POWER UNIT

DEYA' ABUL-NADI
CIRCUITS I

MOHAMMED ZIAD

POWER UNIT-JU
http : // POWER UNIT-JU.COM

CH1:

- Charge:-

$Q, q, q(t)$

coulomb (c).

- current:

$I, i, i(t)$

$$\boxed{I = \frac{dq}{dt}}$$

amperes, A

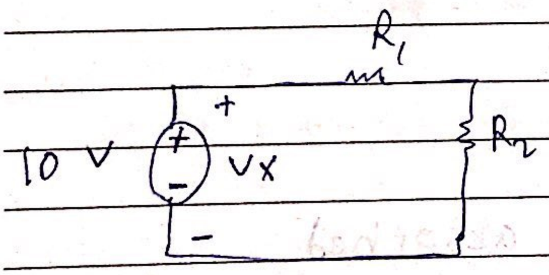
- Voltage (potintial difference):-

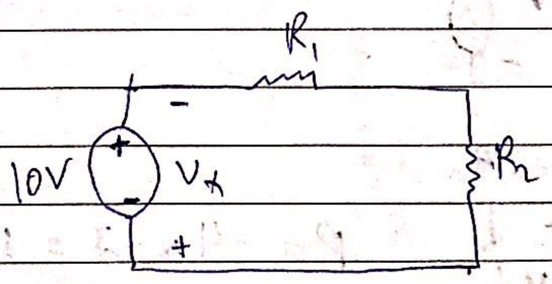
Vab: the energy or work needed to move a unit charg from a to b

$\Delta V = \frac{\Delta W}{\Delta q}$

$$\boxed{V = \frac{dw}{dq}}$$

w: work (energy)





$Vx = 10\ V$ $\qquad$ $Vx = -10\ V$

* Power and Energy :-

P, p, $p(t) \rightarrow$ watt (W)

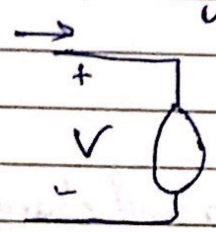
$$\boxed{p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt}} \qquad \boxed{p = v\,i}$$

$$w = \int_{t_0}^{t} p \cdot dt$$

$$\boxed{p(t) = v(t) \cdot i(t)}$$ instantances Power

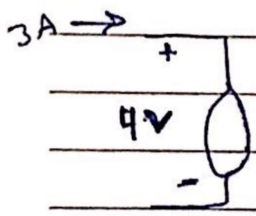
Absorbed or generated power

passive Sign convention



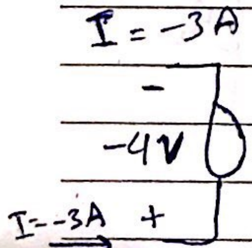
$p = i \cdot v$ watt absorbed

Ex $I = 3\,A$

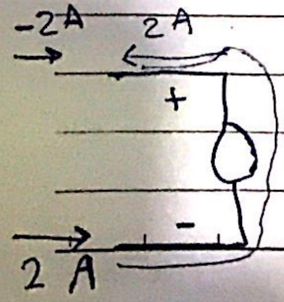


$p = 4 \times 3 = 12$ w absorbed

$I = -3\,A$



$p = -4 \times -3 = 12$ w absorbed



$p = 5 \times -2 = -10$ w absorbed

$= 10$ w generated

Total absorbed = Total genration

energy (work)

$W, w, w(t) \longrightarrow$ jouls , J , KWh , Ws

$w = \int_{t_0}^{t_1} p \cdot dt \qquad w = p\,(t_1 - t_0)$

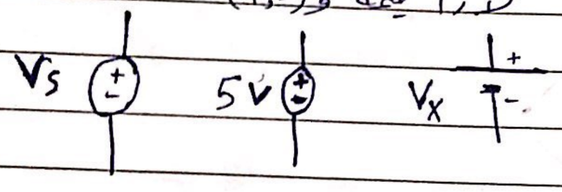
ثابت

Circuit Elements:-

1) Voltage Sources

a) independant voltage sources

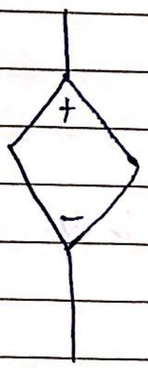
لازم قيمة و (+,-)



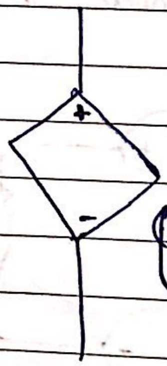
I ↑ or I ↓

التيار ممكن عكسه

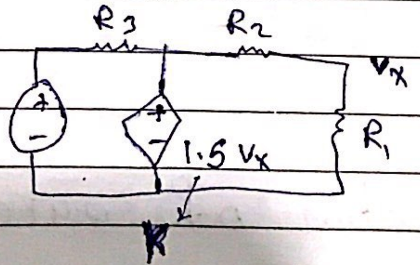
b) dependant voltage source:



$K\,V_x$ (Voltage controlled)
d V S



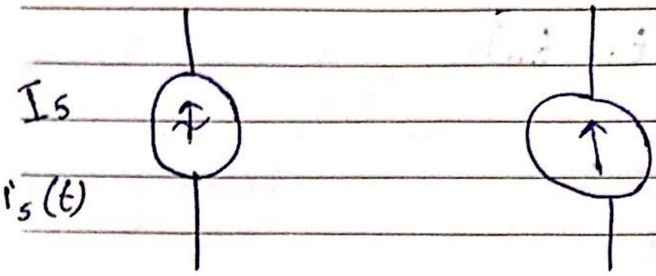
$r\,i_x$
(current controled)
d V S



$V_x \rightarrow$ موجودة أي مكان بالدائرة

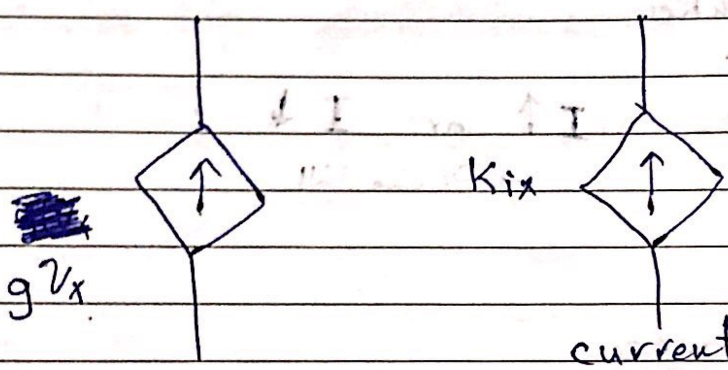
2) Curent source

a) independent current sources :-



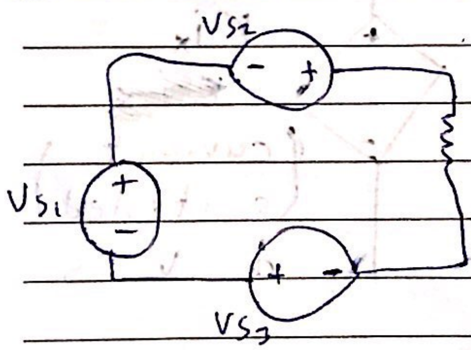
فرق الجهد لا يهم

b) Dependent current source :-

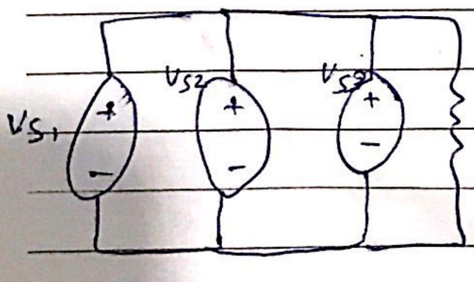


current controlled D C S

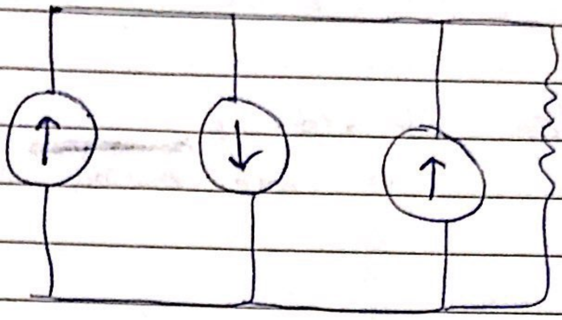
Voltage controlled D C S



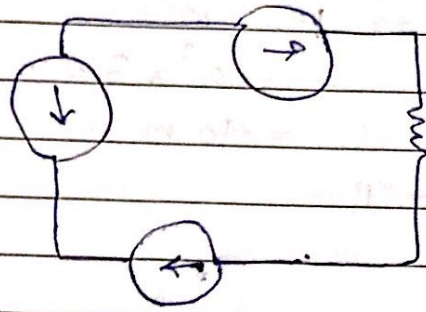
series conection is valid



parallel conection of sources is invalid



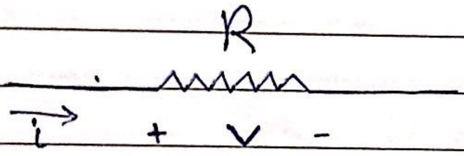
current sources in parallel is valid



current sources in series is invalid

CH2:

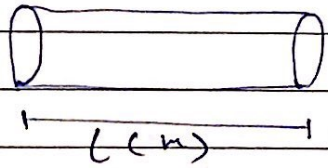
* Ohm's Law :-



$v \propto i$

$$\boxed{v = R i}$$

A: cross sectional area (m^2) R: resistance of the conductor (Ω)



ρ: resistivity ($\Omega \cdot m$)

$$\boxed{R = \frac{\rho L}{A}} \rightarrow R = \frac{L}{\sigma A} \qquad * \frac{1}{\sigma} = \rho$$

نوعية

موصلية

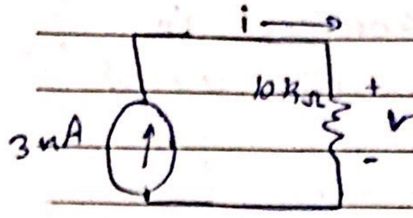
$$\boxed{p = i v = i^2 R = \frac{v^2}{R}}$$

$\left(\frac{1}{\Omega} = \text{siemens} = \Omega^{-1} \right)$

• The Conductance (G)

$G = \frac{1}{R} = \frac{i}{v}$ (siemens, S) $\rightarrow$ (mhos $\mho$)

practice problem 2.2



calculate v, G, p absorbed by $10k\Omega$

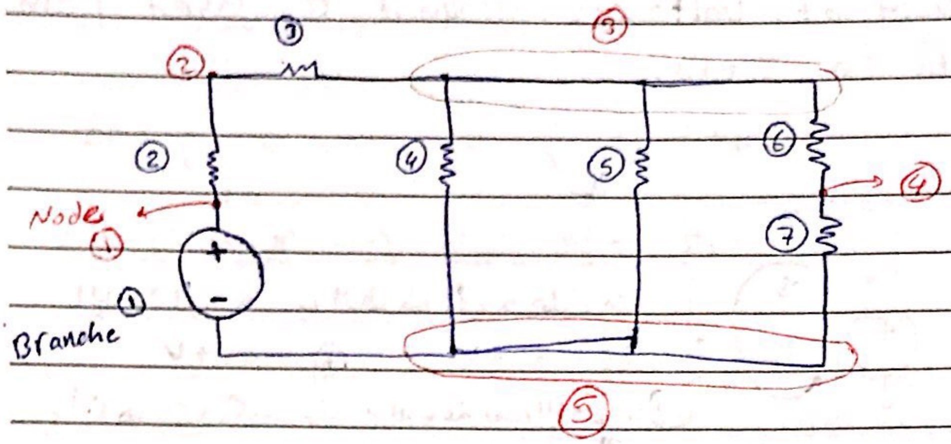
Solu.

$v = iR$
$= 3\times10^{-3} \times 10\times10^{3}$
$= 30$

$G = \frac{1}{R}$
$= \frac{1}{10\times10^{3}}$
$= 10^{-4}\ S = 100\mu S$

$p = iv$
$= 3\times10^{-3} \times 30$
$= 90\ mW$

• Nodes, Branches, paths and loops:-

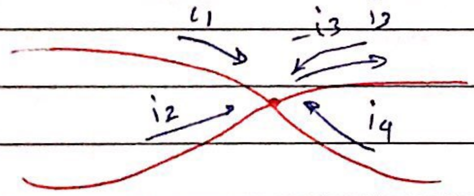


path → مسار من نقطة إلى أي نقطة أخرى

loop → closed path

* Kirchhoff's current Law (KCL) :-

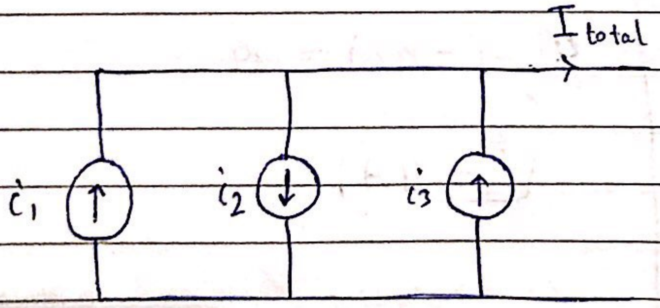
the algebraic sum of current entering a node is equal 0



$i_1 + i_2 - i_3 + i_4 = 0$

$i_1 + i_2 + i_4 = i_3$

التيارات الداخلة = التيارات الخارجة

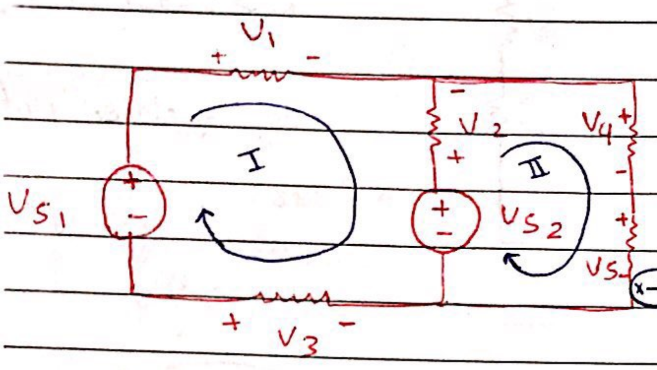


$i_1 - i_2 + i_3 - I_T = 0$

$I_T = i_1 + i_3 - i_2$

Kirchhoff's voltage law (KVL) :-

The algebraic sum of voltages around a closed path (loop) is equal to zero.



I

$-V_{S1} + V_1 - V_2 + V_{S2} - V_3 = 0$

الإشارة حسب الطرف الي دخلت منه

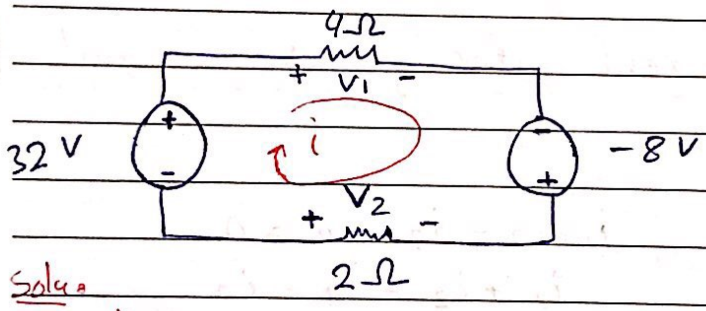
$\ominus \rightarrow -V$ $\quad\quad$ $\oplus \rightarrow +V$

واذا مشيت عكس عقارب الساعة نفس الإشارة بس (x-1)

II

$V_4 + V_S - V_{S2} + V_2 = 0$

Practice problem 2.5 page 41 :-



Find V_1 and V_2 ?

Solu.

apply KVL

$-32 + V_1 - (-8) - V_2 = 0$

$V_1 - V_2 = 24$ ---- ①

by ohms law

$V_1 = 4i$

$V_2 = -2i$

$4i - (-2i) = 24$

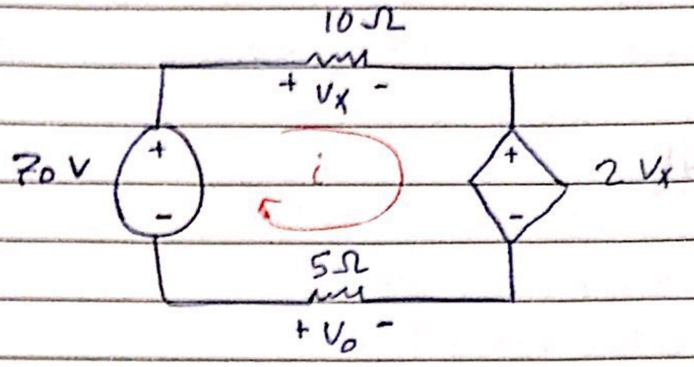
$6i = 24$

$\boxed{i = 4A}$

$V_1 = 4 \times 4 = 16\ V$

$V_2 = -2 \times 4 = -8\ V$

practice 2.6 :-



Find V_x and V_o ?

Solu:

by KVL

$-70 + V_x + 2V_x - V_o = 0$

$3V_x - V_o = 70$ --- ①

by ohm's Law

$V_x = 10i$ $\quad$ $V_o = -5i$

$3(10i) - (-5i) = 70$

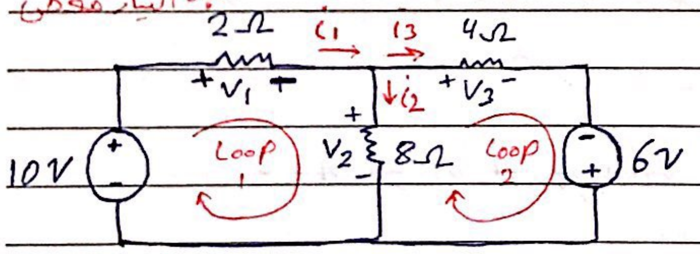
$35i = 70 \longrightarrow \boxed{i = 2A}$

$\therefore V_x = 20\ V$

$V_o = -10\ V$

practice 2.8 Page 43

اتجاه التيار معطى



Find $V_1, V_2, V_3, i_1, i_2, i_3$?

by KCL

$i_1 = i_2 + i_3$ --- $\boxed{1}$

Apply KVL for Loop 1

$-10 + V_1 + V_2 = 0$ --- $\boxed{2}$

Apply KVL for loop 2

$-6 - V_2 + V_3 = 0$ --- $\boxed{3}$

$V_1 = 2i_1$

$V_2 = 8i_2$

$V_3 = 4i_3$

from ②

$-10 + 2i_1 + 8i_2 = 0$

$2i_1 + 8i_2 = 10$

$2(i_2 + i_3) + 8i_2 = 10$

$5i_2 + i_3 = 5$ --- (A)

from ③

$-6 - 8i_2 + 4i_3 = 0$

$-8i_2 + 4i_3 = 6$ --- (B)

A + B

$-8i_2 + 4i_3 = 6$

$+ \quad (5i_2 + i_3 = 5) \times -4$

$-28 i_2 = -14 \longrightarrow i_2 = \frac{-14}{-28}$

$\boxed{i_2 = 0.5\ A}$

$5(0.5) + i_3 = 5$

$\boxed{i_3 = 2.5\ A}$

$\boxed{i_1 = 3A}$

$V_1 = 2 \times 3 = 6\ V$

$V_2 = 8(0.5) = 4V$

$V_3 = 4(2.5) = 10\ V$

للتأكد من الحل :-

$P_{10V} = 10 \times 3 = 30\ w$ generated

$P_{6V} = 6(2.5) = 15w$ "

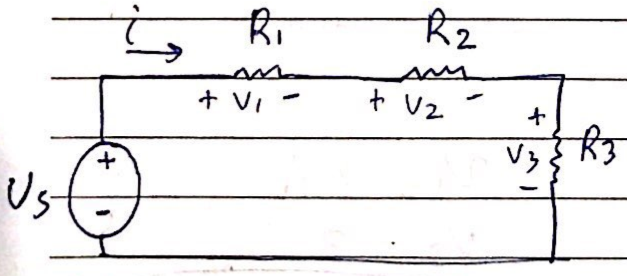
$P_{2\Omega} = 3^2 \times 2 = 18\ w$ absorbed

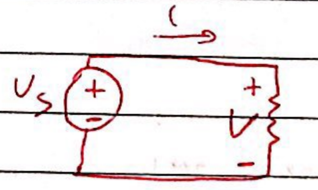
$P_{8\Omega} = (0.5)^2 \times 8 = 2w$ "

$P_{4\Omega} = (2.5)^2 \times 4 = 25\ w$ "

generated = absorbed ∴ ✓

2.5 Series resistors and voltage division :-





Req $(R_1 + R_2 + R_3)$

$-V_s + V = 0$

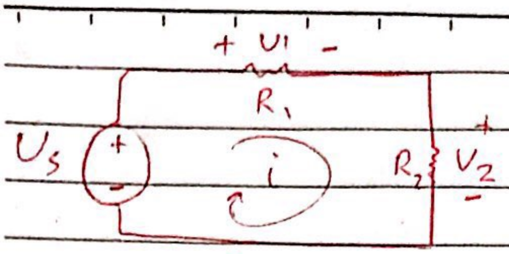
$V_s = i R_{eq}$

$-V_s + V_1 + V_2 + V_3 = 0$

$V_s = iR_1 + iR_2 + iR_3$

$V_s = i(R_1 + R_2 + R_3)$

$R_{eq} = R_1 + R_2 + \dots R_N$ (series Resistors)



$$i = \frac{V_s}{R_1 + R_2}$$

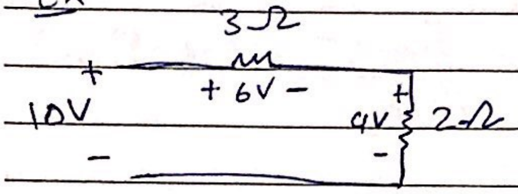
$V_1 = i R_1$
$V_2 = i R_2$ } $\rightarrow V_s = V_1 + V_2$
$V_s = i(R_1 + R_2)$

$$v_1 = \frac{R_1}{R_1 + R_2} \times v_s$$

$$v_2 = \frac{R_2}{R_1 + R_2} v_s$$

} Voltage Division

Ex

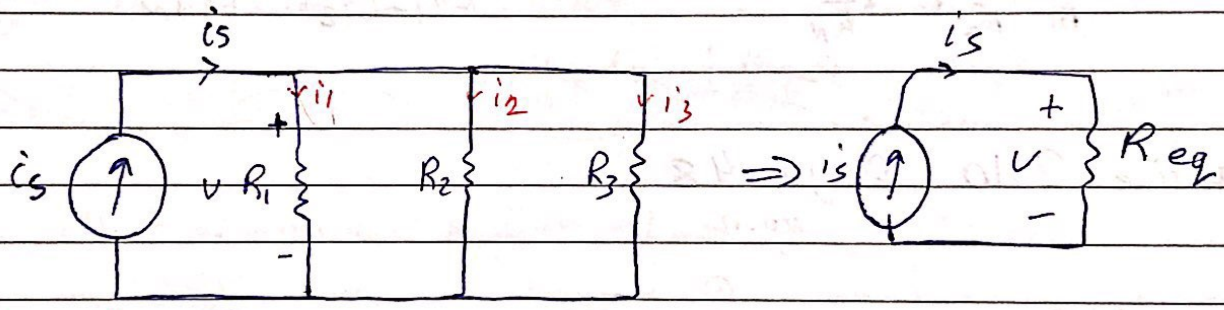


$$\frac{3}{3+2} \times 10 = 6V$$

$$\frac{2}{3+2} \times 10 = 4V$$

(+) } 10V

2.6 parallel Resistors and current Division :-



by KCL :

$i_s = i_1 + i_2 + i_3$

$$i_s = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$i_s = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$i_s = \frac{V}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$ ← أصغر من أصغر مقاومة

$I_1 = \frac{V}{R_1}$ $\qquad V = I_1 R_1 = I_2 R_2$

$I_2 = \frac{V}{R_2}$

$$V = I_s \underbrace{\left(\frac{R_1 R_2}{R_1 + R_2}\right)}_{R_{eq}} \rightarrow I_1 \cancel{R_1} = I_s \left(\frac{\cancel{R_1} R_2}{R_1 + R_2}\right), \quad I_2 \cancel{R_2} = I_s \left(\frac{R_1 \cancel{R_2}}{R_1 + R_2}\right)$$

$\Downarrow$

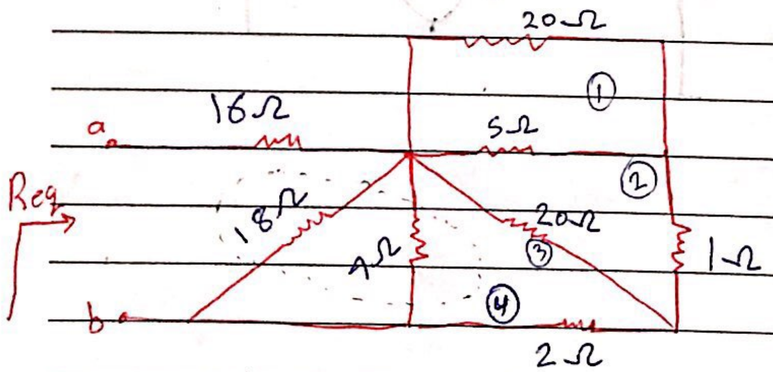
$$I_1 = \frac{R_2}{R_1 + R_2} I_s \quad , \quad I_2 = \frac{R_1}{R_1 + R_2} I_s$$

current Division rule :-

$$I_K = \frac{\frac{1}{R_K}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}} \cdot I_s \quad \text{or} \quad I_K = \frac{G_K}{G_1 + G_2 + \cdots + G_N} \cdot I_s$$

بيوصلوني لـ I_1, I_2 الي فوق.

practice 2.10 page 48



① $\frac{20 \times 5}{25} = 4\,\Omega$ $\qquad$ ③ $\frac{1}{5} + \frac{1}{20} = \frac{1}{4} \Rightarrow 4\,\Omega$ for parallel $\qquad \frac{1}{R_{eq}} = \frac{1}{18} + \frac{1}{4} + \frac{1}{6}$

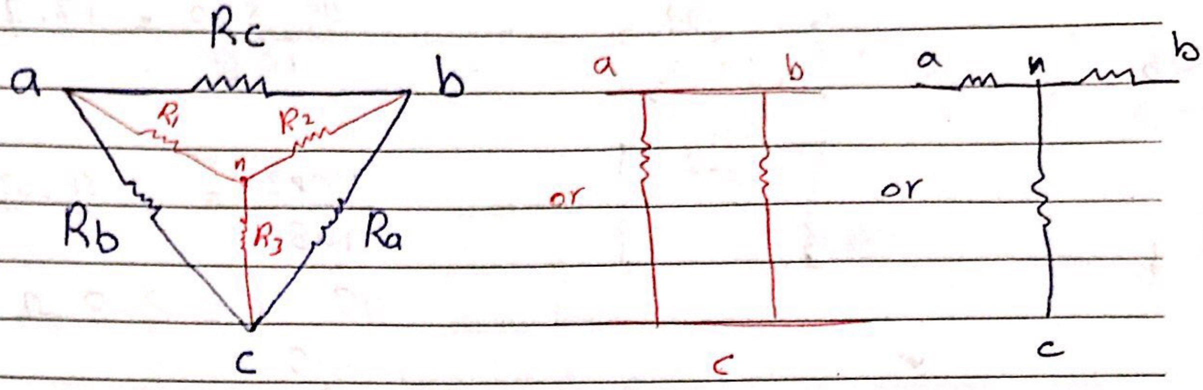
② $4 + 1 = 5\,\Omega$ $\qquad$ ④ $4 + 2 = 6\,\Omega$ $\qquad \frac{1}{R_{eq}} = \frac{1+2+3}{18} = \frac{6}{18} = \frac{1}{3}$

$R_{eq} = 3\,\Omega$

$\boxed{R_{eq} = 3 + 16 = 19\,\Omega}$

* Wye - Delta Transformation :-

T, π, Y



Delta to wye conversion.

