Em 2

No.

maxwell's equations:-
in time varying Fields (in time domain)

electric flux dencity

$\nabla \cdot \bar{D} = \rho_v$ ~> Gauss

$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{dt}$

Sources

$\nabla \times \bar{H} = \bar{J} + \frac{d\bar{D}}{dt} \rightarrow J_d$ (displacment current Density).

$\nabla \cdot \bar{B} = 0$ ~> Gauss → (there is no single pole magnet).

$\rho_v\,(x, y, z, t)$ function of space and time.
→ Ac

$I = \int_S \vec{J} \cdot ds$ → current density A/m^2

$\bar{J}$ funtion of space and time

$J = \sigma \bar{E} = \rho_v \vec{u}$

Constitutive Relations

$D = \epsilon \bar{E}$
$\bar{B} = \mu \bar{H}$
$J = \sigma \bar{E}$

↗

$\nabla \cdot \epsilon \bar{E} = \rho_v$
$\nabla \times \bar{E} = -\mu \frac{d\bar{H}}{dt}$
$\nabla \times \bar{H} = \sigma \bar{E} + \epsilon \frac{d\bar{E}}{dt}$
$\nabla \cdot \mu \bar{H} = 0$

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maxwells Equations in Phasor domain

$\nabla \cdot \epsilon E_s = \rho_{vs}$

$\nabla \times \overline{H}_s = (\sigma + j\omega\epsilon)\overline{E}_s$

$\overline{E}(x, y, z, t)$

$\overline{E}_s(x, y, z)$

$\nabla \times \overline{H}_s = 0$

$\nabla \times E_s = -j\omega\mu\overline{H}_s$

Time Varying potetials

in Dc : Q

$V = \int \frac{\rho_v \, dv}{4\pi\epsilon R}$ ✓

electric potentiaal

$\overline{A}$ vector magnetic potential $= \int \frac{\mu J \, dv}{4\pi R}$ ✓

$\nabla^2 V = -\frac{\rho_v}{\epsilon}$ (scalar)

$\nabla^2 \overline{A} = -\mu\overline{J}$ (vector)

$\nabla^2 A_x = -\mu J_x$

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$\nabla \cdot \bar{D} = \rho_v$ $\quad$ $\nabla \times \bar{H} = J + \frac{d\bar{D}}{dt}$

$\nabla \times \bar{E} = \frac{d\bar{B}}{dt}$ $\quad$ $\nabla \cdot \bar{B} = 0$

(+) $\bar{D} = \epsilon \bar{E}$ $\quad$ $J = \sigma \bar{E}$

$\bar{B} = \mu H$

(+) $\nabla \times (-\nabla V) = 0$

$\nabla \times \nabla \times \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$

(+) $\bar{B} = \nabla \times \bar{A}$ & $(\bar{E} = -\nabla V)$ for DC just

magnetic flux density $\quad$ magnatic vector potential

$$\nabla \times \bar{E} = \frac{-d}{dt}(\nabla \times \bar{A})$$

$$\nabla \times \left(E + \frac{d\bar{A}}{dt}\right) = 0 \qquad E + \frac{d\bar{A}}{dt} = -\nabla V$$

$$\boxed{\bar{E} = -\nabla V - \frac{d\bar{A}}{dt}} \rightarrow (1)$$

$\nabla \cdot \epsilon \bar{E} = \rho_v \Rightarrow \nabla \cdot \bar{E} = \frac{\rho_v}{\epsilon}$

take $\nabla \cdot$ to equation 1

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$$\nabla\cdot\bar{E} = \nabla\cdot(-\nabla V) - \frac{d}{dt}(\nabla\cdot\bar{A})$$

$$\nabla\cdot E = -\nabla^2 V - \frac{d}{dt}(\nabla\cdot\bar{A})$$

$$\boxed{\nabla^2 V + \frac{d}{dt}(\nabla\cdot\bar{A}) = \frac{-\rho_v}{\epsilon}} \quad (2)$$

$$\left(\nabla\times\bar{H} = \bar{J} + \frac{d\bar{D}}{dt}\right)\mu$$

$$\nabla\times\bar{B} = \mu\bar{J} + \mu\frac{d\bar{D}}{dt}$$

$$= \mu\bar{J} + \mu\epsilon\frac{d\bar{E}}{dt}$$

Sub equation 1

$$\nabla\times\bar{B} = \mu\bar{J} + \mu\epsilon\frac{d}{dt}\left(-\nabla V - \frac{d\bar{A}}{dt}\right)$$

$$\nabla\times\bar{B} = \mu\bar{J} - \mu\epsilon\nabla\frac{dv}{dt} - \mu\epsilon\frac{d^2\bar{A}}{dt^2}$$

$$\nabla\times\nabla\times A = $$

$$\nabla\cdot(\nabla\bar{A}) - \nabla^2 A = \mu\bar{J} - \mu\epsilon\nabla\frac{dv}{dt} - \mu\epsilon\frac{d^2 A}{dt}$$

We may choose:

$$\nabla(\nabla\bar{A}) = -\mu\epsilon\nabla\frac{dv}{dt} \quad \left(\nabla\cdot\bar{A} = -\mu\epsilon\frac{dv}{dt}\right)$$

$$-\nabla^2\bar{A} = \mu\bar{J} - \mu\epsilon\frac{d^2\bar{A}}{dt^2}$$

SUB in Equation #2

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$$\boxed{\nabla^2 V - \mu\epsilon \frac{d^2V}{dt^2} = -\frac{\rho_v}{\epsilon}}$$ ⇒ Scalar poissions equation for time varying field.

sol $V = \int_V \frac{[\rho_v]\, dv}{4\pi\epsilon R}$

$$\nabla^2 A = -\mu\bar{J} + \mu\epsilon \frac{d^2 A}{dt^2}$$

$$\boxed{\nabla^2 A - \mu\epsilon \frac{d^2 \bar{A}}{dt^2} = -\mu J}$$ → vector ~~Scalar~~ poissions equations equations for time varying field.

$$\int_V \frac{\mu [J]\, dv}{4\pi R}$$

$\rho_v\ (x, y, z, t)$ $\quad$ $[\rho_v]\ (x, y, z, t')$

$J\ (x, y, z, t)$ $\quad$ $[J]\ (x, y, z, t')$

t' = Retarted time

lag, hysterisies

$t' = t + \frac{R}{u}$ delay time, u → speed

$$u = \frac{1}{\sqrt{\mu\epsilon}}$$

if free-space $u = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 = C$

$u = C$ in free space

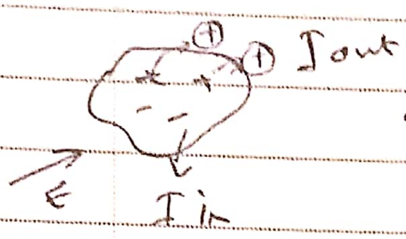
$u \leq$ speed of light (C)

$\mu \epsilon = \frac{1}{u^2}$

Continuity Equation (5.8)

relaxation time

$T_r = \frac{\epsilon}{\sigma}$



$I_{out} = I_{in}$

$\oint_S \bar{J} \cdot d\bar{s} = -\frac{dQ_{in}}{dt}$, $Q = \int_V \rho_v \, dv$

, $\nabla \cdot \bar{D} = \rho_v$

$\oint \bar{J} \cdot ds = \int_V \frac{d\rho_v}{dt} dv$

$\oint \bar{J} \cdot ds = \int_V \frac{d\, \nabla \cdot \bar{D}}{dt} dv$

apply divergence theorem on the left side

$\oint \bar{D} \cdot ds = \int_V \nabla \cdot \bar{D} \, ds$

$\int_V \nabla \cdot \bar{J} dv = \int_V -\frac{d}{dt} \nabla \cdot \bar{D} \, dv$

$\boxed{\nabla \cdot \bar{J} = -\frac{d\rho_v}{dt}} = -\frac{d\, \nabla \cdot \bar{D}}{dt}$

Continuity Equation

$\bar{J} = -\frac{\nabla \cdot \bar{D}}{dt}$ ~~electric~~

$J_d = -\bar{J} = \frac{d\bar{D}}{dt}$ (~~Star~~)

$\epsilon = \epsilon_0 \epsilon_r$ Types of media:
$\mu = \mu_0 \mu_r$

① lossy Dielectric $\epsilon = \epsilon_0 \epsilon_r$ $\sigma \neq 0$
$\mu = \mu_0 \mu_r$

② lossless Dielectric
$\epsilon = \epsilon_0 \epsilon_r$ $\sigma_d = 0$
$\mu = \mu_0 \mu_r$

③ free Space media
$\epsilon = \epsilon_0$ $\sigma = 0$
$\mu = \mu_0$

④ perfect Conductor medium:- (non magnetic material $\mu_r = 1$).
$\epsilon = \epsilon_0$ $\mu = \mu_0 \mu_r$ $\sigma \simeq \infty$
or $\mu = \mu_0$

Wave equation:
sterting from the time varying potential

$$\nabla^2 V - \mu\epsilon \frac{d^2 V}{dt^2} = \frac{-\rho_v}{\epsilon}$$

assume source free region ($\rho_v = 0$

$$\nabla V^2 - \mu\epsilon \frac{d^2 V}{dt^2} = 0$$

assume the wave is travelling in One direction only (let it Z axis)

$$\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d}{dz^2} \quad \text{in Cart}$$

$$\frac{d^2 V}{dz^2} - \mu \epsilon \frac{d^2 V}{dt^2} = 0$$

$$\mu \epsilon = \frac{1}{u^2} \qquad u = \frac{1}{\sqrt{m\epsilon}}$$

$$\frac{d^2 V}{dt^2} - u^2 \frac{d^2 V}{dz^2} = 0 \Rightarrow \frac{d\epsilon^2}{dt^2} - u^2 \frac{d\epsilon^2}{dz^2} = 0$$

Ed Ed

convert to frequency domain.

$$E(x,y,z,t) = E_s(x,y,z)\, e^{j\omega t}$$

$$\frac{-\omega^2 E_s e^{j\omega t}}{-\omega^2} - \frac{u^2 d^2 E_s e^{j\omega t}}{\frac{dz^2}{-\omega^2}} = 0$$

$$\frac{dE_s^2}{dz^2} + \left(\frac{\omega}{u}\right)^2 E_s = 0$$

constant

let $\beta = \frac{\omega}{u}$ (Gauss constant)

rad/m
phase constant

$$\frac{d^2 E_s}{dz^2} + \beta^2 E_s = 0$$

let $\frac{d}{dz} = m$

$$m^2 E_s + \beta^2 E_s = 0$$

$m = \pm j\beta$ pure imaginary

if the solution is pure imagenary, the

Solution is Sin, cos

If the solution is pure Real, the Solution is Sinh & Cosh

Wave equation :-

$$\frac{d^2 E_s}{dz^2} + \beta^2 E_s = 0$$

Solution to wave equation

$$E_s = \underbrace{E_0^+ \quad E_0^-}_{\text{amplitudes}}$$

$$E_s = E_0^+ e^{-j\beta z} + E_0^- e^{+j\beta z}$$

$$E_s = \underbrace{E_0^+ e^{-j\beta z}}_{\text{forward wave}} + \underbrace{E_0^- e^{+j\beta z}}_{\text{backward wave}}$$

+ve z direction — -ve z direction

choose the forward solution (assume are symm)

$$E(z,t) = E_0^+ \cos(\omega t - \beta z) \; V/m$$

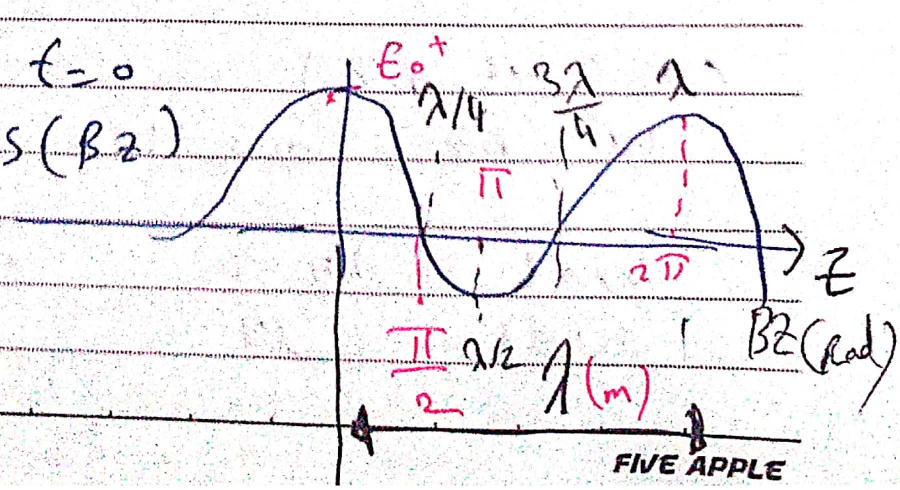
Speed, time

Space domain — time is constant

time domain — Space is constant

in Space domain let $t = 0$

$$E(z) = E_0^+ \cos(\beta z)$$



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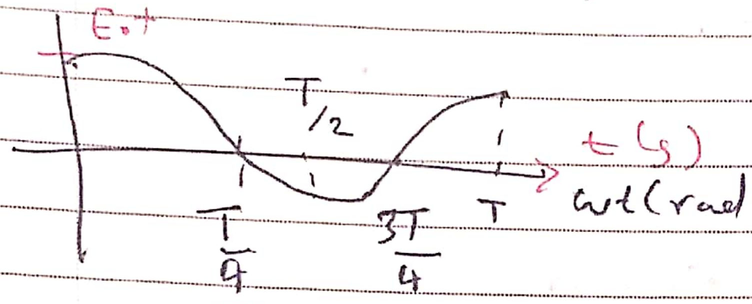
in time domain let $z=0$

$E(t) = E_0^+ \cos(\omega t)$ V/m



$u = \frac{\lambda}{T} = \lambda f$ $\qquad \omega = 2\pi f = \frac{2\pi}{T}$

$\beta = \frac{\omega}{u} = \frac{u}{\frac{\lambda \omega}{2\pi}} = \frac{2\pi}{\lambda}$

$\lambda = \frac{2\pi}{\beta}$ is also a sol

$c = \lambda f$ in free space

constant

$\sin(\varphi \pm 180) = -\sin(\varphi)$

$\cos(\varphi \pm 180) = -\cos(\varphi)$

$\sin(\varphi \pm \frac{\pi}{2}) = \pm \cos(\varphi)$

$\cos(\varphi \pm \frac{\pi}{2}) = \mp \sin(\varphi)$

$\cos(\omega t - \beta z)$

$\omega t - \beta z =$ constant

$\omega dt - \beta dz = 0$ $\qquad \frac{dz}{dt} = \frac{\omega}{\beta}$

$\omega dt = \beta dz$

$u = u$

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No. $\frac{2\pi}{T}$

Ex given $\bar{E} = 50\cos(10^8 t + Bx)\,\hat{a}_y$ V/m in free space :-

a- Find the direction of the wave propagation

b- calculate B and time it take to travel a distance of (λ/ω)

c- sketch the wave at $t = 0, \frac{T}{4}, \frac{T}{2}$.

(a) in the negative X direction $(-\hat{a}_x)$

(b) $B = \frac{\omega}{u} = \frac{1}{3}$ rad/s $= \frac{10^8}{3\times10^8}$

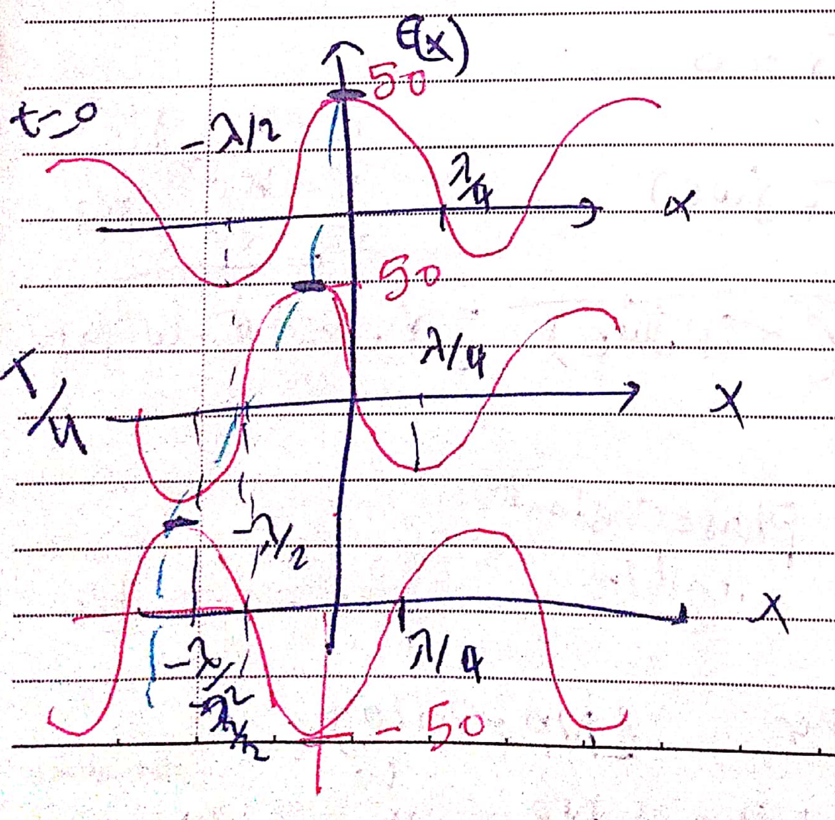
$t = \frac{T}{2}$ s

$\omega = \frac{2\pi}{T}$, $T = \frac{2\pi}{\omega}$, $t = \frac{T}{2} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega} = T = 31.4$ ns

(c) at $t = 0$, $E = 50\cos\beta x\,\hat{a}_y$

at $t = \frac{T}{4}$ $E = -50\sin(\beta x)\,\hat{a}_y$

at $t = \frac{T}{2}$ $E = -50\cos(\beta x)\,\hat{a}_y$



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Wave propagation in Lossy dielectric

$\epsilon = \epsilon_0 \epsilon_r$, $\mu = \mu_0 \mu_r$, $\sigma \neq 0$

Maxwell's equation in Source free region ($\rho_v = 0$, $J = 0$)

$\nabla \cdot \frac{\epsilon_0 E}{\epsilon_0} = 0$ $\nabla \cdot \bar{E} = 0$

$\nabla \cdot \bar{H} = 0$ always

① $\nabla \times \bar{E}_s = -j\omega\mu H_s \leftarrow -\frac{dB}{dt}$

② $\nabla \times \bar{H}_s = (\sigma + j\omega\epsilon) E_s \leftarrow \begin{matrix} J + J_d \\ \sigma E + \frac{\partial D}{dt} \end{matrix}$

take the curl to Either equation 1 or eq 2

eq (1)

$\nabla \times \nabla \times E_s = -j\omega\mu \nabla \times H_s$

$\cancel{\nabla (\nabla \cdot \bar{E}_s)} - \nabla^2 \bar{E}_s = -j\omega\mu (\sigma + j\omega\epsilon E_s)$

$\nabla^2 \bar{E}_s - j\omega\mu(\sigma + j\omega\epsilon) \bar{E}_s = 0$

$\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0$

$\bar{Z} = x + jy$

$Z = K e^{j\theta}$

$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$

$\sqrt{Z} = \sqrt{r}\, e^{j\theta/2}$

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{j\omega\mu\sigma - \omega^2\mu\epsilon}$

$\gamma = \alpha + j\beta$

γ: propagation (/m) permeter

α: attenuation constant Np/m

β: Phase Constant rad/m

$1\,Np = 8.66\,dB$

NP = neper $1\,NP = 20 \log_{10} e$

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dB Np لـ الجرا ب لا عملين بال نحو من د ال

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No.

Wave propagation in lossy Dielectric

$\nabla \times \bar{E}_s = -j\omega\mu\bar{H}_s$

$\nabla \times \bar{H}_s = (\sigma + j\omega\epsilon)\bar{E}_s$

$\epsilon = \epsilon_0\epsilon_r$
$\mu = \mu_0\mu_r$
$\sigma \neq 0$

$\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0 \rightarrow$ Helmholtz's wave equation

$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$

$\gamma = \alpha + j\beta$

$\alpha \rightarrow$ zero in free region

$\boxed{1\,NB = 8.686\,dB}$

in free Space

$\nabla^2 \bar{E}_s + \beta^2 \bar{E}_s = 0$

$\gamma^2 = -\omega^2\mu\epsilon + j\omega\mu\sigma$

$\gamma^2 = \alpha^2 + 2j\alpha\beta - \beta^2$

* equating the Real part

$-\alpha^2 + \beta^2 = \omega^2\mu\epsilon \longrightarrow (1)$

* equating the magnitude of γ^2

$|\gamma^2| = \sqrt{(\text{Real})^2 + (\text{Imag})^2}$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}} \Rightarrow \text{attenuation constant}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)}$$

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Solution to wave equation

assume the wave is travelling in the Z direction (1-direction only).

$$\frac{\partial^2 \overline{E_s}}{dz^2} - \gamma^2 \overline{E_s} = 0$$

Let $\frac{d}{dz} = m \Rightarrow m^2 E_s - \gamma^2 E_s = 0$

$$\boxed{m = \pm \gamma}$$

Sinh, Cosh
or $e^{\gamma}/e^{-\gamma}$
solution

$$\overline{E_s}(z) = E_o^+ e^{-\gamma z} + E_o^- e^{+\gamma z}$$

forward wave — +ve Z-direction

Backward wave — -ve Z-direction

assume $\hat{a_x}$ → "علاقة باتجاه الموجة".

- choose the forward wave

Converting to time domain

$$\overline{E}(z,t) = Re\{\overline{E_s}\, e^{j\omega t}\}$$
$$= Re\{E_o^+ e^{-\gamma z} e^{j\omega t}\}$$
$$= Re\{E_o^+ e^{-\gamma z} \cdot e^{-j\beta z} \cdot e^{j\omega t}\}$$

$$\overline{E}(z,t) = Re\{E_o^+ e^{-\alpha z} e^{j(\omega t - \beta z)}\}\hat{a_x}$$

$$\overline{E}(z,t) = E_o^+ e^{-\alpha z} \cos(\omega t - \beta z)\,\hat{a_x} \ V/m$$

by similar procedure

$$\nabla^2 \overline{H_s} - \gamma^2 \overline{H_s} = 0$$

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$$\frac{d^2 H_s}{dz^2} - \gamma^2 H_s = 0$$

sol $\Rightarrow$ $\bar{H}(z,t) = H_0^+ e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_H$ $\quad \hat{a}_y$

$\hat{a}_H$ $\quad \hat{a}_E \to \boxed{\hat{a}_x}$

$\hat{a}_z$ $\quad \hat{a}_y \quad \boxed{\hat{a}_z} \leftarrow \hat{a}_k \quad \hat{a}_H \to$ magnetic field direction

unit vector in the wave direction

$\hat{a}_H = \hat{a}_k \times \hat{a}_E$

$\hat{a}_z \times \hat{a}_x$

$= \hat{a}_y$

define ; Intrinsic Impedance

$$(\Omega)\ \eta = \frac{E}{H} \quad \frac{V/m}{A/m} = \frac{V}{A} = \Omega$$

could be complex.

$$\eta = |\eta| \angle \theta_\eta$$

η derivation from maxwell's equation

$\nabla \times \bar{E}_s = -j\omega\mu \bar{H}_s$ $\qquad E_s = E_0^+ e^{-\alpha z} e^{-j\beta z}\, \hat{a}_x$

$\bar{H}_s = H_0^+ e^{-\alpha z} e^{-j\beta z}\, \hat{a}_y$

$$\Big[\qquad \Big] = \qquad \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

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$$\eta = \frac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{\frac{1}{4}}} \angle \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)$$

$|\eta|$ θ_η

$\tan(2\theta_\eta) = \frac{\sigma}{\omega\epsilon}$

Loss tangent → $\tan(\theta) = \frac{\sigma}{\omega\epsilon}$

$\boxed{\theta = 2\theta_\eta}$ → loss angle

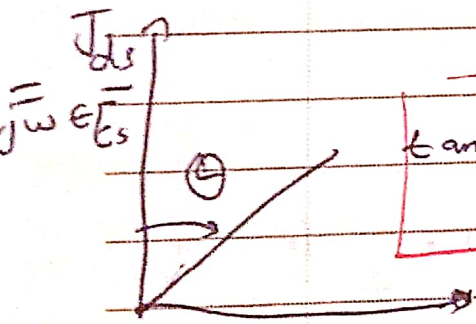
$\lambda = \frac{2\pi}{\beta}$

$u = \frac{\omega}{\beta}$

$= \lambda f$ بتتغير

$u = \frac{1}{\sqrt{\mu\epsilon}}$ ⊗ lossless or free space only

loss tangent $\tan\theta = \frac{\sigma}{\omega\epsilon}$



$$\boxed{\tan\theta = \frac{|J_s|}{|J_{ds}|}} = \frac{|\sigma \cancel{E_s}|}{|j\omega\epsilon \cancel{E_s}|} = \frac{\sigma}{\omega\epsilon}$$

$J_s = \sigma E_s$

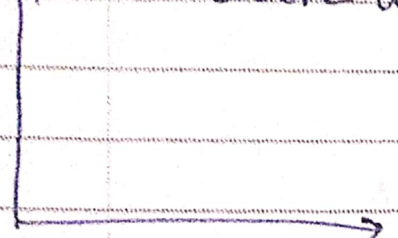
$0 < \theta < 90^\circ$

lossless or free space — lossy — perfect conductor material

$0 < \theta_\eta < 45^\circ$

Lossless or free space — lossy — perfect conductor

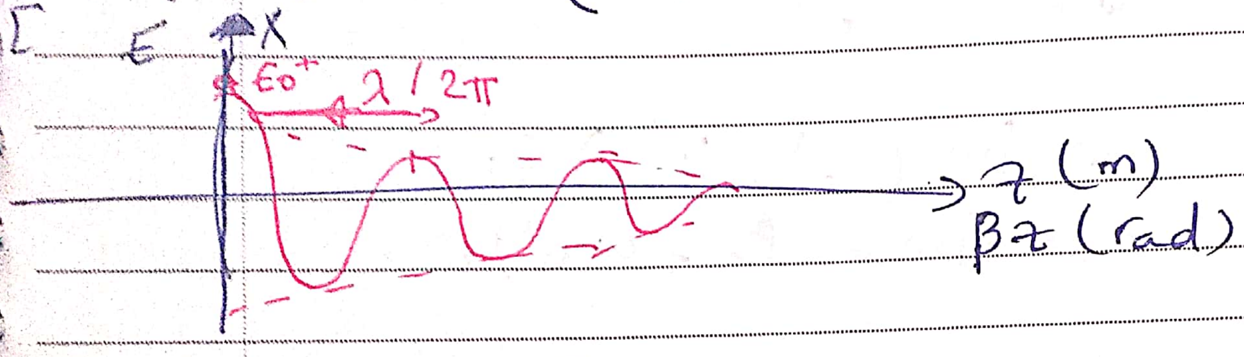
Sketch the wave in lossy media



$$\bar{E} = E_o^+ e^{-\gamma z} \cos(\omega t - \beta z)\, \hat{a_x}$$

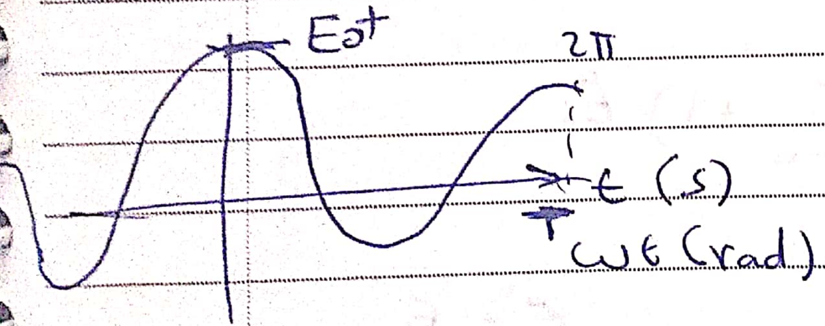
- in space domain $\rightarrow t=0$

$$E(z) = E_o^+ e^{-\gamma z} \cos(\beta z)\, \hat{a_x}$$

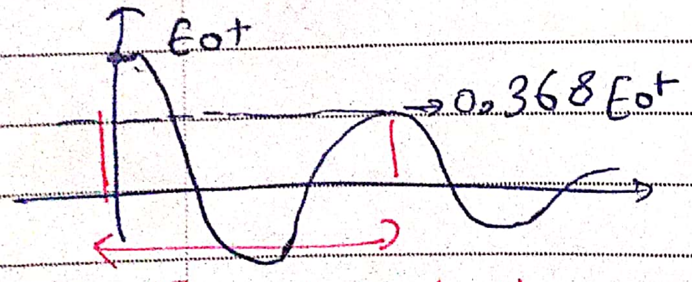


- in time domain $z=0$

$$\bar{E}(t) = E_o^+ \cos(\omega t)\, \hat{a_x}$$



$e^{-1} = 0.368 = 36.8\%$



$\delta \rightarrow$ skin depth

e^{-2} $2\delta \rightarrow 14\%$

e^{-3} $3\delta \rightarrow 5\%$

e^{-4} $4\delta \rightarrow 1.8\%$

e^{-5} $5\delta \leq 0.1\%$

No.

$$\delta = \frac{1}{\alpha} \ (m)$$

$$\nabla \times \bar{H}_s = (\sigma + j\omega\epsilon)\bar{E}_s$$

in Some media ϵ is complex

$$\epsilon_s = \underbrace{\epsilon'}_{Real} - j\underbrace{\epsilon''}_{imag}$$

$$\boxed{\tan\theta = \frac{\sigma}{\omega\epsilon} = \frac{\epsilon''}{\epsilon'}}$$

$$\cancel{\nabla \times \bar{H}_s = \sigma + j\omega(\epsilon' - j\epsilon'')\bar{E}_s}$$

$$\cancel{= (\sigma + j\omega\epsilon' \bar{E}_s + \omega\epsilon'')\bar{E}_s}$$

$$= \frac{\omega\epsilon'}{\cancel{\sigma + \omega\epsilon''}}$$

$$\nabla \times \bar{H}_s = (\sigma + j\omega\epsilon)\bar{E}_s$$

$$\nabla \times \bar{H}_s = j\omega\epsilon\left(\frac{-\sigma}{j\omega\epsilon} + 1\right)\bar{E}_s$$

$$\nabla \times \bar{H}_s = j\omega\epsilon_s\bar{E}_s$$

$$\epsilon' = \epsilon$$

$$\epsilon'' = \frac{\sigma}{\omega}$$

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1

Wave propagation in lossy media

$\epsilon = \epsilon_0 \epsilon_r$

$\mu = \mu_0 \mu_r$

$\sigma \neq 0$

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$

~~$\alpha =$~~

$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}$

$$\boxed{\lambda = \frac{2\pi}{\beta} \qquad u = \frac{\omega}{\beta}}$$

$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)}$

$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \frac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{\frac{1}{4}}} \angle \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)$

$\eta = \frac{E}{H}$

$\bar{E} = E_0^+ e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x$

$\bar{H} = H_0^+ e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_y \quad A/m$

$\bar{H} = \frac{E_0^+}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \phi_\eta)\, \hat{a}_y \quad A/m$

$\delta = \frac{1}{\alpha}$

2 Wave propagation in lossless media

$\epsilon = \epsilon_0 \epsilon_r \qquad \mu = \mu_0 \mu_r \qquad \sigma = 0$

$\alpha = 0 \qquad \delta \rightarrow \infty \qquad \gamma = j\omega\sqrt{\mu\epsilon} = j\beta$

$\beta = \omega\sqrt{\mu\epsilon}$

$\eta = \sqrt{\frac{\mu}{\epsilon}} \angle 0^\circ$

$E = E_0^+ \cos(\omega t - \beta z)\, \hat{a}_x \quad V/m$

$H = \frac{E_0^+}{|\eta|} \cos(\omega t - \beta z)\, \hat{a}_y \quad A/m$

Inphase

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$\lambda = \frac{2\pi}{\beta}$, $u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}$

③ wave propagation in free Space :-

$\epsilon = \epsilon_0 \qquad \mu = \mu_0 \qquad \sigma = 0$

$\alpha = 0 \qquad \delta \to \infty \qquad \gamma = j\beta = j\omega\sqrt{\mu\epsilon}$

$\beta = \omega\sqrt{\mu_0\epsilon_0} \qquad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \angle 0 = 377\,\Omega$

$120\pi\,\Omega$

$\bar{E}$ and $\bar{H}$ are Inphase

$\lambda = \frac{2\pi}{\beta}$, $u = \frac{1}{\sqrt{\mu_0\epsilon_0}} = c = 3\times10^8\, m/s$

④ wave propagation in good Conductor media -

$\epsilon = \epsilon_0 \quad \mu = \mu_0\mu_r$ Usually , $\sigma = \infty$

$\mu = \mu_0$ ↳ non-magnetic material.

NB/m

$\alpha = \sqrt{\frac{\sigma\mu\omega}{2}}$, $\omega = 2\pi f \qquad \alpha = \sqrt{\pi f \mu \sigma}$

rad/m $\beta = \sqrt{\frac{\sigma\mu\omega}{2}} = \sqrt{\pi f \mu\sigma} = \alpha \qquad \boxed{\alpha = \beta}$

$\gamma = \alpha + j\beta = \beta(1+j)$

$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$

$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu\sigma}} \qquad \lambda = \frac{2\pi}{\beta} \qquad u = \frac{\omega}{\beta}$

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No. if equal ⇒ Perfect conductor

$$\bar{E} = E_o^{+} e^{-\alpha z} \cos(\omega t - \beta z)\hat{a}_x \quad V/m$$

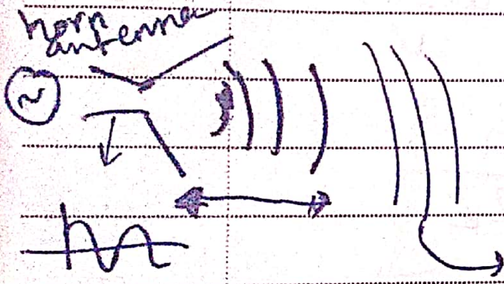
$$\bar{H} = \frac{\bar{E_o}}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - 45^\circ)\hat{a}_y$$

$\bar{E}$ leads $\bar{H}$ by θ_η inductances.

$\bar{E} \perp \bar{H} \perp \hat{a}_k$ TEM mode

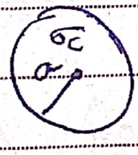
propagation direction Transverse electro magnetic

UPW ⇒ Uniform plane wave

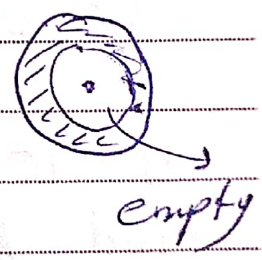


Skin effect

in DC
$f = 0$
$\delta = \infty$
[illegible]



in Ac Frequency
$f > 50 Hz$
$\delta =$ ~~[illegible]~~



let $\mu = \mu_o$ $\sigma = 5.7 \times 10^7$ S/m

f (Hz)	δ (mm)
60	8.6 mm
100 HZ	6.6 mm
500 HZ	3 mm
10^{10} HZ	6.6×10^{-4} m

δ: 63.2% in this small region from Signal

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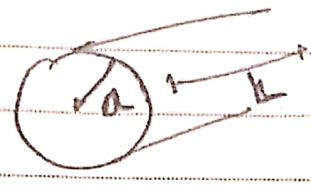
Dc and Ac Resistances :-

$R_{dc} = \frac{L}{\sigma A}$

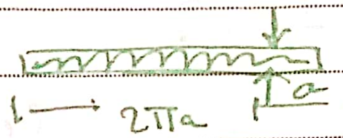


$A = \pi a^2$

R_{AS} = Surface Resistance

$\frac{1}{\sigma \delta}$

$R_{AC} = \frac{R_s l}{w} = \frac{l}{\sigma \delta w}$



$R_{AC} = R_{DC}$ at $f=0$ and at low frequency
But at higher frequency

$R_{AC} >>> R_{DC}$ at higher frequency

$$\frac{R_{AC}}{R_{DC}} = \frac{\frac{L}{\delta \sigma \underbrace{w}_{2\pi a}}}{\frac{L}{2\pi a^2}} = \boxed{\frac{a}{2\delta}}$$

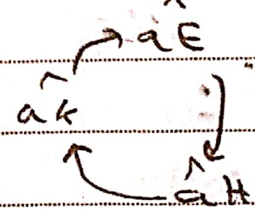
Example a lossy Dielectric has $\eta = 200\angle 30^\circ\ \Omega$
if the ~~phase~~ plane wave in this media has

$\bar{H} = 10 e^{-\alpha x} \cos(\omega t - \frac{1}{2} x)\hat{a}_y$ A/m

Find $\bar{E}$, α, δ, polarization.

polarization = $\hat{a}_E = \hat{a}_H \times \hat{a}_k$

$\hat{a}_y \times \hat{a}_x = -\hat{a}_z$



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No.

$\bar{E} = E_0 \, e^{-\alpha x} \cos(\omega t - \frac{1}{2}x + 30^\circ) \, \hat{a}_z \, V/m$

$E = \eta H$ (ohm's law) 2000

$10 * 200 = 2000$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right)}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)} = \frac{1}{2}$$

$\boxed{\text{let } \frac{\sigma}{\omega\epsilon} = X}$

$$\frac{\alpha}{\beta} = \sqrt{\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}} = 2\alpha \rightarrow (1)$$

$$\eta = \frac{\mu/\epsilon}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{\frac{1}{4}}} = 200$$

$\theta_\eta = \frac{1}{2}\tan^{-1} X = 30^\circ \Rightarrow \boxed{X = \sqrt{3}}$

sub in (1)

$2\alpha = \frac{1}{\sqrt{3}} \qquad \alpha = \frac{1}{2\sqrt{3}}$ NB/m

$\boxed{\delta = 2\sqrt{3}\, m \simeq 3.4 m}$

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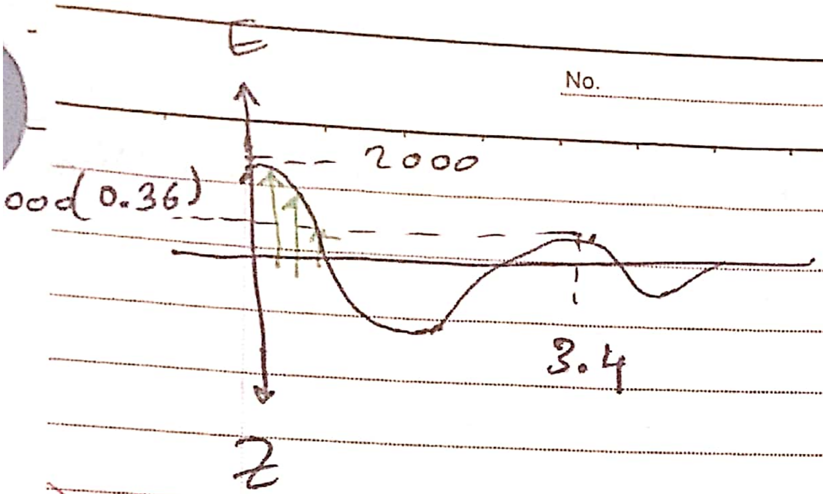
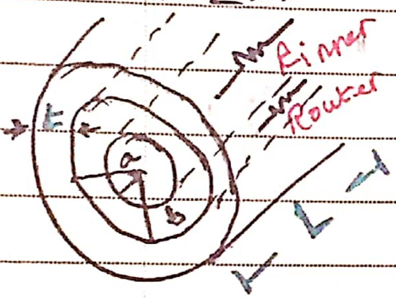
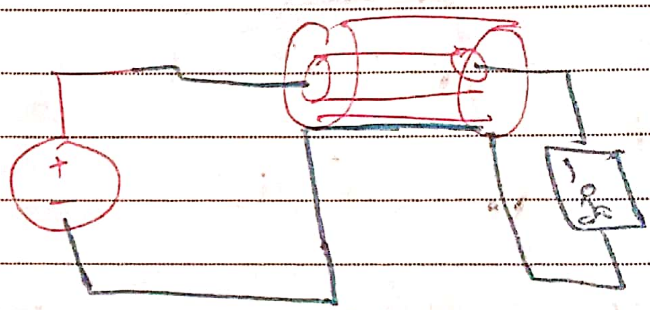


Ex : For a copper Coaxial Cable find the resistance at DC and at 100 MHz if

$a = 2$ mm $b = 6$ mm $t = 1$ mm

$L = 2$ m





at dc

$R_{DC} = \frac{L}{\sigma A}$, $\delta \to \infty$

~~$A = \pi a^2$~~

R_{DC} total = R_{inner} + R_{outer} series

$R_i = \frac{2}{5.7 \times 10^7 * \pi (2 \times 10^{-3})^2} = \boxed{2.744 \text{ m}\Omega}$

$R_o = \frac{2}{5.7 \times 10^7 * \pi * (13 \times 10^{-3})^2} = \boxed{0.8429 \text{ m}\Omega}$

$\boxed{R_{DC} = 3.587 \text{ m}\Omega}$

$\pi((b+t)^2 - b^2)$

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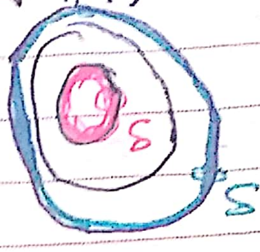
No.

at 100MHZ $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 6.6\,\mu m$

$R_{AC} = \frac{L}{\sigma \delta w}$



$R_{inner} = \frac{a}{\sigma \delta (2\pi a)}$

$R_{outer} = \frac{L}{\sigma \delta (2\pi (b+t))}$

$R_{ac} = R_{in} + R_{out} = 0.5484\,\Omega$
$= \boxed{548.4\,m\Omega}$

$\frac{R_{AC}}{R_{DC}} = \frac{a}{2\delta} = 150$ times.

$\boxed{H.\omega \quad f = 300MHZ}$

*power:
poynting vector

$\boxed{\nabla \times \bar{E} = -\frac{d\bar{B}}{dt} = -\mu \frac{d\bar{H}}{dt}}$ $\boxed{\nabla \times \bar{H} = \sigma \bar{E} + \epsilon \frac{d\bar{E}}{dt}}$

from $\nabla \times \bar{H}$ (dot product both sides) with E

$E.(\nabla \times H) = \sigma \bar{E}^2 + \epsilon \frac{dE}{dt} . \bar{E}$

for any vector $\bar{A}$ and $\bar{B}$

$\nabla.(\bar{A} \times \bar{B}) = \bar{B}.(\nabla \times \bar{A}) - \bar{A}.(\nabla \times \bar{B})$

let $B = \bar{E}$
$A = \bar{H}$

$\nabla (\bar{E}.(\nabla \times \bar{H}) = \nabla.(\bar{H} \times \bar{E}) + \bar{H}.(\underbrace{\nabla \times \bar{E}}_{-\mu \frac{d\bar{H}}{dt}})$

$= \sigma \bar{E}^2 + \bar{E} \epsilon \frac{d\bar{E}}{dt}$

No.

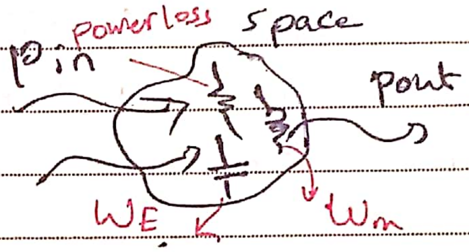
$$\nabla\cdot(\bar{H}\times\bar{E}) + \bar{H}\cdot\left(-\mu\frac{d\bar{H}}{dt}\right) = \sigma E^2 + \bar{E}\cdot\epsilon\frac{d\bar{E}}{dt}$$

$$\nabla\cdot(\bar{H}\times\bar{E}) = \sigma E^2 + \bar{E}\cdot\epsilon\frac{d\bar{E}}{dt} + \bar{H}\cdot\mu\frac{d\bar{H}}{dt}$$

$$\nabla\cdot(\bar{E}\times\bar{H}) = \sigma E^2 + \frac{\epsilon}{2}\frac{dE^2}{dt} + \frac{\mu}{2}\frac{dH^2}{dt}$$

Integrate all terms over the entire Volume.

$$\int_V \nabla\cdot(\bar{E}\times\bar{H})\,dv = -\int_V \underbrace{\sigma E^2\,dv}_{P_L} - \frac{d}{dt}\int_V$$



$$-\frac{d}{dt}\int_V\left[\underbrace{\frac{1}{2}\epsilon E^2}_{W_E} + \underbrace{\frac{1}{2}\mu H^2}_{W_m}\right]dv \rightarrow \text{poynting theorem}$$

applying the divergence theorem

$$\int_V \nabla\cdot(\bar{E}\times\bar{H})\,dv = \oint_S \bar{E}\times\bar{H}\cdot d\bar{s}$$

$$\oint_S \bar{E}\times\bar{H}\cdot ds = \;---$$

$P = \bar{E}\times\bar{H}$ → Poynting vector w/m^2
in $\hat{a}_k$ direction.

in lossy media

$$\bar{E}(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\ V/m\ \hat{a}_x$$

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No.

$$\bar{H}(z,t) = \frac{E_o}{\eta} e^{-\alpha z} \cos(wt - \beta z - \theta_\eta)\, a_y \ \text{A/m}$$

$$P = \frac{E_o^2}{2\eta} e^{-2\alpha z} \left(\cos(2wt - 2\beta z - \theta_\eta) + \cos(\theta_\eta) \right) \hat{a_z}$$

↳ time varying power

② time average poynting vector ($\bar{p}_{avg}$) (S_{avg})

$$\frac{1}{T}\int_T \bar{P}\, dt = \frac{1}{T}\int_T \bar{E} \times \bar{H}\, dt$$ (in time domain)

peak values

OR $$\bar{P}_{avg} = \frac{1}{2} Re\{ \bar{E_s} \times \bar{H_s}^* \}$$ in freq domain

$$\overline{P_{avg}} = \frac{E_o^2}{2\eta} e^{-2\alpha z} \cos(\theta_\eta)\, \hat{a_z} \ (w/m^2)$$

in Loss less $$p_{avg} = \frac{E_o^2}{2\eta} \hat{a_z} \ (w/m^2)$$

③ Total power (P_{avg}).

$$P_{avg} = \int_S \bar{p}_{avg} \cdot ds$$

→ Reciever Area

Example In a non magnetic medium ~~[illegible]~~

$\bar{E} = 4 \sin(2\pi \times 10^7 - 0.8x)\, \hat{a_z}$ V/m

Find
1- E_r, η
2- Time average power
3- Total power crossing 100 cm of plane $2x + y = 5$

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No.

