lecture 1:-

No.

*Trig Identities:-

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	0.5	$\sqrt{3}/2$	$1/\sqrt{3}$
$\frac{\pi}{3}$	$\sqrt{3}/2$	$\frac{1}{2}$	$\sqrt{3}$
$\pi/2$	1	0	∞

$\circledast$ $\sin(x\pm y) = \sin x\cos y \pm \cos x\sin y$

$\cos(x\pm y) = \cos x\cos y \mp \sin x\sin y$

$\circledast$ $\sin(x)\cdot\overset{\sin}{\cancel{\cos}}(y) = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

$\cos(x)\cdot\cos(y) = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$

$\sin x\cdot\cos(y) = \frac{1}{2}[\sin(x-y) + \sin(x+y)]$

$\circledast$ $\sin(\frac{\pi}{2}\pm x) = \cos x$

$\cos(\frac{\pi}{2}\pm x) = \mp\sin x$

$\sin(\pi\pm x) = \mp\sin x$

$\cos(\pi\pm x) = -\cos x$

$\circledast$ $\sin^2(\theta) + \cos^2\theta = 1$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$\sin^2\theta = \frac{1}{2}(1-\cos\theta)$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

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No.

* Maclurian's Series:-

$$* \; e^{\theta} = 1 + \theta + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} \cdots\cdots \frac{\theta^k}{k!} = \sum_{k=0}^{\infty} \frac{e^k}{k!}$$

$$* \; \cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots$$

$$= \sum_{k=0}^{\infty} (-1)^k \frac{\theta^{2k}}{2k!}$$

$$* \; \sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k \theta^{2k+1}}{(2k+1)!}$$

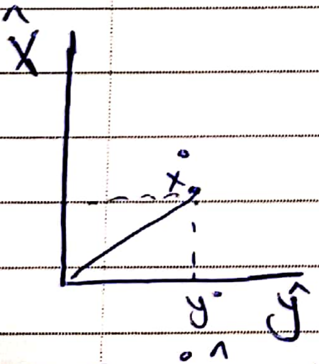
*complex numbers:-

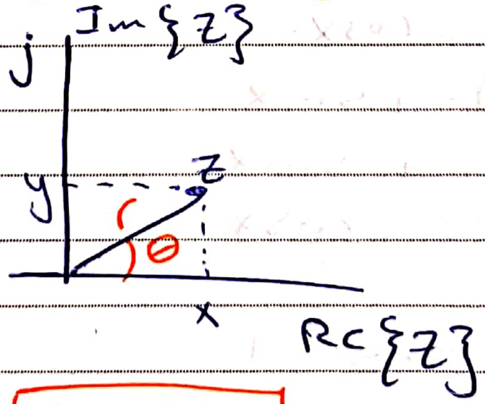
let $Z = x + j y$, where x and y are real

$j = \sqrt{-1}$

j

$\mathrm{Re}\{Z\}$ $\quad$ $\mathrm{Im}\{Z\}$





$Z = \dot{x}\,\hat{x} + y.\hat{y}$

$\boxed{Z = x + jy} \rightarrow$ cartesian form.

* Polar form:- $r \angle \theta$

$$re^{j\theta} \longrightarrow \tan^{-1}\left(\frac{y}{x}\right)$$

$$\longrightarrow r = \sqrt{x^2+y^2}$$

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$x = r\cos\theta$
$y = r\sin\theta$ } → from Polar to cartesian.

No.

$z = re^{j\theta} = x + jy = r\cos\theta + jr\sin\theta$

$\boxed{re^{j\theta} = r\cos(\theta) + jr\sin(\theta)}$ → Eulers Identity.

Proof:-

L.H.S

$$e^{j\theta} = 1 + (j\theta) + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \text{----} \frac{(j\theta)^4}{4!}$$

$$= 1 + j\theta - \frac{\theta^2}{2!} \cancel{th} - j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} \cdots$$

$$= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!}\right) + j\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots\right)$$

$$= \cos(\theta) + j\sin(\theta)$$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$ → ①

$e^{-j\theta)} = \cos(-\theta) + j\sin(-\theta) = \boxed{\cos\theta - j\sin\theta}$ → ②

① + ② → $\cancel{2}\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2}$

$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$, $\cos(j\theta) = \frac{e^{j(j\theta)} + e^{-j(j\theta)}}{2}$

$$= \frac{e^{-\theta} + e^{\theta}}{2} = \cosh(\theta)$$

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$e^{j\theta} = 1$

$e^{j2\pi} = 1$

No.

$\boxed{e^{j2\pi k} = 1}$ → for integer values

* $e^{j\pi} = -1$ → $e^{j\pi} + 1 = 0$ → queen of all equations

$e^{-j\pi} = -1$

$e^{j\frac{\pi}{2}} = j$

$e^{-j\frac{\pi}{2}} = -j$

$e^{j(\theta+2\pi k)} = e^{j\theta}$, for integer values of k.

↳ $e^{j\theta} \cdot \underbrace{e^{j2\pi k}}_{\downarrow (1)}$

* operation on two complex numbers:-

let $z_1 = x_1 + jy$, $z_2 = x_2 + jy_2$

$= r_1 e^{j\theta_1}$, $= r_2 e^{j\theta_2}$

① $z_1 \pm z_2 = (x_1 \pm x_2) \pm j(y_1 \pm y_2)$

② $z_1 \cdot z_2 = (x_1 + jy_1) \cdot (x_2 + jy_2)$

$= \underbrace{(x_1 x_2 - y_1 y_2)}_{\downarrow \, Re\{z_1 z_2\}} + j\underbrace{(y_1 x_2 + x_1 y_2)}_{\downarrow \, Im\{z_1 z_2\}}$

or $r_1 e^{j\theta_1} \cdot r_2 e^{j\theta_2} = r_1 r_2 e^{(j\theta_1 + j\theta_2)}$

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③ No. lec (2)

let $Z_1 = x_1 + jy_1$, $Z_2 = x_2 + jy_2$

$= r_1 e^{j\theta_1}$ $\qquad = r_2 e^{j\theta_2}$

(*) $$\frac{Z_1}{Z_2} = \frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$$

or $$\frac{x_1+jy_1}{x_2+jy_2} \cdot \frac{x_2-jy_2}{x_2-jy_2} = \frac{x_1+jy_1(x_2-jy_2)}{x_2^2+y_2^2}$$

$$= \frac{x_1x_2 + jy_2y_1 - jx_1y_2 + y_1y_2}{y_2^2 + y_2^2} = \underbrace{\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}}_{Re} + j\underbrace{\frac{(y_2y_1 - x_1y_2)}{x_2^2+y_2^2}}_{Im}$$

④ $$|Z_1| = |r_1 e^{j\theta_1}| = r_1 \underbrace{|e^{j\theta_1}|}_{(1)} = r_1$$

$e^{j\theta}$ $\overset{(1)}{\text{why?}}$ because $e^{j\theta} = \cos(\theta) + j\sin\theta$

$|e^{j\theta}| = |\cos(\theta) + j\sin\theta|$

$= \sqrt{\cos^2\theta + \sin^2\theta} = \boxed{1}$

⑤ $|Z_1 \cdot Z_2| = |Z_1| \cdot |Z_2|$

$Z_1 Z_2 = r_1 r_2 e^{j(\theta_1+\theta_2)}$

$|Z_1 Z_2| = r_1 r_2 |e^{j(\theta_1+\theta_2)}| = r_1 r_2$

⑥ $$\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$$

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$z_1 = x_1 + jy_1 = r_1 e^{j\Theta_1}$

$z_2 = x_2 + jy_1 = r_2 e^{j\Theta_2}$

Def:- the complex conjugate of complex number $z = x + jy_1$ is given by:-

$z^* = x - jy = re^{-j\Theta}$

1) $z_1 \cdot z_1^* = (x_1 + jy_1) \cdot (x_1 - jy_1) = x_1^2 + y_1^2 = (\sqrt{x_1^2 + y_1^2})^2$

$= (r_1)^2 = |z_1|^2$

then $\boxed{|z_1| = \sqrt{z_1 z_1^*}}$

2) $\boxed{(z_1 + z_2)^* = z_1^* + z_2^*}$

3) $\boxed{(z_1 \cdot z_2)^* = z_1^* \cdot z_2^*}$

4) $\boxed{\left(\frac{z_1}{z_2}\right)^* = \frac{z_1^*}{z_2^*}}$

* بيصحون عالضرب والجمع والقسمة

* example:- let $z = \frac{1}{1+jw}$

Find z^* and $|z|$

$z^* = \frac{z_1^*}{z_2^*} = \frac{1}{1-jw}$

$|z| = \frac{1}{\sqrt{1+w^2}}$

or $|z| = \sqrt{z \cdot z^*} = \sqrt{\frac{1}{1+jw} \cdot \frac{1}{1-jw}} = \sqrt{\frac{1}{1+w^2}}$

$= \frac{1}{\sqrt{1+w^2}}$

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(H.W) examples:-

① Is $(e^{z})^{*}$ e^{z*} ??

② let $z = e^{j\pi\theta} + e^{j3\pi\theta}$, find $|z|$

* nth power of a complex number ?

$$z = x + jy = re^{j\theta}$$

$$= (x+jy)^{n}$$

$$= (re^{j\theta})^{n} = r^{n}e^{nj\theta}$$

$$\boxed{(a+b)^{n} = \sum_{k=0}^{n} \binom{n}{k} a^{k} b^{n-k}}$$ → binomial theorm.

$$(re^{j\theta})^{n} = (r\cos(\theta) + jr\sin(\theta))^{n} = r^{n}(\cos\theta + j\sin\theta)^{n}$$

$$= r^{n} e^{j\theta n}$$

$$= r^{n}(\cos(\theta n) + j\sin(\theta n))$$

then $\boxed{(\cos(\theta) + j\sin(\theta))^{n} = \cos(\theta n) + j\sin(\theta n)}$

let $n=1$ L.H.S

$(\cos(\theta) + j\sin\theta)^{1}$

$n=2 \rightarrow ((\cos(\theta + j\sin\theta)^{2} = \cos(2\theta) + j\sin(2\theta)$

$\rightarrow \underline{-\sin^{2}(\theta) + \cos^{2}\theta} + j\,\underline{2\sin\theta\cos\theta} = \underline{\cos(2\theta)} + j\,\underline{\sin(2\theta)}$

⇒ Identities are improved.

No.

* the nth root for a complex number

let

$w = \sqrt[n]{z}$

$* z = re^{j\theta}$

$w = z^{1/n} = (re^{j\theta})^{1/n} = \left(re^{j(\theta+2\pi k)}\right)^{1/n}$ (for integer k)

$= r^{1/n} e^{\frac{j(\theta+2\pi k)}{n}}$, $k = 0, 1, ---- n-1$

$\boxed{\text{let } n=4}$

$w = r^{\frac{1}{4}} e^{j\frac{(\theta+2\pi k)}{4}}$

$w_0 = r^{\frac{1}{4}} e^{\frac{j\theta}{4}}$

$w_1 = (r)^{\frac{1}{4}} e^{j\left(\frac{\theta+2\pi}{4}\right)}$

$w_2 = (r)^{\frac{1}{4}} e^{j\frac{(\theta+4\pi)}{4}}$

$w_3 = (r)^{\frac{1}{4}} e^{j\frac{(\theta+6\pi)}{4}}$

$w_4 = (r)^{\frac{1}{4}} e^{\frac{j\theta}{4}}$ → they will be repeated

so I stop at $\boxed{n-1} = k$

Find $\sqrt[3]{1}$ $\boxed{n=3}$

$w_k = r^{1/n} e^{\frac{j(\theta+2\pi k)}{n}}$

$w_0 = (1)^{1/3} e^{\frac{(0+0)}{n}} = 1$

$w_1 = (1)^{\frac{1}{3}} e^{(0+2\pi)} = e^{\frac{2\pi}{3}}$

$w_2 = 1 \cdot e^{\frac{4\pi}{3}}$

so $\boxed{\sum_{k=0}^{n-1} e^{\frac{j2\pi k}{n}} = 0}$

$\boxed{w_0 + w_1 + w_2 = 0}$

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* log of complex numbers :-

let Z = A complex number $= re^{j\theta}$

then $\ln(Z) = \ln(re^{j\theta + 2\pi k}) = \ln(r) + j(\theta + 2\pi k$

if $k=0$ $\quad \ln(Z) = \ln(r) + j\theta$ → principle value of $\ln(Z)$

* example : $\ln(-1)$ → $\ln(-1 + 0*j)$

$r = 1$

$\theta = \pi$

Principle value → $\ln(1) + \pi j = \pi j$

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Ex: Find j^j

$$X = e^{\ln X}$$

then $j^j = e^{\ln(j^j)} = e^{j \ln j}$

let $y = j \ln j$

$$y = j\left[\underset{\boxed{0}}{\cancel{\ln(1)}} + j(\pi/2 + 2\pi k)\right] = -(\pi/2 + 2\pi k)$$

* geometric series

$$S_N = \underbrace{a + a.b + a.b^2 + a.b^3 + \dots . a.b^{N-1}}_{N \text{ turns}}$$

$$S_N = \begin{cases} \dfrac{a(1-b^N)}{1-b}, & b \neq 1 \\ N \cdot a, & b = 1 \end{cases} \leftarrow \text{finite } \underline{\underline{N}}$$

AS $N \longrightarrow \infty$

$$\lim_{n \to \infty} S_N = \begin{cases} \boxed{\lim_{N \to \infty} \dfrac{a(1-b^n)}{1-b}}, & b \neq 1 \\ \lim_{n \to \infty} N \cdot a . \end{cases}$$

$$\longrightarrow S_N = \frac{a}{1-b}, \quad |b| \overset{\Delta}{=} -1 < b < 1$$

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Ex:- $S = e^{\frac{j\pi}{2}} + \frac{1}{2} e^{j\pi} + \frac{1}{4} e^{\frac{j\pi}{2}} + \cdots + 2^{\frac{1}{a}} e^{\frac{10j\pi}{2}}$

Find |S|

$$\boxed{\sum_{k=0}^{N-1} e^{\frac{j2\pi k}{N}} = 0}$$

$$\rightarrow S_N = 1 + e^{\frac{j2\pi}{N}} + e^{j\frac{4\pi}{N}} \cdots\cdots e^{\frac{j(N-1)\pi}{N}}$$

$a=1 \quad , \quad b = e^{\frac{j2\pi}{N}}$

$$S_N = \frac{\left(1 - e^{\left(\frac{j2\pi}{N}\right)N}\right)}{1 - e^{j\left(\frac{2\pi}{N}\right)}} = 0$$

* Integrals of complex functions:-

Ex: $$\int_0^{\frac{\pi}{4}} e^{j2t}\, dt = \frac{e^{j2t}}{2j}\Bigg|_0^{\frac{\pi}{4}} = \frac{e^{j\pi/2} - 1}{2j}$$

$$= \frac{j-1}{2j} = \frac{1}{2} + \frac{1}{2}j$$

* $$\int_a^b e^{-at}\, dt = \frac{e^{-at}}{-a}\Bigg|_a^b = \frac{1-e^{-a}}{a}$$

* Integration by parts:-

$$\int_{\cancel{a}0}^{b} u(x)\, dv(x) = u(x)\, v(x) \Big|_{0\cancel{a}}^{b} - \int_{\cancel{a}0}^{b} v(x)\, d(u(x))$$

$$\int_{0}^{1} \underbrace{t}_{u}\, \underbrace{e^{-jwt}\, dt}_{dV}$$

$u = t$ $\qquad$ $dV = e^{-jwt}$

$du = dt$ $\qquad$ $V = \dfrac{e^{-jwt}}{-jw}$

$$\longrightarrow -t \cdot \frac{e^{-jwt}}{jw} \Big]_{0}^{1} - \int_{0}^{1} \frac{e^{-jwt}}{-jw}\, dt$$

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No. lecture (4)

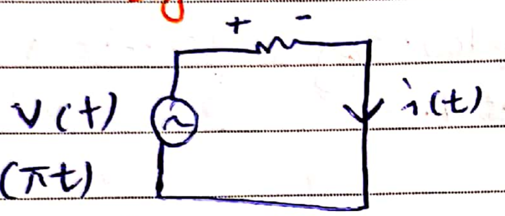
*chapter 1:-

*what is a signal ??

A signal is a function of an independent variable (time) carries some information or describes physical phenomenans.

Ex: $x(t) = \cos(w_0 \overset{\text{indp. variable.}}{\boxed{t}})$

↳ signal

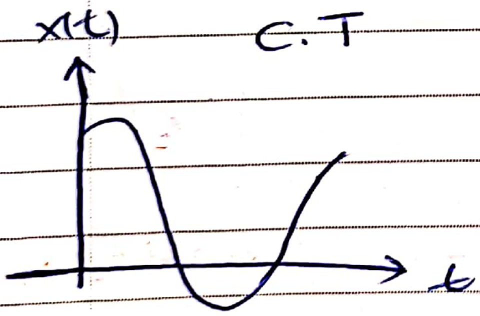


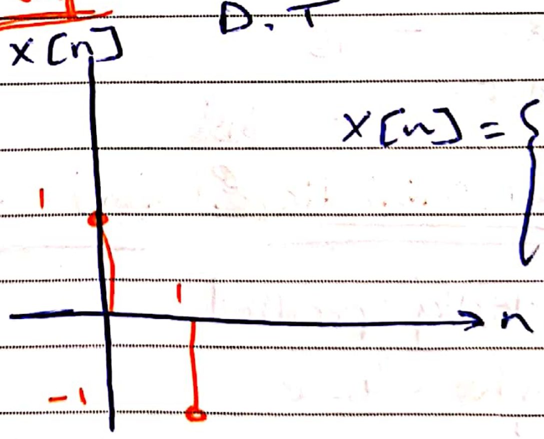
$v(t) = \cos(\pi) = -1$

↳ signal value taken at t=1

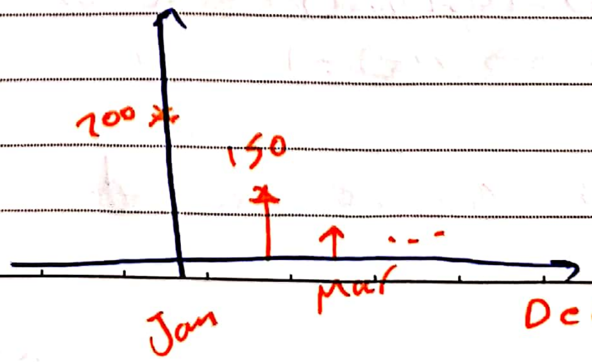
** Classification of Signals.

1- continous time signals & $\boxed{VS}$ Discret time signals.

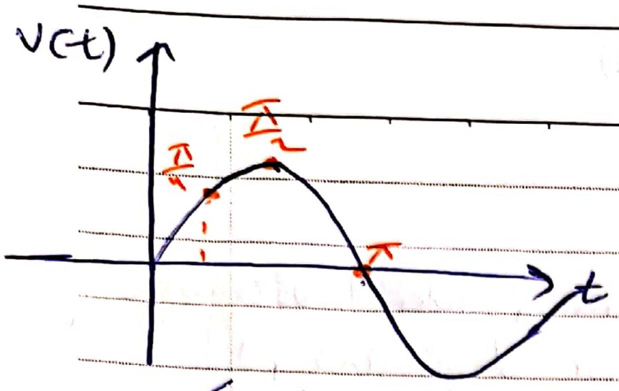




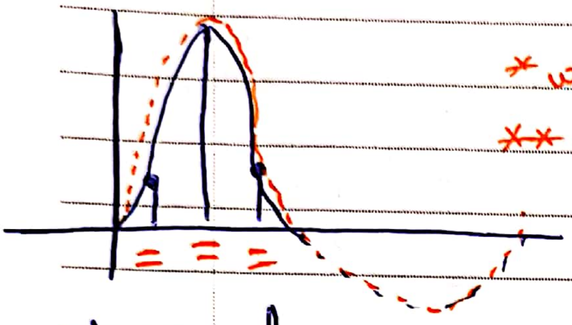
$$x[n] = \begin{cases} 1 & , n=0 \\ -1 & , n=1 \end{cases}$$



$= \sin(t) \; , \; 0 \leq t < 2\pi$

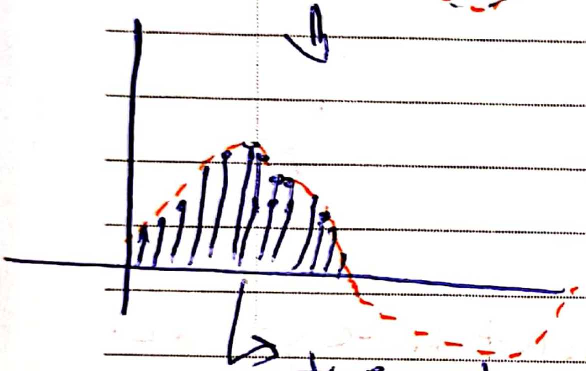


$V = [\, 0 \;\; \frac{1}{\sqrt{2}} \;\; 1 \;\; \frac{1}{\sqrt{2}} \;\; 0 \; \cdots$



* we'll have alost information

** so we must increase the number of samples. by decreasing the # Sampletime.



the lost information is smaller.

2- Deterministic $\boxed{\text{vs}}$ & Random signals.

completely specified at any given time.

ex: $x(t) = \cos(2\pi t) + \sin(\pi t)$

at $t = 1 \rightarrow x(t) = 1$

⋮

$x(t_0) \equiv$ can be specified.

$x(t) = \sin(2\pi(t + \frac{\theta}{2}))$

$x(t) =$ ~~$\sin((2\pi t) + \theta)$~~

where θ is random variable takes $\{0, 1\}$ with a probability $\boxed{\frac{1}{2}}$.

$$x(t) = \begin{cases} \sin(2\pi t) \; , \; \theta = 0 \\ \text{~~}\sin(2\pi t) + 1\text{~~} \\ \sin(2\pi t + \pi) \;\; \theta = 1 \end{cases}$$

then $x(\frac{1}{4}) = \begin{cases} 1 \, , \, \theta = 0 \\ -1 \;\; \theta = 1 \end{cases}$

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the tue expeted value $E[x(1/4)] = \frac{1}{2} * (1) + \frac{1}{2}(-1) = 0$

3- Real vs complex signals.

let x(t) be a complex signal

then

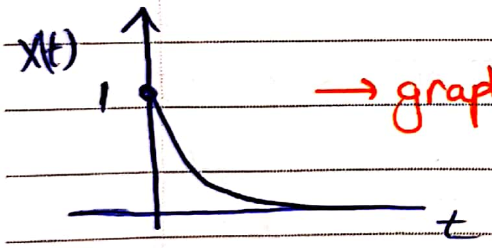
$$x(t) = x_1(t) + j\,x_2(t) \quad , j = \sqrt{-1}$$

$x_1(t)$ & $x_2(t)$ are Real signals.

Ex:- $x(t) = \cos(t) + j\sin(t) = e^{jt}$

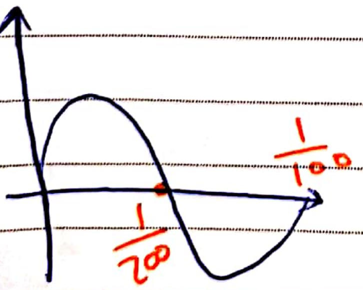
* How to specify signals?

$x(t) = e^{-t}$, $t \geq 0$ ⇒ mathematically



→ graphically.

$x(t) = \sin(200\pi t)$



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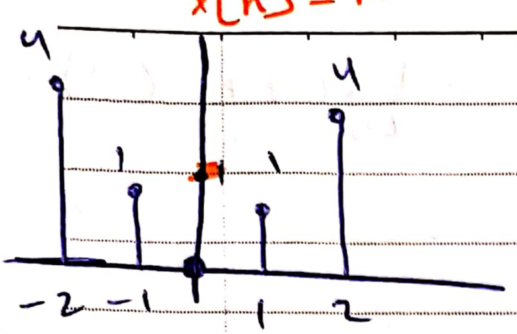
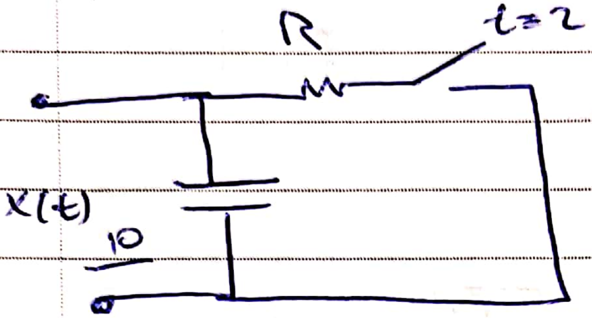
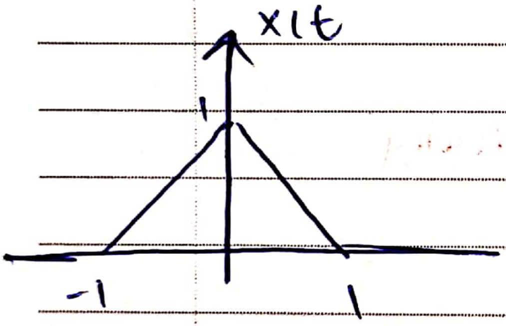
$x[n] = n^2$





$$x(t) = \begin{cases} 10 & , t<2 \\ 10e^{\frac{-t}{RC}} & , t \geq 2 \end{cases}$$



$$x(t) = \begin{cases} 1-|t| & , 1>t>-1 \\ 0 & , o.w \end{cases}$$

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No. lecture(5)

*periodic signals

Def: if a signal x(t) is said to be periodic with period "T" if there is a positive non-zero value of T

such that

$x(t+T) = x(t)$ for all t

Ex:- $x(t) = \sin(t)$

when $T = 2\pi, 4\pi, 6\pi, \ldots, 2\pi k$

ex: $x(t) = e^{\sin(t)}$ is it periodic?

$x(t+T) = e^{\sin(t+T)}$

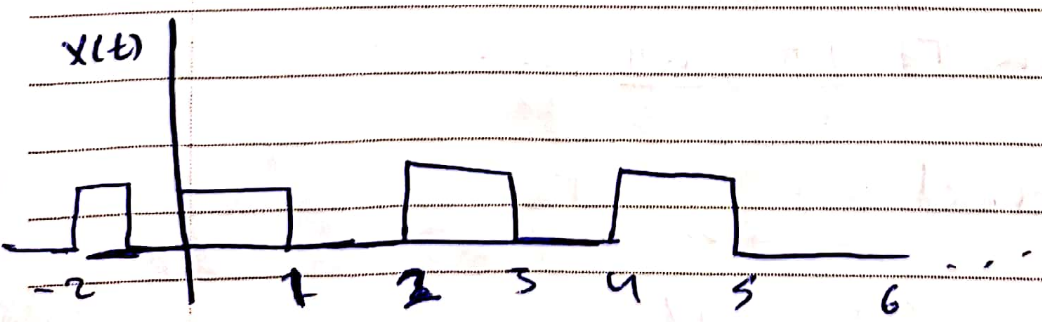
what are the values of "T"

such that

$e^{\sin(t)} = e^{\sin(t+T)}$

$T = 2\pi, 4\pi, 6\pi, \ldots$

* fundemental period "T_0" is the minimum of all values of $\boxed{T}$.



x(t) is periodic

$T = 2, 4, 6 \ldots$

$T_0 = 2$

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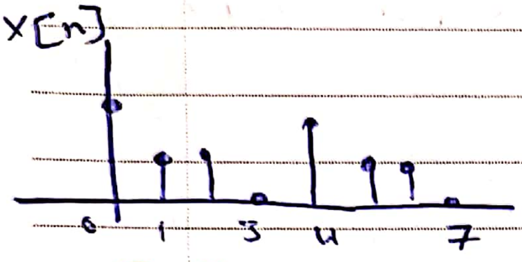
** A D.T signal is periodic if there is a positive integer N such that $x[n+N] = x[n]$



$x[n+4] = x[n]$

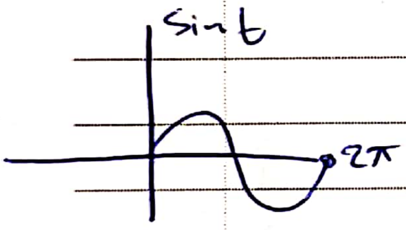
Period

$\boxed{N=4}$

* N : number of samples in on period.

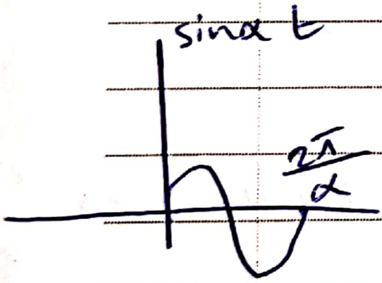
let $x(t) = \sin(\omega_\circ t + \Theta)$

what is the fundemental period "$T_\circ$" ??



since $x(t)$ is periodic then there is $\underline{T}$

$\boxed{x(t+T) = x(t)}$



$x(t+T) = \sin(\omega_\circ(t+T) + \Theta)$

$= \sin(\omega_\circ t + \Theta + \omega_\circ T)$

for $\sin(\omega.t+\Theta) = \sin(\omega.t+\Theta+\omega_\circ T)$

$\omega_\circ T = 2\pi, 4\pi, \ldots$

then $T = \frac{2\pi}{\omega_\circ}, \frac{4\pi}{\omega_\circ}, \ldots$

fundemental period $= \frac{2\pi}{\omega_\circ} = T_\circ$

$\omega_\circ = \frac{2\pi}{T_\circ}$ rad/s

fud frequency

Ex:- $x(t) = \cos(200\pi t)$

$$T_0 = \frac{2\pi}{200\pi} = \frac{1}{100}$$

$$\omega_0 = 200\pi$$

*periodicity of a sum of two sine waves:-

let $x(t) = \sin(\omega_1 t) + \cos(\omega_2 t)$

then x(t) is pelliodic if $\frac{T_{01}}{T_{02}} = \frac{\text{integer}}{\text{integer}} \overset{\triangle}{=}$ ratios of integers

let $x(t) = \sin(\omega_1 t) + \sin(\omega_2 t) + \cos(\omega_3 t)$

x(t) is periodic if $\frac{T_{01}}{T_{02}}$ = ratio of integers

& $\frac{T_{01}}{T_{03}}$ also ratio of integers.

