

power electronics

No.

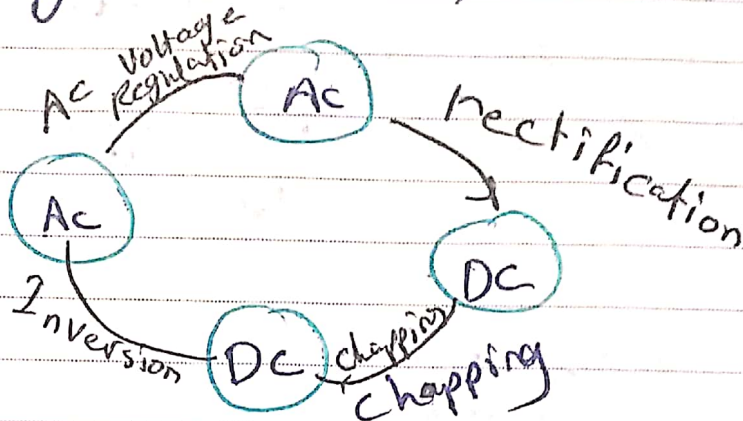
power electronics



Power Engineering + Electronics Eng. + Control Eng.

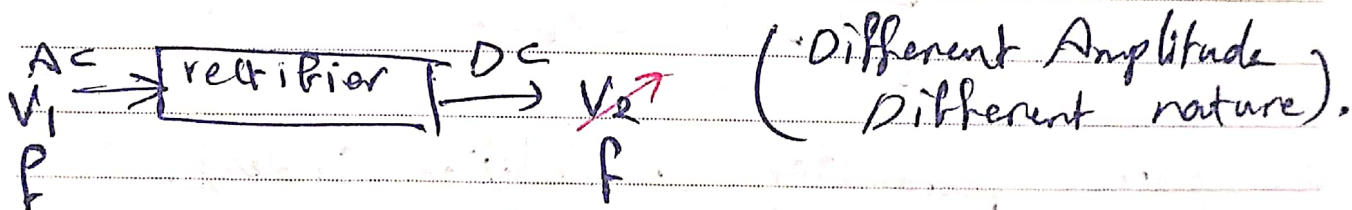
Note

why AC? \Rightarrow AC to change the Frequency by Reshaping the DC into AC



Types of converters in P.E:-

PE Converter: a device that can vary the voltage, current, freq, nature of a certain signal.

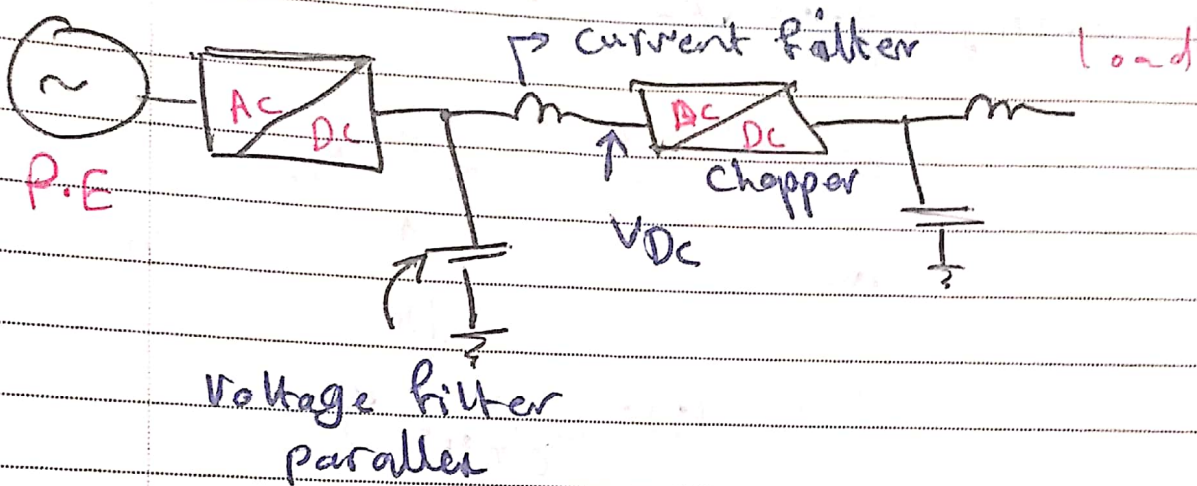


① Rectification: change of nature of the signal + vary the amplitude (A_{avg}) of the signal



Dc signal: can be variable in Amplitude or fixed.

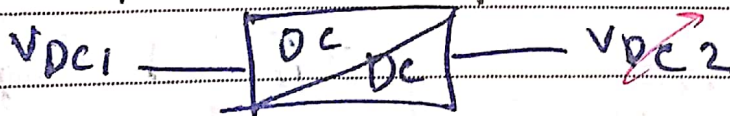
Depends ON:- the rectifier which can be fully controlled, Semi Controlled or Uncontrolled.



note: usually there is a filter at the end of rectifier (to smooth).

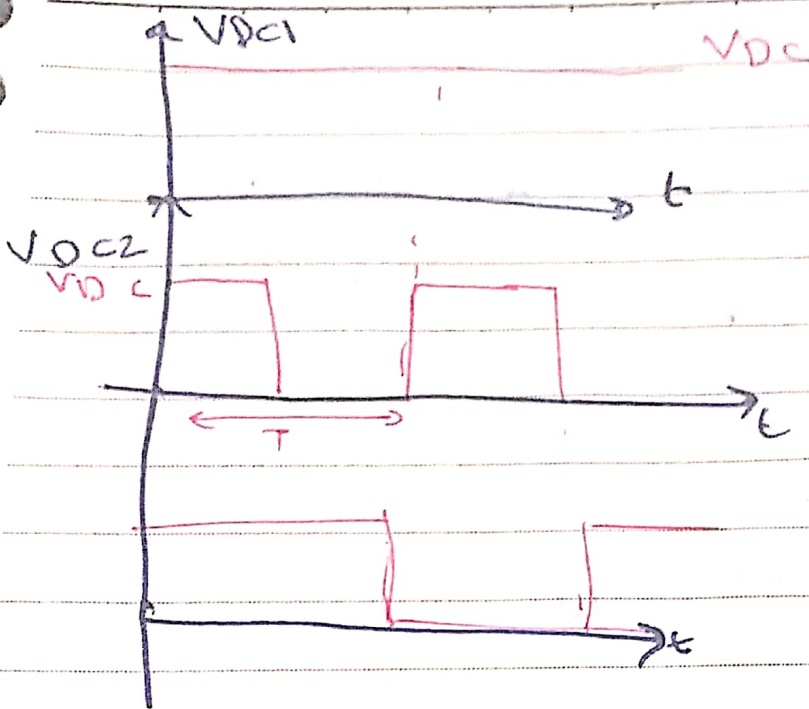
② Chopping :-

DC to DC chopper

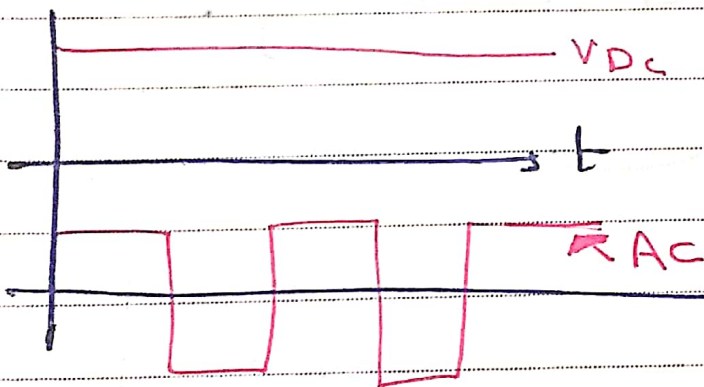


S : control parameter (modulation Index).

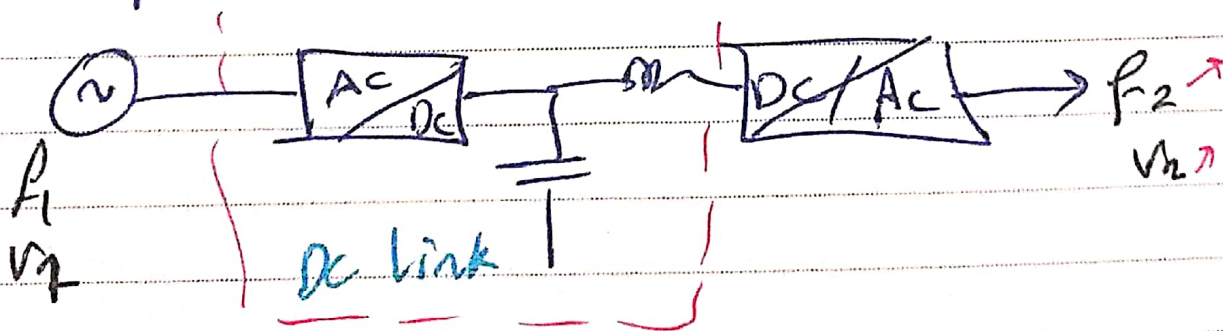
fch: filtration of harmonics



③ Inverters (DC to AC) converters

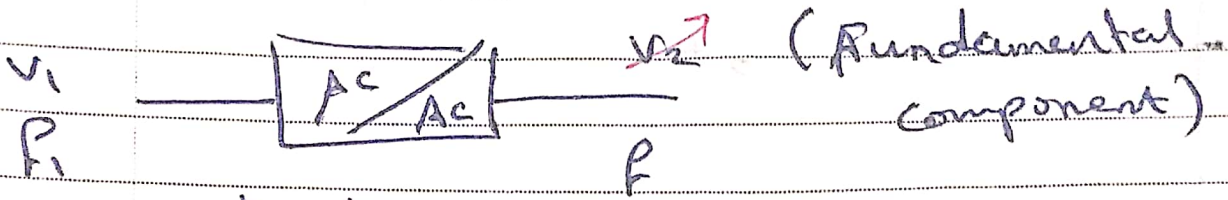


freq. converter



No. _____

④ Ac voltage Regulator



Amplitude

Ac Fundamental Dc avg

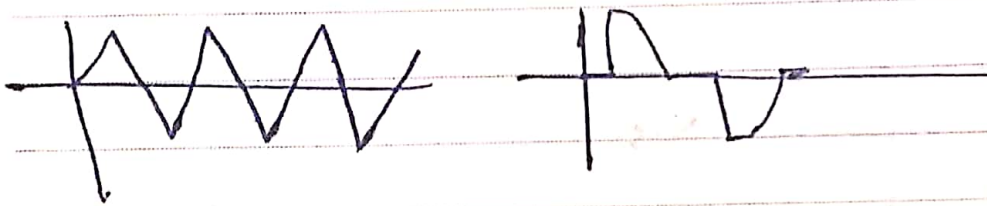
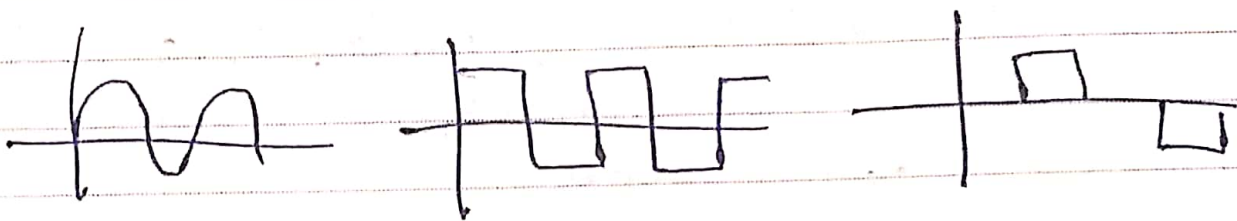
The 4 converters are referred to as the family of power conditioning.

all converters in Electric power Conditioning

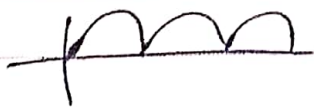
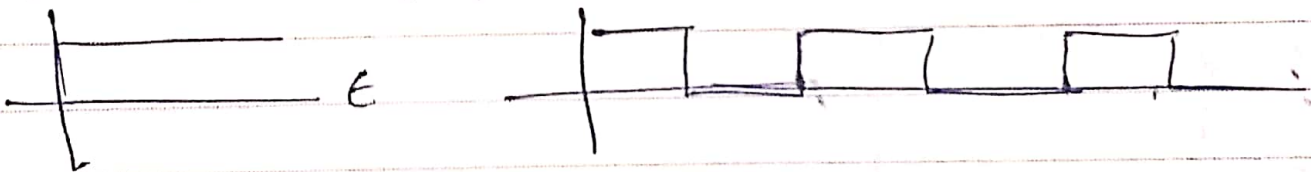
power conditioning

- ① Change of the nature of electric signal
AC \rightarrow DC or DC \rightarrow AC

AC signal \Rightarrow any signal that has zero average amplitude



DC signal \Rightarrow any signal that has non zero average



- ② change in frequency in case of AC signals.

frequency involved in AC motor speed control $N_s = \frac{60f}{P}$

③ Amplitude Control

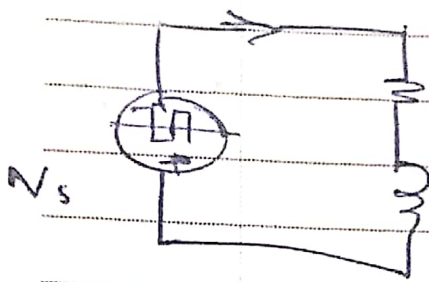
a) average control → Dc systems

in Dc
(Rectifiers + Choppers)

Fundamental

④ ~~amplitude~~ Control

in AC inverters + AC regulators

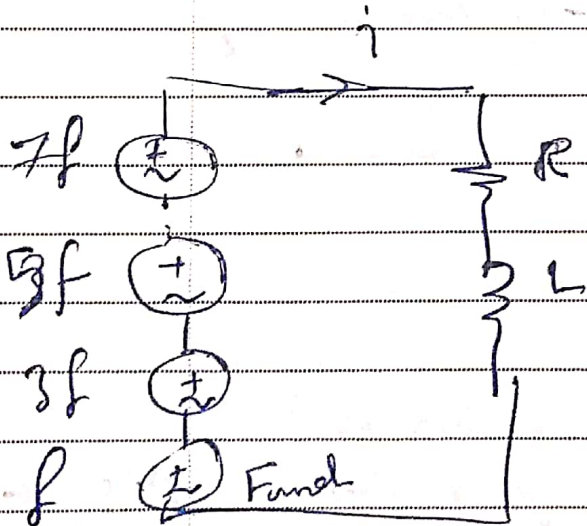
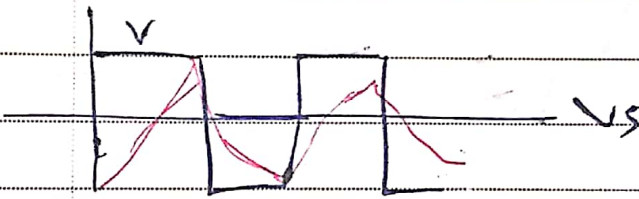


discrete

$$V_s(t) = \begin{cases} +V & 0 < \omega t \leq \pi \\ -V & \pi < \omega t \leq 2\pi \end{cases}$$

$$V_s(t) \approx \sum_{n=1,2,3} V_m(n) \sin(n\omega t + \phi(n))$$

Fourier series



Dc Signal Quality \Rightarrow Ripple Factor $= \frac{\sqrt{V_{rms}^2 - V_{avg}^2}}{V_{avg}}$

$0 \leq R_f \leq \infty$
 $V_{rms} = V_{avg}$ \sim very bad
 For pure Dc pure Ac??

Excellent

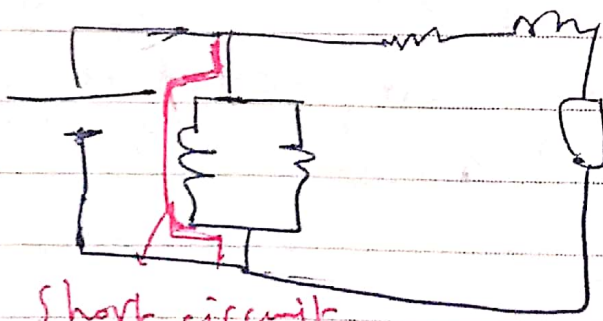
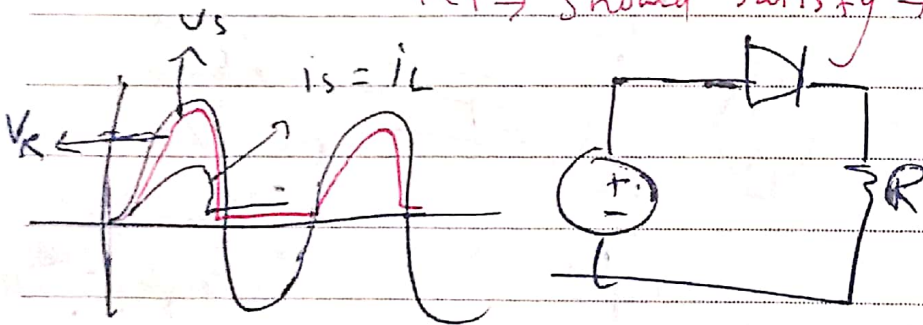
Ac Signal Quality: Define total harmonic Distortion factor

$$THDF = \frac{\sqrt{V_{rms}^2 - V_{(f)}^2(rms)}}{V_{(f)}(rms)}$$

$$\hookrightarrow \frac{THDF}{PF} V_f(rms)$$

THDF

should satisfy the Power Supplier Regulations
 PF \rightarrow should satisfy the load (Consumer).

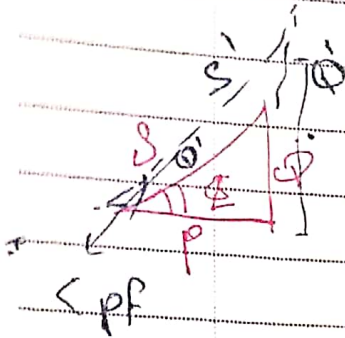


$$PF = DTF * DPF \leq \cos \phi$$

$$\frac{I_C(rms)}{I_C(rms)} * \cos(\psi_C - \psi_i)$$

ϕ
displacement angle

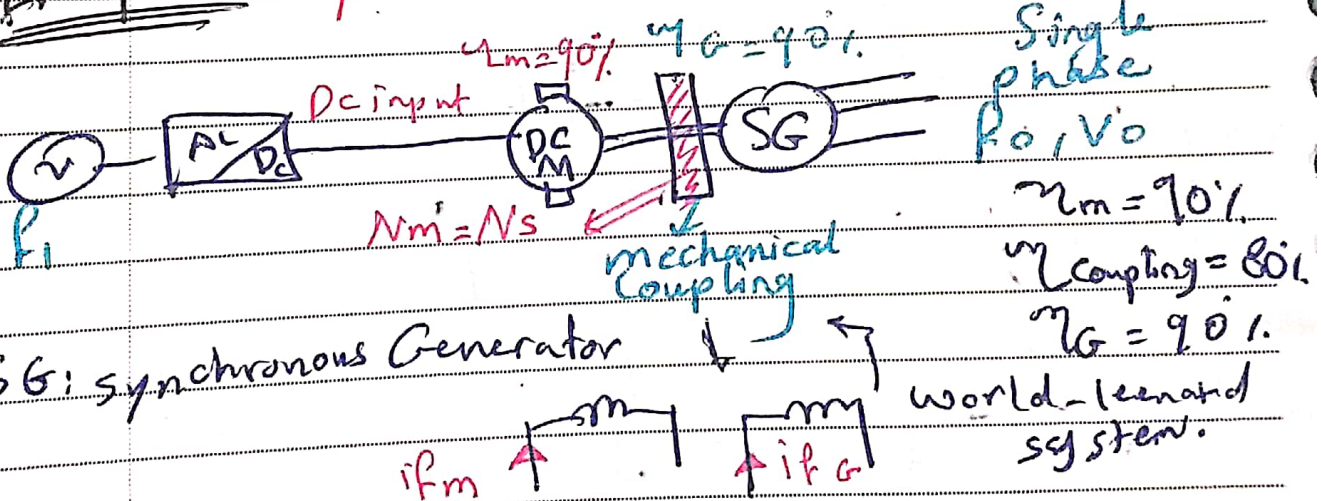
PF is usually reduced with power electronics inverters, this is a drawback.



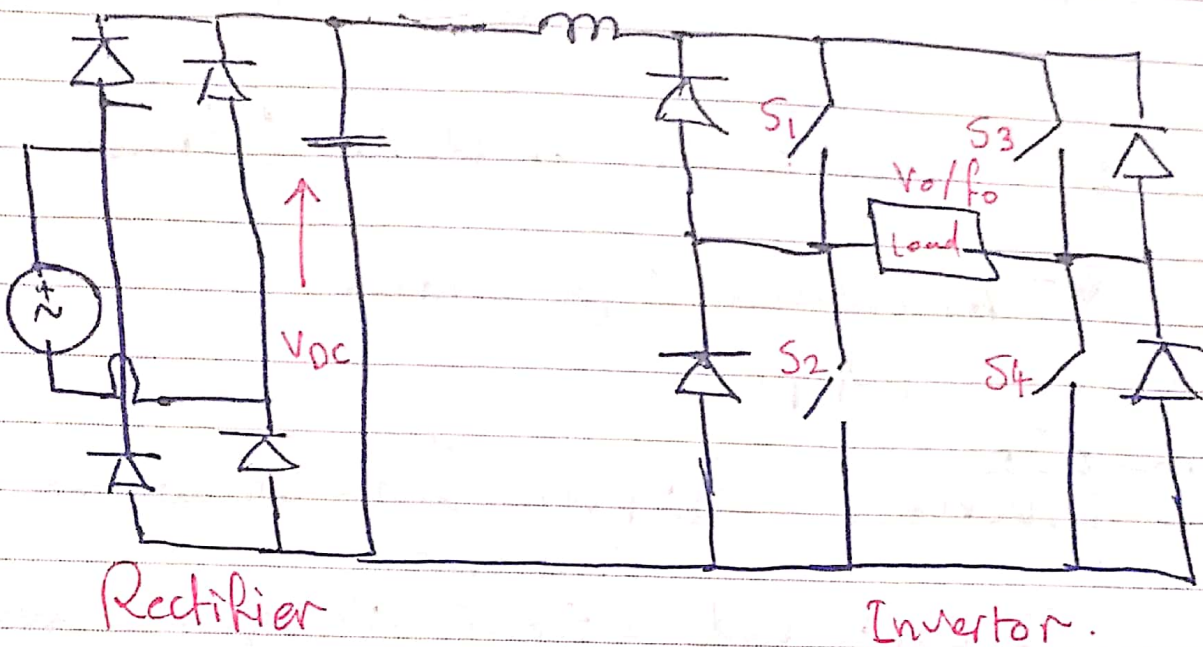
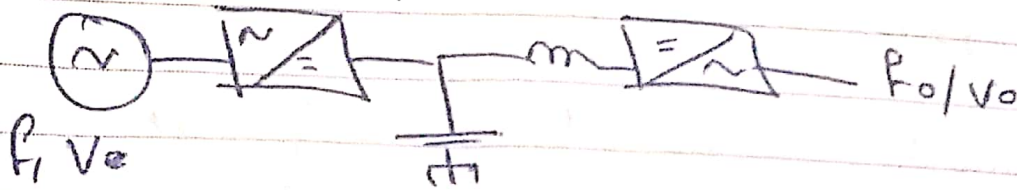
- 1- more losses
- 2- more installed capacity

Merits and drawbacks of P.E compared to conventional systems.

Example freq control: "Conventional"



PF alternative rectifier



Rectifier

Inverter.

Merits of P.F.E

- * Much lower initial (capital cost).
- * Much lower maintenance requirements (maintenance free)
- * Efficiency. 96%.
 ↓ 60% Conventional.
- * No special foundations.
- * More Reliable
- * Less space, size & weight.
- * Faster Response (efficient control).
- * No Caustic Noise.

$P_o \rightarrow$ inverter, controlled by switching pattern of $S_1 \rightarrow S_4$

$P_o \rightarrow$ conventional $\rightarrow P_o = \frac{P N_s}{60} = \frac{P}{60} N_m$

$N_m \begin{matrix} \uparrow + \\ \downarrow - \end{matrix}$ by field weakening of DCM. (I_{fm})
Armature voltage control.

$$N_m = \frac{V_t}{k I_f} \quad (\text{drop})$$

$V_o \rightarrow$ inverter : ① PWM with uncontrolled rectifier

② Controlled rectifier with square wave.

$V_o \rightarrow$ conventional system

$$V_{oph} = 4.44 N_p h a \frac{\phi_m}{k_f \times I_{FG}} \times P_o \quad (\text{controlled by } I_f)$$

Drawbacks of P.E.:-

1- No overload capacity; even for (very short time) this means design to the worst case of the running.

ex: starting condition of motor Extra Cost

2 | Harmonics generation, No pure Dc or pure Dc signals are generated by p.e

∴ Waveforms distorted
↓
Harmonics Impurities beside to useful signal.

$$v(t) = V_{avg} + \sum_{n=1,2,3 \dots}^{\infty} V_m(n) (\sin(n\omega t + \psi(n)))$$

$$i(t) = I_{avg} + \sum_{n=1,2,3}^{\infty} I_m(n) \sin(n\omega t + \psi(n) - \phi(n))$$

In Dc application.