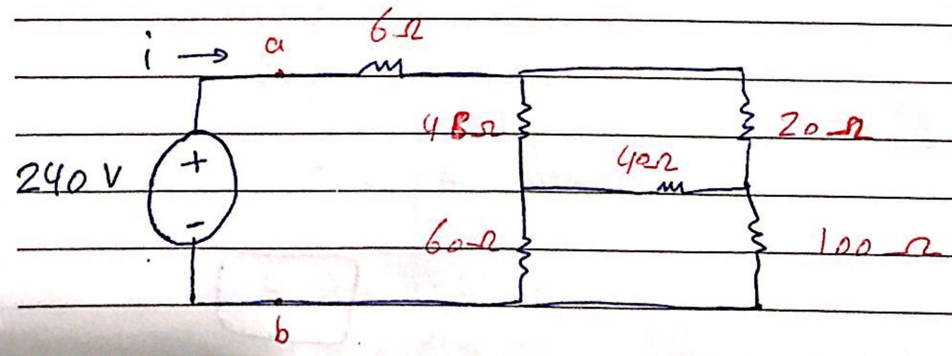
$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$, $R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$, $R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$

wye to Delta conversion :-

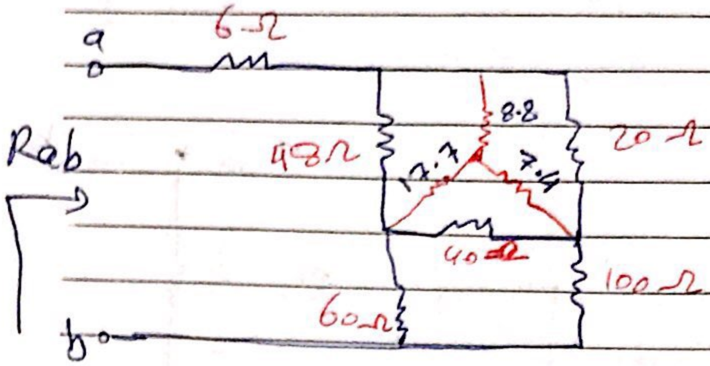
$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$, $R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$, $R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$

دائماً بالمقام بأخذ المقاومة البعيدة الي مش مشتركة بالـ Node

practice 2.15 page 57



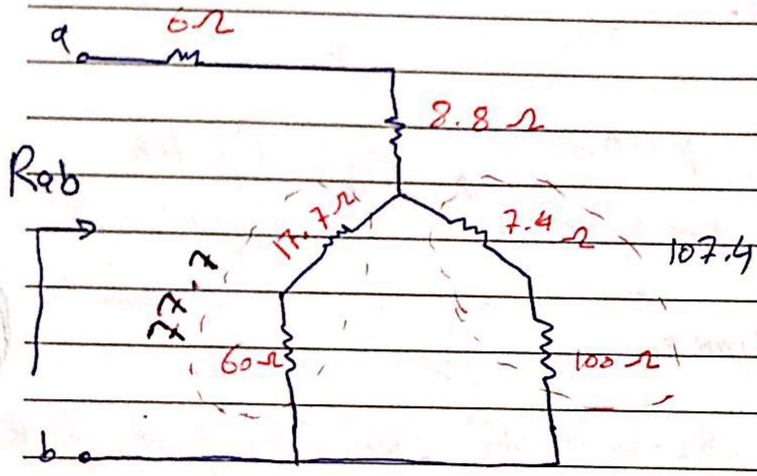
Find R_{ab} , i ?



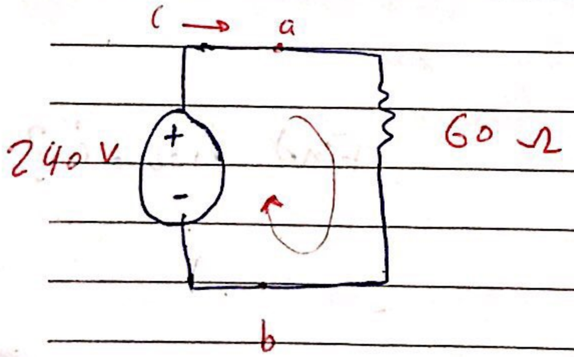
$$\frac{48 \times 40}{108} = 17.7\ \Omega$$

$$\frac{40 \times 20}{108} = 7.4\ \Omega$$

$$\frac{48 \times 20}{108} = 8.8\ \Omega$$



$$R_{ab} = \left(\frac{77.7 \times 107.4}{77.7 + 107.4}\right) + 8.8 + 6 = \boxed{60\ \Omega}$$

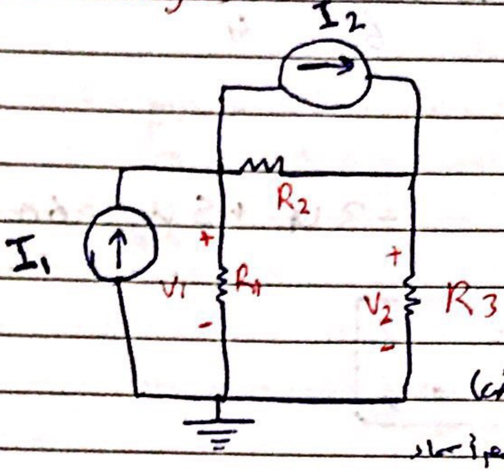


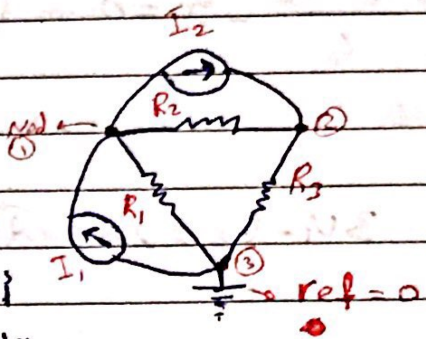
$$-240 + 60i = 0$$

$$i = \frac{240}{60} = \boxed{4\ A}$$

Chapter 3 "Methodes of analysis" :-

*Nodal Analysis





أول خطوة ببسط
الدائرة لأقل عدد Node
و بحدد ref=0 (الأرض)
وباقي الـ Node بعطيهم أسماء

Nod ① → V_1
Nod ② → V_2
Nod ③ → ref = 0

by KCL at Node1 :-

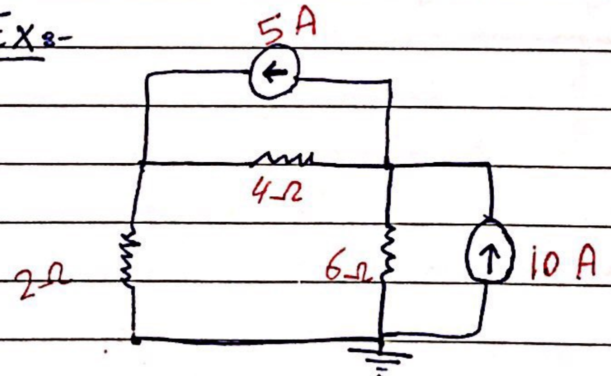
$$-I_1 + \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + I_2 = 0 \quad \ldots \boxed{1}$$

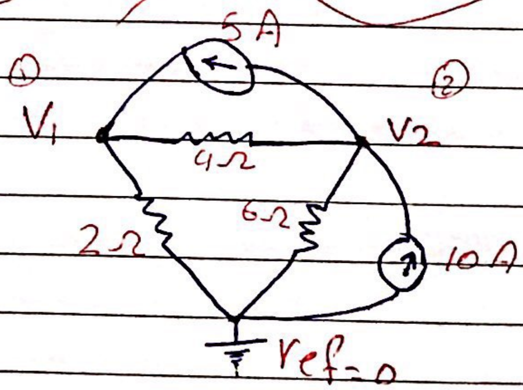
⊖ التيار الي داخل على الـNode1
⊕ " خارج من " "

by KCL at Node 2:-

$$\frac{V_2}{R_3} + \frac{V_2 - V_1}{R_2} - I_2 = 0 \quad \text{——} \boxed{2}$$

Ex:-





إعادة الرسم لتسهيل الحل مش خطوة ضرورية

Find V_1, V_2 :-

Five Apple

Scanned with CamScanner

KcL at Node 1 :-

$$\frac{V_1}{2} + \frac{V_1 - V_2}{4} - 5 = 0 \quad \text{---} (1)$$

(1) ×4 $\;= 2V_1 + V_1 - V_2 - 20 = 0$

$$3V_1 - V_2 = 20 \quad \text{---} (A)$$

KcL at Node 2 :-

$$-10 + \frac{V_2}{6} + \frac{V_2 - V_1}{4} + 5 = 0 \quad \text{---} (2)$$

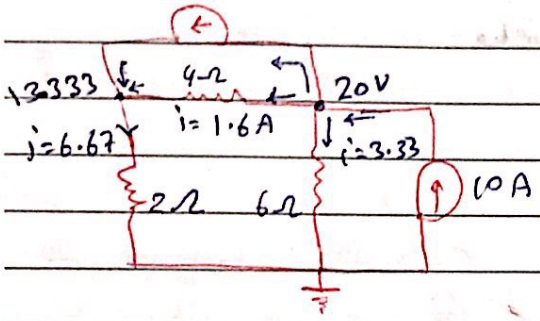
(2) ×12 $\;\; 2V_2 + 3V_2 - 3V_1 = 60$

$$-3V_1 + 5V_2 = 60 \quad \text{---} (B)$$

(A+B)

$$4V_2 = 80 \rightarrow \boxed{V_2 = 20\text{ V}}$$

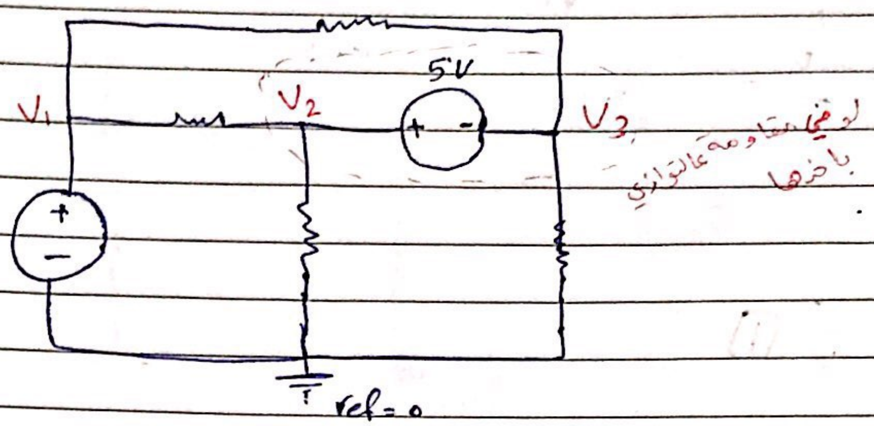
$$3V_1 = 20 + 20 \rightarrow \boxed{V_1 = 13.333\text{ V}}$$



لازم التيار الداخل يساوي الخارج ونقدر نشوف التيار على كل مقاومة من $i = \frac{V}{R}$

وبعدين بنقدر نطلع أي إشي power / energy

Nodals with Voltage sources :-



Case 1 → Voltage source between ref and nonref Node (V_1)

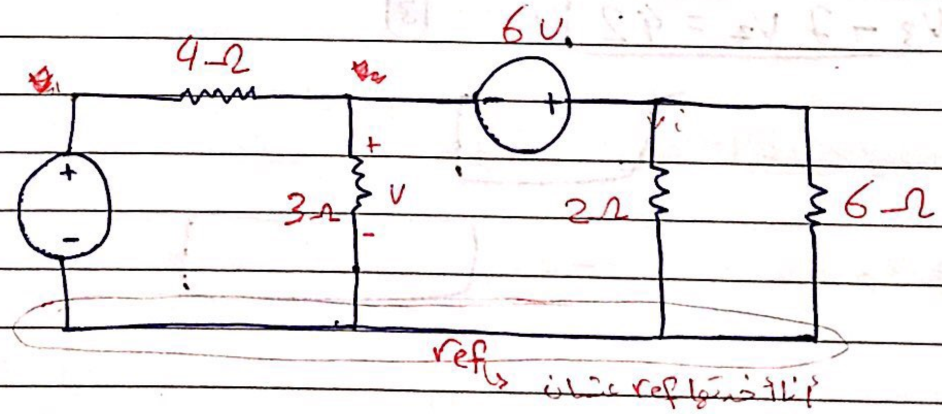
then $\boxed{V_1 = 10\,V}$ --- (1)

case 2 → Voltage source between tow non ref node (V_2, V_3)

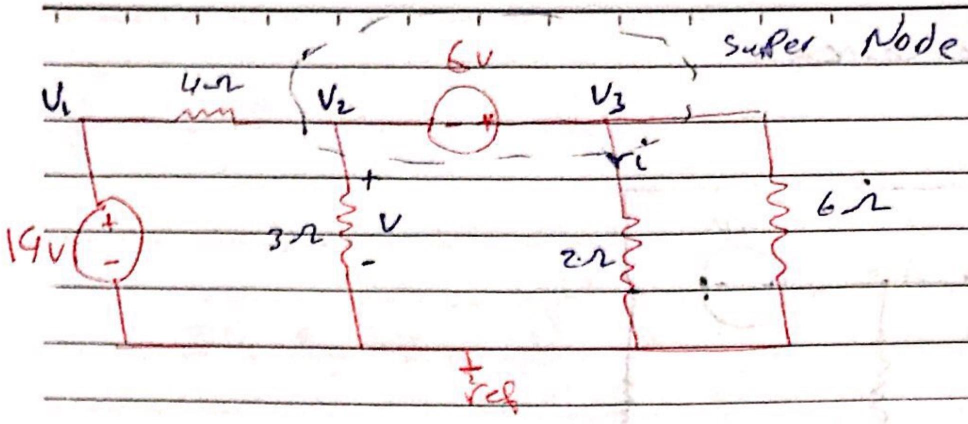
$\boxed{\text{Super node}}$ ---- (2) $\boxed{V_2 - V_3 = 5}$ --- (3)

↓ KCL

Ex : practice Problem 3.3 page 88



Find V and i :-



$\boxed{V_1 = 14\ V}$ --- [1]

KCL on the super Node

$$\left(\frac{V_2 - V_1}{4} + \frac{V_2}{3} + \frac{V_3}{2} + \frac{V_3}{6} = 0\right) \times 12$$

$$3V_2 - 3V_1 + 4V_2 + 6V_3 + 2V_3 = 0$$

$$7V_2 - 3(14) + 8V_3 = 0$$

$\boxed{7V_2 + 8V_3 = 42}$ --- [2]

$(V_3 - V_2 = 6) \times 7$

$\boxed{7V_3 - 7V_2 = 42}$ --- [3]

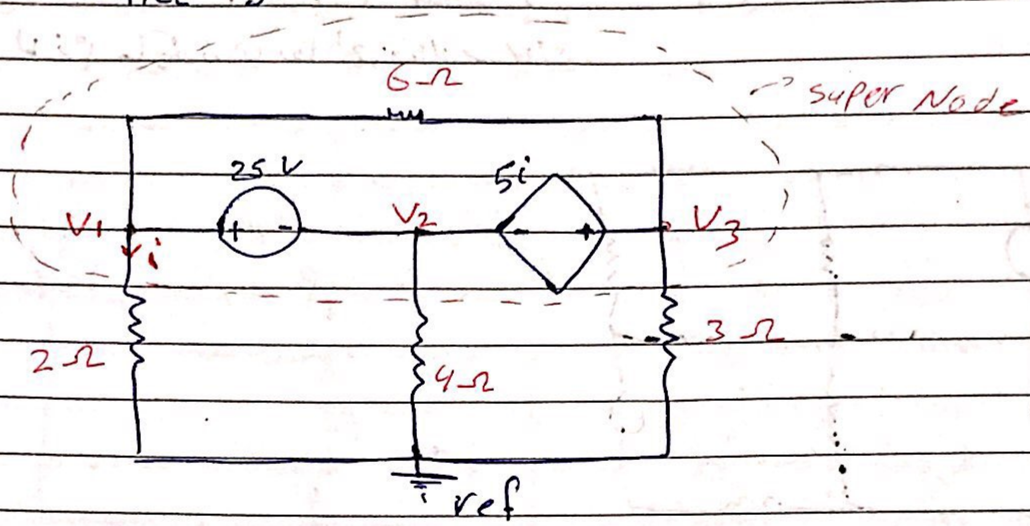
[2]+[3] $15V_3 = 84 \rightarrow \boxed{V_3 = \frac{84}{15}}$

$V_2 = V_3 - 6 = 5.6 - 6 = \boxed{-0.4V = V_2}$

$\boxed{i = 2.8A}$

Five Apple

Ex: Practice 3.4



KCL at the superNode:

$$\left(\frac{V_1}{2} + \frac{V_2}{4} + \frac{V_3}{3} = 0\right) 12$$

$$6V_1 + 3V_2 + 4V_3 = 0 \quad --- \boxed{1}$$

$$\boxed{V_1 - V_2 = 25} \quad --- \boxed{2}$$

$$V_3 - V_2 = 5i \rightarrow V_3 - V_2 = 5\left(\frac{V_1}{2}\right)$$

$$(2.5V_1 + V_2 - V_3 = 0)\, 4 \quad + \boxed{1}$$

$$16V_1 + 7V_2 = 0$$

+

$$7(V_1 - V_2 = 25)$$

$$23V_1 = 175$$

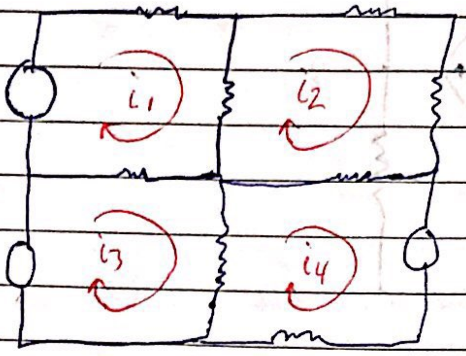
$$V_1 = \frac{175}{23}\ V \Rightarrow \begin{cases} V_1 = 7.609V \\ V_2 = -17.39\ V \\ V_3 = 1.63\ V \end{cases}$$

بقدر أطلع كل التيارات والجهود والطاقات

Five Apple

* Mech Analysis :- → "planar circuit"

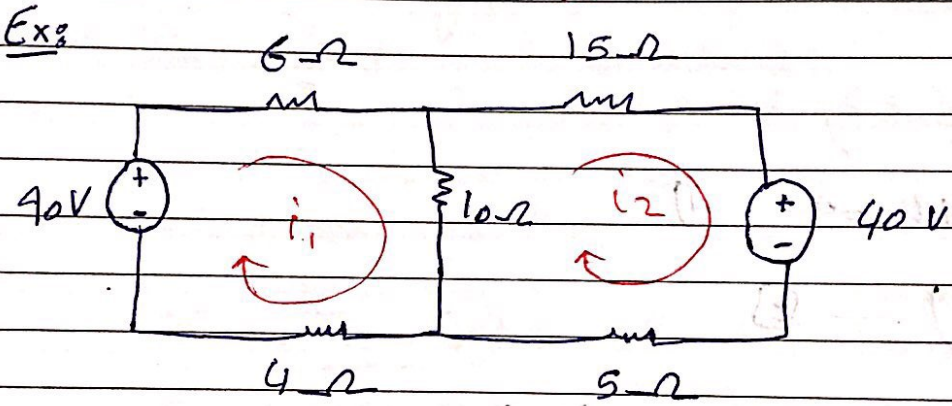
لازم مايكون في تقاطع بين الأسلاك



بنوجد ٤ ضفور loop وبنحدد اتجاهاتها،
وبطلع هنا ٤ loop نفس عدد i

Ex:

Find i_1, i_2 ?