Ex: $x(t) = \underbrace{\cos\left(\frac{10\pi}{3}t\right)}_{x_1(t)} + \underbrace{\sin\left(\frac{5\pi}{3}t\right)}_{x_2(t_2)}$

$$\frac{T_{01}}{T_{02}} = \frac{\frac{2\pi * 3}{10\pi}}{\frac{2\pi * 3}{5\pi}} = \frac{3/5}{8/5} = 3/8$$ => is a rational number

=> x(t) is periodic

$T_0 \overset{\triangle}{=}$ the fund of the sum x(t)

$$T_0 = \underbrace{LCM}_{\text{least common mulipple.}}(T_{01}, T_{02}) = LCM\left(\frac{3}{5}, \frac{8}{5}\right) = \boxed{24/5}$$

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$T_{01} = \frac{2\pi}{5}$ $\quad$ $T_{02} = \frac{2\pi}{3}$

No.

$x(t) = \underbrace{\sin(5t)}_{} + \cos(3t) + \sin(\pi t)$

is x(t) periodic signal ??

$\frac{T_{01}}{T_{02}} = \frac{3}{5}$, $\frac{T_{01}}{T_{02}} = \frac{2\pi}{5*2} = \frac{\pi}{5}$ → not a ratio of integers.

x(t) is $\underbrace{\text{aperiodic}}_{}$
↓
not periodic.

** is $x(t) = \sin\left(\frac{5\pi t}{6}\right) + \cos\left(\frac{3\pi t}{4}\right) + \sin\left(\frac{\pi t}{3}\right)$ periodic ??

$\boxed{\text{if yes find } T_0}$

$T_{01} = 12/5$

$T_{02} = \frac{2\pi}{5\pi/6} = \frac{8}{3}$

$T_{03} = \frac{2\pi}{\pi/3} = 6$

$\frac{T_{01}}{T_{02}} = \frac{12/5}{8/3} = \frac{9}{10}$

$\frac{T_{01}}{T_{03}} = \frac{2}{5}$

→ x(t) is periodic

$T_0 = LCM\left(\frac{12}{5}, \frac{8}{3}, 6\right) =$ ~~[illegible]~~ $= LCM\left(\frac{36}{15}, \frac{40}{15}, \frac{90}{15}\right)$

$= \frac{360}{15} = 24$

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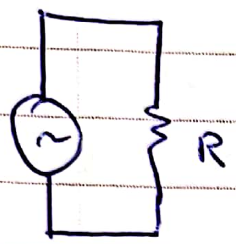
*Energy Associated by signal x(t).



$P(t)=V^2(t)/R$

if $R=1\Omega$

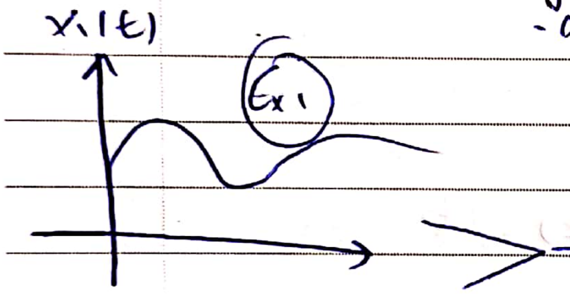
$P(t)=V^2(t)=I^2(t)$

$$\frac{dE(t)}{dt}=P(t)$$

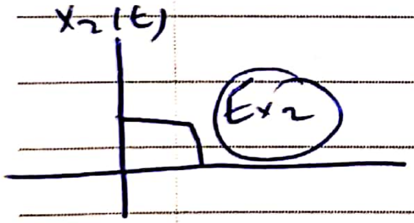
$$E(t)=\int P(t).dt \longrightarrow E=\int_{-\infty}^{\infty} V^2(t)dt=\int_{-\infty}^{\infty} I^2(t)d$$

$$E_x=\int_{-\infty}^{\infty} x^2(t)\,dt$$

$\hookrightarrow E_x \in [0,\infty)$



→ to compare between two signals.



$0<E_x<\infty \Rightarrow$ then x(t) is an energy signal.

*If x(t) is a complex signal then $E_x=\int_{-\infty}^{\infty}|x(t)|^2dt$

for

*energy & power signals we <u>can't</u> substitute in the integral a complex signal.

$$\Longrightarrow \boxed{E_x=\int_{-\infty}^{\infty} x(t)\,\overset{*}{x}(t)\,dt} \rightarrow$$ valid for complex & Real signals.

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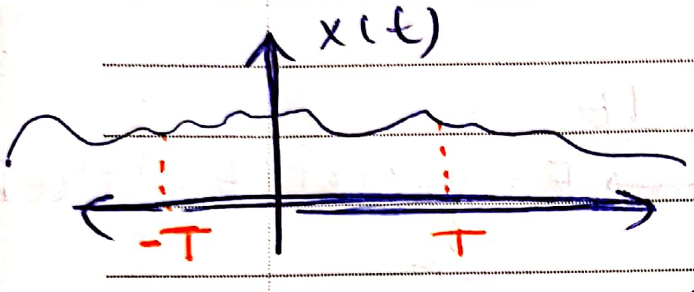
* If we have a D.T signal x[n]

$$\boxed{E_x = \sum_{-\infty}^{\infty} |x[n]|^2}$$

** power:-



f(T)

Average power $P_x = \int_{-T}^{T} x^2(t)\,dt$

$$\Rightarrow P_x = \lim_{T\to\infty} \frac{\int_{-T}^{T} x^2(t)\,dt}{2T}$$

*If x(t) is a complex signal

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt$$

*unit step function is a power signal.

* for D.T signals

$$P_x = \lim_{N\to\infty} \frac{\sum_{n=-N}^{N} |x[n]|^2}{2N+1}$$

smile

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~~** If the power of a signal is zero then the signal must be an energy signal. the $0 < E_x < \infty$~~

** If the signal is an energy signal then the Power must be zero.

~~> If the power of the~~

proof:-

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\,dt$$

$$\le \lim_{T\to\infty} \frac{\underbrace{\int_{-\infty}^{\infty} x^2(t)\,dt}_{E_x}}{2T} = 0$$

$$P_x \le 0$$

$\boxed{\text{then } P_x = 0}$ because it can't be less than zero.

??

Proof that if that if the power is zero then the energy is finite & its energy signal.

**** energy of $x(t)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

$x(t)$ is an energy signal if

$$\infty > E_x > 0$$

Power of $x(t)$

$$P_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\,dt$$

$x(t)$ is a power signal if

$$\infty > P_x > 0$$

$x[n]$

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

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** if $P_x = 0 \Rightarrow 0 < E_x < \infty$ (energy signal)

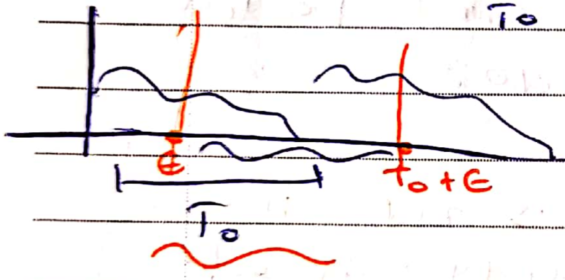
** if $0 < P_x < \infty \Rightarrow E_x = \infty$ (power signal)

** if $P_x = \infty \Rightarrow$ the signal is nietner energy Nor power signal.

$$P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

* what if $x(t)$ is periodic?

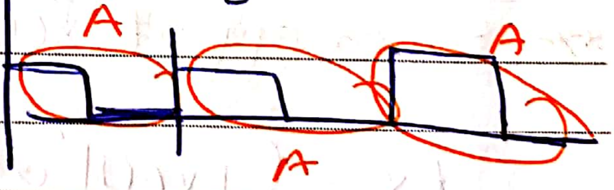
$$\Rightarrow P_x = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt$$



$$P_x = \lim_{n \to \infty} \frac{1}{2nT_0} \int_{-nT_0}^{nT_0} |x(t)|^2 dt$$

$$\lim_{n \to 0} \frac{1}{2nT_0} \cdot \underline{(2n)} \int_{0}^{T_0} |x(t)|^2 dt$$

$$= \frac{1}{T_0} \int_{0}^{T_0} x^2(t) dt$$

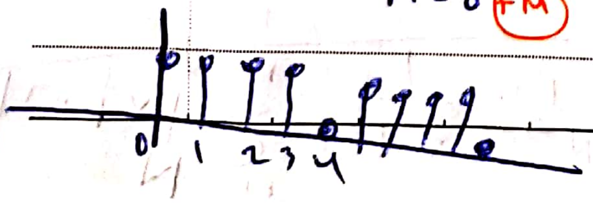


* for D.T (periodic)

$$P_x = \frac{1}{N_0} \sum_{n=0 \, \boxed{+M}}^{N_0-1 \, \boxed{+M}} |x[n]|^2$$

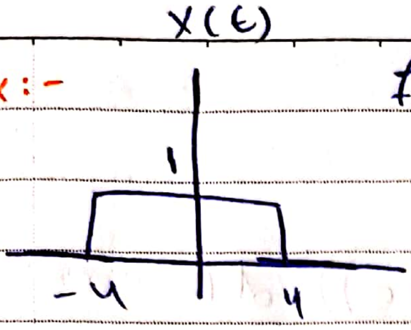
اهم الفرق بضل

$\boxed{N_0 - 1}$



* periodic signals Are power

Ex:-



find the energy of the signal x(t).

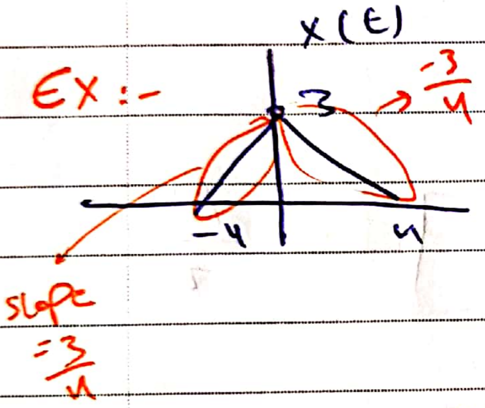
sol. $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$= \int_{-\infty}^{-4} (0)^2 dt + \int_{-4}^{4} (1)^2 d(t) + \int_{4}^{\infty} (0)^2 d(t) = 4+4 = 8$$

↳ energy signal

power for this signal is (zero).

Ex:-



Find Ex

$$E_x = \int_{-\infty}^{-4} (0)^2 dt + \int_{-4}^{0} \left(3\left(1+\frac{t}{4}\right)\right)^2 + \int_{0}^{4} \left(3\left(1+\frac{t}{4}\right)\right)^2 dt$$

$$= 9\left[\frac{\left(1+\frac{t}{4}\right)^3}{3\cdot\frac{1}{4}}\Bigg]_{-4}^{0} + \frac{\left(1-\frac{t}{4}\right)^3}{3\cdot\left(\frac{-1}{4}\right)}\Bigg|_{0}^{4}\right]$$

$$= 9\left(\frac{4}{3} + \frac{4}{3}\right) = (3)(8) = 24$$

* $x(t) = e^{jw_0 t}$ find the power of x(t)

$$P_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} (1)^2 dt$$

$$|e^{jw_0 t}| = \sqrt{e^{jw_0 t} \cdot e^{-jw_0 t}} = e^0 = 1$$

$\sqrt{x(t) \cdot x^*(t)}$

$$\lim_{T\to\infty} \frac{2T}{2T}$$

$= 1$

its periodic ≡ power signal

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* $x(t) = A\cos(\omega_0 t + \theta)$

Find P_x .

$x(t)$ is periodic

then $P_x = \frac{1}{T_0}\int_{\epsilon}^{T_0+\epsilon} (A\cos(\omega_0 t+\theta))^2 \, dt$

$$= \frac{A^2}{T_0}\int_{\epsilon}^{T_0+\epsilon} \cos^2(\omega_0 t+\theta) \, dt$$

$$= \frac{A^2}{T_0}\int_{\epsilon}^{T_0+\epsilon} \frac{1+\cos(2\omega_0 t+2\theta)}{2} \, dt$$

$$= \frac{A^2}{T_0}\left[\frac{T_0}{2} + \frac{1}{2}\int_{\epsilon}^{T_0+\epsilon} \cancelto{zero}{\cos(2\omega_0 t+2\theta) \, dt}\right] = \frac{A^2}{2}$$

lecture(8):-

Ex:- $x(t) = \cos(\omega_1 t) + \cos(\omega_2 t)$, Find P_x

→ Ex:- $x(t) = \cos(\omega t + \theta_1) + \cos(\omega t + \theta_2)$ Find P_x

Ex:- $x[n] = \left(\frac{1}{3}\right)^n$, $n \geq 0$. Find E_x

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=0}^{\infty}\left(\left(\frac{1}{3}\right)^n\right)^2 = \sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n$$

$$= \frac{1}{1-\frac{1}{9}} = \frac{9}{8}$$

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Ex:- $x(t) = e^{-at}$, $t \geq 0$, Find Ex

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{0}^{\infty} (e^{-at})^2 dt = \int_{0}^{\infty} e^{-2at}\,dt = \frac{e^{-2at}}{2a}\Big|_{\infty}^{0}$$

$$= \begin{cases} \infty & a \leq 0 \\ \frac{1}{2a} & a > 0 \end{cases}$$

$x(t) = e^{-at}$, $t \geq 0$ is an energy signal if $a > 0$

Ex:- $x(t) = e^{-(t+jt^2)}$, $t \geq 0$ find Ex

try to solve.

$x(t) = e^{-at}$ $t \geq 0$ $a < 0$

$$P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} e^{-at}\,dt = \lim_{T \to \infty} \frac{1}{2T} \int_{0}^{T} e^{at}\,dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[1 - e^{-2aT}\right] = \lim_{T \to \infty} \frac{2a\,e^{-2aT}}{2} = \infty$$

then the signal is neither energy nor power signal.

** Even and odd signals.

Def: A real signal x(t) is said to be even if $x(t) = x(-t)$

" " " " " " " odd if $x(t) = -x(-t)$

$\Leftrightarrow$ $x(-t) = -x(t)$

No.

real

* Any signal $x(t)$ can be expressed as the sum of even and odd signals

$$x(t) = x_e(t) + x_o(t) \quad — ①$$

$$x(-t) = x_e(-t) + x_o(-t)$$

$$\boxed{x(-t) = x_e(t) - x_o(t)} \quad — ②$$

⊛ $$\frac{x(t) + x(-t)}{2} = x_e(t)$$

⊛ $$\frac{x(t) - x(-t)}{2} = x_o(t)$$

Ex :- $x(t) = t^2 + te^{-t}$ Find $x_e(t)$, $x_o(t)$

$$\frac{x(-t) + x(t)}{2} = \frac{t^2 + te^{-t} + t^2 - te^{t}}{2} = x_e(t)$$

$$= \frac{2t^2 + 2(e^{-t} - \cancel{s}e^{t}) * t}{2} = t^2 - t\left(\frac{e^{-t} + e^{t}}{2}\right)$$

sinh(t) ← ~~cosh(t)~~

$$x_o(t) = te^{-t} + t^2 - (t^2 - te^{t})$$

$$= \frac{te^{-t} + te^{t}}{2} = t\left(\frac{e^{-t} + e^{t}}{2}\right) = t\cosh(t)$$

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Ex:- $x(t) = \underbrace{\sin(t)}_{o} + \underbrace{\overset{\cos(t)}{\cancel{\sin(t)}}}_{e} + \underbrace{\cos(t)\sin(t)}_{o}$.

$x_o(t) = \cos(t)\sin(t) + \sin(t)$

$x_e(t) = \cos(t)$

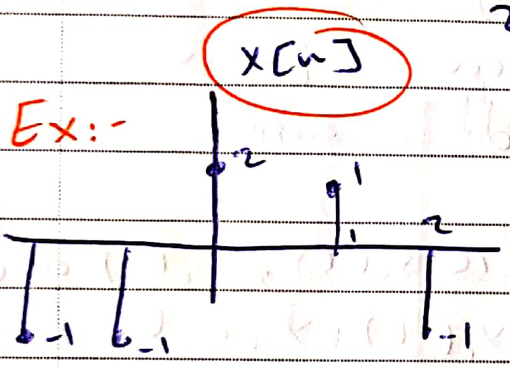
for D.T signal

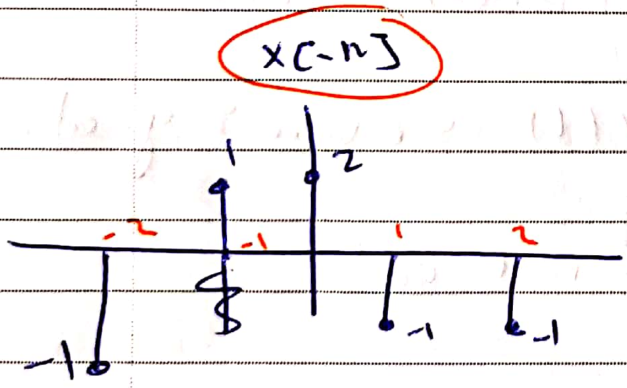
$x[n] = x_e[n] + x_o[n]$

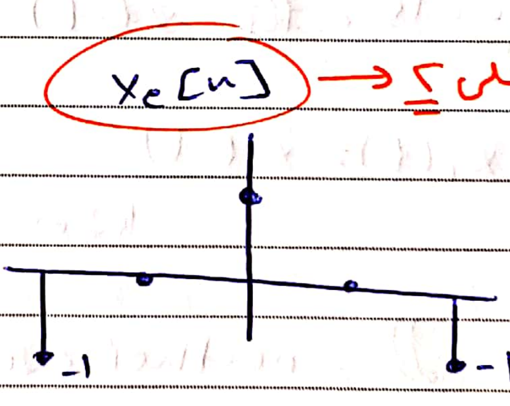
$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

Ex:-

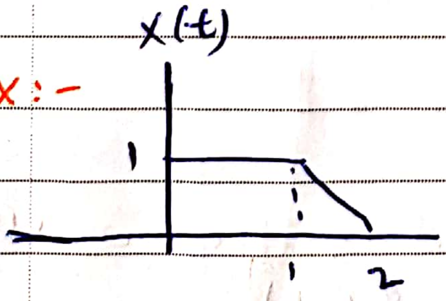




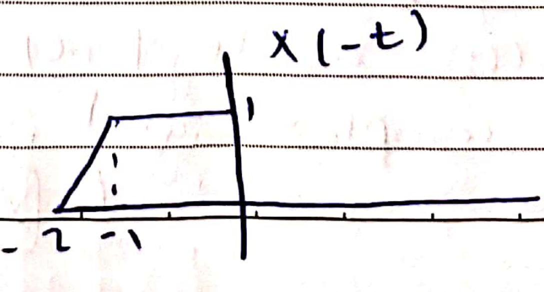


→ جمعت وقسمت على ٢

Ex:-



Find $x_e(t)$



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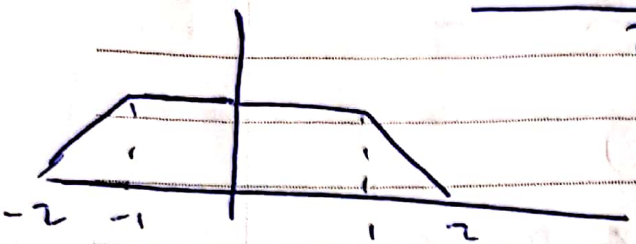
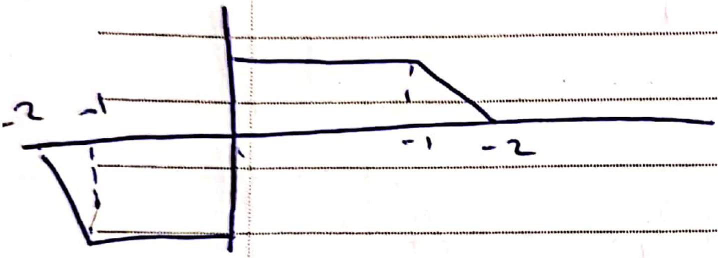
No.

$x_e(t) = \frac{x(t)+x(-t)}{2}$



$x_o(t)$ (2-P)

$\int_{-T}^{T} x_o(t) = 0$



lecture(9)

***Notes:-** * Even signal ± Even = Even

* odd ± odd = odd

let $x(t) = x_1(t) + x_2(t)$, where $x_1(t), x_2(t)$ are even signals. for $x(-t) = x_1(-t) + x_2(-t)$

$= x_1(t) + x_2(t)$

because $x_1(t) = x_1(-t)$

then $x(t)$ is even signal.

* Even ± odd = neither even nor odd

E × E = E

E × O = O

O × O = E

** let $x(t) = a(t) + j\,b(t)$

Def:- A complex signal is said to be conjugate symmetric signal, if $\boxed{x(t) = x^*(-t)}$

$x(t) = a(t) + j\,b(t)$

$x(-t) = a(-t) + j\,b(-t)$

$x^*(-t) = \left(a(-t) + j\,b(-t)\right)^* = a(-t) - j\,b(-t)$

The Signal $x(t)$ is conjugate symmetric signal if $x(t) = x^*(-t)$

$\Longrightarrow \underset{\sim}{a(t)} + j\,\underset{\sim}{b(t)} = a(-t) \ominus j\,b(-t)$

$\Rightarrow a(t) = a(-t)$

$b(t) = -b(t)$

$\Rightarrow$ $a(t)$ must be even &

$b(t)$ must be odd.

Ex: $x(t) = t^2 + j\sin t$

$t^2 \rightarrow$ even

$\sin t \rightarrow$ odd $\Big\}\rightarrow$ conjugate symmetric signal.

C.S

Def:- A Signal $x(t)$ is said tobe conjugate Aunti symmetric signal if $\boxed{x(t) = -x^*(-t)}$

$x(t) = a(t) + j\,b(t)$, $a(t)$ & $b(t)$ are real-signals.

$a(t)$ must be odd

$b(t)$ must be even.

$x(t) = \sin t + j\,t^2$

$\sin t \rightarrow$ odd

$t^2 \longrightarrow$ even $\Big\}\rightarrow$ C.A.S (↗ Aunti)

* Any complex signal can be expessed as

$$x(t) = \underbrace{x(t)}_{C.S} + \underbrace{x(t)}_{C.A.S}$$

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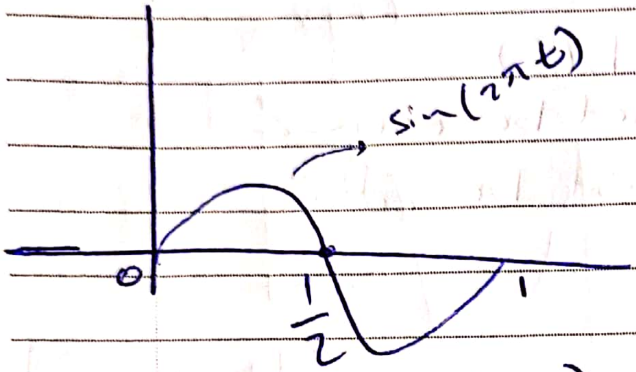
$$X_{c.s}(t) = \underline{x(t) + x^*(-t)}$$

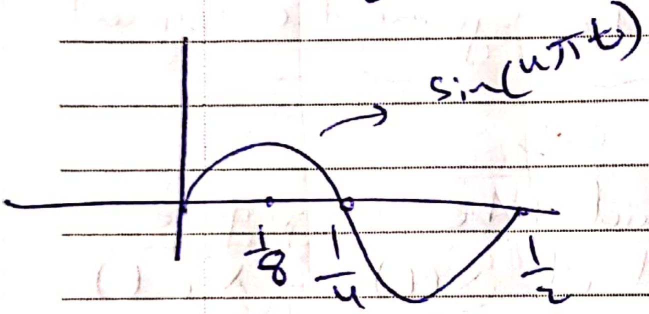
No:2

$$x_{C.A.S}(t) = \frac{x(t) - x^*(-t)}{2}$$

** Time transformation:- of C.T Signals.

we are given x(t) and we want to find X(at+b) wher a and b are real.



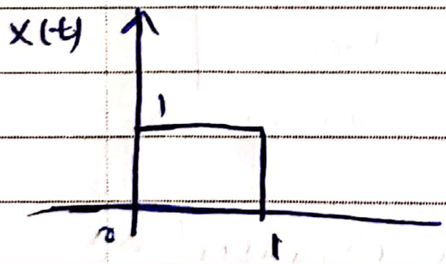


-Time shift:-

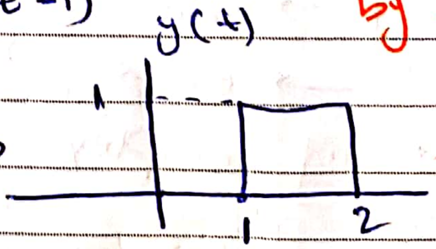
Suppose we have x(t) and we are intrested in $y(t) = x(t \ominus t_o)$

↳ if $t_o > 0$ we shift x(t) to the right by t_o

↳ if $t_o < 0$ shift to the left by t_o



$y(t) = x(t-1)$

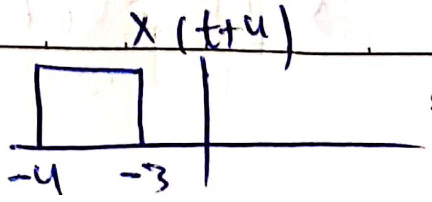


$y(0) = x(0-1) = x(-1) = 0$

$y(1) = x(1-1) = x(0) = 1$

$y(2) = x(2-1) = x(1) = 1$

$y(3) = $ ~~x~~ $X(2) = 0$



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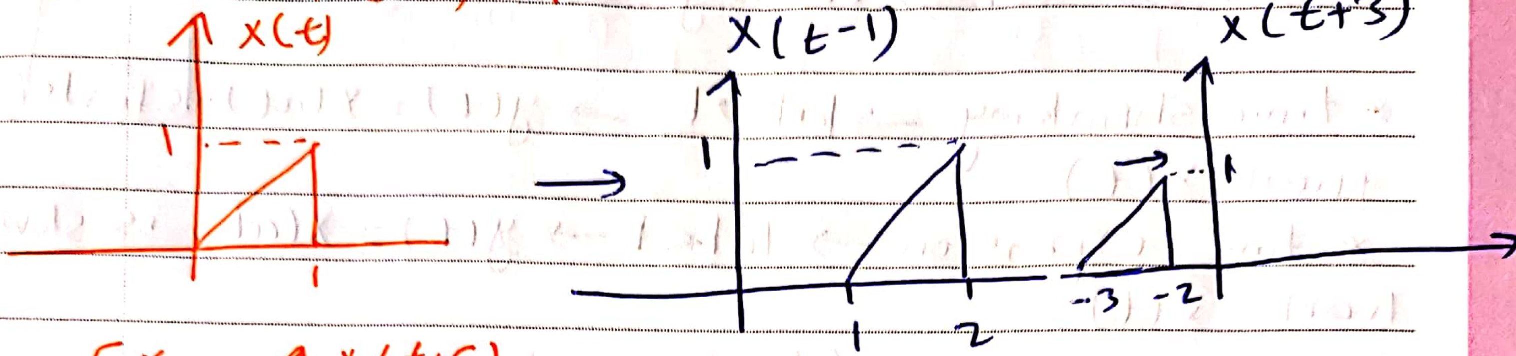
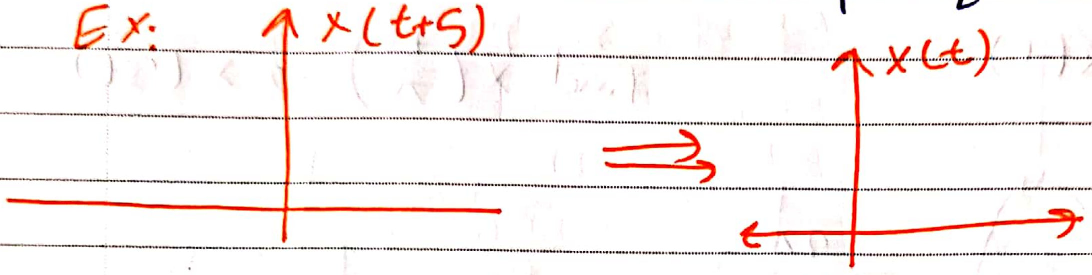
Ex: find $x(t-1)$ if

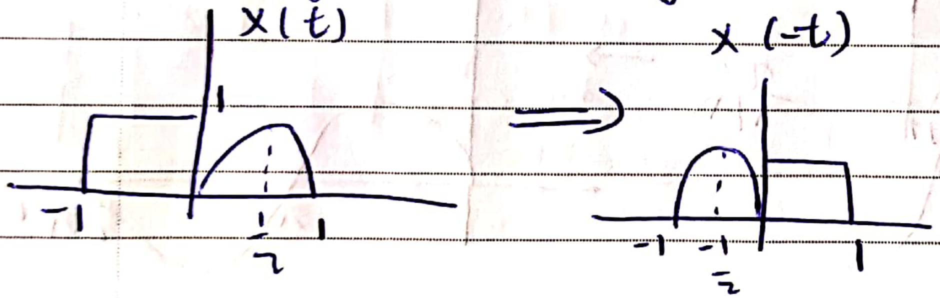


Ex:



* Time revased (reflection)

- $x(t)$ is given, find $y(t) = x(-t)$



$y(t) = x(-t)$

$y(\frac{1}{2}) = x(\frac{-1}{2}) = 1$

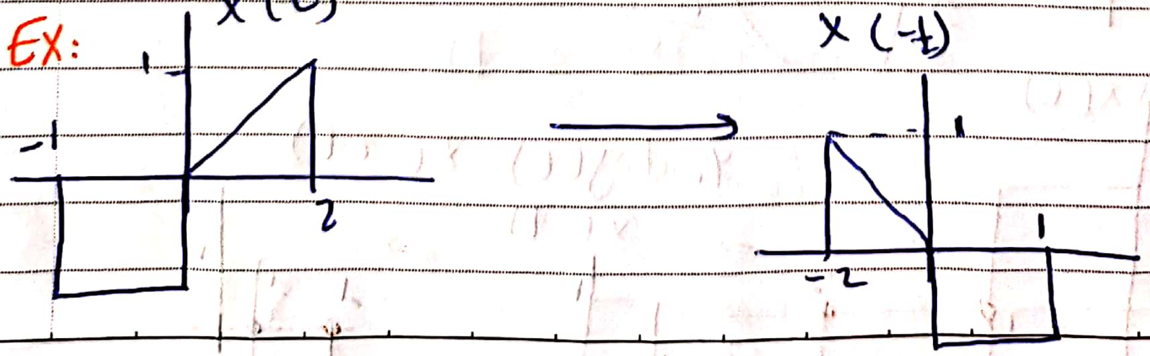
$y(1) = x(-1) = 1$

$y(2) = x(-2) = 0$

$y(\frac{-1}{2}) = x(\frac{1}{2}) = 1$

$y(-1) = x(1) = 0$

Ex:



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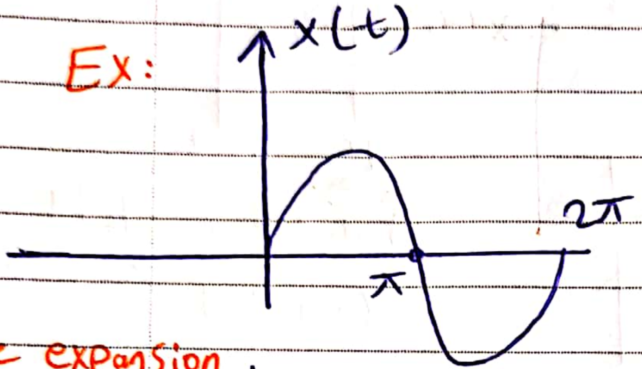
• **Time Scaling:** $x(t)$ is given, find $y(t) = x(at), a \neq 0$

* time shrinking → $|a| > 1$ → $y(t) = x(at)$ is faster than $x(t)$

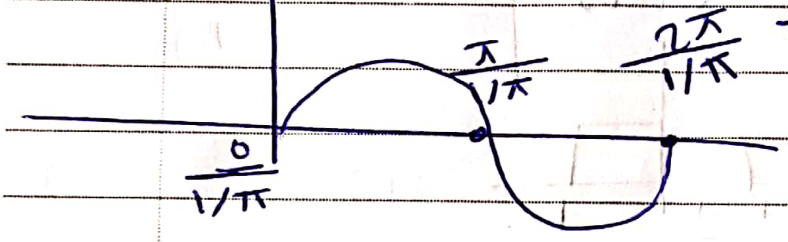
* time expansion → $|a| < 1$ → $y(t) = x(at)$ is slower than $x(t)$

Ex:

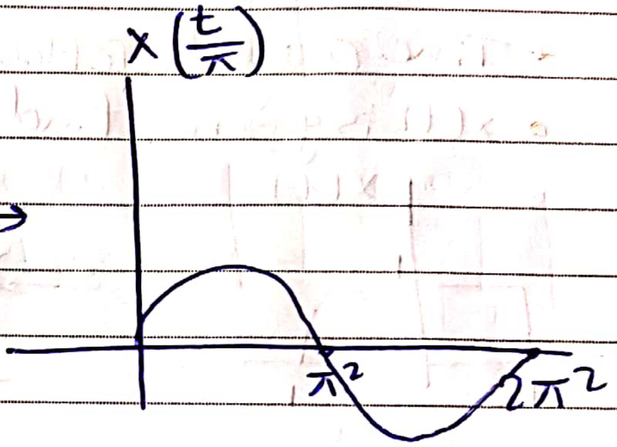


find $x\left(\frac{t}{\pi}\right)$, $x(3t)$

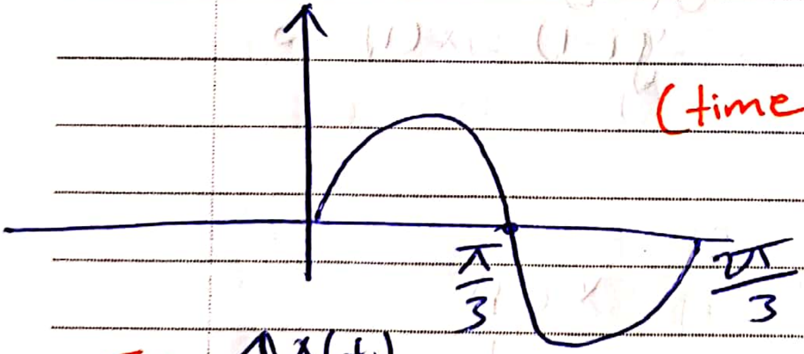
time expansion ⇒



→

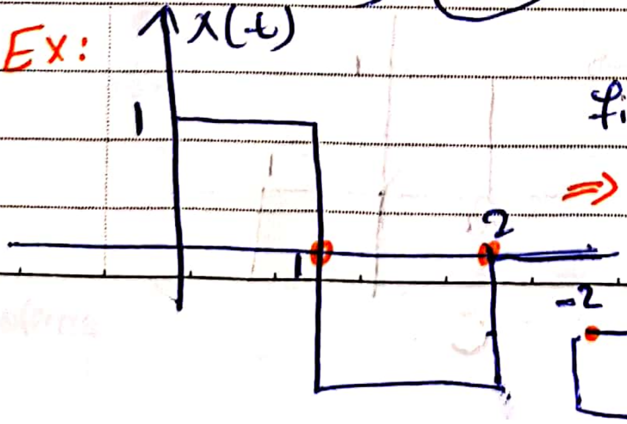


$x(3t)$



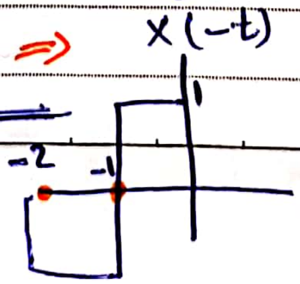
(time shrinking)

Ex:



find $y(t) = x(-2t)$

⇒



⇒

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No.

** General time transformation :-

x(t), find $y(t) = x(at+b)$, $a \neq 0$, (a,b) are constants.

$x(t) \longrightarrow x(at+b)$ shift by

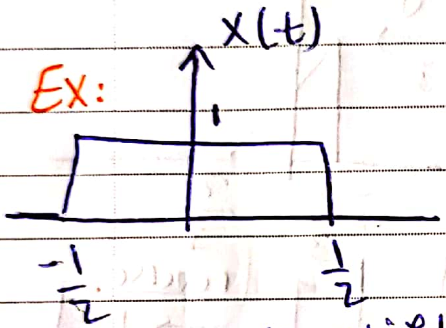
• method (1): $x(t) \xrightarrow{(b)} y_1(t) = x(t+b) \xrightarrow[\text{by } \boxed{a}]{\text{time Scaling}} y_2(t) = x(at+b)$

timeshift

• method 2: $x(t) \xrightarrow{\text{Scaling}} y_1(t) = x(at) \xrightarrow{\text{by}(b/a)} y_2(t) = y_1\left(t+\frac{b}{a}\right)$

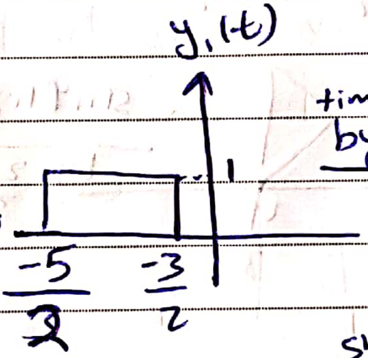
$$= x\left(a\left(t+\frac{b}{a}\right)\right)$$

Ex:

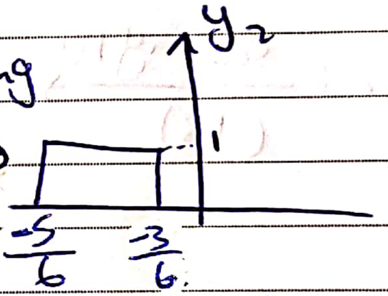


find $y(t) = x(3t+2)$

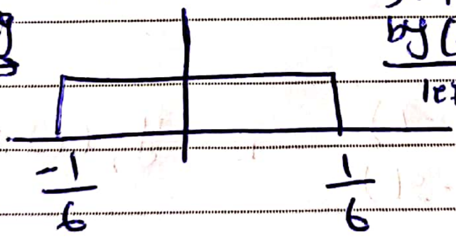
→ M1: $\xrightarrow[\text{(left)}]{\text{timeshift by}(2)}$



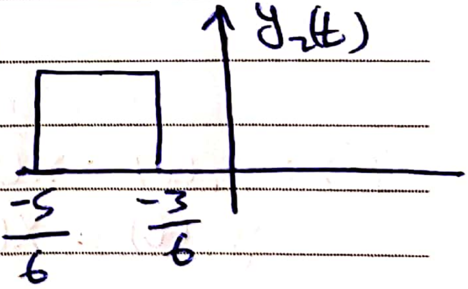
$\xrightarrow[\text{by}(3)]{\text{time scaling}}$



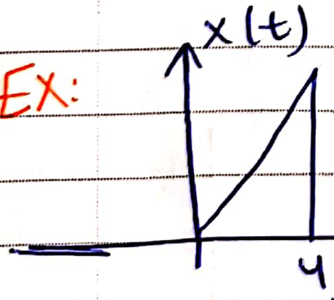
→ M2: $\xrightarrow{\text{time scaling}}$



$\xrightarrow[\text{left}]{\text{shifting by}\left(\frac{2}{3}\right)}$

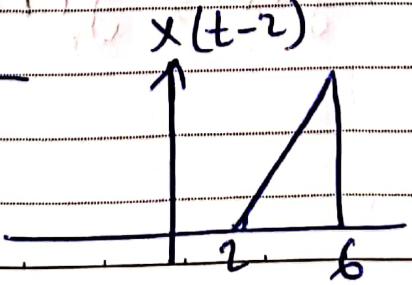


Ex:

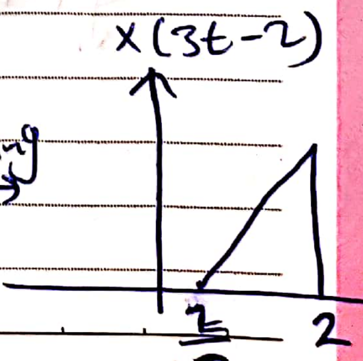


find $x(3t-2)$

M1:- $\xrightarrow[\text{right by}(2)]{\text{time shifting}}$



$\xrightarrow[\text{by}(3]{\text{time scaling}}$



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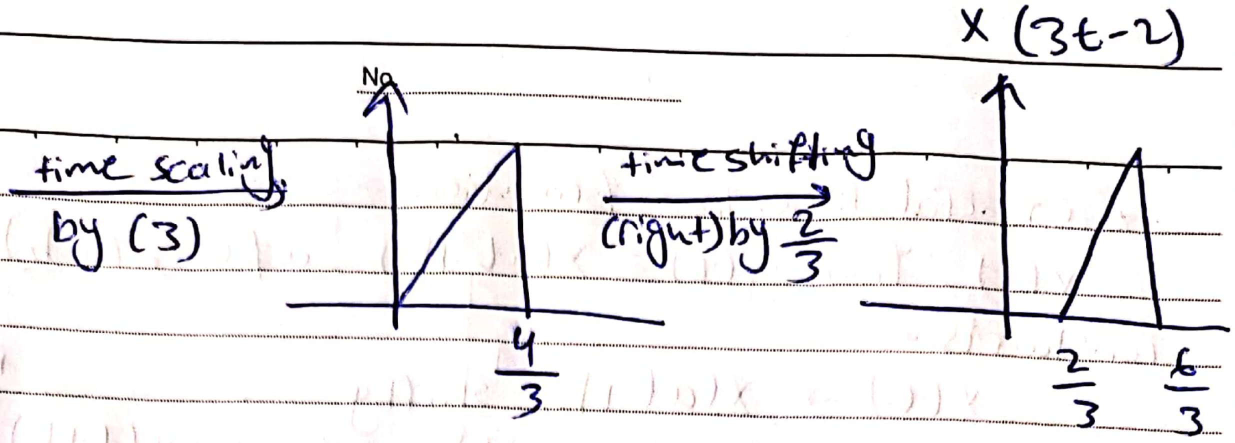
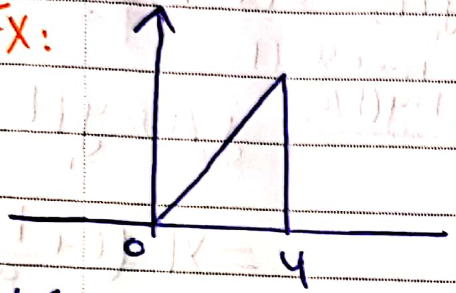
M2:-

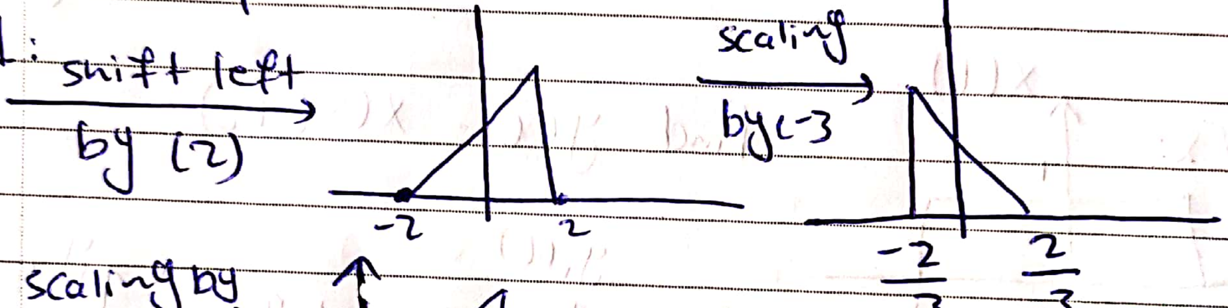


Ex:

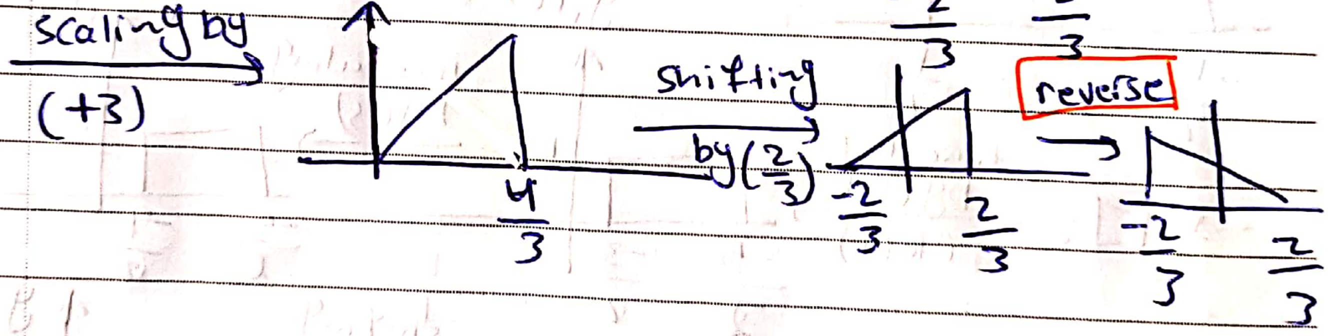


, find $x(2-3t)$

M1:



M2:



Ex: $y_1(t) = x(3t-2)$ is $y_2(t) = y_1(-t)$??
$y_2(t) = x(2-3t)$

$y_1(-t) = x(-3t-2) \neq x(-3t+2)$
then $y_1(t) \neq y_2(-t)$

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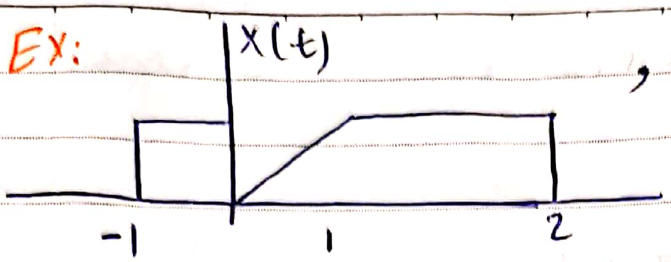
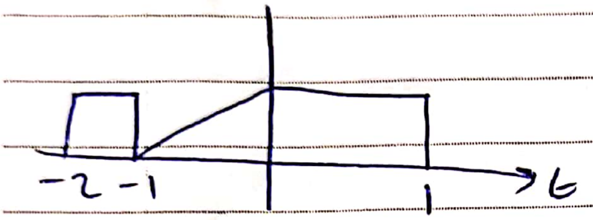
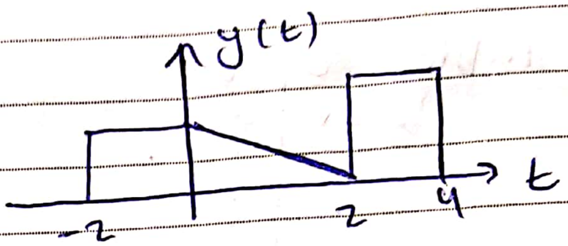
Ex:



, find $y(t) = x(1-t/2)$

M1: $\boxed{①}$ shift left by $\underline{①}$, $\boxed{②}$ scale by $\frac{1}{2}$





M2: $y(t) = x(1-t/2) = x(\frac{-1}{2}(t-1))$

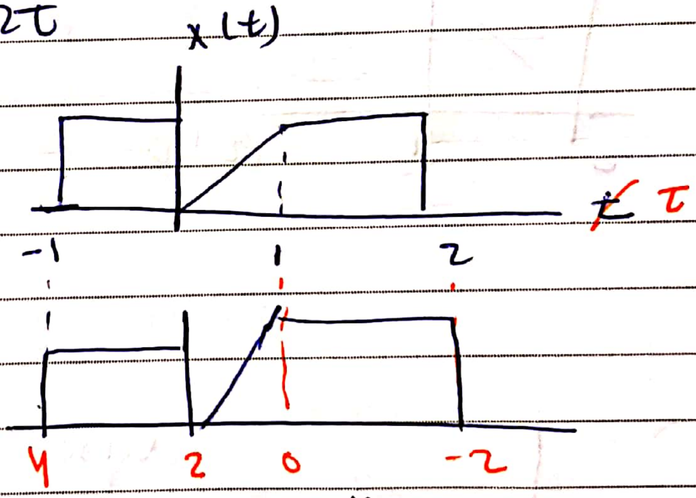
** or (Genric method)

$y(t)=x(\underbrace{1-t/2}_{\tau}) \longrightarrow y(2-2\tau) = x(\tau)$

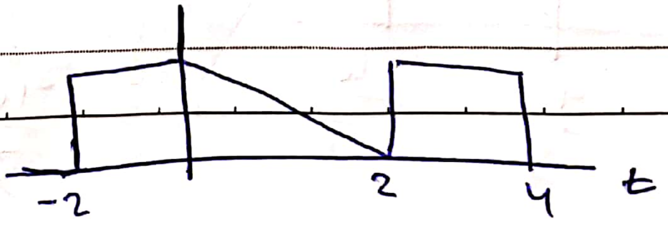
$\tau = 1 - t/2$

$t = 2-2\tau$

t	τ
4	-1
2	0
0	1
-2	2



⇓ reverse axis y(t)



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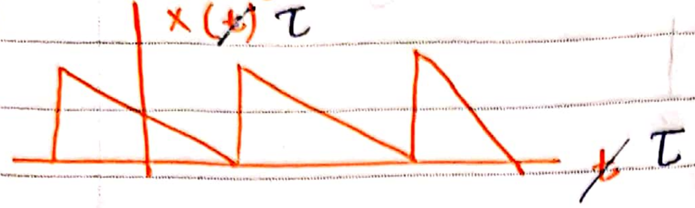
- Given $y(t) = x(at+b)$

①

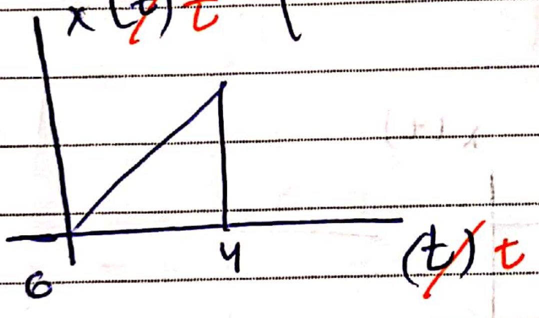


② assume that $(at+b) = \tau \longrightarrow t = \frac{\tau - b}{a}$

③ table

τ	t
⋮	⋮

Ex: $y(t) = x\left(\frac{-3t+4}{5}\right)$



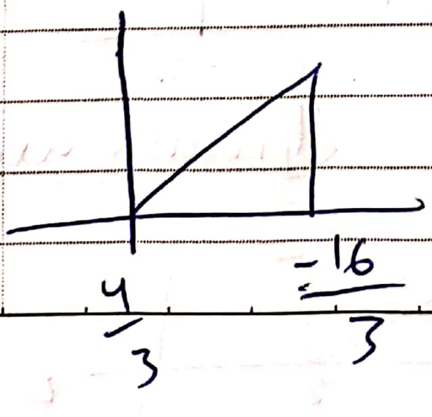
① $\tau = \frac{-3t+4}{5}$

$\rightarrow t = \frac{5\tau - 4}{-3}$

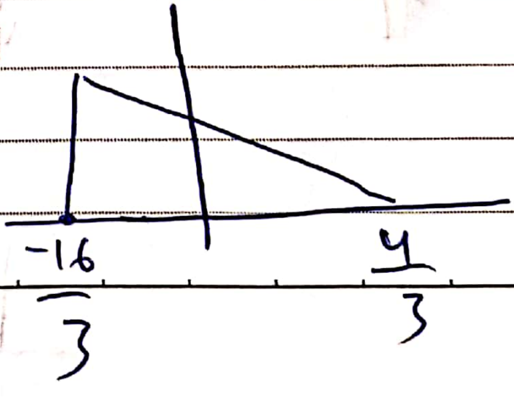
②

τ	t
4	$\frac{-16}{3}$
0	$\frac{4}{3}$

③



reverse =>



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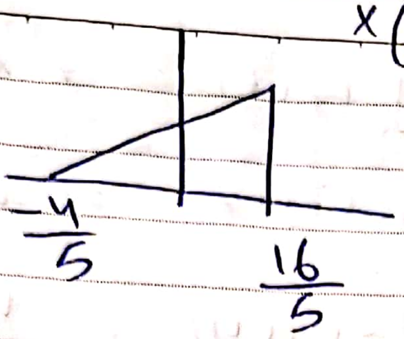
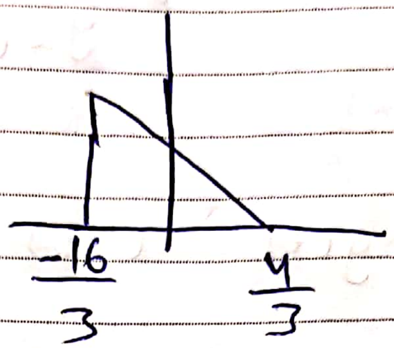
① Shift by $\frac{4}{5}$

using M1:

$x(t+\frac{4}{5})$



② Scaling by $\frac{-3}{5}$



Ex:
$$x(t) = \begin{cases} 2t, & t>0 \\ -t, & t<0 \\ 0, & t=0 \end{cases}$$
, Find

Find $y(t) = x(1-2t)$

$$x(u) = \begin{cases} 2u, & u>0 \\ -u, & u<0 \\ 0, & u=0 \end{cases} \rightarrow u = 1-2t$$

$$y(t) = \begin{cases} 2(1-2t), & 1-2t>0 \\ -(1-2t), & 1-2t<0 \\ 0, & 1-2t=0 \end{cases} = \begin{cases} 2-4t, & t<\frac{1}{2} \\ 2t-1, & t>\frac{1}{2} \\ 0, & t=\frac{1}{2} \end{cases}$$

***** In general:-**
$$\begin{cases} g_1(t), & h_1(t)>0 \\ g_2(t), & h_2(t)>0 \\ \vdots \\ g_n(t), & h_n>(t)>0 \end{cases}$$

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$\Rightarrow$ then $x(f(t)) = \begin{cases} g_1(f(t)), & h_1(f(t)) > 0 \\ g_2(f(t)), & h_2(f(t)) > 0 \\ \vdots & \\ g_n(f(t)), & h_n(f(t)) > 0 \end{cases}$

$\boxed{\text{Find}}$ $y(t) = x(3t) = \begin{cases} 1+\cos(3t), & 3t > 0 \\ (3t+1)^3, & 3t < 0 \end{cases}$

if $x(t) = \begin{cases} 1+\cos(t), & t > 0 \\ (t+1)^3, & t < 0 \end{cases}$

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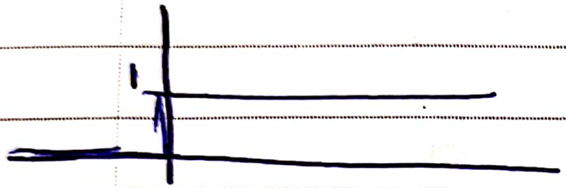
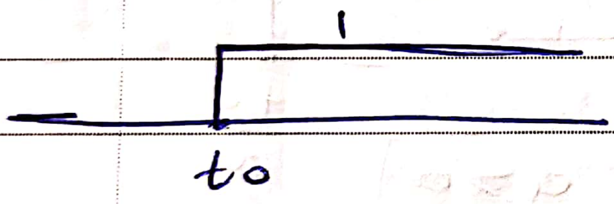
* Basic C.T Signal

1- unit Step function



$$u(t) = \begin{cases} 1 , & t>0 \\ 0 , & t<0 \\ un , & t=0 \end{cases}$$

$$* \ u(t-t_o) = \begin{cases} 1 , & t \geq t_o \\ 0 , & t < t_o \end{cases}$$



①

$$* \left(u(t-t_o)\right)^N = u(t-t_o)$$

②

$$* \ u(at) = ?$$

$$u(t) = \begin{cases} 1 , & t>0 \\ 0 , & t<0 \end{cases} \Rightarrow u(at) \begin{cases} 1 , & at>0 \\ 0 , & at<0 \end{cases}$$

if $a>0 \Rightarrow u(at) = \begin{cases} 1 , & t>0 \\ 0 , & t<0 \end{cases}$

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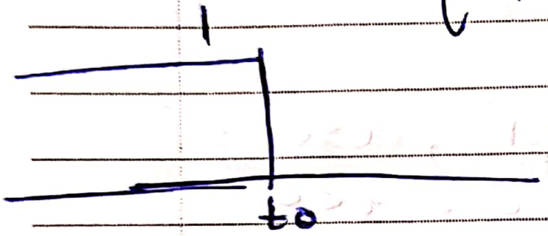
No.

$$u(at) \text{ if } a<0 = \begin{cases} 0 , & t>0 \\ 1 , & t<0 \end{cases} = u(-t)$$

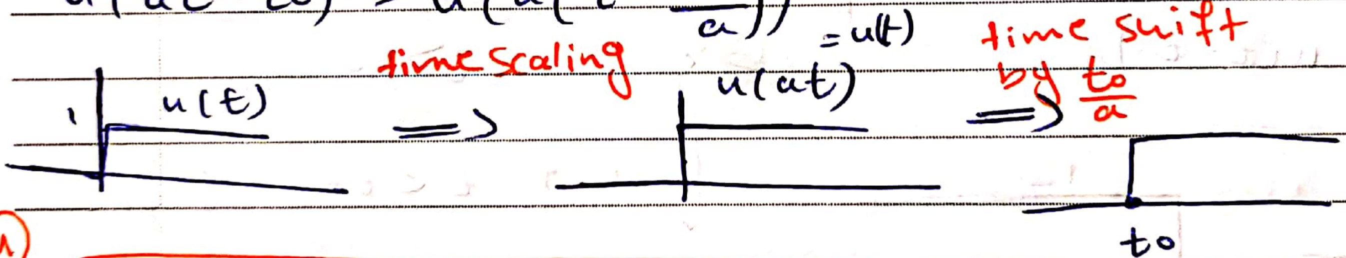
③ $u(at-t_0)$, $a>0$

$= u\left(t-\frac{t_0}{a}\right)$

$$* \; u(-t+t_0) = \begin{cases} 0 & t>t_0 \\ 1 & t<t_0 \end{cases}$$



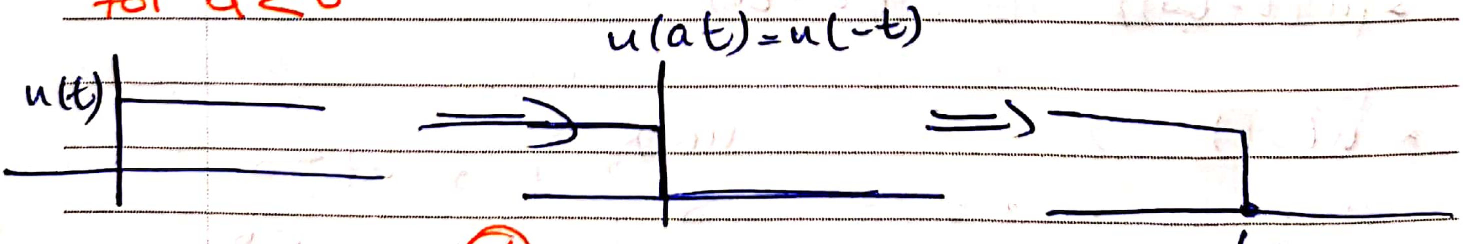
$$u(at-t_0) = u\left(a\left(t-\frac{t_0}{a}\right)\right)$$



④

$$\boxed{* \quad u(at-t_0) = u\left(t-\frac{t_0}{a}\right) \text{ for } a>0}$$

for $a<0$



a

$$\boxed{* \quad u(at-t_0) = u\left(\frac{t_0}{a}-t\right) \text{ when } a<0}$$

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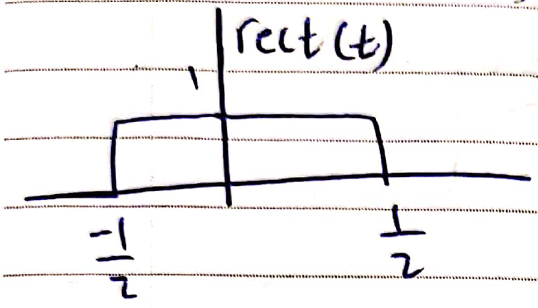
$\boxed{2}$

No.

