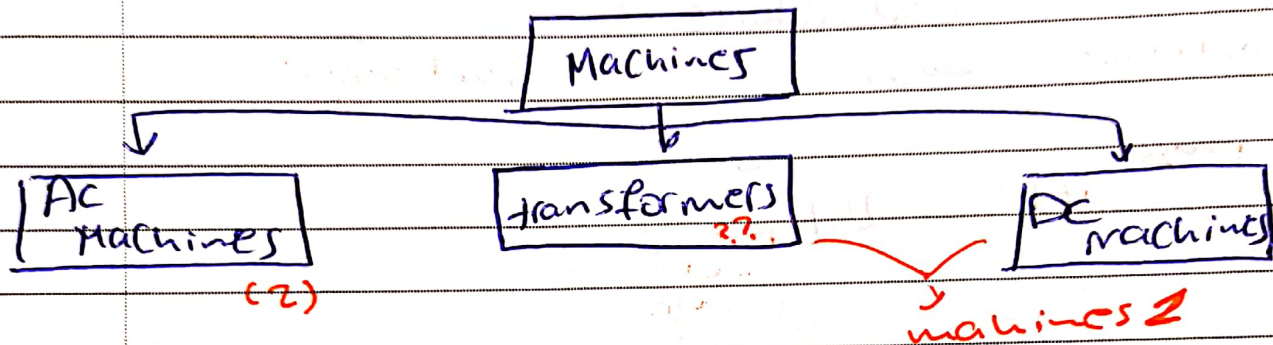


\* electrical machines 2 :-

- This includes :-
- electromagnetic principles (to review)
- power transformers.
- Direct Current Machines.



\* Review of electromagnetics

electrical : Voltage, current, power, resistance (Impedance) capacitance.

magnetic : MMF  
 ↑  
 magnetomotive force.

$$F = \overset{\text{EMF}}{i} \cdot N$$

no magnetic structure with out current except its a permanent magnet } to make a machine from a permanent magnet it will be very expensive.

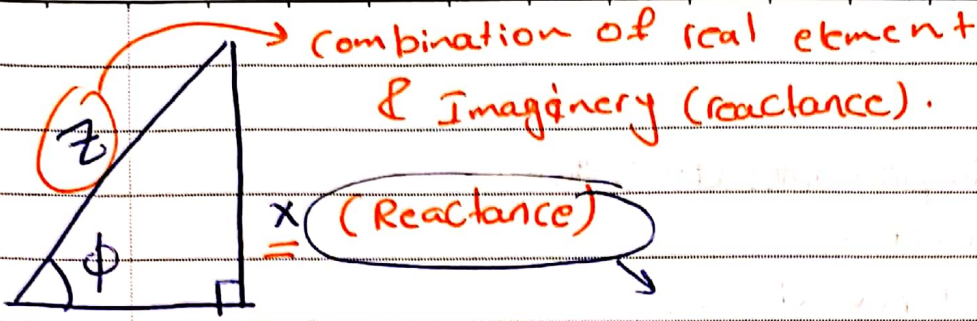
$i \equiv i_e$  (electrical) → response.

$\phi \equiv \phi_m$  (magnetic) →  $\phi$  (flux) Wb (forcing function)

\* between the forcing function & the response in electric structures,

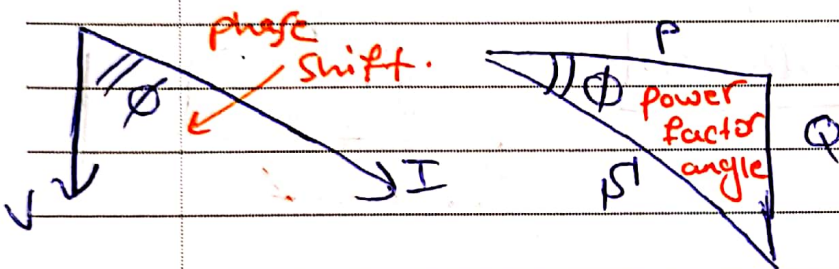
The resistance.

No. \_\_\_\_\_

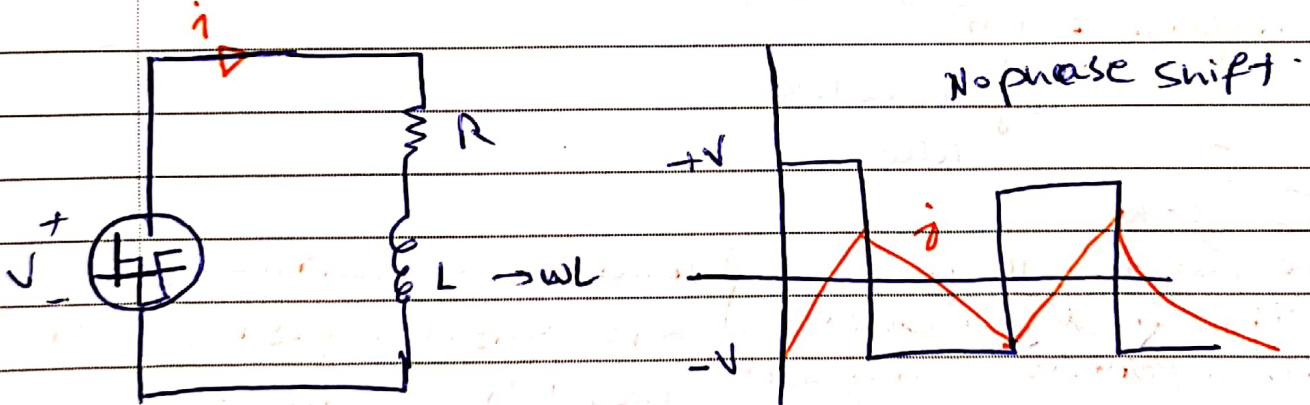


(R) resistance.

Capacitance & inductance  $\rightarrow \sin \phi \rightarrow$  load angle.



$\phi$ : PF angle & phase shift and load phase  
 $\rightarrow$  same if  $v$  and  $i$  are pure sinusoidal.



$$V(t) = \begin{cases} +V, & 0 \leq \omega t \leq \pi \\ -V, & \pi < \omega t < 2\pi \end{cases}$$

$$V(t) = \sum V_m(n) \sin(n\omega t + \phi(n))$$

$$V_m(n) = \sqrt{A(n)^2 + B(n)^2}$$

$$\phi(n) = \tan^{-1} \left( \frac{A(n)}{B(n)} \right)$$



$$PF = X \cos \phi$$

$$\text{non sinusoidal} = DTF \times \overset{\text{No.}}{DPF}$$

Distortion factor

Displacement angle.

$$\frac{I_{Fund}}{I_{total}}$$

$$\cos(\phi_{(1)} - \phi_{i(1)})$$

Fund n=1      Fund n=1

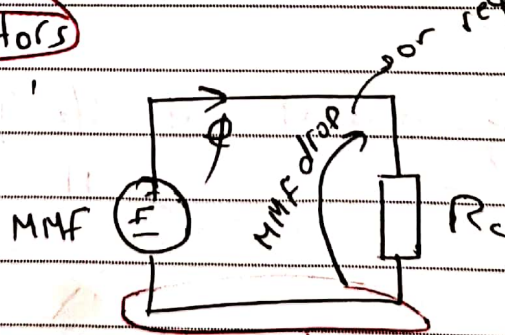
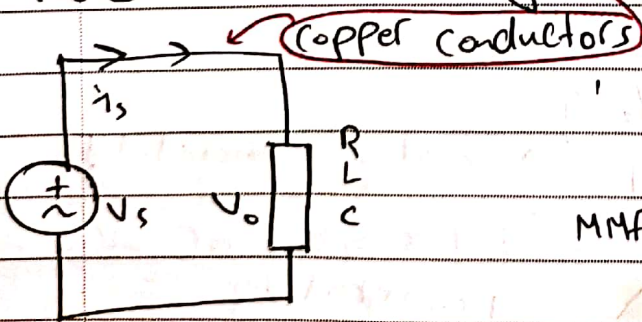
lec(2) :-

	Forcing function	Response	Factor of response.
Electric	V	i	Resistance (Impedence) $R = \frac{\rho l}{A} = \frac{l}{\sigma A}$
Magnetic	$F, i$	$\phi$	Reluctance ( $R_m$ ) $R_m = \frac{l}{\mu A}$ (cross sectional area.)

$\sigma \equiv$  conductivity (electric)  
 $\mu \equiv$  Magnetic conductivity

\* Resistance (Impedence)  $\rightarrow$  complex (imag + Real)

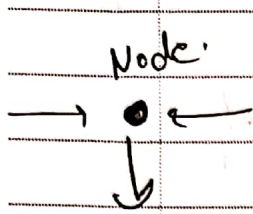
\* Reluctance  $\rightarrow$  (only real)



\* series & parallel connections [same Rules]

Laws, Theorems, Principles, Techniques, ... [same]

**Laws** Ohm's law  $i = \frac{V}{R} \rightarrow \phi = \frac{F}{R_c}$



KCL :  $i = \sum_{n=1}^k i(n)$  Kirchhoff's  
node. branches.  $\rightarrow$  Flux law.

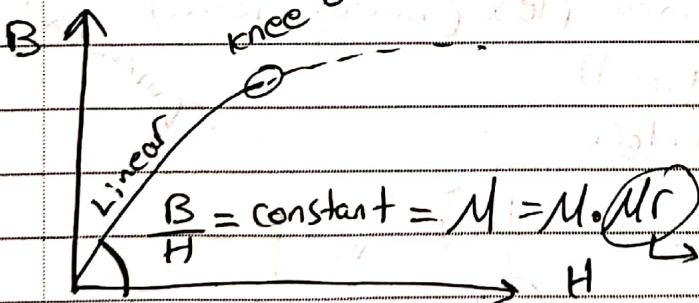
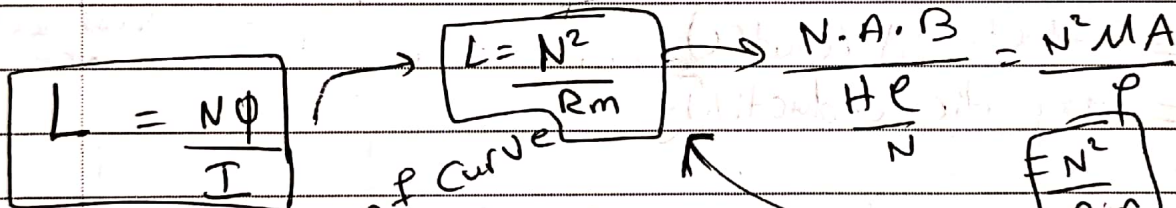
$\sum \phi(n) = 0$   
 $n=1, 2, 3, \dots$

KVL:  $\sum V$  in a closed Path = 0  $\rightarrow \sum F$  in a closed Path = 0

Ampere's law  $N \cdot i = \oint \vec{H} \cdot d\vec{l}$

then

$F = NI = Hl = \phi R_m$



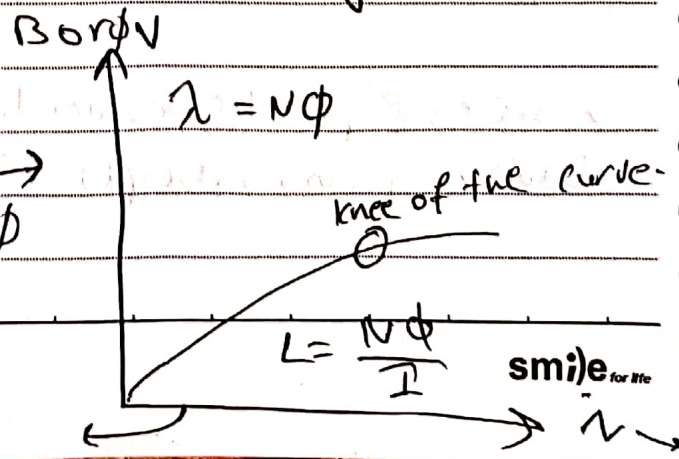
$\mu_r$ : relative permeability  
 $\mu_0$ : free space or air permeability

$1 \leq \mu_r \leq 400,000$  (upto)

$V = k \cdot \omega_m \cdot \phi$

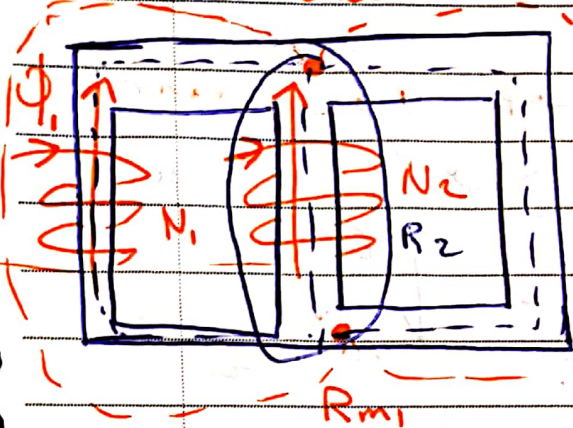
$B = \frac{\phi}{A}$

$i = \frac{Hl}{N}$





**\* Inductance :-**

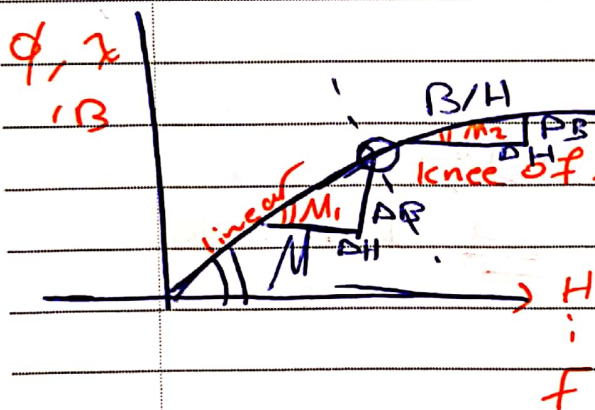
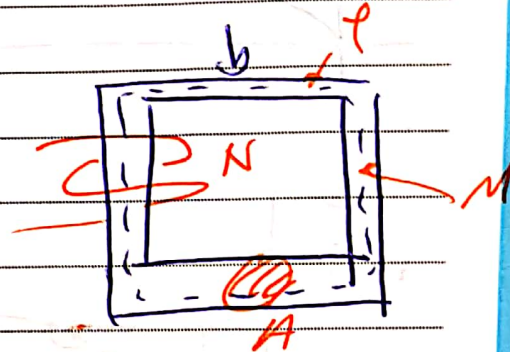


• self inductance  $L = \frac{N^2}{R_m}$   
 $R_m = \frac{l}{\mu A}$

$\Rightarrow \frac{N^2 \mu A}{l} \rightarrow$  I can use it only when I have one coil

$L = \frac{\lambda}{I}$  (for linear)

or  $\frac{d\lambda}{di}$  (for non-linear)



$\mu_1 > \mu_2$

when saturation increases  $\frac{dB}{dH} = \mu$   
 $\rightarrow$  decrease.

**\*\*** Saturation of magnetic core leads to (almost short-circuit) electric

① decrease

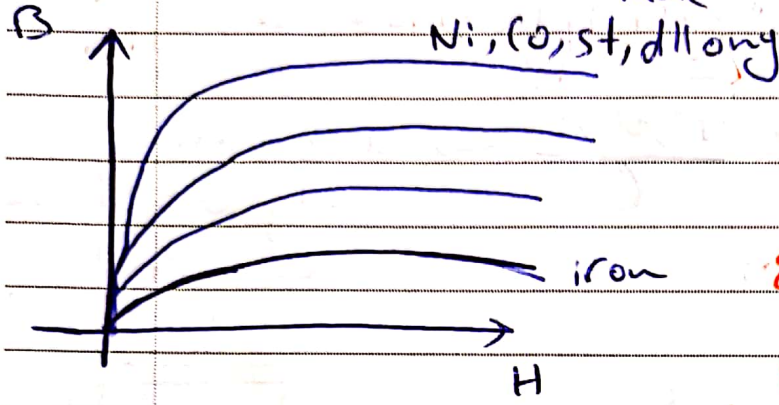
why?  $\mu$  increase then from  $L = \frac{N^2 \mu A}{l}$

then  $\mu$

$\rightarrow$  low  $\omega L = X \rightarrow$

leads to short circuit

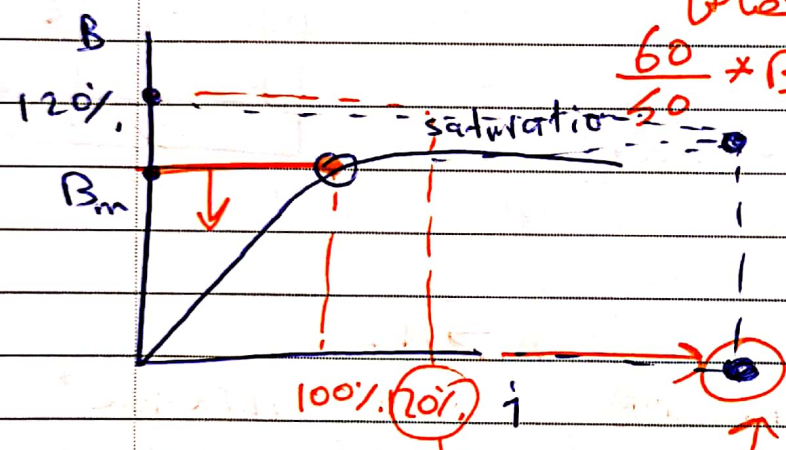
$E \equiv V \equiv 4.44 * N_{ph} * \phi_{max} * f \rightarrow V = K \cdot B_m * f$



If  $V = \text{constant}$   
 $B_m * f = \text{constant}$   
 60 Hz  
 ↓  
 50 Hz

بترتفع  
 بنفس  
 النسبة

للاختصاص



$\frac{60}{50} * B_m$

بصرفهون

so I'll have losses

linear كارلوكان  
 saturation من لينة

Ex

$f \ 50 \rightarrow 60$   
 then  $B = B_m * \frac{50}{60} \rightarrow B \rightarrow 80\%$  (linear)  
 then I will decrease.



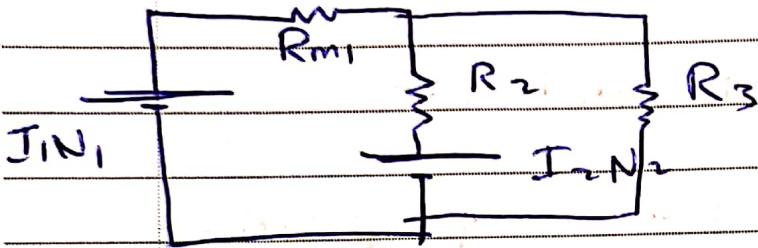
\* Back to the (self inductance)

to calculate  $L_1 = N_1^2$

$R_m$  →  $R_m$ : as see

Don't use  $\frac{N^2 \mu A}{\rho}$

- eq ckt



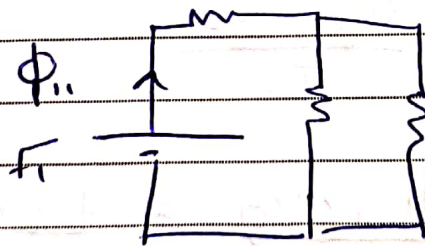
$$R_m = \frac{\ell}{\mu A}$$

$$R_m (\text{seen by } N_1) = R_{m1} + (R_{m2} // R_{m3})$$

باعتبار  $N_1$  كمنفذ موجود

$$\text{then } L_{11} = \frac{N_1^2}{R_{meq}} = \frac{\lambda_{11}}{I_1} = \frac{N_1 \Phi_{11}}{I_1}$$

\* to find  $\Phi_{11} \equiv$  flux due to coil (1)



while or other

sources are killed

$$\Phi_{11} = \frac{I_1 N_1}{R (\text{seen by coil (1)})}$$

Killing  $\equiv$  Super position theorem is applied.

in linear ckt

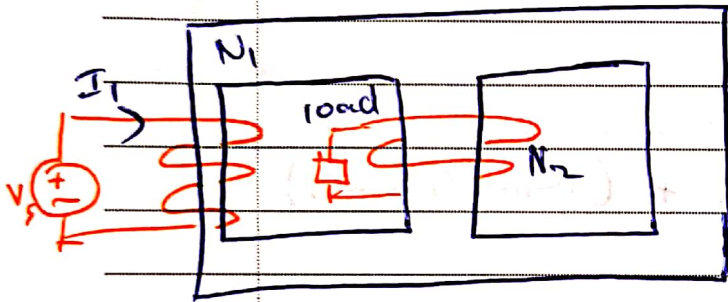
$$L_{11} = \frac{N_1^2}{R_m (\text{eq})} \quad \text{must equal} \quad \frac{N_1 \Phi_{11}}{I_1}$$

$$L_{22} = \frac{N_2^2}{R_{m\text{eq}}^{(2)}}$$

coil No  $\nearrow$   
coil current  $\nearrow$

$$R_{\text{eq}}^{(2)} = R_{m2} + (R_1 // R_3)$$

**\*\* Mutual Inductance / coil coupling :-**



(also super position is applied)

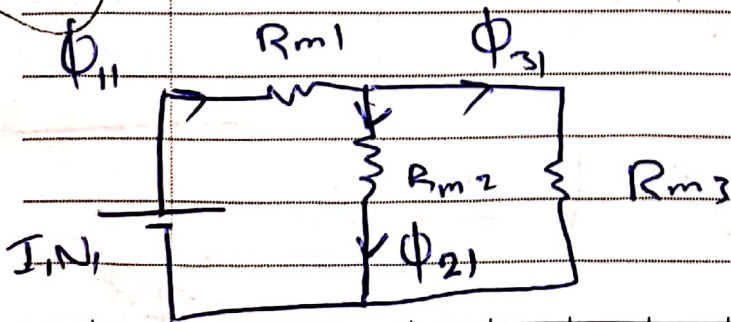
$M_{21}$   $\rightarrow$  mutual inductance of coil (2) due to current (1)

$M_{12}$   $\rightarrow$  " " " " (1) " " " (2)

$$M_{21} = \frac{\lambda_{21}}{I_1} = \frac{\phi_{21} \times N_2}{I_1}$$

$\phi_{21}$

$$M_{12} = \frac{\lambda_{12}}{I_2} = \frac{\phi_{12} \times N_1}{I_2}$$

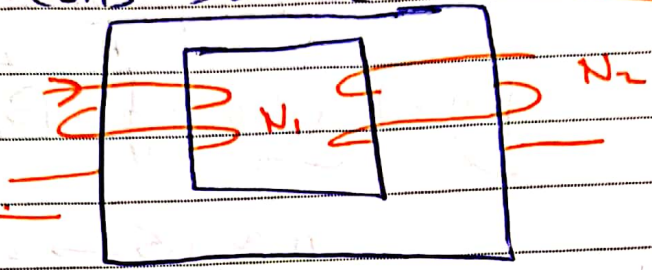
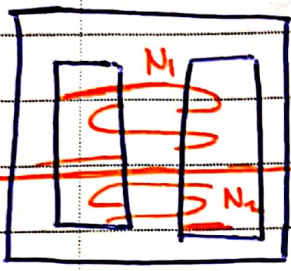




$M_{12} = M_{21}$  ?

↑ if both coils share common

homogeneous core



core type

if the

shell type.

$M = k \sqrt{L_{11} L_{22}}$

k : coupling coefficient

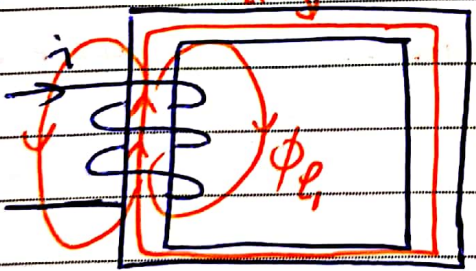
$1 \leq k \leq \dots \rightarrow$  Small value.

poor coupling. (high leakage flux)

↑ tight coupling

then  $\phi_{11} = \phi_{21} + \phi_{e1}$

↓  
≈ zero leakage flux



\*\* core losses

Copper loss  $\rightarrow P_{cu} = I^2 R_{circuit}$

core loss is due to flux  $\phi$  in the core.

$P_{core} \equiv P_{magnetic} \equiv P_{iron} = P_{eddy-current} + P_{hysteresis}$

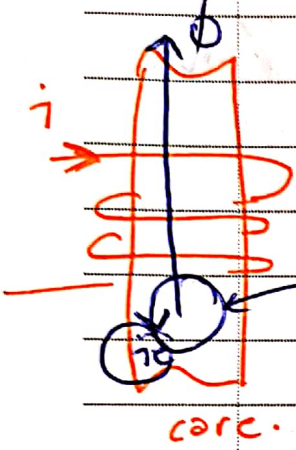
Eddy current loss  $\rightarrow \phi$  should be time varying.

$$\phi \rightarrow \frac{d\phi}{dt} \neq 0$$

volume of the core.

$$\hookrightarrow \frac{d\phi}{dt} \neq 0$$

$$P_e = k \times Vol \times f^n \cdot B_m^2$$



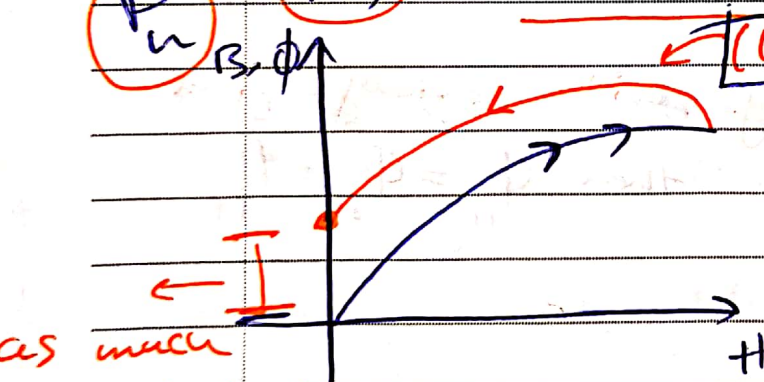
$$n \approx 1.5 \rightarrow 2.5$$

$n \equiv$  Steimets factor

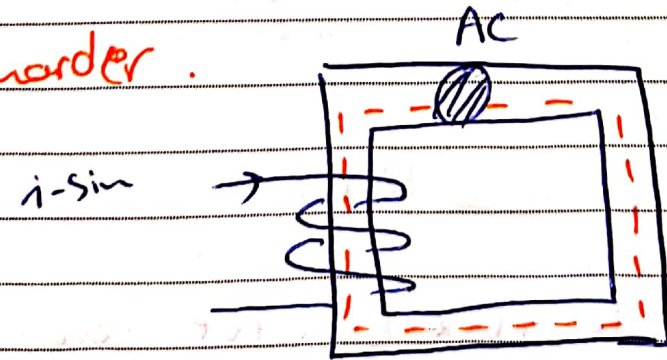
$$P_e = k_n \cdot f^n \cdot B_m^2$$

$\uparrow$  voltage dependent.

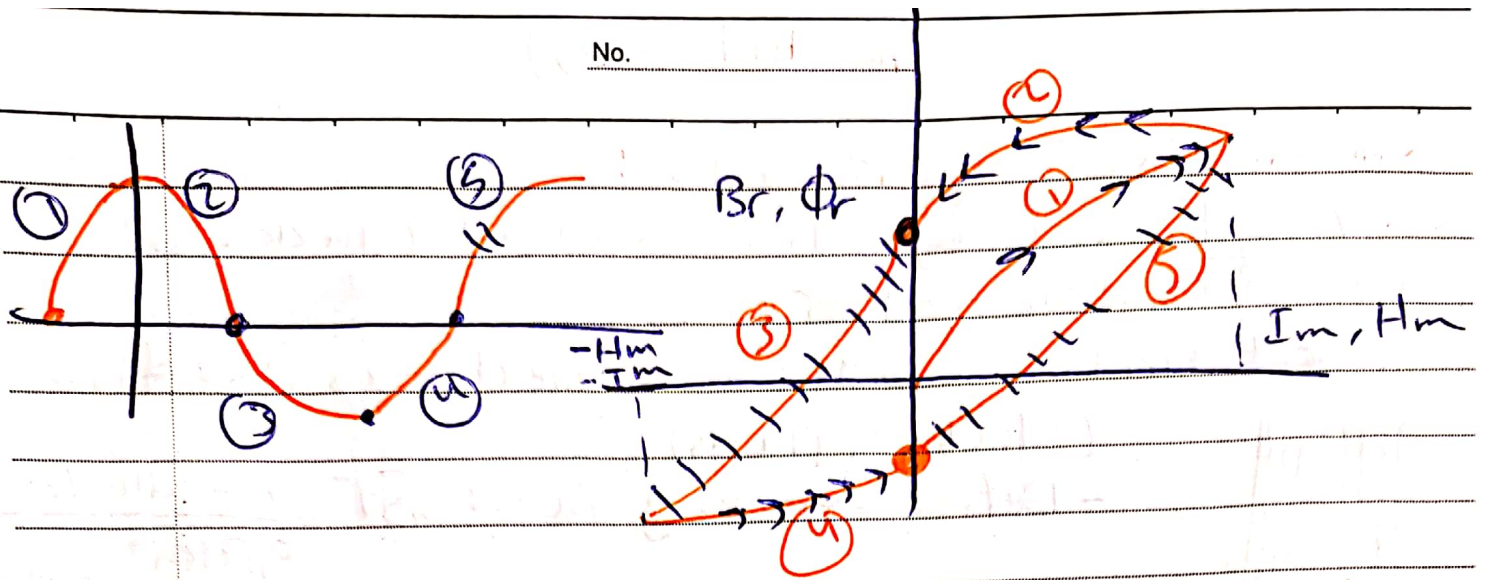
$P_w$  (AC) because as shown the (back current) is not the same of the first in direction amount.



as much this distance is bigger the material is harder.







$\Delta \text{Area} = \Delta B \times \Delta H$   
 $= \frac{\Phi}{A} \times \frac{NI}{l} = \frac{V \cdot I \cdot \text{Sec}}{\text{Vol}} = \frac{\text{Joule}}{\text{Vol}}$

\* This implies that the area of the hysteresis loop represents the hysteresis loss per volume unit.

$$P_h = \text{Area} \times \text{Vol} \times f$$

$$P_h = k \cdot \text{Volume} \cdot f \cdot B_m^2$$

$$= K_h \cdot f^2 \cdot B_m^2$$

$$P_{\text{core}} = P_h + P_e$$

Prore  $\rightarrow$  fixed losses as for as the voltage is fixed.

$P_h$  is usually less than  $P_{cu}$   $P_{cu} =$  variable losses.

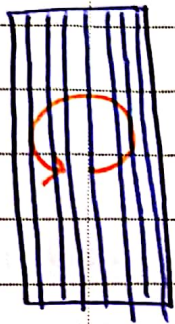
When  $P_h = P_{cu}$

$\downarrow$  leads to Max. efficiency of the machines.

\* How to reduce eddy current loss?

\*\* Core lamination (thin sheets of  $\tau$  thickness isolated from each other).

Sheet direction is in the same direction as flux flow.  
(stacking process)



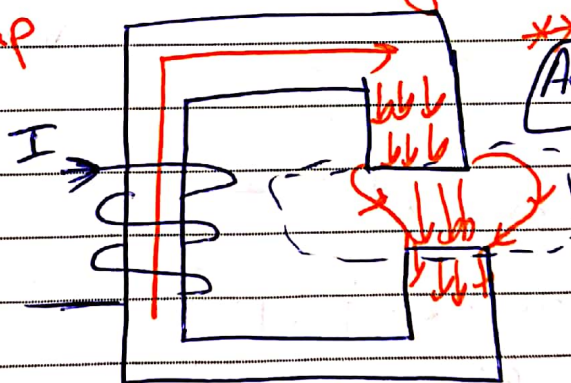
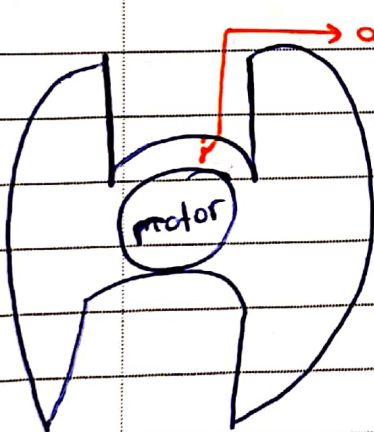
- Define stacking factor  $SF = \frac{\text{Actual core vol.}}{\text{apparent core vol.}}$

$SF < 1$

$SF = 1 \rightarrow$  No sheets

$\uparrow \phi$  (time varying)

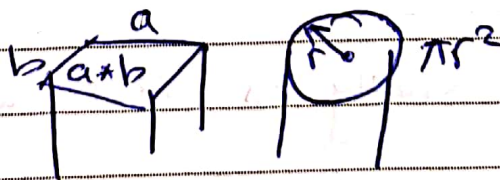
\* if not AC no need for laminating.



\*\* Air gap correction (fringing)

\* Flux distribution in the air gap area is not uniform as that inside core.

$B_c = \frac{\phi}{A_c}$



$B_g = \frac{\phi}{A_g(\text{app})}$  if no fringing

$B_g = \frac{\phi}{A_g(\text{eff})}$   $A_g(\text{eff}) > A_g(\text{app})$



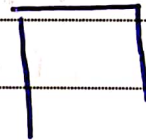
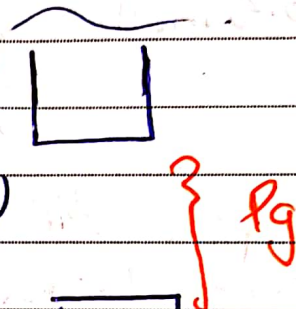
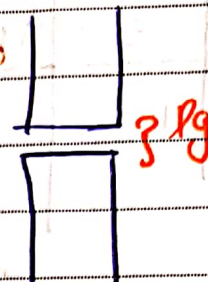
→ Fringing factor

$$A_g(\text{eff}) = \underbrace{FF}_{\text{fringing factor}} \times A_g(\text{app})$$

~~FF < 1~~  $FF \geq 1$

$$FF = \frac{A_g(\text{eff})}{A_g(\text{app})}$$

$$= \frac{(a+r_g) \times (b+r_g)}{a \times b} \quad (\text{rectangular})$$



(circular section)

$$FF = \frac{\pi(r+r_g)^2}{\pi r^2}$$

$$R_g = \frac{\phi}{A_g(\text{eff})} = \frac{\phi}{FF \times (A_g(\text{app}))}$$



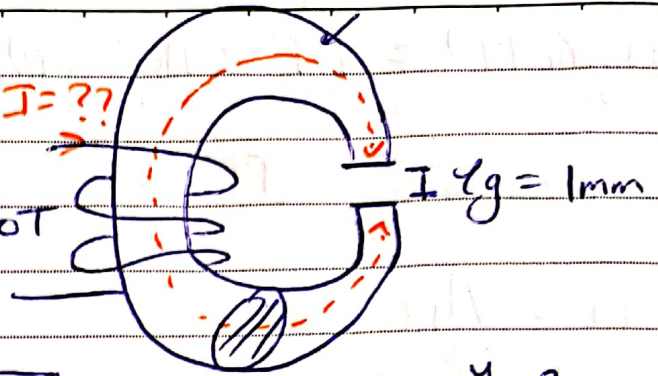
(B/H)

$R_c = 30 \text{ cm}$

H	2500	3000	3500	4000
B	1.55	1.59	1.6	1.615
T/(wb/m <sup>2</sup> )				

$I = ??$

$N = 500$



$\phi = 0.5 \text{ mwb}$

$D = 2 \text{ cm}$

$A_c = \pi r^2$

$A_c = \pi \times 10^{-4} \text{ m}^2$

$M_r = 4000$

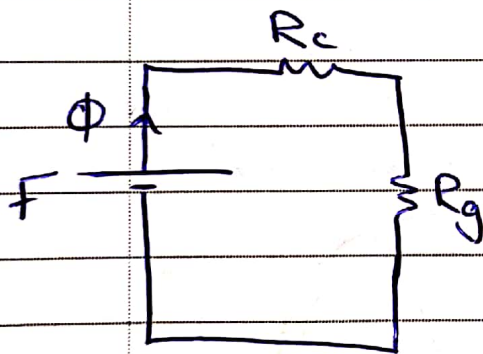
$I = ?$

a) No air gap.  $M_r = 4000$  (linear)

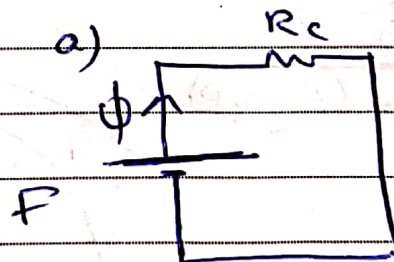
b)  $M_r = 4000$  with air gap (linear)

c) non linear (B/H) curve given.

(مس) و (ب) و (ج)



in a)  $R_g = 0$  because  $R_g = \frac{l_g}{\mu_0 \mu_r A}$



$R_c = \frac{l_c}{\mu_0 \mu_r A_c}$

$= \frac{30 \times 10^{-2}}{4\pi \times 10^{-7} \times 4000 \times \pi}$

$A_c = \pi r^2 = (\frac{2}{2} \times 10^{-2})^2 \times \pi$

$R_c = 190 \times 10^3 \left(\frac{1}{H}\right)$

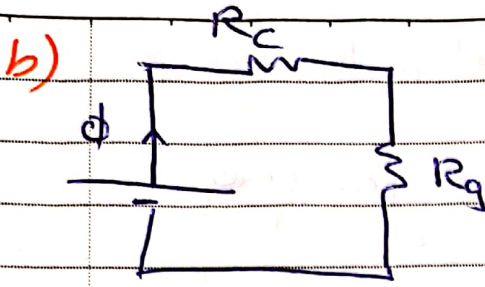
$F = IN = \phi \times R_m$

$I = \frac{\phi \times R_m}{N} = \frac{0.5 \times 10^{-3} \times 190 \times 10^3}{500} = 0.19 \text{ A}$

stacking factor.  $\mu_r$   $A_c$   $\mu_0$   $\mu_r$   $A_c$   $\mu_0$   $\mu_r$   $A_c$

correct the lamination factor.





$$R_g = \frac{l_g}{\mu_0 \mu_r A_g} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 4000 \times \pi \times 10^{-4}} = 2.5 \times 10^6 \frac{1}{H}$$

× لو حڪ خيال فرينجنگ فكتور  
 ان مقام بل فرينجنگ فكتور

$A_g = A_c \rightarrow$  No fringing.

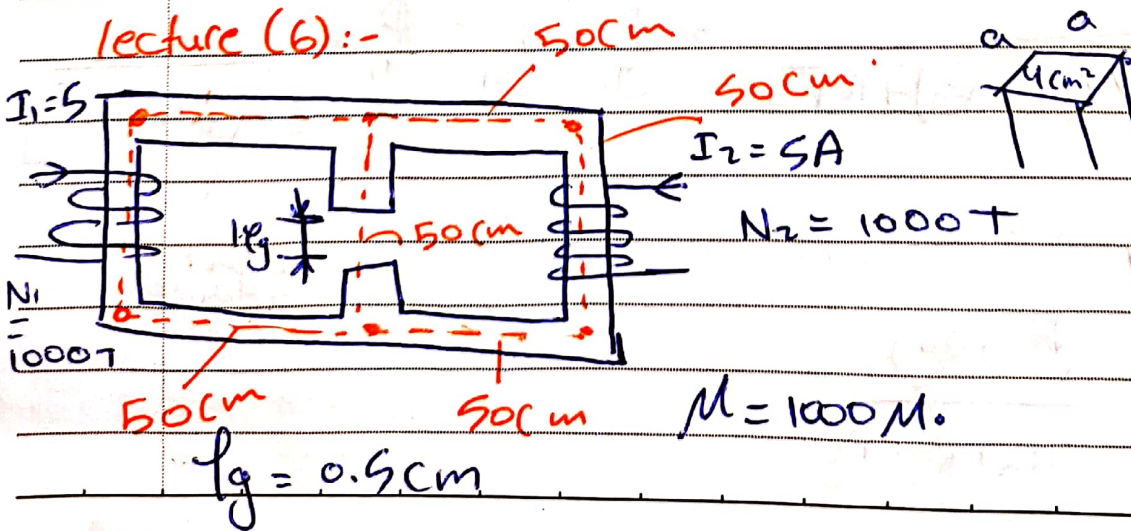
$$I N = \frac{\phi (R_c + R_g)}{N} = \frac{0.5 \times 10^{-3} (2.5 + 0.19) \times 10^6}{500} = 2.7 A$$

c)  $H_g = \frac{B_g}{\mu_0 \mu_r} = \frac{\phi}{A_g \mu_0 \mu_r}$

$B_c = \frac{\phi}{A_c} \rightarrow$  from curve  $\rightarrow$  find  $H_c$

$I N = H_c l_c + H_g l_g$

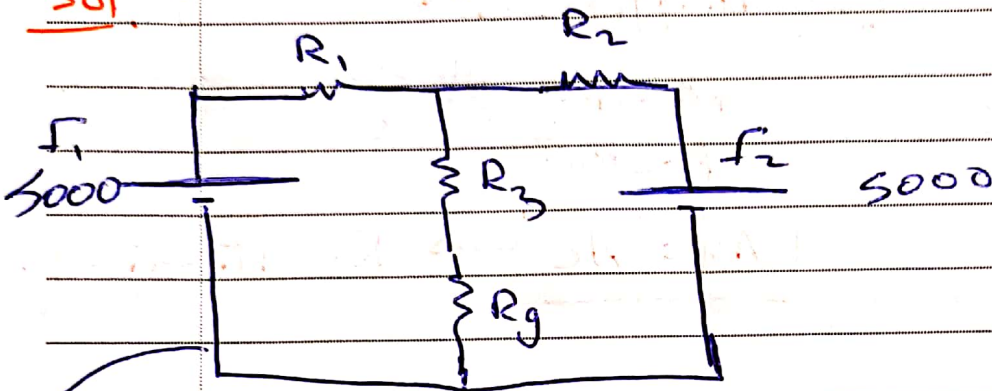
lecture (6):-



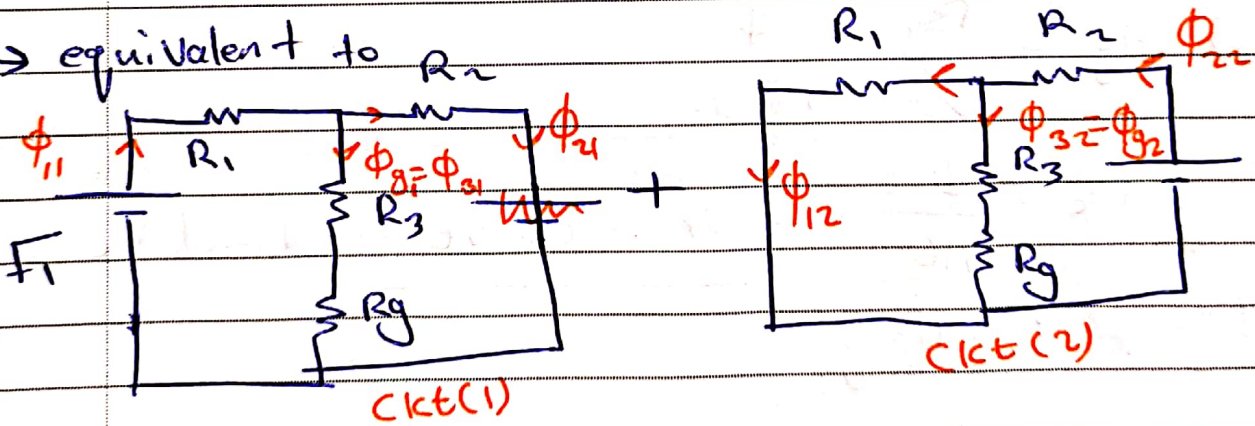
$\phi_g, B_g, H_g = ??$  self & Mutual inductance.