① $u = \frac{\omega}{\beta} \neq c$ (lossless media),

$u = \frac{\omega}{\beta} = \frac{c}{\sqrt{\epsilon_r}}$ $\boxed{\epsilon_r = 14.59}$ $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$\frac{1}{\sqrt{\mu\epsilon}}$

$\eta = \sqrt{\frac{\mu}{\epsilon}} = \eta_0 \sqrt{\frac{1}{\epsilon_r}} = \boxed{10\pi^2 \, \Omega}$

② $P_{avg} = \frac{E_0^2}{2\eta} \hat{a}_k \ W/m^2 = \frac{16}{2(98.7)} \hat{a}_x \ W/m^2$

$= 81 \hat{a}_x \ W/m^2$

c) $P_{avg} = \int_S 81 \times 10^{-3} \hat{a}_x \cdot ds$

$\hat{a}_n = \frac{\nabla f}{|\nabla f|} = 2x + y - 5$

$\hat{a}_n = \frac{2}{\sqrt{5}} \hat{a}_x + \cancel{\frac{1}{\sqrt{5}} \hat{a}_y}$

$\nabla f = \frac{(2, 1, 0)}{\sqrt{5}}$

$= \int_S 81 \times 10^{-3} \times 100 \times 10^{-4} \times \frac{2}{\sqrt{5}} = \boxed{724.5 \, \mu W}$

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No.

Polarization of Em waves:-

↳ its the locus of the electric field as it propagates.

assume Types of polarization:-

1. Linear polarization

Linearized polarization Wave (LPW)

Assume LPW has:-

$$\bar{E} = E\, e^{j(\omega t - \beta y)}\, \hat{a}_x \ V/m$$

$$E = E_1 [\cos(\omega t - \beta y) + j\sin(\omega t - \beta y)]\, \hat{a}_x \ V/$$

Take the real or imagenary part

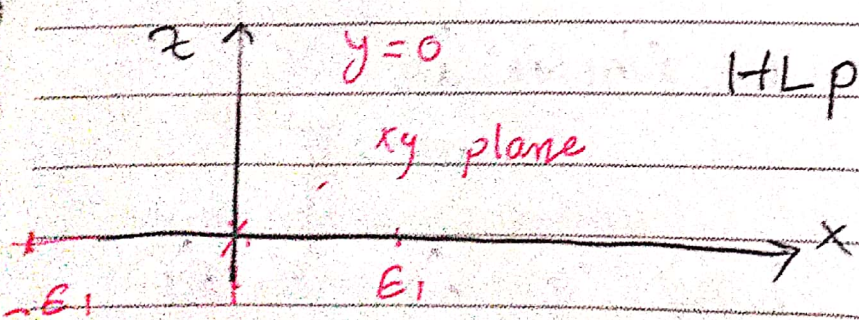
$$\bar{E} = E_1 \sin(\omega t - \beta y)\, \hat{a}_x \ V/m$$

in time domain ⟶ let $y = 0$

$$E = E_1 \sin \omega t\, \hat{a}_x \ V/m$$

$\omega t = 0$ | $\omega t = \pi/2$

$\bar{E} = 0$ | $\bar{E} = E_1 \hat{a}_x$



HLPW (horizontal linear polarized wave)

Ie for a Uniform planewave has

$$E = E_z\, e^{j(\omega t - ky)}\, \hat{a}_z$$

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Polarization:

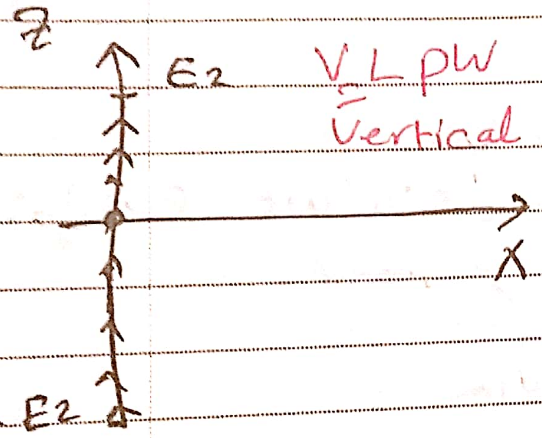
$\hat{E} = E_2 \sin(\omega t - ky)\, \hat{a_z}$ V/m

Let $y = 0$

$\hat{E} = E_2 \sin(\omega t)\, \hat{a_z}$ V/m

$\omega t = 0$ $\qquad\qquad$ $\omega t = \frac{\pi}{2}$

$\bar{E} = 0$ $\qquad\qquad$ $\bar{E} = E_2\, \hat{a_z}$ V/m



V L P W

Vertical Linear polarized wave.

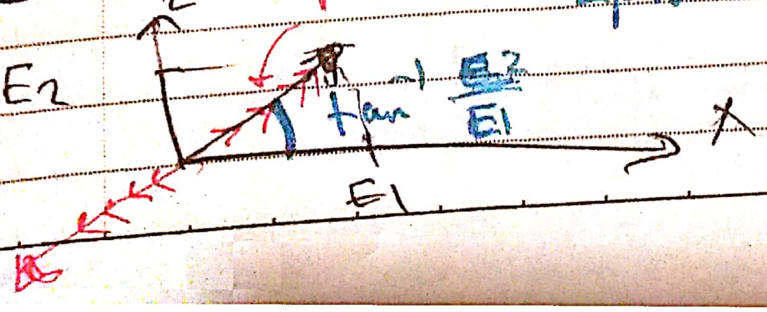
i.e for $\hat{E} = E_1 e^{j(\omega t - ky)}\, \hat{a}x + E_2 e^{(j(\omega t - ky))}\, \hat{a_z}$

Take imag part and let $y=0$

$\hat{E} = E_1 \sin \omega t\, \hat{a}x + E_2 \sin(\omega t)\, \hat{a_z}$

$\omega t = 0$ $\qquad\qquad$ $\omega t = \frac{\pi}{2}$

$E = 0$ $\qquad$ $\sqrt{E_1^2 + E_2}$ $\qquad$ LPW $\qquad$ $E_1 \hat{a}x + E_2 \hat{a}y$



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2) Circular polarization (cpw)

A UPW has $E = E_1 e^{j(\omega t - kz)} \hat{a_x} + E_1 e^{j(\omega t - kz \pm \frac{\pi}{2})} \hat{a_y}$

$\hat{a_z}$ الاتجاه

take the imag part and let $z = 0$

$\bar{E} = E_1 \sin(\omega t)\hat{a_x} + E_1 \sin(\omega t \pm \frac{\pi}{2}) \hat{a_y}$ V/m

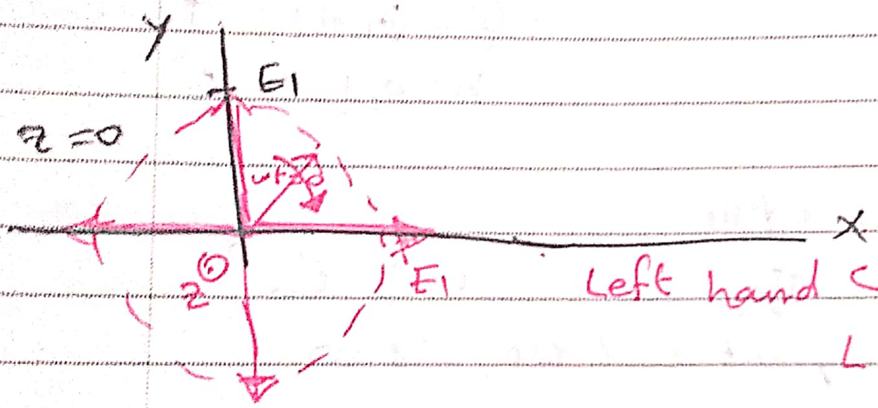
at $\omega t = 0$

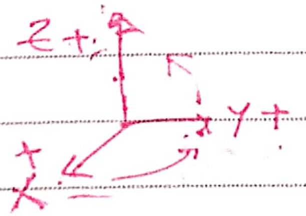
$\boxed{\text{let } \varphi = +\frac{\pi}{2}}$ →

$E = E_1 \hat{a_y}$

at $\omega t = \pi/2$

$\bar{E} = \bar{E_1} \hat{a_x}$





Left hand cpw

LHCPW

if $\varphi = -\pi/2$

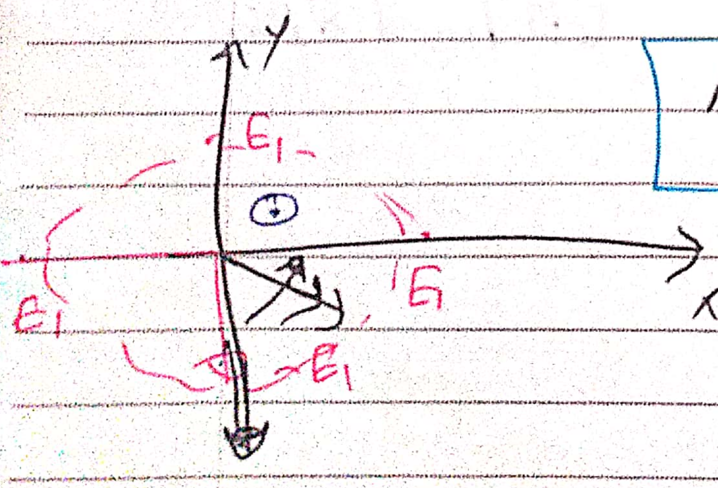
$\bar{E} = \sin \omega t \hat{a_x} + E_1 \sin(\omega t - \frac{\pi}{2}) \hat{a_y}$ V/m

at $\omega t = 0$

$\bar{E} = -E_1 \hat{a_x}$

at $\omega t = \frac{\pi}{2}$

$\bar{E} = E_1 \hat{a_x}$



$\boxed{\text{RHCPW}}$

$a_k = +\hat{a}_z$

③ Elliptically polarized wave

EPW → generally

A Upw has:

$\bar{E} = E_1 e^{-j(\omega t - kz)} \hat{a}_x + E_2 e^{j(\omega t - kz \pm \psi)} \hat{a}_y$

Could be Linear if

$\psi = n\pi \quad n = 0, 1, 2$

$E_2 = 0$

$E_1 = 0$

Could be circular

① $E_1 = E_2$ and

$\varphi = (n+1)\frac{\pi}{2}$

$n = 0, 1, 2, 3$

Other wise ⟹ EPW

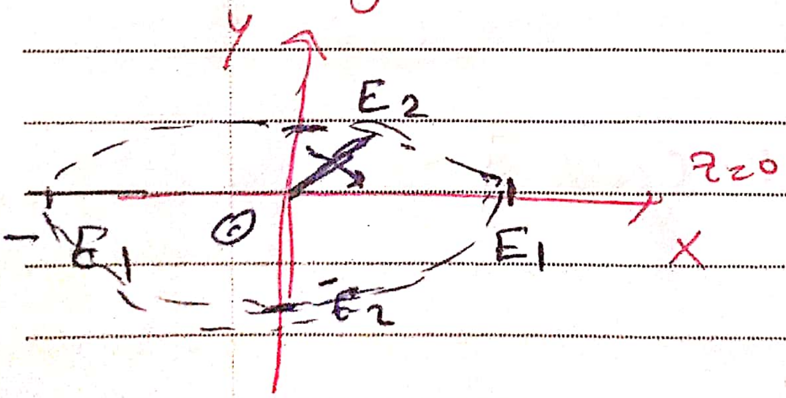
if $E_1 \neq E_2 \quad \psi = +\pi/2 \qquad E_1 > E_2$

take the imag. part and let $z = 0$

$\bar{E}_1 = E_1 \sin\omega t \, \hat{a}_x + E_2 \sin(\omega t + \frac{\pi}{2}) \hat{a}_y$

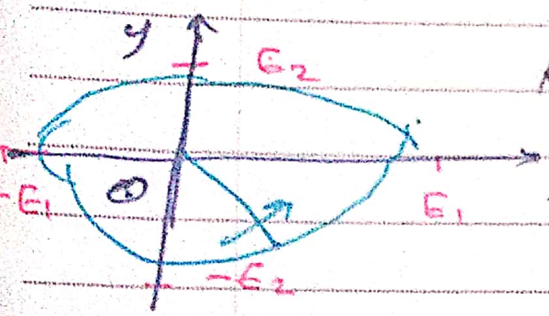
$\omega t = 0 \qquad\qquad \omega t = \frac{\pi}{2}$

$\bar{E} = E_2 \hat{a}_y \qquad\qquad \bar{E} = E_1 \hat{a}_x$



left hand EPW

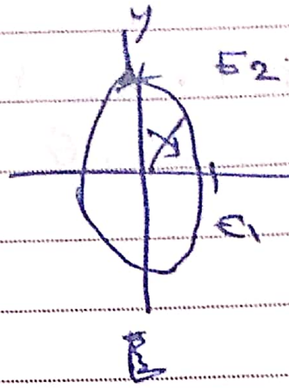
if $\psi = -\pi/2$



RHEPW

$\tau = 0^\circ$

if $E_2 > E_1 \rightarrow \psi = +\frac{\pi}{2}$



LHEPW

$\tau = 90^\circ$

Axial Ratio = $\dfrac{\text{Major axis}}{\text{minor axis}}$

AR = $\dfrac{\text{Major axis}}{\text{minor axis}}$

Tilt angle (τ) τ (with Reference to the ~~vertical~~ Horizontal axis).

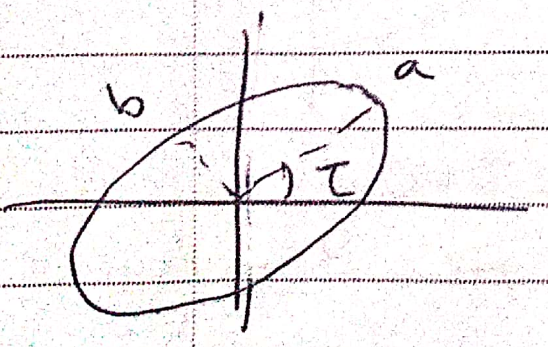
τ = angle between Refrence and the Major axis.

$1 \le AR < \infty$ (linear)
↳ (circular)

$0 \le AR(dB) < \infty$

$AR \rightarrow 0-3\ dB$ (circular)

$AR > 10\ dB$ (Linear)



$\tau \neq 0$

$\tau \neq 90^\circ$

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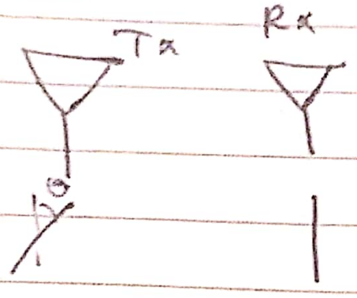
Example:



PLF polarization loss factor

$\therefore PLF = \cos\theta$

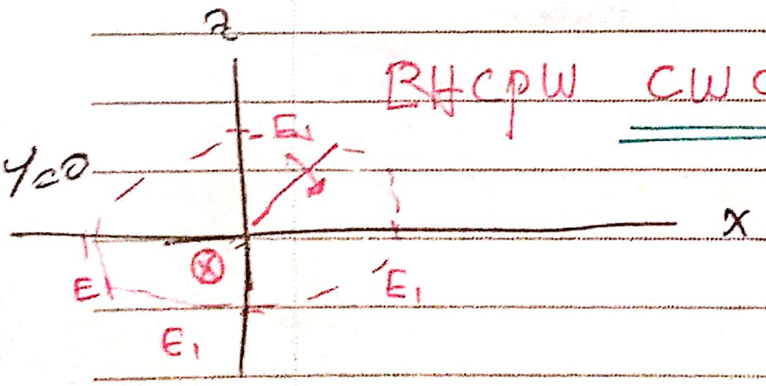
Ex For a Upw along +ve y-direction has

$\bar{E} = E_1 \sin(\omega t - \beta y)\,\hat{a}_x + E_2 \cos(\omega t - \beta y)\,\hat{a}_z$ V/m

Find the polarization AR and τ if

ⓐ $E = E_2$ at $y=0$

ⓐ ~~E~~ = $\omega t = 0$ $\quad E = E_1\,\hat{a}_z$ $\qquad \omega t = \frac{\pi}{2}$ $\quad E = E_1\,\hat{a}_x$



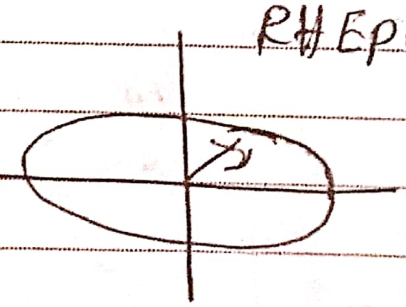
RHCPW CW cpw

$AR = 1$

$\tau =$ doesn't exist.

ⓑ $E_1 = 2E_2$

RHEPW



$\tau = 0^\circ$

$AR = 2 = \frac{2E/2}{E/2}$

Ex. the $\bar{E}$-field for a Upw in free space is

$$\bar{E}_s = (3\hat{a}_x + j4\hat{a}_y)\, e^{-j0.5\pi z} \quad V/m$$

j ↳ 90° phase shift.

Find :- a- frequency and $\bar{H}$

b- it's polarization and (AR) and (τ)

From $\bar{E}$ ← c- pave and the total power crossing a $2\times2\,m^2$ plane along xy plane.

(a) $\beta = \frac{\omega}{c}$ $\quad \omega = \beta c$

$\omega = 0.5\pi * 3\times10^8 = 1.5\pi\times10^8$

$f = \frac{\omega}{2\pi} = 75\,MHz$

$$\bar{H} = \frac{3}{\eta_0}\hat{a}_y - \frac{j4}{\eta_0}\hat{a}_x\, e^{-j0.5\pi z} \quad A/m.$$

$\hat{a}_H = \hat{a}_k \times \hat{a}_E = \hat{a}_y$

(b) $\bar{E} = \text{Imag}\{E_s e^{j\omega t}\}$

$$3\sin(\omega t - 0.5\pi z)\hat{a}_x + 4\sin\left(\omega t - 0.5\pi z + \frac{\pi}{2}\right)\hat{a}_y$$

Let $z=0 \Rightarrow 3\sin(\omega t)\hat{a}_x + 4\sin(\omega t + \pi/2)$

$\omega t = 0$ $\qquad\qquad$ $\omega t = \pi/2$

$\bar{E} = 4\hat{a}_y$ $\qquad\qquad$ $\bar{E} = 3\hat{a}_x$

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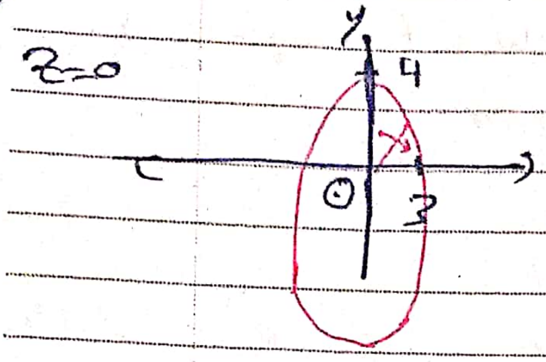


LHEPW

$AR = 4/3$

$\tau = 90^\circ$

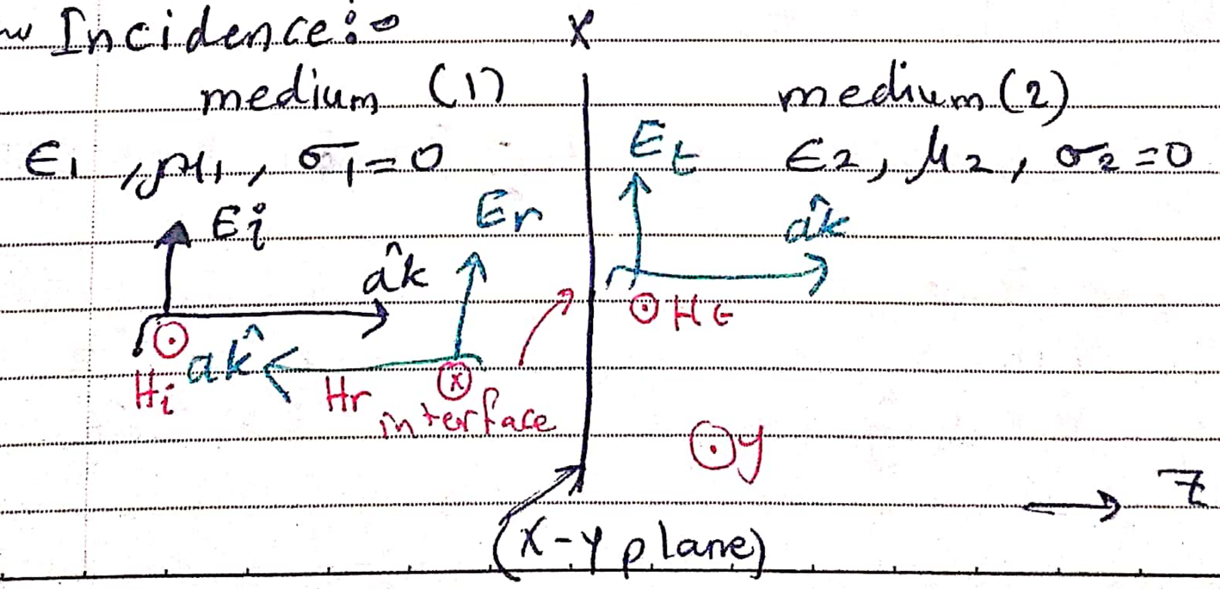
ⓒ $P_{ave} = \frac{1}{2} Re \{ \bar{E}_s \times \bar{H}_s^* \}$

$= \frac{E_o^2}{2\eta_o} e^{-2\alpha z} \cos\theta_\eta$

$= \frac{25}{2(120\pi)} \hat{a}_z = \frac{5}{48\pi} \hat{a}_z \ w/m^2$

$P_{ave} = \int_s \mathcal{P}_{ave} \cdot ds = \frac{5}{48\pi} * 4 = 0.417 W$

Reflection of a plane wave at normal Incidence:



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The incident fields :- For lossless

$$E_i = E_{io}\, e^{-\gamma_1 z}\, \hat{a_x}\ \text{V/m} = E_{io}\, e^{-j\beta_1 z}\, \hat{a_x}\ \text{V/m}$$

$$H_i = \frac{E_{io}}{\eta_1}\, e^{-\gamma_1 z}\, \hat{a_y}\ \text{A/m}$$

~~$\alpha_1, \beta_1, \eta_1$~~ $\rightarrow$ $\alpha_1, \beta_1, \eta_1$ (calculated from $(\epsilon_1, \mu_1, \sigma_1)$.

The Reflected field

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$$\bar{E}_r = E_{ro}\, e^{+\gamma_1 z}\, \hat{a_x}\ \text{V/m}$$

$$H_r = \frac{E_{ro}}{\eta_1}\, e^{\gamma_1 z}\, (-\hat{a_y})\ \text{A/m}$$

The transmitted fields.

$$\hat{E}_t = E_{to}\, e^{-\gamma_2 z}\, \hat{a_x}$$

$$\bar{H}_t = \frac{E_{to}}{\eta_2}\, e^{-\gamma_2 z}\, \hat{a_y}\ \text{A/m}$$

Apply the boundary conditions to find at the interface $(z=0)$

$\tilde{E}_{ro}, \tilde{E}_{to}$

$$\left.\begin{aligned} \bar{E}_{1t} &= \bar{E}_{2t} \\ \bar{H}_{1t} &= \bar{H}_{2t} \end{aligned}\right\}$$

$(H_{1t} - H_{2t} = K) \Rightarrow$ if there is a current at the surface.

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$$E_1 = E_i + E_r$$

$$E_2 = E_t$$

$$E_{1t}(0) = E_{2t}(0) \qquad (t \rightarrow \text{tangent}).$$

$$E_{io}\, a_x + E_{ro}\, \hat{a}_x = E_{to}$$

$$\boxed{E_{io} + E_{ro} = E_{to}} \text{ — } ①$$

from $H_{1t} = H_{2t}$

$$\frac{E_{io}}{\eta_1}\, \hat{a}_y - \frac{E_{ro}}{\eta_1}\, \hat{a}_y = \frac{E_{to}}{\eta_2}\, \hat{a}_y$$

$$\boxed{E_{io} - E_{ro} = \frac{\eta_1}{\eta_2} E_{to}} \text{ — } ②$$

$$E_{ro} = E_{io}\left(\frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}\right)$$

$$E_{to} = E_{io}\left(\frac{2\eta_2}{\eta_1 + \eta_2}\right)$$

$$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}$$ (Reflection coeff.

$$E_{ro} = \Gamma E_{io} \qquad E_{to} = \tau E_{io}$$

$$\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$ (transmission coeff.)

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No.

$0 < |\Gamma| < 1 \rightarrow$ total Reflection (no transmission

Total Transmission (no reflection).

$1 < \boxed{\tau = 1 + \Gamma} < 2$

Special Case

medium(1) is a lossless media and medium(2) is a good Conductor media

$\eta_2 = 0 \quad \tau = 0 \quad \Gamma = -1$

$\bar{E}_{i_0} = E_{io}\, e^{-j\beta_1 z}\, \hat{a}_x$

$E_r = \Gamma E_{io}\, e^{+j\beta_1 z}\, \hat{a}_x$, $\Gamma = -1$

$\bar{E}_t = \tau E_{io}\, e^{-j\beta_2 z}\, \hat{a}_x = 0$

$E_1 = E_i + E_r$

$E_1 = E_{io}\left(e^{-j\beta_1 z} - e^{j\beta_1 z}\right)\hat{a}_x$

$-E_{io}\left(e^{j\beta_1 z} - e^{-j\beta_1 z}\right)\hat{a}_x$

$= -2j\, E_{io} \sin(\beta_1 z)\, \hat{a}_x$

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$\bar{E}(z,t) = Re\{E_s\, e^{j\omega t}\}$

$\hat{E}(z,t) = 2E_{io}\sin(\beta_1 z)\sin(\omega t)\hat{a}_x$

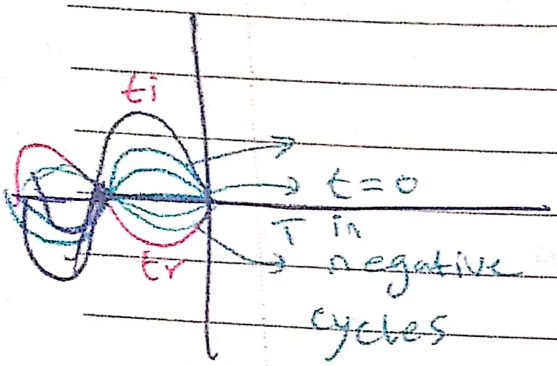
standing wave

$t=0$

$t=T/8$

$t=T/4$

$t=T/2$



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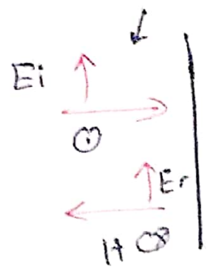
* lossless

good conductor

$\eta_2 = 0$



$\overline{E}_i = E_{io} e^{-jz\beta_1} \hat{a}_x$

$\overline{E}_r = E_{ro} e^{j\beta_1 z} \hat{a}_x$

$E_{ro} = -E_{io}$ $\quad \boxed{\Gamma = -1}$

$\overline{E}_1 = \overline{E}_i + \overline{E}_r$

$\overline{E}_1 = 2 E_{io} \sin(\beta_1 z) \sin(\omega t) \hat{a}_x$

for standing wave

$\overline{H}_i = \dfrac{E_{io}}{\eta_i} e^{-j\beta_1 z} \hat{a}_y$

$\overline{H}_r = -\dfrac{E_{ro}}{\eta_1} e^{j\beta_1 z} \hat{a}_y$

$\Gamma_H = \dfrac{H_r}{H_i} = \dfrac{E_{io}}{E_{io}} = 1$

$\overline{H}_1 = \overline{H}_i + \overline{H}_r$

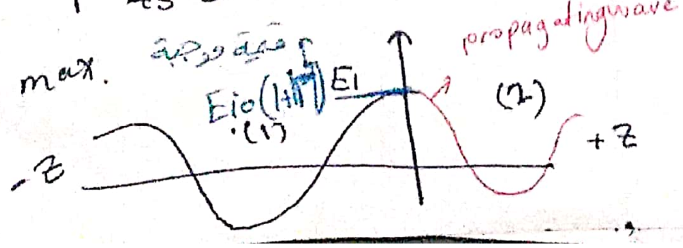
$= \dfrac{E_{io}}{\eta_1} \left(e^{-j\beta_1 z} + e^{j\beta_1 z} \right) \hat{a}_y$

$= H(z,t) = 2 \dfrac{E_{io}}{\eta_1} \cos(\beta_1 z) \cos(\omega t) \hat{a}_y$

* Special Cases :-

$\boxed{\text{for } \eta_1 > \eta_2}$

Γ is -ve but $\neq -1$



①

propagating waves

E_1 is a standing wave

$\left. \begin{matrix} E_i \\ E_r \end{matrix} \right] E_1$

$E_t \,]\, E_2$

maxima locations

↳ when $\sin \beta_1 z$ (max) $= 1$

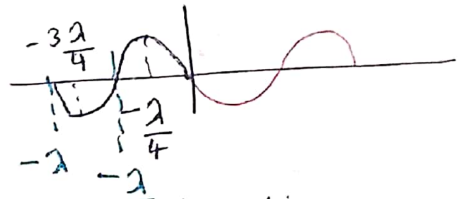
$\beta_1 z = (2n+1)\dfrac{\pi}{2}$

$n = 0, 1, 2, \ldots$ $\quad \beta = \dfrac{2\pi}{\lambda}$

$z = \dfrac{(2n+1)\frac{\pi}{2}}{\frac{2\pi}{\lambda}}$

medium (1) is in the negative z region

$z = -(2n+1)\dfrac{\lambda}{4}$



minima locations

$\beta_1 z = n\pi$

$n = 0, 1, 2, \ldots$

$z = \dfrac{n\pi}{\frac{2\pi}{\lambda}}$ $\qquad z = -\dfrac{n\lambda}{2}$

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$\boxed{\text{if } \eta_2 > \eta_1}$

Γ is positive

E_1 is a standing wave

$E_2 \neq 0$ it is a propagating wave

maxima location

$$z = \frac{-n\lambda}{2} \quad n = 0, 1, 2, \ldots$$

minima Location

$$z = -(2n+1)\frac{\lambda}{4} \quad n = 0, 1, 2, \ldots$$

Standing Wave Ratio (SWR) or (s)

$$s = \frac{E_{max}}{E_{min}} = \frac{H_{max}}{H_{min}}$$

$$= \frac{|E_i| + |E_r|}{|E_i| - |E_r|}$$

$$= \frac{|E_i| + |\Gamma||E_i|}{|E_i| - |\Gamma||E_i|}$$

$$= \boxed{E_i\left(\frac{1+|\Gamma|}{1-|\Gamma|}\right)}$$

$$1 \leq s < \infty$$

total transmission $\Gamma = 0$

total Reflection $\Gamma = \pm 1$

mag. only

$$|\Gamma| = \frac{s-1}{s+1}$$

in free space and lossless only.

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$s(dB) = 20 \log_{10} s$$

$$0 < s_{dB} < \infty$$

$\boxed{\text{Example}}$

In free space $(z \leq 0)$ a Uniform UPW has $\overline{H}_i = 10\cos(10^8 t - \beta z)\hat{a}_x$ mA/m is incident normally in a lossless medium ($\epsilon = 2\epsilon_0$, $\mu = 8\mu_0$) in Region $(z \geq 0)$

Determine $\overline{H}_r$, $\overline{E}_r$, $\overline{H}_t$, $\overline{E}_t$

$$\eta_1 = \eta_0 = 120\pi\,\Omega$$

$$\eta_2 = \eta_0\sqrt{\frac{\mu_r}{\epsilon_r}} = 2\eta_0 = 240\pi\,\Omega$$

$$\eta_2 > \eta_2$$

$$\Gamma = \frac{2\eta_0 - \eta_0}{2\eta_0 + \eta_0} = \frac{1}{3}$$

$$\boxed{\Gamma = \frac{1}{3}}$$

$$\Gamma_H = -\frac{1}{3}$$

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$$\bar{E}_i = 10\eta_o \cos(10^8 t - \beta_1 Z)\, \hat{a}_y$$

$$\bar{E}_r = -\frac{10\,\eta_o}{3} \cos(10^8 t + \beta_1 Z)\, \hat{a}_y$$

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$$H_r = \frac{-10}{3} \cos(10^8 t + \beta_1 Z)\, \hat{a}_x$$

$\Gamma_H H_i$

or $-\Gamma H_i$

$$\boxed{T = \frac{4}{3}}$$

$$\bar{E}_t = -\frac{40}{3}(\eta_o) \cos(10^8 t - \beta_2 Z)\, \hat{a}_y$$

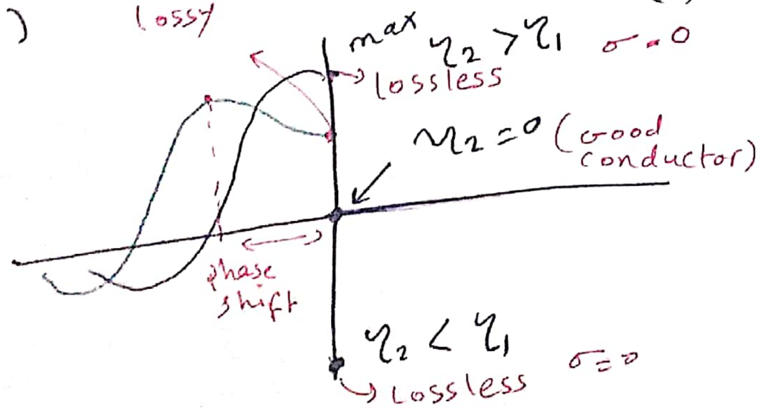
$$\bar{H}_t = \frac{20}{3} \cos(10^8 t - \beta_2 Z)\, \hat{a}_x$$

$$\beta_1 = \frac{\omega}{c} \quad \text{free Space} = \boxed{\frac{1}{3}\ \text{rad/m}}$$

$$\beta_2 = \frac{\omega}{c}\sqrt{\mu_r \epsilon_r} = \boxed{\frac{4}{3}\ \text{rad/m}}$$

(1) lossy

(2)



Example

A given Uniform Upw plane in air has

$$\bar{E}_i = 40\cos(\omega t - \beta Z)\hat{a}_x + 30\sin(\omega t - \beta Z)\hat{a}_y \quad \text{V/m}$$

Find ⓐ $\bar{H}_i$

ⓑ if the wave encounters a perfectly conducting plane normal to the Z-axis at $z=0$ find $\bar{E}_r$, $\bar{H}_r$

ⓒ calculate $\bar{E}_1$, $\bar{H}_1$, $\bar{E}_2$, $\bar{H}_2$

ⓓ calculate the time average poynting vector for $z \le 0$ $\quad z > 0$

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$\eta_1 = \eta_0$ (4)

ⓐ $\bar{H_i}$ = $\hat{a}_{H1} = \hat{a}_z \times \hat{a}_{\cancel{x}} = \hat{a}_y$

$\hat{a}_{H}^{2} = \hat{a}_z \times \hat{a}_y = -\hat{a}_x$

$$H_i = \frac{-30}{\eta_0} \sin(\omega t - \beta_1 z)\,\hat{a}_x + \frac{40}{\eta_0}\cos(\omega t - \beta_1 z)\,\hat{a}_y \quad A/m$$

ⓑ Air η_0 | Conductor $\eta_2 = 0$ $\Gamma = -1$

$E_2 = 0$, $H_2 = 0$ > no transmitted

$\bar{E_1} = \bar{E_i} + \bar{E_r}$

$\hookrightarrow$ ( ) $\hat{a}_x$ + ( ) $\hat{a}_y$

ⓒ $P_{avg_1} = \dfrac{E_p^2}{2\eta}\,\hat{a}_k$

~~E_{p}~~ $\cancel{|E_i|^2 + |E_r|^2}$ $\underline{\underline{= 0}}$

$P_{ave_2} = 0$

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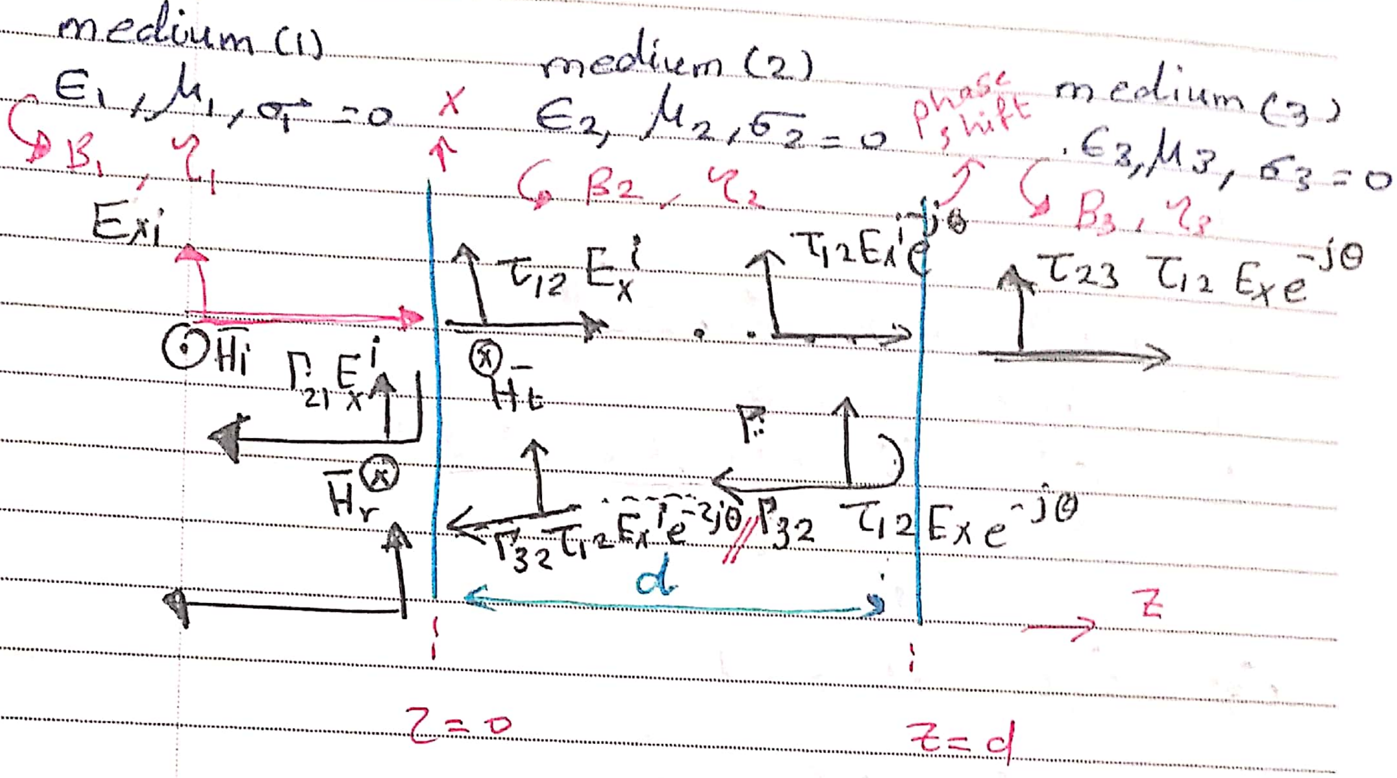
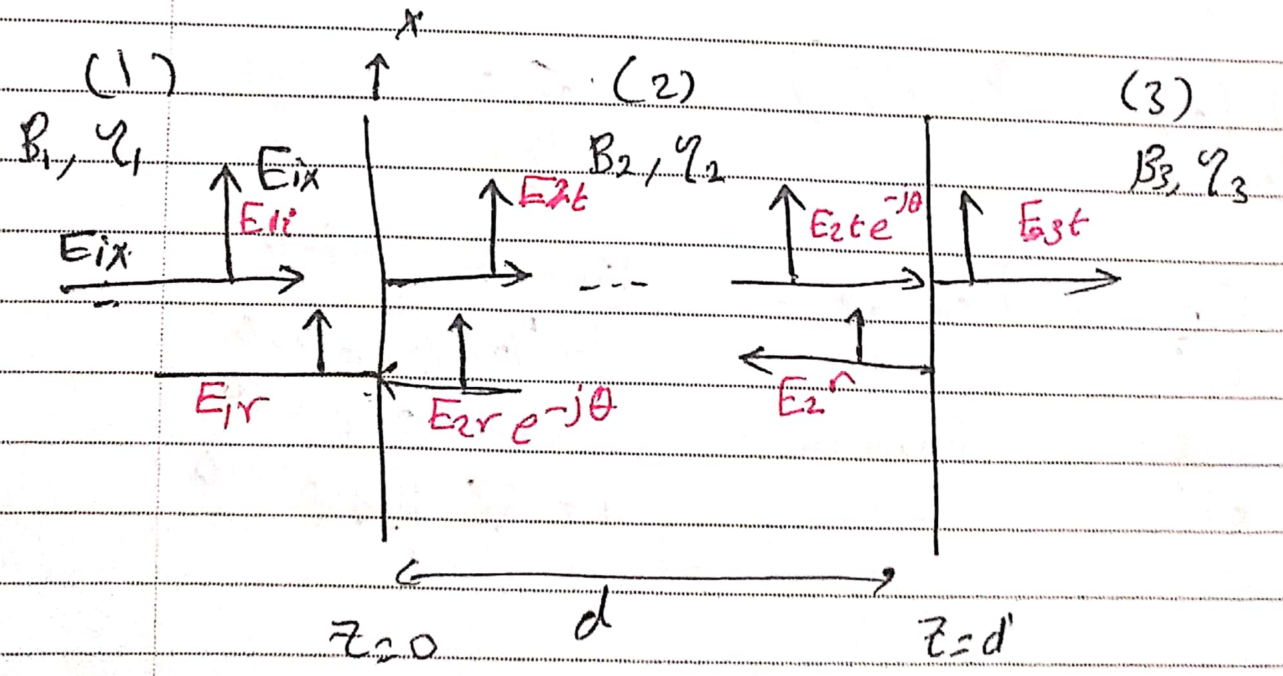
25-6

Reflection of Em Wave at normal Incidence on Multiple reflections



$\theta = \beta_2 d$



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No.

Unknowns are

E_2t

E_2^r

E_3^t

E_r

Boundary Conditions

at $z=0$

at $z=d$

tangent

$E_1(t) = E_2(t)$ at $z=0$

? ? ?

$E_1^i + E_1^r = E_2t + E_2r e^{-j\theta}$ — (1)

$H_1^t = H_2t$ at $z=0$

$\frac{1}{\eta_1}(E_1^i - E_1^r) = \frac{1}{\eta_2}(E_2^t - E_2^r e^{-j\theta})$ — (2)

? ? ? ?

$E_2t = E_3t$ at $z=d$

? ? ?