* Rectangular function . rect(·)

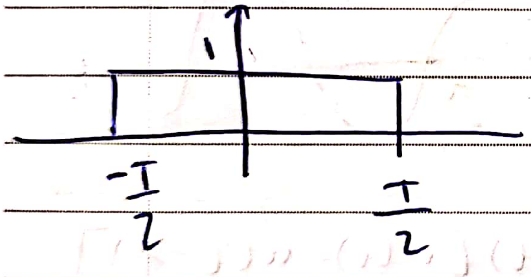
Def:

$$\mathrm{rect}(t) = \begin{cases} 1, & \frac{1}{2} > t > -\frac{1}{2} \\ 0, & \text{o.w} \end{cases}$$

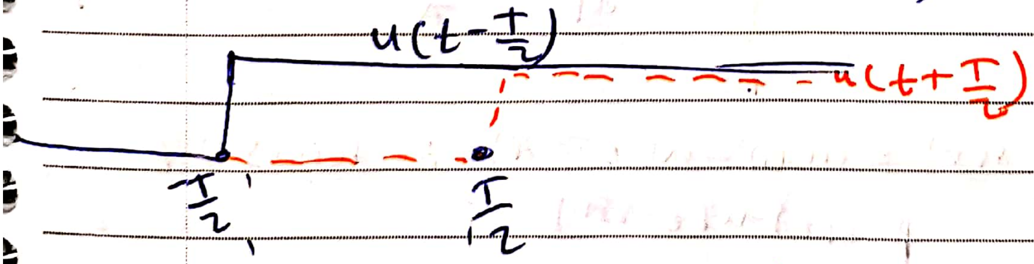


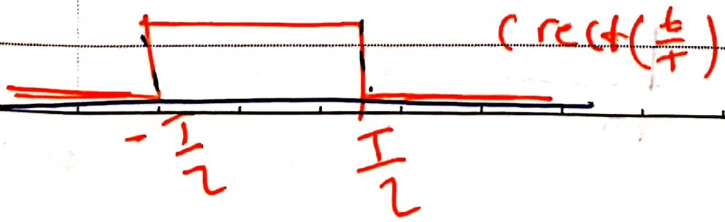
$$\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1, & \frac{1}{2} > \frac{t}{T} > -\frac{1}{2} \\ 0, & \text{o.w} \end{cases}$$

$$\Rightarrow \mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1, & \frac{T}{2} > t > -\frac{T}{2} \\ 0, & \text{o.w} \end{cases}$$



$$u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right) = \mathrm{rect}\left(\frac{t}{T}\right)$$





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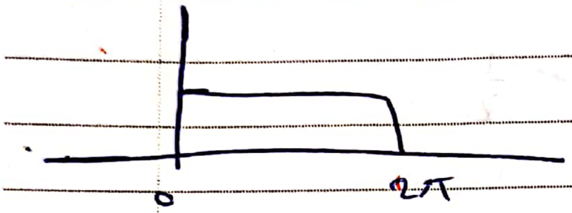
No.

$$\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} u(t+T/2) - u(t-T/2) & \text{—①} \\ u(-t+T/2) - u(-t-T/2) & \text{—②} \\ u(t-T/2) \cdot u(T/2-t) & \text{—③} \end{cases}$$

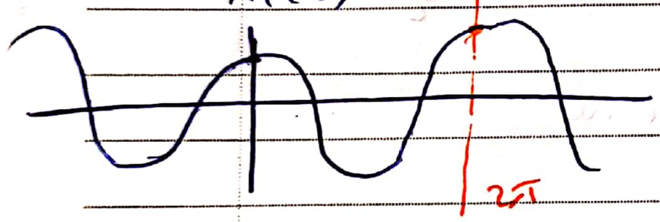
lecture.

EX: let $x(t) = \underbrace{\cos(t)}_{x_1(t)} \underbrace{[u(t) - u(t-2\pi)]}_{x_2(t)}$

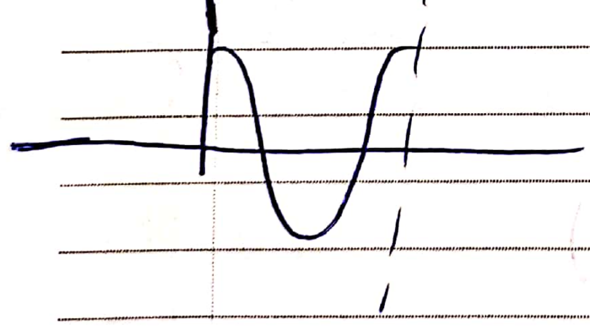
$x_2(t) = u(t) - u(t-2\pi)$



$x_1(t)$

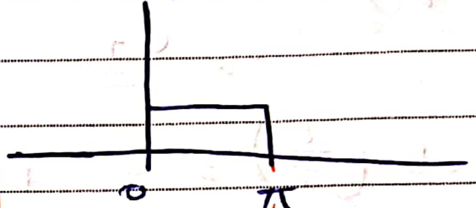


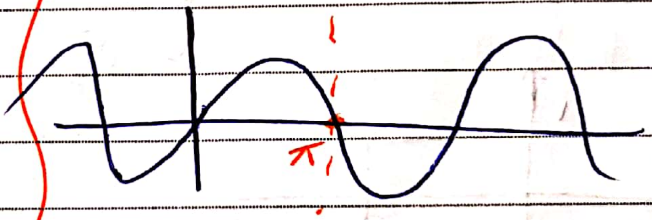
$x(t)$



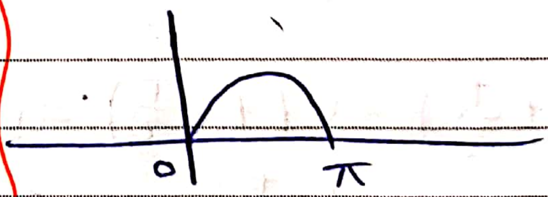
* let $x(t) = \sin(t)[u(t) - u(t-\pi)]$

$u(t-\pi)$

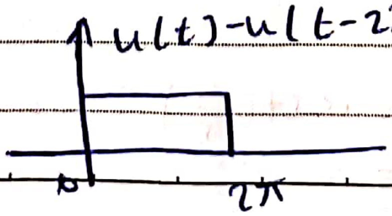




$\sin(t)[u(t) - u(t-\pi)]$



* EX: express $x(t) = u(t) - u(t-2\pi)$ in terms of rect function.



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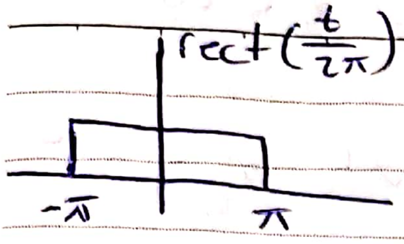
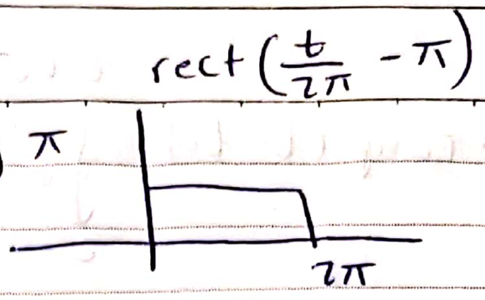


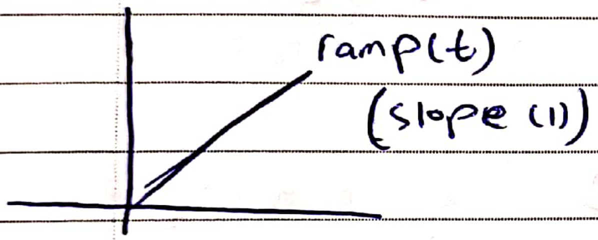
shift rigut by π $\Longrightarrow$



then $x(t) = u(t) - u(t-2\pi) = rect(\frac{t}{2\pi} - \pi)$

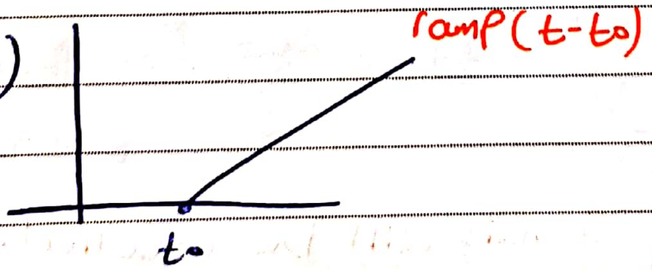
3 Ramp function. $\boxed{ramp(t)}$

$$ramp(t) = \begin{cases} t, & t \geqslant 0 \\ 0, & t<0 \end{cases} = t \cdot \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases} = \boxed{t\,u(t)}$$



$$* \; ramp(t) = \int_0^t u(\tau)\, d\tau = \int_0^t 1 .\, d\tau = \begin{cases} t, & t>0 \\ 0, & t<0 \end{cases} = t\,u(t)$$

$* \, ramp(t-t_0)$



$$* \; ramp(t-t_0) = (t-t_0)\, u(t-t_0)$$

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$$* \text{ramp}(t-t_0) = \int_0^{t-t_0} u(\tau)\, d\tau = \begin{cases} 0, & t<t_0 \\ \int_0^{t-t_0} 1 \cdot d\tau, & t>t_0 \end{cases}$$

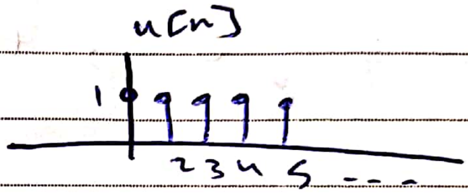
$$= \begin{cases} 0, & t<t_0 \\ t-t_0, & t>t_0 \end{cases} = t-t_0 \begin{cases} 0, & t<t_0 \\ 1, & t>t_0 \end{cases}$$

$$= (t-t_0)\, u(t-t_0)$$

* D.T signals

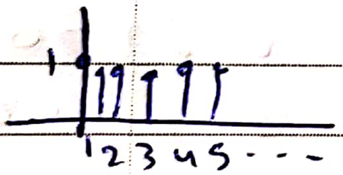
* $u[n]$ is the D.T unit step function.

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n<0 \end{cases}$$

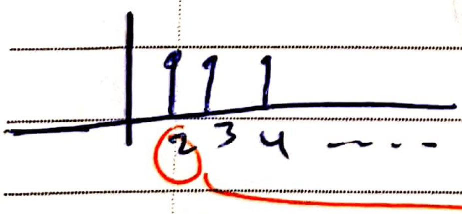


Plot $x[n] = u[n] - u[n-2]$

$u[n]$

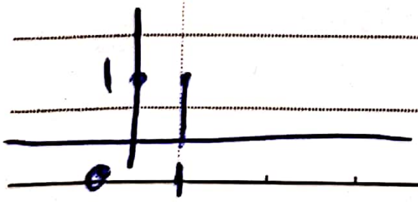


$u[n-2]$



this will be cancelled.

$u[n]-u[n-2]$



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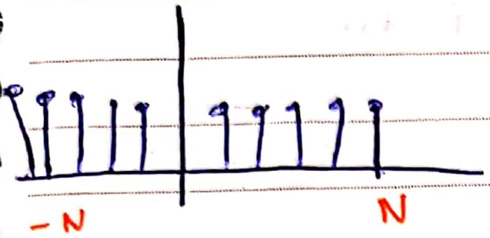
* D.T rect function.

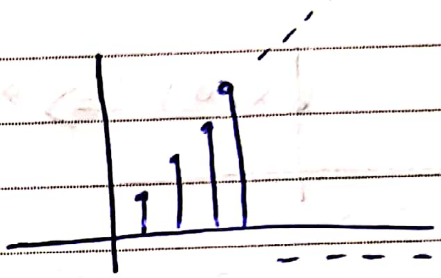
$\text{rect}_N[n]$



$* \ \text{rect}_N[n] = u[n+N] - u[n-(N+1)]$

* D.T Ramp function.

$$\text{ramp}[n] = \begin{cases} n , & n \geq 0 \\ 0 , & n < 0 / \text{ow} \end{cases}$$



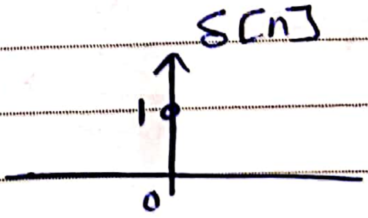
$$\text{ramp}[n] = \sum_{n=-\infty}^{n} u[n]$$

or

$n=0$

** Discrete time ⊛ Impulse function (kronecker Delta function).

$$\delta[n] = \begin{cases} 1 , & n = 0 \\ 0 , & n \neq 0 \end{cases}$$



** $\delta[n]$ is used to model events that happen at one particular time.

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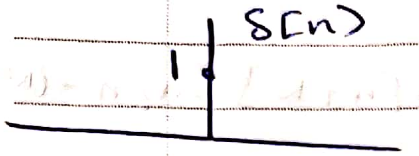
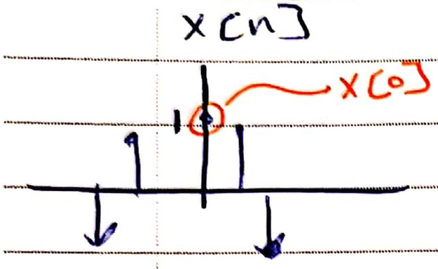
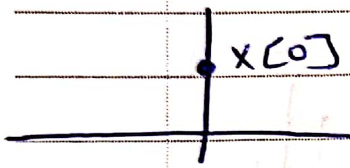
**properties:-

$$1)\ x[n]\cdot\delta[n] = \boxed{\underbrace{x[0]\cdot\delta[n]}}$$

↳ new function of time.







⟹ * new delta function with amplitude of x[0]

* $x[n] = [1\ \ 2\ \ 3\ \ 4]$
$\delta[n] = [1\ \ 0\ \ 0\ \ 0]$

$x[n]\cdot\delta[n] = [1\ \ 0\ \ 0\ \ 0]$ (not scalar). (function of time).

$$\boxed{2)\ \sum_{n=-\infty}^{\infty} x[n]\cdot\delta[n] = x[0]}$$

*proof: $\sum_{n=-\infty}^{\infty} x[0]\cdot\delta[n] = x[0]\underbrace{\sum_{n=-\infty}^{\infty}\delta[n]}_{1} = x[0]$

$x[n]\cdot\delta[n] \neq x[0]$

$$3)\ \delta[n-n_0] = \begin{cases} 1, & n=n_0 \\ 0, & n\neq n_0 \end{cases}$$

a) $x[n] \cdot \delta[n-n_0] = x[n_0]\,\delta[n-n_0]$

b) $\sum_{n=-\infty}^{\infty} x[n]\,\delta[n-n_0] = x[n_0]$

Ex: $\cos[\pi n]\,\delta[n] = \cos(\pi \cdot 0)\,\delta[n] = \delta[n]$

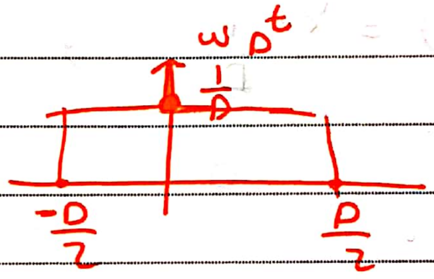
Ex: $e^{-3n}\delta[n+1] = e^{-3(-1)}\delta[n+1] = e^{3}\delta[n+1]$

- $\sum_{n=-\infty}^{\infty} \cos\left(\frac{n\pi}{8}\right)\delta[n-2] = \cos\left(\frac{2\pi}{8}\right) = \cos\left(\frac{\pi}{4}\right)$

** C.T impulse function

- Dirac - Delta -

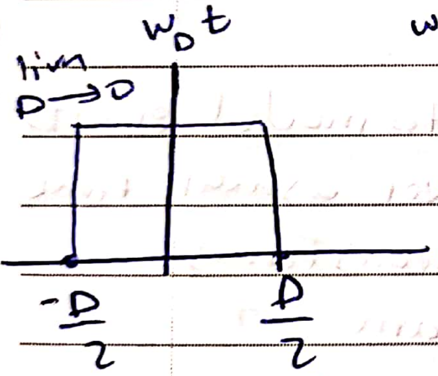
- let's define $w_D(t)$ as:



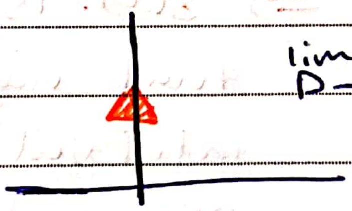
as $D \to 0$

width $\to 0$

hight $\to \infty$



$\xrightarrow{D \to 0}$



$\lim_{D \to 0} w_D(t) = \delta(t)$

$$\boxed{\delta(t) = \lim_{D \to 0} w_D(t)}$$

$$\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1$$

$\boxed{1}$ $x(t) \cdot \delta(t) = x(0)\,\delta(t)$

$\boxed{2}$ $\int_{-\infty}^{\infty} x(t) \cdot \delta(t)\,dt = x(0)$

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• proof:- $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} x(0)\,\delta(t)\,dt$

$= x(0) \underbrace{\int_{-\infty}^{\infty} \delta(t)\,dt}_{1} = x(0)$ ⁂

** So the Def: $\delta(t) = \lim_{D \to 0} w_D(t)$, where

- $w_D(t) = \begin{cases} \frac{1}{D} , & \frac{D}{2} > t > -\frac{D}{2} \\ 0 , & \text{o.w} \end{cases}$

- $\int_{-\infty}^{\infty} \delta(t) = 1$ ⟹ $\delta(t)$: is used to model events that happen over asmall time interval but transferms non-negligible am;ant of physical quantity.

1) $x(t)\,\delta(t) = x(0)\,\delta(t)$

2) $\int_{-\infty}^{\infty} x(t)\,\delta(t) = x(0)$

1) $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

2) $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

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Ex: 1) $(t^3+3)\,\delta(t) = 3\,\delta(t)$

2) $\sin\left(t^2-\frac{\pi}{2}\right)\delta(t) = \sin\left(-\frac{\pi}{2}\right)\delta(t) = -\,\delta(t)$

3) $$\int_{-\infty}^{\infty}\underbrace{\frac{w^2+1}{w^2+9}}_{X(w)}\,\delta(w+1)\,dw = X(-1) = \frac{1+1}{1+9} = \frac{2}{10}$$

↳ $t_0=-1$

4) $$\sum_{-\infty}^{\infty}\cos\left(\frac{n\pi}{8}\right)\delta[n-2] = x(2) = \cos\left(\frac{\pi}{4}\right)$$

5) $$\sum_{n=3}^{\infty}\cos\left(\frac{n\pi}{8}\right)\delta[n-\underset{\sim}{2}] = 0$$

$t_0=\textcircled{2}$

2 مش داخل ←

6) $$\int_{-\infty}^{-1}(t^3+3)\,\delta(t)\,dt$$

→ $=0$

→ zero undefined

$\int_{-\infty}^{0}$ لازم من

أقل إشي

7) $$\int_{2}^{4}\cos\left(\frac{\pi t}{2}\right)\delta(t+2)\,dt = 0$$

↳ $\textcircled{-2}$ undefined.

8) $$\int_{-3}^{4}\cos\left(\frac{\pi t}{2}\right)\delta(t+2)\,dt = \cos(-\pi) = -1$$

*unit step and Impulse function:-

$$\int_{-\infty}^{t}\delta(t)\,dt = \begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases}$$

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No.

$$\therefore u(t) = \int_{-\infty}^{\infty} \delta(\tau)\, d\tau$$

$$\frac{du(t)}{dt} = \delta(t)$$

→ back to Second property:-

$$\int_{-\infty}^{\infty} x(t)\, \delta(t) = x(0) \quad , \quad \delta(t) = \frac{du(t)}{dt}$$

$$\Longrightarrow \int_{-\infty}^{\infty} x(t) \frac{du(t)}{dt} dt = u(t)\, x(t) \Big|_{-\infty}^{\infty} - \int u(t)\, x'(t) dt$$

$$= x(\infty) - \left(\int_{0}^{\infty} x'(t)\, dt \right)$$

$$= x(\infty) - \left((x(\infty) - x(0) \right))$$

$$= x(0)$$

Summery:- let $x(t)$ be a cont function on (C.T) at $t = t_0$

① $x(t)\delta(t-t_0) = x(t_0)\, \delta(t-t_0)$

② $\displaystyle\int_{-\infty}^{\infty} x(t)\, \delta(t-t_0)\, dt = x(t_0)$

③ $\displaystyle\int_{-\infty}^{\infty} x(t-t_1)\, \delta(t-t_0)\, dt = x(t_0 - t_1)$

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④ a) $u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$

$\equiv$

c) $u(t-t_0) = \int_{-\infty}^{t-t_0} \delta(\tau) d\tau$

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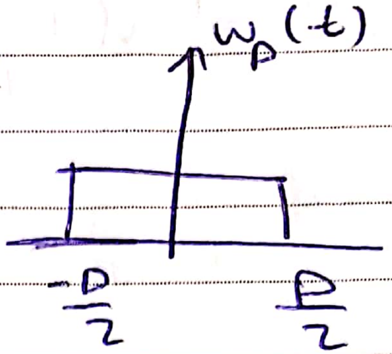
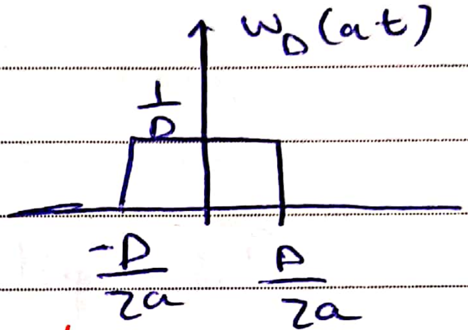
-ixy

5. a) $\delta(-t) = \delta(t)$

b) $\delta(at) = \frac{1}{|a|} \delta(t) \xrightarrow{\text{proof}}$



$\Rightarrow \delta(at) = w_D a(t) = \lim_{D \to 0} w_D(at$



↳ new delta function with amplitude $\frac{1}{a}$ becaus the area is $\frac{1}{a}$

general property

c) $\delta(at - to)$

$\delta(t) \longrightarrow \delta(at) = \frac{1}{|a|} \delta(t)$

then $\delta(at - to) = \frac{1}{|a|} \delta\left(t - \frac{to}{a}\right)$

$\Rightarrow \delta(at - to) = \frac{1}{|a|} \delta\left(t - \frac{to}{a}\right)$

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- show that $\delta(t-t_0) = \frac{1}{|a|}\,\delta\left(t-\frac{t_0}{a}\right)$.

first evaluate $\displaystyle\int_{-\infty}^{\infty} x(t)\,\delta(\underbrace{at-t_0}_{v})\,dt$

let $v = at - t_0$

$\frac{dv}{dt} = a$

for $a>0$ $\displaystyle\int_{-\infty}^{\infty} \delta(v)\cdot x\left(\frac{v+t_0}{a}\right)\frac{dv}{a}$

$a<0$ $\displaystyle\int_{-\infty}^{\infty} \delta(v)\cdot x\left(\frac{v+t_0}{a}\right)\frac{dv}{-a}$

$$\Rightarrow \int_{-\infty}^{\infty} x(t)\,\delta(at-t_0)\,dt = \frac{1}{|a|}\int_{-\infty}^{\infty} x\left(\frac{t_0+v}{a}\right)dv$$

$$= \frac{1}{|a|}\,\underbrace{x\left(\frac{t_0}{a}\right)} =$$

↳ $\displaystyle\int_{-\infty}^{\infty} x(t)\cdot\delta\left(t-\frac{t_0}{a}\right)$

$$\Rightarrow \int_{-\infty}^{\infty} x(t)\,\delta(at-t_0)\,dt = \int_{-\infty}^{\infty} \frac{1}{|a|}\,x(t)\,\delta\left(t-\frac{t_0}{a}\right)dt$$

then

$$\delta(at-t_0) = \frac{1}{|a|}\,\delta\left(t-\frac{t_0}{a}\right)$$

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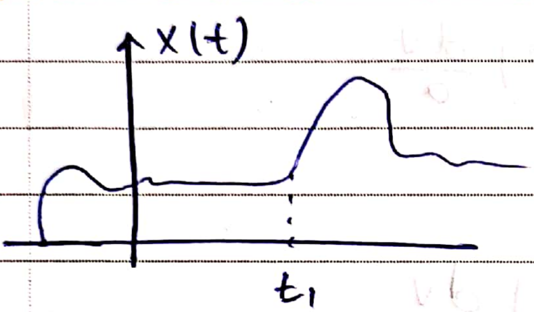
** important result :- $\int_{-\infty}^{\infty} x(t)\,\delta(at-t_0)\,dt = \frac{1}{|a|}\,x\left(\frac{t_0}{a}\right)$

Ex: $\int_{-\infty}^{\infty} \delta(2t-2)\,e^{-2t}\,.dt = \frac{1}{|2|}\,x\left(\frac{2}{2}\right) = \frac{1}{2}\,x(1) = \frac{1}{2}\,e^{-1}$

* For any given signal $x(t)$ such that $x(t)$ is discontinous at $t = t_1$



then $\frac{dx(t)}{dt} = \acute{x}(t)$

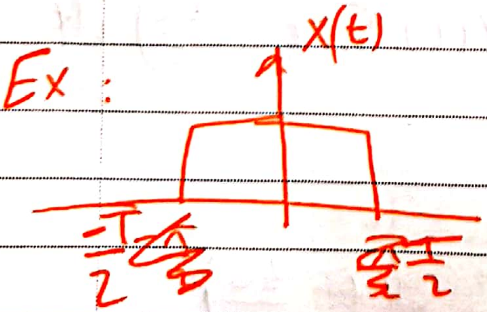
$\boxed{\mp}\left(x(t_1^+) - x(t_1^-)\right)\delta(t-t_1)$

حسب ال function

if I have many discontinuity points.

$$\frac{dx(t)}{dt} = \acute{x}(t) + \sum_{i=1}^{n}\left(x(t_i^+) - x(t_i^-)\right)\delta(t-t_i)$$

Ex :



find $\acute{x}(t)$

~~$\frac{dx(t)}{dt} = 0 + 1\cdot\delta(t+\frac{T}{2}) - 1\cdot\delta(t-\frac{T}{2})$~~

~~$= \delta(t+\frac{T}{2}) - \delta(t-\frac{T}{2})$~~

$$\frac{dx}{dt} = 0 + 1\cdot\delta\left(t+\frac{T}{2}\right) - 1\cdot\delta\left(t-\frac{T}{2}\right)$$

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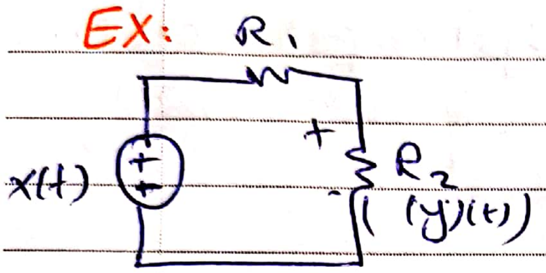
* what is a system??

input signal "x(t)" $\longrightarrow \boxed{\text{system}} \longrightarrow$ output signal "y(t)"

- A system is a process that transforms an input signal into an output signal.

EX:



$$y(t) = \frac{R_2}{R_1+R_2} \cdot x(t) \implies V_{out} = V_{in} \cdot \frac{R_2}{R_1+R_2}$$

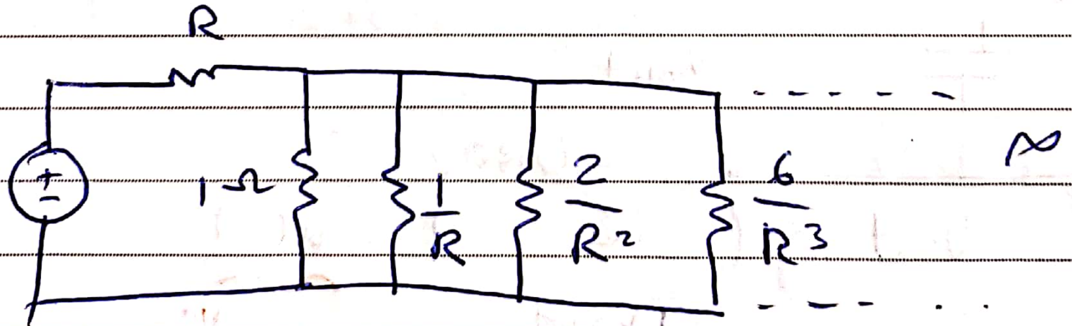
$$x(t) \longrightarrow \boxed{T} \longrightarrow y(t)$$

$$y(t) = T\{x(t)\}$$

EX: $y(t) = \alpha x(t)$

$y(t) = x^2(t)$

$y(t) = \cos(x(t))$



$$G = 1 + \frac{1}{R} + \frac{2}{R^2} + \frac{6}{R^3} \ldots.$$

by series I'll have G

smi)e

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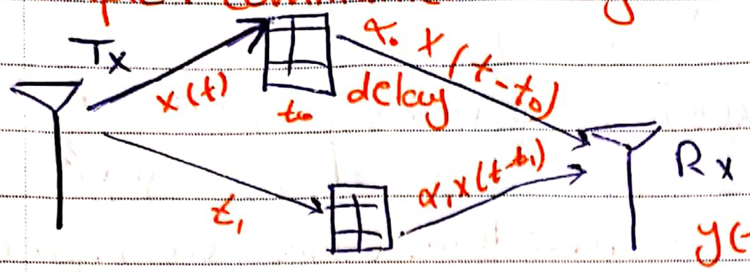
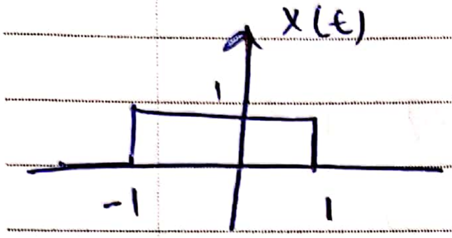
* example: Communication System.



$$y(t) = \underbrace{\alpha_0}\, x(t-\underbrace{t_0}) + \underbrace{\alpha_1}\, x(t-\underbrace{t_1})$$

system parameters

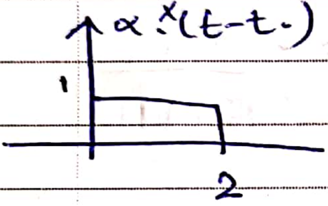


Find y(t)?

I need system parameters.