Neglect leakage but correct for fringing.

Sol



- since  $L_1, L_2, M_1, M_2$  are req, super position is to be applied.



$$\phi_{11} = \frac{F_1}{[R_2 \parallel (R_3 + R_g) + R_1]} \quad \rightarrow \quad \phi_{g1} = \phi_{s1} = \phi_{11} \times \frac{R_2}{[R_2 \parallel (R_3 + R_g)]}$$

$$\phi_{22} = \phi_{11} - \phi_{s1}$$

$$\underline{OR} \quad \phi_{11} \times \frac{(R_3 + R_g)}{R_2 + R_3 + R_g}$$

Current divider rule.



$$\phi_{22} = \frac{\sqrt{2}}{R_2 + [R_1 \parallel (R_3 + R_g)]} = \phi_{11} \quad \text{only in this case} \\ \text{— Substituição}$$

$$\phi_{g2} = \phi_{32} = \phi_{22} * \frac{R_1}{R_1 + R_3 + R_g}$$

$$\phi_{12} = \phi_{22} - \phi_{32}$$

$$* \phi_g = \phi_{g1} + \phi_{g2}$$

$$B_g = \frac{\phi_g}{A_{\text{eff}}} \rightarrow (\text{correct for fringing}).$$

$$-A_{\text{eff}} = (2 + 0.5) * (2 + 0.4) = 6.25 \text{ cm}^2 = 6.25 * 10^{-4} \text{ m}^2$$

$$* B_g = \frac{\phi_g}{6.25 * 10^{-4}}$$

$$H_g = \frac{B_g}{M}$$

$$R_1 = R_2 = \frac{1.5}{4 * 10^{-4} * 1000 * 4\pi * 10^{-7}}$$

$$R_3 = \frac{0.4 * 5}{[ \quad ]}$$

$$R_g = \frac{0.5 * 10^{-2}}{6.25 * 10^{-4} * 4\pi * 10^{-7}}$$

$$L_{11} = \frac{N_1^2 \phi_{11}}{M_{12} I_1}$$

$$M_{12} = \frac{N_1 \phi_{12}}{I_2}$$

$$L_{22} = \frac{N_2 \phi_{22}}{I_2}$$

$$M_{21} = \frac{N_2 \phi_{21}}{I_1}$$

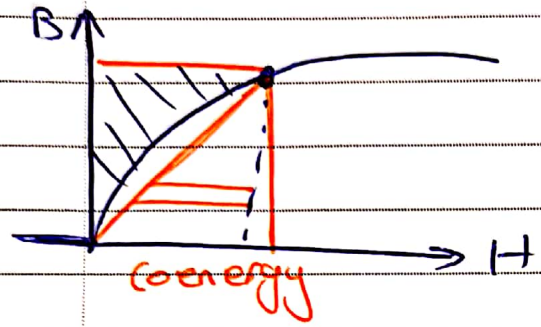
if  $M_{12} = M_{21}$  then it's homogeneous. smile for life



$$w_e = \int P \cdot dt = \int e_i \cdot dt = \int N \frac{d\phi}{dt} \frac{H \ell}{N} dt$$

$$\oint N = H \ell$$

$$P \int H \ell dB \rightarrow \text{Vol.} \cdot M \int H dH$$



$$w_f = \int H dB$$

if the integration is complicated we can calculate the Coenergy & then subtract it from the rectangle.

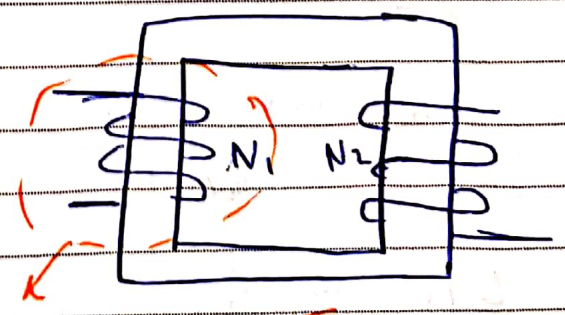
(Electrically isolated).

**\*\* Transformers:-**

a Transformer:- static, Electromagnetic device, where at two coils are electromagnetically coupled by a core that is usually ferromagnetic. & this is a Pure AC ~~(machine)~~ / device, (flux should be time varying  $\rightarrow \frac{d\phi}{dt} \neq 0$ ).

\* core: two major forms:

① core type C-I



\* shell type E-I

أفضل من C-I core  
يكون ال Coils  
بعض (tight coupling).  
smile for life



E-I Core.



No. \_\_\_\_\_

→ could be vertical or horizontal  
→ best coupling (tight) when the two coils are put here.

E-I Core.



No. \_\_\_\_\_

→ could be vertical or horizontal  
 → best coupling (tight) when the two coils are put here.

\*lecture (7):-

$\tau \approx 0.1 \sim 0.5$

↳ in power applications.

$\tau \approx 0.05 - 0.01$

↳ communication

Stacking Factor =  $\frac{\text{Vol (actual)}}{\text{Vol (app)}}$   
 $\bar{c} \bar{b}$   
 (unity)

$\tau \rightarrow \bar{c} \bar{b}$   
 eddy current loss will decrease.

Homework (1)

$$W_g = \frac{1}{2} Vol \times B_g \times H_g$$

$$= \frac{1}{2} Vol \frac{B_g^2}{\mu}$$

$$= \frac{1}{2} \phi^2 \frac{B_{max}}{g}$$

$$= \frac{1}{2} Vol \times H_g \mu$$

$$W_c = \frac{1}{2} \phi^2 R_m$$

$$= \frac{1}{2} B_c \times H_c$$

$$= \frac{1}{2} B \times A \times \frac{l}{\mu A}$$

$P_r = k_r \times f \times B_m^n$

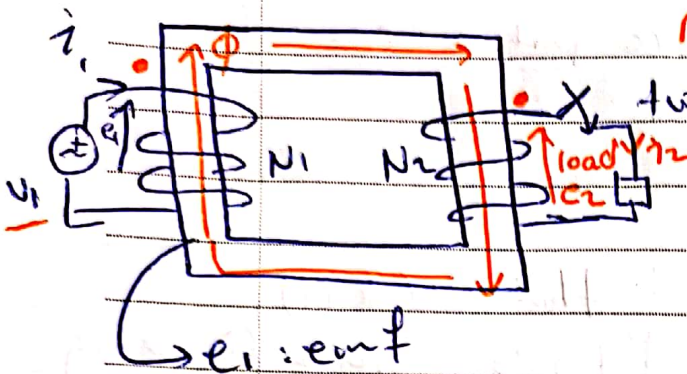
$P_e = f^2 \times k_e \times B_m^2 \times \tau^2$

$\tau$ : lamination thickness



lecture (8) :-

Back to transformers :-



Basic TR :- Common core + at least two coils or windings.

one coil is excited by an AC current. This coil is referred as the primary coil.

The second coil is usually connected to the load, to supply it by power (indirect), is referred to the secondary coil.

$i_1 \rightarrow$  flux ( $\phi$ )  $\rightarrow$  emf against the supply.

$$e_1 = -N_1 \frac{d\phi}{dt} \quad (\text{counter emf}), \text{ usually } e_1 < V_1$$

$\downarrow$   
because of the leakage.

\*\* due to  $\phi$  link coil (2)  $e_2$  is created according to Faraday's law.

$$e_2 = -N_2 \frac{d\phi}{dt}$$

\*\* The direction of  $e_2$  is such that when a current flows into the load, it should create a flux that opposes the original flux.

$$\frac{e_1}{e_2} = \frac{-N_1 \frac{d\phi}{dt}}{-N_2 \frac{d\phi}{dt}} = \frac{N_1}{N_2} \implies e_1 \text{ and } e_2 \text{ are in phase (if no losses)}$$

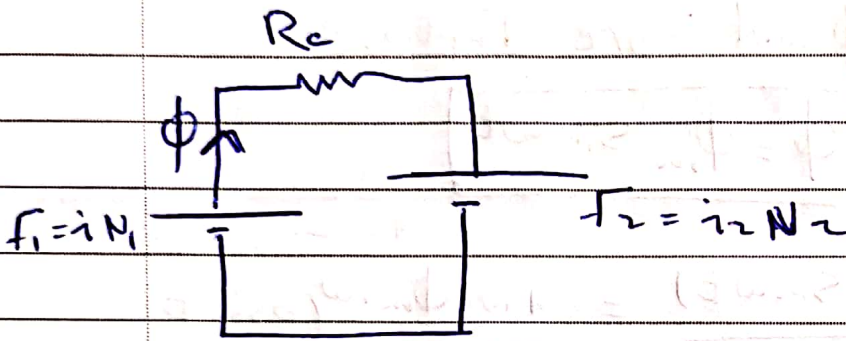
$$\implies \frac{E_{m1}}{E_{m2}} = \frac{N_1}{N_2} \implies \frac{E_1(\text{rms})}{E_2(\text{rms})} = \frac{N_1}{N_2}$$

smile for life

Since  $V_1 \approx E_1$ ,  $V_2 \approx E_2 \therefore \frac{V_1(\text{rms})}{V_2(\text{rms})} = \frac{N_1}{N_2} = a$

Transformation ratio  
or  
Transformation ratio.

Magnetic eq. Circuit



Assuming ideal core.

$\mu_r \rightarrow \infty$

$R_c = \frac{l}{\mu A} = 0 \rightarrow \phi \neq R_c \rightarrow 0$

$\mu = \frac{B}{H} \rightarrow H = \frac{B}{\mu} \rightarrow \frac{B}{\infty} \rightarrow H \sim 0$

$i_1 N_1 - H \phi - i_2 N_2 = 0 \rightarrow i_1 N_1 = i_2 N_2$

~~$\phi \cdot R_c$~~   
zero (ideal)

$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$

$a = \frac{i_2}{i_1} = \frac{V_1}{V_2}$

$i_1 V_1 = i_2 V_2$

$\phi_1 = \phi_2$



$$E_{(capita)} \equiv rms \text{ value}$$

No.

$$e_1 = N_1 \frac{d\phi}{dt}$$

$$i_1 = I_m \sin \omega t$$

$$i_1 N_1 = Hl = \frac{B}{\mu} l = \frac{\phi l}{AM}$$

$$i = \frac{l}{MAN} \phi$$

→  $\phi$  and  $i$  are Inphase

$$\phi = \phi_m \sin \omega t$$

$$\begin{aligned} &= N_1 \frac{d(\phi_m \sin \omega t)}{dt} = +N_1 \phi_m \omega \cos \omega t \\ &= N_1 \phi_m \omega \sin \left( \omega t - \frac{\pi}{2} \right) \end{aligned}$$

$$E_m(i) = N_1 \phi_m \omega = N_1 \phi_m (2\pi f)$$

$$E(i) = E_m(i) = \frac{2\pi}{\sqrt{2}} N_1 f \phi_m = 4.44 N_1 f \phi_m$$

↓ similarity

$$E_2 = 4.44 N_2 f \phi_m$$

\*The effect of running a transformer at lower freq than the rated value

(Assuming  $V \times E$  is same)

$$E_1 \approx V_1 = 4.44 \times N_1 \times f_1 \times \phi_m$$

$f_2$  (new freq)

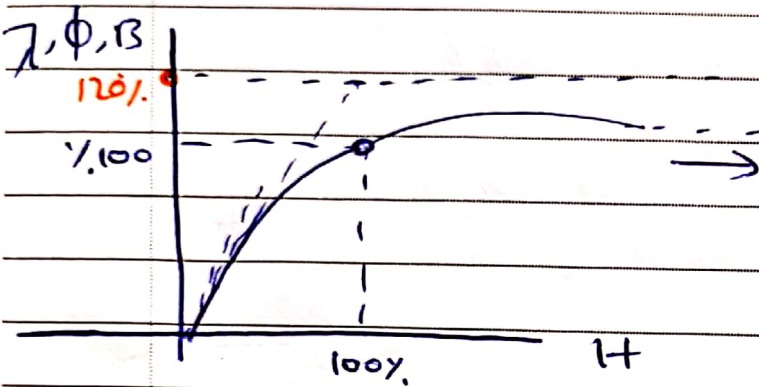
$$E_1 \approx V_1 = 4.44 \times N_1 \times f' \times \phi_m'$$

\*\* Reduction of  $f$  to  $f'$  → leads to increase of  $\phi_m$  to  $\phi_m'$  (since  $V_r = \text{constant}$ )

\* Increasing  $\phi_m$  to  $\phi_m'$  → leads to core saturation.

ex:  $\frac{\phi_m'}{\phi_m} = \frac{f}{f'} = \frac{60}{50} = 1.2$

$f$  60Hz  
↓  
50Hz



\* which mean very much current is required to match the new flux.

\* This current may be in the range of hundreds of the original value leading to transformer destruction.

\*\* If  $f > f_r$  No danger, but bad design from the economic point of view.

lecture 81: -

\* If freq is to be changed, then:-

a) If  $f < f_r$  to keep  $\phi_m = \text{constant} \rightarrow \frac{V}{f'} = \frac{V_r}{f}$

$$\phi_m = \frac{V_r}{4.44 N f_r} = \text{const} = \frac{V}{4.44 N f'}$$

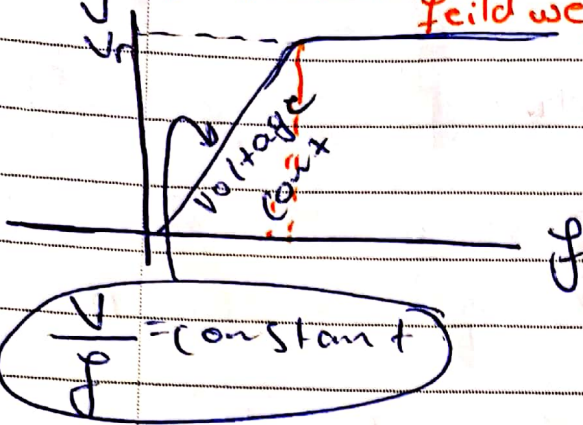
$\frac{V_r}{f_r} = \frac{V}{f'}$  if  $\phi_m$  is to be constant and

to avoid any saturation problems

(VVVF control) smile for life



b)  $I_f \uparrow \Rightarrow \phi \rightarrow \phi_r$   
 field weakening



**Ideal Tr**

$$\left. \begin{aligned} \frac{E_1}{E_2} &= \frac{V_1}{V_2} = a \\ \frac{I_2}{I_1} &= a \end{aligned} \right\}$$

①, ②, ③, ④ for an Ideal Tr

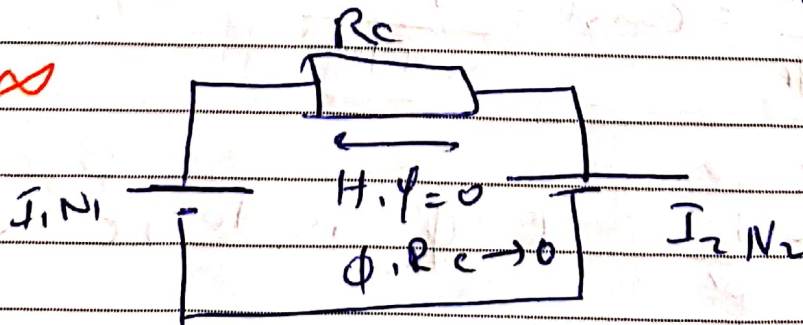
① Coil resistances = 0

$R_1, R_2 \rightarrow 0$  (No voltage drop, No copper power loss).

② zero-leakage flux  $\rightarrow \phi_f \rightarrow 0 \rightarrow$  tight coupling,

leakage reactances = 0  $\rightarrow X_{e1}, X_{e2} \rightarrow 0$

③  $M \rightarrow \infty$



$R_c = \frac{\phi}{M A} \rightarrow 0$

then  $H = \frac{B}{\mu} \Rightarrow 0$

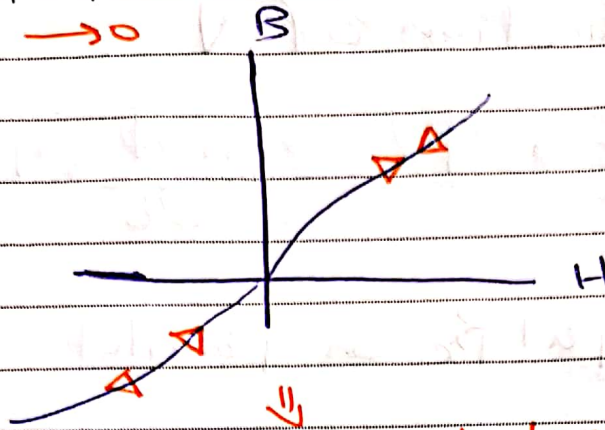
then  $I_1 N_1 = I_2 N_2$



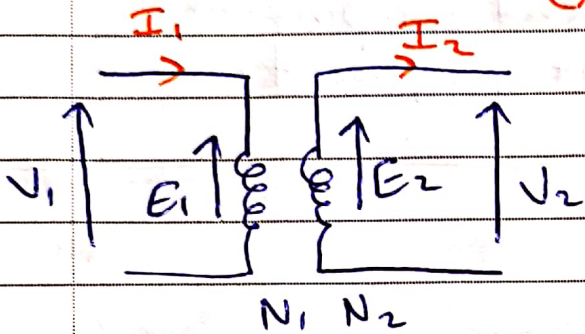
[4] Core loss  $\rightarrow 0$

$P_h = 0$

$P_e = 0$

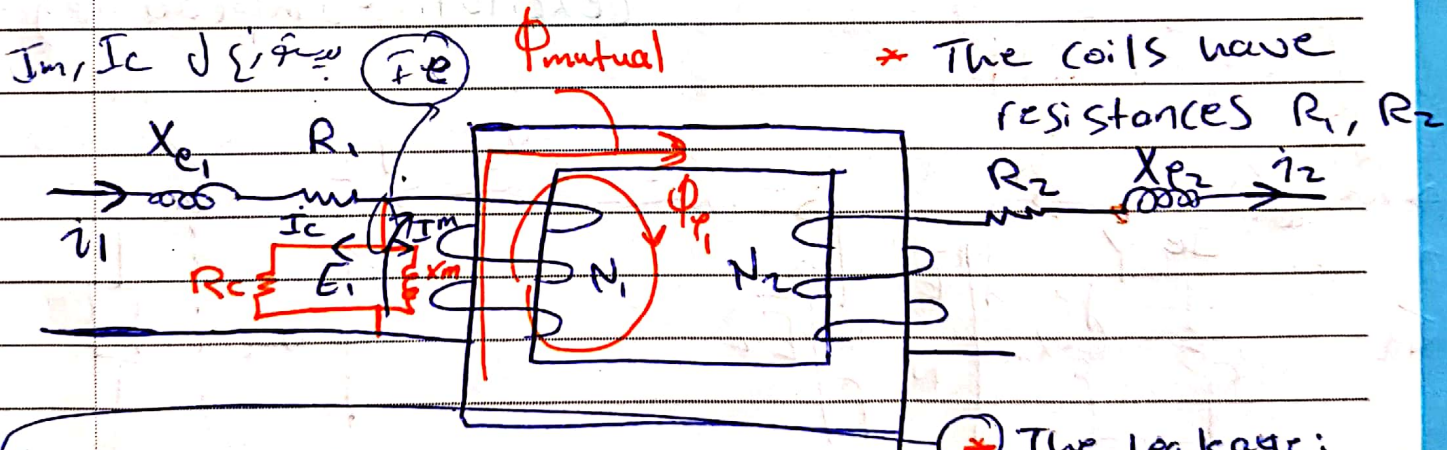


excellent lamination.



$\frac{E_1}{E_2} = \frac{V_1}{V_2} = a$

\*\* Practical Transformer



\* The coils have resistances  $R_1, R_2$

$E_1$  here less than in ideal because there is leakage in flux.

\* The leakage:  $\Phi_e$  leads to  $X_l$

\* Core loss: can be represented by a core resistance.

\*  $X_m$   $\rightarrow$  Mutual reactance (Parallel with  $R_c$ )

by which  $E_1$  and  $E_2$  are produced.



smile for life



**Core loss**  $\rightarrow$  function of  $V$

$P_w \dots B^2 \rightarrow \Phi \rightarrow \frac{N d\Phi}{dt} = V$   
 $P_e \dots B^2$

then we put  $R_c$  on parallel

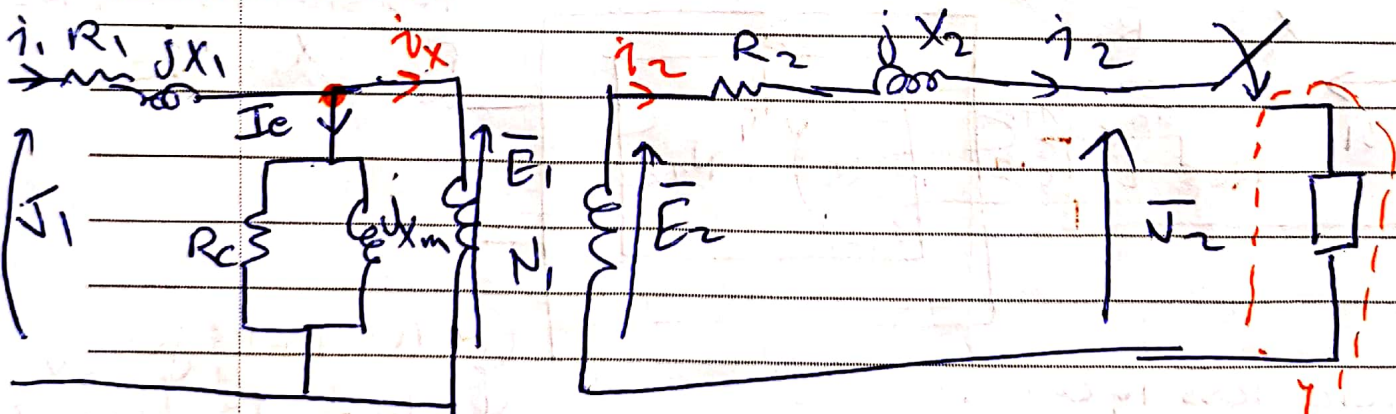
- \*  $I_m$ : Magnetization current component.
- $I_c$ : core-loss current component.

$I_e = I_c + I_m$

excitation current.

$Z_0 = (R_c // jX_m) \rightarrow$  core equivalent Impedance.  
(excitation Impedance).

**eq. circuits**



$Z_1 = R_1 + jX_1$

transformer at load.

$i_{load} = i_2$



$i_x = ?$   
 $i_x = i_2$

$i_x N_1 = i_2 N_2$

$i_x = \frac{N_2}{N_1} i_2 = \frac{i_2}{a} \rightarrow$  (equivalent load current)

$\rightarrow$  current of the supply to compensate for  $i_2$  in the load.

$\therefore$  Supply current  $i_1$  has two components  $i_1 = i_e + i_x$

$= i_e + \frac{i_2}{a}$   
 compensate for losses and magnetization  
 load component

\* At No-load or at open circuit

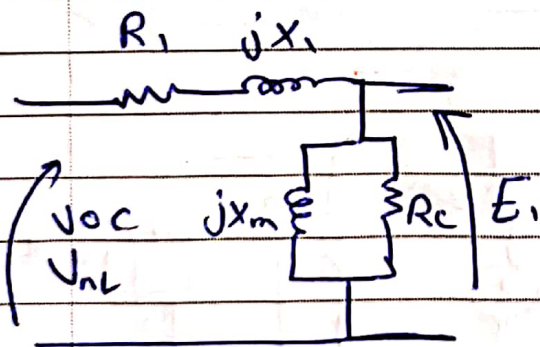
$i_2 = 0, i_x = 0$

$i_1 = i_e$

$i_1$  at No-load =  $i_e =$

$i_{oc} = i_{(no\ load)} = i_{AL}$

Eq. circuit of Tr at No load.

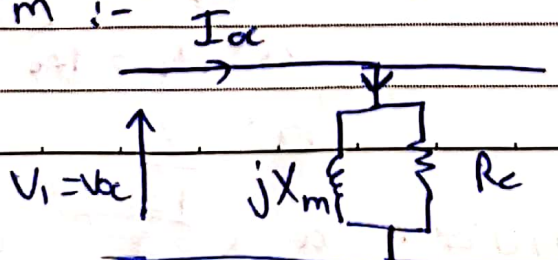


$R_1 \lll R_c$

$X_1 \lll X_m$

$Z_0 = (R_c \parallel jX_m) \rightarrow Z_0 = jX_1 + R_1$

\*\* At no-load the eq. circuit is reduced to the form :-

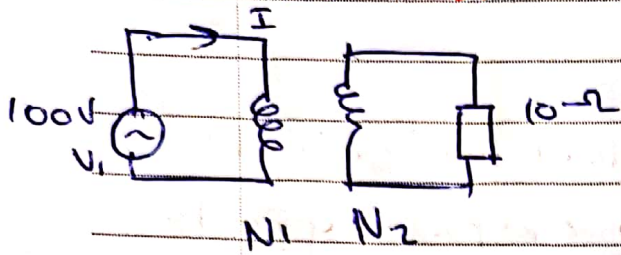


$Z_{oc} = R_c \parallel jX_m$

(open ckt impedance or no-load impedance)



\* In ideal transformer in circuits (2)



$$I_2 N_2 = I_1 N_1$$

$$I_1 = I_2 \frac{N_2}{N_1} = \frac{I_2}{a} \rightarrow I_1 = I_2' \rightarrow \text{the (load current referred to the supply or primary)}$$

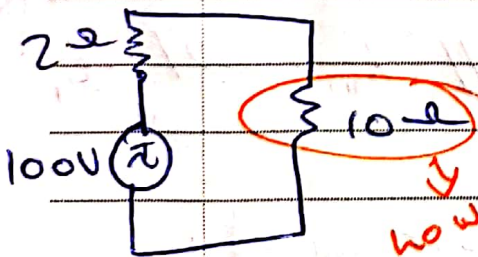
$$Z_L' = \frac{V_1}{I_1} \rightarrow \text{as seen by the supply.}$$

$$Z_L' = \frac{V_2 \times a}{\frac{I_2}{a}} = \frac{V_2^2}{I_2} \times a^2 = Z_L \times a^2$$

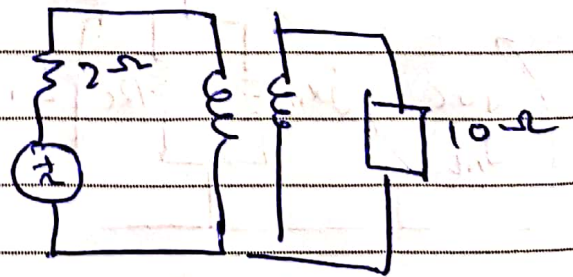
$$V_2' = a V_2 \rightarrow \text{(load voltage as seen by the primary or the supply)}$$

\* Supply 100V,  $R_1 = 2\Omega$

$R_{load} = 10\Omega$



how to make power be maximum.



$$R_1' = 2\Omega$$

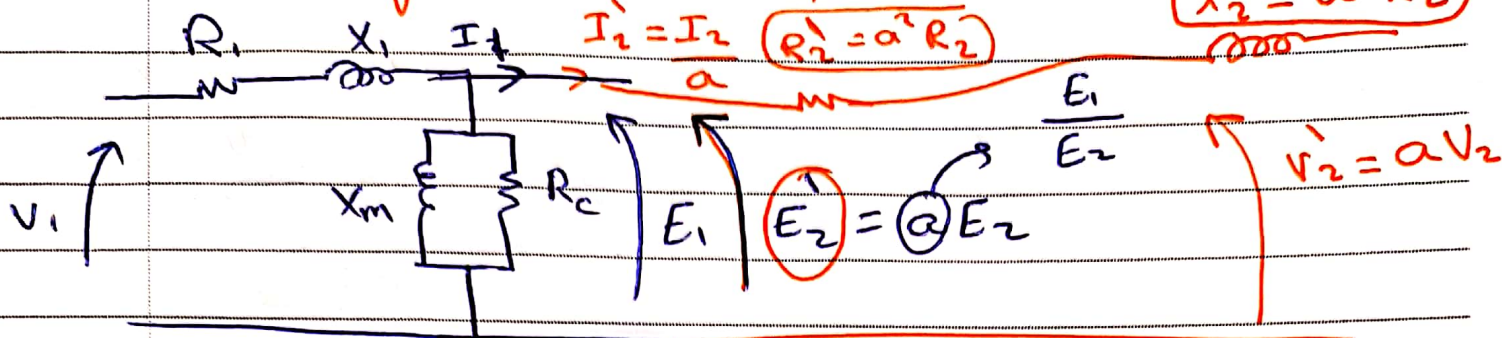
$$a^2 R_L = R_1'$$

$$a^2 = 2$$

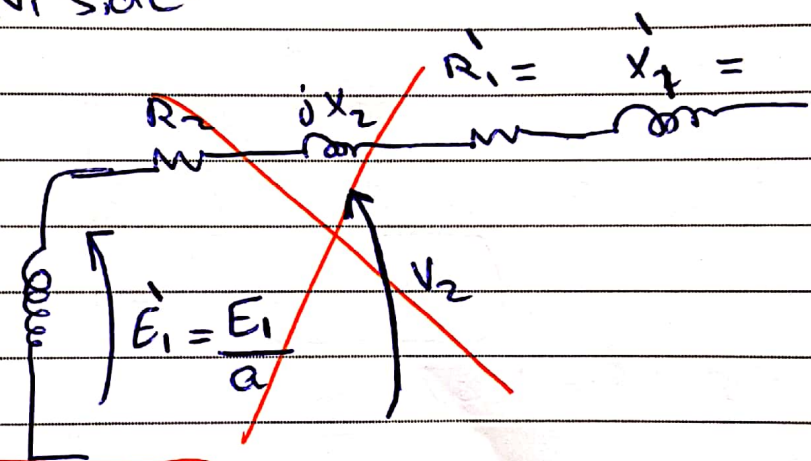
$$a = \sqrt{\frac{10}{2}} = 0.45 = \frac{N_1}{N_2}$$

(maximum power) لى 100V لى جالە بوسا \*

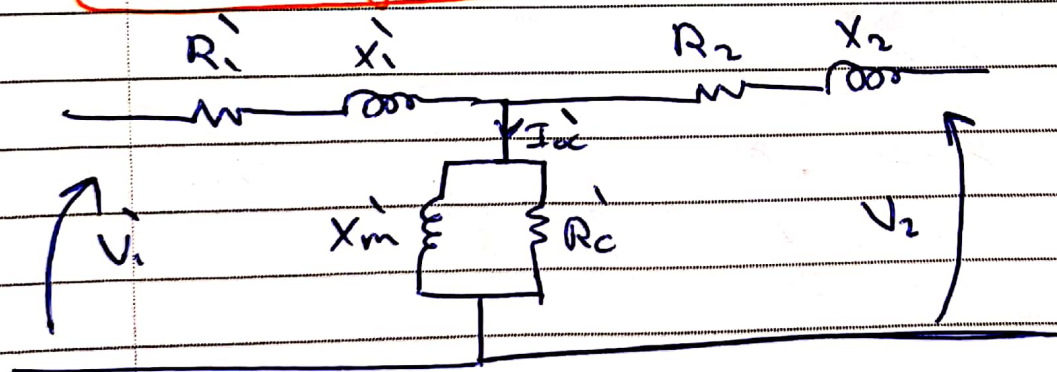
\* Referred equiv circuit to the  $N_1$  side.



the Exact or T equivalent circuit referred to  $N_1$  side.



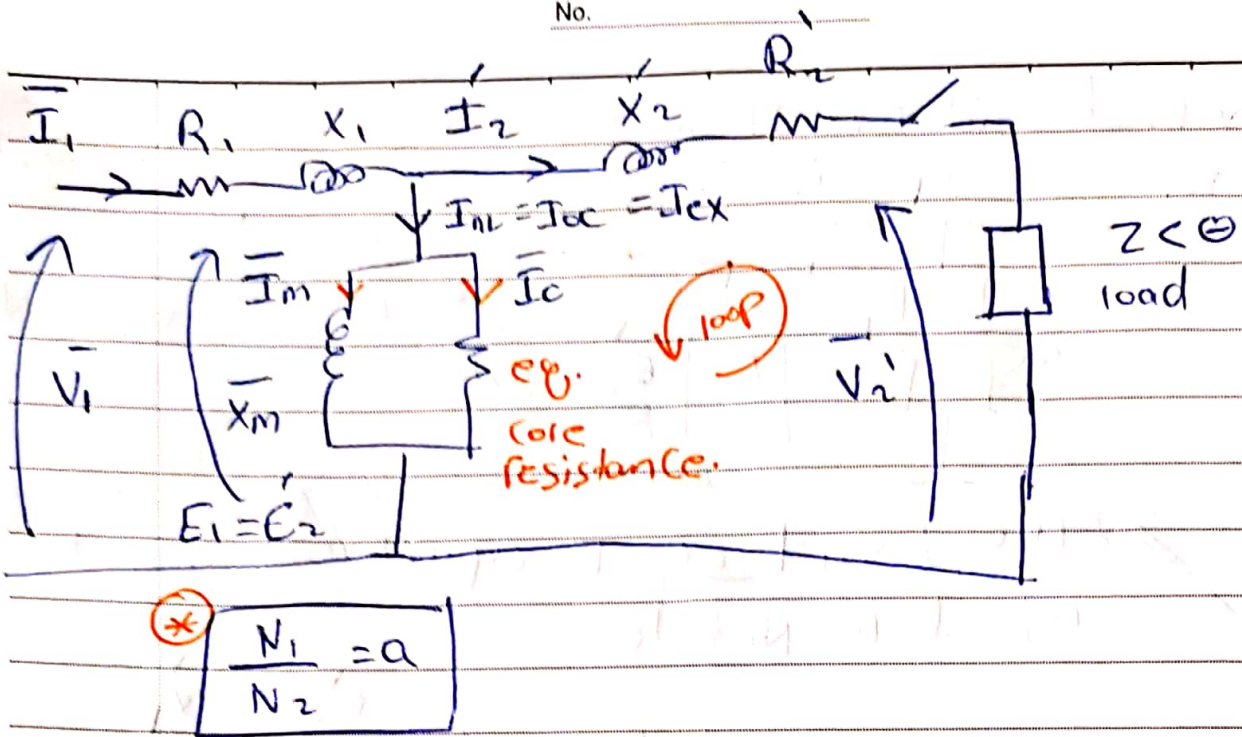
\* referring to  $N_2$  side :- (T equiv circuit)



$$V_1' = \frac{V_1}{a}, \quad I_1' = aI_1, \quad R_1' = \frac{R_1}{a^2}, \quad X_1' = \frac{X_1}{a^2}$$

$$X_m' = \frac{X_m}{a^2}$$



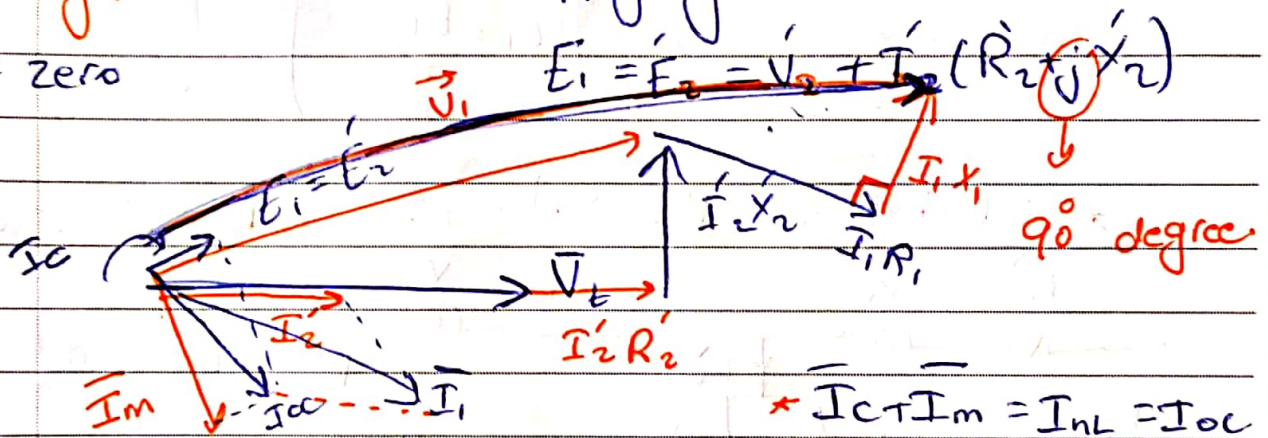


\* phasor 3-cases:-

① unity PF

$\phi = \text{zero}$

Applying KVL on the loop:-



\*  $\vec{I}_c + \vec{I}_m = I_{NL} = I_{oc}$

\*  $\vec{I}_c = \frac{E_1}{R_c} = \frac{E_2}{R_c}$

\*  $\vec{I}_m = \frac{E_1}{jX_m} = \frac{E_2}{jX_m} = -j \left( \frac{E_2}{X_m} \right)$

\*  $\vec{I}_2 + \vec{I}_{oc} = \vec{I}_1$

\*  $\vec{V}_1 = \vec{E}_1 + \vec{I}_1 (R_1 + jX_1)$

\* Voltage Reg =  $\frac{V_1 - \vec{V}_2}{\vec{V}_2} (\times 100\%)$   
or per unit

usually  $\vec{V}_2 = \vec{V}_2(\text{r})$

$-6\% \leq V.R_{eg} \leq +6\%$

$V_1 \text{ Reg} = \frac{\text{voltage at no load} - \text{voltage at full load}}{\text{voltage at full load}}$

(Inductive)

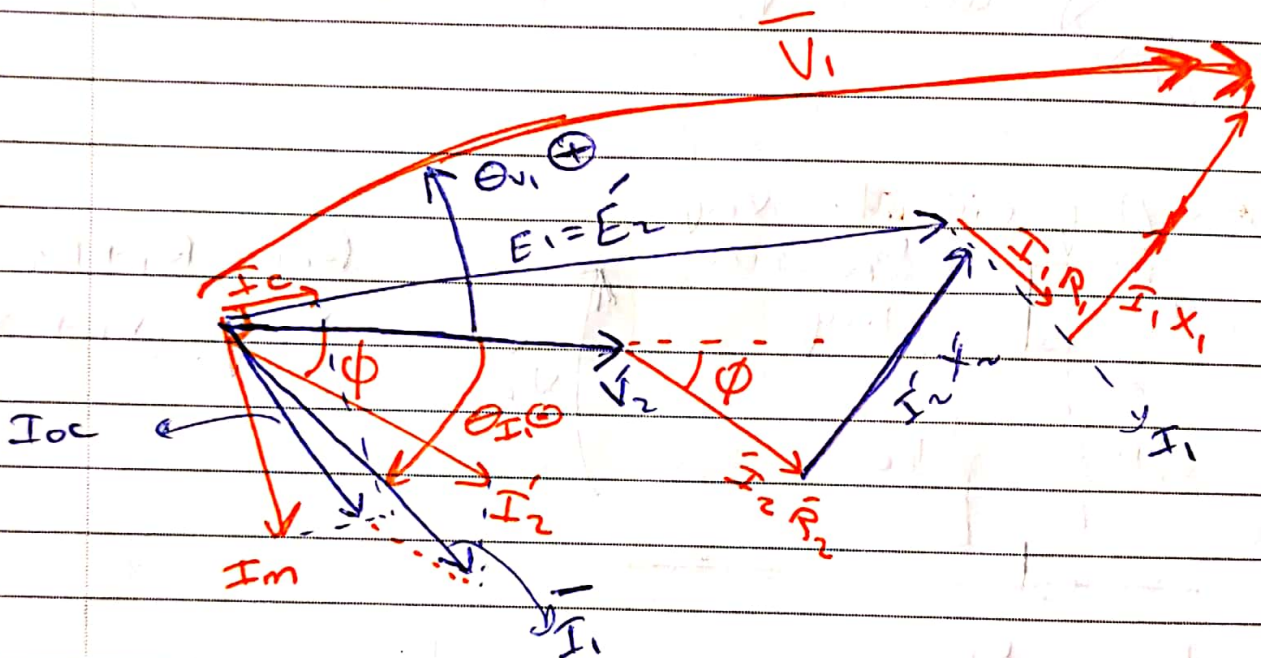
No.

② lagging PF  $\angle \phi \rightarrow \ominus \rightarrow I \angle -\phi$   $Z \angle +\phi$   
 $I = \frac{V}{Z} \angle -\phi$

$V \angle \phi \rightarrow$  angle is positive.

