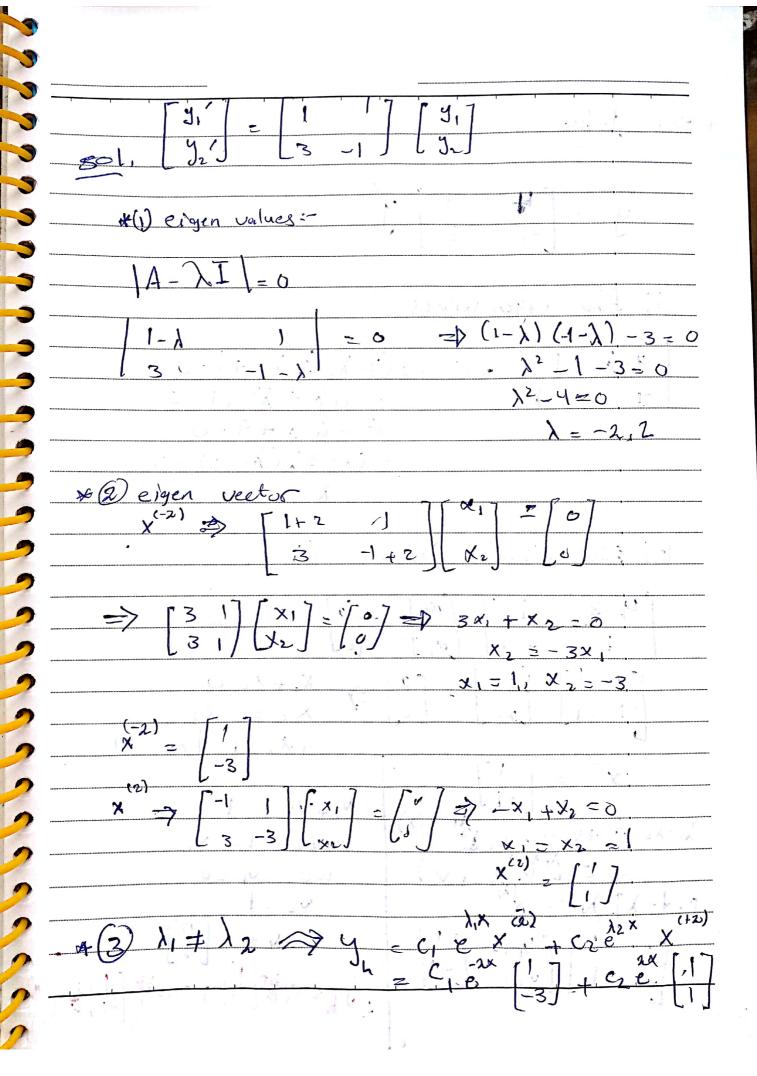


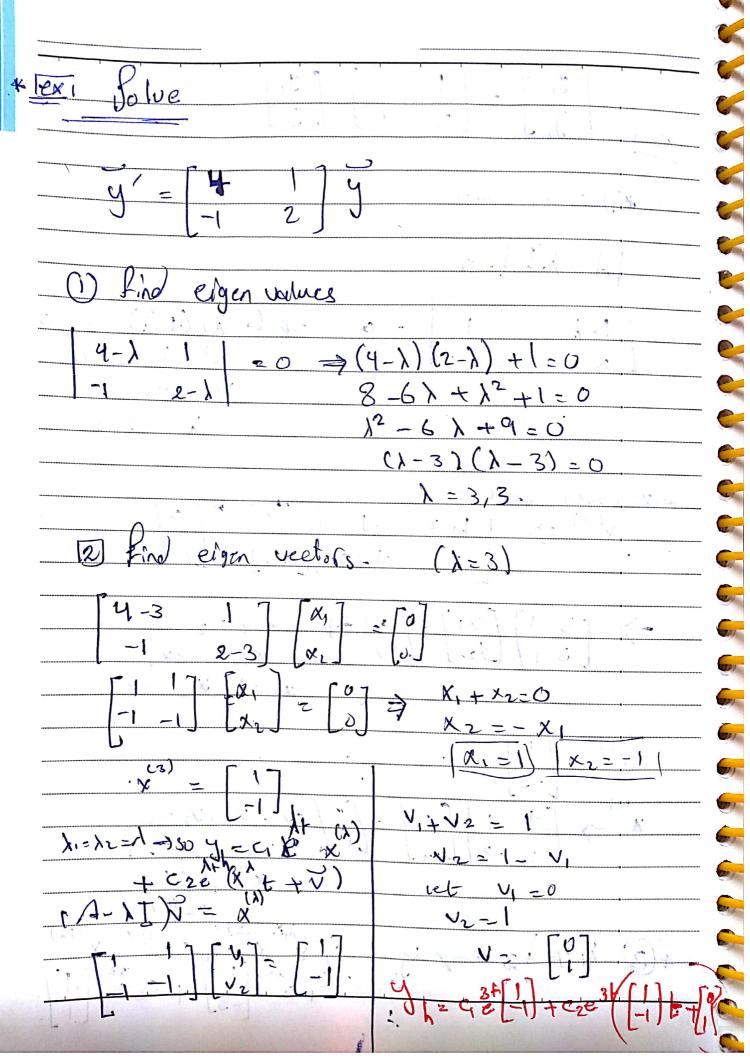


$\Rightarrow (-4-\lambda)(1.2-\lambda)+1.6(4)=0$
-4.8 +4 \ 1.2\ + \ 2 +6.61 =0
$\lambda^2 + 2.8 \lambda + 1.6 = 0$
$(\lambda + 2)(\lambda + 0.8) = 0$
$\lambda = -2, -0.8$
+ eigen valutions
(-2) X
$(A-\lambda I) \times = \delta b \cup A \cup$
[-4] +2 +] [x] = [0]
$\begin{bmatrix} 1.6 & 1.2-2 \end{bmatrix} \begin{bmatrix} x_2 \\ 0 \end{bmatrix}$
$\begin{bmatrix} -2 & +4 \end{bmatrix} \begin{bmatrix} \alpha_1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$ $\begin{bmatrix} 1.6 & 3.2 \end{bmatrix} \begin{bmatrix} \alpha_2 \end{bmatrix} \begin{bmatrix} \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$
$2X_1 - 4X_2 = 0$
$X_1 = 2X_2$
$X_2 = +X_1$
2
1et N1 = 1
$\alpha_2 = -\frac{1}{2}$
So $\begin{pmatrix} -2 \\ \chi \end{pmatrix} = \begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix}$