$\alpha_0 = 1$ $\quad t_0 = 1$

$\alpha_1 = 2$ $\quad t_1 = 2$



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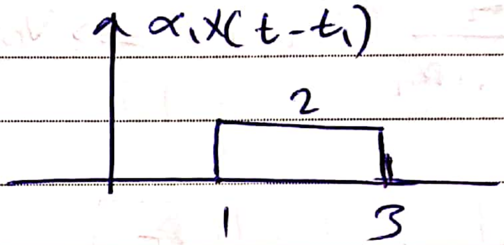
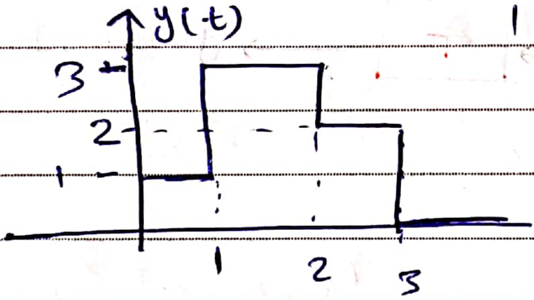
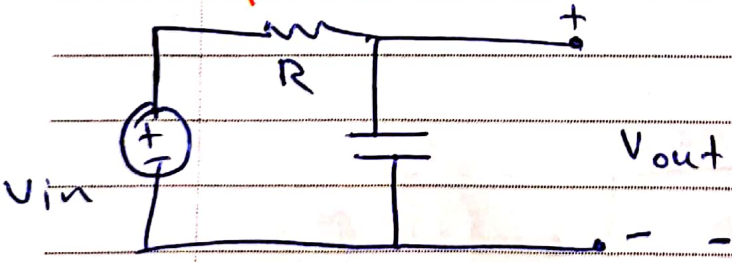
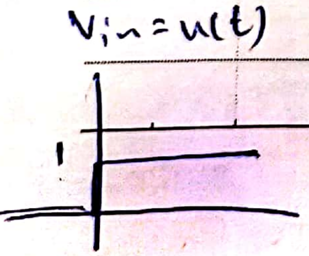


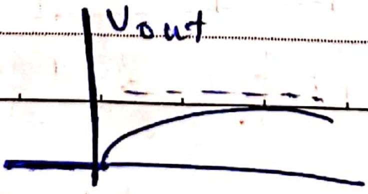


* example:-



$$V_{out} = \left(1 - e^{-t/RC}\right) \cdot \underbrace{u(t)}_{V_{in}}$$





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* system Calssification :-

$\boxed{1}$ memoryless systems and systems with memory.

Def: a system is said to have memory if the output at $((t=t_0) \equiv y(t_0))$ depends on the input at $t=t_1$ $(x(t_1))$ where $t_1 \neq t_0$

Ex: $y(t) = x(t-1)$

$y(t_0) = x(t_0 - 1)$

Since the output at t_0 depends at the input at $(t_0 - 1)$ then the system has memory.

Ex$_2$: $\left.\begin{array}{l} y(t) = x^2(t+1) \\ y(t_0) = x^2(t_0+1) \end{array}\right\} \rightarrow x^2(t_0+1)$ with memory

* $y(t) = x^3(t)$

memoryless system.

* $y(t) = \cos(x(t))$, memoryless system.

* $y(t) = \cos(t+1) \cdot x(t)$

$y(t_0) = \cos(t_0+1) \cdot \underbrace{x(t_0)}$ → memoryless system.

بطلع عليِه

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EX: D.T

let $y[n] = y[n-1] + x[n]$

where $y[0]=0$, $x[0]$

is the system memoryless?

$\Rightarrow$ $y[1] = y[0] + x[1] = x[1]$

$y[2] = y[1] + x[2] = x[1] + x[2]$

$\therefore$ this is a system with memory.

** BIBO stability:-

<< Bounded input, Bounded output >>

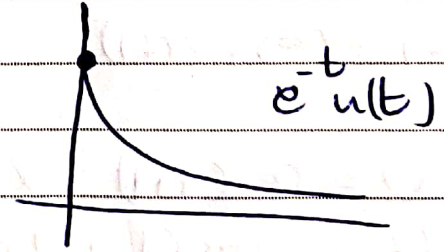
Def: a signal $x(t)$ is said to be bounded if $|x(t)| \le M_x < \infty$ for all $\underline{\underline{t}}$.

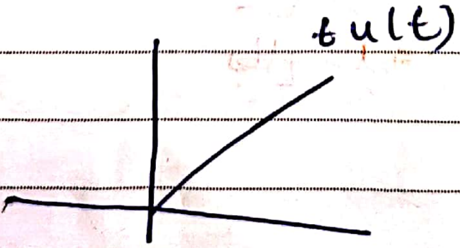
EX: $x(t) = \cos(t) \rightarrow |x(t)| \le 1$

$x(t) = \cos(t) + 2 \rightarrow |x(t)| \le 3$

$x(t) = e^{-t} u(t)$

$\therefore$ bounded.



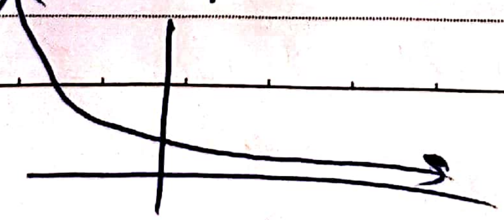


$x(t) = t\,u(t)$

$\rightarrow$ unbounded

$x(t) = e^{-t}$

$\Rightarrow$ unbounded.



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Def: A system is said to be BIBO stable system if every bounded input produces a bounded output.

No.

, mathematically consider a system $x(t) \longrightarrow y(t)$

$$|x(t)| \leq M_x < \infty$$

$$|y(t)| \leq M_y < \infty$$

, then its BIBO stable system.

EX: $y(t) = t^2 x_1(t)$

→ Suppose $x(t) = 1$ (bounded)

then $y(t) = t^2$ unbounded

∴ the System is unstable.

EX: $x(t) = \frac{1}{1+t^2}$ bounded.

$y(t) = \frac{t}{1+t^2}$ bounded.

$$\lim_{t \to \infty} \frac{t^2}{1+t^2} = 1$$

EX: $y(t) = e^{x(t)}$

for any $x(t)$ such that $|x(t)| \leq Mx < \infty$

then $y(t) = e^{MX} \leq M_y < \infty$

∴ this system is stable.

EX: $y(t) = x(t) \underbrace{e^{-t} u(t)}$ max → 1

→ assuming $x(t)$ is bounded

$y(t) \leq Mx \cdot 1$

→ System is stable.

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* $y(t) = x(t)\, e^{-t}$? → unstable.

No.

** Causality:-

$$x(t) \longrightarrow \boxed{\quad\quad} \longrightarrow y(t)$$

Def: A system is a non causal if the output at $t=t_0$ $(y(t_0))$ depends on any future value of the input $(x(t_1), t_1 \neq t_0)$

$y(t) = x(t)$ → causal system

$y(t) = x(t-1)$ → Causal System.

$y(t) = x(t) + x(t+1)$ → non Causal system

$$x(t) \longrightarrow \boxed{T_1} \longrightarrow y_1(t) = x(t)+x(t+1) \longrightarrow \boxed{T_2} \longrightarrow \begin{array}{l} y(t) \\ = y_1(t-1) \\ = \underbrace{x(t-1)+x(t}_{\text{Causal sys}} \end{array}$$

EX: $y(t) = x(-t)$

if $t>0$

$y(t_0) = x(-t_0)$ → Causal

if $t<0$

$y(t_0) = x(t)$ → non Causal

∴ in General its non causal.

fst exam material : // done.

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* Linearity:-

$$x(t) \longrightarrow \boxed{T} \longrightarrow y(t)$$

- Additivity (Super position):

$$x_1(t) \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow y_2(t)$$

then $x_3(t) = x_1(t) + x_2(t) \longrightarrow y_3(t) = y_1(t) + y_2(t)$

Ex: $y(t) = t\,x(t)$. Is the system additive.

$$x_1(t) \longrightarrow y_1(t) = t \cdot x_1(t)$$

$$x_2(t) \longrightarrow y_2(t) = t \cdot x_2(t)$$

let

$x_3(t) = x_1(t) + x_2(t)$

$$x_3(t) \longrightarrow y_3(t) = t \cdot x_3(t)$$

$$= t(x_1(t) + x_2(t)) = t x_1(t) + t x_2(t)$$

$$= y_1(t) + y_2(t$$

Ex: $\boxed{y(t) = x^2(t)}$ → not additive.

$x_1(t) \longrightarrow y_1(t) = x_1^2(t)$

$x_2(t) \longrightarrow y_2(t) = x_2^2(t)$

$x_3(t) \longrightarrow y_3(t) = (x_1(t) + x_2(t))^2$

$$= \underbrace{x_1^2(t)}_{y_1(t)} + \underbrace{x_2^2(t)}_{y_1(t)} + \underbrace{2x_1(t)x_2(t)}_{\downarrow}$$

because of this the system is not additive.

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Ex: $y(t) = x(t)x(t-1)$. Is this system additive ??

- Homogenity :-

$$x(t) \longrightarrow \boxed{T} \longrightarrow y(t)$$

let $x_1(t) \rightarrow y_1(t)$

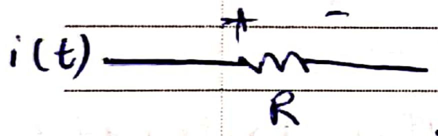
then $a x_1(t) \rightarrow a y_1(t)$

Ex: $y(t) = t\,x(t)$

$x_1(t) \rightarrow y_1 = t\,x_1(t)$

$a x_1(t) \rightarrow t(a x_1(t)) = a\,t\,x_1(t) = a y_1(t)$

=> Homogenous.



$$\underset{\text{output}}{v(t)} = \overset{\text{constant.}}{R}.\underset{\text{input}}{i(t)}$$

Ex: $y(t) = Ax(t) + B$

$x_1(t) \rightarrow \cancel{a} y_1(t)$

$a x_1(t) \rightarrow y_2(t) = A(a x_1(t)) + B$

$a y_1(t) = A a x_1(t) + aB \neq y_2(t)$

$\neq a y_1(t)$

$y(t) \equiv$ not homogenous

but for $B=0$

$a x_1(t) \rightarrow A a x_1(t) = a y(t)$

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$\Rightarrow$ A system is linear if it is both Additive and Homogenous.

$$x(t) \longrightarrow y(t)$$

$$\left.\begin{aligned} a_1 x_1(t) &\longrightarrow a_1 y_1(t) \\ a_2 x_2(t) &\longrightarrow a_2 y_2(t) \end{aligned}\right\} \text{Homogenity}$$

$$\boxed{a_1 x_1(t) + a_2 x_2(t) \longrightarrow a_1 y_1(t) + a_2 y_2(t)} \rightarrow \text{Linearity}$$

Ex: $x(t) \longrightarrow y(t)$ (Linearity)

let $a_1 x_1(t) \longrightarrow a_1 y_1(t)$

$a_2 x_2(t) \longrightarrow a_2 y_2(t)$

$\vdots$

$a_n x_n(t) \longrightarrow a_n y_n(t)$

$$x(t) = \sum_{i=1}^{N} a_i x_i(t) \longrightarrow \sum_{i=1}^{N} a_i y_i(t)$$

* a linear system $y(t) = T\{x(t)\}$, $x(t) \longrightarrow y(t)$

for linear system. $y(t) = T\left\{\sum_{i=1}^{N} a_i x_i(t)\right\} = \underbrace{\sum_{i=1}^{N} T\{a_i x_i(t)\}}$ additive system.

also

$$= \sum_{i=1}^{N} a_i T\{x_i(t)\} \rightarrow \text{homgenity}$$

$$= \sum_{i=1}^{N} a_i y_i(t)$$

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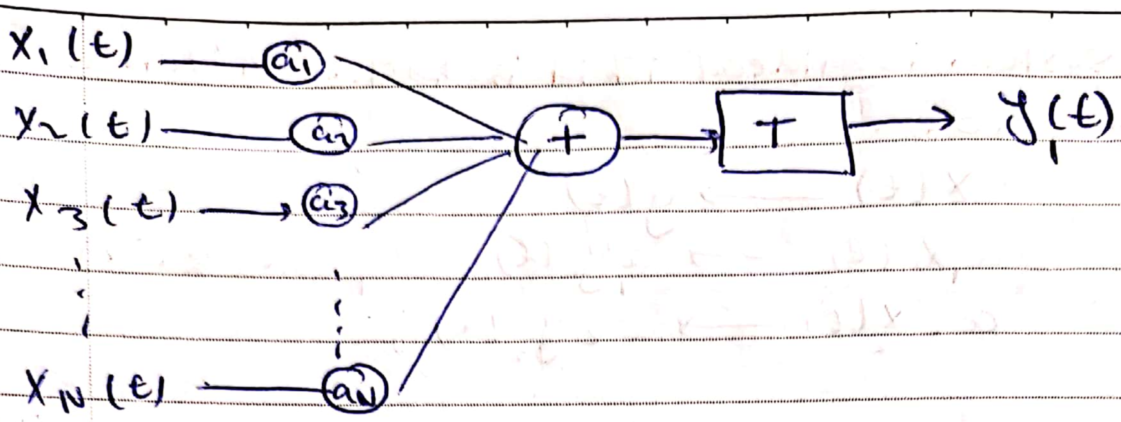
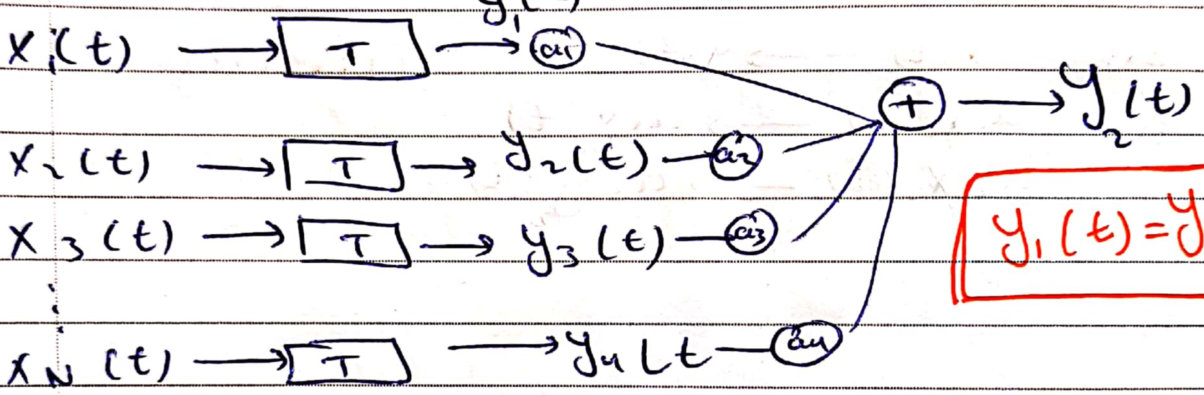
No.





$$\boxed{y_1(t) = y_2(t)}$$

* linear systems reduce number of systems neended to produce the same output from N inputs,

Ex: consider a linear system :-

$$x(t) \longrightarrow \boxed{T} \longrightarrow y(t)$$

given that $x_1(t) = e^{j2\pi t} \longrightarrow y_1(t) = e^{j3\pi t}$

$x_2(t) = e^{-j2\pi t} \longrightarrow y_2(t) = e^{-j3\pi t}$

what is the output "$y_3(t)$" when the input $x_3(t) =$
$= \sin(2\pi t)$

$\longrightarrow x_3(t) = \sin(2\pi t) \longrightarrow y_3(t) = ?$

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$x_3(t) = \sin(2\pi t) \longrightarrow y_3(t) = ?$

$$x_3(t) = \sin(2\pi t) = \frac{e^{j2\pi t} - e^{-j2\pi t}}{2j}$$

$$y_3(t) = T\{x_3(t)\} = T\left\{\underbrace{\frac{e^{j2\pi t}}{2j}}_{\alpha_1 x_1(t)} + \underset{\alpha_2 x_2(t)}{\frac{e^{-j2\pi t}}{-2j}}\right\}$$

- Since the system is linear, then:-

$$y_3(t) = T\{x_3(t)\} = T\left\{\frac{e^{j2\pi t}}{2j} + \frac{e^{-j2\pi t}}{-2j}\right\}$$

$$= \frac{1}{2j} T\{e^{j2\pi t}\} + \frac{-1}{2j} T\{e^{-j2\pi t}\}$$

$$= \frac{1}{2j} e^{j3\pi t} - \frac{1}{2j} e^{-j3\pi t} = \sin(3\pi t)$$

** Summary:-

$$x(t) \longrightarrow \boxed{T} \longrightarrow y(t)$$

$$y(t) = T\{x(t)\}$$

- Homogenity $\rightarrow T\{a\,x(t)\} = a\,T\{x(t)\} = a\,y(t)$
- Additivity $\rightarrow T\{x_1(t) + x_2(t)\} = T\{x_1(t)\} + T\{x_2(t)\}$
- linearity $\rightarrow T\{a_1x_1(t) + a_2x_2(t)\} = T\{a_1x_1(t)\} + T\{a_2x_2(t)\}$

$$= a_1T\{x_1(t)\} + a_2T\{x_2(t)\}$$

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*Time Invariance :-

$$x_1(t) \rightarrow \boxed{T} \rightarrow y_1(t) = T\{x_1(t)\}$$

↳ time invariance implies that system parameters do not change with time.

* $y_1(t) = k\, x_1(t)$ → time invariant

* $y_2(t) = k(t)\, x_1(t)$ → time Variant.

****Time Invariance:-** if output corresponding to a delayed version of the input signal "$x(t)$" is the output corresponding to $x_1(t)$ delayed by the Same amount.

$$x_1(t) \rightarrow \boxed{T} \rightarrow y(t)$$

$$y(t) = T\{x_1(t)\}$$

$$y_1(t) = T\{x(t-t_0)\}$$

$$y_1(t) = y(t-t_0)$$

$$x_1(t) \rightarrow \boxed{\text{delay } t_0} \rightarrow x_2(t) = x_1(t-t_0) \rightarrow \boxed{T} \rightarrow y_2(t)$$

$$x_1(t) \rightarrow \boxed{T} \xrightarrow{y_1(t) = T\{x_1(t)\}} \boxed{\text{delay } t_0} \rightarrow y_1(t-t_0)$$

* $\boxed{\text{if } y_1(t-t_0) = y_2(t) \rightarrow \text{time invariance.}}$

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Ex:- $y(t) = \cos(x(t))$

$\longrightarrow x_1(t) \longrightarrow \boxed{S}^{t_0} \longrightarrow x_2(t) = x_1(t-t_0) \rightarrow \boxed{T} \rightarrow y_2(t) = \cos(x_2(t)) = \cos(x_1(t-t_0$

$\rightarrow x_1(t) \rightarrow \boxed{T} \rightarrow y_1(t) = \cos(x_1(t)) \rightarrow \boxed{S^{t_0}} \rightarrow y_1(t-t_0) = \cos(x_1(t-t_0)$

=> they are equal → T.I

Ex: $y(t) = x(2t)$

$\rightarrow x_1(t) \Rightarrow \boxed{S^{t_0}} \rightarrow x_2(t) = x_1(t-t_0) \rightarrow \boxed{T} \rightarrow y_1(t) = x_2(2t) = x_2(2t-t_0)$

$\longrightarrow x_1(t) \rightarrow \boxed{T} \rightarrow y_1(t) = x_1(2t) \rightarrow \boxed{S^{t_0}} \rightarrow y_1(t-t_0) = x_1(2(t-t_0)) = x_1(2t-2t_0)$

=> they are not equal → time variant.

** linear time invariant Systems:-

$\boxed{a}$ impulse response "h(t)"

- consider a system $x(t) \rightarrow \boxed{T} \rightarrow y(t)$

the impulse is the output $y(t)$ when the input signal $x(t) = \delta(t)$

$$\cdot\ h(t) \overset{\Delta}{=} y(t)\Big|_{x(t)=\delta(t)}$$

Ex: $y(t) = x(t) + x(t-1)$, find the impulse response "h(t)"

$\rightarrow h(t) = y(t)\Big|_{x(t)=\delta(t)}$ $\quad\rightarrow h(t) = T\{x(t)\}$

$h(t) = \delta(t) + \delta(t-1)$

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* the impulse response completley choretrizes an LTI system . we can find the output y(t) for any input signal x(t) if h(t) is known.

* consider an LTI D.T system.

$$x[n] \rightarrow \boxed{LTI} \rightarrow y[n]$$

$$h[n] = y[n]\Big|_{x[n]=\delta[n]} \qquad h[n]=T\{\delta[n]\}$$

$$y[n] = T\{x[n]\}$$

$\Longrightarrow$ using sifting property $\rightarrow x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$

$$\rightarrow y[n] = T\left\{\sum_{k=-\infty}^{\infty} x[k]\cdot\delta[n-k]\right\}$$

$\boxed{\rightarrow}$ remember that the sys is linear :-

$$y[n] = \sum_{k=-\infty}^{\infty} T\{x[k]\,\delta[n-k]\} \quad \text{(by additivity)}$$

$$= \sum_{k=-\infty}^{\infty} x[k]\cdot T\{\delta[n-k]\} \quad \text{(by homogenity)}$$

$\boxed{\rightarrow}$ the system is time invariant :-

$$h[n] = T\{\delta[n]\}$$

$$T\{\delta[n-1]\} = h[n-k]$$

$$* \boxed{y[n] = \sum_{k=-\infty}^{\infty} x[k]\cdot h[n-k]}$$

$$* \boxed{y[n] = x[n] * h[n]}$$

↳ convulution sum

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* for D.T LTI system

$$x[n] \longrightarrow \underset{LTI}{\boxed{h[n]}} \longrightarrow y[n]$$

$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

** Consider C.T LTI system:-

$$x(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$$

$$h(t) = T\{\delta(t)\} \quad , \quad y(t) = T\{x(t)\}$$

⇒ replacing $x(t)$ with $\int_{-\infty}^{\infty} x(\tau)\cdot\delta(t-\tau)\,d\tau$

$$y(t) = T\left\{\int_{-\infty}^{\infty} \underbrace{x(\tau)\,\delta(t-\tau)}_{f(\tau)}\, d\tau\right\}$$

given function $f(\tau)$

$$\int_{-\infty}^{\infty} f(\tau)\,d\tau \approx \sum_{n=\infty}^{\infty} D\cdot f(D\cdot k) \quad , \quad \text{for } D \longrightarrow 0$$

$$\longrightarrow y(t) = \sum_{k=-\infty}^{\infty} D \cdot k(k\cdot D) \cdot T\{\delta(t-k\cdot D)\}$$

⟶ by time in Voriance

$$y(t) = \sum_{k=-\infty}^{\infty} x(D\cdot k)\cdot h(t-k\cdot D))\cdot D$$

where $D \longrightarrow 0$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

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for C.T systems:-

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

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LTI system

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$$

Properecties of convelution:-

1- $x(t) * h(t) = h(t) * x(t)$, the convolution is commutitive.

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(\underbrace{t-\tau}_{s})\, d\tau$$

$s = t - \tau$

$\frac{ds}{d\tau} = -1$

$$= \int_{-\infty}^{\infty} x(t-s) \,.\, h(s)\, ds$$

↳ replace $\boxed{s}$ by τ

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = h(t) * x(t)$$

2- Associative property:-

$$[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$$

Proof: let $g(t) = x(t) * h_1(t)$

L.H.S

$$g(t) * h_2(t) = \int_{-\infty}^{\infty} g(\tau)\, h_2(t-\tau)\, d\tau \quad —①$$

$$\text{but } g(\tau) = x(\tau) * h_1(\tau) = \int_{-\infty}^{\infty} x(s)\, h_1(\overset{\tau-s}{\cancel{s-\tau}})\, ds \quad — ②$$

$$g(t) * h_2(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(s)\, h_2(t-\tau)\, h_1(\underset{\tau-s}{\cancel{s-\tau}})\, d\tau\, ds \quad —③$$

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let $t-\tau = v$

$dv = -d\tau$

No.

→ ③ can be expresse as

$$g(t) * h_2(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(s)\, h_2(v)\, h_1(t-v-s)\, dv\, ds$$

$$= \int_{-\infty}^{\infty} x(s) \underbrace{\int_{-\infty}^{\infty} h_2(v)\, h_1(t-v-s)\, dv}_{} \qquad ds$$

$\leftarrow f(t-s)$

let $f(t) = h_1(t) * h_2(t)$

$$= \int_{-\infty}^{\infty} h_2(v)\, h_1(t-v)\, dv$$

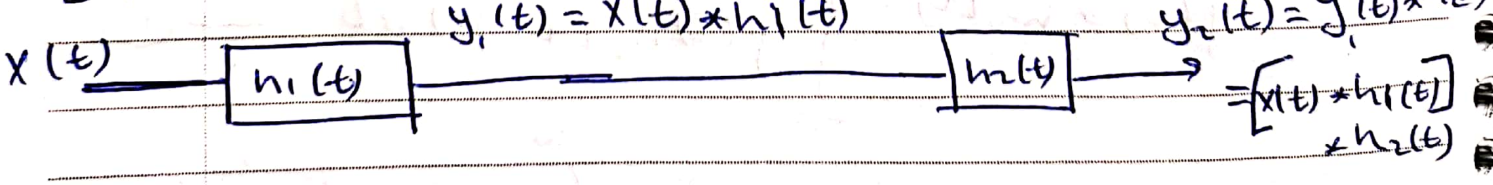
$$f(t-s) = \int_{-\infty}^{\infty} h_2(v)\, h_1(t-s-v)\, dv$$

then, $\int_{-\infty}^{\infty} x(s)\, f(t-s)\, ds = x(t) * f(t)$

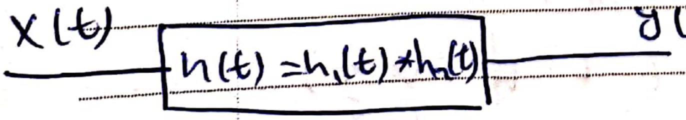
$= x(t) * [h_1(t) * h_2 t]$

※

L.H.S



R.H.S



$y(t) = x(t) * (h_1(t) * h_2(t))$

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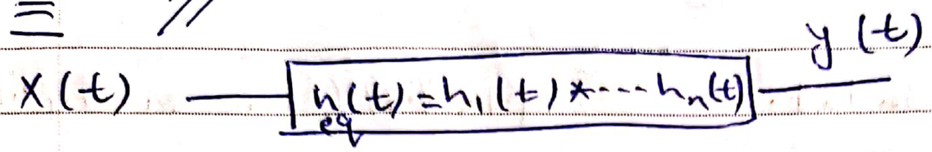
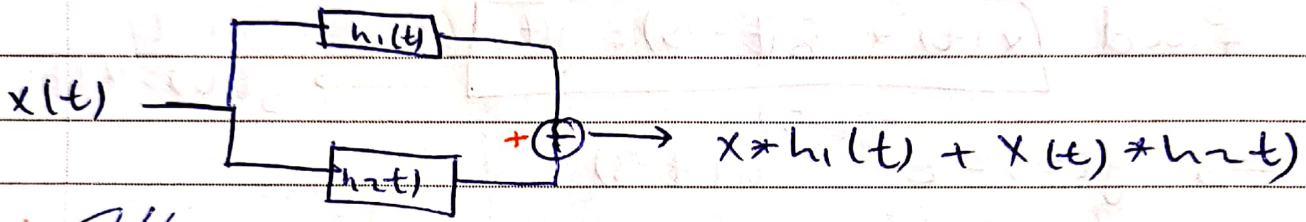
⇓

≡



3- Distributive property.

$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$



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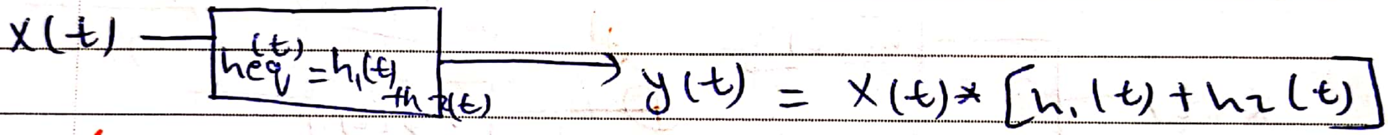
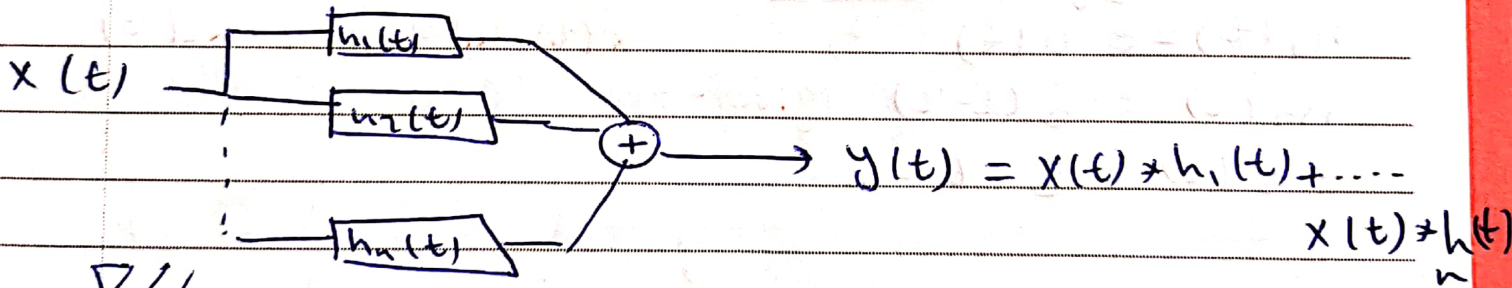


⊛ for n



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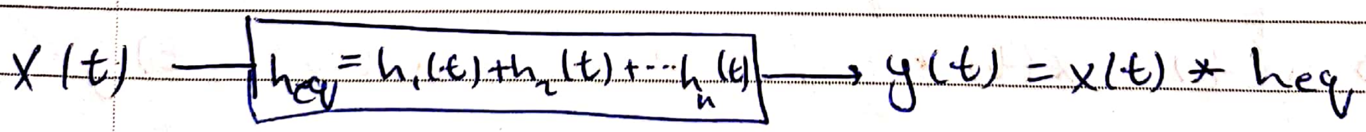


* Notes:- The convolution is Homogenous.