Apply KVL for mesh 1 :-

$$-40 + 6i_1 + 10(i_1 - i_2) + 4i_1 = 0$$

$$20i_1 - 10i_2 = 40 \quad --- \boxed{1}$$

Apply KVL for Mesh 2 :-

or $-10(i_1 - i_2)$

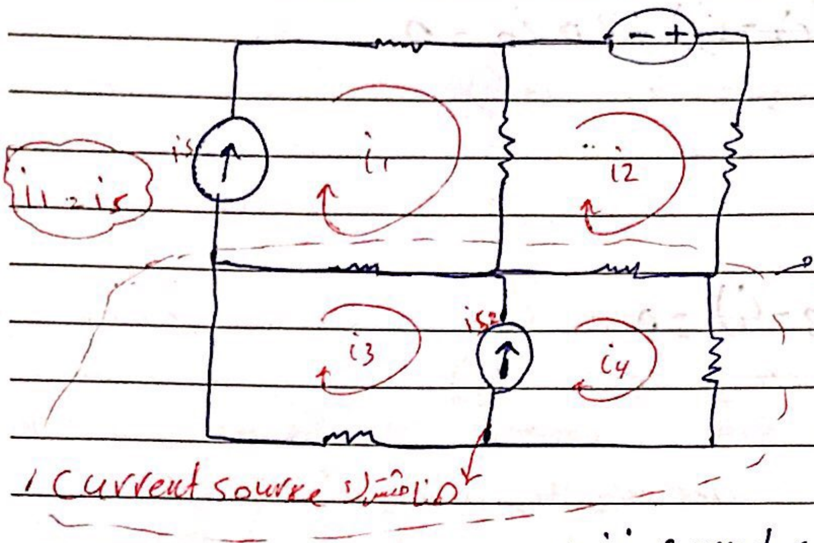
$$40 + 5i_2 + 10(i_2 - i_1) + 15i_2 = 0$$

$$-10i_1 + 30i_2 = -40 \quad --- \boxed{2}$$

$\boxed{2} \times 2$ وحل المعادلتين ⟹ $i_2 = \frac{1}{5} = 200\ mA$ $\quad$ $i_1 = \frac{92}{20} = 4.6\ A$

Power, V ⟵ لما نجد بنقدر نطلب أي اشي

Mesh with current source :-



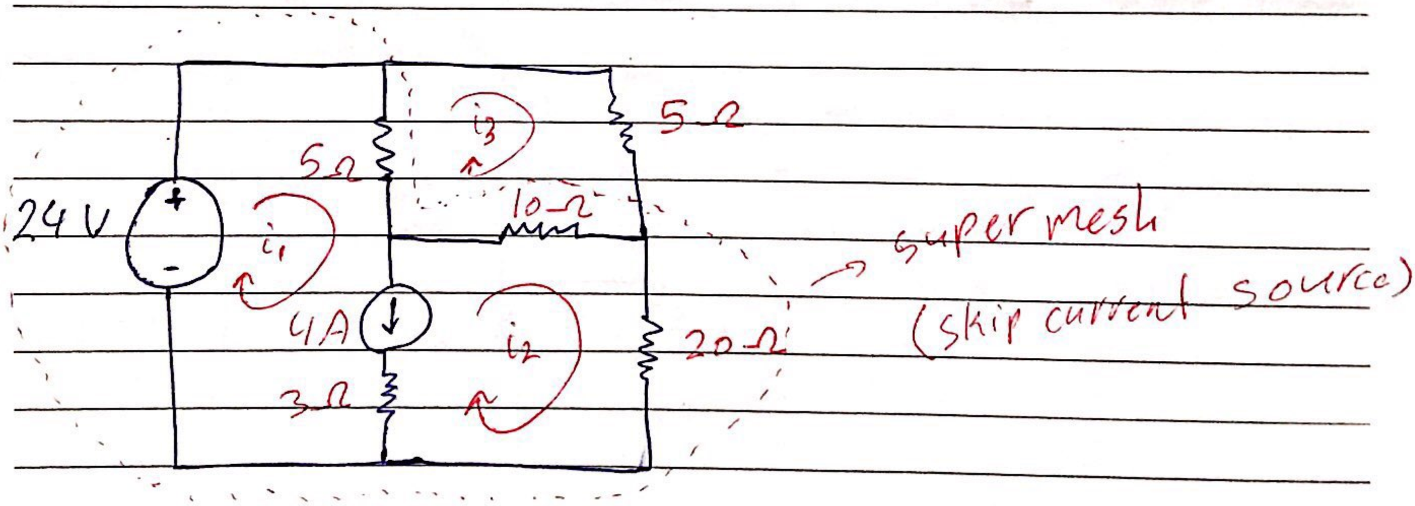
(super mesh) باخذ
ونسى الـ current source
وبمشي موكل مقاومة والتيار
اي بعرفيها بطلع معادلة بمجهولين

المعادلة الثانية بجيبها من الـ current source نفسه

~~$i_3 - i_2 = i_s$~~

$$i_4 - i_3 = i_{s_2}$$

Practice 3.7 Page 98



Find i_1 و i_2 و i_3 :-

"الإجابات النهائية بالكتاب خاطئة"

وضعهم بطلع أي اشي بالدارة

KVL at the sper mesh:-

$$-24+5(i_1-i_3)+10(i_2-i_3)+20\,i_2=0$$

$$5i_1+30\,i_2-15\,i_3=24 \quad ---\ (1)$$

KVL at mesh 3 :-

$$5i_3+10\,(i_3-i_2)+5\,(i_3-i_1)=0$$

$$-5i_1-10\,i_2+20\,i_3=0 \quad ---\ (2)$$

$i_1-i_2=4 \quad ---\ (3)$ والمعادلة الأخيرة من نفس ال currensource

$i_1=4.8\,A$ $\quad i_2=0.8\,A$ $\quad i_3=1.6\,A$ وبحل المعادلات

CH4: Linearity and superposition

- superposition is applied if the circuit is linear

- The circuit is linear if all element in the circuit are linear.

R: $V = Ri$ linear

Voltage and current sources:

all independent sources are linear (Voltag, current)

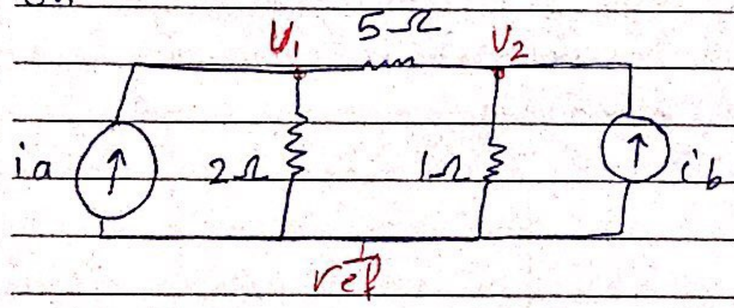
- Dependent sources:

$V_s = K V_x \longrightarrow$ linear dependent Voltage source

$i_s = K\sqrt{i_x} \longrightarrow$ non linear $\quad V_s = K i_x i_z \longrightarrow$ nonlinear

$V_s = K_1 V_{x_1} + K_2 V_{x_2} \longrightarrow$ linear $\quad V_s = K V_x^2 \Rightarrow$ non linear

Ex



by Use Nodal ANAlysis

KCL at Node 1

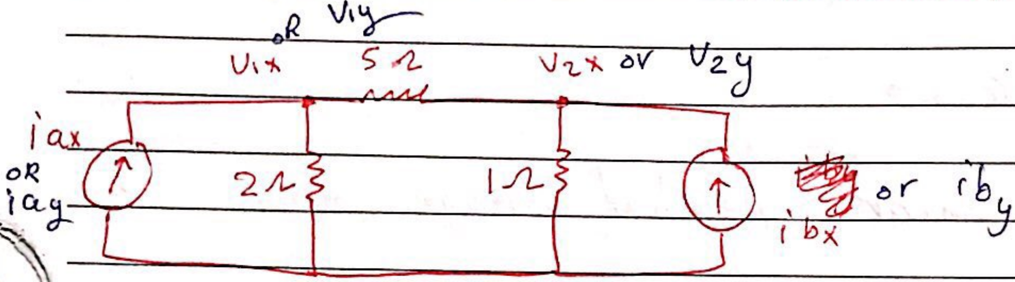
$$-i_a + \frac{V_1}{2} + \frac{V_1 - V_2}{5} = 0$$

$$0.7\,V_1 - 0.2\,V_2 = i_a \text{ — } (1)$$

by KCL at Node 2

$$-i_b + \frac{V_2}{1} + \frac{V_2 - V_1}{5} = 0 \longrightarrow -0.2\,V_1 + 1.2\,V_2 = i_b \text{ --- } (2)$$

*الطريقة الأخرى (super position)



Experiament (x)

$$0.7\,V_{1x} - 0.2\,V_{2x} = i_{ax}$$
$$-0.2\,V_{1x} + 1.2\,V_{2x} = i_{bx}$$

Experiament (y)

Exp x + Exp y

$$0.7\,V_{1y} - 0.2\,V_{2y} = i_{ay}$$
$$-0.2\,V_{1y} + 1.2\,V_{2y} = i_{by}$$

$$0.7(V_{1x} + V_{1y}) - 0.2(V_{2x} + V_{2y}) = (i_{ax} + i_{ay})$$
$$0.7\,V_1 \quad - 0.2\,V_2 \quad = \quad i_a$$

$$-0.2(V_{1x} + V_{1y}) + 1.2(V_{2x} + V_{2y}) = (i_{bx} + i_{by})$$
$$-0.2\,V_1 \quad + 1.2\,V_2 \quad = \quad i_b$$

الـ (Super position) عبارة عن تجزئة القيم الي عندي مثلاً

$i_a = 5A$ → $i_{ax} = 3$, $i_{ay} = 2$ oR $i_{ax} = 4$, $i_{ay} = 1$

المهم يعطيني نفس القيمة

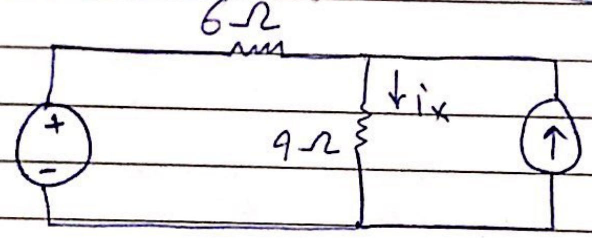
أو أحسن طريقة أصفّر جزء وأعطي جزء كل القيمة

$V_a = 5V$ → $V_{ax} = 5$, $V_{ay} = 0$ عشان تصير short circuit

Short circuit → killing the source

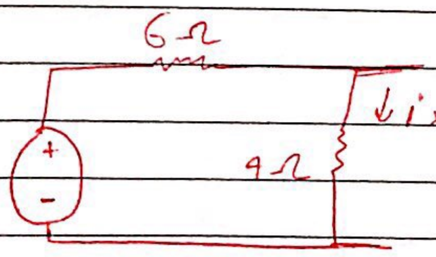
الـ dependant ما بنعمله Kill

Ex



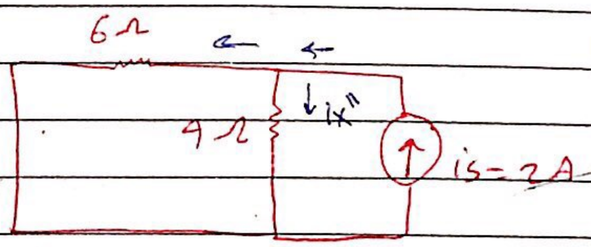
$V_S = 3V$, $i_s = 2A$ → Linear

use (super Position) to find i_x in the circuit ?



→ Killing the source

$$i_x' = \frac{3}{15} = \frac{1}{5} = 0.2A$$



killing the source

خليناه لحاله

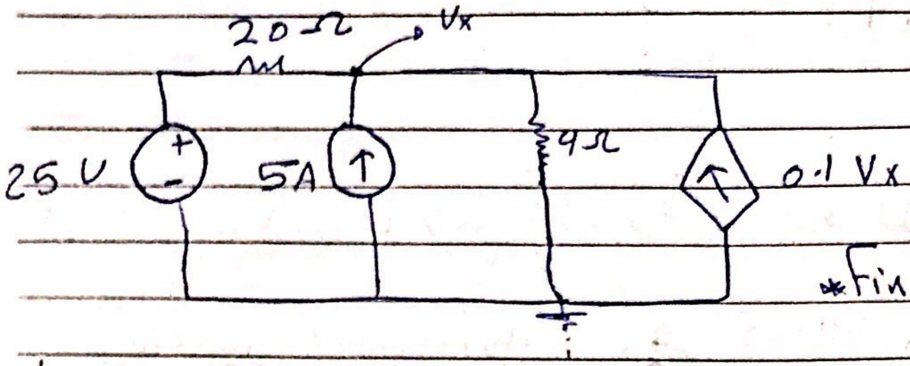
by current Division

$$i_x'' = \frac{6}{15} \times 2 = \frac{12}{15} = 0.8 A$$

$$\boxed{i_x = i_x' + i_x'' = 1 A}$$

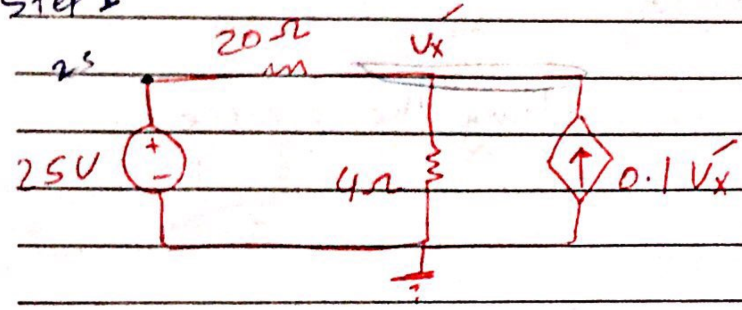
* الـ Voltage Source بعد الـ Kill بتصير short cir أما الـ current source بتصير open cir

practice problem 4.4 Page 132



*Find Vx using superposition

step1



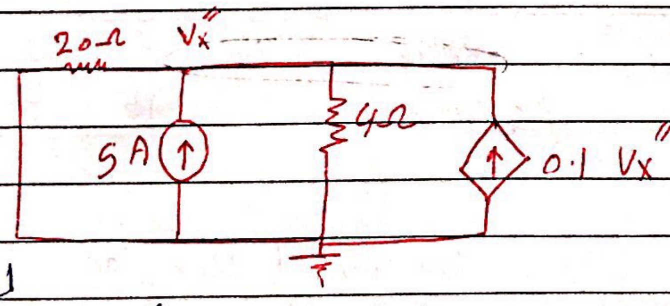
by Nodal

$$\frac{V_x' - 25}{20} + \frac{V_x'}{4} - 0.1V_x' = 0 \qquad \times 20$$

هاد جزء من ال V_x

$$V_x' - 25 + 5V_x' - 2V_x' = 0 \rightarrow 4V_x' = 25 \rightarrow \boxed{V_x' = 6.25V}$$

step 2



by Nodal

$$\frac{V_x''}{20} - 5 + \frac{V_x''}{4} - 0.1V_x'' = 0 \qquad \times 20$$

هاد الجزء الثاني من ال V_x

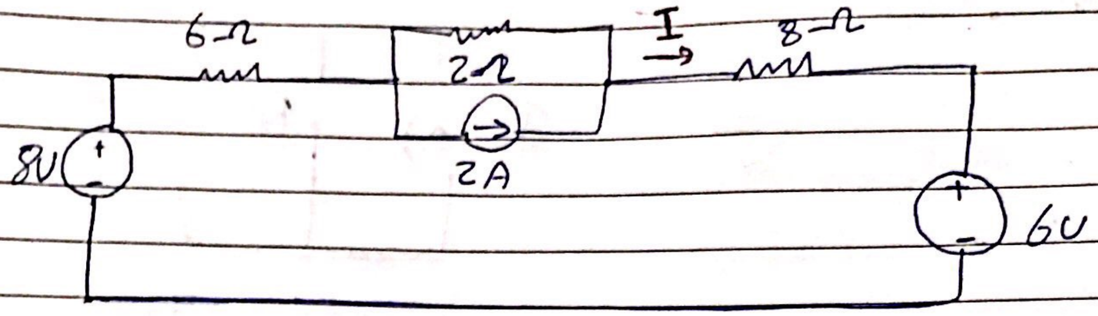
$$V_x'' - 100 + 5V_x'' - 2V_x'' = 0 \rightarrow \boxed{V_x'' = 25V}$$

$$\therefore V_x = V_x' + V_x'' = 6.25 + 25 = \boxed{31.25V}$$

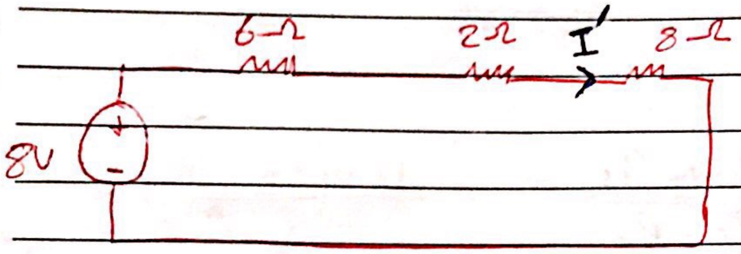
Five Apple

لو حلينا نودال من أول بطلع نفس الجواب

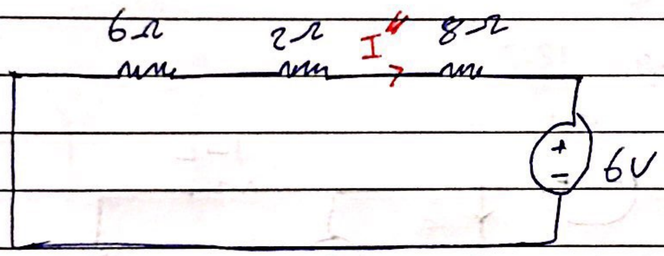
Practice problem 4.5



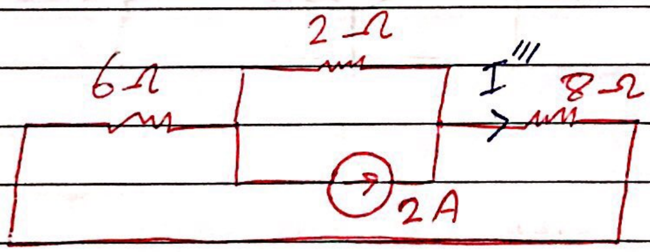
Find "I" using super position :—



$$I' = \frac{8}{8+2+6} = 0.5\ A$$



$$I'' = \frac{-6}{16} = -0.375\ A$$



by current Division :-

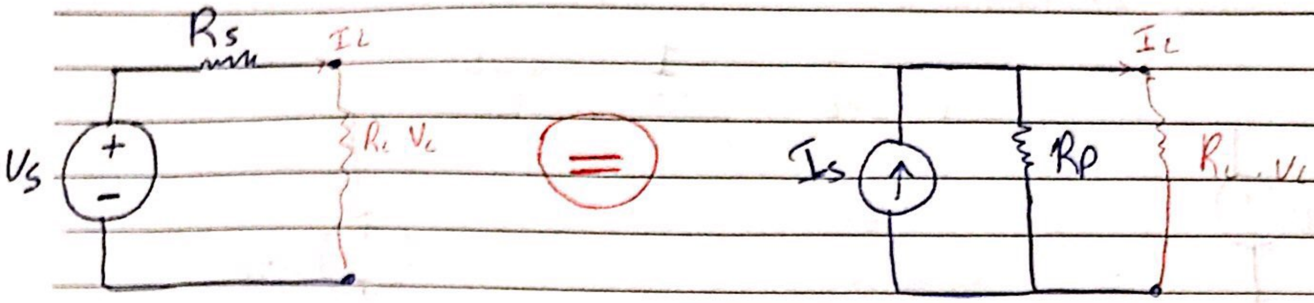
$$I''' = \frac{2}{16} \times 2 = 0.25\ A$$

$$I = I' + I'' + I'''$$

$$0.5 - 0.375 + 0.25$$

$$= 0.375\ A = \boxed{375\ mA}$$

* Source transformation:-

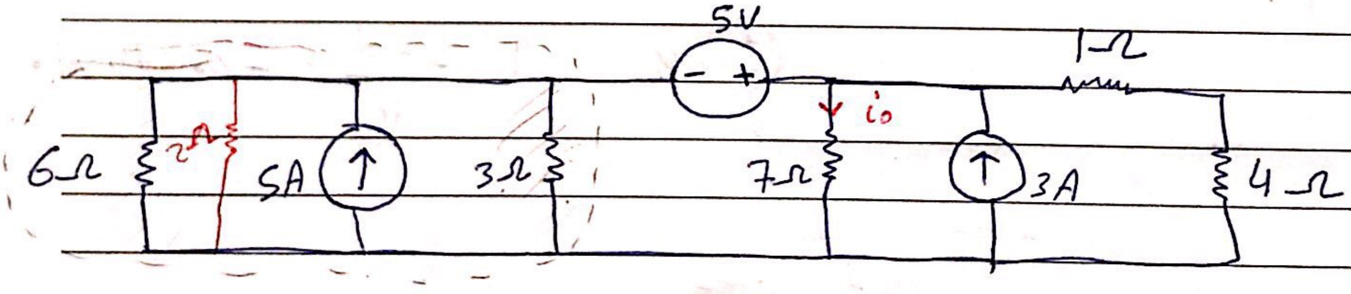


بقدر أعوض Voltag source ومقاومة عالتوالي
ب current source ومقاومة عالتوازي

$$V_L = \frac{R_L}{R_L + R_s} V_s \qquad V_s = R_p I_s \qquad V_L = \left(\frac{R_L \cdot R_p}{R_p + R_L}\right) I_s$$

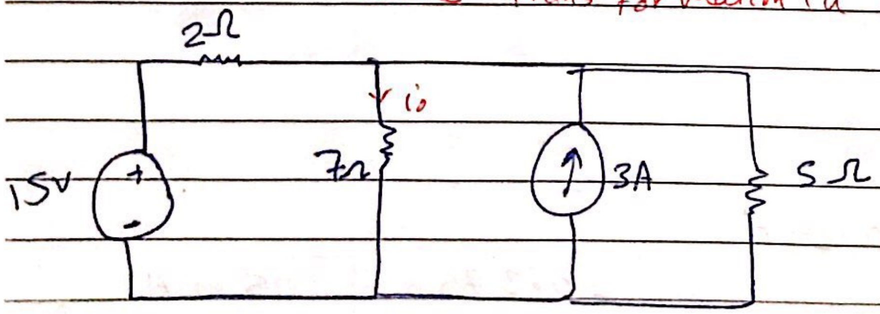
$$R_p = R_s$$

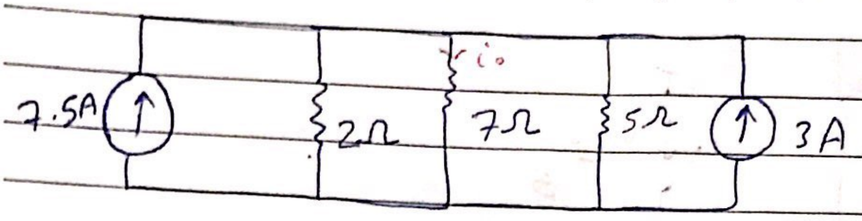
Practice problem 4.6 Page 135

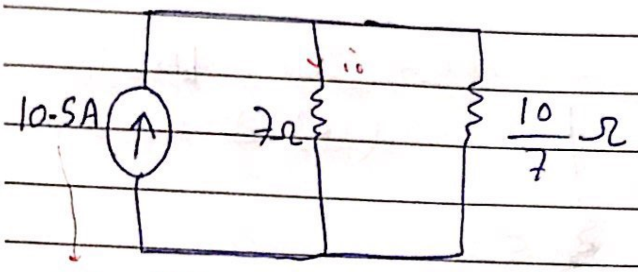


Find i_o :-

source transformation in current source







جمعناهم عشان
نفس الإتجاه

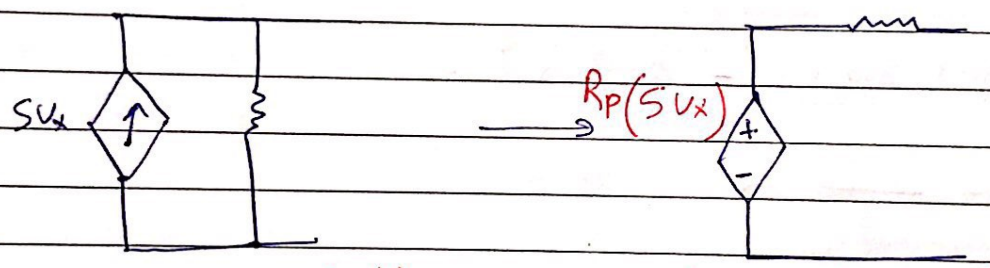
by current division

$$i_0 = \left(\frac{\frac{10}{7}}{\frac{10}{7} + 7}\right) \times 10.5$$

$$= \boxed{1.78\ A}$$

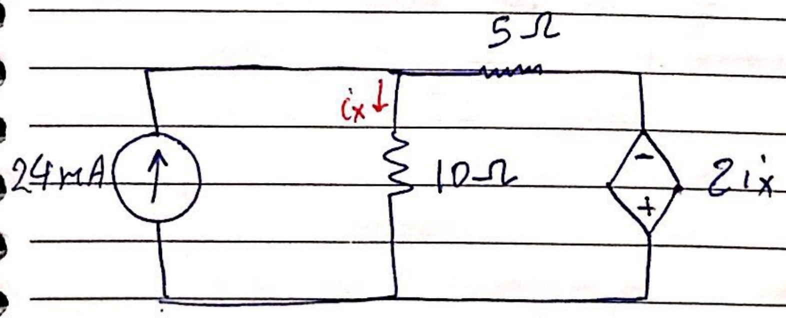
* Note:

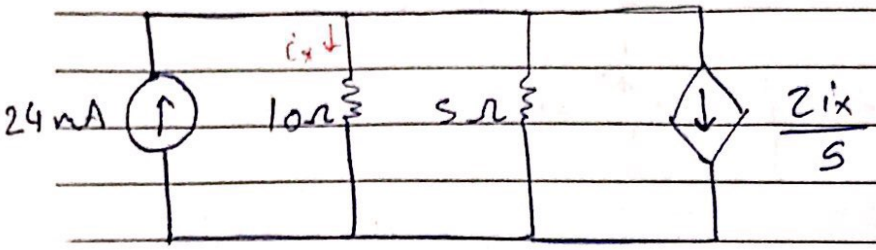
بقدر أحول الـ dependent source



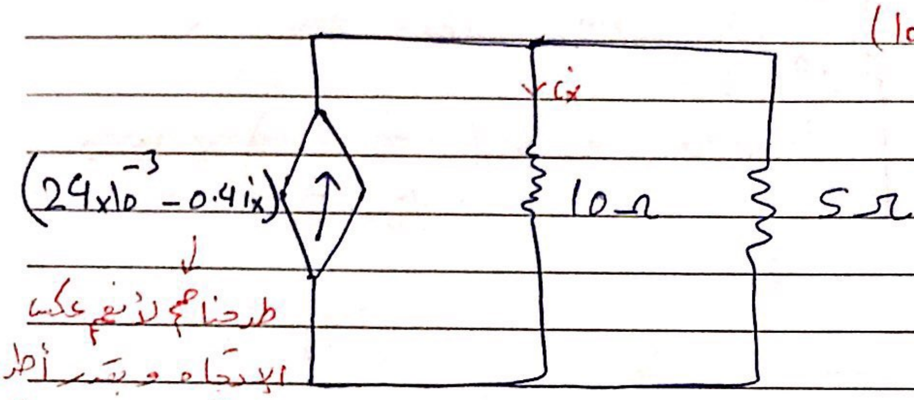
ممنوع أغير أو أحول الـ Bransh المتعلقة بالـ (Vx)

practice problem 4.7





* ما اشتغلنا على مقاومة (10Ω) لأنو عليها الـ (i_x)



طرحنا لأنهم عكس الاتجاه وبصير أطرح العكس $(0.4 i_x - 24\times10^{-3})$

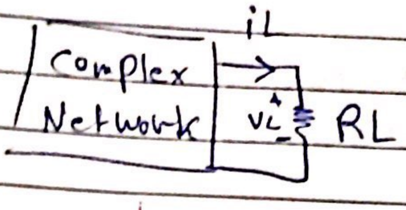
$$i_x = \left(\frac{5}{15}\right)\left(24\times10^{-3} - 0.4\, i_x\right)$$

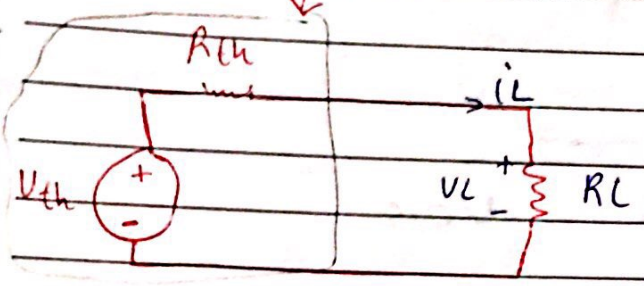
$$i_x = 8\times10^{-3} - \frac{0.4}{3} i_x$$

$$\frac{34}{30} i_x = 8\times10^{-3}$$

$$i_x = 8\times10^{-3} \times \frac{30}{34} = \boxed{7.06\ mA}$$

* Thevenin and Norton Equivalents *



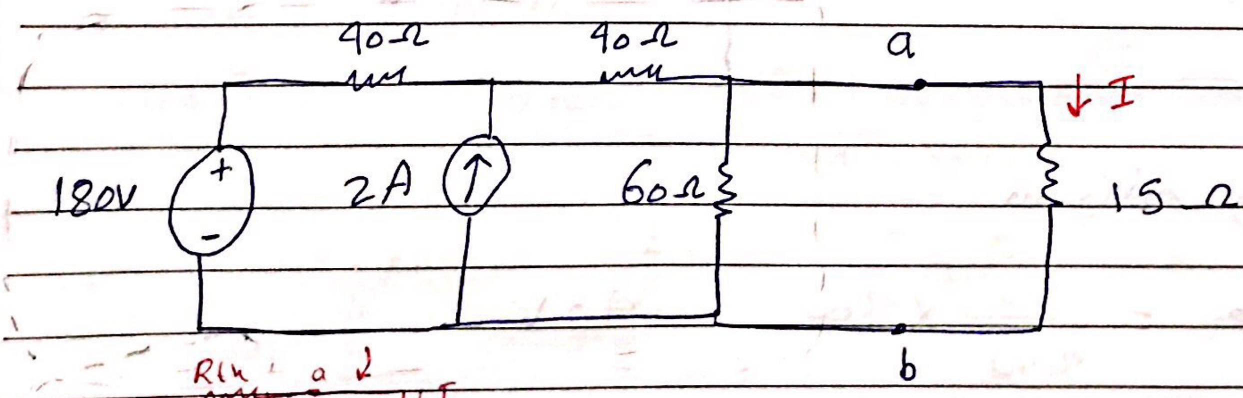


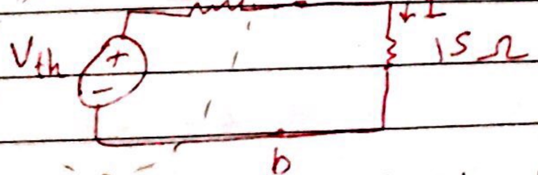
Case 1 : all independent sources :-

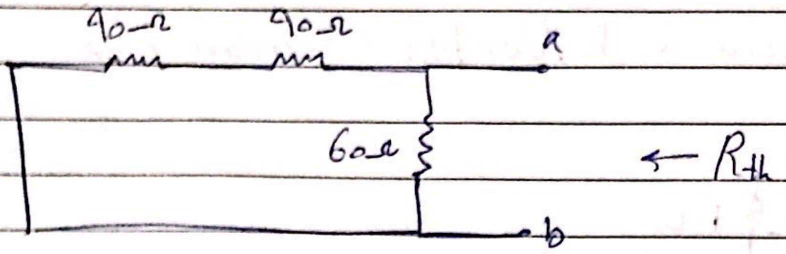
$R_{th} \rightarrow$ kill all sources and find $R_{eq} = R_{th}$

$V_{th} = V_{\text{open circuit}}$ $V_{th} = V_{o.c}$

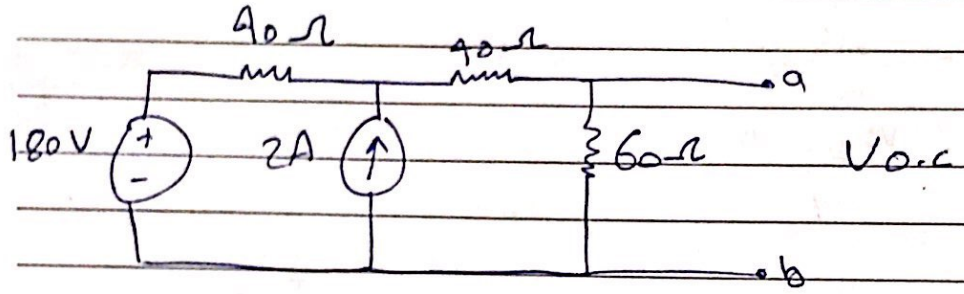
Ex: practice problem 4.8 page 140

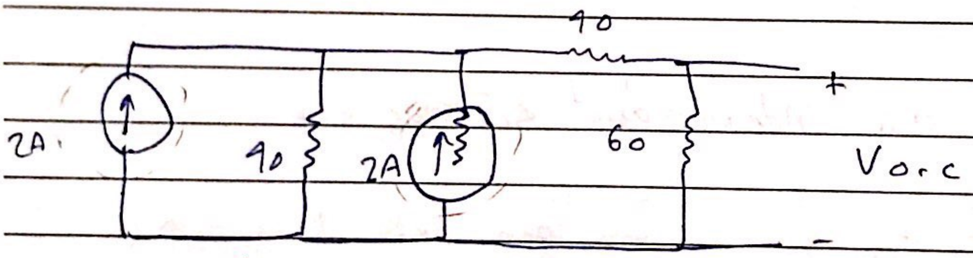




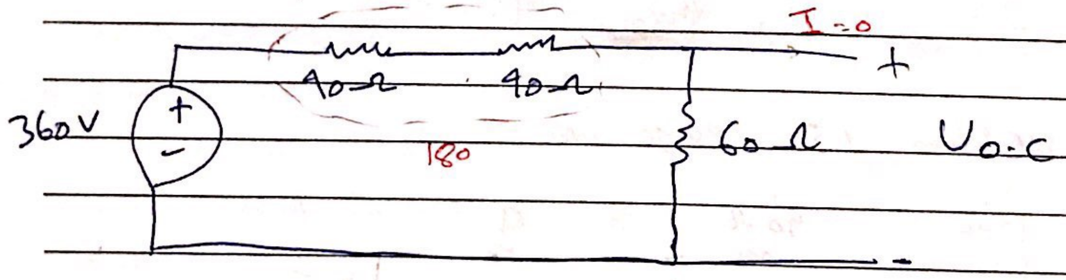


$90+90 = 180\ \Omega \rightarrow \frac{180 \times 60}{240} = 45\ \Omega = R_{th}$





current transformation (ندمج و نحول)



Voltage division

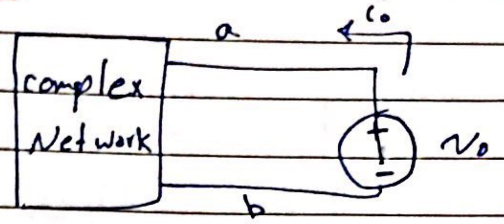
$V_{o.c} = \frac{60}{240} \times 360 = 90\,V$

$I = \frac{90}{60} = 1.5$

Case 2: independent and dependent sources :-

1) Kill all independent source
2) induce a voltage source V_o between the tow terminals
3) calculate i_o generated by the source V_o
4) calculate R_{th}

$$R_{th} = \frac{V_o}{i_o}$$

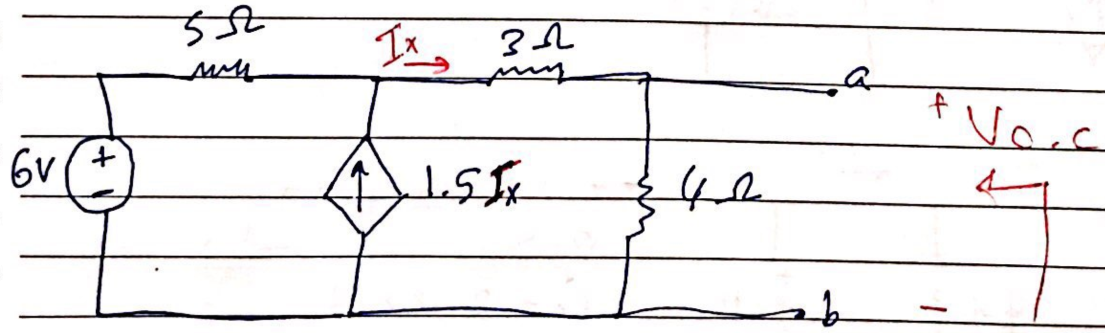


- Alternative method :-

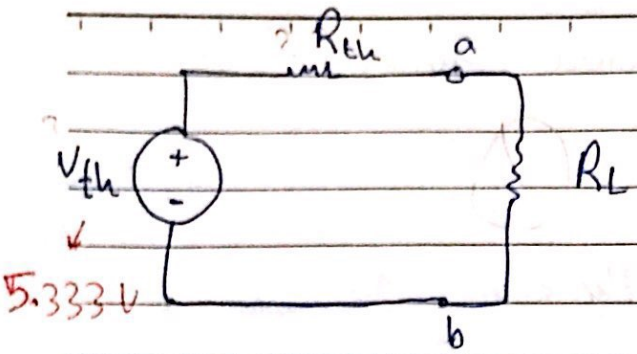
1) Kill all independent source
2) induce a current source i_o btween the tow terminals
3) calculate V_o across the current source
4) calculat $R_{th} = \frac{V_o}{i_o}$

for simplicity let $V_o = 1V$
let $i_o = 1A \Rightarrow V_o$?
to find $V_{th} \Rightarrow$ proceed as in case 1.

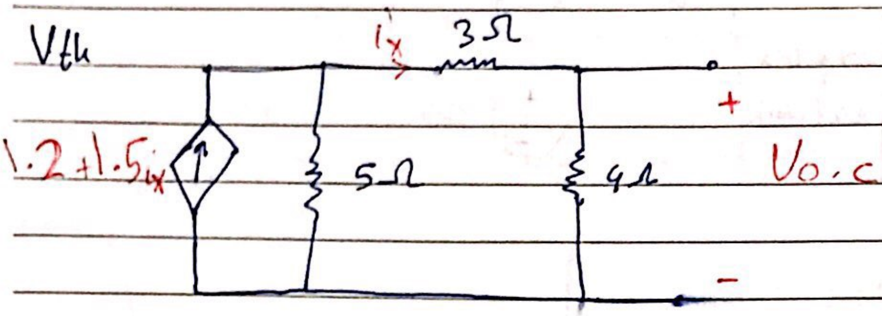
Practice problem 4.9

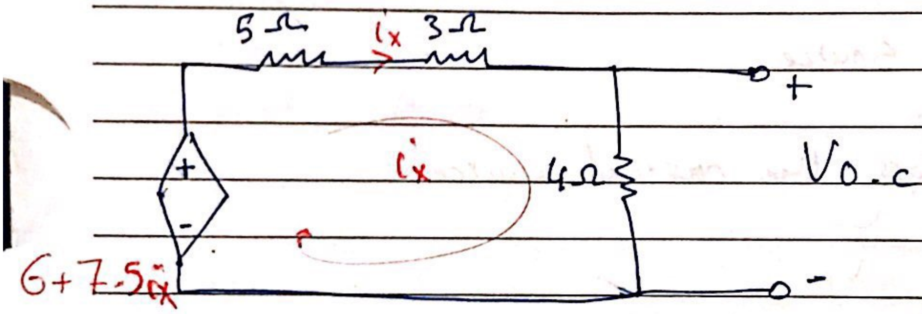


find the thevenin equivalent of the circuit above :-

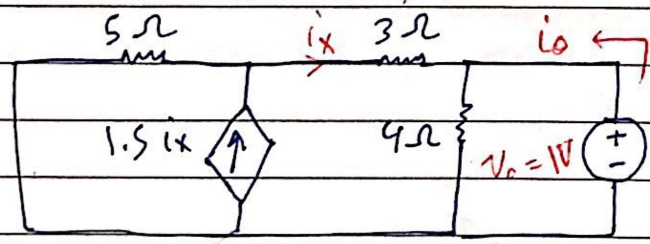


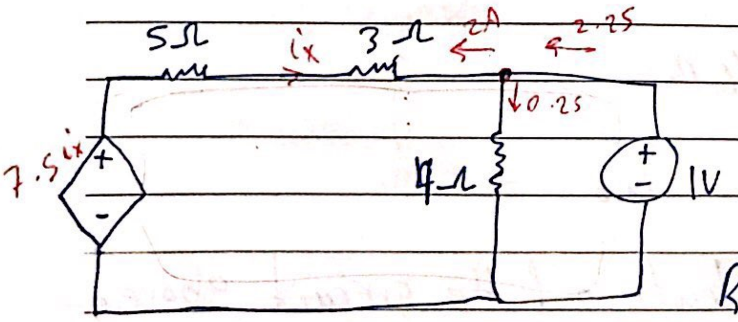
Vth





$$i_x = \frac{6+7.5i_x}{12} \longrightarrow i_x = \frac{4}{3}\,A \quad V_{o.c} = 4\left(\frac{4}{3}\right) = \frac{16}{3} = 5.333\,V$$





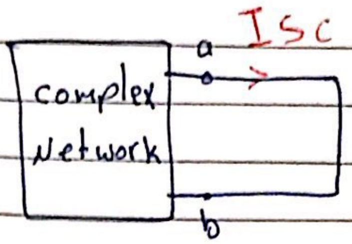
$$-7i_x + 5i_x + 3i_x + 1 = 0$$

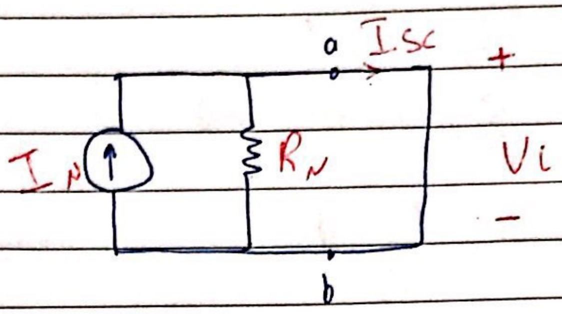
$$0.5i_x = -1 \rightarrow \boxed{i_x = -2}$$

$$i_o = 2.25\,A$$

$$R_{th} = \frac{V_o}{i_o} = \frac{1}{2.25} = \boxed{0.4444\,\Omega}$$

Norton equivalent:-

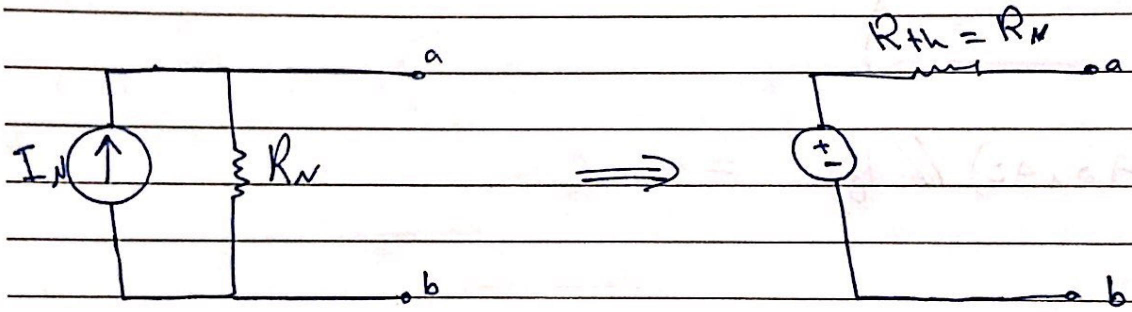




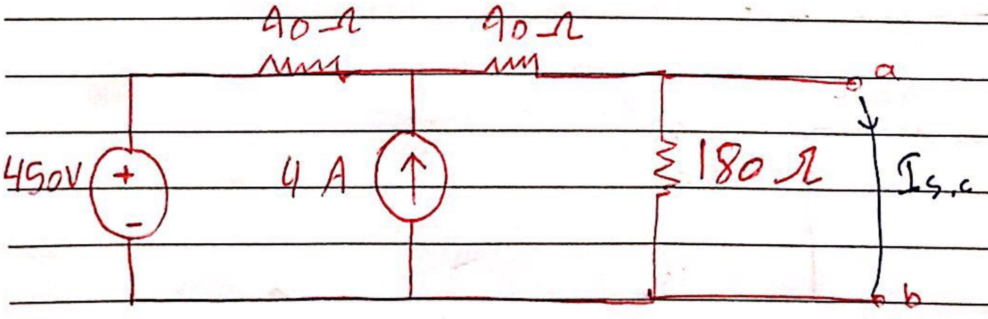
$R_N = R_{th}$ (case1 and case 2)

$V_{th} = I_N \cdot R_N = I_N \cdot R_{th}$

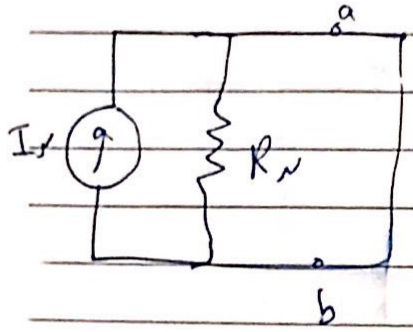
source tranformation



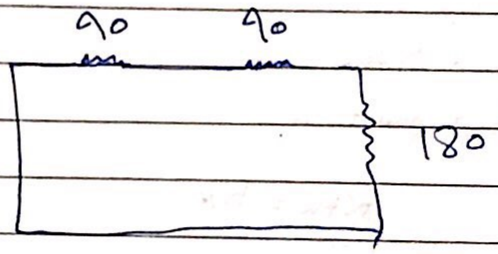
practice problem :- (4.11)



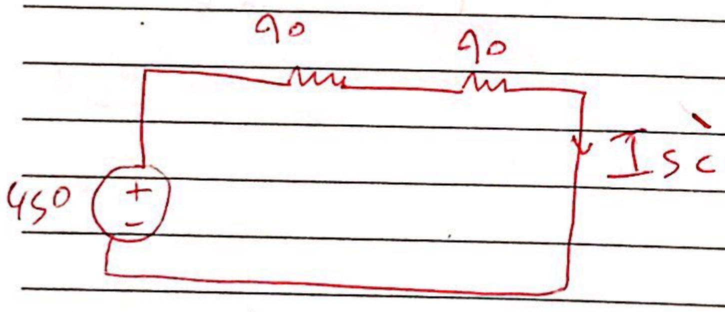
Find Norton equivelant:-



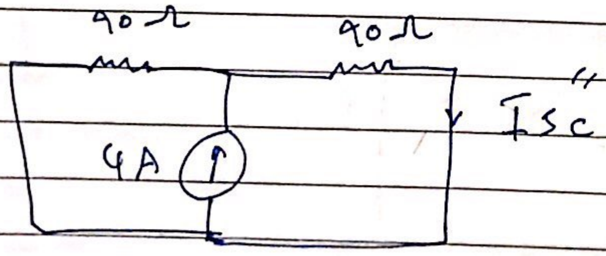
R_N:



$(90+90) \,//\, 180 = 90\ \Omega$



$I_{sc}' = \frac{450}{180} = 2.5\ A$



$I_{sc}'' = 2A$

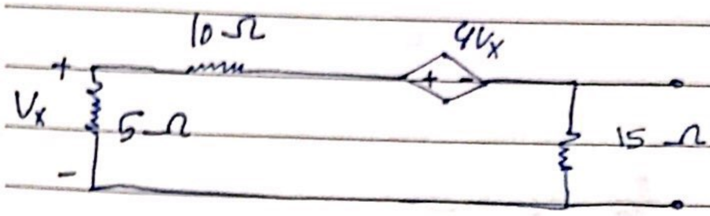
$I_{sc} = I_{sc}' + I_{sc}''$

$= 2.5 + 2$

$= 4.5\ A$

$\boxed{I_N = 4.5\ A}$

practice problem 4.10



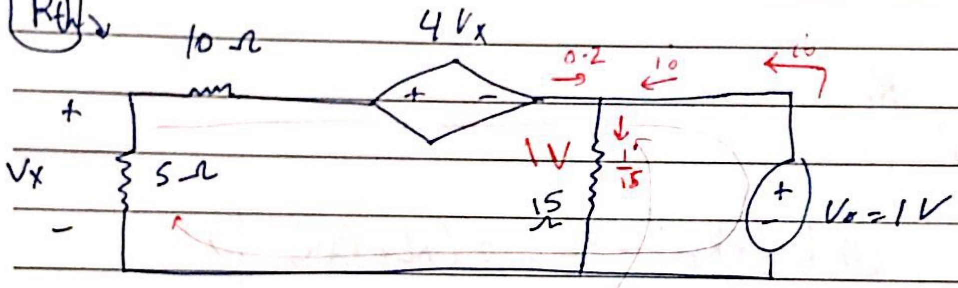
Find the thevenin equ:-

$5i + 10i + 4V_x + 15i = 0 \qquad (V_x = -5i)$

$5i + 10i + 4(-5i) + 15i = 0 \rightarrow \boxed{i = 0}$

$V_{o.c} = V_{th} = 0$

$\boxed{R_{th}}$

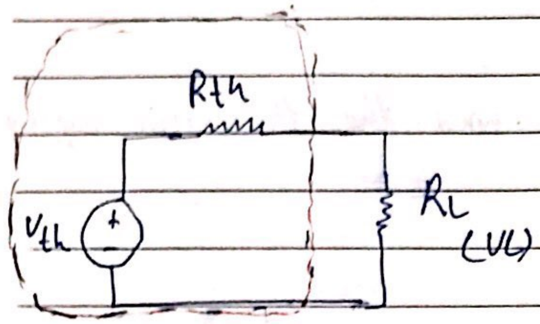


$5i + 10i + 4V_x + 1 = 0$

$V_x = -5i \longrightarrow 5i + 10i - 20i + 1 = 0 \rightarrow \boxed{i = +\frac{1}{5}\,A}$

