UNIVERSITY OF JORDAN
Electrical Engineering Department

EE 221– FIRST EXAM
Signals and Systems

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March 6, 2018

Name: ____________________
Student Number: ____________________
Section: ____________________

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1. (2 points) Find the even and odd parts of the signal

$$x(t) = \sin\left[-\cos\left(t\right)\right] - \cos\left[\sin\left(t\right)\right] + \left(t-\pi\right)\left(t-2\,\pi\right).$$

$x_e(t) =$ [ ]

$x_o(t) =$ [ ]

2. (2 points) Find the energy of the signal $x(t) = e^{-(\alpha+j\beta)t}\, u(t)$, where α and β are real constants, $\alpha > 0$.

3. (4 points) Consider the signal $x(t) = e^{j\alpha_1 t} + e^{j\alpha_2 t}$, where α_1 and α_2 are real constants.

   (a) (2 points) Evaluate the power of the signal $x(t)$.

(b) (2 points) Show that $|x(t)| = 2\left|\cos\left[\left(\frac{\alpha_1-\alpha_2}{2}\right)t\right]\right|$.

4. (5 points) Evaluate the following integrals

(a) (1 point)

$$\int_{-\infty}^{\infty} \frac{t^2-1}{t-1}\delta(3t-3)dt.$$

(b) (1 point)

$$\int_{-\infty}^{t} \delta(2v-4)dv.$$

(c) (1 point)

$$\int_{-\infty}^{\infty} e^{-\alpha(t-\tau)}\delta(2\tau)d\tau.$$

(d) (2 points)

$$\lim_{T\to\infty} \frac{1}{2\,T} \int_{-T}^{T} u\left(\cos(t) - \sin(t)\right) dt,$$

where $u(t)$ is the unit step function.

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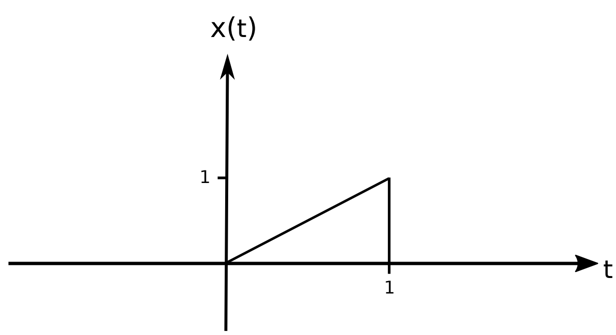
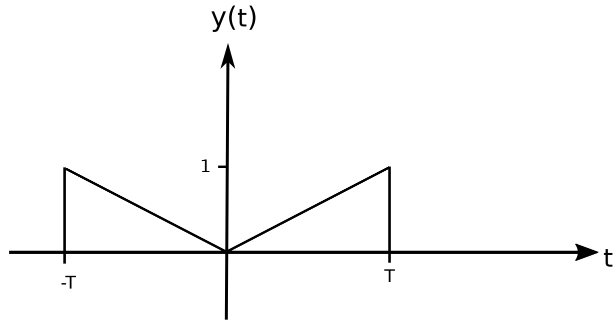
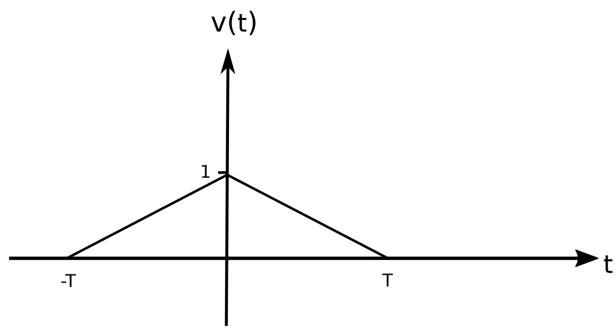
5. (6 points) Let $x(t)$ be defined as below. Express the signals $y(t)$, $g(t)$ and $h(t)$ below, in terms of the signal $x(t)$ and its time-shifted, time scaled, or time reversed versions.

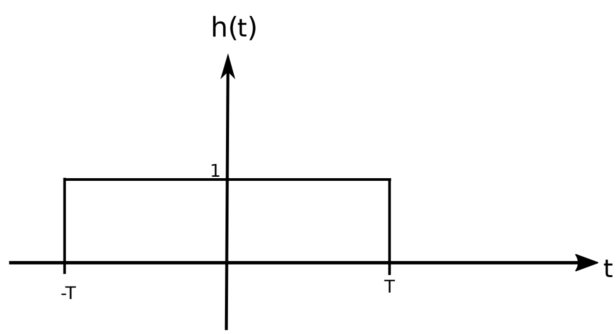




(a) (2 points)



(b) (2 points)



(c) (2 points)

6. (2 points) Consider a system whose input- output relationship, $y(t) = T\{x(t)\}$, is given by

$$y(t) = x(a - bt) + x^2(\alpha - b(t + c)),$$

where a, b and c are real constants. Obviously, the system $y(t) = T\{x(t)\}$ has memory for arbitrary values of a, b and c. If the output, $y(t)$, serves as an input to another system whose input-output relationship, $h(t) = T\{y(t)\}$, is given by

$$h(t) = y\left(f(t, a; b; c)\right),$$

Find an expression for the function $f(t, a; b; c)$ such that the system $h(t) = T\{x(t)\}$ is memoryless.

Hint: $f(t, a; b; c)$ is a function of the independent variable t and the constants a, b and c.

7. (2 points) Let $f(t)$ be defined as

$$f(t) = \sum_{i=1}^{N} A_i \cos(w_i t),$$

where A_i and w_i are non-zero real numbers. Show that the power of the signal $f(t)$ is given by

$$P_f = \sum_{i=1}^{N} \frac{A_i^2}{2}.$$

8. (2 points) Show that

$$\sin(t)\delta'(t) + \delta(t) = 0,$$

where $\delta'(t) = \frac{d[\delta(t)]}{dt}$.

Exam Formula Sheet

Even and odd Parts of a signal

For any signal $x(t)$, the even part, $x_e(t)$, is given by

$$x_e(t) = \frac{x(t) + x(-t)}{2},$$

and the odd part, $x_o(t)$, is given by

$$x_o(t) = \frac{x(t) - x(-t)}{2}.$$

Energy and power of a signal

The energy in the signal $g(t)$ is given by

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt$$

The power in the real signal $g(t)$ is given by

$$P_g = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |g(t)|^2 dt$$

Properties of Unit Impulse Function

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_0)dt = x(t_0)$$

$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$$

$$\delta(at - t_0) = \frac{1}{|a|}\delta\left(t - \frac{t_0}{a}\right)$$

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$