* $a.(x(t)) * y(t) = a.(x(t) * y(t))$

No.

$$* \quad x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau = x(t)$$

$\delta(t)$ in convalution $\equiv$ ~~multipy~~ (1) in multiplication

$$* \quad x(t) * \delta(t-t_0) = x(t-t_0)$$

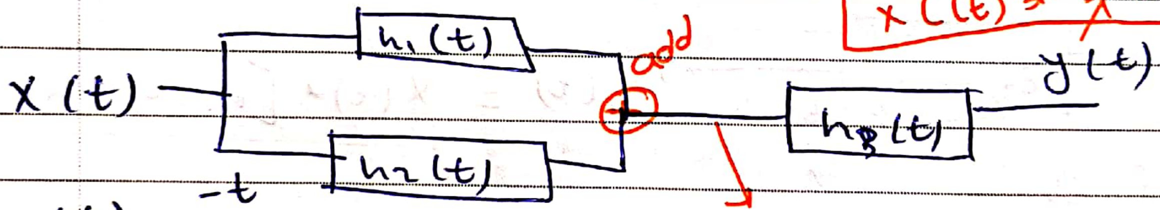
Ex: $x(t) = e^{-t} u(t)$

find $\boxed{x(t) * \delta(t-3) = y(t)}$ sol $\longrightarrow y(t) = x(t-3)$

$\Rightarrow y(t) = e^{-(t-3)} u(t-3)$

Ex: (for 2nd exam):

$y(t) = \boxed{x(t) * h_1(t) * h_3(t) + x(t) * h_2(t) * h_3(t)}$



$x(t) * h_1(t) + x(t) * h_2(t)$

$h_1(t) = e^{-t} u(t)$

$h_2(t) = \delta(t-2)$

$h_3(t) = \delta(t-1)$

} given

① Find the $h_{eq}(t)$ of this system.

$\hookrightarrow y(t) = x(t) * [h_1(t) * h_3(t) + h_2(t)\, h_3(t)]$

$= x(t) * \Big[h_3(t) * [h_1(t) + h_2(t)]\Big]$

$h_{eq} = h_3(t) * [h_1(t) + h_2(t)]$

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$\boxed{\text{Calculation}}$: $\delta(t-1)\left[e^{-t}u(t)+\delta(t-2)\right]$

$h_{eq}=\delta(t-1)*e^{-t}u(t)+\delta(t-1)*\delta(t-2)$

$=e^{-(t-1)}u(t-1)+\delta|(|t|-1+$ [illegible]$)$ $\delta(t-3)$

$*\int_{-\infty}^{\infty}x(t)\,\delta(t-t_0)=x(t_0)$

$x(t)*\delta(t-t_0)=x(t-t_0)$

Ex: let $y(t)=\int_{-\infty}^{t}x(\tau)\,d\tau$

1- find the impulse respons "$h(t)$" of this system.

$h(t)=\int_{-\infty}^{t}\delta(\tau)\,d\tau=u(t)$

2- find $y(t)$ when $x(t)=u(t)$

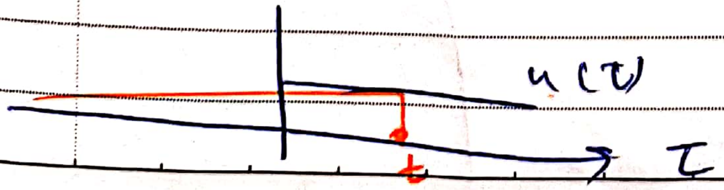
the output $y(t)$ corresponding to $x(t)=u(t)$

$y(t)=\int_{-\infty}^{t}u(t)\,d\tau=t\,u(t)$

Checking if its LTI sys.

$y(t)=x(t)*h(t)$

$=u(t)*u(t)=\int_{-\infty}^{\infty}u(\tau)\cdot u(t-\tau)\,d\tau$



$=\int_{0}^{t}(1)\,d\tau=\underline{\underline{t\,u(t)}}$

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Convolution:- EX: $x(t) = \text{rect}\left(\frac{t-1}{2}\right)$, $h(t) = \text{rect}\left(\frac{t-1}{2}\right)$

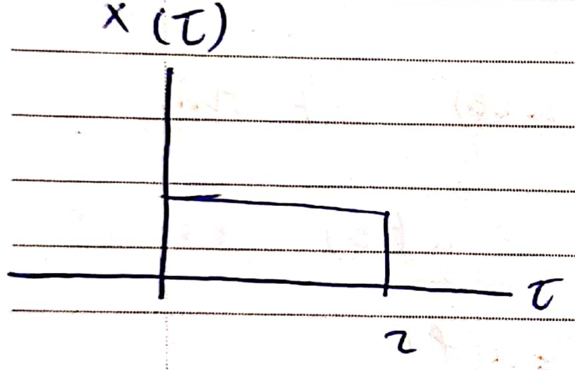
No.

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

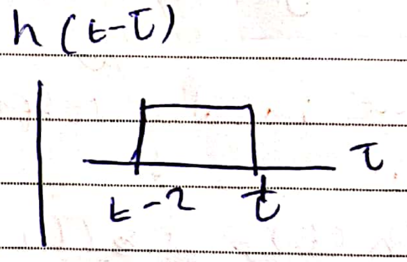
$$= \int_{-\infty}^{\infty} \text{rect}\left(\frac{\tau-1}{2}\right) \cdot \text{rect}\left(\frac{t-\tau-1}{2}\right) d\tau$$

$\Rightarrow$ $\boxed{*}$ its hard to be computed in general we resort to graphical method.

① Plot $x(\tau)$



②



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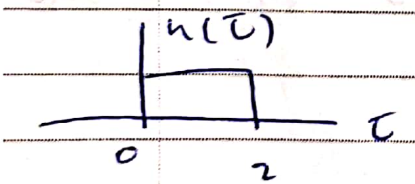

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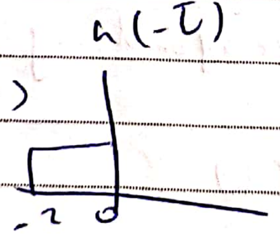

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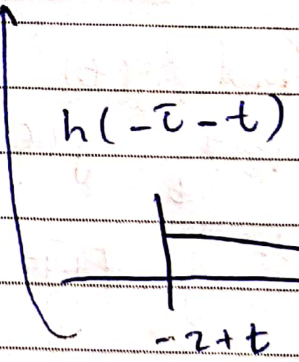
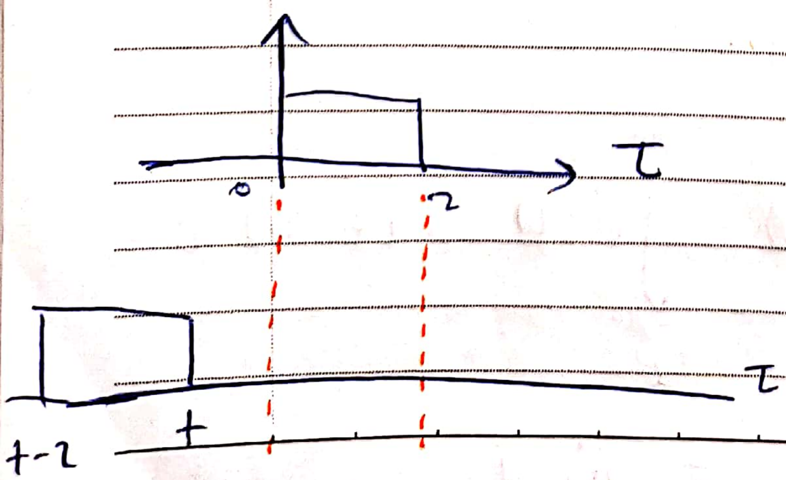
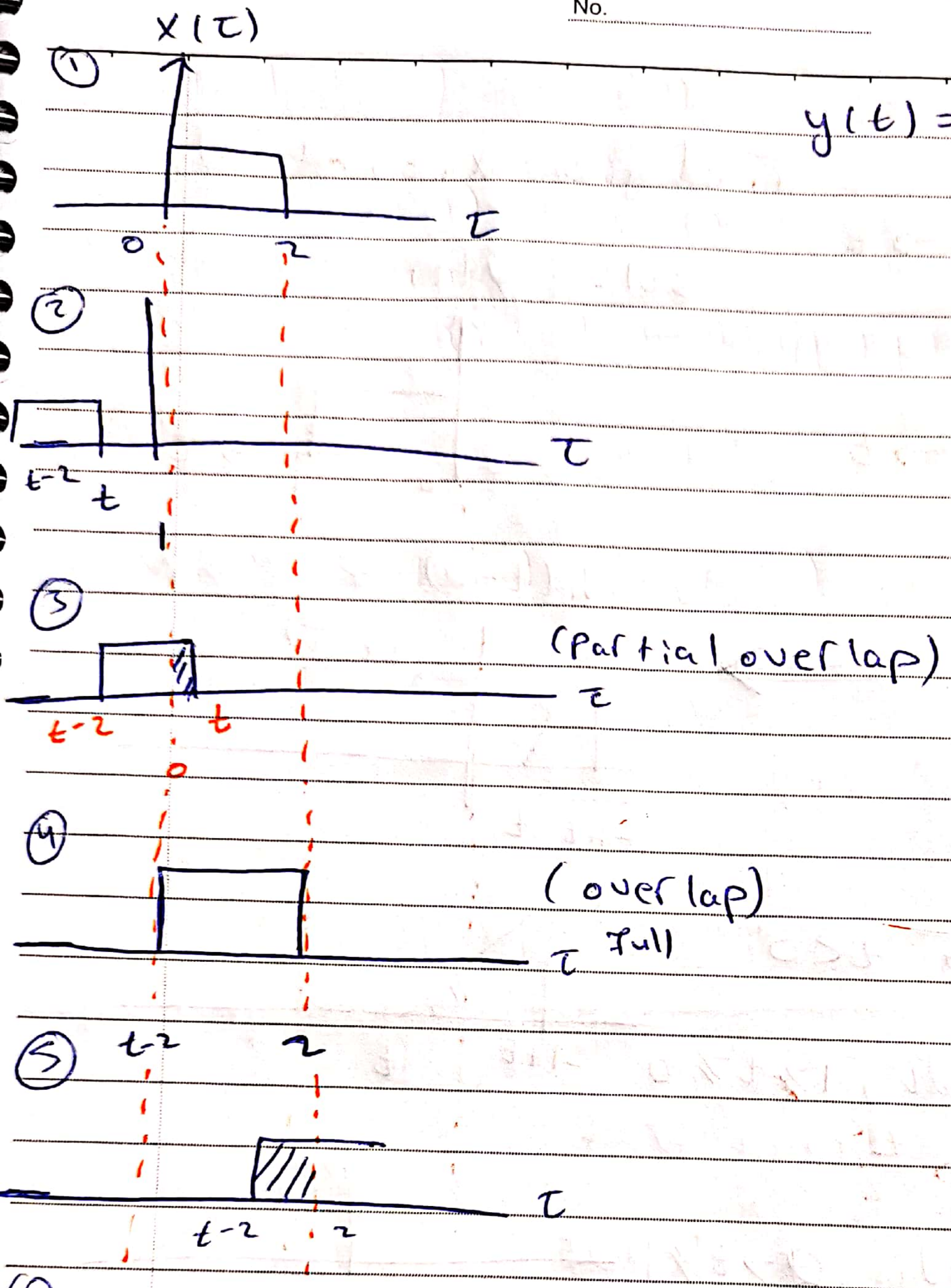




$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

$$y(t) = \begin{cases} 0 , & t < 0 \end{cases}$$

No.



⑥ no overlap

$$y(t) = \begin{cases} 0 & , t < 0 \\ \int_0^t 1.dt = t & , 2 > t \geq 0 \\ \int_0^2 1.d\tau = 2 & , t = 2 \\ \int_{t-2}^2 1.d\tau = 4-t & , 4 \geq t \geq 2 \\ 0 & , t > 4 \end{cases}$$

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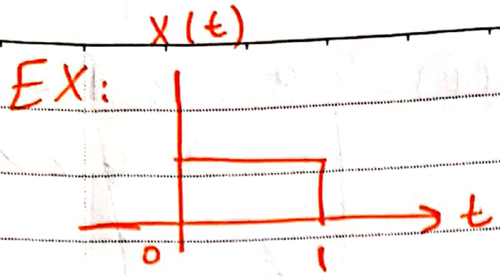
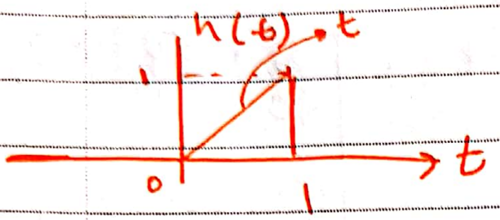
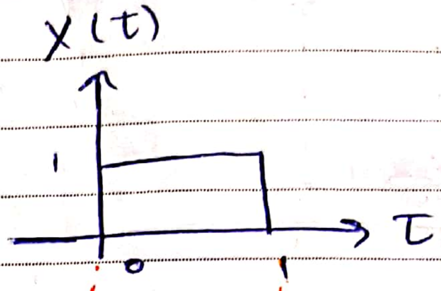
EX:

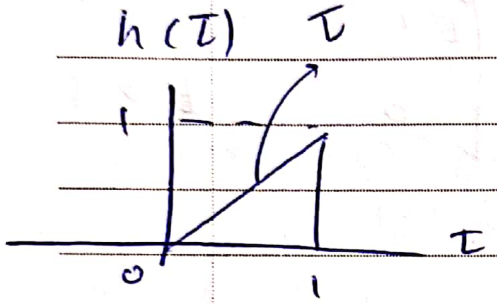


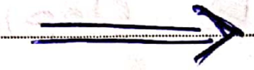
find $y(t) = x(t) * h(t)$

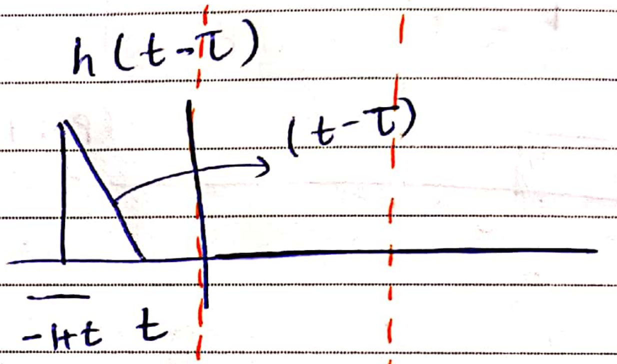


sol.







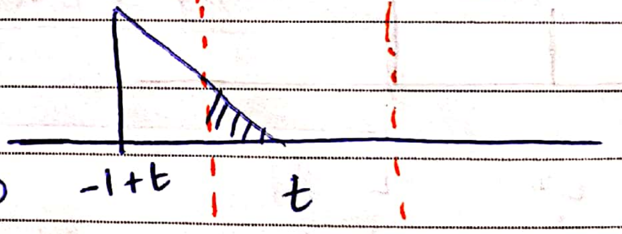


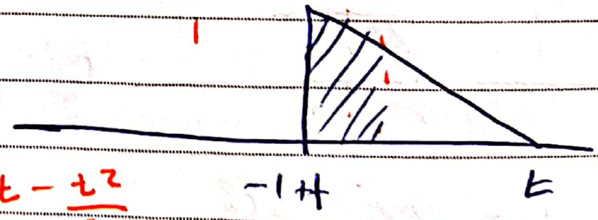
$$y(t) = \begin{cases} 0 & , t<0 \\ \int_0^t 1\cdot(t-\tau)\,d\tau & , 1 > t \geq 0 \\ \int_{t-1}^{1}(t-\tau)\,d\tau & , 2>t\geq 1 \\ 0 & , t\geq 2 \end{cases}$$

$\hookrightarrow \frac{t^2}{2}$

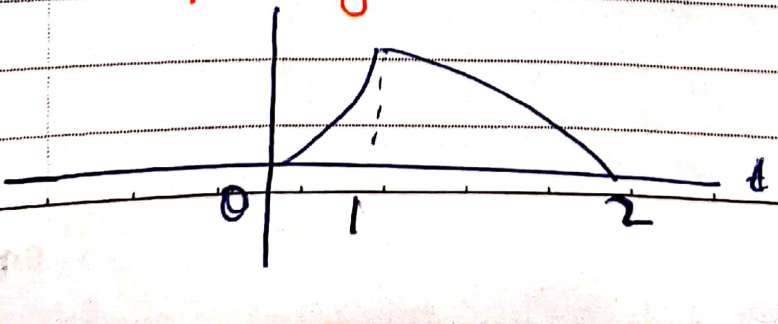
$\hookrightarrow \left[\frac{(t-1)^2-1}{-2}\right] \rightarrow t-\frac{t^2}{2}$

* إذا كان ال two functions لهم نفس ال width I can neglect the overlap.

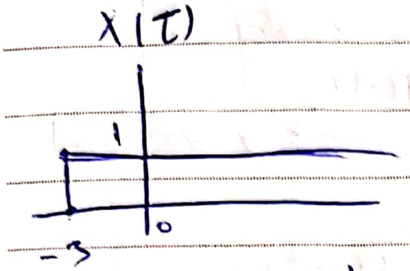


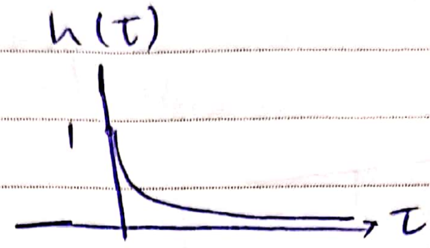


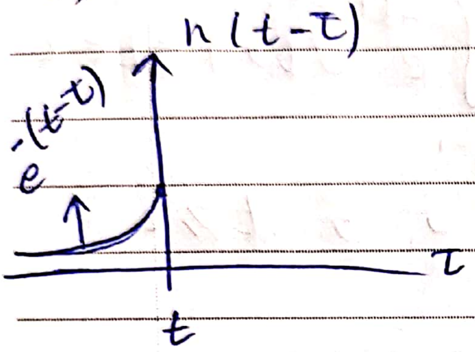
plot y(t)



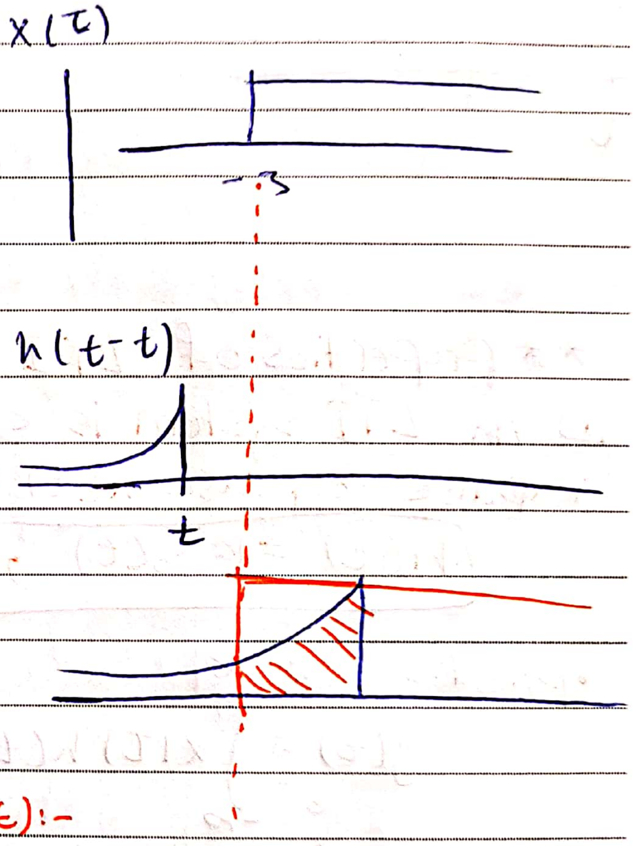
EX: $x(t) = u(t+3)$, $h(t) = e^{-t} u(t)$





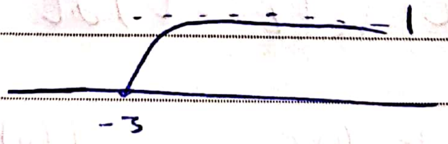


$\Rightarrow$



$$y(t) = \begin{cases} 0 \; , & t < -3 \\ \int_{-3}^{t} 1 \cdot e^{-(t-\tau)} \; , & t \geqslant 3 \end{cases}$$

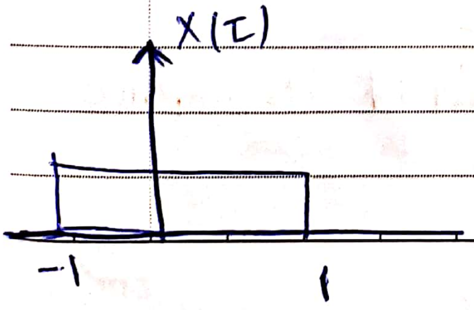
Plot y(t):-

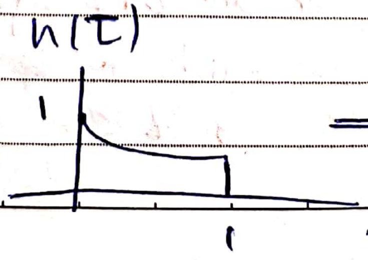


EX:

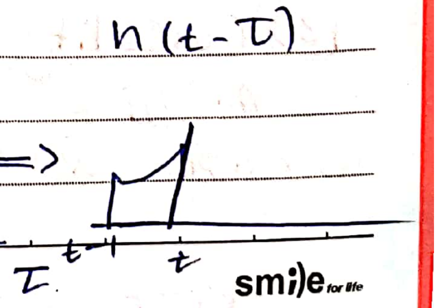
↳ $x(t) = u(t+1) - u(t-1)$

$h(t) = e^{-t}\big(u(t) - u(t-1)\big)$





$\Rightarrow$



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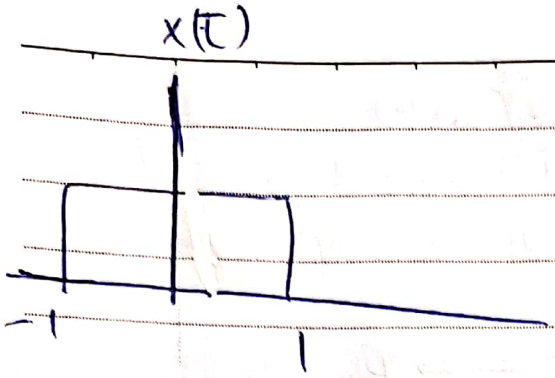

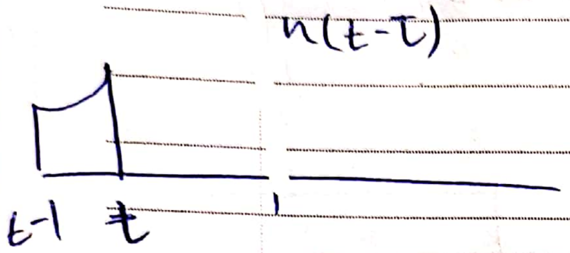

$$y(t) = \begin{cases} 0 & , t<-1 \\ \int_{t-1}^{t} e^{-(t-\tau)} d\tau & , 0 \le t<1 \\ \int_{t-1}^{1} e^{-(t-\tau)} d\tau & , 1 \le t<2 \\ 0 & , t \geq 2 \end{cases}$$

** Properties of LTI systems:-

1 An LTI system is said to be memoryless if the impulse response h(t) is given by:-

$$\boxed{h(t) = k\,\delta(t) \quad , \ k \text{ is constant}}$$

• proof:-

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t_0) = \int_{-\infty}^{\infty} x(\tau)\, h(t_0-\tau)\, d\tau$$

if $h(t) = k\,\delta(t) \rightarrow y(t_0) = \int_{-\infty}^{\infty} x(\tau) . k\delta(t_0-\tau) d\tau$

$$= k\, x(t_0)$$

2 Causality of an LTI System:-

An LTI sys is said to be causal if its impulse response $h(t)=0$, $t<0$

Proof ⟹ follows

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$$y(t) = \int_{-\infty}^{\infty} x(t-\tau) \cdot h(\tau)\, d\tau$$

for this LTI to be Causal , $h(\tau) = 0$

$$y(t) = \int_{0}^{\infty} x(t-\tau) \cdot h(\tau)\, d\tau \rightarrow$$ output of a Causal LTI system.

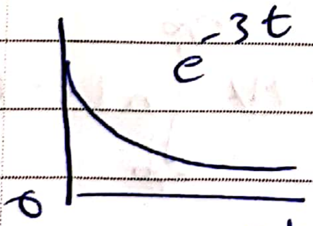
or $$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

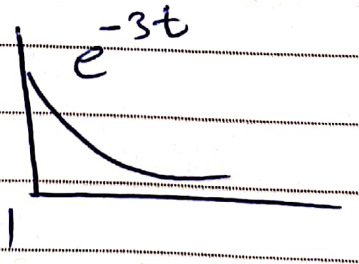
this system is Causal if → $h(t-\tau) = 0$, $t-\tau < 0$
, $t < \tau$

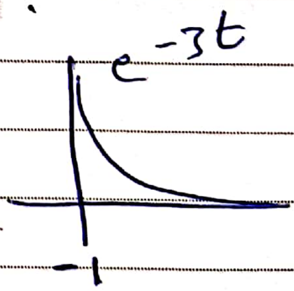
$$\boxed{y(t) = \int_{-\infty}^{t} x(\tau) \cdot h(t-\tau)\, d\tau}$$

Ex: Consider an LTI system whoese inpulse responce is $h(t) = e^{-3t} u(t)$, is the system Causal??

if Causal → $h(t) = 0$, $t < 0$







- $h(t) = e^{-3t} u(t-1)$ → causal
- $h(t) = e^{-3t} u(t+1)$ → non Causal
- this system is with memory.

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3) BIBO Stability of an LTI system :-

An LTI is BIBO stable if h(t) is absolutly integrable

$$\int_{-\infty}^{\infty} |h(t)| < \infty$$

Proof :- $y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \, d\tau$

this system is stable given that

for $|x(t)| \leq M_x < \infty$

$|y(t)| \leq M_y < \infty$

$$|y(t)| = \left| \int_{-\infty}^{\infty} x(\tau) \, h(t-\tau) d\tau \right|$$

$$\leq \int_{-\infty}^{\infty} |x(\tau) \, h(t-\tau) d\tau| = \int_{-\infty}^{\infty} |x(\tau)| |h(t-\tau)| d\tau$$

- Consider $|x(t)| \leq M_x \rightarrow |y(t)| \leq \int_{-\infty}^{\infty} |x(\tau)| . h(t-\tau) dt$

$$\leq M_x \int_{-\infty}^{\infty} h(t-\tau) d\tau$$

for $y(t) \leq M_y \rightarrow \int_{-\infty}^{\infty} h(t-\tau) \, d\tau < \infty$

$$\rightarrow \int_{-\infty}^{\infty} h(t) \, d\tau < \infty$$

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No.

Ex: $h(t) = e^{-3t} u(t)$

is the LTI system BIBO stable?

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty \rightarrow \text{BIBO stable}$$

$$\int_{0}^{\infty} e^{-3t}\, dt = \frac{1}{3}\left[e^{-3t}\right]_{\infty}^{0} = \frac{1}{3} \Rightarrow \text{BIBO stable}$$

** unit step responce:-

is the output "y(t)" when $x(t) = u(t)$

$$\underbrace{S(t)} = T\{u(t)\} = y(t)\Big|_{x(t)=u(t)}$$

مش دش

⊛ ①

$$s(t) = u(t) * h(t) = \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau = \boxed{\int_{0}^{\infty} h(t-\tau)\, d\tau}$$

or $s(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau = \int_{-\infty}^{t} h(\tau)\, d\tau$

②

$$\boxed{\frac{d\, s(t)}{d(t)} = h(t)}$$

s(t) given.

Find y(t) when $x(t) = e^{-t} u(t)$

↳ s(t) بنشتق ①

h(t) نجيب ②

convolution ③

with $x(t) = y(t)$.

No.

Ex:- $h(t) = e^{-3t} u(t)$, find the step response $s(t)$.

$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau = \int_{-\infty}^{t} e^{-3\tau} u(\tau)\, d\tau = \int_{0}^{t} e^{-3\tau}\, d\tau$$

$$= -\frac{1}{3} e^{-3\tau} \Big|_{t}^{0} = \frac{1}{3}\left[1 - e^{-3t}\right] \underline{u(t)}$$

لانه عشان zero
مش محدد .

$$h(t) \overset{??}{=} \frac{ds(t)}{dt}$$

$$\frac{d}{dt}\left[\frac{1}{3}\left[1-e^{-3t}\right]u(t)\right] = \overset{0}{\cancel{\frac{1}{3}\left[1-e^{-3t}\right]\delta(t)}} + e^{-3t}\, u(t)$$

$$= e^{-3t}\, u(t)$$

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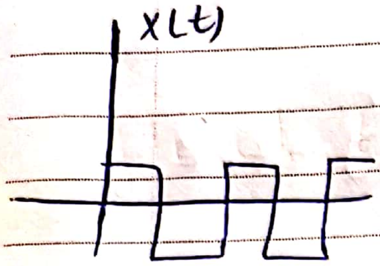
** Fourier Series:-



can be expressed by :- $X(t) = \sum_{k=1}^{\infty} C_k \sin(k\omega_0 t + \theta$

- orthognal signals:-

Def:- A set of N signals $\{g_1(t), g_2(t), \dots g_N(t)\}$ defined over a time period $0 \le t \le T$, is said to be orthognal if:-

$$\int_0^T g_i(t)\, g_j^*(t)\, dt = \begin{cases} \overset{\text{constant}}{v_i}, & i=j \\ 0, & i \neq j \end{cases}$$

* if $v_i = 1 \Rightarrow$ orthonormal set

real valued signals $x(t)$ and $y((t)$ $(0,T)$ are orthognal if $\int_0^T x(t)\, y(t)\, dt = 0$

* Linear Combination of N orthognal signals is a signal of the form:-

$$X(t) = \sum_{i=1}^{N} c_i g_i(t)$$

weigted sum

Fourier Series :- $\sum_{k=1}^{\infty} a_k \sin(k\omega_0 t + \theta_k) \overset{P}{=} \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$

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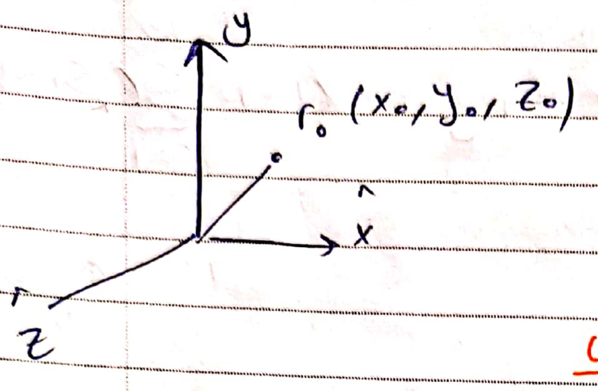
$e^{jk\omega_0 t}$ ⇒ orthognal set.

$$\int_0^{T_0} e^{j\omega_0 t} \cdot e^{j2\omega_0 t}\, dt = \frac{e^{j3\omega_0 t}}{j3\omega_0}\Bigg|_0^{T_0} = 0$$



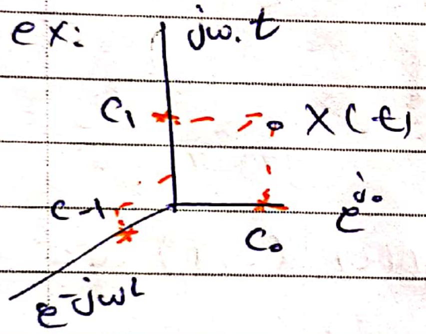
$$\underset{\downarrow}{\underline{r_0}} = x_0\hat{x} + y_0\hat{y} + z_0\hat{z}$$

linear combination

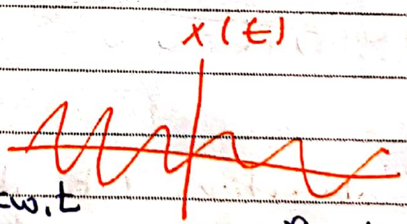
$\hat{x}, \hat{y}, \hat{z}$ → orthognal set.