$E_2t e^{-j\theta} + E_2^r = E_3t$ — (3)

$H_2t = H_3(t)$ at $z=d$

$\frac{1}{\eta_2}(E_2t e^{-j\theta} - E_2^r) = \frac{E_3t}{\eta_3}$ — (4)

$\frac{E_1r}{E_1^i} = \Gamma_{overall} = \frac{\Gamma_{21} + \Gamma_{32} e^{-j2\theta}}{1 + \Gamma_{21}\Gamma_{32} e^{-j2\theta}}$

$\frac{E_3t}{E_1^i} = \tau_{overall} = \frac{\tau_{12}\tau_{23} e^{-j2\theta}}{1 + \Gamma_{21}\Gamma_{32} e^{-j2\theta}}$

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Shielding Effectiveness (S.E)

$S.E = 20\log_{10} T_{overall}$.

from E 2701

$$\Gamma_{21} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \Gamma_{32} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2}$$

$$\tau_{12} = \frac{2\eta_2}{\eta_1 + \eta_2} \qquad \tau_{23} = \frac{2\eta_3}{\eta_2 + \eta_3}$$

$\theta = \beta_2 d$

Example: If a singal is incident normally from medium (1) find ~~overall~~ $\Gamma_{overall}$, $T_{overall}$, S.E

(1)	(2)	(3)
$\epsilon = \epsilon_0$ $\mu_1 = \mu_0$ $\sigma = 0$	$\epsilon_1 = 16\epsilon_0$, $\mu_2 = \mu_0$, $\sigma_2 = 0$	$\epsilon_3 = 4\epsilon_0$ $\sigma_3 = 0$, $\mu_3 = \mu_0$

$\longleftarrow d \longrightarrow$

ϵ_2

for a) $d = \lambda/4$

b) $d = \lambda/2$

c) Find (ϵ_2) to have $\Gamma_{overall} = 0$ if $d = \frac{\lambda}{4}$, $\mu_2 = \mu_0$

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$\eta_1 = 120\pi\ \Omega$ $\qquad$ $\eta_3 = \eta_0/2 = 60\pi\ \Omega$

$\eta_2 = \frac{\eta_0}{4} = 30\pi\ \Omega$

$\Gamma_{21} = -0.6$ $\qquad$ $\Gamma_{32} = \frac{1}{3}$ $\qquad$ $\tau_{12} = 0.4$

$\tau = \frac{4}{3}$

a) $d = \frac{\lambda}{4}$ $\qquad$ $\theta = \beta_2 d$

$$\theta = \frac{2\pi}{\lambda} * \frac{\lambda}{4} = \frac{\pi}{2}$$

$\Gamma_{overall} = 0.78$

$\tau_{overall} = -0.44$

$S.E = 20 \log \tau_{overall}$

β

b) $\theta = \frac{2\pi}{\lambda} * \frac{\lambda}{2} = \pi$

$\Gamma_{overall} = -0.33$ $\qquad$ $\tau_{overall} = 0.67$

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(c) $\Gamma_{overall} = 0$

$$\Gamma_{21} + \Gamma_{32} \underbrace{e^{-j2\theta}}_{=-1} = 0$$

$$\Gamma_{21} = \Gamma_{32}$$

$$\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2}$$

η_2 $\boxed{\eta_2 = \sqrt{\eta_1 \eta_3}}$ ← after simplification

→ Matching condition.

$$\eta_2 = 60\pi\sqrt{2}\ \Omega = \frac{\eta_0}{\sqrt{\epsilon_r}} = 2 \qquad \boxed{\epsilon_2 = 2}$$

* Reflection of Em Wave at Oblique incidence

in general a Upw has

$$\bar{E}(r, t) = \bar{E}_0 \cos(\bar{k} \cdot \bar{r} - \omega t)$$

$$\bar{k} = k_x \hat{a}_x + k_y \hat{a}_y + k_z \hat{a}_z$$

$$\bar{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

$$\bar{k} \cdot \bar{r} = k_x x + k_y y + k_z z$$

maxwells equation Can be written as:

$$k \cdot \bar{E} = 0$$
$$k \cdot \bar{H} = 0$$
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$$k \times \bar{E} = \omega \mu \bar{H}$$
$$k \times \bar{H} = -\omega \epsilon \bar{E}$$

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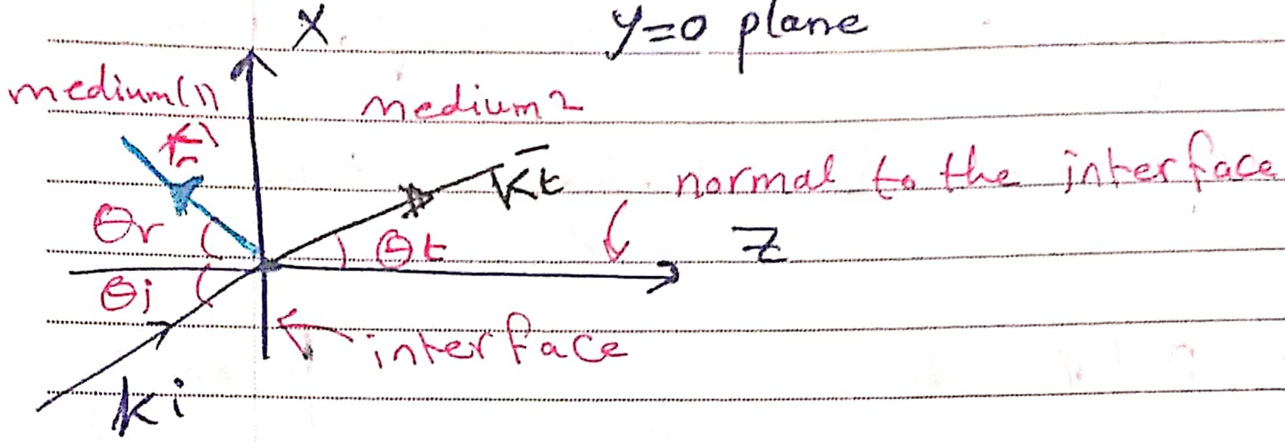
$$\bar{H} = \frac{k \times \bar{E}}{\omega\mu} = \frac{k\hat{a}_x \times \bar{E}}{\omega\mu}$$

$k=\beta$ for lossless $\qquad k = \omega\sqrt{\mu\epsilon}$

$$\bar{H} = \sqrt{\frac{\epsilon_0}{\mu_0}}\;\hat{a}_k \times \bar{E} = \hat{a}_k \times \frac{E}{\eta}\hat{a}_E$$



$$E_i = E_{io}\cos(k_{ix}X + k_{iy}Y + k_{iz}Z - \omega_i t)$$

$$E_r = E_{ro}\cos(k_{rx}X + k_{ry}Y + k_{rz}Z - \omega_r t)$$

$$E_t = E_{to}\cos(k_{tx}X + k_{ty}Y + k_{tz}Z - \omega_t t).$$

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τ: tangent

$E_t(z=0^-) = Z_t(z=0^+)$

plane of incidence is the XY plane ($\bar{k}$ and $\hat{a}_n$)

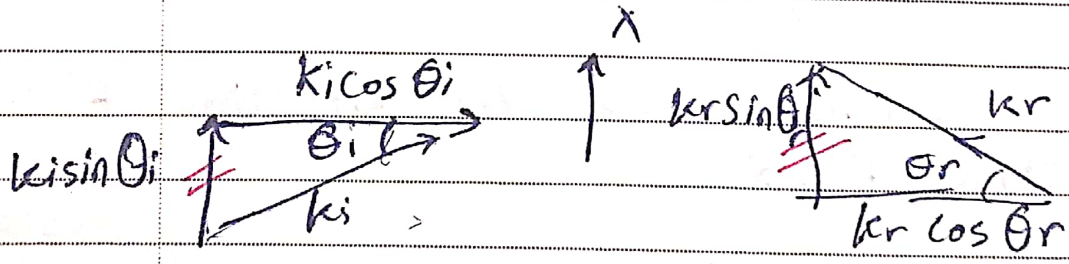
$E_i(z=0^-) + E_r(z=0^-) = Z_t(z=0^+)$

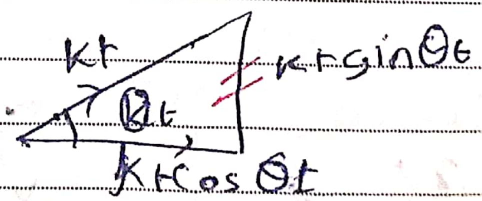
There are several conditions.

(1) $\omega_i = \omega_r = \omega_t = \omega$ (freq. Unchanged).

(2) $k_{ix} = k_{rx} = k_{tx} = k_x$ ∥ x,y — parallel to the interface — the tangential phase constant are matched

(3) $k_{iy} = k_{ry} = k_{ty} = k_y$





$k_i \sin\theta_i = k_r \sin\theta_r$

$k_i = \beta = \omega\sqrt{\mu_1 \epsilon_1}$

$k_i = k_r = \omega\sqrt{\mu_1 \epsilon_1}$

$\therefore \boxed{\theta_i = \theta_r}$

Snell's law for reflection.

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$$\boxed{k_i \sin\theta_i = k_t \sin\theta_t}$$ snell's law for reflection.

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{k_t}{k_i} = \frac{\omega\sqrt{\mu_2 \epsilon_2}}{\omega\sqrt{\mu_1 \epsilon_1}}$$

$$= \frac{\sqrt{\mu_{r2}\epsilon_{r2}}/\cancel{c}}{\sqrt{\mu_{r1}\epsilon_{r1}}/\cancel{c}} \Rightarrow \frac{\sin\theta_i}{\sin\theta_t} = \frac{\sqrt{\mu_{r2}\epsilon_{r2}}}{\sqrt{\mu_{r1}\epsilon_{r1}}} = \frac{n_2}{n_1}$$

n: reflection index.

$$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$$

since $z=0$ and no propagation in y direction.

$$\cancel{E_{io}\cos(k_i x - \omega t) + E_{ro}\cos(k_r x - \omega t) = E_{to}\cos(k_t x - \omega t)}$$

There are two cases on the direction of electric field

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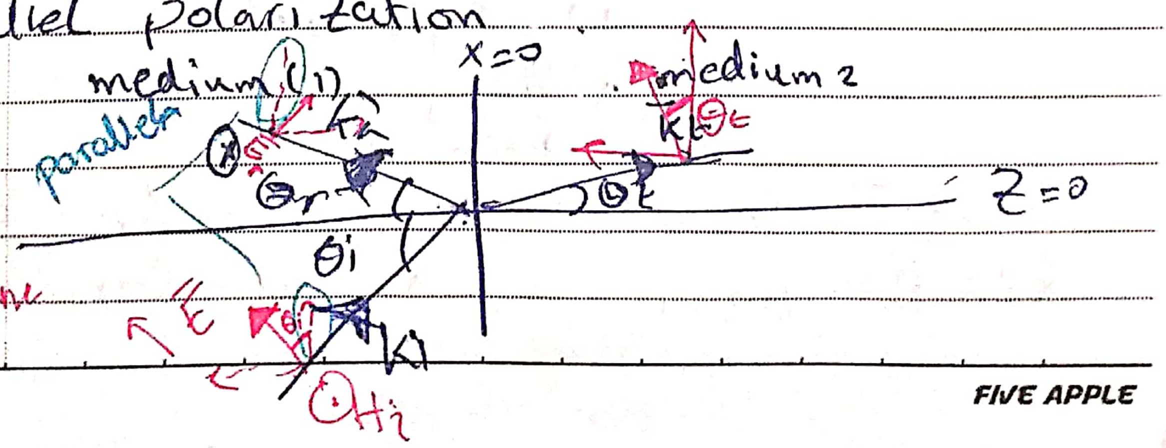
E in the same plane

parallel polarization

$\hat{a}_H = \hat{a}_k \times \hat{a}_E$



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The incident fields

$$E^i = E_{io}\left(\cos\theta_i\,\hat{a}_x - \sin\theta\,\hat{a}_z\right)_i \quad V/m$$
$$\cdot\, e^{-j\beta_1(\sin\theta_i x + \cos\theta_i z)}$$

$$H^i = \frac{E_{io}}{\eta_1} e^{-j\beta_1(\sin\theta_i x + \cos\theta_i z)} \quad \hat{a}_y \; A/m$$

* * * * * * *

The Reflected fields

$$\bar{E}_r = E_{ro}\left(\cos\theta_i\,\hat{a}_x + \sin\theta_i\,\hat{a}_z\right) e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

$$\bar{H}_r = \frac{E_{ro}}{\eta_1} \cdot e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \quad -\hat{a}_y \; A/m$$

* * * * * *

The transmitted fields

$$\bar{E}_t = E_{to}\left(\cos\theta_t\,\hat{a}_x - \sin\theta_i\,\hat{a}_z\right) e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad V/m$$

$$\bar{H}_t = \frac{E_{to}}{\eta_2} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \quad \hat{a}_y \quad A/m$$

$\boxed{\text{Unknowns} \quad E_{ro}, E_{to}\ ?}$ B.C.s

$E_{1t} = E_{2t}$

$H_{1t} = H_{2t}$

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$$E_i(z=0) + E_r(z=0) = E_t(z=0)$$

$$e^{-j\beta_1 x \sin\theta} = e^{-j\beta_2 x \sin\theta_t}$$

$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t \rightarrow$ snell's law

$$E_{io}\cos\theta_i \cancel{a_x} + E_{ro}\cos\theta_i \cancel{a_x} = E_{to}\cos\theta_t \cancel{a_x}$$

$$(E_{ro} + E_{io})\cos\theta_i = E_{to}\cos\theta_t \rightarrow 1$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{E_{to}}{\eta_2} - 2$$

$$\frac{E_{ro}}{E_{io}} = \Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\frac{E_{to}}{E_{io}} = \tau_{\parallel} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\boxed{1 + \Gamma_{\parallel} = \tau_{\parallel}\frac{\cos\theta_t}{\cos\theta_i}}$$

$E_{ro} = \Gamma_{\parallel} E_{io}$ / $E_{to} = \tau_{\parallel} E_{io}$

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no reflection $\rightarrow$ $\Gamma_{\parallel} = 0$

$\theta_i = \theta_{B\parallel} \equiv$ Brewster angle
(Polarizing angle)

$\eta_2 \cos\theta_t = \eta_1 \cos\theta_{B\parallel}$

$\eta_2 (1 - \sin^2\theta_t) = \eta_1 (1 - \sin^2\theta_{B\parallel})$

$$\frac{\sin\theta_{B\parallel}}{\sin\theta_t} = \frac{k_t}{k_i} = \frac{\sqrt{\mu_{r2}\,\epsilon_{r2}}}{\sqrt{\mu_{r1}\,\epsilon_{r1}}}$$

$$\sin\theta_t = \frac{\sqrt{\mu_{r1}\,\epsilon_{r1}}}{\sqrt{\mu_{r2}\,\epsilon_{r2}}} \sin\theta_{B\parallel}$$

$$\boxed{\sin\theta_{B\parallel} = \frac{1 - \frac{\mu_2 \epsilon_1}{\mu_1 \epsilon_2}}{1 + \left(\frac{\epsilon_1}{\epsilon_2}\right)^2}}$$

Special Cases ; for non Magnetic media
$(\mu_1 = \mu_2 = \mu_0)$

$$\sin^2\theta_{B\parallel} = \frac{1}{1 + \frac{\epsilon_1}{\epsilon_2}} = \frac{\epsilon_2}{\epsilon_1 + \epsilon_2}$$

$$\boxed{\tan\theta_{B\parallel} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

$$\boxed{\sin\theta_2 = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1 + \epsilon_2}}}$$

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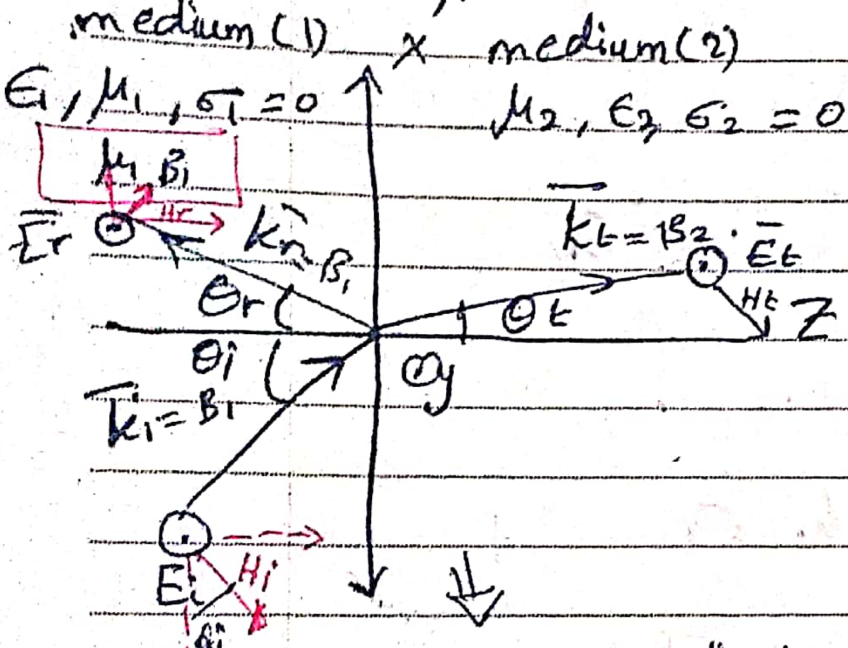
* perpendicular polarization

($\bar{E}$ is perpendicular to the plane of incidence).



$\boxed{\beta_2, \mu_2}$

$\hat{a}_H = \hat{a}_k \times \hat{a}_E$

$(\hat{a}_x, \hat{a}_z) \times \hat{a}_y$

$\hat{a}_H = \hat{a}_z, -\hat{a}_x$

perpendicular component $\hat{a}_z$

** the incident fields:-

$$\bar{E}_i = E_{io}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\, \hat{a}_y \text{ V/m}$$

$$\bar{H}_i = \frac{E_{io}}{\eta_1}(-\cos\theta_i\, \hat{a}_x + \sin\theta_i\, \hat{a}_z)\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \text{ A/m}$$

** The Reflected fields:-

$$\bar{E}_r = E_{ro}\, e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}\, \hat{a}_y \text{ V/m}$$

$$\bar{H}_r = \frac{E_{ro}}{\eta_1}(\cos\theta_i\, \hat{a}_x + \sin\theta_i\, \hat{a}_z)\, e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \text{ A/m}$$

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$\hat{a}_z \Rightarrow$ Normal

No.

The transmitted fields ..

$$\bar{E}_t = E_{to}\, e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)}\, \hat{a}_y \quad V/m$$

$$\bar{H}_t = \frac{E_{to}}{\eta_2} \left(-\cos\theta_t \hat{a}_x + \sin\theta_t \hat{a}_z \right) e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)} \quad A/m.$$

To find E_{To}, E_{ro} apply B.C at the interface $(z=0)$

$$\bar{E}_i(z=0) + \bar{E}_r(z=0) = \bar{E}_t(z=0)$$

$$\left(E_{io}\, \cancel{\hat{a}_y} + E_{ro}\, \cancel{\hat{a}_y}\right) \cancel{e^{-j\beta_1 x\sin\theta_i}} = E_{to}\, \cancel{e^{-j\beta_2 x\sin\theta_t}}\, \cancel{\hat{a}_y}$$

By Snell's equation

$$E_{io} + E_{ro} = E_{to} \longrightarrow (1)$$

$$\frac{(-E_{io} + E_{ro})}{\eta_1} \cancel{\cos\theta_i\, \hat{a}_x} = \frac{-E_{to}}{\eta_2} \cancel{\cos\theta_t\, \hat{a}_x}$$

$$(E_{io} - E_{ro})\cos\theta_i = \frac{\eta_1}{\eta_2} \cos\theta_t\, E_{to}$$

$$E_{to} = \frac{\eta_2}{\eta_1} \frac{\cos\theta_i}{\cos\theta_t} (E_{io} - E_{ro}).$$

$$\frac{E_{ro}}{E_{io}} = \Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\boxed{E_{ro} = \Gamma_\perp E_{io}}$$

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No.

$$\frac{E_{to}}{E_{io}} = \tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$E_{to} = \tau_\perp E_{io}$$

$$\boxed{1+\Gamma_\perp = \tau_\perp}$$

for No Reflection Case

$\Gamma_\perp = 0$ occurs at $\theta_i = \theta_{B\perp}$

$$\eta_2\cos\theta_i = \eta_1\cos\theta_t$$

$$\eta_2^2\cos^2\theta_{B\perp} = \eta_1^2\cos^2\theta_t$$

$$\eta^2(1-\sin^2\theta_{B\perp}) = \eta_1(1-\sin^2\theta_t)$$

$$\beta_2\sin\theta_t = \beta_1\sin\theta_{B\perp}$$

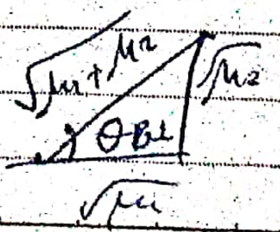
$$\sin^2\theta_{B\perp} = \frac{1-\frac{\mu_1\epsilon_2}{\mu_2\epsilon_1}}{1-\left(\frac{\mu_1}{\mu_2}\right)^2}$$

if $\mu_1 = \mu_2 = \mu_0$

$\theta_{B\perp}$ doesn't exist.

if $\epsilon_1 = \epsilon_2$

$$\sin\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_1+\mu_2}}$$



$$\boxed{\begin{aligned}\tan\theta_{B\perp} &= \sqrt{\frac{\mu_2}{\mu_1}}\\ \epsilon_2 &= \epsilon_1\end{aligned}}$$

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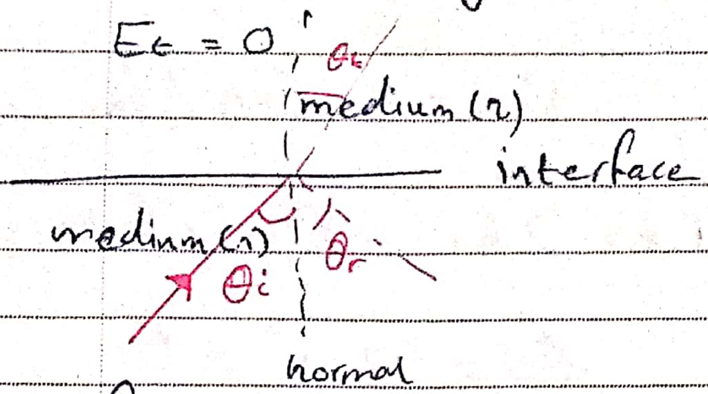
Critical angle: θ_c

Total Reflection angle



$\theta_i = \theta_c$ $\qquad \theta_t = 90$ or above.

$n_1 \sin\theta_c = n_2 \sin 90^\circ$ $\qquad \boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$

$n_1 = \sqrt{\mu_1 \epsilon_1} = \sqrt{\mu_{r_1} \epsilon_{r_1}}/c$

Ex a U.P.W in Air with $E = 8\cos(\omega t - 4x - 3z)\hat{a}_y$ V/m is incident in a dielectric slab ($z > 0$) with ($\mu_r = 1$, $\epsilon_r = 2.5$, $\sigma = 0$)

Find

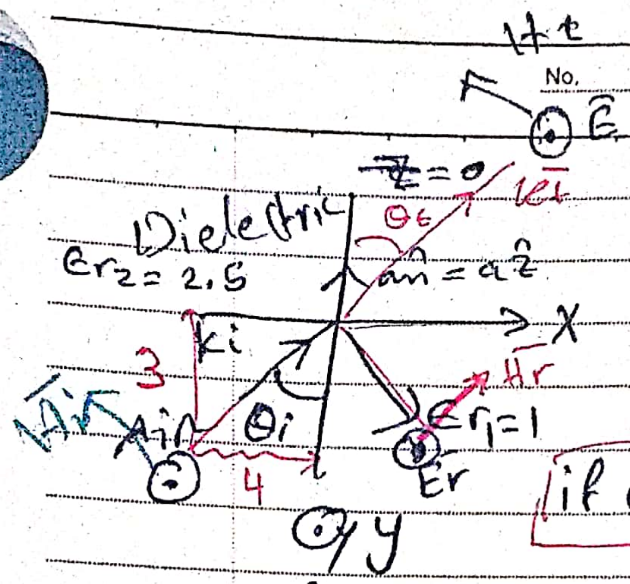
a) polarization

b) θ_i

$\overline{E_r}$

$\overline{H_t}$

a = perpendicular $\Rightarrow$ E has one component.



$k_i = 4\hat{a}_x + 3\hat{a}_z$

$|k_i| = \beta_1 = 5$ rad/m

$\boxed{\text{if } \epsilon_2 > \epsilon_1 \quad \theta_i > \theta_t}$

$\sin\theta_i = 4/5$ $\qquad \theta_i^\circ = 53.13^\circ = \theta_r$

(c) $\bar{E}_r = \Gamma_\perp \bar{E}_i$ $\qquad \eta_1 = \eta_0$

$\eta_2 = \frac{\eta_0}{\sqrt{2.5}}$

$$\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$n_1\sin\theta_i = n_2\sin\theta_t$

$$\frac{\sin\theta_i}{c} = \frac{\sqrt{\epsilon_{r2}}\sin\theta_t}{c}$$

$\theta_t = 30.39^\circ$

$\Gamma_\perp = -0.389$ $\qquad \bar{E}_r = -3.112\cos(\omega t - 4x + 3z)\,\hat{a}_y$ V/m

(d) $\frac{E_{t0}}{\eta_2}(-\cos\theta_i \hat{a}_x + \sin\theta_i \hat{a}_z)\, e^{-j\beta_2(x\sin\theta_i + z\cos\theta_i)}$

$T_\perp = 1 + \Gamma_\perp = 0.611$

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No.

$B_2 = \sqrt{k_{tx}^2 + k_{tz}^2}$ $\qquad E_{to} = 4.888\ V/m$

4

$\tan\theta_t = \frac{k_{tz}}{k_{tx}}$

$\tan(80.39^\circ) = \frac{k_{tz}}{4}$ $\qquad k_{tz} = 6.819$

$\beta_2 = 7.906$

$\beta_1 = \frac{\omega_1}{c}$ $\qquad \omega = \beta_1 \cdot c$

$\omega = \boxed{13 \times 10^8\ rad/sec.}$

$= 4.888\ (-\cos\theta_i \hat{a}x + \sin\theta_i)\hat{a}z \cdot \cos(\omega t - 4x - 6.819z)$

A/m

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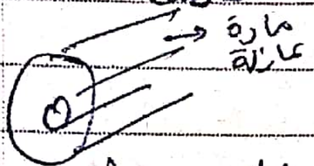
No.

CH.11 Transmission Lines (T.L)

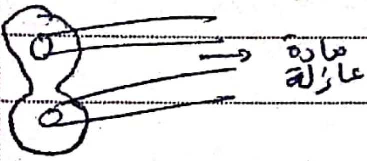
two or more conductors. T.L | one conductor T.L

2 wire transmission Line

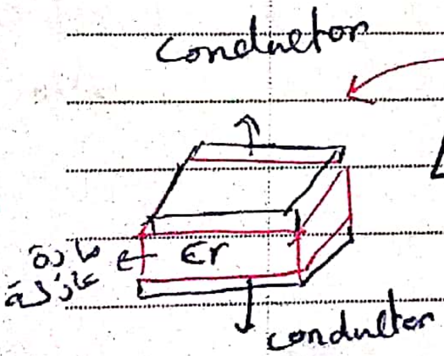
① Coaxial Cable



② twin wire Cable



③ planner Lines → slot lines
→ strip lines
→ microstrip lines.
→ Fin Line



wave guides

CH. 12

T.L parameters. (Distributive elements).

Source

Load

Resistance per Unit Length Ω/m

R/L & G/L & C/L & L/L

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No.

* for a coaxial cable

$$R/l = \frac{1}{2\pi \sigma_c \delta}\left(\frac{1}{a}+\frac{1}{b}\right) \quad \Omega/m$$

$$G/l = \frac{2\pi \sigma_d}{\ln(b/a)} \quad S/m$$

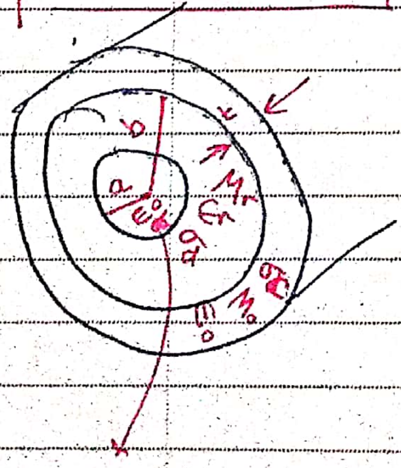
$$G = \frac{1}{R_d} \leftarrow \text{dielectric Resistance.}$$

$$C/l = \frac{2\pi \epsilon}{\ln(b/a)} \quad F/m$$

$$L_{ext}/l = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \quad H/m$$

$L_{in} \to 0$ as $\omega \to \infty$ $\quad L = \frac{X_L}{\omega}$

$$\boxed{L_{ext} C = \mu \epsilon} \qquad R_d C = \frac{\epsilon}{\sigma} \qquad \boxed{\frac{C}{G} = \frac{\epsilon}{\sigma}}$$



ϵ_0 σ μ_0

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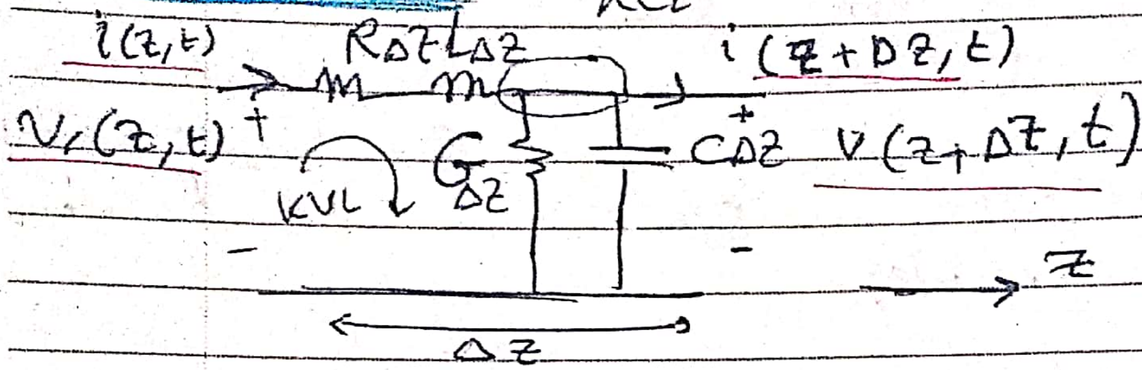
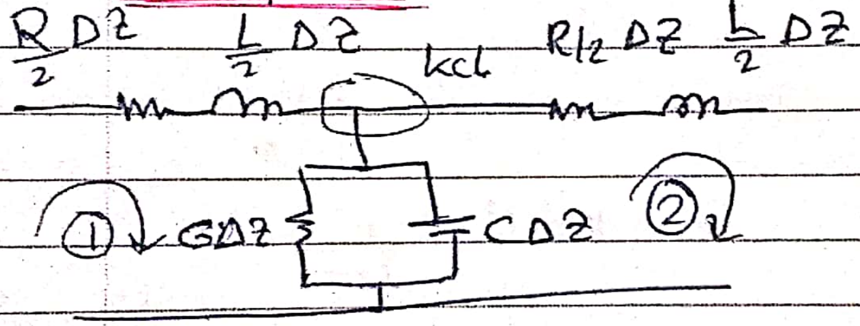
No.

T.L eq. ckt.

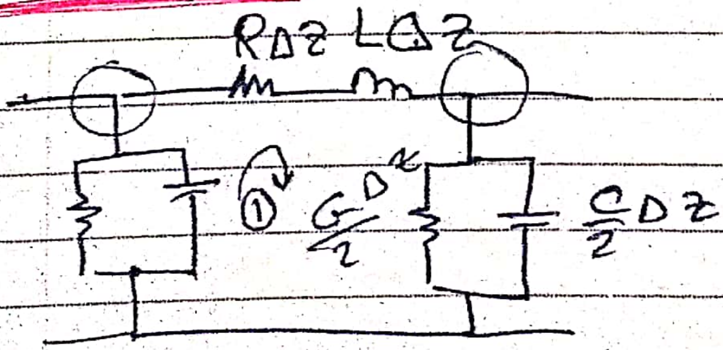
↳ L-eq. ckt



↳ T-eq. ckt.



↳ Π-eq. ckt



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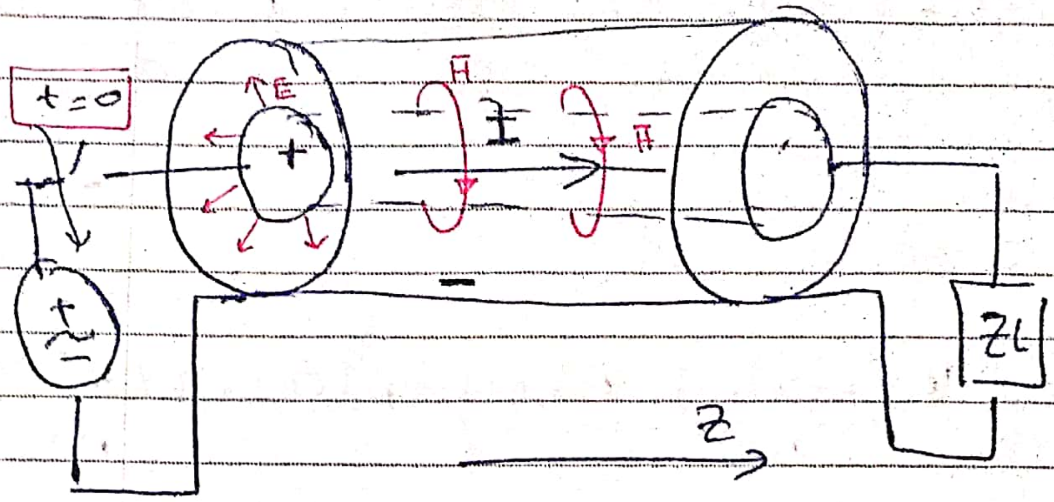
No.

T.L equations. (it is a TEM wave).



$V = -\int E \cdot dl$ faraday's low

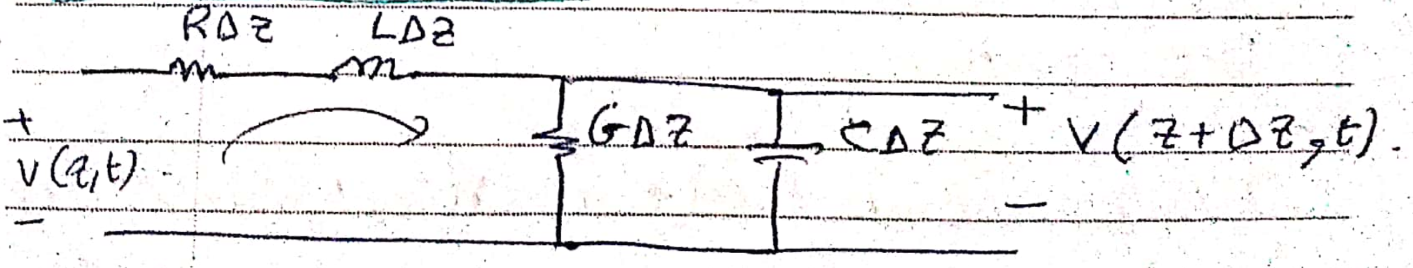
$\bar{E} = \frac{\rho_L}{2\pi \epsilon \rho} \hat{a}_\rho$

$I = \oint H \cdot dl$. ampere's law

$\bar{H} = \frac{I}{2\pi \rho} \hat{a}_\phi$

$P = \bar{E} \times \bar{H} = \hat{a}_z$

$\boxed{\text{L Type eq. ckt.}}$



KVL

$-V(z,t) + R\Delta z + L\Delta z \frac{dI(z,t)}{dt} + V(z+\Delta z, t) = 0$

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No.

$$-\left(\frac{V(z+\Delta z,t)-V(z,t)}{\Delta z}\right) = \frac{R\cancel{\Delta z}\, I(z,t) + L\cancel{\Delta z}\, \frac{dI}{dt}(z,t)}{\Delta z}$$

$\hookrightarrow$ division

Take limit
$\Delta z \to 0$

$$\lim_{\Delta z \to 0} -\left(\frac{V(z+\Delta z,t)-V(z,t)}{\Delta z}\right) = RI(z,t) + L\frac{dI}{dt}(z,t)$$

$$-\frac{dV(z,t)}{dz} = RI(z,t) + L\frac{dI(z,t)}{dt}$$

convert to phasor

$V(z,t) = Re\{V_s(z)e^{j\omega t}\}$.

$$-\frac{dV_s(z)}{dz} = R I_s(z) + j\omega L\, I_s(z)$$

$$-\frac{dV_s(z)}{dz} = (R+j\omega L)(I_s(z)) \rightarrow 1$$

$\boxed{\text{kcl equation}}$

$$I(z,t) = I(z+\Delta z,t) + G\Delta z V(z+\Delta z,t) + C\Delta z \frac{dV(z+\Delta z,t)}{dt}$$

$$-\left(\frac{I(z+\Delta z,t)-I(z,t)}{\Delta z}\right) = GV(z+\Delta z,t) + C\frac{dV(z+\Delta z,t)}{dt}$$

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No.

$\lim_{\Delta z \to 0}$

$$-\frac{dI(z,t)}{dz} = GV(z,t) + C\frac{dv(z,t)}{dt}$$

$\boxed{\text{phasor}}$

$$-\frac{d\hat{I}_s(z)}{dz} = (G + j\omega C)V_s(z) \rightarrow (2)$$

Derive (1) and sub (2) in (1)

$$-\frac{d^2V_s(z)}{dz^2} = (R + j\omega L)\frac{dI_s(z)}{dt}$$

Constant γ^2

$$\frac{d^2V_s(z)}{dz^2} = \underbrace{(R + j\omega L)(G + j\omega C)}\,V_s(z).$$

$\mu, \epsilon, \sigma_c, \sigma_d, \omega$

$$\boxed{\frac{d^2V_s(z)}{dz^2} - \gamma^2 V_s(z) = 0}$$ voltage wave equation

$$\boxed{\frac{d^2I_s(z)}{dz^2} - \gamma^2 I_s(z) = 0}$$ current wave equation

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

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No.

$$\frac{d^2 V_s}{dt^2} - \gamma^2 V_s(z) = 0$$

roots let $\frac{d}{dz} = m$

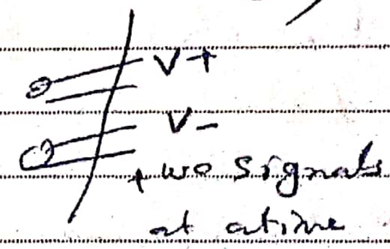
$$m^2 - \gamma^2 = 0 \qquad \text{Sinh} \quad \text{Cosh}$$

$$m = \pm\gamma \quad \text{real Roots.} \quad e^{-\gamma z}, e^{\gamma z}$$

Sol

$$V_s(z) = \underbrace{V_o^+ e^{-\gamma z}}_{\text{forward wave}} + \underbrace{V_o^- e^{\gamma z}}_{\text{backward wave}} \quad (V)$$

$$I_s(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z} \quad (A)$$



Types of T.L :-

The One describes befor is Called general lossy T.L

a) Loseeless T.L (free space).

$R = 0 \quad G = 0 \rightarrow$ if $\sigma_d = 0$

if $\sigma_c = \infty$

FIVE APPLE

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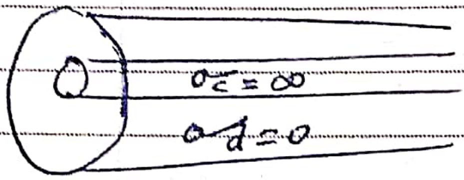
No.

$$R = \frac{1}{2\pi\sigma_c}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)}$$

$$\gamma = j\omega\sqrt{LC}, \quad \alpha = 0$$
$$= j\beta\sqrt{\mu\epsilon}$$



$$\beta = \omega\sqrt{LC} \text{ or } \omega\sqrt{\mu\epsilon}$$
(rad/m)

(real) $Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu}{\epsilon}} \quad [\Omega]$

$$\lambda = \frac{2\pi}{\beta}, \quad u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{LC}}$$

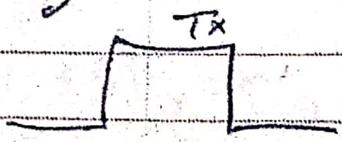
$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

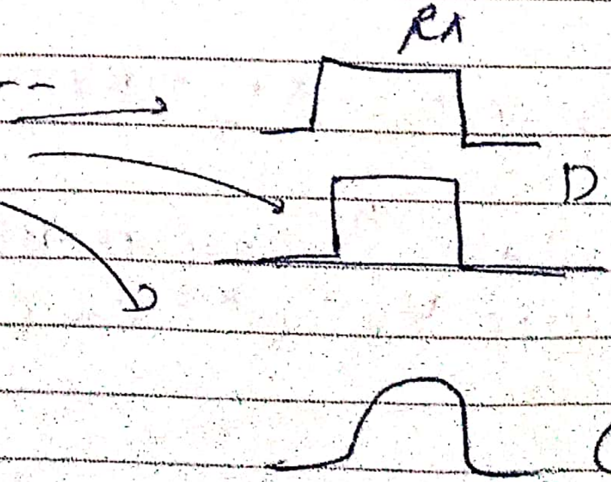
$$V(z,t) = V_0^+ \cos(\omega t - \beta z) + V_0^- \cos(\omega t + \beta z)$$

Same for

$I(z,t) = ---$

b) Distortionless ~~Distributed~~ Lines :-





lossless

Distortionless

* $\frac{R}{L} = \frac{G}{C}$

$RC = LG$

$\frac{R}{G} = \frac{L}{C}$

lossy (distributed)

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No.

$$\gamma = \sqrt{R\left(1+\frac{j\omega L}{R}\right) G \left(1+\frac{j\omega C}{G}\right)}$$

$$\sqrt{RG}\left(1+\frac{j\omega L}{R}\right)$$

$$\sqrt{RG} + j\omega\sqrt{\frac{RGL^2}{R^2}}$$

$$= \sqrt{RG} + j\omega\sqrt{\frac{GLC}{G}} = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\alpha = \sqrt{RG} \qquad \beta = \omega\sqrt{LC} = \omega\sqrt{\mu\epsilon}$$

$$\underset{(\text{real})}{Z} = \sqrt{\frac{R\left(1+\frac{j\omega L}{R}\right)}{G\left(1+\frac{j\omega C}{G}\right)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} =$$

$$= R_0 = \sqrt{\frac{\mu}{\epsilon}} \qquad \lambda = \frac{2\pi}{\beta} \qquad u = \omega/\beta = \frac{1}{\sqrt{LC}}$$

$$V_1(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{+\alpha z} e^{j\beta z}$$

$$V(z,t) = V_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_0^- e^{\alpha z} \cos(\omega t + \beta z)$$

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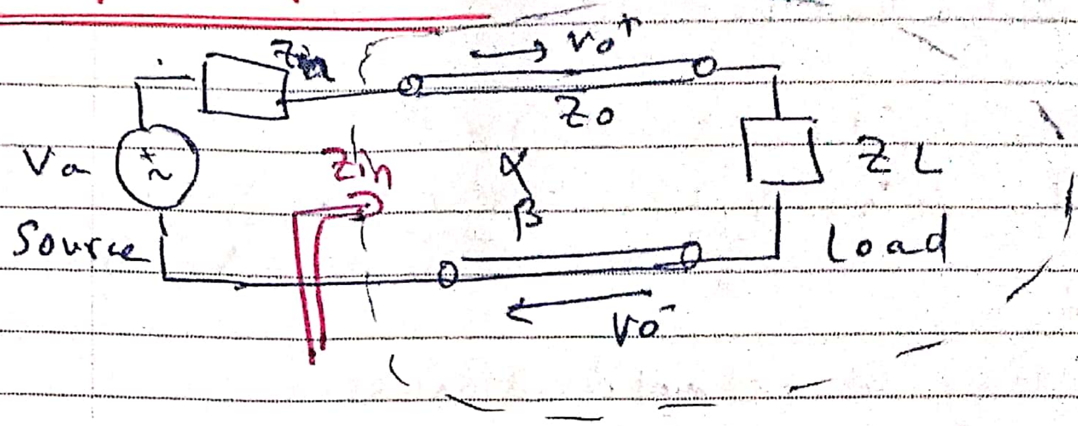
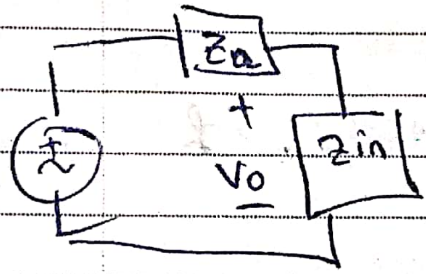
No.

Input Impedence



and it is a lossy T.L $\left.\begin{matrix}\alpha \\ \beta\end{matrix}\right] \to \gamma$



$$V_0 = \frac{V_a Z_{in}}{Z_{in} + Z_a}$$

$$I_0 = \frac{V_a}{Z_{in} + Z_a}$$

at the Source at $z=0$ $\quad V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$

$V_s(0) = V_0 = V_0^+ + V_0^- \Rightarrow ①$

$I_s(0) = I_0 = I_0^+ + I_0^-$

$I_0 = \frac{V_0^+ - V_0^-}{Z_0} \Rightarrow ②$

$V_0^+ = \frac{1}{2}(V_0 + I_0 Z_0)$

$V_0^- = \frac{1}{2}(V_0 - I_0 Z_0)$

at the end $(z=d)$

at

$V_s(z=l) = V_L = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \Rightarrow ③$

$I_s(z=l) = I_L = \frac{V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}}{Z_0} \Rightarrow ④$

FIVE APPLE

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No.

$$V_0^+ = \frac{1}{2}(V_L + I_L Z_0)e^{\gamma L}$$

$$V_0^- = \frac{1}{2}(V_L - I_L Z_0)e^{-\gamma L}$$

and $Z_{in} = \frac{V_S(Z)}{I_S(Z)} = \frac{V_S(0)}{I_S(0)} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}(Z_0)$

for lossy T.L and ∵ distortionless

$$Z_{in} = \frac{Z_0 Z_L + Z_0 \tanh \gamma L}{Z_0 + Z_L \tanh \gamma L}$$ Valid for all L

for lossless line $\gamma = j\beta$ $\tanh(j\beta L) = j\tan\beta L$

$$Z_{in} = \frac{Z_0 Z_L + jZ_0 \tan\beta L}{Z_0 + jZ_L \tan\beta L}$$

~~tan(...)~~

Reflection coefficient

Γ = reflected signal / Incident signal

$$\Gamma_L = \frac{V_0^-}{V_0^+} = \frac{V_L - I_L Z_0}{V_L + I_L Z_0} e^{-2\gamma L}$$

$$\boxed{\begin{array}{l} V_0^- = \Gamma_L V_0^+ \\ \Gamma_{Li} = -\Gamma_{Lv} \end{array}}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$ Z_L = medium 2 Γ_2

$\Gamma_L \angle \theta_{\Gamma_L}$

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SWR $\equiv$ S = standing wave Ratio.

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}}$$

$$= \frac{|V_0^+| + |V_0^-|}{|V_0^+| - |V_0^-|} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} \qquad \Gamma = \frac{S-1}{S+1}$$

power $\longrightarrow$ $P = V i$ (in time domain)

$$V_s(z) = V_0^+ e^{-\gamma z} + \Gamma V_0^+ e^{\gamma z}$$

$$I_s(z) = \frac{V_0^+ e^{-\gamma z} - \Gamma V_0^+ e^{\gamma z}}{Z_0}$$

for lossless $\gamma = j\beta$

$$P = \frac{1}{2} \{ Re \; V_s(t) \; . \; I_s(z)$$

$\leftarrow$ peak $\rightarrow$

for lossless

$$V_s(z) = V_0^+ e^{-j\beta z} + \Gamma V_0^+ e^{j\beta z}$$

$$I_s(z) = (V_0^+ e^{-j\beta z} + \Gamma V_0^+ e^{j\beta z}) / Z_0$$

$$P = \frac{1}{2} Re \left\{ \frac{(V_0^+)^2}{Z_0} - \frac{\Gamma (V_0^+)^2}{Z_0} e^{-j2\beta z} + \Gamma \frac{V_0^+}{Z_0} e^{-j2\beta z} - \frac{\Gamma^2 (V_0^+)^2}{Z_0} \right]$$

$$P = \frac{(V_0^+)^2}{2Z_0} - \frac{\Gamma_2^2 (V_0^+)^2}{2Z_0}$$

$$P_t = P_i - P_r$$

P_t → transmitted P; P_i → incident; P_r → reflected.