V_{avg} & I_{avg} are useful

$\sum_{1/2}$ --- are set of impurities, that should be filtered, if not a problem will occur

B

IN AC Signals

$$v(t) = V_m(t) \sin(\omega t + \psi(t))$$

$$i(t) = I_m(t) \sin(\omega t + \psi(t) - \phi(t))$$

are the Useful Signals

Remaining are Imperities that should be avoided as much as possible

Drawbacks of power electronics :-

② Harmonics Generation (Distortion of the electric (Voltage and current) waveforms).

$$\begin{aligned}
 v(t) &= V_{av} + \sum V_m(n) \sin(n\omega t + \psi(n)) \\
 &= V_{av} + V_m(1) \sin(\omega t + \psi(1)) + V_m(2) \sin(2\omega t + \psi(2)) \\
 &\quad + \dots + V_m(n) \sin(n\omega t + \psi(n)) + \dots + \infty
 \end{aligned}$$

DC \rightarrow V_{av} useful \rightarrow all remaining are Impurities.
 AC \rightarrow $V_m(1) \sin(\omega t + \psi(1))$ useful \rightarrow all remaining are Impurities.

* To improve the quality of the signal

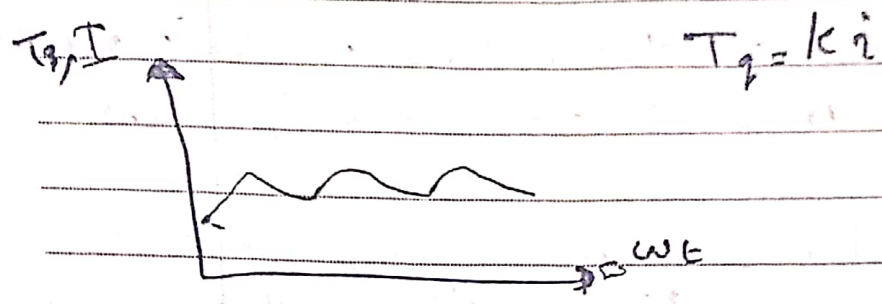
- Use better control topology. example multi-level wave form rather than square wave.
- Filtration (either voltage filtration by capacitors or current filtration by inductors).

Quality parameter : THDF \downarrow

Harmonics Impacts

- electromagnetic distortion (communication systems) ^{badly affects}
- badly affect nearby consumers (supply voltage distortion)
- Overheating of electrical machines (motors in specific).

In Dc motors → Torque developed due to Average current
 other components cause (a) Torque pulsation badly realized at low speeds
 (b) motor Overheating.



$P_{cu} = I^2 R_a$
 Copper loss

$I = I_{av} + I_m \sin(\omega t + \phi(i) - \phi(I)) + \dots$

$I_{rms} = \sqrt{I_{av}^2 + \sum_{n=1}^{\infty} \left(\frac{I_m(n)}{\sqrt{2}} \right)^2}$

P_{cu} without harmonics = $I_{av}^2 R_a$ (pure Dc)

P_{cu} with harmonics = $I_{rms}^2 R_a$

Since $I_{rms} > I_{av}$ then P_{cu} with harmonics $>$ P_{cu} without harmonics

$\frac{P_{cu} \text{ without}}{P_{cu} \text{ with harmonics}} = \left(\frac{I_{av}}{I_{rms}} \right)^2$

this leads to overheating by the same Ratio. The motor may not tolerate such an increase of heat and may be damaged

Solutions:- ① motor oversizing
leads to More motor cost + lower efficiency
due to Running Underloaded

② Motor derating, implying that the motor is loaded
by less load power by average $\left(\frac{I_{av}}{I_{rms}} \right)$

Harmonics

in Ac $\left(\frac{I_F(rms)}{I_T(rms)} \right)^2 = \frac{P_{cu \text{ without harmonics}}}{P_{cu \text{ (with harmonics)}}$

derating factor = $\frac{I_F(rms)}{I_{total}(rms)}$

Harmonics effects on P_f in

$PF \neq \cos \phi$

power factor reduction is one of the drawbacks of
power electronics

PF can only be defined as $\frac{\text{Real power}}{\text{Apparent power}}$

Rectifier $\rightarrow pf_{in} = DPF * DTF < \cos \phi$

Merits more efficient than the drawbacks
 Research activities are directed towards improving the performance evaluation of power electronics systems.

Classifications of power electronics switches,

- 1- diode family → (Rectifier) diodes
 ① Conventional diodes (up to 400 Hz)
 ② Fast Recovery diodes (high freq)
 ↳ (Inverter/Chopper) diodes

2- Thyristors family (1st one SCR) Father of P.E

3. power transistors (main switch IGBT)

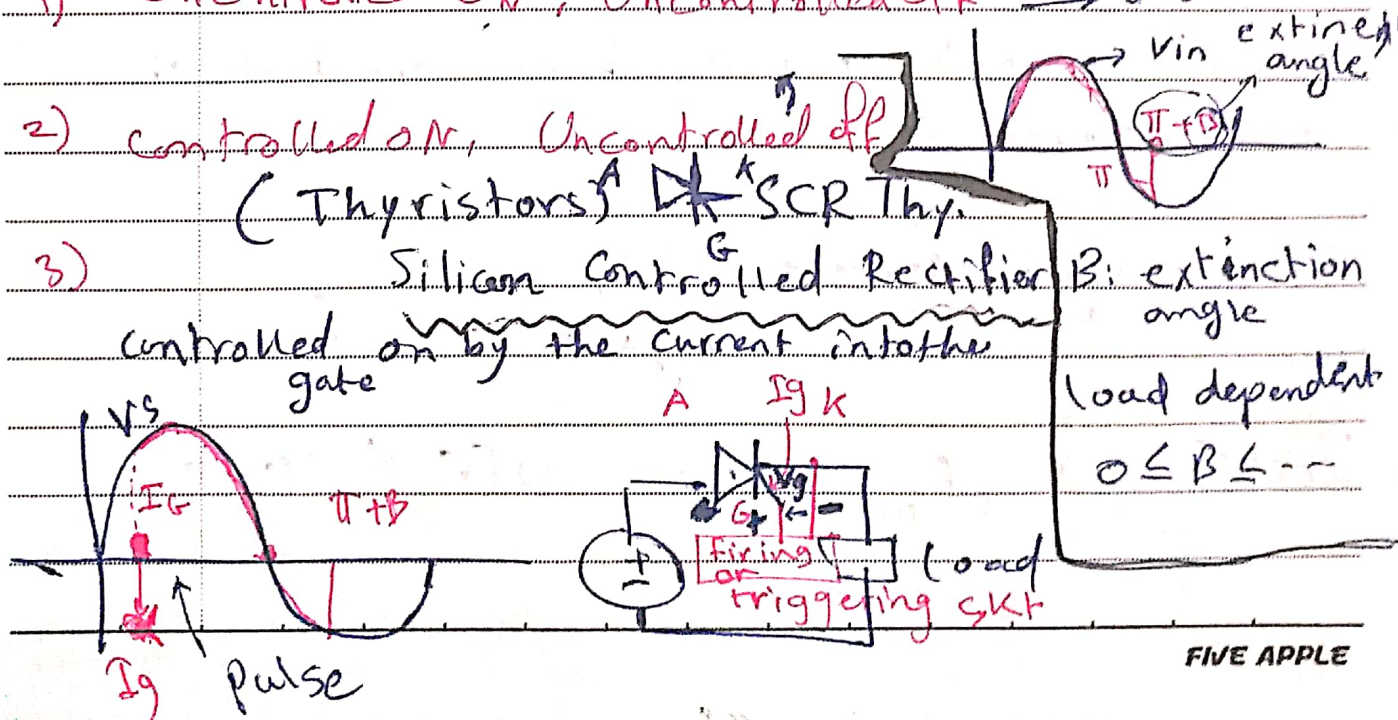
Ⓐ Classification According to the Controllability of switches

Supply → load

1) Uncontrolled ON, Uncontrolled OFF → diodes

2) controlled ON, Uncontrolled OFF
 (Thyristors) SCR Thy.

3) Silicon Controlled Rectifier
 controlled on by the current into the gate



$V_G = 1.5 \rightarrow 5V$

I_G switch power dependant scale: few to
severals tens of mA

SCR can be switched on when:

- a. FIB
- b. Apply gate current at an angle α known as the firing angle / the triggering angle or the phase delay angle

$$0 \leq \alpha \leq \pi$$

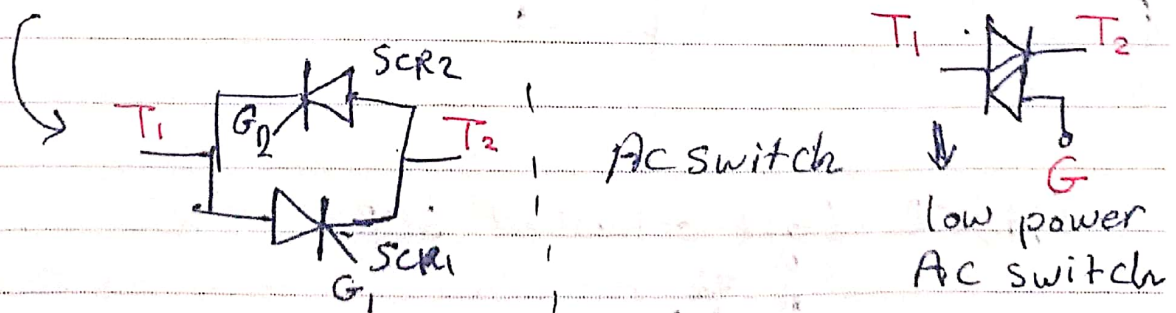
* Switch off is Uncontrolled (depends on the load nature & α)

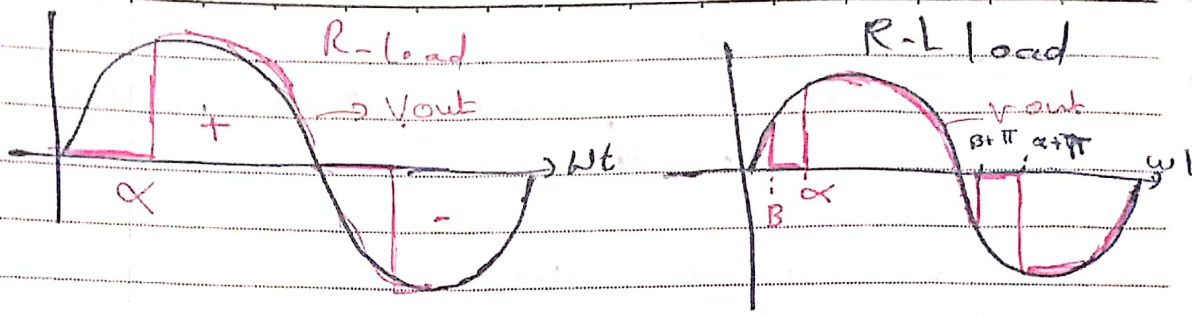
Once the SCR is ON, it behaves like a diode (the gate loses control)

Frequency ω is same as the Supply Freq. This implies α is synchronized with V_s .

TRIAC: TRIOD For AC

equivalent to two Antiparallel connected SCRs





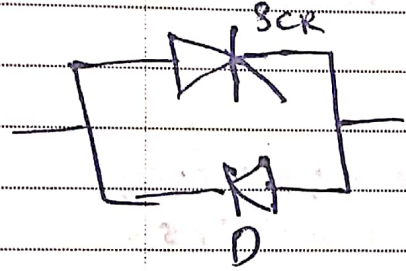
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classifications of PE switches

* Controlled ON, uncontrolled off

SCR, TRIAC, RCT

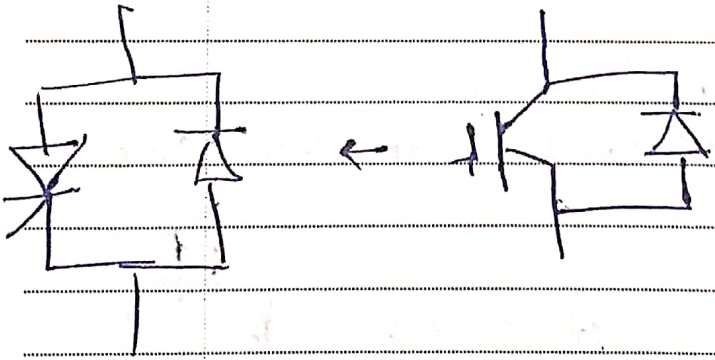
RCT : Reverse conducting Thyristor (Thyristor)



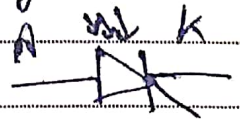
was used in SCR-controlled inverters

SCR : main switch

D : freewheeling / feedback switch



LASCR : Light activated SCR

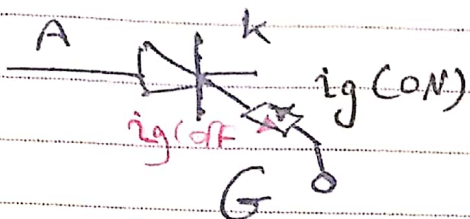


very large power Applications.

(HVDC T)

High voltage Direct Current transmission

3) Controlled ON, Controlled off

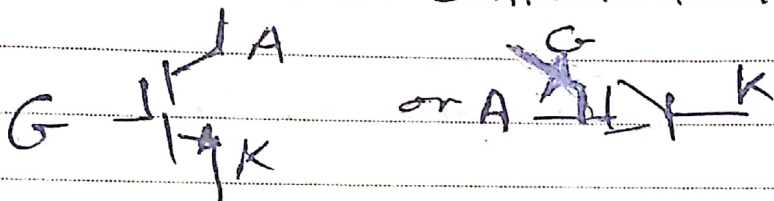


① GTO: Gate Turnoff Thyristor

to switch ON: Same as SCR

to switch off: -ve gate current pulse, but its amplitude must be very large, $I_{gate(off)} \approx 80\% I_{load}$.

② MCT: MOS controlled Thyristor



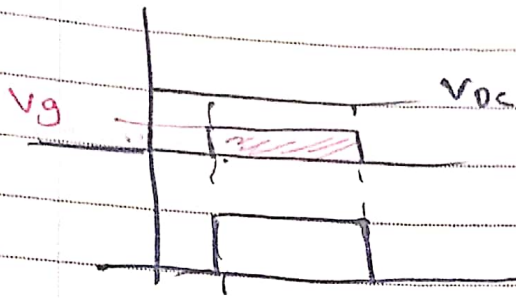
switch ON: short positive pulse to the Anode with respect to the gate $\Rightarrow G-$
A+

switch off: short negative pulse $= = =$
G+
A-

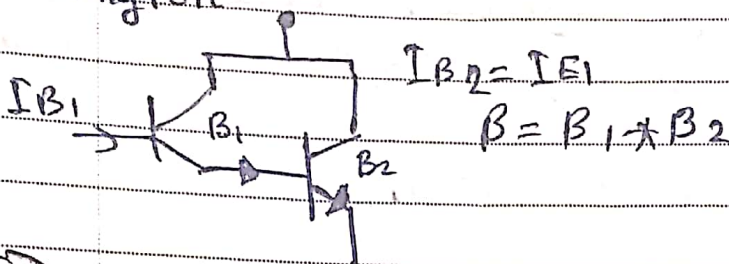
almost same amplitude (small pulse)

③ All Transistors are controlled ON, Controlled off

<p>up to 10kHz, 200A, 600V</p> <p>BJT Bipolar junction Transistor (current controlled)</p>	<p>up to 200V 600A</p> <p>up to 10⁶ Hz S-speed</p> <p>MOSFET (voltage controlled switch)</p> <p>20V V_{GS} < +20V</p>	<p>insulated Gate IGBT Bipolar Transistor</p> <p>99% inverters and choppers use IGBT</p> <p>- voltage controlled</p> <p>up to 100kHz</p> <p>FIVE APPLE 2000V 2000A</p>
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Darlington



(B) classification in Terms of nature of the Control signal.

- a) Latching switches
- b) continuous switches, (Trigger)

@ latch switching : single short pulse is required to switch on and off if controlled off all Thyristors are latching switches.

(b) continuous switching :- all Transistors (Drive)

(c) Voltage with Standing Capability

- (1) Unipolar
- (2) bipolar

tolerate one polarity either positive or negative all transistors,

Certain Types of GTO
SCR, TRIAC, LASCR

Certain Types of GTO, MCT

Charge control

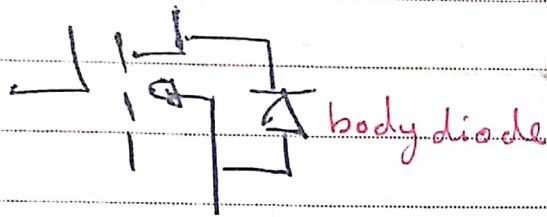
D) Current with Standing Capability.

a) Unidirectional

all remaining switches are bidirectional.

b) bidirectional

TRIAC, RCT (AC), MOSFET module



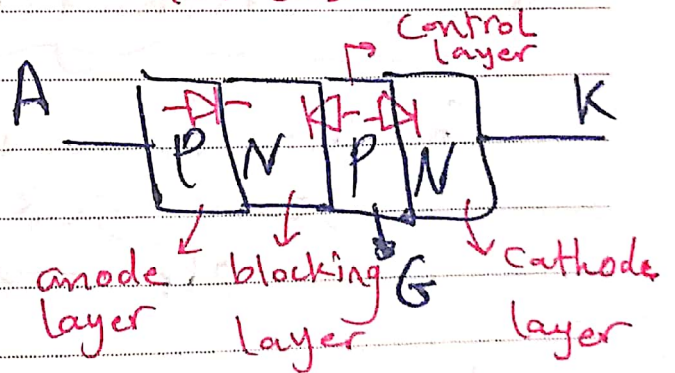
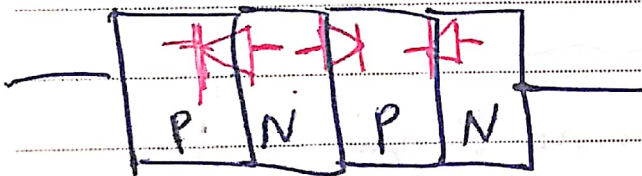
SCR: Silicon Controlled Rectifier.

Defined:-

it is a 4-layer, 3-Terminals, Bi-stable "tri-stable", Semiconductor switch.

-ve case

+ve case

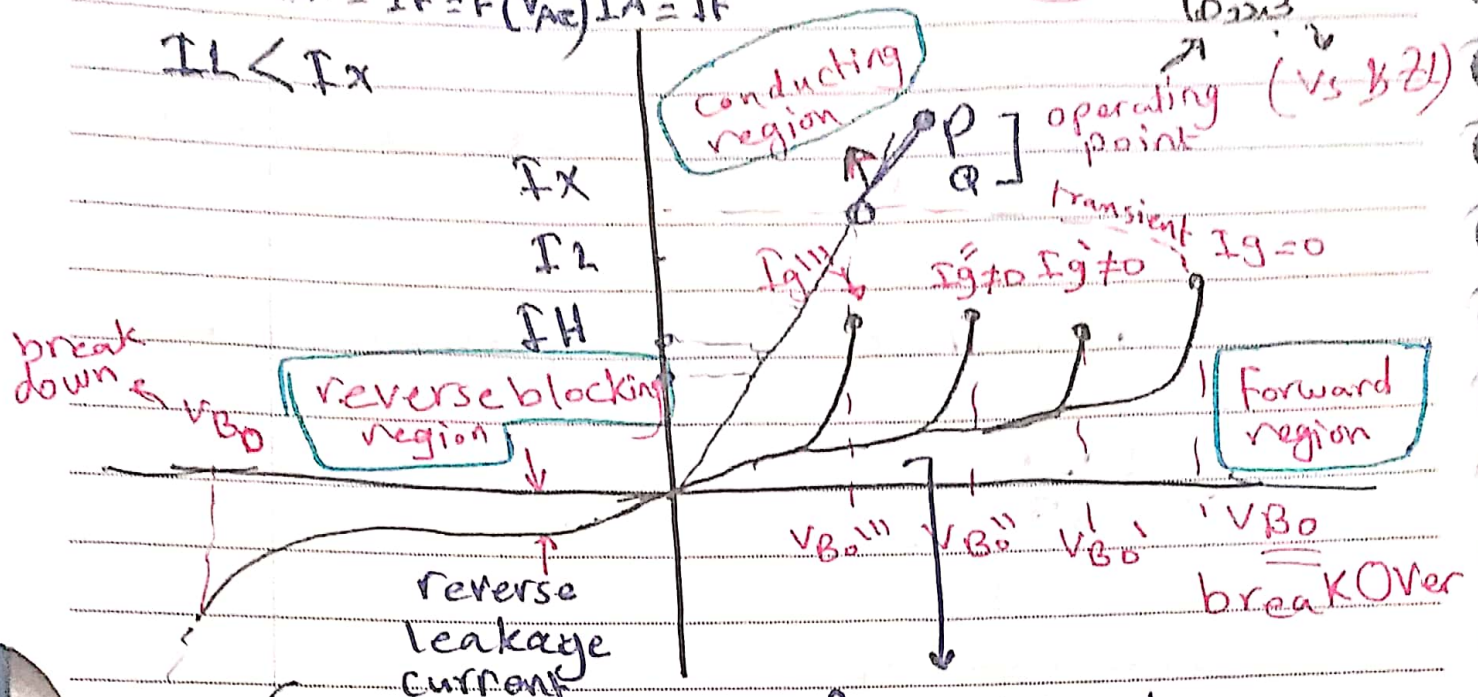


* to be forward bias! (+ve) voltage to be connected to anode with respect to the Cathode.

* V/A characteristics (steady state).

$I_A = I_F = f(V_{Ae}) I_A = I_F$

$I_L < I_H$



Reverse leakage current (few several uncontrolled break down)

Forward leakage current (greater than Rev. leakage current)

* I_L : Latching Current

The min. Forward Current Required to start SCR conduction off \rightarrow ON state

I_H \rightarrow Holding Current

The min Forward Current required to keep the SCR ON

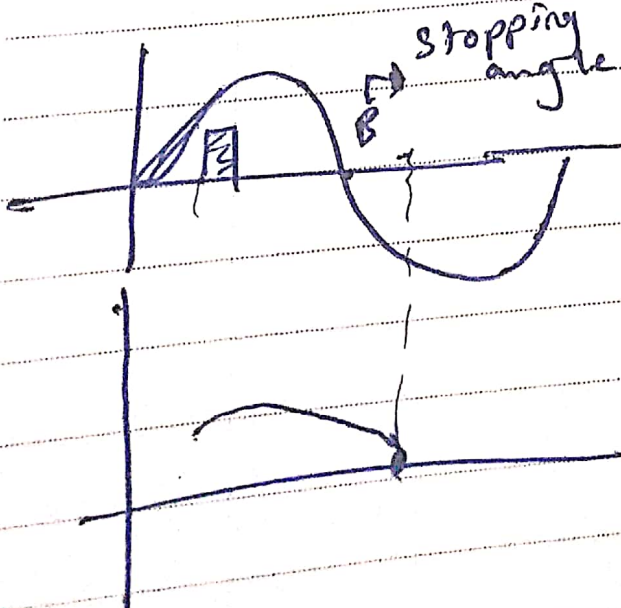
above $I_H \rightarrow$ ON
below $I_H \rightarrow$ off

Switching off process = Commutation process

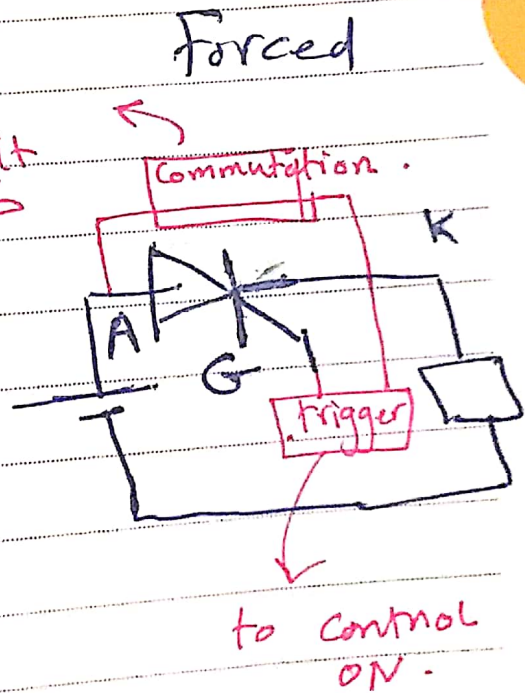
Note:- Load highly inductive \rightarrow pulse ^{wider}

Commutation

Natural
Supply Ac



Circuit diagram



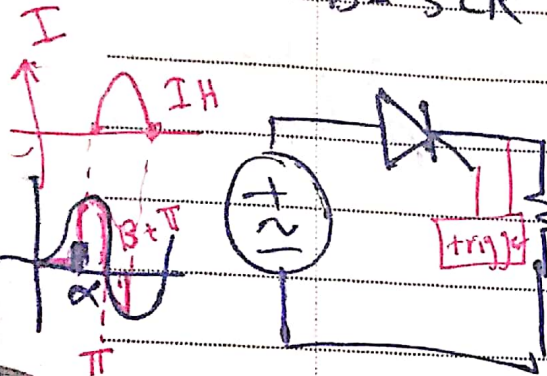
Sunday is 23/6

to switch off (commutation) SCR

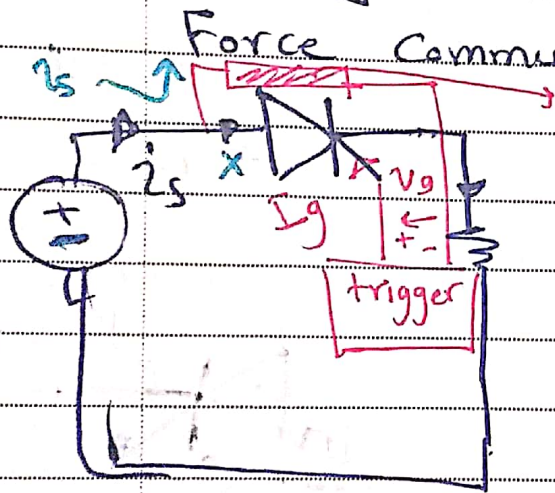
a - $I_f < I_H$

and

b - SCR to be reverse biased



Commutation can be natural in AC excited circuits.



