UNIVERSITY OF JORDAN
Electrical Engineering Department

EE 221– FIRST EXAM
Signals and Systems

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October 31, 2018

- Write you name on the exam right now.
- The exam is closed books and closed notes. The maximum time allowed - 90 mins.
- Partial Credit Policy: Partial credit is awarded provided you show clearly all the steps in solving a question. Answers to question without work shown will earn no credit.

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1. Consider the system
$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau.$$
   (a) (15 points) Discuss why this system is linear, has memory, and time invariant. (Justify your answer)
   (b) (15 points) What is the impulse response of this LTI system?
   (c) (10 points) What is the impulse response of the LTI system described by $v(t) = \int_{t}^{t+1} x(\tau)d\tau$?

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2. Let
$$x(t) = A\cos\left(\frac{2\pi t}{T}\right)\sum_{n=-\infty}^{\infty} rect\left(\frac{t - \frac{T}{4} - nT}{\frac{T}{2}}\right).$$
   (a) (10 points) Sketch the signal $x(t)$.
   (b) (10 points) Find the power in the signal $x(t)$
   (c) (10 points) Let $y(t) = B + \sqrt{\frac{3}{8}}A\cos\left(\frac{2\pi t}{T}\right)$, find B (in terms of A) such that the power in the signal $y(t)$ is equal to the power in the signal $x(t)$.

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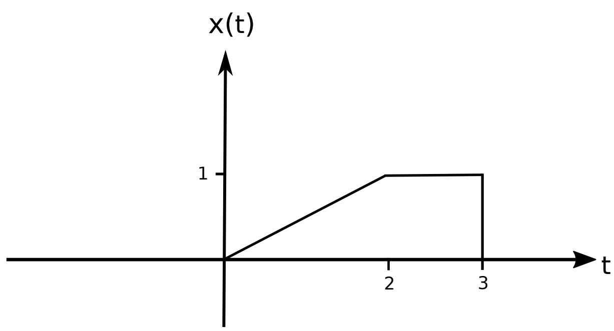
3. (a) (10 points) Let $x(t) = g(at - b)$, where $a \neq 0$. Show that $E_x = \frac{1}{|a|} E_g$, where E_x and E_g are the energies in $x(t)$ and $g(t)$, respectively.

   (b) (25 points) Let $x(t) = g\left(\frac{-t+1}{2}\right)$, where $x(t)$ is given below (note that $x(t)$ is given not $g(t)$). Consider the signal $f(t) = g(\frac{5t-3}{2})$:

   i) Use the result of part (a) to evaluate the energy in $f(t)$.
   ii) Find and sketch $f(t)$.



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4. (a) (15 points) Evaluate the follwoing integrals:

   i) $\int_3^\infty e^{-3t}\, \delta(2t-4)dt$.

   ii) $\int_{-\infty}^\infty e^{-t^2}\, \delta'(t-1)dt$, where $\delta'(t) = \frac{d}{dt}\delta(t)$.

   (b) (10 points) Let

$$\int_{-\infty}^{\infty} x(t)\,[\delta(t+t_1) - \delta(t-t_2)]\,dt = \alpha$$

   and

$$\int_{-\infty}^{\infty} 2\,x(-2t)\left[\delta(2t-t_2) - \frac{1}{2}\delta\left(t - \frac{t_1}{2}\right)\right]dt = \beta.$$

   Express $\int_{-\infty}^\infty x_o(t)\delta(t-t_2)dt$ in terms of α and β, where $x_o(t)$ is the odd part of $x(t)$.

   (c) (10 points) Express the following in terms of unit step function

$$\int_{\tau=-\infty}^{t-t_0} \int_{v=-\infty}^{\infty} \frac{\sin(w\,v)}{w\,v}\,\delta(\tau)\,\delta(\tau - v)dv\,d\tau.$$

The End

Exam Formula Sheet

Even and odd Parts of a signal

For any signal $x(t)$, the even part, $x_e(t)$, is given by

$$x_e(t) = \frac{x(t) + x(-t)}{2},$$

and the odd part, $x_o(t)$, is given by

$$x_o(t) = \frac{x(t) - x(-t)}{2}.$$

Energy and power of a signal

The energy in the real signal $g(t)$ is given by

$$E_g = \int_{-\infty}^{\infty} g^2(t)dt$$

The power in the real signal $g(t)$ is given by

$$P_g = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} g^2(t)dt$$

Properties of Unit Impulse Function

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_0)dt = x(t_0)$$

$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$$

$$\delta(at - t_0) = \frac{1}{|a|}\delta\left(t - \frac{t_0}{a}\right)$$

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

The definition of Unit rectangular pulse

$$rect\left(\frac{t}{T}\right) = \begin{cases} 1, & -\frac{T}{2} \leq t \leq \frac{T}{2} \\ 0, & \text{otherwise} \end{cases} \quad .$$