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No.

* $Z_{in} = Z_0 \dfrac{(Z_L + jZ_0 \tan \beta l)}{Z_0 + jZ_L \tan \beta l}$ lossless

* $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$ $\quad 0 \leq |\Gamma| \leq 1$

* $S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$ $\quad 1 \leq S \leq \infty$

* $P_L = \dfrac{V_0^{+2}}{2Z_0} \left(1 - |\Gamma_L|^2\right)$

* $|\Gamma| = \dfrac{S - 1}{S + 1}$

* $S\cancel{max} = \dfrac{V_{max}}{V_{min}} = \dfrac{I_{max}}{I_{min}}$

* $Z_{in\,max} = \dfrac{V_{max}}{I_{min}} \cdot \dfrac{V_{min}}{V_{min}}$

* $Z_{in_{max}} = S Z_0$

* $Z_{in_{min}} = \dfrac{V_{min}}{I_{max}} \cdot \dfrac{I_{min}}{I_{min}} = \dfrac{Z_0}{S}$

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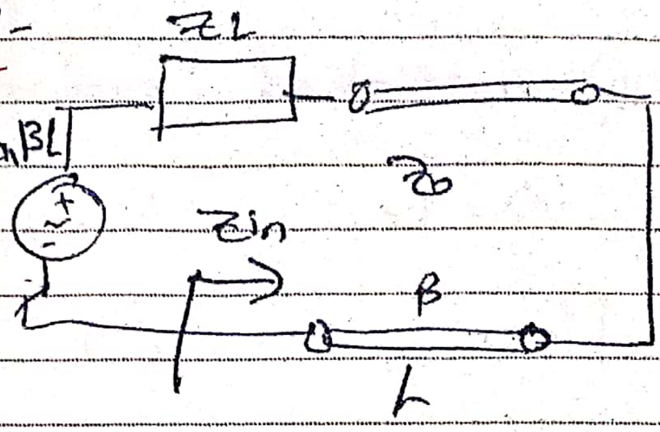
No.

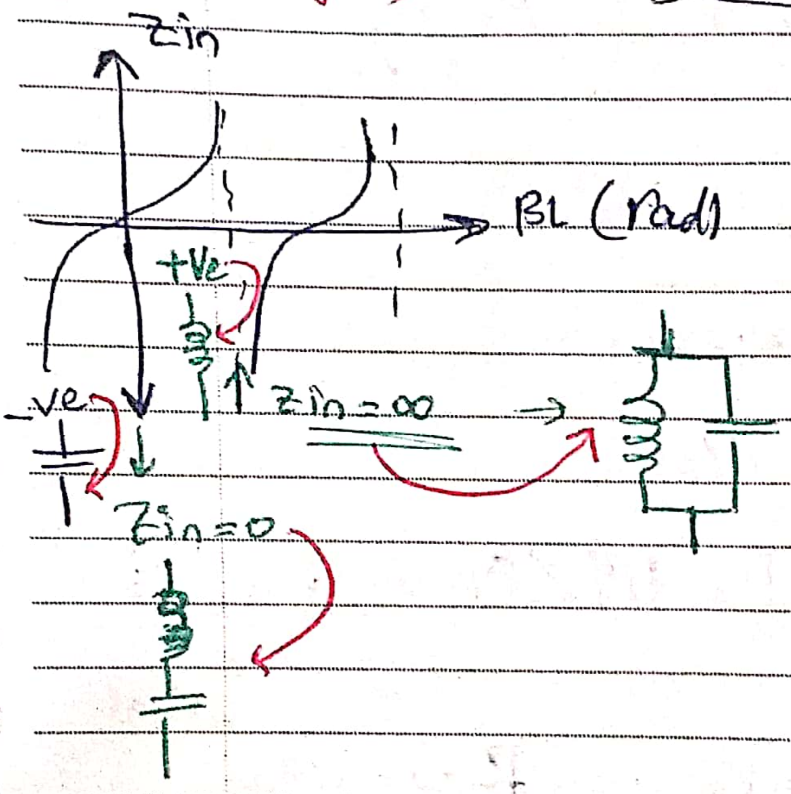
special cases for load :-

1) short circuit load :-



$Z_L = 0$ $\quad$ $Z_{in} = \underbrace{j Z_0 \tan \beta L}$

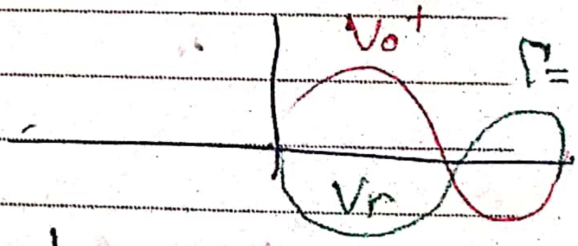
pure imaginary



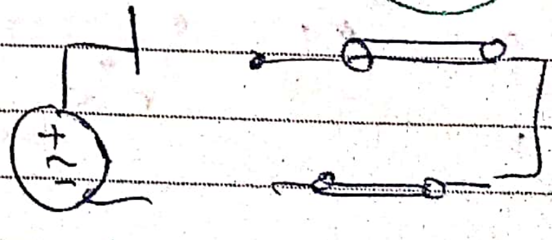
$\Gamma = -1$

$S = \infty$

$\rho = 0$



2) Open circuit Load



$Z_L = \infty$

$Z_{in} = -j Z_0 \cot(\beta L)$

↑ pure imaginary

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$\Gamma = +1$ $\quad$ $P = 0$ (current = 0)

$S = \infty$ $\quad$ $\Gamma_{L_i} = -1$

3) matched Load.

$Z_L = Z_0$ $\quad$ $Z_{in} = Z_0$

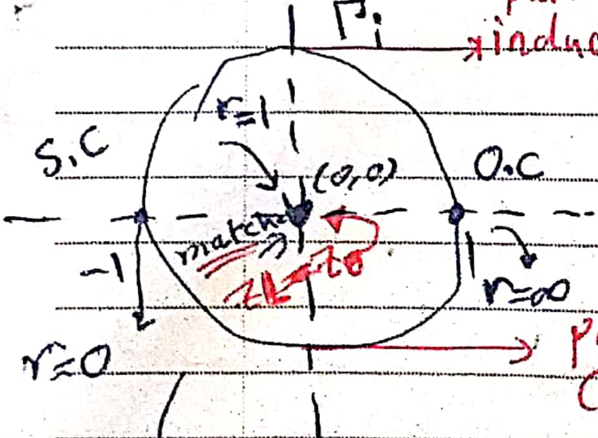
$\Gamma = 0$ $\quad$ $S = 1$ $\quad$ $P = \dfrac{V_0^{+2}}{2Z_0}$ maximum power transfer

$Z_{in\,SC} \cdot Z_{in\,O.C} = Z_0^2$

$Z_0 = \sqrt{Z_{in\,sc} \cdot Z_{in\,Oc}}$

Smith Chart : 1939

$\Gamma = |\Gamma| \angle \theta_r = \Gamma_r + j\Gamma_i$



$\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$ real

for lossless line

No.

Impedence on the chart must be normalized

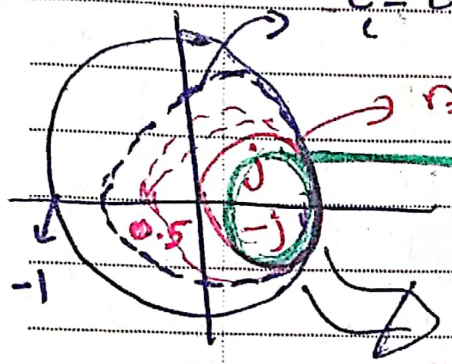
if $Z_0 = 50\,\Omega$ $\quad Z_L = 50\,\Omega$

$z_L = \frac{Z_L}{Z_0} = 1 + j0$

if $Z_L = 100\,\Omega$

$z_L = 2$

$z_L = 0.3$



r=1 circle

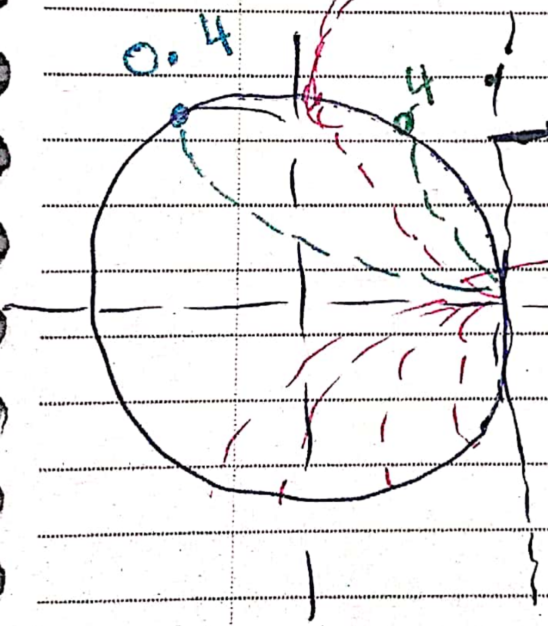
r=2 circle

Resistance circles.

$Z_L < Z_0$

$Z_L = 25$

$z_L = 0.5$



centre of imaginary circles.

$x_L = 1$ circle

Reactance circles.

if $Z_L = 100 + j200$

$Z_0 = 50$

$z_L = 2 + j4$

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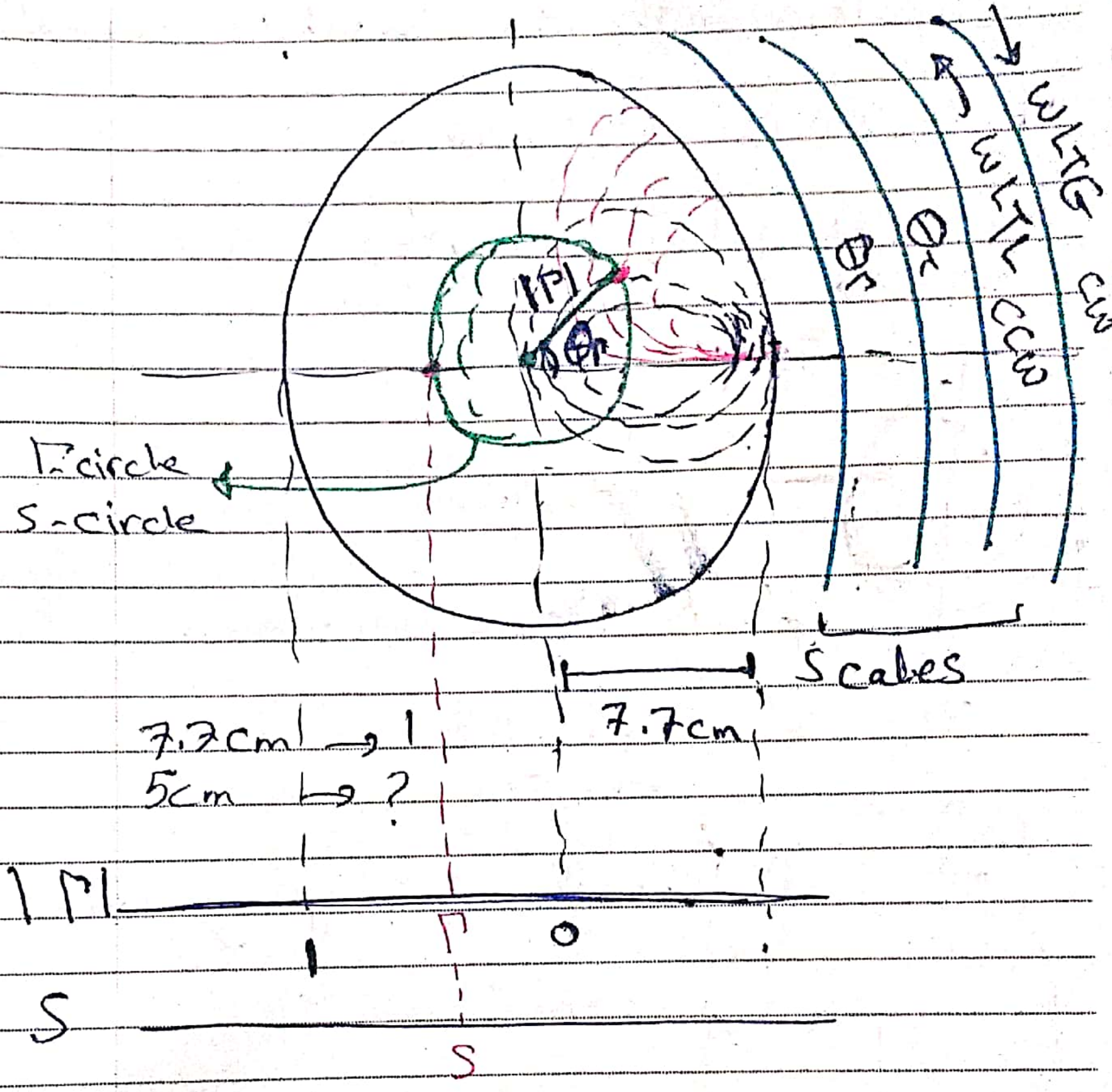
No.



Z_{in} max = S
Z_{in} min = $1/S$

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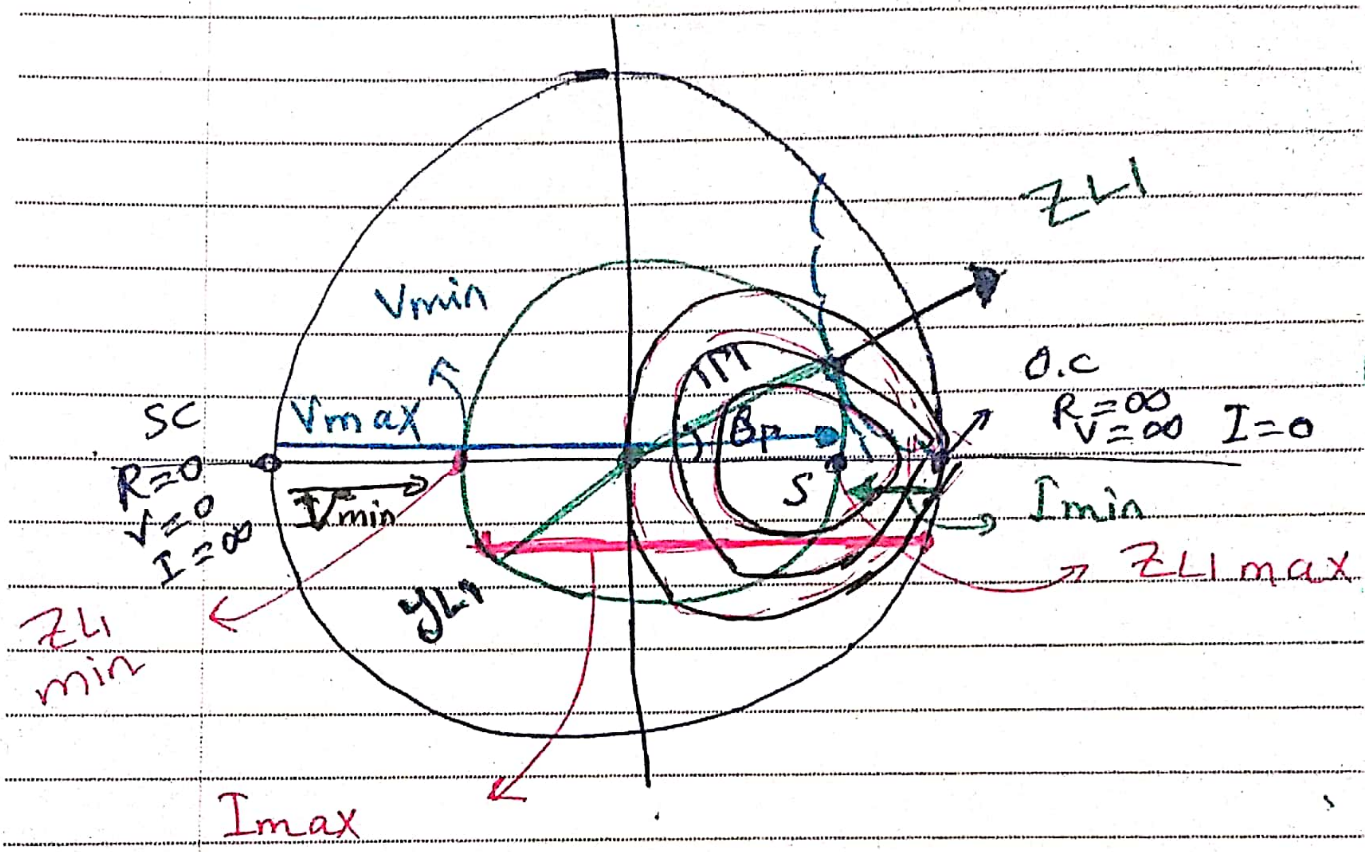
No.



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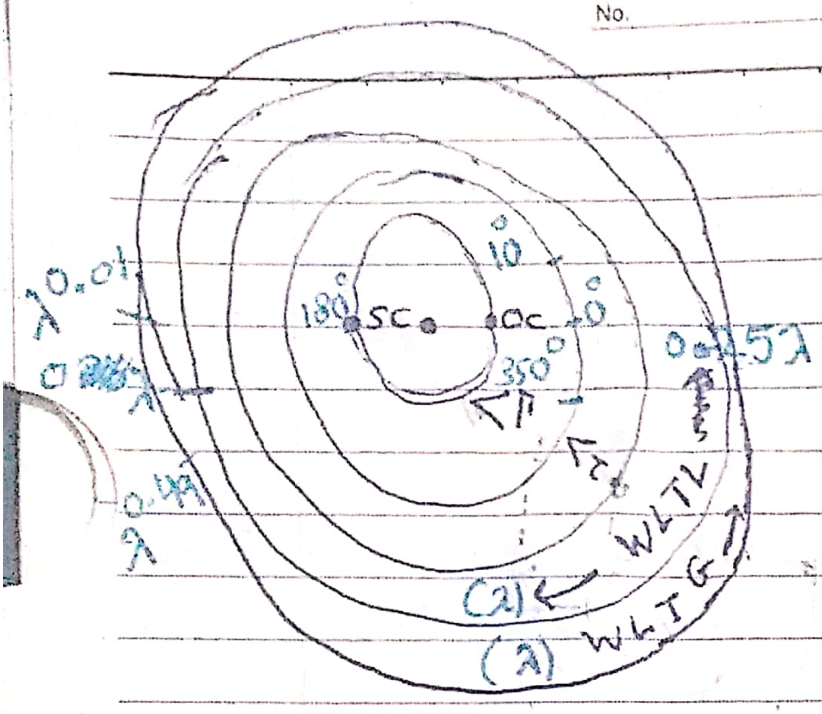
الأحد

No.



$360° \rightarrow 0.5\lambda$

$1\lambda = 720°$

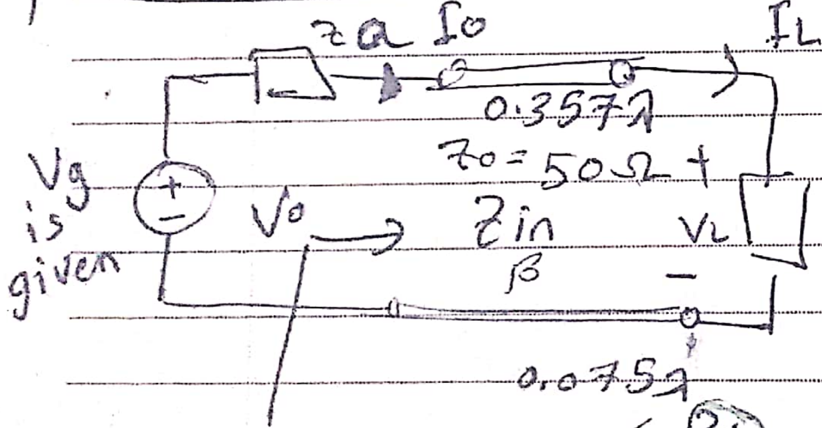
$Z_{in.sc} =$

$Z_{in\,oc}$

S

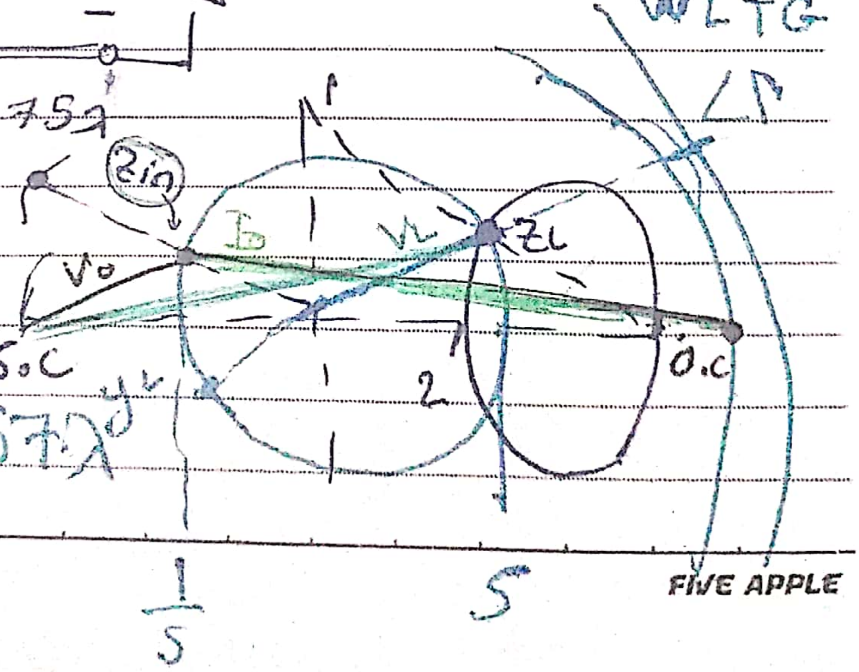
Γ

T



$Z_L = 100 + j50\ \Omega$

$Z_L = 2 + j1$



WLTG + 0.357 λ

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No.

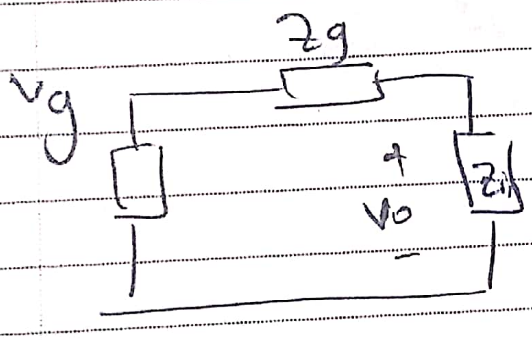
assume WTG = 0.2λ

$+ 0.375\lambda$

$= 0.575\lambda$

$= 0.075\lambda$ ⇐ to this WTG

$Z_{in} = z_{in} * Z_o$

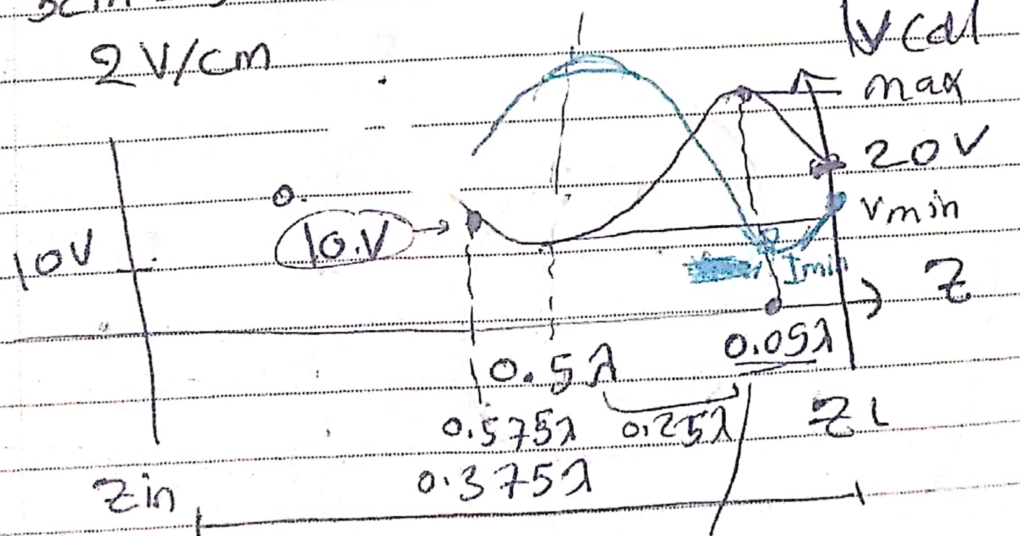


$V = \frac{V_g Z_{in}}{Z_{in} + Z_g} = (10V)$

$I_o = \frac{V_o}{Z_{in}} = \frac{V_g}{Z_{in}+Z_g} = (3mA$

5cm → 10V

2V/cm



الفرق بين ال max وال ZL

voltage voltage

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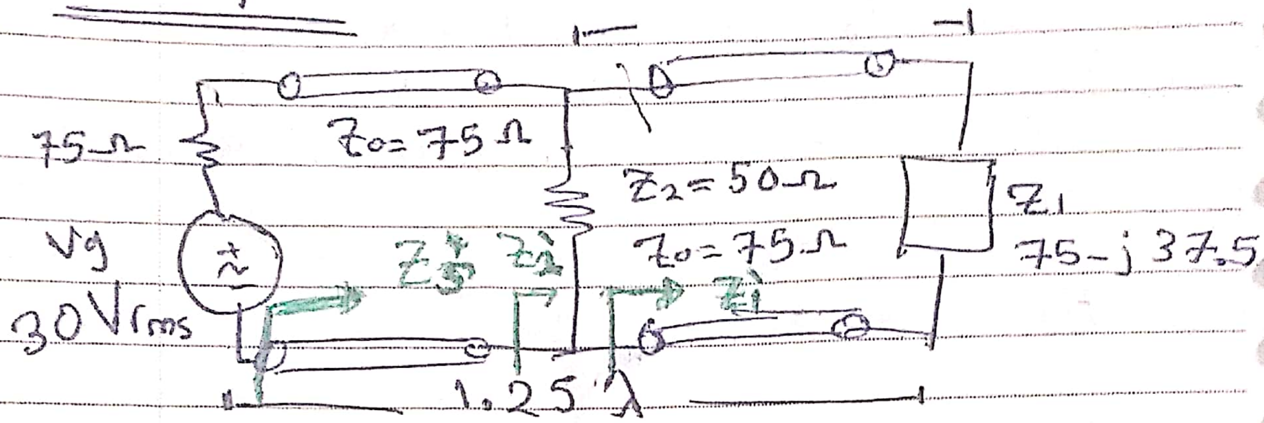
No.

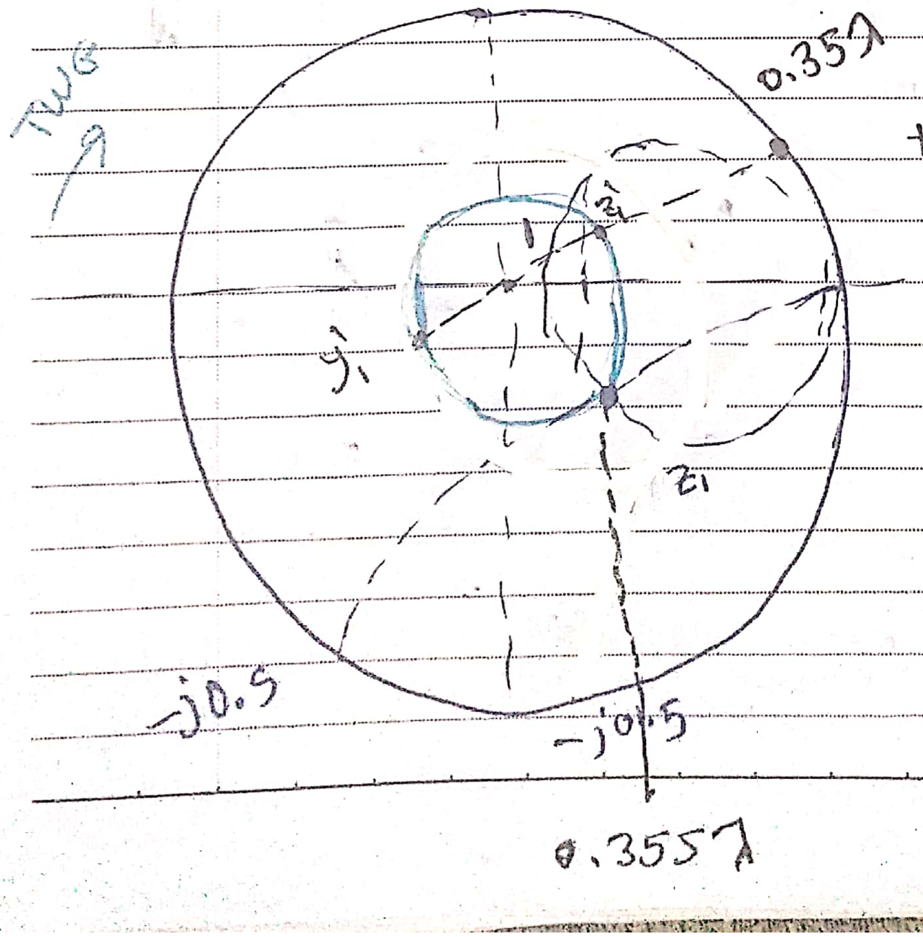
Example :



ⓐ Z_1' , Y_1' , Z_2' , Y_2' , Z_s , Y_s

ⓑ V_1, I_1, V_2, I_2, V_s, I_s

$$z_1 = \frac{Z_1}{Z_0} = 1 - j0.5$$



$$\begin{array}{r} 0.355\lambda \\ +\,0.375\lambda \\ -\,0.5\lambda \\ \hline 0.235\lambda \end{array}$$

No.

$z_i' = 1.6 + j0.2$

$Z_1' = 75 z_i' = 120 + j15\,\Omega$

$Y_1 = \dfrac{1}{Z_1'} = 0.0107 + j0.0053\,\Omega^{-1}$

$Y_1' = \dfrac{1}{Z_1'} = 0.008 - j0.001\,\Omega^{-1}$

$y_1'\ Y_0 y_1' = \dfrac{y_1'}{Z_0}$

$\cancel{Y_2}\quad y_2 = \dfrac{1}{Z_2} \Rightarrow \dfrac{50}{75} = Z_2$

$Y_2 = 0.02 = \dfrac{1}{50}$

$Y_2' = \dfrac{Y_1' + Y_2}{1}$

$y_2' = Y_2' * (Z_0) = 2.11 - j0.075$ ($Z_0 = 75$)

Smith Chart . على طول

$z_2' = 0.5 + j0.03$

$Z_2' = (0.5 + j0.03)\,75$

$$\boxed{Z_2' = 37.5 + j2.25\,\Omega}$$

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No.

$Z_S = 0.765 - j0.575$

$Z_S = 57.4 - j43.1\ \Omega$

$Y_S = \frac{1}{Z_S} = 0.011 + j0.008\ (S)$

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EM2

$Z_1 = 1 - j0.5$

$Z_1' = 1.6 + j0.2$

$y_1 = 0.8 + j0.4$

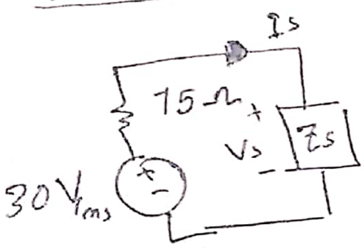
$y_1' = 0.61 - j0.075$

$y_2' = 2.11 - j0.075$

$Z_s = 0.765 - j0.575$

$Z_2' = 0.5 + j0.03$

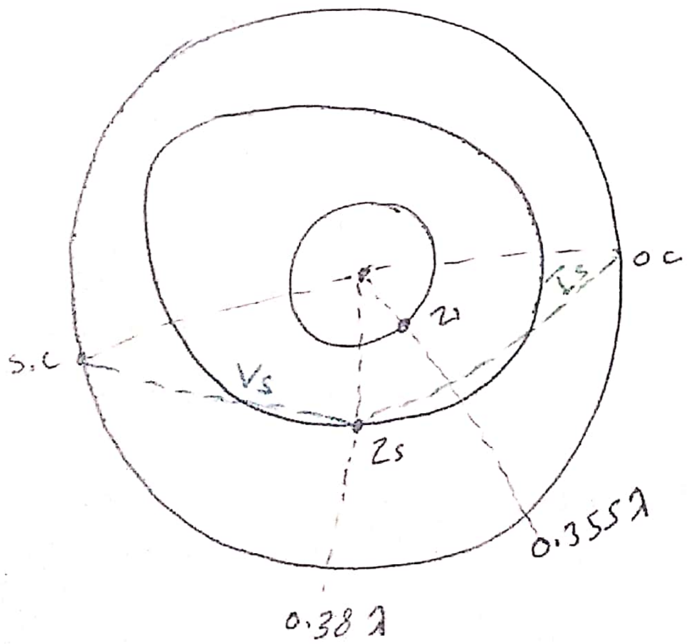
$V_2 = V_2' = V_1'$



$Z_s = 75.4 - j43.1\ \Omega$

$V_s = 30 * \frac{Z_s}{Z_s + 75} = 15.5 \angle -18.9\ V(rms)$

$I_s = \frac{30}{Z_s + 75} = 0.216 \angle 18^\circ\ A(rms)$



Voltage Scale

$\frac{15.5V}{8cm} = 1.93 V/cm$

8/7/...

Current Scale

$\frac{0.216A}{8.2cm} = 0.026 A/cm$

$V_2' = 5.1cm * (1.9375 \frac{V}{cm})$

$V_2' = 9.9V = V_2 = V_1'$

$I_2' = 10.25cm * 0.026 \frac{A}{cm}$

$I_2' = 0.27A$

$I_2 = \frac{V_2}{Z_2} = \frac{9.9}{50} = 0.198 A(rms)$

$I_1' = \frac{V_1'}{Z_1'} = \frac{9.9}{|75(1.6 + j0.2)|}$

⇓ take the magnitude

$I_1' = \frac{9.9}{120.9} = 0.082\ A(rms)$

For the inner circle Z_1 circle take another Scale

$V_1' = V_2' = 9.9V$

$9.9V \Rightarrow 9.5cm = 1.04 V/cm$

$I_1' \Rightarrow \frac{0.082}{5.9cm} = 0.0139 A/cm$

$V_1 = 1.04 \frac{V}{cm} * 8.3 cm = 8.65V$

$I_1 = 0.0139 \frac{A}{cm} \cdot 7.4 cm$

$I_1 = 0.1A$

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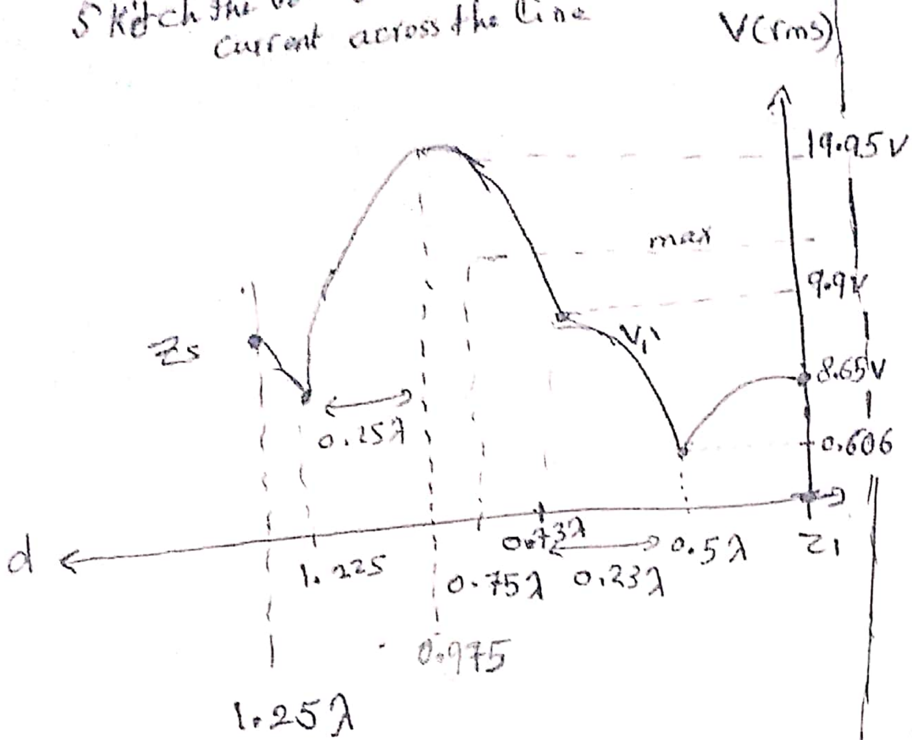
Sketch the Voltage and current across the Line



$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right)$$

$$\beta = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

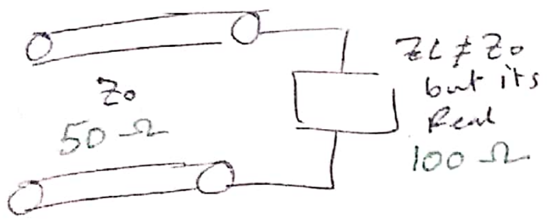
$$Z_{in} = \frac{Z_0'^2}{Z_L} = Z_0$$

$$Z_0' = \sqrt{Z_0 Z_L}$$

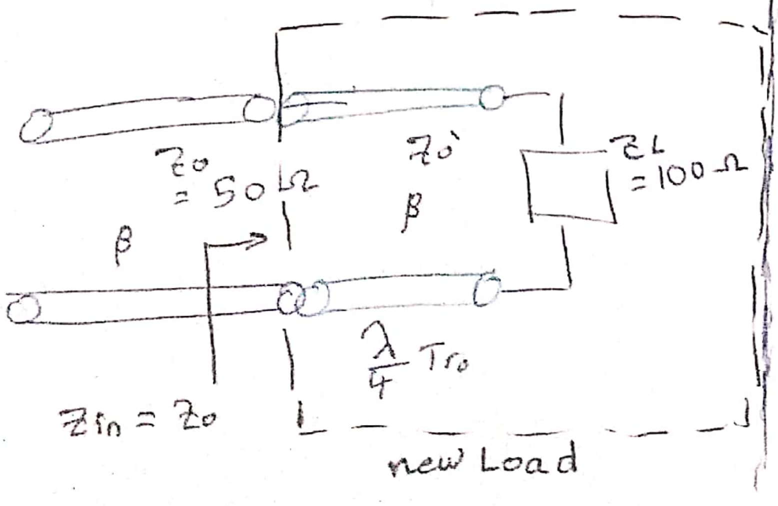
$$Z_0' = \sqrt{50 \cdot 100} = \underline{\underline{70.1\,\Omega}}$$

* How to achieve matching.

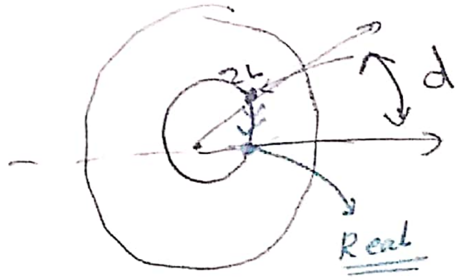
1) Quarter Wave length Transformer
($\lambda/4$ Trap)

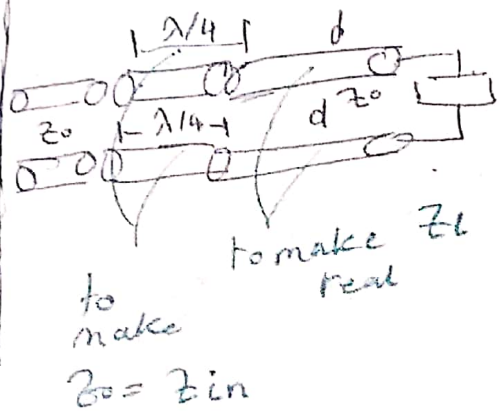


$$\Gamma = \frac{100-50}{100+50} = \frac{1}{3}$$



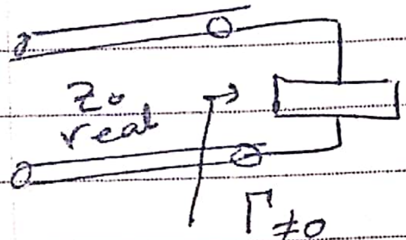
if Z_L is Complex
find the closest distance
to the Real load.





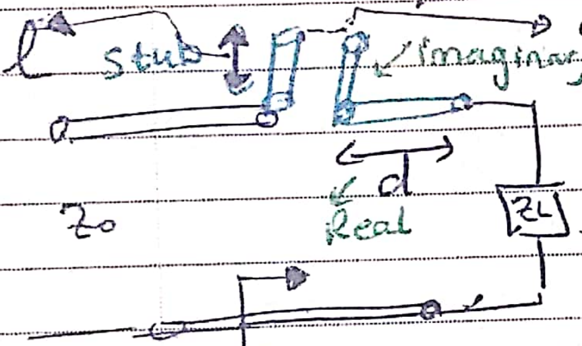
9/7 . الثلاثاء

b) Single Stub Matching (Tunner).



$Z_L = R_L + jX_L$
$\neq Z_o$

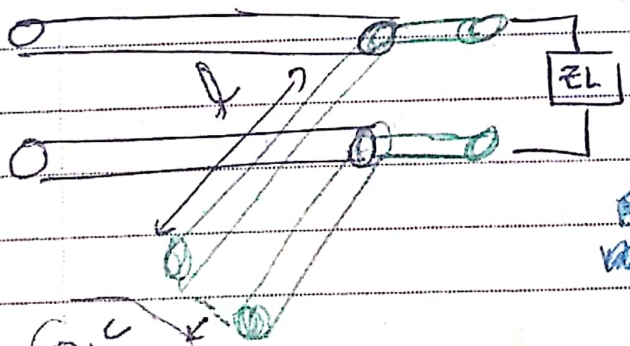
$\Gamma \neq 0$



→ O.C or short circuit.

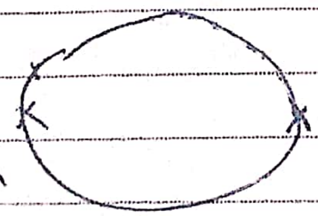
$Z_{in} = Z_o = R_o + j0$

series stub → S.C
→ O.C



→ diffrence = 0.25 λ

Shunt stub
S.C → Best way.
O.C → causes Radiation



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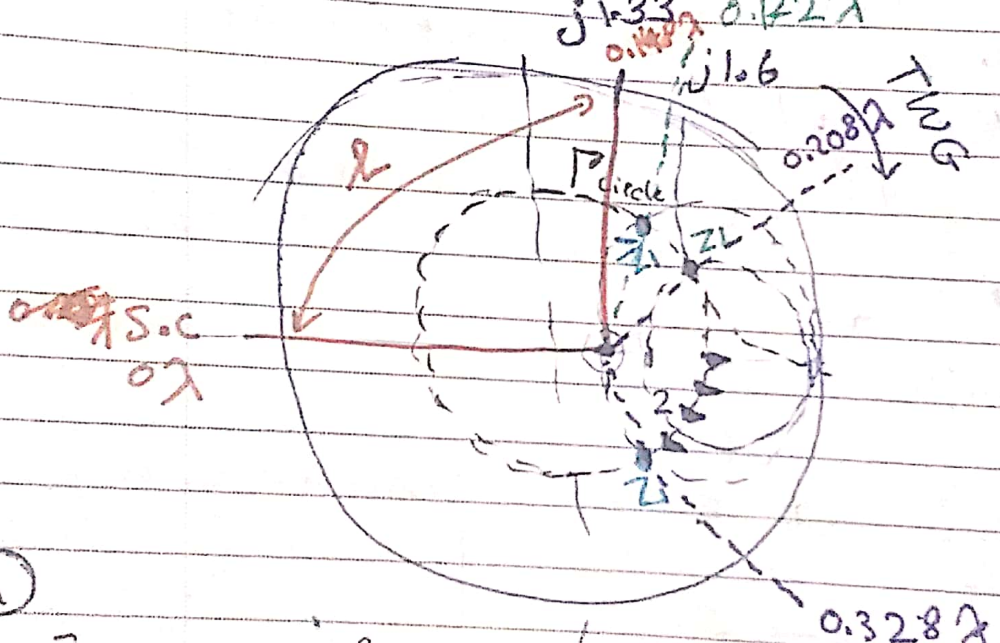
No.

Example :- Match a load impedence

$Z_L = 100 + j80\,\Omega$ to a $50\,\Omega$ transmission line Using Single Stub Tunner.

(A) using series S.C stub.



(1)

$$z_L = \frac{100 + j80}{50} = 2 + j1.6$$

(2) Find the Γ circle (draw).

(3) Find the intersection of Γ circle with $1 + jX$ circle (start from Load TWG)

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No

$Z_1 = 1 - j1.33$

$Z_1' \quad 1 + j1.33$

$0.328 - .0208 = 0.12\lambda = d$

$d' = 0.5\lambda - (0.208\lambda + 0.172\lambda) = 0.465\lambda$

from $Z_1, \overline{\overline{J}}$

④ find the stub length.

for $Z_1 = 1 - j1.33$ to have it $= 1 + j0$

$Z_{in} = Z_i' + Z_s$

$Z_i' \to Z_1$

after moving d TWG

Z_s: stub impeedence.

$1 - j0.133 + j0.133 = 1 + j0$

$\boxed{Z_s = j0.133}$

$\boxed{l = 0.148\lambda}$

∴	$Z_1 = 1 - j1.33$	$d = 0.12\lambda$	$l = 0.148\lambda$
∴	$Z_1' = 1 + j1.33$	$d' = 0.465\lambda$	$l' = 0.352\lambda$

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No.