Force Commutation in DC Excited Circuits

force commutation circuit

↳ provides alternative path for SCR current

b - to apply Reverse biasing to the SCR

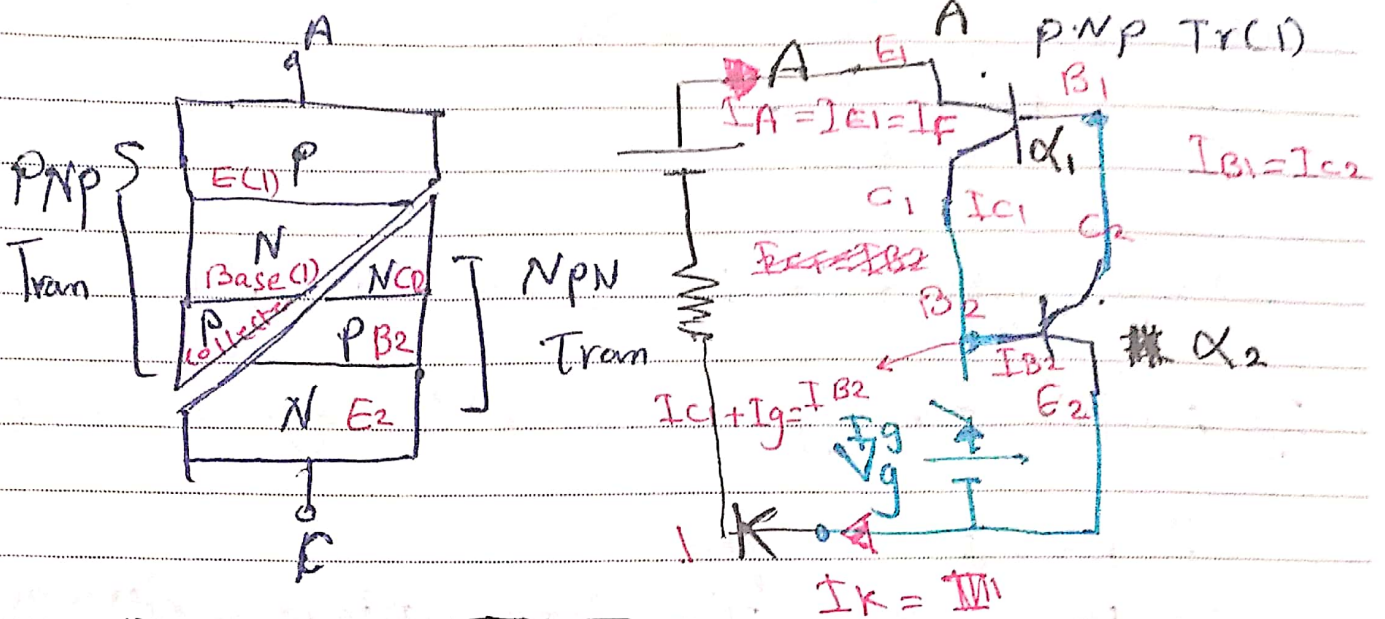
(by using commutation

Ro B to be applied for a period αT that is slightly greater than T_{off}

(Turn off time, usually given by the manufacturer).

Circuitry Capacitors + Auxiliary Thy. + ...)

The two transistor model of SCR



$$I_C = I_B = I_E$$

$$I_{E1} = \alpha_1 I_{E1} + I_{CBO(1)}$$

$$I_{C2} = \alpha_2 I_{E2} + I_{CBO(2)}$$

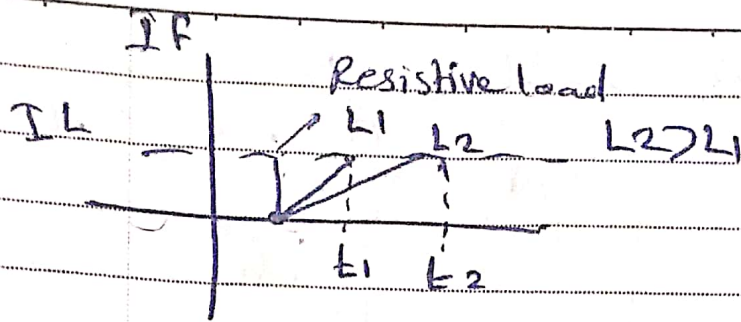
$$I_F = \frac{I_{CBO(1)} + I_{CBO(2)}}{1 - (\alpha_1 + \alpha_2)}$$

$$(\alpha_1 + \alpha_2) < 1 \quad (0.95)$$

* if I_F is ^{made} greater than I_C then the SCR is turned ON successfully

How can I_F increased to switch ON SCR??

↳ by applying a sufficient gate signal.
natural method to switch ON SCR



2) by Light Activation (For LASCR)

3) switch ON by increased Leakage Current

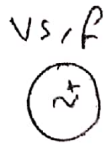
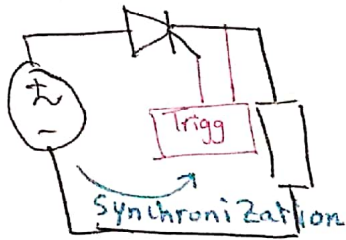
I_{CBO}

a) by applying High V_{AK} Voltage externally
Such that $V_{AK} > V_{BO}$ (not a proper method)

b) by Temperature rise over the Recommended
Value

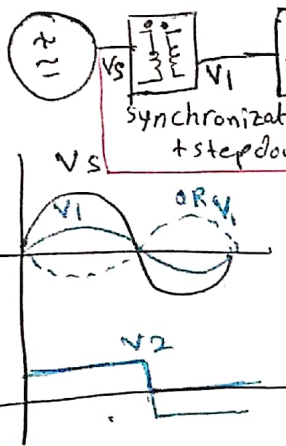
Firing Circuits

Firing → triggering

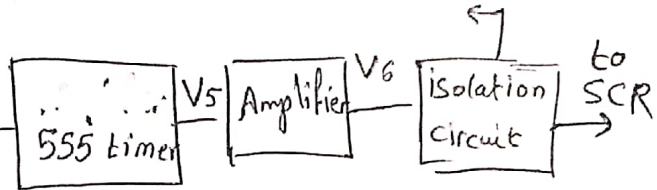
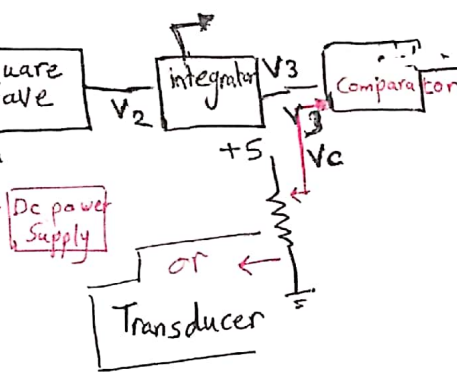


Core material never saturate.
 ↓
 pulse transformer
 or opto Coupler

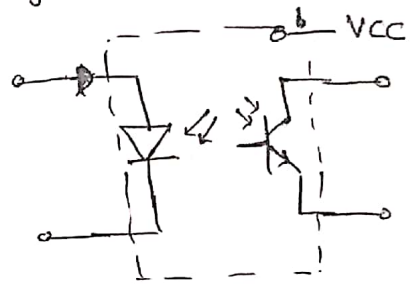
Step-down T



Sawtooth Gen.



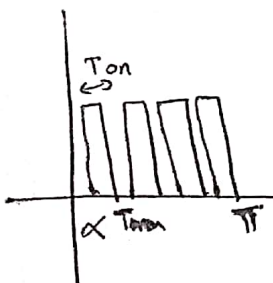
opto Coupler



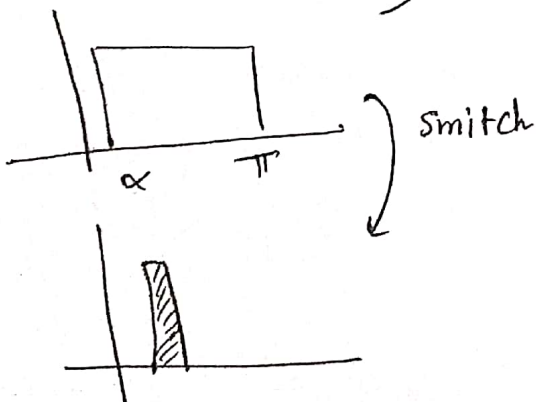
Carrier Frequency (10-20) kHz

$$f_{ch} = \frac{1}{T}$$

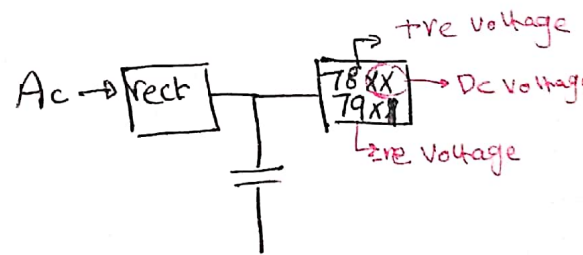
$$\delta = \frac{T_{on}}{T} \approx 0.6$$



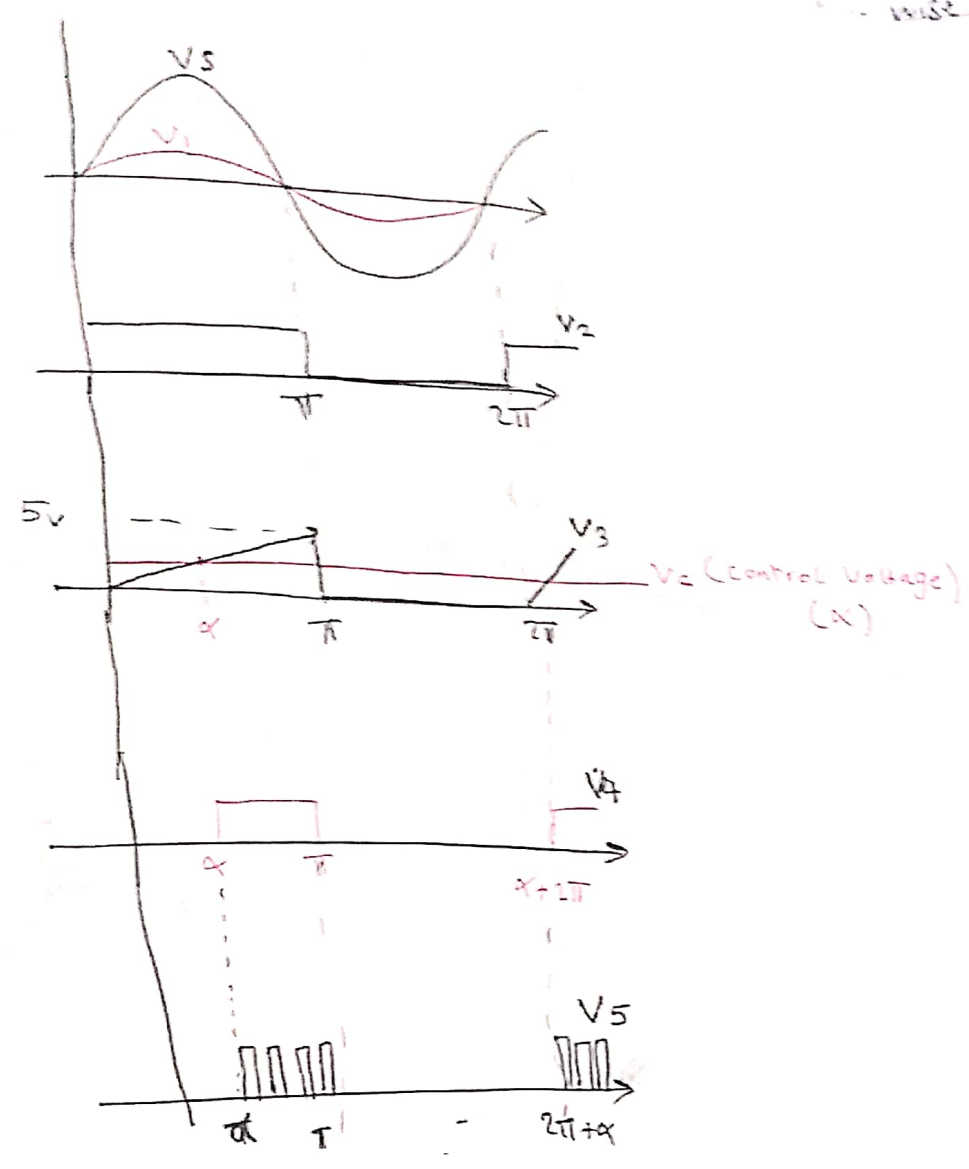
555 Timer



Dc power Supply.

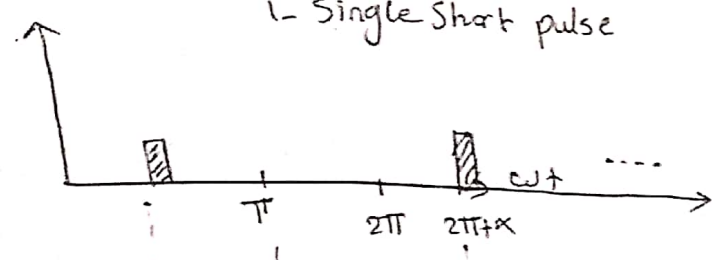


XX must be > minimum



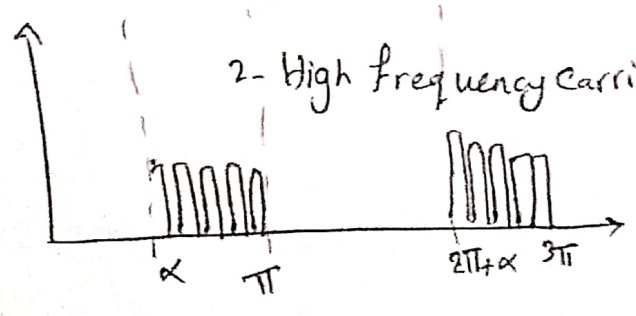
Types of Triggering Signals

1- Single Short pulse



- less losses
- = Uncertain Trig especially for higher Inductive Loads.

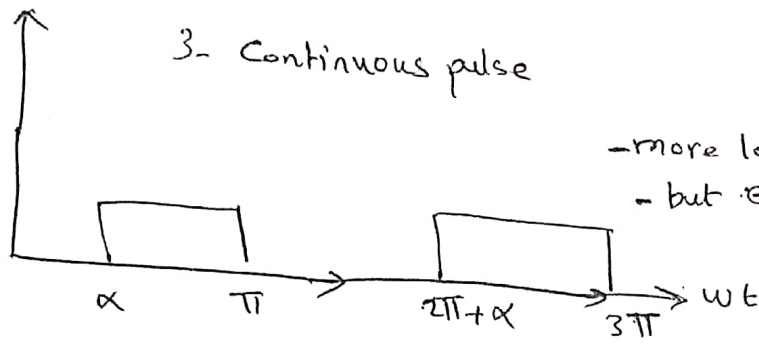
2- High Frequency Carrier Signal



- = medium losses
- ensure trig

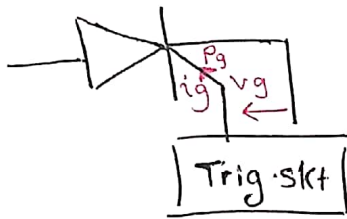


MOST Common



- more losses (high losses),
- but ensures Trig.

Gate characteristics :-

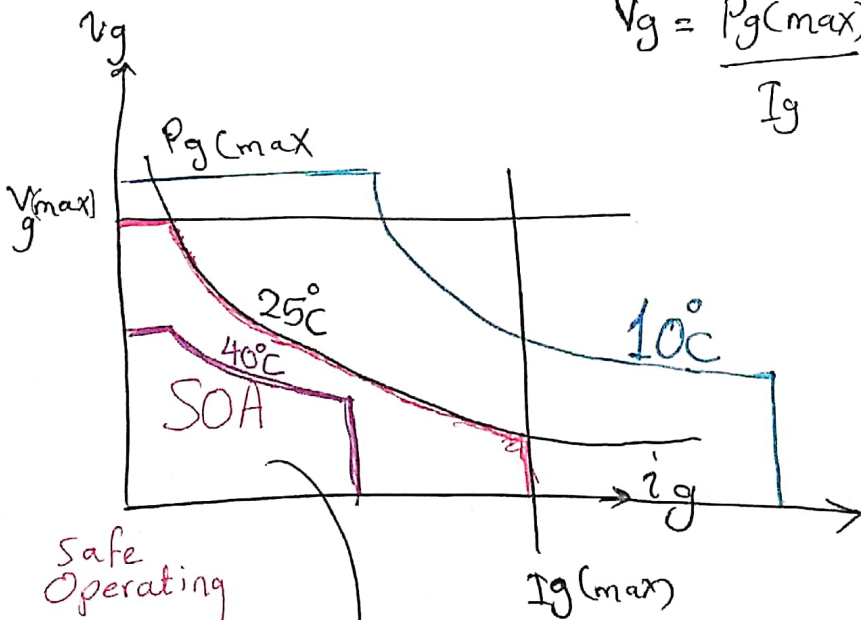


$$V_g = f(i_g)$$

To start with the limitations.
(given by the Manufacturer).

- $V_g(\max)$
- $I_g(\max)$
- $P_g(\max) = V_g * I_g$

$$V_g = \frac{P_g(\max)}{I_g} = \frac{\text{constant}}{I_g}$$



Safe Operating Area

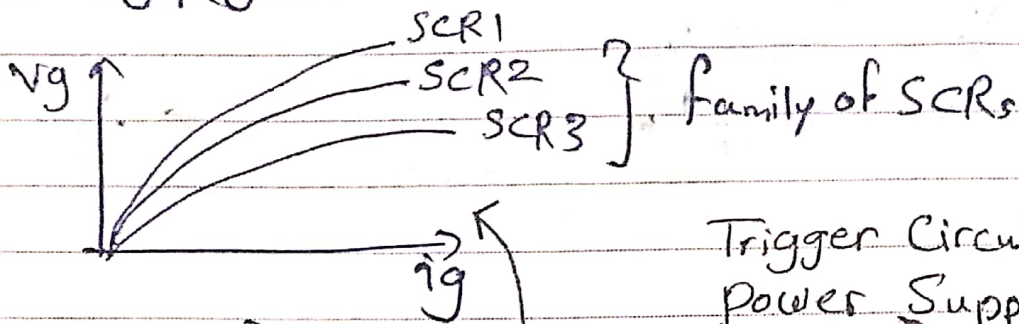
Usually given as a function of Temperature
↓
Ambient

25-6-2009

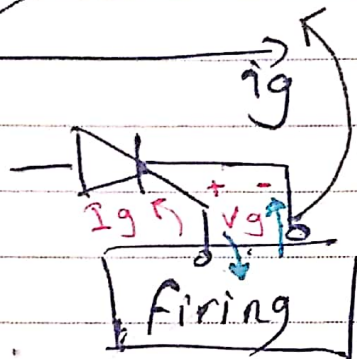
Gate characteristics:-

(A) Safe Operating Area SOA
(Temperature dependent, Given by the manufacturer).

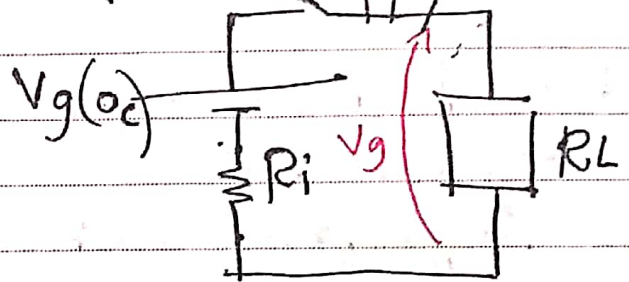
(B) $V_g = f(I_g)$ of the SCR itself (Manufacturer data).



(C)

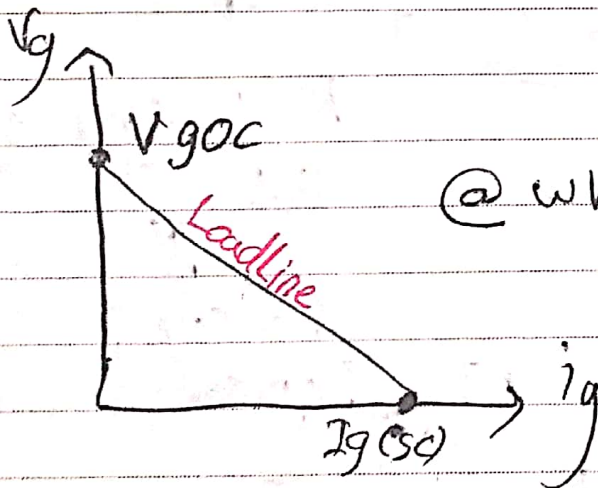


Trigger Circuit is Dc Power Supply



Triggering circuit power supply SCR

$$V_g(oc) = V_g + I_g R_i$$



@ when Open circuiting, $I_g = 0$
 $V_g = V_g(oc)$

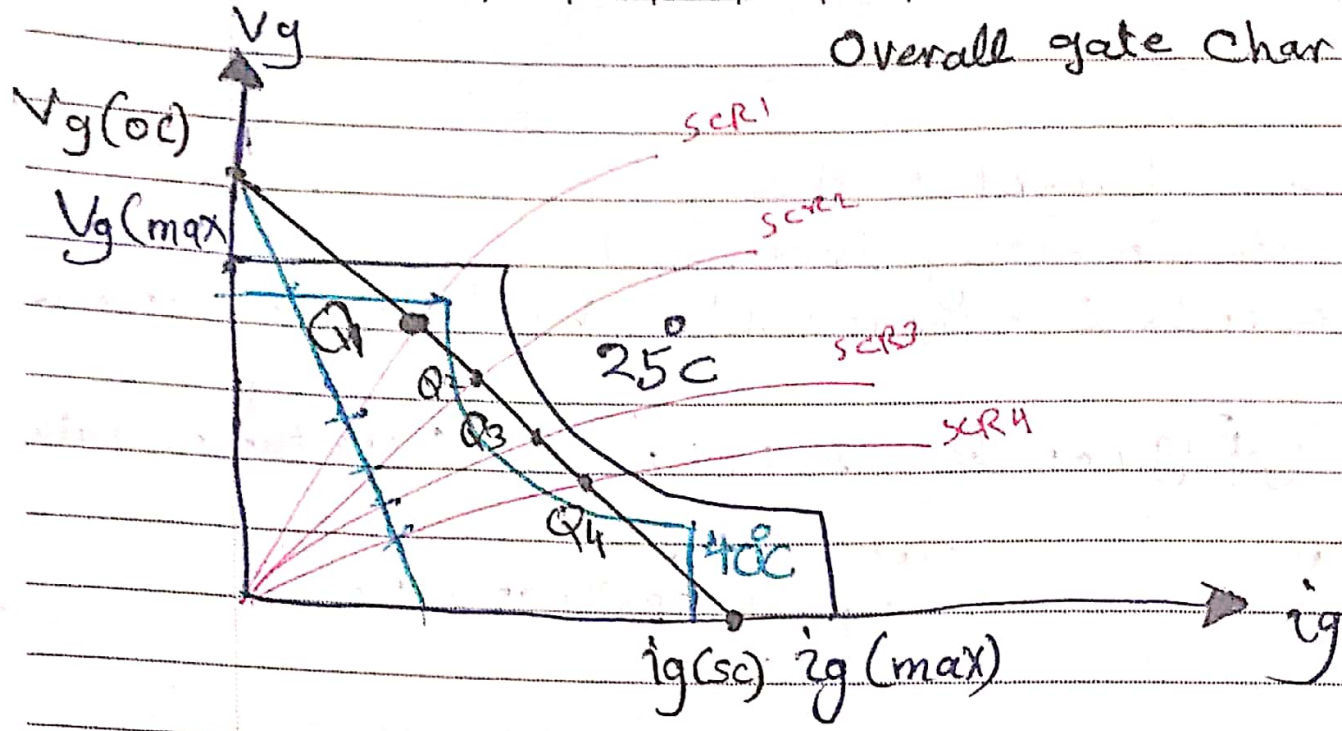
(b) when Short

circuiting

$$V_g = 0 \quad V_g(oc) = I_{g(sc)} R_i$$

$$I_{g(sc)} = \frac{V_g(oc)}{R_i}$$

Overall gate Char.