$e^{jk\omega_0 t}$ → linear combonation of infinte number of orthognal signals

ex:



For any periodic signal :-



F.S ⇒ $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$, ω_0: fundemantal frequency.

$$\Rightarrow x(t) \cdot e^{-jn\omega_0 t} = \sum_{n=-\infty}^{\infty} c_k e^{jk\omega_0 t - jn\omega_0 t}$$

$$\int_0^{T_0} x(t) e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} c_k \underbrace{\int_0^{T_0} e^{j\omega_0 t(k-n)} dt}_{I} = T_0 c_n$$

$$I = \begin{cases} T_0 , & k = n \\ 0 , & k \neq n \end{cases}$$

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~~x(t)~~

$$C_n = \frac{1}{T_0}\int_{0}^{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$\circledast\quad C_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$\left.\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} \\ \text{where } C_k &= \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt \end{aligned}\right\} \longrightarrow \text{Complex exponential form.}$$

$$\boxed{\text{For signal } x(t) \text{, we have } C_{-k} = C_k^*}$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

$$= \sum_{k=-\infty}^{-1} C_k e^{jk\omega_0 t} + \sum_{k=1}^{\infty} C_k e^{jk\omega_0 t} + C_0$$

$$= C_0 + \sum_{k=1}^{\infty}\left(\overset{C_{-k}}{C_k^*}\, e^{-j\omega_0 t} + C_k e^{j\omega_0 t}\right)$$

$$= C_0 + \sum_{k=1}^{\infty}\left(|C_k| e^{-j\theta_k} e^{-jk\omega_0 t} + |C_k| e^{j\theta_k} e^{jk\omega_0 t}\right)$$

$$= \boxed{C_0 + \sum_{k=1}^{\infty} 2|C_k| \cos(k\omega_0 t + \theta_k)}$$

Combined trigonometric form

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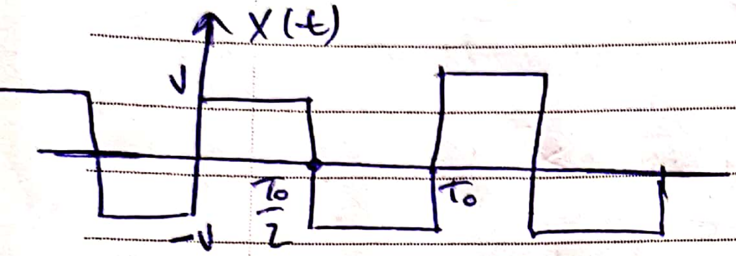
$$C_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt$$

F.S of a square wave:-



express x(t) interms of complex exponential F.S?

Sol.

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

$$C_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T_0}\left[\int_0^{T_0/2} V\cdot e^{-jk\omega_0 t}\, dt + \int_{T_0/2}^{t_0} -V\, e^{-jk\omega_0 t}\, dt\right]$$

$$= \frac{V}{T_0}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\Big|_0^{T_0/2} + \frac{e^{-jk\omega_0 t}}{jk\omega_0}\Big|_{T_0/2}^{T_0}\right]$$

$$= \frac{V}{jk\omega_0 T_0}\left[\left(1 - e^{\frac{-jk\omega_0 T_0}{2}}\right) + \left(e^{-jk\omega_0 T_0} - e^{\frac{-jk\omega_0 T_0}{2}}\right)\right]$$

$$\boxed{{}_k = \frac{V}{jk2\pi}\left[1 + e^{-j2\pi k} - 2e^{-j\pi k}\right]}$$

$\boxed{\text{*calculate } C_0 \text{ from area}}$

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$$e^{-j\pi k} = \begin{cases} -1, & k \text{ odd} \\ 1, & k \text{ even.} \end{cases}$$

$$C_k = \begin{cases} \dfrac{2V}{j\pi k}, & k \text{ odd} \\ 0, & k \text{ even} \end{cases}$$

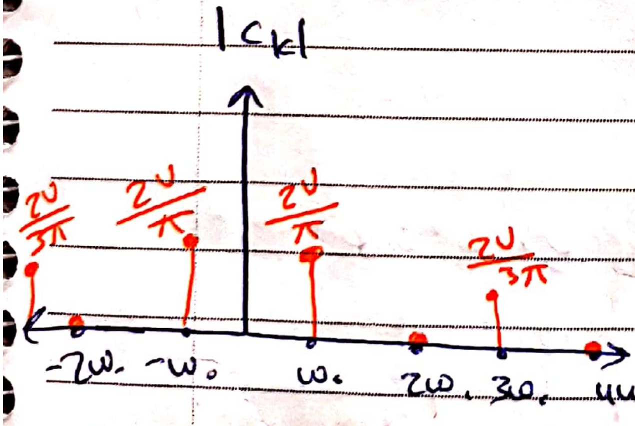
$$x(t) = \sum_{\substack{k=-\infty \\ k \text{ odd}}}^{\infty} \frac{2V}{j\pi k} e^{jk\omega_0 t}$$

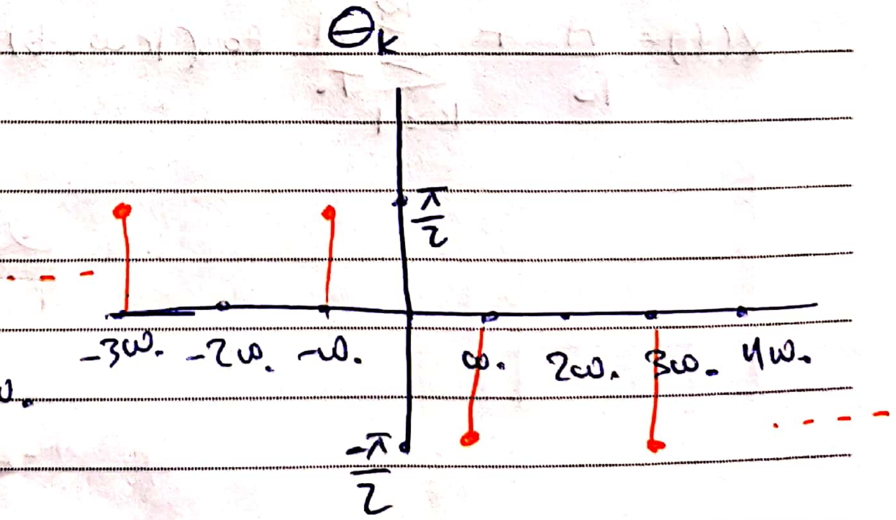
$$|C_k| = \frac{2V}{\pi k}, \quad \theta_k = \pi/2 \quad \rightarrow \text{combined trigonometric}$$

$$C_0 = \text{zero}, \quad x(t) = \sum_{k=1}^{\infty} 2\left(\frac{2V}{\pi k}\right) \cos(k\omega_0 t + \cancel{\theta} - \tfrac{\pi}{2})$$

$$= \sum_{k=1}^{\infty} \frac{4V}{\pi k} \sin(k\omega_0 t)$$

Fourier spectra:-





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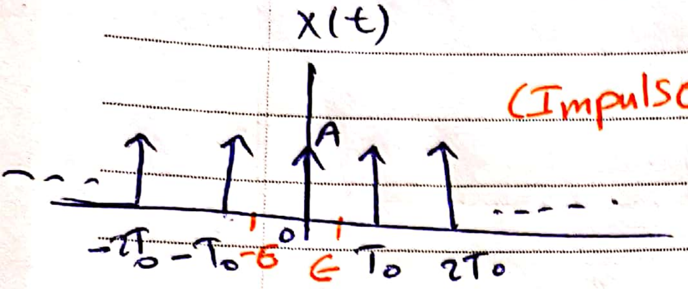
No.

Ex:- $x(t) = \sum_{k=-\infty}^{\infty} A\delta(t-kT_0)$



$$C_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$C_k = \frac{1}{T_0}\int_{-\epsilon}^{\epsilon} \underset{\sim}{A\delta(t)}\, \underset{x(t)}{\underset{\sim}{e^{-jk\omega_0 t}}}\, dt = \boxed{\frac{A}{T_0} = C_k} \rightarrow \text{Valid for any } \underline{\underline{k}}.$$

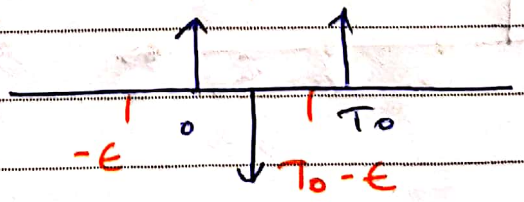
$$x(t) = \sum_{k=-\infty}^{\infty} \frac{A}{T_0} e^{-jk\omega_0 t}$$

in combined trigonometric.

$\theta_k = 0$, $|C_k| = \frac{A}{T_0}$, $C_0 = \frac{A}{T_0}$

$$x(t) = \frac{A}{T_0} + \sum_{k=1}^{\infty} \frac{2A}{T_0} \cos(k\omega_0 t)$$

* ممكن ييجي زي هيك



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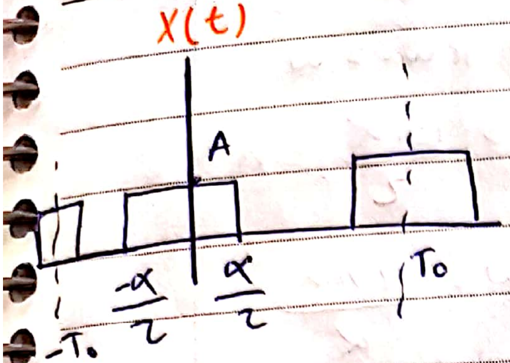
No.



, $\alpha < T_0$

$$C_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} A\cdot e^{-jk\omega_0 t}\, dt$$

$$= \frac{A}{T_0}\frac{\left[e^{-jk\omega_0\frac{\alpha}{2}} - e^{+jk\omega_0\frac{\alpha}{2}}\right]}{-jk\omega_0} = \frac{A}{T_0}\frac{\left[e^{jk\omega_0\frac{\alpha}{2}} - e^{-jk\omega_0\frac{\alpha}{2}}\right]}{jk\omega_0} \cdot \left(\frac{\alpha}{2}\right) \cdot \left(\frac{\alpha}{2}\right)$$

$$= \frac{A\alpha}{T_0} \cdot \frac{\boxed{\frac{e^{jk\omega_0\frac{\alpha}{2}} - e^{-jk\omega_0\frac{\alpha}{2}}}{2j}}}{k\omega_0\frac{\alpha}{2}} \rightarrow \sin\left(k\omega_0\frac{\alpha}{2}\right)$$

$$= \frac{A\alpha}{T_0} \cdot \frac{\sin\left(k\omega_0\frac{\alpha}{2}\right)}{k\omega_0\frac{\alpha}{2}} = \frac{A\alpha}{T_0}\, \mathrm{Sinc}\left(k\omega_0\frac{\alpha}{2}\right)$$

$C_0 = \frac{A\alpha}{T_0}$ from the figure or $\lim_{k\to 0} C_k$

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{A\alpha}{T_0}\, \mathrm{Sinc}\left(k\omega_0\frac{\alpha}{2}\right) e^{jk\omega_0 t}$$

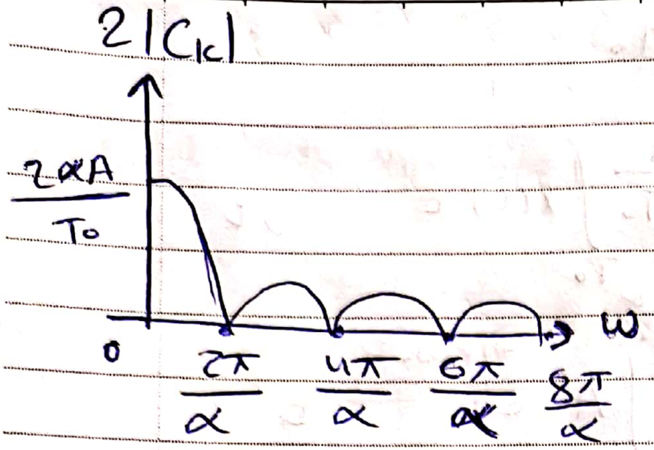
trigonometric

$$= \frac{\alpha A}{T_0} + \sum_{k=1}^{\infty} \frac{2\alpha A}{T_0}\left|\mathrm{sinc}\left(\frac{k\omega_0\alpha}{2}\right)\right| \cos(k\omega_0 t + \theta_k)$$

$$\theta_k = \begin{cases} 0 & , 0 < \mathrm{Sinc}\left(\frac{k\omega_0\alpha}{2}\right) \\ \pi & , 0 > \mathrm{Sinc}\left(\frac{k\omega\alpha}{2}\right) \end{cases}$$

$\text{sinc}\left(\frac{k\omega_0\alpha}{2}\right)=0$ if $\sin\left(\frac{k\omega_0\alpha}{2}\right)=0$

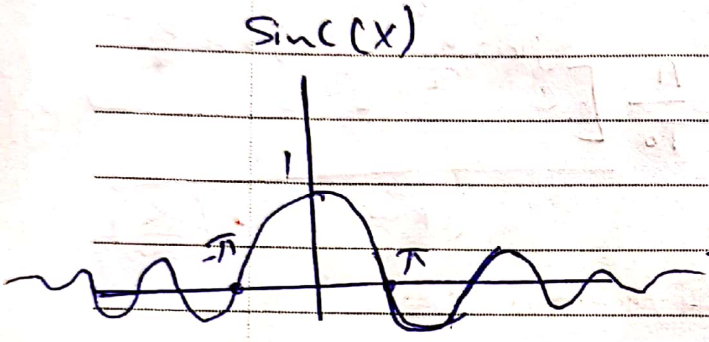
then $\frac{k\omega_0\alpha}{2}=n\pi$

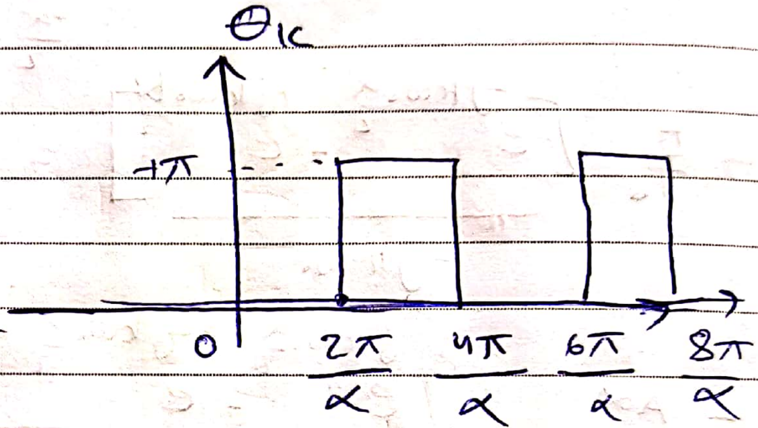


let $k\omega_0=\omega$

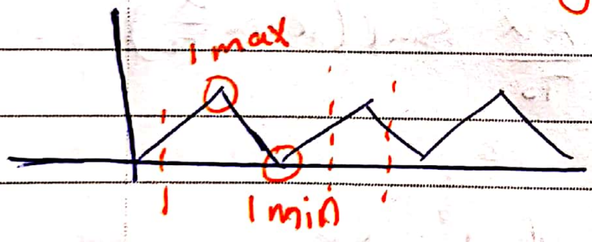
$\Rightarrow \frac{\omega\cdot\alpha}{2}=n\pi$

$\omega=\frac{2\pi n}{\alpha}$

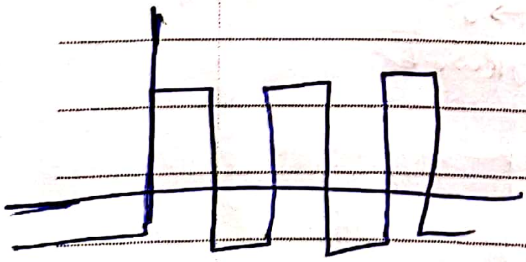




-conditions for having a fourier series: ~~for~~ a signal:-

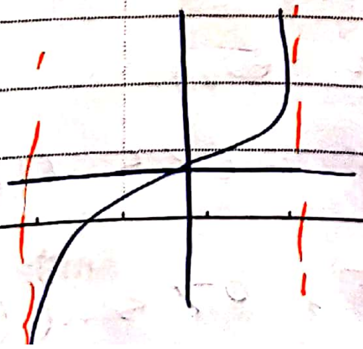


① - finite number of maximum and minimum in one period.



② - finite number of disronuties in one period.

③ - must be bounded.



→ unbonded => No F.S expansion.

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EX: $x(t) = \sin(\omega_0 t) + \sin(3\omega_0 t)$

Find F.S expansion for $x(t)$.

$$x(t) = \frac{e^{j\omega_0 t}}{2j} - \frac{1}{2j} e^{-j\omega_0 t} + \frac{1}{2j} e^{j3\omega_0 t} - \frac{1}{2j} e^{-j3\omega_0 t}$$

$C_0 = 0$

$C_1 = \frac{1}{2j}$, $C_{-1} = \frac{-1}{2j}$

$C_2 = 0$, $C_{-2} = 0$

$C_3 = \frac{1}{2j}$, $C_{-3} = \frac{-1}{2j}$

$C_4, C_5 \cdots = 0$

** Parsaval's power theorm:-

let $x(t)$ be periodicsignal with fundemental period "T_0"
then the Power of $x(t)$

$$P_x = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt$$

using F.S expansion
↳ let $x(t)$ has fourier Series expansion

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}}$$

then $P_x = \sum_{k=-\infty}^{\infty} |C_k|^2$

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Proof:- $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{+jk\omega_0 t}$

$$P_x = \frac{1}{T_0}\int_{T_0} x(t)\cdot x^*(t)\,dt = \frac{1}{T_0}\int_{T_0}\left(\sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}\right)\left(\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}\right)^*$$

$$= \sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} C_k C_n^* \underbrace{\left[\frac{\int_0^{T_0} e^{j\omega_0 t(k-n)}\,dt}{T_0}\right]}_{I}$$

$$I = \begin{cases} 0, & k \neq n \\ 1, & k = n \end{cases} \Rightarrow P_x = \sum_{k=-\infty}^{\infty} C_k C_k^*$$

$$= \sum_{k=-\infty}^{\infty} |C_k|^2$$

Ex: A Sin(2πt), find P_x using integral $\rightarrow P_x = \frac{A^2}{2}$

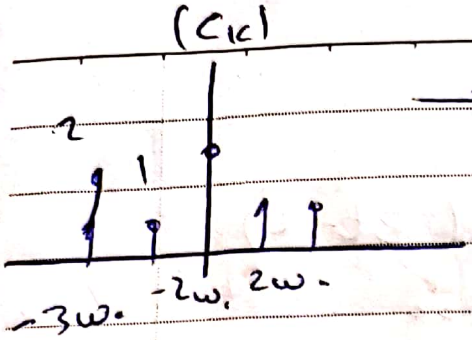
- using F.S

$$x(t) = \frac{A}{2j} e^{j(\overset{\omega_0}{2\pi})t} + \frac{A}{-2j} e^{-j(\overset{\omega_0}{2\pi})t}$$

C=1 C-1

$$P_x = \sum_{k=-\infty}^{\infty} |C_k|^2 = |C_{-1}|^2 + |C_1|^2 = \left(\frac{A}{2}\right)^2 + \left(\frac{A}{2}\right)^2$$

$$= \frac{A^2}{2}$$



→ باخد ال Summation
لتربيع ال magnitude
عشان أحسب
ال power

let $x(t)$ has F.S expansion

$$x(t) = \sum_{k=-\infty}^{\infty} C_{kx} e^{jk\omega_0 t}$$

what is the F.S expansion for $y(t) = x(at+b)$

$$\Longrightarrow \text{using } x(t) = \sum_{k=-\infty}^{\infty} C_{kx} e^{jk\omega_0 t}$$

$$x(at+b) = \sum_{k=-\infty}^{\infty} C_{kx} e^{jk\omega_0 (at+b)}$$

$$= \sum_{k=-\infty}^{\infty} \left(C_{kx} e^{jk\omega_0 b} \cdot e^{jk\omega_0 at} \right)$$

$$C_{ky} = = \begin{cases} C_{kx} e^{jk\omega_0 b} , & a>0 \\ \left(C_{kx} e^{jk\omega_0 b} \right)^* , & a<0 \end{cases}$$

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Ex: $x(t) = \sum_{k=-\infty}^{\infty} \frac{-j2v}{\pi k} e^{jk\omega_0 t}$

Find [illegible] C_{ky} for $y(t) = x(\underset{a>0}{3}t+\underset{b}{2})$

$\Longrightarrow C_{ky} = \frac{-j2v}{\pi k} \cdot e^{jk\omega_0 2}$

if $y(t) = x(1-t)$

$b=1$ $\quad a=-1<0$

$$C_{ky} = \left(\frac{-j2v}{\pi k} \cdot e^{jk\omega_0 1} \right)^* = \frac{j2v}{\pi k} \cdot e^{-jk\omega_0}$$

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$A' \; x(at+B)$

No.

Ex: $x(t) = \sum_{k=-\infty}^{\infty} \frac{-j2V}{\pi k} e^{jk\omega_0 t}$

Find [illegible] C_{ky} for $y(t) = x(\underset{a>0}{3}t+\underset{b}{2})$

$\Rightarrow C_{ky} = \frac{-j2V}{\pi k} \cdot e^{jk\omega_0 2}$

if $y(t) = x(1-t)$

$b=1$ $\quad a=-1<0$

$C_{ky} = \left(\frac{-j2V}{\pi k} \cdot e^{jk\omega_0 1}\right)^* = \frac{j2V}{\pi k} \cdot e^{-jk\omega_0}$

$y(t) = A\,x(t) + \boxed{B}$

$x(t)$ has F.S coeff C_{kx}

$y(t)$ has F.S coff $C_{ky} =$

$$y = A \sum_{k=-\infty}^{\infty} C_{kx} e^{jk\omega_0 t} + B$$

$$= \underbrace{(A \cdot C_{0x} + B)}_{C_{0y}} + \underbrace{\sum_{k=-\infty}^{\infty} A \cdot C_{kx} e^{jk\omega_0 t}}_{C_{ky}}$$

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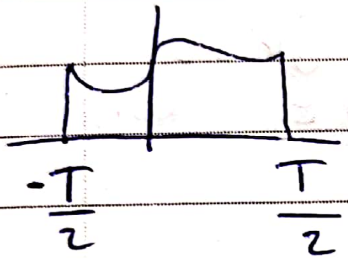
** Fourier transform:-

F.S ← periodic signals

F.T ← aperiodic

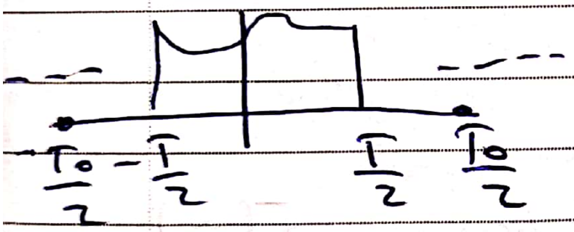
f(t)



aperiodic

$$f(t) = \lim_{T_0 \to \infty} f_{T_0}(t)$$

$f_{T_0}(t)$



periodic

$f_{T_0}(t)$ is periodic signal

$$f_{T_0}(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

$$C_k = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f_{T_0}(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T_0}\int_{-\infty}^{\infty} f(t)\, e^{-jk\omega_0 t}\, dt$$

$\boxed{\text{let us define}}$:- $F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$

$$\frac{F(k\omega_0)}{T_0} = C_k$$

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$$f_{T_0}(t) = \sum_{k=-\infty}^{\infty} \frac{F(k\omega_0)}{T_0} e^{jk\omega_0 t}$$

$$= \frac{1}{2\pi} \sum_{k=\infty}^{\infty} F(k\omega_0) \cdot e^{jk\omega_0 t} \cdot \omega_0$$

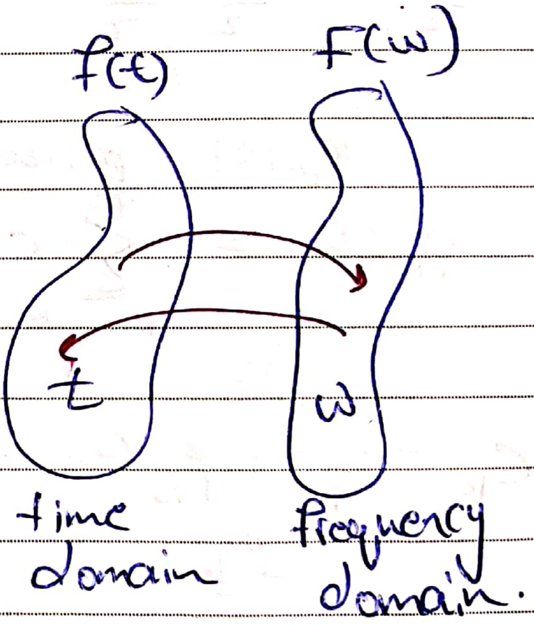
$$T_0 \to \infty , \quad \omega_0 \to 0$$

$$= \lim_{T_0 \to \infty} f_{T_0}(t) = \lim_{\omega_0 \to 0} \frac{1}{2\pi} \sum_{k=-\infty} F(k\omega_0) e^{jk\omega_0 t} \cdot \omega_0$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} F(k\Delta W) e^{jk\Delta W t} \cdot \Delta W$$

as $\Delta W \to 0$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$



* $f(t) \xleftrightarrow{F} F(\omega)$

* $\mathcal{L}\{f(t)\} = F(\omega)$

* $f(t) = \mathcal{F}^{-1}\{F(\omega)\}$

$$\{f(t)\} = \int_{-\infty}^{\infty} f(t) \cdot e^{-st} . dt$$

$$s = \sigma + j\omega$$

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VI

$$x(t) = e^{j\omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau = \int_{-\infty}^{\infty} e^{j\omega_0(t-\tau)}\, h(\tau)\, d\tau$$

$$= e^{j\omega_0 t} \underbrace{\int_{-\infty}^{\infty} h(\tau)\, e^{-j\omega_0 \tau}\, d\tau}_{H(\omega)\big|_{\omega=\omega_0}}$$

Ex: $f_1(t) = e^{j\omega_0 t}$, find $F(\omega)$.

$$F(\omega) = \int_{-\infty}^{\infty} \underbrace{e^{j\omega_0 t} \cdot e^{-j\omega t}}_{e^{-jt(\omega-\omega_0)}}\, dt = \frac{e^{-j(\omega-\omega_0)}}{\omega-\omega_0}\Bigg|_{-\infty}^{\infty}$$

$$= \frac{e^{-j(\omega-\omega_0)} - e^{+j(\omega-\omega_0)}}{\omega_0-\omega} = \text{undefined}.$$

let $F(\omega) = 2\pi\, \delta(\omega-\omega_0)$

$$f(t) = \frac{\cancel{2\pi}}{\cancel{2\pi}} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega = \int_{-\infty}^{\infty} \delta(\omega-\omega_0)\, e^{j\omega t}\, d\omega$$

$$= \frac{e^{j\omega_0 t}}{}$$

Ex: let $f(t) = A\,\delta(t-t_0)$, find $F(\omega)$.