(5) find the Length from the (s.c or o.c) point Until you reach to imaginary circle.

(B) series O.C Stub

z_1 = Same	$d = 0.12\lambda$	$\ell = 0.148 + 0.25\lambda$
z_1' = Same	$d' = 0.4865\lambda$	$\ell = 0.352\lambda - 0.25\lambda$

(C) shunt S.C stub.

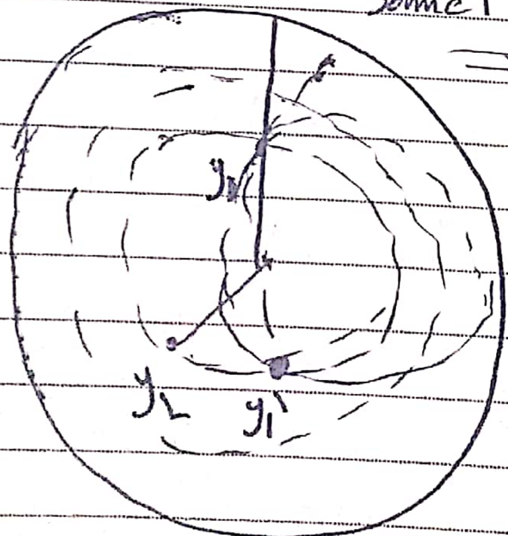
$z_L = 2 + j1.6$

$y_L = 0.3 - j0.24$

Same circle

$y_1 = 1 + j1.33$

$y_1' = 1 - j1.33$



$d_1 = (0.172 + 0.5 - 0.458)\lambda$

$= 0.214\lambda$

$d' = (0.328 + 0.5 - 0.458)\lambda = 0.37\lambda$

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No.

for y_1

$y_s = -j1.33$

(note that S.C point for admittances is O.C).

$l_1 = 0.352\lambda - 0.25\lambda = 0.102\lambda$

$l_1' = 0.25\lambda + 0.148\lambda = 0.398\lambda$.

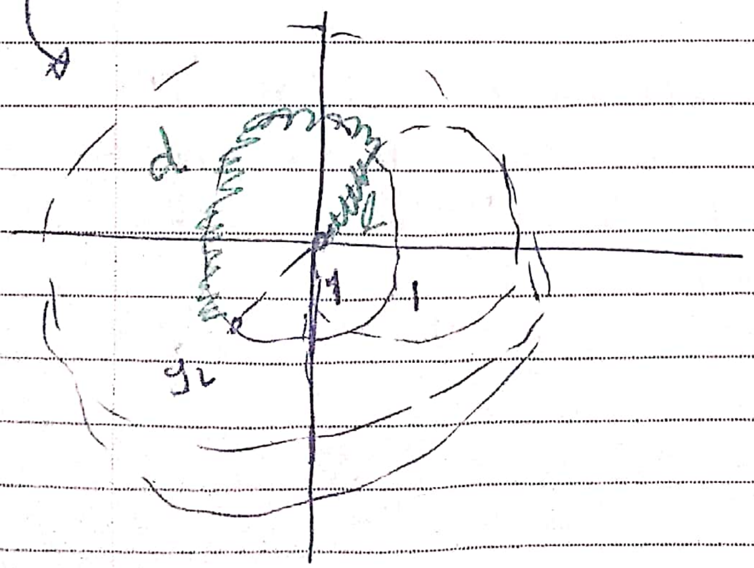
$y_s = +j1.33$.

d) shunt open.

$d = 0.214\lambda$ $d' = 0.37\lambda$

$l = 0.102\lambda + 0.25\lambda = 0.352\lambda$.

$l' = 0.398\lambda - 0.25\lambda = 0.148\lambda$



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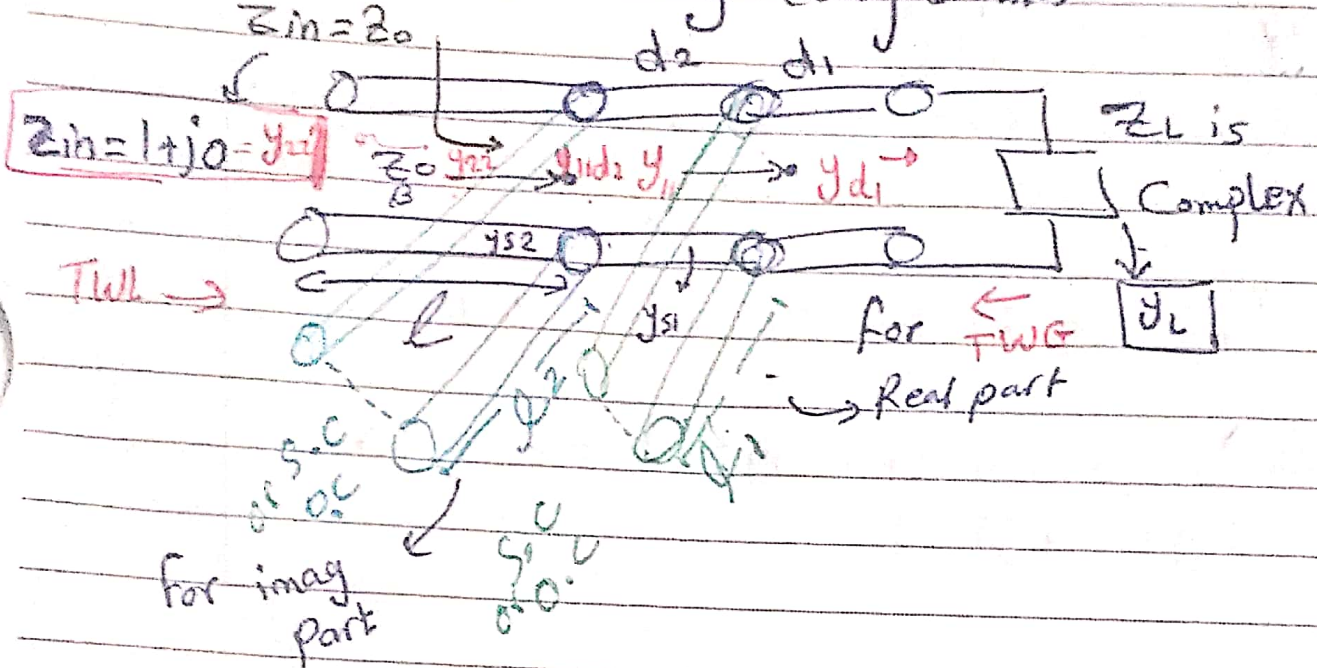
No.

③ double stub Matching (only shunt) :



(d_1) and (d_2) are fixed

d_1 could be zero.

$y_{11} = y_{d_1} \pm y_{s1}$

$y_{s1} = \pm jB$

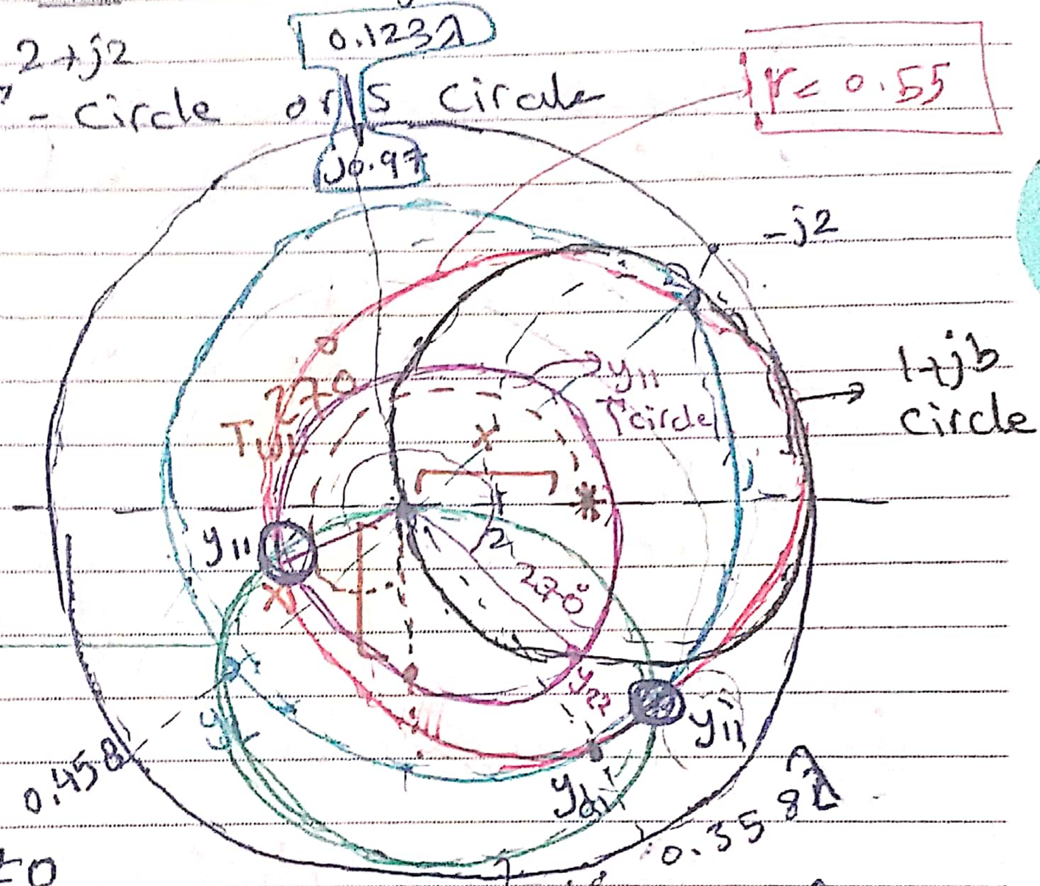
Example 80 Match the Load $Z_L = 100 + j100$ to a 50Ω T.L using double short circuited shunt stubs with 50Ω ~~...~~ if $d_1 = 0.4\lambda$ and $d_2 = 3\lambda/8$

No.

① $z_L = \frac{100+j100}{50} = 2+j2$

$y_L = \frac{1}{2+j2} = 0.25 - j0.25$

② draw Γ - circle or S circle



③ if $d_1 \neq 0$

move y_L a distance (d_1) TWG and find y_{d1}

$0.458 + 0.4 = 0.858 - 0.5 = \boxed{0.358}$

$y_{d1} = 0.55 - j1.08$

④ draw the Spacing circle or the Rotated (1+jb) circle

by rotating 1+jb circle a distance d_2 in TWL scale.

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No.

$d_2 = \frac{3\lambda}{8} * 720^{\circ} = 270^{\circ}$

(5) Find the intersections between the Rotated Circle (spacing circle) and the r-circle of y_{d_1} $(r = 0.55)$.

if $d_1 = 0$
we find the intersection between Rotated circle with y_L circle.

(6) $y_{11} = 0.55 - j0.11$

$y_{11}' = 0.55 - j1.88$

$y_{11} = y_{d1} + y_{s1}$

$y_{11}' = y_{d1} + y_{s2}$

$y_{s1} = -j0.11 + j1.08 = j0.97$

$y_{s2} = -j1.88 + j1.08 \quad -j0.8$

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No.

(7) locate y_{s1} and measure L_1 (distance) from (s.c point).

$l_1 = 0.25 + 0.123 = 0.373\lambda$

$l_1' = 0.2525\lambda$

(8) Draw the Γ-circle for (y_{11}) or (y_{11}')

$y_{22} = 1 - j0.61$

$y_{22}' = 1 + j2.6$

$y_{22} = y_{11} + y_{s2}$

(9) $y_{s2} = +j0.61$ $\quad l_2 = 0.087\lambda + 0.25\lambda$

$= 0.337\lambda$

$y_{s2}' = -j2.6$ $\quad l_2' = 0.065\lambda$

solution

$$\boxed{\begin{array}{ll} l_1 = 0.373\lambda & l_2 = 0.337\lambda \\ \text{or} & \\ l_1 = 0.225\lambda & l_2 = 0.065\lambda \end{array}}$$

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d_1 ⇒ the load must be out of the forbidden Region.

دائرة محاسبة لدائرة ال Spacing

ده بيحسب O.C

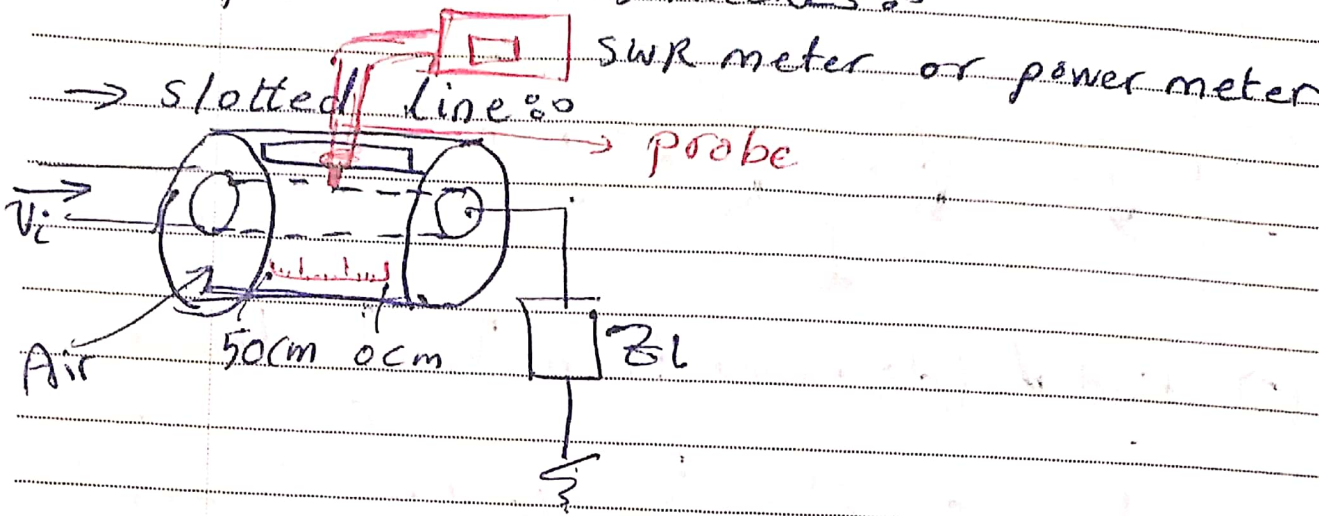
No.

الاثنين

Applications of T.L :-

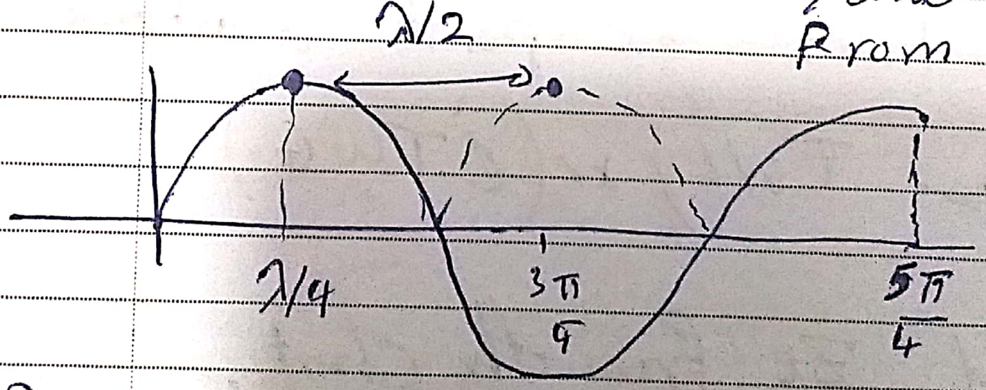
Impedence measurments :-

→ Slotted line :-



To find Z_L

(1) Connect the Load and Locate the ~~minimum~~ Voltage minima (more sharp to locate), and read (S) from SWR meter



$\frac{\lambda}{2} = 7cm$ i.e. 5cm, 12 cm .

(2) Replace Z_L by a short circuit .

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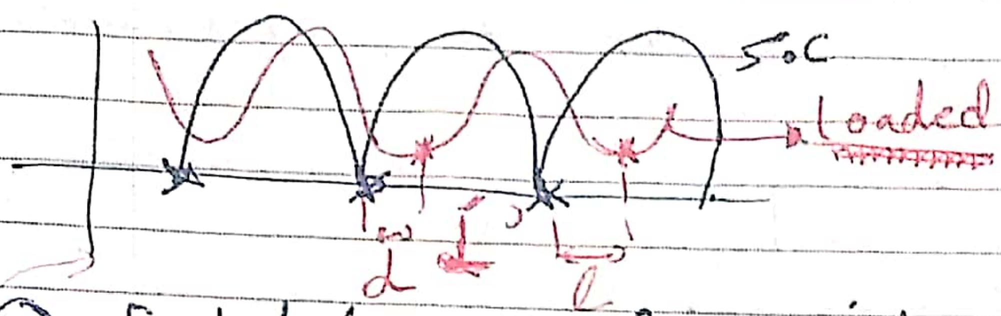
and locate the Voltage minimum



③ Find l (distance from $V_{min}|_{S.C}$ toward $V_{min}|_{Load}$ or $l' = (V_{min\,load} \longrightarrow V_{min}|_{S.C})$

④ On smith chart Draw S-circle

→ move from S.C point a distance

(l ~~towa~~ TWL) or (l' TWG)

5) read Z_L from the Chart.

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No.

air

Ex Unknow Load connected to a slotted line has $S=2$ and minima are found at 11 cm, 19 cm, —
When the load is replaced by a S.C the minima are at 16 cm, 24 cm
if $Z_0 = 50\,\Omega$, find λ, f, Z_L

$l = 24 - 19$ OR $16 - 11 = 5\text{cm}$.

$l' = 3\text{ Cm}$

$\lambda/2 = 24 - 16$ or $19 - 11$

$\lambda/2 = 8$

$\underline{\underline{\lambda = 16\text{cm}}}$

$l + l' = \frac{\lambda}{2}$

$u = c \rightarrow f = \frac{c}{\lambda} = \frac{3 \times 10^8}{16 \times 10^{-2}} = 1.875\text{GHz}$

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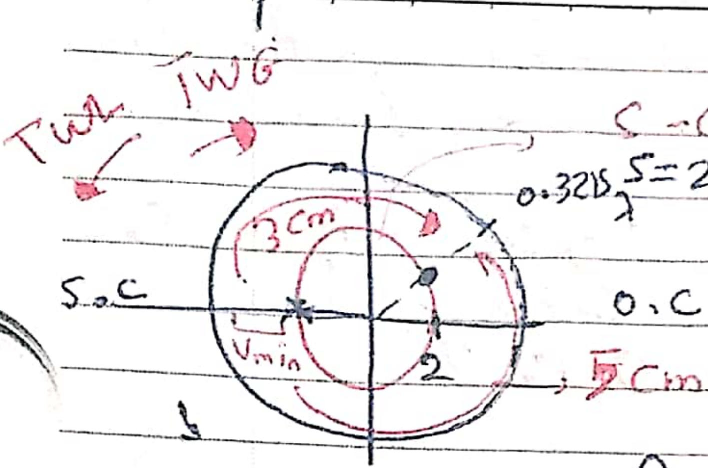
22/

No.



$$\frac{\lambda}{2} \Longrightarrow 8$$
$$? \longleftarrow 5$$

~~5cm~~

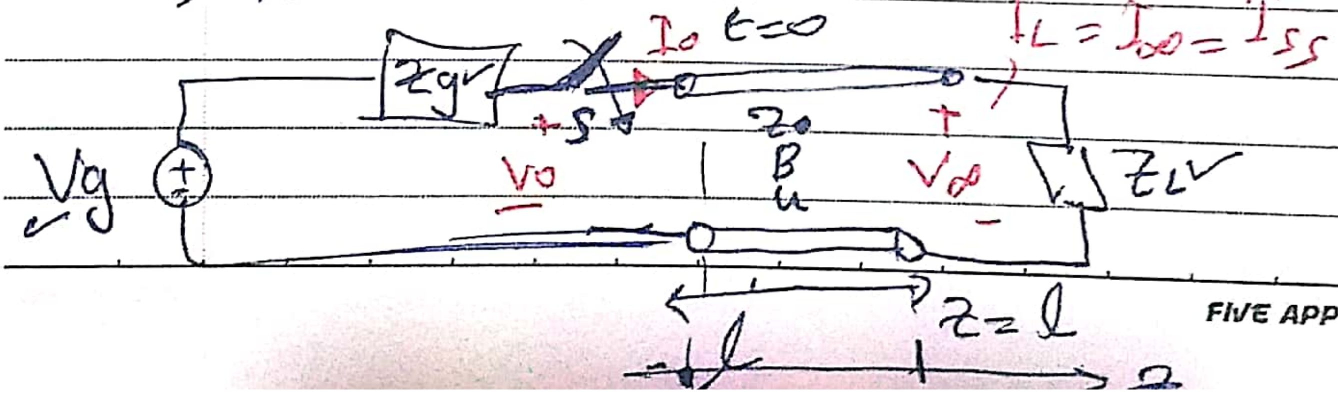
$$\ell = 0.3125\,\lambda$$

$$z_L * Z_0 = Z_L$$
$$(1.4 + j0.75)\,50 = 70 + j\,37.5\,\Omega$$

إذا عرفنا ال load وين واقع عالدائرة
ما بنحتاج نعمل S.C

11.7 Transient on T.L

→ in Time domain



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No.

at $t = 0^+$ and $Z = 0^+$

$V(0, 0^+) = V_0$

$I(0, 0^+) = I_0$

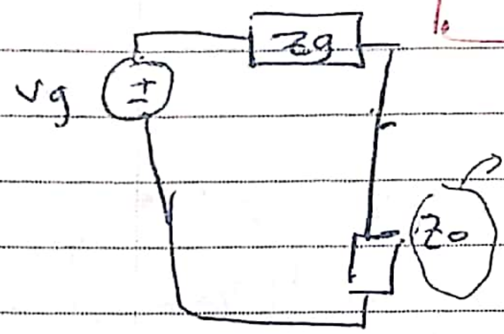
$V(l, t_1) = V_L$

$I(l, t_1) = I_L$

$t_1 \equiv$ transient time

$t_1 = \frac{l}{u}$

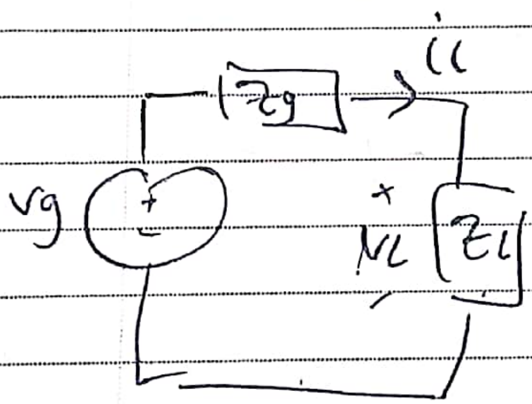
$\boxed{\text{at } z = 0^+}$



Signal

ما تشوف ال Z_L

$$V_0 = \frac{V_g Z_0}{Z_0 + Z_g} \qquad I_0 = \frac{V_g}{Z_0 + Z_g}$$



$$V_L = \frac{V_g Z_L}{Z_L + Z_g}$$

$$I_L = \frac{V_g}{Z_g + Z_L}$$

$\boxed{Z = l \qquad t = t_1}$

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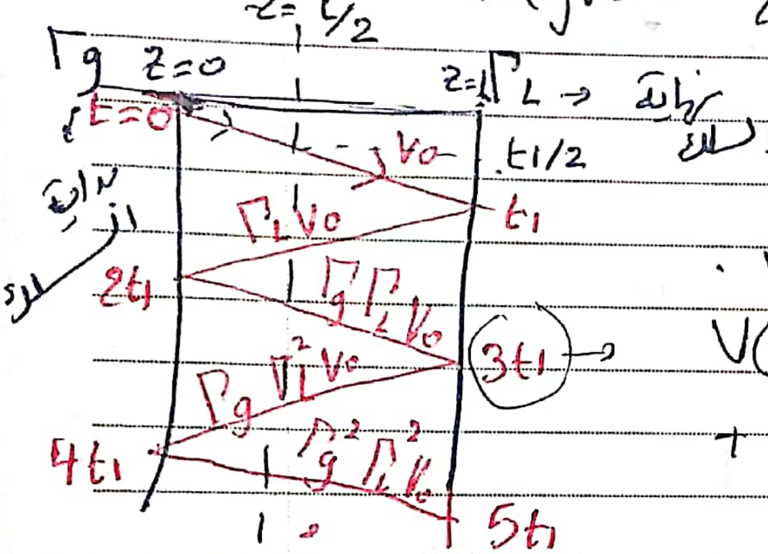
No.

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} \qquad \Gamma_{Li} = -\Gamma_{LV}$$

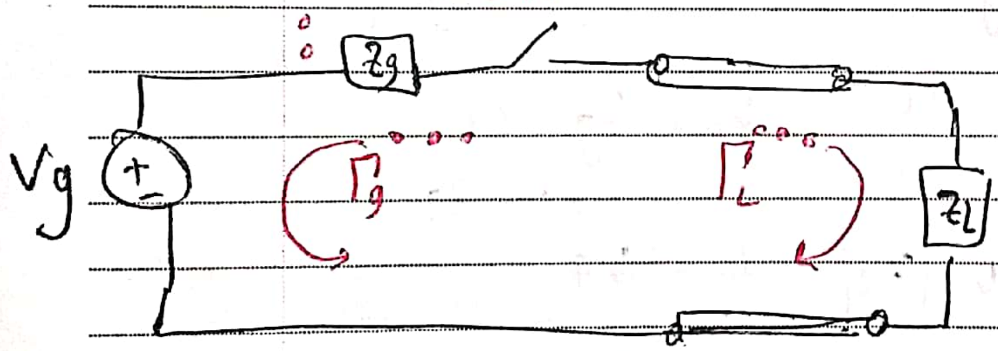
$$\Gamma_g = \frac{Z_g - Z_o}{Z_g + Z_o} \qquad \Gamma_{gi} = -\Gamma_{gV}$$

Lattice diagram / space-time diagram
Bounce diagrame / Zig Zag.



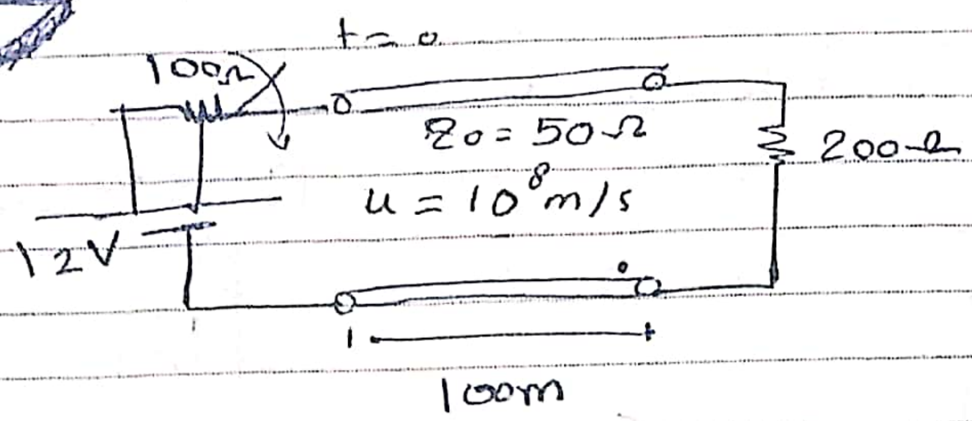
at بنجمع كل اللي قبل دايماً .

$$V(3t_1) = V_o + \Gamma_L V_o + \Gamma_g \Gamma_L V_o + \Gamma_g \Gamma_L^2 V_o$$



No.

*** Ex ***



* Calculate and sketch.

a) The Voltage at the load end and Source end for $0 < t < 6\mu s$

b) repeat for the current

$t_1 = 1\,\mu s$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{150}{250} = \frac{3}{5}$$

$$\Gamma_g = \frac{50}{150} = \frac{1}{3}$$

$$V_0 = \frac{12 \times 50}{150} = 4V \qquad V_L = \frac{12\,(200)}{250} = 8V$$

$$I_0 = \frac{12}{150} = 80\,mA \qquad I_L = \frac{12}{250} = 40\,mA$$

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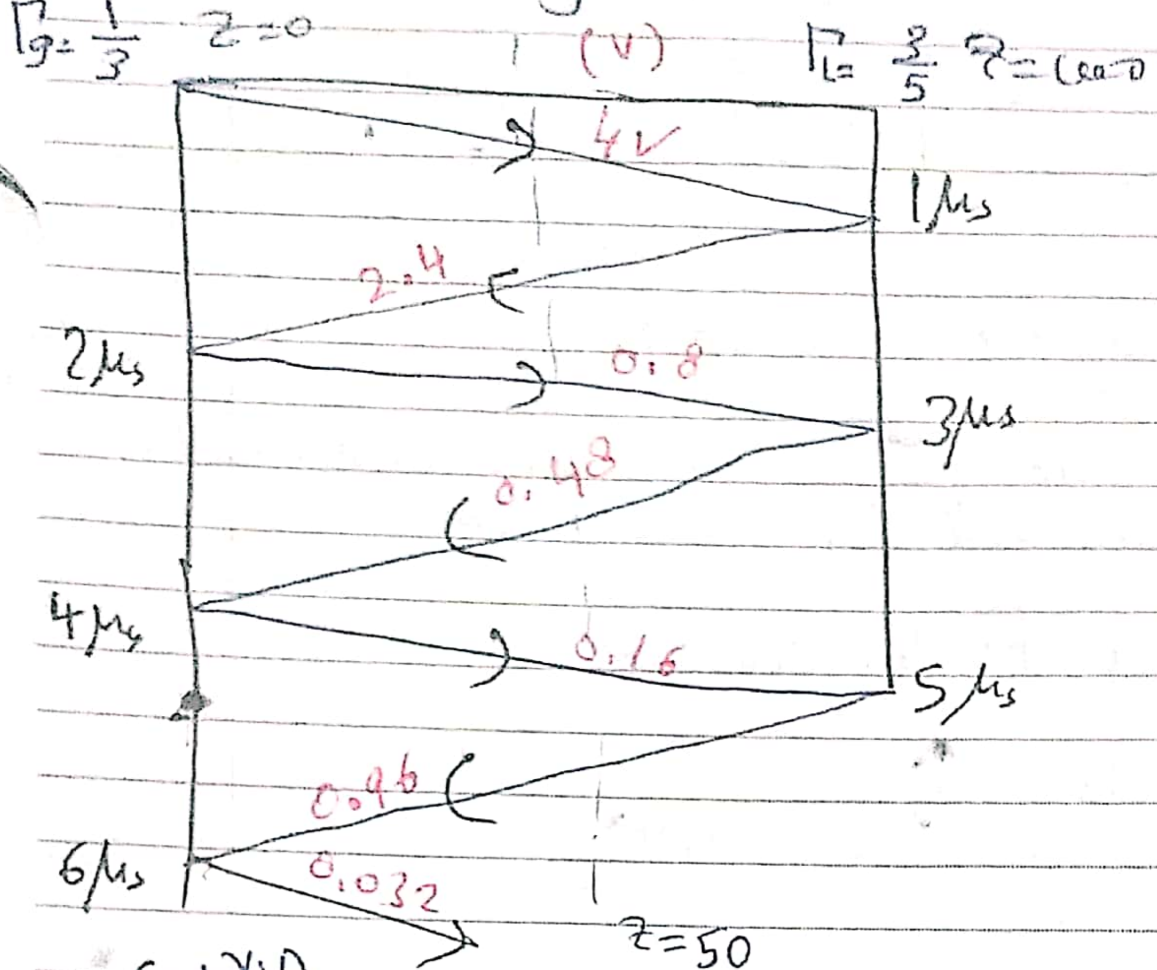
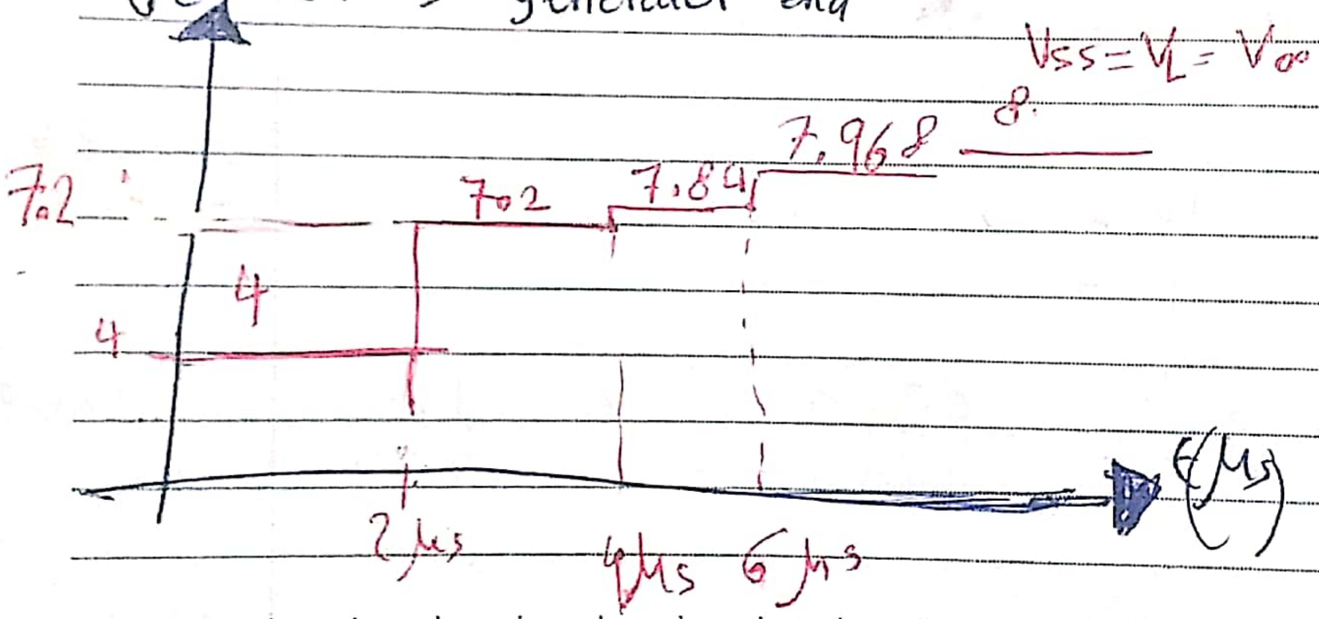
Lattice diagram



$v(0,t)(V) \rightarrow$ generator end



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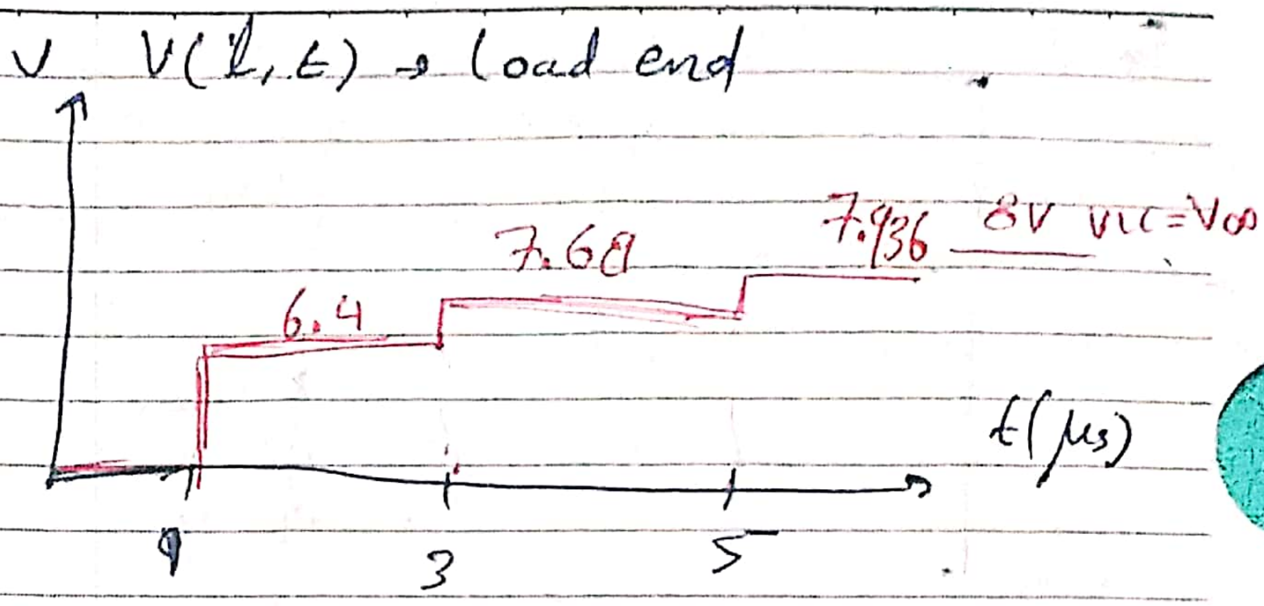

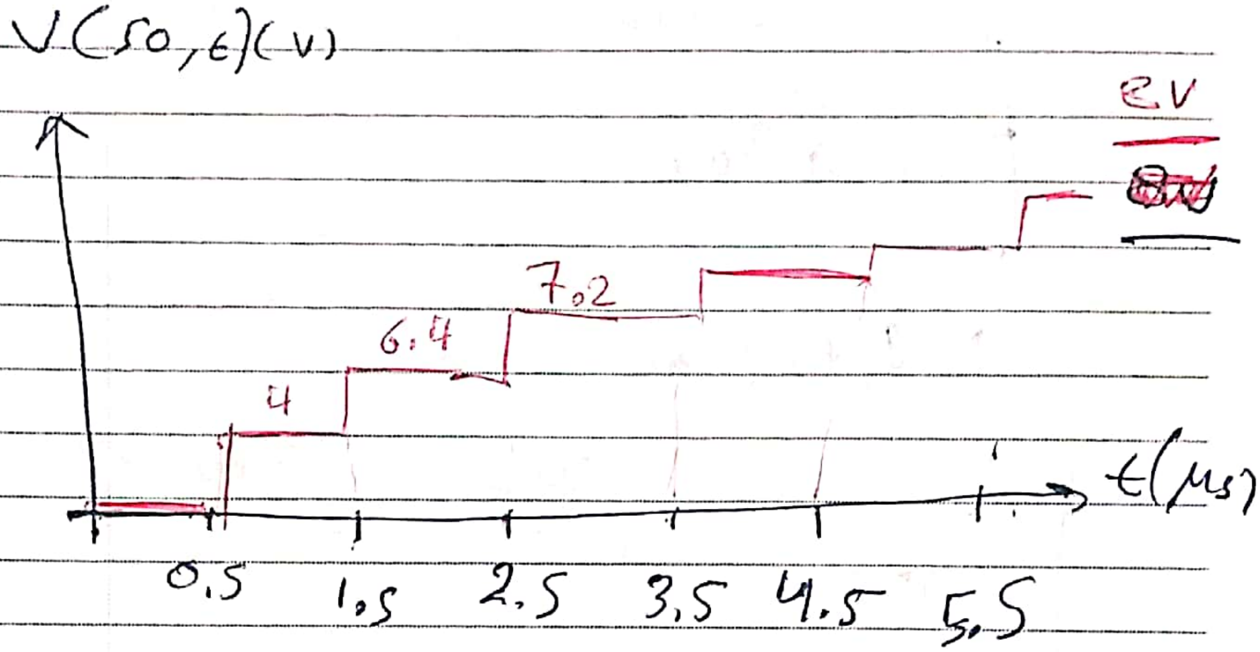

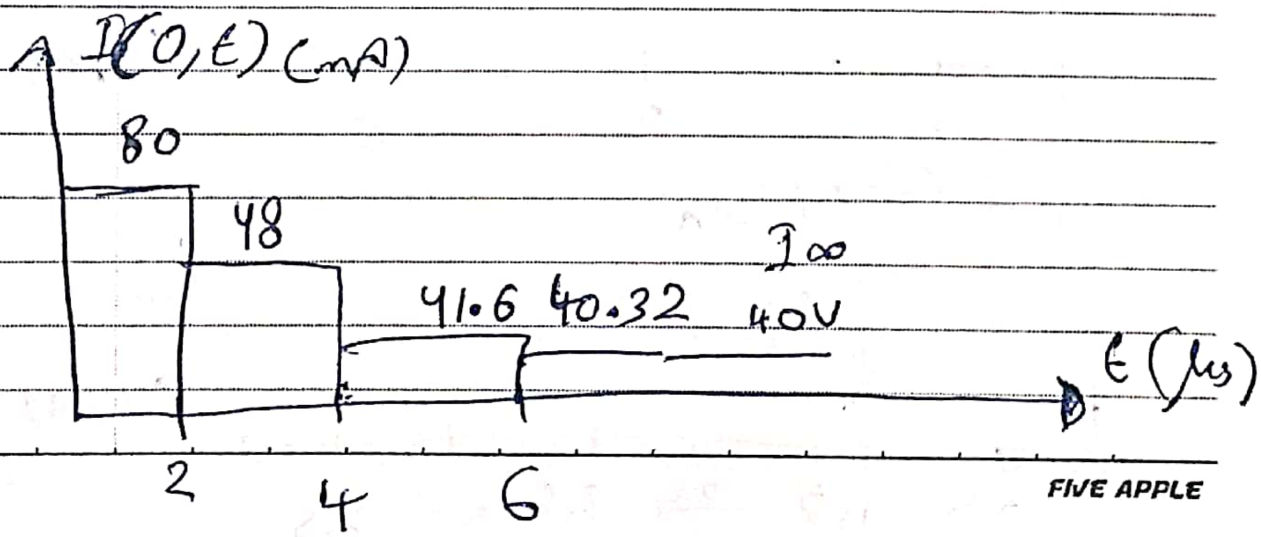

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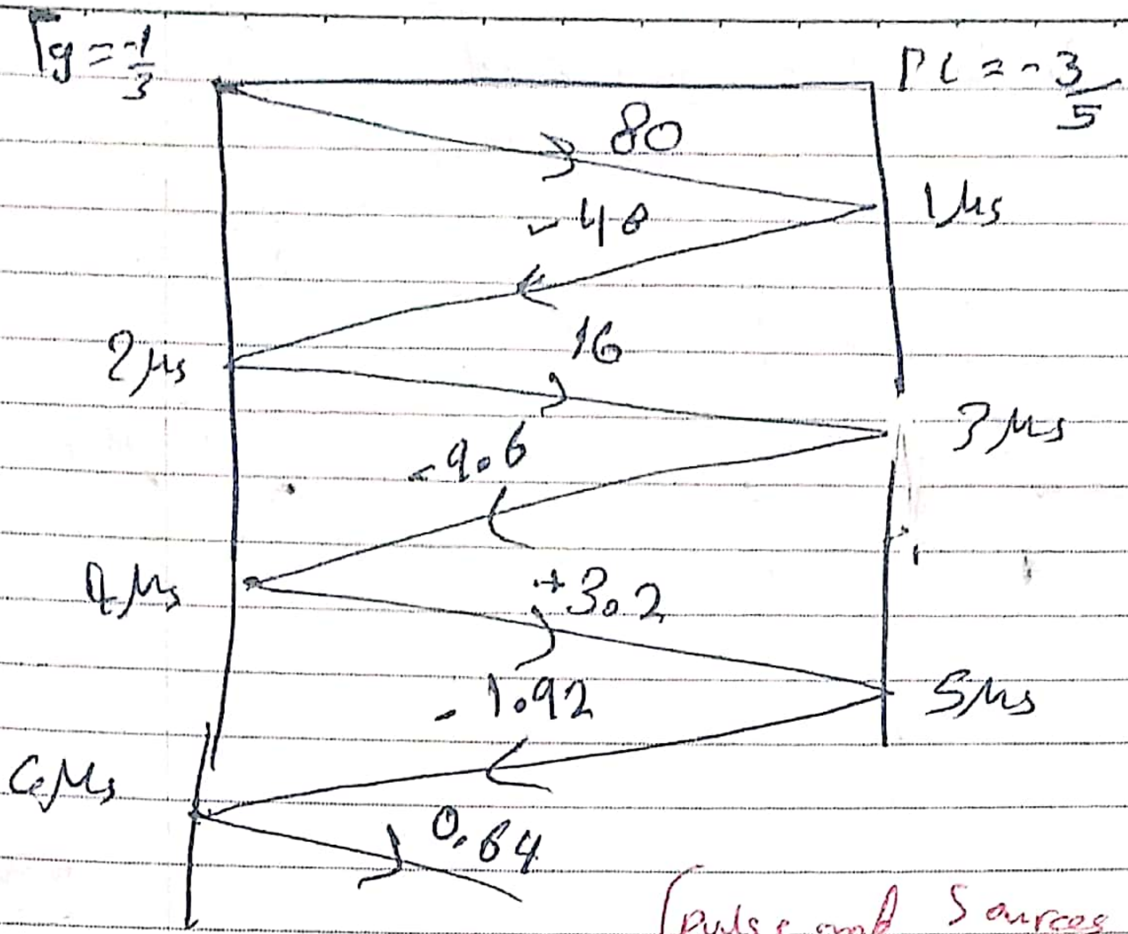
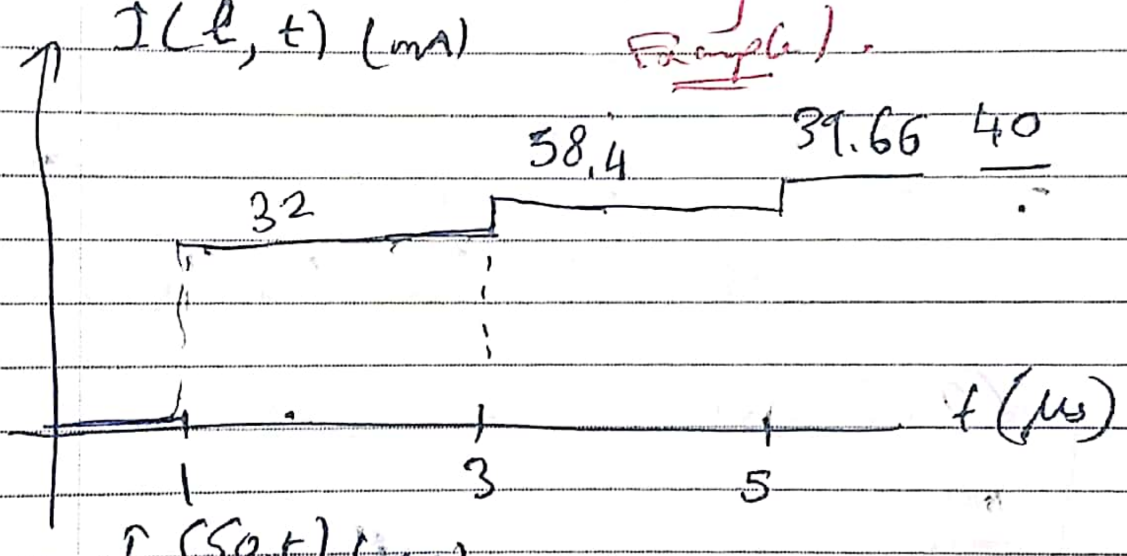
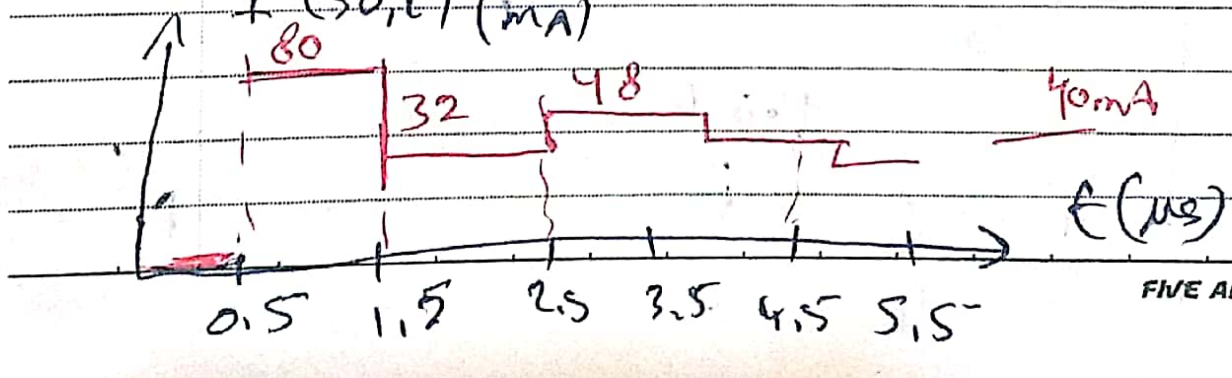


(pulse and sources triangle ~~test~~ Example).