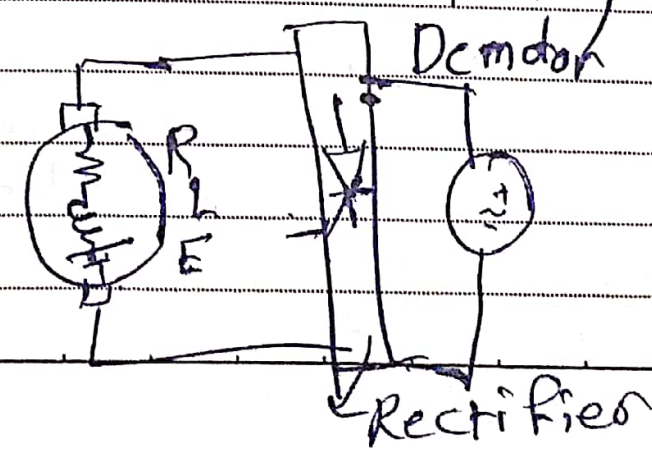
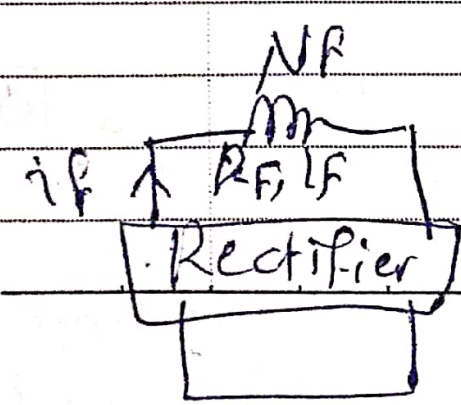
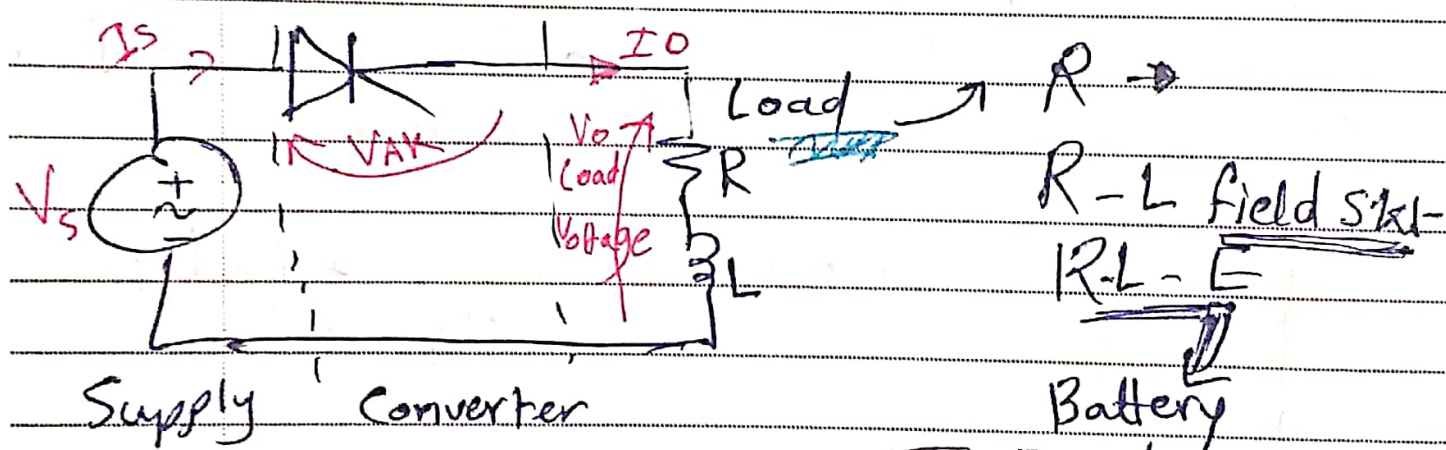


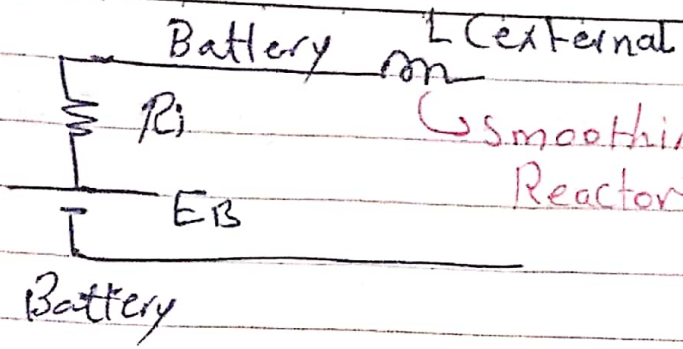
Circuits analysis :-

Rectifiers :- Controlled

to start with

Single phase - half wave Controlled Rectifier





Smoothing for the Current Reactor

Supply Side (Line Side)

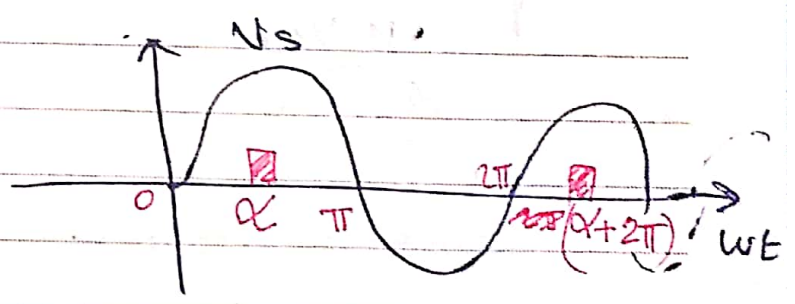
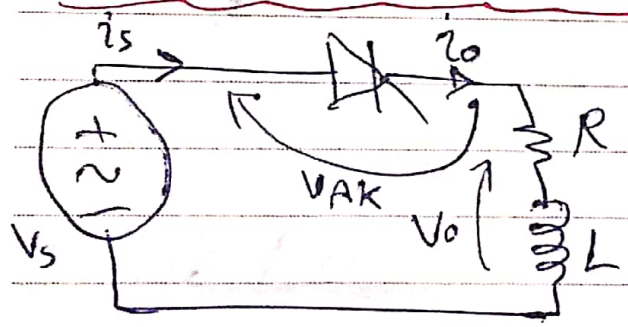
$V_s = V_m \sin(\omega t)$ \rightarrow 2 π f [f: Supply or fundamental frequency.]
 \hookrightarrow peak per phase (max).

$\tau = \frac{L}{R} \rightarrow$ Load Time Constant.

$\tau \rightarrow 0$ For pure Resistive load.

$\tau \rightarrow \infty$ for pure inductive load.

Single phase, half-wave, controlled Rectifier



α : The angle at which the SCR Triggered, measured with respect to the zero-crossing point, after which the SCR becomes forward Biased

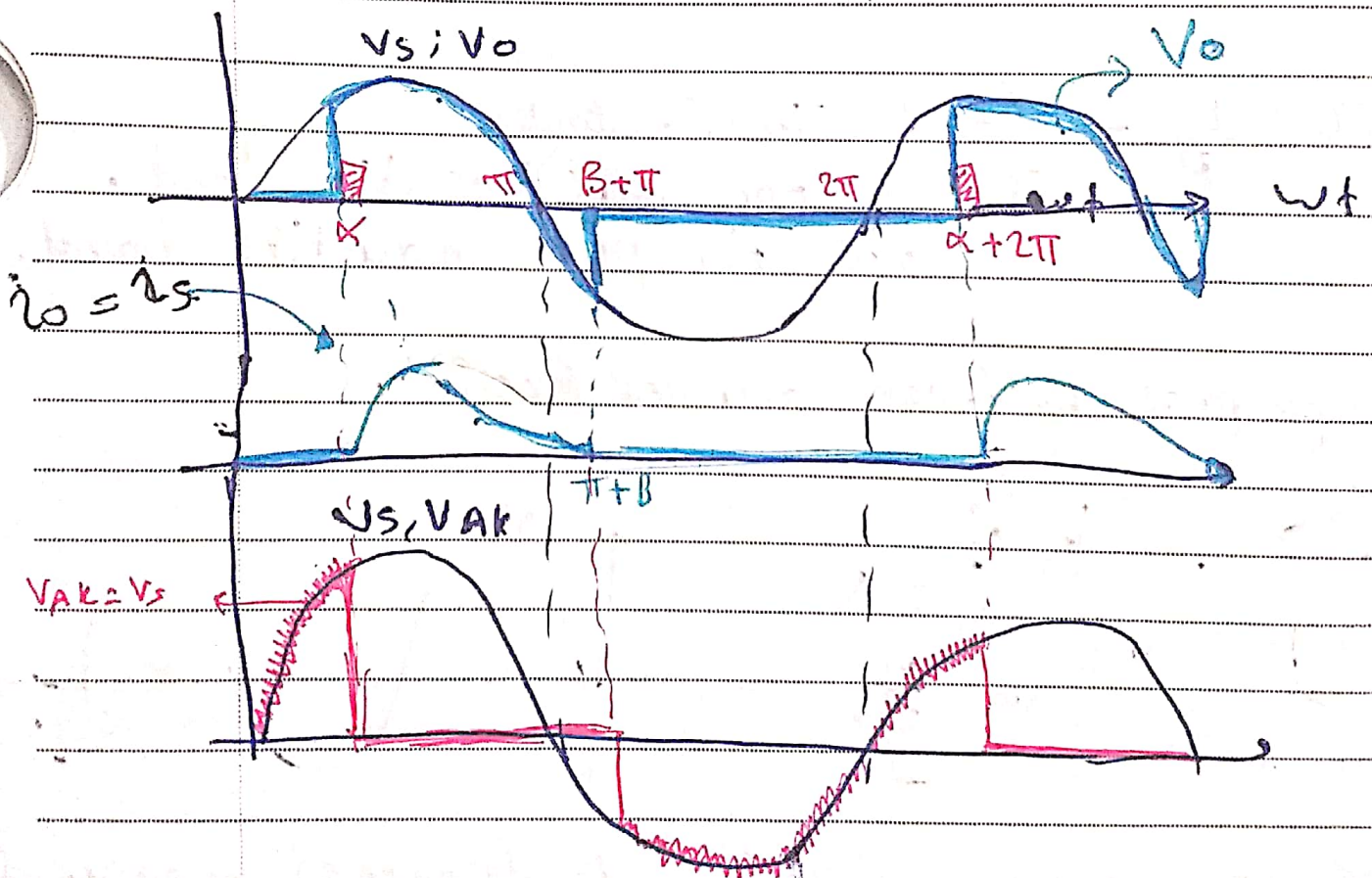
$0 \leq \omega t \leq \alpha$ SCR is Forward biased But Untriggered. \Rightarrow (Forward blocking Region) only small negligible leakage current is expected.

$$i_{out} + V_{AK} + (-V_s) = 0$$

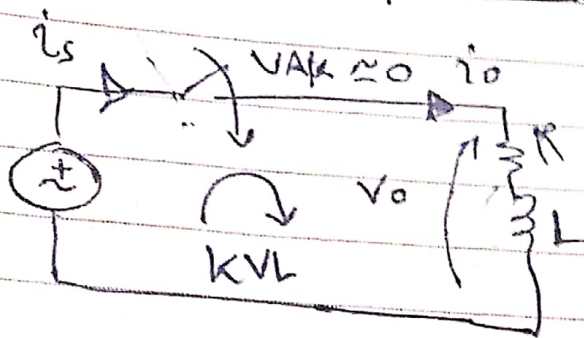
Zero \downarrow

$$i_{out} = 0$$

$$\therefore V_{AK} = V_s$$



$$\alpha \leq \omega t \leq \pi$$

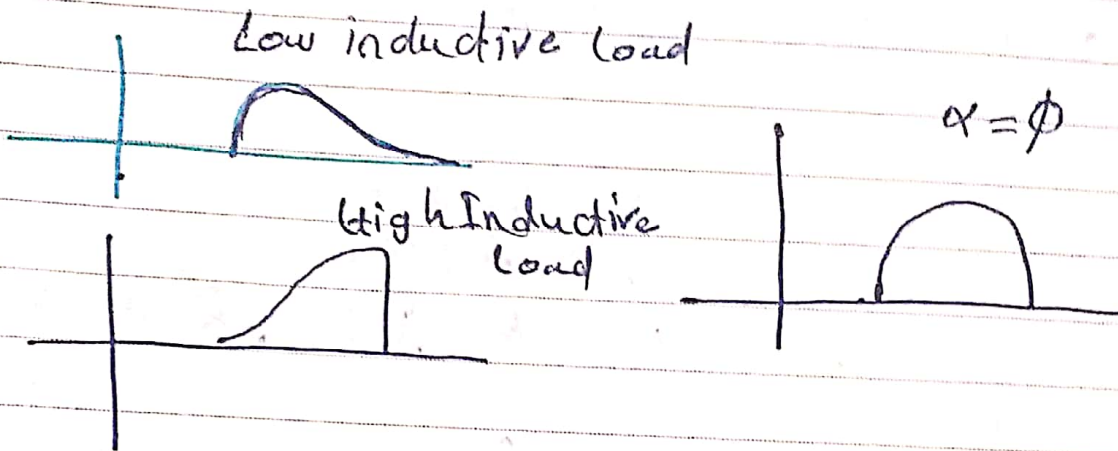


1st order non-linear Homog. D.E

$$V_o = V_m \sin \omega t = IR + L \frac{di}{dt}$$

$$i = i_s = i_o$$

Current shapes



PIV = V_m (peak inverse Voltage).

SCR Voltage Rating = $(2 \rightarrow 3)$ PIV
safety factor.

$V_s = 240V$ Rms

SCR Rating $(2 \rightarrow 3) \times \sqrt{2} \times 240$

$= (2-3) \cdot 339$

$680 \rightarrow 1020 V.$

Solution for i

$$V_s = V_m \sin \omega t = iR + L \frac{di}{dt}$$

$$i = \frac{V_{\max}}{Z} (\sin \omega t - \phi) \quad (+) \text{ steady state solution.}$$

$$A e^{-t/\tau}$$

$$i = \frac{V_{\max}}{Z} \sin(\omega t - \phi) + A e^{-t/\tau}$$

integration constant

natural response

steady state
solution

solution.

$$\frac{V_m}{Z} = I_m$$

Load Impedance
at supply freq.

$$Z = R + j\omega L = \sqrt{R^2 + (\omega L)^2} \angle \phi$$

$$\phi = \tan^{-1} \left(\frac{\omega L}{R} \right) \quad \# \text{ (load phase angle at supply freq.)}$$

$$i = I_m \sin(\omega t - \phi) + A e^{-t/\tau}$$

$$\tau = L/R$$

$$e^{-t/\tau} = e^{-\frac{Rt}{L}} = e^{-\frac{R\omega t}{\omega L}} = e^{-\frac{R}{\omega L} \omega t} = e^{-\phi \omega t}$$

$$\phi = \frac{-R}{\omega L} = \frac{-1}{\tan \phi} = -\cot \phi$$

Apply I.C.C, where at $\omega t = \alpha \rightarrow i = 0$

$$0 = \text{Im}(\sin \alpha - \phi) + A e^{\rho \alpha}$$

$$\therefore A = -\text{Im}(\sin \alpha - \phi) * e^{-\rho \alpha}$$

$$\therefore \boxed{i = \text{Im} \sin(\omega t - \phi) - \text{Im}(\sin \alpha - \phi) e^{-\rho \alpha} e^{\rho \omega t}}$$

to Evaluate β :-

β = Extinction angle

The angle that takes the forward current to drop to $i \approx 0$ after the point at which SCR becomes R. Biased.

Applying final current condition

at $\omega t = \pi + \beta$, $i = 0$

$$0 = \text{Im} \sin(\pi + \beta - \phi) - \text{Im} \sin(\alpha - \phi) e^{-\rho \alpha} e^{\rho(\pi + \beta)}$$

$$0 = -\sin(\beta - \phi) - \sin(\alpha - \phi) e^{-\rho(\pi - \alpha)} e^{\rho \beta}$$

Constant

assuming

$$k = -\sin(\alpha - \phi) e^{\rho(\pi - \alpha)}$$

$$\therefore 0 = -\sin(\beta - \phi) + k e^{\rho\beta}$$

How to evaluate β in such a
Transcendental Equation.

Solution.

- a) Trial and Error.
- b) Graphical Solution.
- c) Numerical Solution, (ex. Newton Raphson method).

assuming

$$k = -\sin(\alpha - \phi) e^{\rho(\pi - \alpha)}$$

⊖	$\alpha = \phi$
⊕	$\alpha < \phi$
⊖	$\alpha > \phi$

$$\therefore 0 = -\sin(\beta - \phi) + k e^{\rho\beta}$$

How to evaluate β in such a Transcendental Equation

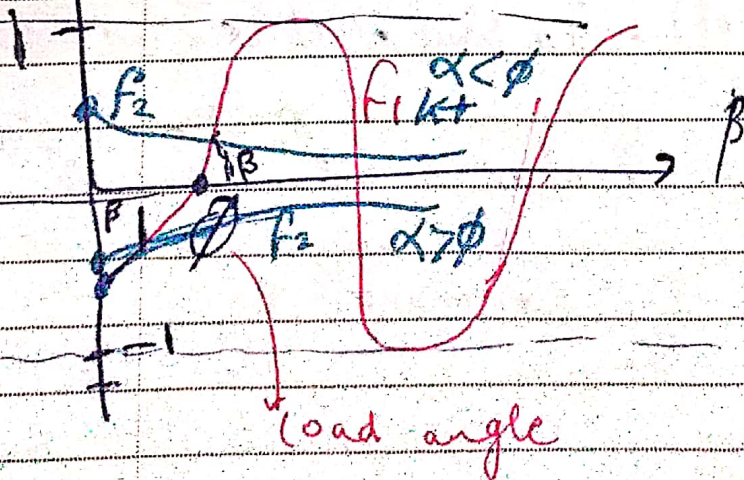
Solution.

- Trial and Error.
- Graphical Solution.
- Numerical Solution. (ex. Newton Raphson method).

$$\sin(\beta - \phi) = k \cdot e^{\rho\beta}$$

$$f_1(\beta) = f_2(\beta)$$

f_1, f_2



$\beta < \phi$
 k is negative

$\beta > \phi$
 k is positive

$\alpha = 30^\circ$

$\phi = 45^\circ$

(a) $\beta = 30^\circ$

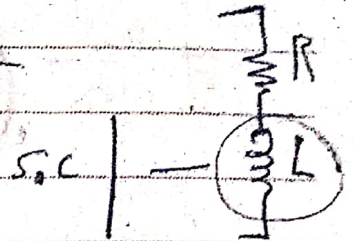
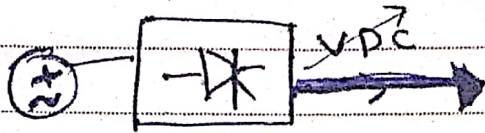
(b) $\beta = 45^\circ$

(c) $\beta = 46^\circ$

(d) $\beta = 32^\circ$

$V_{DC} = V_{AV} = P$

$I_{DG} = I_{av} = \frac{V_{av}}{R}$



Ripple F_v R_{Fi}
 Usually ($R_{Fi} \leq R_{Fv}$)

R_{Fi} for inductive ~~0.65~~

$R_{Fi} =$ (a) 0.65

(b) 0.4

(c) 0.8

(d) 0

$$V_o(\text{av}) = \frac{1}{2\pi} \int_{\alpha}^{\pi+\beta} V_o(t) d\omega t = \int_{\alpha}^{\pi+\beta} V_m \sin \omega t d\omega t$$

$$V_o(\text{rms}) = \frac{V_m}{2\pi} (-\cos \omega t) = \frac{V_m}{2\pi} (\cos \alpha + \cos \beta)$$

↳ to calculate R_{Fi}

$$V_{orms} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi+\beta} (V_o)^2 d\omega t}$$

$$= \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi+\beta} V_m^2 \sin^2 \omega t \, d\omega t} = \sqrt{\frac{V_m^2}{2\pi} \left[\frac{1 - \cos 2\omega t}{2} \right]}$$

$$= \sqrt{\frac{V_m^2}{4\pi} \left(\omega t - \frac{\sin(2\omega t)}{2} \right)}$$

$$= \sqrt{\frac{V_m^2}{4\pi} \left[\underbrace{\pi + \beta - \alpha}_{\text{rad}} - \frac{1}{2} \sin(2\beta) + \frac{1}{2} \sin(2\alpha) \right]}$$

Pure Resistive Diode H.w Rectifier $\alpha=0$
 $\beta=0$

$$R.F.V = \frac{\sqrt{V_{rms}^2 - V_{oav}^2}}{V_{oav}}$$

$R_f(i,0) = \frac{V_{rms}}{I_{or} R}$ (its not pure sinusoidal).

$$V_o = \frac{120}{\sqrt{2}} + 120 \left[\sum_{n=2,3,4} \frac{1}{n} \sin(n\omega t) \right] \dots$$

$$V_{av} \quad V(1) = \frac{120}{\sqrt{2}} \sin(\omega t) = \frac{120}{\sqrt{2}}$$

$$V(2) = \frac{120}{2\sqrt{2}} \quad \dots \quad V(3) = \frac{120}{3\sqrt{2}}$$

$$V(3) = \frac{120}{3\sqrt{2}}$$

$$i_2 = \frac{120}{2\sqrt{2} Z(2)}$$

$$Z_2 = \sqrt{R^2 + (2\omega L)^2}$$

$$i_3 = \frac{120}{3\sqrt{2} Z(3)}$$

$$Z_3 = \sqrt{R^2 + (3\omega L)^2}$$

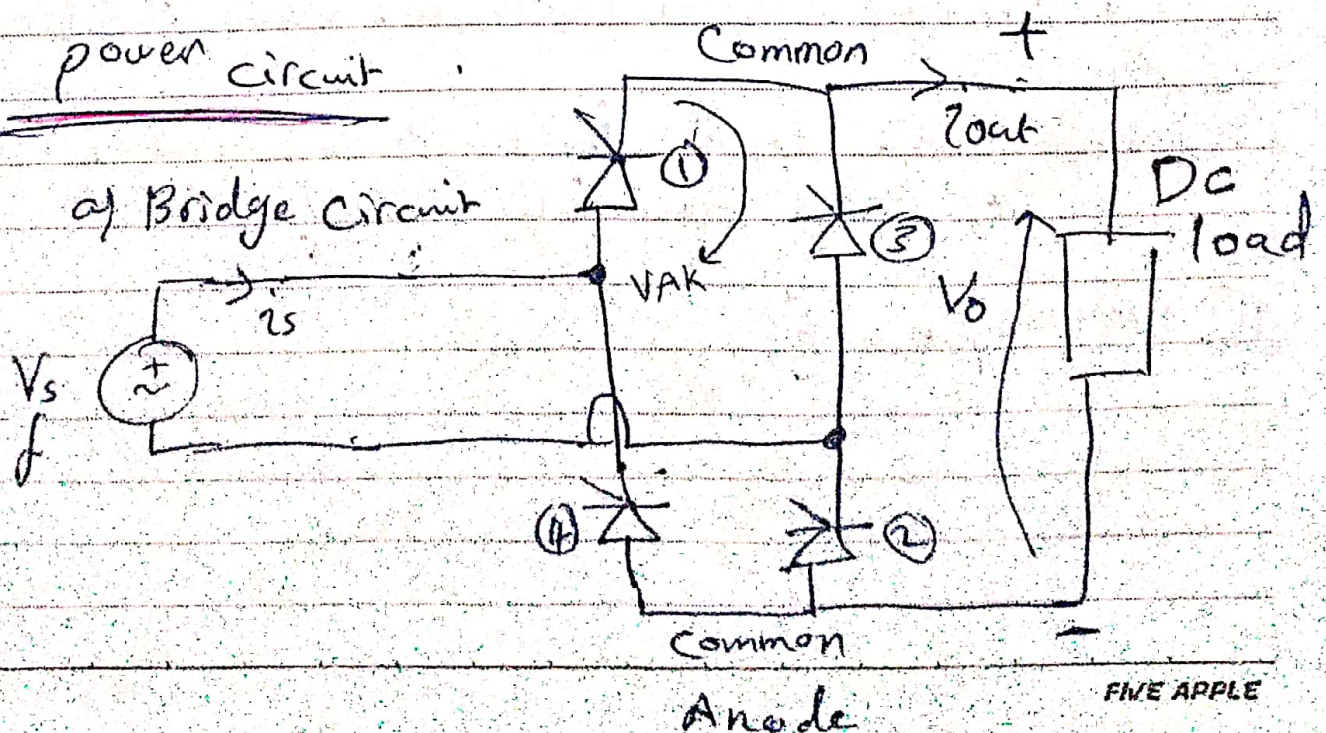
$$I_{rms} = \sqrt{I_{av}^2 + I_{(1)}^2_{rms} + I_{(2)}^2_{rms} + I_{(3)}^2_{rms}}$$

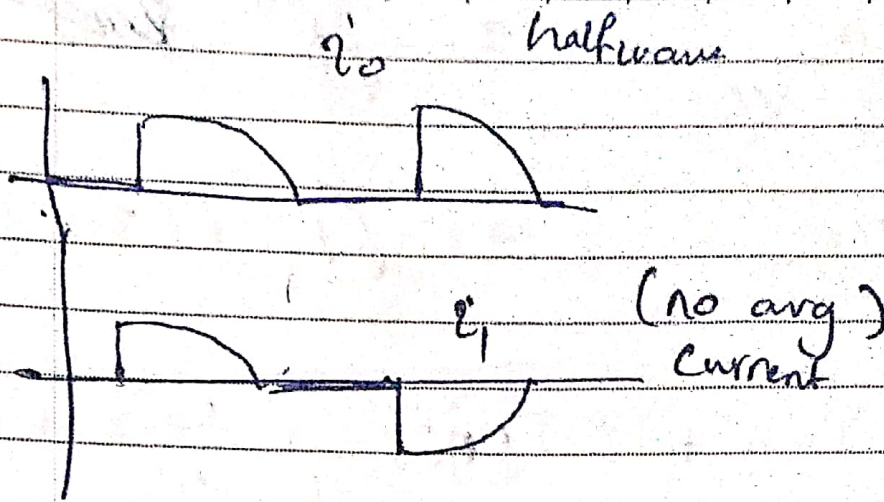
~~*****~~

$\beta = 0$ for pure R

$$V_o(\omega) = \frac{V_m}{2\pi} (1 + \cos\alpha)$$

* single phase - full wave, fully - controlled Rectifier





• 1/7/2019.