$0.2 + i_o - \frac{1}{15} = 0 \rightarrow i_o = \frac{-1}{5} + \frac{1}{15} = \boxed{\frac{-2}{15}}$

$R_{th} = \frac{V_o}{i_o} = \frac{1}{\frac{-2}{15}} = -\frac{15}{2} = \boxed{-7.5\,\Omega}$



$i_L = \dfrac{V_{th}}{R_{th}+R_L}$

$P_{R_L} = V_L \cdot i_L$

OR

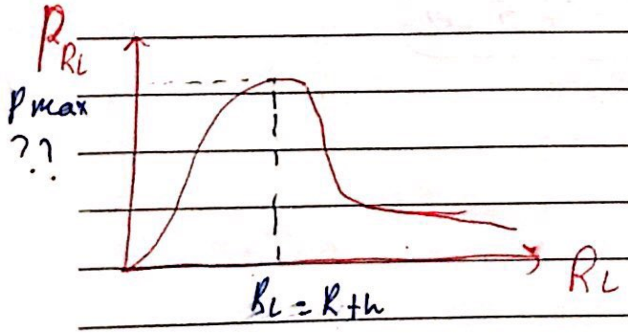
$P_{R_L} = i^2 R_L$

$P_{R_L} = \dfrac{V_{th}^2}{(R_L+R_{th})^2} R_L$

$\dfrac{d P_{R_L}}{d R_L} = V_{th}^2 \left[\dfrac{(1)(R_L+R_{th})^2 - R_L \cdot 2(R_L+R_{th})}{(R_L+R_{th})^{4\,3}} \right]$

$= V_{th}^2 \left[\dfrac{R_L + R_{th} - 2R_L}{(R_L+R_{th})^3} \right] = 0$

$R_L + R_{th} - 2R_L = 0 \longrightarrow \boxed{R_L = R_{th}}$



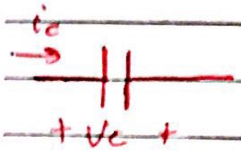
$P_{max} = \dfrac{V_{th}^2 \cdot R_{th}}{(2R_{th})^2}$

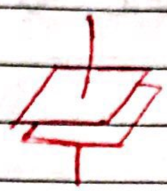
$P_{max} = \dfrac{V_{th}^2}{4R_{th}}$

CH 5 $\longrightarrow$ ملغي

CH6: capacitors and inductors :-

* capacitor (C) :-





$$i_c(t) = C \frac{d v_c(t)}{dt}$$

C: capacitance in Farad (F).

$$C = \frac{\varepsilon \cdot A}{d}$$ ε: permittivity of the insulater

$\varepsilon = \varepsilon_0 \varepsilon_r$

$\hookrightarrow$ permittivity of free space (8.85×10^{-12} F/m)

$$i(t) = \frac{dq}{dt} , \quad i = C \frac{dv}{dt} \longrightarrow q(t) = C\, v(t)$$

$$\int_{t_0}^{t} dv = \int_{t_0}^{t} \frac{1}{C} i(t)\, dt$$

$$V(t) = \frac{1}{C} \int_{t_0}^{t} i(t')\, dt' + V(t_0)$$

Assume $t_0 \longrightarrow -\infty$ and $v(t = -\infty) = 0$

$$v(t) = \frac{1}{C} \int_{-\infty}^{t} i\, dt'$$

Ex: determin the current flowing through a 5 μF capacitor in response to avoltage v equal to

a) $-20\ v$ b) $2e^{-5t}\ v$

Solve

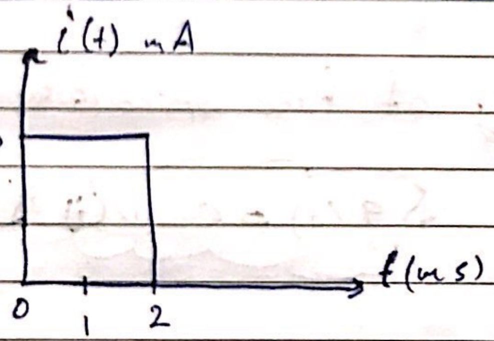
a) $i = c\frac{dv}{dt} = 0$

b) $i = 5\times10^{-3} \times -10\,e^{-5t}$

$i(t) = -50e^{-5t}\ mA$

$\boxed{\text{* A capacitor is open circuit to dc}}$

Ex



find $v(t)$ for all t :-

$$v(t) = \frac{1}{c}\int_{t_0}^{t} i(t')\,dt' + v(t_0)$$

we have three intervals $t \leq 0$, $0 \leq t \leq 2ms$, $t \geq 2ms$

$t \leq 0$

$v(t_0) = v(-\infty) = 0$

$$v(t) = \frac{1}{5\times10^{-6}}\int_{-\infty}^{t} 0 \cdot dt' + 0 = 0 \quad , \ t \leq 0$$

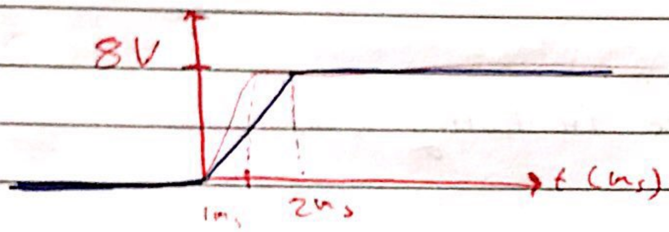
$0 \leq t \leq 2\,ms$

$$v(t) = \frac{1}{5\times10^{-6}} \int_0^t 20\times10^{-3}\, dt + v(0) \qquad 0 \leq t \leq 2\,ms$$

$$v(t) = 4000\,t \qquad 0 \leq t \leq 2\,ms$$

$t \geq 2\,ms$

$$v(t) = 0 + v(2m) = 8V$$



$$\boxed{\text{* the voltage across a capacitor cannot be changed abruptly.}}$$

Energy storage in the capacitor (J)

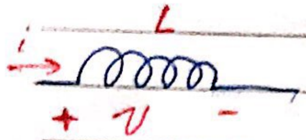
$$P = iv = c\,\frac{v\,dv}{dt}$$

$$w_c(t) - w_c(t_0) = \int_{t_0}^{t} p\,dt' = c\int_{t_0}^{t} v\,\frac{dv}{dt'}\,.dt'$$

$$= c\int_{v(t_0)}^{v(t)} v.\,dv = \frac{1}{2}c\left[v^2(t) - v^2(t_0)\right] \rightarrow w_c(t) = \frac{1}{2}c\,v^2(t) \rightarrow J$$

* A finite amount of energy can be stored in a capacitor even if the current through of capacitor is zero, when $v_c(t) =$ constant

* The inductors:- (L)

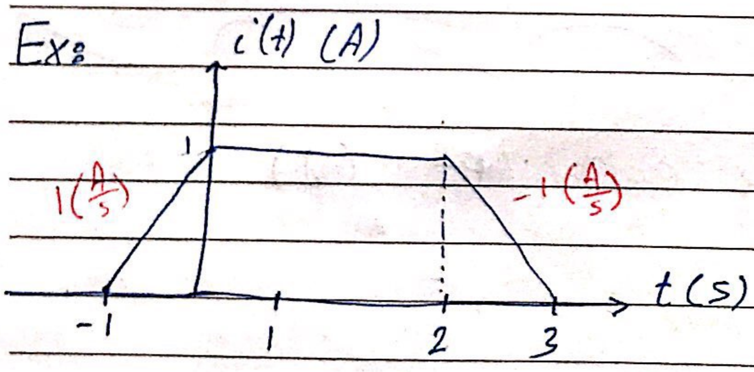


$v = L\frac{di}{dt}$ L: inductance in henry (H).

* for constant current $\frac{di}{dt} = 0 \Rightarrow v_L = 0$

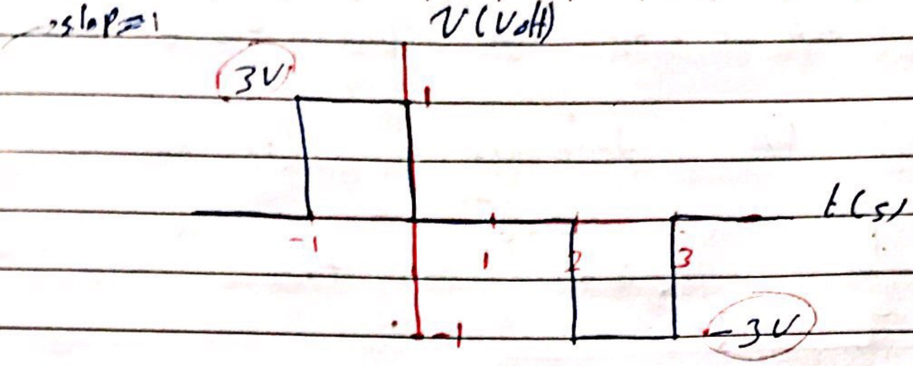
The inductor is short circuit to dc

Ex:

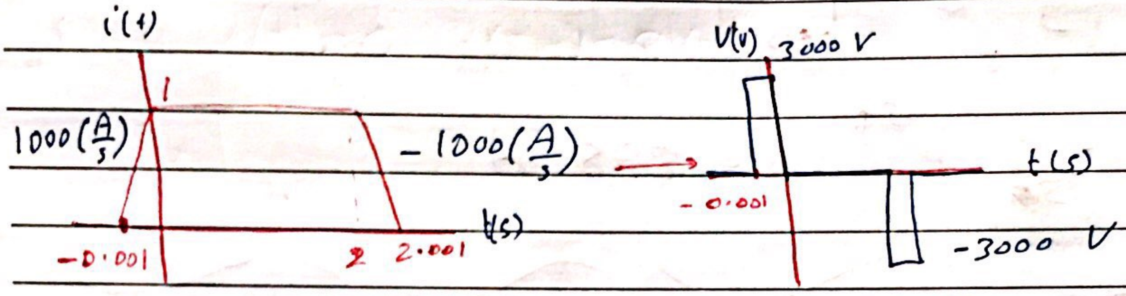


Given the current waveform above passing through 3H inductor, determin the inductor voltage:-

$v = 3\frac{di}{dt}$



another example..



$\boxed{\text{* The current passing through an inductor cannot be changed abruptly.}}$

$$v_L = L\frac{di}{dt}$$

$$\int_{i(t_0)}^{i(t)} di = \frac{1}{L}\int_{t_0}^{t} v_L \, dt'$$

$$i(t) - i(t_0) = \frac{1}{L}\int_{t_0}^{t} v_L \, dt' \longrightarrow i(t) = \frac{1}{L}\int_{t_0}^{t} v_L \, dt' + i(t_0)$$

$$i(t_0) = 0 \quad \text{at} \quad t_0 = -\infty$$

$$\boxed{i(t) = \int_{-\infty}^{t} v_L \, dt'}$$

Ex: A 100 mH inductor has $v_L(t) = 2e^{-3t}$ V and $i_L(-0.5) = 1A$, determine $i_L(t)$:-

$$i(t) = \frac{1}{L}\left[\int_{t=-0.5}^{t} 2e^{-3t} \cdot dt\right] + 1$$

$$= \frac{1}{0.1}\left[\frac{-2}{3} e^{-3t}\Big|_{-0.5}^{t}\right] + 1$$

$$i(t) = \frac{-20}{3} e^{-3t} + 30.878 \text{ A}$$

* Energy storage :-

$$P = vi = \left(L\frac{di}{dt}\right) i = Li\frac{di}{dt}$$

$$\int_{t_0}^{t} p\,dt = L\int_{t_0}^{t} i \frac{di}{\cancel{dt}} \cdot \cancel{dt}$$

$$w_L(t) - w_L(t_0) = L\int_{i(t_0)}^{i(t)} i\,di = \frac{1}{2}L\left[i^2(t) - i^2(t_0)\right]$$

$$w_L(t) = \frac{1}{2} L i^2(t)$$

Ex8 $L = 25\,mH$ a) find v_L at $t = 12\,ms$ if $i_L = 10te^{-100t}$ A

b) find i_L at $t = 0.1\,s$ if $v_L = 6e^{-12t}$ V. and $i_L(0) = 10$ A

* if $i_L = 8(1-e^{-40t})$ mA, find :-

c) The power being delivered to the inductor at $t = .50\,ms$

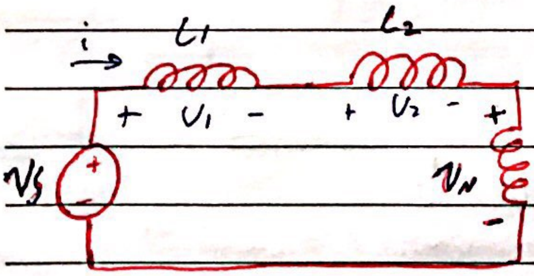
d) The energy stored in the inductor at $t = 40\,ms$.

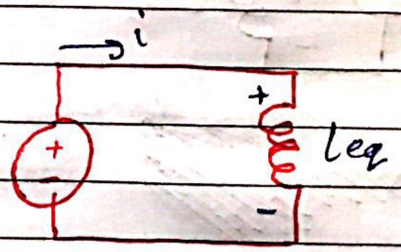
Solu: a) $-15.06\,mV$ b) $23.48\,A$

c) $7.49\,\mu W$ d) $0.51\,\mu J$

* inductance and capacitance combinations:-

1) inductors in series





$$v_s = v_1 + v_2 + \dots + v_N$$

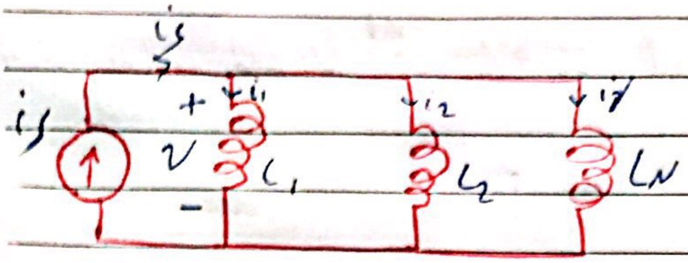
$$= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \dots + L_N \frac{di}{dt}$$

$$v_s = L_{eq} \frac{di}{dt}$$

$$v_s = \underbrace{(L_1 + L_2 + L_N + \dots)}_{L_{eq}} \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + \dots + L_N$$

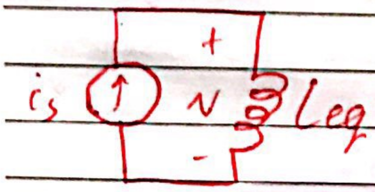
inductors in parallel :-



$i_s = i_1 + i_2 + \dots + i_N$

$$= \frac{1}{L_1} \int v\,dt + \frac{1}{L_2} \int v.\,dt + \dots + \frac{1}{L_N} \int v.\,dt$$

$$i_s = \left[\frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N} \right] \int v.\,dt$$

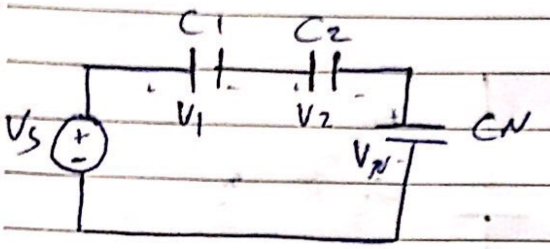


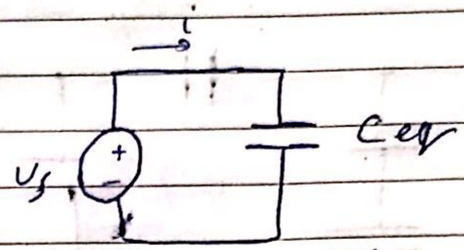
[redacted] $i_s = \frac{1}{L_{eq}} \int v.\,dt$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}$$

* $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \longrightarrow L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$

* Capacitors in series :





$$V_s = V_1 + V_2 + \dots + V_N$$

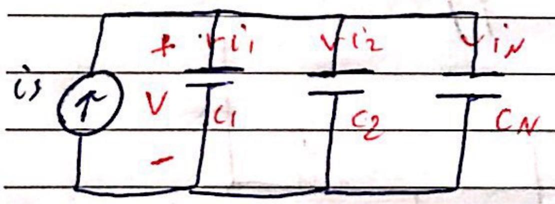
$$= \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt + \dots + \frac{1}{C_N}\int i\,dt$$

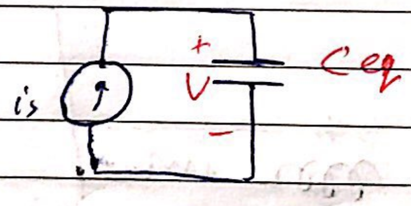
$$V_s = \left[\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}\right]\int i\,dt$$

$$V_s = \frac{1}{C_{eq}}\int i\,dt$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

* Capacitors in parallel :-





$$i_s = i_1 + i_2 + \dots + i_N$$

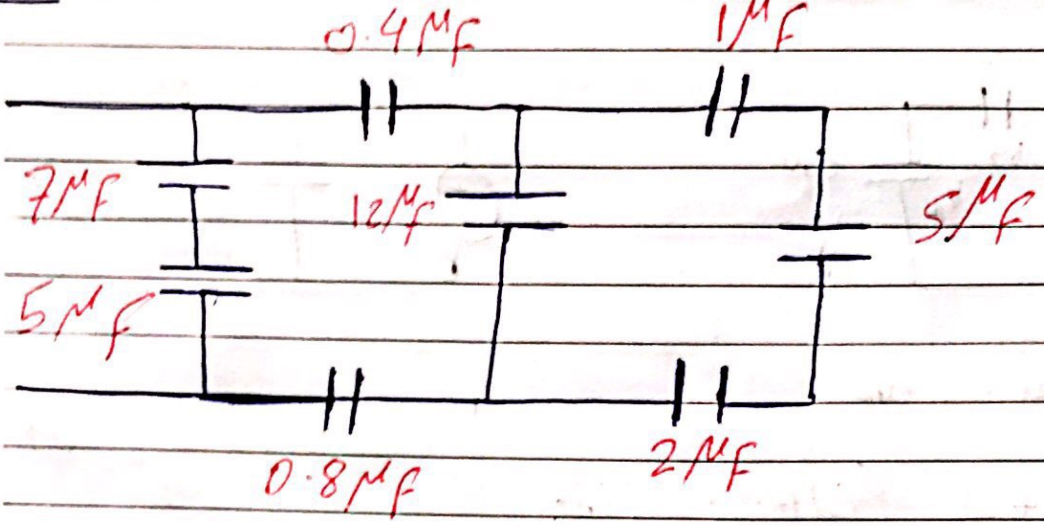
$$= C_1\frac{dV}{dt} + C_2\frac{dV}{dt} + \dots + C_N\frac{dV}{dt}$$

$$= \left[C_1 + C_2 + \dots + C_N\right]\frac{dV}{dt}$$

$$i_s = C_{eq}\frac{dV}{dt}$$

$$C_{eq} = C_1 + C_2 + \dots + C_N$$

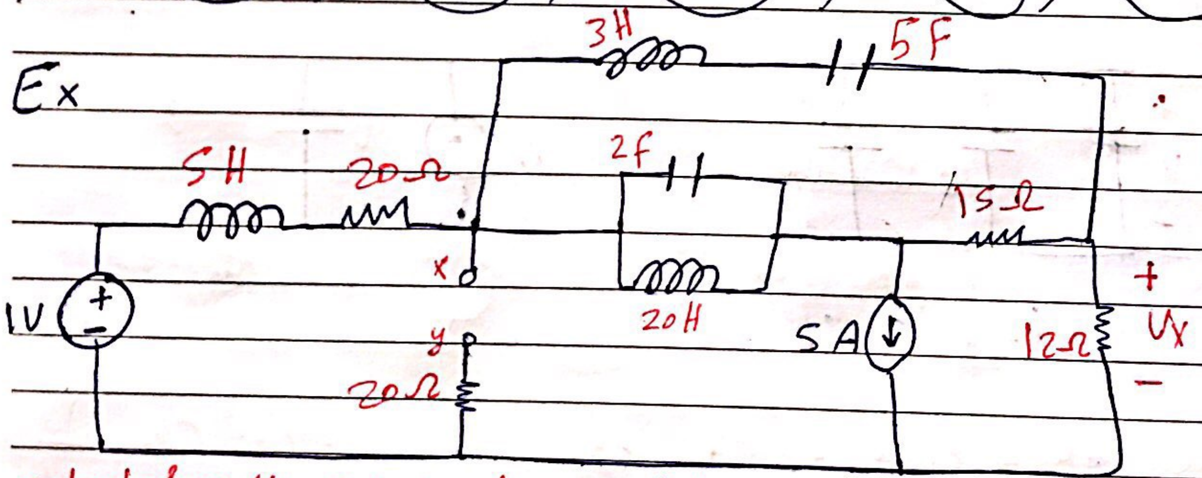
Ex:



① $\frac{1\times5}{6} = \frac{5}{6}\,\mu F$ ② $\left(\frac{\frac{5}{6}\times2}{\frac{5}{6}+2}\right)\mu F = \frac{10}{17}\,\mu F$

continue the solution $\boxed{C_{eq} = 3.178\ \mu F}$

Ex



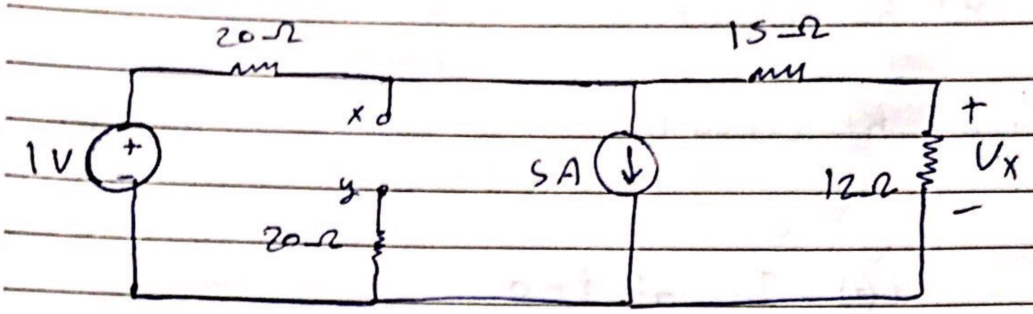
calculate V_x assuming the circuit has been running a long time if:-

a) a 10 Ω resistor is connected between x and y

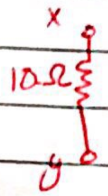
b) a 1 H inductor " " " " " "

c) a 1 F capacitor " " " " " "

d) a 4 H in parallel with 1 Ω is conected between x and y



a)



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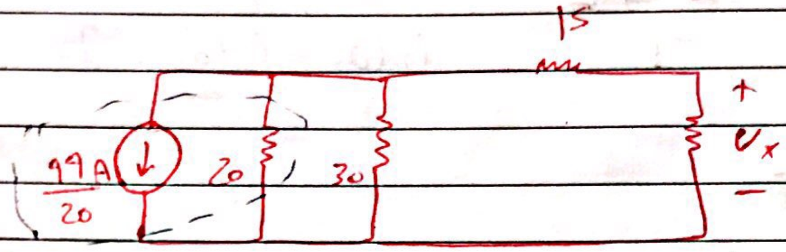
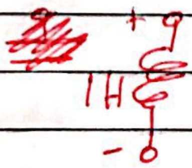


source transformation and addition current sources

$$\frac{30 \times 20}{30+20} = 12\ \Omega \qquad V_x = 12\ \frac{99}{20}\left(\frac{12}{34}\right)$$

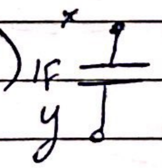
$$= -18.64\ V$$

b)



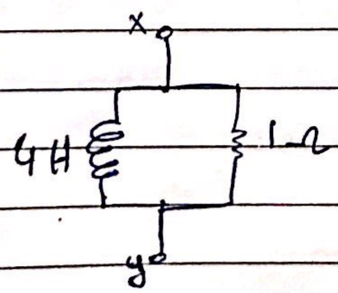
→ short circuit Solu : −16.38

c)



→ open circuit Solu : −25.79

d)



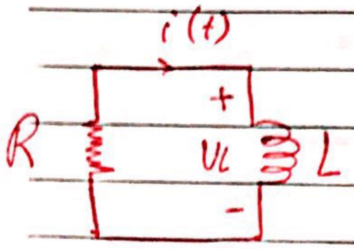
→ short circuit → same b

Solu : −16.38

CH 7: first order circuits

* Basic RL and RC circuits :-

- The source-free RL circuit :-



$i(t) = I_o$ at $t=0$

$i_L(0^-) = i_L(0^+) = I_o$

by KVL

$$L\frac{di}{dt} + iR = 0 \longrightarrow \frac{di}{dt} + \frac{R}{L} i = 0$$

$$i(t) = A e^{s_1 t} \longrightarrow \frac{di}{dt} = A s_1 e^{s_1 t}$$

$$A s_1 e^{s_1 t} + \frac{R}{L} A e^{s_1 t} = 0$$

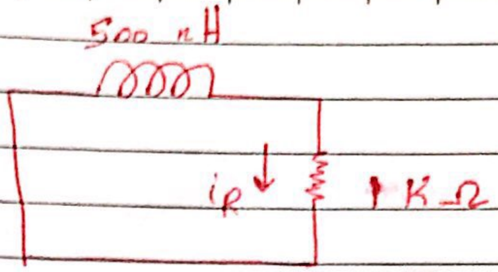
$$A e^{s_1 t}\left[s_1 + \frac{R}{L}\right] = 0 \longrightarrow s_1 + \frac{R}{L} = 0$$

$$\boxed{s_1 = -\frac{R}{L}}$$

$$i(t) = A e^{-\frac{Rt}{L}} \longrightarrow i(0) = A = I_o$$

$$i(t) = I_o e^{-\frac{Rt}{L}}$$

Ex8



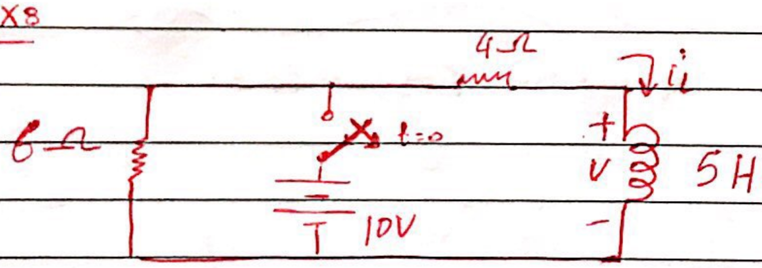
Determin the current i_R through the resistor at $t = 1ns$ if $i_R(0) = 6A$

$$i_R = i_L = I_0 e^{-\frac{Rt}{L}}$$

$$= 6e^{\frac{-1000t}{500\times10^{-9}}} = 6e^{-2\times10^{9}t}\Big|_{t=1ns}$$

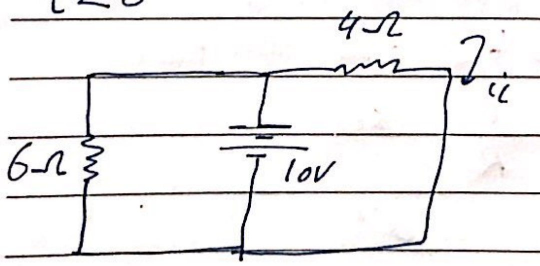
$$= 6e^{-2} = 0.812A = 812 nA$$

Ex8



Determin the inductor Voltage v in the circuit shown for $t > 0$.