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} A\,\delta(t-t_0)\, e^{-j\omega t}\, dt = Ae^{-j\omega t}$$

$$* \quad A\,\delta(t-t_0) \xleftrightarrow{\mathcal{F}} Ae^{-j\omega t_0}$$

$$\delta(t) \xleftrightarrow{\mathcal{F}} 1$$

$\delta(t) \xleftrightarrow{\mathcal{F}} 1$

$A\delta(t-t_0) \xleftrightarrow{\mathcal{F}} Ae^{-j\omega t_0}$

$1 \xleftrightarrow{\mathcal{F}} 2\pi\,\delta(\omega)$

$e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} 2\pi\,\delta(\omega-\omega_0)$

* properties of F.T

1- linearity:-

$f(t)= a_1 f_1(t) + a_2 f_2(t), \xleftrightarrow{\mathcal{F}} F(\omega)\ a_1 F_1(\omega) + a_2 F_2(\omega)$

2- time shift:-

let $f(t) \xleftrightarrow{\mathcal{F}} F(\omega)$

then $f(t-t_0) \xleftrightarrow{\mathcal{F}} F(\omega)\,e^{-j\omega t_0}$

$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(\tau)e^{-j\omega t}\,.dt$

$$\mathcal{F}\{f(t-t_0)\} = \int_{-\infty}^{\infty} f(t-t_0)\,e^{-j\omega t}\,dt = \Big(\underbrace{\int_{-\infty}^{\infty} f(\tau)e^{-j\omega t}\,d\tau}_{F(\omega)}\Big)\,e^{-j\omega t_0}$$

3- $f(t) \xleftrightarrow{\mathcal{F}} F(\omega)$

then $f(at-t_0) \longleftrightarrow \dfrac{F\left(\frac{\omega}{a}\right)e^{-j\omega\frac{t_0}{a}}}{|a|}$

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4- Frequency shift:-

$$f(t) \xleftrightarrow{\mathcal{F}} F(\omega)$$

then

$$f(t)\, e^{j\omega_0 t} \longleftrightarrow F(\omega - \omega_0)$$

5- Convolution:-

let $f_1(t) \xleftrightarrow{\mathcal{F}} F_1(\omega)$

$f_2(t) \xleftrightarrow{\mathcal{F}} F_2(\omega)$

then $f_1(t) * f_2(t) \xleftrightarrow{\mathcal{F}} F_1(\omega) \cdot F_2(\omega)$

6- $f_1(t) \cdot f_2(t) \xleftrightarrow{\mathcal{F}} \frac{1}{2\pi} F_1(\omega) * F_2(\omega)$

** Duality property:-

let $f(t) \xleftrightarrow{\mathcal{F}} F(\omega)$

then

$$F(t) \xleftrightarrow{\mathcal{F}} 2\pi f(-\omega)$$

$$f(t) \longleftrightarrow \frac{1}{a^2+\omega^2} \quad (F\omega)$$

then

$$\frac{1}{a^2+t^2} \xleftrightarrow{\mathcal{F}} 2\pi f(-\omega)$$

Proof:-

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \, d\omega$$

$$\Rightarrow \quad 2\pi f(-t) = \int_{-\infty}^{\infty} F(\omega) e^{-j\omega t} \, d\omega$$

No.

$$= \int_{-\infty}^{\infty} F(t') e^{-j\omega' t} \, dt'$$

$$\Rightarrow 2\pi f(-\omega) = \int_{-\infty}^{\infty} F(t') e^{-j\omega t'} \, dt'$$

$$= \mathcal{F}\{F(t)\}$$

ex.: $f(t)$ $F(\omega)$

$\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\delta(-\omega) = 2\pi\delta(\omega)$

2- time Differentiation.

let $f(t) \xleftrightarrow{\mathcal{F}} F(\omega)$

then $$\frac{d(f(t))}{dt} \xleftrightarrow{\mathcal{F}} j\omega F(\omega)$$

$$\frac{d^n(f(t))}{dt^n} \xleftrightarrow{\mathcal{F}} (j\omega)^n F(\omega)$$

Proof:-

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \, . d\omega$$

$$\dot{x}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \underbrace{j\omega F(\omega)}_{Y(\omega)} e^{j\omega t} \, . d\omega$$

$$\Rightarrow f'(t) \longleftrightarrow j\omega F(\omega)$$

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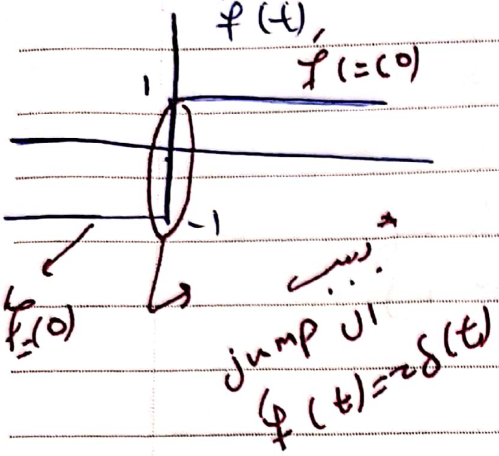
No.

* examples:-



find $F(\omega)$.

$$f(t) = \text{Sgn}(t)$$

$$f'(t) = \frac{d[\text{sgn}(t)]}{dt} = 2\,\delta(t)$$

$$\mathcal{F}\{f'(t)\} = \mathcal{F}\{2\,\delta(t)\}$$

$$j\omega F(\omega) = 2\cdot 1$$

$$\Rightarrow F(\omega) = \frac{2}{j\omega}$$

but $f(t) = 2u(t) - 1$

take F.T for both sides

$$F(\omega) = 2\,\mathcal{F}\{u(t)\} - 2\pi\,\delta(\omega) = \frac{2}{j\omega}$$

then $\mathcal{F}\{u(t)\} = \frac{1}{j\omega} + \pi\,\delta(\omega)$

⊛
$$\boxed{u(t) \xleftrightarrow{\mathcal{F}} \frac{1}{j\omega} + \pi\,\delta(\omega)}$$

Ex:- $f(t) = \cos(\omega_0 t)$.

$F(\omega) = ?$

$$\cos(\omega_0 t) = \frac{e^{j\omega_0 t}}{2} + \frac{e^{-j\omega_0 t}}{2}$$

$$\mathcal{F}\{\cos(\omega_0 t)\} = \frac{1}{2}\mathcal{F}\{e^{j\omega_0 t}\} + \frac{1}{2}\mathcal{F}\{e^{-j\omega_0 t}\}$$

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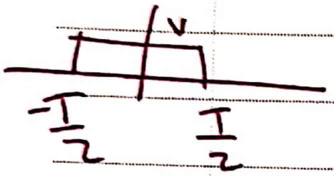
$$= \frac{1}{2}\left(2\pi\delta(\omega-\omega_0) + 2\pi\delta(\omega+\omega_0)\right)$$

$$= \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

$$f(t) = \sin(\omega_0 t)$$

$$F(\omega) = \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

Ex: $f(t) = V\,\text{rect}\left(\frac{t}{T}\right)$



$$F(\omega) = \int_{-T/2}^{T/2} V e^{-j\omega t}\cdot dt$$

$$= \frac{V}{j\omega\frac{T}{2}}\left[\frac{-e^{-j\omega T/2} + e^{j\omega T/2}}{2}\right]$$

$$= V\left[\text{sinc}\left(\frac{\omega T}{2}\right)\right]$$

* $V\text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{\mathcal{F}} V\cdot T\,\text{Sinc}\left(\frac{\omega T}{2}\right)$

using duality property:-

$VT\,\text{Sin}\left(\frac{T}{2}t\right) \xleftrightarrow{\delta} 2\pi VT\,\text{rect}\left(\frac{\omega}{T}\right)$

Set ω.

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No.

** time integration :-

let $f(t) \xleftrightarrow{f} F(\omega)$

then

$$g(t) = \int_{-\infty}^{\infty} f(\tau)\, d\tau \xleftrightarrow{f} \frac{F(\omega)}{j\omega} + \pi F(0)\, \delta(\omega)$$

where $F(0) = F(\omega)\Big|_{\omega=0}$

$$= \int_{-\infty}^{\infty} f(t)\, dt$$

Proof:- $g(t) = \int_{-\infty}^{t} f(\tau)\, d\tau$

$$= \int_{-\infty}^{\infty} u(t-\tau)\, dt$$

$$= u(t) * f(t)$$

$$G(\omega) = F(\omega) \cdot \underbrace{\left[\frac{1}{j\omega} + \pi\, \delta(\omega)\right]}_{u(\omega)}$$

$$= \frac{F(\omega)}{j\omega} + \pi F(0)\, \delta(\omega)$$

$$G(\omega) = f\{u(t)\}$$

$$= \frac{1}{j\omega} + \pi\, \delta(\omega)$$

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Ex: $f(t) = e^{-at} u(t)$, $Re\{a\} > 0$

find $F(\omega)$

$$F(\omega) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} dt = \underbrace{\frac{e^{-(a+j\omega)t}}{a+j\omega}}_{\downarrow \; \frac{e^{-a} \cdot e^{-j\omega}}{a+j\omega}} \Bigg|_{\infty}^{0} = \frac{e^{0}}{a+j\omega} = \frac{1}{a+j\omega}$$

$\frac{e^{-a} \cdot e^{-j\omega}}{a+j\omega}$ ⇝ bounded.

$$* \quad \frac{1}{a+j\omega} \xleftrightarrow{f} \underbrace{2\pi e^{+at} u(-t)}_{f(-\cancel{\omega}\, t)}$$

(Duality)

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$\frac{1}{a+j\omega} = \int_{-\infty}^{\infty} \underline{e^{-at} u(t)} e^{-j\omega t} dt$$

$$-\frac{d}{da}\left[\frac{1}{a+j\omega}\right] = \int_{-\infty}^{\infty} -t e^{-at} u(t) e^{-j\omega t} dt$$

$$\frac{1}{(a+j\omega)^2} = \int_{-\infty}^{\infty} t e^{-at} u(t) e^{-j\omega t} dt$$

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$$F(w) = \int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt$$

$$\grave{F}(\omega) = \int_{-\infty}^{\infty} -jt\, F(t)\, e^{-jwt}\, dt$$

$$j\grave{F}(\omega) = \int_{-\infty}^{\infty} t\, f(t)\, e^{-jwt}\, dt$$

$$\mathcal{F}\{t f(t)\} = j\grave{F}(\omega)$$

---

let $f(t) = \cos(\omega_0 t) \cdot g(t)$

$$f(t) = \frac{g(t)\, e^{-jw_0 t}}{2} + \frac{g(t)\, e^{-jw_0 t}}{2}$$

$$= \frac{1}{2} G(\omega - \omega_0) + \frac{1}{2} G(\omega + \omega_0)$$

$y(t) = 1 \Rightarrow G(\omega) = 2\pi$

$$\Longrightarrow \mathcal{F}\{\cos(\omega_0 t)\} = \pi\,[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

$f(t) = e^{-a|t|}$, $a > 0$

Find $F(\omega)$

$$f(t) = \underbrace{e^{-at} u(t)}_{g(t)} + \underbrace{e^{at}\, u(-t)}_{g(-t)}$$

$$G(\omega) = \mathcal{F}\{e^{-at} u(t)\} = \frac{1}{a + j\omega}$$

$$F(\omega) = G(\omega) + G(-\omega) = \frac{1}{a + j\omega} + \frac{1}{a - j\omega} = \frac{2\cancel{\pi} a}{a^2 + \omega^2}$$

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$$\frac{1}{a^2+t^2} \longleftrightarrow \frac{\pi}{a} e^{-a|w|}$$

$$\int_{-\infty}^{\infty} \frac{1}{a^2+t^2} dt = ? \quad \boxed{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} \frac{1}{t^2+a^2} e^{-jw.t} dt$$

$$= \frac{\pi}{a} e^{-a|w|}$$

---

*let $F(\omega) = \frac{1}{(a+jw)^2}$, find $f(t)$

$$F(\omega) = \left(\frac{1}{a+jw}\right) \cdot \left(\frac{1}{a+jw}\right)$$

$f(t) = g(t) * g(t)$, where $g(t) = \mathcal{F}^{-1}\left\{\frac{1}{a+jw}\right\}$

$$\int_{-\infty}^{\infty} e^{-a\tau} u(\tau) e^{-a(t-\tau)} u(t-\tau) d\tau$$

$$= \int_{0}^{t} e^{-at} .dt = t e^{-at} u(t)$$

Ex: let $f(t) = e^{-at^2}$. find $F(\omega)$

$$f(t) = e^{-at^2}$$

$$f'(t) = -2^{at}at^2$$

$$f'(t) + 2at\,f(t) = 0$$

$$\mathcal{F}\{f'(t) + 2at f(t)\} = \mathcal{F}\{0\} = 0$$

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$$jw\, F(w) + 2\pi j\, F'(w) = 0$$

$$jwF + 2aj\frac{dF}{dw} = 0$$

$$\frac{dF}{dw} = \frac{-wF}{2a}$$

$$\frac{dF}{F} = \frac{-w}{2a}\, dw$$

$$\ln(F) = \frac{-w^2}{4a^2} + C$$

$$\boxed{F(w) = A \cdot e^{\frac{-w^2}{1+a^2}}}$$

$$F(0) = A = \int_{-\infty}^{\infty} e^{-at^2}\, dt$$

$$I = \int_{-\infty}^{\infty} e^{-at^2}\, dt \quad , \quad I = \sqrt{\left(\int_{-\infty}^{\infty} e^{-at^2}\, dt\right)^2}$$

$$= \sqrt{\left(\int_{-\infty}^{\infty} e^{-at^2}\, dt\right) \cdot \left(\int_{-\infty}^{\infty} e^{-av^2}\, dv\right)}$$

$$I = \sqrt{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-a(t^2+v^2)}\, dt\, dv}$$

let $r = t^2 + v^2$

$dt\, dv = r\, dr\, d\theta$

$$I = \sqrt{\int_{0}^{2\pi}\int_{r=0}^{\infty} r e^{-ar^2} \cdot dV\, d\theta} = \sqrt{\frac{\pi}{a}}$$

$$e^{-at^2} \xleftrightarrow{\mathcal{F}} \sqrt{\frac{\pi}{a}}\, e^{\frac{-a^2}{4a^2}}$$

* let $f(t) \xleftrightarrow{\mathcal{F}} F(\omega)$

$$\overset{*}{f}(t) \xleftrightarrow{\mathcal{F}} ??$$

$$(f(t))^* = \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega\right)^*$$

$$\overset{*}{f}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{\overset{*}{F}(\omega)}_{F(-\omega)}\, \underbrace{e^{-j\omega t}}_{e^{+j\omega t}}\, d\omega$$

$$\overset{*}{f}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overset{*}{F}(-\omega)\, e^{j\omega t}\, d\omega)$$

then

$$\overset{*}{f}(t) \xleftrightarrow{\mathcal{F}} \overset{*}{F}(-\omega)$$

if $f(t)$ is real $\boxed{F(\omega) = \overset{*}{F}(-\omega)}$

or

$F(-\omega) = \overset{*}{F}(\omega)$ for real signal

* Parsaval's theorm :-

for the signal $f(t)$

$$E_f = \int_{-\infty}^{\infty} |f(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2\, d\omega$$

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Proof:-

L.H.S.

$$E_f = \int_{-\infty}^{\infty} f(t)\, f^*(t)\, dt = \int_{-\infty}^{\infty} f(t) \left[\frac{1}{\pi} \int_{-\infty}^{\infty} F^*(\omega)\, e^{j\omega t}\, d\omega \right] dt$$

$$\int_{-\infty}^{\infty} \frac{1}{2\pi} F^*(\omega) \left(\underbrace{\int_{-\infty}^{\infty} f(t)\, e^{j\omega t}\, dt}_{F(-\omega)} \right) d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F^*(-\omega)\, F(\omega)\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, F^*(\omega)\, d\omega$$

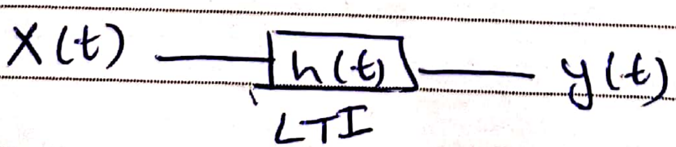
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \underbrace{|F(\omega)|^2}\, d\omega.$$

↳ energy spectral density.

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathcal{E}_f(\omega)\, d\omega$$

$$= \frac{1}{2\pi} \int_{0}^{\infty} \mathcal{E}_f(\omega)\, d\omega$$

* Applications in LTI systems.



Find the energy spectral density of y(t).

No.

$\mathcal{E}_y(\omega) = \left| Y(\omega) \right|^2$

$y(t) = x(t) * h(t)$

$Y(\omega) = X(\omega) \cdot h(\omega)$

$\mathcal{E}_y(\omega) = \left| X(\omega) \cdot H(\omega) \right|^2 = \underbrace{\left| X(\omega) \right|^2}_{\mathcal{E}_x(\omega)} \cdot \left| H(\omega) \right|^2$

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No.

Ex: $\int_{-\infty}^{\infty} \frac{1}{t^2+4} dt = \frac{\pi}{4} e^{-4|w|}$

$F(0) = \int_{-\infty}^{\infty} f(t)$

$= \int_{-\infty}^{\infty} \left(\frac{1}{t^2+4}\right)^2 dt$

$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\pi^2}{a^2} e^{-2a|w|} dw$

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Ex:
([illegible])

$$\int_{-\infty}^{\infty} \frac{1}{2\pi} F^{*}(-\omega) \Big(\underbrace{\int_{-\infty}^{\infty} f(-t)\, e^{j\omega t}\, dt}_{F(-\omega)} \Big)\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F^{*}(-\omega)\, F(-\omega)\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, F^{*}(\omega)\, d(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^{2} d\omega .$$

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^{2} d\omega$$

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathcal{E}_{f}(\omega)\, d\omega = \frac{1}{\pi} \int_{0}^{\infty} \mathcal{E}_{f}(\omega)\, d\omega$$

$$\underset{e^{-t}}{x(t)} \longrightarrow \overset{e^{-2t}}{\boxed{h(t)}} \longrightarrow y(t)$$

$$E_g = \left| X(\omega) \right|^2 \cdot \left| H(\omega) \right|^2$$

$$= \left(\frac{1}{1+\omega^2} \right) \cdot \left(\frac{1}{1+\omega^2} \right)$$

$$\cancel{E_g} \; E_y = \frac{1}{\pi} \int_0^{\infty} \frac{1}{1+\omega} \cdot \frac{1}{4+\omega} \cdot d\omega = \frac{1}{12}$$

* auto-correction function :-

$$\psi_g(\tau) = \int_{-\infty}^{\infty} g(t) \cdot g(t-\tau)\, dt$$

1- $\psi_g(0) = E_g$

2- $\psi_g(\tau) \leq \psi_g(0) = E_g$

3- $\psi_g(\tau)$ is an even function.

↳ Proof:- $\psi_g(-\tau) = \int_{-\infty}^{\infty} g(t) \cdot g(\underbrace{t-\tau}_{t'})\, dt$

$$= \int_{-\infty}^{\infty} g(t'+\tau)\, g(t') = \psi_g(\tau)$$

4- $\mathcal{F}\{\psi_g(\tau)\} = |G(\omega)|^2 = \varepsilon_g(\omega)$

$$L.H.S = \int_{-\infty}^{\infty} \psi_g(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} g(t) \left(\int_{-\infty}^{\infty} g(\tau+t)\, e^{-j\omega\tau}\, d\tau \right) dt$$

$$= G(\omega) \left(\int_{-\infty}^{\infty} g(t)\, e^{j\omega t}\, dt \right).$$

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for Final *

No.

$\boxed{\int_{-\infty}^{t} \rightarrow \text{integration property}}$

$\int_{-\infty}^{\infty}$ or $\int_{-5}^{5}$

→ defenition of F.T

<u>or</u>

using parsaval's theorim.

$\boxed{\text{* F.T properties}}$
→ integratio
→ convolution
→ <u>duality</u> <u>صعب</u>

→ Sampling question ?
→ ± convolution.
→ ± <u>F.S</u>

} First & second (10-12) mark

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EX: find the F.T of

$$y(t) = \int_{-\infty}^{\infty} v\,\mathrm{rect}\left(\frac{t}{T}\right) dt$$

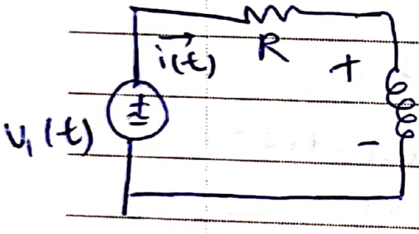
$$f(t) = V\,\mathrm{rect}\left(\frac{t}{T}\right)$$

$$F(w) = VT\,\mathrm{Sinc}(wT/2)$$

$$G(w) = \frac{F(w)}{jw} + \pi F(0)\,\delta(w)$$

$$= \frac{VT\,\mathrm{Sin}(wT/2)}{jw} + \pi VT\,\delta(w)$$

Application to ckt.



$$V_2(t) = L\frac{di(t)}{d(t)}$$

$$-V_1(t) + R\,i(t) + L\frac{di(t)}{dt} = 0$$

take the F.T

$$-V_1(w) + R\,I(w) + Ljw\,I(w) = 0$$

$$V_1(w) = I(w)\cdot(R + jwL)$$

$$I(w) = \frac{V_1(w)}{R + jwL}$$

$$V_2(w) = jwL\,.\,I(w) = \frac{jwL\,.\,V_1(w)}{R + jwL}$$

frequency responce

$$H(w) = \frac{V_2(w)}{V_1(w)} = \frac{jwL + R}{R + jwL}$$

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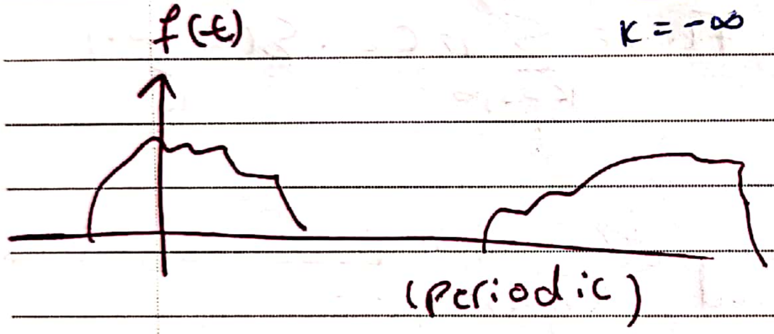
F.T of periodic signals :-

any periodic signal $$f(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

$$F(\omega) = \mathcal{F}\left\{ \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} \right\}$$

$$= \sum_{k=-\infty}^{\infty} C_k . \mathcal{F}\left\{ e^{jk\omega_0 t} \right\}$$

$$F(\omega) = \sum_{k=-\infty}^{\infty} 2\pi C_k \cdot \delta(\omega - k\omega_0)$$



let us define $$y(t) = \begin{cases} f(t) , & \frac{T_0}{2} \geqslant t \geqslant -\frac{T_0}{2} \\ 0 , & o.w \end{cases}$$

(consedring one period).

$$= \sum_{k=-\infty}^{\infty} g(t) * \delta(t - kT_0) = g(t) * \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$$

$$F(\omega) = G(\omega) \cdot \mathcal{F}\left\{ \sum_{k=-\infty}^{\infty} \delta(t - kT_0) \right\}$$

$$= \frac{2\pi}{T_0} \sum_{k=-\infty}^{\infty} G(\overset{k}{\omega_0}) \, \delta(\omega - k\omega_0)$$

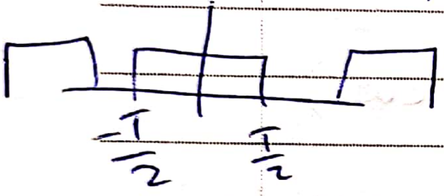
then $C_k = \frac{G(k\omega_0)}{2\pi}$

if $f(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$

$$F(\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T_0} \delta(\omega - k\omega_0)$$

---

find the F.T



① find C_k

② then $F(\omega) = \sum_{k=-\infty}^{\infty} 2\pi C_k \cdot \delta(\omega - \underset{k}{\omega_0})$

<u>or</u>

for one period $g(t) = ---$

$G(\omega) = ---$

then substitute بدل $G(\omega)$

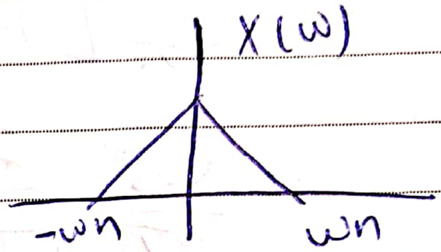
$G(k\omega_0)$

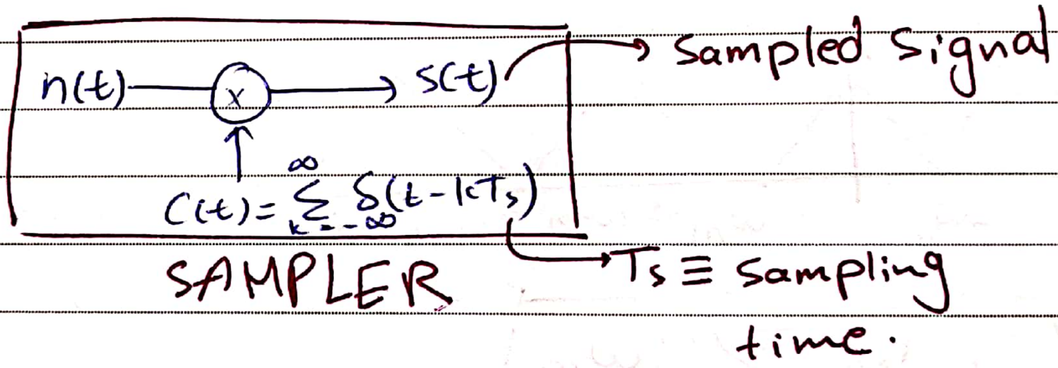
إذا طالب بعدها ال F.S ← بجيب C_k من $G(k\omega_0)$

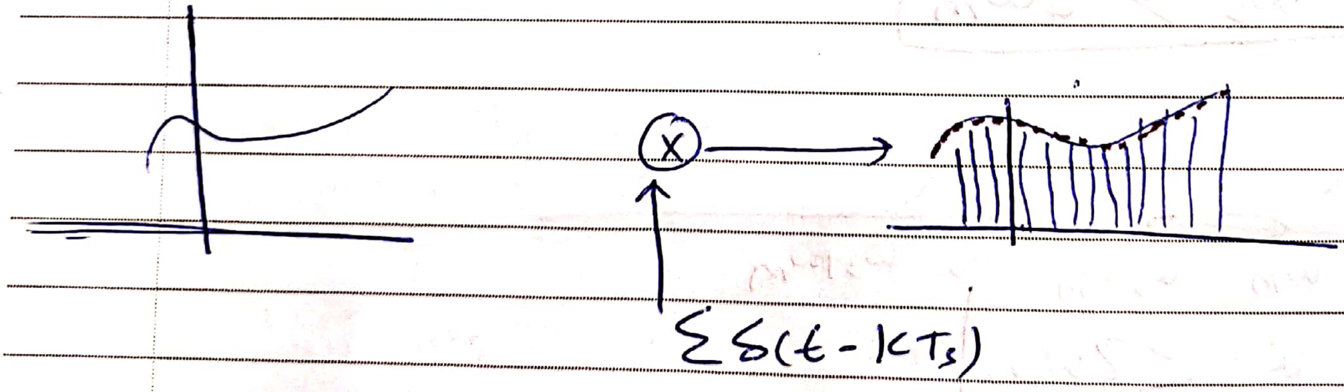
**** Sampling:-** reduction of a C.T signal into D.T signal.

-Def: band limited signal: A signal $x(t)$ is B.L whose F.T is Non-Zero over a small range of frequencies.

$x(t)$ is a B.L to $[-w_n, w_n]$ if $X(\omega)$ is zero for $\omega \notin [-w_n, w_n]$







we have to look at the spectrum of $s(t)$ $(S(\omega)$

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$$s(t) = m(t) \cdot \sum_{k=-\infty}^{\infty} \delta(t-kT_s)$$

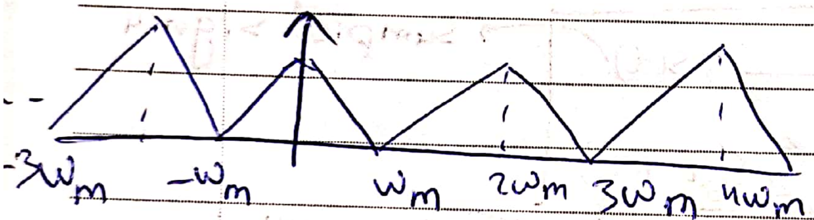
$$S(\omega) = \frac{1}{2\pi} M(\omega) * \mathcal{F}\left\{\sum_{k=-\infty}^{\infty} \delta(t-kT_s)\right)$$

$$= \frac{1}{2\pi} M(\omega) * \left[\sum_{k=-\infty}^{\infty} \overset{\omega_s}{\left(\frac{2\pi}{T_s}\right)} \cdot \delta(\omega-k\omega_s\right]$$

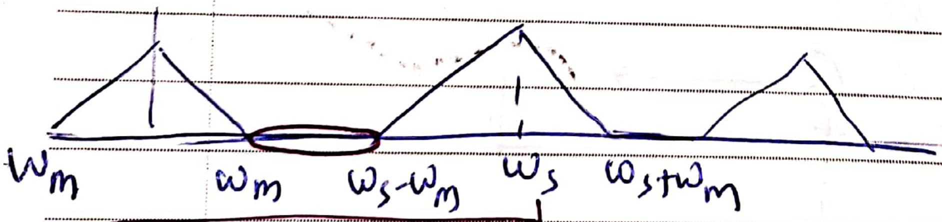
$$= \frac{\omega_s}{2\pi}\left[M(\omega) * \left(\sum_{k=-\infty}^{\infty} \delta(\omega-k\omega_s)\right)\right]$$

$$= \frac{\omega_s}{2\pi} \sum_{k=-\infty}^{\infty} M(\omega-k\omega_s)$$

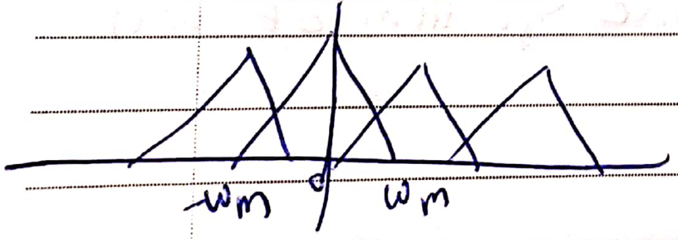
$\boxed{1)\ \omega_s = 2\omega_m}$



$\boxed{2)\ \omega_s > 2\omega_m}$



$\boxed{3)\ \omega_s < 2\omega_m}$



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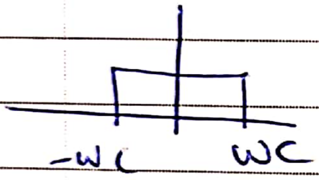
⟹ Sampling theorm:- A signal can be represented in its samples and can be recovered back when the sampling freq is greater than or equal to twice of the maximum frequency component (ω_m)

$$\omega_s \geq 2\omega_m$$

↳ recovered m(t)

$H(\omega) \rightarrow$ LPF



$\omega_s - \omega_m \geq \omega_c \geq \omega_m$ → for Perfect reastruction

---

سؤال

→ ① T_s نجيب

② $\frac{2}{T_s} = \omega_s$

if $\omega_s \geq 2\omega_m$ → if Yes ✓
if No

what conditions should be in the LPF ?

$\omega_s - \omega_m \geq \omega_c \geq \omega_m$

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