Full-wave Rectifier : Merits over half wave rectifier (Zero Average)

- ① The Line (Supply or Input) current is AC
This will avoid any problems connected to the average Dc components.
- ② More power can be generated with only small extra cost compared to half wave rectifier
- ③ Less Distortion to the Input line Current.
(Less total harmonic distortion) ~~Current~~
THDF
- ④ Better input PF

SCR 1 and SCR 2 should be Simultaneously triggered at α
 Same as for SCR 3 and SCR 4

at $(\omega t = \alpha + \pi)$

$0 \leq \omega t \leq \pi$ SCR 1 and 2 are Forward B
 SCR 3 and 4 are Reverse B

$\alpha \leq \omega t \leq \pi + \beta$

KVL

$$V_s - V_{AK1} - V_{out} - V_{AK2} = 0$$

\downarrow Neg \downarrow Neg.

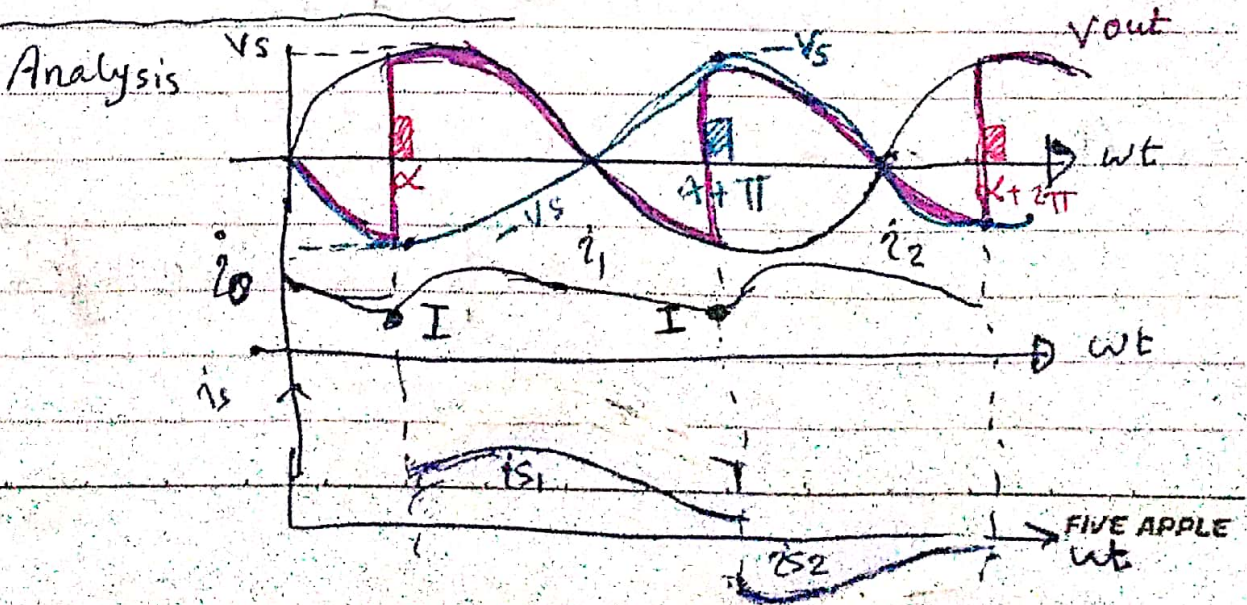
$V_o = V_s$

$\pi \leq \omega t \leq 2\pi$ - SCR 1 and SCR 2 RB
 SCR 3 and SCR 4 FB

$\pi + \alpha \leq \omega t \leq 2\pi + \beta$ KVL

$$V_s + V_o = 0$$

$V_o = -V_s$



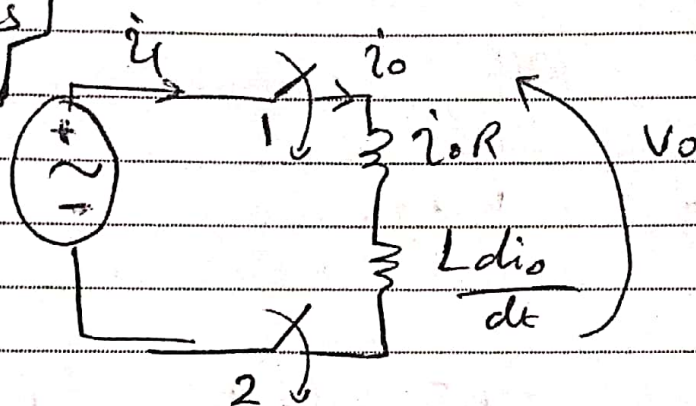
Full wave Rectifier

The expected current Wave form for Load can be

a- Continuous (Continuous Current mode CCM).

assume
Continuous
C.C.M

b- Discontinuous (Discontinuous DCM)



$$V_s = i_o R + L \frac{di_o}{dt}$$

$$i_o = I_m \sin(\omega t - \phi) + A e^{p\omega t}$$

Dec i at $\omega t = \alpha$ $i_o = I$

$$I = I_m \sin(\alpha - \phi) + A e^{p\alpha}$$

$$A = \left[I - \frac{I_m}{m} \sin(\alpha - \phi) \right] e^{-p\alpha}$$

$$i_o = I_m \sin(\omega t - \phi) + \left[I - \frac{I_m}{m} \sin(\alpha - \phi) \right] e^{-p\omega t}$$

apply final current condition F.C.C

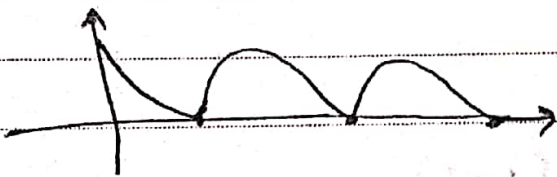
where at $\omega t = \pi + \alpha$ $i = I$

$$I = I_m \sin(\pi + \alpha - \phi) + (I - I_m \sin(\alpha - \phi)) e^{-\rho \alpha} e^{\rho(\pi + \alpha)}$$

$$I = -I_m \sin(\alpha - \phi) + [I - I_m \sin(\alpha - \phi)] e^{\rho \pi}$$

$$I [1 - e^{\rho \pi}] = -I_m \sin(\alpha - \phi) [1 + e^{\rho \pi}]$$

$$I = \underbrace{(-I_m \sin(\alpha - \phi))}_{\text{C.C.M}} \cdot \left[\frac{1 + e^{\rho \pi}}{1 - e^{\rho \pi}} \right]_{\text{D.C.M}}$$



when $I = 0$
critical case

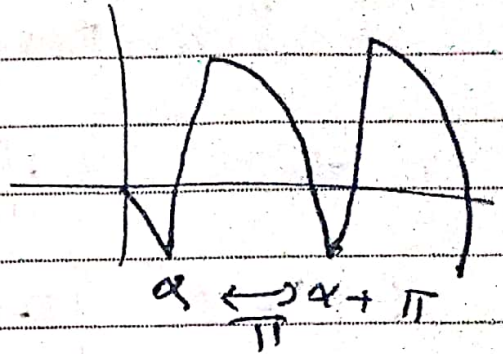
$$I \begin{cases} 0 & \alpha = \phi \rightarrow \text{critical case} \\ + & \alpha < \phi \rightarrow \text{C.C.M} \\ - & \alpha > \phi \rightarrow \text{D.C.M} \end{cases}$$

I will never be negative practically.

CCM

$$V_o(\text{av}) = \frac{1}{\pi} \int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t) d\omega t$$

$$= \frac{V_m}{\pi} \left[-\cos \omega t \right]_{\alpha}^{\alpha+\pi}$$



$$= \frac{V_m}{\pi} \left[-\cos(\pi + \alpha) + \cos \alpha \right]$$

$$V_o(\text{av})_{\text{CCM}} = \frac{2V_m}{\pi} \cos \alpha$$

$$I_o(\text{av}) = V_o(\text{avg}) / R$$

$$V_o(\text{rms}) = \frac{V_m}{\sqrt{2}} = V_s(\text{rms})$$

$$V_o(\text{rms})_{\text{CCM}} = \sqrt{\frac{V_m^2}{2\pi} \left(\pi + \beta - \alpha - \frac{1}{2} \sin(2\beta) + \frac{1}{2} \sin 2\alpha \right)}$$

~~$V_o(\text{rms})$~~ Special case

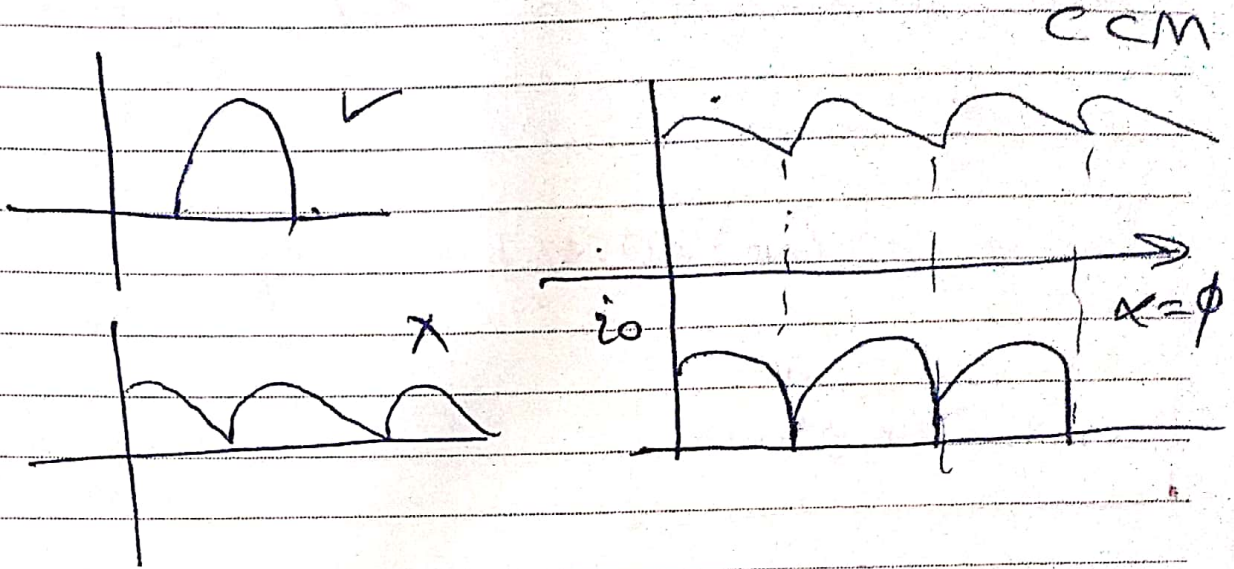
$\alpha = \phi \rightarrow$ critical case

$$i_o = I_m \sin(\omega t - \phi) + [I - I_m \sin(\alpha - \phi)]$$

$$e^{-s\alpha} e^{s\omega t}$$

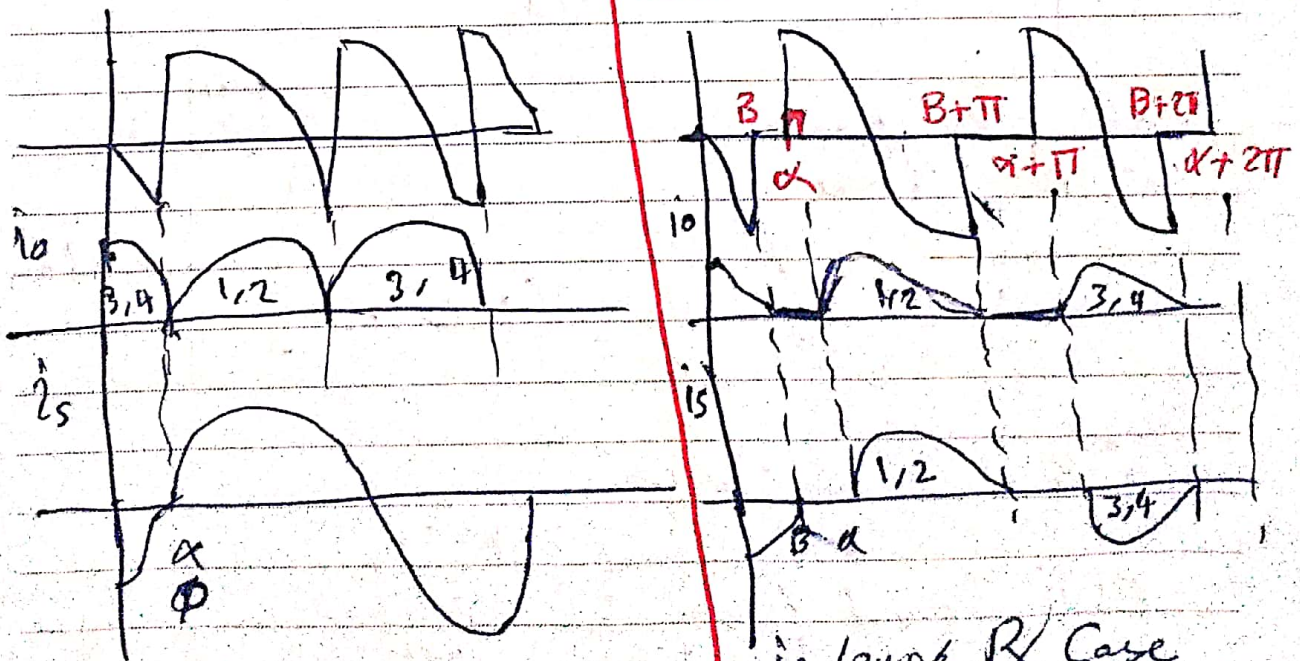
No. _____

$$i_o = I_m \sin(\omega t - \phi) \quad I = 0$$



in Critical Case (CT)

$$V_o(\text{av}) = \frac{2V_m}{\pi} \cos \alpha$$



P Fm = cos alpha } pure sin
 THD F = 0
 is

in pure R Case
 R > 0
 $\phi = \tan^{-1} \frac{\omega L}{R}$
 $\alpha \rightarrow \alpha \neq 0$

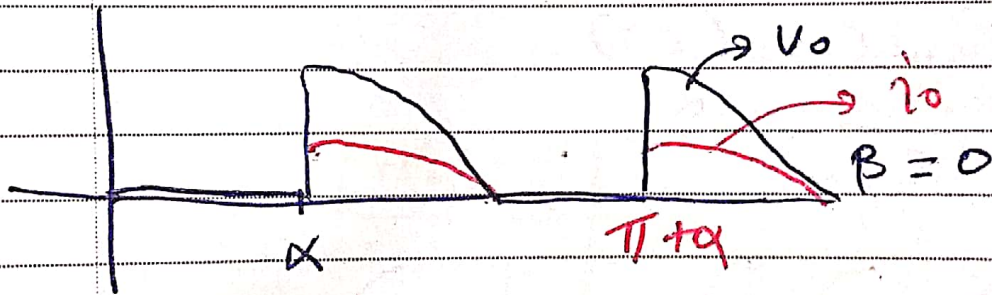
In pure Resistive case
 $R \neq 0 \quad L \rightarrow X = 2\omega FL = 0 \quad \phi = \tan^{-1} \frac{\omega L}{R} \rightarrow 0$

$$f = \frac{-R}{\omega L} \rightarrow -\infty$$

$$i_o = I_m \sin(\omega t) + 0 \Rightarrow I_m = \frac{V_m}{R}$$

$$i_o = I_m \sin \omega t$$

$$V_o = V_m \sin \omega t$$



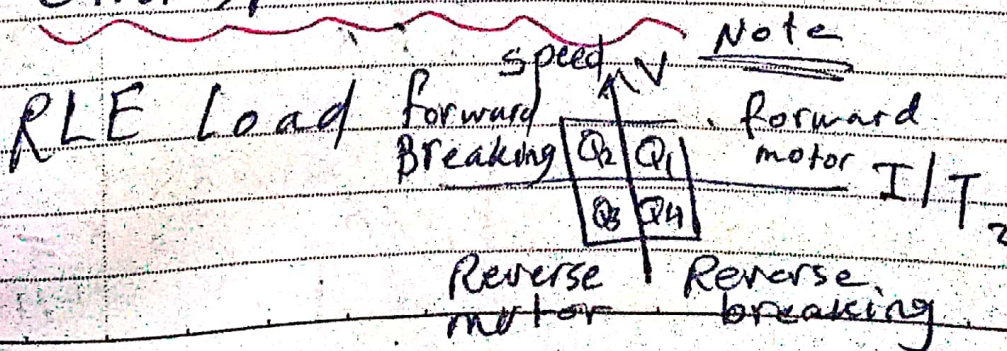
Mode of operation
 CCM :- Never

CT Case : $\alpha = \beta = 0$

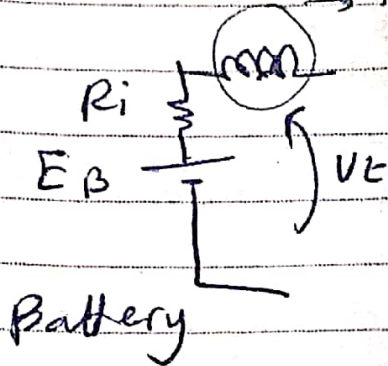
DCM: $V_o(\text{avg}) = \frac{V_m}{\pi} (1 + \cos \alpha)$

pure Resistive loading condition

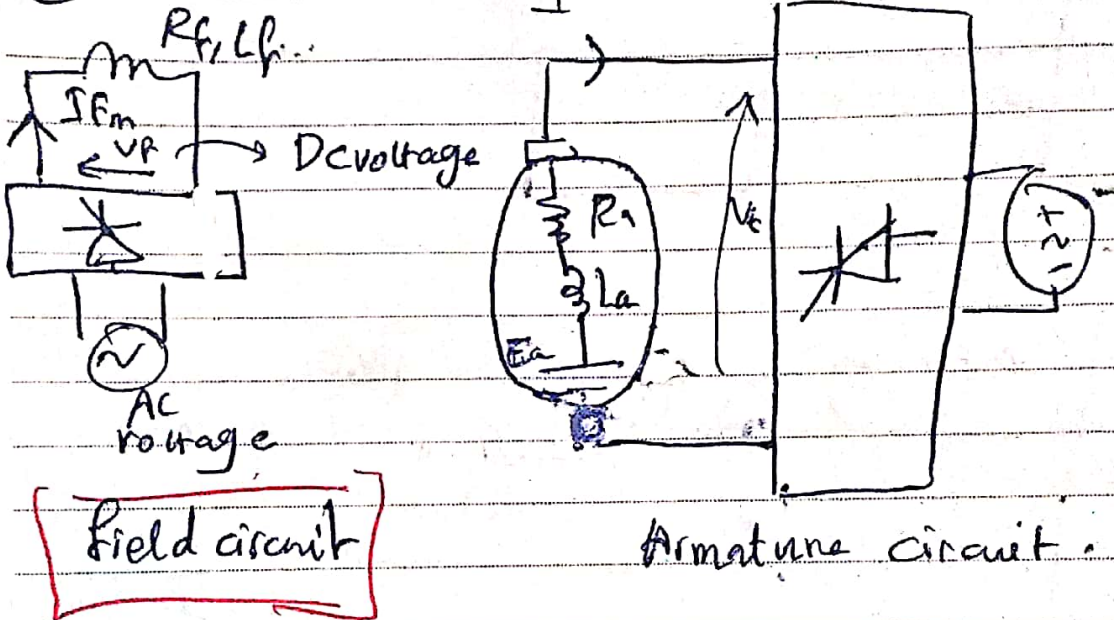
Other Special Case



(a) Battery with inductive filter
 → to Reduce harmonics



(b) Dc motor



Field circuit

Armature circuit

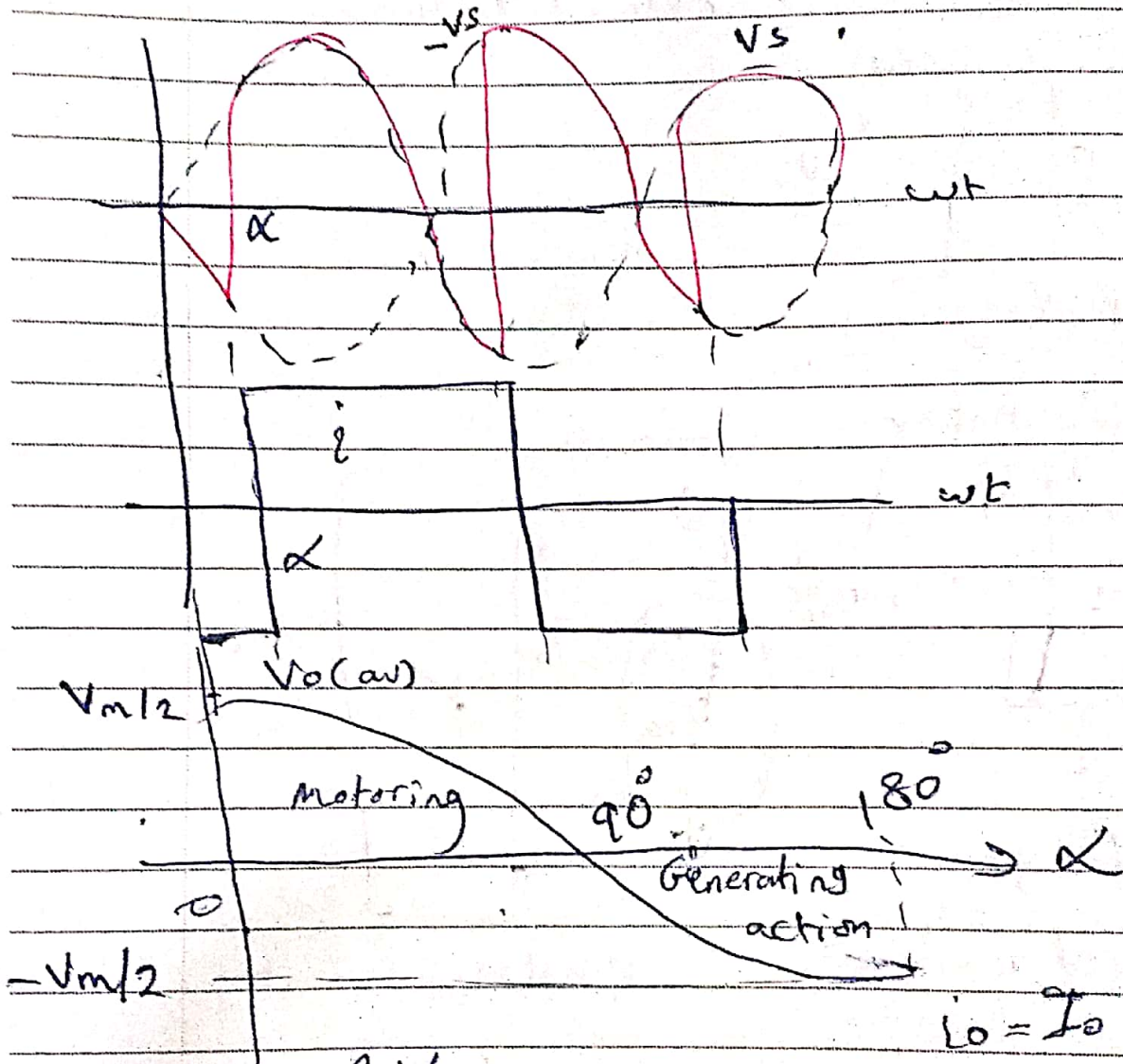
assume that the armature Inductance is so large such that the load current is almost ripple free or (virtually constant).

$$\frac{L \frac{di_a}{dt} \rightarrow 0$$

mode of Operation : CCM pure DC

$$V_o (av) = \frac{2 V_m \cos \alpha}{\pi} \quad 0 \leq \alpha \leq 180^\circ$$

3-7-2018



$$V_o(av) = \frac{2 V_m}{\pi} \cos \alpha$$

CCM

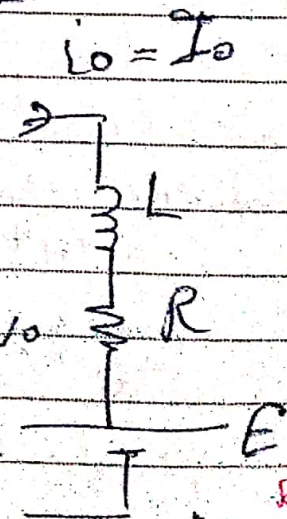
$$V_o(av) = R I_o + 0 + E$$

torque development

$$T_q = K_a \phi I_a = 1/c_a I_o$$

speed dependent

$$E = k \phi \omega_m = K_e \phi N_m$$



$$\omega_m = \frac{2\pi \cdot N_m}{60}$$

$$P_{\text{shaft}} = \omega_m \cdot T_{\text{sh}}$$

$$V_o(\text{av}) \rightarrow \frac{2 V_m \cos \alpha}{\pi} = R I_o + E$$

$$P_{\text{out}} = \underbrace{I_o^2 R}_{\text{copper loss}} + E I_o \quad \rho_o = I_{a0} V_o(\text{av})$$

↑ the control parameter

↓ developed Armature power to be converted into mechanical power.

$$= I_o (I_o R + E) = \bar{I}_{o(\text{av})} V_o(\text{av})$$

★ Load Voltage Ripple:-

$$\Delta V_o(\text{av}) = \frac{2 V_m \cos \alpha}{\pi}$$

$$\Delta V_o(\text{rms}) = V_s(\text{rms})$$

$$\Delta R F_{v0} = \dots$$

★ Load current Ripple = $R \cdot f_{i0} = 0$

★ Supply voltage (pure THDF = 0 sinusoidal)

★ Supply current

↓
(square wave)

THDF = 48.43% fixed irrespective of α

I_s harmonics only odd due to its symmetry

1st, 3rd, 5th, ... harmonics

Impurities.