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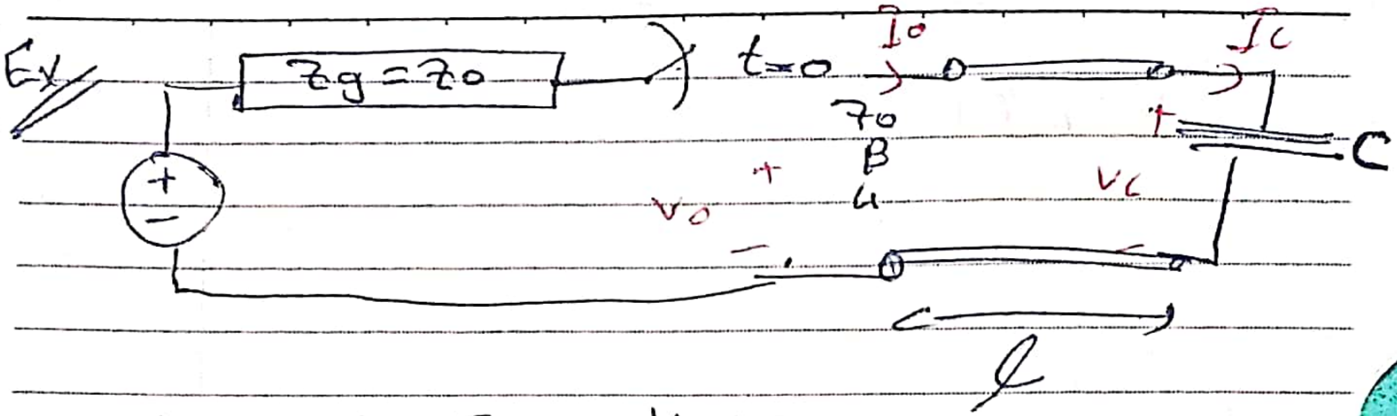
No.

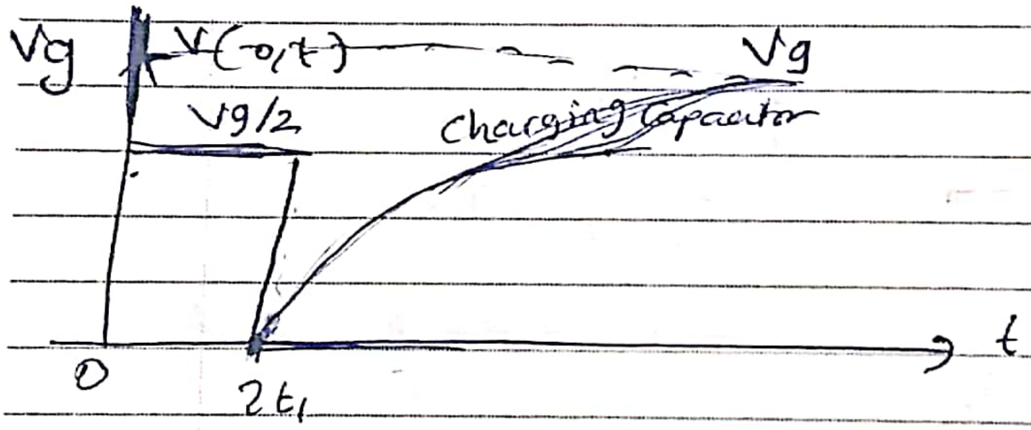
Ex/



$V_o = \frac{V_g Z_o}{2Z_o} = V_g/2$ $\Gamma_g = 0$

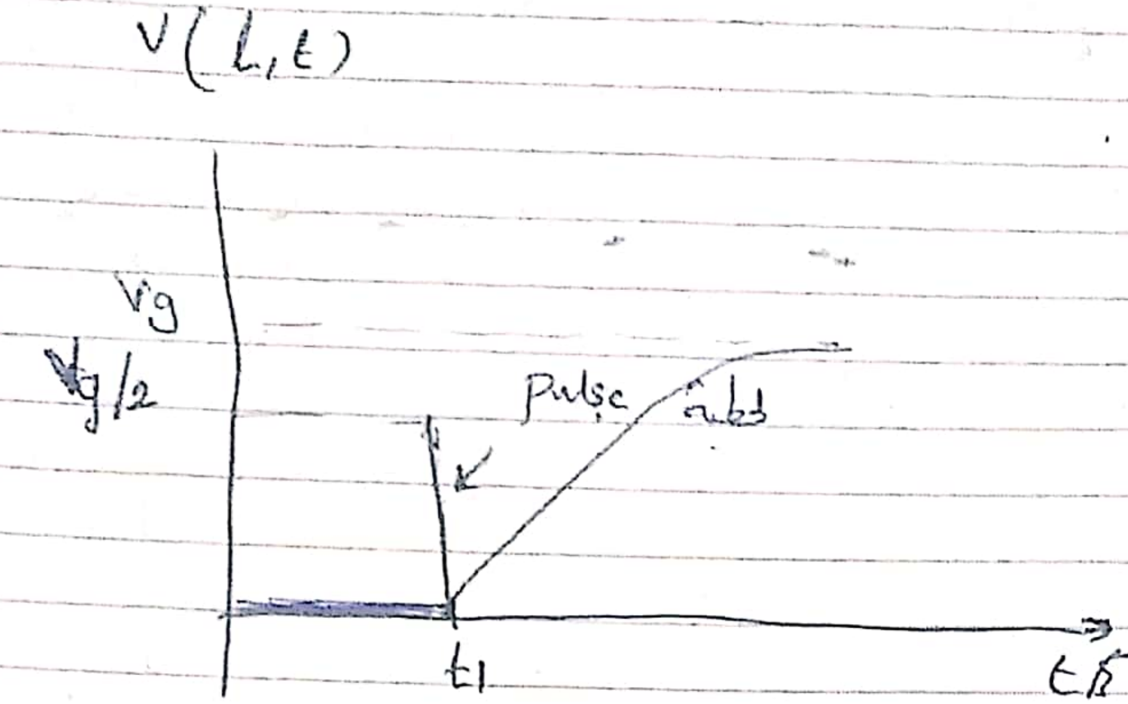
$I_o = \frac{V_g}{2Z_o}$. $t_1 = \frac{\ell}{u}$

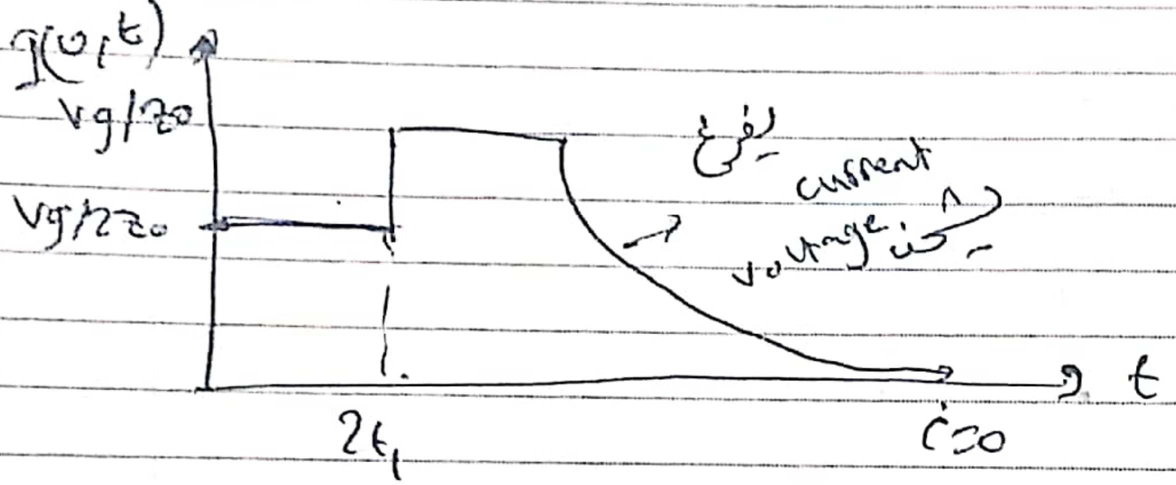
at $t < t_1$ $V_L = 0 \rightarrow$ S.C $\Gamma = -1$

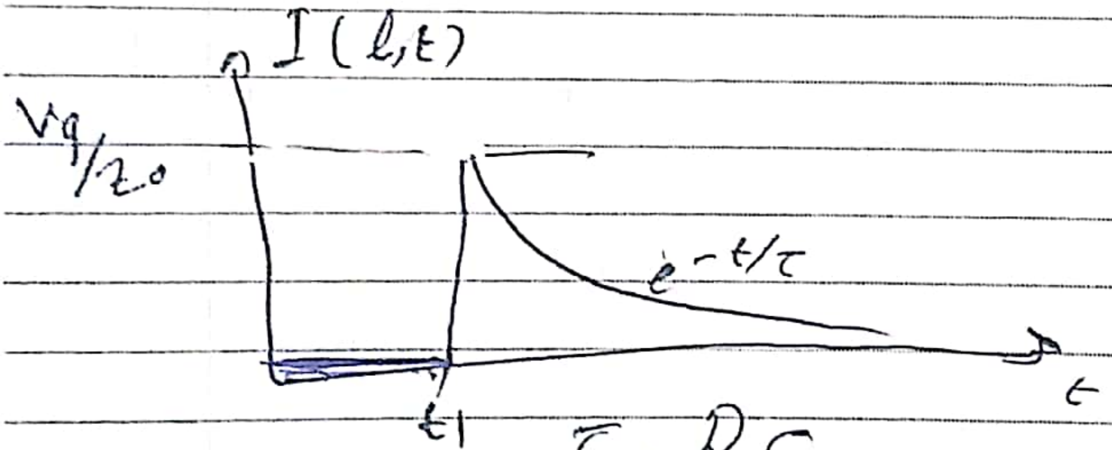


at $t = \infty$ $I = 0$ $\Gamma_{Li} = -1$ $\Gamma_{Lv} = 1$

$V_L = V_g$







$\tau = RC$

$= Z_g\,C$

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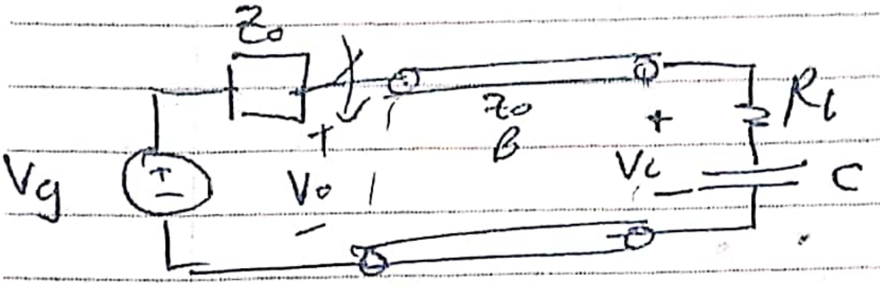


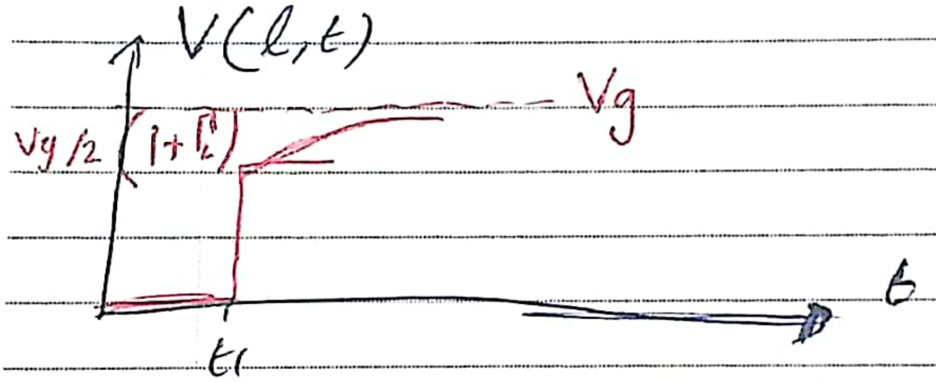
at $t = 0^-$ → C is s.c load is R_L

$$\Gamma = \frac{R_L - Z_0}{R_L + Z_0} \qquad \Gamma_g = 0$$

$$V_0 = \frac{V_g Z_0}{2Z_0} = V_g/2$$

$$t_1 = \frac{l}{u} \qquad u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{LC}} = \frac{\omega}{\beta}$$

$$I_0 = \frac{V_g}{2Z_0}$$



$$\tau = R_{eq} C = (R_L + Z_g) C$$

* after the capaciter is chared load o.c $\Gamma = 1$

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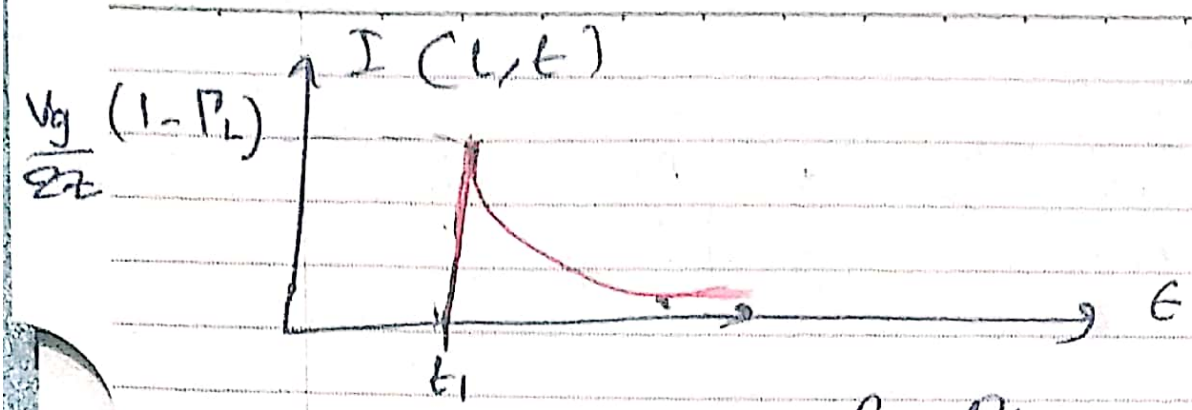
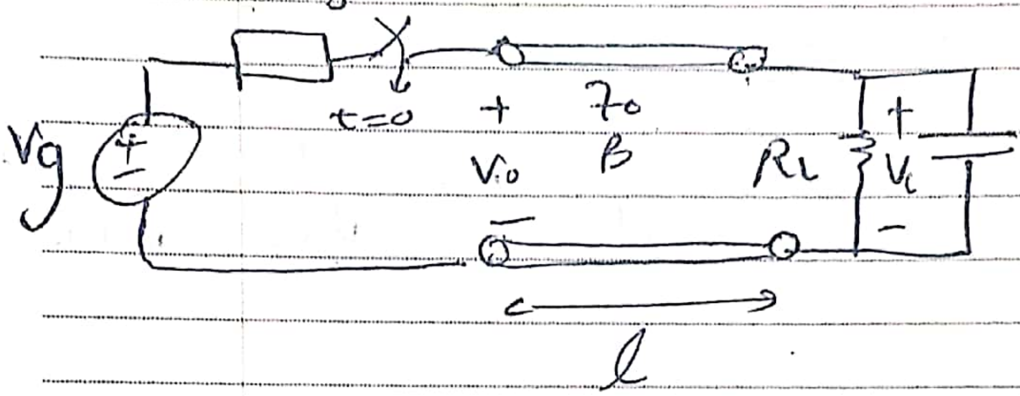


for RL parallel

$V(0,t)$, $I(0,t)$ $\Rightarrow$ V → I نفس حل ال

I → V نفس حل ال

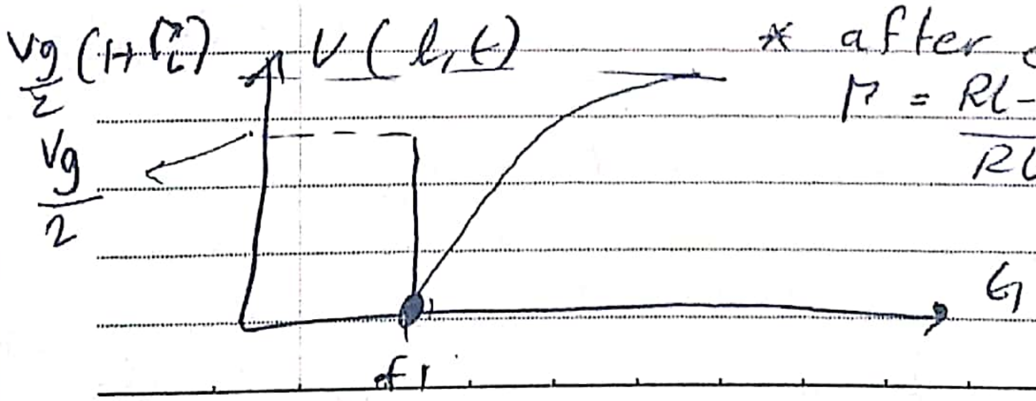
Ex/



$$V_o = \frac{V_g}{2}$$

$$I_o = \frac{V_g}{2Z_o}$$

at $t = 0^+$ → $\Gamma = -1$ S.C



* after charging

$$\Gamma = \frac{R_L - Z_o}{R_L + Z_o}$$

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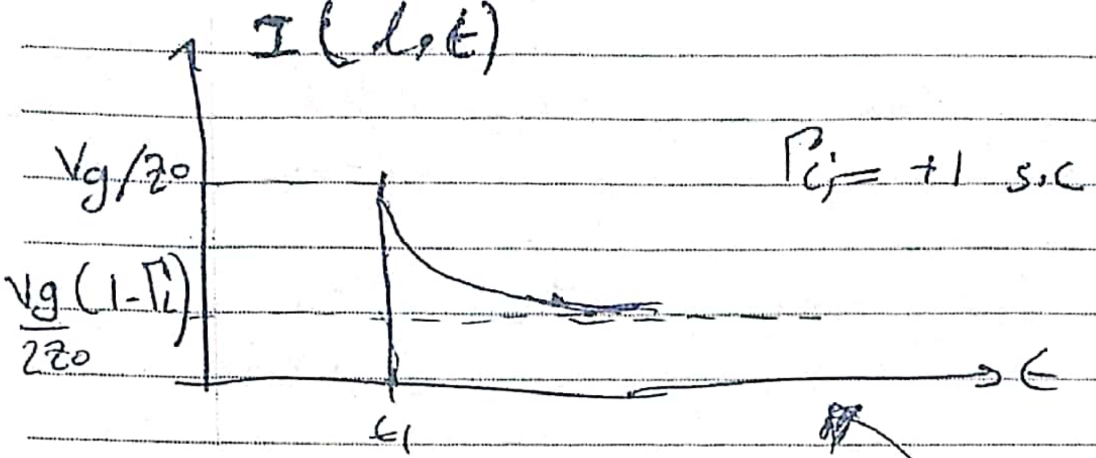
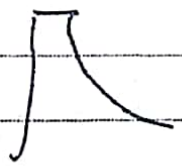
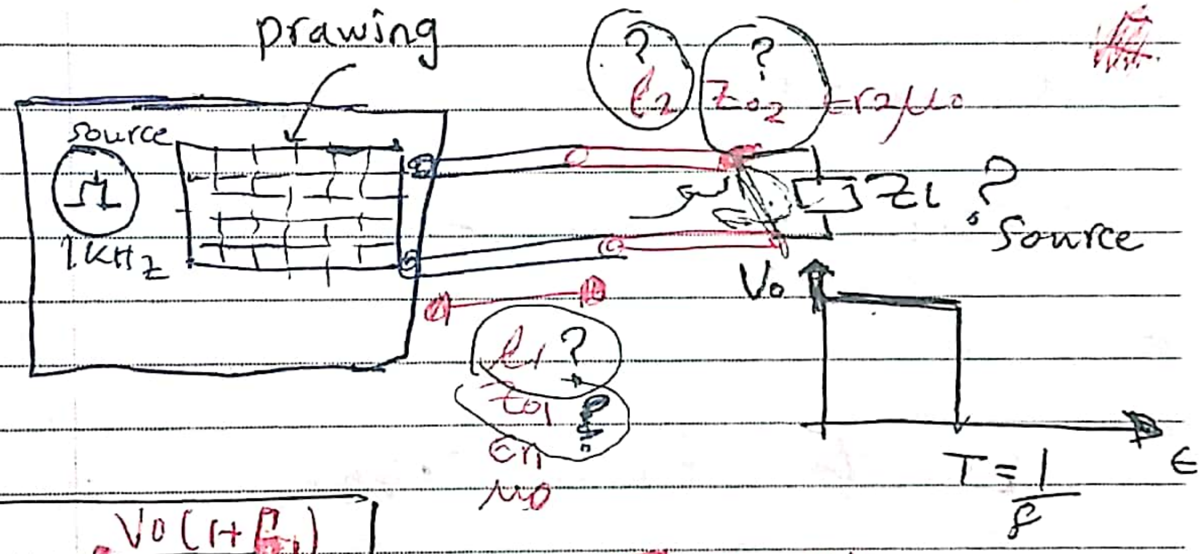


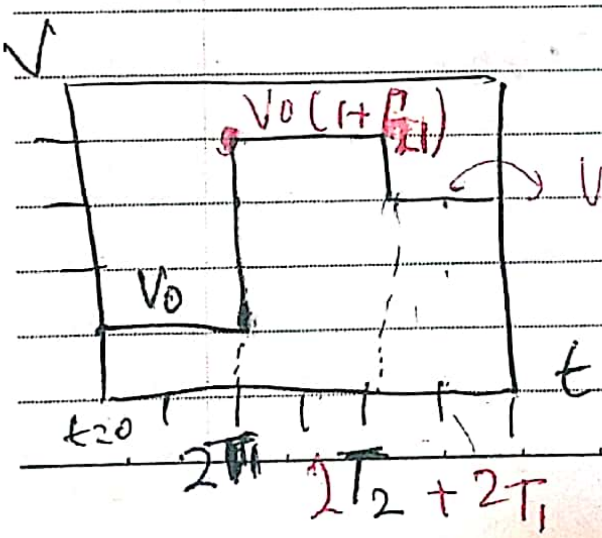
$\Gamma_L = +1$ s.c

$V(0,t)$? $I(0,t)$
and
$V(\frac{l}{2},t)$ $I(\frac{l}{2},t) \rightarrow$



(*) Time domain Reflectometer (TDR)





$V_0(1+T_{12}\,T_{21}\,\Gamma_L)$

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lossless TDR

$\Gamma_L = \dfrac{Z_L - Z_{02}}{Z_L + Z_{02}}$ $\qquad \tau_{12} = \dfrac{2Z_{02}}{Z_{01} + Z_{02}}$

$\Gamma_{21} = \dfrac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$ $\qquad \tau_{21} = \dfrac{2Z_{01}}{Z_{01} + Z_{02}}$

$\Gamma_{12} = \dfrac{Z_{01} - Z_{02}}{Z_{01} + Z_{02}}$

$l_1 = u_1 T_1$ $\qquad u_1 = \dfrac{c}{\sqrt{\epsilon_{r_1}}}$

from the graph.

l_1 ✓

$l_2 = u_2 T_2$ ✓

Γ_{21} ✓ $\Rightarrow Z_{02}$ ✓

$\Rightarrow$ ✓ τ_{12} $\Rightarrow$ τ_{21} ✓

Γ_L ✓ $\Rightarrow$ from the graph.

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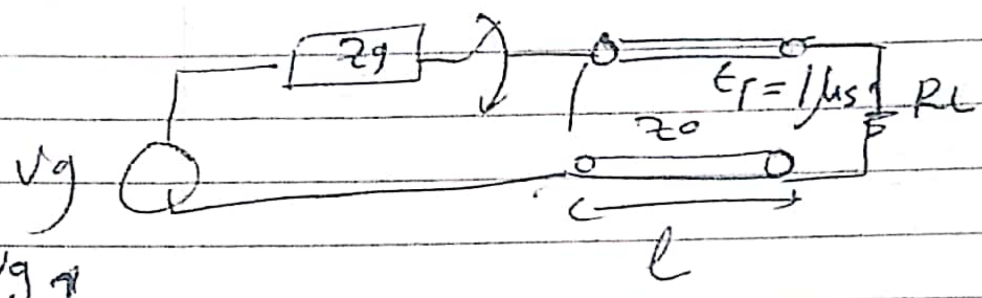
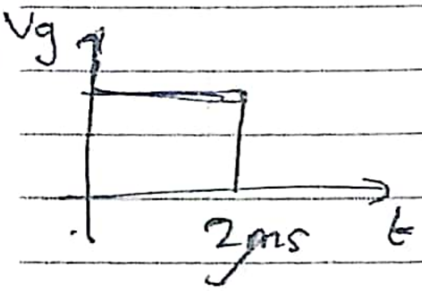
No.

Meow

Example

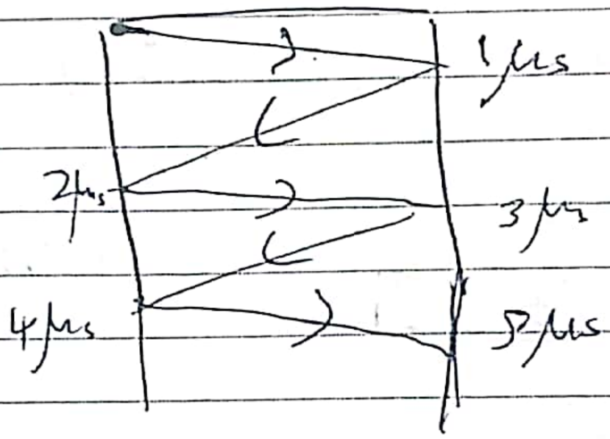
t=0





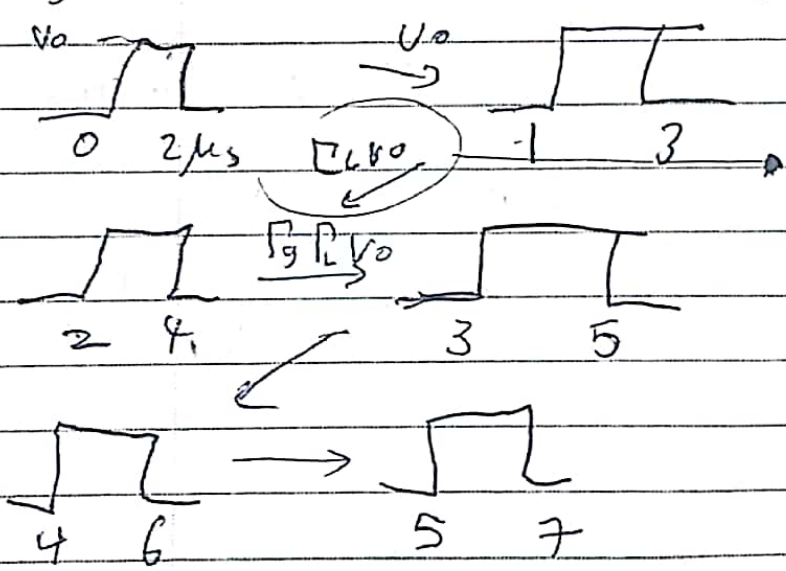
Vov

Iov



generator

Load



at 2μs sparke

$V_o + \Gamma_L V_o$

at 2μs

$\Gamma_L V_o$ not $(\Gamma_L + 1) V_o$

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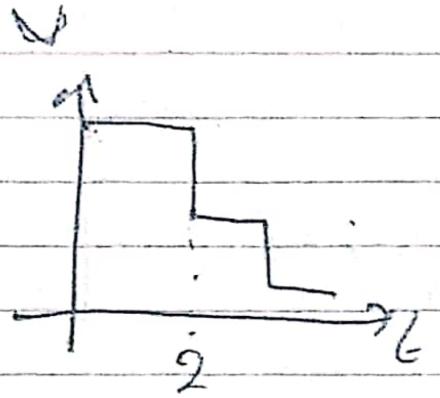
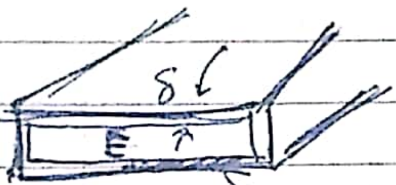


(2) Wave guides: CH.12

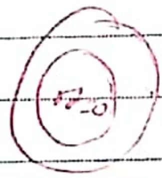
(one conductor T.L)



$\delta \to 0$ if $\sigma \to \infty$

RWG Rectangular wave guide

→ good conductor.



CH10:

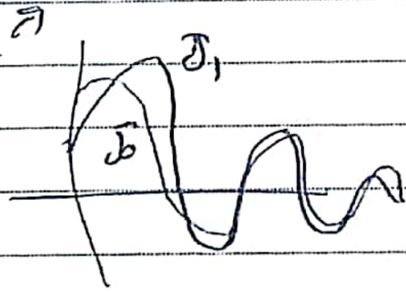
$$\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0$$

$$\nabla^2 \bar{H}_s - \gamma^2 \bar{H}_s = 0$$

CWG

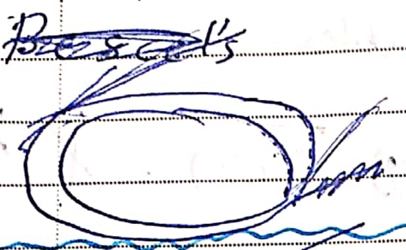
Bescel's functions

circular $\bar{E}, \bar{H}$

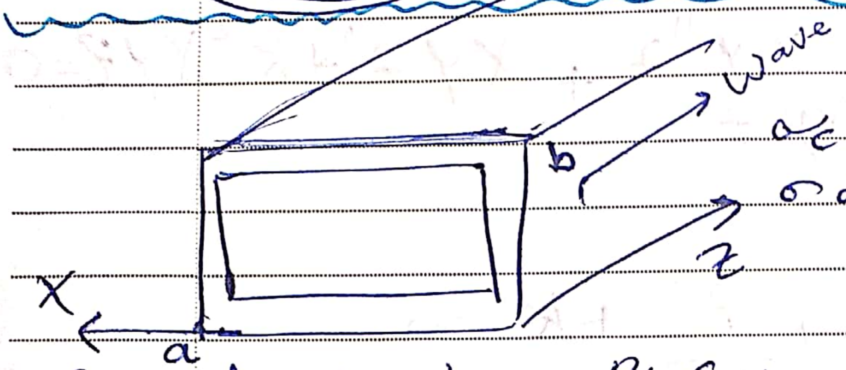


ideal $\sigma_c = \infty$

$\sigma_d = 0$ (Air).



EWG.



$\sigma_c = \infty$] Ideal
$\sigma_d = 0$

Consider lossless RWG :-

$\alpha = 0$

Source free $\rho_v = 0$

$J = 0$

Solving the wave equation :-

$$\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0$$

$$\gamma = \alpha + j\beta$$

$$\nabla^2 \bar{E}_s + k^2 \bar{E}_s = 0 \qquad k = \beta \text{ if lossless.}$$

Wave guide is Not a TEM

let $\bar{E}_s = (\bar{E}_{xs}, \bar{E}_{ys}, \bar{E}_{zs})$

Take The z - component

Solve (6) scalar Equations

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_{zs}}{\partial y^2} + \frac{\partial^2 E_{zs}}{\partial z^2} + k^2 E_{zs} = 0 \qquad (1)$$

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No.

use Separation of Variables.

assume $E_{zs} = X(x)\,Y(y)\,Z(z) = XYZ$

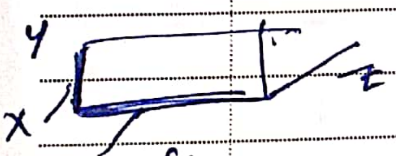
$$X''YZ + XY''Z + XYZ'' + k^2 XYZ = 0$$

devide by XYZ

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} + k^2 = 0$$

Choose $k^2 = k_x^2 + k_y^2 + k_z^2$ propagation [illegible]

$k^2 = \underbrace{k_x^2 + k_y^2}\, - \gamma^2$



Standing wave

wave guide ال [illegible]

reflection $\Rightarrow$ SW

$e^{-\gamma z}$

$E \to 0$

as $z \to \infty$ ✓

must $\to 0$

$z \to \infty$

$e^{\gamma z}$

$E \to \infty$

as $z \to \infty$

X

No.

$$\frac{X''}{X} = -k^2 X \rightarrow X'' + k_x^2 X = 0$$

$$m^2 X + k_x^2 X = 0$$

$$m^2 = -k_x^2$$

$$m = \pm j k_x \quad \left(\begin{array}{c}\text{roots are}\\ \text{imaginary}\end{array}\right)$$

$$X(x) = A_1 e^{-jx k_x} + A_2 e^{j k_x X}$$

$$= C_1 \cos k_x X + C_2 \sin k_x X$$

$$\frac{Y''}{Y} = -k_y^2$$

$$\frac{Z''}{Z} = +\gamma^2$$

$$Y(y) = C_3 \cos k_y Y + C_4 \sin k_y Y$$

$$\frac{Z''}{Z} = \gamma^2$$

$$m = \pm \gamma \quad (\text{real Roots})$$

$$Z'' - \gamma^2 Z = 0$$

$$m^2 - \gamma^2 = 0$$

$$Z(z) = C_5 e^{-\gamma z} + C_6 e^{+\gamma z}$$

$$\text{or} = C_5 \cosh(\gamma z) + C_6 \sinh \gamma z$$

$$E_{zs} = X(x)\, Y(y)\, Z(z)$$

$$E_{zs} = \left(C_1 \cos k_x X + C_2 \sin k_x X\right) \cdot$$

$$\left(C_3 \cos k_y Y + C_4 \sin k_y Y\right) \cdot$$

$$\left(C_5 e^{-\gamma z} + \cancel{C_6} e^{\gamma z}\right).$$

$C_6 = 0$ since $E \rightarrow 0$, $z \rightarrow \infty$

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$$E_{zs} = \left(A_1 \cos k_x X + A_2 \sin k_x X\right) \cdot \left(A_3 \cos k_y Y + A_4 \sin k_y Y\right) e^{-j\beta z}$$

$A_1 = C_1 \cdot C_5$

$A_2 = C_2\, C_5$

$A_3 = C_3\, C_5$

$A_4 = C_4\, C_5$

By Duality

$$\nabla^2 H_s + k^2 H_s = 0$$

$$H_{zs} = \left(B_1 \cos k_x X + B_2 \sin k_x X\right) \left(\cancel{B_3 \cos}\right.$$

$$\left(B_3 \cos k_y Y + B_4 \sin(k_y Y)\right) e^{-\gamma z}$$

to find E_x, E_y, H_x, H_y

Use maxwell's equations

$\nabla \cdot D_s = \rho_{vs} = 0$ source free Region

$\nabla \cdot B_s = 0$

$\nabla \times \bar{E} = -j\mu\omega H_s$

$\nabla \times \bar{H} = j\omega\epsilon E_s \quad (\sigma = 0)$

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No.

$$\nabla \times \bar{E}_s = -j\omega\mu H_s$$

$$\bar{H}_s = \frac{-1}{j\omega\mu} \nabla \times E_s$$

$$\bar{H}_s = \begin{bmatrix} H_{xs} \\ H_{ys} \\ H_{zs} \end{bmatrix} = \frac{j}{\omega\mu} \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_{xs} & E_{ys} & E_{zs} \end{bmatrix}$$

$$H_{xs} = \frac{j}{\omega\mu}\left(\frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z}\right) \quad \text{— (a)}$$

$$H_{ys} \frac{-j}{\omega\mu}\left(\frac{\partial E_{zs}}{\partial x} - \frac{\partial E_{xs}}{\partial z}\right) \quad \text{— (b)}$$

$$H_{zs} = \frac{j}{\omega\mu}\left(\frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y}\right) \quad \text{— (c)}$$

from $\nabla \times \bar{H}_s = j\omega\epsilon E_s$

$$\bar{E}_s = \frac{1}{j\omega\epsilon} \nabla \times \bar{H}_s$$

$$\begin{bmatrix} E_{xs} \\ E_{ys} \\ E_{zs} \end{bmatrix} = \frac{1}{j\omega\epsilon} \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_{xs} & H_{ys} & H_{zs} \end{bmatrix}$$

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No.

$$E_{xs} = \frac{1}{j\omega\epsilon}\left(\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z}\right) \quad - (d)$$

$$E_{ys} = \frac{1}{j\omega}\left(\frac{\partial H_{zs}}{\partial x} - \frac{\partial H_{xs}}{\partial z}\right) \quad - (e)$$

$$E_{zs} = \frac{1}{j\omega\epsilon}\left(\frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y}\right) \quad - (f)$$

Solving for E_{xs}

solve Eq (d) with (β)

$$j\omega\epsilon E_{xs} = \frac{\partial H_z}{\partial y} - \frac{\partial}{\partial z}\left(\frac{-j}{\omega\mu}\left(\frac{\partial E_{zs}}{\partial x} - \frac{\partial E_{xs}}{\partial z}\right)\right)$$

$$j\omega\epsilon E_{xs} = \frac{\partial H_{zs}}{\partial y} + \frac{j}{\omega\mu}\frac{\partial^2 E_{zs}}{\partial x \partial x} - \frac{j}{\omega\mu}\frac{\partial^2 E_{xs}}{\partial z^2}$$

$$\frac{\partial E_{zs}}{\partial z} \rightarrow -\gamma(\quad)(\quad)e^{-\gamma z}$$

$$= -\gamma E_{zs}$$

E_{xs} in terms of $z \rightarrow e^{-\gamma z}$

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No.

$$\frac{\partial^2 E_{xs}}{\partial z^2} = \gamma^2 E_{xs}$$

$$j\omega\epsilon E_{xs} = \frac{\partial H_{zs}}{\partial y} - \frac{j\gamma}{\omega\mu}\left(\frac{\partial E_{zs}}{\partial x}\right) - \frac{j\gamma^2 E_{xs}}{\omega\mu}$$

$$E_{xs}\left(j\omega\epsilon + \frac{j\gamma^2}{\omega\mu}\right) = \frac{-j\gamma}{\omega\mu}\frac{\partial E_{zs}}{\partial x} + \frac{\partial H_{zs}}{\partial y}$$

let $h^2 = \gamma^2 + \omega^2\mu\epsilon$

$$E_{xs} = \frac{\omega\mu}{jh^2}\left(\frac{-j\gamma}{\omega\mu}\frac{\partial E_{zs}}{\partial x} + \frac{\partial H_{zs}}{\partial y}\right)$$

$$E_{xs} = \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}$$

$$E_{zs} = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y)e^{-\gamma z}$$

$$H_{zs} = (B_1 \cos k_x x + B_2 \sin k_y y)(B_3 \cos k_y y + B_4 \sin k_y y)e^{-\gamma z}$$

$$E_{xs} = \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}$$

$$E_{ys} = \frac{-\gamma}{h^2}\frac{E_{zs}}{\partial y} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x}$$

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No.

$$H_{xs} = \frac{j\omega\epsilon}{h^2}\frac{\partial E_{zs}}{dy} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{dx}$$

$$H_{ys} = -\frac{j\omega\epsilon}{h^2}\frac{\partial E_{zs}}{dx} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{dy}$$

$$\boxed{h^2 = \gamma^2 + \omega^2\mu\epsilon}$$

Cases :-

1. $E_{zs} = 0$, $H_{zs} = 0$ (TEM mode)
   all Components are zeros X
   RWG doesn't support TEM ~~mo~~.

2. $E_{zs} = 0$, $H_{zs} \neq 0$
   $E_{xs}, E_{ys}, H_{xs}, H_{ys} \neq 0$
   TE Mode

3. $E_{zs} \neq 0$ $H_z = 0$
   TM mode $(E_{x_s}, E_{y_s}, H_{x_s}, H_{y_s}) \neq 0$

4. $E_{zs} \neq 0$, $H_z \neq 0$
   Hybride mode
   HE mode
   ~~or~~ EH mode
   (Ex) $\boxed{\text{optical fiber}}$

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No.

$$\eta = \frac{E}{H} = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

$$\eta = \frac{\gamma}{j\omega\epsilon} = \frac{j\omega\mu}{\gamma} = \frac{j\beta}{j\omega\epsilon} = \frac{\beta}{\omega\epsilon}$$

$\hat{a}_z \quad \hat{a}_x \quad \hat{a}_y$

$$\frac{\cancel{\omega}\sqrt{\mu\epsilon}}{\cancel{\omega}\epsilon} = \sqrt{\frac{\mu}{\epsilon}} \quad \text{if TEM}$$

* ٢٠١٩/٧/٢٢ الاثنين

No.

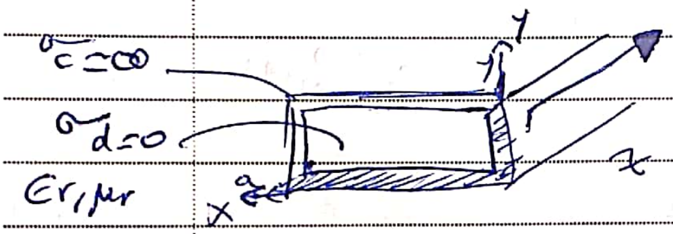
$$h^2 = \gamma^2 + k^2 = \boxed{k_x^2 + k_y^2} \checkmark$$

$$k^2 = k_x^2 + k_y^2 - \gamma^2$$

TM mode

$H_z = 0$ $\quad (\vec{H} \perp \hat{a}_k)$

E_z, E_x, E_y, H_x, H_y ?



$\hat{E}_z$

لكل الجدران 4

$E_{zs} = 0$ at $x = 0$

$E_{zs} = 0$ at $x = a$

$E_{zs} = 0$ at $y = 0$

$E_{zs} = 0$ at $y = b$

$E_{zs} = 0$ at $x = 0 \rightarrow A_1 = 0$

$E_{zs} = 0$ at $y = 0 \rightarrow A_3 = 0$

$$E_{zs} = \overset{A_2 * A_4}{C_0} \sin k_x x \, \sin k_y y \, e^{-j\beta z}$$

$E_{zs} = 0$ at $x = a$

$\sin k_x a = 0$ $\qquad k_x a = m\pi$

$m = 1, 2, 3, \ldots \infty$ $\qquad \boxed{k_x = \frac{m\pi}{a}}$

$m \neq 0$

since $k_x \neq 0$

(الموجة موجودة في اتجاه الـ x)

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No.