$I = \frac{V}{Z} \angle -\phi$

$S = \bar{V} I^* = VI \angle +\phi$



$PF = \cos(\theta_{v_1} - \theta_{i_1})$

ex:  $V_1 = 200 \angle +15$   
 $I_1 = 10 \angle -30 \rightarrow PF = \cos(15 - (-30)) = \cos(45)$

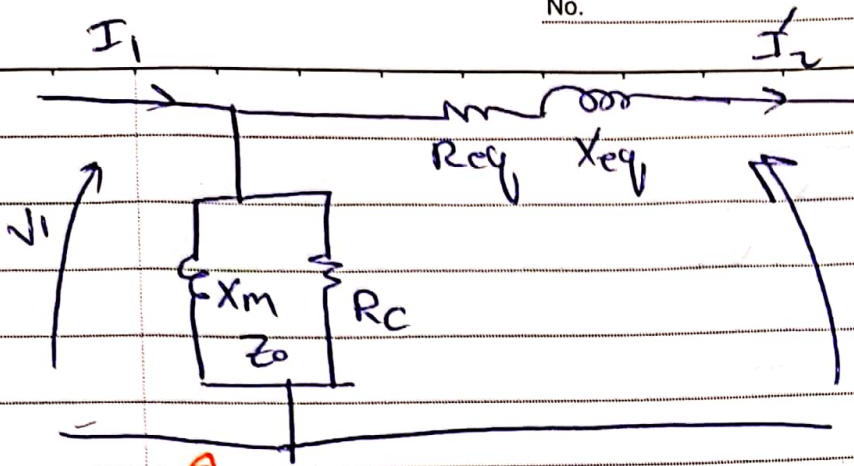
\* Efficiency =  $\frac{P_o}{P_{in}} = \frac{V_2 \times I_2 \times \cos\phi}{V_1 \times I_1 \times \cos(\theta_{v_1} - \theta_{i_1})}$







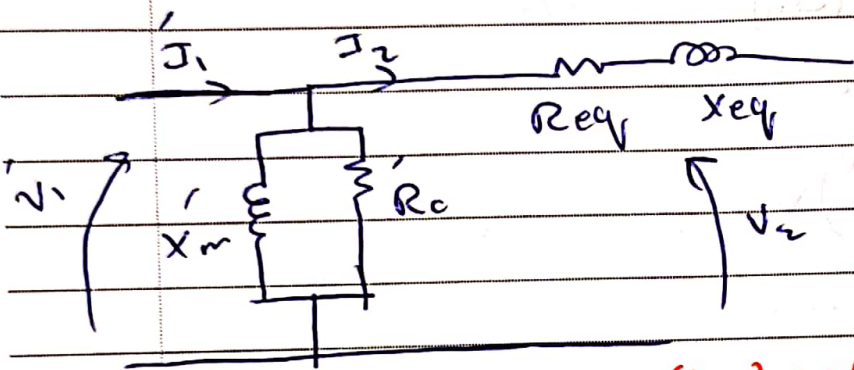
No.



$$* R_{eq} = R_1 + R_2'$$

$$X_{eq} = X_1 + X_2'$$

↙ referred to  $V_1$  side ( $N_1$ )



\* Note

$$\square X = \frac{1}{B}$$

$$B = \frac{1}{X} \rightarrow \text{susceptance}$$

↙ refer to ( $N_2$ ) side.

$$* R_{eq} = R_1 + R_2 \quad , \quad R_1 = \frac{R_1}{a^2}$$

$$X_{eq} = X_1 + X_2 \quad , \quad X_1 = \frac{X_1}{a^2}$$

$$V_1 = \frac{V_1}{a}$$

$$\square Y = \frac{1}{Z}$$

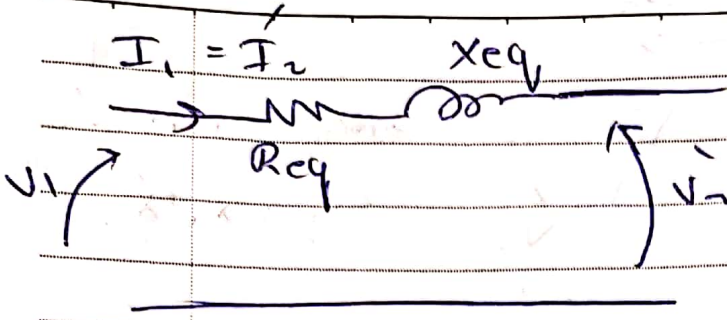
↳ Admittance

\*\* Approximate linear circuit

$$I_{oc} \ll I_{rated}$$

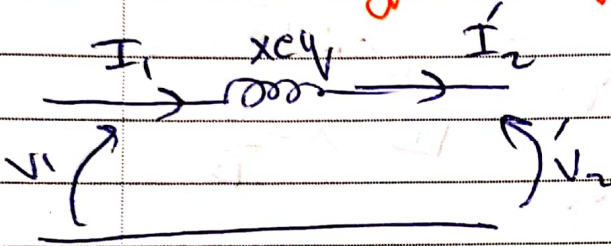
$$I_1 = \cancel{I_{oc}} + I_2' \quad , \quad I_{oc} \approx (1-s)I_1 \text{ of } I_{rated} \text{ approaches to zero}$$





Since  $R_{eq} < X_{eq}$

↓ neglect  $R_{eq}$



\* Rated values

$V_1(r)$ ,  $V_2(r)$ ,  $I_1(r)$ ,  $I_2(r)$

$V_r, I_r \leftarrow P_0, f$

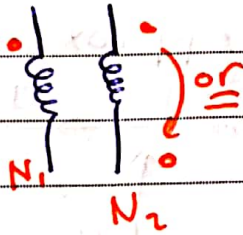
**\*\* Transformer testing :-**

**Aim:-** to determine the eq circuit parameters.

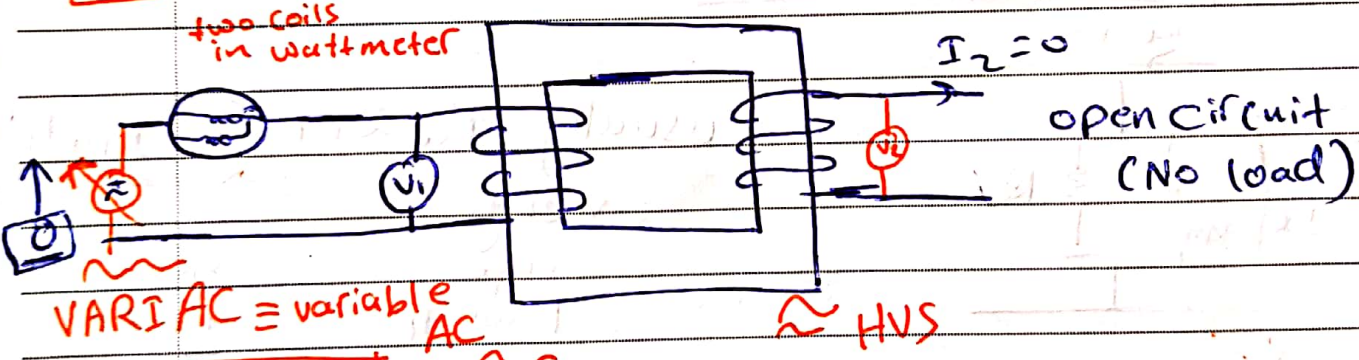
$T_r \rightarrow R_1, R_2, X_1, X_2, X_m, R_f$  --- I need to know them.

**\* Three types of test :-**

- open-circuit Test (No-load test)
- short-circuit test
- Polarity test  $\Rightarrow$



**No-load test**



**N1: supply**

$V_{oc}$	0	10%	100%	120%
$V_2$				
$I_{oc}$				
$P_{oc}$				

التيار بالسيارة \*

$V_{rated}$

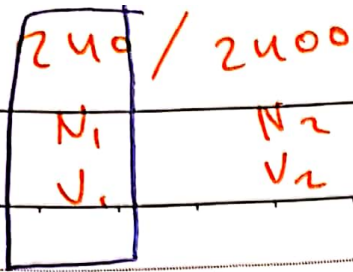
افضل تر اسلوب

**\*\* gradually increase  $V_1(oc)$  in steps of in each step Record  $P_{oc}, I_{oc}, V_2(oc)$  --- (approx)**

**\*\* preferred to be conducted in the LVS.** smile for life



No



$N_2$   
 $V_2$

No.

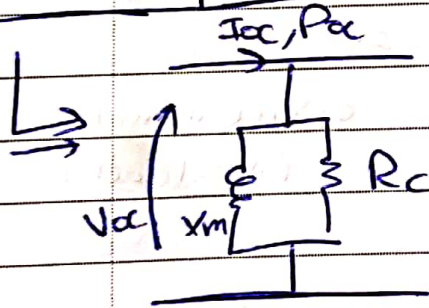
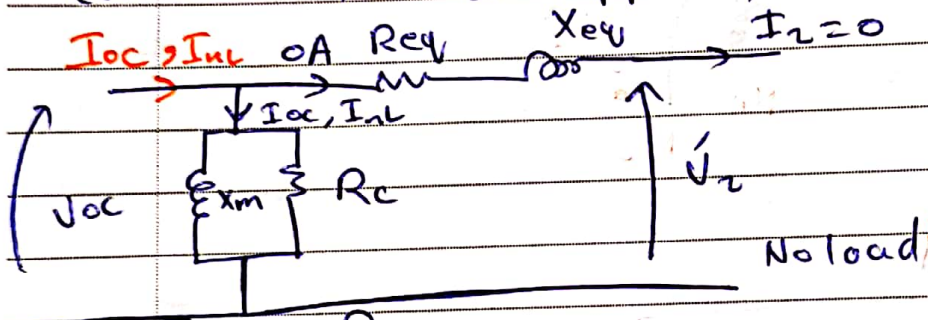
LVS



Prefered to make test on it.

Eq. circuit at No-load.

consider the L- Approximate circuit.



\* usually  $I_{oc} \approx (1-5\%) I_{rated}$   
 very large Power Tr.  $\rightarrow$  Small power Tr.

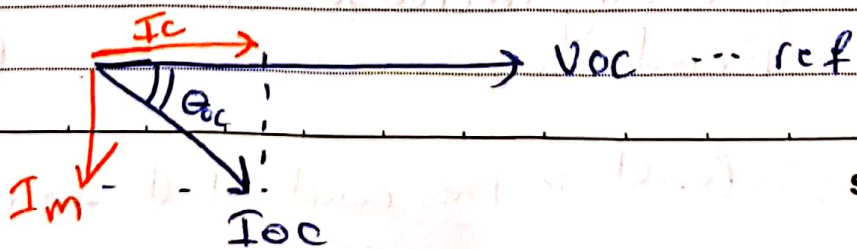
LVS  
Prefered.

$V_{oc} = V_{rated} \text{ (LVS)}$

$P_{oc} \Rightarrow$  at rated voltage.

$I_{oc} \rightarrow$  at  $V_1$  side.

\* phasor diagram.

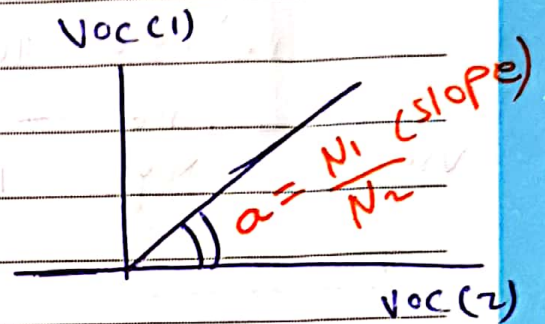
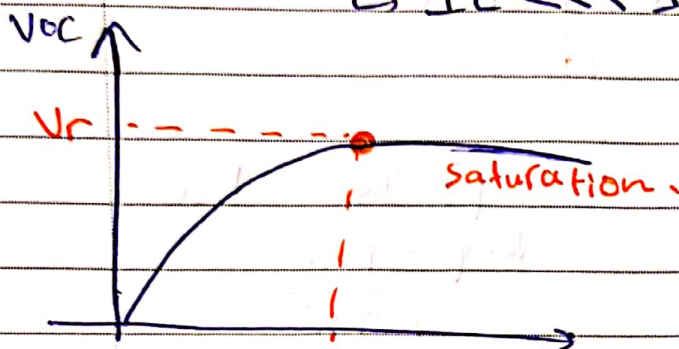


$$R_c = \frac{V_{oc}}{I_c} = \frac{V_{oc}}{I_{oc} \cos \phi_{oc}}, \quad X_m = \frac{V_{oc}}{I_m} = \frac{V_{oc}}{I_{oc} \sin \phi_{oc}}$$

$$\phi_{oc} = \cos^{-1} \left( \frac{P_{oc}}{V_{oc} I_{oc}} \right)$$

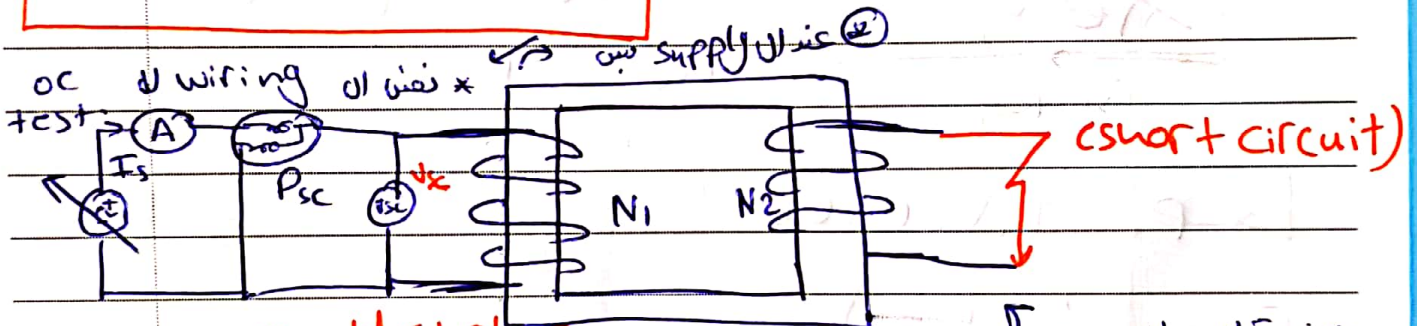
→ real  
→ apparent

usually  $X_m \ll R_c$   
 $\hookrightarrow I_c \ll I_m$



$I_{oc}$   
 $I_m$  } →  $\frac{N_1}{N_2}$

## 2 - short circuit test



\* we should start from zero. → use  $\frac{N_1}{N_2}$

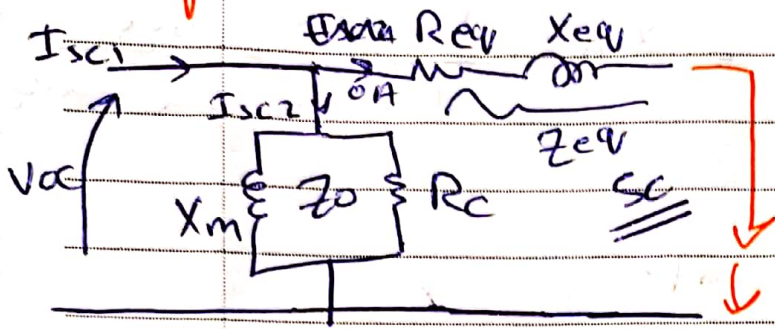
$I_{sc}$	0	10% $I_r$	20% ...	100% $I_r$
$V_{sc}$				(3-8)% $V_r$
$P_{sc}$				

\* carefully and gradually increase  $I_{sc}$  in steps up to  $I_{sc} = I_{rated}$

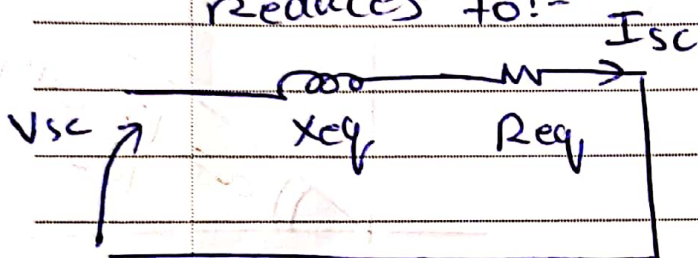


usually  $V_{sc}$  (to give  $I_{sc} = I_{rated}$ ) =  $(\beta - 8)\% V_{rated}$

Eq. Circuit at short circuit conditions.



→ eq. circuit at short ckt conditions reduces to:-

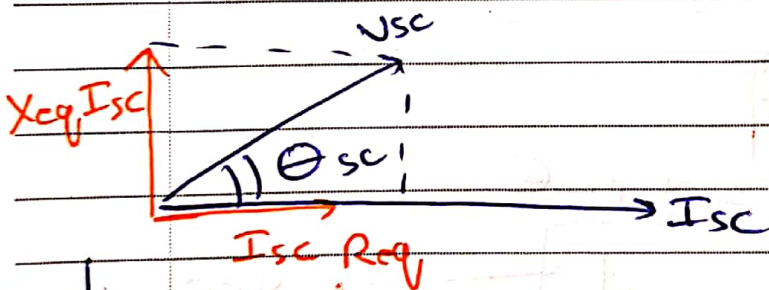


$$\therefore R_{eq} \times I_{sc}^2 = P_{sc}$$

$$R_{eq} = \frac{P_{sc}}{I_{sc}^2}$$

$$Z_{eq} = \sqrt{R_{eq}^2 + X_{eq}^2} = \frac{V_{sc}}{I_{sc}}$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$



$$P_{sc} = I_{sc} V_{sc} \cos \theta_{sc}$$

$$\theta_{sc} = \cos^{-1} \left( \frac{P_{sc}}{I_{sc} V_{sc}} \right)$$

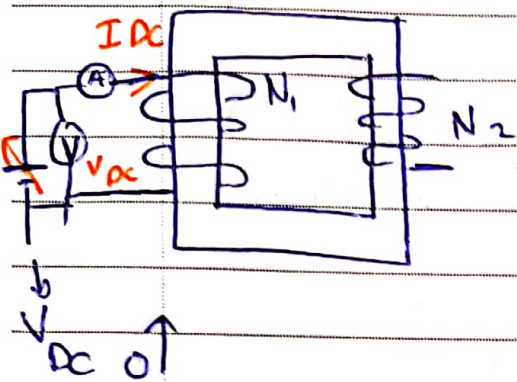
$$V_{sc} \cos \theta_{sc} = I_{sc} R_{eq}$$

$$R_{eq} = \frac{V_{sc}}{I_{sc}} \cos \theta_{sc}$$

$$V_{sc} \sin \theta_{sc} = I_{sc} X_{eq}$$

$$X_{eq} = \frac{V_{sc}}{I_{sc}} \sin \theta_{sc}$$

DC Test

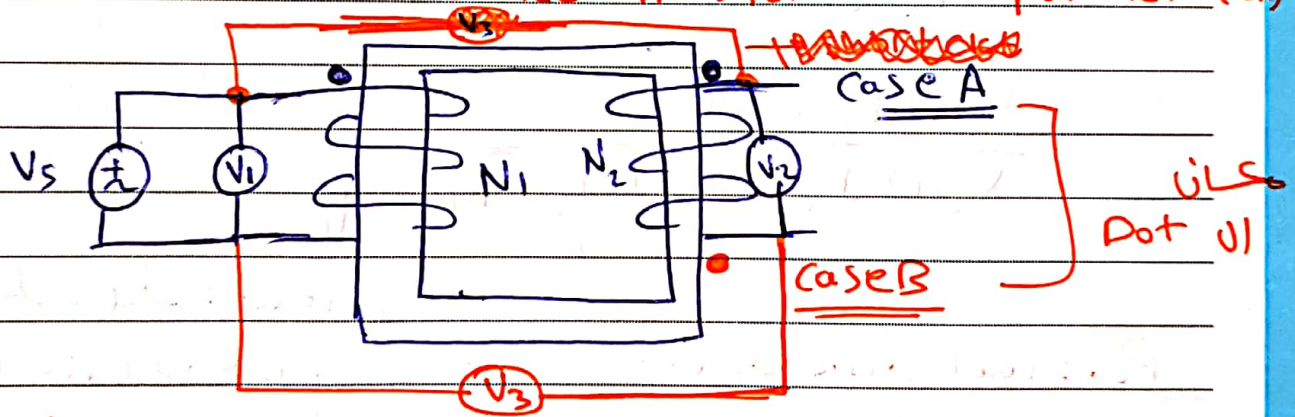


$I_{DC} < I_{rated}$

$R_1 = R_1' + R_2 + R_3'$

$V_{DC}$	$I_{DC}$	$R_1$
		$R_1'$
		$R_2$
		$R_3'$

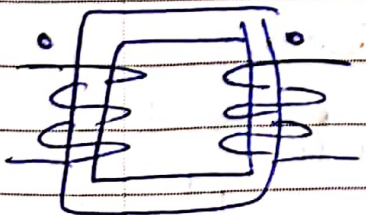
3- Polarity test :- (for dot notation) // required to connect transformers in parallel. (ex)



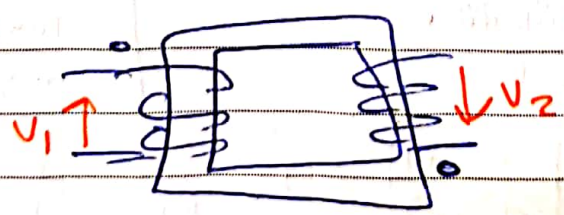
$V_3 = V_1 - V_2$     or     $V_3 = V_1 + V_2$

↳ case A  
Dot up

↳ case B  
Dot



(Subtractive Polarity)

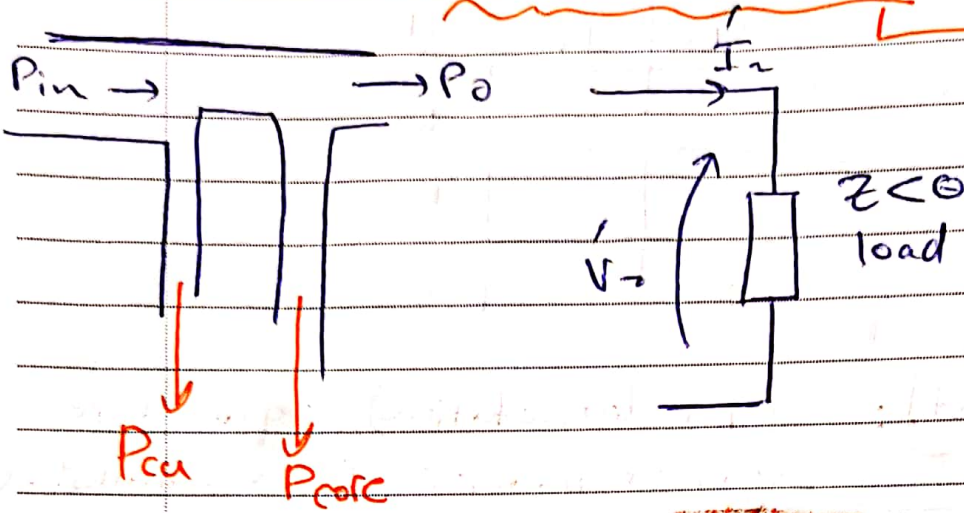


(additive Polarity)



**\*\* Performance evaluation / Efficiency :-**

$$\eta = \frac{P_o}{P_{in}} = \frac{V_2 I_2 \cos \theta_2}{\underbrace{V_2 I_2 \cos \theta_2 + P_{core} + P_{cu}}_{= V_1 I_1 \cos \theta_1}} = \frac{S_o P_{f_o}}{S_{in} P_{f_{in}}}$$

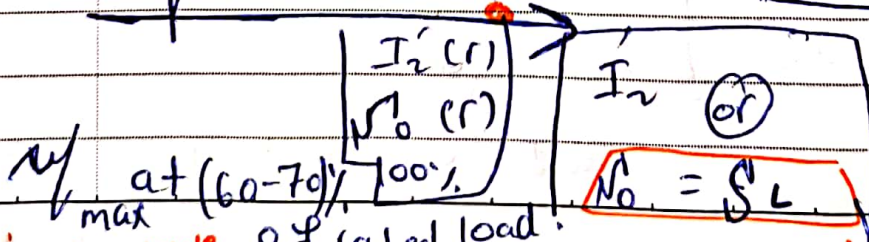
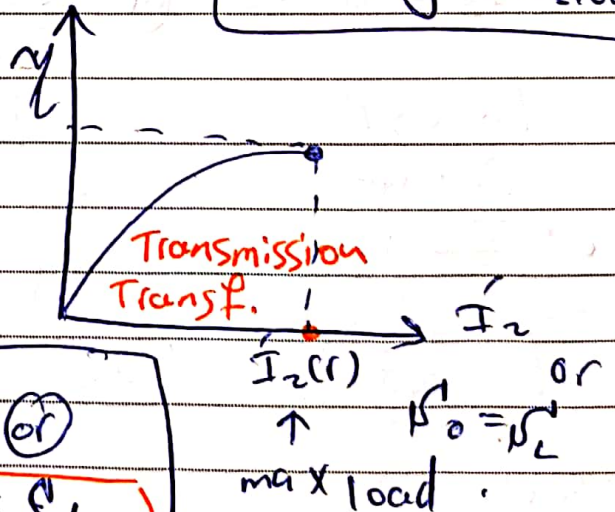
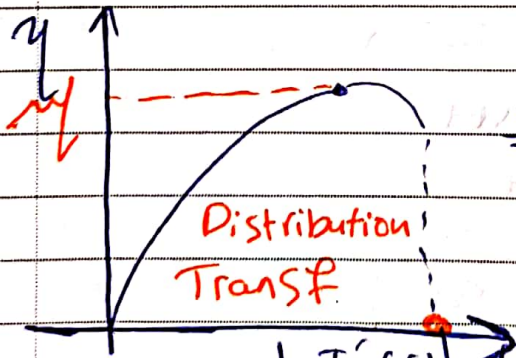


$$\eta = \frac{V_2(r) I_2 \cos \theta_2}{V_2(r) I_2 \cos \theta_2 + I_2^2 R_{eq} + P_{core}}$$

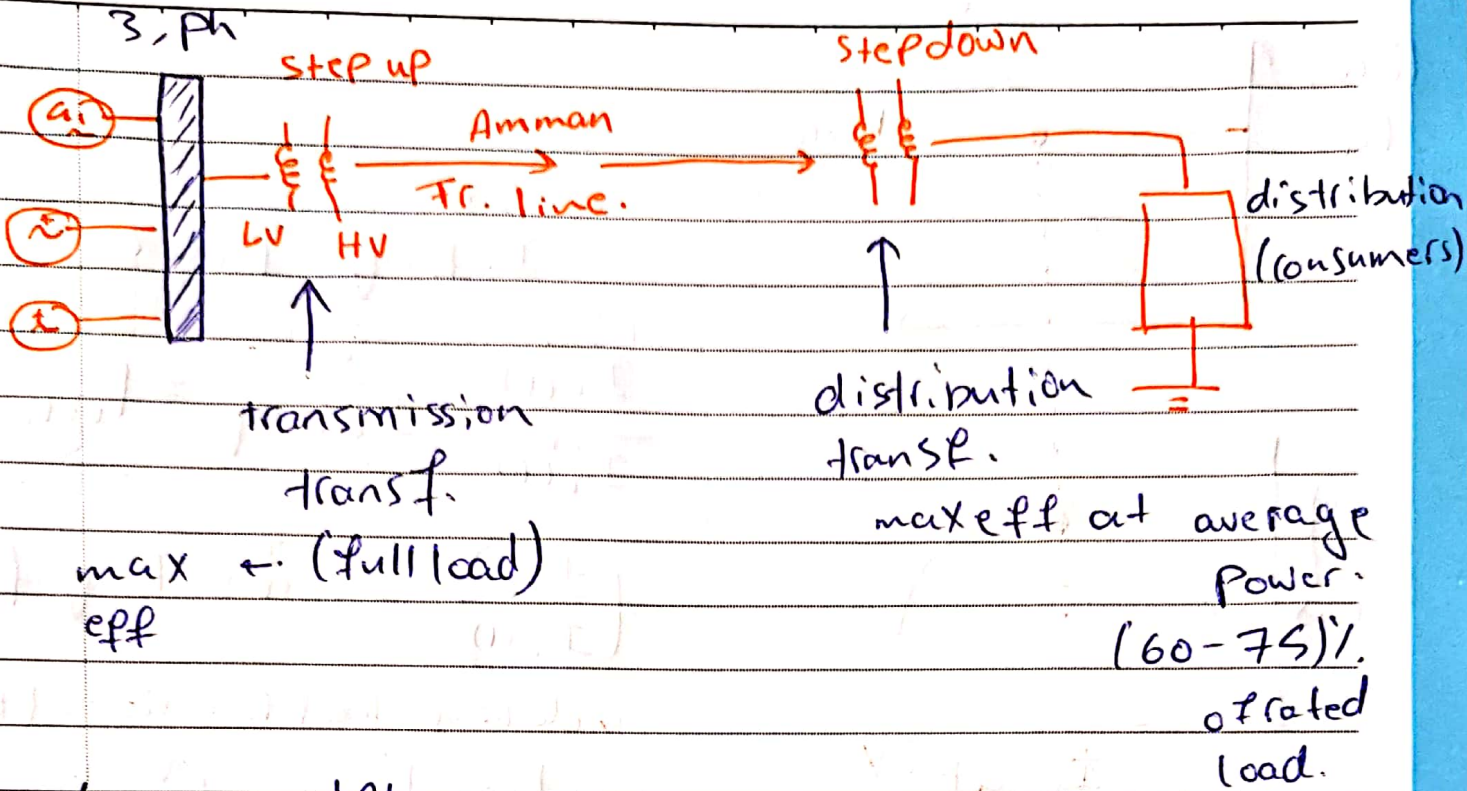
load independent.  
constant (fixed) losses

$P_{cu}$ : load dependent.  
Variable losses

$$P_n + P_{eddy} = P_{nload}$$



at (60-70%) max  
Amplitude of rated load.  
له عايوفتي سلكي اعني بس.



$$\eta_{\max} : \frac{d\eta}{dI_2} = 0$$

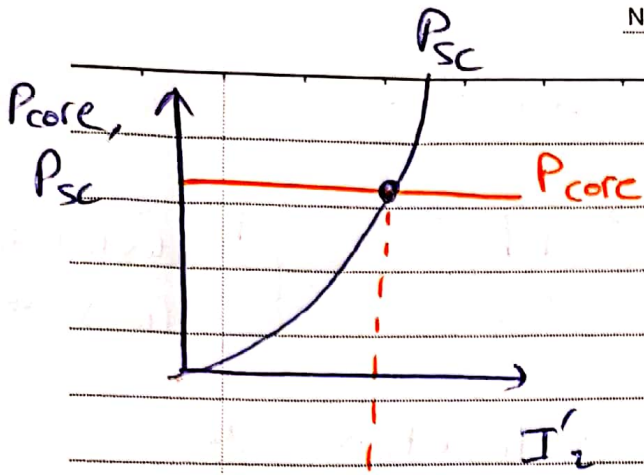
$$0 = V_2(r) \cos(\theta_2) (V_1(r) I_2 \cos \theta_2 + I_2 R_{eq}) + P_{core} - (V_2(r) I_2 \cos \theta_2 (V_2(r) \cos \theta_2 + 2 R_{eq} I_2 + P_{core}))$$

$$I_2 R_{eq} = P_{core}$$

$$P_{cu} = P_{core}$$

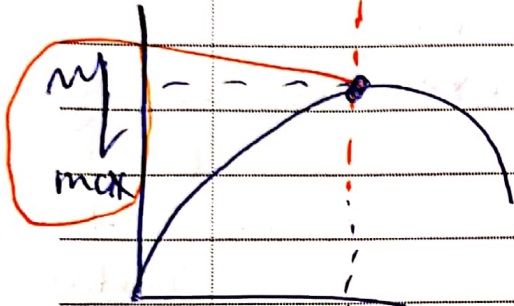
\* Max  $\eta$  occurs when the copper (variable) loss equals the constant core loss.





$$I_2 Req = P_{core}$$

$$\frac{I_2^2(r)}{I_2^2(r)} * I_2 Req = P_{core}$$



$$\left(\frac{I_2}{I_2(r)}\right)^2 * I_2(r) Req = P_{core}$$

\* define load function (k)

$$k = \frac{I_2}{I_2(r)} \text{ or } \frac{I_2}{I_r}$$

(at which the eff is max)

$$k_{\eta_{max}} = \left(\frac{I_2}{I_2(r)}\right) = \frac{I_r}{I_r}$$

$$\Rightarrow k_{\eta_{max}}^2 * P_{cu}(r) = P_{core}$$

$$k_{\eta_{max}} = \sqrt{\frac{P_{core}}{P_{cu}(r)}} = \sqrt{\frac{P_{oc}}{P_{sc}}}$$

$$\Rightarrow k_{\eta_{\max}} * P_{cu(r)} = P_{core}$$

$$k_{\eta_{\max}} = \sqrt{\frac{P_{core}}{P_{cu(r)}}} = \sqrt{\frac{P_{oc}}{P_{sc}}}$$

\*\*

$$P_{cu} = k^2 P_{cu(r)}$$

$$P_{core} = P_{oc} \rightarrow (V_{oc} = V_{rated})$$

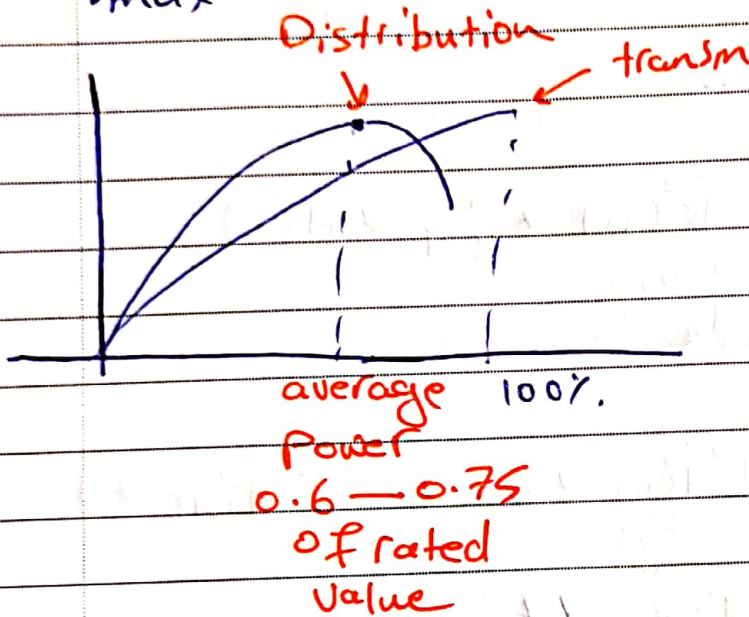
$$P_{cu(r)} = P_{sc} \rightarrow (I_{sc} = I_{rated})$$



$$\eta = \frac{V_2 I_2 \cos \phi}{V_2 I_2 \cos \phi + 2 P_{core}}$$

$$V_2 I_2 \cos \phi + 2 P_{core} \quad \text{or} \quad 2 P_{cu} \quad \left. \vphantom{V_2 I_2 \cos \phi + 2 P_{core}} \right\} \rightarrow \text{at max eff.}$$

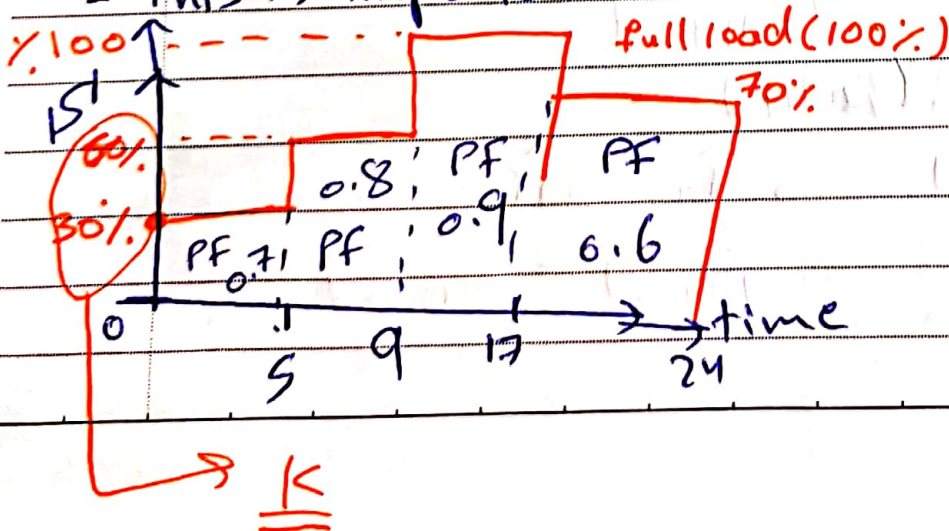
at  $\eta_{max} \rightarrow P_{cu} = P_{core} = k^2 P_{cu}$



**\*\* Energy (or All-day) Efficiency.**

$$\eta_{\text{energy}} = \eta_{\text{all day}} = \frac{W_o}{W_{in}}$$

- This is important in distribution trans.



No.  $S_o$

PF

$$|W_o| = \sum_{n=1,2,3,4} k(n) * S_r * \cos \phi(n) * H(n)$$

$$= 0.3 * S_r * 0.7 * 5 + 0.6 * S_r * 0.8 * 4 + 1 * S_r * 0.9 * 8 + 0.7 S_r * 0.6 * 7 \quad (\text{KW.H})$$

$$W_{in} = W_{out} + W_{Loss}$$

$$W_{Loss} = P_{core} * 24 + \sum_{n=1,2,3..} k^2(n) * P_{cur} * H(n)$$

$P_{core} = P_{oc}$

\*  $P_{core}$  is voltage dependent.

$$W_{Loss} = 24 P_{core} + (0.3)^2 P_{cur} * 5 + (0.6)^2 * P_{cur} * 4 + 1 * P_{cur} * 8 + (0.7)^2 * P_{cur} * 7$$

\*\* voltage regulation: (relative difference in voltage between [no-load] & [full-load] conditions).

$$V_o \text{ Reg \%} = \frac{V_{no\ load} - V_{full\ load}}{V_{full\ load}} * 100\%$$

standard

$$+7\% \leq V_{reg} \leq -7\% \quad (\text{quality})$$

Full-load:  $k=1 \rightarrow I_2 = I_2'$

$$S_{load} = S_{rated}$$







**\*\* per unit system.**

{	$Tr_1$	}	$Tr_2$
	500 kVA		375 kVA
	losses: 1000 W		losses: 900 W

$\eta = 0.95$

$\eta = 0.94$

و الأما الأما الأما  
eff. ال

↳ this Tr is better → eff is higher → then losses will be smaller.  
P.u.

- per unit system: - better for calculation, better to discriminate between machine in one field.

\* Select two base values <sup>out</sup> of:  $V, I, S$

usually:  $S_r = S_B = \text{base power.}$

$V_r = V_B = \text{base voltage.}$

Calculate  $I_B = \frac{S_B}{V_B} = \frac{S_r}{V_r} = I_r$

$Z_B = \frac{V_B}{I_B}$  or  $\frac{V_B^2}{S_B}$

$S (pu) = \frac{S}{S_B}$

$V (pu) = \frac{V}{V_B}$

$I (pu) = \frac{I}{I_B}$

\* No base values for angles.

$Z (pu) = \frac{Z}{Z_B}$  or  $\frac{R}{Z_B}$  or  $\frac{X}{Z_B}$

smile



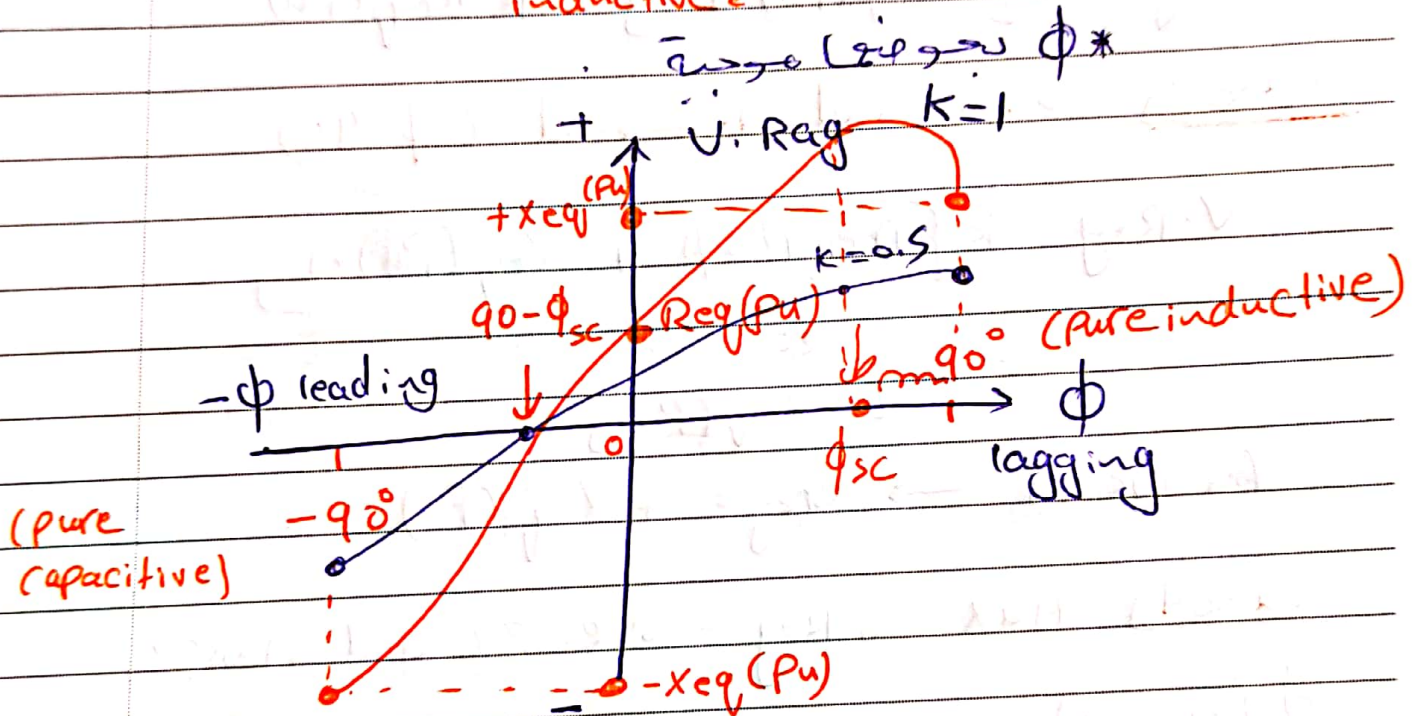
$$\Rightarrow V_{Reg} = k \left[ \frac{R_{eq}}{Z_B} \cos \phi + \frac{X_{eq}}{Z_B} \sin \phi \right]$$

$$V_{Reg} = k \left[ R_{eq}(Pu) \cos \phi + X_{eq}(Pu) \sin \phi \right]$$

\* To generalize: (for lag or lead PF) :-

$$V_{Reg} = k \left[ R_{eq}(Pu) \cos \phi \mp X_{eq}(Pu) \sin \phi \right]$$

inductive
capacitive



Cases

\* Zero Voltage Reg = (??) at what phi??

$$0 = R_{eq}(Pu) \mp X_{eq}(Pu) \sin \phi$$

∴ only with leading PF,  $V_{Reg} = 0$

$$R_{eq}(Pu) \cos \phi = X_{eq}(Pu) \sin \phi$$

$$\frac{\cos \phi}{\sin \phi} = \frac{X_{eq}(Pu)}{R_{eq}(Pu)}$$

at SCC

$$\phi_{sc} = \tan^{-1} \left( \frac{X_{eq}}{R_{eq}} \right)$$

$$\Rightarrow \cot \phi = \tan \phi_{sc}$$

$$\phi = 90 - \phi_{sc}$$

Case 2)

at  $\phi = 0 \rightarrow$  pure resistive load  
 $V_{Reg} = ?$

$$V_{Reg} = k [R_{eq} (P_u) \mp 0] = k R_{eq} (P_u)$$

Case 3)  $\rightarrow$  pure inductive load ( $\phi = 90$ )

$$V_{Reg} = k [ \underbrace{R_{eq} \cos \phi}_{\text{zero}} \mp X_{eq} (P_u) * 1 ]$$

$$= k [ \mp X_{eq} (P_u) ]$$

$$\text{for } k=1 \rightarrow V_{reg} = \mp X_{eq} (P_u)$$

Case 4) Max.  $V_{Reg}$  = when ?? and How??

$$\frac{d(V_{Reg})}{d\phi} = 0 \quad (\text{Self evaluation})$$

$$\frac{d(k [R_{eq} \cos \phi \mp X_{eq} \sin \phi])}{d\phi} = k [ -R_{eq} \sin \phi \mp X_{eq} \cos \phi ]$$

$$0 = k [$$

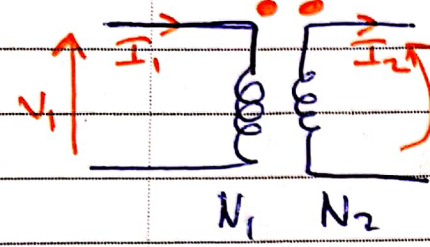
$$\tan \phi = \frac{X_{eq}(P_u)}{R_{eq}(P_u)} = \frac{X_{eq}(P_u) \sin \phi}{R_{eq}(P_u) \cos \phi} = \tan \phi_{sc}$$



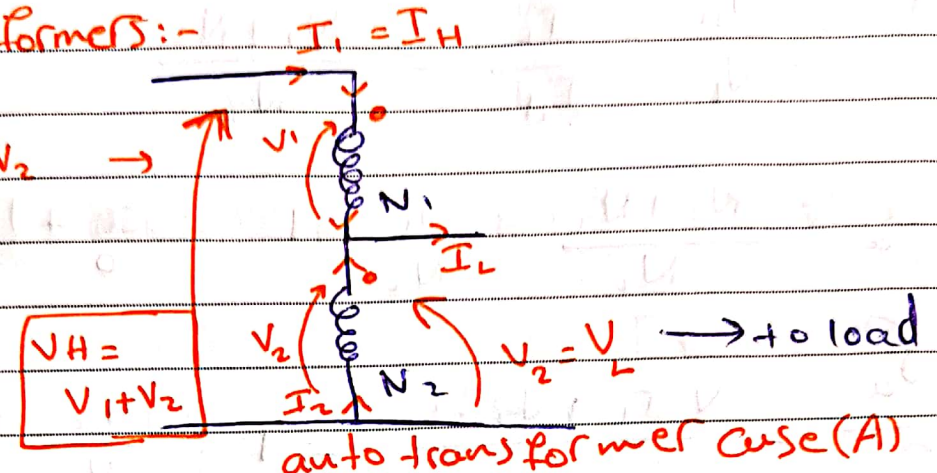
result) Max V. Reg  $\rightarrow \tan \phi = \tan \phi_{sc}$

$\phi = \phi_{sc} \rightarrow V_{reg} (max) = R_{eq}(Pu) \cos \phi_{sc} + x_{eq}(Pu) \sin \phi_{sc}$

Auto-Transformers:-



ordinary Tr



$V_H = V_1 + V_2$

auto transformer case (A)

$V_H = V_1 + V_2$   
 $I_H = I_1$   
 $V_L = V_2$   
 $I_L = I_1 + I_2$

Case (A) ordinary and auto Tr are physically identical (No extra copper or core material).