useful fundamental

$\psi(n)$

HW show that $I_{supply} = \sum_{n=1,3,5,\dots}^{\infty} \frac{4 I_0}{n\pi} \sin(n\omega t - \phi_n)$

and prove the number 48.43%

$$C(n) = \sqrt{A(n)^2 + B(n)^2}$$

$$THDF = \frac{\sqrt{I_s^2(rms) - I_s(F)^2}}{I_s(F)}$$

$$I_s(rms) = I_0 = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} I_0^2 d\omega t}$$

$$I_s(rms) \approx \sqrt{\left(\sum_{n=1,3,5,\dots} \frac{4 I_0}{n\pi\sqrt{2}} \right)^2}$$

$$I_s(F)_{rms} = \frac{4 I_0}{\pi\sqrt{2}}$$

Input PF

$$PF_{in} = \frac{P_{in \text{ real}}(\omega)}{S_{in}(VA)} = \frac{P_o(\omega)}{S_{in}(VA)}$$

$$= \frac{I_o V_o(\text{av})}{V_s(\text{rms}) \cdot I_s(\text{rms})} = \frac{V_o(\text{av})}{V_s(\text{rms})}$$

alternatively

$$V_s = V_m \sin(\omega t) + 0 + 0$$

$$I_s = \frac{4 I_o}{\pi} \sin(\omega t - \alpha) + \frac{4 I_o}{3\pi} \sin(3\omega t - 3\alpha)$$

$$+ \frac{4 I_o}{5\pi} \sin(5\omega t - 5\alpha) + \dots$$

$$P_s = \left(\frac{V_m}{\sqrt{2}} \cdot \frac{4 I_o}{\pi \sqrt{2}} \neq 0 + 0 + \dots \right) \cos(\theta_{v1} - \theta_{i1})$$

$$P_s = V_s(\text{rms}) \cdot I_{sF}(\text{rms}) \cdot \cos(\alpha) \psi(1)$$

$$PF = \frac{V_s(\text{rms}) \cdot I_{sF}(\text{rms}) \cdot \cos(\alpha) \psi(1)}{V_s(\text{rms}) \cdot I_s(\text{rms})}$$

$$\frac{I_{sF}(\text{rms})}{I_s(\text{rms})} \cdot \cos \psi(1) \quad \text{displacement factor}$$

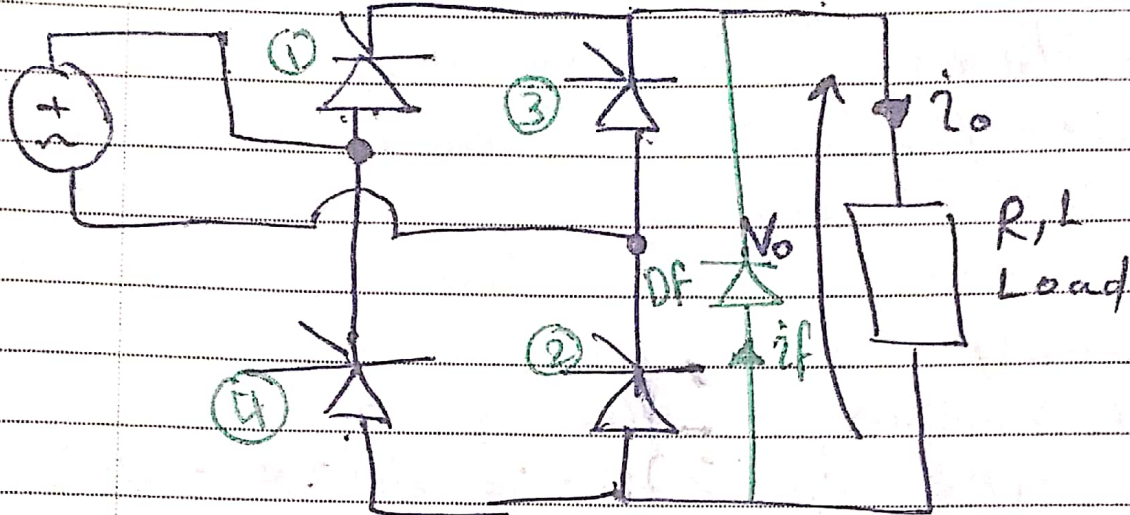
$$\downarrow < 1 \quad \downarrow < 1$$

$$\text{distortion factor } DTF \times DPF < 1$$

PF, $\cos \phi$ DTF, DPF

7/7 10/11

Semi controlled Rectifier



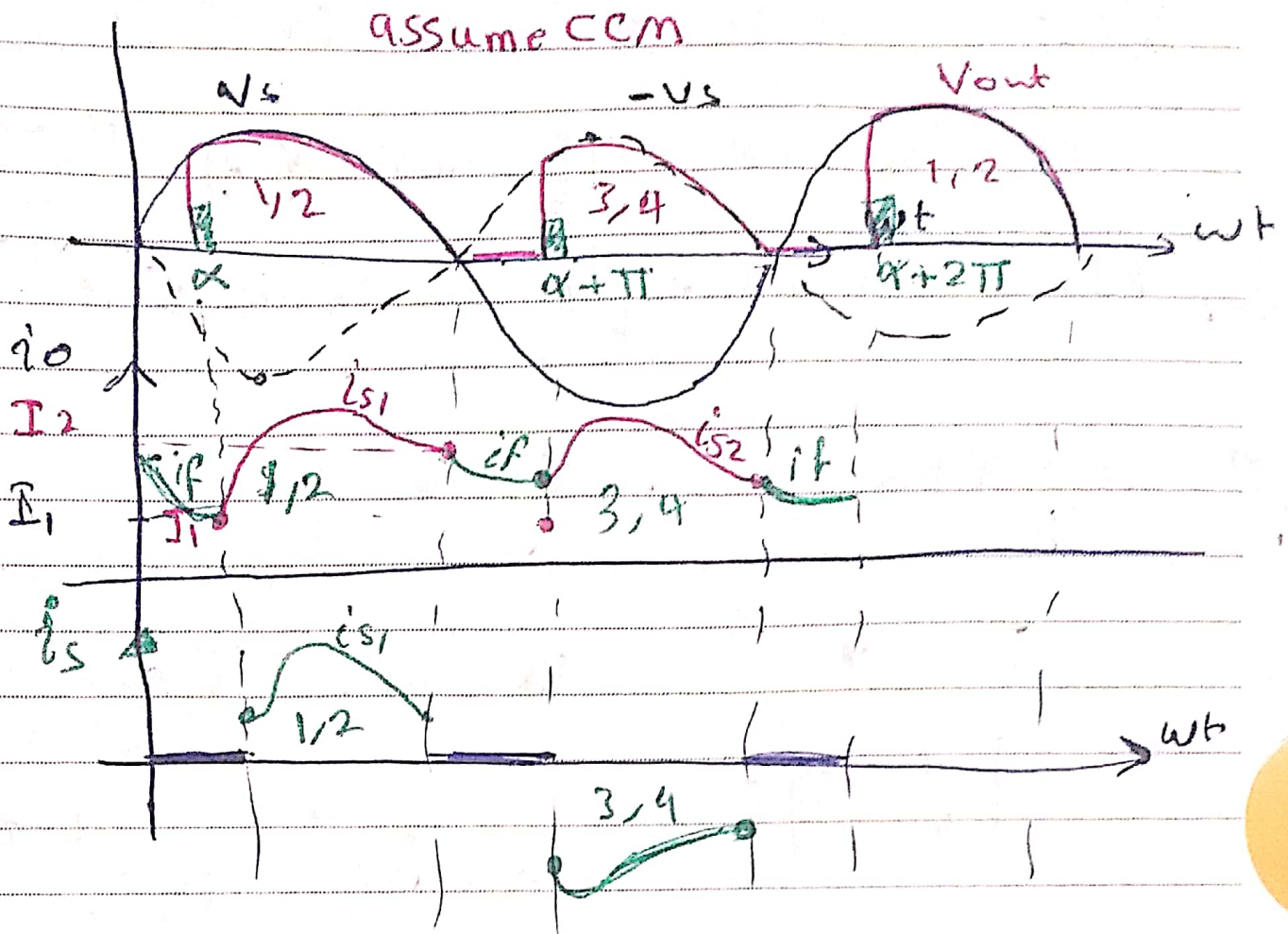
advantage:-

to clip any negative part of the V_o

This Results in higher average Voltage for the same value of α .

* disadvantage: there is no possibility for Negative Output Voltage in this case. No possible motor control in the 4 quadrants of T_q / speed characteristics.

Df, free wheeling Diode
if existed \Rightarrow semi-controlled
if not \Rightarrow fully controlled.



$$\alpha \leq \omega t \leq \pi$$

SCRs : 1, 2, F.B and Triggered

$$v_o = V_m \sin \omega t = i_s R + L \frac{di_s}{dt}$$

$$i_s = I_m \sin(\omega t - \phi) + A e^{p \omega t}$$

ICCB at $\omega t = \alpha$, $i_s = I_1$

$$\therefore I_1 = I_m \sin(\alpha - \phi) + A e^{p \alpha}$$

$$A = [I_1 - I_m \sin(\alpha - \phi)] e^{-p \alpha}$$

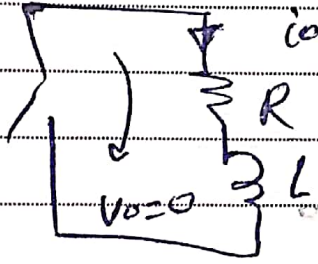
$$i_s = I_m \sin(\omega t - \phi) + [I_1 - I_m \sin(\alpha - \phi)] e^{-p \omega t} e^{p \alpha}$$

FCC:- at $\omega t = \pi$ $i_{s1} = I_2$

$$i_2 = I_m \sin(\pi - \phi) + [I_1 - I_m \sin(\alpha - \phi)] e^{-\rho \alpha} e^{-\rho \pi}$$

$$\textcircled{1} \Rightarrow I_2 = I_m \sin \phi + [I_1 - I_m \sin(\alpha - \phi)] e^{\rho(\pi - \alpha)}$$

$\pi \leq \omega t \leq \pi + \alpha$ Freewheeling Area



$$0 = i_o R + L \frac{di_o}{dt}$$

$$i_o = B e^{\rho \omega t} = B e^{-t/\tau}$$

2cc:-

at $\omega t = \pi$, $i_o = I_2$

$$I_2 = B e^{\rho \pi} \rightarrow$$

$$B = I_2 e^{-\rho \pi}$$

$$i_o = B I_2 e^{-\rho \pi} e^{\rho \omega t}$$

FCC

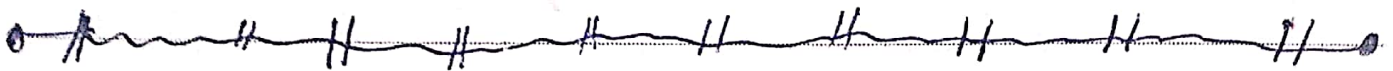
at $\omega t = \pi + \alpha$
 $i_o = I_1$

$$I_1 = I_2 e^{-\rho\pi} e^{\rho(\pi+\alpha)} = I_2 e^{\rho\alpha} \quad (2)$$

$$I_2 = I_m \sin \phi [I_1 - I_m \sin(\alpha - \phi)] e^{+\rho(\pi - \alpha)}$$

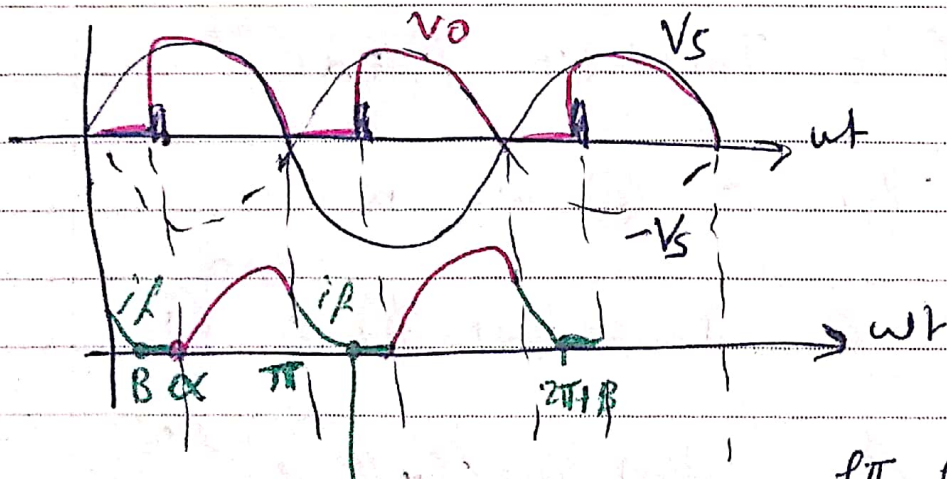
$$I_1 =$$

$$I_2 =$$



- if $I_1 > I_H \rightarrow$ CCM
- if $I_1 \approx I_H \rightarrow$ Critical Case
- if $I_1 < I_H \rightarrow$ DCM

DCM



$i_o = i_r$ (Free Wheeling Area) = $I_2 e^{-\rho\pi} e^{\rho(\pi+B)}$ $f\pi$ pwt
 at $wt = \pi + B$ $i_r = I_H$ Holding Current

$I_H = I_2 e^{-\rho\pi} e^{\rho(\pi+B)}$ $I_H = I_2(\rho_{cm}) e^{\rho B}$
(DCM)

$$P_B = I_H$$

$$e = \frac{I_H}{I_2(\text{DCM})}$$

$$P_B = \ln \left(\frac{I_H}{I_2(\text{DCM})} \right)$$

$$\beta = \frac{1}{P} \ln \left(\frac{I_H}{I_2(\text{DCM})} \right) \rightarrow \textcircled{3}$$

$$\alpha = \beta \rightarrow \text{(critical case)}$$

$$\beta > \alpha \rightarrow \text{CCM}$$

$$\beta < \alpha \rightarrow \text{DCM}$$

For DCM

$$i_{s1} = I_m \sin(\omega t - \phi) + A e^{p\omega t}$$

$$i_{s1} = 0 \text{ at } \omega t = \alpha$$

$$0 = I_m \sin(\omega t - \phi) e^{p\alpha} + A$$

$$A = -I_m \sin(\alpha - \phi) e^{-p\alpha}$$

$$i_{s1} = I_m \sin(\omega t - \phi) + I_m \sin(\alpha - \phi) e^{p\alpha} e^{p\omega t}$$

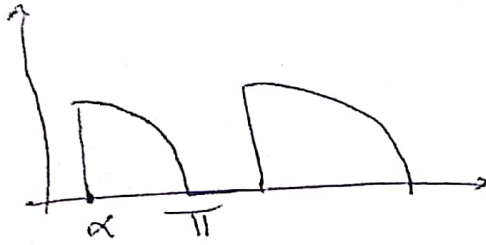
at $\omega t = \pi$, $I_2(\text{DCM})$

No.

$$I_2(Dcm) = \text{Im} \sin(\pi - \phi) - \text{Im} \sin(\alpha - \phi) e^{-\rho\alpha} e^{\rho\pi}$$
$$= \text{Im} \sin \phi - \text{Im} \sin(\alpha - \phi) e^{-\rho\alpha} e^{\rho\pi}$$

$$I_2(Dcm) = \text{Im} [\sin \phi - \sin(\alpha - \phi)] e^{\rho(\pi - \alpha)}$$

$$* V_{o(av)}_{S.C.R} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin(\omega t) d\omega t$$



$$= \frac{V_m}{\pi} [-\cos \omega t]_{\alpha}^{\pi}$$

$$V_{o(av)}_{S.C.R} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

For both CCM DCM
Semi controlled

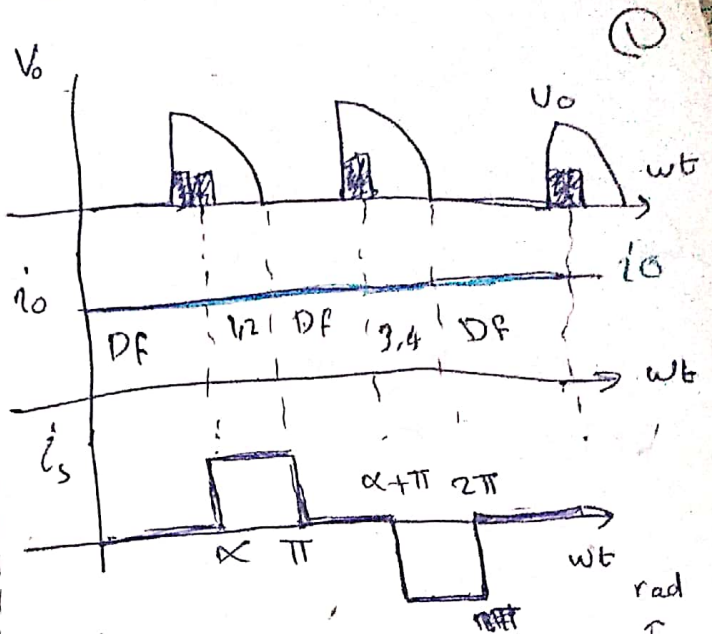
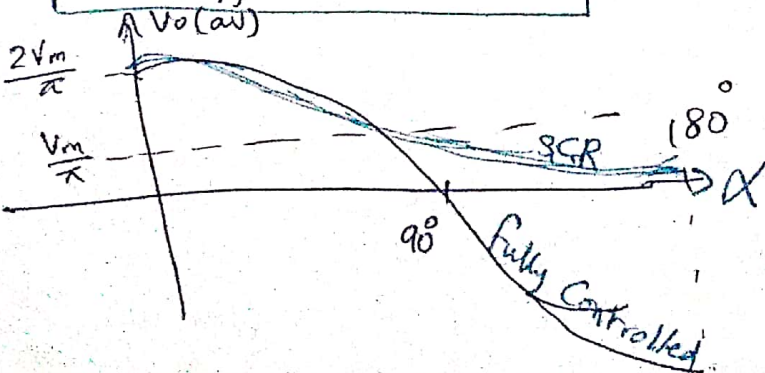
$$V_{o(av)} = \begin{cases} \frac{2V_m}{\pi} \cos \alpha & \text{CCM} \\ \frac{V_m}{\pi} (\cos \alpha + \cos \beta) & \text{DCM} \end{cases}$$

Fully Controlled

$$I_{av} = \frac{V_{o(av)}}{R}$$

$$V_o(rms) = \sqrt{\frac{(V_m)^2}{2\pi} \left[\pi - \alpha + \frac{1}{2} \sin(2\alpha) \right]}$$

Case: Ripple Free $i_o = I_o$



$$I_o(rms) = I_o$$

$$I_o(av) = I_o$$

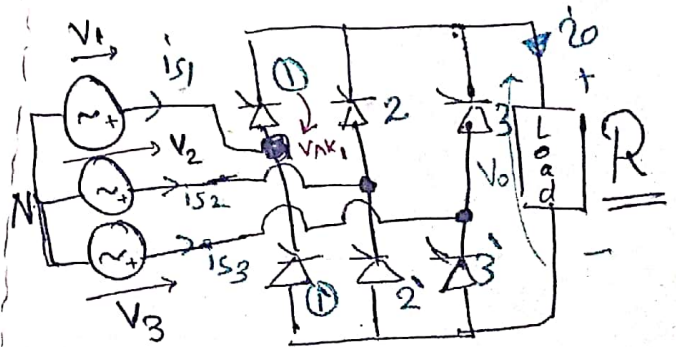
$$I_s(av) = 0$$

$$I_s(rms) = \sqrt{\frac{\pi - \alpha}{\pi}} I_o$$

$$I_o = \frac{V_m}{\pi} (1 + \cos \alpha) - \frac{E}{R}$$

3-phase Rectifier Circuits

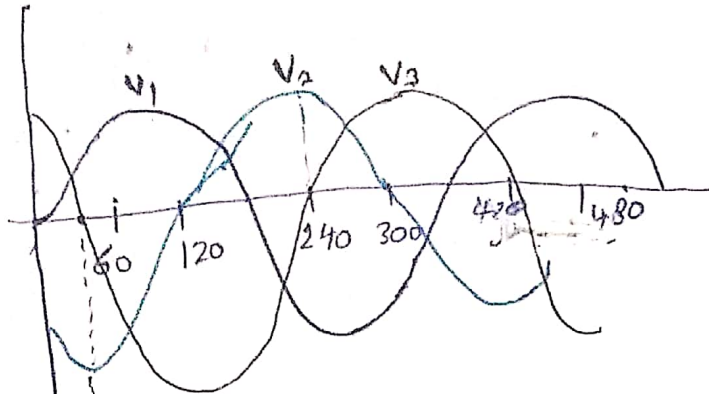
* Full Wave [Bridge] [6-pulse]



Supply:- 3-phase, Sequence,

Balanced Supply

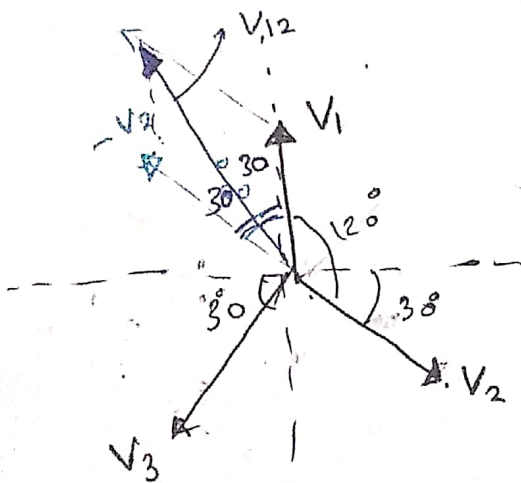
- equal Amplitude (Rms value or peak)
- equal Frequency
- phase shift 120°



$$V_1 = V_m \sin(\omega t)$$

$$V_2 = V_m \sin(\omega t - 120^\circ)$$

$$V_3 = V_m \sin(\omega t + 120^\circ)$$



3 voltages wave forms are involved in wave form constructions.

$$V_{12} = V_1 - V_2 = \sqrt{3} V_m \sin(\omega t + 30^\circ)$$

$$V_{23} = \sqrt{3} V_m \sin(\omega t - 90^\circ)$$

$$V_{31} = \sqrt{3} V_m \sin(\omega t + 150^\circ)$$

$$V_{21} = \sqrt{3} V_m \sin(\omega t - 150^\circ)$$

$$V_{32} = \sqrt{3} V_m \sin(\omega t + 90^\circ)$$

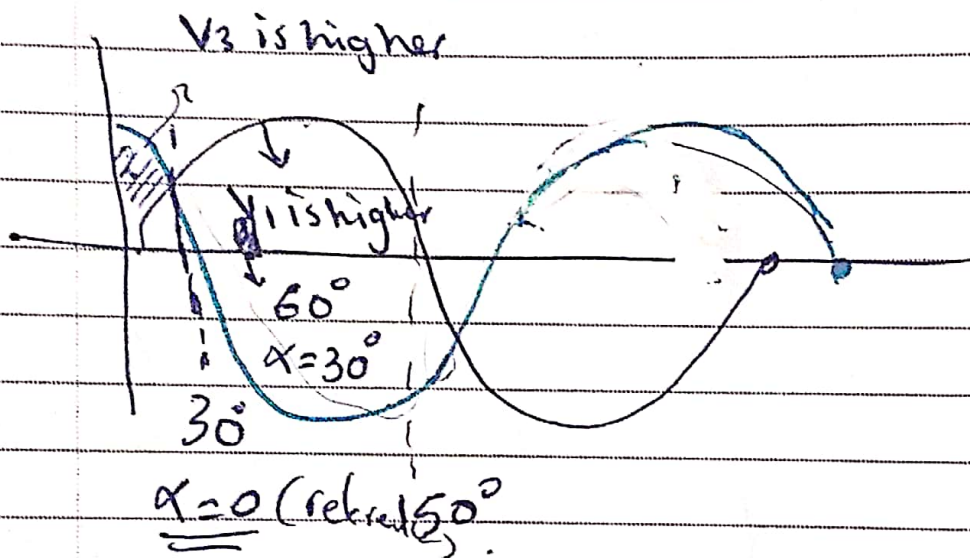
$$V_{13} = \sqrt{3} V_m \sin(\omega t - 30^\circ)$$

9-7-2019

→ UJWJ

① Reference of measuring α

wt=0 isn't the good Reference.



Reference of measuring α is the angle at which V_1 becomes highest in polarity compared to V_2, V_3 .

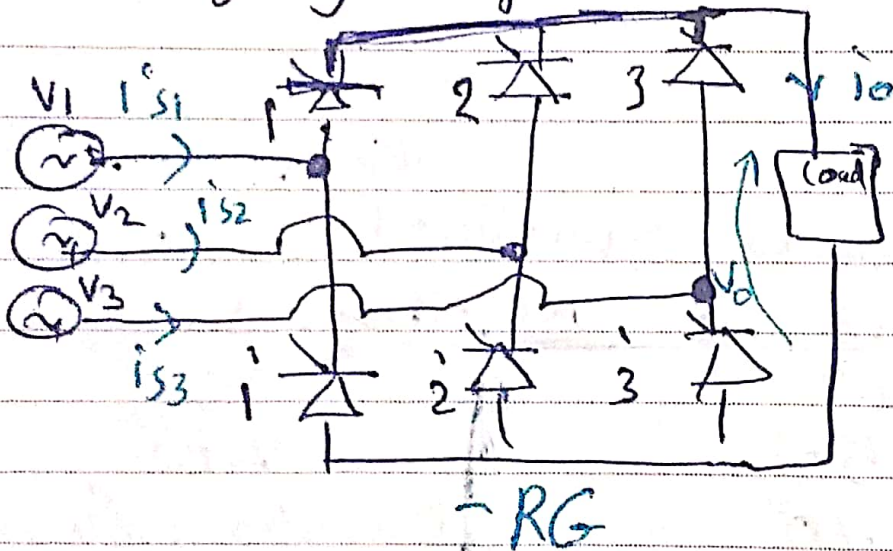
$$\omega t = 30^\circ = \pi/6$$

if $\alpha = 30^\circ$ SCR1 is triggered at $\omega t = \underline{\underline{60^\circ}}$

if SCR₁ is triggered at $\omega t = 120^\circ$

$$\alpha = 90^\circ$$

② two SCRs should simultaneously conduct one from the positive Rectifying Group, the second from the negative rectifying Group. +RG



SCR₁ → SCR₂ or SCR₃ , SCR₁ X → Short circuit

③ ^{② +} The providing that the two ~~two~~ SCRs that belong to the same phase will never conduct simultaneously.