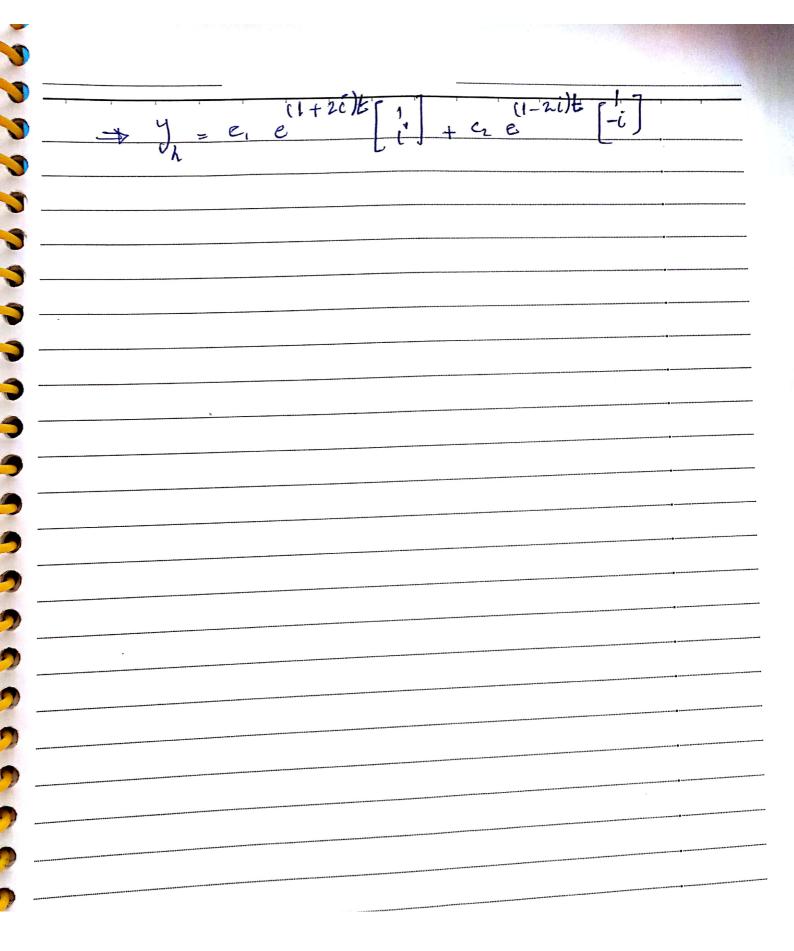


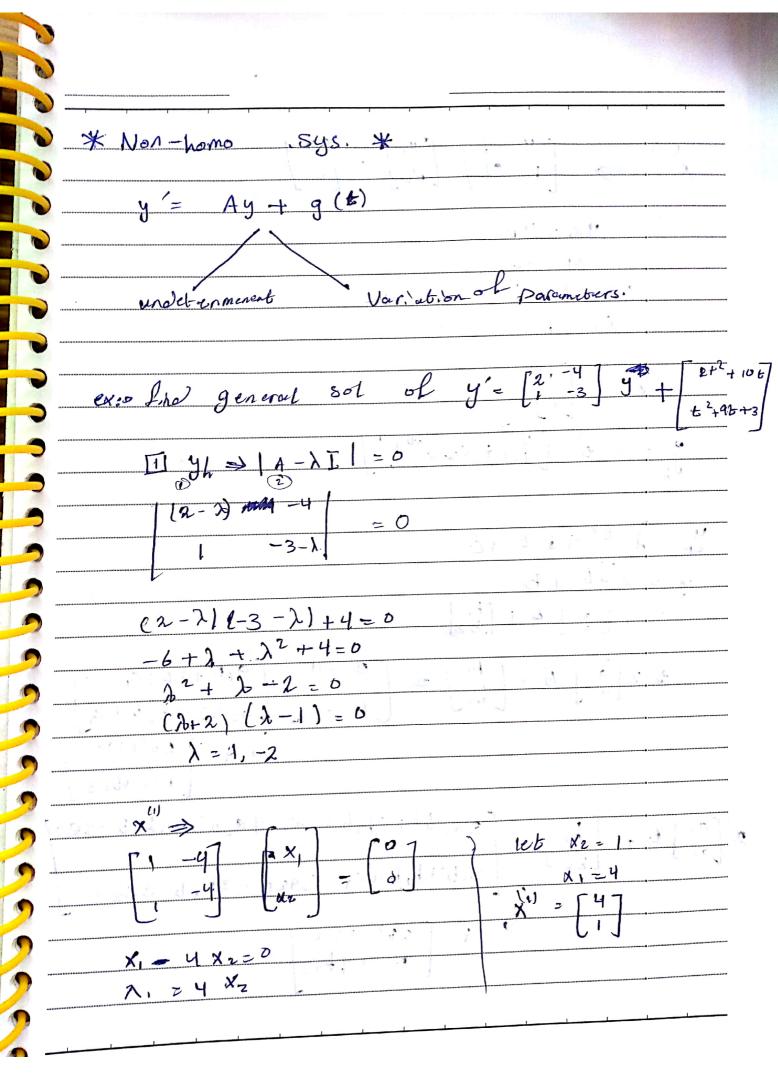




$c_{\alpha}:-y'=\begin{bmatrix}-1\\1\end{bmatrix}$
A
Ofinal eigen values
$\frac{1A - \lambda II = 0}{1 - 1 - \lambda} = 0$
$\frac{\left(-1-\lambda\right)\left(-1-\lambda\right)}{1+2\lambda+\lambda^2+1=0}$
2 0 1 1 1 2 1
@Finel eigen veetors.
[-1-(-1+c)] [x1] [0]
$\begin{bmatrix} -1 & \cdots & -1 - (-1+i\alpha) \end{bmatrix} \begin{bmatrix} x_1 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix}$
$\frac{L}{-c'} \times + \times 2 = 0 \qquad \Rightarrow \qquad x = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
$x^{2} = +i \alpha,$ $ et x_{1} = 1$
X22C