$P_o = V_1 I_1 = V_2 I_2$

$S_a = V_H I_H = V_L I_L$

$\Rightarrow S_a = V_H I_H = (V_1 + V_2) * I_1 = V_1 I_1 + V_2 I_1$

or

$S_a = V_L I_L = V_2 (I_1 + I_2)$

$S_a = V_2 I_2 + V_2 I_1$

$V_2 I_1 \rightarrow$  extra Power gain  
 $\downarrow$   
 Power by conduction

smile for life



\*  $V_2 I_2$  or  $V_1 I_1 \rightarrow$  power by induction

$$a_0 = \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

$$a_a = \frac{V_H}{V_L} = \frac{I_L}{I_H} = \frac{N_1 + N_2}{N_2}$$

$$a_a = \frac{N_1 + N_2}{N_2} = \frac{N_1}{N_2} + 1 = a_0 + 1$$

$$\frac{\sum a}{\sum a_0} = \frac{V_1 I_1 + V_2 I_1}{V_1 I_1} = 1 + \frac{V_2}{V_1} + \frac{1}{a_0}$$

\* More power gain can be obtained if  $a_0 < 1$

\* lower  $a_0 \rightarrow$  More power gain

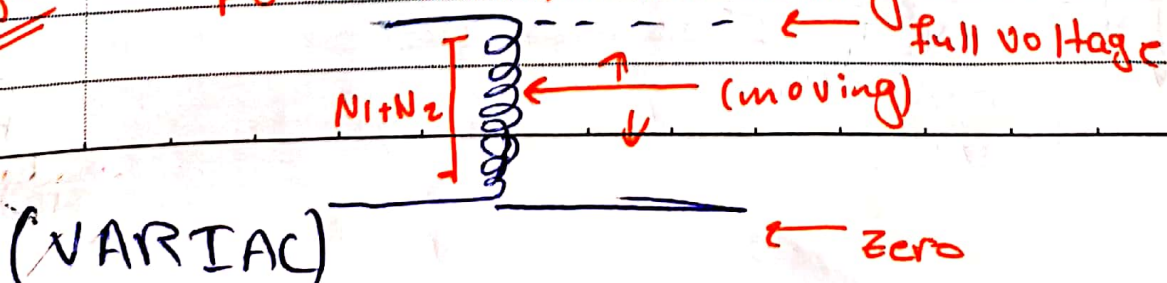
\*\* Due to safety precautions, auto-transformers use is limited

⊙ dangerous SC conditions, where  $N_1$  should receive  $(V_1 + V_2) = V_H$ .

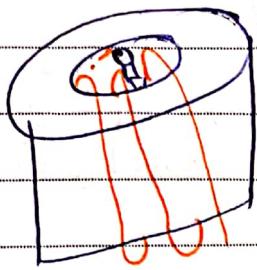
• operators safety.

\* usually used in the lab to gain extra power variable AC voltage.

$V_1 + V_2$   
 $V_2$   
 $N_1 + N_2$





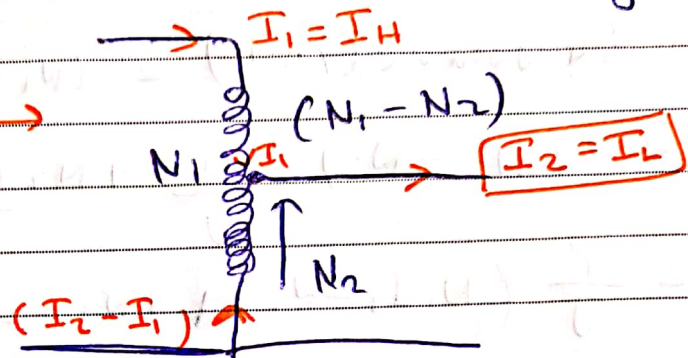
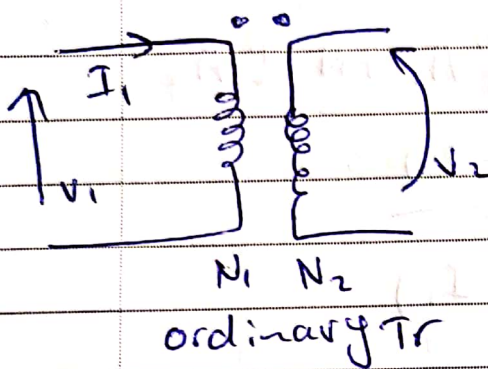


(VARIAC)

(Same electric features)

\* Case (B) → Same Power ratings ( $S_a = S_o$ ), (Same electric features)

Select the coil whose number of turns is higher



$N_2$  Physically doesn't exist.

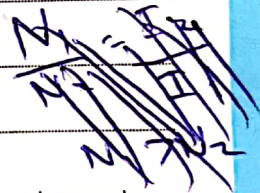
\* In case (B),

Case (B)

Copper required lower in auto-Tr than that of equivalent ordinary Tr.

① coil  $N_2$  is completely saved.

② cross-section of  $N_2$  part of coil  $N_1$  has lower cross section



since  $I(N_2) = I_2 - I_1$

smile for life

\* Copper Saving  $\Delta$  Cu volume??

$$\text{Cu Vol (o)} = N_1 \times \rho \times A_1 + N_2 \rho \times A_2$$

length of each turn  $\rho$  coil cross section Area.

$$A = \frac{I}{J} \quad , \quad J \equiv \text{current density}$$

$$\text{Cu Vol (o)} = N_1 \rho \frac{I_1}{J} + N_2 \rho \frac{I_2}{J}$$

$J_1 = J_2 = J$   
(constant)

$$\text{Cu Volume (o)} = \frac{\rho}{J} (N_1 I_1 + N_2 I_2) \quad \text{--- ①}$$

\*\*

$$\begin{aligned} \text{Cu Volume (Auto)} &= (N_1 - N_2) \rho A_1 + N_2 \rho A_2 \\ &= (N_1 - N_2) \rho \frac{I_1}{J} + N_2 \rho \frac{(I_2 - I_1)}{J} \\ &= \frac{\rho}{J} (N_1 I_1 - N_2 I_1 + N_2 I_2 - N_2 I_1) \end{aligned}$$

$$= \frac{\rho}{J} (N_1 I_1 + N_2 I_2 - 2 N_2 I_1) \quad \text{--- ②}$$

\* compare ① with ②

$$N_1 I_1 = N_2 I_2$$

$$\text{Cu Volume saving} = \frac{2\rho}{J} N_2 I_1$$

$$\% \text{ saving} = \frac{2\rho}{J} \times N_2 I_1$$

$$\frac{\frac{2\rho}{J} \times N_2 I_1}{\frac{\rho}{J} (N_1 I_1 + N_2 I_2)} = \frac{2 N_2 I_1}{N_1 I_1 + N_2 I_2}$$

$$\Rightarrow \% \text{ Cu Saving} = \frac{1}{2} \times 100\%$$

\*  $N_1 > N_2$   
\* More saving is possible when a. is small but  $> 1$  smile for life



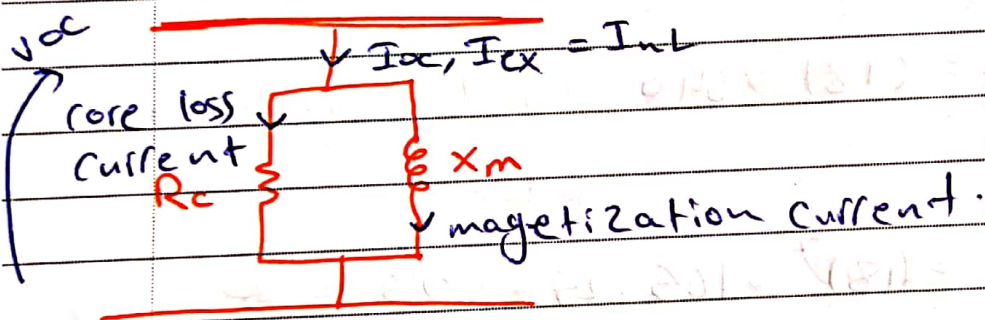
EX: No load test conducted on 20 KVA, 7200/400 V, 50 Hz Tr

$$\underline{V_{oc}} = 400 \text{ V}, \quad \underline{I_{oc}} = 2.5 \text{ A}, \quad \underline{P_{oc}} = 250 \text{ W}$$

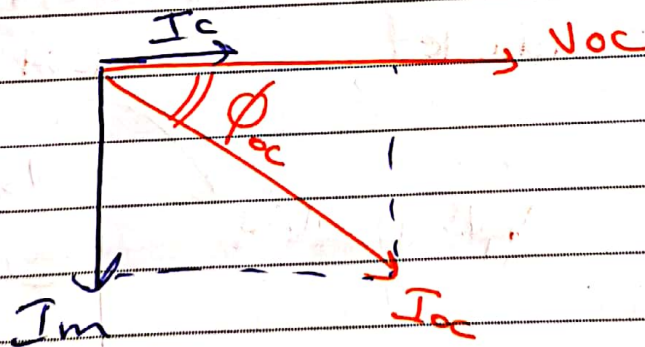
Neglect  $Z_{eq}$  and calculate:-

- core loss current.
- magnetizing current
- elements of the excitation branch referred to HVS and LVS.
- No load power factor.

SOL.



Req. circuit at no load.



$$d) \text{PF}_{in(nL)} = \cos(\phi_{oc})$$

$$P_{oc} = V_{oc} I_{oc} \cos \phi_{oc} \rightarrow \frac{P_{oc}}{V_{oc} I_{oc}} = 0.25 \text{ lag}$$

$$\text{then } \phi_{oc} = \cos^{-1}(0.25) = 75.5^\circ$$

smile for life

a)  $I_c = I_{oc} \cos \phi_{oc} = 0.25 \times 2.5 = 0.625 A$

b)  $I_m = I_{oc} \sin \phi_{oc} = 2.4206 A$

or

$I_m = \sqrt{I_{oc}^2 - I_c^2}$

c)  $R_c = \frac{V_{oc}}{I_c} = 640 \Omega$

$X_m = \frac{V_{oc}}{I_m} = 165.25 \Omega$

$R_c \gg X_m \checkmark$

referred + LV side.  
 \* رفته بزنه ال test  
 اصل على 400V  
 بجز عند ال low

$R_{c_{HVS}} = a^2 R_{c_{LVS}} = (18)^2 \times 640$

$X_{m_{HVS}} = a^2 X_{m_{LVS}} = (18)^2 \times 165.25 = 53 k \Omega$

\* In SC test its better to make the test in the LVS

EX: SC test conducted on a 20 kVA, 7200/400V, 50 Hz Tr.

$V_{sc} = 360V$

$I_{sc} = 2.777 A$

$P_{sc} = 400W$

بختار ال HVS

\* the test is connect to the HVS →

$I_{sc} = I_r$

$I_r \rightarrow LVS = \frac{20 \times 10^3}{400} = 50 A$

$V_{sc} \approx (3-8) \cdot V_r$

a)  $\frac{7200 \times 5}{100} = 360V$   
HVS

b)  $\frac{400 \times 5}{100} = 20$   
LVS

$I_{sc} = I_{rated}$

HVS →  $\frac{20 \times 10^3 \text{ (smile) for Wc}}{7200} = 2.7$





$$R_{eq, LVS} = \frac{R_{eq, HVS}}{18^2} = 0.16 \Omega$$

$$X_{eq, LVS} = \frac{X_{eq, HVS}}{18^2} = 0.36 \Omega$$

T- eq

In well-designed:

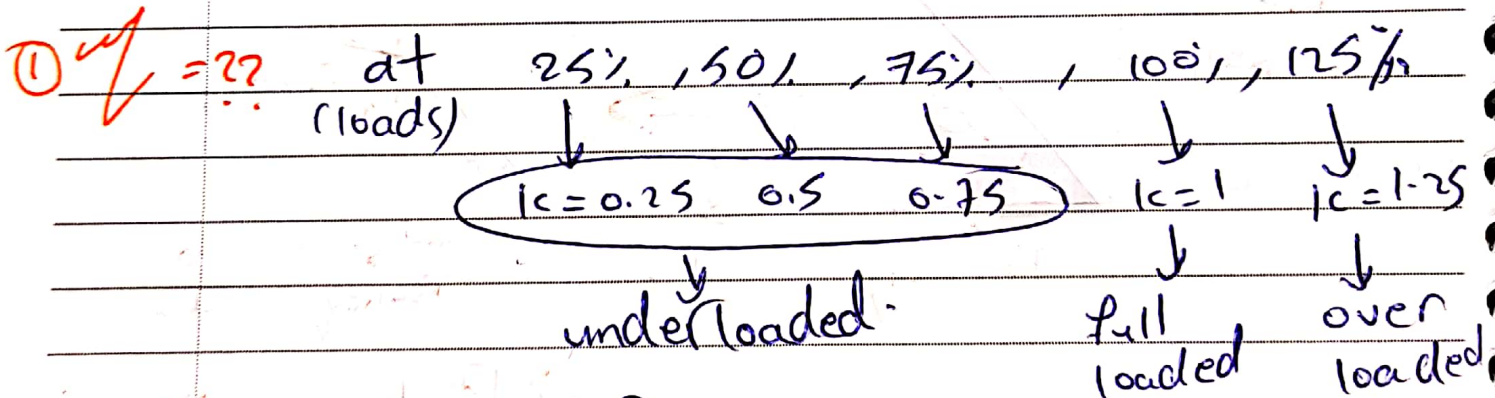
$$\left. \begin{matrix} R_1, R_2 \\ X_1, X_2 \end{matrix} \right\} \begin{matrix} R_1 = R_2 = \frac{1}{2} R_{eq} \\ X_1 = X_2 = \frac{1}{2} X_{eq} \end{matrix}$$

$$R_{eq} = R_1 + R_2$$

rated voltage

EX: transformer 300 kVA, has full voltage core loss 1.5 kW and a full load copper loss 4.5 kW

PF=1  $\frac{P_{core}}{I_{rated}}$



$$\eta = \frac{P_o}{P_o + P_{loss}} = \frac{P_o}{P_{core} + P_o + k^2 P_{cu}}$$

$$P_{out} = V_r I \cos \phi = V_r (k I_r) \cos \phi = k S_r \cos \phi$$

$$\eta = \frac{k S_r \cos \phi}{k S_r \cos \phi + P_{core} + k^2 P_{cu}}$$

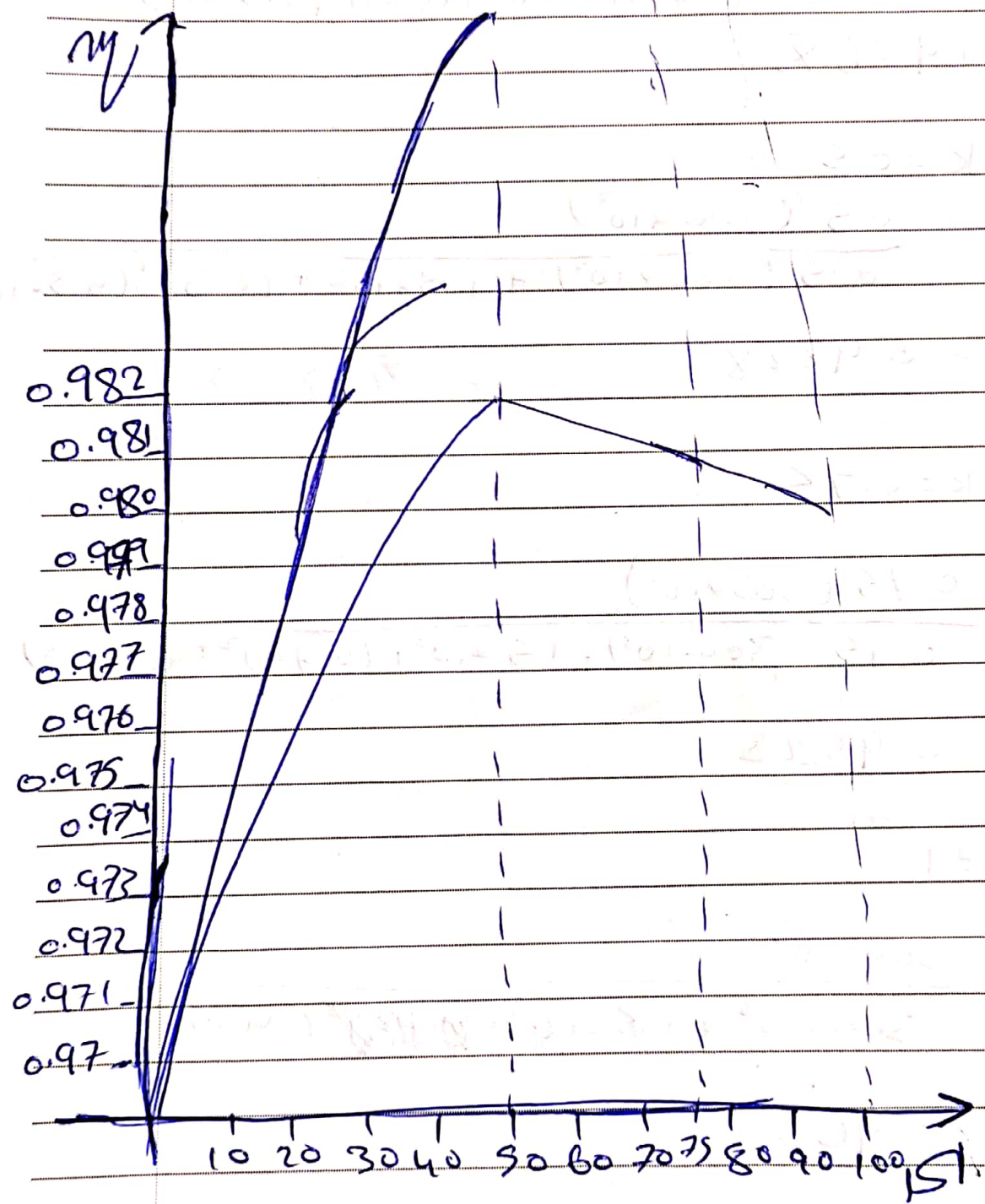


حل واریان  
مستوی در ارتفاع

No. \_\_\_\_\_

at  $k = 1.2$

$$\eta = 0.9777$$



$$\eta_{max} = 0.982$$

at 50%

$k=50$   
smile for life

**Home work:-**

500 kVA, 2400/480 V, 50 Hz power Tr  
 has:  $R_c = 2000 \Omega$ ,  $X_m = 500 \Omega$  (R to HVS)  
 $R_1 = 0.06 \Omega$ ,  $R_2 = 0.003 \Omega$   
 $X_1 = 0.3 \Omega$ ,  $X_2 = 0.012 \Omega$

?? القارة بل قارة  
 HVS دلة

**load**: connected to LVS, draws rated current at 0.8 PF lagging, while voltage is at rated value.

- Sketch the eq circuit r. to. HVS and LVS.
- Calculate the voltage Reg, Input power factor and  $\eta$  of the transformer.

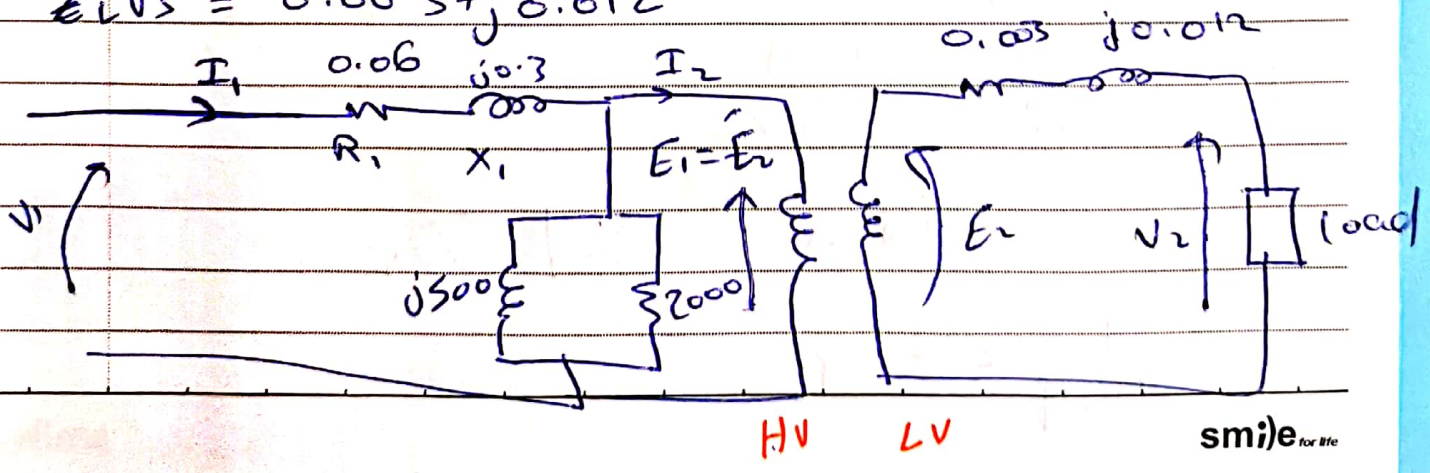
Use the T-eq circuit R. to HVS

Repeat by using L-eq circuit r. to HVS.  
 (compare results).

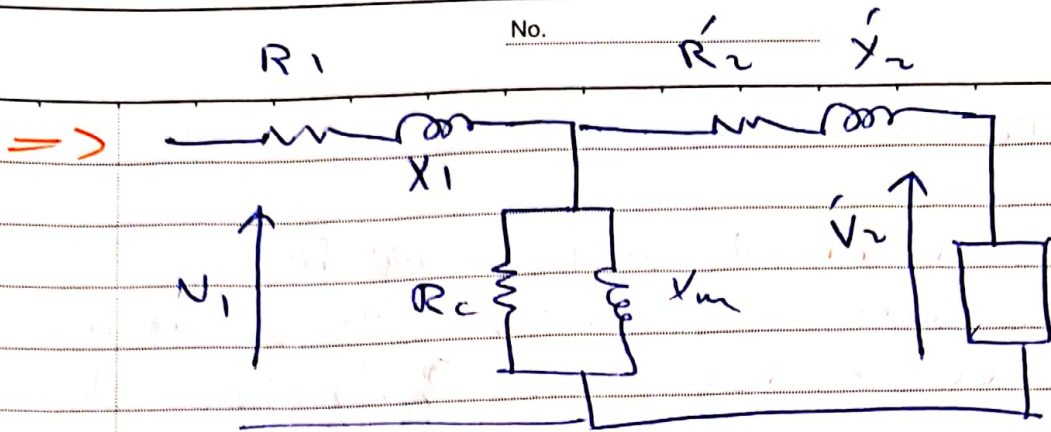
\* Since  $R_1 \gg R_2 \rightarrow R_1$  (2400V) HVS  
 $R_2$  (480) LVS

$Z_{HVS} = 0.06 + j0.3$

$Z_{LVS} = 0.003 + j0.012$







$$R'_2 = R_1 \left( \frac{2400}{480} \right)^2$$

$$I'_2(r) = \frac{S_r}{V_2(r)} = \frac{500 \times 10^3}{2400} = 208.3 \text{ A}$$

or  $I_2(r) = \frac{S_r}{V_2}$

$$\begin{aligned} \vec{E}_1 = \vec{E}_2 &= \vec{V}_2 + \vec{I}_2 (R'_2 + jX'_2) \\ &= 2400 \angle 0^\circ + 208.3 \angle -\cos^{-1}(0.8) * (R'_2 + jX'_2) \end{aligned}$$

lagging

$$Z_{oc} = \frac{R_c + jX_m}{R_c + jX_m} = \frac{R_c}{\sqrt{R_c^2 + X_m^2}} \cdot \angle 90^\circ - \tan^{-1} \left( \frac{X_m}{R_c} \right)$$

$$\begin{aligned} Y_{oc} &= \frac{1}{R_c} + \frac{1}{jX_m} \\ &= G_c - jB_m = |Y_{oc}| \angle \theta_Y \end{aligned}$$

$$Z_{oc} = \frac{1}{|Y_{oc}|} \angle -\theta_Y$$

$$\vec{I}_1 = \vec{I}_{oc} + \vec{I}_2$$

$$\vec{V}_1 = E_1 = (\vec{E}_2) + \vec{I}_1 * (R_1 + jX_1)$$

$$V_{reg} = \frac{V_1 - V_2}{V_2}$$

$$PF_{in} = \cos(\theta_{V_1} - \theta_{I_1})$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V_2 I_2 PF_o}{V_1 I_1 PF_{in}}$$

\* 3kVA, 220/110, 60 Hz Tr.

OCT: 200V, 1.4A, 50W (conducted on the HVS)  
 SCT: 4.5V, 13.6A, 30W

$M = ??$  at 2kVA, 0.85 lag PF

Current in LVS or HVS

$$P_{core(r)} = P_{nl} = \frac{V_r(\infty)^2}{R_c}$$

\* but 200V at OCT  $\neq$  220 (rated)

$$P_{nl}' = 50W = \frac{V_{oc}^2}{R_c}$$

$$P_{core(r)} = P_{nl}' * \left(\frac{V_r(\infty)}{V_{oc}}\right)^2 = 50 \left(\frac{220}{200}\right)^2 = 60.5 \text{ watt}$$

$$I_r(H) = \frac{3000}{220} = 13.64$$

the SCT on HVS

$$I_r(L) = \frac{3000}{110} = 27.28$$



$$P_{sc} = I_{sc}^2 R_{eq}$$

$$P_{cu}(I) = I_r^2 R_{eq}$$

$$\frac{P_{cu}(I)}{P_{sc}} = \left( \frac{I_r}{I_{sc}} \right)^2 \rightarrow P_{cu}(I) = P_{sc} \quad \text{since } I_{sc} = I_r$$

$$\eta = \frac{k S_r \cos \phi}{k S_r \cos \phi + k^2 P_{cu}(I) + P_{core}(I)}$$

$\downarrow$                        $\downarrow$   
 प्रसारण क्षमता    प्रसारण क्षमता

$$k = \frac{S_L}{S_r} = \frac{2}{3}$$

is it (Dist or trans transformer)??

$$k_{\eta_{max}} = \sqrt{\frac{P_{core}(I)}{P_{cu}(I)}}$$

$k \geq 1 \rightarrow$  transmission

$k < 1 \rightarrow$  distributive

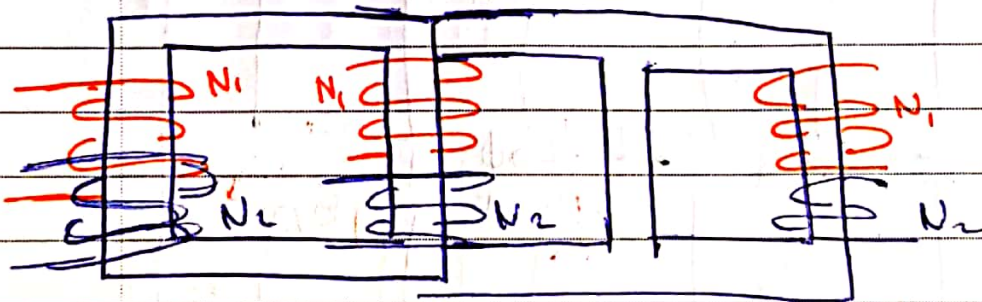
## 3-Ph Transformers

Tr upto 10 GVA

\* Two forms :-

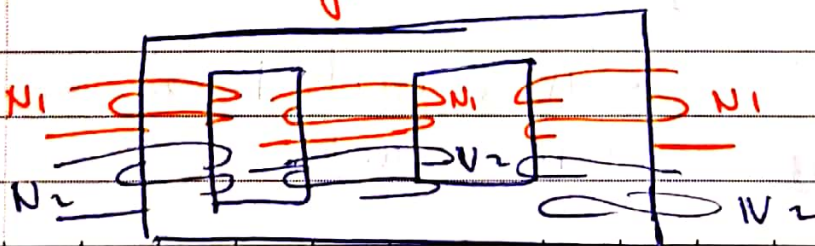
- a) Bank of three single-ph Transformers.
- b) single unit 3-ph transformer: The most common
  - a) • easy transportation
  - less stand by requirements (in terms of cost)
  - easily allows open delta connection.
  - b) • less connection, cheaper for same power ratings.
  - less weight, size for same power ratings.

connections: \* core type.  
\* shell type.



X not Actual

Actual core type.

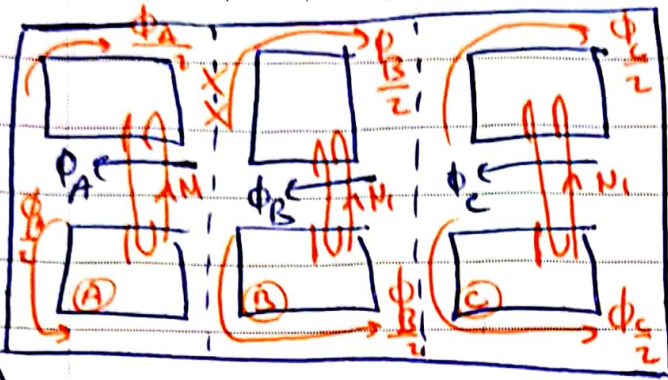


Actual core type.



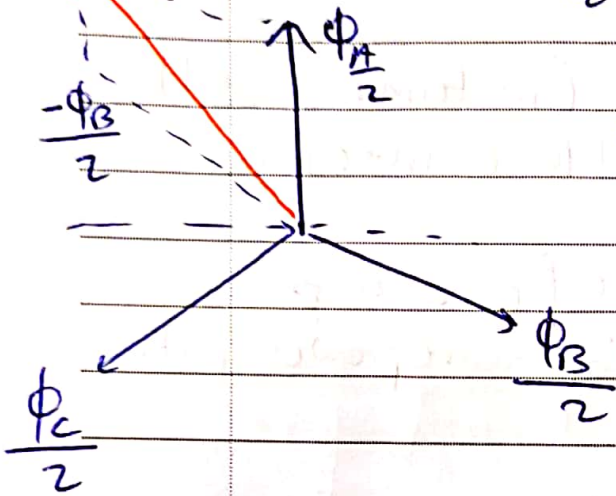
\* Shell type.

No. \_\_\_\_\_



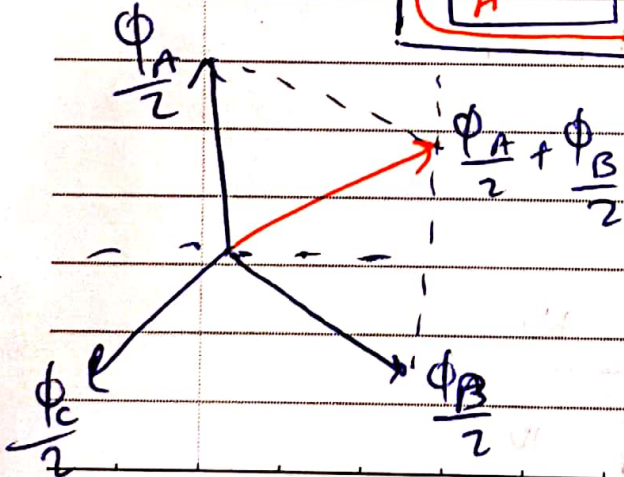
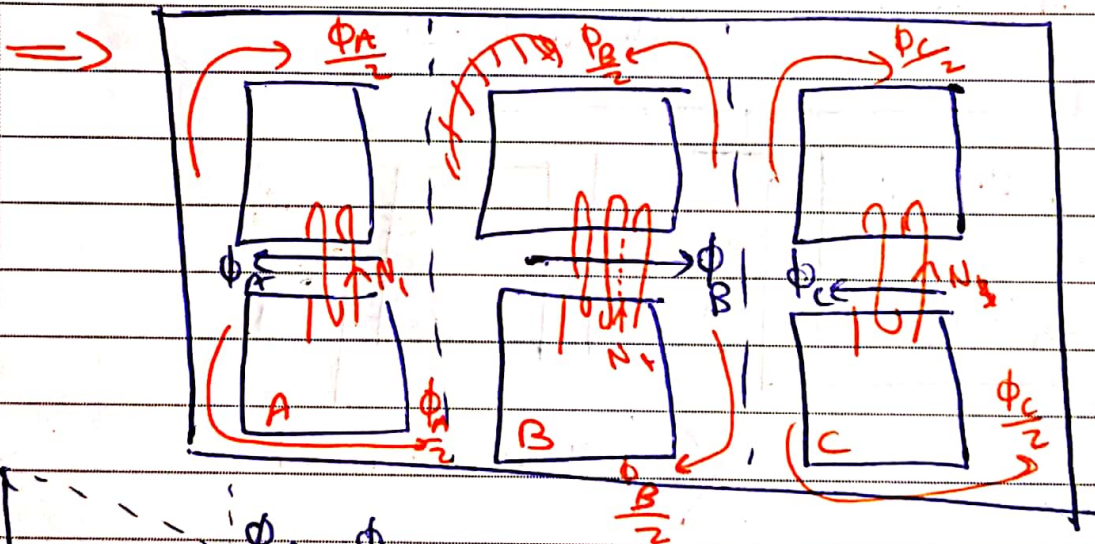
$$= \frac{\sqrt{3}}{2} \frac{\Phi_A}{2}$$

at (XX)  $\phi = \frac{\Phi_A}{2} - \frac{\Phi_B}{2} = \frac{\sqrt{3}}{2} \frac{\Phi_A}{2} = 0.866$



limp needs more cross sectional Area ..

\* To avoid this:- the center phase is wound in opposite direction



\* This leads to 15% less core weight.



Shell type:- More expensive than core type. it gives more mechanical strength under sc conditions and it allows for open- $\Delta$  connection.

Connections :- Y-y

D-y

Y-d

D-y

Y - d  
2

\* In distribution transformers  $\rightarrow$  the secondary must be y

\* high current  $\rightarrow$   $\Delta$  connection.

\* Groups of 3-ph power transformer:-

Group ①  $\rightarrow$  the phase between any secondary line to voltage and the corresponding primary due to voltage is  $0^\circ$  (In phase).

Group ②  $\rightarrow$  ||||| is  $180^\circ$  (out of phase)

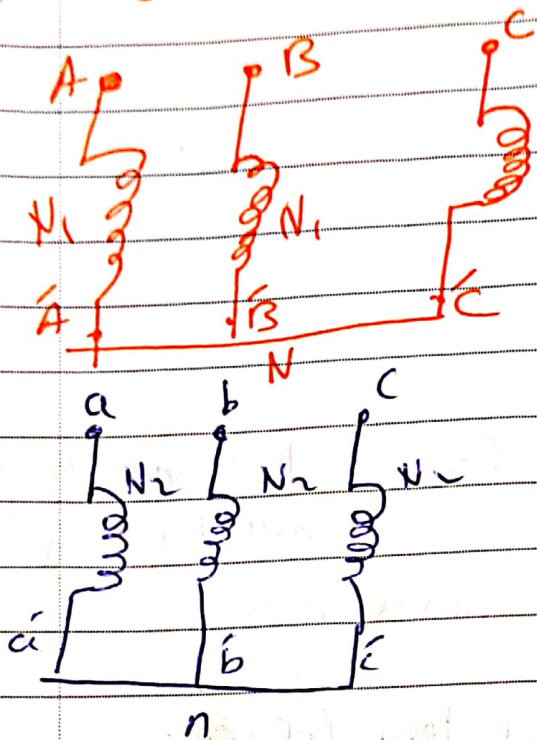
Group ③  $\rightarrow$  ||||| is  $-30^\circ$  (secondary line voltage lags primary line voltage by  $30^\circ$ )

Group ④ |||||  $30^\circ$  (||||| leads by  $30^\circ$ )

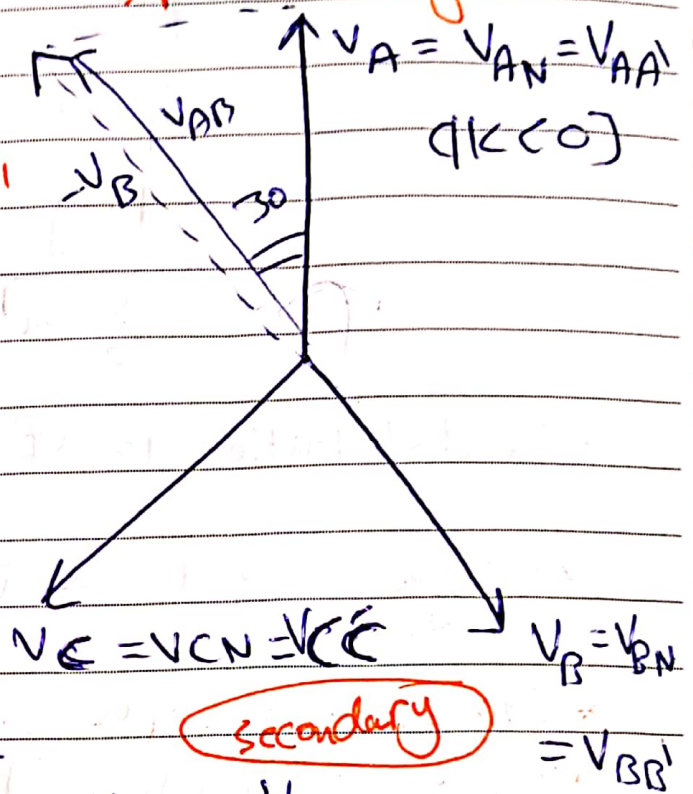


Aim of groups:- to say if certain 3ph Tr is safely connected in parallel with another 3-ph Tr.

① Y-Y transformer



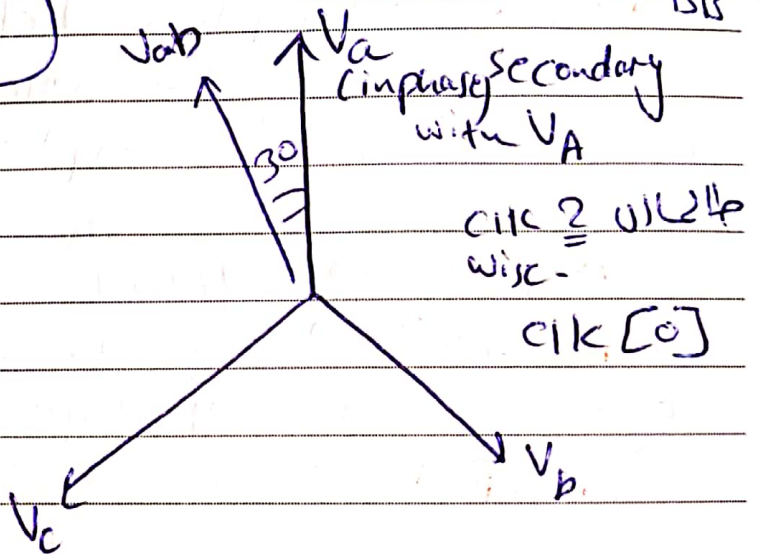
\* Phasor diagram



② 6 line voltages

$$V_{AB} = V_A - V_B$$

- $V_A$  in phase  $V_a$
- $V_B$  " "  $V_b$
- $V_C$  " "  $V_c$

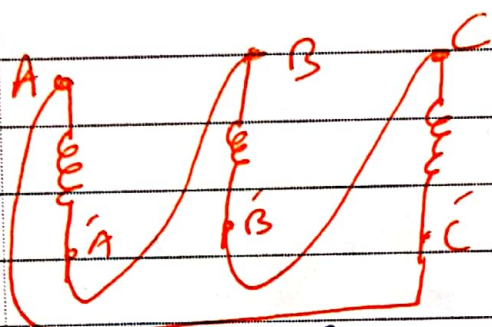


$V_{ab}$  In-phase with  $V_{AB}$  (Phase shift 200)

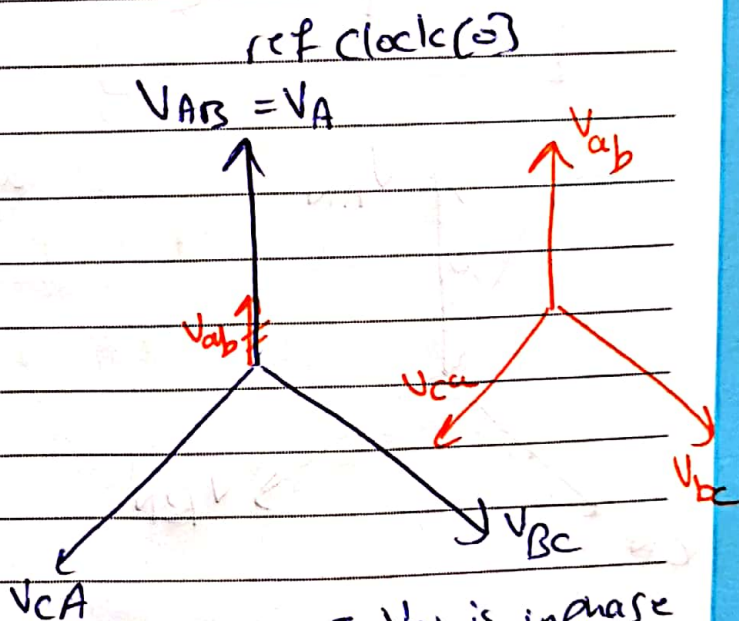
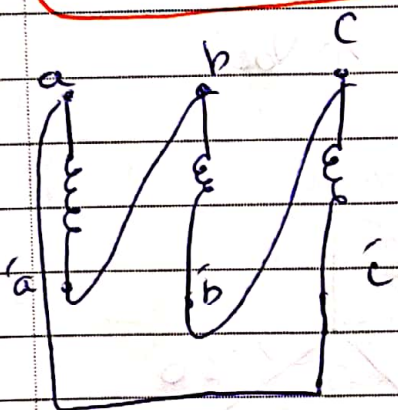
Tr Y-Y can be group No.1

Y-y connection can also be. Y-y belongs to G<sup>2</sup> ( $V_{ab}$  lag  $V_{AB}$  by 180)

**D-d Transformer:-**



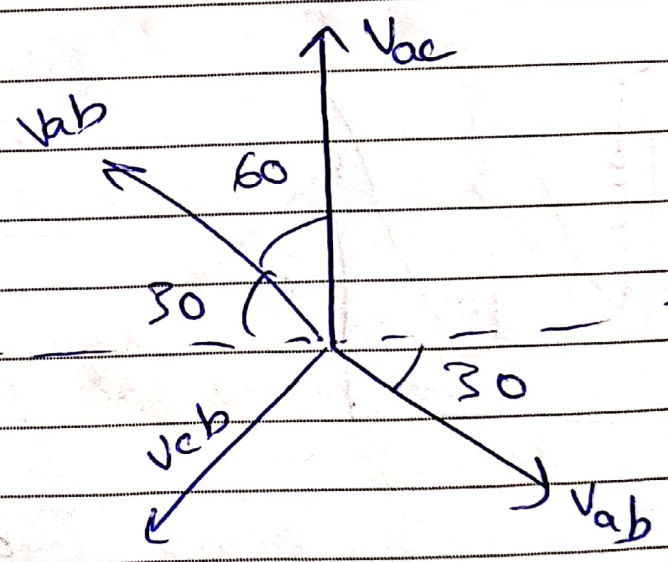
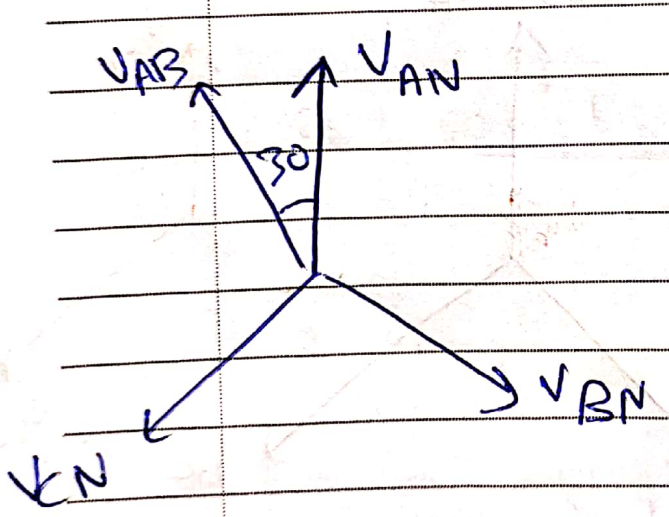
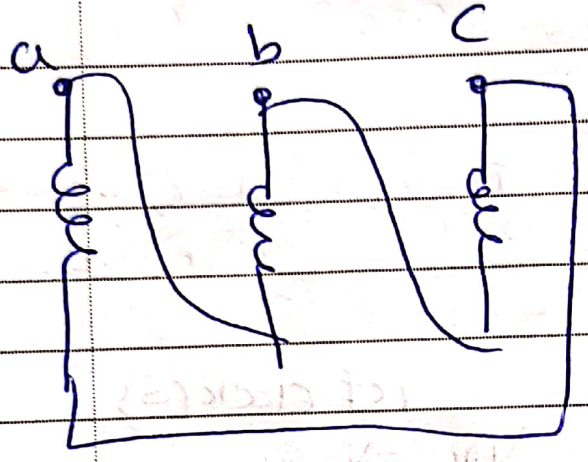
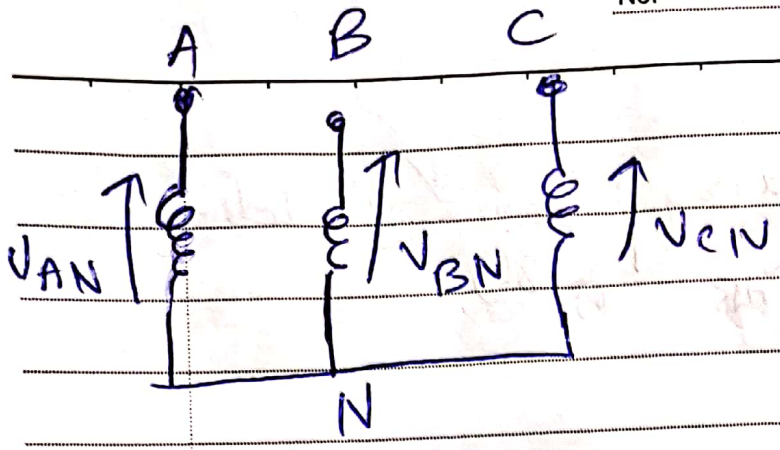
من بداية ملف الى آخره  
ملف الثاني



-  $V_{ab}$  is in phase with  $V_{AB}$   
group No ① D-d

Y-y and D-d can be connected on parallel.  
group ①      group





$V_{ab}$  leads  $V_{AB}$  by  $(30^\circ)$  group Y



100 kVA, 120/120 Volt, 60 Hz Tr PF=1

$\eta = 95.75\%$  at rated conditions

its leakage impedance =  $(2.5 + j5.0) \mu\Omega$

OCT data : 100V, 1400W, 50 Hz

find

(1)  $P_{eddy}$  &  $P_w$  at 60 Hz, 120V

(2) Can the transformer deliver rated power at rated voltage and 50 Hz.