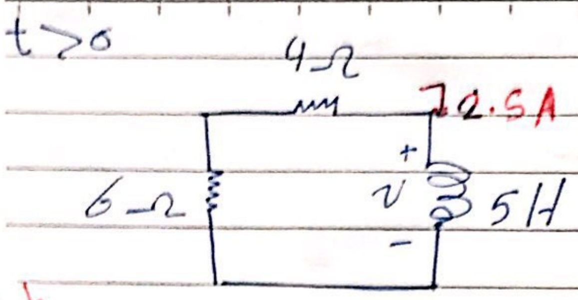
$t < 0$



$$i_L = \frac{10}{4} = 2.5A$$

$$i_L(0^-) = i_L(0^+) = 2.5A$$

$t > 0$



طريقة 1

$$i_L(t) = I_o \, e^{-\frac{Rt}{L}} = 2.5\, e^{-\frac{10t}{5}} = 2.5\, e^{-2t}$$

$$v_L(t) = L\frac{di}{dt} = 5(2.5)(-2)\, e^{-2t} = -25\, e^{-2t} \ V$$

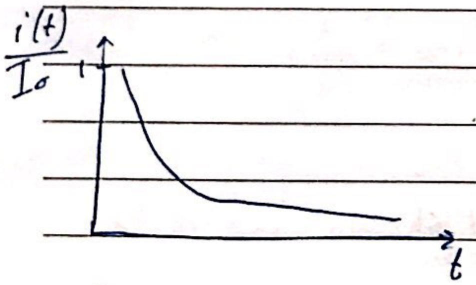
طريقة 2

$$v(t) = A\, e^{-2t} \qquad (V(0) = A)$$

$$v(0^+) = (-2.5)(6+4) = -25V \longrightarrow \text{mesh loop}$$

$$v(t) = -25\, e^{-2t} \ V \longrightarrow \text{same answer}$$

* properties of the exponential responses:-



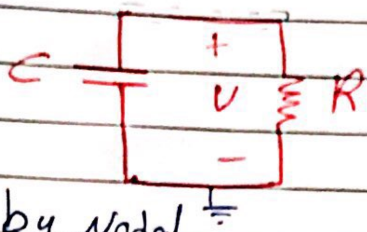
$$i(t) = I_o\, e^{-\frac{Rt}{L}} = I_o\, e^{-\frac{t}{\tau}}$$

τ = the time constant

$\tau = \frac{L}{R}$ sec

$$\frac{i(t)}{I_o} = e^{-\frac{t}{\tau}} \longrightarrow \frac{i(t=\tau)}{I_o} = e^{-1}$$

* The Source-Free RC Circuits :-



by Nodal

$$\frac{v}{R} + C\frac{dv}{dt} = 0 \qquad (v(0) = V_0)$$

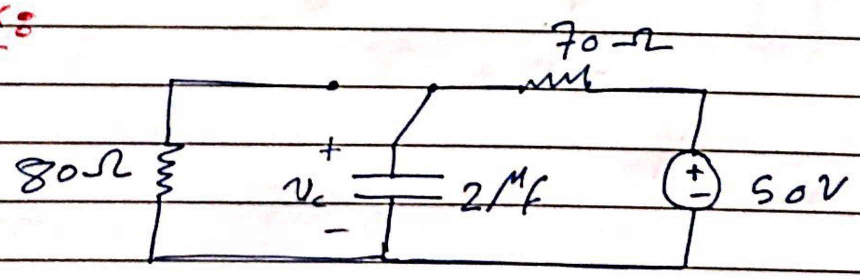
$$\frac{dv}{dt} + \frac{1}{RC} v = 0 \qquad \boxed{v(t) = A e^{s_1 t}}$$

نفس الإشتقاق السابقة بطلع معنا

$$v(t) = V_0 e^{\frac{-t}{RC}}$$

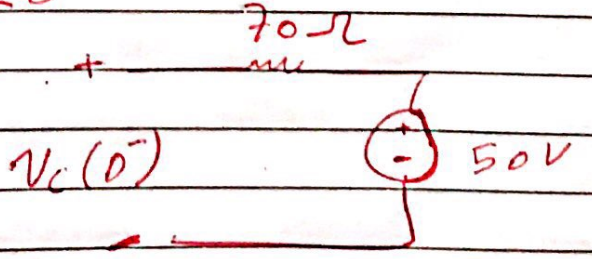
* the time constant $\tau = RC$ sec

Ex:



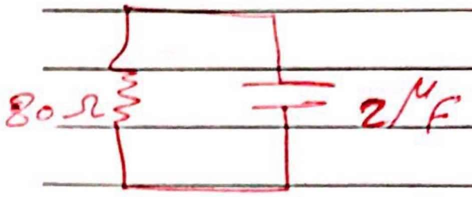
Determin $v(t)$ at $t=0$ and $t = 160\,\mu s$.

$t < 0$



$$V_c(0^-) = V_c(0^+) = 50V$$

$t > 0$



$V_c(0) = 50\,V$

$$v(t) = V_0 e^{-\frac{t}{RC}} = 50\, e^{\frac{-t}{80 \times 2 \times 10^{-6}}} = 50\, e^{-6250t}$$

$v(0) = 50\,V$

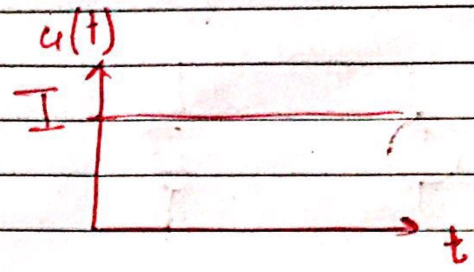
$$v\,(t = 160\,\mu s) = 50\, e^{-6250(160 \times 10^{-6})} = 18.34\,V$$

Five Apple

Scanned with CamScanner

* The unit step Function:-

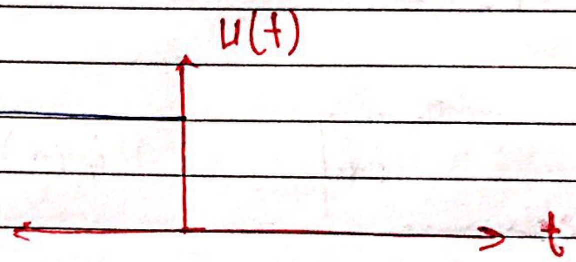
$$u(t) = \begin{cases} 1 & , \; t > 0 \\ 0 & , \; t < 0 \end{cases}$$



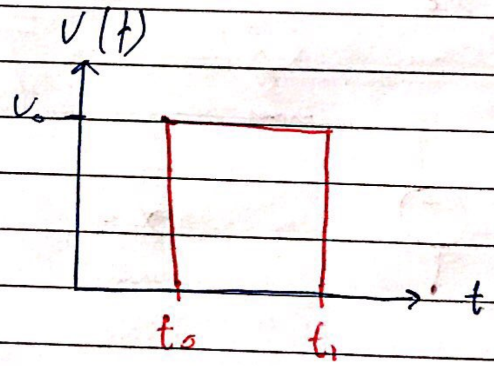
$$u(t - t_0) = \begin{cases} 1 & , \; t > t_0 \\ 0 & , \; t < t_0 \end{cases}$$

$t - t_0 > 0 \qquad\qquad t > t_0$

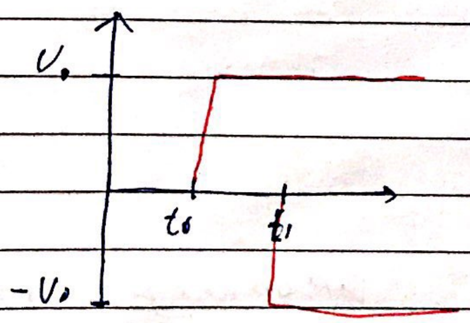
$$u(-t) = \begin{cases} 0 & , \; t > 0 \\ 1 & , \; t < 0 \end{cases}$$

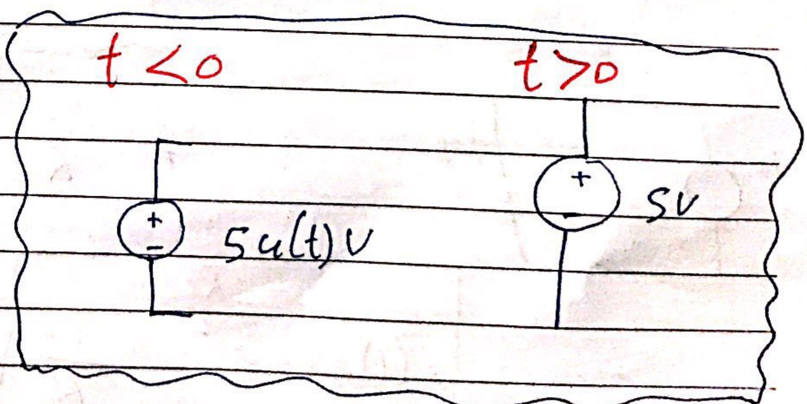


$$v(t) = \begin{cases} 0 & , \; t < t_0 \\ V & , \; t_0 \leq t \leq t_1 \\ 0 & , \; t > t_1 \end{cases}$$

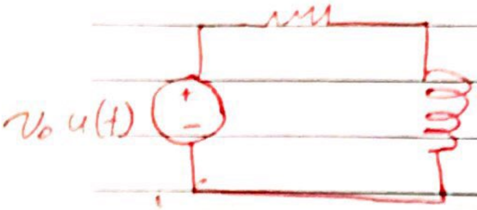


$$v(t) = V_0 \left[u(t - t_0) - u(t - t_1) \right]$$





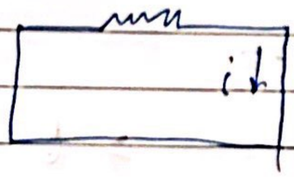
Driven RL circuit



$i(t) = i_n + i_f$

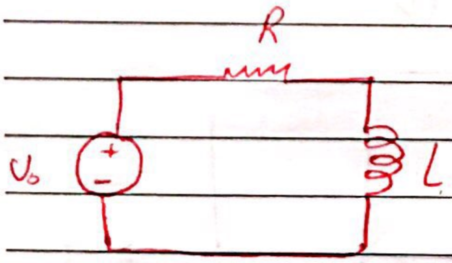
i_n : Natural current
i_f : forced current

if $t < 0$



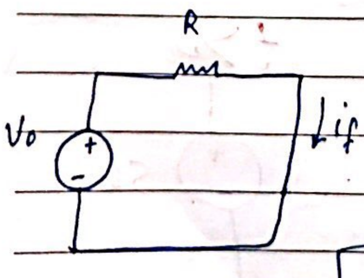
$i_L(0^-) = i_L(0^+) = 0$

$t > 0$



$i_n = A e^{-\frac{Rt}{L}}$

$t \longrightarrow \infty$



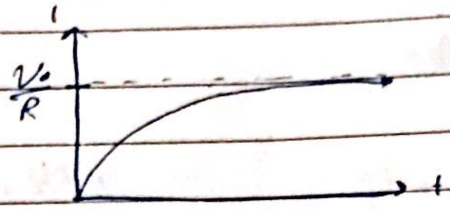
$i_f = \frac{V_0}{R}$

$$\boxed{i(t) = A e^{-\frac{Rt}{L}} + \frac{V_0}{R}}$$

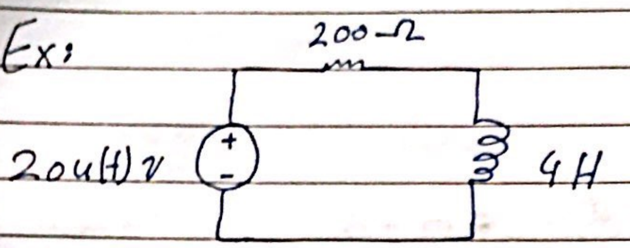
لإيجاد A بعوض بصفر

$0 = A + \frac{V_0}{R}$ $\therefore A = \frac{-V_0}{R}$

$$i(t) = \frac{V_0}{R}\left[1 - e^{-\frac{Rt}{L}}\right]$$

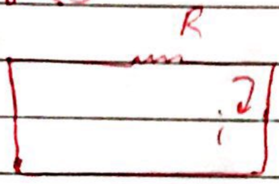


Ex:



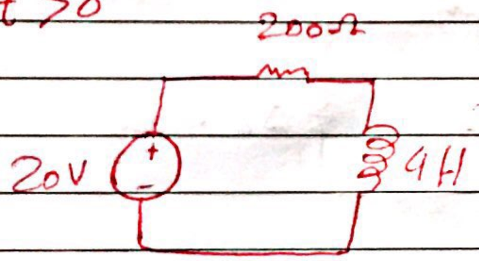
- Find $i_L(0^-)$, $i_L(0^+)$ and $i_L(15ms)$

$t<0$



$i_L(0^-) = i_L(0^+) = 0$

$t>0$

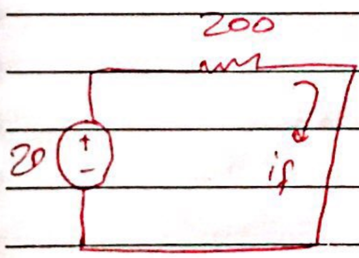


$i(t) = i_n + i_f$

$i_n = Ae^{-50t}$

$i_f = \frac{20}{200} = 0.1\,A$

$t \rightarrow \infty$



$i(t) = Ae^{-50t} + 0.1$

$0 = A + 0.1 \rightarrow A = -0.1$

$\therefore i(t) = 0.1\left[1 - e^{-50t}\right] A$

$i(15ms) = 0.1\left[1 - e^{-50(15\times10^{-3})}\right]$

$= \boxed{52.8\ mA}$

نفس السؤال بإذا لدي أوجد $V(t)$

$v(t) = v_n + v_f$

$= Ae^{-50t} + v_f$ $\qquad v(0^+) = i(0^+) * 200 \quad = 0$

at $t \to \infty$

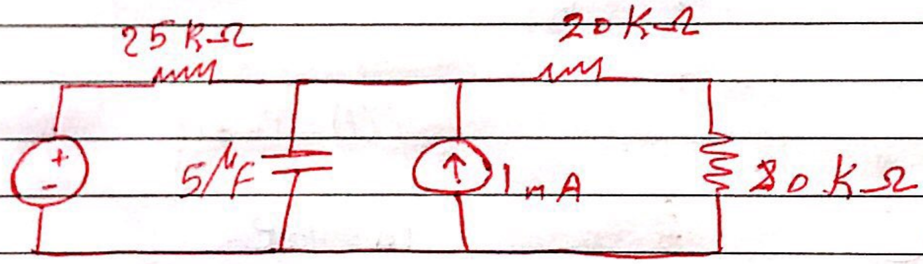
$v_f = 20V$

$v(t) = Ae^{-50t} + 20 \quad \to v(0^+) = 0 = A + 20 \quad \to A = -20$

$v(t) = 20(1 - e^{-50t})$

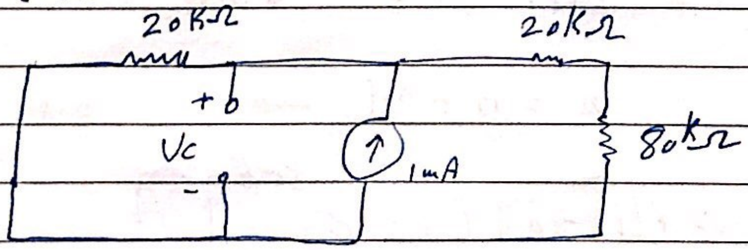
Driven RC circuit :-

Ex:



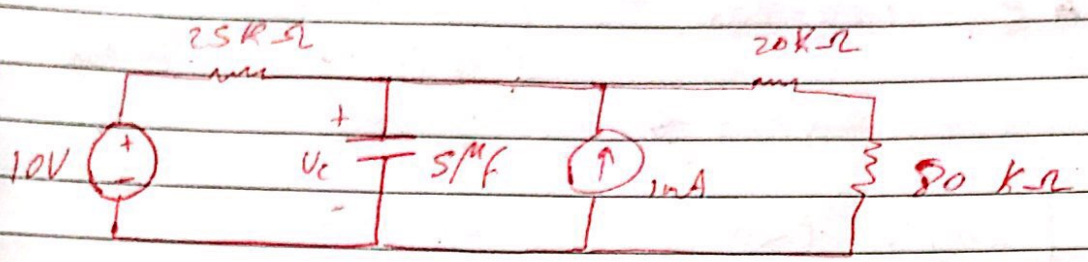
Find, $v_c(0^-)$, $v_c(0^+)$, $v_c(\infty)$ and $v_c(0.08 \text{ sec})$:-

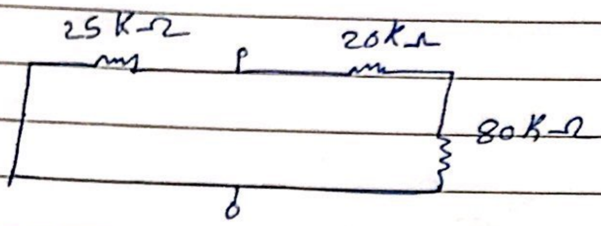
$t < 0$



$v_c(0^-) = v_c(0^+) = \left(\frac{100K}{125K} \cdot 1mA\right) 25K \quad = 20V$

$t > 0$



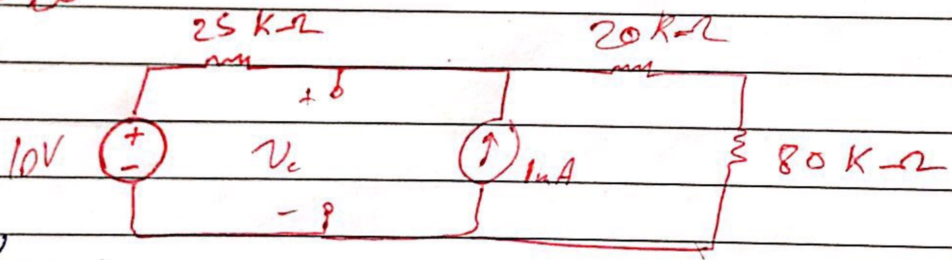


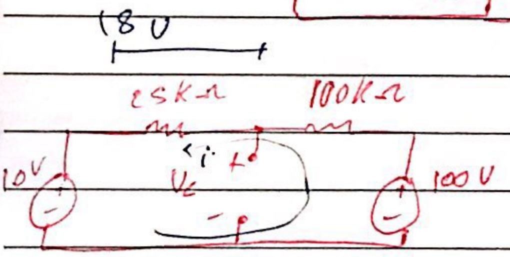
$(20k + 80k) \parallel 25k = 20k\Omega$

$V_c(t) = V_{cN} + V_{cf}$

$V_{cn} = A e^{-\frac{t}{R_{eq}C}} = A e^{\frac{-t}{(20 \times 10^{3} \times 5 \times 10^{-6})}} = A e^{-10t}$

$t \to \infty$





$i = \frac{100 - 10}{125k} = 0.72\ mA$

KVL

$-10 - 18 + V_c = 0 \longrightarrow V_c = 28V$

$v_c(t) = A e^{-10t} + 28 = 20V$

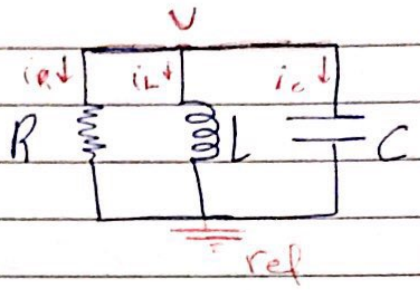
$v_c(t) = 20 + 8(1 - e^{-10t})\, u(t)$

$20 = A + 28 \longrightarrow A = -8$

$$v_c(t) = \begin{cases} 20 , & t < 0 \\ 28 - 8e^{-10t} , & t > 0 \end{cases}$$

* The RLC circuit:

- the source-Free parallel RLC circuit:



KCL:-

$$\frac{v}{R} + \frac{1}{L}\int_{t_0}^{t} v\,dt - i_L(t_0) + C\frac{dv}{dt} = 0 \quad \Big] \frac{d}{dt}$$

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

with $i_L(0^+) = I_0$ and $v_C(0^+) = V$

and I Need

$$i_C(t) = C\frac{dv}{dt} \longrightarrow \frac{dv}{dt}(0^+) = \frac{i_C(0^+)}{C}$$

$$\frac{dv}{dt} = \frac{i_C}{C}$$

$$= \frac{-[i_L(0^+) + i_R(0^+)]}{C} \quad , \quad i_R(0^+) = \frac{V_0}{R}$$

The form of solution

$v = Ae^{st}$

$$\frac{dv}{dt} = sAe^{st} \longrightarrow \frac{d^2v}{dt^2} = s^2Ae^{st}$$

$$s^2Ae^{st} + \frac{1}{Rc}sAe^{st} + \frac{1}{Lc}Ae^{st} = 0$$

$$Ae^{st}\left[s^2 + \frac{1}{Rc}s + \frac{1}{Lc}\right] = 0$$

$$s^2 + \frac{1}{Rc}s + \frac{1}{Lc} = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$s_{1,2} = \frac{-1}{2Rc} \pm \sqrt{\left(\frac{1}{2Rc}\right)^2 - \left(\frac{1}{Lc}\right)^2}$$

1. if s_1 and s_2 are real and distinct $\rightarrow$ (overdamped system).

$$v(t) = A_1e^{s_1t} + A_2e^{s_2t}$$

2. if $s_1 = s_2$ and real $\rightarrow$ (critically damped system)

$$v(t) = e^{st}\left[A_1t + A_2\right]$$

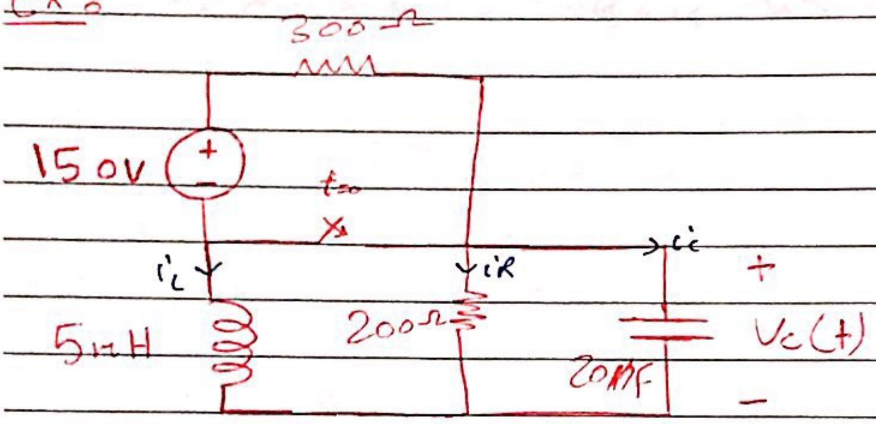
③ if s_1 and s_2 are complex conjugates → underdamped

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$$s_{1,2} = -\alpha \pm j\omega_d \qquad \alpha = \frac{1}{2RC}, \quad \omega_o = \frac{1}{\sqrt{LC}}$$

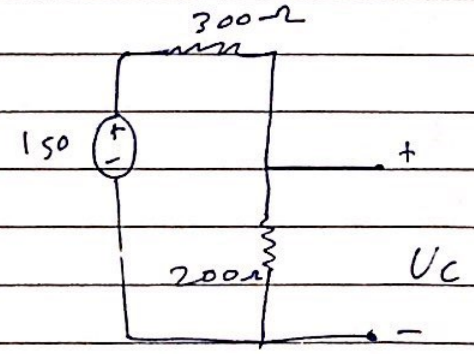
where $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

Ex 8



Find expression for $V_c(t)$ for $t > 0$?