$E_{z_s} = 0$ at $y = B$

$\sin k_y b = 0$

$k_y = \frac{n\pi}{b}$ $n = 1, 2, --- \infty$

$n \neq 0$

$$E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$H_{zs} = 0$

$$E_x = -\frac{\gamma}{h^2} \frac{m\pi}{a} E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$$h^2 = k_x^2 + k_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$E_y = -\frac{\gamma}{h^2} \frac{n\pi}{b} E_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$$H_x = \frac{j\omega\epsilon}{h^2} \frac{n\pi}{b} E_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$$H_y = -j\frac{\omega\epsilon}{h^2} \frac{m\pi}{a} E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

The lowest mode in TM is TM_{11}

TM_{mn}

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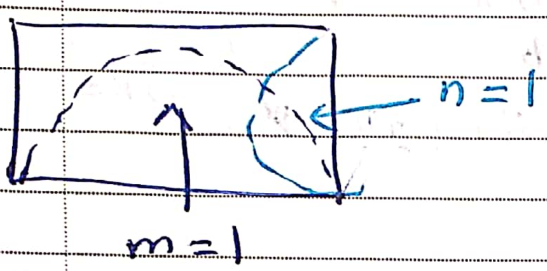
No.

variations

$m \equiv$ No. of half cycles in x-direction

$n =$ No " " " y-direction



$E_{zs} \rightarrow E(z,t)$

$$h^2 = \gamma^2 + k^2$$

$$\gamma^2 = h^2 - k^2$$

$$\gamma^2 = \left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon$$

3-cases $\rightarrow$ $h = k$

$h > k$

$h < k$

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No.

a) cutoff mode $h = k$ — frequency at this case is $\underline{\underline{f_c}}$ or W_c

$$\frac{m\pi}{a^2} + \frac{n\pi}{b^2} = \omega^2 \mu \epsilon$$

$$\gamma = 0 = \alpha + j\beta \qquad \alpha = 0, \beta = 0$$

$$\omega_c^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\boxed{\omega_c = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

cutoff wavelength $\lambda_c = \frac{u_p}{f_c}$ Largest Wave Length

at TM_{11} $f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2}$

$$f_{c11} = \frac{u}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

phase velocity for TEM mode

No.

b) Evanescent mode (attenuated).

$h^2 > k^2$

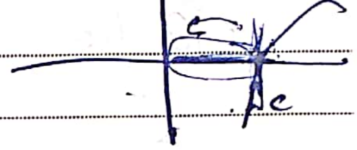
$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 > \omega^2\mu\epsilon$

γ is Real $\quad \alpha \neq 0 \quad \beta = 0$

No propagation.

at frequencies $< f_c$

high pass filter



c) propagation mode

$h^2 < k^2$

$\omega^2\mu\epsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \qquad f > f_c$

$\alpha = 0$

$\gamma = j\beta$

$\gamma = j\beta = \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b^2}\right) - \omega^2\mu\epsilon}$

$=$

$j\beta = j\sqrt{\omega^2\mu\epsilon - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}$

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No.

$$\beta = \sqrt{\omega^2 \mu \epsilon - \underbrace{\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = (2\pi f_c)^2 \mu\epsilon$$

$$\beta = \sqrt{\omega^2 \mu \epsilon - \left[(2\pi f_c)^2 \mu \epsilon\right]} \quad \times \frac{\omega^2}{\omega^2}$$

$$\beta = \omega\sqrt{\mu\epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \rightarrow \text{cutoff}$$

$$\beta = \beta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \qquad \beta' = \omega\sqrt{\mu\epsilon}$$

$\underline{\text{CH10}}$

$$\boxed{f_{c_{mn}} = \frac{c'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$\lambda = \frac{2\pi}{\beta} \qquad u_p = \frac{\omega}{\beta} = \boxed{\frac{1}{\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}}$$

$$u_p = \frac{c'}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

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In Evanescent mode $\beta = 0$

$\alpha \neq 0$

$$\alpha = \beta' \sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$$

$$\eta_{TM} = \frac{\gamma}{j\omega\epsilon} = \frac{j\beta}{j\omega\epsilon} = \eta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

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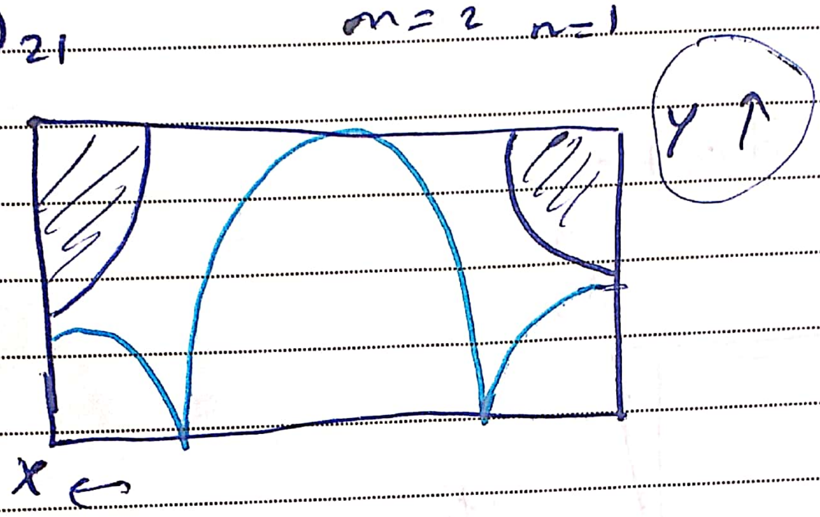
No.

TM_{21} $m=2$ $n=1$



TE mode →

إذا كان عنده أطراف مسن

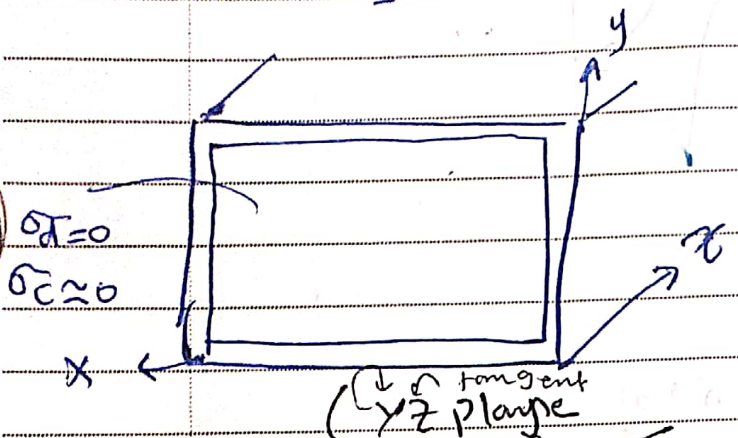
TM mode

رسمته لا أطراف

No.

* TE modes

$E_{zs} = 0$, $H_{zs} \neq 0$



$E_{zs} = 0$ at $x = 0$ & $E_y = 0$

$E_{zs} = 0$ at $x = a$ & $E_y = 0$

$E_{zs} = 0$ at $y = 0$ & $E_x = 0$

$E_{zs} = 0$ at $y = b$ & $E_x = 0$

$E_{zs} = 0$

$$H_{zs} = (B_1 \cos(k_x x) + B_2 \sin(k_x x)) * (B_3 \cos k_y y + B_4 \sin k_y y) e^{-j\beta z}$$

$$E_{xs} = \frac{-j\omega\mu}{h^2} \frac{dH_{zs}}{dy}$$

$$E_{ys} = \frac{-j\omega\mu h^2}{h^2} \frac{dH_{zs}}{dx} \qquad H_{ys} = \frac{-\gamma}{h^2} \frac{dH_{zs}}{dy}$$

$$H_{xs} = \frac{-\gamma}{h^2} \frac{dH_{zs}}{dx}$$

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at $X=0$ and $X=a$

$E_{ys}=0$

$\downarrow$

$$\frac{\partial H_{zs}}{dx}=0$$

at $y=0$ and $y=b$

$E_{xs}=0=0$

$$\frac{\partial H_{zs}}{dy}=0$$

at $\underline{X=0}$. مشتقة

$$B_1 K_x \cancel{\underbrace{\sin(K_x X)}_{=0}} - B_2 K_x \underbrace{\cos(K_x X)}_{=0} = 0$$

$\boxed{B_2=0}$

at $y=0$ $\boxed{B_4=0}$

$H_{zs}=H_0 \cos K_x X . \cos K_y y \, e^{-j\beta z}$

at $X=a$ $\frac{\partial H_{zs}}{dx}=0$

$B_1 . K_x \sin K_x(a)=0$

لأنها

cosine

صيرت تعوض صفر

$\sin(K_x(a))=0=\frac{m\pi}{a}$ $m=0,1,2$

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No.

at $y=b$

$$\frac{dH_{zs}}{dy}=0$$

$$\sin k_y \beta = 0$$

$$k_y = \frac{n\pi}{b} \qquad n=0,1,2 ----$$

(6)

$$H_{zs} = H_o \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$E_{zs}=0$

$$E_{xs} = \frac{+j\omega\mu}{h^2} \frac{n\pi}{b} H_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$E_{ys}=0$

$$\frac{+j\omega\mu}{h^2} H_o \cdot \frac{m\pi}{a} \sin(\quad) \cos(\quad) e^{-j\beta z}$$

$$H_{xs} = \frac{+\gamma}{h^2} H_o \cdot \frac{m\pi}{a} \sin(\quad) \cos(\quad) e^{-j\beta z}$$

$$H_{ys} = \frac{+\gamma}{h^2} H_o \frac{n\pi}{b} \cos(\quad) \sin(\quad) e^{-j\beta z}$$

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No.

$m=0$, $n=0$ Cannot be Zero at the Same time

The lowest mode in TE is

TE_{01} or $\boxed{TE_{10}}$ → domenant mode

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a^2}\right)+\left(\frac{n}{b^2}\right)}$$

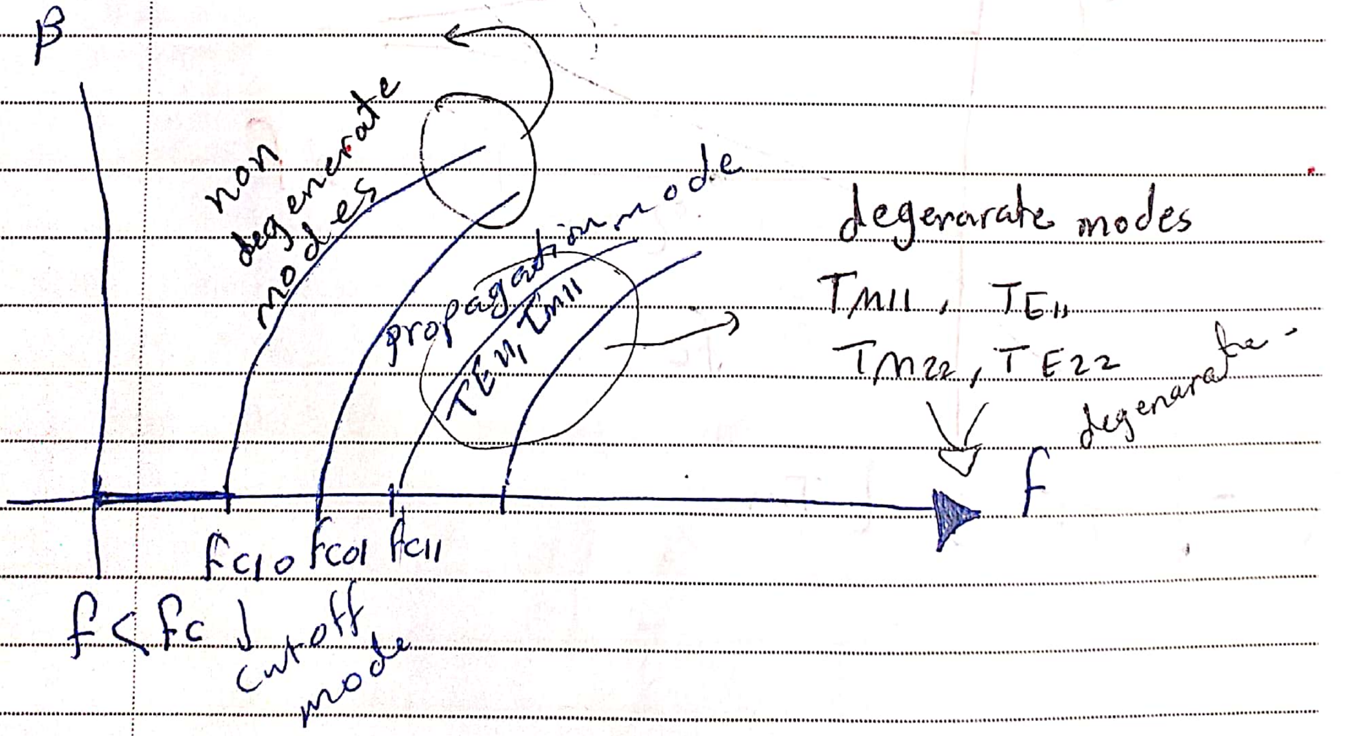
$$\boxed{a>b}$$

$f_{c10} = \frac{u}{2a}$ $f_{c01} = \frac{u}{2b}$

$f_{c10} < f_{c01}$

$TE_{10} \rightarrow TE_{01} \rightarrow TM_{11}$

ترتيب الـ Modes في



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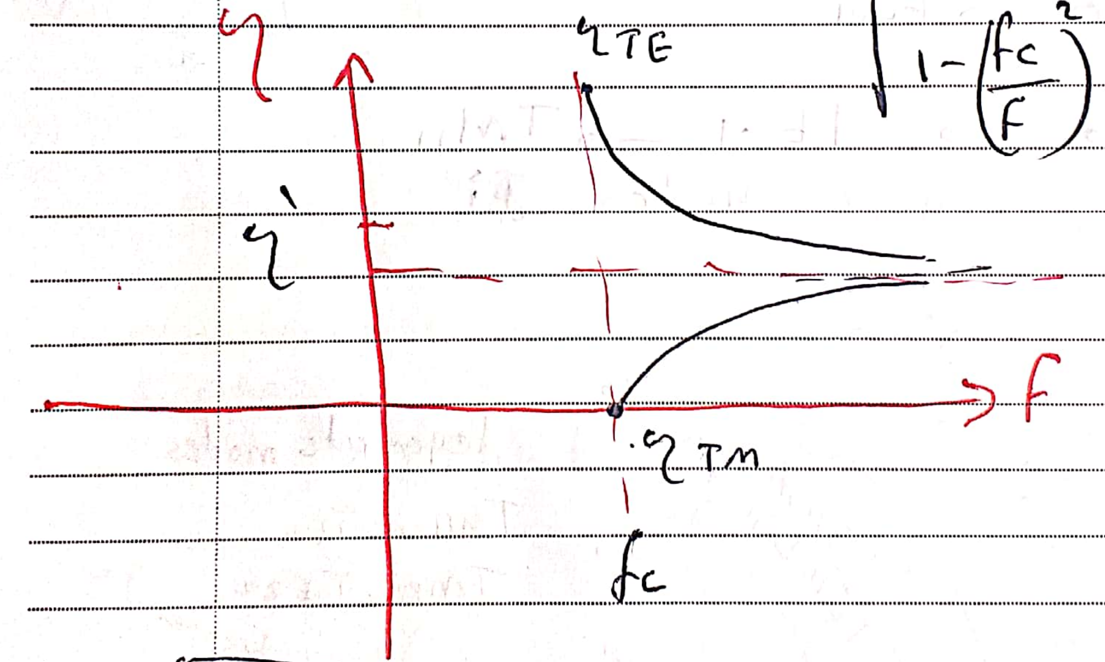
No.

$f_c =$, $\lambda_c =$; u_p , β , α) same

$$\eta_{TE} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{j\omega\mu}{\gamma}$$

$$\eta_{T\cancel{M}E} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{j\,\omega\mu}{j\beta}$$

$$\eta_{TE} = \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{F}\right)^2}}$$



$$\eta' = \sqrt{\eta_{TE} * \eta_{Tm}}$$

No.

for the domenemt mode

TE_{10}

$$H_{zs} = H_o \cos\left(\frac{m\pi x}{a}\right) e^{-j\beta z} \quad A/m$$

$$E_{ys} = \frac{j\omega\mu}{h^2} H_o \left(\frac{\pi}{a}\right) \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} \quad V/m$$

$$H_{xs} = \frac{\gamma}{h^2} H_o \left(\frac{\pi}{a}\right) \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

$E_{xs} = 0 \quad H_{ys} = 0 \quad E_{zs} = 0$

in time domain

$$H_z(z, t) = H_o \cos\left(\omega t - \beta z + \frac{\pi x}{a}\right) \quad A/m$$

Example

In a R.W.G for which $a = 1.5\,cm$
$b = 0.8\,cm$, $\sigma = 0$, $\mu = \mu_o$, $\epsilon = 4\epsilon_o$
has $H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\sin\left(\pi \times 10^{11} t - \beta z\right)$ A/m
Determine
a) the mode of operation
b) f_c
c) β
d) γ
e) η
f) other field components.

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No.

a) TM13 or TE13

assume TM13

b) $f_{c13} = \frac{u'}{2}\sqrt{\left(\frac{1}{a^2}\right)^2 + \left(\frac{3^2}{b^2}\right)}$

$u' = \frac{c}{2}$

$$\boxed{f_{c13} = 28.57\,GHz}$$

$\omega = 2\pi f \qquad f = \frac{\pi * 10^{11}}{2\pi} \qquad f = \boxed{50\,GHz}$

c) $\beta = \beta' \sqrt{\left(1 - \left(\frac{f_c}{f}\right)^2\right.}$

$\beta' = \frac{\omega \sqrt{\epsilon_r}}{c \cancel{\mu_r}} = 2\omega c = \boxed{1718.91\ rad/m}$

d) $\gamma = j\beta \qquad \alpha = 0$

$$\boxed{\gamma = j1718.91\ /m}$$

e) $\eta_{TM} = \eta' \sqrt{1 - \left(\frac{f_c}{f_n}\right)^2} = \frac{\eta_0}{2}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

$$\boxed{\eta_{TM} = 154.7\,\Omega}$$

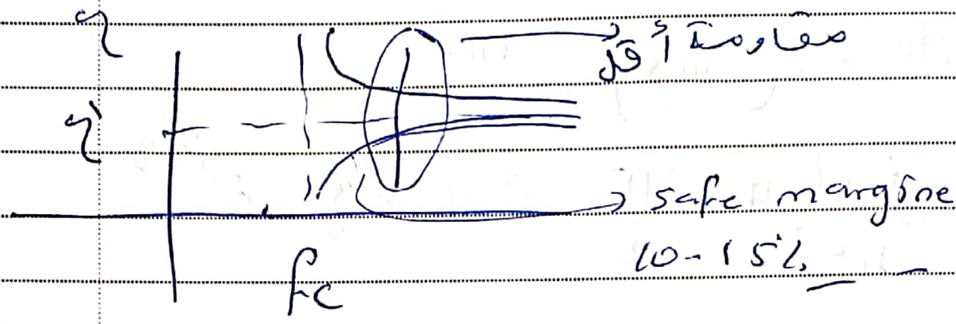
لازم أقل من >

η

FIVE APPLE

No.

$$\boxed{\eta_{TE} = 229.69\,\Omega \qquad \text{لازم أكبر من } \eta'}$$



$$Z = \frac{j\omega\epsilon}{h^2} E_0 \left(\frac{n\pi}{b}\right)$$

$$h^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2$$

E_0 ✓

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No.

Wave propagation in the Wave guide.

For TE_{10} mode

$$H_{zs} = H_0 \cos\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

$$E_{ys} = \frac{+j\omega\mu}{h^2} H_0 \frac{\pi}{a} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

$$H_{xs} =$$

$E_{zs} = 0 \quad E_{xs} = 0 \quad H_{ys} = 0$

$$h^2 = \left(\frac{\pi}{a}\right)^2$$

$$E_{ys} = \frac{j\omega\mu}{(\pi/a)^2} H_0 (\pi/a)$$

$$E_{ys} = \frac{j\omega\mu a}{\pi} H_0 \sin(\pi x/a) e^{-j\beta z}$$

Using Euler's Identity.

$$\sin\theta = (e^{j\theta} - e^{-j\theta}) / 2j$$

$$E_{ys} = \frac{\cancel{j}\omega\mu a H_0}{\pi} \left(\frac{e^{\frac{j\pi x}{a}} - e^{-j\frac{\pi x}{a}}}{2\cancel{j}} \right) e^{-j\beta z}$$

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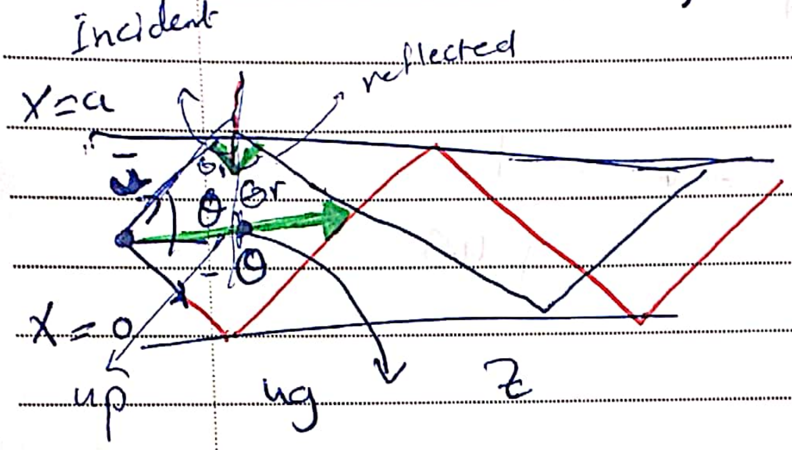
$$= \frac{\omega \mu a H_0}{2\pi}\left(e^{j\beta\left(\frac{\pi x}{\beta a} - z\right)} - e^{j\beta\left(\frac{-\pi x}{\beta a} - z\right)} \right)$$

Wave travels in +ve Z direction

with Phase shift $\theta = \tan^{-1}\left(\frac{\pi}{\beta a}\right)$

with phase shift $-\theta$



$u' = \frac{1}{\sqrt{\mu\epsilon}}$ TEM velocity

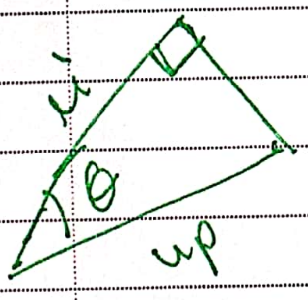
$u_p = \frac{u'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$ phase velocity

3 fields for TE_{10} →

كل واحد منهم بيعمل موجتين Zig-Zag.

⇓

6 waves in the wave guide.



$\cos\theta = \frac{u'}{u_p}$

$u_p = \frac{u'}{\cos\theta}$

$$\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \cos\theta$$

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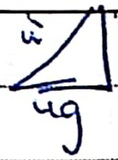


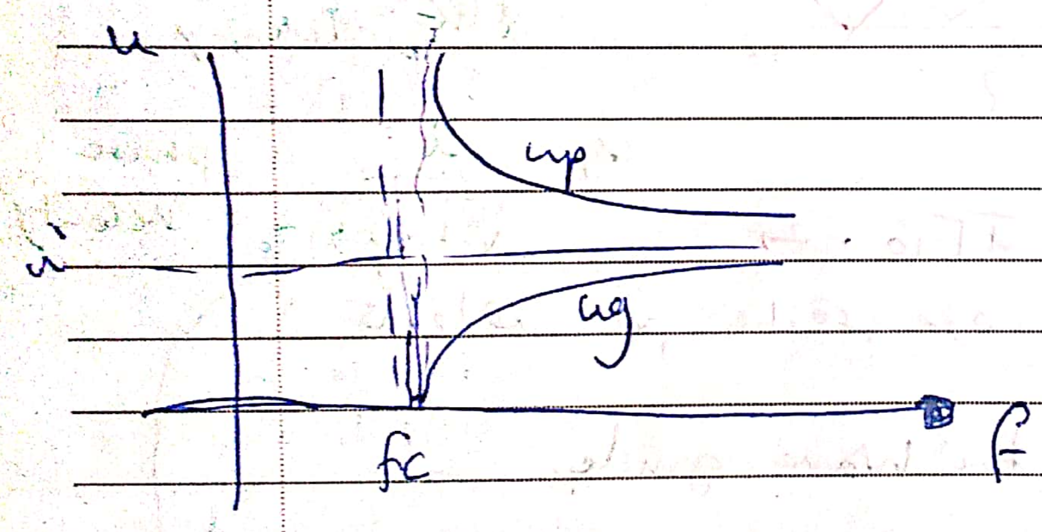
$\cos\theta = u_g / \grave{u}$

$$u_g = \grave{u}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$u_g < \grave{u}$

$u_g \rightarrow$ group velocity.

$$\sqrt{u_g u_p} = \grave{u} \rightarrow = \frac{1}{\sqrt{\mu \epsilon}}$$



No.

* power transmission and attenuation :-

$\bar{P} = \bar{E} \times \bar{H} \quad w/m^2$ (power مع اتجاه الموجة)

$\bar{P}_{avg} = \frac{1}{2} Re\{\bar{E}_s \times \bar{H}_s^*\} = \ldots \; w/m^2$

$= \frac{1}{T} \int \bar{E} \times \bar{H} \, dt$

$P_{ave} = \int \bar{P}_{ave} \cdot ds \; (w)$

For $TE_{10} \to H_z$
$\to E_y$
$\to H_x$

$H_s = (H_x, 0, H_z) \qquad E_s = (0, E_y, 0)$

$H_{xs} = \frac{E_y}{\eta_{TE}}$

ال power اللي بتحرك هي ال power اللي باتجاه ال propagation

in general

$\bar{E} = E_{xs}, E_{ys}, E_{zs}$

$\bar{H} = \bar{H}_{xs}, H_{ys}, H_{zs}$ either E_{zs} or $H_{zs} = 0$

x, y, standing wave power ما بتحرك في

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$$P_{ave} = \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\eta}$$

Effesk

لا تنتج ال power اللي باتجاه ال $\hat{a}_z$

$$P_{ave} = \int_0^b \int_0^a \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\eta} \cdot dx\,dy \quad (W)$$

---

attenuation

$\sigma_d \neq 0 \qquad \sigma_c \neq \infty, \; \delta \neq 0, \; \alpha \neq 0$

$$\alpha = \alpha_c + \alpha_d$$

$$\epsilon_c = \epsilon' - j\epsilon'' \qquad \epsilon'' = \frac{\sigma}{\omega}$$

for α_d:

$$\gamma = \alpha_d + j\beta_d = \sqrt{h^2 - k^2}$$

$$= \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon_0}$$

$$\alpha_d = \frac{\omega \mu \sigma_d}{2\beta_d}$$

$$\beta_d = \beta = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$\alpha_c \rightarrow$ given (0.55)

No.

Bandwidth of Operation

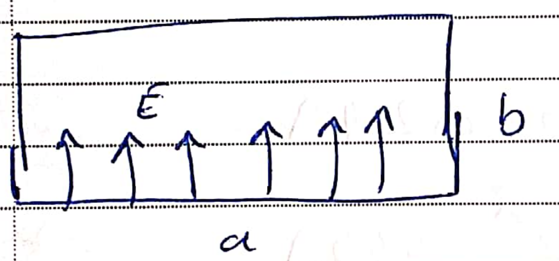
TE_{10}, TE_{01}, TE_{11}, TM_{11}

f_{c10} $\quad$ f_{c01} $\quad$ f_{c11}

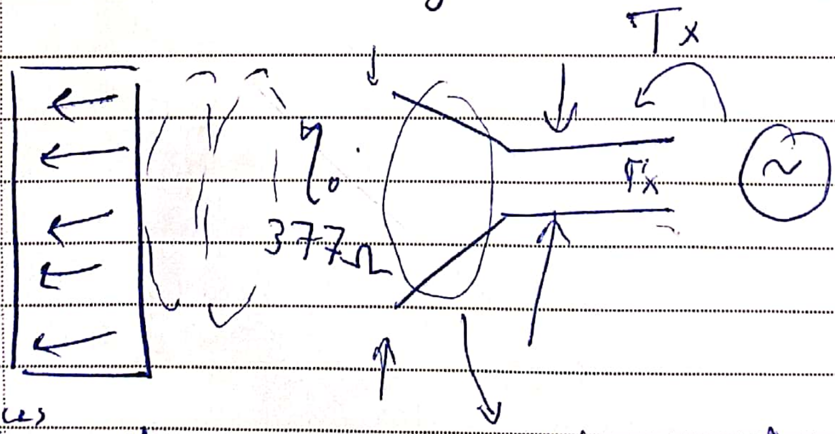
safe margine:-

$$BW = (f_{c11} - 10\% f_{c01}) - (f_{c01} + 25\% f_{c01})$$

$$BW = f_H - f_L$$



electric field is always parallel to the narrower Edge.



Impedances [illegible] Reflection [illegible] matching.

No.

PLF (polarization loss factor) = $\cos\theta$

EX// a double stub match this load using double shunt o.c stubs

$d_2 = \frac{\lambda}{8}$, ($d_1 = 0$) default

$f = 2\,GHz$

$Z_L = 60 - j80\,\Omega$

sol

$l_1 = 0.146\lambda$ ✓

$l_1' = 0.489$ ✓

$l_2 = 0.204\lambda$ ✓

$l_2' = 0.35\lambda$

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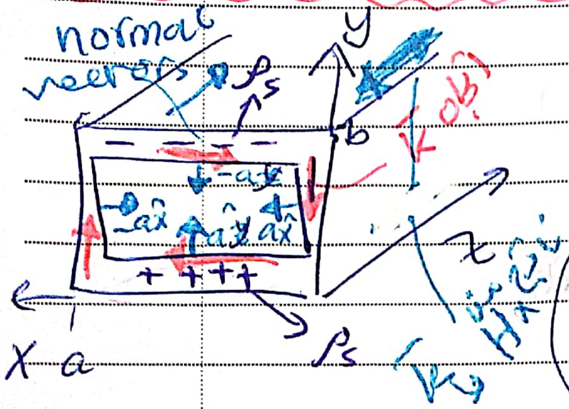
* Wave guide current :- ملاحظه



Recall from EM1

$$\boxed{\begin{aligned} \rho_s &= \bar{D} \cdot \hat{a}_n \\ \bar{K} &= \vec{H} \times \hat{a}_n \end{aligned}}$$

$\hat{a}_n$ inside

$\hat{a}_n$ Outside

$\bar{K} = (\bar{H}_1 - \bar{H}_2) \hat{a}_{n12}$

$\bar{K} = \hat{a}_n \times \vec{H}$ — $\hat{a}_n$ inside

for TE10 mode :-

$H_z = H_0 \cos\left(\frac{\pi x}{a}\right) e^{-jBz}$ A/m

$E_y =$ - - - - - - - - — ρ_s → $\bar{K}$

$H_x =$ - - - - - - -

$\rho_s = \bar{D} \cdot \hat{a}_n = \epsilon \bar{E} \cdot a_n$

at $y = 0$ $\hat{a}_n = \hat{a}_y$ (iside normal)

$\rho_s = \epsilon |E_y|$

at $y = b$ $\hat{a}_n = -\hat{a}_y$ (≠ inside).

mode ال ركّز *

fields ال ركّز *

ρ_s بنجيب ← E من ال

$\bar{K}$ منها بنجيب H من ال

for $H\hat{z}$ in (z direction).

$\bar{K} = \hat{a_n} \times \bar{H}$

$\hat{a_x} \times \hat{a_z} \quad (x=0)$

$= -\hat{a_y}$

$\bar{K} = -\hat{a_x} \times \hat{a_z} \quad (x=a)$

$= \hat{a_y}$

$\bar{K} = \hat{a_y} \times \hat{a_z} \quad (y=0)$

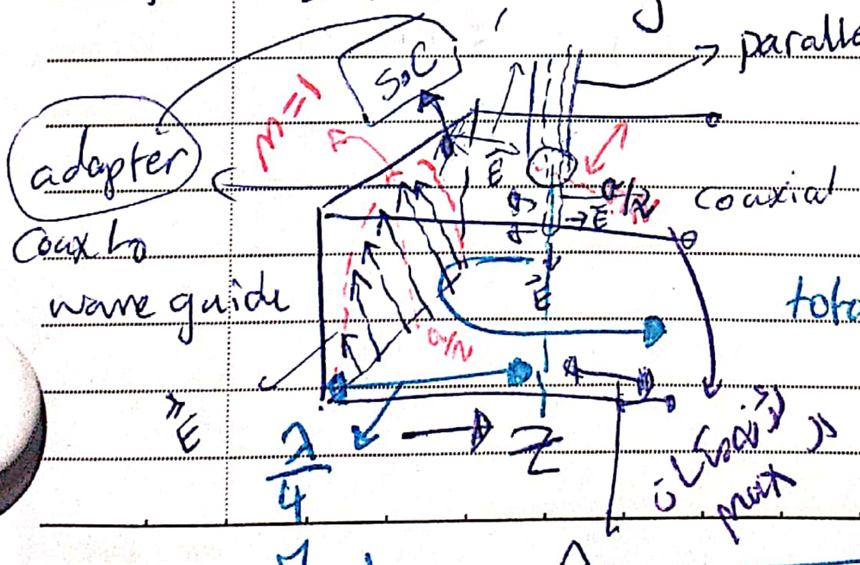
$= \hat{a_x}$

$\bar{K} = -\hat{a_y} \times \hat{a_z} \quad (y=b)$

$= -\hat{a_x}$

Wave guide Excitation

How to insert a signal in RWG from Source Using a Coaxial cable



→ parallel to $\vec{E}$

for TE_{10} mode

coaxial cable

$m=1$

$n=0$

total Phase shift $360°$

↓

Phase shift

$90°$ $90°$ $180°$ s.c

$\frac{\lambda}{4}$ $\Gamma = -1$

$\frac{\lambda}{8}$

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No.

TE20

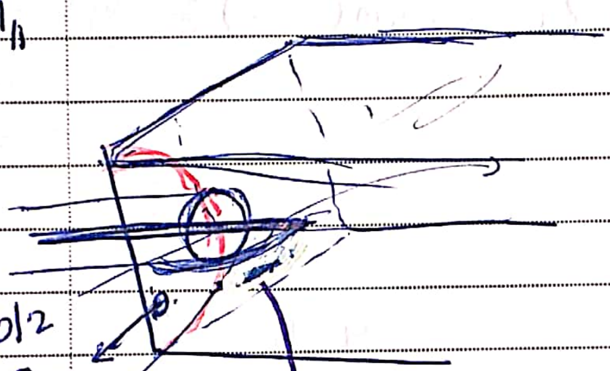
$m = 2$

3a/4 } max

TE10
TE01
TE11
TM11

كلهم
لا يكونوا
موجودين

TM11



B/2

SOC

inner conductor

Magnetic dipole

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Wave guide Resonators :-

TM mode $(H_z = 0, E_z \neq 0)$.

$E_{zs} = X(x)\, Y(y)\, (Z(z))$

$X(x) = C_1 \cos(k_x X) + C_2 \sin(k_x X)$

$C_3 \cos(k_y Y) + C_4 \sin(k_y Y)$

$Z(z) = C_5 \cos k_z z + C_6 \sin(k_z) z$

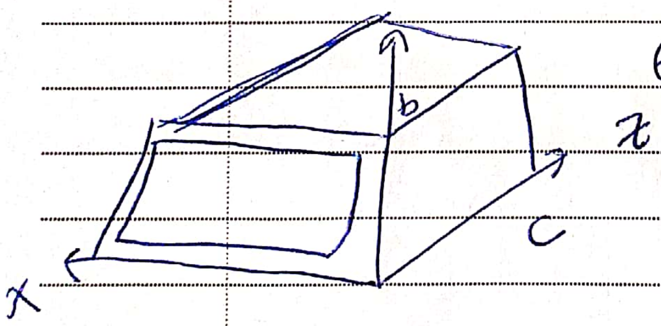
standing wave

لأنه مسكر عند

$z = c$

$E_{zs} = 0$ at

$x = 0$	$y = b$
$x = a$	$z = 0$
$y = 0$	$z = c$



6 .S,Cs

$K_x = \frac{m\pi}{a}$ $m = 1, 2, \ldots$

$k_y = \frac{m\pi}{b}$ $n = 1, 2, \ldots$

$K_z = \frac{p\pi}{c}$ $p = 0, 1, 2, \ldots$

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No.

$$E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \cdot \sin\left(\frac{n\pi y}{b}\right)$$

$$\cdot \cos\left(\frac{p\pi z}{c}\right)$$

$$\boxed{E_0 = C_2 * C_4 * C_5}$$

$TM_{110} \longrightarrow$ Resonators (lowest mode)

for TE mode

$E_z = 0 \qquad H_z \neq 0$

at $x = 0 \rightarrow E_y = 0 \rightarrow \frac{\partial H_{zs}}{\partial x} = 0$

$x = a \rightarrow E_y = 0$

at $y = 0 \rightarrow E_x = 0 \rightarrow \frac{\partial H_{zs}}{\partial y} = 0$

$y = b \rightarrow E_x = 0$

at $z = 0 \rightarrow E_x = 0, \qquad \frac{\partial H_{zs}}{\partial x} = 0$ &

at $z = c \rightarrow E_y = 0$

$\frac{\partial H_{zs}}{\partial y} = 0$

$k_x = \frac{m\pi}{a} \qquad k_y = \frac{n\pi}{b} \qquad k_z = \frac{p\pi}{z}$

$m = 0, 1, 2 \ldots \qquad n = 0, 1, 2 \qquad p = 1, 2, \ldots$

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$m = 0$
$n = 0$ ممنوع بنفس الوقت

No.

lowest mode
dominant mode

a>b
is TE_{101}

$$\boxed{H_0 = C_1 \cancel{*} C_3 \cancel{*} C_6}$$

$$H_{zs} = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{c}\right)$$

$\boxed{29/7}$

⊛ Wave length Resonators :-
for TM mode $E_{zs} \neq 0$ $H_{zs} = 0$

$$E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{c}\right)$$

$$\beta^2 = k_y^2 + k_x^2 + k_z^2 = \omega^2 \mu \epsilon$$

$$\beta = \omega\sqrt{\mu\epsilon} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}$$

$$B = \frac{\omega r}{u'} = \frac{2\pi f r}{u'} \qquad u' = \frac{1}{\sqrt{\mu\epsilon}}$$

$$f_r = \frac{u'\beta}{2\pi}$$

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No.

$$f_r = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$ tripple incidies

$f_{r110} = f_{c11}$

$$\lambda_r = \frac{2\pi}{\beta} = \frac{u'}{f_r}$$

resonant wave length

$$\lambda_r = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}$$

* for TE mode $E_{zs} = 0$ $H_{zs} \neq 0$

$$H_{zs} = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{c}\right)$$

dominent mode is :-

TE101 if $a > b$

$\beta, f_r, \lambda_r \rightarrow$ Same for Tm mode

as apractical waveguide

$\sigma_c \neq \infty$ $\sigma_d \neq 0$

Quality factor

$$Q = 2\pi * \frac{\text{Energy stored in Time domain } [J]}{\text{power Loss per cycle of oscillation}}$$

$$Q = 2\pi \frac{W_E + W_m}{P_L / f} \qquad Q = \omega \cdot \frac{W}{P_L}$$

$$Q = \frac{2\pi W}{P_L \cdot T}$$

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No.

$$W_E = \frac{1}{2}\int_V \epsilon E^2 \, dV$$

$$W_m = \frac{1}{2}\int_V \mu H^2 \, dV$$

Ex for dominant mode

TE_{101}

$$Q_{TE101} = \frac{(a^2+c^2)\,abc}{\delta\,[2b(a^3+c^3)+ac(a^2+c^2)]}$$

$$\delta = \frac{1}{\sqrt{\pi f_{r_{101}} \mu_0 \sigma_c}}$$

Ex An air filled Resonant cavity with dimentions

$a = 5cm$, $b = 4cm$ $c = 10cm$

is made of Copper

$(\sigma_c = 5.8 * 10^7 \; S/m)$

Find ⓐ the five lowest order modes

ⓑ Q

ⓒ BW for the dominant mode if the safe margin is 10% from both sides.

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No.

Solutions:-

a) $fr = \frac{v'}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$

$c > a > b$

TE	Fr_{101}	$= 3.333$ GHz	not degerate
TE	Fr_{011}	4.04 GHz	
TM	Fr_{110}	4.85 GHz	
TE	Fr_{102}	4.243 GHz	
	Fr_{111}	5.031 GHz	degenerate TM_{111}, TE_{111}

b) Q given

$\delta = 1.14176 \mu m$

$Q\ TE_{101} = 14358$

c) $BW = f_H - f_L$

$= (F_{r011} - 10\%\ Fr_{101}) - (Fr_{101} + 10\%\ Fr_{101})$

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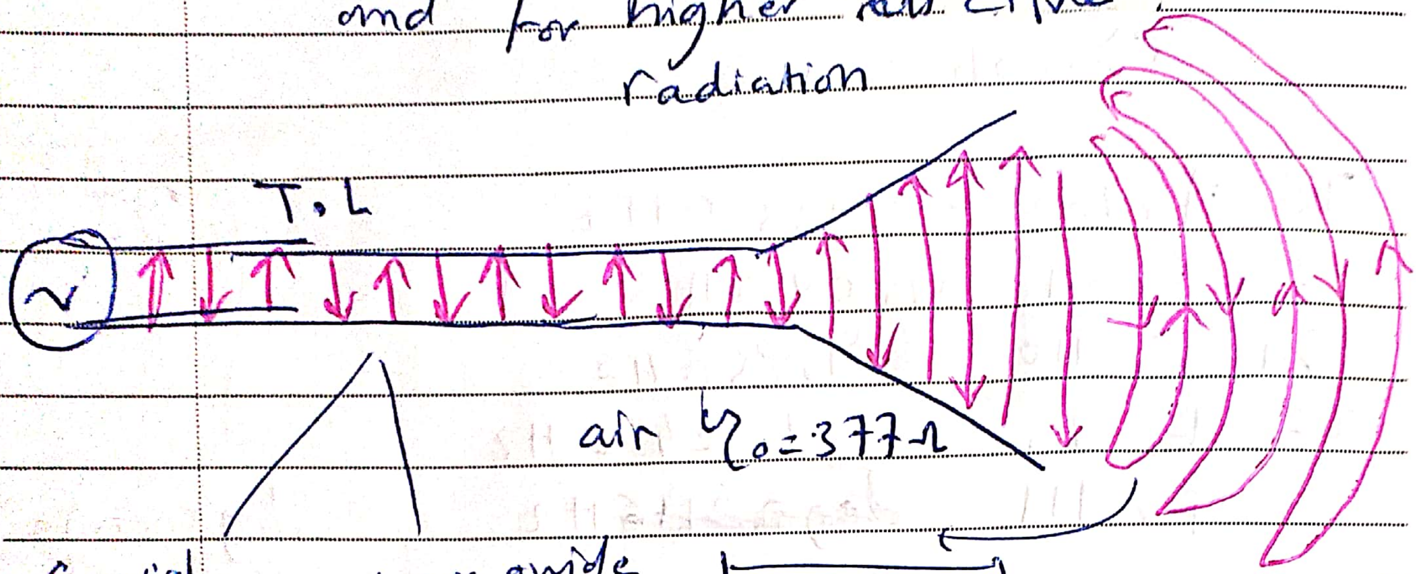
Ch. 13

Antennas

↳ used for matching impedences and for higher affective radiation



Coaxial
V, I

wave guide
E, H

antenna

wire antenna

Loop antenna

$$\nabla^2 V - \mu\epsilon \frac{d^2 V}{dt^2} = \frac{-\rho_v}{\epsilon}$$

$$\nabla^2 A - \mu\epsilon \frac{d^2 A}{dt^2} = -\mu \bar{J}$$

$$\bar{V} = \int_V \frac{[\rho_v]\, dV}{4\pi\epsilon_o r}$$

$$\bar{A} = \int_V \frac{[\bar{J}]\, \mu\, dv}{4\pi r}$$

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No.

/ lagging

[J] : retarted hysteresis current density

$\bar{J}(x, y, z, t')$ $\quad t' = t - \frac{r}{u}$

retarted used in waves

$\bar{A}$

$\downarrow$

$\bar{B} = \nabla \times \bar{A}$

$\hat{H} = \frac{\bar{B}}{\mu}$

$\vec{E} \rightarrow (\nabla \times \bar{H}) = \cancel{J}^{\,0} + \frac{dD}{dt}$

$\bar{E} = \frac{\bar{D}}{\epsilon}$

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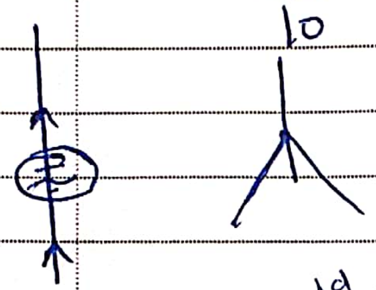
No.