Safe side operation :  $\frac{V}{f} = \text{constant}$

$$\frac{V_r}{f_r} = \frac{120}{60} \text{ must equal } = \frac{V_s}{50} = \frac{100}{50}$$

$2 = 2$

$V = 100$  equal the  $\phi$  constant } no  $B$  constant } Problem

$V_{oc} \neq V_r$

~~$P_{oc} = P_{core} = P_e + P_w$  when  $V_{oc} = V_{rated}$~~   
 ~~$P_{oc}(r) = 1400 \times \left(\frac{120}{100}\right)^2 = 2016$~~

$P_{oc} = P_{wL} = 1400 \text{ watt at } 50 \text{ Hz, } 100 \text{ V}$

$$\eta = \frac{k S_r PF}{k S_r PF + k^2 P_{cu}(r) + P_{core}}$$

95.75%

$$100k + k^2 P_{cu}(r) + P_{core}$$

smile for life



$$P_{cu} (1) = R_{eq} (P_u)$$

$L_{10} \text{ plus } *$

No. \_\_\_\_\_

$$Z_{eq} (P_u) = R_{eq} + j X_{eq}$$

$$P_{cu} (1) = 0.025 * \frac{100 \text{ k}}{N_r}$$

$$P_{cu} (1) = 2500$$

$$0.9575 = \frac{100 * 10^3}{100 * 10^3 + 2500 + P_{core} (1)}$$

$$P_{core} = 1938 \text{ watt}$$

Rated conditions.

$$P_{core} (1) = P_h + P_e$$

$$= k_h * f * B^2 + k_e \frac{f^2 * B^2}{\omega B}$$

$$P_{cor} = k_h 60 + k_e (3600)$$

rated frequency.

$$1938 = 60 k_h + 3600 k_e$$

Test conditions

7400

$$1400 = k_h 50 + 2500 k_e$$

$$k_e = 0.43, \quad k_h = 6.5$$

$$P_e = k_e (50)^2$$

new

$$S_r = V_r I_r$$

$$\text{delivered power} = \frac{S}{S_r} \times 100\%$$

(rated)  $P_{rated}$

$$\bar{B} = \frac{\sum B}{6}$$



HW:-

$$P_{cu}(r) P_u \stackrel{??}{=} R_{eq}(P_u)$$

$$P_{cu}(r) = I_r^2 R_{eq}$$

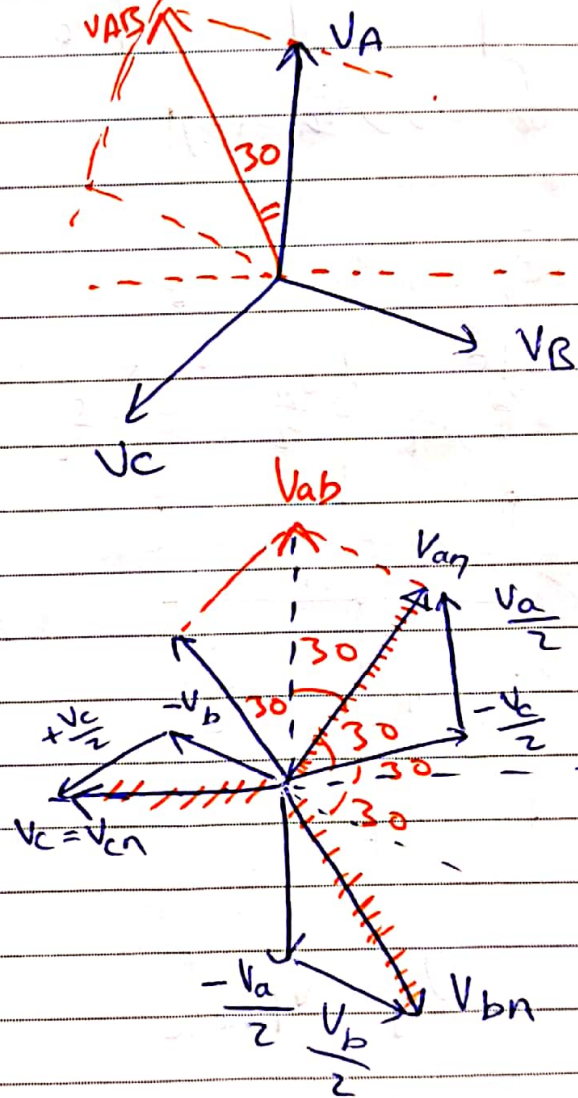
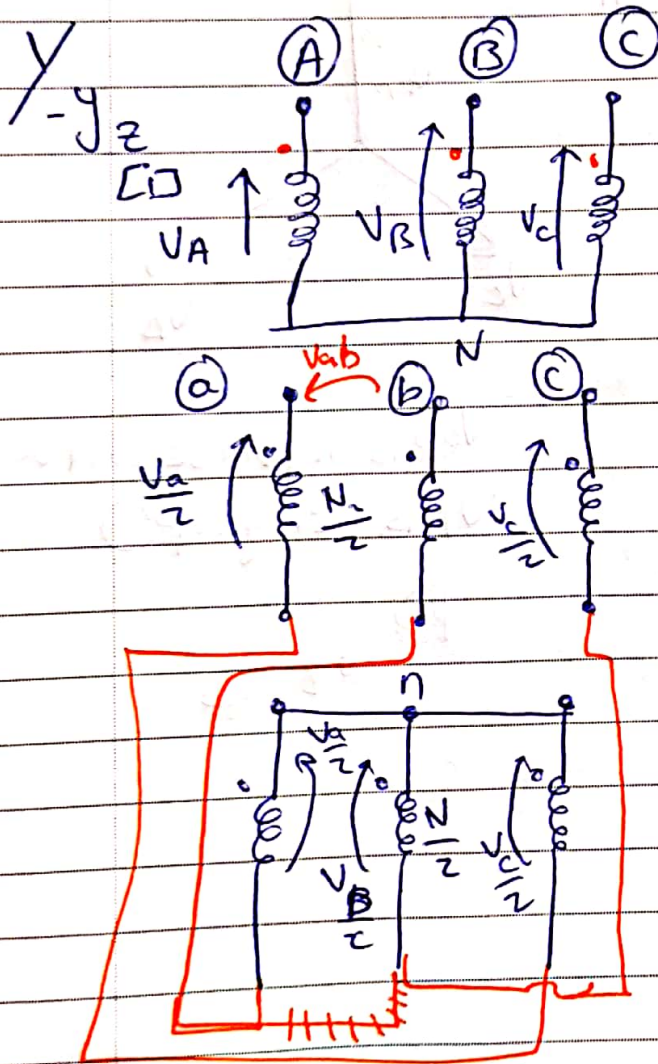
$$P_{cu} P_u = I_r^2(r) P_u R_{eq}(P_u)$$

$$= \left( \frac{I_r^2(r)}{I_r^2(r)} \right)^2 R_{eq}(P)$$

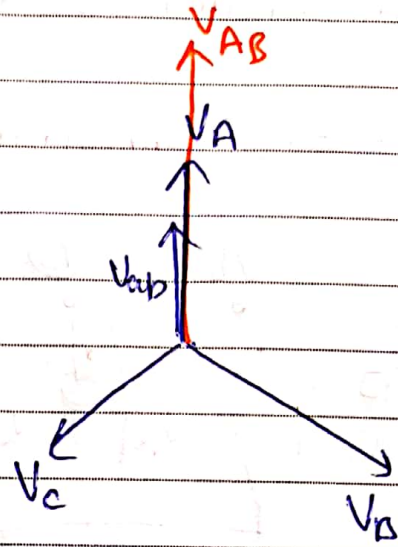
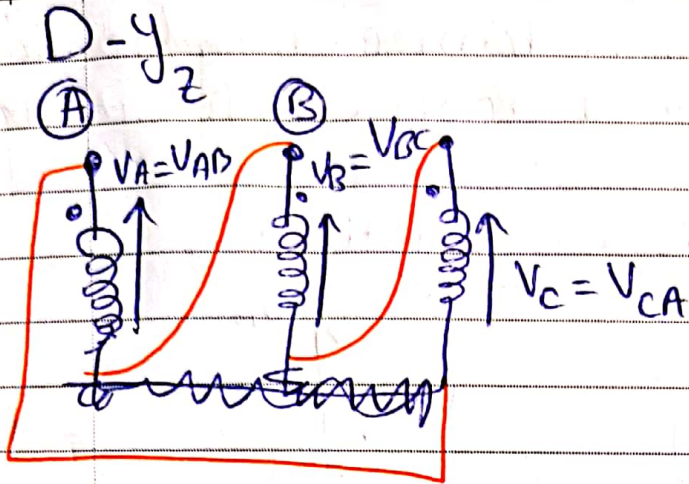
$$= 1 * R_{eq} \underline{P_u} = R_{eq}(P_u)$$

\* To reduce harmonics in transformers and to allow 3-ph parallel operation of Tr with different groups, Zig-Zag connection is the solution.

Zig-Zag connection is the solution.

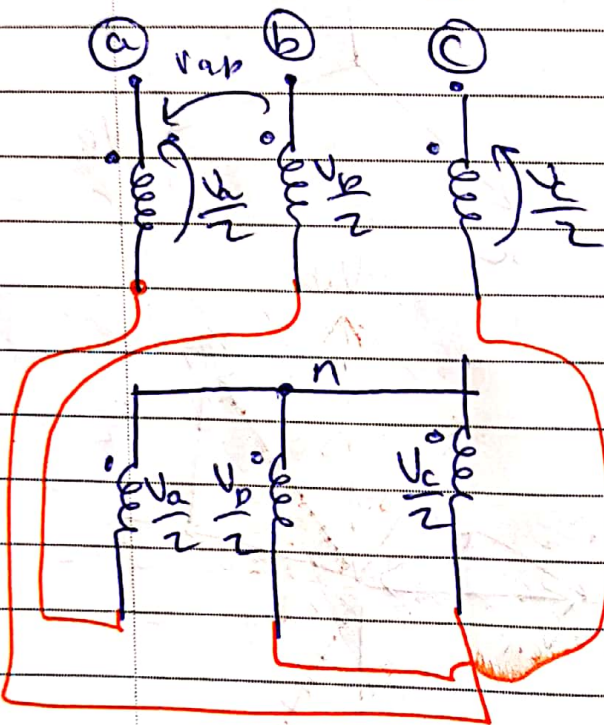






$V_{ab}$  is in phase with  $V_{AB}$   $\therefore$  group ①

D-y  $Z [0]$



$$V_y = V_{\text{Phase}}$$

~~$V_{\text{Phase}}$~~

Y-y :-

$$a_T = \frac{V_{pny}}{V_{phy}} = \frac{N_1}{N_2}$$

$$Y - Y_2 \text{ :-}$$

$$V_{an} = \sqrt{3} \frac{V_{ph}}{2} = \frac{\sqrt{3}}{2} V_{ph}$$

$$V_{ab} \frac{1}{2} = \sqrt{3} \left( \frac{\sqrt{3}}{2} V_{ph} \right) = \frac{3}{2} V_{ph}$$

$$V_{any} = \frac{2V_{ph}}{2} = V_{ph}$$

$$V_{aby} = \sqrt{3} V_{ph}$$

$$\frac{V_{aby}}{V_{abz}} = \frac{\sqrt{3} V_{ph}}{\frac{3}{2} V_{ph}} = \frac{2}{\sqrt{3}} \Rightarrow \boxed{\frac{V_{aby}}{V_{abz}} = 1.15}$$

$\Rightarrow$  This implies that  $Y_2$  loses 15% of the voltage compare to  $Y$  connection.

$\rightarrow$  to obtain same voltage as for  $Y$  connected  $Tr$ ,  $Y_2$  should have 15% extra turns in the windings

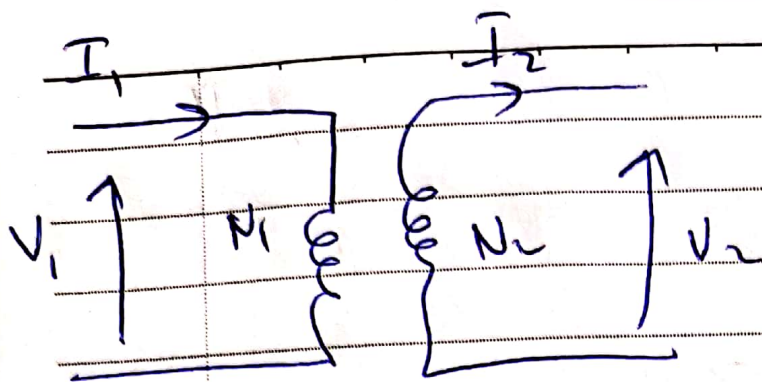
$\Rightarrow Y_2$  have more weight, more expensive.

at  $\epsilon$ -transformation ratio:

$a$ : turns ratio

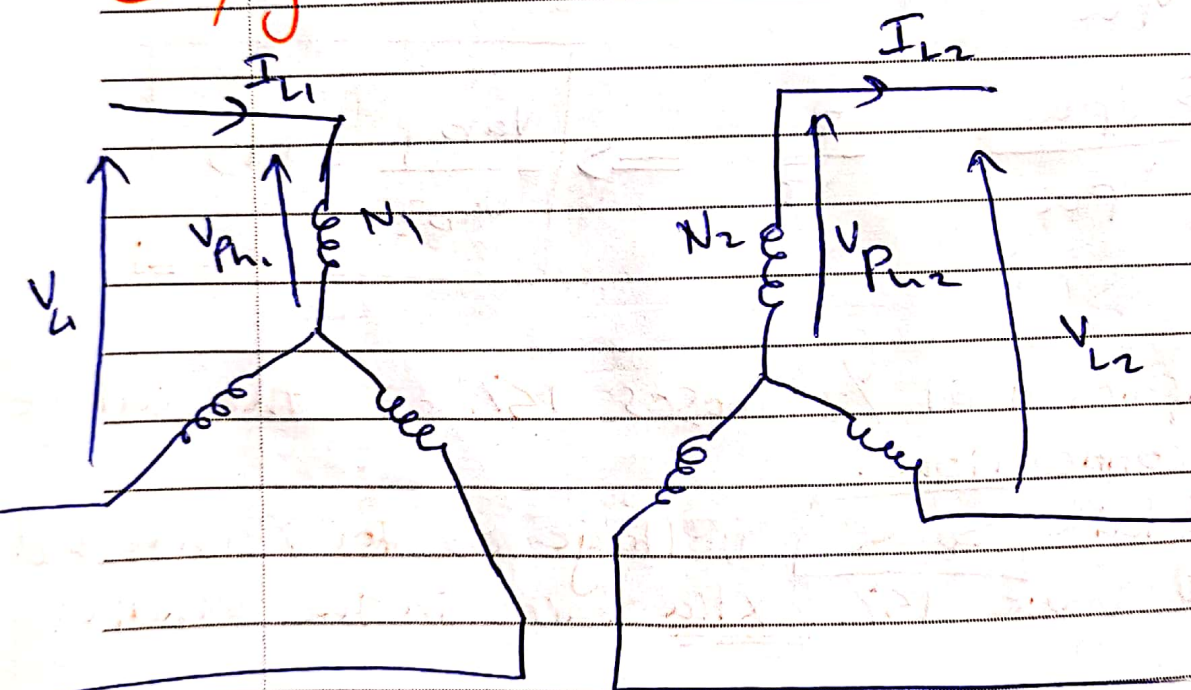
$$a = \frac{N_1}{N_2} = \frac{V_{ph1}}{V_{ph2}} = \frac{I_2 P_{h2}}{I_1 P_{h1}} \rightarrow \text{single phase.}$$





AT : ??

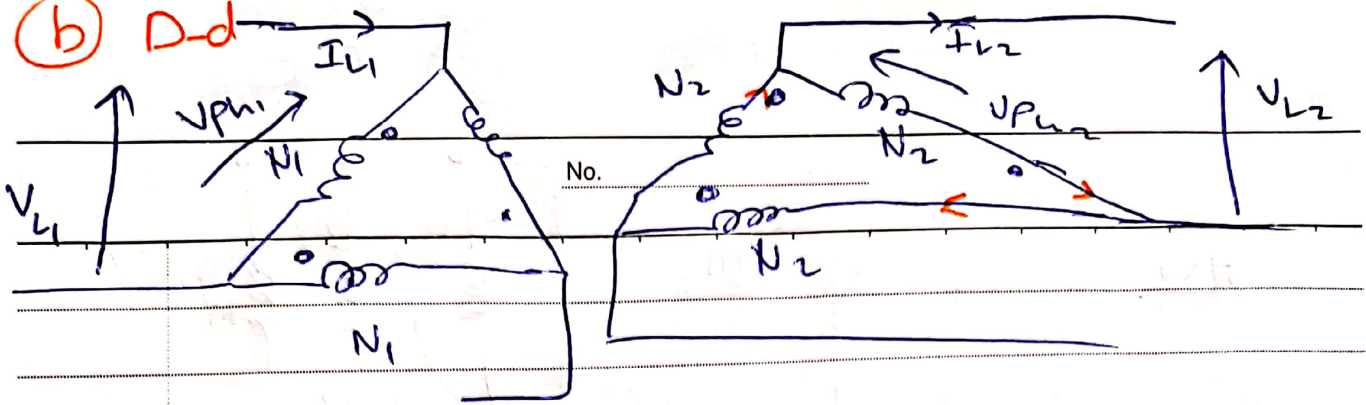
(a) Y-y



$$AT = \frac{V_{L1}}{V_{L2}} = \frac{V_{ph1} \sqrt{3}}{V_{ph2} \sqrt{3}} = \frac{V_{ph1}}{V_{ph2}} = \frac{N_1}{N_2} = a$$

\*\* So in Y-y connection  $AT = a$

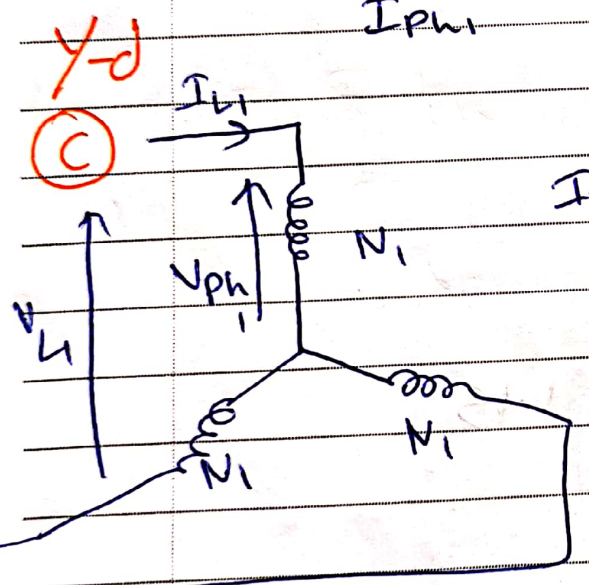
$$\frac{I_{ph1}}{I_{ph2}} = \frac{I_{L1}}{I_{L2}} = \frac{N_2}{N_1} = \frac{1}{a}$$



$$a = \frac{N_1}{N_2} = \frac{V_{ph1}}{V_{ph2}} = \frac{V_{L1}}{V_{L2}} = a_T$$

\*  $a_T = a$  in D-d

$$a = \frac{I_{ph2}}{I_{ph1}} = \frac{I_{L2}/\sqrt{3}}{I_{L1}/\sqrt{3}} = \frac{I_{L2}}{I_{L1}} = a_T$$



$$a = \frac{N_1}{N_2} = \frac{V_{ph1}}{V_{ph2}} = \frac{V_{L1}/\sqrt{3}}{V_{L2}}$$

$$a = \frac{1}{\sqrt{3}} \frac{V_{L1}}{V_{L2}} \Rightarrow \underline{\underline{a_T = a}}$$

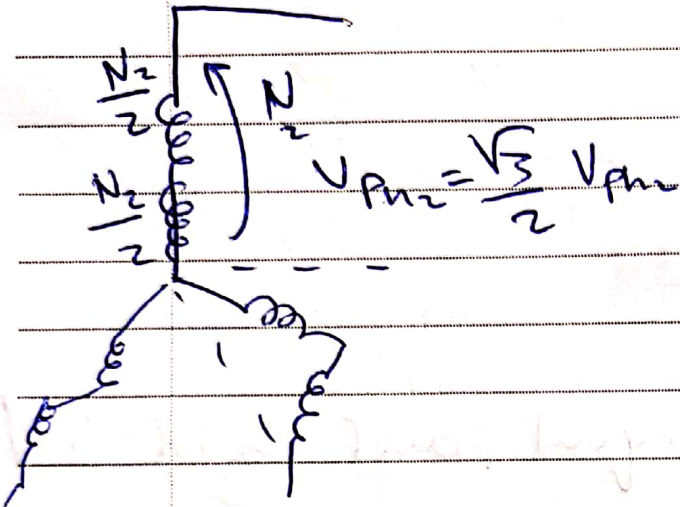
\*\*  $a = \frac{1}{\sqrt{3}} a_T$   
 $a_T > a$   
 at Y-d

\*\* at D-y

$$a_T = \frac{a}{\sqrt{3}}$$



H.W

D-Y<sub>z</sub>

$$a = \frac{2}{\sqrt{3}} \frac{V_{ph1}}{V_{ph2}}$$

$$a_z = \frac{2}{\sqrt{3}} \frac{N_1}{N_2}$$

$$= \frac{2}{\sqrt{3}} \cdot a$$

$$= 1.15 a$$

$$a_T = \frac{V_{LD}}{V_{Lz}} = \frac{V_{ph1}}{\frac{\sqrt{3} \cdot \sqrt{3}}{2} V_{ph2}} = \frac{2}{3} \frac{V_{ph1}}{V_{ph2}}$$

$$a_T = \frac{2}{3} a$$

Parallel operation of transformers:-

Why??

- ① More reliable system.
- ② To face growth of the network.

How?

Conditions to be satisfied:-

- ① Same voltage Ratings and Tr. ratio.
- ② Same time

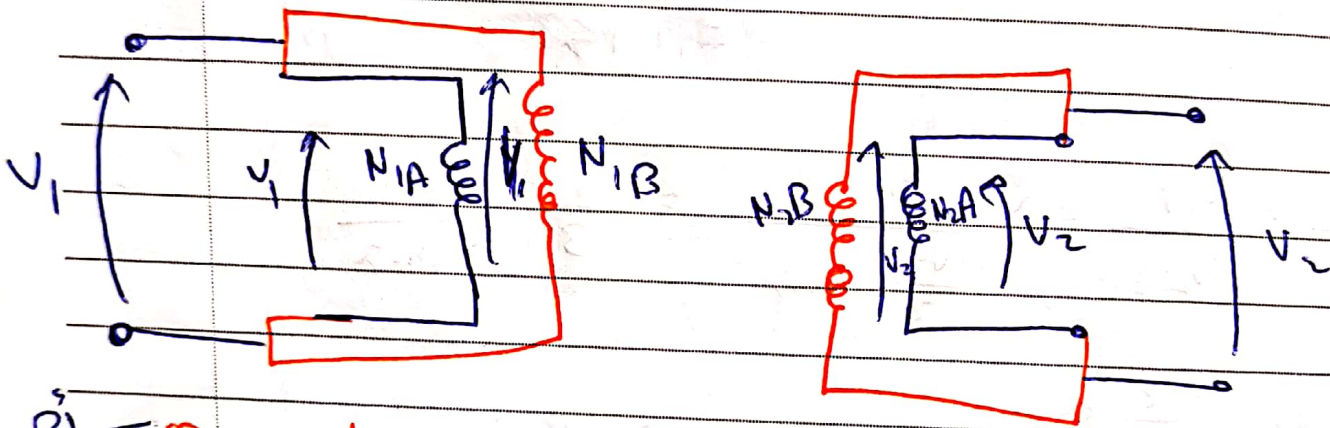
↳ the (two or more) should belong to the same group.

③ for good power sharing :-

$$Z_{eq, (Pu)}_A = Z_{eq, (Pu)}_B$$

$$\frac{X_{eq}}{R_{eq}} \Big|_A = \frac{X_{eq}}{R_{eq}} \Big|_B$$

⇒ this leads to equal power factor (PF)



- تشریح
- ① equal voltage ratings
  - ② P<sub>r</sub> - per polarity (same group in 3ph Tr)
- $\sum V = 0$  (closed path).

$$V_2 = \frac{V_1}{a_B} \quad (\text{Transformer B})$$

$$V_2 = \frac{V_1}{a_A} \quad (\text{Transformer A})$$

if  $a_A = a_B$

$$\frac{V_1}{a_A} = \frac{V_1}{a_B}$$

$$I_{cc} = \frac{V_1}{a_A} - \frac{V_1}{a_B} = 0$$

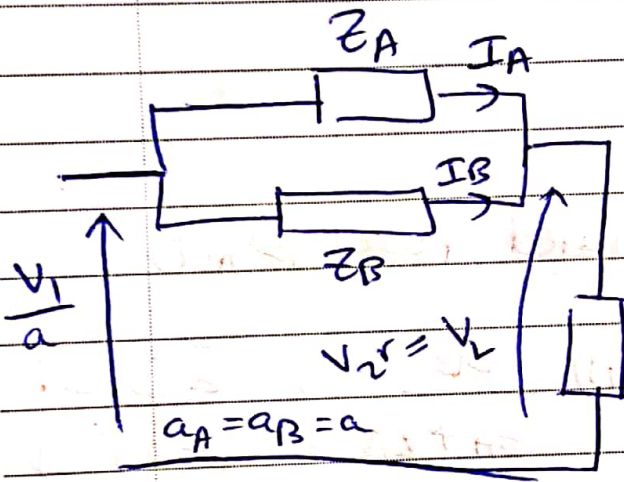
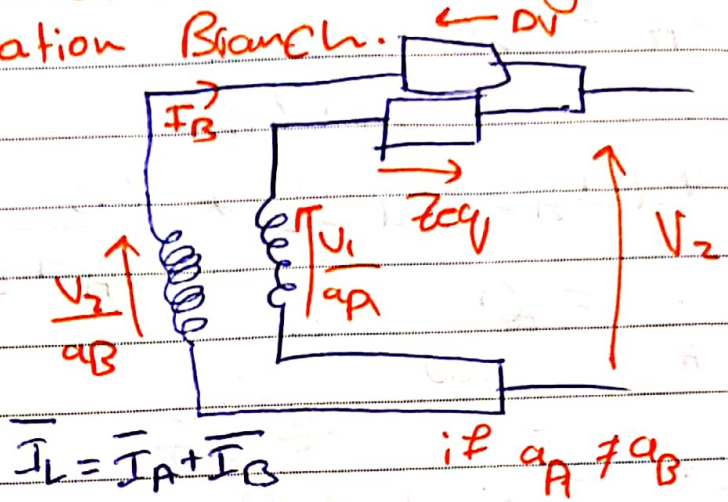
$$Z_{eq,1} + Z_{eq,2}$$

if  $a_A \neq a_B \rightarrow \frac{V_1}{a_A} \neq \frac{V_1}{a_B}$



$I_{cc} \neq 0 \rightarrow$  problem

$\rightarrow$  Equivalent circuit referred to secondary (load).  
 \*\* Neglect excitation Branch.



$I_{cc} = 0$  (proper connection)

$$I_A Z_A = I_B Z_B$$

$$\frac{I_A}{I_B} = \frac{Z_B}{Z_A}$$

$$\frac{V_{2r} I_A}{V_{2r} I_B} = \frac{Z_B}{Z_A}$$

$$\frac{P_A}{P_B} = \frac{Z_B}{Z_A}$$

$$\frac{P_A (r)}{P_B (l)} = \frac{Z_B}{Z_A}$$

Proper Power Sharing

$\rightarrow$  with condition for power connection.

3d:-  $\frac{X_{eq}}{R_{eq}} \Big|_A = \frac{X_{eq}}{R_{eq}} \Big|_B$   
 for same PF

Power sharing:-

$$I_A = I_L \times \frac{Z_B}{Z_A + Z_B}$$

$$V_{2r} I_A = V_{2r} I_L \times \frac{Z_B}{Z_A + Z_B}$$

$$S_A = S_{load} \times \frac{Z_B}{Z_A + Z_B}$$

$$S_B = S_{load} \times \frac{Z_A}{Z_A + Z_B}$$

→ if  $T_{rA}$  is to provide its rated power ( $S_{Acr}$ )

$$S_A(r) = S_{load} (\text{max due to A}) \frac{Z_B}{Z_A + Z_B}$$

$$S_B(r) = S_{load} (\text{max due to B}) \frac{Z_A}{Z_A + Z_B}$$

أكبر قدرة  
لا يمكن  
تجاوزها  
في  $T_{rA}$   
هذه الـ  
القدرة

→ for  $T_{rA}$  not to be over loaded:-

$$S_{load \max A} = S_A(r) \times \frac{Z_A + Z_B}{Z_B}$$

→ for  $T_{rB}$  not to be over loaded:-

$$S_{load \max B} = S_B(r) \times \frac{Z_A + Z_B}{Z_A}$$



(A)

$$\text{If } S_{\text{load max A}} = S_{\text{load max B}}$$

Proper sharing occurs

$$\text{ex: } S_L = 100 \text{ M} \rightarrow S_A(r) = 60 \text{ M} \\ S_B(r) = 40 \text{ M}$$

$$S_A(r) + S_B(r) = S_{\text{load}}$$

$$\text{ex: } S_L = 80 \text{ M} \rightarrow S_A(r) = 60 \\ S_B(r) = 40 \rightarrow S_A = 80 \times \frac{60}{100} = 48 \text{ M}$$

$$S_B = 80 \times \frac{40}{100} = 32 \text{ M}$$

$$\text{ex: } S_{\text{load}} = 110 \rightarrow S_A(r) = 60 \\ S_B(r) = 40$$

على مستوى  
التيار

$$(B) S_L(\text{max A}) \neq S_L(\text{max B})$$

$S_L(\text{max}) = \text{either } S_L(\text{max A}) \text{ or } S_L(\text{max B})$   
which of them is lower.

\* In this case one Tr will run at rated power  
second will be under rated.

$$\frac{S_A(r)}{S_B(r)} \neq \frac{Z_0(B)}{Z_0(A)}$$

last common +

$$\frac{I_A Z_A}{V_{2r}} = \frac{I_B Z_B}{V_{2r}}$$

$$\frac{Z_A}{\left(\frac{V_{2r}}{I_A}\right)} = \frac{Z_B}{\left(\frac{V_{2r}}{I_B}\right)}$$

\* if two Tr share rated power :-

$$\frac{Z_A}{\left(\frac{V_{2r}}{I_{A(r)}}\right)} = \frac{Z_B}{\left(\frac{V_{2r}}{I_{B(r)}}\right)}$$

$$\frac{Z_A}{Z_{Base(A)}} = \frac{Z_B}{Z_{Base(B)}}$$

$$Z_A (P_n) = Z_B (P_n)$$



\* if two Tr share rated power :-

$$\frac{Z_A}{\left(\frac{V_{2V}}{I_A(r)}\right)} = \frac{Z_B}{\left(\frac{V_{2V}}{I_B(r)}\right)}$$

$$\frac{Z_A}{Z_{Base(A)}} = \frac{Z_B}{Z_{Base(B)}}$$

$$Z_A (pu) = Z_B (pu)$$

Ex:

$S_A = 25 \text{ kVA}$  } Transformer connected on parallel.  
 $S_B = 75 \text{ kVA}$  }

$$Z_B = 0.0125 < 53.13$$

$$Z_A = 0.375 < 53.13$$

load :- 100 kVA.

$$a) S_A = S_L \times \frac{Z_B}{Z_A + Z_B} = 100 \times \frac{0.0125}{0.05} = \frac{100 \times 0.0125}{0.05} = 25 \text{ kVA}$$

$$S_B = 100 \times \frac{0.0375}{0.05} = 75 \text{ kVA}$$

← excellent sharing.

$$\frac{S_A(r)}{S_B(r)} = \frac{Z_B(r)}{Z_A(r)} = \frac{1}{3}$$



$$\text{If } Z_A = 0.075 < 53.13 \Omega$$

$$Z_B = 0.05 < 53.13 \Omega$$

$$\frac{S_A(r)}{S_B(r)} = \frac{1}{3}$$

no good sharing.

$$\frac{Z_B(r)}{Z_A(r)} = \frac{2}{3}$$

$$S_{L \max A} = S_A(r) \times \frac{Z_A + Z_B}{Z_B} = \frac{100 (0.125)}{0.05} = 62.5 \text{ kVA}$$

$$S_{L \max B} = S_B(r) \times \frac{Z_A + Z_B}{Z_A} = \frac{100 (0.125)}{0.075} = \frac{500}{3} \text{ kVA}$$

$$= \frac{500}{3} \text{ kVA} = 167.67 \text{ kVA}$$

125 kVA

$$S_A = 100 \times \frac{Z_A}{Z_A + Z_B} = 40 \text{ kVA (over loaded)}$$

$$S_B = 100 \times \frac{Z_B}{Z_A + Z_B} = 60 \text{ kVA (under loaded)}$$

\*  $\rightarrow S_{L \max} = 62.5$  Can be served

\* EX:-

3-phase, 230V, 27 kVA, 0.9 PF lag load is supplied by 3 single phase Tr (10 kVA, 1330/230V 50 Hz) connected in Y-d

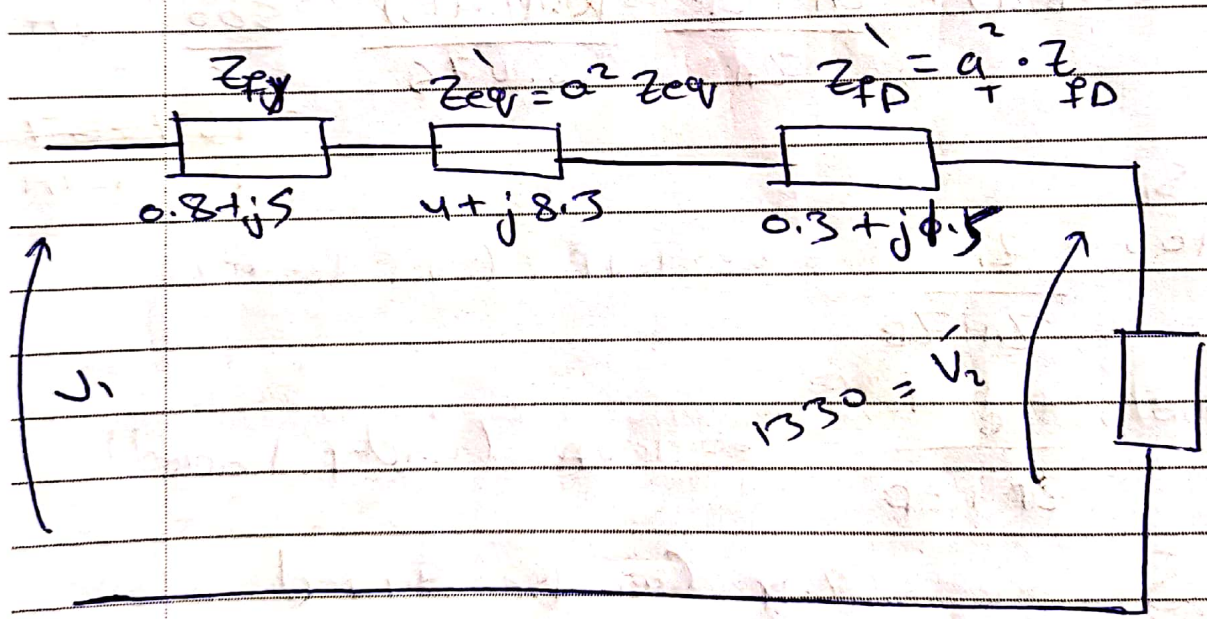
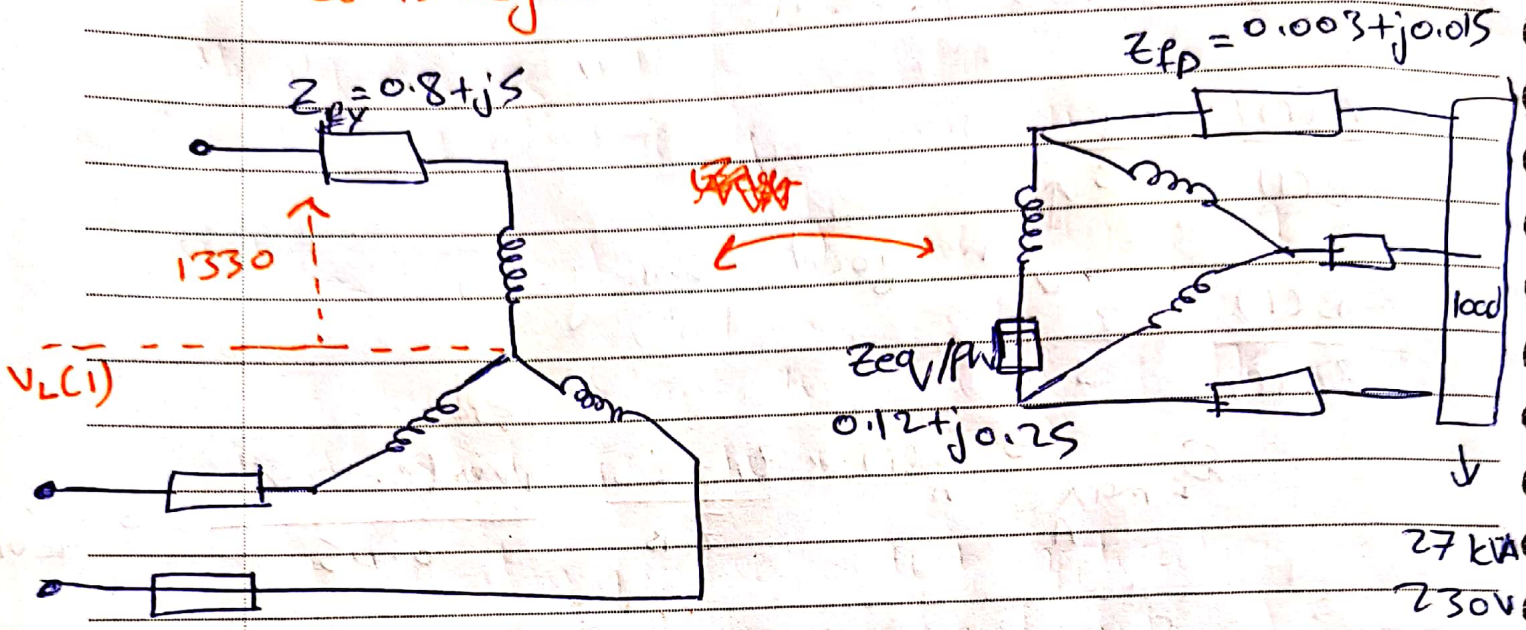
Tr Secondary is connected to the load through a feeder  $(0.003 + j0.015) \Omega / \text{ph}$



Tr. primary is connected to 3-ph supply through a feeder  $(0.8 + j5) \Omega/\text{ph}$

Eq. Impedance :-  $(0.12 + j0.25) \Omega/\text{ph}$  R. to LVS

$Z_0$  is neglected.



$$a = \frac{1330}{230} = 5.78$$

$$Z_{eq}' = (5.78)^2 (0.12 + j0.25) = 4 + j8.3$$

$$Z_{LD}' = (aT)^2 Z_{LD} = 100 Z_{LD}$$

$$aT = 1330 \text{ V} = 10$$



$$\hat{V}_2 = V_2 \times a = 230 \times 5.78 = 1330 \angle 0$$

Per phase.

$$P = 27 \times 10^3 = \sqrt{3} V_L I_L \cos \phi$$

$$I_{LD} = \frac{27 \times 10^3}{\sqrt{3} \times 230 \times 0.9} = 75.$$

$$I_2 = \frac{75 \cdot 3}{a_T} = 7.5$$

$$\Rightarrow R_{eq} = 0.8 + 4 + 0.3 = 5.1 \Omega$$

$$X_{eq} = 5 + 8.3 + 1.5 = 14.8 \Omega$$

$$\left. \begin{array}{l} R_{eq} = 5.1 \Omega \\ X_{eq} = 14.8 \Omega \end{array} \right\} Z_{eq} = 15.65 \angle 70.9^\circ$$

$$\cos^{-1}(0.9)$$

$$\bar{V}_1 = 1330 \angle 0 + 7.53 \angle -25.84^\circ (15.65 \angle 71)$$

$$= 1413 \angle 3.3^\circ \text{ V/Ph.}$$

$\rightarrow 1415 \sqrt{3} = V_L$

$$V \cdot R_{eq} = \frac{1415 - 1330}{1330}$$

$$\eta = \frac{P_o}{P_{in}} = \frac{27 \times 10^3}{7.53 \times 1415 \cos(29)}$$

$$PF_{in} = \cos(29)$$



## \*\*\* DC Machines :-

Mainly used as Motors.