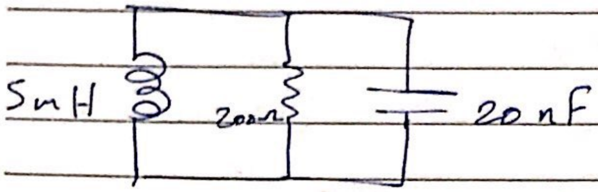
$t < 0$



$$V_c(0^-) = \frac{200}{500} \times 150 = 60\ V = V_c(0^+)$$

$$i_L(0^-) = \frac{-150}{500} = -300\,mA = i_L(0^+)$$

$t>0$



$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 200\times 20\times 10^{-9}} = 125,00 \ s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\times 10^{-9}\times 20\times 10^{-9}}} = 100,000 \ s^{-1}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 + \omega_0^2} = -50,000$$

$$s_2 = -\alpha - \sqrt{\alpha^2 + \omega_0^2} = -200,000$$

$$v_c(t) = A_1 e^{-50000t} + A_2 e^{-200000t}$$

$$v_c(0^+) = 60 = A_1 + A_2 \quad --\ (1)$$

$$C\frac{dv}{dt} = i_c \longrightarrow \frac{dv}{dt}(0^+) = \frac{i_c(0^+)}{C}$$

$$i_R(0^+) = \frac{60}{200} = 300\,mA$$

$$i_c(0^+) = -\frac{[i_R(0^+) + i_L(0^+)]}{C} \longrightarrow \frac{dv}{dt} = 0 \ V/s$$

Five Apple

$$\frac{dV_c(t)}{dt}\Big|_{t=0} = -50000\, A_1\, e^{-50000t} - 200000\, A_2\, e^{-200000t}$$

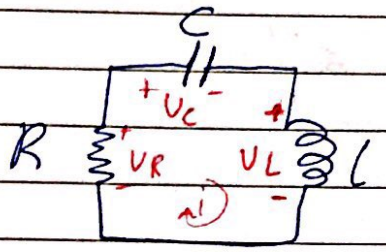
$$-50000\, A_1 - 200\,000\, A_2 = 0$$

① $A_1 + 4A_2 = 0$

② $A_1 + A_2 = 6 \longrightarrow A_1 = 80 \quad A_2 = -20$

* The source free RLC circuit :-

↳ series



$$L\frac{di}{dt} + iR + \left(\frac{1}{C}\int_{t_0}^{t} i\,dt - v_c(t_0)\right) = 0$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}\, i = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}\, i = 0$$

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (over damped system)

$$S_{1,2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{R}{2L} \quad \text{and} \quad w_o = \frac{1}{\sqrt{LC}}$$

critically damped system $\alpha = w_o$

$$i(t) = e^{-\alpha t}(A_1 t + A_2)$$

for under damped system:

$$i(t) = e^{-\alpha t}\left[B_1 \cos w_d t + B \sin w_d t\right]$$

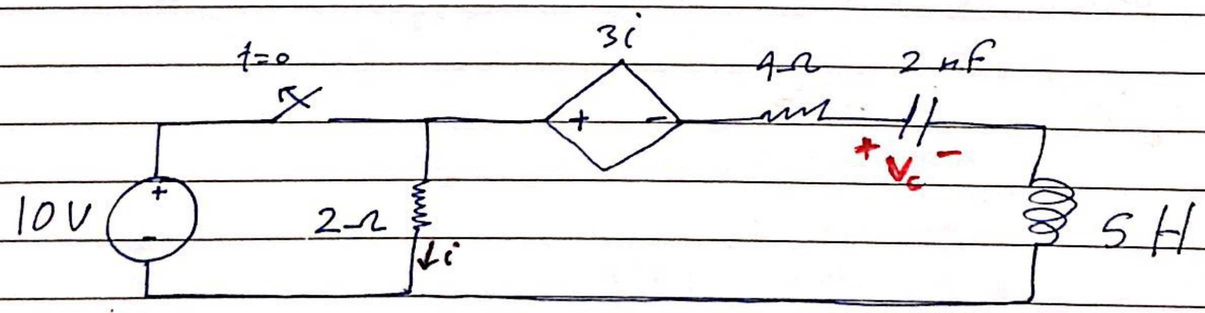
where $w_d = \sqrt{w_o^2 - \alpha^2}$

remember:-

important $i(0^+)$, $\frac{di}{dt}(0^+)$

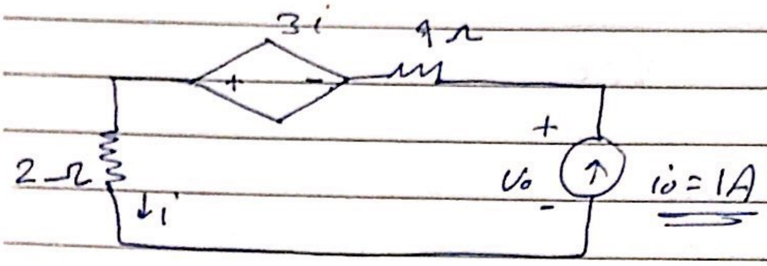
$v_L(0^+) = L \frac{di}{dt}(0^+) \rightarrow \frac{di}{dt}(0^+) = \frac{V_c(0^+)}{L}$

Ex:



Find an expresion for $v_c(t)$ for $t>0$?

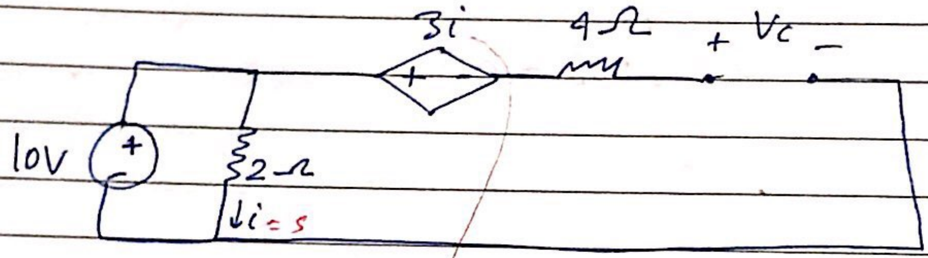
to find Rth :-



$4(1) - 3(1) + 2(1) - V_o = 0 \longrightarrow V_o = 8\ V$

$R_{th} = \frac{8}{1} = 8\,\Omega$ ✓

t<0



$i = \frac{10}{2} = 5A$ $3i = 15$ volt

KVL

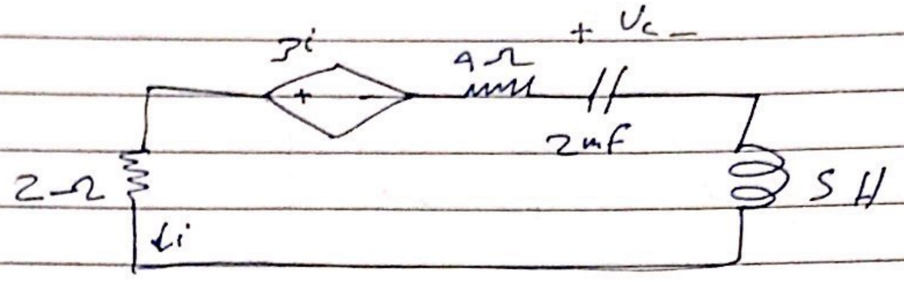
$-10 + 15 + V_c = 0 \longrightarrow V_c = -5V$

$V_c(0^+) = V_c(0^-) = -5\ V$

$i_L(0^+) = i_L(0^-) = 0$

open circuit

$t > 0$



$$\alpha = \frac{R}{2L} = \frac{8}{10} = 0.8 \ s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 2 \times 10^{-3}}} = 10 \ \text{rad/sec}$$ (under damped)

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 9.968 \ \text{rad/sec}$$

$$* \ v_c(t) = e^{-0.8t}\left[B_1 \cos(9.968t) + B_2 \sin(9.968t)\right]$$

$$v_c(0^+) = -5 = e^{0}\left[B_1 + 0\right] \longrightarrow B_1 = -5$$

$$\left.\frac{dv_c}{dt}\right|_{t=0} = -0.8 e^{-0.8t}\left[B_1 \cos(9.9866) + B_2 \sin(9.968 t)\right] + e^{-0.8t}\left[-9.986 B_1 \sin(9.968t) + 9.968 B_2 \cos(9.986t)\right]$$

$$0 = -0.8 B_1 + 9.986 B_2$$

$$C \frac{dv_c(0^+)}{dt} = i_c(t)$$

$$\frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C}$$

$$= \frac{0}{C} = 0$$

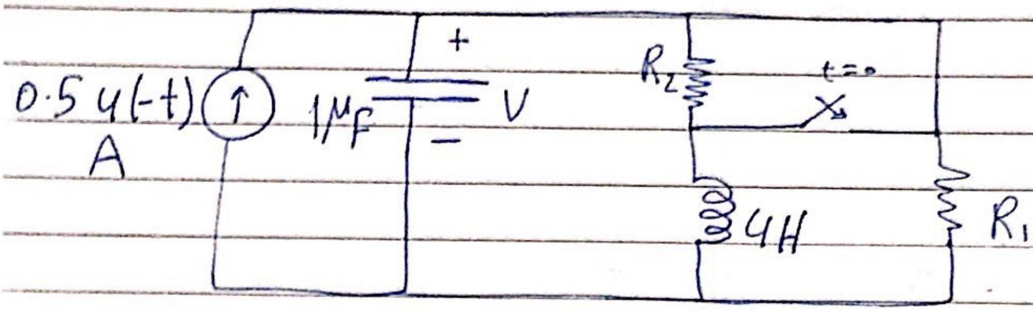
$$-0.8(-5) + 9.968 B_2 \Rightarrow B_2 = -0.4013$$

نعوض في ✱ لإيجاد المعادلة النهائية

$$v_c(t) = -e^{-0.8t}\left[5 \cos(9.968t) + 0.4013 \sin(9.968t)\right]$$

Five Apple

Ex8

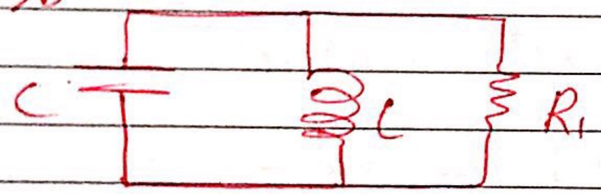


a) R_1 such that critically damped system

b) R_2 to obtain $v(0) = 100V$

c) find $v(t)$ at $t = 1ms$

$t > 0$



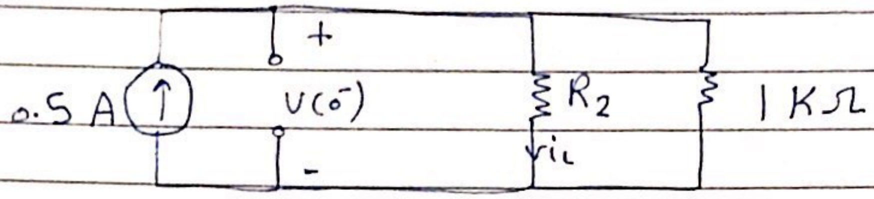
$\alpha = \frac{1}{2R_1C}$, $\omega_o = \frac{1}{\sqrt{LC}}$

$\omega_o = \frac{1}{\sqrt{4 \times 1 \times 10^{-6}}} = 500 \ rad/sec$

$500 = \frac{1}{2R_1(1 \times 10^{-6})} \longrightarrow R_1 = \frac{1}{500\,(2 \times 10^{-6})} = 1 \, k\Omega$

$v(t) = e^{-500t}\left[A_1 t + A_2\right]$

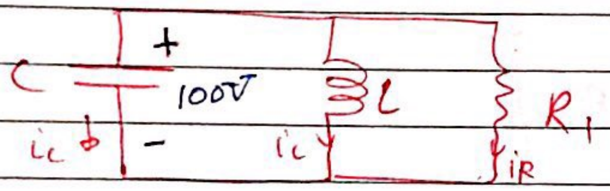
$t<0$



$$100 = (R_1 // R_2)(0.5) \longrightarrow 100 = \frac{1000 R_2}{1000 R_2}(0.5)$$

$\boxed{R_{eq} = 200\,\Omega}$ $\boxed{R_2 = 250\,\Omega}$

Back to ~~t<0~~ $t>0$



$$i_L = \frac{1000}{250} \times 0.5 = 0.4A$$

$$\frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C} = -\frac{[i_c(0^+) + i_L(0^+)]}{C}\ 5A$$

$$= -\frac{[0.1 + 0.4]}{1\times10^{-6}} = -500 \text{ rad/s}$$

$$\frac{dv(0^+)}{dt} = -0.5\times10^6 \text{ V/s}$$

$v(0) = 100 = A_2$

$$\left.\frac{dv}{dt}\right|_{t=0} = -500e^{-500t}(A_1 t + A_2) + A_1 e^{-500t}\Big|_{t=0}$$

$$= -500 A_2 + A_1 = -0.5\times10^6$$

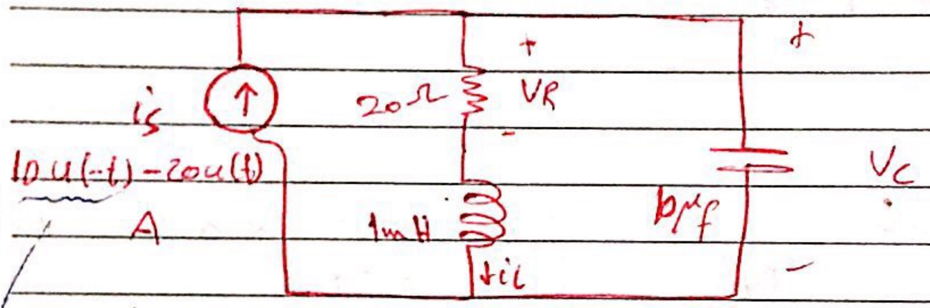
$A_1 = -450,000$

$v(t) = e^{-500t}(100 - 450,000t)$

$t = 1ms \quad = e^{-0.5}(100 - 450) = -2123 \text{ V}$

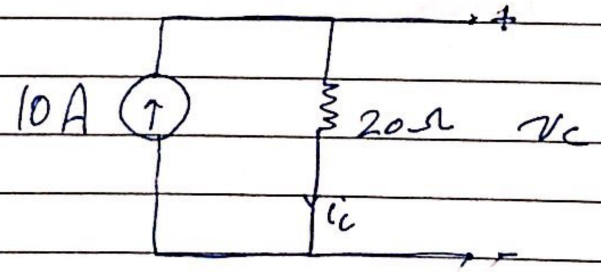
*** Driven Series RLC circuit :-**

Ex :



Find : $i_L(0^-)$, $V_C(0^+)$, $V_R(0^+)$, $i_L(\infty)$ and ~~$i_L(...)$~~ $i_L(0.1\ ms)$

$t < 0$



$i_L(0^+) = i_L(0^-) = 10 \text{ A}$

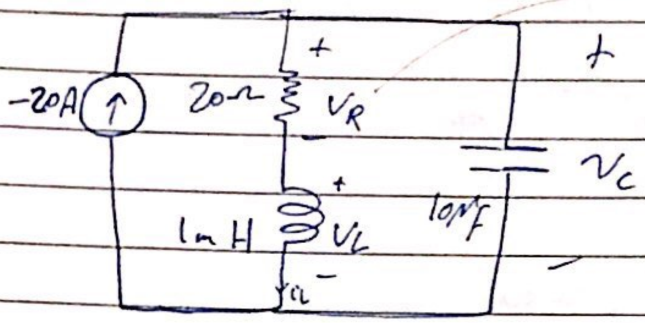
$V_C(0^-) = 20 \times 10 = 200 \text{ volt} = V_C(0^+)$

Five Apple

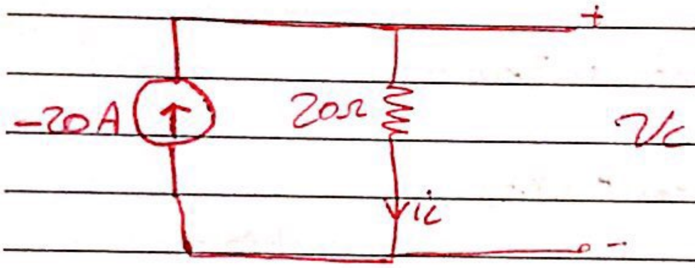
$t > 0$

بعوض عن نفس تيار $i_L = 10A$

$V_R(0^+) = i_L(0^+) \times 20 = 200$ Volt



$t \longrightarrow \infty$



$i_L(\infty) = -20\ A$

لوطلب ← $V_C(\infty) = -20 \times 20 = -400\ V$

$$\alpha = \frac{R}{2L} = \frac{20}{2 \times 1 \times 10^{-3}} = 10{,}000\ s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10^{-3} \times 10 \times 10^{-6}}} = 10{,}000\ \text{rad}/s$$

$\alpha = \omega_0 \longrightarrow$ critically damped

$i_L(t) = i_n + i_f$

$= e^{-10000t}[A_1 t + A_2] - 20$

$i_L(0^+) = 10 = A_2 - 20 \longrightarrow \boxed{A_2 = 30}$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = -10000\, e^{-10000t}[A_1 t + A_2] + e^{-10000t} A_1 \Big|_{t=0^+}$$

$$\frac{di_L(0^+)}{dt} = -10000 A_2 + A_1 = 0$$

$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{0}{L} = 0$$

$v_C(0^+) = 200$

$v_R(0^+) = 200$

$v_C = v_R + v_L$ parallel

$A_1 = (10{,}000)(30) \longrightarrow A_1 = 300000$

$i_L(t) = -20 + e^{-10000t}[300000\, t + 30]$

$t > 0$

$i_L(0.1\ ms) = -20 + e^{-1}(30 + 30)$

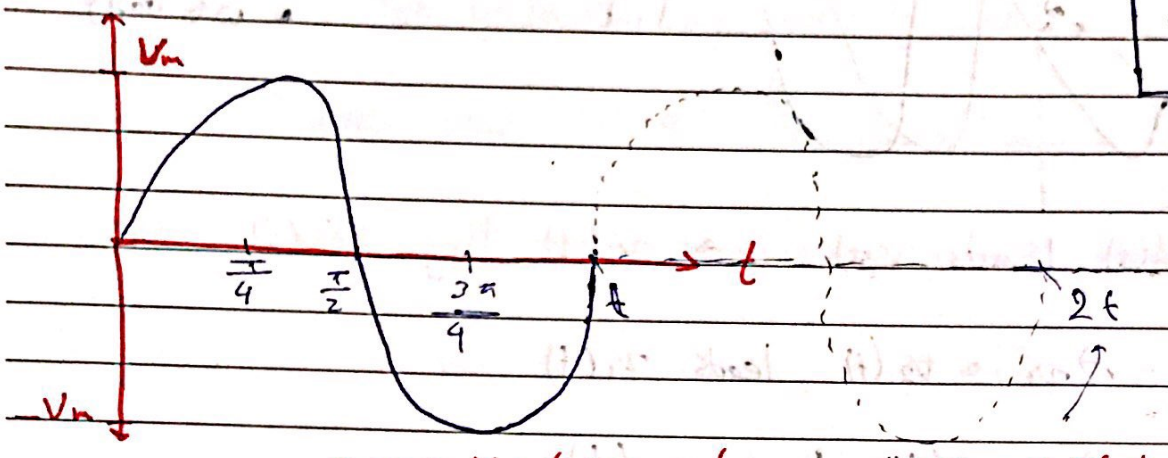
$i_L(0.1\ ms) = 2.07\ A$

CH9-CH10 :- Sinusoidal Steady-state Analysis :-

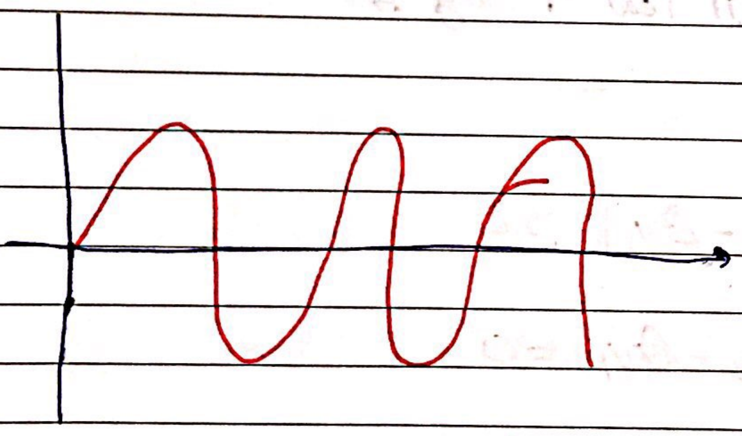
characteristics of sinusoidal :-

$\sin\theta$
$\sin(\omega t)$
ωt (rad)
ω (rad/sec)

$v(t) = V_m \sin(\omega t)$



periodical signal $\rightarrow v(t) = v(t+t)$



$\omega T = 2\pi$

$\omega = \frac{2\pi}{T}$ rad/sec

(radian frequency)

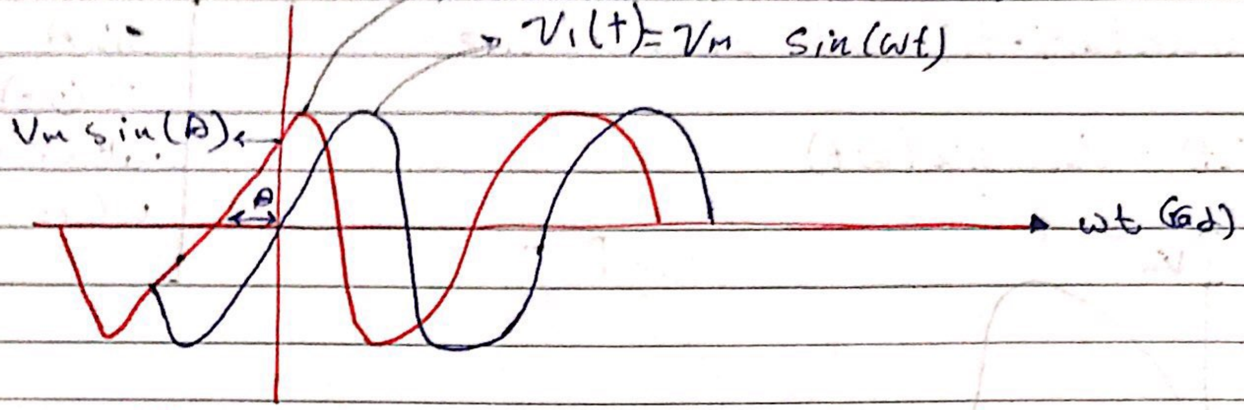
The frequency :-

$f = \frac{1}{T}$ (Hertz) (Hz)

$\omega = 2\pi f$

$v_2(t) = V_m \sin(\omega t + \theta)$

$v_1(t) = V_m \sin(\omega t)$



* $v_2(t)$ leads $v_1(t)$ * $v_1(t)$ lags $v_2(t)$

$\theta_{v_2} - \theta_{v_1}$: $v_2(t)$ leads $v_1(t)$

$\theta_{v_1} - \theta_{v_2}$: $v_1(t)$ leads $v_2(t)$

$\theta - 0 = \theta$: $v_2(t)$ lead v_1 by θ

phase angle:

out of phase $|\theta_{v_2} - \theta_{v_1}| > 0$

in phase $\theta_{v_2} - \theta_{v_1} = 0$

Tow sinsoidal waves whose phases are to be compared much:-

1) Both be written as: sin waves, or Both as cosin waves.