SCR₂ → SCR₃ or SCR₁

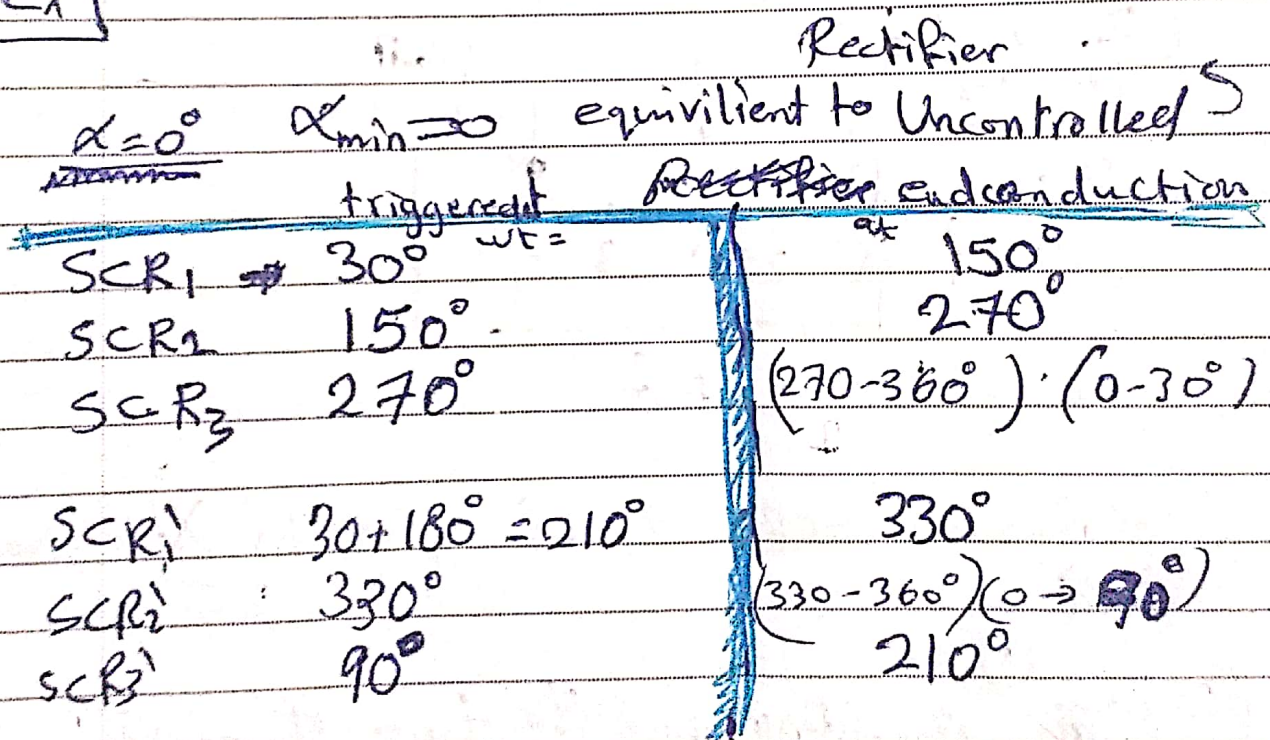
Since in the positive group or negative group

3 SCRs to share conduction in one period of symmetry

each SCR is expected to conduct for 120°

Sharing two other ~~the~~ SCRs from the opposite group 60° each.

Ex



sequence of triggering

- ① SCR₁ at 30°) 60°
- ② SCR_{3'} at 90°)

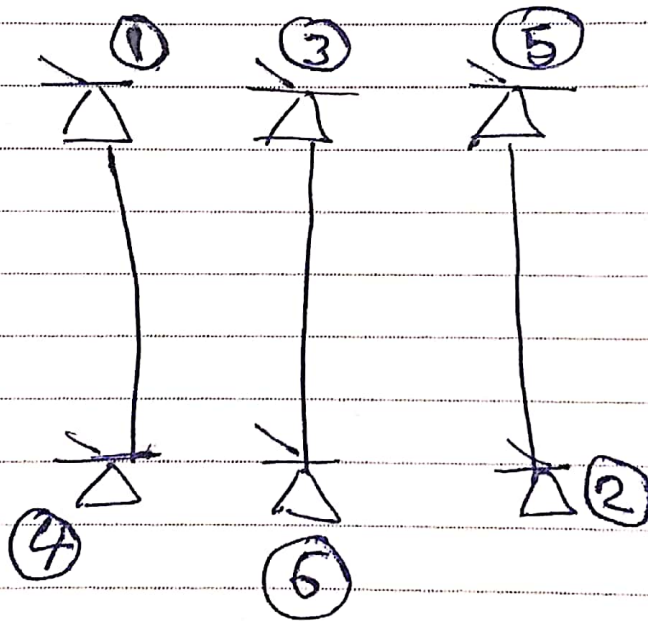
③ SCR₂ at 150°) 60°

④ SCR₁' at 210°) 60°

⑤ SCR₃ at 270°) 60°

⑥ SCR₂' at 330°) 60°

according to which SCRs are numbered in its standard form.



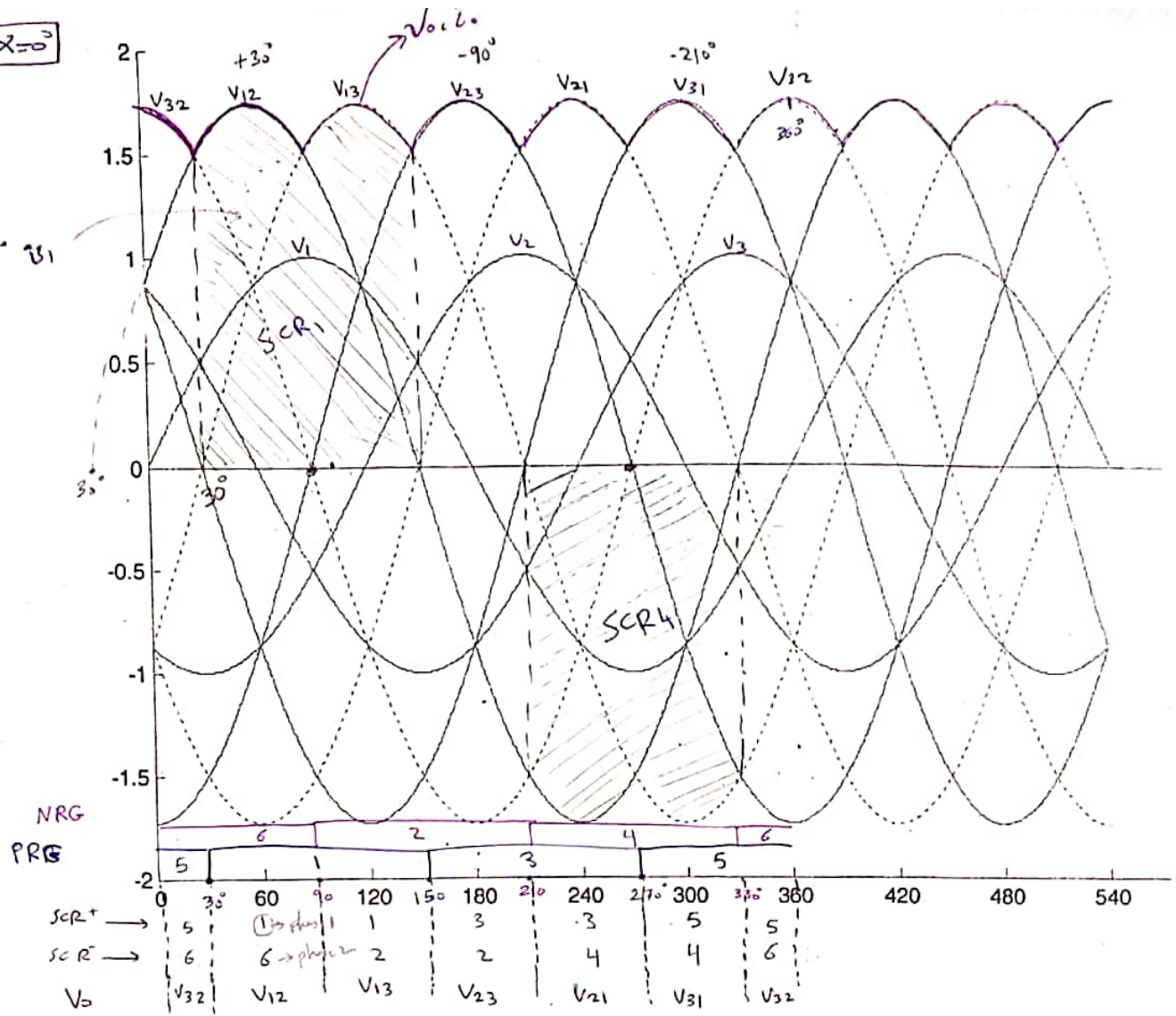
ex if $\alpha = 60^\circ$, SCR 5 is triggered

at

1	→	90
2	→	150
3	→	210
4	→	270
5	→	330
6	→	

$$\boxed{\begin{aligned} &(\text{SCR} \times 60) \\ &+ (\alpha - 30^\circ) \end{aligned}}$$

$\alpha = 0^\circ$



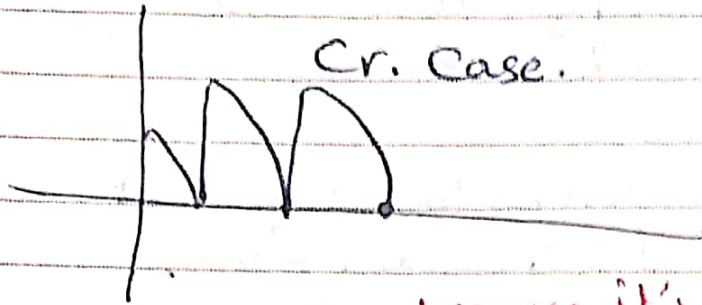
$\alpha = 60^\circ$

HW

$\rightarrow \alpha = 60^\circ$

Critical Case

$\alpha = 90^\circ$



$\alpha = 90^\circ$

second trigger because it's DCM

SCR

Trig* \downarrow END

1

$120^\circ \rightarrow 180^\circ$ 240° ?

2

180° 360

3

$240^\circ \rightarrow 300^\circ$ 420 360°

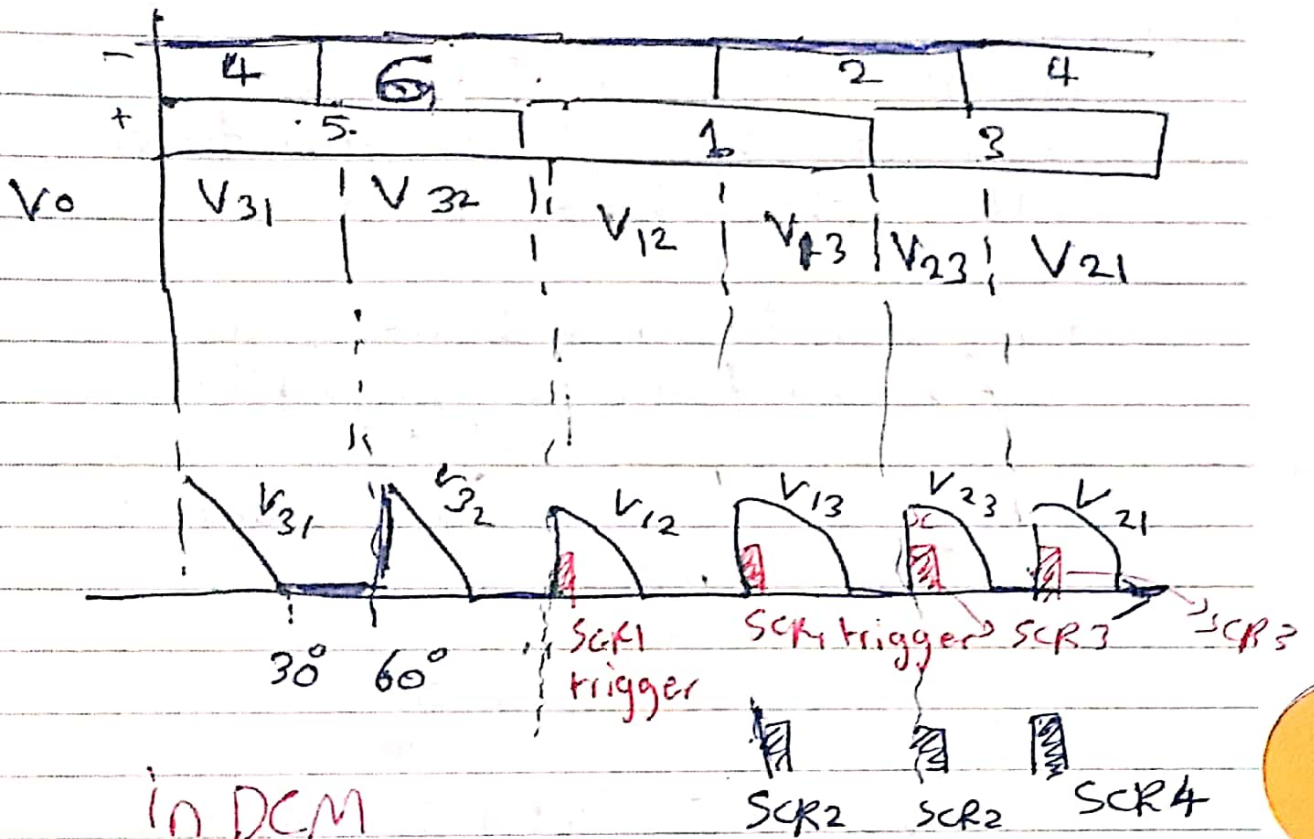
4

300° 260°
0 \rightarrow 60°

5

360° \rightarrow 120°

420° \rightarrow 120°
60°



in DCM
2 Trigger

$\alpha + 30^\circ$
and $\alpha + 60^\circ$

Q if $\alpha = 75^\circ$

SCR4 triggered at

- | | |
|---|----------|
| | SCR4 |
| 1 | 105 |
| 2 | 165 |
| 3 | 225 |
| 4 | 285, 345 |

- a) 105
- b) 165
- c) 165, 105 ✓
- d) ---

due to load current discontinuity,
each SCR should be triggered twice
with a phase shift of 60°

if not, the Rectifier behaves as
if being half wave Rectifier

$$V_o(\text{av})_{\text{CCM}} = \frac{1}{\left(\frac{2\pi}{6}\right)} \int_{\alpha + \frac{\pi}{6}}^{\alpha + \frac{\pi}{6} + \frac{\pi}{3}} V_m \sin \omega t \, d\omega t$$

$$\Rightarrow \frac{3}{\pi} \int_{\alpha + \frac{\pi}{6}}^{\alpha + \frac{\pi}{6} + \frac{\pi}{3}} \sqrt{3} V_m \sin(\omega t + \frac{\pi}{6}) \, d\omega t$$

$$\Rightarrow \frac{-3\sqrt{3} V_m \cos(\omega t + \frac{\pi}{6})}{\pi} \Bigg|_{\alpha + \frac{\pi}{6}}^{\alpha + \frac{\pi}{6} + \frac{\pi}{3}}$$

$$\frac{3\sqrt{3} V_m}{\pi} \left(-\cos\left(\alpha + \frac{\pi}{2} + \frac{\pi}{6}\right) + \cos\left(\alpha + \frac{\pi}{6} + \frac{\pi}{6}\right) \right)$$

$$V_{o(av) \text{ CCM}} = \frac{3\sqrt{3} V_m}{\pi} \cos \alpha$$

Constant. \int $\frac{5\pi}{6}$
DCM.

$$V_{o(av) \text{ DCM}} = \frac{1}{2\pi/6} \int_{\alpha + \pi/6}^{5\pi/6} \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right) d\omega t$$

$$\frac{3\sqrt{3} V_m}{\pi} \left[-\cos\left(\omega t + \frac{\pi}{6}\right) \right]_{\alpha + \pi/6}^{5\pi/6}$$

$$= \frac{3\sqrt{3} V_m}{\pi} \left[-\cos\left(\frac{5\pi}{6} + \frac{\pi}{6}\right) + \cos\left(\alpha + \frac{\pi}{6} + \frac{\pi}{6}\right) \right]$$

$$V_{o(av) \text{ DCM}} = \frac{3\sqrt{3} V_m}{\pi} \left[1 + \cos\left(\alpha + \frac{\pi}{3}\right) \right]$$

$$V_{o(av) \text{ CCM}} = V_{o(av) \text{ DCM}} \rightarrow \text{critical case.}$$

$$\alpha = \alpha_c = 60^\circ$$

$$\frac{3\sqrt{3} V_m}{2\pi} \stackrel{?}{=} \frac{3\sqrt{3} V_m}{2\pi}$$

✓

What is the max possible firing angle if Resistive load.

$$\alpha_{\max} \rightarrow ?? = V_{o(\text{av})} = 0^\circ$$

DCM

$$V_{o(\text{av})} = 0 = \frac{\sqrt{2} V_m}{\pi} (1 + \cos(\frac{\pi}{3} + \alpha))$$

$$1 + \cos(\alpha + \frac{\pi}{3}) = 0$$

$$\cos(\alpha + \frac{\pi}{3}) = -1$$

$$\alpha + 60^\circ = 180^\circ$$

$$\alpha = 120^\circ$$

~~Why~~ $\alpha = \begin{cases} 0 \leq \alpha < 60^\circ, & \text{for CCM} \\ 60^\circ, & \text{for critical} \\ 60^\circ < \alpha \leq 120^\circ, & \text{for DCM.} \end{cases}$

HW

$$V_o(\text{Rms}) = ? \quad \text{in CCM}$$

DCM

$$V_o(\text{rms}) = \frac{1}{\sqrt{2\pi}} \int_{\alpha + \frac{\pi}{6}}^{\alpha + \frac{\pi}{2}} V_{12}^2 dt$$

$$V_o(\text{rms}) = \frac{1}{\sqrt{2\pi/6}} \int_{\alpha + \frac{\pi}{6}}^{\alpha + \frac{\pi}{2}} \left[\sqrt{3} V_m \sin(\omega t + \pi/6) \right]^2 dt$$

$$V_o(\text{rms}) = V_{\text{cm}} \quad \text{Same} \quad \left[\alpha + \frac{\pi}{6} \quad \dots \quad 5\pi/6 \right]$$

$$V_{\text{cm}} = \frac{3V_m}{\sqrt{\pi}} \sqrt{\frac{\pi}{3} - \frac{1}{4} \left[\sin\left(2\alpha + \frac{4\pi}{3}\right) - \sin\left(2\alpha + \frac{2\pi}{3}\right) \right]}$$

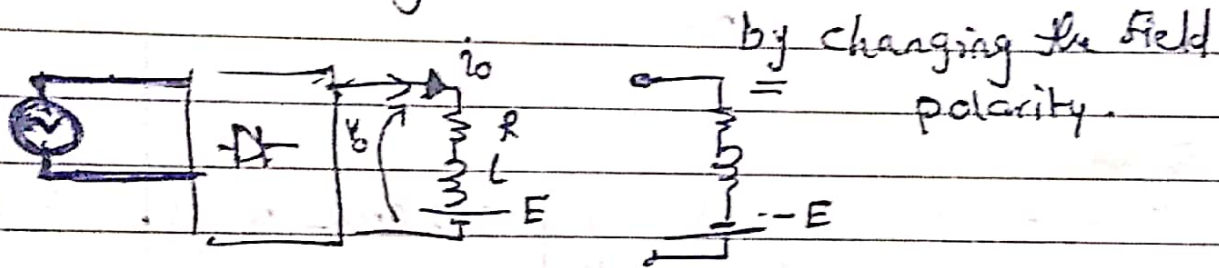
$$V_{\text{cm}} = \frac{3V_m}{\sqrt{\pi}} \sqrt{\frac{1}{2} \left(\frac{2\pi}{3} - \alpha \right) + \frac{1}{4} \sin\left(2\alpha + \frac{2\pi}{3}\right)}$$

* controller single phase, Full wave fully controlled Bridge Rectifier

Supply 100 V (Vrms), 60 Hz, sinusoidal

Load Series RLE Load $R = 0.5 \Omega$
 L is sufficiently large
 such that the load current is ripple free and equal to 10 A

$E = \begin{cases} 65 \text{ V} \uparrow & \text{motoring action} \\ -75 \text{ V} \downarrow & \text{Braking action} \end{cases}$



a) find α

b) P_{in}

c) which source is delivering power to the other

(D) THDF of ~~E~~ is

e) RF of F V_o and I_o

F) ~~sketch~~ ^{all wave} sketch ~~of~~ ~~the~~ ~~forms~~ of interest.

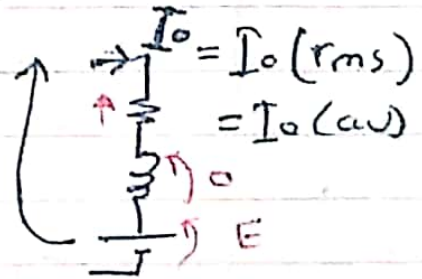
$$\alpha \Rightarrow V_o(\text{av}) = \frac{2V_m}{\pi} \cos \alpha$$

CCM \rightarrow because it is Ripple free pure DC

$$V_o(\text{av}) = \frac{2 \times (110) \sqrt{2}}{\pi} \cos \alpha$$

$$V_o(\text{av}) = I_o(\text{av}) R + E$$

$I_o(\text{av}) = I_o(\text{rms}) = I_o = 10 \text{ A}$



$$10 \times 0.5 + 65 = 70 \text{ V}$$

$$\alpha = \cos^{-1} \left(\frac{70 \times \pi}{2\sqrt{2} (110)} \right) = \underline{\underline{45^\circ}}$$

$$\alpha = 45^\circ$$

$$PF_{in} = \frac{P_{input} (\text{W})}{I_{input} (\text{VA})} =$$

$P_{in} (\text{W}) = P_{out} (\text{W}) \rightarrow$ lossless Rectifier.