Recently  $\approx$  load sharing no more than 10%.

Motor expensive (2-3) cost of eq. AC  
control  $\rightarrow$  so easy.

\* power electronics ease the control of AC motor  
(which is cheaper than eq. DC)

$\Leftrightarrow$  still used in precise control applications, & in  
certain mech load (ex: traction system).

# DC Machines Classification

the machine excites itself.

**Self excited machines**

- one extra power source is required to create excitation field.

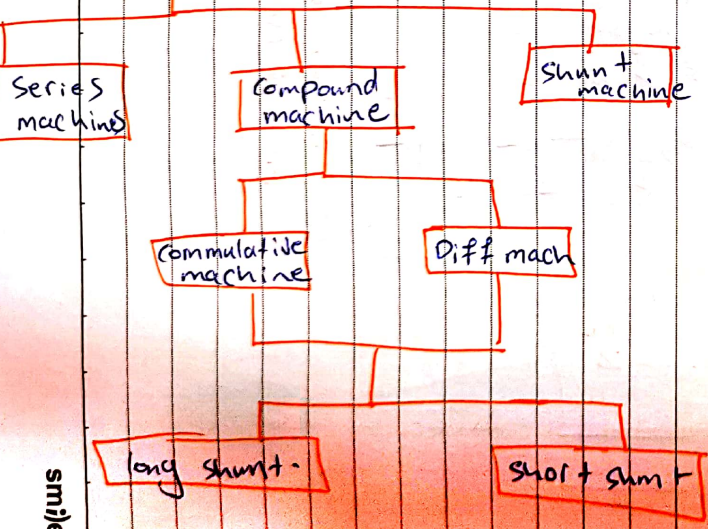
**Permanent magnet machine.**

- Small-power machine.

**seperately excited machine**

more controllability since field and power circuit are physically independent.

- one extra power source is required to create excitation field (extra cost)



smile



→ (field excitation)

Construction :- Stationary elements - (Stator)

Rotating elements (Armature)

Part where mech power is converted into electrical (generator)

or electric to mech power (motor)

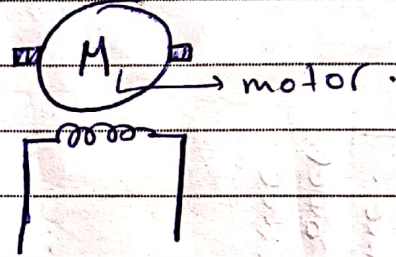
فصل في الآلات الكهربائية



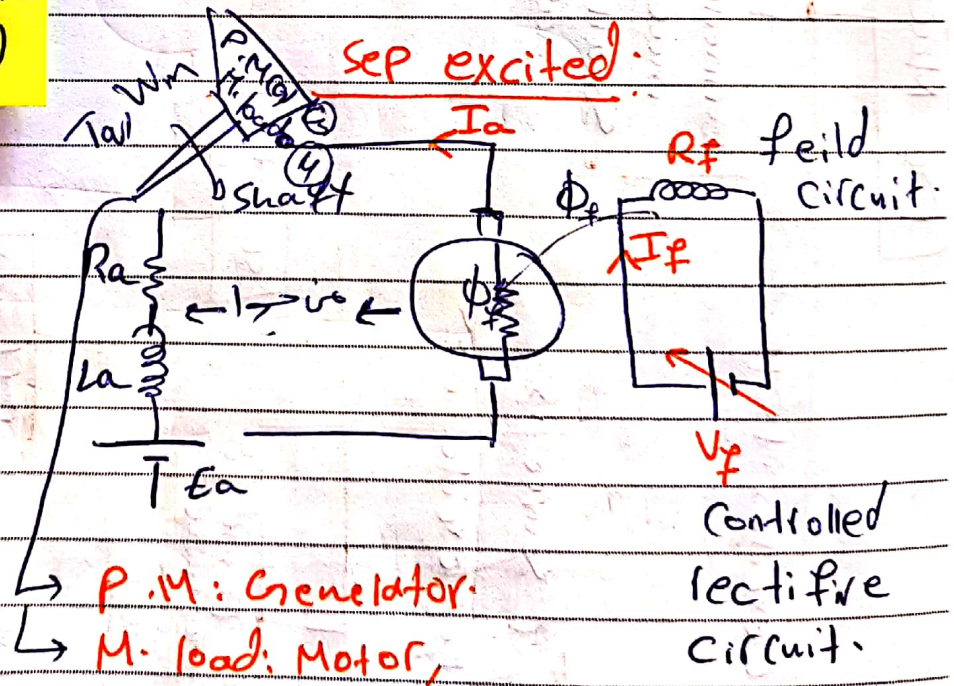
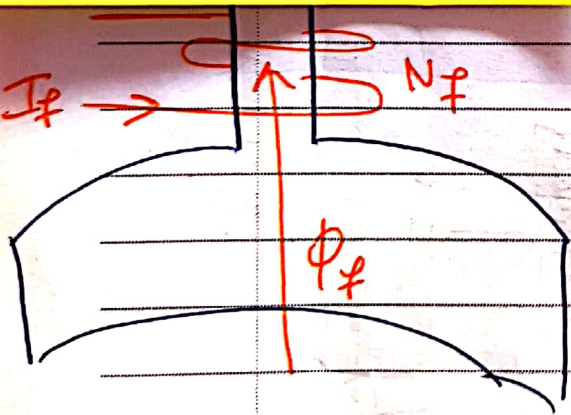
Stator (ferromagnetic yoke core)

DC Machine Symbol

G → generator



sep excited:



P.M: Generator

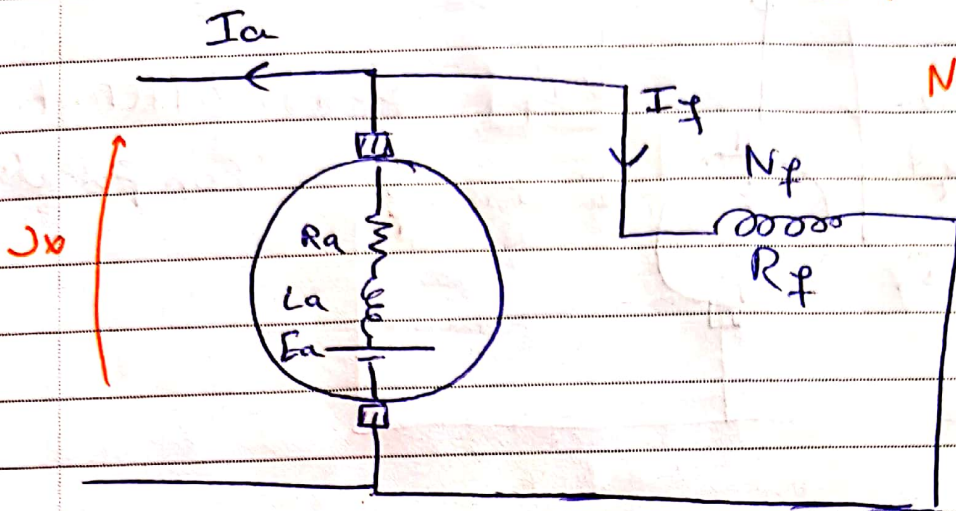
M-load: Motor

Controlled rectifier circuit

Rf very large.



Self Excited Shunt (Self)



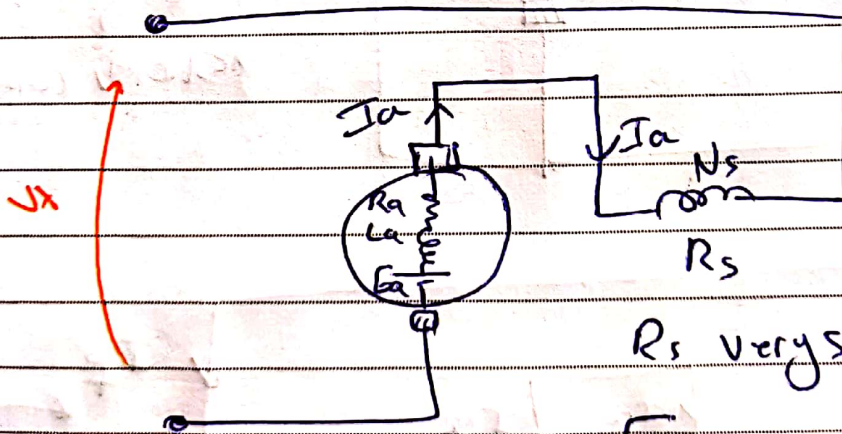
$N_f$ : large.

$A_f$ : thin

$I_f \ll I_a$   
thin  $R_f = \text{huge.}$

$F_f = MMF = I_f \cdot N_f$

Series (Self)



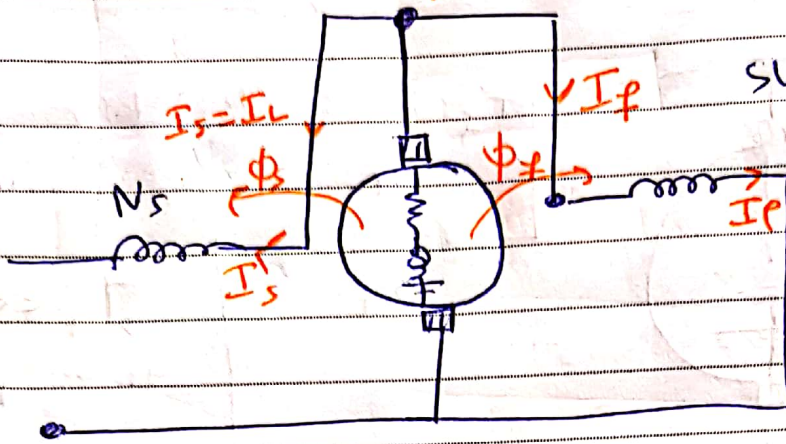
$R_s$  very small ( $\approx R_a$ )

$F_f = I_a * N_s$

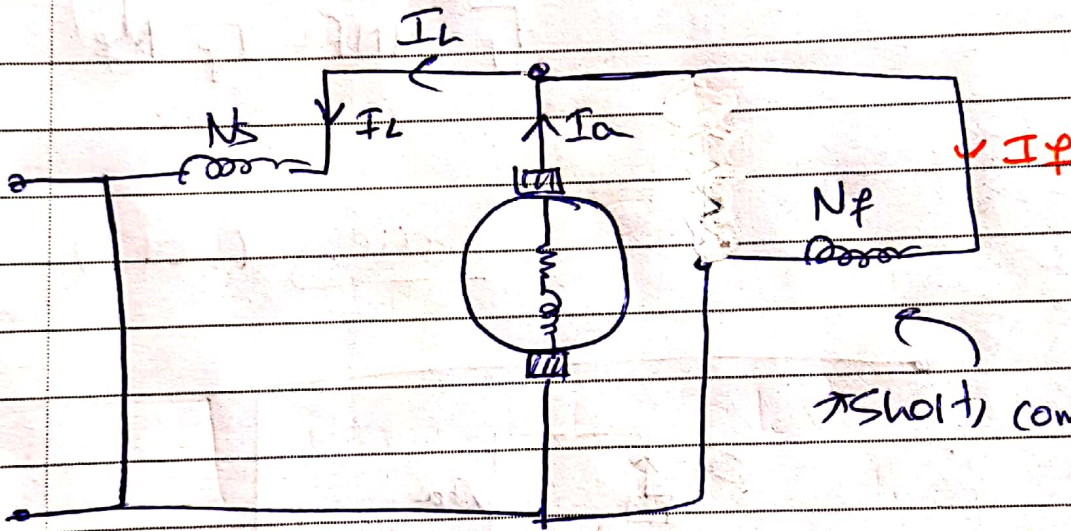
$N_f = \text{few turns.}$



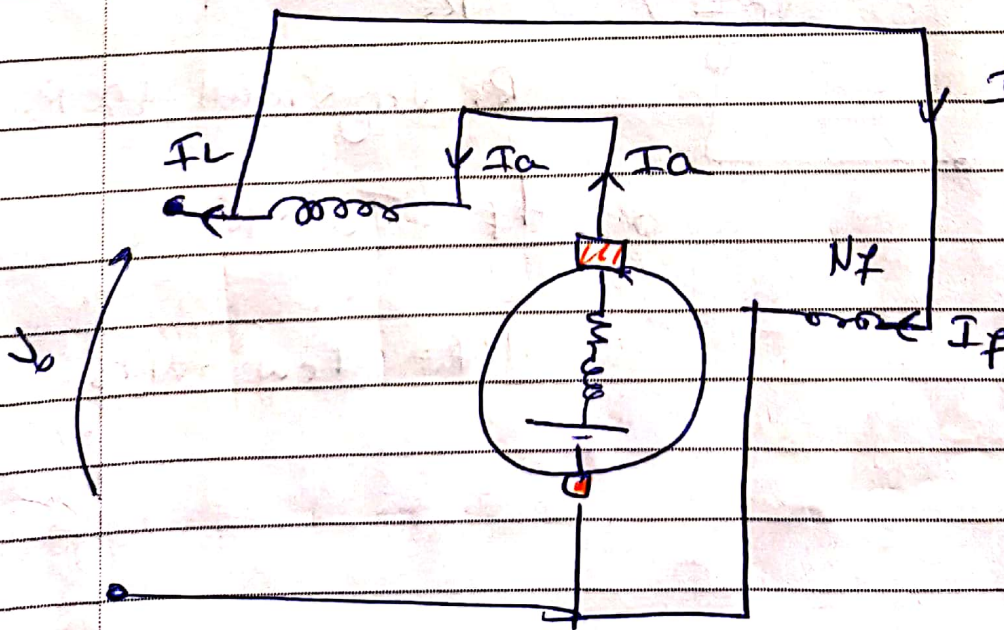
Compond (self).



short, differential Compond.

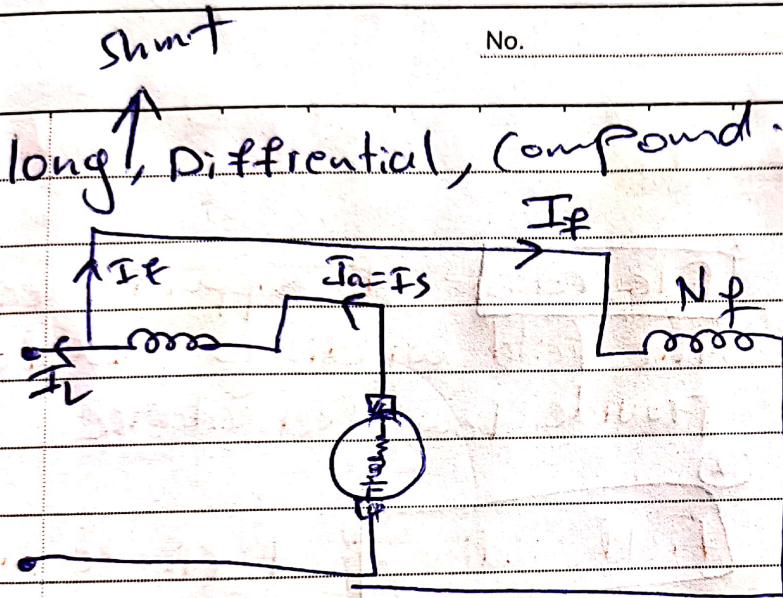


short, commulative



long, commulative

No. \_\_\_\_\_



\* Construction of DC machine.

- Stationary elements

• Stator Yoke:-

- To mech. support + the field (excitation) poles.
- to provide a low reluctance closed path for the flux.



AR

The funct

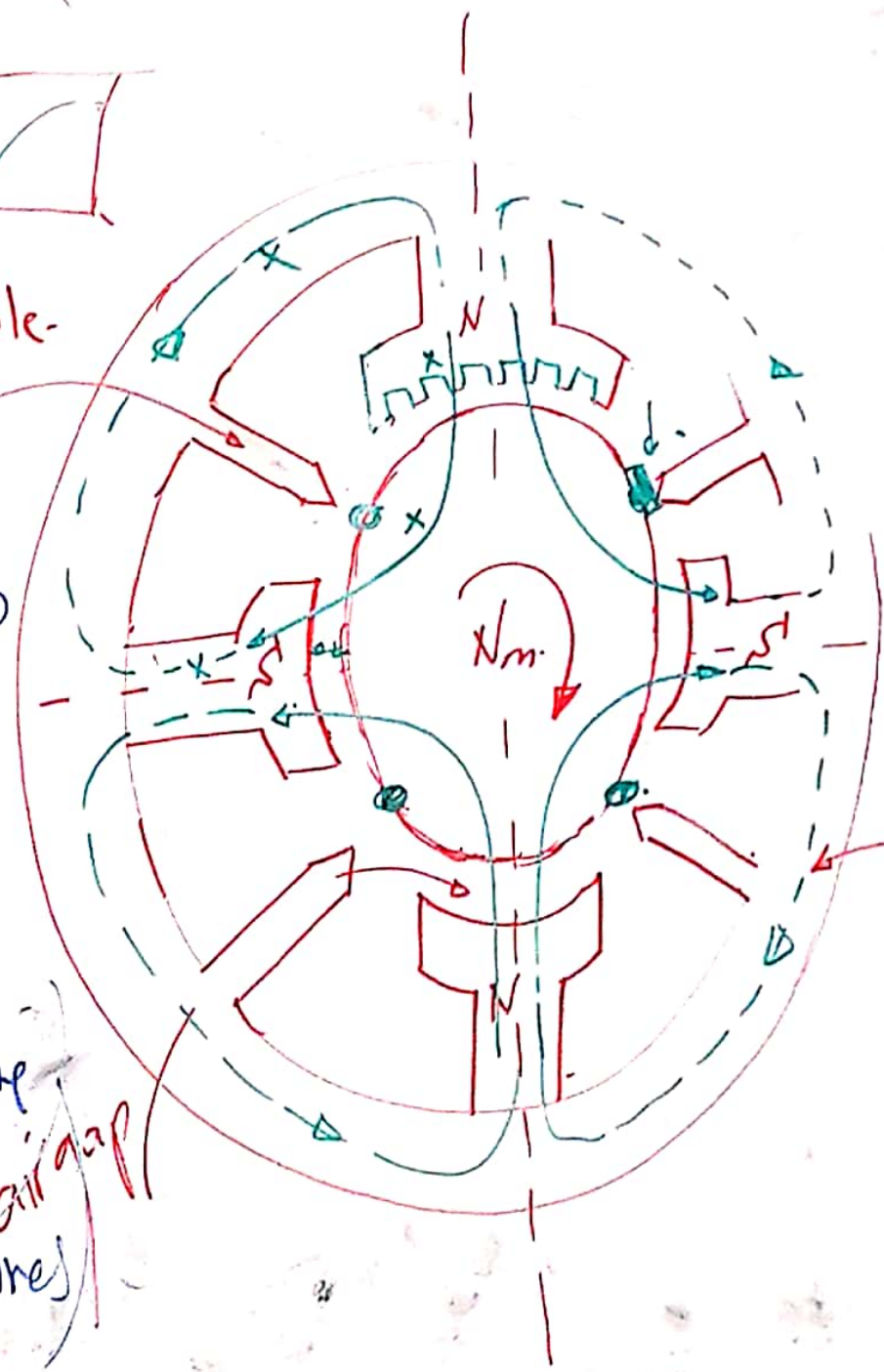
Compo-

velop

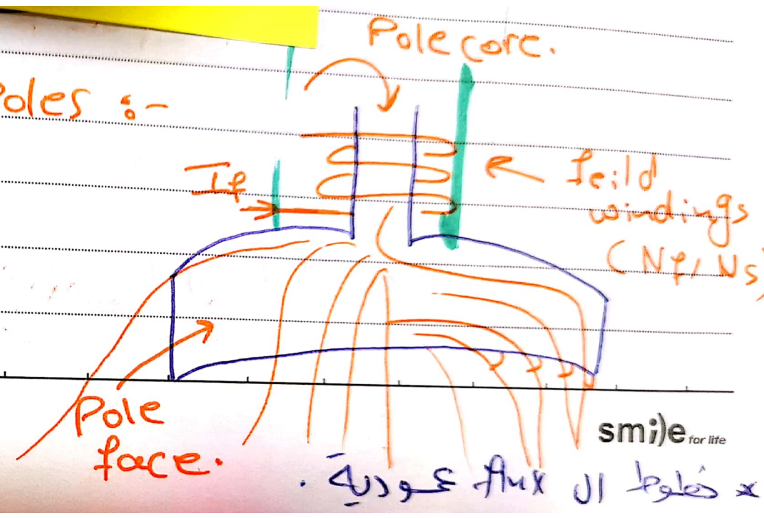
f Machine  
s Machines

air gap

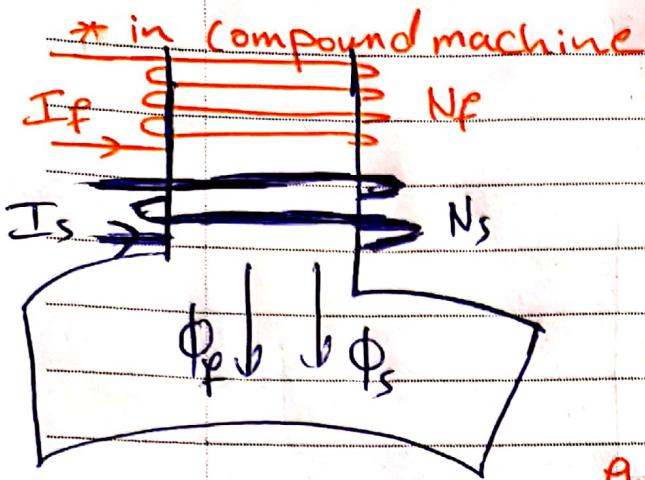
Yoke: (Ferromagnetic



- excitation (field) poles :-
- pole core
- pole face
- field windings.







⊗ Pole cores:- to support mech the field windings and to provide low-Reluctance

⊗ Field windings: to create main flux that is required to develop

EMF in the Armature. ⊖ could be separated from the rotor (Sep. excited).

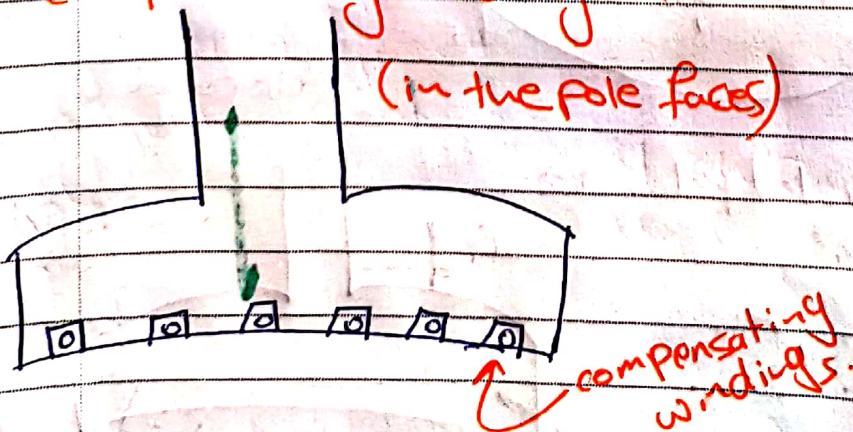
⊖ could be connected in parallel to the rotor (shunt machine).

⊖ could be connected in series to the rotor (series machines).

• **Compoles**:- (commutating poles) to avoid problems of the commutation process.

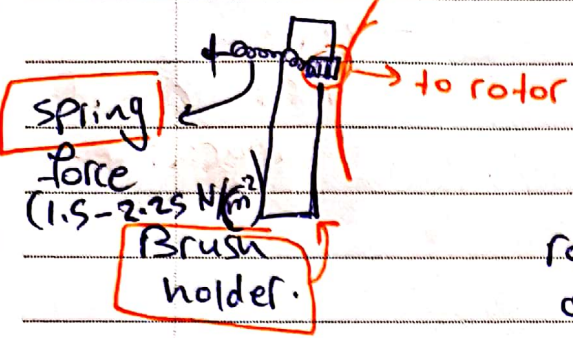
~~Comps~~

• compensating windings.





# • Brush Gear



- Spring
- brush holder
- brush, → graphite.

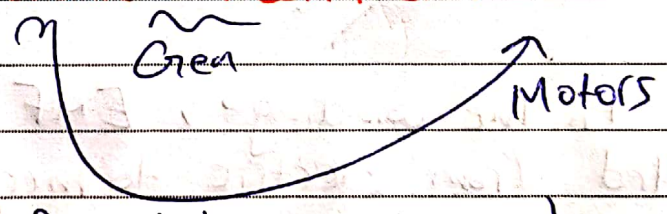
the function → interfacing between rotational and stationary elements of the machine.

## \*\*Rotating elements:-

### a) Armature:-

- Armature core
- shaft [for interconnection between the motor and PM (a) or mech load (M)].
- Armature windings.

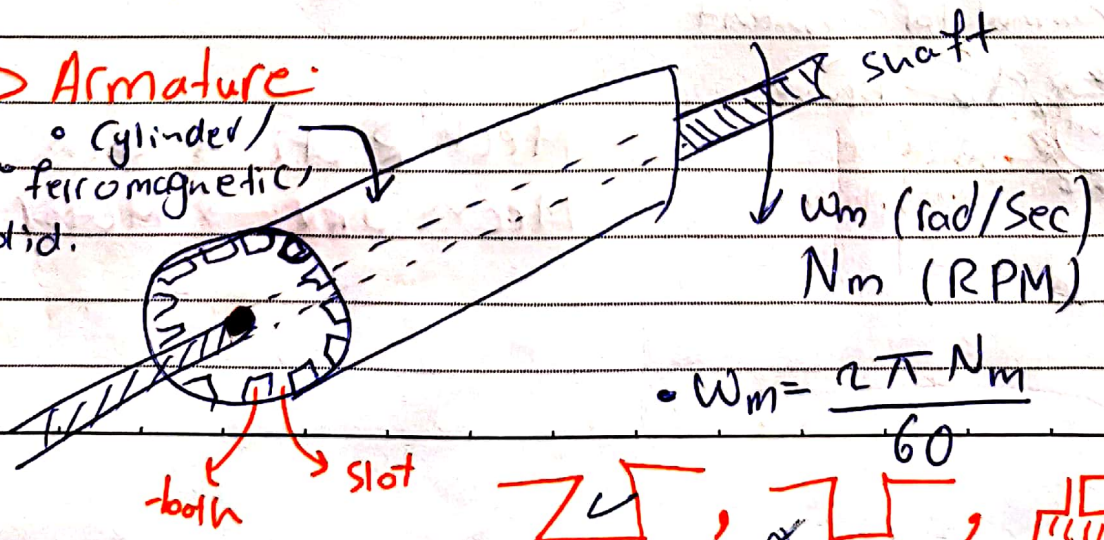
### b) Commutator :- Mechanical Rectifier / Inverter



### c) Bearings :- (to facilitate rotations)

#### → Armature:

- Cylinder / ferromagnetic
- Solid.



$$W_m = \frac{2\pi N_m}{60}$$





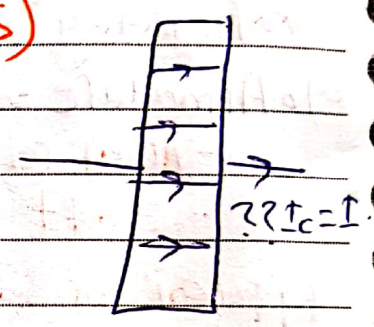
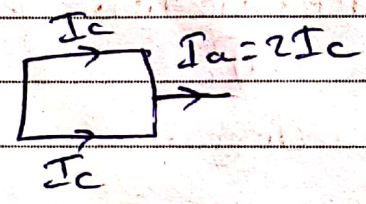
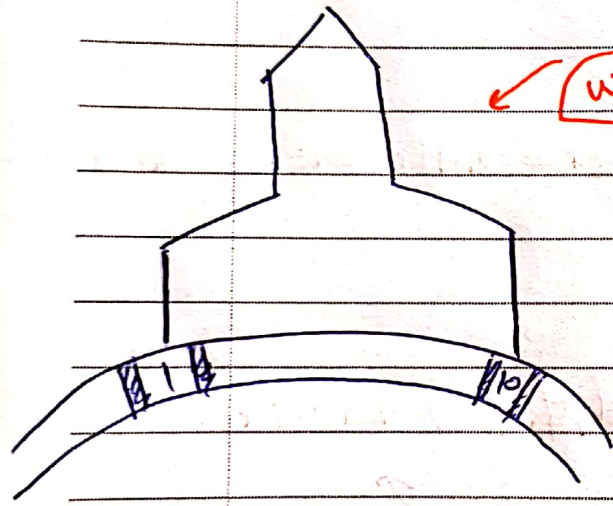
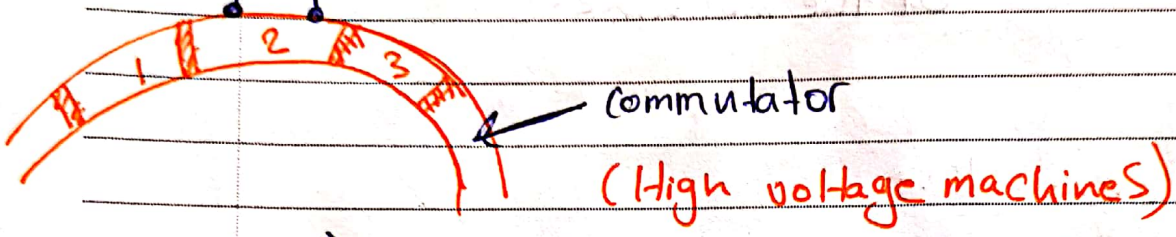
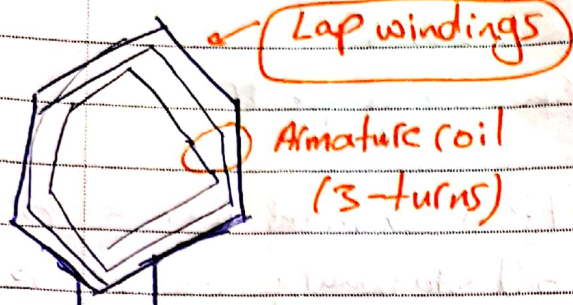
No. \_\_\_\_\_

for high current machine ← type ① - Lap windings  
لفظ تطابق

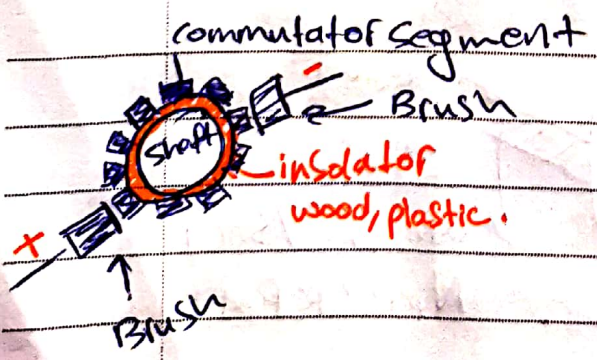
Inside Slots → Armature windings

type ② = Wave windings  
تفریق

type ③ for low current machines

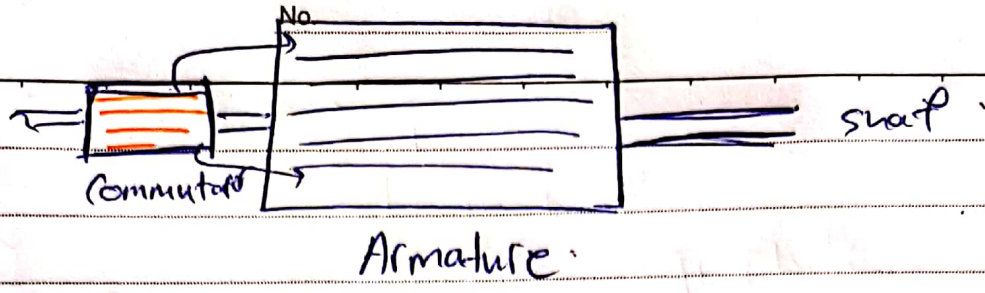


\*\* In the windings, EMF induced and power converted from electric to mechanical (Motor) or from mech to electrical (generator).

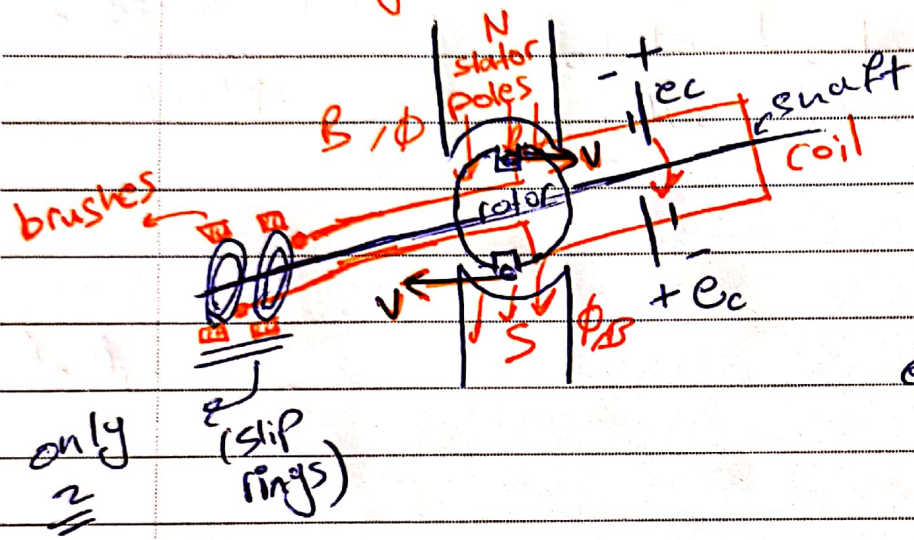


Mech Recti → Gen  
Mech Inv → Motor.





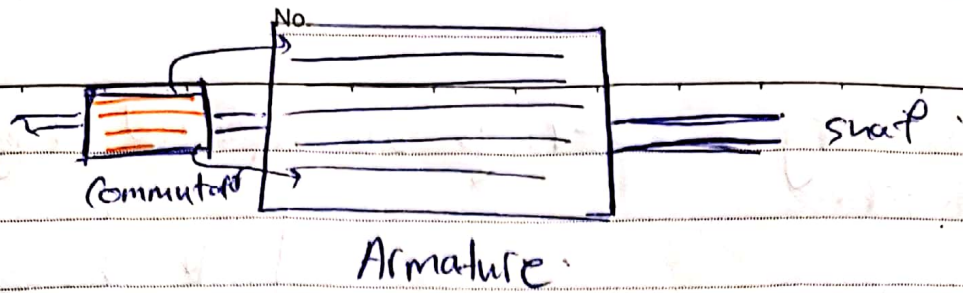
\* Principle of operation:-  
Elementary machine. (Generator)



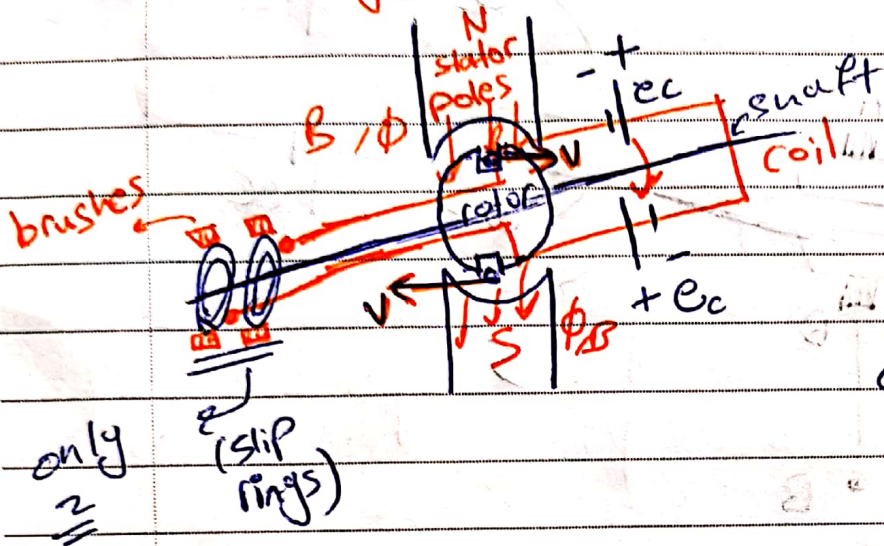
**R.H.R \***  
Flux  $\phi$  \*

تاریخ عریسی دہانہ  
Velocity  $v$   $\phi$   
موتور جبر ارتیہ  $e_c$





\* Principle of operation:-  
Elementary machine. (Generator)

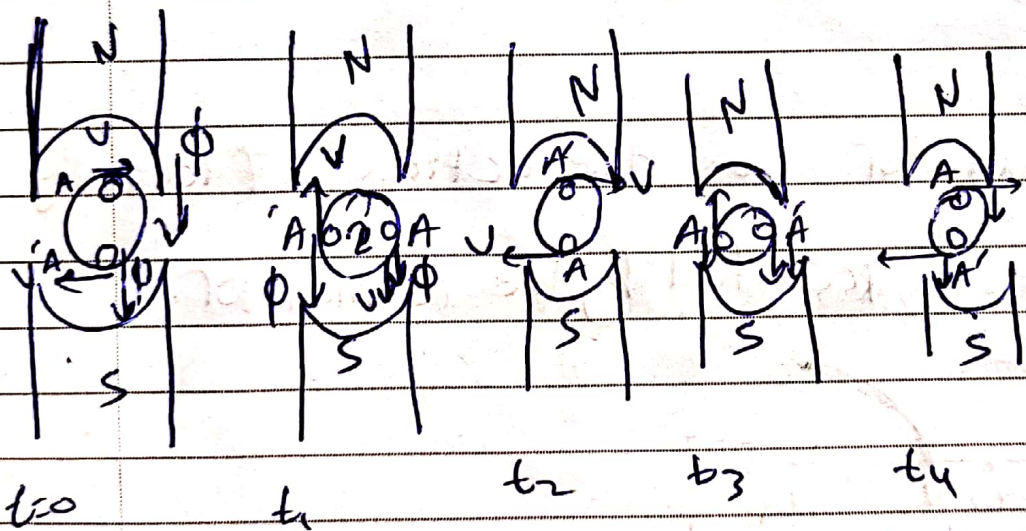


R.H.R \*

Flux ال \*

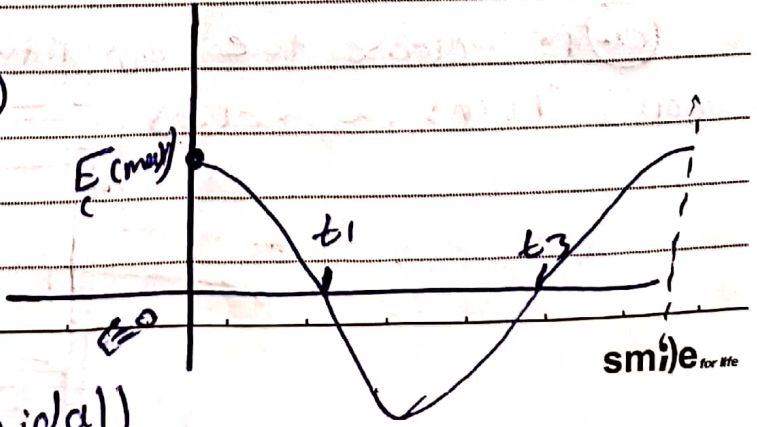
تأثر عدي في ال  
التيه مع ال Velocity  
مما يجر ال

only  
= (slip rings)



$$e_c = \phi B V \sin \theta$$

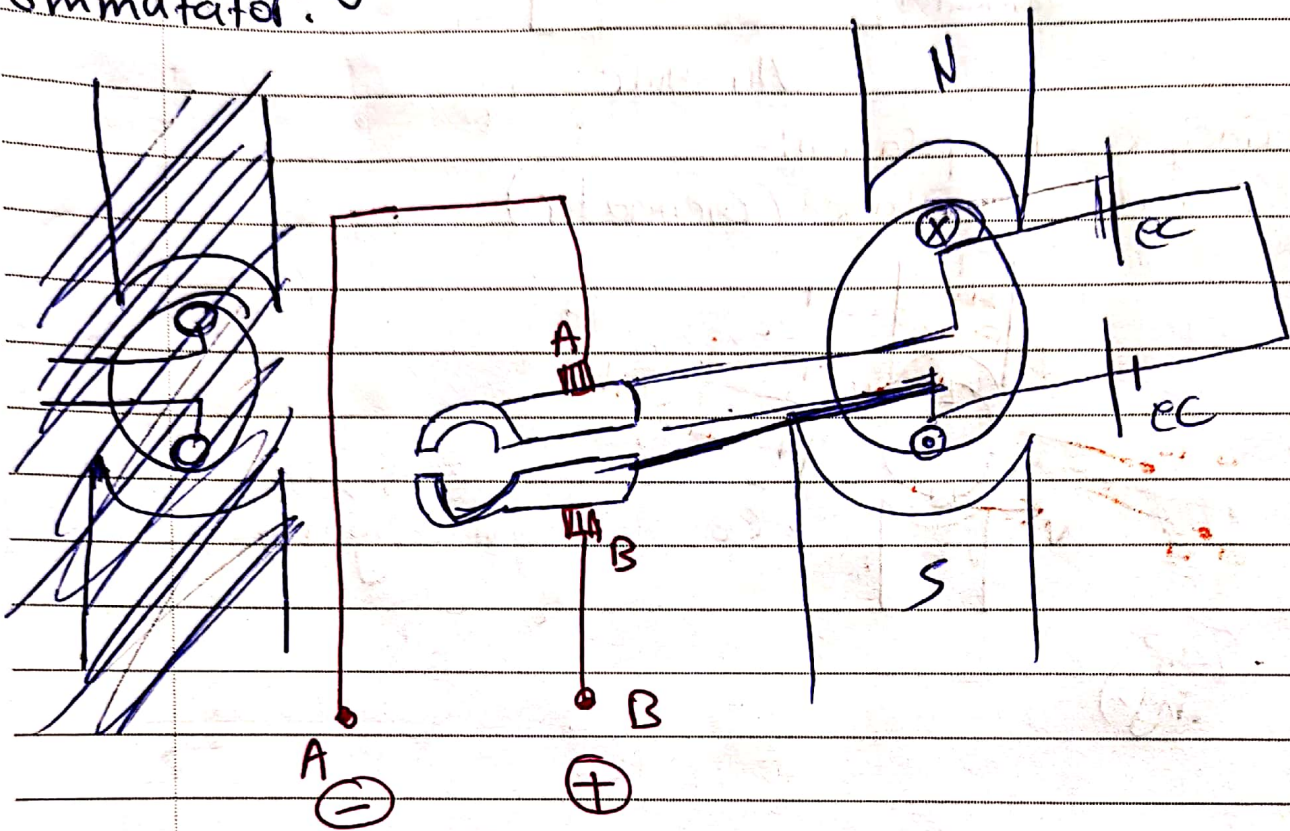
↑ (BV)



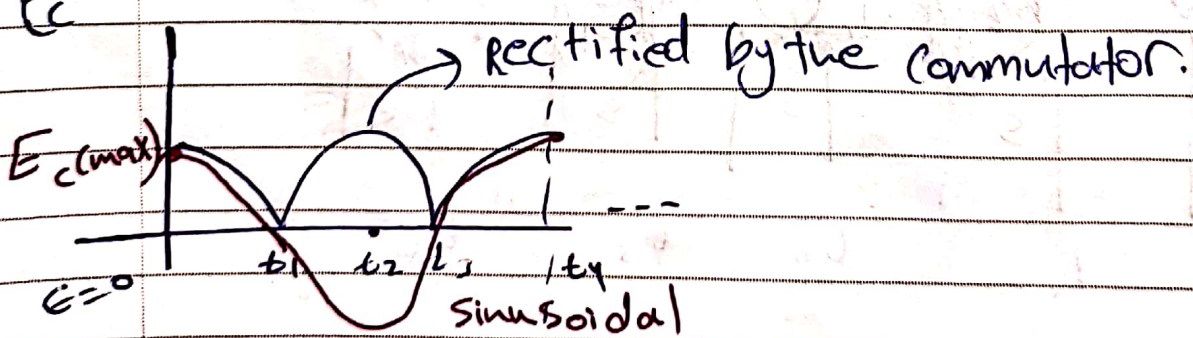
(sinusoidal)



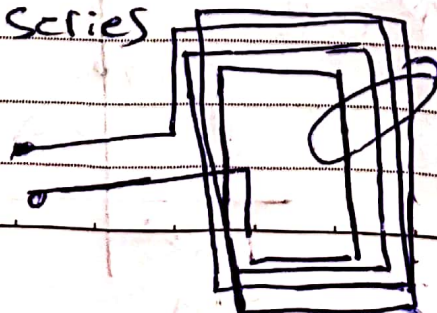
to Rectify : Sliprings are replaced by a two-segment commutator.