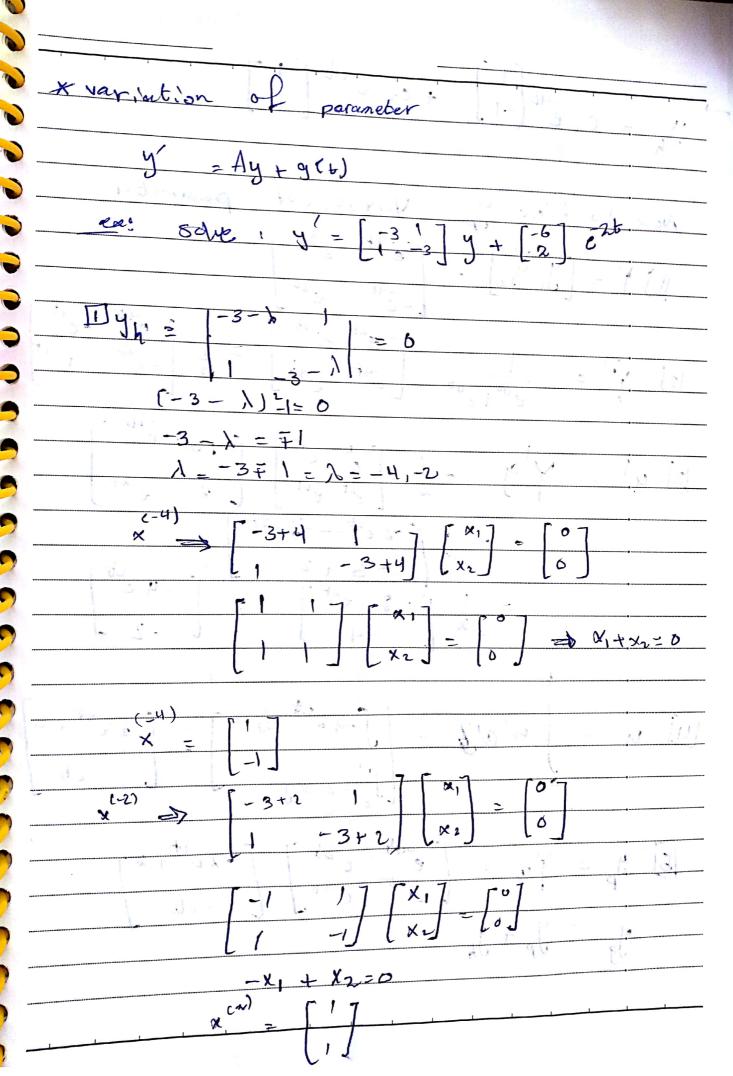
(a+cH)t 1x-iH) (x+cH) 6 (2+cH) + C2 C (-1+c)t Ofind eigen values = ±261 1-(H20) W12 1, 02= ?