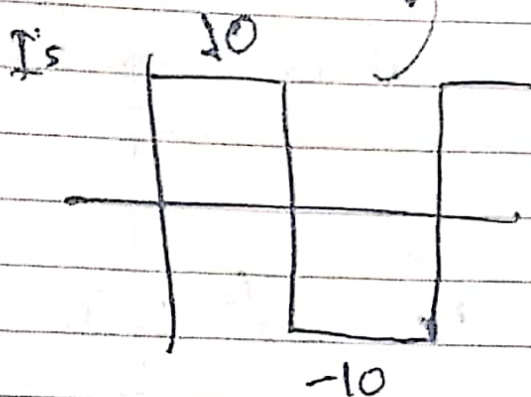
$$P_o = \underline{I_o^2 (R)} + \underline{I_o(\text{av}) E}$$

$$P_o = I_o^2 (R) + I_o E = 700 \text{ W}$$

$$S_{in} = V_{supply(rms)} I_s(rms)$$

$$S_{in} = 110V \cdot 10 = 1100$$

$$I_s(rms) = 10A$$



$$p_{in} = \frac{700}{1100} = 0.636 \text{ lagging}$$

(inductive load)

OK $P_{Fin} = DPF + DTF$

$$DPF \Rightarrow \cos \phi_1 = -n\alpha$$

$$\phi_1 = -45^\circ = 0.707$$

$$C(n) = \frac{4I_0}{n\pi}$$

$$DTF \Rightarrow \frac{I_s(n)_{rms}}{I_s(1)_{rms}} = \frac{9}{10} = 0.9$$

$$C(1) = \frac{4I_0}{\pi}$$

$$P_{Fin} = 0.707 \cdot 0.9 = 0.636$$

(lagging PF)

$$I_s(n)_{rms} = \frac{C(n)}{\sqrt{2}}$$

$$= \frac{4I_0}{\sqrt{2}\pi} = 9A$$

the Ac source $\overset{\text{supplies power to load}}{\text{delivers}} \left(\begin{array}{l} \text{Motoring} \\ \text{Charging} \end{array} \right)$

$$\text{THDF} \% = \frac{\sqrt{I_{(s)}^2 - I_{(A)}^2(\text{rms})}}{I_{(A)}(\text{rms})} = 48.99\%$$

$$= \frac{10^2 - 9^2}{9} =$$

$$Rf_{v_o} = \frac{\sqrt{V_o(\text{rms})^2 + V_o(\text{av})^2}}{V_o(\text{av})} = \frac{\sqrt{110^2 - 70^2}}{70}$$

$$Rf_{i_o} = 0\%$$

$$Rf_{v_o} = 121\%$$

$$V_o(\text{rms}) = V_o(\text{supply rms}) = 110 \text{ V}$$

CCM

$$V_o(\text{av}) = 70 \text{ V}$$

(b)

$$V_o(\text{av}) = I_o R + E_{mf}$$

$$10 \cdot 0.5 - 75 = -70 \text{ V}$$

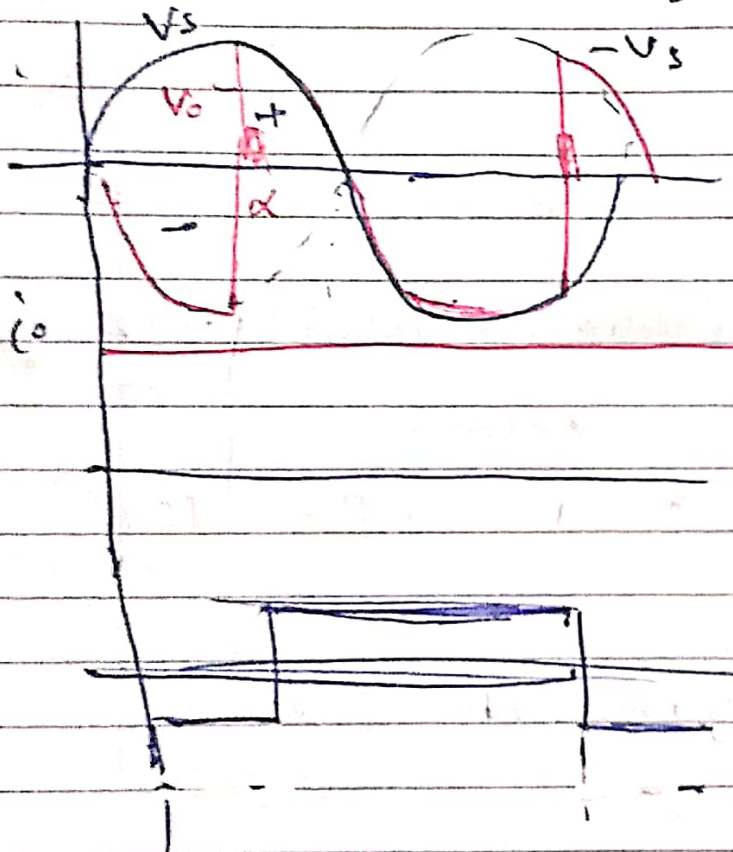
$$\alpha = \cos^{-1} \left(\frac{-70}{2\sqrt{2}(110)} \right) = 135^\circ \quad 790^\circ \checkmark$$

$$P_{Fin} = \frac{P_{In} (W)}{\sin(VA)} = \frac{-V_o I_o}{\sin}$$

$$= \frac{-70 \times 10}{110 \times 10} = -0.636$$

the supply is consuming P
←

lagging.



$$TADDF = 48.43\%$$

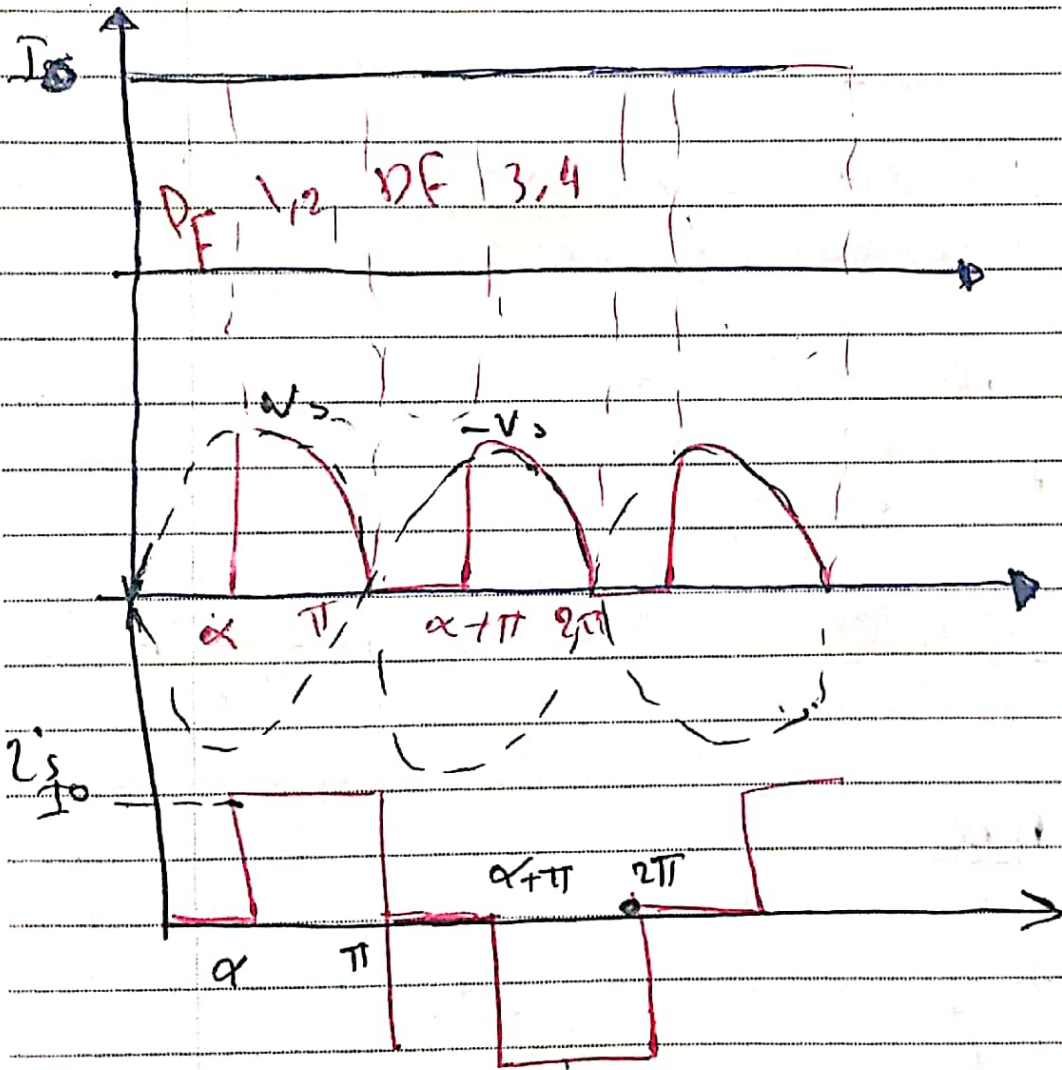
$$R_{F_{V_o}} = 121\% \quad R_{F_{i_o}} = 0$$

V/17 . > Biji

Q) if a free-wheeling diode is connected to the circuit while $E = 65V$ and all parameters still the same (DF parallel to the load).

Find ~~Complete~~ all performance parameters.

$I_0 = 10A$



$$V_o(\text{av}) = \frac{V_m}{\pi} (1 + \cos \alpha)$$

$$I_o R + E_{\text{mf}} = 5 + 65 = 70$$

$$70 = \frac{V_m}{\pi} (1 + \cos \alpha)$$

$$\alpha = 65.5^\circ$$

$$PF_{\text{in}} = \frac{P_{\text{out}}(\omega)}{S_{\text{in}}(\text{VA})} = \frac{V_o(\text{av}) I_o}{V_s(\text{rms}) \cdot I_s(\text{rms})}$$

$$I_s(\text{rms}) = \sqrt{\frac{180 - 85.5}{180}} I_o = 7.97 \text{ A}$$

$$PF = \frac{70 \cdot 10}{110 \cdot 7.97} = 0.798 \approx 0.8 \text{ lagging PF}$$

Better than fully controlled.

$$\psi(\alpha) = -\frac{\pi \alpha}{2}$$

$$C(\alpha) = P$$

$$THDF = \frac{\sqrt{I_s(\text{rms})^2 - \frac{1}{3} I_{c0}(\text{rms})^2}}{I(\alpha)(\text{rms})}$$

No.

$$R_p V_o = \sqrt{V_o^2(\text{rms}) - V_o^2(\text{av})}$$

$V_o(\text{rms})$ in fully controlled = $V_s(\text{rms})$

⊛ is semi controlled $V_{o\text{rms}} < V_s(\text{rms})$

$$V_o(\text{rms}) = \sqrt{\frac{V_m^2}{2\pi} \left(\pi + \beta - \alpha + \frac{1}{2} \sin(2\alpha) - \frac{1}{2} \sin(2\beta) \right)}$$

$V_o(\text{rms}) =$ ~~_____~~

R_{pro} (in semi controlled is better) = ~~_____~~

Given controller single phase fully controlled Rectifier Bridge Supply 220V 50Hz sinusoidal

Load R-L Load ($R=10\Omega$, $L=55.133\text{mH}$)
for $\alpha=60^\circ$ evaluate performance parameters.

⊛ $\alpha \neq \phi$

$$\phi = \tan^{-1} \left(\frac{WL}{R} \right) = \phi \quad WL = 17.3\Omega$$

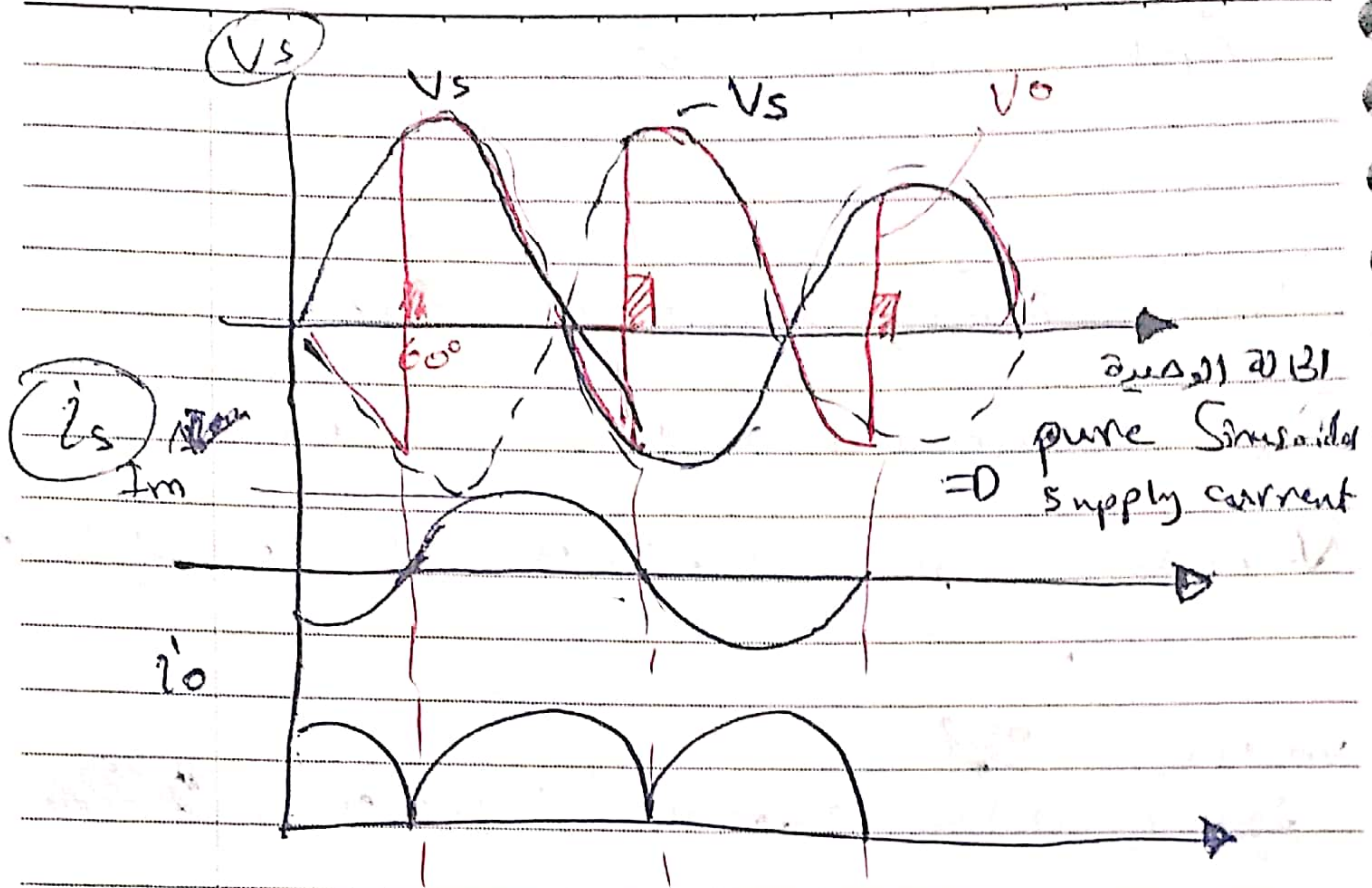
$$\tan^{-1} \left(\frac{17.3}{10} \right) \approx 60^\circ \quad \phi = 60^\circ \quad \boxed{\text{Critical mode}}$$

FIVE APPLE

$$\pi - \beta = \pi + \alpha$$

No.

~~27~~ 1



$$i_s = I_m \sin(\omega t - \phi)$$

$$V(\text{rms}) = 220 \text{ V}$$

$$V_o(\text{av}) = \frac{2V_m}{\pi} \cos(60^\circ) = \frac{2(220\sqrt{2})}{\pi} \left(\frac{1}{2}\right)$$

$$R_p I_o =$$

$$I_o(\text{av}) =$$

$$I_m = \frac{V_m}{Z}$$

$$Z = \sqrt{10^2 + (17.13)^2} = \frac{V_o(\text{av})}{R}$$

No. _____

$$I_0(\text{rms}) = \frac{I_m}{\sqrt{2}}$$

$$PF = \cos \alpha = 0.5 \text{ lagging}$$

$$\text{WDF} = 0$$

17-7

17-7

Q) Controlled ∴ Single-phase, full wave
Center-tapped Rectifier

Supply | 230V, 50Hz, Sinusoidal

Transformer | (230V / 115-115V), 2kVA, 50Hz

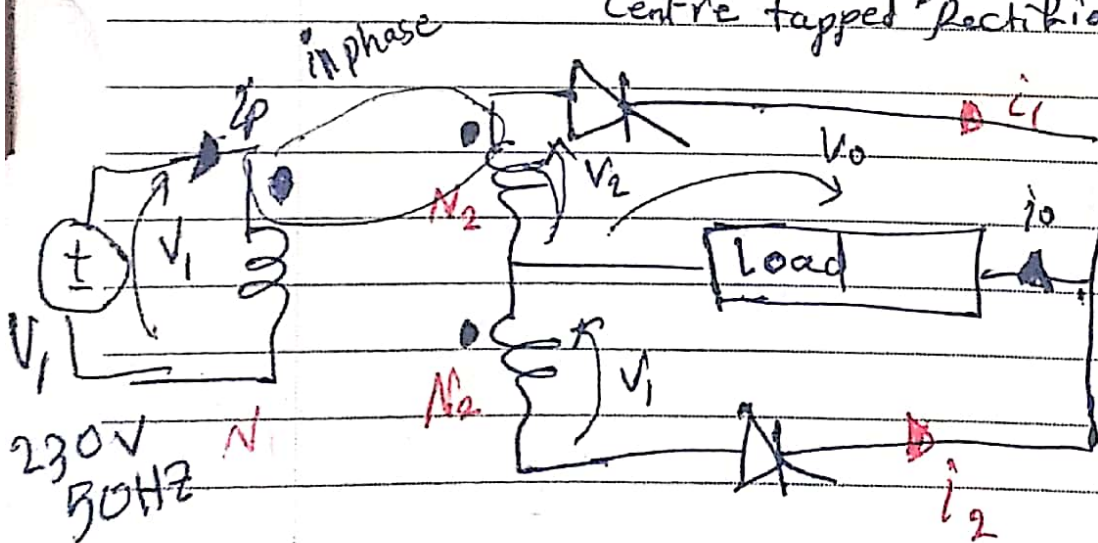
Load | pure Resistive 15Ω

- 1) Max possible load average current
- 2) α? For $I_o(\text{av}) = 5A$

3) $\alpha = 45^\circ$ $p/f = ??$

5) select suitable voltage & current Rating of the SCR

4) State the major difference between Bridge and SCR
Centre-tapped Rectifier.



$$a (\text{Turns Ratio}) = \frac{N_1}{N_2} = 2:1$$

$$V_p = 230\sqrt{2} \sin(\omega t)$$

$$V_s = 115\sqrt{2} \sin(\omega t)$$

$$\frac{V_p}{V_s} = \frac{N_1}{N_2} = a$$

$$V_s = \frac{V_p}{a} \Rightarrow V_p = \frac{230\sqrt{2}}{2} \sin \omega t$$

$$V_s = 115\sqrt{2} \sin \omega t \dots$$

Resistive Load $\Rightarrow V_o(\text{av}) = I_o(\text{av}) R$

$$I_o(\text{av}) = \frac{V_o(\text{av})}{R} \text{ max}$$

max

$$V_o(\text{av}) = \frac{V_m}{\pi} (1 + \cos \alpha)$$

DCM

$\beta = 0$ Resistive Load

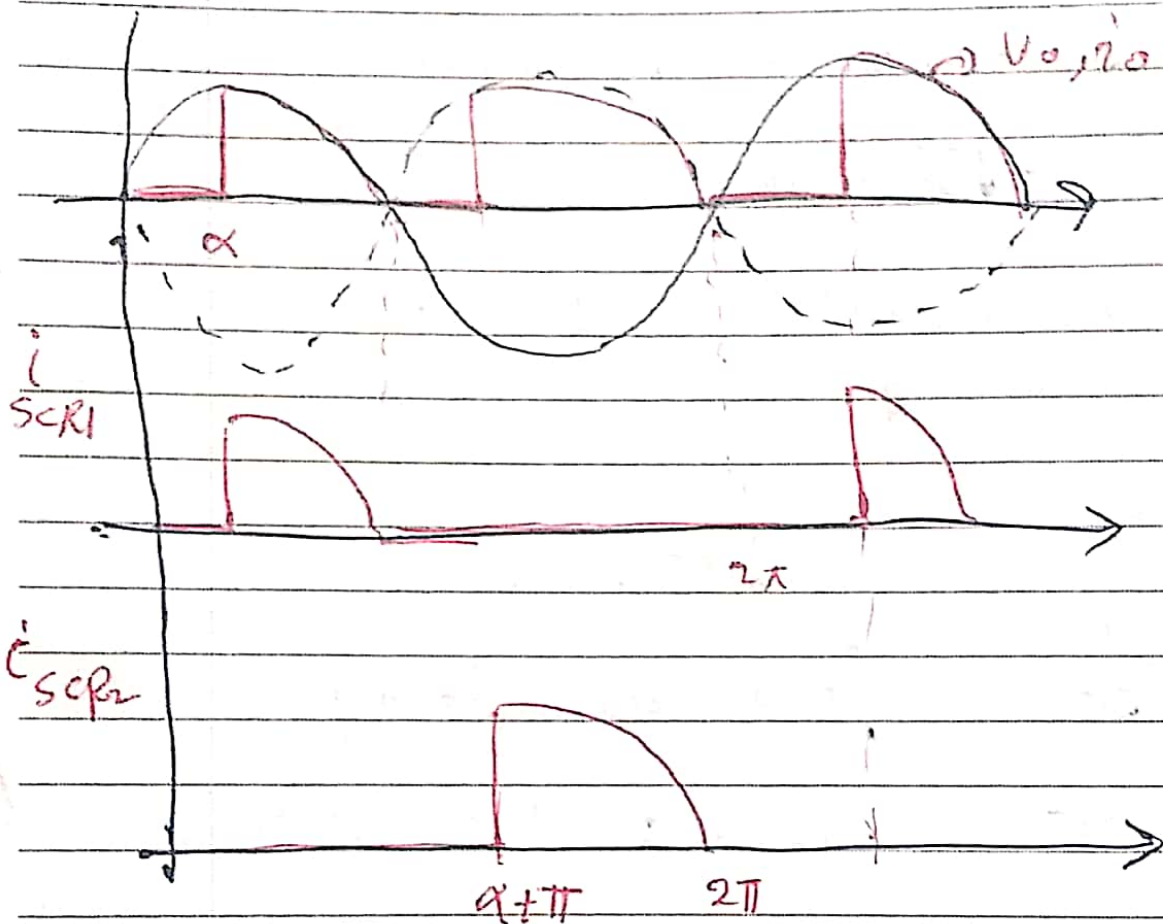
$$V_o(\text{av})_{\text{max}} = \frac{115\sqrt{2}}{\pi} (1 + \cos 0^\circ) \text{ to find max}$$

$$V_o(\text{av})_{\text{max}} = \frac{2 \cdot 115 \cdot \sqrt{2}}{\pi} = 103.5 \text{ V}$$

$$I_{o(\text{av}) \text{ max}} = \frac{103.5}{15} = 6.9 \text{ A}$$

Rated Values \rightarrow at maximum possible case.

$$I_{rms} = \frac{V_o(rms)}{R} \quad (\text{Load is Resistive}).$$



$$I_{SCR(av)} = \frac{I_o(av)}{2} = 3.45A$$

$$V_o(rms)_{Resistive} = \sqrt{\frac{V_m^2}{2\pi} \left(\pi - \alpha + \frac{1}{2} \sin(2\alpha) \right)}$$

$$I_{SCR} (rms) = \sqrt{\frac{I_m^2}{4\pi} (\pi - \alpha + \frac{1}{2} \sin 2\alpha)}$$

Form factor ?

$$I_m = \frac{V_m}{R} \sqrt{2} \text{ (rms)}$$

$$\alpha = 0, I_{SCR} = \frac{I_m}{2} = 5.42 \text{ A}$$

(rms)
rating

$$I_{SCR} \text{ peak} = I_m = 10.84 \text{ A}$$

PIV (peak inverse Volt)

Rect (PIV)

HW single-phase	V_m
FW single-ph	V_m
centre tapped	$2V_m$
3-phase Rect.	$\sqrt{3}V_m$

SCR Voltage Ratings

Ex (2+3) PIV \rightarrow $2V_m$
 Centre tapped

$$V_m = \sqrt{2} (115) (2)$$

$$V_{SCR} = (2+3) \times (\sqrt{2}) (115) (2)$$

$$V_{SCR} (648 \rightarrow 972)$$

$$\alpha = ?$$

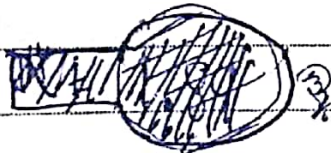
$$I = 5A$$

$$V_o(av) = R I_o(av) = 10 \times 15 = 150V$$

$$150 = \int \frac{V_m}{\pi} (1 + \cos \alpha)$$

$$150 = \frac{V_m}{\pi} (1 + \cos \alpha)$$

$$\alpha = \cos^{-1} \left(\frac{150 \pi}{\sqrt{2} \times 115} - 1 \right)$$



$$\cos \alpha = 1.89$$

IF 5A

$$\cos \alpha = 0.448$$

$$\alpha = 63.8^\circ$$

IF $\alpha = 45^\circ$

Find pf

$$V_o(\text{av}) = \frac{115\sqrt{2}}{\pi} (1 + \cos 45^\circ)$$

$$V_o(\text{av}) = 88.3 \text{ V}$$

$$I_o(\text{av}) = V_o(\text{av}) / R = 5.8 \text{ A}$$

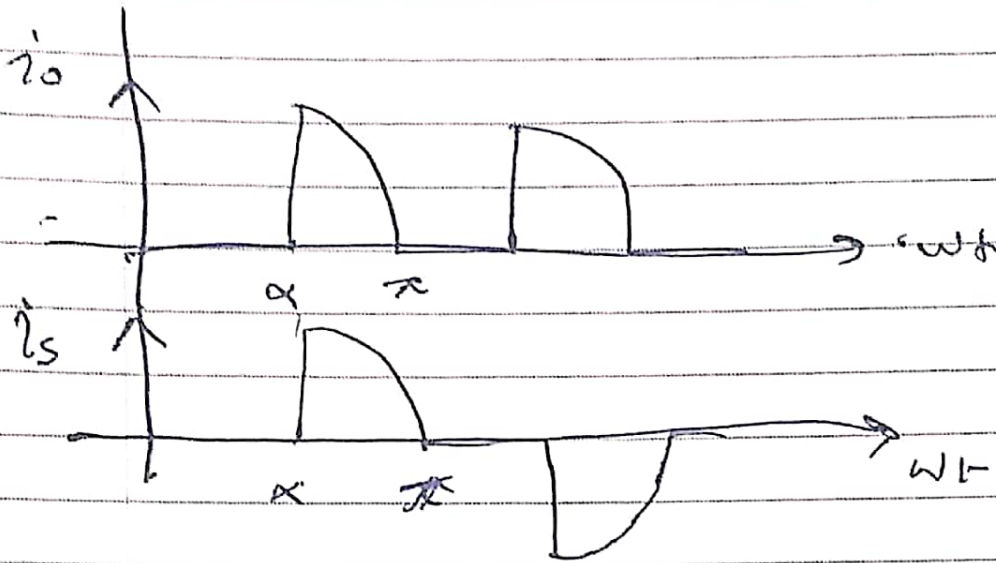
$$PF = \frac{\boxed{I_o^2(\text{rms}) \cdot R}}{P_{\text{out}}(\omega)} = \frac{S(\text{VA})}{P_{\text{out}}(\omega)}$$

$$I_s(\text{rms}) V_s(\text{rms})$$

$$I_o(\text{rms}) = \sqrt{\frac{I_m^2}{2\pi} (\pi - \alpha + \frac{1}{2} \sin 2\alpha)}$$

$$I_o(\text{rms}) =$$

$$I_s(\text{rms}) = I_o(\text{rms})$$



Center Thyristor is more expensive
 triggering circuit is cheaper

Transformer is a must (drawback).

They are same core, but the copper is more
 so, its weight is larger.

Power ~~loss~~ \Rightarrow Bridge Better.