\* The commutator has the action of rectifying  $E_c$



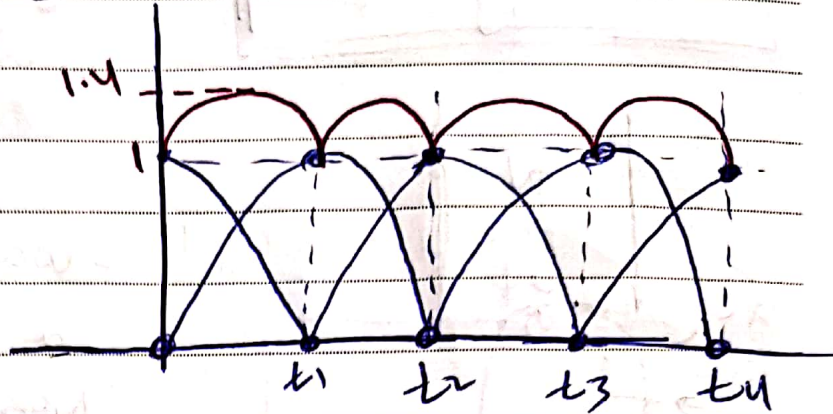
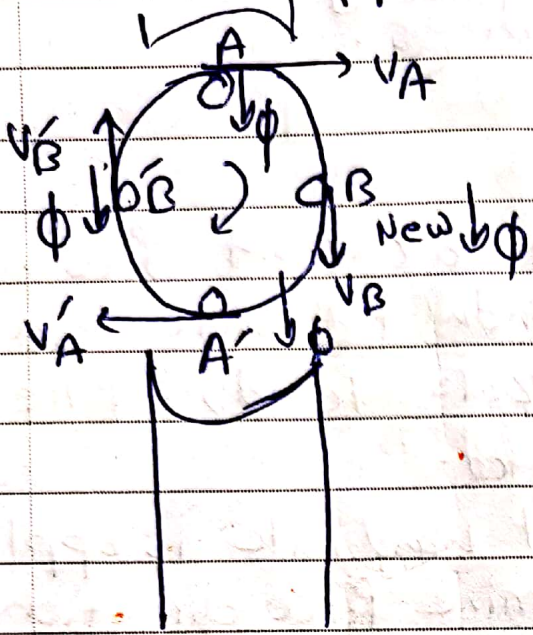
Ⓐ To increase the amplitude of EMF : Add more Turns in series



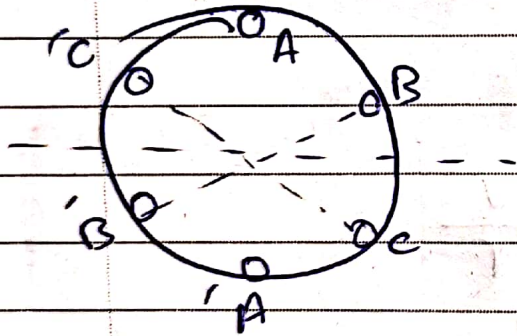
$N=4$   
(means  $E_c$  Now 4 time that of  $N=1$ ) smile for life



(b) to reduce ripple of ec  $\Rightarrow$



3-Coils



To reduce the ripple add one more coil (B-B') in series with (A-A') and phase shifted by  $90^\circ$

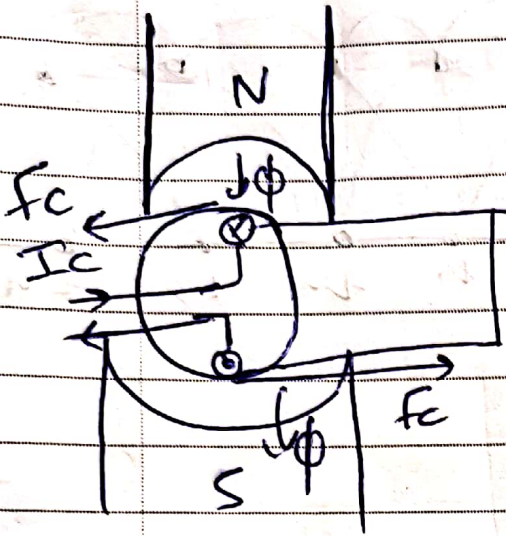
If more coils are added (ex: 3 coils with  $60^\circ$  phase an so on...) then less ripple is exuted with reasonable core, almost pure DC is obtained.



(\*)

**Motoring action**

= you supply current to the windings.



- when a current flow in a conductor within a non-zero field, a force is created.

(left hand rule is applied to determine force direction, and hence the direction of rotation).

**$F = l B I$**  → force

$T_q = F \cdot r$       ,  $\omega_m = \frac{v}{r}$

$P_{\text{mech}} = T_q \cdot \omega_m = F v$   
(mechanical)

$T_q = \frac{P}{\omega_m}$

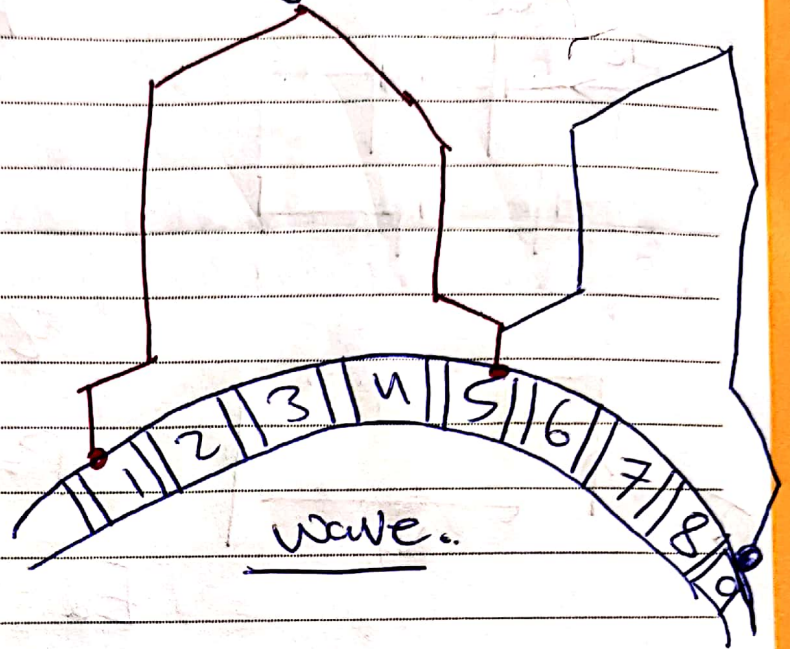
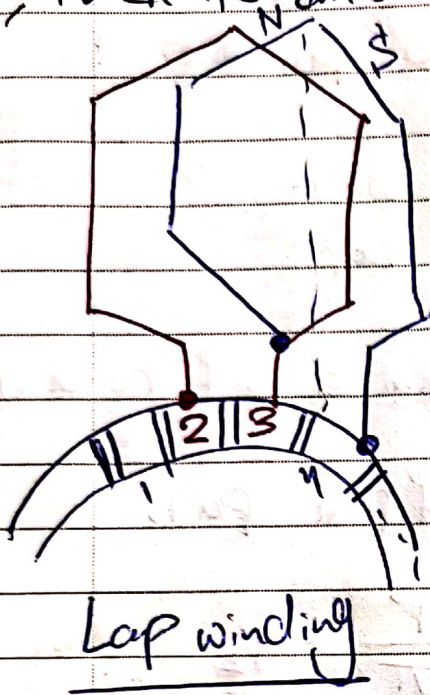
\* Windings :- two types  
 (A) lap-winding  
 (B) wave winding

Armature

\* In all cases :-  
 each coil starts at one commutator segment to slot under certain pole polarity, then back



to another slot under opposite pole polarity, then to another commutator segment.



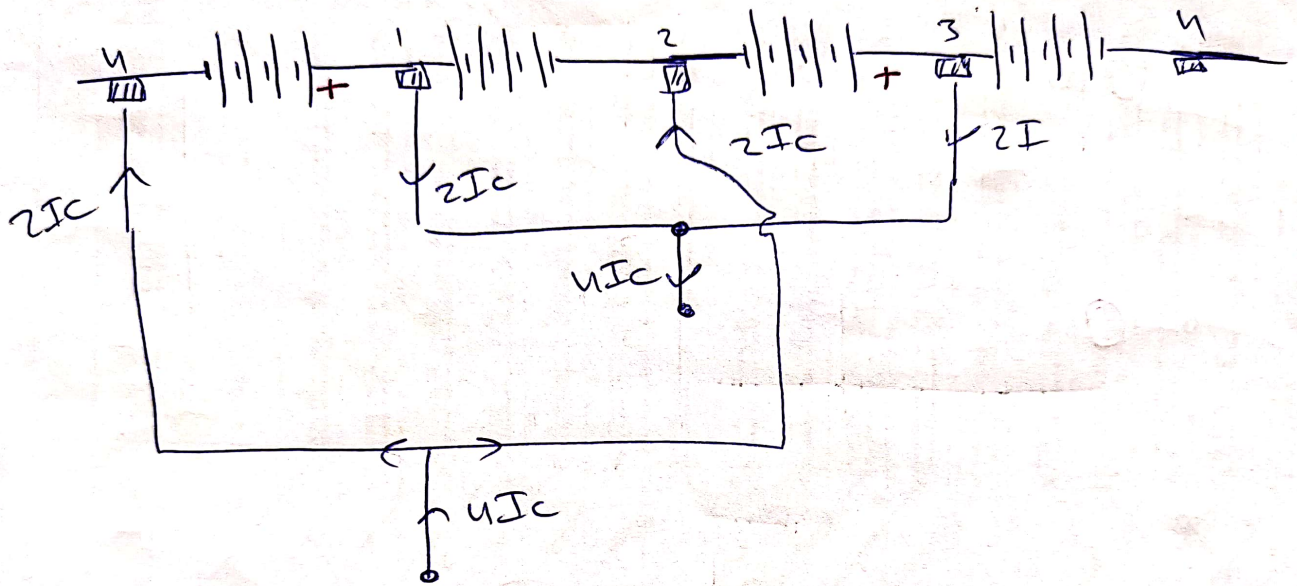
Lap winding : No parallel paths (High current machine).

wave windings : only 2 current paths  
by more coils in series (high voltage machine)





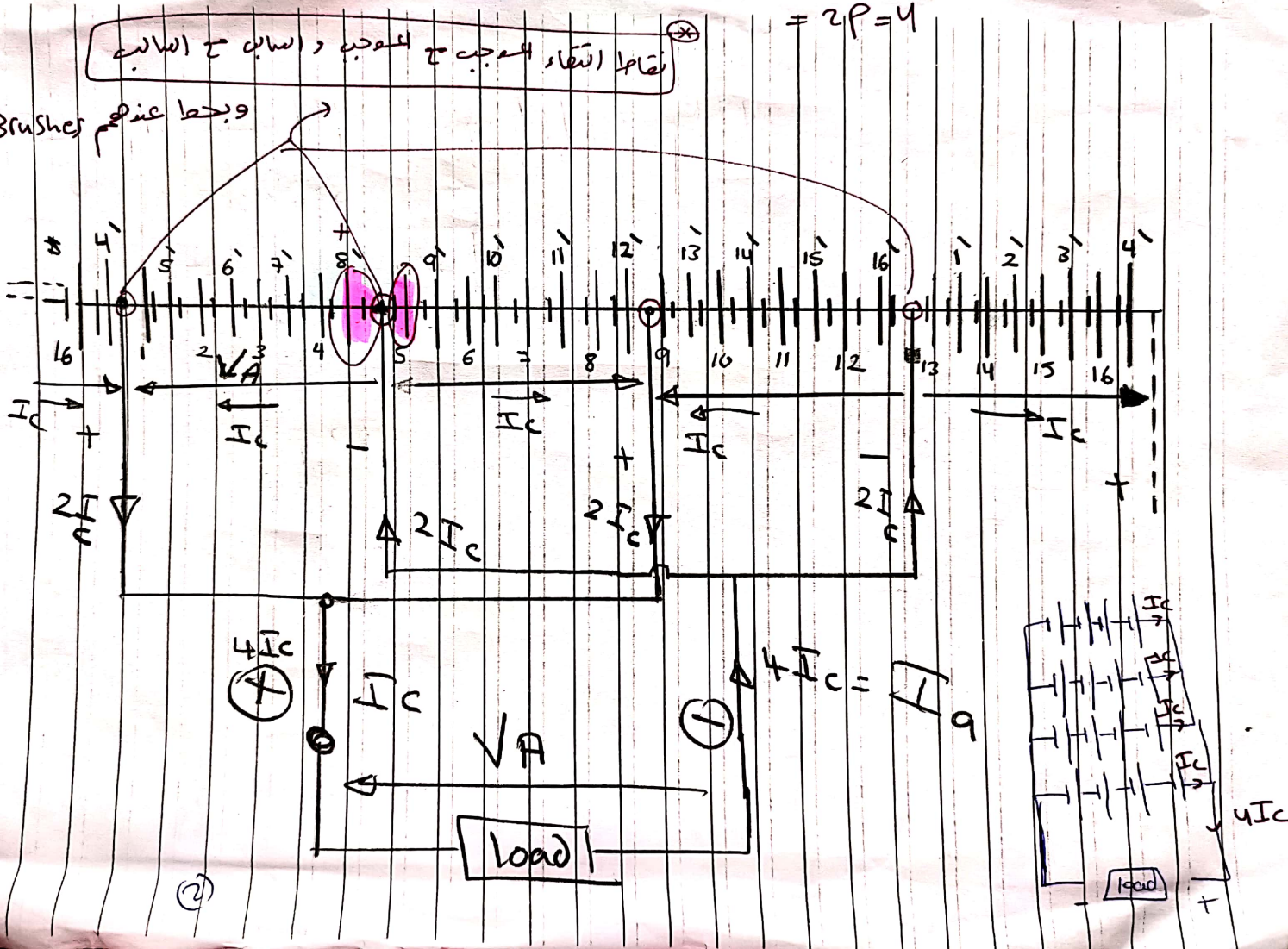




No. of parallel paths  
 $= 2P = 4$

نقاط التقاء المعجب 2 للمعجب 2 السابق - السابق

ويعمل عندهم Brushes





(3)

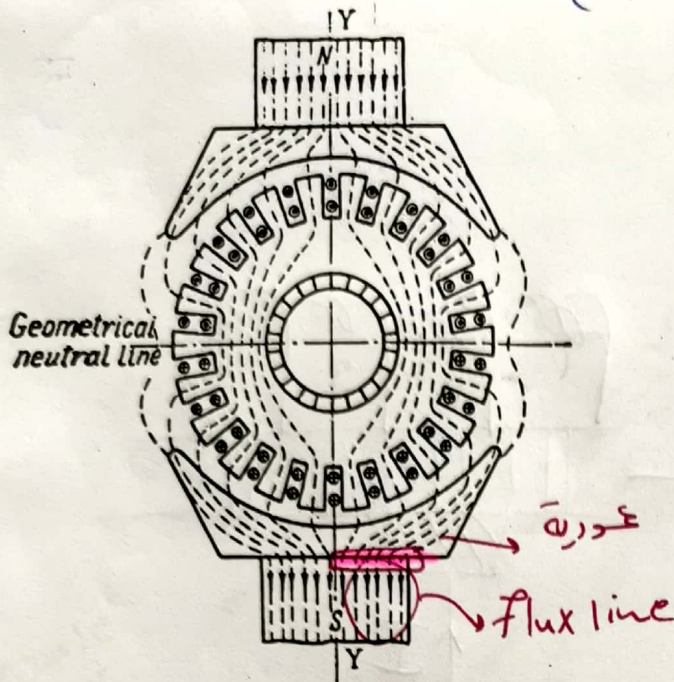


Fig. Main field

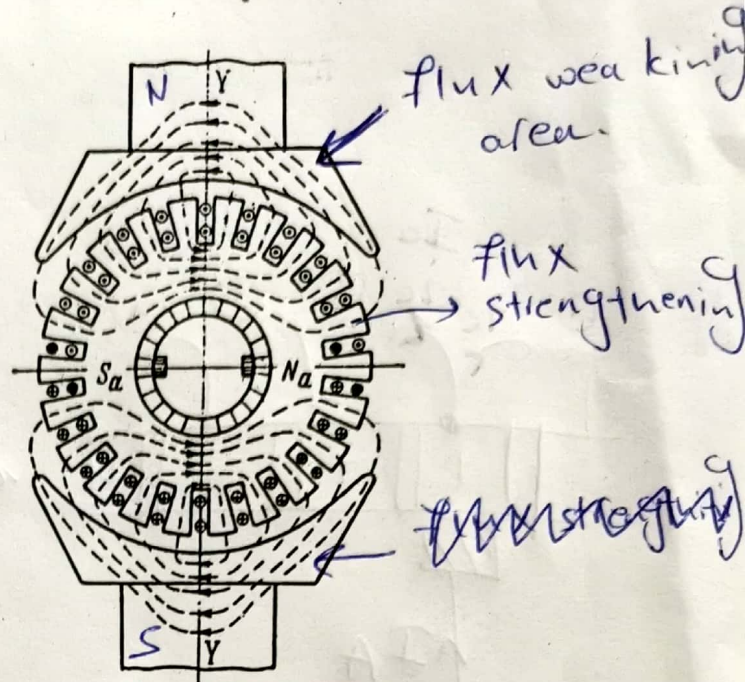


Fig. Armature field (reaction)

due to the current in the armature, the armature flux cause distortion to the main flux leading to shift in the MNA from GNA. This is called Armature reaction.

\* weakening & concentration of flux in the Armature field let the magnetic neutral axis by  $\alpha$ .

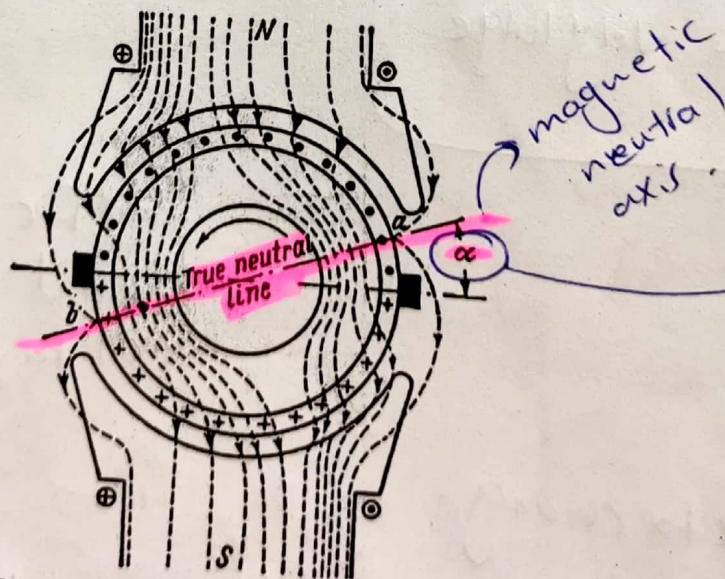
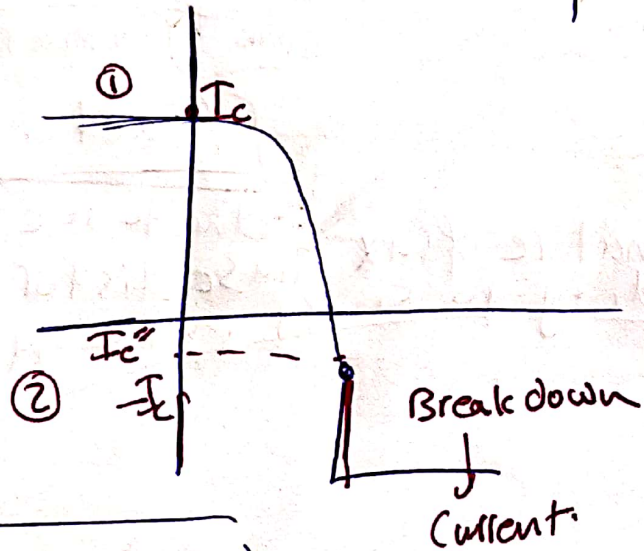
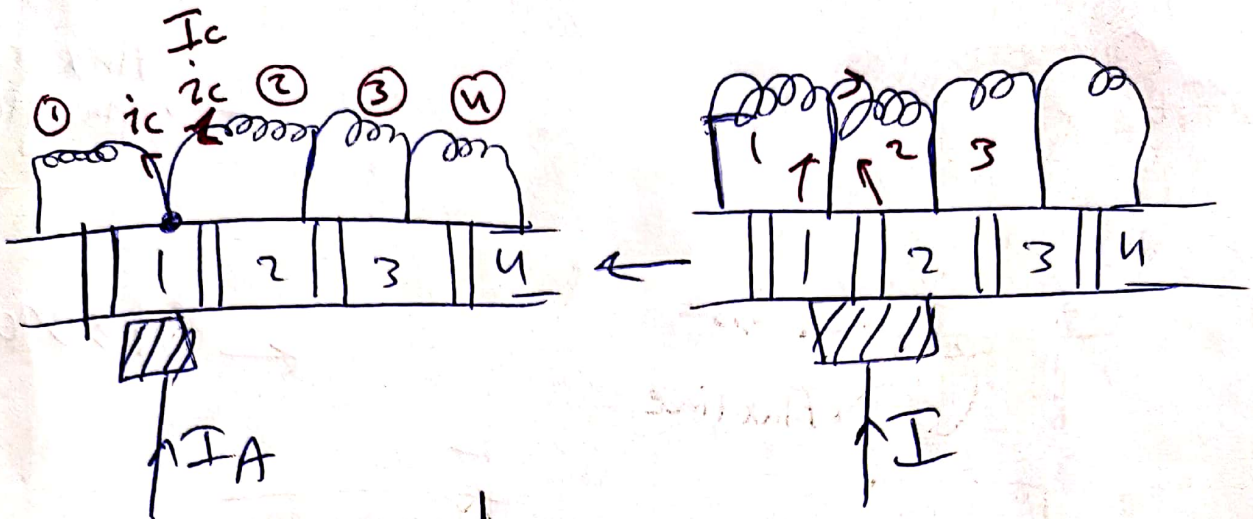


Fig. Distribution of resultant magnetic field of machine when brushes are on geometrical neutral line





$\frac{di}{dt}$  very large

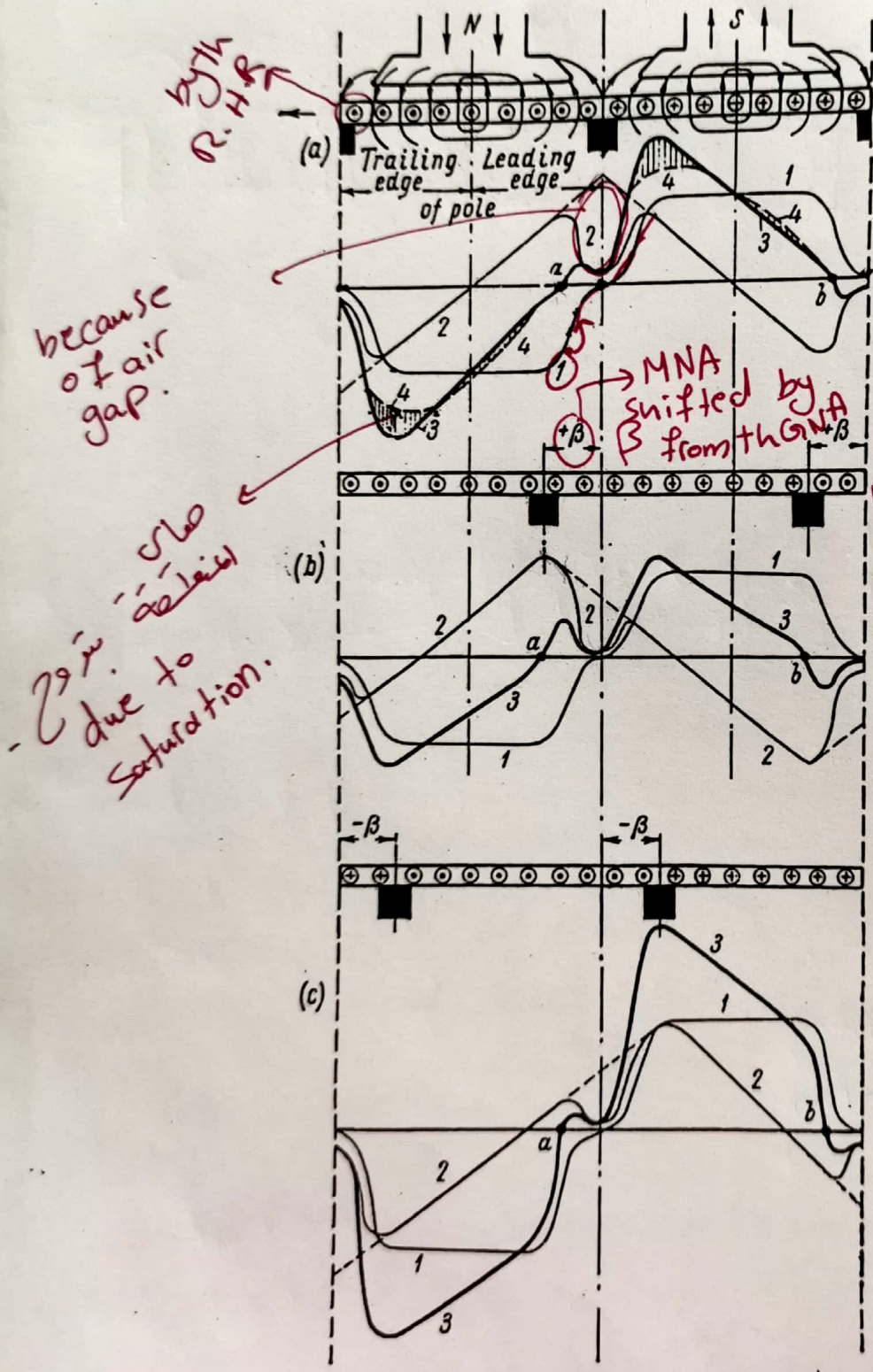
of Dc Motor above rated speed limitation  $n_p$  (2-3)

\* due to change in  $i_c$  from  $I_c''$  in  $I_c$  very large  $e_{is}$  induced, ionize the surrounding area, causing harmful spark

[Commutation Process]. \* sparks increase as speed increases, this is a limit to speed control



To explain the Armature reaction



1: flux distribution

2: due to Armature Current.

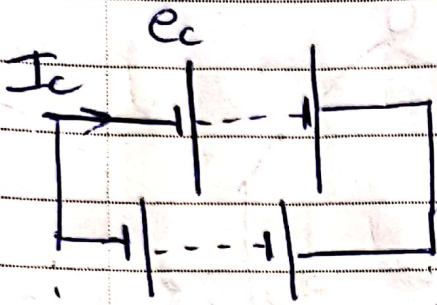
3: due to Armature reactions & main poles

↳ overall average flux under one pole is reduced, leading to reduced  $E_a$ .

Fig. Curves of generator fields for different positions of brushes



## EMF, Torque equations:-



A: No. of parallel paths.

$A = 2$  wave

$A = 2P$  Lap.

$Z$  total number of conductors

$P =$  No of pole pairs

$$E_a = \left( \frac{Z}{A} \cdot \frac{P}{\pi} \right) \cdot \phi_f \cdot \omega_m$$

$$= k_a \cdot \phi_f \cdot \omega_m$$

$k_a$ : Design constant.

$\phi_f$ : Flux per pole.

$\omega_m =$  Rotor speed (rad/sec)

$$\omega_m = \frac{2\pi N_m}{60}$$

where  $N_m$ : speed in revolution per minute.  
(R.P.M)

$$E_a = k_a \cdot k_f \cdot I_f \cdot \frac{2\pi N_m}{60}$$

$$E_a = k I_f \cdot N_m$$



EMF developed.

$$E_a = \left( \frac{Z}{A} \cdot \frac{P}{\pi} \right) \cdot \phi_f \cdot \omega_m$$

$$= k_a \phi_f \omega_m$$

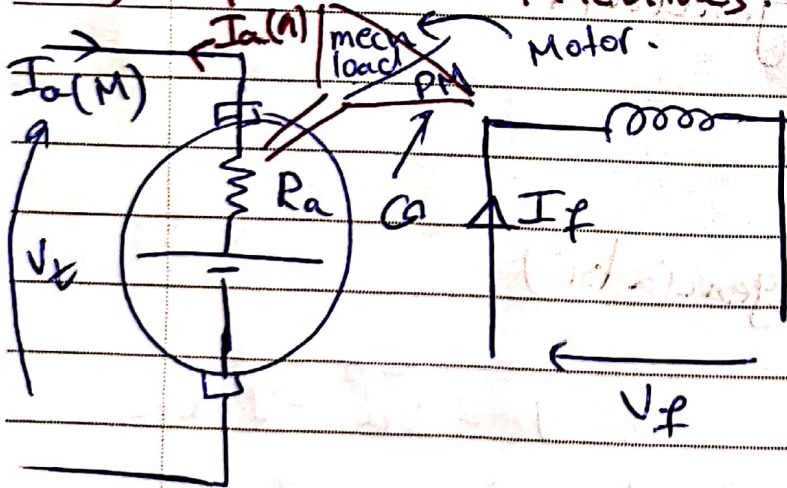
$T_a$  : developed armature torque.

$$T_a = T_d = k_a \cdot \phi_f \cdot I_a$$

$$T_a = T_d = \frac{P_a = P_d}{\omega_m}$$

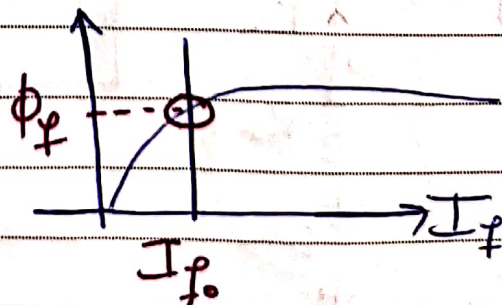
Circuit model

a) Sep-excited Machines.



$$I_f = \frac{V_f}{R_f}$$

$$\phi_f = k_f I_f$$



Cren

$$V_t = E_a + I_a R_a$$

Motor



$$P_d = P_a = E_a I_a$$

$$P_{sh} = P_o = P_a - P_{rot}$$

$P_{rot}$  = rotational mechanical losses  
 =  $P_{mech}$  friction +  $P_{windings}$

$$P_{cu} = I_a^2 R_a$$

→ copper loss

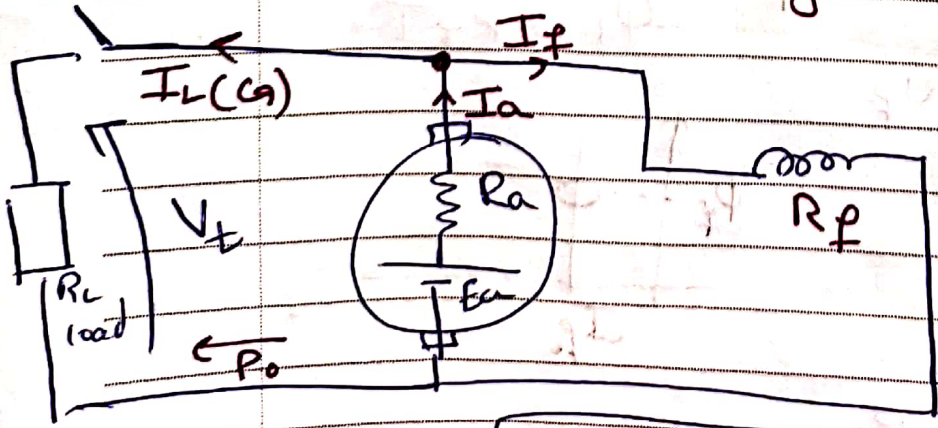
$$P_{in} = V_t I_a \text{ (Motor)}$$

$$P_a = P_{in} - P_{cu} \text{ (Motor case)}$$

$$P_o = I_a V_t \text{ (generator)}$$

$$P_a = P_o \text{ generator}$$

B \* Shunt machine. (generator)



$$V_t = E_a - I_a R_a$$

$$P_o = V_t I_L$$

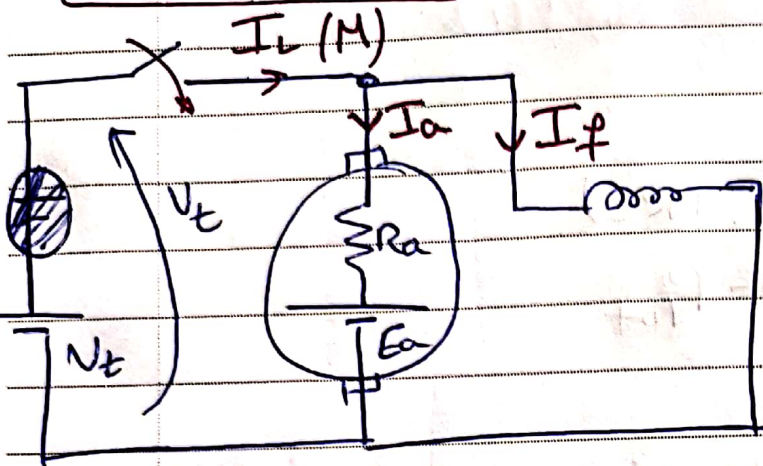
$$P_d = E_a I_a$$

$$P_d = P_o + I_a^2 R_a$$

$$I_a = I_L + I_f$$



**shunt motor**



$$V_t = E_a + I_a R_a$$

$$P_a = E_a I_a$$

$$P_{in} = V_t I_L$$

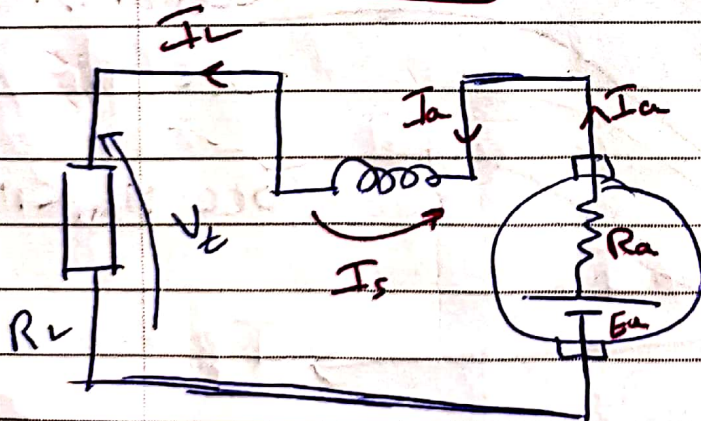
$$P_a = P_{in} - I_a^2 R_a$$

$$P_o = P_a - P_{rot}$$

$$P_a = P_{in} - I_a^2 R_a$$

$$\frac{V_f}{R_f} = I_f$$

**Series generator**



$$V_t = E_a - I_a R_a - I_s R_s$$

$$= E_a - I_a (R_a + R_s)$$

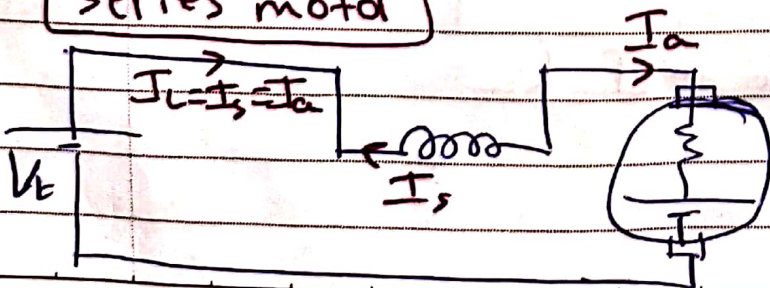
$$I_a = I_s = I_L$$

$$P_d = E_a I_a$$

$$P_o = I_L V_t$$

$$P_o = P_d - I_a^2 (R_s + R_a)$$

**Series motor**



$$I_s = I_a = I_L$$

$$V_t = E_a + I_a (R_a + R_s)$$



$$P_{in} = V_t I_a$$

$$P_a = P_d = E_a I_a$$

$$P_a = P_d = P_{in} - I_a^2 (R_s + R_a)$$

$$P_o = P_{sh} = P_a - P_{rot}$$

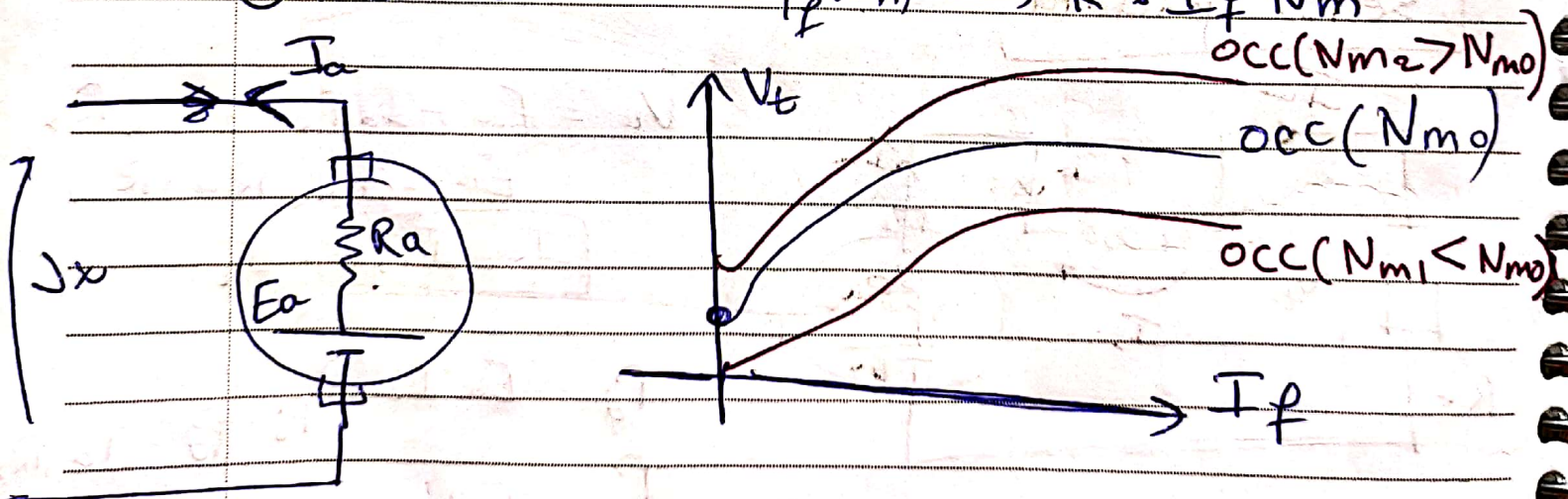
\*\* Generators characteristics: -

① OCC (open-circuit ch)  $\Rightarrow V_t = f(I_f)$

② load characteristics  $\rightarrow V_t = f(I_a)$

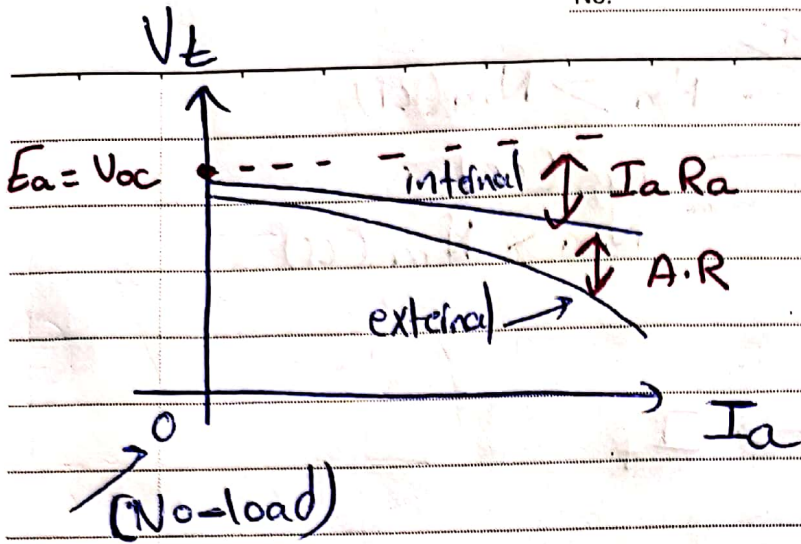
Self excited

$$① V_{oc} = E_a = K_a \phi_f \omega_m \rightarrow K \cdot I_f N_m$$



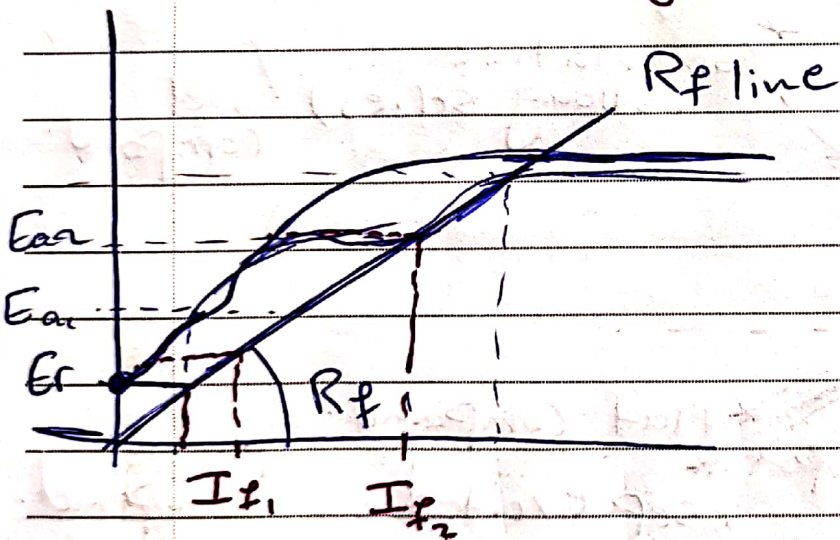
$$② V_t = E_a - I_a R_a$$





Shunt generator :-

Residual flux is very important, since if  $\phi_r = 0, E_r = 0$   
 & no building of  $E_a$



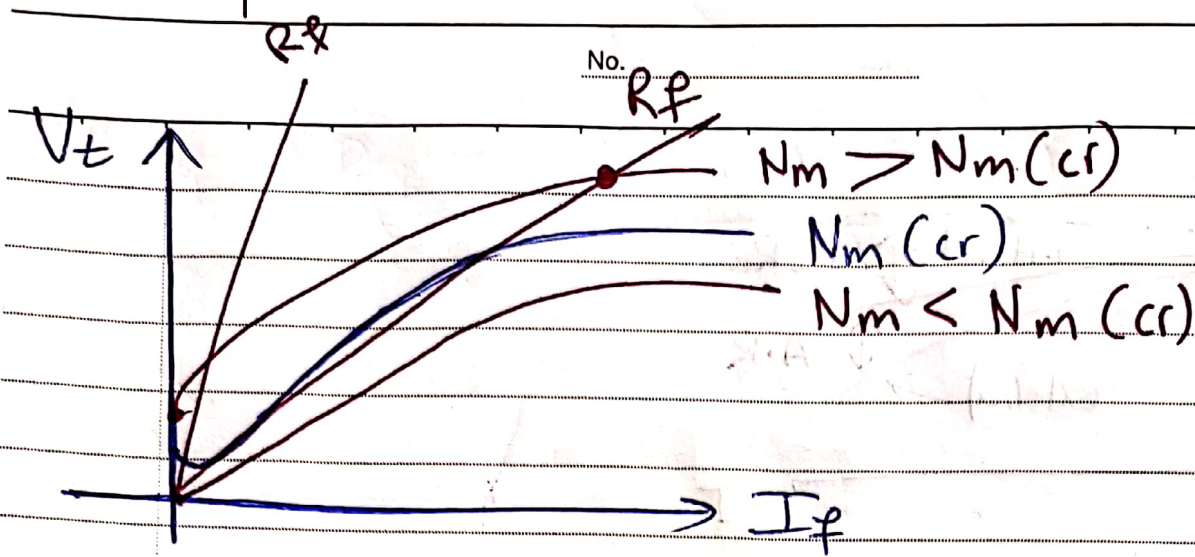
\* Failure to build EMF is due to the following reasons :-

① No residual flux.

in this case, reexcite separately the field coils by external DC source.  
 but  $I_{f1}$   $E_a$  fails to build ??