* Wire antenna : 1D segment

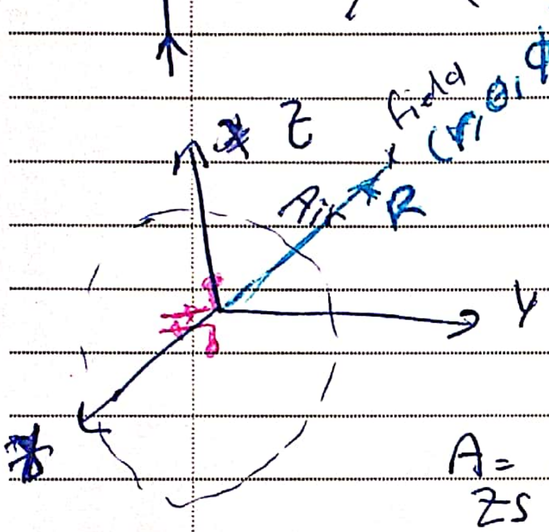
Hertzian Dipole

(Infinitesmal Dipole)

$dl <<< \frac{\lambda}{10}$



$\nabla^2 A - \mu\epsilon \frac{d^2 A}{dt^2} = -\mu J$



$A = \int_L \frac{\mu [I] dl}{4\pi R}$ for line current

$A_{s} = \frac{\mu [I_s] dl}{4\pi R}$

$I(x, y, z, t')$, $t' = t - \frac{R}{u}$

$\boxed{\text{Assume}}$ $I = I_o \cos(\omega t')$

$[I] = I_o \cos\left(\omega\left(t - \frac{r}{u}\right)\right)$

$\beta = \frac{\omega}{u}$

$[I] = I_o \cos(\omega t - \beta r)$

$$\boxed{I_s = I_o e^{-j\beta r}}$$

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$$A_{zs} = \frac{\mu_0 I_0 e^{-j\beta r}\, dl}{4\pi R}$$

to convert $A_{zs} \Rightarrow (A_{rs}, A_{\theta s}, A_{\phi s})$

$A_{rs} = A_{zs}\cos\theta$

$A_{\theta s} = -A_{zs}\sin\theta$

$A_{\phi s} = 0$

$\bar{B} = \nabla \times \bar{A}_s$ $\qquad \bar{B}_s = \mu H_s$

$\vec{H} = \frac{1}{\mu}\nabla \times \bar{A}_s$

$$H_s = \frac{1}{\mu_0}\left(\frac{1}{r^2\sin\theta}\right)\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \partial/\partial r & \partial/\partial\theta & \cancel{\partial/\partial\phi} \\ A_{rs} & rA_{\theta s} & \cancel{r\sin\theta A_{\phi s}} \end{vmatrix}$$

مضروبة
بال matrix

$H_{rs} = 0$

$H_{\theta s} = 0$

$$H_{\phi s} = \frac{I_0\, dl}{4\pi}\sin\theta\left(\frac{j\beta}{r} + \frac{1}{r^2}\right)e^{-j\beta r}$$

from Maxwell's eq

$\nabla \times H_s = j\omega\epsilon \bar{E}_s$, $(J=0)$

$\bar{E}_s = \frac{1}{j\omega\epsilon} \nabla \times H_s$

$\frac{1}{j\omega\epsilon} \frac{1}{r^2 \sin\theta} \begin{bmatrix} - & - & - \\ - & - & - \\ 0 & 0 & H_{s\phi} \end{bmatrix}$

$E_{rs} = \frac{(\eta I_o dl)}{2\pi} \cos\theta \left(\frac{1}{r^2} - \frac{j}{\beta r^3}\right) e^{-j\beta r}$

$E_{\phi s} = 0$, TEM

$E_{\theta s} = \frac{\eta I_o dl}{4\pi} \sin\theta \left(\frac{j\beta}{r} + \frac{1}{r^2} - \frac{j}{\beta r^3}\right) e^{-j\beta r}$

$\eta = \sqrt{\frac{\mu}{\epsilon}}$

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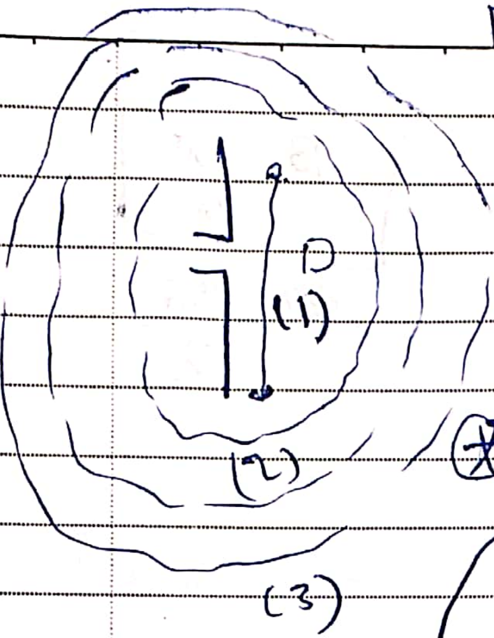
No.

Frensnal's Zones



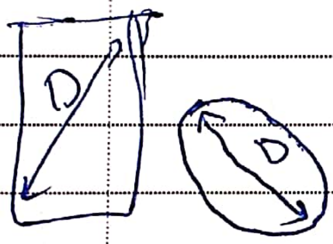
(1) reactive near field

(2) radiating near field

(3) radiating far field

$(\beta r \gg 1)$

$R \geq \frac{2D^2}{\lambda}$ → largest dimension of the antenna.



* منطقة رقم 2 بنستغني عن ال r^3
لأنه R كبيرة ف $\frac{1}{r^3} \simeq 0$

* منطقة رقم 3 بنستغني عن ال r^2 و r^3

$$\frac{1}{r^2} \simeq \frac{1}{r^3} \simeq 0$$

in far field Region

$H_{rs} = 0 \quad H_{\phi s} = 0 \quad E_{rs} = 0 \quad E_{\phi s} = 0$

$$H_{\phi s} = \frac{I_0 dl}{4\pi} \sin\theta \left(\frac{j\beta}{r}\right) e^{-j\beta r}$$

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No.

مشغول أصلياً بدلالة λ

وحيكون تحوي

$$H_{\phi s} = \frac{j I_0 \beta d\ell \sin\theta}{4\pi R} e^{-j\beta r}$$

$$\boxed{\beta = \frac{2\pi}{\lambda}}$$

$$\cancel{\beta m} = \frac{\cancel{2\pi}}{\lambda}$$

$$E_{\theta s} = \eta H_{\phi s}$$

Power

$$\bar{p} = \vec{E} \times \vec{H}^{*} \quad (\hat{a_r})$$

$$\bar{p} = \eta |H_{\phi s}|^2 \hat{a_r}$$

$$\bar{p}_{ave} = \frac{1}{2} Re\{E_s \times H_s^{*}\}$$

$$\bar{p}_{ave} = \frac{\eta}{2} |H_{\phi s}|^2$$

$$p_{rad} = p_{ave} = \int_s \bar{p}_{ave} \cdot ds$$

Constant

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{2} \frac{I_0^2 \beta^2 d\ell^2}{(16\pi^2 r^2)} \sin^2\theta \, \cancel{\hat{a_r}} \cdot \cancel{\hat{a_r}} \, r^2 \sin\theta \, d\theta \, d\phi$$

$$\int_0^{\pi} \sin^3\theta \, d\theta = \frac{4}{3}$$

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$= \frac{I_o^2 \pi (\eta)}{3} \left(\frac{dl}{\lambda}\right)^2$ (ω) $\beta = \frac{2\pi}{\lambda}$

$\frac{1}{2} I_o^2 R_{rad} = P_{rad}$

$\eta_o = 120\pi$

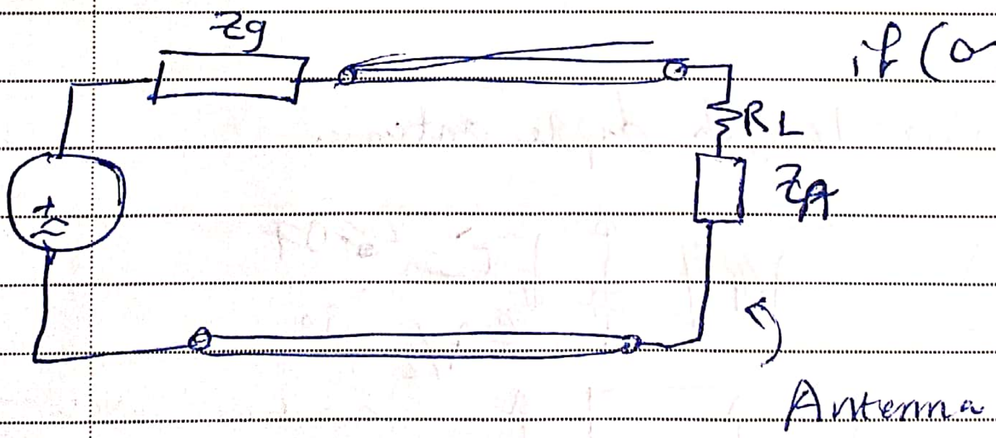
$R_{rad} = \frac{2 P_{rad}}{I_o^2} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$ in free space

$40\pi^2 \left(\frac{dl}{\lambda}\right)^2 I_o^2 (\omega) = P_{rad}$

in freespace

if $\frac{dl}{2} = \frac{\lambda}{20}$ $R_{rad} = 2\Omega$

if $(\sigma \approx \infty)$. $\frac{l}{\sigma A}$

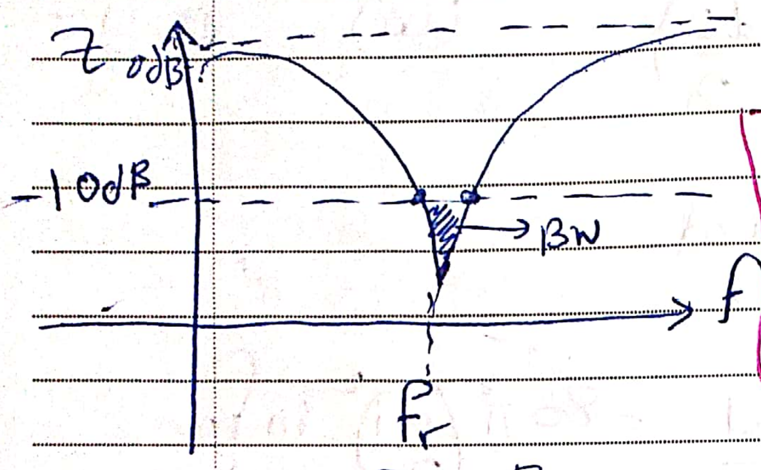


$Z_A = R_{rad} + jX_A$

P_{rad} [illegible]

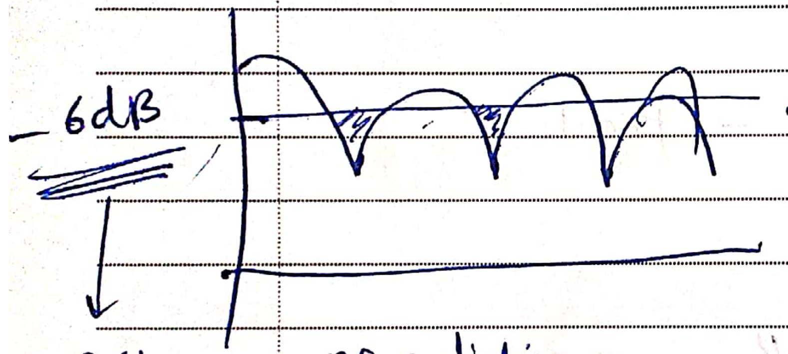
[illegible]

No.



$Z_A = R_{rad}$

90% From power
رشعشت
10% .. رضعت

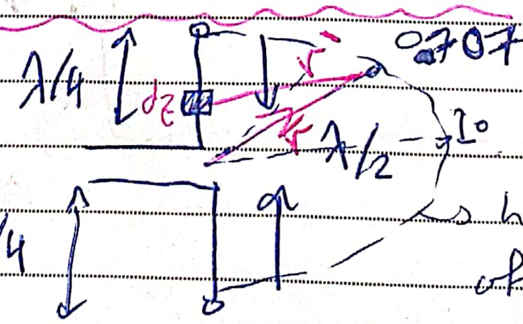


Multi Resonance antenna

80% power radiation.

Half wavelength dipole antenna

$l = \frac{\lambda}{2}$



half cycle of current

$dA = \frac{\mu_0 [I] dl}{4\pi R}$

No.

$I_s = I_o \cos(\beta z)\, e^{-j\beta r'}$

$[I] = I_o \cos(\beta z)\, e^{-j\beta r'}\, e^{j\omega t}$

Field

$x(r, \theta, \phi)$

E?

H?

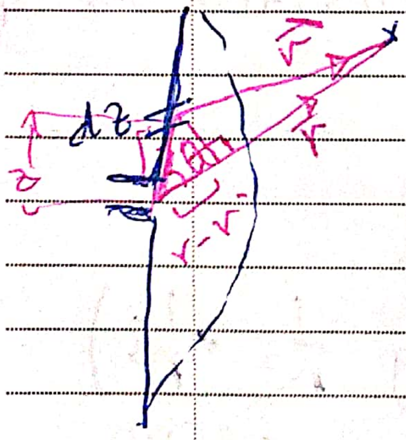
$A_{zs} = \int_L \frac{\mu_o [I]\, dL}{4\pi R} \longrightarrow d\vec{L} = dz\, \hat{a}_z$

$dA_{zs} = \frac{\mu_o\, dz}{4\pi r'}\, I_o \cos(\beta z)\, e^{-j\beta r'}$

$= \frac{\mu_o I_o\, dz}{4\pi r'} \cos(\beta z)\, e^{-j\beta r'}$

$\boxed{\text{Approximations :-}}$

$A_{zs} = \int_{-\lambda/4}^{\lambda/4} \frac{\mu_o I\, dz \cos(\beta z)\, e^{-j\beta r'}}{4\pi r'}$



r' in domenator $\approx r$

$\cos\theta = \frac{r - r'}{z}$

$r - r' = z\cos\theta$

$r' = r - z\cos\theta \longrightarrow$ in $e^{-j\beta r'}$

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No.

$$A_{zs} = \frac{\mu_o I}{4\pi R} \int_{-\lambda/4}^{\lambda/4} \cos(\beta z)\, e^{-j\beta(r - z\cos\theta)}\, dz$$

from an integral table

$$\int e^{az} \cos(bz)\,dz = e^{az} \frac{(a\cos(bz) + b\sin(bz))}{a^2 + b^2}$$

$a = j\beta\cos\theta$

$b = \beta$

$\Downarrow$

$$A_{zs} = \frac{\mu I_o e^{-j\beta r} \cos(\frac{\pi}{2}\cos\theta)}{2\pi r \beta \sin^2\theta}$$

$\boxed{\text{Convert to spherical } \cancel{\text{Coor}} \ (A_{rs}, A_{\theta s}, A_{\phi s})}$

$B = \nabla \times \bar{A}$

$H = \frac{B}{\mu}$ $\quad\quad$ $H = \frac{1}{\mu} \nabla \times A$

in far field (neglecting $\frac{1}{r^2}$ & $\frac{1}{r^3}$ terms)

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No.

$$H_{\phi s} = \frac{j I_o e^{-j\beta r} \cos\left(\frac{\pi}{2} \cos\theta\right)}{2\pi r \sin\theta}$$

$$E_{\theta s} = \eta \, H_{\phi s}$$

$$\bar{P}_{ave} = \frac{1}{2} Re \{ \bar{E}_s \times \bar{H}_s^* \}$$

$$= \frac{1}{2} \eta \, | H_{\phi s} |^2 \, \hat{a}_r$$

$$P_{rad} = P_{ave} = \int \bar{P}_{ave} \cdot d\bar{s}$$

$$= \frac{1}{2} \eta \int_0^{2\pi} \int_0^{\pi} \frac{I_o^2 \cos^2\left(\frac{\pi}{2} \cos\theta\right)}{4\pi^2 \cancel{r^2} \sin^2\theta} \cancel{r^2} \sin\theta \, d\theta \, d\phi$$

$$= \cancel{\frac{1}{2}} \int_0^{\pi} \frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \, d\theta$$

$$\boxed{P_{rad} = 36.5 \, I_o^2 \, (W)}$$ in free space

$$\eta_o = 120\pi$$

$$P_{rad} = 0.5 \, I_o^2 \, (R_{rad})$$

$$R_{rad} = \frac{2 P_{rad}}{I_o^2} = 73 \, \Omega$$

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No. ....

$Z_A = R_{rad} + jX_a$

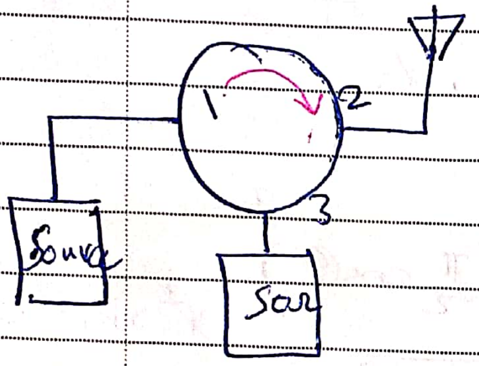
$\boxed{Z_A = 73 + j42.5\ \Omega \rightarrow \neq\ l = 0.5\lambda}$

if $l = 0.485\lambda \rightarrow Z_A = 73\ \Omega$

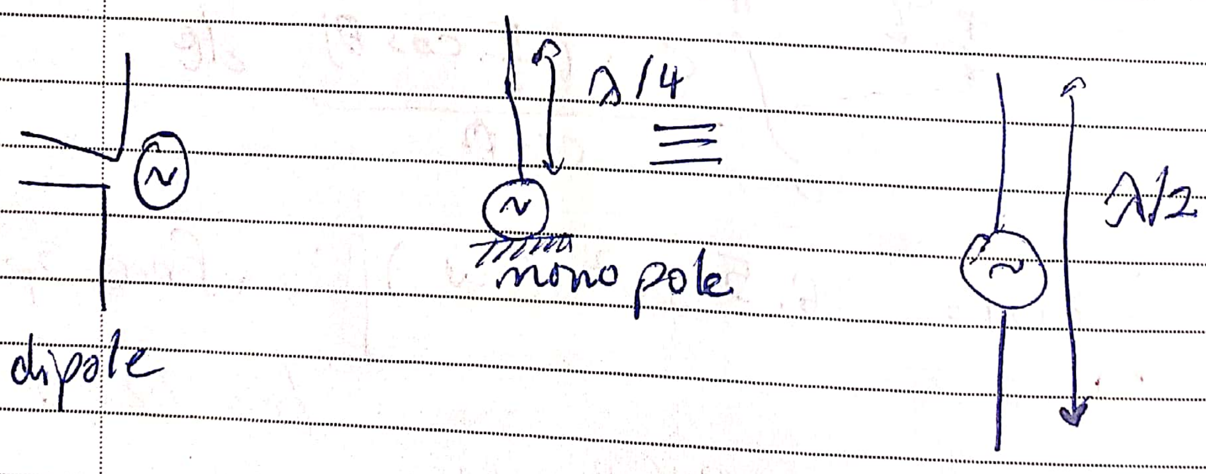
$X_A = 0$

Real كله 0

الطول المراد part



Quartor Wave length monopole Antenna



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No.

$H_{\phi s}$ = same for $\lambda/2$ dipole $(z>0)$

$E_{\theta s} = \eta H_{\phi s}$

$P_{rad} = 18.28 I_0^2$ (W)

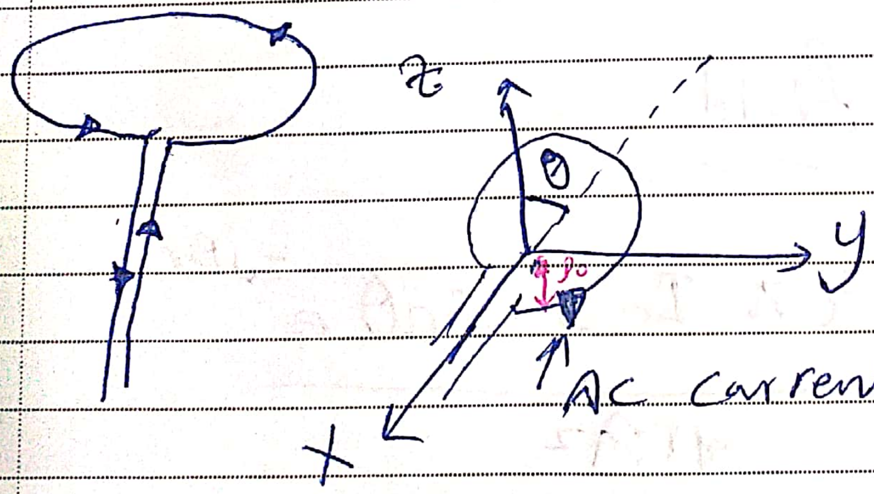
$R_{rad} = 36.5 \Omega$

$Z = 36.5 + j21.25 \Omega \Rightarrow l = 0.25\lambda$

$l = 0.2425\lambda \rightarrow Z_A = R_{rad} = 36.5 \Omega$

* Small loop Antenna

→ as a directional finder



$r \gg\gg \rho_0$

magnetic dipole

CH. 8 $A = \dfrac{\mu I \pi \rho_0^2 \sin\theta}{4\pi r^2} \hat{a}_\phi$

$\boxed{\text{in DC}}$

$\hat{a}_z \times \hat{a}_r = \sin\theta \hat{a}_\phi$

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No.

$$[I] = I_o \cos(\omega t')$$

$$= I_o \cos\left(t - \frac{r}{u}\right)$$

$$= I_o \cos(\omega t - \beta r)$$

$$I_s = I_o e^{-j\beta r}$$

$$\bar{A}_s = \int_0^{2\pi} \frac{\mu_o I_o e^{-j\beta r'}}{4\pi r^2} \, \overline{dl}$$

$\overline{dl} \rightarrow \rho_0 \, d\phi \, \hat{a}_\phi$

$$\bar{A}_{s\phi} = \frac{\mu_o I S \sin\theta}{4\pi r^2} \cdot (1 + j\beta r) e^{-j\beta r}$$

AC current

Propagation [illegible]

$$S = \pi \rho_o^2$$

in far field

$$E_{\phi s} = \frac{\eta \pi I_o S \sin\theta \, e^{-j\beta r}}{r\lambda^2}$$

$$H_{\theta s} = -\frac{E_{\phi s}}{\eta}$$

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No.

$$\bar{P}_{ave} = \frac{1}{2\eta} |E_{\phi s}|^2 \hat{a_r}$$

$$P_{rad} = \int_s \bar{P}_{ave} \cdot d\bar{s}$$

$d\bar{s} \rightarrow r^2 \sin\theta \, d\theta \, \partial\phi$

$$P_{rad}\Big|_{\text{free Space}} = \frac{320 \pi^4 S^2 I_o^2}{2 \lambda^4} \, (w)$$

$S = \pi \rho_o^2$ — for $N = 1$

$S = N \pi \rho_o^2$ — for N-terms

$$R_{rad} = \frac{2 P_{rad}}{I_o^2} = \frac{320 \pi^4 S^2}{\lambda^4}$$

FIVE APPLE

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4/8 الاثنين

No.

Ex A magnetic field strength of 5 μA/m is required at a point on $\theta = \pi/2$, $r = 2$ Km from an antenna in air neglecting ohmic losses, how much power must the Antenna transmits if it is

a) a Hertzian Dipole of length $\lambda/20$?

b) half wave length dipole ?

c) a quarter wave length Monopole

d) a 10-turn loop antenna of radius $\frac{\lambda}{20}$?

a) $|H_{\phi s}| = 5 \mu A/m$

$$\left|\frac{I_o \beta \, dl \, \sin\theta}{2\pi r}\right|$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\boxed{I_o = 0.5 A}$$

$$P_{rad} = 40\pi^2 \left(\frac{dl}{\lambda}\right)^2 I_o^2$$

$$= \boxed{158\,mW}$$

b) $|H_{\theta s}| = 5\mu A/m$

$$= \frac{I_o\left(\cos\left(\frac{\pi}{2}\cos\theta\right)\right)}{2\pi r \sin\theta}$$

$I_o = 20\pi\, mA$

$P_{rad} \approx 36.5\, I_o^2$

$$= \frac{1}{2} I_o^2 \underline{R_{rad}} (73\,\Omega) = \boxed{144\,mW}$$

c) $P_{rad} = \boxed{72\,mW}$

D) $|H_{\theta s}| = 5\mu A/m$

$$= \frac{\pi I_o S \sin\theta}{r\lambda^2}$$

$S = N\pi\rho_o^2$

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No.

$I_o = 40.53\,mA$

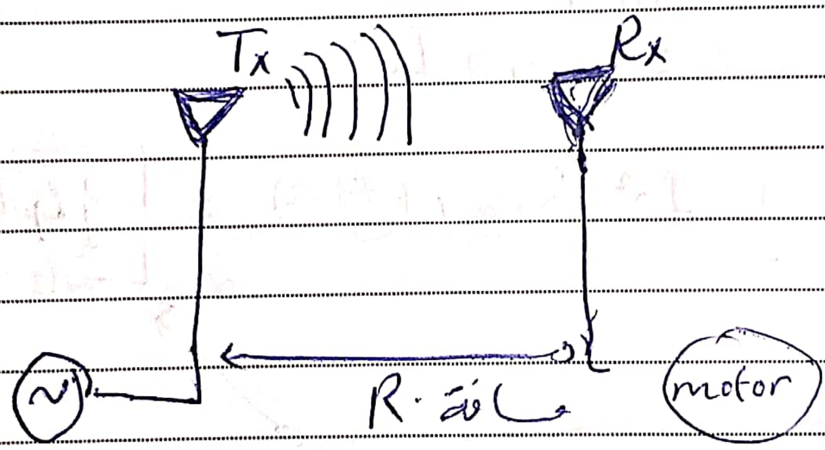
$R_{rad} = \dfrac{320\pi^4 S^2}{\lambda^4} = 192.3\,\Omega$

$P_{rad} = 0.5\, I_o^2\, R_{rad} = \boxed{158\,mW}$

Antenna Characterstics:

① Raddiation pattern

Exp.



ⓐ field pattern → E-plane
→ H plane

ⓑ power pattern

No.

for a Hertzian Dipole

بتتغير

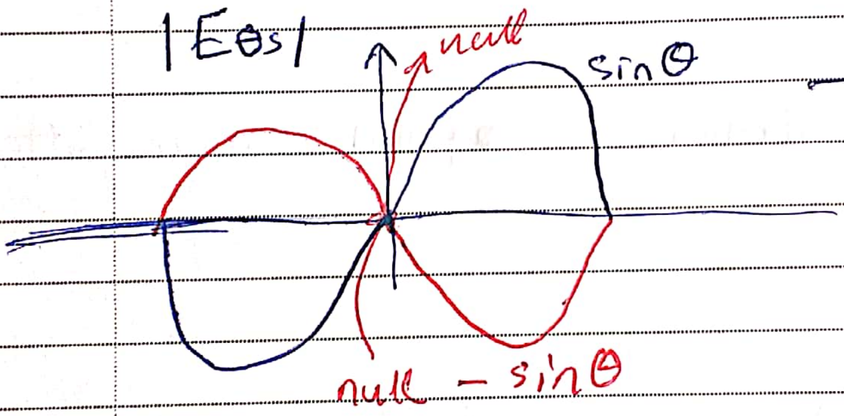
$E_{\theta s} = \eta H_{\phi s}$

$$E_{\theta s} = \frac{j\eta I_o \beta dl}{4\pi r} \sin\theta \, e^{-j\beta r}$$

constants

* Normalize the field

$$|E_{\theta s}| = |\sin\theta| \qquad \frac{E_{\theta s}}{\text{constants}}$$



→ E plane

→ function of θ

θ : $\phi = 0°$ → θ متغيرة

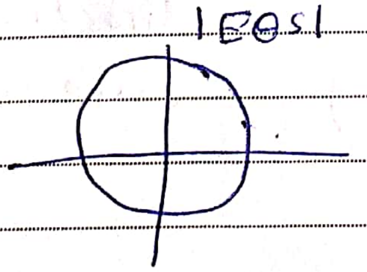
XZ plane

$\phi = 90°$

θ متغيرة → YZ plane

for H-plane

as a function of ϕ for a constant θ, usnally $\theta = \frac{\pi}{2}$ → Xy plane



H-plane

ϕ

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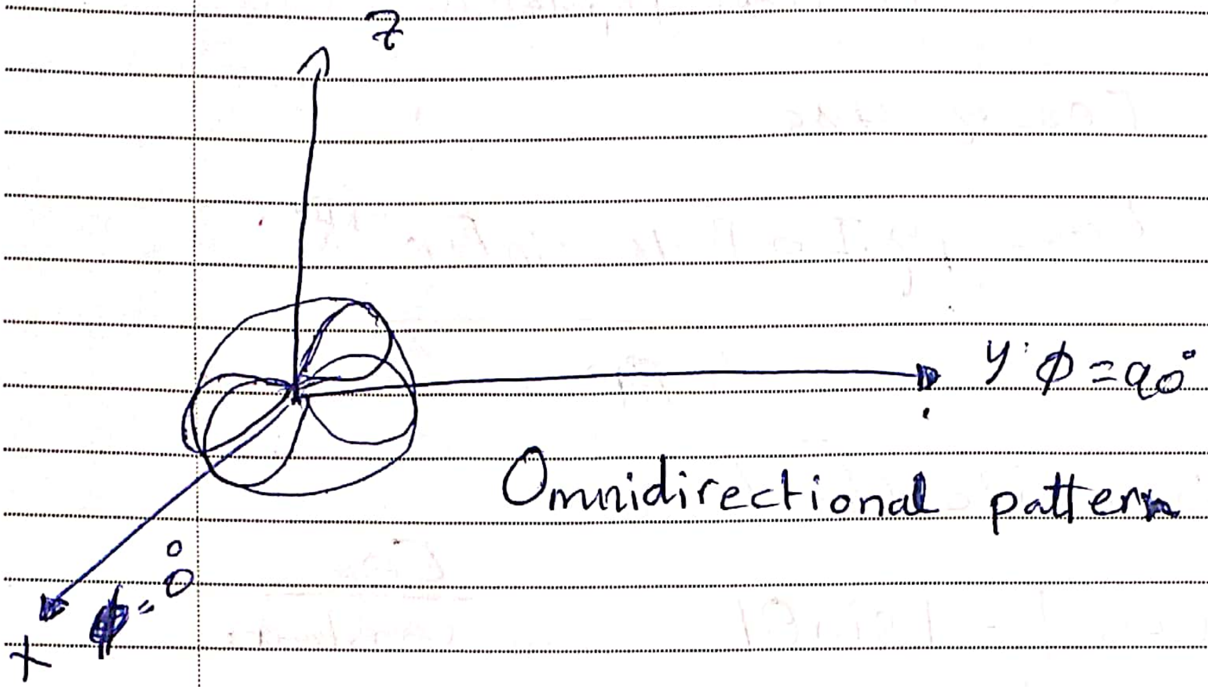
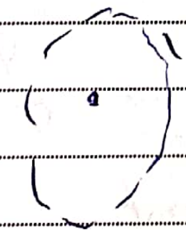
No.



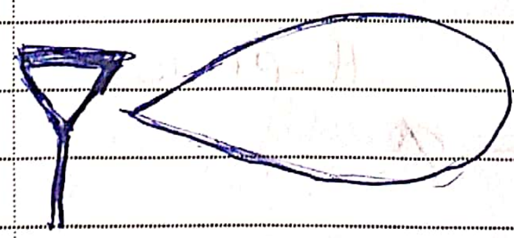
Omnidirectional pattern

Isotropic antenna → point source pattern

افتراضي



→ as arefrence antenna.



directional antenna

Max power at $\theta = \frac{\pi}{2}$

half power $|E_{\theta s}| = |\sin\theta|$

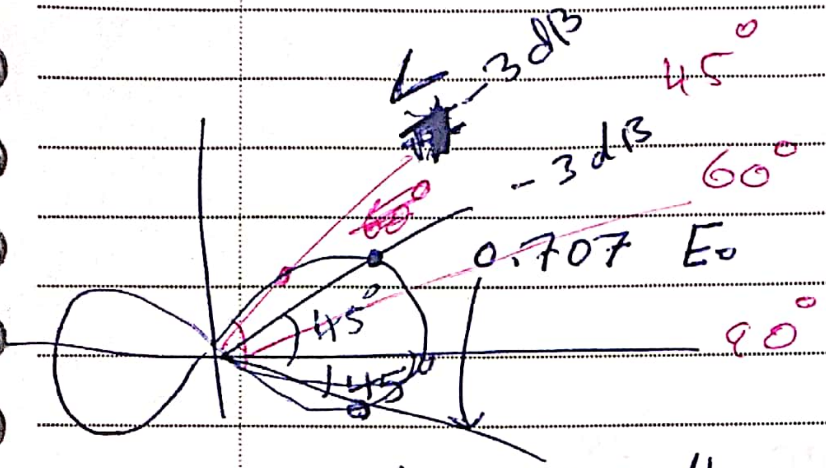
at $\theta = \underline{\underline{45^\circ}}$

$\sin\theta = \frac{1}{\sqrt{2}}$

$|E_{\theta s}|^2 = \frac{1}{2}$

↑

ال E تربيع . بال power



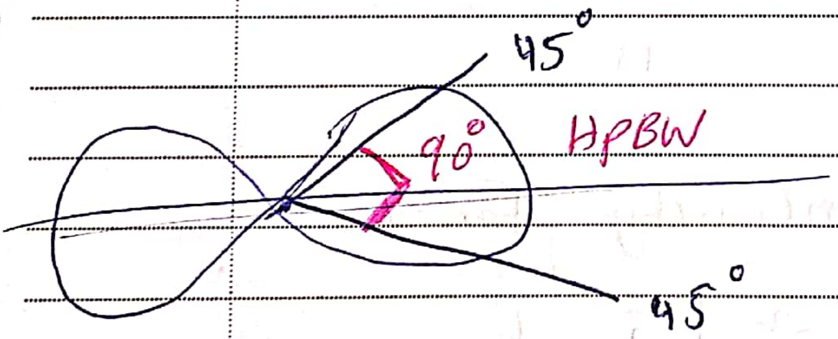
in power pattern $= 0.5\ (P)$

or 3dB in $\underline{\underline{3}}$-D

HPBW: half power Beam width.

$= 90^\circ = 2(45)$

60° power أقل من 45°

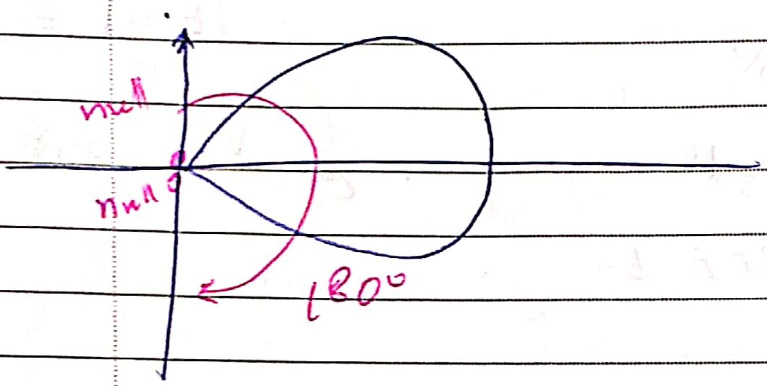


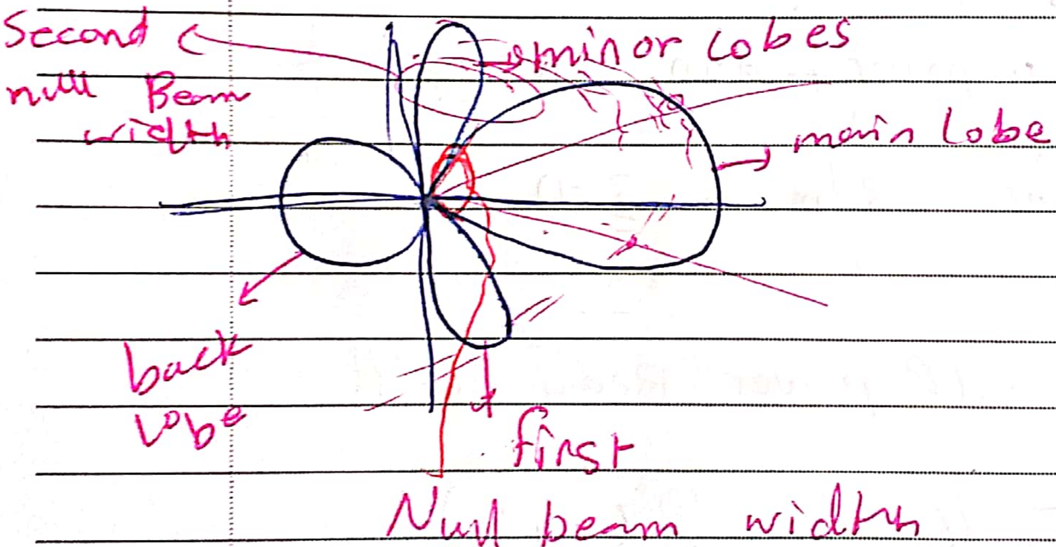
No.

First null Beam width FNBW

$\theta_n = 0^\circ$ $\quad$ $\theta_n = 180^\circ$ $\quad$ FNBW = 180°

+





main

2) Radiation Intensity $\boxed{\text{Far-field}}$

$$U(\theta,\phi) = r^2 \, |\bar{P}_{ave}|$$

↳ Independent on r

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No.

$E \propto \frac{1}{r} \rightarrow p_{ave} \propto \frac{1}{r^2}$

$$P_{rad} = \int_{S} \bar{p}_{ave} \cdot ds$$

$$= \int_0^{2\pi}\int_0^{\pi} |\bar{p}_{ave}| \, r^2 \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi$$

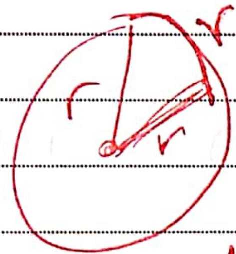
$$P_{rad} = \int_{\Omega} U(\theta, \phi) \, d\Omega$$

$$\Omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi$$

solid angle

1 rad

circle



arc طوله
بطول نصف القطر

Sphere

$r^2 =$ $\boxed{}$ r

$\equiv r^2$

Area $= 4\pi r^2$

زاوية بين أنصاف الأقطار ; Ω

لما نعطي مساحة r^2

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No.

$$P_{rad} = 4\pi \, U_{ave}$$

for isotropic antenna

تشع بالتساوي في جميع الاتجاهات

$$U_{ave} = \frac{P_{rad}}{4\pi}$$

وحدة الصلبه

Ω (sr) steradian.

$$U(ave) = W/sr$$

③ Directive Gain $G_d(\theta,\phi)$ or $D(\theta,\phi)$

$$D(\theta,\phi) = \frac{u(\theta,\phi)}{U_{ave}} \qquad U_{ave} = \frac{P_{rad}}{4\pi}$$

$$D = \frac{4\pi \, u(\theta,\phi)}{P_{rad}}$$

$$u(\theta,\phi) = r^2 |P_{ave}|$$

$$P_{ave} = \frac{D}{4\pi r} P_{rad}$$

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for isalropic antenna

$D = 1$

$D_o \equiv$ maximum Directivity

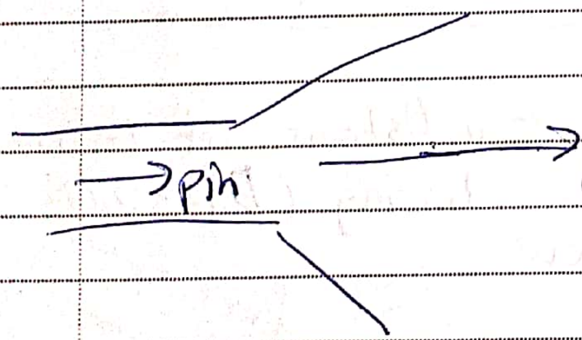
$$D_o = \frac{U_{max}}{U_{ave}} = \frac{4\pi U_{max}}{P_{rad}}$$

$$D\Big|_{dB} = 10 \log(D)$$

4) power Gain $G_p(\theta,\phi)$

Gain $G(\theta,\phi)$

$PL = \frac{l}{GP}$ $PL \Rightarrow$ loss power



$P_{out} = P_{rad}$

$P_{in} = P_{loss} + P_{rad}$

$$\frac{1}{2} I_{in}^2 (R_L + R_{rad})$$

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$$G = \frac{4\pi\, u(\theta,\phi)}{P_{in}} \qquad \eta_r = \frac{G}{D}$$

η_r = radiation efficiency

$$\eta_r = \frac{G}{D} = \frac{\frac{4\pi u(\theta,\phi)}{P_{in}}}{\frac{4\pi u(\theta,\phi)}{P_{rad}}} = \frac{P_{rad}}{P_{in}} = \frac{P_{rad}}{P_{rad} + P_{loss}}$$

$$\eta_r = \frac{R_{rad}}{R_{rad} + R_L}$$

$$G_0\Big|_{dB} = 10\log(G_0)$$

Ex// Determine (E) at a distance of 10km from an antenna having $D = 5\,dB$ and $P_{rad} = 20kw$ → free space

$$5dB = 10\log_{10} D$$

$$D = 10^{5/10} = 10^{0.5} = 3.162$$

$$P_{ave} = \frac{D\, P_{rad}}{4\pi}$$

No.

$\checkmark\ p_{ave} = \dfrac{E_0^2}{2\eta_0}$ $\boxed{E_0 = 0.1948\ V/m}$

Ex// show that the directivity of the hertzial dipole is $D = 1.5 \sin^2\theta$

$$D = \frac{4\pi\, u(\theta,\phi)}{p_{rad}}$$

for 1

$$u(\theta,\phi) = r^2 |\overline{p}_{ave}|$$

for Hertzial dipole $E\theta s \checkmark = \eta H\phi s$

$$|\overline{p}_{ave}| = \frac{|E\theta s|^2}{2\eta_0} = \frac{1}{2\cancel{\eta_0}}\left(\frac{\eta^{\cancel{2}} I_0^2 \beta^2 dl^2}{10\pi^2 r^2}\right)\sin^2\theta$$

$$|\overline{P}_{ave}|_{normalized} = \sin^2\theta$$

$$p_{rad} = \int_s p_{ave} \cdot ds = \int_S \sin^2\theta\, r^2 \sin\theta\, d\theta\, d\phi$$

$$\overline{D} = \frac{4\pi \cancel{r^2}\ \sin^2\theta}{\int_0^{2\pi}\int_0^{\pi} \sin^2\theta\, \cancel{ds}\, \cancel{r^2}\, \sin\theta\, d\theta\, d\phi}$$

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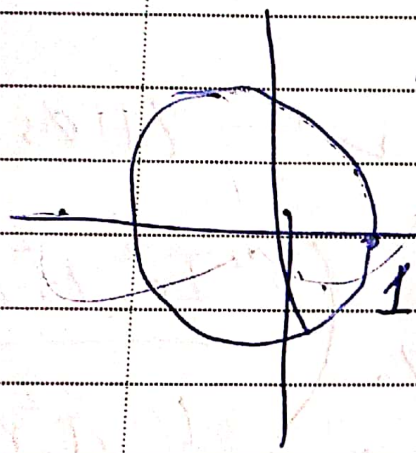
$$D = \frac{\sin^2\theta \cdot \cancel{4\pi}}{\cancel{2\pi}\left(\frac{4}{3}\right)}$$

$$D = 1.5 \sin^2\theta \quad \#$$

$$\underline{\underline{D_o = 1.5}}$$

$$D_o dB = 10 \log_{10} (1.5) = 1.76 dB$$

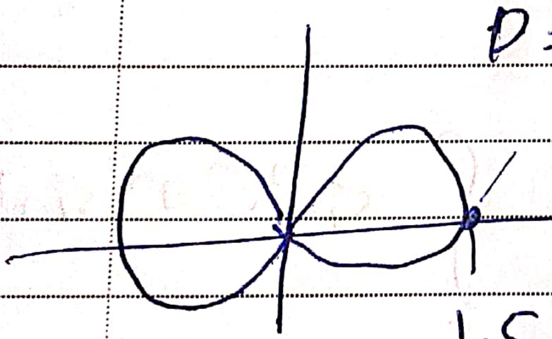


$D = 1$

for point source antenna

$D = 1.76\ dB =$

$D_o = 1.5$



Hertzian dipole

for $\frac{\lambda}{2}$ dipole anttena

$$D = \frac{1.64 \cos^2\left(\frac{\pi}{2} \cos\theta\right)}{\sin^2\theta} \qquad D_o = 1.64$$

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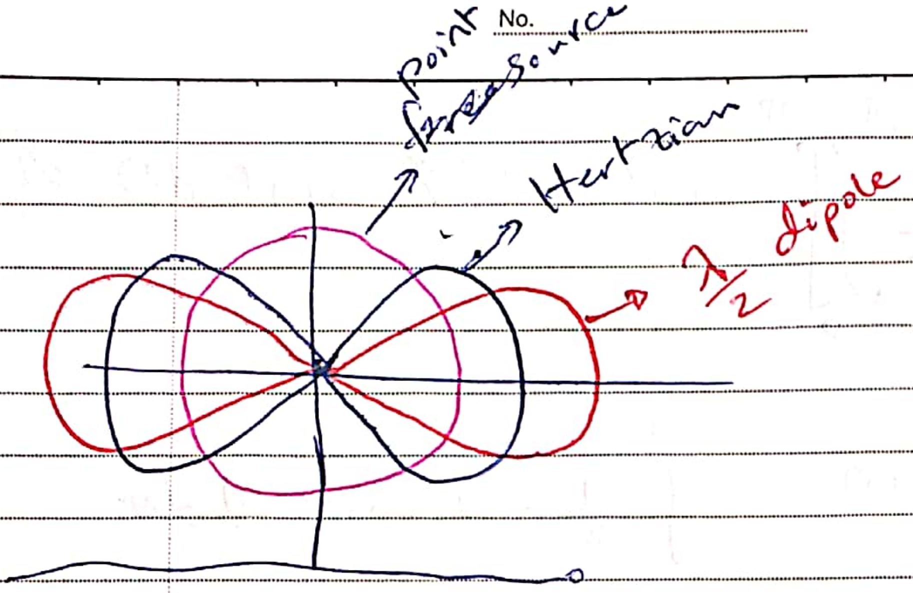
No.



$D \uparrow \qquad HPBW \downarrow$

Ex// The radiation intensity of a certain antenna is

$$u(\theta,\phi) = \begin{cases} 2\sin\theta \sin^2\phi, & 0 \le \theta \le \pi \\ & 0 \le \phi \le \pi \\ 0, & \text{Otherwise} \end{cases}$$

Determine the directivity (D_o)

$$D = \frac{u(\theta,\phi)}{u_{avg}}$$

$$D_o = \frac{u_{max}}{u_{ave}} \qquad \boxed{U_{max} = 2}$$

$$u_{ave} = \frac{P_{rad}}{4\pi} = \frac{\int u \, d\Omega}{4\pi}$$

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No.

$$= \frac{1}{4\pi} \int_0^{\pi} \int_0^{\pi} 2 \sin\theta \ \sin^3\phi \ \sin\theta \ d\theta \, d\phi$$

$$\int_0^{\pi} \sin^2\theta \ d\theta = \int_0^{\pi} \frac{1}{2} - \frac{1}{2}\cos(2\theta) \, d\theta$$

$$= \frac{\pi}{2}$$

$$\int_0^{\pi} \sin^3\phi \ d\phi = \frac{4}{3}$$

$$D = \frac{\cancel{2}}{\frac{1}{\cancel{4\pi}} (\cancel{2}) \left(\frac{\cancel{\pi}}{2}\right) \left(\frac{\cancel{4}}{3}\right)}$$

$D_0 = 6$ $\qquad$ $D_0 = 10 \log_{10} 6 = 7.78 \ dB$

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