2) Both be written with positive amplitudes.

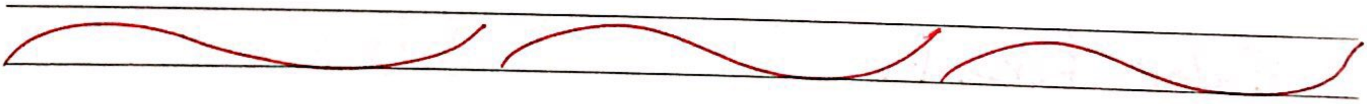
3) Each have the same frequency.

$-\sin\omega t = \sin(\omega t \pm 180^\circ) = \sin\omega t \underbrace{\cos 180}_{-1} \pm \cos\omega t \underbrace{\sin 180}_{zero}$

$-\sin(\omega t - 70^\circ) = \sin(\omega t - 70 + 180^\circ) = \sin(\omega t + 110^\circ)$

$-\cos\omega t = \cos(\omega t \pm 180)$, $\pm\cos(\omega t) = \sin(\omega t \pm 90^\circ)$

$\mp \sin\omega t = \cos(\omega t \pm 90^\circ)$



Ex: $v_1(t) = 120 \cos(120\pi t - 40^\circ)\ V$.

(a) $i_1 = 2.5\cos(120\pi t + 20^\circ)A$ (b) $i_1 = 1.4\sin(120\pi t - 70^\circ)A$

Find the angle at wich v_1 leads i_1 :-

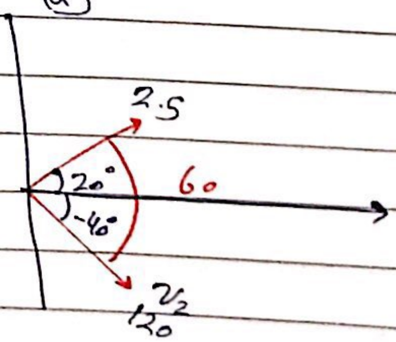
a) $\theta_{v_1} - \theta_{i_1} = -40 - 20 = -60^\circ$

b) $i_1 = 1.4\cos(120\pi t - 70^\circ - 90^\circ)$

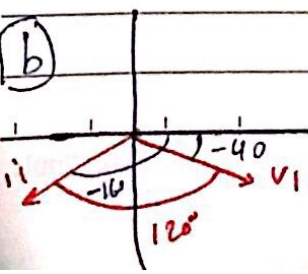
$= 1.4 \cos(120\pi t - 160^\circ)$

$\theta_{v_2} - \theta_{v_1} = -40 - (-160) = 120^\circ$

a



b



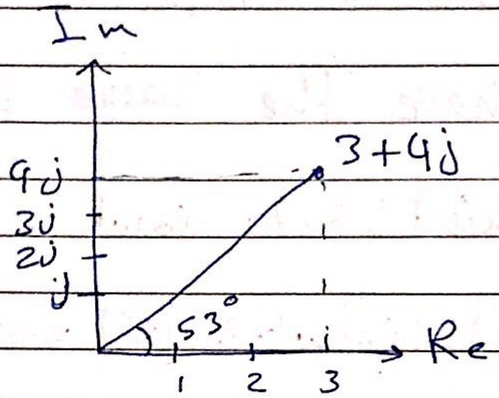
complex Numbers:-

a + jb ⟶ imaginery part

↓

the real part

$A = 3 + j4$



$A = 3 + j4 = 5e^{j53}$

$= 5\left[\cos(53°) + j\sin(53°)\right] = 3 + j4$

Euler fermola

$e^{j\theta} = \cos\theta + j\sin\theta$

rectangular ⟶ Polar form

$A = 3 + j4 = 5\angle 53°$

$B = -3 + j4 = 5\angle 127°$

$A+B = 0 + j8 = 8 \angle 90^\circ$ / $A-B = 6 + j0 = 6 = 6 \angle 0^\circ$

$$\frac{A}{B} = \frac{3+j4}{-3+j4} \times \frac{-3-j4}{-3-j4} = \frac{(3+j4)\ (-3-j4)}{9+16}$$

$$= \frac{-9-j12-j12+16}{25} = \frac{7-j24}{25} = \frac{7}{25} - \frac{j24}{25}$$

$$\frac{A}{B} = \frac{5\angle 53^\circ}{5\angle 127^\circ} = \frac{5e^{j53^\circ}}{5e^{j127^\circ}} = \frac{5}{5}\angle 53-127$$

$$= 1\angle -74^\circ \rightarrow 1\cos(-74^\circ) + j\,1\sin(-74^\circ)$$

$$A \cdot B = (3+j4) \cdot (-3+j4) = -9 + j12 - j12 - 16$$

$$= -25 = 25 \angle 180^\circ$$

$Rea\ (5, 53^\circ) = 3 + j4$

$Pol\ (3, 4) = 5\angle 53^\circ$

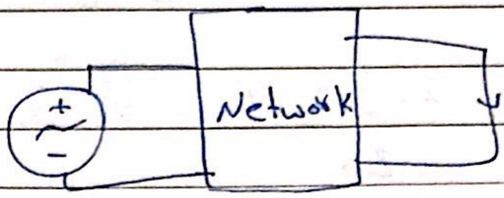
$$A \cdot B = 5\angle 53^\circ \cdot 5\angle 127^\circ$$

$$= 5e^{j53} \cdot 5e^{j127}$$

$$= 25\angle 53^\circ + 127^\circ = 25\angle 180^\circ$$

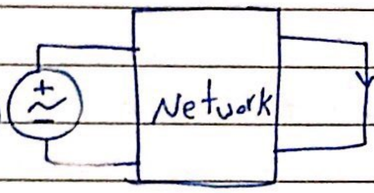
The complex forcing function:-

$V_m \cos(\omega t + \theta)$



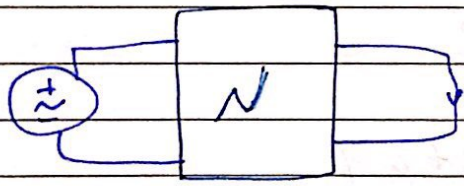
$I_m \cos(\omega t + \phi)$

$jV_m \sin(\omega t + \theta)$



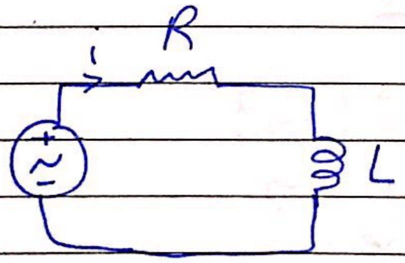
$jI_m \sin(\omega t + \phi)$

$V_m e^{j(\omega t + \theta)}$



$I_m e^{j(\omega t + \phi)}$

$V_s = V_m \cos(\omega t)$



$$V_m \cos(\omega t) = Re\{V_m e^{j\omega t}\}$$

$$I = I_m e^{j(\omega t + \theta)}$$

$$V_m e^{j\omega t} = V_m [\cos \omega t + j \sin \omega t]$$

$$= V_m \cos(\omega t) + j V_m \sin \omega t.$$

$$Ri + L\frac{di}{dt} = V_s$$

$$R I_m \cancel{e^{j(\omega t+\phi)}} + j\omega L I_m \cancel{e^{j(\omega t+\phi)}} = V_m \cancel{e^{j\omega t}}$$

$$R I_m e^{j\phi} + j\omega L I_m e^{j\phi} = V_m$$

$$I_m e^{j\phi} = \frac{V_m}{R + j\omega L}$$

$$I_m \angle\phi = \frac{V_m}{R + j\omega L}.$$

$$= \frac{V_m \angle 0^\circ}{\sqrt{R^2 + \omega^2 L^2} \angle \tan^{-1}\left(\frac{\omega L}{R}\right)}$$

$$= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \left(\cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)\right) A$$

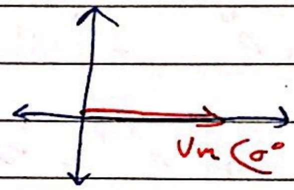
The ~~Phasor~~ Phasor :-

$v(t) = V_m \cos(\omega t) = V_m \cos(\omega t + 0^\circ)$

$V = V_m \angle 0^\circ \quad V.$

$V_m \cos(\omega t) = Re \left\{ V_m e^{j(\omega t + 0^\circ)} \right\}$

$V_m e^{j0^\circ} = V_m \angle 0^\circ \quad V$



$i(t) = I_m \cos(\omega t + \Phi)$

$I = I_m \angle \Phi \quad A.$

<u>Ex:</u> $V = 115 \angle -45^\circ$ V with $\omega = 500$ rad/sec Find $V(t)$:

$v(t) = 115 \cos(500t - 45^\circ) \; V.$

* the resistor

time domain — phasor domain

$V = iR$

$v(t) = R\,i(t) \longrightarrow V(t) = V_m e^{j(\omega t + \theta)}$

$i(t) = I_m e^{j(\omega t + \phi)}$

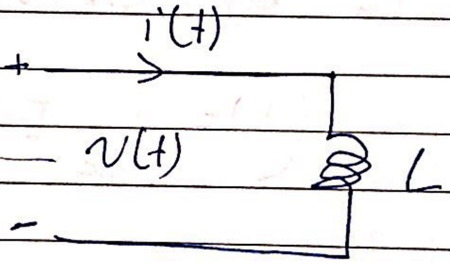
$$V_m \cancel{e^{j(\omega t+\theta)}} = R\,i(t) = R\,I_m \cancel{e^{j(\omega t+\phi)}}$$

$$V_m e^{j\theta} = R\,I_m e^{j\phi}$$

$$V_m \angle\theta = R\,I_m \angle\phi \qquad \boxed{V = RI}$$

*The inductors:-

$$v(t) = L\frac{di(t)}{dt}$$



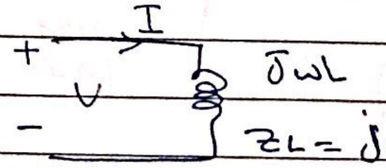
$$v(t) = V_m e^{j(\omega t+\theta)} = L\frac{d}{dt}\left(I_m e^{j(\omega t+\phi)}\right)$$

$$V_m \cancel{e^{j(\omega t+\theta)}} = j\omega L\,I_m \cancel{e^{j(\omega t+\phi)}}$$

$$V_m e^{j\theta} = j\omega L\,I_m e^{j\phi}$$

$$V_m\angle\theta = j\omega L\,I_m\angle\phi$$

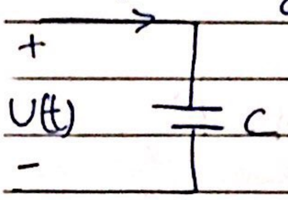
$$\boxed{V = j\omega L\,I}$$



The impedance of $L \Rightarrow Z_L = j\omega L\ \Omega$ $\qquad \boxed{Z = \frac{V}{I}}$

The capacitor :-

$i(t) = C\frac{dV}{dt}$



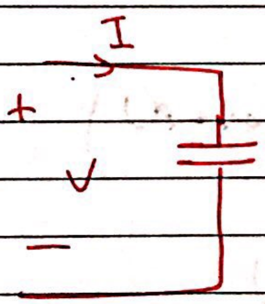
$$i = C\frac{dV}{dt}$$

$$I_m e^{j(\omega t+\phi)} = C\frac{d}{dt}\left(V_m e^{j(\omega t+\theta)}\right)$$

$$I_m e^{j(\omega t+\phi)} = j\omega C\, V_m e^{j(\omega t+\theta)}$$

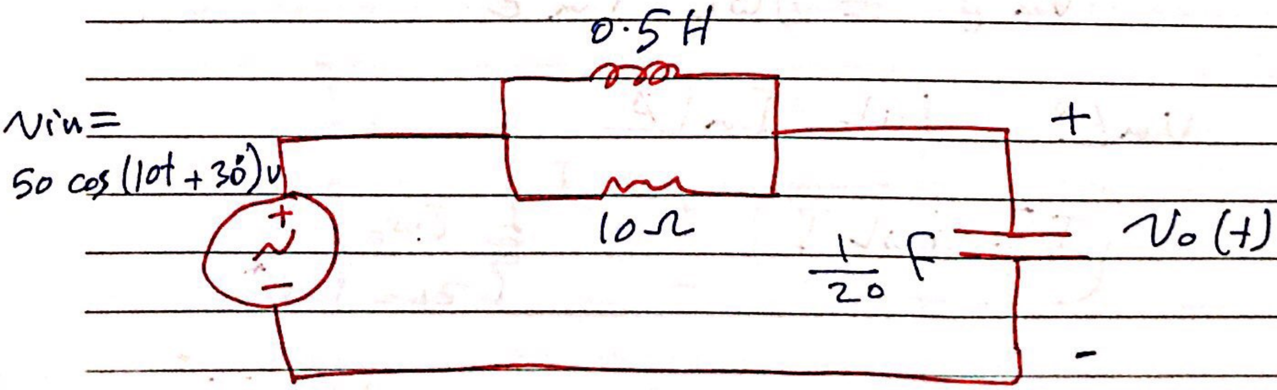
$$I_m \angle\phi = j\omega C\, V_m \angle\theta$$

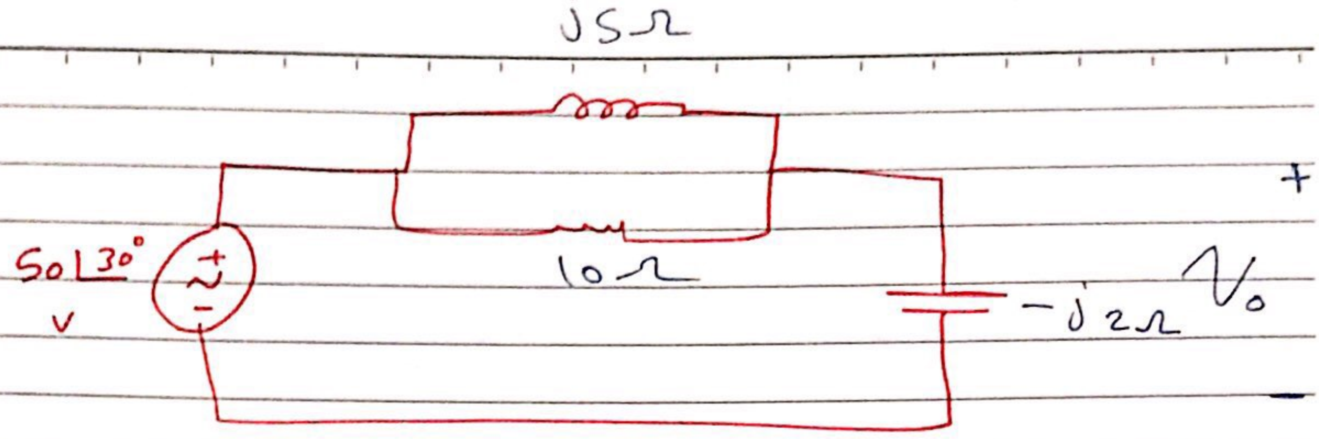
$I = j\omega C V$ $\qquad$ $\boxed{V = \frac{1}{j\omega C} I}$



$\frac{1}{j\omega C}\,\Omega = \frac{-j}{\omega C}\,\Omega$

Ex Find $v_o(t)$ in the circuit shown:-





$Z_L = j10(0.5) = j5\,\Omega$

$Z_C = \frac{-j}{10\frac{1}{20}} = -j2$

parallel // $\longrightarrow \frac{j5 \times 10}{10 + j5} = \frac{j50}{11.18\angle 26.57^\circ} = 4.47\angle 63.43^\circ\,\Omega$

$j50 \to 50\angle 40^\circ$

$= 2 + j4\,\Omega$

Voltage Division

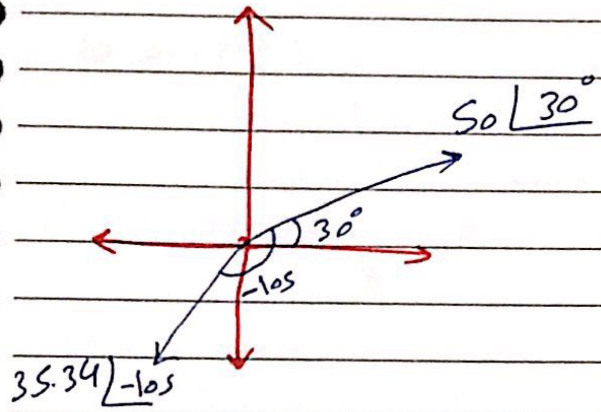
$$V_o = \frac{-j2}{\underbrace{2 + j4 - j2}_{2+j2}} \times 50\angle 30^\circ$$

$$V_o = \frac{2\angle -40^\circ \cdot 50\angle 30^\circ}{2.83\angle 45^\circ} = 35.34\angle -105\ V.$$

Back from phasor to time domain

$v_o(t) = 35.34\cos(10t - 105^\circ)\ V$

Draw phasor diagram for $v_{in}(t)$ and $V_o(t)$



$v(t)$ lead $V_o(t)$ by 135°

The impedance $Z = \frac{V}{I}$

$Z = R \pm jX$

resistance

reactance

The Admittance Y

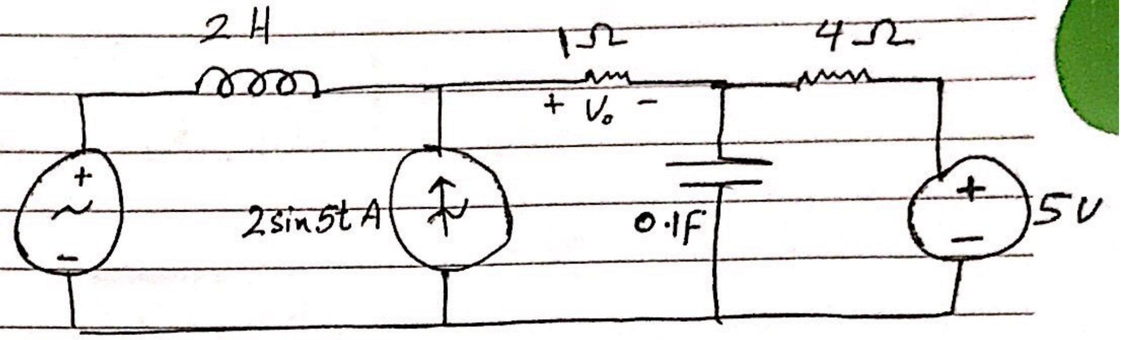
$Y = \frac{I}{V} = \frac{1}{Z}$ Siemens (S)

$Y = G \pm jB$

condactance (S)

susceptance (S)

Ex:-



Find $v_o(t)$:- (use superposition)

Note:-

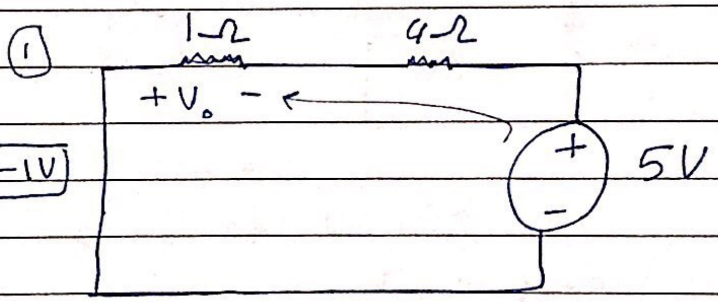
$v(t) = V_m \cos(\omega t + \theta)$

DC: $\omega = 0$

$R \rightarrow R$ $\quad L \rightarrow j\omega L$ $\quad C \rightarrow \frac{-j}{\omega C} = \frac{-j}{(0)C} = \infty$

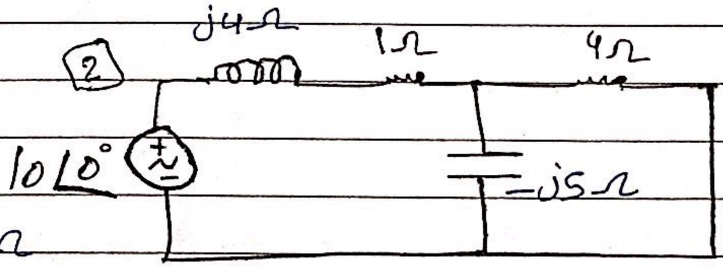
$j(0)L = 0\,\Omega$

Solu:-



$v_{o1} = -5 \times \frac{1}{4+1} = \boxed{-1V}$

$\boxed{v_{o1}(t) = -1\ V}$



$\omega = 2$ rad/sec

$2H \rightarrow j(2)(2) = j4\ \Omega$

$0.1F \rightarrow \frac{-j}{0.1 \times 2} = -j5\ \Omega$

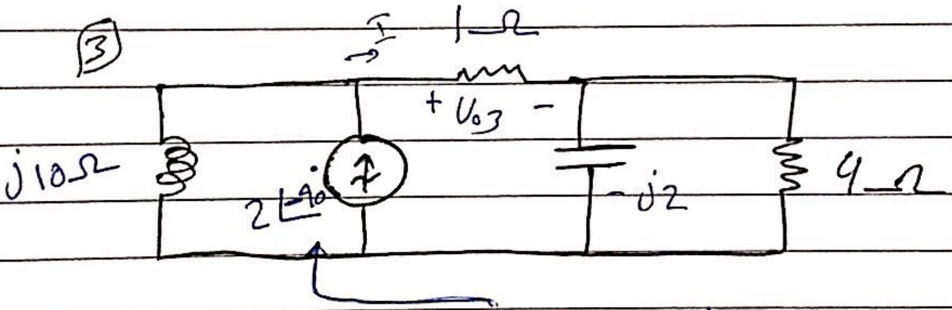
$\parallel \rightarrow \frac{4(-j5)}{4-j5} = \frac{-20j}{4-j5}$ after polar form calculater $= 2.439 - j\,1.951\,\Omega$

$$V_{o2} = \left(\frac{1}{1 + j4 + 2.434 - j1.951}\right)(10\angle 0^\circ)$$

$$= \frac{10\angle 0^\circ}{3.434 + j2.044} = 2.498\angle -30.79^\circ \text{ V}$$

$$\boxed{V_{o2}(t) = 2.498\cos(2t - 30.79^\circ)\text{ V}}$$

3



$2\sin 5t \longrightarrow 2\cos(5t - 40^\circ)$ بالطاقات

$(\omega = 5 \text{ rad/sec})$

$2H \longrightarrow j(5)(2) = j10\,\Omega$

$0.1F \longrightarrow -\frac{j}{0.1\times 5} = -j2$

$-j2 \parallel 4 \longrightarrow \frac{-j2\times 4}{4 - j2} = 0.8 - j1.6\,\Omega$

$$I = \left(\frac{j10}{j10 + 1 + 0.8 + -j1.6}\right) 2\angle -40^\circ = \frac{10\angle 90 \quad 2\angle 40}{1.8 + j8.4} = 2.328\angle -80 \text{ A}$$

$V_{o3} = 2.328\angle -80^\circ \times 1 = 2.328\angle -80^\circ \text{ V} \Rightarrow \boxed{V_{o3}(t) = 2.328\cos(5t - 80^\circ)\text{V}}$

$V_o(t) = V_{o1}(t) + V_{o2}(t) + V_{o3}(t)$

$= -1 + 2.498\cos(2t - 30.79^\circ) + 2.328\cos(5t - 80^\circ)$ V.

Five Apple