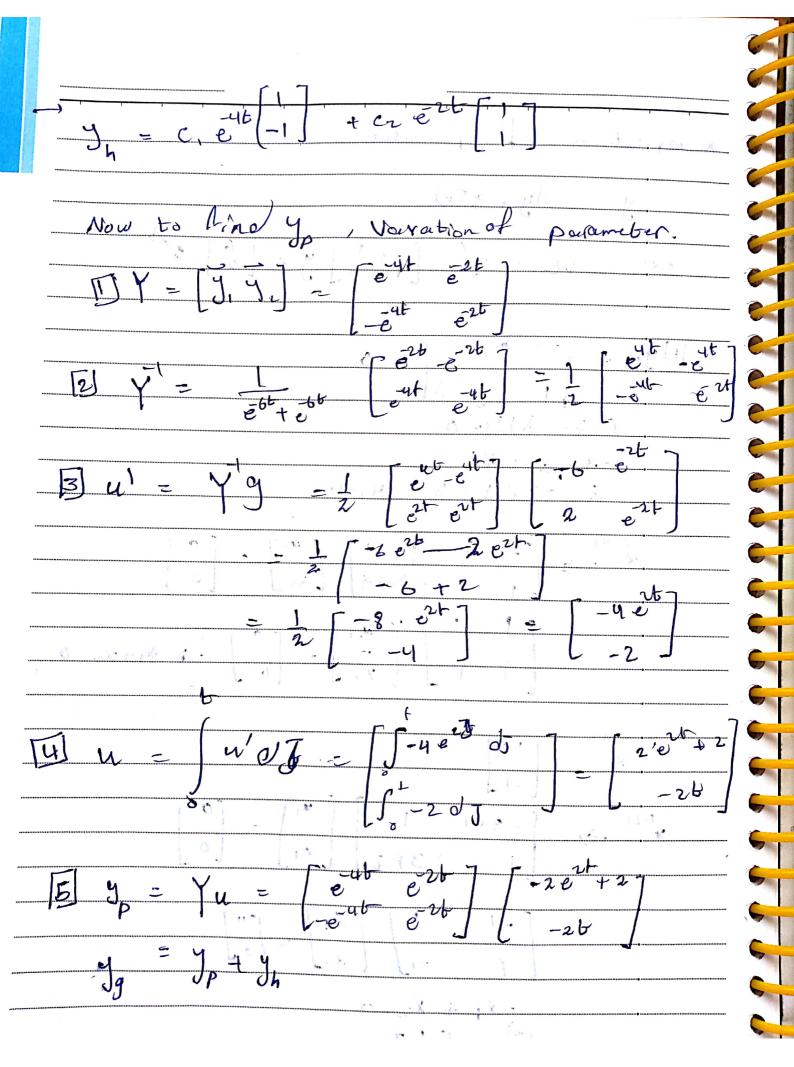


$\Rightarrow 2 \begin{bmatrix} a_{1} \end{bmatrix} + \begin{bmatrix} b_{1} \end{bmatrix} = \begin{bmatrix} 2a_{1} - 4a_{1} \\ a_{2} \end{bmatrix} + \begin{bmatrix} b_{2} \end{bmatrix} = \begin{bmatrix} a_{1} - 3a_{2} \\ a_{1} - 3a_{2} \end{bmatrix}$	$\begin{bmatrix} 2b_1 - 4b_2 \\ b_1 - 3b_2 \end{bmatrix} + \begin{bmatrix} 2c_1 - 4b_2 \\ b_1 - 3b_2 \end{bmatrix} + \begin{bmatrix} 2c_1 - 4b_2 \\ c_1 - 3b_2 \end{bmatrix}$
2 a, - 4a2 = -2	
$-2(a_1-3a_2=-1)$	7 b1 = 0 = 2 c1 - 4 c2
$2a_2=0 \Rightarrow a_2=0$	b2=3=c1-3c2+3
9,	-2(0=c1-3c2)
Λ 'Γ.1.7	0 = 2 C2 = C2 C
$A = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$	C1 = 3 C2 = 0
* 2a, = 2b-4b2+10	2 = [3]
$-2-10=2b_1-4b_2=-12$	y = [-] 1-2 + [0] t
2012 = b1 = 3b2 +9	Jp 13 J
4	Ja = y'p + 4.
$-2(b_1-3b_2=-9)$	
2 b2 = 6	
[b2=3]	,
b, = 9+9=6 -> [b,=0]	constitution of the second
18= 10	
L J	
	0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	1 /

* Find the form of you Solve * example =>) = V1



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Past. Perper THE EXAMPLES corres ponding eign vector X,=0