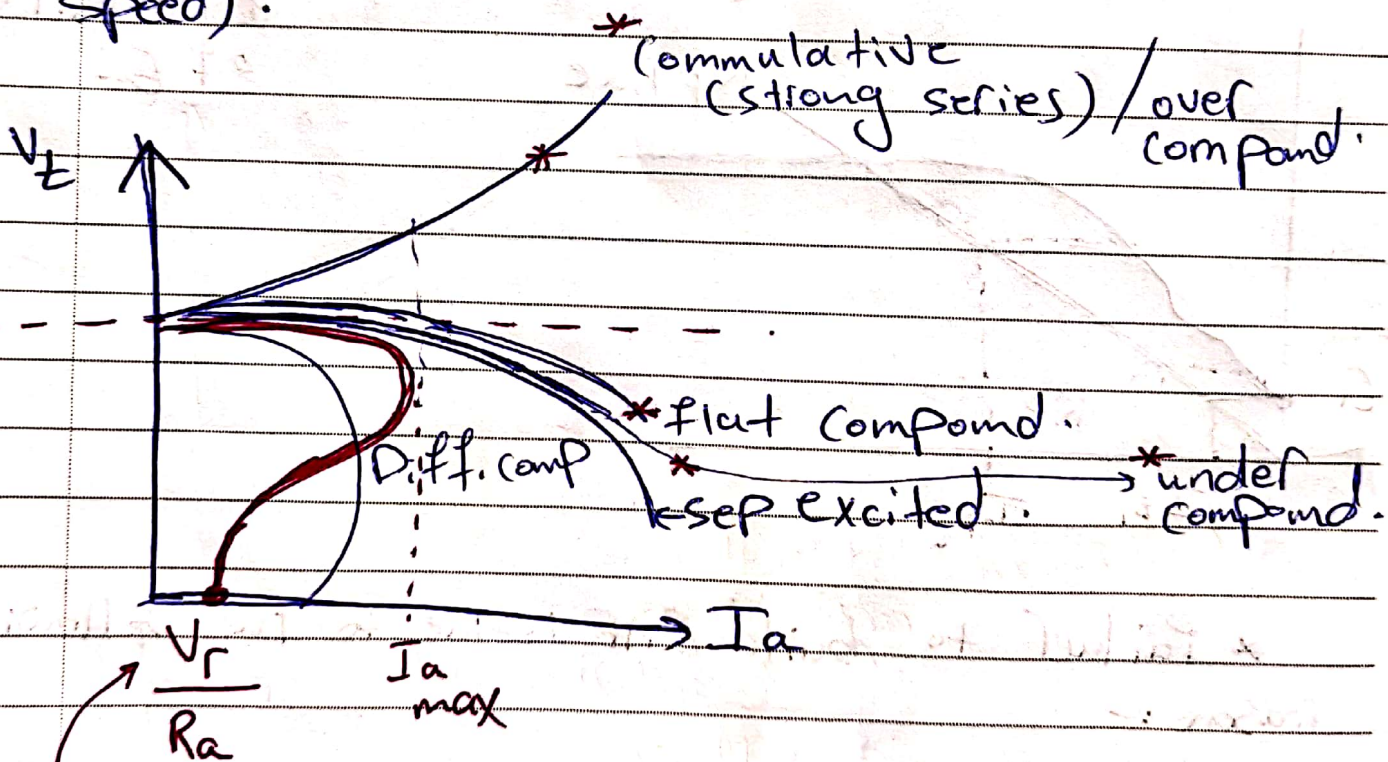
②  $N_m < N_m(Cr)$  . For certain  $R_f$





$I_{f_{not}} \underline{\underline{??}}$

③  $I_f R_f > R_f$  (critical) (for certain speed).



$I_{sc}$   
short  
circuit  
current

- short ckt of shunt Gen is not a problem.

- cumulative generator to compensate for voltage regulation, (flat compound commutative

results in zero V. Reg at full load).

smile<sub>for</sub>me



## Motors 8-

$$N_m = \frac{f}{w_m} (T_q)$$

$$N_m = \frac{f}{I_a} (\frac{I_a}{k_a \phi})$$

### sep excited Motor

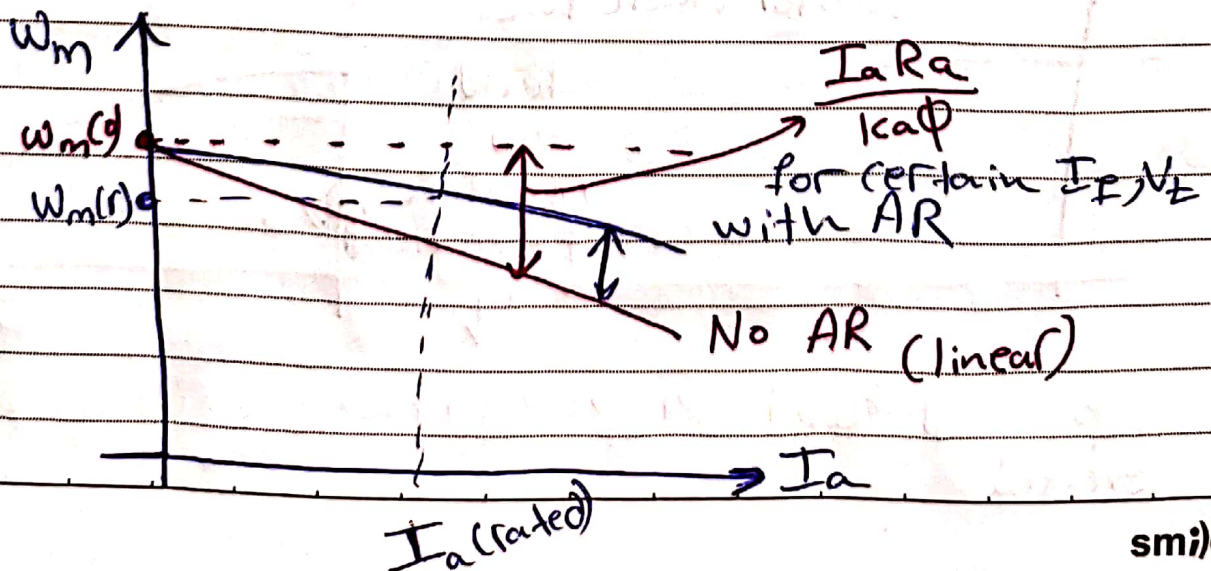
$$E_a = V_t - I_a R_a$$

$$k_a \phi w_m = V_t - I_a R_a$$

$$w_m = \frac{V_t}{k_a \phi} - \frac{R_a}{k_a \phi} \cdot I_a$$

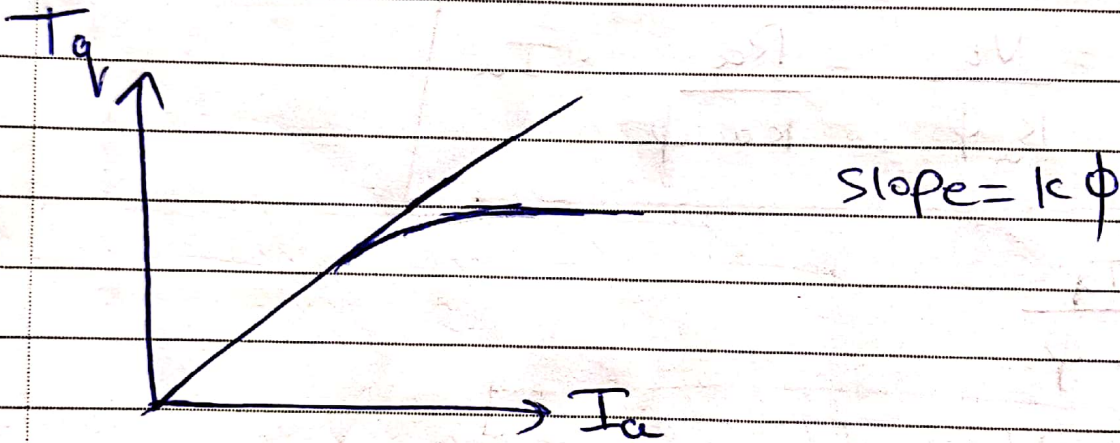
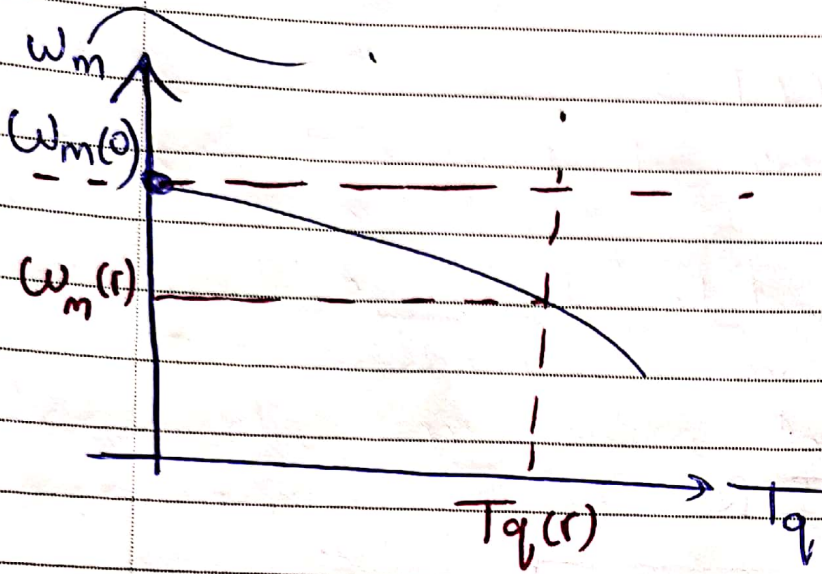
$$I_a = \frac{T_q}{k_a \phi}$$

$$w_m = \frac{V_t}{k_a \phi} - \frac{R_a \cdot T_q}{(k_a \phi)^2}$$

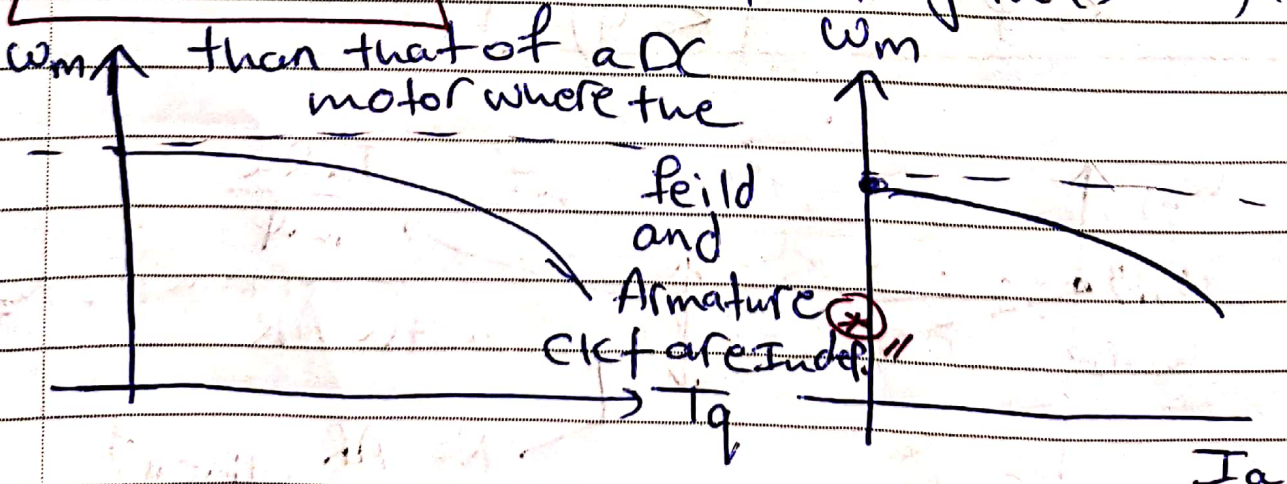




$$\text{Speed Reg} = \frac{\omega_m(nL) - \omega_m(r)}{\omega_m(r)}$$



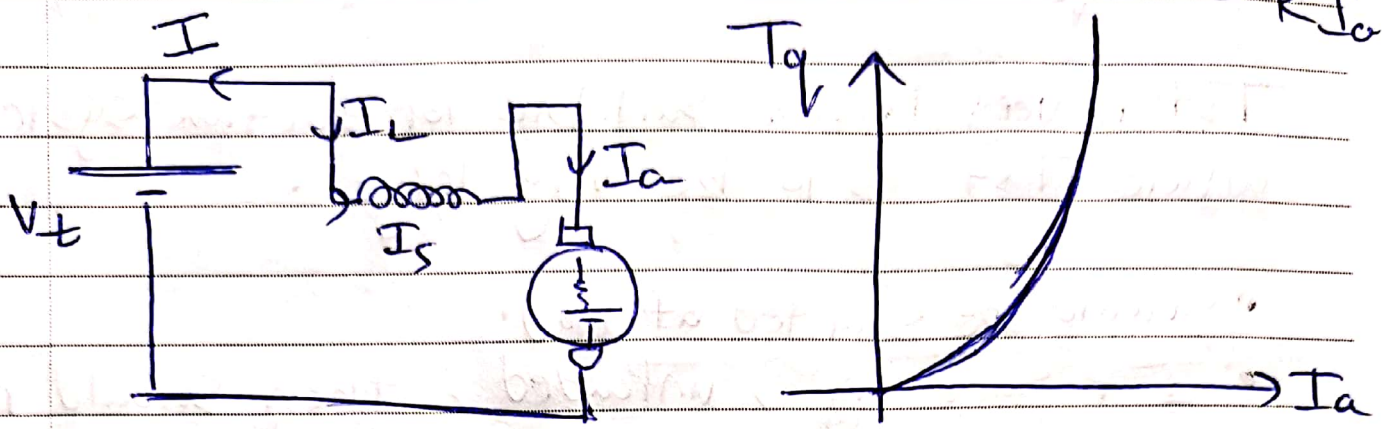
**Shunt motor** - controllability in (shunt) less than that of a DC motor where the



sep. dī qīp k ar k hāḡāḡ wā - excited

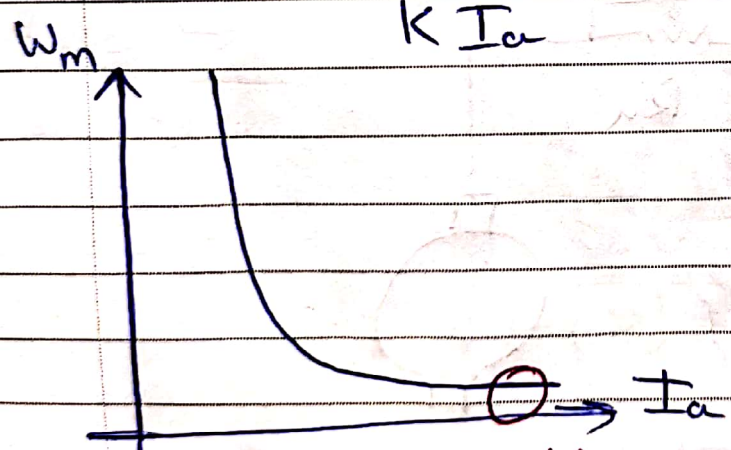


**Series motor** :-  $T_q = k_a \phi I_a = k_a (k I_a) I_a = k I_a^2$



$$E_a = V_t - I_a (R_s + R_a) \rightarrow k I_a \omega_m = V_t - I_a (R_a + R_s)$$

$$\omega_m = \frac{V_t}{k I_a} - \frac{(R_a + R_s)}{k}$$



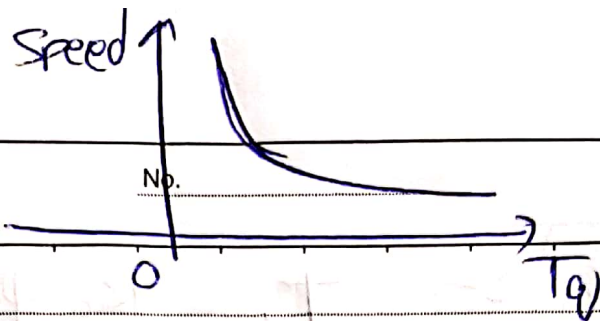
No load  
light load

$\omega_m$  Very ~~low~~ low  
 $I_a$  high.

$$\omega_m = \frac{V_t}{k I_a} - \frac{(R_a + R_s)}{k}$$

$$T_q = k I_a^2 \rightarrow I_a = \sqrt{\frac{T_q}{k}}$$

$$\omega_m = \frac{V_t}{k \sqrt{\frac{T_q}{k}}} - \frac{(R_a + R_s)}{k}$$



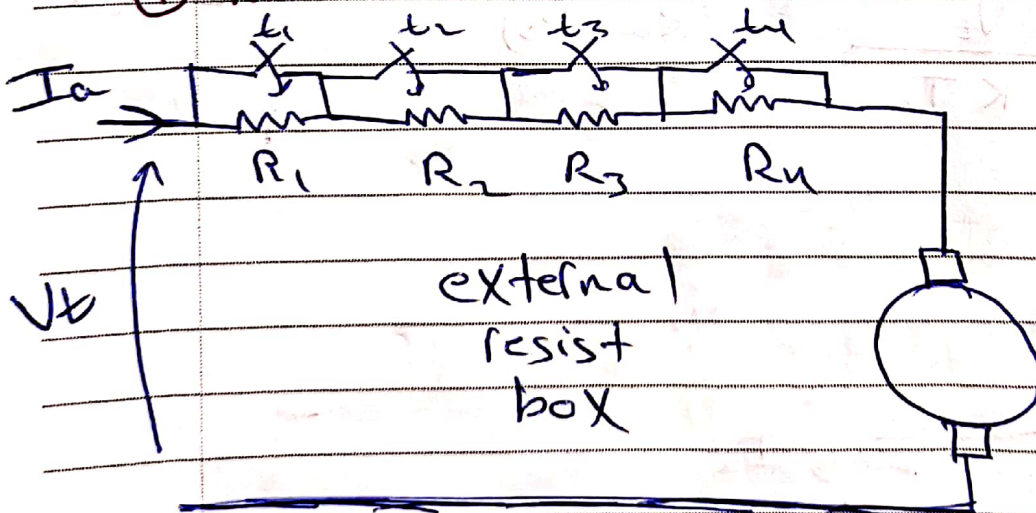
starting torque

$T_{st}$  is very large. Suitable for traction system where  $T_{st}$  is to be very large.

- Should be started at load.
- If suddenly, unloaded, then should be disconnected from source to avoid dangerous acceleration.

\*\* Speed control method:-

(1) Armature Resistance Control.



$$I_a = \frac{V_t - E_a}{R_a}$$

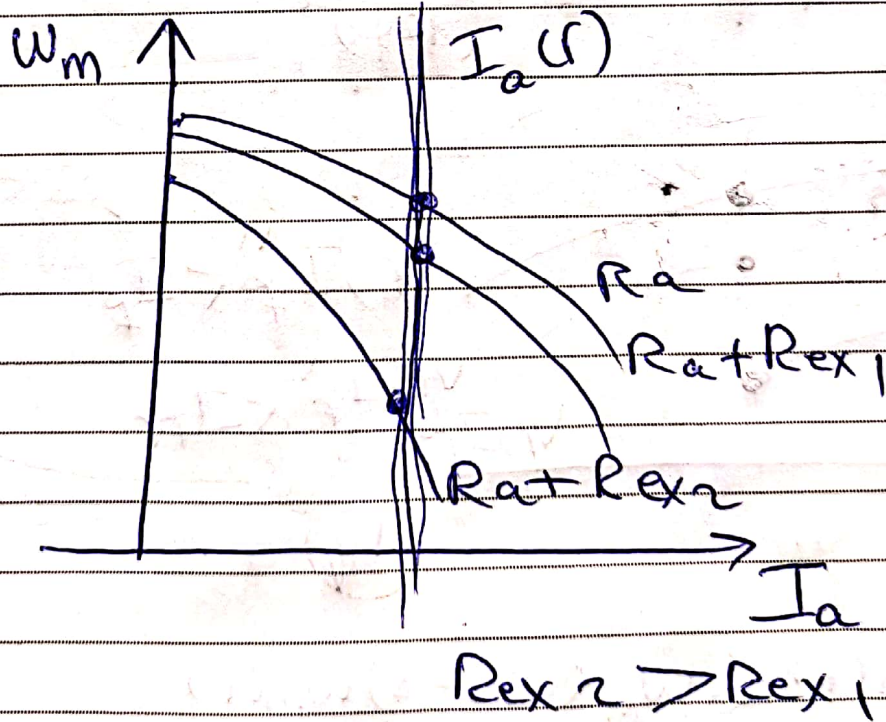
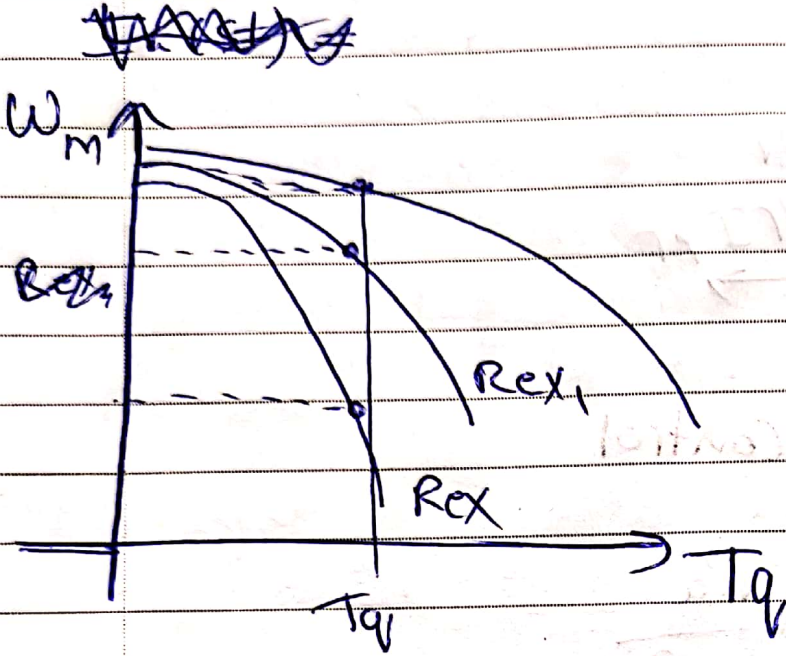
• at starting  $\omega_m = 0$

↓  $E_a \rightarrow 0$

$$I_a(st) = \frac{V_t}{R_a} \text{ (Very large)}$$

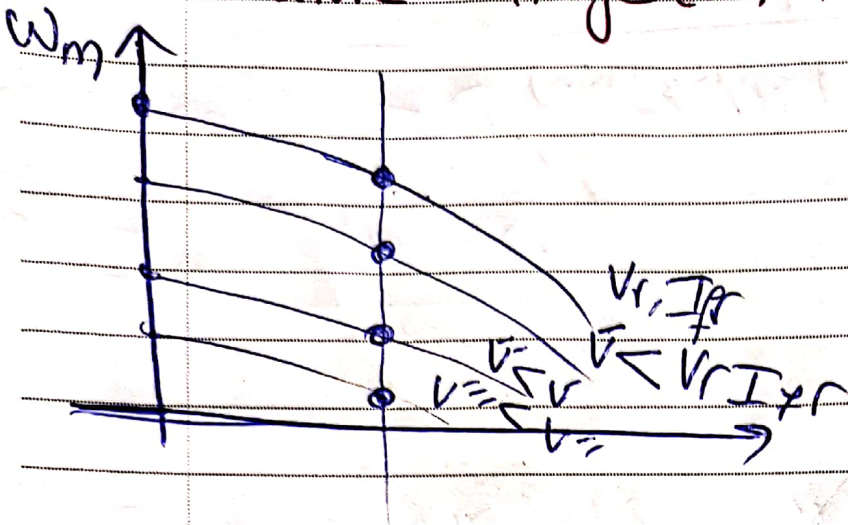


$$I_{st}' = \frac{V_t}{R_{at} + (R_1 + R_2 + R_3 + R_w)}$$



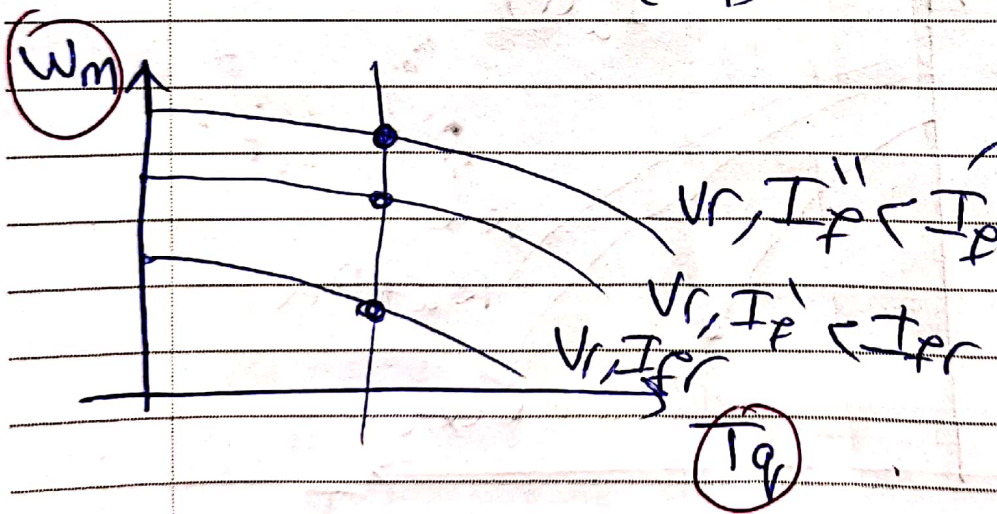
\*Inefficient due to copper losses.

## ② Armature voltage control



## ③ Field weakening control

$$\omega_m = \frac{V_t}{k\phi} - \frac{T_a R_a}{(k\phi)^2}$$



$$\omega_{mcr} \leq \omega_m \leq (2-3)\omega_{mcr}$$

limit?? due to what?

- ① due to commutation
- ② " " mech design



- \*\* given a DC machine with the following parameters
- Sep. excited
  - $V_r = 250 \text{ V}$ ,  $N_m(r) = 500 \text{ RPM}$
  - $\phi = 30 \text{ mWb/pole}$ .
  - No. of Armature slots = 120 slot
  - No. of conductors per slot = 8 C/slot.
  - $R_c$  (conductor resistance) =  $0,015 \text{ } \Omega/\text{cond}$
  - Winding: Lap, double layer,  $2P = 6$
- Required.

a) Machine Motor or generator?

b)  $I_a$  at rated speed and voltage

c) developed at (b) above.

Sol.

Motor:  $V_t > E_a$

Generator:  $V_t < E_a$

$$E_a = k_a \phi \times \omega_m$$

$$k_a = \frac{Z}{A} \cdot \frac{P}{\pi}$$

$$Z = 120 \times 8 = 960 \text{ cond}$$

$$P = 3, A = 6 \left[ \frac{P}{A} = \frac{3}{6} = \frac{1}{2} \text{ Always} \right]$$

$$k_a \rightarrow \frac{480}{\pi}$$

$$\omega_m(r) = \frac{2\pi N_m(r)}{60} = 52,36 \text{ rad/sec}$$

$$\phi = 30 \times 10^{-3} \text{ Wb}$$

$$E_a(r) = \frac{180}{\pi} \times 52.36 \times 30 \times 10^{-3} = 240 \text{ V}$$

Since  $V_t > E_a \rightarrow$  Motor

$$b) I_a = \frac{V_t - E_a}{R_a}$$

$$R_a = ??$$

$$R_{\text{path}} = 0.015 \times 16 = 2.4 \Omega$$

$$c) P_a = \frac{E_a I_a}{W_m} = 116$$



$$\frac{960}{6} = 16 \text{ cond/Path}$$



- given : Dc shunt motor (10 hp, 230 V,  $R_a = 0.3 \Omega$ ,  $R_f = 160 \Omega$ ).

\* The motor draws a line current of 3.938 A on No-load, and a speed of 1200 RPM

- at full load : the armature current is 40

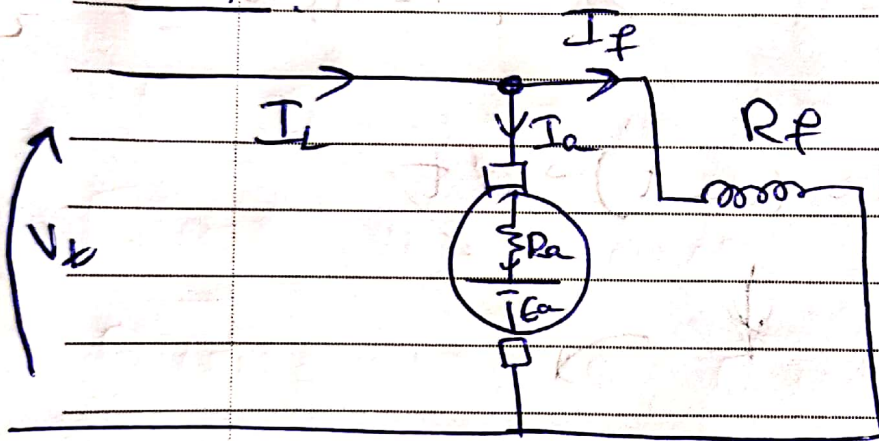
\* Find Arm current at No load.

\* developed power at No load.

\*  $\eta_{FL}$

\* Full load speed of the motor.

Sol



$$I_{(NL)} = I_f + I_{a(NL)}$$

$$I_f = \frac{V_t}{R_f} = \frac{230}{160} = 1.4375 \text{ A}$$

$$I_{a(NL)} = I_{L(NL)} - I_f = 3.938 - 1.4375 = 2.5 \text{ A}$$

$$P_d = P_a = E_a(NL) \times I_{a(NL)} = 229.25 \times 2.5 = 573.125 \text{ W}$$

$$E_a(NL) = V_t - I_{a(NL)} \times R_a = 230 - 2.5 \times 0.3 = 229.25$$



→ (0) at no-load.

$$P_a = P_d = P_{rot} + P_o$$

$$P_{rot} = 573.125 \text{ W}$$

$$\eta = \frac{P_o (FL)}{P_{in} (FL)}$$

$$I_a (FL) = 40 \text{ A}$$

$$I_L (FL) = I_a (FL) + I_f \\ = 40 + 1.4375 = 41.4375$$

$$P_{in} (FL) = V_b * I_L (FL) = 230 (41.4375) \\ = 9530.625 \text{ watt}$$

$$P_{out} (FL) = P_a (FL) - P_{rot} = E_a (FL) I_a (FL) - P_{rot} \\ = 218 \text{ volt}$$

$$P_a (FL) = E_a (FL) * I_a (FL) = 218 * 40 = 8720 \text{ watt}$$

$$P_o (FL) = 8750 - 573.125 = 8146.875 \\ \text{Volt}$$

$$\eta = \frac{P_o (FL)}{P_{in} (FL)} = 85.88 \%$$

$$3) E_a (FL) = k \Phi_p N_m (FL) =$$

$$E_a (nL) = k \Phi_p N_m (nL)$$

$$\frac{N_m (FL)}{N_m (nL)} = \frac{E_a (FL)}{E_a (nL)} \Rightarrow N_m (FL) = 1141.11 \text{ RPM}$$

$$* \text{ Speed Reg} = \frac{N_m (nL) - N_m (FL)}{N_m (nL)} = \frac{1200 - 1141.11}{1200}$$



230 V, 25 hp, DC shunt motor.

draws an armature of 90 A at full-load.

$$R_a = 0.2 \Omega, R_f = 216$$

$$P_{rot} = ?? \text{ at full-load and } \eta_{EL} = ??$$

$$I_L = I_a (FL) + I_f = 90 + 1.0648 = 91.0648 \text{ A}$$

$$I_f = \frac{230}{216} = 1.0648 \text{ A}$$

$$P_{in} = 230 \times 91.0648 = 20944.9 \text{ W}$$

$$P_d = ? = P_a \rightarrow P_{rot} = P_a - P_o$$

$$P_a = E_a (FL) * I_a (FL)$$

$$E_a (FL) = V_t - I_a (FL) * R_a = 230 - 90 * 0.2 = 212 \text{ V}$$

$$P_a (FL) = 212 * 90 = E_a (FL) * I_a (FL) = 19080 \text{ W}$$

$$P_o = 25 * 746 = 18650 \text{ W}$$

$$P_{rot} = P_a (FL) - P_o (FL) = 19080 - 18650 = 430 \text{ watt}$$

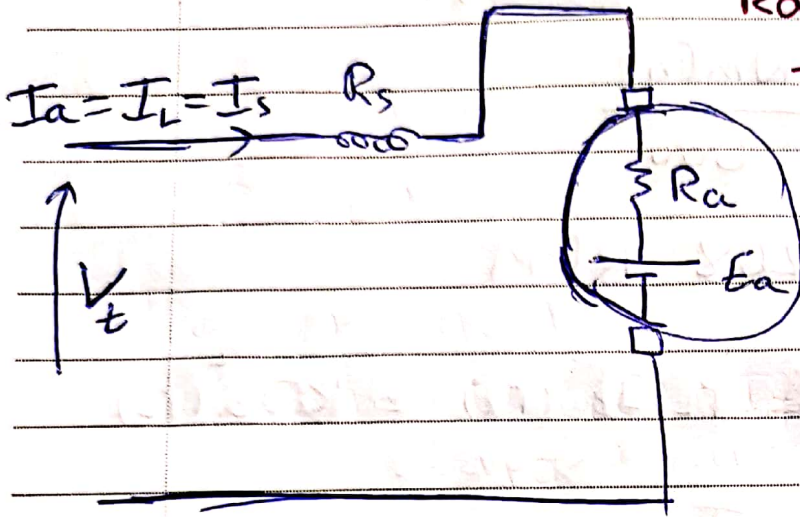
$$\eta = \frac{25 * 746 = 18650}{20944.9} = 89\%$$



volt

**Series - Motors** :- 600 HP, 150 HP, DC Series motor at full load  $N_m(r) = 600$  RPM

$R_a = 0.12 \Omega$ ,  $R_s = 0.04 \Omega$   
the motor draws 200 A at full load.



a)  $E_a(nl) = ??$

b)  $P_a(FL)$ ,  $T_d(FL)$

c) change in the load:

$I_L = 150 A \rightarrow$

$N_m(n)$ ,  $T_d(w) = ??$

a)  $E_a(FL) = 600 - 200(R_s + R_a) = 568 \text{ V}$

b)  $P_a(FL) = E_a(FL) * I_a = 568 * 200 = 113.6 \text{ kW}$

$T_a(FL) = \frac{P_a(FL)}{\omega_m} = \frac{113600}{\frac{2\pi * 600}{60}} = 1808 \text{ Nm}$

c)  $E_a(n) = 600 - 150(R_a + R_s) = 576 \text{ V}$

$$\frac{E_a(n)}{E_a(FL)} = \frac{k_a \cdot \phi(n) \omega_m(n)}{k_a \cdot \phi(r) * \omega_m(r)} = \frac{k_a \cdot k_s I_a * 2\pi N_m(n)}{60}$$

$$\frac{k_a \cdot k_s I_a(r) * 2\pi N_m(r)}{60}$$



$$\frac{E_a(n)}{E_a(r)} = \left( \frac{I_a(n)}{I_a(r)} \right) * \frac{N_m(n)}{N_m(r)}$$

$$\frac{576}{568} = \left( \frac{150}{200} \right) * \frac{N_m(n)}{600}$$

$$\rightarrow N_m(n) = 811.268 \text{ RPM}$$

$$T_a(\text{FL}) = k_a \cdot k_s I_a(\text{FL}) I_a(\text{FL}) = k I_a^2(\text{FL})$$

$$\begin{aligned} T_a(\text{new}) &= k_a \cdot k_s I_a(n) * I_a(n) \\ &= k I_a^2(n) \end{aligned}$$

$$\frac{T_a(\text{new})}{T_a(\text{FL})} = \left( \frac{I_a(\text{new})}{I_a(\text{FL})} \right)^2$$

$$T_a(\text{new}) = 1808 \left( \frac{150}{200} \right)^2 = 1017 \text{ N.m}$$

\* Same motor above is to be started such

that the  $I_{st} = 150\% I_{rated}$ .

a)  $R_{st} = ??$  and  $T_{st} = ??$

b) If  $R_{st}$  still connected and  $I_L = 200A$ ,

$E_a = ??$   $N_m = ??$



$$I_a = \frac{V_t - E_{ca}}{R_a + R_s}$$

$$I_{st} = \frac{V_t - 0}{R_a + R_s} = \frac{600}{0.16} = 3750 \text{ A}$$

$$\left( \frac{I_{st}}{I_a} \right) = \frac{3750}{200} = 18.75$$

direct start

\*\* to start safely reduced  $I_{st}$

a) reduced  $V_t \rightarrow$  using controlling rectifiers

$$I_{st}' = 150\% = 1.5 \times 200 = 300 \text{ A}$$

$$V_{st} = 300 \times 0.16 = 48$$

b)  $R_{st} = ??$

$$I_{st}' = \frac{V_t(r)}{R_a + R_{st} + R_s} \rightarrow 300 = \frac{600}{0.16 + R_{st}}$$

1.84

صحيح

$$\frac{T_{st}'}{T_r} = \left( \frac{1.5}{1} \right)^2 = 2.25 \checkmark \text{ good}$$



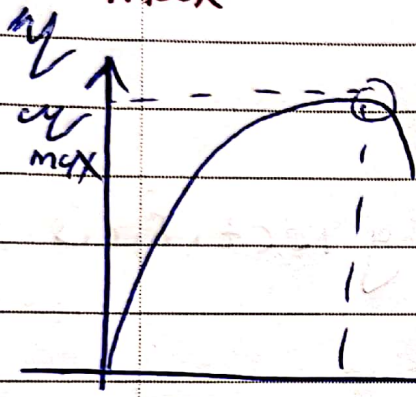
\* Same motor parameters

$$P_o = 150 \text{ hp}, I_L = I_a = 200 \text{ A}$$

Find:  $P_{rot} = ?$

Find the Armature Current at Max Efficiency

$$\eta_{\max} = ??$$



$$P_{acr} = E_{acr} \times I_{acr} = 568 \times 200 = 568 \text{ W}$$

$$P_o = 150 \times 746 = \text{---}$$

$$P_{rot} = P_{acr} - P_{oc} = 1700 \text{ W}$$

$$I_a(\eta_{\max}) = ??$$

$$\eta = \frac{P_o}{P_{in}}$$

$$P_{out} = V_t I_a - I_a^2 (R_s + R_a) - P_{rot}$$

$$P_{in} = V_t I_a$$

$$\eta = \frac{V_t I_a - I_a^2 (R_s + R_a) - P_{rot}}{V_t I_a}$$

$$\left. \frac{d\eta}{dI_a} \right|_{=0} = \frac{[V_t - 2I_a (R_s + R_a)] V_t I_a - V_t [V_t I_a]}{(V_t I_a)^2}$$

No.

$$I_a (\eta_{\max}) = \sqrt{\left(\frac{P_{\text{rot}}}{R_a + R_s}\right)}$$

$$I_a (\eta_{\max}) = \sqrt{\frac{1700}{0.16}} = 103.07 \text{ A}$$

$$E_a = 600 - 103.07(0.16) = 583.5 \text{ V}$$

$$~~P_o = P_m~~$$

$$P_a = E_a I_a = 583.5 \times 103.07 = 58446.584 \text{ watt}$$

$$P_o = P_a - P_{\text{rot}} = 58446.584 \text{ watt}$$

$$\eta_{\max} = \frac{58446.584}{103.07 \times 600} = 94.5 \%$$

done

